MULTI-OBJECTIVE TRAJECTORY OPTIMISATION ON ENVIRONMENTAL IMPACTS

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Mengying Zhang
School of Mechanical, Aerospace, and Civil Engineering
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With the rapid development of modern aviation industry, how to make a balance between gradually increased aviation activities and the life quality of residents living within the vicinities of the airports becomes an increasingly prominent challenge.

Research efforts have been made on single or multiple objective trajectory optimisations using gradient-based methods. Despite the effectiveness of the gradient-based methods, their applicability is limited by high standard requirements for problem formulation with smooth differentiable dynamics models and explicit expressions of optimisation objectives. An alternative option is using gradient-free algorithms capable of solving optimisation problems with discrete models, facilitating the integration of “black box” models that lack gradient information. However, the goals are generally achieved at the price of extensive computational burden. Moreover, adding further environmental parameters or different noise attributes, an optimal solution cannot be selected from the solution set obtained in the multi-objective optimisation problems, unless further algorithms are introduced.

The present work focuses on the development of a multi-objective optimisation framework for departure and arrival aircraft to minimise multiple environmental impacts (noise and exhaust emissions). This framework includes a user-defined input module, a optimisation module, and flight performance module. The trajectory optimisation module includes a set of of nonlinear models: aircraft dynamics, trajectory constraints and objective functions. A method to parameterise movement in the lateral plane based on a Bézier curve has been proposed to decrease the number of free parameters. The environmental objectives are modeled by a comprehensive flight mechanics program FLIGHT and the ANP database from EUROCONTROL. Two posterior selection strategies based on a preference function and monetisation approaches are used to evaluate the resulting Pareto solution set.

A simple single flight trajectory optimisation problem is identified and formulated as a multi-objective optimal control problem with a discontinuous problem formulation, solved by non-gradient algorithms. Among the three different non-gradient algorithms used, a non-dominated sorting genetic algorithm is identified as the most widely used method. However, two PSO-based multi-objective optimisers are explored to overcome some of the drawbacks of the GAs technique. Complex multiple flight events trajectory optimisation problem is identified and formulated as a mixed-integer non-linear programming problem. A time-based separation rule is applied to approach the real flight assignment scenario. Finally, this application is conducted with a simple scenario to demonstrate its functionality. We demonstrate that this simulation framework is capable of solving trajectory optimisation problems with multiple simultaneous environmental objectives.

**Keywords** Aircraft trajectory; Noise; Exhaust Emissions; Multi-objective Optimisation.
Declaration

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List of Publications

The list of publications resulting from this PhD work is given in inverse chronological order as follows:


**Nomenclature**

**Acronyms and Abbreviations**

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<tr>
<th>Acronym</th>
<th>Definition</th>
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<tr>
<td>ACARE</td>
<td>Advisory Council for Aeronautics Research in Europe</td>
</tr>
<tr>
<td>AFE</td>
<td>above field elevation</td>
</tr>
<tr>
<td>AG</td>
<td>Multi-Objective Particle Optimisation based on Adaptive Grids</td>
</tr>
<tr>
<td>ANOPP</td>
<td>Aircraft Noise Prediction Program</td>
</tr>
<tr>
<td>ANSI</td>
<td>the American National Standard Institute</td>
</tr>
<tr>
<td>BEIS</td>
<td>Department for Business, Energy and Industrial Strategy</td>
</tr>
<tr>
<td>CAS</td>
<td>calibrated airspeed</td>
</tr>
<tr>
<td>CDA</td>
<td>Continuous Descent Approach</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CNP</td>
<td>cost for noise protection</td>
</tr>
<tr>
<td>CO</td>
<td>Carbon monoxide</td>
</tr>
<tr>
<td>CO₂</td>
<td>Carbon dioxide</td>
</tr>
<tr>
<td>DALY</td>
<td>Disability-Adjusted Life Year</td>
</tr>
<tr>
<td>DM</td>
<td>decision maker</td>
</tr>
<tr>
<td>DOF</td>
<td>degree-of-freedom</td>
</tr>
<tr>
<td>EA</td>
<td>Evolutionary algorithms</td>
</tr>
<tr>
<td>ETS</td>
<td>Emissions Trading System</td>
</tr>
<tr>
<td>FMS</td>
<td>Flight Management System</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>GATAC</td>
<td>Green Aircraft Trajectories under ATM Constraints</td>
</tr>
<tr>
<td>ICAO</td>
<td>International Civil Aviation Organisation</td>
</tr>
<tr>
<td>IGCB(N)</td>
<td>Interdepartmental Group on Costs and Benefits Noise Subject Group</td>
</tr>
<tr>
<td>JAXA</td>
<td>Japan Aerospace Exploration Agency</td>
</tr>
<tr>
<td>MOPSO</td>
<td>Multi-objective Particle Swarm Optimisation</td>
</tr>
<tr>
<td>MOTO</td>
<td>Multi-Objective Trajectory Optimisation</td>
</tr>
<tr>
<td>MPPA</td>
<td>million passengers per annum</td>
</tr>
<tr>
<td>INM</td>
<td>Integrated Noise Model</td>
</tr>
</tbody>
</table>
NBI = Normal Boundary Intersection method
NC = Normal Constraints method
NLP = Non-Linear Programming
NOx = Oxides of nitrogen
NPD = Noise-Power-Distance
NSA = Noise Sensitive Area
NSGA-II = Nondominated Sorting Genetic Algorithm II
OCP = Optimal Control Problem
ONERA = Office National d’Etudes et de Recherches Aérospatiales
PE = Multi-objective Particle Optimisation based on Pareto Entropy
PSO = Particle Swarm Optimisation
RF = radius-to-a-fix
RNAV = Area Navigation
SID = Standard Instrument Departure
SCC = social cost of carbon
TBVP = Two-Point-Boundary-Value Problem
UCNPP = unit cost of noise protection by a population
YLD = Years Lost due to Disability
YLL = Years of Life Lost

Roman Symbols

\[ b = \text{Bernstein basis polynomials} \]
\[ c = \text{PSO parameters} \]
\[ f = \text{fuel flow rate, kg/s; criterion} \]
\[ g = \text{gravitational acceleration, m/s}^2 \]
\[ h = \text{altitude, m} \]
\[ k = \text{drag induced parameter} \]
\[ m = \text{aircraft mass, kg} \]
\[ n = \text{load factor, preference value function} \]
\[ p = \text{preference value} \]
\( t \) = flight time, s
\( V \) = true airspeed, m/s; total preference value
\( x \) = flight time, horizontal distance from aircraft to threshold of the runway, m
\( y \) = lateral distance from the aircraft to the centreline of the runway, m
\( C \) = aerodynamic force coefficient
\( D \) = aerodynamic drag
\( F_N \) = net thrust from all engines, N
\( Ma \) = Mach number
\( N_1 \) = engine rpm, %
\( P \) = percentage
\( R \) = Radius, m
\( S \) = wing area, m\(^2\)
\( B \) = Bézier curve function
\( P \) = coordinate of control points
\( u \) = vector of control variables
\( x \) = vector of state variables

**Greek Symbols**

\( \alpha \) = angle of attack, rad; image of preference value function
\( \beta \) = coefficient of preference value function
\( \gamma \) = flight path angle, rad
\( \delta \) = pressure ratio
\( \epsilon \) = error
\( \theta \) = temperature ratio
\( \rho \) = density, kg/m\(^3\)
\( \sigma \) = density ratio
\( \phi \) = bank angle, rad
\( \chi \) = heading angle, rad
\( \omega \) = inertia weight
Subscripts

0 = initial
f = final
c = control
L = lift
$L_0$ = lift coefficient at zero angle of attack
$L_\alpha$ = slope of lift coefficient curve
$D = $ drag
$D_0$ = drag coefficient at zero lift
max = maximum
min = minimum
Chapter 1

Introduction

As one of the most significant traffic services in the world, it is challenging for air transportation to make a balance between maximising the utilisation of aviation growth and minimising the environmental impacts it exerts. A vision goal exists for the future aviation industry: a 65% reduction in perceived noise in 2050 set by Council for Aeronautics Research in Europe (ACARE) [9] and a 75% reduction in CO$_2$ emissions per passenger kilometre as well as a 90% reduction in NOx emissions [10], which is a big challenge from both theoretical and practical aspects. This research discusses a flight trajectory optimisation framework to minimise noise and emission subject to different environmental constraints.

The first chapter provides a general introduction to the topic of this research project and specifies the context in which the subject is going to be carried out. It furthermore describes the objective as well as the scope and a brief outline of this work.

1.1 Background

With the rapid growth of commercial aviation, increasing public concern about the environmental issues of air traffic which consists of noise, air pollution and climate changes has caused attention widely. Although the past several decades have witnessed policies and regulations adopted to control the noise exposure and gaseous emissions as well as novel technologies applied, meeting community expectations along with the sustainable development of a commercial aviation market is a big problem that cannot be ignored. As is shown in Table 1.1, though the population within the 57 dBA contour for the four largest airports in the UK has fallen by 28.11% during the past 17 years (i.e. 1998-2015) [11–13], the public still tends to believe that the noise impacts on the environment have become worse rather than better [14]. From this point of view, the problem is not merely how to achieve a quieter, cleaner and more efficient aircraft flight but how to compensate the additional environmental cost to
Table 1.1: Changes to designated airport noise contours.

<table>
<thead>
<tr>
<th>Airport</th>
<th>1998</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of aircraft movements</td>
<td>Area of 57 dBA contour [km²]</td>
</tr>
<tr>
<td>Heathrow</td>
<td>441,200</td>
<td>163.7</td>
</tr>
<tr>
<td>Gatwick</td>
<td>240,200</td>
<td>76.8</td>
</tr>
<tr>
<td>Manchester</td>
<td>161,800</td>
<td>53.5</td>
</tr>
<tr>
<td>Stansted</td>
<td>102,200</td>
<td>64.5</td>
</tr>
<tr>
<td>TOTALS</td>
<td>945,400</td>
<td>358.5</td>
</tr>
</tbody>
</table>

support the development of the entire aviation industry.

Another fact is the increasing demand for finding solutions and strategies to achieve the biggest efficiency of fuel yet with reduced emissions. From the forecast of 2011 International Civil Aviation Organisation (ICAO), worldwide passengers will witness a steady growth of about 4%-5% annually till the year 2030. According to the Airport Commission’s forecasts, the total potential capacity demand for UK airports is likely to experience a significant increase achieving 450 million passengers per annum in 2050 due to unconstrained policies towards airports capacity and aviation emissions [15]. The increasing requirement for air traffic capacity leads to a rise in carbon trades, especially when CO₂ emissions from UK aviation currently occupies 5%-6% of total UK CO₂ emissions. Effective and efficient control of carbon emissions will directly determine the sustainable development of the UK aviation industry. Despite carbon dioxide, other gaseous emissions including oxides of nitrogen (NOx), water vapour, particulates, carbon monoxide (CO), unburned hydrocarbons (HC), soot and oxides of sulphur (SOx) also add to the environmental problems. Compared with the data recorded in 2000, two future goals set by the Advisory Council for Aeronautics Research in Europe (ACARE) considering both CO₂ and NOx are listed below [16]. By the year 2020:

- To reduce fuel consumption and CO₂ emissions by 50%;
- To reduce NOx by 80%.

In order to meet the gap between the target of emission reduction and the continuous development of the aviation industry, attempts should be made through technological innovation and improvement. On one side, novel techniques of engine and aircraft configuration design have the potential to eliminate the environmental pressure from aviation activities. On the other side, considering the existing aircraft will be on active service continuously, it is preferable to find solutions with the existing fleet and the
technology we have already possessed. One potential program is to improve aircraft flight trajectory for minimum noise and minimum environmental emissions.

1.2 Context

The issue to develop a greener and quieter approach to commercial air traffic has received considerable critical attention. The endeavour to reduce aircraft noise, carbon dioxide, oxides of nitrogen and other substances has been made for many decades. Although great efforts have been made to achieve a reduction in these three objectives separately from both technological aspects and policy aspects, it is still a challenge to ensure all three targets decrease simultaneously. For example, there are significant trade-off issues between noise, NOx and CO$_2$, primarily related to the thrust-settings that may be employed at various stages of the departure procedure in the effort of noise reduction. Another case is during the en-route phase. For instance, the cruise speed impacts the whole fuel at the same time determines occasion of the holding or diversion before a curfew, which will exert a trade-off between local noise issues and flight energy consumed [17]. Although we know that measures such as Continuous Descent Approach have the potential to reduce all three, there are still limitations from both techniques and safety issues. Therefore, operational practices or potential technologies to obtain a sustainable aviation development are increasingly recognised as a global economic and environmental issue.

The possibility of minimising noise and pollutant emissions by trajectory optimisation has generated wide interest during the past few years. New air traffic management concepts have been aroused in the United States under the name of NextGen [18], in Europe under the name of SESAR [19] and in Japan under the name of CARATS (Collaborative Actions for Renovation Air Traffic Systems) [20]. One main aim is to satisfy the increasingly diverse requirements of airlines and passengers to deal with global environmental issues. As a consequence, each commercial flight aims to be more environmentally friendly and produce less negative impacts towards the environment. Previous studies have shown that aircraft noise annoyance of communities near the airport can be significantly reduced by trajectory optimisation [21–23]. In the search for flight trajectories, the CleanSky [16] European research project has developed a flight planning tool with lateral and vertical trajectories decoupled and optimised to meet constraints of routes and flight limitations, resulting in a full optimal trajectory for various criteria, like noise influence, fuel consumption, flight cost, and time [21]. Similarly, in JAXA’s new traffic management system called DREAMS, advanced flight trajectory control technologies will provide data for selecting routes to minimise noise exposure area making it possible to guide the aircraft along the optimal routes for noise reduction [24].

Despite quiet aircraft technology adopted by manufacturers and applied on engines
and other components, the way to reduce aircraft noise impact is to put restrictions on
noise emissions resulting from propulsion systems and aerodynamics, and to regenerate
flight procedures at departure, arrival and approach. In the 1980s, noise abatement
flight procedures (NAPs) for departure and approach were developed [25]. For ex-
ample, it was shown in a case conducted in work from Menéndez that a nearly 11%
reduction in maximum noise annoyance could be achieved by NAP flight path [26].
However, incomplete noise reduction potential has been realised due to unsatisfactory
engine performance and noise levels in addition to other factors embedded in aircraft
flight, guidance and control procedure. Several studies investigating noise abatement
of different flight phases have been carried out. Measurement in Ref [27] shows that
noise abatement can be achieved with an optimised approach procedure.

Table 1.2: Summary of noise mitigation opportunities with aircraft departure.

<table>
<thead>
<tr>
<th>Departure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical noise mitigation</td>
</tr>
<tr>
<td>• Continuous climb</td>
</tr>
<tr>
<td>• Climb thrust management</td>
</tr>
<tr>
<td>Horizontal noise mitigation</td>
</tr>
<tr>
<td>• Off-set Standard Instrument Departures (SIDs)</td>
</tr>
<tr>
<td>• Runway alternation</td>
</tr>
<tr>
<td>• Defined SIDs</td>
</tr>
<tr>
<td>• Noise preferential routes (NPRs)</td>
</tr>
<tr>
<td>• Runway directional preference</td>
</tr>
<tr>
<td>Aircraft operational practice</td>
</tr>
<tr>
<td>• Noise management such as NADP1 or NADP2.</td>
</tr>
</tbody>
</table>

Table 1.3: Summary of noise mitigation opportunities with aircraft arrivals.

<table>
<thead>
<tr>
<th>Arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical noise mitigation</td>
</tr>
<tr>
<td>• Continuous descents</td>
</tr>
<tr>
<td>• Displaced threshold</td>
</tr>
<tr>
<td>• Steeper approaches and segmented steeper approaches</td>
</tr>
<tr>
<td>Horizontal noise mitigation</td>
</tr>
<tr>
<td>• Curved approaches</td>
</tr>
<tr>
<td>• Adjusted joining point</td>
</tr>
<tr>
<td>• Runway alternation</td>
</tr>
<tr>
<td>• Defined Standard Arrivals Routes (STARS)</td>
</tr>
<tr>
<td>• Runway directional preference</td>
</tr>
<tr>
<td>Aircraft operational practice</td>
</tr>
<tr>
<td>• Low power low drag</td>
</tr>
<tr>
<td>• Managed approach speeds</td>
</tr>
<tr>
<td>• Avoiding reverse thrust on landing</td>
</tr>
</tbody>
</table>
Table 1.4: Summary of aircraft ground noise mitigation opportunities

<table>
<thead>
<tr>
<th>Ground noise</th>
<th>Ground noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical noise</td>
<td>N/A</td>
</tr>
<tr>
<td>Vertical noise</td>
<td></td>
</tr>
<tr>
<td>Horizontal noise</td>
<td>Siting of aircraft engine test facilities at airports</td>
</tr>
<tr>
<td>Horizontal noise</td>
<td></td>
</tr>
<tr>
<td>Aircraft operational</td>
<td>Reduced engine taxi</td>
</tr>
<tr>
<td>practice</td>
<td>Use of Fixed Electrical Ground Power and</td>
</tr>
<tr>
<td></td>
<td>Pre-Conditioned Air</td>
</tr>
</tbody>
</table>

Other operational noise mitigation opportunities can be summarised as it is shown in the Table 1.2 to Table 1.4. Distinctive methods have been under exploration, and some have been applied for specific flight phases in order to contribute to noise reduction. Besides, it is also important to note that other environmental factors can be influenced by operational techniques as well since any technique that affects the thrust required would, in turn, have an impact on the gaseous emission. In general, to operate quieter and greener cannot be achieved without trajectory optimisation techniques.

Many research groups have made contributions to multi-objective trajectory optimisation to reduce emissions and noise. By fixing ground routes, vertical departure profiles are optimised with a departure optimisation tool named NOISHHH developed by the University of Delft [28, 29]. Likewise, a research group from Cranfield University has developed a multi-disciplinary flight trajectory optimisation tool called GATAC founded by the project Clean Sky aiming at addressing the environmental impact and commercial implications [30, 31]. Similar efforts have been made including Fernandes’ [21] optimal control base framework for on-board optimal flight planning, Prats’ [32] multi-criteria optimisation framework for computing optimal NAP, Serafino and Fanti’s [33–35] method based on Dijkstra algorithm to reduce aircraft emissions in case of weather hazard, and so on.

1.3 Objectives

Meeting community expectations on aircraft noise reduction and environmental emission has always been a challenge to the aeronautic industry. It is during the take-off, descent and landing phases that the aircraft noise and exhaust emissions become more significant because the aircraft flies at a very low altitude with a relatively high power condition. Based on the discussion above, the main objective of this work is to develop a multi-objective trajectory optimisation framework for minimum noise and exhaust
emissions, subject to multiple constraints such as weather hazard, flight speed, and so on.

For commercial aircraft, there usually exist certain routes to follow which come from the guidance of Instrumental Flight Rules (IFN). With the help of navigation instruments and systems, aircraft operate standard procedures containing departure, arrival and approach manoeuvres. These standardised procedures are designed for the sake of aviation safety and optimal air traffic management. Therefore, to meet the need of designing noise/emission-optimised trajectories, limitation and constraints from real flight procedure should be taken into consideration. Otherwise, extreme manoeuvres would add less reliability to aviation events.

The noise index and emission index needed in the simulation and optimisation problems in this study are calculated from two sources. One is the existing flight performance software named FLIGHT [36] and the other is the International Aircraft Noise and Performance Database from Eurocontrol [4]. With the multi-objective optimisation problem constructed, an optimisation algorithm will be applied to compute the best departing or approaching trajectory where constraints are determined by the real flight environment. To select the best solution, analyses based on the decision making process are needed. In general, the objective of this work can be summarised as follows:

- Provide a comprehensive review of state of art in aircraft noise and gaseous emissions.
- Design a trajectory optimisation model needed for noise and emission abatement with necessary constraints models.
- Develop a mechanism to integrate the noise index model and exhaust emission model into optimisation framework.
- Apply and test the optimisation framework in a real scenario environment.

The most important contributions to knowledge of this work include four main aspects:

- A multi-objective trajectory optimisation framework using the FLIGHT program to calculate environmental indexes more accurately is developed to make up for the shortcomings in the use of INM model.
- The traditional non-gradient algorithm to optimise the trajectory has the disadvantage of expensive computational cost. In the model developed in the thesis, the parameterisation method based on Bèzier curve is adopted to parameterise lateral tracks, which reduces the number of free parameters and improves the efficiency.
- Considering the fact that the decision making in the multi-objective optimisation on environmental impacts is not intuitive, two posterior selection strategies are proposed to quantify and evaluate different environmental objectives.
Since the existing environmental impact-based trajectory optimisation can only deal with the optimisation of a single flight operation, a method to optimise multiple flight operations for minimal noise and emissions is proposed.

1.4 Thesis Structure

The thesis is presented in eight Chapters.

- Chapter 1 provides a general introduction to the research background, the objective and its outline.

- Chapter 2 provides an overview of the aircraft noise and emissions as well as fundamental knowledge of aircraft trajectory optimisation. It also includes a review of different noise modelling methods and their applications in flight trajectory optimisation problems. Furthermore, Chapter 2 discusses the concept of multi-objective trajectory optimisation as well, including the optimisation techniques and the applications.

- Chapter 3 establishes the generic formulation of the trajectory optimisation problem of single aircraft. A blueprint of a multi-objective trajectory optimisation framework is built under generic considerations, assumptions and statements on the basis of the aircraft performance model and optimisation process.

- Chapter 4 contains the posterior selection strategy for decision making. It includes the single metric value assessment evolving subjective factors and monetisation method to provide objective decisive criteria.

- Chapter 5 develops a multi-objective trajectory optimisation method focusing on the departure procedure. The result of applying the proposed method is shown in a scenario with one noise sensitive area and emission impact considered. This chapter allows us to prove the proposed optimisation framework.

- Chapter 6 studies the optimal arrival trajectory considering environmental impacts where noise sensitive areas (NSAs) and emission impacts are found. A combined objective concerning the characteristic of the aircraft arrival is assessed.

- Chapter 7 defines the problem of optimising multiple flight operations for minimal noise and emission impacts. A mixed-integer nonlinear programming problem is built to deal with the multi-objective trajectory planning problem considering noise impacts and emission exhausted. The method is proved by a multiple-arrivals re-planning scenario.
• Chapter 8 gives the conclusions that are drawn from this work and points out some limitations. Some future work that could be done is also concluded in this chapter.
Chapter 2

Literature Review

2.1 Aircraft Noise Modelling

Aircraft noise is sound pressure disturbance generated by propulsion systems and the aerodynamic interaction during the various phase of a flight [37]. As for aircraft noise prediction, different research groups aim at dealing with various aspects of this problem, which leads to a diversity in the researches conducted and the tools developed.

On the component level, Office National d’Etudes et de Recherches Aéronautiques (ONERA) from France focuses more on the aeroacoustic hybrid methods to achieve aircraft noise prediction. Computational tools, especially CFD solvers for CFD-CAA coupled approaches, have been developed greatly through the past decade. A time-domain CAA solver with high accuracy called sAbrinA [38–40] and its derivatives are applied in real complex aeroacoustic examples. Three different stages for aircraft noise emission have been taken into consideration in ONERA’s work: 1) unsteady turbulence in near-field region for noise generation; 2) steady yet heterogeneous flow in mid-field region for noise propagation including interactions between noise sources; 3) steady homogenous flow in far-field propagation [41]. Based on the hybrid aeroacoustic approach, both isolated and installed aircraft noise problems have been studied, ranging from noise emission by landing gear in LAGOON project [42–44] to aft fan noise emission by a partly installed engine [38,45]. One advantage of the aeroacoustic hybrid approach is that it provides an advanced approach to solve realistic aircraft noise problems and might be the best alternative to direct numerical simulation due to its relatively “economic” cost performance regarding computational time and memory consumption [41]. The research area covers both theoretical aspect and empirical aspects in order to make a combination to provide solid support for further validations of codes and numerical methods about aeroacoustic problems.

On the systems level, NASA’s Aircraft Noise Prediction Program (ANOPP) was designed to predict the total aircraft noise signature from propulsion and airframe
noise sources and to propagate total noise to arbitrary ground observers. This program has undergone major development in the past decade and an updated successor of this program, ANOPP2 [46], has been developed. Compared with its predecessor (ANOPP), ANOPP2 is expected to provide more robust, higher fidelity and physics-based noise prediction rather than empirical or semi-empirical prediction methods. Besides, another advantage that ANOPP2 offers is its ability to provide noise prediction from various aircraft design. Options ranging from fast computations for system level studies, unconventional aircraft type study by a mixture of method fidelity, and model scale predictions using high-fidelity and reduced-order methods are all available in ANOPP2. Still, the whole system is waiting for the experimental data to be accessible for validation.

Another noise prediction model which is widely used is the Integrated Noise Model (INM) [47] released by the Federal Aviation Authority. With noise metrics provided for a range of common commercial aircraft, INM is able to predict a noise contour of single flight operation or multiple flight operations. The model it uses to calculated noise is by means of noise-power-distance (NPD) tables, where noise levels are calculated by interpolation and extrapolation with net corrected thrust and the aircraft’s location. Moreover, adjustments like lateral attenuation and noise fraction are taken into account by INM. The advantage of INM is that the database it uses is from experiments or industry supply, so it is a fully validated model. However, as an empirical correlation based method, INM has the problem of coarse precision.

The last model introduced here is the flight performance software FLIGHT [36]. This multi-disciplinary software has the ability of environmental analysis including aircraft noise prediction and exhaust emissions prediction. One of its advantages comes from its noise propagation model: all the routines that are used to calculate the external effects on the noise source are taken into account including atmospheric absorption, atmospheric thermos-physics (temperature, density and humidity distributions), wind and turbulence, and ground effects. Besides, a sub-module that deals with the signal analysis is available, which means all the propagation effects from source to the receiver can be tracked for validation. What’s more, the time consumed for the noise evaluation for one trajectory is no more than 1 minute, which proves it to be a useful tool for aircraft noise prediction.

Due to the high physical complexity of aircraft noise emission and propagation, the adoption of the accurate models is often limited to off-line simulation processes. Propagation effects, including spherical spreading, the atmospheric absorption, ground absorption and reflection, refraction or scattering and terrain effects, are taken into consideration to provide a more accurate noise footprint on the ground. However, in the real-time trajectory optimisation application involved with aircraft noise emission and propagation, especially those performed to optimise departure and approach trajectories, prediction methods adopted need to be accurate yet simple enough to enable integration in real-time simulation for noise evaluation of the trajectory generated. In
this study, FLIGHT is mainly used to support trajectory optimisation on noise index.

2.2 Aircraft Emission Modeling

Components of emissions from aircraft engines depend on the flight power condition and the combustion efficiency. The emissions may contain carbon dioxide (CO$_2$) (∼70%), water vapour (H$_2$O) (30%) and other pollutants consist of: nitrogen oxides (NOx) as the by-products of high pressure and high temperature combustion; sulphur oxides (SOx) from combustion of sulphur containing fossil fuels; unburned hydrocarbons (HC); and carbon monoxide (CO) from incomplete combustion of fossil fuels, particulate matters and so on [48].

Since combustion is a complex reaction flow process, it is difficult to simulate the reaction process accurately in the combustion chamber and to predict emissions. Although it is challenging to quantify air pollutants from the aircraft engine, three different levels of prediction methods are capable of providing quantitative information of emissions. They are high fidelity models with detailed CFD calculations, physics-based stirred reactor [48] and empirical correlations.

The first one involves high fidelity CFD calculations using the simulation of the chemical reactions and flow dynamics within the combustion chamber. Its application to real-time prediction is limited for the time consuming and expensive computational characteristics. The second one, developed by Cranfield University, can predict emissions from current and future engine configurations concerning degradation. For more detail on emission prediction with this method, the reader may refer to Ref. [49].

The third one is a kind of semi-empirical approach which is simple with easy flexibility to apply for fast evaluation of air pollution. The method predicts Emission Indexes (EI) specific to each air pollutant from the fuel flow. The general expression to calculate the total emission of the pollutant species is given by

$$m_{\text{airpollutant}} = \int_{t_0}^{t_f} EI(t)\dot{m}_f dt$$

where $EI$ is the emission index for the general air pollutant in [g/kg], $\dot{m}_f$ is the fuel flow [kg/s].

As the most widely used methods for emission prediction, these semi-empirical approaches are applied by ICAO [50] and Eurocontrol’s Base of Aircraft Data (BADA) [51]. Based on the database collected from different entities independently, the ICAO established an extensive and periodically updated databank for emission prediction with verified data on several specific exhaust emissions measured under reference throttle settings for most modern jet engines. Note that Eq.(2.1) still needs corrections when applied off-reference conditions.

A more accurate semi-empirical model using interpolation to estimate emission
indexes is used in the aircraft flight performance simulation software FLIGHT [36]. This method expresses both the fuel flow and the emission indexes with a function of the engine speed \( N_1 \). The emission indexes are obtained by interpolation from the reference value in the ICAO databank. Therefore, the mass of gaseous emission below 3,000 ft can be expressed by

\[
m_{\text{air pollutant}} = \int_{t_0}^{t_f} EI(N_1)m_f(N_1)dt
\]

where \( n \) is the number of the engine. In a word, the exhaust emissions can be estimated more precisely with an updated engine condition under specific atmosphere in each time step, which is the current method adopted in the Chapter 5.

### 2.3 Trajectory Optimisation

#### 2.3.1 General review

Complex factors, including involved airports and their surroundings, atmospheric and meteorological environment, air traffic management issues set by official organisations, economic considerations and public concerns, need to be taken into account for the flight path design of a commercial airline. These considerations, together with the operational limitations, constitute the optimisation criteria and constraints of the flight dynamics and performance problem. Requirements above lead to a nonlinear multiphased optimal control problem (OCP) with multiple constraints. The optimisation objectives include minimising fuel consumption, minimising noise annoyance and exhaust emissions and so on.

For example, the departure phase contains a take-off, climbs and accelerations before reaching the 10,000 feet above field elevation (AFE) cruise altitude. ICAO A,

![Figure 2.1: Flight procedure.](image-url)
CHAPTER 2. LITERATURE REVIEW

Table 2.1: Recommendatory departure procedures.

<table>
<thead>
<tr>
<th>STANDARD Procedure</th>
<th>ICAO A</th>
<th>ICAO B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take off at MTOT.</td>
<td>Take off at MTOT.</td>
<td>Take off at MTOT.</td>
</tr>
<tr>
<td>Climb at 1,000 feet altitude</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pitch over and cutback to climb</td>
<td>Climb at constant KCAS to 1,500 feet.</td>
<td>Climb to 1000 feet and pitch-over to</td>
</tr>
<tr>
<td>power.</td>
<td></td>
<td>accelerate at full power to clean</td>
</tr>
<tr>
<td>Accelerate to zero flaps.</td>
<td></td>
<td>configuration.</td>
</tr>
<tr>
<td>Climb at constant speed to 3000</td>
<td>- Reduce thrust to climb power.</td>
<td>At Clean Configuration,</td>
</tr>
<tr>
<td>feet altitude.</td>
<td>- Climb at KCAS to 3,000 feet.</td>
<td>cutback top climb power.</td>
</tr>
<tr>
<td>Upon achieving 3,000 feet altitude,</td>
<td>- Accelerate while retracting flaps to Zero.</td>
<td>Climb at constant speed to 3000 feet.</td>
</tr>
<tr>
<td>accelerate to 250 knots.</td>
<td>- Continue accelerating to 250 knots.</td>
<td></td>
</tr>
<tr>
<td>Upon achieving 250 knots, climb out</td>
<td>Upon achieving 250 knots, climb to 10,000</td>
<td>Upon achieving 250 knots, climb to 10,000</td>
</tr>
<tr>
<td>10,000 feet.</td>
<td>feet.</td>
<td>feet.</td>
</tr>
</tbody>
</table>

ICAO B and STANDARD procedures are recommendatory procedures for commercial aircraft take-off procedures. Table 2.1 summarises the guidelines for these three procedures.

Note that the referential flight procedures shown in Table 2.1 provide a loosely guidance for departures, which means the potential to optimise the flight path aiming at a certain objective does exist. Based on the referential flight procedure described above, optimised procedures with different objectives have already been proposed by international aviation organisations (e.g. Noise Abatement Departure Procedure (NADP) developed by ICAO). This gives rise to the study of flight trajectory optimisation for different objectives.

From the previous researches, methods to solve aircraft trajectory optimisation problems involving both noise and emission objectives can be classified into two groups. The first category establishes the optimisation problem with continuous sub-models and chooses to convert the original problem into a continuous optimal control problem whose gradient information towards state variables is available (e.g. NOISHHHH). Common features include:

- Aircraft Performance Model (APM) is based on differential algebraic equations
CHAPTER 2. LITERATURE REVIEW

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• Cost function can be expressed by aircraft flight states and controls;

• The optimisers can be built either with calculus-based algorithms (direct or indirect methods) or derivative-free methods (e.g. evolutionary algorithms);

• Optimal results consist of minimised objective and dynamic variables (states and controls) which are continuous variables of time.

While the second category focuses on the optimisation problem built with sub-models that lack gradient information (e.g. GATACT). Frameworks from this category share the characteristics below:

• Some design variables are selected as optimisation parameters (e.g. altitudes of endpoints along flight the profile);

• Cost functions are calculated by modules that are “black boxes” and have no explicit relationship with input variables;

• Optimisers are usually based on evolutionary algorithms (e.g. Genetic Algorithms (GAs) and Artificial Neural Network);

• According to the algorithms selected, the optimal results include the set of optimal design variables and the optimal value of the cost function (or a set of Pareto optimal solutions).

For example, the trajectory optimisation framework NOISHHH [23, 28, 52] developed by Delft University of Technology falls into the classification of the first category. From the last version of NOISHHH, this tool takes a variety of environmental performance criteria into account, including gaseous emission based on the ICAO Engine Exhaust Emissions Data Bank [53], fuel burn, and noise exposure based on the well-known Integrated Noise Model (INM) [47].

From the previous work of NOISHHH, studies on both 2D [23,28,54], and 3D [55] trajectory optimisation have been achieved. Either 2D or 3D problems apply a direct method to convert this optimal control problem into a finite-dimensional parameter optimisation problem that can be solved by non-linear programming. Since the flight path is piecewise due to a priori sequence, the trajectory optimisation problem solved by the optimiser also has a multi-phase optimisation formulation. Most of the 2D optimisation problems that NOISHHH dealt with are in the vertical plane with a point mass model formulation, yet efforts were also made on 3D trajectory optimisation [56] studying ground routes optimisation. Besides, all the 2D trajectory optimisation are conducted either within the horizontal plane with the vertical profile pre-defined or within the vertical plane with the lateral track fixed. Moreover, a common drawback
shared by previous work done by NOISHHH is that sometimes the lateral manoeuvre and other variations might be so extensive that the Flight Management System (FMS) can hardly navigate the aircraft along these routes. Therefore, noise reduction of Area Navigation (RNAV) departure and arrival has become a new trend [55]. Similarly, Prats [22, 32, 57, 58] and Fernandes [21] have also studied aircraft trajectory optimisation that minimises noise annoyance based on optimal control theory by using the noise model from the NPD table in INM program.

While efforts have also been made with a trajectory optimisation framework based on self-developed noise/emission models, which leads to a different direction in this study field. One of the representatives is the Green Aircraft Trajectories under ATM Constraints (GATAC). As a multi-disciplinary flight trajectory optimisation tool, GATAC is equipped with Genetic Algorithm (GA) optimiser, aircraft airframe and system models as the core as well as objective models such as noise and emission models. Two system-level components cooperate like this: the GATAC core makes up of optimisation suite for parameter defining and constrains and criteria analysis as well as a evaluation handler for data transfer and interfacing, the model suite to support simulation calculation. Data is transferred between these two components, which forms the data flow for the optimisation process.

The real physical model of this system is a nonlinear, non-smooth and non-differentiable system with complex input parameters and a cluster of complex outputs such as flight performance, noise, emission and so on. Yet the trajectory problem can be simplified into a vertical motion of aircraft along the symmetric plane, and then a 2D trajectory optimisation is obtained.

The common feature which GATAC shares with NOISHHH is that they both apply a set of nodes to divide the trajectory into segments. Then the optimisation object becomes the newly formed segmented flight trajectory. There are two points which are worth noticing:

1. GATAC chooses the parameters at the intermediary points as the optimisation variables, while NOISHHH regards these points as the points at two fixed ends of each phase;

2. GATAC’s flight dynamic model in the optimisation problem is a 2D one without looking into details of the models itself, while NOISHHH builds the mathematical model for each subsystem to present an integral optimal control problem, which can be solved by calculus-based optimisation methods.

Yet, both of these two softwares show less capability to cover the whole three target referred in the introduction part: NOISHHH focuses more on fuel and noise impact but does not cover the emission objects; while GATAC currently has not integrated noise
evaluation into its framework. What is more, although NOISHHH and GATAC have considered some path constraints, neither of them adds complex multiple constraints such as no-fly zones, weather hazard avoidance and so on.

2.3.2 Trajectory optimisation algorithms

Previous studies have provided reviews of methods to solve trajectory optimisation problems [59, 60]. There are many approaches to obtain optimal trajectories targeted at various objectives such as minimising fuel, minimising time, minimising noise and minimising emissions. In general, two different categories of algorithms are available for solving this kind of optimal control problems:

- Indirect methods involving the calculus of variations or the Maximum Principle of Pontryagin.
- Direct methods which transform the original optimal control problem into a nonlinear programming (NLP) problem.

Indirect methods are characterized by explicitly solving the optimality conditions stated in terms of the adjoined differential equations, the maximum principle, and associated boundary conditions. An indirect method for optimising a function of \( n \) variables requires analytically computing the gradient and then locating a set of variables using a root-finding algorithm such that the gradient is zero.

A direct approach can transcribe a continuous problem into a discrete optimisation problem with algebraic constraints. Betts [61] defined the steps of applying direct methods in 2001:

- Convert the dynamic system into a problem with a finite set of variables;
- Solve the finite-dimensional problem using a parameter optimisation method (i.e. solving a NLP problem);
- Assess the accuracy of the finite-dimensional approximation and repeat transcription and optimisation steps if necessary.

Therefore, an infinite-dimensional original problem is converted into a finite-dimensional optimisation problem.

Compared with indirect methods, direct methods do not need an analytic expression for the necessary conditions as well as the initial guesses for the adjoint variables, which lead to an advantage: direct methods can solve very complex problems with minimum efforts of mathematical analysis. There are two categories of direct methods: shooting methods and collocation (or transcription) methods which have been
reviewed in Betts’s survey [60] in 1998 from which introductions of software packing solving optimal control problems are also provided.

Shooting methods guess the initial conditions and then propagate the differential equations from initial time to the terminal time. After the first guess, the error in the boundary conditions is evaluated and by using an NLP algorithm the control variables and the initial guess are adjusted to satisfy the constraints.

The direct collocation methods provide the time histories of control inputs and state variables as a set of nodal points at each time step. The unknowns are the values of the controls and the states at these nodes. The cost function and the state equations can be expressed regarding these parameters which effectively reduce the optimal control problem to an NLP that can be solved by standard nonlinear programming code. The time histories of both control and state variables can be obtained by using an interpolation scheme [62,63]. There are other discretisation and interpolation methods including Trapezoidal discretisation, Runge-Kutta or Hermite-Simpson polynomials [64]. More complicated methods include pseudo-spectral techniques [65,66]. For example, the state variables are interpolated using a basis of Lagrange polynomials collocated at the Legendre-Gauss-Radau nodes in Gauss Pseudospectral method [67–69]. Compared with the shooting method, collocation methods have a faster computation performance [26]. Therefore, it has been applied widely as a generic method to solve optimisation problems with continuous formulation.

However, the two classes of numerical algorithms introduced above categorised as gradient-based methods have their limitation when dealing with optimisation problems with discontinuous models: the objective function and constraints need to be differentiable. However, with the increasing complexity and integration of current optimisation problem formulation, not all the integrated problems can be constructed with continuous models and functions that have accessible derivatives. Exploration to deal with discrete problems leads to the boom of heuristic algorithms which are generally not computationally competitive, yet do not need gradient information, making it suitable to search global optimal solution of the specific types of problems described above. Those methods include dynamic programming [70–72], rapidly-exploring random tree algorithm [73,74], receding horizon [75,76], genetic algorithms, particle swarm methods [77] and so on. The theory of this kind of methods is summarised by Betts [60]:

“The basic notion of genetic algorithms (GA), simulated annealing (SA), tabu search and evolutionary or Monte Carlo methods is to randomly select values for the unknown problem variables. After a finite number of random samples the “best” value is considered the answer.”

As one well-known class of heuristic methods, genetic algorithms can deal with optimal control problems with functions that have discontinuous derivatives. Although Betts
considered GAs is not a competitive method to solve trajectory optimisation problem because “trajectory optimisation problem is not characterized by discrete variables and there is no reason to use a method which incurs the penalty associated with this assumption” [60]. As the derivative-free or gradient-free methods, GAs have the capability to deal with optimisation problems formulated with discrete models which may have lower access to their analytical expressions. Due to their easy access and flexibility, GA based algorithms have become popular and gained preferences among trajectory optimisation applications, such as NASA’s attempt to study multi-objective spacecraft trajectory optimisation design [78], RLV re-entry problems based on NSGAII [79], multi-disciplinary optimisation design problem [80] and so on. For commercial aircraft trajectory optimisation, GAs based methods also have effective application [81, 82]. Note that although the heuristic optimisation algorithms have a better performance solving discrete problems, their applications are not limited to this kind of problems only. Optimisation problems with continuous and smooth formulation can be solved by heuristic optimisation algorithms, for instance, GAs, when numerical measures are undertaken to convert the OCP into a finite-dimensional parametric optimisation problems [83,84].

2.3.3 Multi-objective optimisation algorithms

Although the approach varies in different literature, numerous authors [21, 58, 60, 85–87] tend to define trajectory optimisation process into a constrained optimal control problem or a constrained multi-objective optimal control problem. The optimisation regarding multiple conflicting criteria results in a large number of optimised solutions each of which from some point of view can be considered as optimal. This leads to the need for a trade-off selection strategy to identify the best optimal solution conforming to the multi-objective optimisation theory. This section resumes the techniques that are particularly suitable for aircraft trajectory with multiple objectives and constraints.

2.3.3.1 Methods with a priori articulation of preferences

The methods in this section allow the user to construct a combination of various objectives at the beginning of the optimisation process. Most of these priori methods will introduce parameters (e.g. coefficients and exponents) to define the combined objective, which leads to analysis on the ways to define the combined objectives from various possible preference articulations of the user. Methods includes 1) weighted global criterion methods, 2) lexicographic method, 3) weighted min-max method, 4) exponential weighted criterion, 5) weighted product method and 6) physical programming [88,89] and so on. Since it is not within the intentions of this work to introduce every technique available for multi-objective optimisation, only two representatives (weighted global criterion methods and physical programming) that have been widely
used in aircraft trajectory optimisation are presented here.

A. Weighted global criterion methods
Among all the methods to solve multi-objective optimisation problems, scaling is the most widely used one. By adding a weight to each objective function \( J_i(z) \) and summing them together, the multi-objective problem is transferred into a single objective optimisation problem.

Define \( J_i^0 \) as the utopia point, which means \( J_i^0 = \min_z \{ J_i(z) | z \in Z \} \), then the combined weighted objective function could be expressed by

\[
\tilde{J} = \left\{ \sum_{i=1}^{n} w_i (J_i(z) - J_i^0) \right\}^{1/s}
\]

(2.3)

where \( w_i \neq 0 \ \forall i \) is the weight assigned to each single objectives, \( n \) is the number of objectives. Note that \( \sum_{i=1}^{n} w_i = 1 \) is the general situation, but it is not strictly necessary.

There are two important subcases of Eq. (2.3). One is when \( w_i = 1 \forall i, r = 1 \) and \( s = 2 \). Then the objective function has a physical meaning of the geometric distance of the optimal solution from the utopia point. The other one is the weighted sum method using a zero utopia point and a linearly weighted sum of the objective functions \( J_i \).

\[
\min_{z \in Z} \{ J_1(z), J_2(z), ..., J_n(z) \} = \min_{z \in Z} \sum_{i=1}^{n} w_i J_i(z)
\]

(2.4)

Since the objective function does not change linearly according to the weighting vectors, it becomes more difficult to select final weights and control the direction of the solutions by weighting coefficients. Another drawback of scalarization method is that the solution in a non-convex region of the Pareto front cannot be found if weighted formulae have a linear combination of different criteria [90]. However, this method has still been broadly used in the application of engineering problems. In the work of the University of Delft, the researchers apply the scalarization method to deal with multiple objectives such as flight time, fuel, and noise index and so on [91].

B. Physical programming
This method allows the decision maker to express preference based on intuitive considerations and unstructured information to construct the combined objective function. Objective functions, constraints and goals are treated as design metrics. A tag to rank the objective in the form of highly desirable, desirable, tolerable, undesirable, highly undesirable and unacceptable is applied to quantify the preference the objective. The user-introduced information is then used to normalise each objective leading to a single objective function as
\[
J = \log \left\{ \frac{1}{dm} \sum_{i=1}^{dm} \bar{J}(J_i(z)) \right\}
\]

(2.5)

where \(dm\) is the number of design metrics considered.

The main advantage of physical programming is that it formulates the objectives in a way that the user can understand directly, which makes it very suitable for multi-objective engineering design and optimisation.

### 2.3.3.2 Methods with a posteriori articulation of preferences

As for posteriori methods, preferences may be used at the end to select an optimal solution from the Pareto front. The posteriori techniques focus on the determination of the shape of Pareto front with an even distribution of solution points, in which way the user can make a decision from a set of sufficiently diversified solutions. Two different methods are developed: 1) mathematical programming methods with one Pareto optimal solution obtained after a run and 2) evolutionary algorithms with one set of Pareto optimal solutions obtained after one run.

The most widely used mathematical programming methods include normal boundary intersection method (NBI) [92], normal constraints methods (NC), successive Pareto Optimisation (SPO) and so on [90]. These methods share the similarity of yielding a Pareto optimal solution by constructing scalarization to obtain an even distribution of Pareto optimal points.

Evolutionary algorithms have become popular to solve multi-objective problems from which the NSGA-II [93] and those applying Pareto-based ranking schemes have become the standard methods. In fact, multi-objective evolutionary algorithms have gained popularity because they deal with a set of possible solutions at the same time and can find multiple Pareto-optimal solutions in one single simulation run [94]. Moreover, EAs are less dependent to the shape or continuity of the Pareto front which are problems in other calculus-based methods [95–97]. Although they still suffer from the low computational speed and the lack of guarantee of global Pareto optimality, their good performance in finding best approximation of the Pareto solution make them widely welcome. In this way, this study applied NSGA-II as the main optimiser when using the posterior methods.

Finally, it is worth emphasising that for some methods, whether they are categorized as priori methods or posteriori methods depends on their applications. One example is physical programming. One advantage of a posteriori physical programming approach is that the search domain can be restricted in the solution region so that the computational resources would not be wasted in calculating Pareto solutions which are not within the acceptable portion of the Pareto front.
CHAPTER 2. LITERATURE REVIEW

2.4 Concluding remarks

With the increasing expansion of commercial flight, solutions must be found to keep a balance between steady aviation increase and controllable and acceptable noise emission. Otherwise either the development of aviation market or the appeal for the environment will fall into a dilemma.

Credible reasons have been provided to show the feasibility of minimising environmental emissions by improving the flight trajectory in Chapter 1. In this chapter, more details were demonstrated to introduce the current development of aircraft noise modelling, emission modelling and trajectory optimisation. From all the methods and models discussed above, the conflict between accuracy and computational efficiency is the key topic. For some problems, the cost resulting from computational deviation could be overwhelming. As is shown in the reference [98], even one dBA underestimation of the overall 57 dBA $L_{eq}$ contours in London Heathrow summer 2000 contours leads to a reduction of 75,000 in people annoyed, which is equivalent to about 30% of the baseline population. This is a case which prefers high fidelity rather than computational efficiency. However, some urgent cases with short-term effects may prefer efficiency because the long simulation time may invalidate the initial condition. For example, flight simulation based on real-time weather data is largely constrained by time because atmospheric conditions are transient and vary across regions. Therefore, it is less easy to choose a method to meet the needs of all kinds of problems.

Since this work has just no intention to explore either the noise or emission in the microcosmic scale, CFD/CAA methods for the noise prediction model and computational chemistry for the emission prediction model currently are less achievable in this study. Models applied here must be fully validated at the same time robust and effective, which leads to the decision for each model constituting the framework in the Chapter 5-7.

As for the optimisation, two different categories of methods - namely calculus-based methods and heuristic algorithms - were introduced in this chapter. The preference of optimisation algorithm largely depends on the formulation and feature of the optimisation problem. Again, similar comparison has been done in the Section 2.3.2 to offer a rough sketch of these two methods and their representative applications. Analysis has shown that one obvious gap shared by these two applications is that they both fail to reduce noise and exhausted emission at the same time, which gives birth to the trajectory optimisation framework for environmental multiple objectives in the Chapter 5.
Chapter 3

Problem Formulation

In this chapter, the theoretical framework of the trajectory optimisation problem is introduced. Problem formulation including optimal control problem construction, dynamics model, constraints model, objective models are constructed for the departure phase of the aircraft motion.

3.1 Optimal Control Problem

As was introduced in Section 2.3.3, the process of trajectory optimisation can be defined as a constrained optimal control problem (OCP) or a constrained multi-objective OCP. The aircraft dynamics and performance parameters are defined as the state and control variables of this OCP. The goal is to minimise single or multiple objectives which are described in a continuous Bolza formulation:

\[
\min_{t, \mathbf{x}(t), \mathbf{u}(t)} J(t, \mathbf{x}(t), \mathbf{u}(t)) = \int_{t_0}^{t_f} L(\mathbf{x}(t), \mathbf{u}(t)) \, dt + \Phi(t_0, \mathbf{x}_0, \mathbf{u}_0, t_f, \mathbf{x}_f)
\]  

where \( \mathbf{x}(t) \in \mathbb{R}^n \) is the state vector, \( \mathbf{u}(t) \in \mathbb{R}^m \) is the control vector, \( t_f \) and \( t_0 \) are respectively the final time and the initial time, \( \mathbf{x}_0 = \mathbf{x}(t_0) \), \( \mathbf{x}_f = \mathbf{x}(t_f) \). The dynamic system defined by a set of ordinary differential equations explicitly by both the state and the control variables is given as:

\[
\dot{\mathbf{x}} = f(\mathbf{x}(t), \mathbf{u}(t), \mathbf{p}, t) \quad t \in [t_0, t_f]
\]  

Note that the dynamics system may incorporate the time-independent parameter \( \mathbf{p} \).

The cost function is then minimised subject to a set of constraints and boundary conditions:
CHAPTER 3. PROBLEM FORMULATION

\[ \Phi(x_0, t_0, x_f, t_f) = 0 \] (3.3)
\[ C(x_0, t_0, x_f, t_f) \leq 0 \] (3.4)
\[ u(t) \in [u_{\text{min}}, u_{\text{max}}] \] (3.5)

where \( \Phi \) is the boundary condition and \( C \) represents the path inequality and equality constraints.

For multi-objective optimal control problem, the aim is to find the optimal solutions that minimises a set of criteria simultaneously. Then the cost function is expressed as:

\[
\min_{z \in Z} \mathbf{J}(z) = \min_{z \in Z} [J_1(z), J_2(z), \ldots, J_{n_j}(z)]^T
\]

where \( z = [x, u, t_f] \) is the decision variable, \( Z \subseteq \mathbb{R}^{n+m+1} \) is the set of decision variables, \( \mathbb{R}^{n+m+1} \) is the \( n + m + 1 \) dimensional objective space, and \( J_i(z) \) is the scalar valued cost function.

A solution \( z^* \) of the multi-objective optimisation problem is Pareto optimal if and only if there does not exist another \( z \in Z \) such that \( J_i(z) \leq J_i(z^*) \) for all \( i \in 1, \ldots, n_j \) and \( J_i(z) \leq J_i(z^*) \) for at least one index \( j \). In other words, a solution is Pareto optimal if and only if an objective \( J_i(z^*) \) can be reduced only at the expense of increasing at least one of the other objectives. The Pareto front formed by these optimal solutions can be either non-convex or non-connected.

On the other hand, a decision vector \( z^* \) is said to be weakly Pareto optimal if there does not exist another decision vector \( z \in Z \) such that \( J_i(z) \leq J_i(z^*) \) for all \( i \in 1, \ldots, n_j \). It can be proved that the Pareto optimal set is a subset of weakly Pareto optimal set.

Moreover, an objective vector minimising each of the objective functions is called an ideal objective vector \( \mathbf{J}^* = [J_1(z^*_1), J_2(z^*_2), \ldots, J_{n_j}(z^*_n)]^T \). This vector is obtained by minimising each of the objective functions individually subject to the constraints defined by \( Z \) [99],

\[ J^*_i = J_i(z^*) = \min_{z \in Z} J_i(z) \quad \forall i \in 1, \ldots, n_j \] (3.7)

When the ideal objective vector is feasible, the solution to this multi-objective optimisation problem would be this vector and the Pareto optimal solution set. When it is not feasible, this vector can even be regarded as a reference point in some multi-objective optimisation strategies.
3.2 Aircraft Flight Dynamics Model

Aircraft flight dynamics is fundamental to trajectory analysis and optimisation. In general, an aircraft can be modelled as a rigid body with varying mass, aerodynamic, propulsive and gravitational forces. Several assumptions are made in order to simplify the problem: (1) The earth is considered to be flat and non-rotational, (2) no wind is presented in this work, (3) all forces acting on the aircraft go through its centre of gravity, (4) the angle between the direction of engine thrust and the longitudinal axis of the aircraft is assumed to be zero. A 3-DOF flight dynamics model with a variable mass is given as following:

\[
\begin{align}
\dot{V} &= \frac{F_N \cos \alpha - mg \sin \gamma - D}{m} \\
\dot{\gamma} &= \frac{F_N \sin \alpha \cos \chi + L \cos \phi - mg \cos \gamma}{mV} \\
\dot{\chi} &= \frac{F_N \sin \alpha \sin \chi + L \sin \phi}{mV \cos \gamma} \\
\dot{x} &= V \cos \gamma \sin \chi \\
\dot{y} &= V \cos \gamma \cos \chi \\
\dot{h} &= V \sin \gamma \\
\dot{m} &= -f 
\end{align}
\]

where the state variables are:

- \( V \) true air speed [m/s]
- \( \gamma \) flight path angle [rad]
- \( \chi \) heading angle [rad]
- \( x \) ground distance to the East [m]
- \( y \) ground distance to the North [m]
- \( h \) altitude [m]
- \( m \) mass of the aircraft [kg]

The control variables:

- \( F_N \) thrust [N]
- \( \alpha \) angle of attack [rad]
- \( \phi \) bank angle [rad]

The angles, including the angle of attack \( \alpha \), the flight path angle \( \gamma \), the bank angle \( \phi \) and the heading angle \( \chi \), are depicted in Figure 3.1 to Figure 3.3. Please note that the subscript \( b \) indicates the body frame and the subscript \( f \) indicates the fuselage.
Figure 3.1: Flight path angle.

oriented frame. It is worth noticing that the variables $x$, $y$ and $h$ in Eq. (3.8) are still indicating the position of the centre of aircraft in the local geographic frame, namely the East-North-Up reference frame. The reference frames demonstrated in Figure 3.1 to Figure 3.3 are introduced to define the rotational angles which are used to describe the aircraft motions.

Other variables and parameters:

- $L$ aerodynamic lift [N]
- $D$ aerodynamic drag [N]
- $f$ fuel flow [kg/s]
- $g$ gravitational acceleration, 9.81 [m/s$^2$]

The aerodynamic lift and drag can be written as:

$$L = \frac{1}{2} \rho V^2 C_L S \quad (3.9)$$

$$D = \frac{1}{2} \rho V^2 C_D S = \frac{L}{R} \quad (3.10)$$

where $\rho$ is the atmospheric density [kg/m$^3$], $S$ is the reference area [m$^2$], $C_L$ is lift coefficient, $C_D$ is drag coefficient and $R$ is the lift-to-drag ratio. Note that an atmospheric model based on International Standard Atmosphere (ISA) [100] is integrated in this model. To obtain aircraft aerodynamics and engine propulsion variables, FLIGHT tools [6, 36, 101, 102] are needed with required aircraft configuration and motion parameters as well in this study.

$$F_N = F_N(h, Ma, N_1) \quad (3.11)$$
where $Ma$ is the Mach number, $N_1$ is the engine rpm with value range $[70\%,103\%]$ for the departure phase. Similarly, the calculation of the aerodynamics coefficients $C_L$ and $C_D$ is achieved based on current flight state variables, the configuration of the specific type of aircraft and custom-defined atmosphere parameters.

### 3.3 Constraints

#### 3.3.1 Path constraints

For all state variables $x = (V, \gamma, \chi, x, y, h, m)^T$ and the control variables $u = (F_N, \phi)^T$ there are a number of path constraints on them which can be expressed as:

$$
\begin{align*}
V_{\text{min}} & \leq V \leq V_{\text{max}} \\
\gamma_{\text{min}} & \leq \gamma \leq \gamma_{\text{max}} \\
\chi_{\text{min}} & \leq \chi \leq \chi_{\text{max}} \\
x_{\text{min}} & \leq x \leq x_{\text{max}} \\
y_{\text{min}} & \leq y \leq y_{\text{max}} \\
h_{\text{min}} & \leq h \leq h_{\text{max}} \\
m_{\text{min}} & \leq m \leq m_{\text{max}}
\end{align*}
$$

(3.13)
and

\[
\begin{cases}
F_{\text{Nmin}} \leq F_N \leq F_{\text{Nmax}} \\
\phi_{\text{min}} \leq \phi \leq \phi_{\text{max}}
\end{cases}
\] (3.14)

Particularly, for aircraft in a specific flight procedure or segment, there are other path constraints considered, which will be introduced in the following chapters with corresponding explanations.

### 3.3.2 Boundary conditions

The values of state and control variables of the dynamics system at the initial and final time nodes are defined by the boundary conditions. Equality constraints as well as inequality constraints are both used to define these values. A general expression for the boundary conditions is

\[
B_{\text{min}} \leq B[x_0, x_f, u_f; p] \leq B_{\text{max}}
\] (3.15)

where the equality conditions can be added by applying \(B_{i,\text{min}} = B_{i,\text{min}}\), \(i\) is the subscript indicating the \(i^{th}\) boundary conditions.

### 3.4 Objective Models

Generally, criteria such as flight duration, fuel consumption, noise and gaseous emissions and so on are the focus of people’s attention that can be set as the aircraft operational optimisation objectives.

#### 3.4.1 Single metric objectives

As is introduced in Section 2.1 and Section 2.2, the comprehensive aircraft performance software FLIGHT provides modules, namely a noise module and an emission module, to assess the environmental impacts.

For the aircraft noise module, it consists of three parts: noise modelling, transmission and propagation. Based on trajectory and engine data, this module can calculate noise indexes at specific locations as well as the noise footprint, stacking patterns and noise directivity [36]. Besides, noise generated from different components and sources, for instance propulsive noise, airframe noise and interference noise, are also available from the output of FLIGHT.
For the emission module, emissions exhausted, such as carbon dioxide (CO$_2$), mono oxide (CO), oxides of nitrogen (NOx) and so on, can be calculated with FLIGHT as well. Inputs required include trajectory data as well as the engine data. Representative environmental indexes provided by FLIGHT are listed in Table 3.1.

### Table 3.1: Environmental emission indexes

<table>
<thead>
<tr>
<th>Noise emissions</th>
<th>Exhaust emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPNL [dB]</td>
<td>CO$_2$ [kg]</td>
</tr>
<tr>
<td>SEL [dB]</td>
<td>CO [kg]</td>
</tr>
<tr>
<td>$L_{\text{AeqT}}$ [dBA]</td>
<td>NOx [kg]</td>
</tr>
<tr>
<td>$L_{\text{Amax}}$ [dBA]</td>
<td></td>
</tr>
<tr>
<td>SPL$_{\text{max}}$ [dBA]</td>
<td></td>
</tr>
</tbody>
</table>

Alternative models to provide noise and gaseous emissions are various and of different levels of accuracy. Currently, popular noise prediction and calculation models include the methodology proposed by the third edition of ECAC Doc.29 [103] supported by the Aircraft Noise and Performance (ANP) Database from ICAO [4] and the commercial software Integrated Noise Model (INM) [104] based on the same report. The reason for using the software FLIGHT for noise modelling has been discussed in previous sections. The main reason is that this software can offer highly efficient computational performance as well as acceptable accuracy. While the decision of choosing the exhaust emissions model from FLIGHT is based on its easy access. Alternative models can also be found with the ANP database.

### 3.4.2 Combined objectives

To comprehensively evaluate the effects of multiple objective functions, combined objectives are constructed with the single metric objectives. Note that for all objectives, it is always applicable to implement the weighted method. Studies on multi-objective trajectory optimisation based on the weighted method to integrate objectives are not rare [28, 54]. However, to decide the weighting factor to be assigned to each objective is not easy or intuitive. Concerning this point, though objectives generated by the normal weighted method can be regarded as a kind of combined objectives as well, they are excluded in the following sections due to the lack of concrete physical meanings.

#### 3.4.2.1 Noise objectives

Despite the simple noise metrics introduced in Section 3.4.1, an alternative way to evaluate the noise impacts produced by typical sound events is to introduce non-acoustic factors. For instance, Prats [57] believes that the following elements should be taken into account when valuing the harm caused by noise:
CHAPTER 3. PROBLEM FORMULATION

- Types of affected zones (classified by their sensitivity to noise).
- Time interval during the noise event (day, evening, night).
- Period between two consecutive flights.
- Personal elements (emotional, apprehension to the noise, personal health, age, etc.).
- Cultural aspects (it depends on the different ages and habits of residents).

Other researchers [37] who hold similar views also assess the aircraft noise impact during a period of airport operational time rather than one single noise event with the following factors:

- Number of people within specified noise contour.
- Physical extent of specified noise contour.
- Number of noise complaints received.
- Amount of time communities are exposed to noise above a predetermined level.
- Number of noise events above a predetermined level to the exposed communities.

Therefore, minimising aircraft noise emission requests for assessment from inter-disciplinary factors including engineering ecology, economy and sociology, which makes up a systematic approach. Models and methods used for assessing environmental noise impacts vary from each other, yet they share most of the factors mentioned above.

- Cost of noise protection
  A criterion used widely for noise problems is the unit cost of noise protection for a population (UCNPP). This is an index which focuses on economic cost on individuals living in the noise-sensitive places. The total cost for noise protection (CNP) is obtained by summing up the cost spent on protecting population that are greatly annoyed in each noise-sensitive area [37]. How to group these regions depends on the noise level that they are exposed to. Similar ideas to convert the noise index into a monetary index include the noise charge [105], which is widely adopted by over 60 airports in 16 countries.

- Annoyance
  Instead of minimising measured noise in vicinity of airports, some models introducing non-noise concepts to scale noise impacts have been introduced [21]. Rather than a simple physical phenomenon, noise is also a human experience involved with negative emotions. Affected by a person’s internal and external
environment, those emotions, including discomfort, fear, anger, a feeling of restrict freedom and so on, give rise to annoyance [106]. In order to scale annoyance caused by aircraft noise, a model to estimate population that might be awoke by a flight was built on the study presented in Ref [21] that quoted the function proposed in Ref [107]:

$$%\text{Awakenings} = 0.0087 \cdot (\text{SEL}_{\text{indoor}} - 30 \text{ dB})^{1.79}$$ (3.16)

where %Awakenings is the percentage of the exposed population predicted to be awakened and SEL$_{\text{indoor}}$ is the indoor sound exposure level. According to Ref [108], a house wall exerts 20.50 dB attenuation, which should be considered when calculating indoor noise level from outdoor noise level. Then the affected population by a flight would be estimated by the equation below.

$$%\text{Awakenings} = 0.0087 \cdot (\text{SEL}_{\text{outdoor}} - 50.50 \text{ dB})^{1.79}$$ (3.17)

Another awaking criterion provided by Ref [109] and adopted by Ref [27] is expressed by the maximum A-weighted sound pressure level $L_{A,max}$ at the ear of the sleeping person:

$$%\text{Awakenings} = 0.0019L_{A,max}^2 + 0.04L_{A,max} - 3.3$$ (3.18)

Therefore, the average number of persons awakened per operation ($N_{\text{AWR}}$) would be derived as the function below

$$N_{\text{AWR}} = \sum_{i=1}^{n_{\text{FRG}}} n_{\text{FRG}} \cdot A_i \cdot %\text{Awakenings}_i$$ (3.19)

where $n_{\text{FRG}}$ is the average population density (residents per square kilometre), $A_i$ is the areas of noise level contours.

In the same vein, influence of aircraft noise on specific areas like residential and industrial places was transformed into annoyance and assessed by certain value in order to do trading offs and multi-objective optimisation in Ref [27].

### 3.4.2.2 Emission objectives

Since most researchers pay more attention to the elimination of aircraft gaseous emission, their objectives are usually defined by certain kinds of exhausted products including ones of CO$_2$ and NOx. According to the ICAO engine emission databank [110], engine emission index can be obtained for a given thrust setting in the form of grams of pollutants per kg of fuel. As a result, total emission for each flight segment can be calculated with the emission index and fuel consumption by the cost function below [21]
\[
\min J = \int_{t_0}^{t_f} EI(x(t)) \cdot \dot{f}(x(t), u(t)) dt \tag{3.20}
\]

where EI is the amount of pollutant in unit fuel, \( \dot{f} \) is the fuel consumption rate.

Another emission cost model adopted by Eurocontrol CARE INO III report [111] focuses more on the emission financial impact. Data in the report is to demonstrate the possibility of achieving optimal cost-efficient trajectory at the same time including all the future public expenses to control the pollutants. Illustrative environmental-related taxes for CO\(_2\) and NO\(_x\) emissions are shown in Table 3.2.

<table>
<thead>
<tr>
<th>Pollutant</th>
<th>Cost per tone</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO(_2)</td>
<td>$37.00</td>
</tr>
<tr>
<td>NO(_x)</td>
<td>$6414.00</td>
</tr>
</tbody>
</table>

Then the cost function of the exhausted emissions can be integrated as the environmental cost by summing up the economic expense cost on the environment on flight operations.

### 3.5 Concluding remarks

In this chapter, the problem expected to be applied for this research is displayed. An introduction to the optimal control problem as well as the problem formulation for this general study is provided. It also includes a description of a fundamental flight mechanism model and the current models used for environmental objectives evaluation. The following chapter shows the posterior selection strategies implemented in the study to select the optimal solutions from the results obtained in the multi-objective trajectory optimisation problem.
Chapter 4

Posterior Selection Strategies

Instead of a single optimal solution, the result of Multi-objective Optimisation Problem is a set of Pareto optimal solutions. After obtaining an optimised solution set, it is not easy to define the criterion to select the best solution. In order to evaluate the optimised solutions intuitively and understandably, two posterior selection strategies are introduced to help with decision making. The first one uses the aggregated preference value function based on the theory of physical programming [112] developed from the preferences of the decision makers. While the second strategy is established by monetising individual objectives (e.g. fuel consumption, noise level and pollutant emissions), which depends more on the environmental policies as well as the harm cost brought by them.

4.1 Aggregated preference value function

This section introduces a method of evaluating various solutions using the decision maker (DM)’s preferences. Firstly, preference types of different attributes are defined. Then, essential knowledge and concern from the DMs are transferred into preference functions and ranges. Lastly, an aggregated preference function is built to produce a single metric to support the decision-making process. Therefore, the optimal result will be selected from the search space by comparing the value of the single metric between different designed objectives. The evaluation process is described in the following statement.

4.1.1 Classification of preference

According to the theory of physical programming, the categories of different criteria are associated with the sharpness of the preference. In general, there are two preferences: (1) type-1: smaller is better and (2) type-2: larger is better. For each criterion, there
### Table 4.1: Preference order and its attribution with $N = 5$.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
<th>Type-1</th>
<th>Type-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Highly desirable</td>
<td>$f_0^i &lt; f_i \leq f_1^i$</td>
<td>$f_1^i &lt; f_i &lt; f_0^i$</td>
</tr>
<tr>
<td>B</td>
<td>Desirable</td>
<td>$f_1^i &lt; f_i \leq f_2^i$</td>
<td>$f_2^i &lt; f_i &lt; f_1^i$</td>
</tr>
<tr>
<td>C</td>
<td>Tolerable</td>
<td>$f_2^i &lt; f_i \leq f_3^i$</td>
<td>$f_3^i &lt; f_i &lt; f_2^i$</td>
</tr>
<tr>
<td>D</td>
<td>Undesirable</td>
<td>$f_3^i &lt; f_i \leq f_4^i$</td>
<td>$f_4^i &lt; f_i &lt; f_3^i$</td>
</tr>
<tr>
<td>E</td>
<td>Highly undesirable</td>
<td>$f_4^i &lt; f_i \leq f_5^i$</td>
<td>$f_5^i &lt; f_i &lt; f_4^i$</td>
</tr>
</tbody>
</table>

are $N$ ranges used to express the different levels of desirability. The most common classification is defined with $N = 5$: highly desirable, desirable, tolerable, undesirable and highly undesirable.

Numerical values of the $i$th attribute $f_i(x)$ at the boundaries of these preference ranges, $f_k^i (i = 1, \cdots, M; k = 0, 1, \cdots, N)$, are used to divide the value of objective function into $N$ intervals for quantifying the differentiated desirability levels. For both type-1 and type-2 preference function, the objective value at the $N + 1$ extreme range points, $f_k^i$ are ranked in ascending order. As for type-1, the lower objective value is, the higher desirability is. For instance, the first boundary point value $f_0^i$, or ideal point, is the best value for the decision maker. Any possible solutions fall into the interval $f_i(x) \leq f_0^i$ are regarded as the best value of this objective. As for the type-2 preference function, the extreme points $f_k^i$ are defined in the opposite direction, which means the higher the objective function value is, the higher desirability is. Then the ideal point $f_0^i$ would be the last point of $f_k^i$. Any objective value $f_i(x) \geq f_0^i$ is the ideal solution. One possible classification of desirability with $N = 5$ is described in Table 4.1.

#### 4.1.2 Construction of preference function

Before the establishment of the preference value function, there are three points need to be declared:

- The selection of the best solution for each criterion or objective is translated into a parameter minimisation problem.

- In order to evaluate all objectives of different physical meanings with a universal guideline, the preference value of every preference range in each objective should be presented with the same image. The solution is to assign objectives of the same desirability level with the same value in the preference function. In this way, the original problem is normalised and is transferred to a dimensionless variable space which is independent of the original Multi-objective Optimisation Problem.

- The result of posterior selection for the same range of preferences should be
insensitive to the shape of the preference function. No matter what shape of
the preference function is, the preference rank of the objectives should not differ
accordingly with the change of the shape of the preference value function.

Then in order to weigh the different criteria with a common metric, the same set of
image $\alpha_k$ is set at the range boundaries $f_k^i$ for each criterion:

$$
\alpha_k = \begin{cases} 
0, & \text{if } k = 0 \\
M^{k-1} \cdot \alpha_{\text{ini}}, & \text{if } 1 \leq k \leq N 
\end{cases} \quad (4.1)
$$

where $\alpha_{\text{ini}} > 0$ is a dimensionless variable. Note that alternative methods to define
the preference value at each boundary points do exist including setting the sequence of
$\alpha_k (k = 0, 1, \cdots, N)$ as an arithmetic progression. Here $\alpha_k$ is set to be a geometric pro-
gression in order to enlarge the differentiation degree between the best and the worst.
The more the objective values are preferred, the smaller the value of the preference
function is. This parameter is used later to construct the preference value function
with a piecewise exponential function [113].

(1) Type-1 preference function construction

For the general $k^{\text{th}}$ interval, $f^{k-1}_i < f_i(x) \leq f^k_i$, the preference function $p_k(x)$ with
order $n \in \mathbb{R}$ can be expressed as:

$$
p_k(x) = \alpha_{k-1} + \beta_k \left| 1 - e^{-\frac{n(f_i(x) - f^{k-1}_i)}{f^k_i - f^{k-1}_i}} \right| \quad (4.2)
$$

where $x$ is the vector of design variables. At the boundary point $f^l_i$, $p_k = \alpha_k$, therefore
the coefficient $\beta_k$ can be obtained as:

$$
\beta_k = \frac{\alpha_k - \alpha_{k-1}}{1 - e^{-n}} \quad (4.3)
$$

Then the Eq. (4.2) can be written as:

$$
p_k(x) = \alpha_{k-1} + \frac{\alpha_k - \alpha_{k-1}}{1 - e^{-n}} \left| 1 - e^{-\frac{n(f_i(x) - f^{k-1}_i)}{f^k_i - f^{k-1}_i}} \right| \quad (4.4)
$$

Thus, the type-1 preference function for the $i^{\text{th}}$ criterion can be defined as:

$$
n^1_i(x) = \begin{cases} 
0, & f_i(x) < f^0_i \\
p_k(x), & f^{k-1}_i \leq f_i(x) \leq f^k_i \\
p_N(x), & f_i(x) > f^N_i 
\end{cases} \quad (4.5)
$$
where \( k = 1, 2, \cdots, N \) and \( p_N(x) = \alpha_N \).

(2) Type-2 preference function construction

Similarly, the type-2 preference function for the \( i \)th criterion can be defined as:

\[
    n_i^2(x) = \begin{cases} 
        0, & f_i(x) > f_i^0 \\
        p_k(x), & f_i^k \leq f_i(x) \leq f_i^{k-1} \\
        p_N(x), & f_i(x) < f_i^N
    \end{cases} \tag{4.6}
\]

where \( k = 1, 2, \cdots, N \) and \( p_N(x) = \alpha_N \). \( p_k \) is the same defined in Eq. (4.6). An example of type-1 and type-2 preference function with \( N = 5 \) and \( n = -2 \) are depicted in the Figure 4.1 and Figure 4.2. Note that the piecewise exponential function is less sensitive to the value of parameter \( n \) in the solution obtained at the end of the posterior selection process. Thus the order of the piecewise exponential function can be set into any value \( n \in \mathbb{R} \).

![Figure 4.1: Preference function type-1](image)

![Figure 4.2: Preference function type-2](image)

Figure 4.1: Preference function type-1 Figure 4.2: Preference function type-2 with \( N = 5, n = -2 \) [5].

Once the preference function for each objective is identified, an aggregated preference value function can be built by summing up all the preference value of the criteria:

\[
    \min_x V(x) = \sum_{i=1}^{M} n_i(x) \tag{4.7}
\]

In this way, the preference value function plays the role to associate the related physical value of the real objectives to the domain of DMs’ preference. Then the posterior selection process turns out to be searching for the best solution \( x \) leading to the minimum \( V(x) \) therefore the highest desirability from the aspect of DMs.
4.2 Monetisation method

The alternative method to measure the impacts of noise and other either environmental or non-environmental concerns is to value their economic expenditure. In this way, attributes of different physical units can be weighed with a single concrete metric. This method is called the monetisation method which might be not appropriate to be applied in the early stage of optimisation but should be a reasonable and objective way to evaluate the solutions obtained afterwards.

4.2.1 Economic cost

The economic cost for airlines including the fuel price, the expense paid for exhausted emissions and the surcharge of night noise.

Manchester airport applies a night noise surcharge scheme for night noise during the night period (e.g. 11pm to 7am). For aircraft operating between 11:30pm to 6am, any noise level over 81 dB(A) is £750 plus £150 for each decibel above. For operations between 11pm to 11:30pm and 6am to 7am, the penalty is £750 plus £150 for each decibel above 82 dB(A).

4.2.2 Damage cost of noise

Based on the potential harm exerted on the well-being of the exposed population, one way to monetise the cost of aircraft noise is to quantify the noise impacts by money. Among all the local concerns of the residential communities, annoyance, sleep disturbance, and awakenings are crucial from both physical and psychological aspects. The dose-response relationship between instantaneous single noise event and long-term multiple noise events are quite different.

(1) Damage cost of aircraft noise of single event

Much attention has been paid to the use of single event descriptions such as $L_{A\text{max}}$ and SEL. The American National Standard Institute publication (ANSI) has defined one method that predicts the probability of an individual noise event waking a person as a result of the SEL alone [114].

$$P_{\text{awakening},k} = \frac{1}{(1 + e^{-(-6.8884+0.04444\text{SEL}_{\text{indoor}})})} \quad (4.8)$$

where $\text{SEL}_{\text{indoor}} = \text{SEL}_{\text{outdoor}} - 20.50 \text{ dB}$.

However, monetising the effect of awakening is not easy. WHO has given disability weights to noise-induced annoyance, sleep disturbance and hypertension for a longer-term range, but excluded awakening [115]. Moreover, unless extreme conditions happen, the noise level of an arbitrary single noise event will bear no relationship to
an individual’s long-term health status, whereas a sustained sufficiently high level over a longer period may. Alternative methods should be introduced to fill the gap.

Instead of valuing the noise-caused cost by the damage, the concept of unit cost of noise protection for a population (UCNPP) \[37\] is introduced, which is widely-used for noise problems. In this way, the potential cost caused by aviation noise can be monetised. Then the total cost awakenings by a single noise event can be estimated as:

\[
\text{Cost of awakening} = \sum \text{UCNPP}_k \cdot P_{\text{awakening},k} \cdot \text{population}_k
\]

where \(P_{\text{awakening},k}\) is the probability of awakening of people living in the \(k^{th}\) noise sensitive zone enclosing a population of size \(\text{population}_k\). UCNPP is assigned the average total costs per person affected by noise prior to the implementation of noise reduction measures, which was discussed in the evaluation of Directive 2002/49/EC (the Environmental Noise Directive) \[116\]. The value of the case in Stuttgart Airport, which is €5.66 (£4.39) per person, is set as the reference value for airports with annual operations ranging from 100,000 to 200,000.

(2) Damage cost of aircraft noise of multiple events

A large number of methods to evaluate the aircraft noise impacts on population are established based on the average-level noise metrics over a longer term, such as night-average noise level \(L_{\text{night}}\), community noise equivalent level \(L_{\text{den}}\) and 24-hour average noise level \(L_{\text{Aeq24h}}\).

The day-evening-night level \(L_{\text{den}}\) in decibels is defined by:

\[
L_{\text{den}} = 10 \cdot \log_{10} \left( \frac{12}{24} \cdot 10^{\frac{L_{\text{day}}}{10}} + 4 \cdot 10^{\frac{L_{\text{evening}}+5}{10}} + 8 \cdot 10^{\frac{L_{\text{night}}+10}{10}} \right)
\]

where \(L_{\text{day}}, L_{\text{evening}}\) and \(L_{\text{night}}\) are the A-weighted 12, 4, 8 hours average sound levels with the default time ranges 07:00-19:00, 19:00-23:00 and 23:00-07:00 local time respectively.

This kind of noise-induced impacts focus on the aircraft noise of multiple flight events during a certain period of time towards residential communities surrounding the airport. By introducing the concepts of Disability-Adjusted Life Year (DALY), any noise-induced potential threat to human health can be measured with the lost time of health.

The DALY is considered as the equivalent “healthy” year lost due being in state of poor health or disability. It is calculated as the sum of the Years of Life Lost due to premature mortality(YLL) in the population and the Years Lost due to Disability(YLD) for incident case of the disease or injury \[117\].
DALY = YLL + YLD  \hspace{2cm} (4.11)

The YLLs are essentially calculated as the number of caused specific deaths multiplied by a loss function defining the year lose as a function of age at which the death occurred. This factor is not considered in this study due to the lack of access to the concrete data of aircraft noise induced death. In the case of aircraft noise, only the YLD is considered for its comparatively clear definition. The YLD for a particular cause in a particular time period is estimated as follows \cite{118}:

\[
YLD(c, a, s) = I(c, a, s) \times DW(c, a, s) \times \frac{1 - e^{-rL(c,a,s)}}{r} \hspace{2cm} (4.12)
\]

where \( I \) is the number of incidents in that period as a parameter of cause \( c \), \( a \) is age and \( s \) stands for sex. \( DW \) is the weight factor reflecting the severity of the disease on a scale from 0 (perfect health) to 1 (death). \( L \) is the average duration of the cause, and \( r \) is the discounting rate which means to discount future benefits of the years of healthy life loss or gain. With the discount rate, a year of healthy life gained or lost now is worth more than the one gained or lost years later.

Except for time discounting, note that there are several social value weights considered for the burden of diseases, for example age weights \cite{119}. Yet due to the lack of age and sex information of the residents around the airport, uniform age and sex weights are applied to all considered population so that there is no difference between the senior and the young as well as different sexes in the cause of health burden caused by aircraft noise. Moreover, since the average duration of the aircraft noise events is a small amount compared with the time duration of other injuries or disabilities, Eq. (4.12) can be simplified using a first-order approximation:

\[
YLD(c) = I(c) \times DW(c) \times L(c) \hspace{2cm} (4.13)
\]

Then the parameter DALY can be used to estimate the health value per person caused by specific aircraft noise. Therefore, the noise-induced damage on human health can be expressed by time as a common currency and be monetised by the unit monetary value of DALY \cite{120}:

\[
\text{Damage Value} = \text{population exposed} \times P(c) \times \text{health value} \hspace{2cm} (4.14)
\]

where

\[
\text{health value} = UC_{\text{DALY}} \times \text{DALY} = UC_{\text{DALY}} \times \text{YLD} \hspace{2cm} (4.15)
\]
UC\textsubscript{DALY} is the unit cost of economic health value recommended to be set as £60,000 per DALY by the Department of Health [120]. \( P \) indicates the proportion of population influenced by the cause \( c \), which is introduced in the following section.

- Highly Annoyed

The proportion of highly annoyed can be quantified following the existing IGCB(N) (i.e. the Interdepartmental Group on Costs and Benefits Noise Subject Group) guidance:

\[
\text{Air: } \% \text{HA} = -9.199 \times 10^{-5}(L_{\text{den}} - 42)^3 + 3.932 \times 10^{-2}(L_{\text{den}} - 42)^2 + 0.2939(L_{\text{den}} - 42)
\]

(4.16)

where \%HA indicates the proportion of highly annoyed, the word “Air” means this relationship between annoyance and \( L_{\text{den}} \) only applies to air transportation. Note that it is possible for \%HA to go below 42 dB, yet data below 45 dB were still excluded due to the unreliability of noise data at very low level as well as the absence of a relationship at this low level. \%HA above 75 dB is assumed constant.

Besides, the disability weight for annoyance is between 0.01 and 0.12. According to the report from WHO, here the central estimate 0.02 is selected as the DW to quantify the health outcome.

- Highly Sleep Disturbed

The percentage of “highly sleep disturbed” persons (HSD\%) can be calculated as a function of \( L_{\text{night}} \):

\[
\text{Air: } \% \text{HSD} = 18.147 - 0.956L_{\text{night}} + 0.01482 \times (L_{\text{night}})^2
\]

(4.17)

where \( L_{\text{night}} \) is the equivalent continuous noise levels for the 8 hour period 23:00 to 07:00 local. Note that below 45 dB, the noise impact is ignored for its weak relationship at a low noise level. Moreover, data above 65 dB is assumed to be constant due to a lack of data to establish a robust relationship at high levels. Thus, a result can be derived that a proportion of population less than 5.14\% could be neglected, while a proportion above 18.62\% would remain the same as which is affected by noise level above 65 dB.
As for the disability weight, it follows the *Night Noise Guidelines for Europe* from WHO to set 0.07 as the mean DW of noise-related sleep disturbance [121]. Moreover, since air traffic noise is characterized by high levels per event and low number, the typical estimate of $I(c)$ in Eq.(4.13) is set to be eight which is also set according to *Night Noise Guidelines for Europe*, the average flights per night(23:00-7:00), when the accurate number of sleep disturbance for a specific situation is unavailable.

**Noise-induced Awakening**

Although the fact that people awake once or twice per night is proved by experimental and sociological evidence, any unnatural increase in awakenings is still important and should be taken seriously. A dose-effect relation between the percentage of noise-induced awakenings and civil aviation noise suggested by WHO is presented as [121]:

$$\text{Air: } \% \text{Noise-induced awakenings} = -0.564 + 1.909 \times 10^{-4}(\text{SEL}_{\text{inside}})^2 \quad (4.18)$$

where $\text{SEL}_{\text{inside}}$ is sound exposure level caused by an aircraft noise event in the bedroom. This relation is confined to commercial aircraft noise over the interval $\text{SEL} \in [54 \text{ dB}, 90 \text{ dB}]$ and the number of noise events per night $N \in [1, 10]$. In the report *Night Noise Guidance in Europe*, it is suggested to choose an average level difference of 21 dB between the level inside and outside, assuming that the windows are open most of time in well-insulated houses. Note that it is also suggested that 20.50 dB should be added to $\text{SEL}_{\text{outside}}$ due to house attenuation [108]. In this study, the difference follows a default value of 20.50 dB.

Please note that due to the physiological reactions of the auditory system, sleep is protected so that awakening by a new noise event is a relatively rare occurrence. Although it will lead to some adverse consequence if this reaction is abused, unlike sleep disturbance and annoyance, there is no direct description of the disability weight of noise-induced awakening. Please note that this difference leads to the exclusion of the monetisation of average-noise level induced awakenings by evaluating the equivalent value of DALYs. This index is listed here as a reference metric in the evaluation of aviation noise at night with the population awakened.
4.2.3 Damage cost of gaseous emissions

Gaseous emissions, including NOx and CO₂, have aroused widespread attention due to their economic cost on environment and climate change. It is also of great significance to describe the cost of emissions in economic terms when evaluating optimised trajectories. In the following sections the damage cost of gaseous emissions is introduced.

(1) Damage cost of NOx

The damage cost of NOx which is estimated in the UK is used for the posterior analysis. In September 2015, the Department of Environment, Food and Rural Affairs (Defra) updated the method to valuing nitrogen oxides through its impact-pathway and estimated the damage cost as the discount in life years [122]. The data are estimated for different geographical locations yet damage cost particularly for aviation is not available. Therefore, the central value of the NOx damage cost for average transport is used as the cost of damage caused by NOx emissions from aviation. This estimates as an average damage cost of NOx of £25,252 per tonne at 2015 prices, with a range of £10,101 to £40,404.

(2) Damage cost of CO₂

It is difficult to price carbon dioxide for it has dynamic long-term impacts on global environment. According to BEIS’s research, the updated short-term traded carbon central value used for modelling purposes is £4.13 at 2017 prices [123]. This value is used for purchasing allowances under the European Union Emissions Trading System (EU ETS), so it is suitable for electricity generation and investment across the Government, but perhaps might not be appropriate for evaluation on damage cost of on flight events of limited times. The metric we used to measure the damage is the social cost of Carbon (SCC) in the UK [124]. Same concept has been applied for damage cost analysis of carbon dioxide produced by passengers on airport surface access in the Manchester Airport [125]. Recently, Nordhaus valued US SCC with an estimate of $31 per ton in 2010 USD [126]. This value is therefore employed as the reference of CO₂ unit damage cost in this study. Table 4.2 provides the damage costs of NOx and CO₂ with currency converted into 2015 GBP [127].

<table>
<thead>
<tr>
<th>Pollutant</th>
<th>Unit Price [per ton]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOx</td>
<td>£25,252.00</td>
</tr>
<tr>
<td>CO₂</td>
<td>£14.89</td>
</tr>
</tbody>
</table>

Table 4.2: Unit damage costs (UDC) of NOx and CO₂ (2015 prices).
4.3 Concluding remarks

In this chapter, two posterior selection strategies, namely the aggregated preference value method and the monetisation method have been proposed to evaluate the Pareto optimal solutions obtained from Multi-Objective Optimisation problems. Although for every single environmental attribute, including aircraft noise level and gaseous emissions, each concern has its unique and different physical meaning, it is still feasible for them to be identified, integrated and evaluated on a universal scale. The first method measures the solutions by quantifying DMs’ preferences, while the second method introduces the economic cost to monetise the cost functions. Significant environmental concerns are identified, categorised and assigned with a universal value describing their importance in each method. It has shown that these two scales, namely preference value and money, can act as the intuitive and objective aggregated weighing metrics which provides the DMs with more straightforward strategies in the decision-making process. The methods established in this chapter will be applied for the multi-objective trajectory optimisation in the following chapters. Figure 4.3 depicts the Multi-Objective Trajectory Optimisation (MOTO) Framework with the aggregated weighing metrics included in the posterior process. The arrow on the top indicates the direction of optimisation process, from the MOTO structure to the aggregated weighting matric. While the arrow at the bottom leads to the decomposed system with the submodules displayed in the first frame of MOTO structure.

Figure 4.3: A schematic diagram of the composition of the Multi-Objective Trajectory Optimisation Framework.
Chapter 5

Departure Trajectory Optimisation

In this chapter, the numerical methods to solve the trajectory optimisation problem defined in Chapter 3 and the posterior selection strategy introduced in Chapter 4 are described as well as the construction of the optimisation framework. Numerical techniques including segmentation and parameterisation methods of the motion in the horizontal and vertical planes are presented. As a preliminary study, this chapter will use a simple and hypothetical scenario in order to test the technique and assess the performance of the optimisation framework.

5.1 Segmentation and Parameterisation Method

This section will introduce the method to discretise the trajectory in the time interval. In order to solve the problem, the flight dynamics function is firstly built in the three-dimensional space and then decoupled on the horizontal and vertical motion planes respectively.

5.1.1 Decoupling of the dynamics equations

Several assumptions are made first for decoupling the motion between horizontal and vertical planes:

1. The angle of attack $\alpha$ is small.

   With this assumption, the calculation for the angle of attack $\alpha$ can be simplified to:

\[
\begin{align*}
\sin \alpha &= 0 \\
\cos \alpha &= 1
\end{align*}
\] (5.1)
Then the Eq.(3.8) can be simplified as:

\[
\begin{aligned}
\dot{V} &= \frac{F_N - mg \sin \gamma - D}{m} \\
\dot{\gamma} &= \frac{L \cos \phi - mg \cos \gamma}{mV} \\
\dot{\chi} &= \frac{L \sin \phi}{mV \cos \gamma} \\
\dot{x} &= V \cos \gamma \sin \chi \\
\dot{y} &= V \cos \gamma \cos \chi \\
\dot{h} &= V \sin \gamma \\
\dot{m} &= -f
\end{aligned}
\] (5.2)

2. The force normal to the flight path is in equilibrium.

This assumption is made so that the lift and the portion of the weight normal to the flight path are balanced during each time step climb procedure, which leads to:

\[
\dot{\gamma} = \frac{L \cos \phi - mg \cos \gamma}{mV} = 0
\] (5.3)

Then the lift coefficient can be derived as:

\[
C_L = \frac{2mg \cos \gamma}{\rho V^2 S \cos \phi}
\] (5.4)

As a result of parabolic drag polar assumption, the drag force can be expressed by:

\[
D = (C_{D0}(h, V) + k(h, V)C_L^2) \cdot \frac{1}{2} \rho V^2 S
\] (5.5)

where \(C_{D0}\) is the zero-lift drag coefficient and \(k\) is the parabolic drag polar coefficient, which can be obtained by the AeroTool with different altitude \(h\) and airspeed \(V\). We also assume that the lift increases in a linear relation with the angle of attack:

\[
C_L = C_{L0} + C_{La} \cdot \alpha
\] (5.6)

where \(C_{L0}\) is the section lift coefficient when the angle of attack is zero, and \(C_{La}\)
is the slope of the lift curve. Similarly, the angle of attack $\alpha$ can be derived from Eq.(5.6):

$$\alpha = \frac{C_L - C_{L0}(h, V)}{C_{La}(h, V)} = \alpha(m, h, V, \phi, \gamma) \quad (5.7)$$

Then $\alpha$ is explicitly expressed in terms of $m$, $h$, $V$, $\phi$ and $\gamma$, which makes $\alpha$ no longer a unknown control variable.

However, since $\dot{\gamma}$ is set to be zero within each time step, the value of flight path angle $\gamma$ in each time step should be defined additionally, which makes $\gamma$ a control variable under this assumption. Thus a new state variable vector is formed $x = [V, \chi, x, y, h, m]^T$ with the control vector updated as $u = [F_N, \gamma, \phi]^T$. Then Eq.(5.2) can be decoupled into motion equations in the vertical plane and horizontal plane as is shown in Eq.(5.8).

**Horizontal:**

\[
\begin{align*}
\dot{\chi} &= g \tan \phi / V \\
\dot{x} &= V \cos \gamma \sin \chi \\
\dot{y} &= V \cos \gamma \cos \chi
\end{align*}
\]

**Vertical:**

\[
\begin{align*}
\dot{V} &= \left( F_N - mg \sin \gamma - D \right) / m \\
\dot{h} &= V \sin \gamma \\
\dot{m} &= -f
\end{align*}
\]

**(5.8)**

### 5.1.2 Horizontal plane parameterisation

Waypoints are usually pre-programmed, which makes this method in accordance with the execution of the flight management system (FMS). Two different types of legs-track-to-a-fix (TF) legs and radius-to-a-fix (RF) legs-are highly preferred, which means the lateral trajectories are either constructed with straight legs or constant radius turns [82, 128, 129]. The latter one is regularly used for connecting two straight segments with different heading angles. At the same time, some studies apply contiguous straight-line segmentation to approximate the segmented trajectory when the sections split from those turns are short enough [84] (seen in Figure 5.2).

Assume the ground track is divided by $n$ segment with $n+1$ waypoints. Note that due to safety issues, no turns are permitted below a certain altitude. So it is always assumed that the first segment of the ground track is a straight line. Another assumption which is made to simplify the situation is: for every two straight segments, only one constant-radius turn is used to connect them. There are two situations (seen in Figure 5.1) differentiated by their types of the final segment, which leads to a different number of free parameters needed with the same fixed endpoint.

The parameters needed to define a straight leg include the length of the leg $l$ and the change of the heading angle $\Delta \chi$. The required parameters to define a constant radius turn are the radius $R$ and the angle of fly-by-turn which equals the change of
CHAPTER 5. DEPARTURE TRAJECTORY OPTIMISATION

(a) Final segment with a constant radius turn. 
(b) Final segment with a straight leg.

Figure 5.1: Type-1: ground tracks with straight legs and constant radius turns.

the heading angle $\Delta \chi$ obtained from the comparison to the previous segment. Note that if the last endpoint of the ground track is fixed, whether the final segment is an arc or a straight line, the control parameters to define the final segment can be derived from the coordinates of the endpoint and known parameters of the previous segment. As is shown in Figure 5.1.(a), the control parameters include $R_n$ and $\Delta \chi$ when the final segment is a constant-radius turn. As for the final segment that is a straight leg, control variables are $l_n$ and $\Delta \chi_{n-1}$ (seen in Figure 5.1.(b)).

Figure 5.2: Type-2: ground tracks with straight legs.

Thus, if the last segment is an arc, there are $n/2$ straight legs as well as $n/2$ constant radius turns, where $n$ is an even number; if the last segment is a straight line, $(n+1)/2$ straight lines and $(n-1)/2$ arcs compose this ground track, where $n$ is an odd number. The comparison of the number of parameters required to define a ground track is given in Table 5.1. By comparing the number of required parameters to define an $n$-segment ground track, it is found that with an equivalent number of segments the first type of parameterisation methods requires fewer parameters than the Type-2.

Trajectory models defined by spline interpolation have also been developed and introduced [130]. Conventionally, the first category of methods is adopted for their easy implementation of the constraints from existing operational requirements and air traffic guidance, while the second category might put more pressure on the situational
Table 5.1: Comparison of the number of free parameters for a ground track.

<table>
<thead>
<tr>
<th>Segment No.</th>
<th>Parameter needed</th>
<th>Type-1</th>
<th>Type-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$n = 2k$, $k \in \mathbb{Z}^+$</td>
<td>$n = 2k - 1$, $k \in \mathbb{Z}^+$</td>
</tr>
<tr>
<td>1st</td>
<td>$l_1$</td>
<td>$l_1$</td>
<td>$l_1$</td>
</tr>
<tr>
<td>$i^{th}$</td>
<td>turns: $(R_i, \Delta \chi_{i-1})$</td>
<td>turns: $(R_i, \Delta \chi_{i-1})$</td>
<td>$(l_i, \Delta \chi_{i-1})$</td>
</tr>
<tr>
<td>straight legs: $l_i$</td>
<td>straight legs: $l_i$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(n-1)^{th}$</td>
<td>$l_n$</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>$n^{th}$</td>
<td>none</td>
<td>$R_{n-1}$</td>
<td>$(l_i, \Delta \chi_{i-1})$</td>
</tr>
<tr>
<td>Total number</td>
<td>$N = \frac{3n-4}{2}$</td>
<td>$N = \frac{3n-5}{2}$</td>
<td>$N = 2n - 3$</td>
</tr>
</tbody>
</table>

awareness of pilots. However, with the development of the technology of today’s navigation and guidance, a far more complicated situation could be handled with the increased airspace complexity caused by optimised free flight routes. In the following section, a parameterisation method using the Bézier approximation curve is imposed to obtain the horizontal track with the potential of reducing the dimensions of the search space.

A Bézier curve is defined by a directed sequence of control points $P_0(x_0, y_0)$ to $P_n(x_n, y_n)$, where $n$ is the order of the polynomial. Bézier curve has two properties which make it suitable for the ground track construction. First, the section formed by connecting the first and the second control points is tangent to the curve at the first control point, and the section between the last and the penultimate control points is tangent to the curve at the last Bézier point. This feature allows users to control the tangential direction of the curve more easily by directly controlling the coordinates of the second and penultimate Bézier points. Moreover, since the two end-points are points of tangency of the first and last sections of the Bézier polygon, the curve can be split into two Bézier curves at any point and to be easily connected with another Bézier spline at any end point; this makes it flexible for piecewise ground path planning. Second, the convex-hull of the Bézier polygon composed of control points would wrap around the Bézier curve (seen in Figure 5.3). Given the population distribution around the airport, it is possible to approximate the flight corridor with the convex hull of the Bézier polygons within which the ground track would steer clear of the NSAs. The curve is defined by:

$$B(\tau) = \sum_{i=0}^{n} b_{i,n}(\tau)P_i, \quad \tau \in [0, 1], i = 0, \ldots, n$$  

(5.9)

where the polynomials
are known as Bernstein basis polynomials of degree \( n \). \( P_i \) are the coordinates of the control points and \( \binom{n}{i} = \frac{n!}{i!(n-i)!} \) is the Binomial coefficient. The radius of curvature \( R(\tau) \) on the curve is:

\[
R(\tau) = \frac{(x'^2(\tau) + y'^2(\tau))^{3/2}}{|x'(\tau)y''(\tau) - x''(\tau)y'(\tau)|}
\]  

(5.11)

The control variable bank angle \( \phi \) can be derived as:

\[
|\phi| = \left| \tan^{-1} \left( \frac{V^2}{gR(\tau)} \right) \right|
\]  

(5.12)

where the radius \( R \) can also be expressed as a function of speed \( V \) and load factor \( n \):

\[
R = \frac{V^2}{g\tan \phi} = \frac{V^2}{g\sqrt{n^2 - 1}}
\]  

(5.13)

\( n = \frac{L}{mg} \) is the load factor in steady level turns. Then the bank angle is constrained as \( |\phi| \leq |\phi| (n_{\text{max}}) = \tan^{-1} \sqrt{n_{\text{max}}^2 - 1}, \) \( n_{\text{max}} = 1.10 \). Note that the positive sign of the bank angle represents turning right; the negative indicates turning left; this direction can be determined by the analysis of the direction vector’s change along the curve defined in Eq.(5.9). Therefore, the control variable \( \phi \) is explicitly controlled by the coordinates of the control points \( P_i \).

### 5.1.3 Vertical segmentation

The departure procedure can be separated into several segments of climbs and accelerations. One possible segmented departure procedure is demonstrated in Figure 5.4.
Figure 5.4: Take-off and climb-out, with key steps [6].

According to this flight operation sequence, two accelerations (i.e. AB and CD) and two constant speed climbs (i.e. BC and DE) are displayed. Further explanations of these segments are introduced in the following sections. The primary concern will become the parameterisation of these segments and how changing the values of the free parameters affects the noise perceived at the NSAs.

A. Phase 1: A-B
In this phase, the aircraft is assumed to take off at maximum thrust until it reaches the point B where the speed $V_B$ and altitude ($h_B = 800$ feet) are user-defined. Please note that although the operational sequence of the high lift devices is given in Figure 5.4, in order to reduced the complexity of the problem, it is assumed that the aircraft flies with a clean configuration in the case study of this chapter.

Another assumption is that the flight path angle $\gamma$ is constant within every time step. Then the designed parameters left in the AB phase are ($\gamma_A, V_B$), where $\gamma_A$ is the initial flight path angle. $V_B$ is airspeed need to be achieved at the end point of this phase. Note that, if the aircraft reaches the final altitude of this segment $h_B$ first, then the aircraft will do a level flight with $\gamma = 0$ and accelerate until the target speed $V_B$ is achieved. However, if $V_B$ is reached first, then let $\dot{V} = 0$, which makes the thrust equal to the drag $F_N = D$. In this way, aircraft makes a constant airspeed climb subsequently until it reaches the designed altitude.

It is worth noticing that apart from the proposed variables above, additional parameters defining this segment do exist. For example, according to the ICAO B procedure, the target altitude $h_B$ can be set as any value between 800 feet to 1,500 feet, which makes it a design variable as well.

B. Phase 2: B-C
In this phase, the aircraft is assumed to climb at a constant speed. The terminal condition is to reach a final altitude $h_C = 3,000$ feet. During this constant speed climb
phase, the only free parameter turns to be the flight path angle $\gamma_B$.

The procedure to solve this flight phase is listed below.

1) Assess the value of the free parameter $\gamma_i$;

2) Calculate aerodynamics coefficients:

   $$ C_{L,i} = \frac{2m_i g \cos \gamma_i}{\rho(h_{i-1})V_B^2 S \cos \phi_i} $$  \hspace{1cm} (5.14)

   $$ C_{D,i} = C_{D0} + kC_{L,i}^2 $$  \hspace{1cm} (5.15)

3) Calculate the thrust

   Since $\dot{V} = 0$, then the thrust can be obtained:

   $$ F_{N,i} = m_i g \sin \gamma_i + C_{D,i} \cdot \frac{1}{2} \rho(h_{i-1})V_B^2 S $$  \hspace{1cm} (5.16)

   Then the engine rational speed $N_{1,i}$ required in this time step can be derived by interpolation.

4) Induce $\gamma_i$, $C_{L,i}$, and $N_{1,i}$ into the dynamics equation to calculate $h_i$ and $V_i$ in this time step.

   Then go to the next step 1) and repeat the procedure.

C. Phase 3: C-D

In this phase, the aircraft is expected to do a level acceleration until it reaches a target speed ($V_D$) for the next climb. Conditions in this phase are:

   $$ \dot{V} = \frac{F_N - D}{m} \geq 0 $$  \hspace{1cm} (5.17)

   $$ \dot{h} = 0 $$  \hspace{1cm} (5.18)

   $$ \dot{\gamma} = 0 \quad \text{and} \quad \gamma = 0 $$  \hspace{1cm} (5.19)

   $$ V_D \leq 250 \text{ knots} $$  \hspace{1cm} (5.20)

Therefore, the acceleration rate becomes the key factor that decides the profile of this phase, which leads to the only control variable in this segment: the engine rpm $N_{1CD}$. Assume $N_{1CD}$ keeps constant during this level acceleration, then solving the flight profile of this segment becomes an initial value problem.

Therefore, the vertical climb profile is parameterised by five free parameters shown in Eq.(5.21). A summary of each segment with the vertical free parameters needed is demonstrated in Table 5.2.

$$ \mathbf{P}_{\text{vertical}} = [\gamma_A, V_B, \gamma_B, N_{1CD}, V_D]^T $$  \hspace{1cm} (5.21)
Table 5.2: Description of each segment.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Description</th>
<th>Free Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>full trust acceleration while climbing</td>
<td>$\gamma_A, V_B, h_B$</td>
</tr>
<tr>
<td>BC</td>
<td>constant climb</td>
<td>$\gamma_B$</td>
</tr>
<tr>
<td>CD</td>
<td>level acceleration</td>
<td>$V_D, N_{1CD}$</td>
</tr>
<tr>
<td>DE</td>
<td>climbing/level flight</td>
<td>$\gamma_{CD}$</td>
</tr>
</tbody>
</table>

5.1.4 Coupling of motions in two planes

Previous sections present the trajectory construction of the horizontal track and the vertical profile respectively. From Eq. (5.8), the departure trajectory can be parameterised through two sets of free parameters:

- Horizontal free parameters: $\mathbf{P}_{\text{horizontal}}$;
- Vertical free parameters: $\mathbf{P}_{\text{vertical}}$.

To be specific, the parameter vector $\mathbf{P}_{\text{horizontal}}$ includes the coordinates of the Bézier control points $\mathbf{P}_i$ that derive to the control variable $\phi$, while the parameter vector $\mathbf{P}_{\text{vertical}} = [\gamma_A, V_B, h_B, \gamma_B, V_D, N_{1CD}, \gamma_D]^T$ defines the segmented movement of the aircraft in the vertical plane. After parameterisation, the continuous control vector in time domain $\mathbf{u} = [N_1, \gamma, \phi]^T$ can be fully discretised and replaced by $\mathbf{P}_{\text{horizontal}}$ and $\mathbf{P}_{\text{vertical}}$, which are free parameters in the defined optimisation problem. Figure 5.5 demonstrates the computational flowchart of solving this 3D flight dynamics model.

![Figure 5.5: Computation procedure of 3D flight dynamics model.](image-url)
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5.2 Optimisation Framework

5.2.1 Optimiser

Due to the discrete definition of the flight segments, objectives and constraint functions are not differential in current optimal control problem. Therefore it is preferable to use optimisation algorithms that do not need any gradient information to search for optimum solutions. For this reason, derivative-based methods are excluded in this chapter. Evolutionary methods, including PSO-based algorithm and GA-based algorithms are applied in this chapter. Although both priori and posterior methods to solve multi-objective trajectory optimisation problems are introduced in Chapter 2, so far only posterior methods are adopted in this work.

Particle Swarm Optimisation (PSO) [77] has been widely used in optimisation problems due to its considerable performance in convergence speed and the friendly regime of parameter adjustment. However, problems appear when extending its application from single objective optimisation to multi-objective optimisation, like how to determine the global and local optimal solution to lead the swarm, how to maintain the external archive where stores the non-dominated solutions found, and how to achieve a balance between exploration and exploitation [131]. To solve this problem, many Multi-objective Particle Swarm Optimisation (MOPSO) methods have been proposed, including weighted objective functions aggregated approaches [132], lexicographic ordering approaches based on the importance of objective functions [95], Pareto based approaches using the concept of Pareto dominance [133–139] to select leaders, and combined approaches. Among the MOPSOs the second category arouses the most interest in the researcher. These Pareto-based approaches should at least propose a leader selection technique, a retaining solution for external archive maintenance, and the perturbation mechanism to keep diversity so that prevent premature convergence. One of the most classic Pareto-based approaches is the AGMOPSO which applies adaptive grids and mutation operator to updates external archive and determine global bests [140]. However, few MOPSOs have designed the adjust mechanism towards exploration and exploitation according the feedback from the current evolutionary environment. A novel MOPSO, namely PEMOPSO, based on Parallel Cell Coordinate system (PCCS) and information Entropy of current Pareto optimal solutions was proposed to evaluate the diversity of the swarm and its evolutionary environment. The idea is to adjust the evolutionary strategy with the feedback to balance convergence and diversity. The general framework of PEMOPSO is presented in Algorithm 1. For more details, references [141–143] should be referred to.
Algorithm 1 PEMOPSO algorithm

**Input:**
- A MOP with $M$ objectives;
- A searching space with $D$ design variables;
- Initialization parameters: maximum capacity of the external archive $K$, swarm size $N$, maximum iterations $T_{\text{max}}$;

**Output:**
- The approximate Pareto optimal set in the external archive, $g_{\text{Archive}}$;

1: Initialization: $t=0$;
2: Initialize a swarm $P = \{x^1, x^2, \ldots, x^N\}$;
3: Evaluate the initial objective values $f^i(x^i) = \{f^1(x^i), f^2(x^i), \ldots, f^M(x^i)\}$, and the objective set $f = \{f^1(x^1), f^2(x^2), \ldots, f^N(x^N)\}$;
4: Initialize the $i^{th}$ particle’s personal archive $p_{\text{Archive}}$ using strategy described in ref [142];
5: Let external archive $g_{\text{Archive}} = \emptyset$, use the same strategy to update $g_{\text{Archive}}$;
6: Update iterations: $t = t + 1$;
7: while $t < T_{\text{max}}$ do
8: Evaluate evolutionary environment and adaptively adjust the flight parameters;
9: Project $g_{\text{Archive}}$ to PCCS;
10: Calculate Pareto entropy and entropy difference from the previous $g_{\text{Archive}}$, by which the evolutionary status is determined;
11: Calculate the flight parameters $\omega(t), c_1(t)$ and $c_2(t)$ according to the evolutionary status;
12: Update the swarm:
13: for each $i \in [1, N]$ do
14: Select global best $g_{\text{Best}}$ from $g_{\text{Archive}}$;
15: Select $p_{\text{Best}}$ which is the closest to the $g_{\text{Best}}$;
16: Update $v^i$ and $x^i$ of the $i^{th}$ particle;
17: if $\text{rand} < L_r$ then
18: Apply an Elitism Learning strategy[43] to $x^i$;
19: end if
20: Evaluate the objective function values $f^N = \{f^1(x^N), f^2(x^N), \ldots, f^M(x^N)\}$;
21: Update $p_{\text{Archive}}$ and $g_{\text{Archive}}$;
22: end for
23: end while

5.2.2 Structure and functionality

The framework for multi-objective aircraft trajectory optimisation used in this thesis is comprised of aircraft flight mechanisms, aerodynamics, engine, noise and emission models and an optimisation module. This framework provides interfaces between individual models and the optimiser. A schematic diagram of the optimisation framework is provided as Figure 5.6. According to the user’s selection of objectives and value of free parameters, the framework will conduct calculation to find out optimal trajectories for different single objectives or multi-objective problems. These objectives
can be selected directly from indexes provided in Table 3.1 or from the combined cost functions defined by users. Note that apart from the choices provided in Table 3.1, flight duration and fuel consumption can also be selected as cost functions of single objective OCP or be used to compose a combined objective with any weighting or aggregated method.

Shown in Figure 5.6, there are two main parts of this optimisation framework: Flight Performance Model and Optimisation Module. After the user submits the requirements, an optimisation problem based on customers’ requirements is established accordingly. The optimisation process mainly takes place in the Optimisation Module consisting of parameter translation module, optimiser and post-processing module. In the Flight Performance Model, tools and submodules from FLIGTH [36] and code programmed by the author are integrated to obtain trajectories for environmental emissions estimation. Then from the value of cost function, the optimisation module applies GA-based or PSO-based algorithms to make simulation iteratively in order to find optimal solutions or solution sets. Afterwards, post-processing will be conducted to analyse the results and to deal with the visualisation work.

As is depicted in Figure 5.6, this optimisation framework is established based on the flight mechanism and heuristic algorithms. Firstly, the user defines the optimisation problem with proposed objectives and free parameters. Next, the specific decision variable values are sent to the flight performance model to calculate the flight trajectories. Then the flight mechanism model needs to call sub-models to obtain aerodynamics force, single engine thrust, and atmospheric parameters to support the calculation. After that, the emission calculator is required to predict environmental emission indexes and export them to Optimisation Module again to evaluate the value of objective functions. Based on the evaluation results, a new set of optimised solutions will be generated as a result of evolution. Then the optimisation loop continues until satisfying the termination criterion.
5.3 Case Study

To demonstrate the capability of the method proposed, we consider one Airbus 320-211 aircraft are departing toward the west off Runway 23R in Manchester Airport [144]. The closest residential community near the end of the Runway 23R/05L is Knutsford which is annoyed by the aviation noise most. Thus, trajectory optimisation on departure for minimising noise impact on Knutsford as well as local emission impact are the objectives.

\[ J_1 = EPNL, \quad J_2 = EC \]  \hspace{1cm} (5.22)

where \( EC = UC_{\text{fuel}} \cdot F + UC_{\text{CO}_2} \cdot \text{CO}_2 + UC_{\text{NO}_x} \cdot \text{NO}_x \) denotes the environmental cost from fuel and emissions. \( UC \) is the unit cost: for the fuel consumption, it is the unit jet fuel price; for the gaseous emissions, environment-related taxes are adopted.

The original coordinate is located at the Runway end 23R and the aircraft departures with initial conditions \( x_0 = -916.71 \text{ m}, y_0 = -399.55 \text{ m}, h_0 = 3 \text{ m}, V_0 = 75 \text{ m/s} \) with lading gear retracted and departure flaps selected. Note that \( h \) is the altitude above the mean sea level. The final conditions are selected from an existing SID, with final ground location fixed \( x_f = -18119.20 \text{ m}, y_f = -16708.69 \text{ m}, h_f \leq 1524 \text{ m} \).

5.3.1 Case study 1 : MOTO with straight legs and constant radius turns [1]

In this case, the traditional lateral parameterisation method is applied to prove the robustness of the optimisation framework. The horizontal track to be optimised can be segmented into five segments with three straight legs and two constant radius turns.
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Table 5.3: Free parameters of ground track.

<table>
<thead>
<tr>
<th>Segment No.</th>
<th>Parameter</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$l_1 \in [4000 , m, 6000 , m]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$R_2 \in [2000 , m, 3000 , m]$, $\Delta \chi_2 \in [40^\circ, 90^\circ]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$l_3 \in [2000 , m, 4000 , m]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$R_4 \in [3000 , m, 9000 , m]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>none</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: Vertical profile parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_A$</td>
<td>4$^\circ$</td>
<td>12$^\circ$</td>
</tr>
<tr>
<td>$V_B$</td>
<td>80 m/s</td>
<td>100 m/s</td>
</tr>
<tr>
<td>$\gamma_B$</td>
<td>4$^\circ$</td>
<td>12$^\circ$</td>
</tr>
<tr>
<td>$h_B$</td>
<td>243.84 m</td>
<td>457.20 m</td>
</tr>
<tr>
<td>$N_{ICD}$</td>
<td>86%</td>
<td>103%</td>
</tr>
<tr>
<td>$V_D$</td>
<td>100.00 m/s</td>
<td>128.60 m/s</td>
</tr>
<tr>
<td>$\gamma_D$</td>
<td>2.8$^\circ$</td>
<td>2.8$^\circ$</td>
</tr>
</tbody>
</table>

Free parameters to define the lateral track $\mathbf{P}_{\text{horizontal}} = [l_1, R_2, \Delta \chi_2, l_3, R_4]^T$ and their upper and lower bounds are listed in Table 5.3.

The free parameters for vertical profile $\mathbf{P}_{\text{vertical}} = [\gamma_A, V_B, \gamma_B, h_B, N_{ICD}, V_D, \gamma_D]^T$ and their initial value ranges are shown in Table 5.4. Note that a reference value (i.e. 2.8$^\circ$) according to the existing SID (Standard Instrument Departure) is adopted for $\gamma_D$ in order to decrease the dimension of the search space. Then we have six free parameters to construct the vertical profile.

Therefore, a 3D departure trajectory is fully described with 11 free parameters. Due to the discrete formulation of this optimal control problem, functions of objectives and constraints are not derivable, which leads to a preference to use algorithms that do not need any gradient information. For this reason, a fast and elitist multi-objective genetic algorithm (NSGA-II) is adopted as the optimiser.

The example presents the departure trajectory optimisation aiming at minimising Effective Perceived Noise Level (EPNL) and environmental cost for fuel consumptions and emissions (EC) in the 3D space. Before proceeding to the result, a reference trajectory is randomly picked as the baseline for performance comparison. The comparison result of the four representative cases, namely the reference one (case1), the optimal solution (case2), EPNL-optimal case (case3), and EC-optimal case (case4), are summarised in Table 5.5. Although time and distance were not considered as the optimisation objectives in this example, they are included in Table 5.5 as the performance indicator for the four cases. Figure 5.7 illustrates the comparison of their ground tracks.

With the comparison between the values of two objectives, the multi-objective
Table 5.5: Comparison for the optimised trajectories and the baseline.

<table>
<thead>
<tr>
<th>Case</th>
<th>Time [s]</th>
<th>Ground distance [km]</th>
<th>( \Delta \text{EPNL} [%] )</th>
<th>( \Delta \text{EC} [%] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>360.36</td>
<td>35.1932</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>3</td>
<td>280.40</td>
<td>30.1672</td>
<td>-7.86</td>
<td>-2.86</td>
</tr>
<tr>
<td>4</td>
<td>271.03</td>
<td>27.8373</td>
<td>+8.15</td>
<td>-12.10</td>
</tr>
</tbody>
</table>

Figure 5.7: Departure ground track comparison

(a) Altitude profile comparison. (b) Airspeed profile comparison.

Figure 5.8: Departure vertical profile comparison.

nature of the problem is demonstrated. The extreme points of the Pareto solutions, case 3 and case 4, exhibit significant difference on the environmental indexes. Regarding the EPNL objective, both case 2 and case 3 have made progress in reducing noise impact (-3.69\% and -7.86\% respectively). By comparing the ground tracks of these two cases, it is clear that in both cases, a relatively early turn with a larger radius is preferred when the aircraft circumvents the Knutsford area, which increases the receiver-to-source-distance and reduces the sound energy received. Case 4, on the contrary, has a much closer trajectory to the noise sensitive area resulting in a higher
noise level compared with the other three cases. However, it is not always true that the further aircraft moves away from the receiver the less annoyance it will cause. It is shown in the baseline trajectory that though the ground track is the furthest among others, its EPNL is not the lowest.

Then as for the second objective EC, case 2 and case 4 have a reduction with $9.12\%$ and $12.10\%$ respectively. Considering their difference in EPNL, case 4 spends less on emission and fuel consumption at the price of bringing higher noise level for the reason that the fuel consumption is less with shorter flight distance (see Table 5.5). However, this leads to a consequence of flying over an area that is sensitive to noise. Therefore, case 2 shows a capability of optimising multiple environmental impacts (e.g. noise, fuel consumption and gaseous emissions) compared with the reference trajectory.

The vertical departure procedure described previously is followed by the four cases, which can be seen in Figure 5.8. It was founded that optimised solutions are similar in altitude profile yet differ from each other in the airspeed profile. From the Figure 5.8(a), it follows that there is a preference towards a lower airspeed during the initial climb phase before reaching 3,000 feet, which exerts a quieter operation. After passing 3,000 feet, it can be found in Figure 5.8(b) that the influence of airspeed on the noise level is weakened due to a long distance between the receiver and the noise source.

From the analysis above, distance is the most influential factor for the EC index in a terminal point fixed departure. After comparing the airspeed profiles of case 2 and case 3 (both of which have similar ground distance (see Table 5.5), it is indicated that lower final target speed leads to lower EC. The main reason for this is that EC is directly linked to the fuel consumption. Especially for the last part of the climb, lower speed keeps the engine maintains at a less intensive working condition, which brings in less fuel burn and emissions in the end.
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5.3.2 Case Study 2: MOTO using Bézier Curve [2]

The same scenario is applied in this case. Also, a UK gridded population based on the 2011 Census and the 2015 Land Cover Map [145] is introduced to estimate the exposure impacts by the aviation activities. The gridded population data based on the British National Grid (OSGB36 datum) [7] has a 1 km by 1 km spatial resolution for each square cell. To simplify the problem, it is assumed that all the population enclosed in the grid cell are gathered in the geometric centre of it, so the noise impact on that grid cell will be estimated using the noise level (expressed by SEL) received at that single point. The simplification is a compromise between accuracy and computational cost. Note that regular shapes of the grid cells can be altered (e.g. triangular cells, rectangular cells and hexagonal cells) [84], but here only the squares are utilised.

A noise-sensitive region of 3 km by 3 km in the town of Knutsford (coordinates: 53.30329° N, 2.37316° W) with a population grid cell size of 1 km by 1 km is used for the analysis of departure operation. The population distribution around this region and Manchester Airport is shown in Figure 5.10 with colour shades differentiated to indicate the density of the residents within the noise-sensitive region circled with a red rectangle.

The departure trajectory is optimised for noise minimisation on Knutsford and carbon dioxide emissions:

\[
\min J_1 = \text{EPNL}, \quad J_2 = \text{CO}_2
\]  

(5.23)

Note that \( J_1 \) is the EPNL measured at the centre of the cell of Knutsford. The noise level (i.e. SEL) received at the nine grid cells are used to calculate the cost of awakening in the process of posterior selection.
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With the original coordinate located in the Runway end 23R, the aircraft departs with initial condition at $x_0 = -916.71 \text{ m}$, $y_0 = -399.55 \text{ m}$, $h_0 = 73 \text{ m}$, $V_0 = 75 \text{ m/s}$. The end point $x_f = -18,119.20 \text{ m}$, $y_f = -16,708.69 \text{ m}$, is defined with the reference of an existing instrumental departure routing SANBA IRIY Noise Preferential Routing. Based on the method proposed in Section 5.1.2, the ground track to be optimised is segmented into the initial straight leg and the following Bézier curve segment with one free control point $(x_c, y_c)$. Note that the flight path angle $\gamma_{DE}$ along the vertical profile DE is given a reference value $2.8^\circ$ from the existing routing, and a fixed control point is added to ensure that the final heading of aircraft is $155^\circ$, which heads to SANBA at EGCC. The 3D trajectory is therefore fully described by eight free parameters, and the original optimal control problem is discretised into a parameter optimisation problem. Table 5.6 gives the settings of the free parameters. The path constraints discussed in Section 2.3 are added by applying a penalty to the cost function.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_A$</td>
<td>$3^\circ$</td>
<td>$12^\circ$</td>
</tr>
<tr>
<td>$V_B$</td>
<td>80 m/s</td>
<td>100 m/s</td>
</tr>
<tr>
<td>$h_B$</td>
<td>243.84 m</td>
<td>457.20 m</td>
</tr>
<tr>
<td>$\gamma_B$</td>
<td>$3^\circ$</td>
<td>$12^\circ$</td>
</tr>
<tr>
<td>$V_D$</td>
<td>80 m/s</td>
<td>125 m/s</td>
</tr>
<tr>
<td>$N_{ICD}$</td>
<td>70%</td>
<td>103%</td>
</tr>
<tr>
<td>$x_c$</td>
<td>$-16,000 \text{ m}$</td>
<td>$-6,000 \text{ m}$</td>
</tr>
<tr>
<td>$y_c$</td>
<td>$-4,000 \text{ m}$</td>
<td>$8,000 \text{ m}$</td>
</tr>
</tbody>
</table>

Optimisers used in this study include GA [93] and multi-objective PSO, namely Adaptive-Grid (AG) and Pareto Entropy (PE) PSO mentioned in section 5.2.1. For the three optimisers, the population or swarm sizes are all set as 40, maximum iterations are set to be 20. The PSO parameters $c_1$, $c_2$ and $\omega$ in Table 5.7 are, respectively, set according to their original suggestions. In multi-objective PSOs, the maximum capacity for the global archive is 100. The special parameters of GA are set according to [93].

The preliminary Pareto-optimal solutions obtained by the three optimisers are shown in Figure 5.11 with ground tracks presented in Figure 5.12, Figure 5.13 and Figure 5.14. From Figure 5.11 the GA has the largest size of solution set while PE has the least number of solutions. Optimised solutions obtained by AG and GA dominate quite a few of the solutions of PE after the same number of iterations. By comparing solution sets of GA and AG, it is found that both of the optimisers have solutions dominated by the other yet they in combination display a smoother Pareto front with gaps filled by solutions from the other set. Although the ones obtained by GA dominate
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Table 5.7: Features and parameter settings of multi-objective PSO.

<table>
<thead>
<tr>
<th>Features</th>
<th>PE</th>
<th>AG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archive strategy</td>
<td>Parallel cell distance and entropy feedback</td>
<td>Adaptive grid</td>
</tr>
<tr>
<td>gBest selection</td>
<td>Leader group</td>
<td>Adaptive grid</td>
</tr>
<tr>
<td>pBest selection</td>
<td>pArchive</td>
<td>One</td>
</tr>
<tr>
<td>Perturbation</td>
<td>Elitist learning strategy</td>
<td>pBest+dominance</td>
</tr>
<tr>
<td>Parameter setting</td>
<td>Self-adaptation based on $\Delta$ Entropy</td>
<td>Rapidly decreasing $\omega=0.4$, $c_1 = c_2=1.429$</td>
</tr>
</tbody>
</table>

more solutions obtained by AG, the fact that the AG reaches the extreme values of two objectives and reaches the closest point, namely Solution 3/4 designated by green numbers in Figure 5.11, demonstrates a better potential of exploration and divergence than GA. To get these solutions, each of the three optimisers executed 800 evaluations to reach the stop condition. Note that for PSO-based algorithms, the calculation of cost functions is not executed if the solution is unfeasible. As a result, for the same total number of evaluations, the computation time of AG and PE are 67.67 h and 67.98 h respectively, which cut down 5.29% and 4.86% of the computational cost, compared with the time spent using NSGA-II.

Five representative solutions are selected from the Pareto-optimal solutions obtained by each optimizer for further analysis: Solution 1 stands for min(EPNL); Solution 2 represents the min(CO$_2$), and solutions 3 to 5 are the trajectories closest to the utopia point (i.e.(EPNL$_{\text{min}}, \text{CO}_2\text{min}$), marked with a pentagram in Figure 5.11, which are those of minimum preference value and minimum economic cost respectively. They are all indicated with coloured number in Figure 5.11 and are differentiated with line styles in Figures 5.12-5.14. Table 5.8 provides a possible preference classification based on the value range of cost functions actually obtained. Note that $f^0_i$ are given the values by rounding down the minimum values of the objectives obtained in the optimisation solution set, similarly, $f^5_i$ are obtained by rounding up the maximum values of the objectives. In order to quantify the increasing undesirability along with the increase of the objective’s value and to show the differentiation between each desirability level, the difference between each pair of end points, namely $f^{k+1}_i - f^k_i$, ($k = 1, 2, ..., 4$), is set to be an arithmetic sequence, which leads to the values displayed in Table 5.8.

Table 5.8: Preferences of two objectives.

<table>
<thead>
<tr>
<th>Objective</th>
<th>$f^0_i$</th>
<th>$f^1_i$</th>
<th>$f^2_i$</th>
<th>$f^3_i$</th>
<th>$f^4_i$</th>
<th>$f^5_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPNL [dB]</td>
<td>75.00</td>
<td>76.07</td>
<td>78.20</td>
<td>81.40</td>
<td>85.67</td>
<td>91.00</td>
</tr>
<tr>
<td>CO$_2$ [kg]</td>
<td>797.00</td>
<td>801.13</td>
<td>809.40</td>
<td>821.80</td>
<td>838.33</td>
<td>931.00</td>
</tr>
</tbody>
</table>
From Figure 5.12 (a) to Figure 5.14 (a), it can be gleaned that the three optimisers produce a scattered distribution of ground tracks. By analysing the distribution of the horizontal tracks, it can be found that the greater the horizontal distance between the aircraft and the ground receivers, the lower the noise level generated in the NSA. However, this is not always true because the influence that vertical movements put on noise levels also cannot be ignored. In the three sets of the lateral tracks, there are many tracks clustered close to the NSA, with similar even coincident ground tracks, yet very different corresponding optimal objective values. Further analysis indicates that the main difference reflects in their vertical profiles, which illustrates that there is a great optimisation potential in the operations of the vertical plane. More detailed comparison on the representative solutions shown in Figure 5.12 (b) to Figure 5.14 (b) is given in Table 5.9, with the objectives and the results obtained from the posterior selection.

A detailed analysis of noise impact on the ground area around Manchester Airport is conducted by simulating the EPNL produced by the Solution 1 and Solution 3/4 obtained by AG. The simulation scenario is shown in Figure 5.15 with the blue track indicating Solution 1 and black track indicating Solution 3/4. As is shown in Figure 5.16, the second trajectory causes much higher noise level on Knutsford, while the first one reduces the EPNL in the NSA with a decrease of more than 5 dB at the cost of +3.58% in CO\textsubscript{2} and +14.81% in NO\textsubscript{x} emissions.

The vertical profiles of the representative trajectories are presented in Figures 5.17-5.19 with altitude and airspeed profiles demonstrated. By comparing trajectories with minimum EPNL in Figures 5.17-5.19, we find that in order to eliminate the EPNL in Knutsford, aircraft tend to fly at a lower airspeed below 3,000 feet to cut down the noise source level. Note that this airspeed should not be too small, otherwise it would take the aircraft longer to climb, leading to a longer exposure time, higher SEL and
CHAPTER 5. DEPARTURE TRAJECTORY OPTIMISATION

Figure 5.12: Optimal ground tracks obtained by PE.

more awakenings. This can be found in Table 5.9, where even with a wider detour (black dash-dot line in Figure 5.13(b)) and much lower speed (black dash-dot line in Figure 5.18 (b)), the trajectory from AG still awakens more population (+12.63%) compared with the one obtained by GA in the same trajectory group 2. At 3,000 feet, if the horizontal distance is far enough from the ground residential community, an acceleration segment with larger acceleration will allow for a faster departure from the receiver without significantly increasing the noise impacts. On the contrary, if the aircraft is close to the ground receiver at the horizontal plane, a quicker level acceleration segment creates more noise than a slower acceleration.

Two posterior selection strategies, namely aggregated preference function and monetisation method are both used in this case. Note that the impact of environmental factors includes damage cost of aircraft noise of single event introduced in Section 4.2.2 and damage cost of gaseous emissions introduced in Section 4.2.3. Thus, the posterior selection of monetisation method is to choose the minimum economic costs of obtained optimised solutions, which is given in Eq. (5.24):

\[
\min \text{total cost} = UCNPP \cdot \text{population} + UDC_{NO_x} \cdot NO_x + UDC_{CO_2} \cdot CO_2 + UC_{fuel} \cdot \text{fuel}
\]  

(5.24)

A further comparison is made based on the objectives and the result obtained from
CHAPTER 5. DEPARTURE TRAJECTORY OPTIMISATION

Figure 5.13: Optimal ground tracks obtained by AG.

the posterior selection listed in Table 5.9. It is noted that all the trajectories with minimum EPNL in each solution set turn out to be optimal based on the monetisation method too, according to Eq. (5.24). An analysis of the first group of solutions shows that even for the trajectory with the least awakenings, the resulting economic cost calculated from Eq. (5.24) still occupies a total cost as high as 62.47\%. This shows that it is the high pricing of UCNPP that makes the number of awakenings a decisive factor in the application of the monetisation method. Second, the result shows that the optimal solutions obtained from the minimum preference value tend to be the same as those closest to the utopia point for both PE and AG. It is due to the theoretical difference of these two posterior methods that makes this phenomenon a coincidence, not a conclusion: the preference value function method is highly dependent on the classification of preference levels identified by DM; identification of the closest trajectory from the utopia point is made without the anthropic factor. Finally, by comparing the awakenings of each solution in group 1 (e.g. min(EPNL)) in Table 5.9 and the correspondent ground tracks, a detour around Knutsford can result in a decrease of awakenings, but not always in EPNL as well. With a posterior evaluation of the Pareto solution sets, the Solution 1/5 of PE is selected by the monetisation method, and Solution 3/4 of AG is selected by the preference function value method.
CHAPTER 5. DEPARTURE TRAJECTORY OPTIMISATION

(a) All optimised ground tracks.

(b) Representative optimised ground tracks.

Figure 5.14: Optimal ground tracks obtained by GA.

Figure 5.15: Simulation scenario; colours indicate ground acoustic impedance. Darker shades are built-up areas [8]. Latitude/longitude are degrees. The blue and the black lines indicate the ground tracks of solution 1 and solution 3/4 obtained by AG.
Table 5.9: Comparison of objectives of the representative cases.

<table>
<thead>
<tr>
<th>Solution group No.</th>
<th>Optimiser</th>
<th>EPNL [dB]</th>
<th>CO₂ [kg]</th>
<th>NOx [kg]</th>
<th>Awakenings V(x)</th>
<th>Cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PE</td>
<td>78.85</td>
<td>905.41</td>
<td>3.66</td>
<td>127.93</td>
<td>0.3589</td>
</tr>
<tr>
<td>1</td>
<td>AG</td>
<td>75.30</td>
<td>858.38</td>
<td>3.41</td>
<td>137.48</td>
<td>0.2346</td>
</tr>
<tr>
<td>1</td>
<td>GA</td>
<td>77.50</td>
<td>850.98</td>
<td>3.24</td>
<td>136.76</td>
<td>0.2413</td>
</tr>
<tr>
<td>2</td>
<td>PE</td>
<td>87.75</td>
<td>807.56</td>
<td>2.97</td>
<td>165.29</td>
<td>0.2986</td>
</tr>
<tr>
<td>2</td>
<td>AG</td>
<td>90.38</td>
<td>797.54</td>
<td>3.78</td>
<td>183.27</td>
<td>0.3188</td>
</tr>
<tr>
<td>2</td>
<td>GA</td>
<td>88.05</td>
<td>804.25</td>
<td>3.04</td>
<td>162.72</td>
<td>0.3015</td>
</tr>
<tr>
<td>3</td>
<td>PE</td>
<td>80.62</td>
<td>832.65</td>
<td>3.02</td>
<td>138.54</td>
<td>0.2237</td>
</tr>
<tr>
<td>3</td>
<td>AG</td>
<td>79.78</td>
<td>824.69</td>
<td>2.97</td>
<td>141.97</td>
<td>0.1763</td>
</tr>
<tr>
<td>3</td>
<td>GA</td>
<td>78.82</td>
<td>840.35</td>
<td>3.15</td>
<td>138.20</td>
<td>0.2288</td>
</tr>
<tr>
<td>4</td>
<td>PE</td>
<td>80.62</td>
<td>832.65</td>
<td>3.02</td>
<td>138.54</td>
<td>0.2237</td>
</tr>
<tr>
<td>4</td>
<td>AG</td>
<td>79.78</td>
<td>824.69</td>
<td>2.97</td>
<td>141.97</td>
<td>0.1763</td>
</tr>
<tr>
<td>4</td>
<td>GA</td>
<td>79.20</td>
<td>838.35</td>
<td>3.16</td>
<td>139.51</td>
<td>0.2216</td>
</tr>
<tr>
<td>5</td>
<td>PE</td>
<td>78.85</td>
<td>905.41</td>
<td>3.66</td>
<td>127.93</td>
<td>0.3589</td>
</tr>
<tr>
<td>5</td>
<td>AG</td>
<td>75.30</td>
<td>858.38</td>
<td>3.41</td>
<td>137.48</td>
<td>0.2346</td>
</tr>
<tr>
<td>5</td>
<td>GA</td>
<td>77.50</td>
<td>850.98</td>
<td>3.24</td>
<td>136.76</td>
<td>0.2413</td>
</tr>
</tbody>
</table>

Figure 5.16: Noise footprint of EPNL difference between two optimised trajectories. The blue and the black lines indicate the ground tracks of Solution 1 and Solution 3/4 obtained by AG.
Figure 5.17: Optimal airspeed and altitude profiles obtained by PE.
Figure 5.18: Optimal airspeed and altitude profiles obtained by AG.
Figure 5.19: Optimal airspeed and altitude profiles obtained by GA.
CHAPTER 5. DEPARTURE TRAJECTORY OPTIMISATION

5.4 Concluding remarks

This chapter presented an optimisation method using non-gradient algorithms for departure trajectory of commercial aircraft, aiming at minimising multiple environmental impacts on local communities around the airport. A parameterisation method is conducted to discretise the dynamics models on both the vertical and lateral planes. Two different lateral parameterisation methods are applied and have been verified their functionality. Two posterior selection methods are introduced and built to identify the optimal solution for the multi-objective problem. The proposed methodology is tested in the departure scenario with multiple environmental objectives considered.

In the case study, a departure trajectory of a commercial aircraft is optimised minimising multiple objectives. The comparison between three non-gradient methods indicates that multi-objective PSO has the inherent advantage in fast convergence yet is not as good as GA in achieving a smooth and well distributed Pareto solution set. When solving this kind of multi-objective optimisation problem, it turns out to be wise to apply more than one method.

As for the posterior selection methods, efforts have been made to monetise the concerned environmental impacts yet it has turned out that the result depends on the unit pricing. Unlike carbon and nitrogen oxides emissions, monetising impacts of aviation noise can be controversial where there still exists debate around the suitability of effectiveness of the monetisation method established. Compared with monetisation, it has been found that the preference value function method is more applicable with DM’s knowledge introduced to guide the decision-making process. Regarding the drawback of the monetisation method, there is a need for other authoritative and sophisticated methodologies to monetise aviation noise impact on the basis of flight operations.
Chapter 6

Arrival Trajectory Optimisation

Unlike departure, where the main noise comes from the engine, noise from arriving aircraft has a large proportion from the airframe that includes the noise sources such as trailing edge flaps, leading edge slats and flaps, nacelles and intake air spillage and deployed the landing gear. As a result, the benefit delivered by the breakthrough in reducing departure noise has a smaller impact in lowering arriving noise.

Currently, low-drag/low-power, as well as Continuous Descent Approach (CDA), are the techniques designed to reduce the noise level for landing aircraft. The first approach is aimed at producing minimum noise on arrival by deploying the flaps and landing gear as late as possible. In this case, less drag will be generated so that the thrust required to balance the drag will be reduced accordingly. Thus, the noise level is lowered at the same time with the exposure time shortened as well.

As for CDA, a widely-used technique in most airports is designed in contrast with traditional arrive-in-steps. Conventionally, aircraft follow a series of steps to land towards an airport, in which there need to be a few noisy bursts of the engine to level out. With CDA, pilots receive accurate instructions from air traffic control on the distance to touchdown, which allows for the decision of the best possible descent procedure. This method leads to more flexibility in manoeuvre and the aircraft can stay as long as possible at a higher altitude, therefore, reduce the needs of burst engine to keep the aircraft level.

According to the noise complaints history at Manchester airport, Knutsford, which lies within the 55 $L_{den}$ contour, is the area most disturbed by landing aircraft when the runway is used in an easterly direction [3]. In this case, aircraft fly over Knutsford. Therefore, Knutsford and its neighbouring communities surrounding Manchester airport is the objective for the study on the arrival noise abatement.

One solution to reduce noise is to find the optimal sequence of the deployment of flaps and landing gear. Another solution is to apply improved navigational performance (P-RNAV) on arrival, both of which have potential to be achieved by optimisation on arrival procedures. In this chapter, the efforts are made first to explore the
Table 6.1: Noise complaints history in 2016 [3].

<table>
<thead>
<tr>
<th>Area</th>
<th>Number of complainants/complaints in 2010</th>
<th>Number of complainants/complaints in 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobberley</td>
<td>12/47</td>
<td>8/10</td>
</tr>
<tr>
<td>Knutsford</td>
<td>78/208</td>
<td>45/206</td>
</tr>
<tr>
<td>Plumley &amp; Tably</td>
<td>4/6</td>
<td>2/2</td>
</tr>
<tr>
<td>Lostock</td>
<td>1/2</td>
<td>3/3</td>
</tr>
<tr>
<td>Davenport &amp; Northwich</td>
<td>9/11</td>
<td>11/12</td>
</tr>
</tbody>
</table>

possibility of decreasing landing noise level by optimising the flight path angle, then to find out the possible improvement on navigational performance (RNAV) by ground track planning before entering the Instrument Landing System (ILS).

### 6.1 Arriving Aircraft Flight Profiles

By analysing the approach procedure, the differential equation of approaching aircraft is shown as:

\[
\begin{align*}
\dot{V} &= \frac{F_N - mg\sin\gamma - D}{m} \\
\dot{\gamma} &= \frac{L\cos\phi - mg\cos\gamma}{mV} \\
\dot{\chi} &= \frac{L\sin\phi}{mV\cos\gamma} \\
\dot{x} &= V\cos\gamma\sin\chi \\
\dot{y} &= V\cos\gamma\cos\chi \\
\dot{h} &= V\sin\gamma \\
\dot{m} &= -f
\end{align*}
\]

(6.1)

with path constraints as:

\[
\begin{align*}
\dot{V} &\leq 0 \\
\gamma &\leq 0 \\
|\phi| &\leq \phi_{\text{max}}
\end{align*}
\]

(6.2) (6.3) (6.4)

and boundary constraints:

\[
x(t_f) = x_f, \quad y(t_f) = y_f, \quad h(h_f) = h_f
\]

(6.5)

There are two cases for the arrival:
1. The height and speed profile is identified, the unknown variable is the engine thrust.

2. The aircraft is set at idle thrust for approach, it is the final altitude that needs to be determined.

For the first case, given the initial and final altitudes, initial and final airspeeds (or level segment distances), the deceleration can be calculated as:

\[
\dot{V} = \frac{V_2^2 - V_1^2}{2(S_2 - S_1)/ \cos \gamma} = \frac{V_2^2 - V_1^2}{2(h_2 - h_1)/ \sin \gamma}
\]  

(6.6)

Then the engine thrust level can be estimated as:

\[
F_N = m g \sin \gamma + D + m \dot{V}
\]

(6.7)

For the second case, according to the flight equations described in Eq.(6.1), the acceleration of the aircraft along the direction of flight is given as:

\[
\dot{V} = \frac{F_N - m g \sin \gamma - D}{m}
\]

(6.8)

For any segment with a descent angle (or level segment distance), if the acceleration of the aircraft is known, the distance traveled along the flight path, as well as the descent altitude can be derived given a change in velocity:

\[
S_2 = \frac{(V_2^2 - V_1^2) \cos \gamma}{2 \dot{V}} + S_1
\]

\[
h_2 = \frac{(V_2^2 - V_1^2) \sin \gamma}{2 \dot{V}} + h_1
\]

(6.9)

where \(S_1\) and \(S_2\) are the initial and final ground track distances respectively, \(h_1\) and \(h_2\) are the initial and final altitudes respectively.

A 2D segmented arrival procedure is built by using the default data from the ANP database [4] of A320-211. The default approach procedural steps are displayed in Table 6.2. The procedure starts from the 6,000 feet AFL and ends on the runway ground with four descent-idle segments, two level-idle segments, two descent segments, one landing and two decelerations on the ground. The certificate point is 2,300 m away from the touchdown point where a microphone is located to record the approach noise level. The results show that the EPNL equals 96.62 dB which is 0.52 dB higher than the certification noise level (i.e. 96.10 dB). The error is 0.54 %. Therefore, this model is acceptable from the perspective of optimisation. The vertical profiles are shown below with EPNL contour presented in Figure 6.5. For the 3D arrival, details are provided in Section 6.3.
Table 6.2: Default approach procedural steps of A320-211 [4].

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Descend-Idle</td>
<td></td>
<td>6000</td>
<td>250.0</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Level-Idle</td>
<td></td>
<td>3000</td>
<td>250.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Level-Idle</td>
<td></td>
<td>3000</td>
<td>201.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Descend-Idle</td>
<td></td>
<td>3000</td>
<td>182.2</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Descend-Idle</td>
<td></td>
<td>2614</td>
<td>173.7</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Descend-Idle</td>
<td></td>
<td>1942</td>
<td>141.0</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Descend FULL</td>
<td></td>
<td>1823</td>
<td>132.6</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Descend FULL</td>
<td></td>
<td>50</td>
<td>132.6</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Land FULL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>303.5</td>
<td>40.0</td>
</tr>
<tr>
<td>10</td>
<td>Decelerate</td>
<td></td>
<td></td>
<td>129.6</td>
<td></td>
<td></td>
<td>2732</td>
<td>10.0</td>
</tr>
<tr>
<td>11</td>
<td>Decelerate</td>
<td></td>
<td></td>
<td>30.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.1: Altitude profile.  
Figure 6.2: True Airspeed profile.  
Figure 6.3: Corrected net thrust per engine profile.  
Figure 6.4: Flight path angle profile.
CHAPTER 6. ARRIVAL TRAJECTORY OPTIMISATION

6.2 Final Approach

The second segment is a constant CAS (calibrated airspeed) descent segment plus landing. This segment can be defined with free parameters including initial altitude $h_0$ and descent angle $\gamma$. The first parameter, initial altitude, can be decided by the final flight state of the previous segment. The second parameter can be derived from the remained ground distance from the expected touch down point. Then the only output becomes the engine thrust which can be derived from the force balance equation.

The alternative way to design the second segment is not to fly a constant flight path angle, though this might add the complexity to the problem. Since the flight path angle is small, in order to reduce the order of the optimisation problem, it is assumed in each time step $\dot{\gamma} = 0$, which leads to $L = mg \cos \gamma$. It is noticed that in the final descent segment, there is no lateral manoeuvring, thus $\phi = 0^\circ$. Then this problem is simplified into a 2D flight segment with the dynamics equation shown as

\[
\begin{align*}
\dot{V}_T &= \frac{F_N - mg \sin \gamma - D}{m} \\
\dot{s} &= V_T \cos \gamma \\
\dot{h} &= V_T \sin \gamma
\end{align*}
\]  

(6.10)

From the equation above, we can define the state vector $\mathbf{x} = [V_T, s, h]^T$, and control vector $\mathbf{u} = [\gamma, F_N]^T$ :

The true airspeed $V_T$ can be expressed by the CAS $V_C$:

\[
V_T = \frac{V_C}{\sqrt{\sigma}}
\]  

(6.11)

where $\sigma$ is the air density ratio at the aircraft altitude by the Ideal Gas Law:

\[
\sigma = \frac{\delta}{\theta}
\]  

(6.12)
where $\theta$ is the absolute ISA temperature ratio \cite{47}.

$$\theta = \frac{273.15 + T - 0.0065 \cdot (h - E)}{288.15} \quad (6.13)$$

where $T$ is the airport temperature in [$^\circ$C], $E$ is the airport elevation in [m] MSL. The pressure ratio $\delta$ at the aircraft altitude is \cite{47}:

$$\delta = \left[ \left( \frac{P}{101.325} \right)^{\frac{1}{5.256}} - \frac{0.0065h}{288.15} \right]^{5.256} \quad (6.14)$$

where $P$ is the pressure at the airport in [kPa]. Then the air density ratio is a parameter subject to the aircraft altitude if the temperature, elevation and pressure of the airport are given.

Since the calibrated airspeed $V_C$ keeps constant in the final approach, the acceleration of true airspeed can be derived as:

$$\dot{V}_T = -\frac{1}{2} V_C \cdot \sigma^{-3/2} \cdot \frac{d\sigma}{dh} \cdot \dot{h}$$
$$= -\frac{1}{2} V_C \cdot \sigma^{-3/2} \cdot \frac{d\sigma}{dh} \cdot V_T \sin \gamma$$
$$= -\frac{1}{2} V_C^2 \cdot \sigma^{-2} \cdot \frac{d\sigma}{dh} \sin \gamma \quad (6.15)$$

Thus the thrust is obtained:

$$F_N = m\dot{V}_T + mg \sin \gamma + D$$
$$= m\dot{V}_T + mg \sin \gamma + R_f \cdot mg \cos \gamma \quad (6.16)$$

where $R_f$ is the drag-over-lift coefficient that depends on the flaps setting. Therefore, the dynamics equation of the final approach segment can be simplified as:

$$\begin{cases}
\dot{s} = V_C \cos \gamma \\
\dot{h} = \frac{V_C \sin \gamma}{\sqrt{\sigma}}
\end{cases} \quad (6.17)$$

where the state variables are $x = [s, h]^T$, $s$ is the range of the ground track; the control variable is the fight path angle $u = \gamma$. Thus the coordinate of the lateral track can be calculated as:

$$x = s \cdot \cos \gamma \sin \chi, \quad y = s \cdot \cos \gamma \cos \chi \quad (6.18)$$

In this case, the start and final point are designated with fixed coordinates and airspeed, which makes it a Two-Point-Boundary-Value (TPBV) problem.
6.2.1 Constraints

Apart from the boundary constraints defined in Eq. (6.5), there are path constraints to describe the descending process:

$$\gamma \leq 0^\circ, \quad F_N \geq 0$$ (6.19)

According to the simplification discussed early this section, the constraint on thrust can be derived as:

$$F_N = mg(k_1 \sin \gamma + k_2 \cos \gamma) = mg\sqrt{k_1^2 + k_2^2 \sin(\gamma + \varphi)} \geq 0$$ (6.20)

where:

$$k_1 = 1 - \frac{V^2 C_d \sigma}{2 \sigma^2 g}, \quad k_2 = R_f, \quad \varphi = \tan^{-1}\left(\frac{k_2}{k_1}\right)$$ (6.21)

which leads to the constraint on the flight path angle $$\gamma$$:

$$\gamma \geq -\varphi$$ (6.22)

Therefore, the value range of $$\gamma$$ can be written as $$\gamma \in [\gamma_{\text{min}}, \gamma_{\text{max}}]$$, where $$\gamma_{\text{min}} = -\varphi$$ and $$\gamma_{\text{max}} = 0^\circ$$.

6.2.2 Cost function

In the final approach, an aircraft is guided by the ILS which follows a straight ground path most of the time inevitably flying over the residential communities, which leads to the priority of aircraft noise impact control on that area. In order to reflect this concern, an objective function considering noise impact as well as the boundary constraints is designed as:

$$J = \sum_{k=1}^{N} P_{\text{awakening},k} \cdot \text{population}_k + K_h |h(t_f) - h_f| + K_s |s(t_f) - s_f|$$ (6.23)

where $$K_v$$ and $$K_s$$ are the penalty coefficients, $$P_{\text{awakening},k}$$ is the probability of an individual noise event awakening a person in the $$k$$th area, $$\text{population}_k$$ is the population in that area. $$P_{\text{awakening},k}$$ is expressed by the function of $$\text{SEL}_{\text{indoor}}$$ shown as in Eq. (6.24) [114]:

$$P_{\text{awakening},k} = \frac{1}{1 + e^{(-6.8884+0.04444\text{SEL}_{\text{indoor}})}}$$ (6.24)

where $$\text{SEL}_{\text{indoor}} = \text{SEL}_{\text{outdoor}} - 20.50\,\text{dB}.$$
6.2.3 Handling optimal control problem

With Eq. (6.17) and Eq. (6.23), the optimal control problem can be written as:

\[
\min_{x,u} \quad J = \sum_{k=1}^{N} P_{\text{awakening},k} \cdot \text{population}_k + K_h |h(t_f) - h_f| + K_s |s(t_f) - s_f|
\]

s.t. \quad \dot{x} = f(x, u, t) \quad \gamma_{\text{min}} \leq \gamma(t) \leq \gamma_{\text{max}}

(6.25)

To solve the optimisation problem described in Eq. (6.25), the control variable of time \(\gamma(t)\) is discretised in the time interval to piecewise constant functions \(\hat{\gamma}(t)\). The control variables then are defined as:

\[
\hat{\gamma}(t) = \gamma_j \quad \text{on} \quad [t_{j-1}, t_j], \; i = 0, 1, ..., N
\]

(6.26)

where \(\gamma_j\) is the constant flight path angle in the time sub-interval \([t_{j-1}, t_j]\), \(t_N = t_f\), \(N\) is the integer number of subintervals of the piecewise time interval. Since the control variable is given at each time node, then the problem is transferred into a piecewise initial value problem in each sub-time interval. Thus the free parameters to define this problem include \(N\) controls at the time nodes and the final time \(t_f\):

\[
U = [\gamma_1, \gamma_2, ..., \gamma_N, t_f]^T
\]

(6.27)

By implementing this discretisation, the resultant problem can be solved with non-gradient based algorithms now.

6.2.4 Case study

A case study on the easterly arrival of Manchester Airport is conducted focusing on the final arrival which starts approximately 2,200 feet MSL, seven nautical miles away from the runway end 5L at which the origin point is located. The x-axis is set parallel to the runway 23R/5L and points to the end 23R, so the coordinate of the initial point is \(x_0 = -11,460 \text{ m}, h_0 = 2,200 \text{ feet above the ground}\). The configuration of aircraft in this segment is with full flaps deployed and landing gear down. The number of the time intervals is set to be \(N = 10\). The optimiser dealing with this single objective optimisation problem is Particle Swarm Optimizer (PSO) with swarm size set to be 50 and iteration number set into 40. The weight parameters are set to be \(K_h = K_s = 1\).

Optimal results are shown below with the deviation of position \(|s(t_f) - s_f| = 2.59 \text{ m}\) and the deviation of altitude \(|h(t_f) - h_f| = 7.3703 \times 10^{-4} \text{ m}\), which is an acceptable accuracy for this optimisation problem. The optimal solution of this problem is \(x = [-0.0500, -0.0067, -0.0509, -0.0537, -0.0757, -0.0855, -0.0711, -0.0440, -0.0383, -0.0807, 148.9892]\)\(^7\), where the first ten variables are the optimal flight path angle control variables, the last variable is the flight time required.
CHAPTER 6. ARRIVAL TRAJECTORY OPTIMISATION

Figure 6.6: Altitude profile.

Figure 6.7: True Airspeed profile.

Figure 6.8: Corrected net thrust per engine profile.

Figure 6.9: Flight path angle profile.

Figure 6.10: EPNL contour of the optimized arrival trajectory.
Table 6.3: Comparison of noise levels obtained between default and optimised procedure.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Default</td>
<td>87.37</td>
<td>96.62</td>
<td>3.68</td>
<td>150.11</td>
</tr>
<tr>
<td>Optimised</td>
<td>85.45</td>
<td>99.22</td>
<td>1.08</td>
<td>148.23</td>
</tr>
</tbody>
</table>

The microphone at Knutsford is located at East = −7307.64 m, North = 0 m which is marked with the first red empty dots in Figure 6.10 and 6.11. The right dot in both contours is indicating the location of certification point. The contour of the optimised trajectory shows that the EPNL received in the centre of Knutsford is almost located on the boundary of 85 [dB] contour line, while the EPNL of the defaulted final arrival procedure is inside the boundary of 85 [dB] contour line. From the comparison demonstrated on Table 6.3, the optimised final approach procedure obtains lower ENPL at Knutsford but produces higher noise level at the certification point, which proves the possibility of using vertical manoeuvres to reduce the local noise impact at the price of increased noise level towards areas closer to the airport.

With the optimised final arrival procedure, the population awakened in the Knutsford community is 148.23, which is about one person less than the population awakened by the default final approach procedure. This phenomenon presents the limitation of noise impact reduction by using trajectory optimisation for the NSAs just under the ILS flight path. Although the noise level received at a certain ground receiver can be reduced to some degree by increasing the vertical distance between the aircraft and that receiver on the ground, the effectiveness to decrease the total noise impact on the certain NSAs is limited for a segment of which initial and final points are fixed. Moreover, by increasing the vertical distance while flying over the NSA, the aircraft has to descent with a sharper descent angle subsequently. Then the noise level received at the certification point is not necessarily guaranteed to be reduced, just as is displayed in Table 6.3.
6.3 Initial Approach Segment

As for the initial approach segment, it starts from the initial approach fix (IAF) and ends at the intermediate fix (IF). According to the ICAO 8168 document, the aircraft changes its configuration and engine power to leave the en-route structure and to manoeuvring to enter the intermediate approach segment.

There are many categories of manoeuvres in the initial approach segment. Here we only discuss the example of omnidirectional arrival using a holding pattern. Under this circumstance, aircraft first leave the holding stack and then head for the ILS. The lateral manoeuvres include straight legs and base turns. The vertical procedures include descents and decelerations.

6.3.1 Horizontal track planning

On the horizontal plane, aircraft need to do several manoeuvres to lead its direction to enter the ILS. Although more complex lateral manoeuvres can be conducted in this segment, the first horizontal parameterisation method mentioned in Section 5.1.2, namely lateral parameterisation with straight legs and constant radius turns, is applied. The lateral track is divided into several subsegments to accomplish the approach.

6.3.2 Vertical procedure

Vertical procedures are relatively intuitive for the initial approach segment. As is given in Table 6.2, ICAO has published default approach procedural steps for different types of aircraft, including a series of descent, level fly and deceleration. It is easy to calculate the ground distance using the equations given in Section 6.1 when the problem is a 2D initial value problem. However, to apply the default approach procedural steps, an adjustment should be conducted to make it following different ground tracks. For different ground tracks, an additional step should be added before the aircraft operates following the default procedure. Here, a level flight step with constant speed is applied to meet the gap between the default ground distance and the real ground distance obtained after being preplanned with new lateral tracks. The details are explained and demonstrated in the Section 6.3.3.

6.3.3 Handling constraints

Presented in this section is the computational procedure of how to realise the coupling of motions in both vertical and horizontal planes. Techniques to deal with the boundary and path constraints considered in the initial approach segment are also introduced.

As a two-point boundary value (TPBV) problem with path constraints, it is not
easy to find the feasible solution. It is possible to apply the same method provided in
Section 6.2, namely using a penalty function to constrain the final position, to solve
the trajectory of this initial approach segment. However, compared with the final ap-
proach segment, extra complexities have been added due to the bank angle constraint
resulted from lateral manoeuvring. So efforts should be made to explore methods to
reduce the difficulties of convergence. Also, the discrete formulation of the considered
cost functions, namely noise levels or gaseous emissions, narrows down the range of
algorithm options available. Therefore, despite using non-gradient algorithms, to deal
with the constraint, the main idea is to convert the TPBV problem into an Initial
Value problem of which feasible solutions can then be approached with acceptable
accuracy by conducting iterative method.

Firstly, the 2D vertical profile of a default approach procedure published by ICAO
is applied to roughly estimate the length of the trajectory projected on the ground
$s_{2D}$. With the imported free parameters $\mathbf{P}_{\text{horizontal}} = [l_1, R_2, l_3]^T$, the target distance,
indicated by $s_{\text{plan}}$, from the holding stack to the final point is easy to be calculated.
Thus the missed segment, namely a constant airspeed level flight to fill the distance
gap can be obtained. In this case, by connecting the initial extra level flight segment
and the subsequent default approach procedure, a preliminary 2D trajectory without
path constraints and accuracy tested is obtained. Note that this vertical profile cannot
be coupled with the planned lateral track directly and the next step will provide the
solution of coupling the motions on two planes.

Next, an iteration process is undertaken to couple the lateral and vertical move-
ment. Based on the extra ground distance, a new vertical profile with the additional
level constant speed flight segment is calculated. Note that this time, since manoeuvres
within the lateral plane should be considered, Eq. (6.16) will be modified accordingly:

$$ F_N = m\dot{V}_T + mg\sin\gamma + R_f \cdot \frac{mg\cos\gamma}{\cos\phi} \quad (6.28) $$

where $\phi$ is the bank angle that reflecting the impacts of horizontal manoeuvre.

As soon as the calculation is finished, a comparison is conducted between the newly
obtained ground distance $s_{3D}$ and the planned ground track $s_{\text{plan}}$ got from the inputs.
The tolerance criterion:

$$ \epsilon_s = \left| \frac{s_{3D} - s_{\text{plan}}}{s_{\text{plan}}} \right| \times 100\% \leq \text{tolerance} \quad (6.29) $$

where $\epsilon_s$ is the error between the obtained distance and the planned distance, the
tolerance is a user-defined small amount. If the error is within the acceptable range,
then this 3D trajectory is regarded as feasible. If it is not, the missed segment should
be substituted by the difference between the newly obtained ground distance and the
planed lateral track, and the second iteration should be repeated until the tolerance
is satisfied.
Apart from the boundary constraint defined by the final point, there is a path constraint defined by the bank angle. According to the analysis in Eq. (5.12), the bank angle $\phi$:

$$\phi = \tan^{-1} \frac{V_T^2}{gR}$$  \hspace{1cm} (6.30)

is defined by the airspeed $V_T$ and the radius of the turn which describes the geometry of the lateral track. The path constraint $\phi_{\text{min}} \leq \phi \leq \phi_{\text{max}}$ can be written as:

$$\epsilon_\phi = \left| \frac{\phi - \phi_{\text{max}}}{\phi_{\text{max}}} \right| \times 100% \leq \text{tolerance}$$  \hspace{1cm} (6.31)

where $\phi_{\text{min}} = -\phi_{\text{max}}$, tolerance is a user-defined amount to control the accuracy. Then the ground track needs to be re-planned accordingly. Thus, within the iteration loop described above, a judgment about whether the constraint on bank angle is violated is also required. If the path constraint is not satisfied, then the planned lateral track need to be updated by changing the input free parameters:

$$P_{\text{horizontal}} = P_{\text{horizontal, lower}} + (P_{\text{horizontal, upper}} - P_{\text{horizontal, lower}}) \times \text{rand}(0, 1)$$  \hspace{1cm} (6.32)

where the subscripts lower and upper represent the lower and upper bounds of the free parameter vector $P_{\text{horizontal}}$ respectively, $\text{rand}(0, 1)$ indicates a random number belong to the range of $[0, 1]$. The computational flowchart is shown as Figure 6.12. The basic algorithm to handle these two constraints is displayed in Algorithm 2.

### 6.3.4 Case Study

In this case, a scenario that aircraft approach easterly from the holding stack DAYNE (53°14.19′N, 2°1.45′W) is considered. For the initial approach before entering the ILS, the endpoint is set to be at $-11,460$ m away from the runway end 05L (53°20.85′N, 2°17.27′W) on the extension line of runway 05L/23R. Five subsegments, shown as in Figure 6.13, are planned to lead the aircraft to enter the final ILS. In order to simplify the parameterisation and to reduce the number of free parameters, the second straight leg is assumed to be parallel to the last straight leg. Therefore, three instead of five free parameters are required to define a ground track displayed in Figure 6.13, following the parameterisation method introduced in Section 5.1.2. The horizontal free parameters and their bounds are listed in Table 6.4.

For the initial approach segment, the objective is to find the entry to the ILS as quickly and as environmentally friendly as possible. The first concern is about the flight duration which has a significant relationship to the fuel burn. The second concern consists of both noise indexes and gaseous emissions. However, since the source-receiver distance is long enough during the initial approach segment, especially for the vertical attitude, the noise factor does not have the same priority as the other
CHAPTER 6. ARRIVAL TRAJECTORY OPTIMISATION

Lateral track planning

Obtain the missed level flight segment

End

Yes

Yes

No

Input:

- Free parameters: $P_{\text{horizontal}}$
- Default approach procedure: $s_{2D}$

Update $P_{\text{horizontal}}$

Lateral track planning

Bank angle constraint violated?

No

Calculate ground distance of the 3D trajectory

Re-calculate thrust and bank angle with Eq.6.28 and Eq.6.30

Tolerance satisfied?

Yes

End

No

Yes

Figure 6.12: Flowchart of handling constraints.
Algorithm 2 Handling constraints

Input:
Free parameters to define the ground track: $P_{\text{horizontal}} = [l_1, R_2, l_3]^T$
Ground distance from ICAO default approach procedure: $s_{2D}$

Output:
3D arrival trajectory satisfying boundary and path constraints

1: Initialisation: $t=0$, $\epsilon_s > \text{tolerance}$, $\epsilon_\phi > \text{tolerance}$;
2: while $\epsilon_s > \text{tolerance}$, or $\epsilon_\phi > \text{tolerance}$ do
3: \hspace{0.5cm} $t = t+1$;
4: \hspace{0.5cm} while $\epsilon_\phi > \text{tolerance}$ do
5: \hspace{1cm} Calculate the distance of the ground track with $P_{\text{horizontal}}$: $s_{\text{plan}}$;
6: \hspace{1cm} if $t=1$ then
7: \hspace{1.5cm} Let $\epsilon_\phi \leq \text{tolerance}$;
8: \hspace{1.5cm} Estimate the distance of the missed level flight: $s_{\text{gap}} = s_{\text{plan}} - s_{2D}$;
9: \hspace{1cm} else
10: \hspace{1.5cm} Update $P_{\text{horizontal}}$ and $s_{\text{plan}}$;
11: \hspace{1.5cm} Update the distance of the missed level flight: $s_{\text{gap}} = s_{\text{plan}} - s_{3D}$;
12: \hspace{1.5cm} Update $\phi$ and $\epsilon_\phi$;
13: \hspace{1cm} end if
14: \hspace{0.5cm} end while
15: \hspace{0.5cm} Calculate the 3D arrival procedure with $s_{\text{gap}}$, obtain $s_{3D}$, $\phi$ and $F_N$;
16: \hspace{0.5cm} Update $\epsilon_\phi$ and $\epsilon_\phi$;
17: end while

Table 6.4: Free parameters of the ground track.

<table>
<thead>
<tr>
<th>Segment No.</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$l_1 \in [1000 \text{ m}, 10000 \text{ m}]$</td>
</tr>
<tr>
<td>2</td>
<td>$R_2 \in [6000 \text{ m}, 10000 \text{ m}]$</td>
</tr>
<tr>
<td>3</td>
<td>$l_3 \in [10000 \text{ m}, 20000 \text{ m}]$</td>
</tr>
<tr>
<td>4</td>
<td>none</td>
</tr>
<tr>
<td>5</td>
<td>none</td>
</tr>
</tbody>
</table>

concerns have. Therefore, the cost function in this segment is set to be:

$$J = t_f + K_s \cdot \Delta s \quad (6.33)$$

where $t_f$ is the overall duration of the initial approach segment, $K_s$ is the penalty coefficient. This parameter plays an important role in adjusting the direction of optimisation with the magnitude of the difference between $t_f$ and $\Delta s$:

$$K_s = \begin{cases} 
1 & \Delta s \leq 10 \text{ m} \\
100 & \Delta s > 10 \text{ m} 
\end{cases} \quad (6.34)$$

and $\Delta s$ is the penalty function in the form of:

$$\Delta s = \sqrt{|x(t_f) - x_f|^2 + |y(t_f) - y_f|^2 + |h(t_f) - h_f|^2} \quad (6.35)$$
The optimiser adopted in this case study is PSO. The swarm size is set to be 100, and the maximum iteration number is 100. The algorithm stops after the maximum iteration number of evaluation of the objective function. A baseline result is given to compare the performance of a random trajectory and the optimal trajectories obtained by PSO. The optimisation algorithm randomly generates the initial population of each optimisation. The comparison between the performance of a randomly picked trajectory and the optimal solution is presented in Table 6.5. The optimal solution is $l_1 = 1000 \text{ m}, R_2 = 6000 \text{ m}, l_3 = 1.0481 \times 10^4 \text{ m}$.

Table 6.5 presents the comparison between one randomly selected trajectory and the optimal solution obtained by the PSO. As is displayed in Table 6.5, the optimal solution achieves the smallest value of $J$. Although the optimal solution has a bigger deviation of the final position of the aircraft, namely 5.3684 m, this is reasonably acceptable for a trajectory optimisation problem. Note that the total ground distance of the trajectory determines the value of flight time, fuel consumed, gaseous emission exhausted. Compared with the baseline trajectory, the quickest way to find the entry to ILS takes 364.60 s from the DAYNE holding stack, which is 5.50% faster than the previous one. The comparison of the trajectories is displayed in Figure. 6.14-6.18 to provide more details. Note that the shape of the profiles demonstrated in the Figure 6.15-6.18 between the baseline and the optimal are similar to each other. This is because they conduct the same default approach procedural steps. The main difference comes from ground distances.

By connecting the optimised initial approach segment and the final approach segment, a complete arrival procedure is obtained with the segmented optimisation
Table 6.5: Comparison between the baseline result and the optimal result.

<table>
<thead>
<tr>
<th>Case</th>
<th>Time [s]</th>
<th>Ground distance [m]</th>
<th>Fuel [kg]</th>
<th>CO$_2$ [kg]</th>
<th>NOx [kg]</th>
<th>$\Delta s$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>385.83</td>
<td>$4.6837 \times 10^4$</td>
<td>54.13</td>
<td>170.50</td>
<td>1.96</td>
<td>2.60</td>
</tr>
<tr>
<td>Optimal</td>
<td>364.60</td>
<td>$4.3615 \times 10^4$</td>
<td>51.18</td>
<td>161.18</td>
<td>1.85</td>
<td>5.37</td>
</tr>
</tbody>
</table>

Figure 6.14: Comparison of ground track.

method by using PSO. The EPNL contour of the optimal trajectory exerts is demonstrated in Figure 6.19. The ground tracks of the initial approach and final approach are differentiated by their line styles and colours. The markers indicate the microphones located in Knutsford and the certification point of arrival procedure. More details are demonstrated in Figure 6.20, which has shown that the 80 dB EPNL contour extends over 10.05 km to the southwest of the airport, including the certification point and Knutsford community. Results have shown that, compared with the default approach procedure steps depicted in Table 6.2, the optimised trajectory produces lower noise level towards the target area at the same time achieves the quickest way to entering the ILS. Further analysis presents that the difference between the noise level received at Knutsford and the certification point produced by the final approach and the whole optimal segment are 0.0963 % and 0.0145 % respectively. With such small difference on noise level received on the NSAs near the airport, if the concern is focused more on reducing the noise level, then efforts should be put more on the optimisation towards the final approach segment rather than the whole arrival procedure.
Figure 6.15: Altitude profile.

Figure 6.16: True Airspeed profile.

Figure 6.17: Corrected net thrust per engine profile.

Figure 6.18: Flight path angle profile.

Table 6.6: Comparison of noise levels obtained between default, optimal final approach and the optimal whole arrival.

<table>
<thead>
<tr>
<th>Case</th>
<th>EPNL_{Knutsford} [dB]</th>
<th>EPNL_{certification} [dB]</th>
<th>Margin [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default</td>
<td>87.37</td>
<td>96.62</td>
<td>3.68</td>
</tr>
<tr>
<td>Final Approach</td>
<td>85.45</td>
<td>99.22</td>
<td>1.08</td>
</tr>
<tr>
<td>Optimal Arrival</td>
<td>85.53</td>
<td>99.23</td>
<td>1.07</td>
</tr>
</tbody>
</table>
Figure 6.19: EPNL contour of the optimal approach trajectory.

Figure 6.20: Partial enlarged view of the EPNL contour.
6.4 Concluding remarks

It is evident that area close to the airport would be affected by the noise from arrival aircraft most. In order to reduce the noise impacts on the residential communities very close to the airport, a segmented optimisation method is applied in this chapter. For the initial approach segment, the aim is to find the entry to the ILS as soon as possible, so the objective is to minimise the flight duration. For the final approach segment, the objective is to reduce the noise impact on the specific area surrounding the airport. The objectives and profile parameter are constructed into functions of the introduced free parameters; therefore, PSO is used as the optimiser to generate optimal solutions to the parameter optimisation problem. A numerical case is provided in this chapter for the purpose of comparison. Results have shown that the segmented optimisation method can reduce the noise impact on a certain area (i.e. Knutsford in this case), and can also find the quickest way to enter the ILS, which proves to be an effective method.
Chapter 7

Complex Flight Scenario
Optimisation

7.1 Aircraft classification database

A reduced aircraft classification database is built to provide reliable noise and emission indices of representative aircraft types, giving database for the study and optimisation of multiple operations in the complex flight scenario.

7.1.1 Noise and emissions

Based on the representative-in-class approach proposed by Antonio J. Torija and Rod H. Self [146], the UK commercial aircraft fleet is grouped into four aircraft categories according to their physical characteristics, noise and engine emissions, within each group a representative aircraft is identified. The aircraft as well as their classification criteria are given in Table 7.1. Table 7.2 provides part of the performance metrics of noise and exhaust emissions.

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Engine Number</th>
<th>Weight</th>
<th>Representative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regional aircraft</td>
<td>2</td>
<td>DW: 32.5 t</td>
<td>CRJ-900</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LW: 28.9 t</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Short-Medium haul</td>
<td>2</td>
<td>DW: 71.4 t</td>
<td>A321-232</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LW: 62.9 t</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Long haul 1</td>
<td>4</td>
<td>DW: 384.5 t</td>
<td>747-8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LW: 304.8 t</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Long haul 2</td>
<td>2</td>
<td>DW: 176.0 t</td>
<td>A330-343</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>LW: 152.6 t</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1: Representative aircraft for each category.
Table 7.2: Aircraft model database of representatives (part) [4].

<table>
<thead>
<tr>
<th>Type</th>
<th>Weight [t]</th>
<th>Engine Type</th>
<th>Manufacturer</th>
<th>Fuel Flow [kg/s]</th>
<th>NOx [g/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>App*</td>
<td>Idle</td>
</tr>
<tr>
<td>CRJ-900</td>
<td>30.62</td>
<td>CF34-8C5</td>
<td>Bombardier</td>
<td>0.179</td>
<td>0.064</td>
</tr>
<tr>
<td>A321-232</td>
<td>71.12</td>
<td>V2530-A5</td>
<td>Airbus</td>
<td>0.377</td>
<td>0.138</td>
</tr>
<tr>
<td>B747-8</td>
<td>304.41</td>
<td>GEneX-2B67B</td>
<td>Boeing</td>
<td>0.701</td>
<td>0.216</td>
</tr>
<tr>
<td>A330-343</td>
<td>167.47</td>
<td>Trent772B-60</td>
<td>Airbus</td>
<td>0.850</td>
<td>0.280</td>
</tr>
</tbody>
</table>

* “App” denotes the condition in approach procedures.

Table 7.3: Arrival performance metrics of representatives.

<table>
<thead>
<tr>
<th>Type</th>
<th>Range [m]</th>
<th>EPNL [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Certification point</td>
</tr>
<tr>
<td>CRJ-900</td>
<td>3.4898×10^4</td>
<td>93.20</td>
</tr>
<tr>
<td>A321-232</td>
<td>4.5528×10^4</td>
<td>95.60</td>
</tr>
<tr>
<td>B747-8</td>
<td>5.2028×10^4</td>
<td>100.40</td>
</tr>
<tr>
<td>A330-343</td>
<td>5.4402×10^4</td>
<td>97.00</td>
</tr>
</tbody>
</table>

The default segmented arrival procedure of each of the four aircraft is simulated by using the flight performance model from the ANP database [4] and the noise model described in ECAC.CEAC Doc 29 [52]. Note that though models of higher accuracy levels such as FLIGHT [36] are available, for a multi-operational flight scenario optimisation problem, a simple and reliable model is preferred to alleviate the computational burden.

The concerned flight performance indices including the arrival range, the EPNL at the certification point as well as the noise level at Knutsford are shown in Table 7.3. Note that the absolute errors of the EPNLs received by the microphones located at the certification point (i.e. 2,300 m away from the touchdown point) for the four selected are all below 5%, which shows an acceptable accuracy for the following simulation and optimisation. The EPNLs at Knutsford are got by calculating the noise level at the location −7307.64 m away from the touchdown point, and this distance is equal to the distance from the centre of Knutsford community to the runway end 05L. Figure 7.1 provides the arrival noise contours of these four aircraft, where the position of certification point is marked with a red circle, and the equivalent location of Knutsford is designated with a red square in each graph.
Figure 7.1: EPNL contours of four representatives.
### 7.1.2 Time-based separation rules

Apart from being assigned to given routes and planned operational fleet, individual flight still needs to be scheduled in a time sequence such that the resulting fleet assignment is feasible. One factor that cannot be neglected is whether the separation time between the leading aircraft and the following aircraft is sufficient to get avoid of the potential harm from wake turbulence. In order to plan the approaching fleet with multiple flights in a safe and reasonable way, a time-based separation rule is introduced. The minimum separation time is decided by the ability of one aircraft to resist or generate the wake turbulence. The International Civil Aviation Organization (ICAO) has made separation rules to the distance between every two types of consecutive aircraft differentiated by their maximum takeoff weight (MTOW)\(^1\). This distance-based separation rule is used as a reference to design the time-based separation rule for the four categorized aircraft included in the database. Moreover, a linear programming problem is constructed to solve the optimal separation time based on the ICAO’s wake vortex separation rules. The detailed derivation process is presented in Appendix A. Table 7.4 gives the resultant result.

### 7.2 Multi-operational modeling

In the following subsections, the mathematic model to describe the multiple flight operations of different aircraft is built. Firstly, activities of the aircraft from the same type over different ground tracks are identified by a Flight Combination Matrix consist of all the possible partitions. Then by selecting one combination of each kind, the operations of the whole fleet in the complex scenario is expressed by a Multi-operation Index Matrix whose possible composition and number are certain.

#### 7.2.1 Flight combination matrix

Consider a complex arrival or departure scenario: there are $M$ flight paths which would be followed by $N$ types of aircraft. It is assumed that within each type there are $n_j$, $j = 1, 2, ..., N$ aircraft whose configurations and performance metrics are the
same. Each aircraft must follow a planned ground track to land. Then for the \( k \)th aircraft from a random type \( j \), the environmental impacts that it produces are entirely from the different choice of its flight path.

For any aircraft of the same type \( j \), it has \( M \) different options to select its flight path. Since the aircraft from the same type are unlabeled, yet the flight paths or ground tracks are labelled, it can be regarded as the classical \( n \)-multicombinations of \( m \) things problem in the famous the Twelvefold Way discussed by RP Stanley [148]. According to the combinatorics theory, the number of different flight path combinations of the type \( j \) can be derived as:

\[
F_j = \binom{n_j + M - 1}{M - 1} = \frac{(n_j + M - 1)!}{(M - 1)!n_j!} \tag{7.1}
\]

where \( F_j \) is the total number of flight path combinations of type \( j \), \( j = 1, 2, \ldots N \). Then by using lexicographic order and ensuring that the average amount of computation per visit is bounded [149], the partitions of \( n_j \) aircraft could be generated. The combination matrix \( C^j \) can be expressed as:

\[
C^j = \begin{bmatrix}
c_{11}^j & \cdots & c_{1k}^j & \cdots & c_{1F_j}^j \\
c_{21}^j & \cdots & c_{2k}^j & \cdots & c_{2F_j}^j \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
c_{M1}^j & \cdots & c_{Mk}^j & \cdots & c_{MF_j}^j
\end{bmatrix}_{M \times F_j}
\]

where

\[
C_k^j = [c_{1k}^j, c_{2k}^j, \ldots, c_{ik}^j, \ldots, c_{Mk}^j]^T, \quad \sum_{i=1}^{M} c_{ik}^j = n_j \tag{7.3}
\]

is the \( k \)th vector of the combination matrix \( C^j \), indicating the \( k \)th partition of all the available partition forms with the total number of \( F_j \), \( k = 1, 2, \ldots, F_j \). \( c_{ik}^j \in [0, n_j] \) is an integer standing for the number of aircraft that follow the flight path \( i(i = 1, 2, \ldots, M) \) in the \( k \)th combination.

If we express the combinations matrix of type \( j \) into a collection of partitions, then the combinations set can be written as:

\[
C^j = \{C_1^j, \ldots, C_k^j, \ldots, C_{F_j}^j\} \tag{7.4}
\]

Note here \( C^j \) denotes the set of partitions in the type \( j \).

### 7.2.2 Multi-operation index matrix

After generating the partitions of each aircraft type, the complex flight operations of the fleet over multiple paths can be constructed mathematically. The whole combinations of each aircraft type can be expressed by a new set obtained by associating every
element of each set with every element of the other sets, which leads to the Cartesian product of $N$ sets. The Cartesian product of two types is [150]:

$$C^1 \times C^2 = \{C^1_1, C^2_1, \cdots, C^1_{F_1}\} \times \{C^2_1, C^2_2, \cdots, C^2_{F_2}\}$$

(7.5)

where $k_j \in [0, F_j], j = 1, 2, \ldots, N$ is an integer. The elements of that set are ordered pairs. In each ordered pair $(C^1_{k_1}, C^2_{k_2})$, the first component is an element of $C^1$, and the second component is an element of $C^2$, which represents a possible combination of these two types over $M$ ground tracks. Similarly, the Cartesian product of $N$ sets is:

$$C^1 \times C^2 \times \cdots \times C^j \times \cdots \times C^N$$

$$=\{C^1_1, C^2_1, \cdots, C^1_{F_1}\} \times \{C^2_1, C^2_2, \cdots, C^2_{F_2}\} \times \{C^j_1, \cdots, C^j_{k_j}, \cdots, C^j_{F_j}\} \times \cdots \times \{C^N_1, C^N_2, \cdots, C^N_{F_N}\}$$

(7.6)

$$=\{(C^1_{k_1}, C^2_{k_2}, \cdots, C^N_{k_N}) : C^1_{k_1} \in C^1 \text{ and } C^2_{k_2} \in C^2 \cdots \text{ and } C^N_{k_N} \in C^N\}$$

Thus the number of pairs in the new set is derived as:

$$T = \prod_{j=1}^{N} F_j = \prod_{j=1}^{N} \left( n_j + M - 1 \right).$$

(7.7)

To demonstrate the combinations in detail, the generic format of the elements displayed in Eq.(7.6) can be re-written into an operational matrix $O$:

$$O = \begin{bmatrix} C^1_{k_1}, C^2_{k_2}, \cdots, C^j_{k_j}, \cdots, C^N_{k_N} \end{bmatrix}$$

(7.8)

where $C^j_{k_j}$ is a $M \times 1$ column vector defined in Eq.(7.3), and $O$ is a $M \times N$ matrix. To reduce the complexity to describe the problem, the variables which define the multiple path choices over the whole fleet of different aircraft types can be replaced by a simple integer variable $o$ rather than a series of matrix:

$$o \in [1, T] \text{ and } o \in \mathbb{Z}^+$$

(7.9)

where $T$ is the number of total combined pairs of operations in the set obtained by Cartesian product in Eq (7.7). The integer $o$ correspond to a unique multi-operational index matrix $O$. In this way, the dimension of the multi-operation planning problem is decreased. A simple example of a fleet consisting of four different aircraft types over two ground tracks is demonstrated in Figure. 7.2 where $M = 2, N = 4, n_j = 4$. It is shown that there are a large number of possible combinations to allocate four types of aircraft over just two ground tracks, which leads to the requirement to find the optimal one with lowest environmental cost and highest operational efficiency.
Chapter 7. Complex Flight Scenario Optimisation

7.2.3 Fleet operational sequence design

Section 7.2.1 and Section 7.2.2 have built the model of fleet assignment, that is to assign each scheduled flight precisely to one path, resulting in a partition of flights into fleets following the same ground tracks. However, how to decide the flight sequence of all the fleets in time space cannot be obtained from the context described above. This section is aiming at establishing the model to design the operational time sequence.

There are three assumptions declared in advance:

1. For aircraft following the same ground track, the same type of aircraft is grouped in the same fleet and assigned to operate in a consecutive time sequence.

2. For aircraft following the same ground track, the operation sequence for different types of aircraft is decided by the time they spend. In another word, the group of aircraft with the shortest flight time to reach the final target has the priority to operate first. In this way, aircraft operating along the same ground track, no matter what category they belong to, are managed to move in order, therefore, prevent collision between each other.

3. Collision possibility between aircraft following different ground tracks is neglected.

For a given operation matrix given in Eq. (7.8), the \(i\)th row vector indicates the combination of different aircraft type following the \(i\)th flight path:

\[
O_i = [c_{ik_1}^1, c_{ik_2}^2, \ldots, c_{ik_j}^j, \ldots, c_{ik_N}^N]
\]  
(7.10)
where \( i = 1, 2, \ldots, M, j = 1, 2, \ldots, N \). \( k_j \) is an integer indicating the index of the \( k^{th} \) partition of the combination matrix \( C^j \) of the \( j^{th} \) aircraft type.

Based on the first assumption, for all the \( j^{th} \) type aircraft belonging to the \( i^{th} \) path, the minimum flight duration is:

\[
D^i_j = \begin{cases} 
0 & c_{ik_j}^j = 0 \\
\bar{t}_{sep} + (c_{ik_j}^j - 1)t_{jj}^i & c_{ik_j}^j \geq 1 
\end{cases} \tag{7.11}
\]

where \( D^i_j \) stands for the duration for all the type \( j \) aircraft operates following the \( i^{th} \) ground track, \( t_{jj}^i \) is the flight time for a single departure or arrival. The subscript \( sep \) means separation, the superscript of \( t_{jj}^{i} \) which is \( jj \) indicates the types of the two consecutive aircraft are the same, and \( t_{jj}^{i} \) is the time-based separation time which can be looked up in Table 7.4.

Next, based on the second assumption, the group of aircraft with the longest operational time is only allowed to operate in the last place. Besides, to calculate the required time for all the aircraft following the \( i^{th} \) path, connections between sets of the same type of aircraft needs to be considered. So the time-based separation rule is applied as well. Then the total time expense for the \( i^{th} \) path, which is expressed by \( D_i \) can be calculated as:

\[
D_i = (n_{\text{non-zero},i} - 1)\bar{t}_{sep} + \max_{j=1}^{N}(D^i_j) \tag{7.12}
\]

where \( n_{\text{non-zero},i} \) is the number of non-zero elements in the row vector \( O_i \), \( \bar{t}_{sep} \) stands for the average separation time of all the different pairs of aircraft (e.g. 107.99 s got from Table 7.4). Note that to decrease the complexity of the mathematical model, the average separation time \( \bar{t}_{sep} \) is used here instead of the ones specified by typical aircraft type.

Therefore for the whole sets of aircraft partitioned in \( M \) ground tracks, the minimum possible operational duration \( D \) with separation time considered is expressed as:

\[
D = (m_{\text{non-zero}} - 1)\bar{t}_{sep} + \max_{i=1}^{M}(D_i) \tag{7.13}
\]

where \( m_{\text{non-zero}} \) is the number of non-zero row vectors in the operational matrix \( O \) defined in Eq. (7.8).

In order to explain the model clearly, a quick analysis based on the same example shown in Figure 7.2 is provided with a given operational matrix in Eq. (7.14).

\[
O = \begin{bmatrix}
2 & 3 & 3 & 4 \\
2 & 1 & 1 & 0
\end{bmatrix} \tag{7.14}
\]

The detailed explanation is demonstrated in the Figure 7.3. We consider that the larger the number of flights in the set of aircraft is, the more total operational time will be taken. Please note that this assumption is only valid in this simple analysis,
operational priority sorting for simulation in the following section is conducted by comparing \( D_i \) of each type of aircraft.

Then after sorting the operational priority of the aircraft following the same path with the total time spent by the same type of aircraft, the operational sequence for aircraft belonging to each path can be obtained as well as the operation time for the flight, namely \( D_1 \) and \( D_2 \) in this case:

\[
D_1 = (n_{\text{non-zero}, 1} - 1)\bar{t}_{\text{sep}} + \max_{j=1}^{4}(D_{1j}) = 3\bar{t}_{\text{sep}} + D_1^4 = 3\bar{t}_{\text{sep}} + t_1^4 + 3t_{44}^4
\]

\[
D_2 = (n_{\text{non-zero}, 2} - 1)\bar{t}_{\text{sep}} + \max_{j=1}^{4}(D_{2j}) = 2\bar{t}_{\text{sep}} + D_2^1 = 2\bar{t}_{\text{sep}} + t_1^1 + t_{11}^1
\]

Then the whole operational duration for all the flights can be obtained:

\[
D = (m_{\text{non-zero}} - 1)\bar{t}_{\text{sep}} + \max_{i=1}^{2}(D_i) = \bar{t}_{\text{sep}} + D_2 = 3\bar{t}_{\text{sep}} + t_2^1 + t_{11}^1
\]

### 7.3 Objective Models

To evaluate the environmental impacts of operations from the flight fleet, especially the noise impacts, appropriate criteria to reflect the strength of the noise influence, as well as the accumulative effects of multiple operations, is required. Apart from the objectives model introduced in Section 3.4, below we will introduce three different objectives for the multiple flight operations: Community Noise Equivalent Level, Accumulative Awakenings and Minimum Operational Duration. The first one estimates the influence from the aspect of average energy. The second one focuses on the accumulative and random effects of multiple noise events on the human sleep over a certain Duration. The third one refers to a metric considering the operational efficiency.
Table 7.5: Weighting and Averaging Factors.

<table>
<thead>
<tr>
<th>Noise Metric</th>
<th>Flight Multiplier $g$</th>
<th>Day</th>
<th>Evening</th>
<th>Night</th>
<th>Averaging Time [hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEL</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>EPNL</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$L_{den}$</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>$L_{Aeq24h}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

7.3.1 Equivalent noise level

Traditional metrics have already provided approaches to consider the effects caused by multiple flights. The most widely used include the Community Noise Equivalent Level $L_{den}$ and 24-hour average noise Level $L_{Aeq24}$. The day, evening and nighttime weighting factors and the time averaging periods for these metrics are shown in Table 7.5.

Expressed in Eq. (7.17) is a generic formula accounting for all significant sound energy received within a period of time:

\[
L_{eq} = 10\log_{10} \frac{1}{T_0} \int_0^{T_0} 10^{L(t)/10} dt
\] (7.17)

where $T_0$ is the time period where all concerned aircraft noise events occur, $L(t)$ is the short-time average. Then the time-weighted equivalent sound level can be expressed as:

\[
L_{eq,W} = 10 \cdot \log_{10} \left( \frac{t_0}{T_0} \sum_{i=1}^{N} g_i \cdot 10^{L_{e,i}/10} \right) + C
\] (7.18)

the subscript $W$ means weighted, $g_i$ denotes the flight multiplier with value given in Table 7.5, $N$ is the total number of flights, $i$ denotes the $i^{th}$ flight, $t_0 = 1 [s]$ is the reference time and $C$ is a constant stands for normalising constant, seasonal adjustment which is neglected here.

In the following case, we use the $L_{AeqT}$ as the metric to value the average noise impacts over a certain duration, where the subscript $T$ represents the period that is user-defined.

7.3.2 Accumulative awakenings

According to the American National Standard Institute publication (ANSI) [114], the probability of an individual noise event awakening a person as a result of the SEL alone can be calculated as:

\[
P_{\text{awakening,flt}} = P_{\text{awakening,flt}}(SEL_{\text{flt}}) = \frac{1}{(1 + e^{-(6.8884+0.04444SEL_{\text{indoor}})})}
\] (7.19)

where $SEL_{\text{indoor}} = SEL_{\text{outdoor}} - 20.50\text{dB}$. Here we regard $SEL_{\text{flt}}$ as $SEL_{\text{outdoor}}$. 
After experiencing a noise event, one is either woke up or continues sleeping. Then from the view of the theory of probability and statistics, we assume that whether the person is awakened after some flight from the same flight track of the same type aircraft can be regarded as independently repeated trials with only two possible outcomes. Therefore, the experiment that a person is awakened or annoyed by a single noise event from the same type of aircraft following the same flight path is a Bernoulli trial, which means the probability of “awakening” or “being annoyed” is the same every time the same experiment is conducted. Thus, for multiple Bernoulli trials, the accumulative probability that one individual is awakened at least once by noise events from different aircraft following their respective flight paths is derived as:

$$P_{\text{awakening, arpt}} = 1 - \prod_{j=1}^{N} \left(1 - P_{\text{awakening,flt}},i,j\right)^{c_{i}^j}$$  \hspace{1cm} (7.20)

where $c_{i}^j$ is an integer and the element of combination vector defined in Eq. (7.3), meaning the number of flights following the track $i$ in the group of aircraft type $j$.

In this way, the total possible population in the surroundings of the airport awakened by a series of flights can be estimated for given population densities:

$$\text{total awakenings} = \sum_{p=1}^{n_{\text{NSA}}} \text{population}_p \cdot P_{\text{awakening, arpt}}^p$$  \hspace{1cm} (7.21)

where the lower-case letter $p$ is the index of Noise Sensitive Areas (i.e. NSAs), and $n_{\text{NSA}}$ is the total number of NSAs. In this way, a metric considering both the noise level as well as the frequency of flight operations over different paths is constructed. Also, this method attempts to avoid the uncertainty from a single dose-response relationship from the aspect of statistic theory.

### 7.3.3 Possible Minimum Operational Duration

Section 7.3.1 and Section 7.3.2 provide the objectives regarding the noise impact from the aspect of noise energy and the accumulative impact caused by consecutive noise events. This section we will give a metric to weigh the multiple flights from the viewpoint of operational efficiency.

There are two reasons to choose a time metric as an objective for the complex flight scenario. The first one is to take the operational efficiency into account. To decrease the possibility of delay and to increase the capacity of the airport, an efficient and safe fleet assignment is the direction to make an effort. On the other hand, the first two objectives do not consider the duration occupied by aircraft noise. In general, intensive noise events within the same period exerts higher noise level yet might reduce the time people get annoyed by the noise. Based on the analysis above, a time metric
which represents the operational efficiency at the same time also can estimate noise impacts to the environment is needed.

Therefore, the possible minimum operational duration which derived in Section 7.2.3 is considered as the third objective:

\[
D = (m_{\text{non-zero}} - 1)\bar{t}_{\text{sep}} + \max_{i=1}^{M}(D_i) \tag{7.22}
\]

### 7.4 Case study: arrival flight path planning

In this section, the environmental impacts, as well as the operational duration of fleets approaching easterly from the holding stack DAYNE (53°14.19′N, 2°1.45′W) to runway end 05L (53°20.85′N, 2°17.27′W), are simulated and optimised. Two different lateral tracks are considered, which means \( M = 2 \) for the rest of this section. The simulation scenario of the fleet arriving from DAYNE stack is depicted in Figure 7.4. Figure 7.4.(a) displays the locations of Manchester Airport and the major residential communities around it under the OSGB36 coordination system. Figure 7.4.(b) provides the population distribution around Manchester Airport. Two residential communities, Knutsford and Wilmslow, are selected as the NSAs due to their proximity to the airport and relatively large populations. The locations of Manchester airport, holding stack as well as the residential communities are labeled by tags with their names. Two groups of microphones are located at Knutsford and Wimslow to evaluate the noise impact on these two target areas, which are indicated with two blue microphone icons in the diagrams. Please note that although Macclesfield community has a large population, this area is excluded when computing noise level for it is far enough in the vertical distance from the noise source (e.g. aircraft) to the receiver (e.g. ground).

In this case, an arrival scenario considering the noise impact of a fleet consisting of ten A321-232 is simulated and optimised. The aircraft are assumed to approach easterly from the holding stack DAYNE to the runway end 05L along two designed ground tracks, which lead to \( M = 2, N = 1, n_1 = 10 \). In order to evaluate the noise impact on human sleep, this scenario is set to be happened at night (i.e. 23:00-07:00).

<table>
<thead>
<tr>
<th>Segment No.</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( l_{i,1} \in [1000 \text{ m}, 10000 \text{ m}] )</td>
</tr>
<tr>
<td>2</td>
<td>( R_{i,2} \in [6000 \text{ m}, 10000 \text{ m}] )</td>
</tr>
<tr>
<td>3</td>
<td>( l_{i,3} \in [10000 \text{ m}, 20000 \text{ m}] )</td>
</tr>
<tr>
<td>4</td>
<td>none</td>
</tr>
<tr>
<td>5</td>
<td>none</td>
</tr>
</tbody>
</table>

Two different lateral tracks are determined by six lateral free parameters, which has been introduced in details in Section 6.3.4. An extra free parameter is needed to
(a) Residential communities around Manchester Airport (OSGB36) [7].

(b) Population density around Manchester Airport (OSGB36) [145].

Figure 7.4: Simulation scenario of fleet arriving from DAYNE stack.
define the partition of this group of aircraft. In total, there are seven free parameters in total for this parameter optimisation problem:

$$\mathbf{P} = [o, l_{1,1}, R_{1,2}, l_{1,3}, l_{2,1}, R_{2,2}, l_{2,3}]^T$$

(7.23)

where \( o \in [1, T] \) is an integer variable to define the aircraft partition over the two ground tracks. According to the expression given by Eq. (7.7), the value of \( T \), in this case, can be derived as:

$$T = \prod_{j=1}^{1} \left( \frac{10 + 2 - 1}{2 - 1} \right) = 11$$

(7.24)

The other parameters \( l_{i,1}, R_{i,2}, l_{i,3} \) (\( i = 1, 2 \)) are free variables to describe the shape of two planned ground tracks, where the first subscript \( i \) indicates the index of the ground track. The value range of all free parameters are presented in Table 7.6.

To evaluate the environmental impacts of the multi-operational flights, in this case, the objectives of this problem are set to be:

$$J_1 = L_{eq}, \quad J_2 = CO_2$$

(7.25)

The optimiser adopted for this problem is NSGA-II, with the number of generation set to be 60, population size set to be 50. After designated generations, the solutions converge to a discontinuous Pareto front shown in Figure 7.5. Further analysis indicates that the main difference of the solutions on the left side and the solutions on the right side is the value of the first free parameter \( o \), which corresponds to a different operational matrix: \( o = 2 \) for the left ones while \( o = 1 \) for the right ones. The operational matrix corresponding to these two values are \( \mathbf{O} = [1, 9]^T \) and \( \mathbf{O} = [0, 10]^T \) respectively. The first one means only one aircraft choose a different ground track from others, while the second operational matrix stands for a situation that all the aircraft choose the same ground track when alternative option is available.

Posterior selection strategies including aggregated preference value function as well as the monetisation method are applied to evaluate the Pareto-optimal solutions we obtained from the optimisation process. Detailed deduction of these two posterior selection strategies are demonstrated in Chapter 4. Five representative solutions are selected from the Pareto-optimal solutions obtained by the NSGA-II for further analysis. Solution 1 stands for \( \min(L_{eq}) \); Solution 2 represents the \( \min(CO_2) \); Solutions 3 is selected as the closest to the utopia point (i.e. \( (L_{eq_{\text{min}}, CO_2_{\text{min}}}) \), marked with a pentagram in Figure 7.5; Solution 4 is the one selected as the one with the smallest amount of damage cost; and Solution 5 is the one selected by the preference value function method. They are all indicated with different markers in Figure 7.5.

According to the methodology introduced in Chapter 4, the two objectives are both categorised into the same type: type-1, the smaller, the better. Table 7.7 provides a possible preference classification based on the value range of the cost functions
obtained. Please note that \( f_i^0 \) are given the values by rounding down the minimum values of the objectives obtained in the optimisation solution set. Similarly, \( f_i^5 \) are got by rounding up the maximum values of the objectives. In order to increase the degree of discrimination corresponding to the value of cost functions of different solutions, the difference between each pair of the end points, namely \( f_{i+1}^k - f_i^k \), \((k = 1, 2, 3, 4)\), is set to be an arithmetic sequence, leading to the preference classification displayed in Table 7.7.

Table 7.7: Preferences of two objectives.

<table>
<thead>
<tr>
<th>Objective</th>
<th>( f_i^0 )</th>
<th>( f_i^1 )</th>
<th>( f_i^2 )</th>
<th>( f_i^3 )</th>
<th>( f_i^4 )</th>
<th>( f_i^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leq [dB]</td>
<td>51.00</td>
<td>51.06</td>
<td>51.19</td>
<td>51.45</td>
<td>51.97</td>
<td>53.00</td>
</tr>
<tr>
<td>CO(_2) [kg]</td>
<td>7015.00</td>
<td>7021.94</td>
<td>7035.81</td>
<td>7063.55</td>
<td>7119.03</td>
<td>7230.00</td>
</tr>
</tbody>
</table>

Figure 7.5: Pareto Front.

Further explanation should be made for the monetisation method applied towards this multi-operational scenario. A metric, which is the percentage of the Highly Sleep Disturbed (e.g. \%HSD) introduced in Section 4.2.2, is used to evaluate the damage cost of multiple aircraft noise events. Therefore, the posterior selection by using the monetisation method is to choose the one with minimum economic cost, which is given in Eq. (7.26):

\[
\text{min cost} = \%\text{HSD} \cdot \text{population} \cdot \text{health value} + \text{UDC}_{\text{NO}_x} \cdot \text{NO}_x + \text{UDC}_{\text{CO}_2} \cdot \text{CO}_2 + \text{UC}_{\text{fuel}} \cdot \text{fuel} \\
(7.26)
\]

where
health value = UC_{DALY} × DALY
= UC_{DALY} × DW × I × L
= £60{,}000 × 0.07 × D

By applying Eq. (4.17), UC_{DALY} = £60{,}000/year, %HSD = %HSD(L_{night}), DW = 0.07 is the disable weight of highly disturbed sleep. Please note that since this scenario is set to be happened at night, L_{night} equals to L_{eq}. Moreover, the average duration of the disturbance L is assumed to be same as the the possible minimum operational time D over the number of the incidences I happened in this period: L = D/I. The unit of time D should be converted into year when calculating the health value.

Table 7.8 provides the comparison based on the objectives and the posterior selection results. Please note that although the possible accumulative awakenings in Knutsford and Wilmslow communities and the probable minimum operational durations of the five representative solutions are not the criteria helping with the posterior selection, they are still displayed in the Table 7.8 as the additional information for decision-makers. The contribution of each factor to the economic cost is shown in the stacked bar chart in Figure 7.6.

<table>
<thead>
<tr>
<th>Solution No.</th>
<th>L_{eq} [dB]</th>
<th>CO$_2$ [ton]</th>
<th>D [s]</th>
<th>Awakenings</th>
<th>Cost [£]</th>
<th>V(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51.07</td>
<td>7.2295</td>
<td>1191.09</td>
<td>3690</td>
<td>2567.58</td>
<td>0.3409</td>
</tr>
<tr>
<td>2</td>
<td>52.83</td>
<td>7.0145</td>
<td>1255.30</td>
<td>3544</td>
<td>2530.64</td>
<td>0.3096</td>
</tr>
<tr>
<td>3</td>
<td>51.50</td>
<td>7.0382</td>
<td>1179.79</td>
<td>3451</td>
<td>2511.45</td>
<td>0.1429</td>
</tr>
<tr>
<td>4</td>
<td>51.53</td>
<td>7.0319</td>
<td>1179.02</td>
<td>3432</td>
<td>2509.74</td>
<td>0.1420</td>
</tr>
<tr>
<td>5</td>
<td>51.51</td>
<td>7.0359</td>
<td>1179.51</td>
<td>3443</td>
<td>2510.82</td>
<td>0.1393</td>
</tr>
</tbody>
</table>

Since the vertical profiles of the trajectories all follow the ANP segmented procedure, the focus of comparison is put on their partition over different ground tracks. Figure 7.7 to Figure 7.11 present the ground track allocation of these five representative solutions. One common characteristic shared by the solutions with lower $L_{eq}$ is that these solutions tend to assign most aircraft to fly along a shorter ground track while only to assign one aircraft to a longer track when dealing with ground path planning and assignment. This phenomenon reflects the conflicting nature of the two objectives chosen in this problem. On the one hand, it helps to shorten the time to complete the approach and landing process by assigning the fleet to a shorter ground track, leading to less fuel consumption, which benefits to CO$_2$ reduction. On the other hand, with the operational duration decreased, the consecutive multiple noise events produce a higher equivalent noise level received within this period. As is demonstrated in Figure 7.7 to Figure 7.11, except for Solution 2, the other four representatives choose to distribute the fleet into two ground tracks: one with shorter ground range to allow for less fuel
consumption, and the other one with a relatively longer ground range to control the frequency of aircraft noise incidences, therefore decrease the overall equivalent noise level.

Further comparison is conducted among the three Pareto-optimal solutions selected: Solution 3, Solution 4 and Solution 5. From Figure 7.5, we can find that they are located quite close to each other on the Pareto front, which can be seen from the similar ground tracks shown in Figure 7.9 to Figure 7.11 as well. From Table 7.8, it can also be concluded that these three solutions are quite similar in the accumulative awakenings, total cost, and the preference value. The fact that the three solutions are very close is a coincidence because their distribution is determined by the parameter setting of the posterior selection method. With an alternative parameter setting, the result of posterior selection could be different.

Other attributes mentioned in Section 7.3, including possible minimum operational duration $D$ and accumulative awakenings, are listed in Table 7.8. It is worth noticing that different aspect on the problem, even sharing the same concern, will deliver to a remarkably different consequence. For instance, concerning noise impacts, Solution 1 produces the lowest $L_{eq}$ on Knutsford community. However, from the aspect of accumulative effects on the sleep of residents living in the vicinity of Manchester Airport, the amount of population influenced by its planning stand out to be the largest among five representative solutions, which can not be obtained intuitively by the decision maker from the single metrics got.
CHAPTER 7. COMPLEX FLIGHT SCENARIO OPTIMISATION

Figure 7.7: Ground track allocation of Solution 1.

Figure 7.8: Ground track allocation of Solution 2.
Figure 7.9: Ground track allocation of Solution 3.

Figure 7.10: Ground track allocation of Solution 4.
7.5 Concluding remarks

In this chapter, the possibility for introducing multiple operations is investigated instead of a single flight for the estimation of environmental impacts on the area around the airport. An analytical model for optimising environmental impacts of multiple flight operations is developed, which could be achieved by fleet assignment and ground track planning with the existing ICAO Default Approach Procedural Steps.

Based on the aircraft classification database introduced, we can not only simulate the noise and emission impacts of a typical kind of aircraft but can also handle more complex scenario with different types of aircraft. Considering that the aircraft in landing sequence would be exposed to wake vortices, time-based separation rules have been considered to increase the rigour of this model.

Results have shown that the NSAs under the flight path are potentially exposed to lower equivalent noise levels with optimised fleet allocation and ground path planning. This specific noise metric can be reduced because the optimised solution leads the fleet to choose a less time-intensive and more distributed way to accomplish the landing process on the early lateral manoeuvre plane. It is also shown that with different attributes concerned, contradictions appear when choosing the best solution: the solution with lowest $L_{eq}$ at the NSAs is not always the one with smallest awakenings, the solution with lowest CO$_2$ might not be the one with possible shortest operation durations. Although the results obtained from the posterior selection strategies in the case study of this chapter are similar to each other, this is hardly universal for other cases. After all, algorithms deliver to a more efficient and non-biased decision making process than the human being. However, with options selected objectively
from Pareto-optimal solution sets in the posterior procedure, the decision makers still need to decide which one is the best subjectively among the optimal.

In general, the multi-objective trajectory optimisation framework can be applied to deal with complex operational scenario effectively.
Chapter 8

Conclusions

This chapter summarises the main achievements and conclusions of this work as well as points out its limitations. Furthermore, a future research plan for this research is proposed.

8.1 Summary of contributions

During this research project, studies of the accomplishing a greener and quieter aircraft operation have been carried out, resulting in the development of a novel multi-objective trajectory optimisation framework, which can be taken into account the variation of the environmental emissions as the objectives of aircraft operational parameters. The main contributions of this PhD thesis are summarised as follows:

- A study of state of the art in the multi-objective trajectory optimisation was presented in Chapter 2. We reviewed the existing noise prediction and emission prediction methods as well as the strategies used nowadays to design noise and emission friendly trajectories. The majority of applications conducted are typically aiming at reducing noise impacts or emission impacts. A comprehensive assessment of the environmental impacts from the aircraft trajectory and its optimisation is still of great importance. Main difficulties encountered for the problem include the conflicts between the computation efficiency and the accuracy of aircraft noise and emissions modelling, the possibility to model constraints considering a realistic scenario, and the considerable computational burden for complicated simulation scenario.

- A multi-objective trajectory optimisation problem was developed for the purpose of designing an environmentally friendly commercial flight in Chapter 3. A generic formulation of the trajectory optimisation problem is constructed including the flight dynamics model, constraints model and objective model, which led
CHAPTER 8. CONCLUSIONS

to a nonlinear optimal control problem with a hybrid constitution: the dynamics function is differential, yet the cost function lacks the gradient information of the state variables.

- A study on the posterior selection strategies applied to the decision-making process to decide which Pareto-optimal solution is the best was conducted in Chapter 4. Two strategies, namely aggregated preference value function method and the monetisation method are proposed typically to evaluate the optimisation objectives of different properties. It is concluded that these two methods compensate each other when evaluating the Pareto solutions with multiple objectives: the first strategy introduces the DMs’ subjective preference while the second strategy monetises the attributes to provide an objective decisive criterion. A survey on noise-induced non-physical impacts was also included. The effect of a single noise event and the effects of multiple noise events are classified and discussed.

- The application of these two strategies was conducted for the posterior selection of case studies in Chapter 5 and Chapter 7. As is expected, different strategies lead to different choices of the best trajectory. However, depending on the scenario characteristics and the preset parameters defining the strategies, sometimes the different strategies also lead to very similar final trajectories. A first conclusion of the posterior selection methods was that they do serve as an effective, intuitive and objective method to make a final decision, yet the main limitation of this approach was the inevitable dependence on the prior knowledge and experience from DMs when accessing a set of Pareto optimal solutions.

- The flight segments that exert most environmental impacts, especially the noise impacts, were studied in Chapter 5 and Chapter 6 respectively. The 3 DOF flight dynamics model was discretised in two perpendicular motion planes and parameterised with a limited number of free parameters. In the horizontal plane, previous parameterisation methods, as well as a newly proposed approach, were applied in the optimisation of departure trajectory. It can be concluded that it is possible to decrease the dimensions of the search space by applying the parameterisation method using Bézier curve for the motion in the horizontal plane.

- Chapter 5 also showed that it was possible to achieve a complete exploration of the whole Pareto front by using more than one optimiser in solving the hybrid MOTO problem. Each of the heuristic optimisers, namely NSGA-II and PSO-based algorithms, performed well individually yet could not obtain a smooth and well-distributed Pareto front with acceptable computational cost. Therefore, robust gradient-free algorithms could be applied to the problem parallelly to enhance the searching ability and to prevent any critical solution loss.
• Compared with departure optimisation which is an initial value problem, optimisation of arrival trajectory is a TBVP that has much more strict boundary constraints. Optimisation towards the arrival segment was conducted in Chapter 6 with a segmented optimisation methodology to integrate main concerns of the initial and final arrival segments, resulting in a single objective trajectory optimisation problem. By applying penalty functions, the original TBVP problem was converted into an initial value problem. Although it seems to be inevitable to get avoid of noise annoyance for NSAs very close to the airport during lading, it is concluded from the arrival scenario that it is possible to lower the local impacts by optimising the vertical profile.

• In Chapter 7, a complicated real scenario of multiple aircraft operations was considered. A systematic description of multiple flight operations of different aircraft is given. A fleet assignment model is established, including the flight combination matrix to define the partitions of several aircraft over available flight paths and the multi-operation index matrix to identify the grouping mode of different types of aircraft. Moreover, the fleet operation sequence was also designed with time-based separation rules to avoid the wake vortex risks. With the multi-operational modelling proposed, only one extra parameter was introduced to describe the complex multi-operation arrangement of multiple aircraft. By introducing the integer parameter, namely the multi-operation index, the MOTO problem turned out to be a mixed-integer nonlinear programming problem where heuristic algorithms still apply.

• Chapter 7 also showed that it was possible to compromise between noise impacts and CO\textsubscript{2} emissions by multi-objective optimisation even with multiple aircraft operations.

Moreover, it is worth noticing that a detailed sensitivity analysis was not conducted in this thesis. Nevertheless, the involved model to provide the cost function values, namely the FLIGHT software, was tested with a sensitivity analysis to include the effects of various uncertainties in the computational procedure [151, 152]. These uncertainties include the key geometry of the aircraft for noise calculation purposes, engine characteristics, propeller, atmosphere and ground parameters. A summary of integral noise metrics from measurements and predictions for departure and arrival trajectories of A320 is given in Table 8.1.

The result has demonstrated the mean value of the predicted noise level and its confidence level within two standard deviations. The confidence window of the predicted noise metrics overlaps with the window of the variability of the experimental measurements demonstrated the robustness of the models that were implemented. Considering the optimal problem, heuristic algorithms including GAs and PSOs were applied to the NLP problem. Despite the advantages including the fast convergence
Table 8.1: Integral noise metrics from measurements (Exp) and predictions from A320 trajectories [151].

<table>
<thead>
<tr>
<th></th>
<th>EPNL [dB]</th>
<th>L(_{\text{max}}) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival</td>
<td>94.02±0.73</td>
<td>94.93±1.44</td>
</tr>
<tr>
<td></td>
<td>86.35±0.52</td>
<td>85.02±1.64</td>
</tr>
<tr>
<td>Departure</td>
<td>90.57±3.58</td>
<td>93.81±3.64</td>
</tr>
<tr>
<td></td>
<td>80.93±4.30</td>
<td>78.45±3.44</td>
</tr>
</tbody>
</table>

and the lower computational cost, this kind of algorithms does not guarantee that the best solutions found are the global optima. Uncertainties come from the optimisers but not from the paradigms developed.

8.2 Future Research

This segment presents some questions that arose in this study, including discussions and the future works.

This thesis proposed a framework intended to produce solutions that supported decision makers to plan environmentally friendly departure and arrival procedures. The methodology was developed aiming at reducing community noise impacts as well as reducing gaseous emissions when sometimes the environmental objectives are conflicting. The multi-objective trajectory optimisation framework can be applied to real-world, mixed, applied problems with more than two objectives.

- The trajectory optimisation framework proposed can be used to test whether the current arrival and departure procedures go in line with the ICAO’s environmental goals, namely less noise and fewer emissions, for local airports and commercial airlines.

- Moreover, expectations of the improvement (e.g., a reduction in the highly annoyed population and CO\(_2\) emissions) that can be made with the existing fleets and operational routes are able to be estimated by the methodology developed. The models are valid when the flight procedure extends, and the database of aircraft types is enlarged.

- This method can also be used to offer a potential environmentally friendly flight operations planning for new runways and airports construction projects. By integrating the demographic and terrain data around the target airport, a range of ground tracks as well as the corresponding vertical profiles can be identified considering the minimization of environmental impacts. Moreover, an initial decision on the best flight trajectory planning can be done by using the posterior strategies, which extends the application of this thesis: not only serves as a simulation and optimisation tool for researchers but also provides decision support.
to flight path designers considering conflicting objectives and related constraints.

Based on the discussions above, the following work items are considered for the future:

- For the sake of completeness, wind vector is needed to be taken into account in the aircraft dynamics equations. However, the effect of wind was ignored in the presented models and cases studies. A sensitivity analysis of the optimal trajectories considering the wind effect would be a point worth studying.

- Additional objectives could be introduced into the optimisation framework. The FLIGHT program and ANP database have provided abundant options of noise and emission indexes. With one or more objectives added to the current MOTO problem, an interesting problem would arise because the conflicting nature among optimisation objectives of different emissions and noise indexes could be not as intuitive as it was shown in a two-objective trajectory optimisation problem, in which the possibility of designing a greener and quieter trajectory does exist.

- The monetisation method to evaluate the economic cost of noise needs further improvement, especially for the cost caused by a single noise event. Although the monetary metric of single noise event in Chapter 4 is proposed after a comprehensive literature review, results show that it costs too much by a single noise event, making the noise impact the dominant factor when applying the monetisation method in the posterior selection process.

- A danger value model for the complex flight scenario is required. From the aspect of Air Traffic Management (ATM), we only prevent the collision accidents for the aircraft operate along the same flight path, yet the possibility of collision of aircraft following different flight paths was ignored. This simplification is made to reduce the complexity of the multi-operational scenario. However, it is clear that the optimal solutions obtained with this simplification are calculated ideally and do not entirely go in line with the practical situations. From the theoretical and practical point of views, the possibility of collision among different aircraft flows would be a point worth working on.

- A more complicated scenario of multi-operational optimisation with more than one type of aircraft is needed to be studied, based on the aircraft classification databased built in Section 7.1.1. It might be worth studying the difference of environment impacts produced by the fleets consisting of various aircraft types and the ones with only one type.
Bibliography


BIBLIOGRAPHY


Appendix A

Time-based Separation Rules

The International Civil Aviation Organization (ICAO) has made separation rules according to the maximum takeoff weight (MTOW) [147], for which is an important factor of resisting or generating wake turbulence. The classification and definition of different types of aircraft is shown in Table A.1. The categories of rules for particular combinations of the aircraft maximum takeoff weight are categorized by the International Civil Aviation Organisation (ICAO) [153]. Table A.2 gives their values. Please note that for separation distances, there are two different suggested values: instrument flight rules(IFR) and the visual flight rules(VFR). Since the current IFR are stricter than VFR, IFR are applied to keep sufficient distances and to minimize the potential impact of the wake vortices of the leading aircraft on the trailing aircraft.

In order to transform the distance into time, an average landing speed of 136 knots is assumed to apply on all kinds of aircraft [154]. Therefore, the time-based separation rules can be given by Table A.3 which can be further refined if more detailed average landing airspeed is available in respect to different types of aircraft.

| Table A.1: Classification of the aircraft. |
|-----------------|-----------------|
| Classification  | MTOW            |
| Heavy           | MTOW< 136 tons  |
| Medium          | 7 tons<MTOW≤ 136 tons |
| Light           | MTOW≤ 7 tons    |

As for the four representative aircraft in the previous introduction, if we take their landing speeds from the ICAO’s ANP database, then the time-based separation rules for these four typical types of aircraft can be derived. The resultant time-based separation rules displayed in Table A.4 is using the average landing speed of these four aircraft, namely 141.68 knots. The separation time can be considered as the integrated index of the ability that the leading aircraft to induce turbulence and the ability that the following aircraft to resist that turbulence.
Table A.2: ICAO Separation Rules for landing aircraft.

<table>
<thead>
<tr>
<th>Leading aircraft</th>
<th>Following aircraft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small [nm]</td>
</tr>
<tr>
<td>Small</td>
<td>3.0</td>
</tr>
<tr>
<td>Large</td>
<td>4.0</td>
</tr>
<tr>
<td>Heavy</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Table A.3: Time-based separation rules with average landing speed 136 knots.

<table>
<thead>
<tr>
<th>Leading aircraft</th>
<th>Following aircraft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small [s]</td>
</tr>
<tr>
<td>Small</td>
<td>79</td>
</tr>
<tr>
<td>Large</td>
<td>106</td>
</tr>
<tr>
<td>Heavy</td>
<td>159</td>
</tr>
</tbody>
</table>

However, the result shown in Table A.4 is not always the best option. In order to increase the capability of the air route and to decrease the possibility of delay with sufficient separation time, a linear programming problem is constructed to find the optimal separation time satisfying ICAO’s wake vortex separation rules.

Let $T_b$ be the amount of separation time contribution due to the turbulence generated by this specific aircraft, and $T_a$ be the amount of time for the aircraft to resist such turbulence, as is shown in Figure A.1. The superscripts $A$ and $B$ indicate the position of different aircraft. Then the total time $T_a^A + T_b^B$ stands for the separation time of this pair of aircraft. According to the given information of current separation time, this two variables of each type of aircraft can be solve by a linear programming problem.

\[
\begin{align*}
\min & \quad 1^T x \\
\text{s.t.} & \quad Ax \geq b, \\
& \quad x_i \geq 0, \quad i = 1, \ldots, 6
\end{align*}
\] (A.1)

where

Table A.4: Time-based separation rules for four representative aircraft.

<table>
<thead>
<tr>
<th>Leading Aircraft</th>
<th>Category</th>
<th>$V_{\text{landing [knots]}}$</th>
<th>Following aircraft</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRJ-900</td>
<td>Light</td>
<td>140.0</td>
<td>CRJ-900 [s]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A320-232 [s]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B747-8 [s]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A330-343 [s]</td>
</tr>
<tr>
<td>A320-232</td>
<td>Large</td>
<td>133.8</td>
<td>76.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>76.23</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>76.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>76.23</td>
</tr>
<tr>
<td>B747-8</td>
<td>Heavy</td>
<td>157.4</td>
<td>101.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>127.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>101.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>101.64</td>
</tr>
<tr>
<td>A330-343</td>
<td>Heavy</td>
<td>135.5</td>
<td>152.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>127.05</td>
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<td>101.64</td>
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<td></td>
<td></td>
<td></td>
<td>101.64</td>
</tr>
</tbody>
</table>
APPENDIX A. TIME-BASED SEPARATION RULES

vortex separation scheme1.pdf

Figure A.1: Wake vortex separation scheme.

\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 
\end{bmatrix} \tag{A.2}
\]

and \( \mathbf{b} = [76.23 \ 101.64 \ 152.46 \ 76.23 \ 76.23 \ 127.05 \ 76.23 \ 76.23 \ 101.46]^T \). The variable vector \( \mathbf{x} = [x_1, x_2, \ldots, x_6]^T \) is the vector consist of \( T_a \) and \( T_b \) of small, large and heavy aircraft respectively. Please note that B747-8 and A330-343 are both belong to the Heavy category, therefore the number of unknown variables is reduced to 6 rather than 8. The constraint \( A\mathbf{x} \geq \mathbf{b} \) represents the basic time separation criteria from ICAO, namely \( T_a^i + T_b^j \geq T_i^j \), where \( i \) is the index of following aircraft, \( j \) is the index of the leading aircraft. Yet, this linear optimisation problem has infinite optimal solutions with \( 1^T\mathbf{x} = 304.91 \). Here we narrow the constraints and set \( 32 \leq x_i \leq 86 \) to pick up an optimal solution \( \mathbf{x} = [69.64 \ 32.00 \ 44.23 \ 32.00 \ 42.23 \ 82.82]^T \), the reference value of the upper and lower bounds is taken from Wang’s work [155, 156]. Then the separation criteria obtained from linear programming are given in Table A.5 Compared the result in Table A.5 from the ICAO regulation in Table A.4, it is shown that there are two significant differences, which are in bold fonts. This is because Eq.(A.1) is using 6 variables to satisfy 9 constraints. Nevertheless, this result meet the criteria defined by ICAO. If more constraints are given, the optimal result can be altered. The value in Table A.5 is used in Chapter 7 as the time-based separation
Table A.5: Time-based separation rules obtained by linear programming.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CRJ-900</td>
<td><strong>101.64</strong></td>
<td>76.23</td>
<td>76.23</td>
<td>76.23</td>
</tr>
<tr>
<td>A320-232</td>
<td>101.64</td>
<td><strong>76.23</strong></td>
<td>76.23</td>
<td>76.23</td>
</tr>
<tr>
<td>B747-8</td>
<td>152.46</td>
<td>127.05</td>
<td><strong>127.05</strong></td>
<td><strong>127.05</strong></td>
</tr>
<tr>
<td>A330-343</td>
<td>152.46</td>
<td>127.05</td>
<td><strong>127.05</strong></td>
<td><strong>127.05</strong></td>
</tr>
</tbody>
</table>

rules when dealing with optimisation problem in multi-operational flight scenario.