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Document Version
Final published version

Link to publication record in Manchester Research Explorer

Citation for published version (APA):

Published in:
University of Cambridge

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Shape Memory Alloy Shading Systems: An Optimization Model

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Abstract

Smart materials that react to heat energy have a potential of usage as shading system components; shape memory alloys (SMA) are one of those materials. SMAs can transform heat into a change in form, introducing movable shading systems without using electricity. However, only tools targeting micro-scale simulations of SMA materials are present. Thus, the paper focuses on introducing a simulation script for a SMA building envelope based on archsim radiation tool. Using a multi-variable optimization tool, the paper explores variable form changing SMA shading screen designs. The overall resultant shows a flexible macro-scale simulation-optimization tool for SMA shading devices.

Introduction: SMAs and Optimization

The notion of using building envelopes that react to external energy stimuli has become an applicable approach to control the environmental conditions of the built environment. Such envelopes have been classified by Loonen R. as climate adaptive building shells (CABS) that act according to climatic changes; thermal, optical and aerodynamic changes (Loonen et al., 2013). At that point and till the moment, the movements of different parts of a CABS mostly depended on the presence of a sensor-actuator system. These systems include smart materials that have the ability to sense certain energy stimuli and react mostly by producing minor electric energy. Thereby, electricity has been a needed form of energy to actuate a motion in the CABS.

Nevertheless; one smart material, interestingly, has the ability to sense and directly react by changing form without a need of electric energy. Particularly, shape memory alloys (SMAs) can react to a change in temperature by changing their form, and one of the most commercially used SMA materials is Nickel Titanium (NiTi). Such material; NiTi, is a two way shape memory alloy which has two remembered states; assumed to be named as state 1 at a low temperature and state 2 at a higher temperature. In that sense, during heating, a NiTi SMA changes gradually from state 1 to state 2 starting form the austenite start temperature (As) till the austenite finish temperature (Af). Yet, while cooling it reverts from state 2 to state 1 at generally lower temperatures starting form martensite start temperature (Ms) and ending with martensite finish temperature (Mf) as shown in Figure 1.

In fact, the direct implementation of NiTi in building envelopes remains in either a conceptual or a prototype state. An example is the proposal of applying a NiTi self-actuated shading system over the western façade of Al-Midan theatre (Lazarovich et al., 2015). The approach introduces a dual spring mechanical system as shown in Figure 2. The first spring (left side of Figure 2) is an exposed two way NiTi spring that changes its compression state according to the change in temperature; this deems the system a passive one. To assure users have control over such a passive system, an electrical spring actuator is added which changes its compression state based on the users’ demands. Another example includes a physical prototype applied for a manufactured SMA ventilation control façade (Formentini and Lenci, 2018). The idea revolves around the ability of a SMA to mildly bump and allow air ventilation as shown in Figure 3.

Figure 1 SMA typical heat cycle form change (Duerig and Pelton, 1994)

Figure 2 Conceptual Detailing of the Shading System on the Western Façade of Al-Midan Theater (Lazarovich et al., 2015)
Although prototypes and concepts deal with the macro-scale changes of the SMAs, most simulations of SMA focus on the molecular structure’s complexity at a micro-scale. This focus has been facing the challenge that reaching a 3-D simulation results in a large number of complex calculation; which in turn leads to sacrificing accuracy for the feasibility of implementing a simulation (Gao et al., 2000; Juhasz et al., 2002; Patoor et al., 2000). Thus, early simulations only dealt with one dimensional (1-D) applications where the SMA movements are only allowed in one axial direction (Patoor et al., 2000, 1994; Tanaka et al., 1986). With 1-D being a limitation, an approach of abstracting 3-D forms into multiple 1-D or 2-D forms introduced a solution to the complexity issues; an example is simulating wires that are arrayed in a 3-D composition (Burton et al., 2006). Hereby, this approach was used to simulate the self-healing effect of SMA where a finite 1-D micro-scale simulation was implemented to reach a 3-D macro-scale output (Burton et al., 2006).

As stated, macro-scale simulations of SMA shading systems are limited; however, this does not apply to normal shading devices. Shading devices are commonly tested by simulating solar energy loads upon building envelopes using validated tools such as Archsim (Reinhart and Walkenhorst, 2001). Optimizing such simulations to maximize the shading efficiency is a part of such approach where genetic algorithms are used for instance. Some simulation-optimization examples include optimizing shading systems against numerous objectives including internal thermal comfort and external heat loads (Khoroshiltseva et al., 2016); another example includes optimizations while calculating daylight and heat loads (Manzan and Pinto, 2009). Yet, and as previously stated, such macro-scale shading systems simulations did not consider the direct implementations of shape changing SMAs.

Simulation and Optimization

The implementation of an optimized SMA dynamic shading system demands two main phases. First, a simulation of the movement of SMA due to temperature changes is run. Second, an optimization for such movement over a south-western building envelope is implemented. It should be clarified that such phases merge within the final script that 1) simulates the movement, 2) calculates solar loads upon the building envelope and 3) optimizes simultaneously.

General Methodology and Tools

As stated, the methodology generally includes two merged phases for achieving a fully optimized SMA dynamic shading system; and for clarity, each phase (step) is separately introduced as follows:

1) The first step; SMA simulation, is based on using the heat transfer equation and the solar energy upon the SMA to calculate its temperature. Such energy is provided using a Diva simulation to calculate the SMA temperature; which is then transferred into a change in the shading system geometry using a grasshopper and python script.

2) The second step; shading system optimization, is based on modifying the dimensions and the initial state of the SMA shading system. This is achieved using octopus genetic algorithm tool which modifies the SMA shading system and aims to optimize the amount of solar energy falling upon the south-western building façade.

Finally, two optimization processes are introduced and compared. One process assumes the shading system is arrayed with a typical initial rotation angle; a typical initial state. The other assumes the shading system is arrayed with variable initial rotation states.

Simulation and Optimization Models: Assumptions and Limitations

Using the broadly mentioned methodology above, the implementation demanded introducing clear assumptions and limitations to the grasshopper parametric tool. In fact, the constraints needed are confined within 1) the type of SMA used, 2) the shading system’s design, 3) the shading system’s dimensions, 4) the shaded façade dimensions and 5) the simulation dates.

Thereby, the following detailed assumptions are introduced for the simulation and optimization in this paper

1) Nickel Titanium Zirconium (NiTiZr) SMA is used with an Austenite Start temperature (As) of 60 degrees Celsius, Austenite Finish temperature (Af) of 100 degrees Celsius, Martensite Start temperature (Ms) of 20 degrees Celsius, and Martensite Finish temperature (Mf) of 40 degrees Celsius (Evirgen et al., 2016).

2) The variable emissivity values during heating and cooling of NiTiZr are compensated for by using an average emissivity value of 0.17 during heating and 0.21 during cooling (Castro and Paulo, 2015).

3) The variable absorptivity of NiTiZr, dependent on colour and surface finish, is counterbalanced by using the absorptivity within a range of 0.5 and 0.7 during the optimization phase.

4) The shading system is designed as a SMA plate that bends at 10 centimetres of its length (at the
bending line) with an angle named $B_{\text{angle}}$, as shown in Figure 4 according to its heating and cooling cycle temperatures.

5) The width of the SMA plate is constant at 10 cm and the thickness at 2 millimetres, while the plate’s length and initial bending angle are variable genes in the optimization phase.

6) The shading system is arrayed upon a façade to simulate self-shading; the number of modules varies according to the plate’s length (variable panel plate length is shown in Figure 4).

7) The façade, on which the shading system is arrayed, is a south-western façade in Cairo, Egypt, with a length of 1.5 meters and a height of 2.5 meters.

8) The simulation runs over the dates 21st of June and the 22nd of December over an hourly rate starting 6:00 until 18:00. The chosen dates signify the days with most and least daylight hours observed in Cairo, Egypt. At such dates, the solar energy falling upon the façade in June ($\text{SE}_{\text{June}}$) and December ($\text{SE}_{\text{December}}$) are calculated.

9) The SMA state is considered using measurements taken hourly to reduce calculation time.

10) Optimization is implemented twice; a) the arrayed panels are assumed to have equal $B_{\text{angle}}$ values, b) the arrayed panels are assumed to have variable $B_{\text{angle}}$. In both cases, the length of the arrayed panel $L_{\text{panel}}$ is assumed to be equal for all panels. A general introduction to the optimization genomes and objectives is shown below (this is to be further discussed in the optimization section):

a. Genomes:
   i. $B_{\text{angle}}$ (1 or multiple values)
   ii. Panel Length; $L_{\text{panel}}$ (1 value)
   iii. Absorptivity [$a$] (1 value)

b. Objectives:
   i. $\text{SE}_{\text{June}}$ (1 value)
   ii. $\text{SE}_{\text{December}}$ (1 value)

11) The octopus settings for the optimization runs include an elitism of 0.6, a mutation rate of 0.5, a mutation probability of 0.5, a crossover rate of 0.6; and a population size of 50.

a. The elitism value indicates that 60% of the solutions of a population are derived from the elite values from the previous population. A higher value would show a very local optimization where longer time would be needed to discover different elites.

b. The mutation rate indicates that when a solution is mutated (to assure diversity of solutions) a 50% change in the parameters from one population elite values to the next. The value of 0.5 is used (which is considered a high one) to enhance the optimization’s ability to find variable elites; a non-local optimization. This is mainly because the goal is to find numerous fairly optimum solutions rather than a chosen elite solution with a little advantage over other undiscovered solutions.

c. The mutation probability of 0.5 indicates a 50% chance that a solution is mutated in the next population; if not mutated, the algorithm works with its current genetic diversity over the corresponding solution in the next population.

d. The crossover rate indicates that 60% of the solutions in a population are formed by crossover from the previous one; i.e. formed by using parameters (genes) from two previous parent solutions. This average value gives the algorithm the chance to locally enhance found elite solutions through generations.

e. The population of a generation is chosen as 50; this is mainly to find more elites in a minimal time where mutation and crossover can occur in a faster manner. For more local optimization, a population size of 100 is recommended. However, for the sake of this paper’s goal of introducing a less local optimization, the value 50 is used.

![Figure 4 One SMA Shading Unit Diagram Showing Dimensions and Variables](image)

Simulation: SMA Material

The SMA simulation requires three main values to be reached. First, the solar energy falling upon the NiTiZr panel should be calculated. Second, a transformation factor that signifies the SMA state is needed. Third, a rotation angle for the bending state should be concluded (Gamal, 2017).

To begin with, the solar energy (SE) is calculated in KWhours using a Diva component where the panel is considered as a one-sensor grid. Such sensor calculates the exact amount of solar energy falling upon the panel’s surface.

$$E = \rho \cdot v \cdot c \cdot \Delta T$$  \hspace{1cm} (1)
The heat transfer equation (1) is used to calculate the SMA’s temperature change; where $E$ is the solar energy absorbed by the panel is Joules, $\rho$ is the density of NiTiZr (6.45 gm/cm$^3$ for Ni50Ti50), $v$ is the volume of the panel, $c$ is the heat transfer coefficient of NiTiZr (0.32 J/gm°C for NiTi) and $\Delta T$ is the change in the SMA’s temperature.

$$E = E'.f.a.e$$  \hspace{1cm} (2)

Since Diva’s component calculates the solar energy falling, not absorbed, upon the panel in Kwh; equation (2) is used to transfer the units to Joules and compensate for reflected solar radiation by the panel. In such case, $E$ is the radiation absorbed by the panel in joules, $E’$ is the SE falling on the panel in Kwh, $f$ is a factor to transfer from Kwh to Joules ($f = 3,600,000$), $a$ is the absorptivity of NiTiZr ($a$ varies from 0.17 to 0.21) and $e$ is the emissivity of NiTiZr ($e$ varies from 0.17 to 0.21).

For the emissivity and absorptivity values, as previously stated, such values are variable. First, the emissivity value varies during heating cycles (Castro and Paulo, 2015); during heating and cooling, therefore, an if statement is used to introduce the emissivity as 0.17 during heating the SMA and as 0.21 during cooling. Second, the absorptivity value varies based on the material colour and texture; which are design choices. Thus, a range from 0.5 to 0.7 is used as the absorptivity value to showcase the effect of colour and texture during the simulation phase.

$$T_{\text{Final}}= DBT + \Delta T$$ \hspace{1cm} (3)

After calculating $\Delta T$; equation (3) is used to calculate the final temperature of the SMA; where $T_{\text{Final}}$ is the SMA’s final temperature, $DBT$ is the dry bulb temperature at the simulated hour and $\Delta T$ is the SMA’s change in temperature calculated from equation (1).

Thereby, an archsim component is used to extract the DBTs at the simulated hours from Cairo, Egypt’s weather file. After calculating equation (3), the $T_{\text{Final}}$ is compared to the SMA material change temperatures.

$$State = SE_2 – SE_1$$ \hspace{1cm} (4)

Equation (4) is used to identify the cooling or heating state of the SMA panel; where $SE_2$ is the solar energy falling upon the panel at the simulated hour and $SE_1$ is the solar energy at the previous hour. The value $State$ indicates if the material is cooling ($State = -ve$) or heating ($State = +ve$)

$$t_f = (T_{\text{Final}} – As) / (Af – As)$$  \hspace{1cm} (5)

$$t_f = (T_{\text{Final}} – Ms) / (Mf – Ms)$$ \hspace{1cm} (6)

Equations (5) and (6) are used to indicate the transformation degree of the SMA; where $t_f$ is the transformation factor of the SMA, $T_{\text{Final}}$ is the panel’s temperature, $As$ is the austenite start temperature, $Af$ is the austenite finish temperature, $Ms$ is the martensite start temperature and $Mf$ is the martensite finish temperature.

Deciding to use equation (5) or (6) depends upon the value of $State$ in equation (4). On the one hand, if $State$ is a positive value; the SMA is heating up and equation (5) should be followed. On the other hand, a negative value for $State$ indicates the SMA is cooling up and $t_f$ is calculated using equation (6).

For clarity, the idea of equations (5) and (6) is to get a transformation range value between 0 and 1; the $t_f$ value 0 indicates the initial state of the two way SMA panel, and the value 1 indicates the final state of the panel. Such value is remapped to a number that ranges from the initial angle (in degrees) and the final state angle as shown in Figure 5.

**Figure 5 Transformation Illustration for One SMA Shading Unit**

**Optimization: SMA Shading System**

The process of optimizing movable shading systems generally requires introducing a schedule of movement states and calculating heat loads over a building façade. For the particular case of SMA system; first, the schedule itself is simulated for an array of panels using the previously mentioned equations. Second, the total solar energy falling upon a south-western façade is calculated. A detailed explanation of the assumptions and optimization process steps is introduced below.

For starters, the building façade –as stated in the limitations- is assumed to be 1.5 meters long and 2.5 meters high. The façade is assumed to face the south-west direction. This particularly aims at maximizing the solar energy upon the SMA panels to maximize the $t_f$.

Second, from an offset distance of 0.1 meters away from the façade; a SMA panel - of a variable length ($L_{\text{panel}}$), a constant width of 10 centimetres and a variable absorptivity according to the finish design ($a$ varies from 0.5 to 0.7) - is arrayed. The grasshopper definition gives $L_{\text{panel}}$ a variation from 30 centimetres to 50 centimetres. In short; the panel can be elongated from 30 to 50 cm, which in turn leads to a maximum of 65 panels and a minimum of 21 panels arrayed respectively. It is important to note that the initial state per panel can vary where the initial bending angle $B_{\text{angle}}$ (at 10 cm of the panel’s length as shown in Figure 4) can change from 0° to 89°.

Third, a Diva squared grid sensors having a side length 10 cm is introduced over the assumed façade; a total of 375 sensors. Such sensors accurately calculates the SE falling upon the south-western façade while considering the SMA panels as shading objects. In that sense, the heat
falling on the panels transforms them which subsequently affects the SE on the façade.

Fourth, all the previously mentioned steps are repeated to simulate both the 21st of June and the 22nd of December from 6:00 till 18:00. Thus, a complete simulation of the arrayed panels’ states along with the SE on the façade is reached. The input and output of such simulation are used to optimize the SMA design.

Finally, octopus tool uses genetic algorithm to reach an optimized state. It should be noted that octopus tries to maximize “objective” values by changing “Genome” values. In this paper, the goal is to maximize -SE_{June} (i.e. minimize SE_{June}) and at the same time maximize SE_{December}. This is achieved by manipulating the L_{panel}, B_{angle}, and the absorptivity (\alpha) of the SMA. Two optimization cases dealing with B_{angle} differently are introduced and compared using the optimization methodology previously introduced; this is shown in detail, for clarity, as follows:

1) Run 1: all SMA panels are arrayed with a typical B_{angle}.
   a. Genomes:
      i. B_{angle} (1 value)
      ii. L_{panel} (1 value)
      iii. Absorptivity [\alpha] (1 value)
   b. Objectives:
      i. -SE_{June} (1 value)
      ii. SE_{December} (1 value)

2) Run 2: each SMA individual panel has its own initial state (B_{angle}).
   a. Genomes:
      i. B_{angle} (from 21 to 65 values)
      ii. L_{panel} (1 value)
      iii. Absorptivity [\alpha] (1 value)
   b. Objectives:
      i. -SE_{June} (1 value)
      ii. SE_{December} (1 value)

It is important to mention that the Octopus tool genetic algorithm runs, as previously mentioned in the methodology, with an elitism of 0.6, a mutation probability of 0.5, a mutation rate of 0.5 and a crossover rate of 0.6.

Results and Discussion

For consistent results, the optimization process is processed using octopus tool over two runs with the same genetic algorithm settings previously stated. Each run reached 30 populations having a size of 50; i.e. 1500 results are tested per run. The first simulation run took approximately 38 hours to generate the results (Average Run Time = 50.99 seconds) to be stated, while the second run took approximately 43 hours (Average Run Time = 62.21 seconds). The reason for such elongated simulation time is, as previously stated, the merged approach implemented. Such approach demands for every hour of day within the solution, a simulation of the heat loads on every single panel along with the building envelope and the simulation of the SMA shape change. In other words, the panels and elevations are simulated 26 times per solution. This clearly justifies the average solution run time of 50.99 seconds for the first run and 62.21 seconds for the second run.

On the one hand, “Run 1” reached 5 dominant solutions within the 30th generation; for clarity, three variable samples are introduce. Such samples, shown in Table 1, have an initial rotation angle B_{angle} of 65, 76 and 89 degrees and a panel length L_{panel} of 31, 38 and 49 centimetres respectively. This resulted in an SE_{June} of 5.03, 5.58 and 5.99 Kwh, and an SE_{December} of 11.32, 12.31 and 13.15 Kwh respectively. The absorptivity has values of 0.6, 0.5 and 0.6 which can guide a designer when implementing the finish of the NiTiZr panel. It is important to pinpoint the fact that the results show a tendency of an increase of SE_{June} and SE_{December} with the increase of L_{panel}. Furthermore, the increase of the initial rotation angle have the same effect upon SE_{June} and SE_{December}; increases their values.

On the other hand, “Run 2” reached 64 dominant solutions having extremely variable B_{angle} values. For simplification, three sample solutions are taken for “Run 2” with variable objective values. For such solutions, L_{panel} has values of 31, 38 and 44 centimetres, SE_{June} values are 4.97, 5.46 and 5.71 Kwh and SE_{December} are 11.33, 12.25 and 12.71 Kwh respectively. An analysis of “Run 2” results shows a clear trend of an increase of SE_{June} and SE_{December} with the increase of the Panel’s length (L_{Panel}). Yet, by going back to Table 1, the B_{angle} values show an inconsistent relationship with the SE values. For more detailed results on “Run 2” B_{angle}, Table 1 showcases the range of the angles for each sample result. Furthermore, a graphical representation of the case of multiple initial rotations (Run 2) and a single rotation as well (Run 1) is showcased in Figure 6.

By comparing the two runs’ results, first a general trend is visible, it can be argued that the length of the panel and the SE values are directly proportional. This is visible throughout the results of both runs. Second, a contradicting relation between the rotation angle (B_{angle}) and SE values is shown where Run 1 shows a directly proportional relation; unlike the inconsistent relation in “Run 2”. This goes back to the fact that the initial rotation angle values highly affect the SE absorbed by the SMA panels; thus affecting their t_f values. Such argument justifies the consistency of “Run 1” relation where similar initial angles per panel results in almost similar t_f values. It also justifies the inconsistent relation of “Run 2” where the variable initial angles lead to different changes in t_f values; i.e. variable rotational transformations and variable SE values over the building façade.

Conclusion

SMA shading systems introduce the ability of dynamically reacting to solar conditions without the need of any electrical systems. Such ability has been showcased by the variety of results reached during the optimization phase due to the physical transformations of SMAs. The process introduced in itself is highly flexible; it is possible to reuse different geometries and different
SMA materials to further optimize the shading system’s efficiency. However, two main challenges are present. First, the macro-scale simulation process of SMA is immanently inaccurate. This leads us to the second challenge where a proper validation process is needed for the SMA simulation tool. The theoretical implementation proved feasible; yet, observing the actual behaviour of physical SMA prototypes and comparing it with the simulation results is a needed validation procedure. To conclude; although SMA shading systems require further research in physical implementations and prototyping, there is a theoretical possibility of optimizing such energy efficient systems; as showcased in this paper.

Table 1 Optimization results with corresponding objectives and genomes

<table>
<thead>
<tr>
<th></th>
<th>( L_{\text{panel}} ) [Meters]</th>
<th>Initial Rotation ( (B_{\text{angle}}) ) [Degrees]</th>
<th>Number of Values in ( B_{\text{angle}} ) Range</th>
<th>Absorptivity ( (a) )</th>
<th>( SE_{\text{June}} ) [Kwh]</th>
<th>( SE_{\text{December}} ) [Kwh]</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Run 1” Sample 1 Solution</td>
<td>0.31</td>
<td>65</td>
<td>1</td>
<td>0.6</td>
<td>5.03</td>
<td>11.32</td>
</tr>
<tr>
<td>“Run 1” Sample 2 Solution</td>
<td>0.38</td>
<td>76</td>
<td>1</td>
<td>0.5</td>
<td>5.58</td>
<td>12.31</td>
</tr>
<tr>
<td>“Run 1” Sample 3 Solution</td>
<td>0.49</td>
<td>89</td>
<td>1</td>
<td>0.6</td>
<td>5.99</td>
<td>13.15</td>
</tr>
<tr>
<td>“Run 2” Sample 1 Solution</td>
<td>0.31</td>
<td>( 0 &lt; B_{\text{angle}} \leq 30 ) 21</td>
<td></td>
<td>0.6</td>
<td>4.97</td>
<td>11.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 30 &lt; B_{\text{angle}} &lt; 60 ) 14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 60 \leq B_{\text{angle}} &lt; 90 ) 25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Run 2” Sample 2 Solution</td>
<td>0.38</td>
<td>( 0 &lt; B_{\text{angle}} \leq 30 ) 14</td>
<td></td>
<td>0.7</td>
<td>5.46</td>
<td>12.25</td>
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<tr>
<td></td>
<td></td>
<td>( 30 &lt; B_{\text{angle}} &lt; 60 ) 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 60 \leq B_{\text{angle}} &lt; 90 ) 13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Run 2” Sample 2 Solution</td>
<td>0.44</td>
<td>( 0 &lt; B_{\text{angle}} \leq 30 ) 11</td>
<td></td>
<td>0.5</td>
<td>5.71</td>
<td>12.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 30 &lt; B_{\text{angle}} &lt; 60 ) 7</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>( 60 \leq B_{\text{angle}} &lt; 90 ) 6</td>
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</tr>
</tbody>
</table>

Figure 6 Graphical representation of “run 2” multiple rotations angle (left) and “run 1” single rotation angle (right)
References