THE CROSS-CORRELATION
BETWEEN LARGE SCALE
STRUCTURE, HI INTENSITY MAPS
AND CMB MAPS

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Abstract

HI intensity mapping is a new and efficient technique for mapping the large-scale-structures in the Universe and its expansion history in three dimensions. Due to the faintness of HI signal, an effective removal of Galactic foregrounds and a careful control of systematics is essential. One way of mitigating systematics is by cross-correlating multiple surveys issued from complementary datasets, which benefits from cancelled systematics, zero-noise-biased cross-spectrum, and complementary mass bias information.

The first part of the author’s work focuses on the cross-correlation between CMB maps and optical galaxy survey, where the thermal SZ cluster residuals in the Planck 2015 NILC CMB map is detected with $\approx 30\sigma$ significance at cluster scale, and overall $\approx 51\sigma$ significance including large scales. The percentage of thermal SZ emission left over in the NILC CMB map is quantified to be $44 \pm 4\%$, which, however, is proved to have negligible impact on ISW measurement but can potentially challenge upcoming CMB experiments with higher resolution. In contrast, we provide an alternative CMB map, produced with the 2D-ILC component separation technique, which is shown to be free from thermal SZ contamination.

In the second part, the author forecasts the impact of intensity mapping $1/f$ noise on cosmological parameter constraints through a Fisher matrix analysis. Without $1/f$ noise, constraints of $w_0 = -1 \pm 0.06$, $w_a = 0 \pm 0.13$ and HI bias $b_{HI} = 1 \pm 0.02$ are obtained from SKA1-MID Band 2+Planck. A representative $1/f$ noise model degrades the results by $\sim 50\%$. To mitigate this, one requires a minimised $1/f$ noise spectral slope, a low knee frequency and a large telescope slew speed. A correlation in frequency is also preferred.

Finally, forecasts have been made on the cross-correlation between HI intensity maps and the galaxy lensing field. Even with two idealised optimal surveys without any noise and a full-sky coverage, the total detection signal-to-noise ratio and parameter constraints are merely comparable to those obtained from Square Kilometre Array (SKA) auto-correlation with the Planck prior. Therefore, the auto-correlation of intensity maps performs better than cross-correlation with galaxy lensing field, in terms of signal detection and cosmological parameter constraints.

University of Manchester,

Tianyue Chen
Doctor of Philosophy

The cross-correlation between large scale structure, HI intensity maps and CMB maps
12 November 2018
Declaration

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The Author

The author was born in China in 24th April, 1992. She attended Shandong University in 2010 and exchanged to the University of Manchester in 2012 through the 2+2 exchange programme. She got a BSc(Hons) in Physics degree from both universities in July, 2014. She then obtained an MSc by Research degree in Astronomy and Astrophysics in September 2015 at the University of Manchester. After that, she commenced the study of a PhD in Astronomy and Astrophysics, the research aspects of which are presented in this thesis. The author’s future career plan is to extend this research and pursue an academic career.
“I wasn’t like you, I wasn’t the most talented student in school, I wasn’t
the brightest, but I was the best.”

—— Dr. Preston Burke, *Grey’s Anatomy* (S2E23), 2005
Chapter 1

Introduction to Cosmology

In this chapter, the basic theory behind the ΛCDM model to explain the observed cosmic acceleration will be introduced, along with cosmological probes to test the theoretical models. Sect. 1.1 introduces the ΛCDM model to explain the discovered cosmic acceleration. Sect. 1.2 discusses two key cosmological probes that can be used to test the cosmological model and, in particular, cosmic acceleration. Sect. 1.3 introduces astrophysical foregrounds, which can limit the accuracy of cosmological data.

1.1 Theory of the cosmological constant

Several theories have been proposed to explain the observed cosmic acceleration including modifications of general relativity (GR) theory (e.g., Dvali et al., 2000; Capozziello & Fang, 2002; Clifton et al., 2012), quintessence (e.g., Peccei et al., 1987; Ratra & Peebles, 1988; Copeland, 2007), and a cosmological constant, which is the simplest solution to the cosmic acceleration and will be introduced in this section. More details can be found in standard text books such as Dodelson (2003).
1.1.1 Discovery of cosmic acceleration

In 1929, by measuring the redshift and observed flux density from Cepheid variable stars in nearby galaxies, Edwin Hubble deduced that the distances from these galaxies to our own galaxy were increasing, indicating that they were moving away from us (Hubble 1929). He therefore discovered that the Universe is expanding and found that the distance $d$ and apparent recession velocity $v$, which is the velocity at which those galaxies move away from us, are linearly related such that

$$H_0 = \frac{v}{d},$$

(1.1)

where $H_0$ is called Hubble constant. The linear correlation between distance and recession velocity is known as the Hubble law (Hubble 1929). It is common to parameterise the Hubble constant with a dimensionless quantity $h$ such that

$$H_0 = 100h\text{km}\text{s}^{-1}\text{Mpc}^{-1}.$$  

(1.2)

The Hubble constant measured by the Hubble Space Telescope has the value of $h = 0.738 \pm 0.024$ (Riess et al., 2011). This value however relies strongly on the assumption of the calibration for standard “candles”, i.e., astrophysical objects with known luminosity, and the different classes of objects (Jackson, 2015). The latest Planck results suggest a value of $h = 0.6736 \pm 0.0054$ (Planck Collaboration VI, 2018), which however is model-dependent since it is obtained for the cosmological constant model to be introduced in Sect. 1.1.2.

For a distant object, the proper position $r$ is the position of that object at a specific cosmological time $t$, which changes with $t$ due to the Universe expansion. The comoving position $x$ is defined such that it factors out the effect of the Universe expansion and is thus independent of cosmological time. The comoving position might change due to other factors such as the proper motion of the object relative to other objects.
The proper position and the comoving position of an object are related by

\[ r = a(t)x, \]  

(1.3)

where \( a(t) \) is the dimensionless scale factor. The Hubble parameter, which quantifies the rate of the Universe expansion, is related to the scale factor as

\[ H = \frac{\dot{a}}{a}, \]  

(1.4)

where \( \dot{a} \) is the first derivative of the scale factor with respective to time. At the present day \( t = t_0 \), \( H = H_0 \approx 70 \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \), but the expansion rate \( H(t) \) evolves with time.

From Newton’s Principia, there exists a universal attractive gravity force which weakens with distance but never vanishes. Einstein’s theory of GR extends Newton’s Principia into the case of strong spacetime curvature and relativistic velocities. In 1922, Friedmann (1922) introduced a cosmological model as a solution of Einstein’s equations of GR, which accounts for the expansion of the Universe. This model can be formulated through the Friedmann equation

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{R^2 a^2}, \]  

(1.5)

where \( G \) is the Newton’s gravitational constant and \( \rho \) is the total energy density of all fluids in the Universe. The constant \( k \) describes the spatial geometry of the Universe with \( k = 0, \pm 1 \) corresponding to zero, positive and negative curvature of the Universe as an example. The speed of light is denoted as \( c \) and \( R \) is the radius of curvature of the space. From Eq. 1.5, the Friedmann acceleration equation can be deduced that

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3P}{c^2} \right), \]  

(1.6)

where \( P \) is the total pressure of all fluids in the Universe. For a homogeneous and
isotropic Universe filled with matter or radiation, the right-hand-side of Equ. 1.6 should always be negative since \( P > 0 \) and \( \rho > 0 \), indicating that the Universe expansion is expected to be decelerating (\( \ddot{a} < 0 \)). Otherwise, from Equ. 1.6, one has to have

\[
\frac{P}{\rho c^2} < -\frac{1}{3}
\]  

(1.7)

in order to have an accelerating Universe (\( \ddot{a} > 0 \)).

One difficulty of the precise measurement of cosmological distances is due to objects moving away from each other. Primarily there are two ways to measure cosmological distances. The first way is through a standard ladder for scale measurement at different redshift, which will be introduced in detail in Sect. 1.2.2. The second way is through a standard ruler where an object with known luminosity is observed so that from the luminosity-distance relation of that object, the Universe expansion effect can be filtered out when measuring cosmological distances. Due to their high brightness, type Ia supernovae are usually used as standard “candles” (e.g., Weinberg et al. 2013). Riess et al. (1998) and Perlmutter et al. (1999) observed type Ia supernovae at redshift around 0.5 and found that the luminosity of those distant supernovae were fainter than expected, meaning that they were further away than expected, if the Universe was freely expanding. Their result was the first observational evidence that the Universe expansion is accelerating.

### 1.1.2 The dark energy component

The theory of the cosmological constant introduces a dark energy component \( \Lambda \), constant in space and time, to the expanding Universe. The cosmological constant component has a constant energy density \( \rho_\Lambda \) of

\[
\rho_\Lambda = \frac{\Lambda c^2}{8\pi G},
\]  

(1.8)
and a constant pressure $P_\Lambda$ of

$$P_\Lambda = -\rho_\Lambda c^2 = -\frac{\Lambda c^4}{8\pi G}.$$  \hspace{1cm} (1.9)

The dark energy component has the equation of state of

$$w = \frac{P_{de}}{\rho_{de} c^2},$$  \hspace{1cm} (1.10)

where $P_{de}$ and $\rho_{de}$ are the pressure and energy density of the dark energy component. $w$ is known as the equation of state (EoS) parameter. In the cosmological constant model, $w \equiv -1$ (see Equ. 1.9), respecting the condition in Equ. 1.7 for an accelerating Universe. Therefore, the cosmological constant model is a simple candidate for dark energy.

One can incorporate the cosmological constant into the Friedman and acceleration equation through the transformation $\rho \rightarrow \rho + \rho_\Lambda$ and $P \rightarrow P - \rho_\Lambda c^2$. By substituting Equ. 1.8 & 1.9 into Equ. 1.5 & 1.6, the modified Friedmann and acceleration equations become

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{R^2 a^2} + \frac{\Lambda c^2}{3}$$  \hspace{1cm} (1.11)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3P}{c^2} \right) + \frac{\Lambda c^2}{3}.$$  \hspace{1cm} (1.12)

In Equ. 1.12, the last term $\Lambda$ acts as a repulsive term against gravity, contributing to the cosmic acceleration.

### 1.1.3 The components of the Universe

The critical density $\rho_{\text{crit}}(t)$ is defined such that below it the Universe will expand forever and above it the gravity effects will cause it to re-collapse in on itself. At the present day, the critical density $\rho_{\text{crit}}(t_0)$ is defined as (from Equ. 1.5 with a flat space
$k = 0$)

$$\rho_{crit}(t_0) = \frac{3H_0^2}{8\pi G}. \quad (1.13)$$

The Universe has several components contributing to the energy density, including matter, radiation, dark energy and curvature. In order to quantify the sizes of different components at the present day, the density parameter of each component is defined as

$$\Omega_m = \frac{\rho_m(t_0)}{\rho_{crit}(t_0)} = \frac{8\pi G \rho_m(t_0)}{3H_0}, \quad (1.14)$$

$$\Omega_r = \frac{\rho_r(t_0)}{\rho_{crit}(t_0)} = \frac{8\pi G \rho_r(t_0)}{3H_0}, \quad (1.15)$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}(t_0)} = \frac{\Lambda c^2}{3H_0^2}, \quad (1.16)$$

$$\Omega_k = \frac{\rho_k(t_0)}{\rho_{crit}(t_0)} = -\frac{k c^2}{R^2 H_0^2}, \quad (1.17)$$

where $\Omega_m$, $\Omega_r$ and $\Omega_\Lambda$ are the density parameters and $\rho_m$, $\rho_r$ and $\rho_\Lambda$ are the energy densities of matter, radiation and dark energy respectively. The spatial curvature density is $\rho_k$ and $\Omega_k$ is the density parameter of the curvature of the Universe. The density parameters of these components are defined to obey

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1. \quad (1.18)$$

The energy density of matter $\rho_m$ scales with the scale factor as $a^{-3}$. This is because the energy density of many non-relativistic particles is proportional to the number density, which is inversely proportional to the volume, i.e. $a^{-3}$. The energy of a photon is inversely proportional to its wavelength. The energy density of radiation $\rho_r$ is the energy per photon times the number density, and thus scales with the scale factor as $a^{-4}$. Fig. 1.1 shows the energy density as a function of the scale factor for a flat Universe for the matter, radiation and dark energy component respectively. The figure is adopted from Dodelson (2003). It can be seen that at the very early Universe, radiation
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is the dominant component due to the $a^{-4}$ scaling, until the epoch $a_{eq}$ where radiation and matte have equal energy density. After that, matter becomes the dominant component until very late Universe ($a \gtrsim 0.7$, $z \lesssim 0.5$) when it drops below the dark energy component which has constant energy in time.

![Figure 1.1: The energy density as a function of scale factor for matter (solid), radiation (dashed) and the dark energy component (bold) in a flat Universe (Dodelson, 2003). The energy density of each component is divided by the critical density today in the log-algorithm as the y-axis.]

At the present day, observations show that $\Omega_m \approx 0.31$ and $\Omega_\Lambda \approx 0.69$ with other components negligible (e.g. Riess et al. 1998, Perlmutter et al. 1999, Eisenstein et al. 2001, Percival et al. 2002, Komatsu et al. 2011 and Planck Collaboration XIII 2016). Together with Fig. 1.1, the observations confirm that we have come into the dark energy era where the dark energy component dominates the Universe over matter and radiation.

There are two types of matter in the Universe. The first type is the baryonic matter, also known as the “normal matter” that are made from atoms and dust in the Universe (e.g., Dodelson, 2003). All stars, planets, galaxies and dust in the Universe
are classified as the baryonic matter. The density of the baryonic matter is denoted as $\Omega_b$, the best estimated value of which is from the Planck CMB measurement that $\Omega_b h^2 = 0.02237 \pm 0.00015$ (Planck Collaboration VI, 2018). The baryonic matter constitutes $\sim 15\%$ of all matter while the rest is in the form of dark matter, the origin of which is still unclear. However, dark matter must be “cold”, i.e. it has been non-relativistic since the early Universe because otherwise, it would have free-streamed out of over-densities (Taoso et al., 2008; Hannestad et al., 2010). One of the evidence for the existence of dark matter is from the measurement of galaxy rotation curves. The theoretical rotation velocities of galaxies are expected to decrease with increased distance to the galaxy centre, following Kepler’s laws. However, the observed galaxy rotation velocities remain flat with distance (e.g., Volders, 1959). The derived galaxy mass from the observed rotation curve is more than the mass of the observed luminous objects in the galaxy, suggesting the existence of “dark matter” beyond the observed luminous objects (e.g., van de Hulst et al., 1957). Currently the most accurate estimate of the dark matter density is $\Omega_c h^2 = 0.1200 \pm 0.0012$ (Planck Collaboration VI, 2018). The cold dark matter together with the baryonic matter constitutes the total matter in the Universe such that

$$\Omega_m = \Omega_b + \Omega_c.$$  (1.19)

1.1.4 The primordial perturbation power spectrum

The Cosmological Principle states that the Universe is homogeneous and isotropic on large scales ($\sim 100$ Mpc), which means that the Universe does not have a privileged direction or position. It is expected that the fundamental forces of nature act uniformly along each direction and position in the Universe, creating no observable large-scale irregularities (e.g., Coles & Lucchin, 2002; Dodelson, 2003). A key piece of evidence for the Cosmological Principle is the observation of the cosmic microwave background (CMB, see Sect. 1.2.1) where the temperature anisotropies are $\sim 10^{-5}$ times smaller than the mean temperature of $T \approx 2.73$ K (e.g., Planck Collaboration IV, 2018).
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However, there are small-scale structures in the Universe, such as galaxies and clusters, which are formed from the anisotropic distribution of matter. The simplest theory to explain these small-scale anisotropies is the cosmic inflation (Guth, 1981; Linde, 1983), where the early Universe went through accelerated expansion, which expanded its size very quickly. This ultra-rapid exponential expansion of the Universe must have been driven by an initial scalar field, known as the inflaton. The quantum fluctuations in the inflaton field reached macroscopic sizes, leading to primordial density fluctuations during inflation, which later on formed the structures, such as galaxies and clusters, observed today. The primordial scalar perturbation power spectrum $P_s(k)$ can be modelled by a power-law such that

$$P_s(k) = A_s k^{n_s - 1},$$

(1.20)

where $A_s$ is the amplitude parameter and $n_s$ is the scalar spectrum index (e.g., Coles & Lucchin, 2002). Currently the most accurate measurement gives $\ln(10^{10} A_s) = 3.044 \pm 0.014$ and $n_s = 0.9649 \pm 0.0042$ (Planck Collaboration VI, 2018). From the value of $n_s$, the primordial perturbation is nearly scale-invariant, which is one of the geometric predictions of inflation (e.g., Liddle & Lyth, 2000).

As the Universe expands, the primordial perturbation power spectrum scales with time and its shape also changes due to the physical processes encountered during the evolution. The processed spectrum at a given redshift is related to the primordial spectrum through the transfer function, $T(k)$, and the growth factor, $D(z)$. The former takes into account the shape change during the Universe evolution and the latter scales the primordial spectrum with time, such that

$$P(k, z) \propto D^2(z) T^2(k) P_s(k),$$

(1.21)
where $D(z)$ is calculated from the following system of equations (Dodelson, 2003)

$$
D''(a) = -\left(\frac{3}{2} + \frac{1}{2} \frac{d \log E(a)}{da}\right) D'(a) + \frac{3\Omega_m}{2a^5 E(a)} D(a)
$$

$$
E(a) = \frac{H(a)^2}{H_0^2}.
$$

(1.22)

The transfer function $T(k)$ depends on the physical processes under consideration, which depends on the cosmological parameters $(\Omega, h)$ and the form of non-baryonic dark matter. For example, in a cold dark matter model, the transfer function is given by

$$
T(k) \propto \left[1 + \frac{(Ak)^2}{\log(1+Bk)}\right]^{-1},
$$

(1.23)

where $A$ and $B$ are constants depending on $(\Omega, h)$ (e.g., Coles & Lucchin, 2002).

The matter fluctuation r.m.s amplitude at a given scale $R$ can be quantified through the variance $\sigma_R$ such that

$$
\sigma_R^2 = \frac{1}{2\pi^2} \int_{k_{\text{min}}}^{k_{\text{max}}} P(k, z) W^2(kR) k^2 dk,
$$

(1.24)

where $W(kR)$ is the window function such that

$$
W(kR) = \frac{3(\sin kR - kR \cos kR)}{(kR)^3}.
$$

(1.25)

It is common to use $\sigma_8$ as the normalisation factor of the matter perturbation power spectrum, which measures the amplitude of the power spectrum at the scale of $8h^{-1}\text{Mpc}$. The latest Planck result gives the tightest constraint of $\sigma_8 = 0.811 \pm 0.006$ (Planck Collaboration VI, 2018).

### 1.1.5 Simple extension to the $\Lambda$CDM model

In the $\Lambda$CDM model, the cosmic acceleration is driven by a constant parameter, without a theoretical basis behind it to explain the value. The EoS parameter is constant
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at \( w = -1 \) within the \( \Lambda \)CDM model. One theoretical problem of the \( \Lambda \)CDM model is the Cosmological Constant Problem. To drive the cosmic acceleration, the cosmological constant \( \Lambda \) is the order of \( H_0^2 \) so that its energy density has (e.g., Amendola & Tsujikawa, 2010)

\[
\rho_\Lambda^{\text{obs}} \sim 10^{-47} \text{GeV}^4.
\]  
(1.26)

However, expectations from Quantum Gravity suggest that

\[
\rho_\Lambda^{\text{exp}} \sim \frac{c^5}{G^2} \sim 10^{120} \rho_\Lambda^{\text{obs}},
\]  
(1.27)

known as the Cosmological Constant Problem.

A simple extension to the \( \Lambda \)CDM model is the \( w \)CDM model, which seeks for a dynamic mechanism for the cosmic acceleration with the evolving background energy densities close to the \( \Lambda \)CDM model. The \( w \)CDM model brings a more sophisticated scenario to describe the dark energy component, by introducing a dynamical field (e.g., Peccei et al., 1987; Ratra & Peebles, 1988) or modification of the general relativity (e.g., Dvali et al., 2000; Capozziello & Fang, 2002). In the \( w \)CDM model, instead of being fixed to \( w \equiv -1 \), the EoS parameter \( w \) is treated as a free parameter \( w_0 \), so that \( w = w_0 \) (e.g., Lonappan et al., 2017).

The EoS parameter can also be parametrised as a function of the scale factor \( a \). For example, Chevallier & Polarski (2001) and Linder (2003) suggested a model of \( w(a) \) that

\[
w(a) = w_0 + w_a (1 - a),
\]  
(1.28)

where \( w_0 \) is the value of \( w(a) \) at the present day and \( w_a \) is the first derivative of the EoS parameter with respect to \( a \). This is known as the CPL model.

In order to place a tight constraint on \( w \), a survey at low redshift is preferred. This is because the dark energy component dominates at the late Universe, where observations of, e.g. CMB, at a single high redshift does not bring much information since the Universe was dominated by the matter component at that age. In this thesis, we will
focus on the CPL model when constraining cosmological parameter in Chapt. 4, and $w$CDM model in Chapt. 5, since the intensity mapping experiments (to be introduced in Chapt. 2) studied within the scope of this thesis operate mostly at low redshift ($z \lesssim 0.5$).

1.2 Cosmological probes

The discovery and suggested theories of cosmic acceleration have stimulated observational cosmology, where different observables are used to probe the Universe and test the theoretical models. For example, the shape distortion of distant galaxies by gravitational lensing can give information for constraining $\Omega_m$ (e.g., Miller et al. 2013). The cluster abundances are also sensitive to $\Omega_m$ which makes clusters as good tools for constraining the growth structure in the matter distribution (e.g., Tinker et al. 2012). In this section, we introduce two of the primary cosmological probes - cosmic microwave background and baryon acoustic oscillation, and the cross-correlation between multiple probes, since these are the author’s primary fields of research.

1.2.1 The cosmic microwave background

The cosmic microwave background (CMB) was first discovered by Penzias & Wilson (1965). They found an uniform background at radio frequencies when observing and investigating atmospheric noise. The uniform radio background cannot be explained by their instrumental noise or other radio sources. Dicke et al. (1965) soon interpreted this background as the cosmic microwave background.

The CMB radiation was emitted at the recombination epoch at a redshift of $z \approx 1100$, when the Universe was cooled enough due to expansion ($T \sim a^{-1}$) to allow the combination of protons and electrons to form neutral hydrogen (e.g., Weinberg et al., 2013). Before recombination, energetic photons were trapped in the hot hydrogen plasma but were then decoupled and free-streamed at the recombination epoch, becoming the first light of the Universe. After recombination, as stars and galaxies
formed, they became energetic to emit ionizing radiation and ionized the neutral hydrogen. This is known as the Epoch of Reionization (EoR), which happened at a redshift of approximately $6 < z < 20$ (e.g., Zaroubi et al., 2012; Weinberg et al., 2013). The CMB photons will again interact with the free electrons from the re-ionized hydrogen through Thompson scattering, quantified by a reionization optical depth, $\tau$. The CMB photons have been propagating in the Universe since the recombination epoch and can still be observed today. Observations of CMB can reveal the information on the early Universe, as a cosmological probe of the Universe.

The most accurate observation of CMB comes from the Planck satellite and Fig. 1.2 gives the CMB map from the latest Planck results (Planck Collaboration IV, 2018). Note from Fig. 1.2 that CMB temperature fluctuations are at $\mu$K level, i.e., $\sim 10^{-5}$ times smaller than the mean temperature of 2.7255 K (Fixsen, 2009). The isotropy of the CMB map provides the evidence of an isotropic Universe at large scales.

Figure 1.2: The Planck 2018 map of CMB temperature anisotropies (Planck Collaboration IV, 2018). The map has a beam resolution of 5 arcmin.

More interestingly, the small temperature anisotropies of the CMB bear the imprint of the primordial density perturbations that give rise to the star and structure formations
in the late Universe. The statistical properties of these anisotropies can be calculated by computing the angular power spectrum of the CMB map. The observed signal is decomposed into spherical harmonics

$$\frac{\Delta T}{T} = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\theta, \phi),$$

(1.29)

where $a_{\ell m}$ is expansion coefficient, and the positions in the sky are denoted by $\theta$ and $\phi$. The CMB power spectrum is defined as the average of $m$ for each $\ell$ such that

$$C_\ell = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2.$$  

(1.30)

Fig. 1.3 shows the latest CMB temperature power spectrum from Planck Collaboration VI (2018). Note that the observed data (red dots) closely follow the theoretical spectrum (blue curve) from the $\Lambda$CDM model with remarkably tiny error bars at $\ell > 30$. The relatively larger error bars at $\ell < 30$ are caused by cosmic variance, which is caused by the limitation that one can only observe realisation of the Universe. By fitting the cosmological parameters based on the $\Lambda$CDM model, one can thus find the maximum-likelihood values of those parameters and tightly constrain them thanks to the high accuracy of Planck CMB power spectrum.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>0.02237 ± 0.00015</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>0.1200 ± 0.0012</td>
</tr>
<tr>
<td>$h$</td>
<td>0.6736 ± 0.0054</td>
</tr>
<tr>
<td>$\ln(10^{10}A_s)$</td>
<td>3.044 ± 0.014</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.9649 ± 0.0042</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.0544 ± 0.0073</td>
</tr>
</tbody>
</table>

Table 1.1: The constraints of the 6 cosmological parameters based on the $\Lambda$CDM model from Planck Collaboration VI (2018). The values of each parameter and its 68% error are the best fits from the full Planck likelihood, after combining the temperature (TT), temperature-polarisation(TE), and polarisation + lensing (EE+lowE+lensing) power spectra.

Table 1.1 lists the latest constraints of the 6 cosmological parameters based on $\Lambda$CDM model from Planck Collaboration VI (2018). Note that the constraints on
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Figure 1.3: The CMB temperature angular power spectrum from Planck Collaboration VI (2018) (upper panel). The red dots are the measured data points and the blue curve is the theoretical spectrum based on the $\Lambda$CDM model from the best-fit cosmological parameters. The lower panel gives the residuals of the data points with respect to the theoretical model.

theses parameters are already incredibly tight ($\lesssim 1\%$ precision on most parameters), meaning that we have come into an era of 'precision cosmology'. However, since the observations of CMB are made at the high redshift ($z \approx 1100$) when the Universe was dominated by matter, they do not provide much information on dark energy, which dominates at much later times. Therefore, for the study of dark energy with, e.g., the $w$CDM model, one requires to probe the Universe with a lower redshift survey.

1.2.2 The baryon acoustic oscillation

As mentioned in Sect. 1.1.1, one way of measuring cosmological distances is through a standard ruler with a known scale. By measuring the known scale at different redshift, from the distance-redshift relation, one can deduce the Universe expansion history. Compared with standard candle, the standard ruler method is more accurate since it is not affected by uncertainties from, e.g., the observed apparent magnitude or luminosity instabilities of standard candles, which could induce large error bars on cosmological distance measurement.
The baryon acoustic oscillation BAO is the imprint left by the acoustic waves in the early Universe, which was formed due to the coupling of baryons and photons through Thomson scattering (e.g., Coles & Lucchin, 2002; Weinberg et al., 2013). The primordial overdensities during inflation attracted matter towards them under gravitation. During this process, the interactions between photons and baryons created outward pressure. The inward gravity and outward pressure created oscillations in the form of a spherical sound wave in the baryon-photon plasma. The baryon-photon sound wave traveled outward at the speed of sound while the cold dark matter only converged under gravity and stayed at the centre. This process stopped at the recombination epoch when the neutral hydrogen formed so the photons decoupled from the baryons and streamed away. The pressure on the system began to fall and the sound wave stalled when the system pressure reached zero, forming a shell of excess baryons at a fixed radius, known as the sound horizon or the acoustic scale. Due to gravity, matter is attracted by the shell of baryons, and the centre made of dark matter respectively, seeding the gravitational instability and forming overdensities at both the shell and the centre. A large number of galaxies therefore formed in these overdensities, separated by the sound horizon.

The peaks and troughs in the CMB power spectrum as shown in Fig. 1.3 are the imprints of BAOs. The acoustic scale corresponds to the first peak in Fig. 1.3, with a value of $\approx 150$ Mpc (e.g., Anderson et al., 2014), which can be calculated by

$$r_s = \int_{z_s}^{t_*} \frac{c_s(t)}{a(t)} dt = \int_{z_s}^{\infty} \frac{c_s(z)}{H(z)} dz$$

where $t_*$ ($z_*$) is the time (redshift) at recombination and $c_s(z)$ is the sound speed

$$c_s = \sqrt{\frac{1}{3(1+R)}}$$

where $R = 2\rho_b/4\rho_r$, with $\rho_b$ and $\rho_r$ being the baryon and radiation densities respectively (e.g. Hu & Sugiyama 1996 and Eisenstein & Hu 1998).
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By measuring the angular sizes of BAOs at different redshifts, one can use BAOs as a precise standard ruler to study the expansion history of the Universe. The distance measure along the line-of-sight corresponding to differences in redshifts is related to the Hubble parameter $H(z)$ through

$$D_H(z) = \frac{cz}{H(z)}. \quad (1.33)$$

The separation transverse to the line-of-sight, corresponding to differences in angles, is calculated through the angular diameter distance $D_A(z)$ such that

$$D_A(z) = c \int_0^z \frac{dz'}{H(z')} \cdot \quad (1.34)$$

The averaged volume distance $D_V$ is given by

$$D_V(z) = [D_A(z)]^{2/3}[D_H(z)]^{1/3}, \quad (1.35)$$

The power of $\frac{2}{3}$ corresponds to the 2-D angular sizes of BAOs while $\frac{1}{3}$ corresponds to the 1-D along redshift, giving total 3D information (Weinberg et al., 2013). Compared with standard candles such as Type Ia supernovae, which measure only luminosities at different redshifts, BAOs could provide complementary geometric information.

The feasibility of probing dark energy through BAOs was first discussed in 2003 (e.g., Eisenstein, 2002; Blake & Glazebrook, 2003; Hu & Haiman, 2003). Since then, many attempts have been made to detect BAOs with redshifted galaxy surveys. The first detection was made by Eisenstein et al. (2005), who observed 46,748 luminous red galaxies (LRG) from the Sloan Digital Sky Survey (SDSS), covering 3816 deg$^2$ of sky at the redshift of $0.16 < z < 0.47$. They calculated the correlation function between pairs of galaxies, which gives the probability of finding a galaxy separated by a spatial distance $\xi$ to another. Since large numbers of galaxies formed at the acoustic scale on average, a peak is expected at this privileged scale on the correlation function. Fig. 1.4
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Figure 1.4: The correlation function of the SDSS LRG samples from Eisenstein et al. (2005). The points with error bars are the measured data, which peak at a comoving separation of $\sim 100\, h^{-1}\text{Mpc}$ corresponding to the acoustic scale. The solid lines with peaks at the acoustic scale give the theoretical curves based on $\Omega_m h^2 = 0.12$, $\Omega_m h^2 = 0.13$, $\Omega_m h^2 = 0.14$ respectively. The smooth line without the peak is based on a pure CDM model ($\Omega_m h^2 = 0.105$) where there is no BAO. The top right window is the zoom-in of the correlation function at the comoving separation of interest.

shows the correlation function from Eisenstein et al. (2005), which indeed detected a peak at a separation distance of $\sim 100\, h^{-1}\text{Mpc}$, consistent with the acoustic scale at the observing redshifts. Note also that the shapes of the expected theoretical curves (solid lines) change with different values of $\Omega_m$. Therefore, by fitting the theoretical models to the measured data, one can constrain cosmological parameters from the measured correlation function.

A number of more recent redshifted galaxy surveys have detected BAOs such as 2dFGRS (Percival et al., 2007), 6dFGS (Beutler et al., 2011), WiggleZ (Blake et al., 2011), BOSS (Anderson et al., 2012), SDSS main galaxy samples (MGS, Ross et al. 2015), and DES (DES Collaboration, 2017) etc. Table 1.2 summarises the results from these galaxy surveys for the BAO measurements. Beutler et al. (2017) gives the most
precise constraint on the distance measure, with 0.88\% uncertainty at \( z = 0.61 \) and 1\% uncertainty at \( z = 0.38 \), among all galaxy surveys. On average, the constraints from those galaxy surveys have a \( \sim 4\% \) accuracy.

<table>
<thead>
<tr>
<th>Survey</th>
<th>Redshift</th>
<th>Result(s)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>6dFGS</td>
<td>0.106</td>
<td>( r_s/D_{V} = 0.336 \pm 0.015 )</td>
<td>Beutler et al. (2011)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( D_{V} = 457 \pm 27 ) Mpc</td>
<td></td>
</tr>
<tr>
<td>SDSS MGS</td>
<td>0.15</td>
<td>( r_s/D_{V} = 0.2239 \pm 0.0084 )</td>
<td>Ross et al. (2015)</td>
</tr>
<tr>
<td>2dFGRS</td>
<td>0.2/0.35</td>
<td>( r_s/D_{V} = 0.1980 \pm 0.0058 )</td>
<td>Percival et al. (2007)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_s/D_{V} = 0.1094 \pm 0.0033 )</td>
<td></td>
</tr>
<tr>
<td>SDSS LRG(^{[4]})</td>
<td>0.35</td>
<td>( D_{V} = 1370 \pm 64 ) Mpc</td>
<td>Eisenstein et al. (2005)</td>
</tr>
<tr>
<td>BOSS</td>
<td>0.38/0.61</td>
<td>( D_{V} = 1476 \pm 15 ) Mpc</td>
<td>Beutler et al. (2017)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( D_{V} = 2146 \pm 19 ) Mpc</td>
<td></td>
</tr>
<tr>
<td>WiggleZ</td>
<td>0.44/0.60/0.73</td>
<td>( r_s/D_{V} = 0.0916 \pm 0.0071 )</td>
<td>Blake et al. (2011)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r_s/D_{V} = 0.0726 \pm 0.0034 )</td>
<td></td>
</tr>
<tr>
<td>DES</td>
<td>0.81</td>
<td>( D_{A}/r_s = 10.75 \pm 0.43 )</td>
<td>DES Collaboration (2017)</td>
</tr>
</tbody>
</table>

Table 1.2: The summary of BAO detections with redshifted galaxy surveys. The first column gives survey names. The second column gives effective redshifts. The third column gives distance measure of BAOs. The last column gives the corresponding references.

Note that all the listed galaxy surveys in Table 1.2 are in the optical band. It is useful to extend the BAO measurement into radio band, through 21 cm HI line or CO line for example. The radio band has different systematics than the optical band so that observations of BAO in the radio band can provide independent measurements, complementary to optical data.

### 1.2.3 The cross-correlation between cosmological probes

In the cosmology community, there is a growing interest in cross-correlating multiple surveys issued from complementary data sets. The cross-correlation requires the two surveys to trace the same underlying density field, i.e., to be two independent tracers of dark matter, in order to have a cross-correlation signal. The two surveys should also be overlapping in redshift and sky area.

In auto-correlation analysis, the auto-correlation function (ACF) is defined as the correlation of one signal \( f(x) \) with its delayed copy \( f(x - \delta) \) as a function of the delay
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$s$, such that

$$F(s) = \int_{-\infty}^{\infty} f^*(x)f(x-s)dx,$$  \hfill (1.36)

where $f^*(x)$ denotes the complex conjugate of the signal $f(x)$. The ACF quantifies the similarity between observations as a function of their time lag. In cross-correlation analysis, instead of correlating a signal by the delayed copy of itself, the cross-correlation function (CCF) correlates one signal $f(x)$ with a different signal $g(x-s)$ at a displacement $s$, such that

$$F(s) = \int_{-\infty}^{\infty} f^*(x)g(x-s)dx.$$  \hfill (1.37)

The CCF quantifies the similarity of the two signals as a function of their relative displacement.

The cross-correlation between independent surveys of dark matter has the advantage of mitigating instrumental noise and systematics, which will likely be uncorrelated between independent datasets (e.g., Sherwin et al., 2012; Pourtsidou et al., 2017). Therefore, in the cross-spectrum of the two independent tracers, the uncorrelated noise will return zero as the mean noise level, leaving the cross-correlation signal without the noise bias.

The cross-correlation also provides a way to calibrate the “mass biases” (Sherwin et al., 2012; Das et al., 2013; Allison et al., 2015), i.e., the ratio between the observable mass distribution and the underlying dark matter mass distribution. For example, the observed galaxy mass distribution $P_g$ is related to the underlying dark matter mass distribution $P(k,z)$ through the galaxy bias factor $b_g$ such that

$$P_g \propto b_g^2 P(k,z).$$  \hfill (1.38)

In order to recover $P(k,z)$ from the measured $P_g$, one can constrain $b_g$ by, e.g., cross-correlating the galaxy density field with CMB lensing (e.g., Allison et al., 2015), where
the cross-correlation density field has $P_{gK} \propto b_g P(k,z)$. In such a way, an estimation of the bias can be obtained by $b_g = P_g / P_{gK}$. In another scenario, by cross-correlating one tracer (e.g., optical galaxies) with another tracer of known mass bias (e.g., HI gas), one can also derive the constraint of the unknown bias $b_g$ from the cross-spectrum, $P_{sHI} \propto b_{HI} b_g P(k,z)$, given the known bias $b_{HI}$ (Pourtsidou et al., 2017). Therefore by cross-correlating two surveys, it allows a more robust measurement of the underlying matter power spectrum.

In summary, if an overlapping sky area and redshift range exist, one shall seek for potential cross-correlation between two different surveys, especially when each individual survey has a low S/N detection. In the case of cross-correlation, the S/N will take into account the uncertainties of both experiments, in contrast with auto-correlation where the S/N is simply calculated as the ratio of its own uncertainty over signal. Therefore, the low S/N of a single survey can be mitigated through cross-correlation with a survey of high S/N. The S/N calculation will be introduced in more details in Chapt. 4 (Equ. 4.21) and Chapt. 5 (Equ. 5.9). Cross-correlation also has the benefit of a stronger signal detection without the noise bias, a mitigated instrumental systematics, and more information about the mass bias. The overlapping redshift and sky coverage with other surveys is also one of the factors to be considered when proposing a new experiment, in order to maximize the chance of cross-correlation.

1.3 Astrophysical foregrounds

One of the biggest challenges for observational cosmology is the astrophysical foreground emission from our local Galaxy. The astrophysical foregrounds give emission between us and the wanted cosmological signal. The proper removal of astrophysical foregrounds is particularly crucial for cosmological probes using radio data, such as CMB measurement and intensity mapping (to be introduced in Chapt. 2). This challenge is known as the component separation problem. Therefore, it is important to
have a good understanding of the astrophysical foregrounds. In this section, the four components of the Galactic foregrounds will be introduced.

### 1.3.1 Synchrotron emission

_Synchrotron_ radiation is produced by the interaction between relativistic cosmic-ray electrons and the magnetic fields within the interstellar medium (ISM) (Rybicki & Lightman, 1979). Cosmic-ray electrons primarily originate from supernovae remnants (SNR), with contributions from other exotic sources such as nearby pulsars (Atoyan et al., 1995). The electrons are then accelerated through gyration in the ISM magnetic field.

The intensity flux of the synchrotron emission across frequency can be approximated as a power-law so that $I_\nu \propto \nu^\alpha$. The brightness temperature of the synchrotron emission is $T \propto \nu^\beta$, with $\beta = \alpha - 2$ since temperature is related to flux by a factor of $1/\nu^2$ (e.g., Fermi-LAT Collaboration, 2009). The value of $\beta$ varies across the sky with a mean value of $\beta = -2.8 \pm 0.2$ at GHz frequencies (e.g., Platania et al., 1998). Fig. 1.5 shows the brightness temperature of the synchrotron emission with respect to frequency (green), along with other foreground components, taken from the Planck 2016 foreground maps (Planck Collaboration X 2016). It can be seen that synchrotron radiation is the dominant radio emission at frequencies of 10 GHz and lower. The astrophysical foreground emissions dominate the majority of the frequency range over the CMB emission, addressing the importance of component separation for CMB experiments. Using the typical value of $\beta = -2.8$, the full-sky synchrotron emission at 1 GHz and 30 GHz has a mean brightness temperature of 2.8 K and 0.2 mK respectively (Olivari, 2018).

The synchrotron emission also has a relatively high linear polarisation fraction. The typical polarisation fraction from multiple spiraling electrons around a magnetic field is $\sim 75\%$. On the sky, the synchrotron emission has been observed to have a typical
1.3. ASTROPHYSICAL FOREGROUNDS

Figure 1.5: The brightness temperature r.m.s. of foregrounds as a function of frequency taken from Planck Collaboration X (2016). For each component, the brightness temperature r.m.s. is on the angular scale of 40 arcmin, with the width representing the variations when using 81% and 93% of the sky.

polarisation fraction of about 30 percent at high latitude (e.g., Vidal et al. 2015). Therefore, the synchrotron emission has to be treated properly to avoid the contamination of the CMB polarisation signal (Dunkley et al., 2009).

1.3.2 Free-free emission

Free-free emission, also known as the thermal Bremsstrahlung emission, is due to the unbound interactions between ions and free electrons (Rybicki & Lightman, 1979). The interactions come from diffuse ionised media such as HII regions in the interstellar medium of our Galaxy. Free-free emission typically correlates with star-forming regions along the Galactic plane, where the ionised gas is illuminated by nearby stars. Free-free emission is intrinsically unpolarised, and at radio frequencies (∼ 10 MHz—100 GHz), it has a typical electron temperature of $T_e \approx 10^4$ K (Rybicki & Lightman, 1979). From Fig. 1.5, free-free emission (blue) dominates other foreground components at frequencies up to ∼ 70 GHz. However, spatially, free-free emission only
dominates at specific bright HII regions, leaving the synchrotron radiation as the dominant radio continuum emission over the sky at frequencies up to $\sim 30$ GHz (Bennett et al., 2003).

The emission from the optical H\(\alpha\) line (hydrogen \(n = 3 \rightarrow 2\) atomic transition) is a good tracer of free-free emission (Dickinson et al., 2003). The brightness temperature of the radio free-free emission can be related to an optical H\(\alpha\) map through a power law

$$T_{ff} \approx 10\text{mK} \left(\frac{T_e}{10^4\text{K}}\right)^{0.667} \left(\frac{\nu}{\text{GHz}}\right)^{-2.1} \left(\frac{I_{H\alpha}}{R}\right),$$

(1.39)

where \(T_{ff}\) is the free-free emission temperature, \(T_e\) is the electron temperature, \(\nu\) is the desired radio frequency of the free-free emission, and \(I_{H\alpha}\) is the H\(\alpha\) brightness in Rayleighs (Dickinson et al., 2003). At 1 GHz, the mean brightness temperature of the full-sky free-free emission is 65 mK, for an electron temperature of \(T_e = 7000\text{K}\) with a 1$^\circ$ beam width, and drops to 50 $\mu$K at 30 GHz (Olivari, 2018). As also shown in Fig. 1.5, free-free emission is dominated by synchrotron emission at 1 GHz, but becomes comparable to synchrotron at 30 GHz.

1.3.3 Thermal dust emission

Dust grains diffuse throughout the ISM and the effects of them are to absorb the emission from other objects and to re-emit in the infrared band, using the absorbed energy. Thermal dust emission originates from the dust grains in the interstellar medium, that are long enough to be in thermal equilibrium with the interstellar radiation field. The dust grains vibrate through the heating from light emitted by nearby stars, thus emitting their own energy (e.g., Draine & Lee, 1984). Thermal dust emission has been mapped out at several infrared bands by, e.g., the COBE and IRAS missions (Finkbeiner et al., 1999), with the most accurate full-sky mapping given by the Planck high frequency instrument (Planck Collaboration XLVIII, 2016).

The observed flux density of the thermal dust emission is often approximated as a
modified black-body emission such that

\[ I_{dust} = B_\nu(T)\tau_\nu, \]  

(1.40)

where \( B_\nu \) is the black-body emission at a given dust temperature \( T \), and \( \tau_\nu \) is the infrared optical depth, which is linked to the observing frequency by a power law that

\[ \tau_\nu = \tau_0 \left( \frac{\nu}{\nu_0} \right)^{\beta}, \]  

(1.41)

with \( \nu_0 \) and \( \tau_0 \) being a reference frequency and optical depth respectively. At 353 GHz, a typical value of \( \beta \approx 1.6 \) and \( \tau \sim 10^{-6} \) is given by Planck Collaboration XLVIII (2016). From Fig. 1.5, thermal dust emission (red) mainly dominates at high frequencies above 100 GHz.

### 1.3.4 Anomalous microwave emission

Anomalous Microwave Emission (AME) is one of the Galactic emission components, observed between 10 GHz to 60 GHz. It is highly correlated with far-Infrared thermal dust emission (e.g., Kogut et al., 1996; Leitch et al., 1997) in the spatial domain. From Fig. 1.5, it can be seen that in the frequency range of 20 – 50 GHz, the AME frequency spectrum (yellow) has a slope and amplitude that are very similar to that of free-free and synchrotron emission, making it hard to distinguish. The first detection of AME was made by Leitch et al. (1997), who observed the North Celestial Pole (NCP) region at 14.5 GHz and 32 GHz, using the Owens Valley Radio observatory (OVRO) 40 m and 5.5 m telescopes. Their detected dust-correlated foreground emission has a microwave spectral index of \( \beta \sim -2 \), suggestive of free-free emission, but however was at least 60 times stronger than the expected free-free level, when compared with H\( \alpha \) maps of the NCP region. Since then, AME has been observed in many CMB experiments in the frequency range of 10 – 60 GHz (e.g., Banday et al., 2003; Davies et al., 2006; Planck Collaboration X, 2016). The polarisation of AME is expected to be small and indeed
observed to be very low ($\lesssim 1\%$, Dickinson et al. 2018).

The most widely-accepted model for the AME mechanism is the spinning dust model, where the electric dipoles in the small dust grains spin up to higher rotational frequencies, producing radio emission (e.g., Draine & Lazarian, 1998a,b; Dickinson et al., 2018). The spinning dust emission heavily depends on the type, size, distribution and plasma environments of the dust grains, such as the dust grain density and temperature. The AME emissivity, i.e., the rotational emission from a population of spinning dust grains per column density, can be computed using the publicly available SPDUST2 code (Ali-Ha¨ımoud et al., 2009; Silsbee et al., 2011, see Sect. 2.4.3) given specific environment parameters.

AME is one of the dominant foregrounds in the frequency range of 10−60 GHz (e.g., Davies et al., 2006; Planck Collaboration X, 2016; Dickinson et al., 2018). Thanks to its steep falling spectrum above $\approx 20$ GHz and spatial correlation with far-IR data, AME can be separated from the CMB and intensity mapping signal (e.g., HI and CO, to be introduced in Chapt. 2) relatively easily (e.g., Planck Collaboration X, 2016; Olivari et al., 2019).

1.4 Summary and thesis outline

Since the cosmic acceleration was discovered (e.g., Riess et al. 1998), the simplest theory to explain the accelerating expansion of the Universe is the $\Lambda$CDM model (e.g., Einstein, 1917). In order to test these theoretical models, experimental cosmologists have been using several observables to probe the Universe and reveal its expanding history through, e.g., the CMB and large-scale-structures (LSS). The LSS, such as BAOs, can be used as a precise standard ruler to provide 3-D information of the Universe expansion history.

The astrophysical foregrounds place a challenge to cosmological radio surveys. Their emission can be $\sim 10^4$ stronger than the desired cosmological signal (see e.g.,
Fig. 2.10 later). Depending on the observing frequency, different foreground components dominate at different bands. One must be careful in properly subtracting these foreground components in order to detect the desired cosmological signal.

In this thesis, Chapter 2 introduces the intensity mapping techniques in detail along with some of the author's work on intensity mapping systematics. Chapter 3 describes the author's work on cross-correlating CMB maps with LSS to seek for SZ cluster residuals in the used Planck CMB maps. Chapter 4 presents the author's work on auto-correlation intensity mapping forecast, focused on the impact of $1/f$ noise on cosmological parameter constraints. Chapter 5 gives forecasts on cross-correlating intensity mapping with optical surveys. Chapter 6 gives the summary and potential future work.
Chapter 2

Introduction to HI Intensity Mapping

As introduced in Chapt. 1, LSS such as BAOs can be used as a standard ruler to probe the Universe expansion history. A following question is how to effectively detect those LSS, which require observations of large samples of galaxies to overcome the statistical limits. This limitation can be quantified through shot noise, which describes the increased outcome fluctuations with decrease measurements of galaxy samples. Due to the faintness of galaxies, the detection of large samples of individual galaxies is difficult to achieve. Optical surveys with fine resolution optical fibres have been a mature technique for LSS detection, such as the SDSS survey (e.g., Stoughton et al., 2002).

Meanwhile, astronomers have been seeking for an efficient and less costly technique to supplement optical surveys to detect LSS in the radio wave band. Recently, a new technique called HI intensity mapping (e.g., Peterson et al., 2006) has been proposed to fill the gaps in observing galaxies that are inaccessible to current galaxy surveys. Observations in the radio band will have different noise and systematics as the optical band, thus providing independent measurement of LSS.

In this chapter, we introduce the concept of HI intensity mapping in Sect. 2.1. Sect. 2.2 provides an overview of several current intensity mapping experiments. Sect. 2.3 explains the challenges for HI intensity mapping. Sect. 2.3 describes the author’s work on tackling a few systematics for the BINGO and COMAP experiment.
2.1 Concept of intensity mapping

In addition to optical galaxy surveys, it is also possible to detect LSS in the radio waveband, using the emission of spectral lines such as the HI emission line. HI galaxy surveys provide information of the Universe in the radio waveband window as a consistency check with the optical and infrared surveys. However, the largest HI galaxy survey so far, the HI Parkes All-Sky Survey (HIPASS), only achieved a sample size of \( \sim 10^4 \) galaxies (Meyer et al. 2004) due to the faintness of the HI signal (Furlanetto et al. 2006). Although the upcoming Square Kilometre Array (SKA) experiment could potentially increase the sample size to \( \sim 10^9 \) (Abdalla et al. 2015), meanwhile, an alternative way to detect BAO in the radio waveband is through intensity mapping (IM).

The concept of intensity mapping was first proposed by Peterson et al. (2006) but the concept has been discussed earlier (e.g., Madau et al., 1997; Battye et al., 2004). It is a technique that maps a single spectral line emission from large scale structures. It aggregates flux from multiple unresolved individual galaxies, each of which is below the detection limit, but it resolves the large scale structure by measuring intensity fluctuations over cosmological distances. Compared with optical galaxy survey, the advantage of intensity mapping is that it efficiently provides 3-D information where the pixel-to-pixel fluctuation maps the large scale structure and the observation frequency reveals accurate redshift information.

**HI intensity mapping.** Neutral hydrogen was formed during recombination when the Universe was cooled enough and remained the primary baryonic content until reionization. The majority of neutral hydrogen remained in dense gas clouds hosted predominantly within galaxies after reionization. Therefore, neutral hydrogen is a good tracer of galaxy densities and thus reveals the matter density fluctuations in the Universe (e.g., Madau et al., 1997).

The 21 cm emission line comes from the transition of electrons between the two
2.1. CONCEPT OF INTENSITY MAPPING

energy levels, $F = (0,1)$, of the $1s$ ground state in the hyperfine structure of the hydrogen, known as the spin-flip transition. The 21 cm line has a frequency of $\approx 1420$ MHz, equivalent to $\approx 21$ cm in free space, and is redshifted by the expansion of the Universe. It is also the dominant spectral line at frequency below 1420 MHz, isolated as a single emission line with no other bright lines at similar frequencies, allowing a direct translation from frequency to redshift (e.g., Dupays et al., 2003). Therefore, by observing the 21 cm line at different frequencies, one can obtain the 21 cm signal as a function of redshift.

As the most extensively studied spectral line for IM, HI IM is efficient and cheap for detecting BAO and constraining dark energy (e.g., Peterson et al., 2006; Bull et al., 2015; Olivari et al., 2018). The idea is to perform an HI spectral survey in a large voxel to map the fluctuations of HI brightness from unresolved galaxies beyond the beam resolution ($\sim 1^\circ$) (e.g., Dickinson, 2014). This eliminates the need of measuring individual galaxies, while still reveals the large scale cosmic density field behind the galaxy distributions. The HI density field provides the geometric information of the Universe from the mapped LSS. The redshift information of the LSS is given from the observing frequency channels. By combining the 2-D density field and 1-D redshift information, one can deduce the evolution history of the LSS and thus understand the expansion of the Universe by using the LSS as a standard ruler.

**Intensity mapping with other emission lines.** Another abundant molecular species other than HI is CO, of which the emission line is a good tracer for star formation and galaxy evolution (e.g., Lidz et al. 2011; Gong et al. 2011; Vieira et al. 2013; Li et al. 2016). CO emits multiple lines at different frequencies. The lines that are particularly interesting for intensity mapping are $J = 1 \rightarrow 0$ transition at 115 GHz and $J = 2 \rightarrow 1$ transition at 230 GHz, because these redshifted two emission lines can be observed from the ground at, for example, $z \sim 3$ near the peak in the star formation history, and $z \sim 6$ during epoch of reionization (EoR). One advantage of multi-frequency emission
CHAPTER 2. INTRODUCTION TO HI INTENSITY MAPPING

is that by cross-correlating the multiple emission lines, one can separate CO signals from the contaminating foregrounds (e.g., Li et al. 2016).

The CO emission is relatively bright even at high redshift, which enables the detection of CO emission from star forming galaxies at the EoR, which ends at $z \approx 6 \sim 7$. For instance, Carilli & Walter (2013) detected CO emissions from the star-forming gas in bright quasi-stellar objects at redshift up to 6.5. One particular interesting redshift range is during the epoch of galaxy assembly at $z \approx 2 \sim 4$, which is the peak epoch of cosmic star formation (e.g., Carilli, 2011). CO traces the fuel for star formation in galaxies and CO intensity mapping can measure the CO mass density in stars which indicates the star formation rate density. Therefore, by using intensity mapping to detect CO mass densities and applying the CO luminosity — star formation rate relation (e.g., Lidz et al. 2011), it is possible to study the star formation history and constrain galaxy population properties.

Some other potential candidates for IM other than HI and CO lines have also been proposed such as [CII] (e.g., Uzgil et al., 2014; Crites et al., 2014), Ly$\alpha$ (e.g. Pullen et al. 2014), H$\alpha$ (e.g., Silva et al., 2018), and other fine structure lines (e.g., Visbal & Loeb 2010).

2.2 Intensity mapping experiments

Many IM experiments, including both interferometer and single dish, have been proposed, with the first detection made by Chang et al. (2010) who detected the HI signal using IM at $z \approx 0.7$ with the 100 m single dish Green Bank Telescope (GBT) in West Virginia, US. Although an interferometer has a better control of systematics than single dish (e.g., Dickinson 2012), it is however more complicated and cost-consuming in terms of construction. Bull et al. (2015) has shown that for SKA, at the BAO scale, single dish has the advantage, in terms of sensitivity, over interferometer at lower redshift where the dark energy component dominates. In this section, we will introduce a few
2.2. INTENSITY MAPPING EXPERIMENTS

proposed IM experiments, in particular, the BINGO, SKA, and COMAP experiments, which the author has been working on.

2.2.1 BINGO

BAO from Integrated Neutral Gas Observations (BINGO) is a single dish HI IM experiment proposed by Battye et al. (2013) and Dickinson (2014). BINGO will be a pathfinder for the SKA single dish IM experiment. It aims to perform an ultra-sensitive redshifted HI survey at frequencies between 960 MHz and 1260 MHz, which corresponds to a redshift of $0.13 - 0.48$ through the redshift-frequency relation that

$$1 + z = \frac{f_{\text{emit}}}{f_{\text{obsv}}},$$

where $f_{\text{emit}} = 1420$ MHz is the 21 cm line emission frequency, and $f_{\text{obsv}}$ is the observing frequency.

The BINGO telescope is a crossed-Dragone (Compact Range Antenna) configuration with dual mirrors, which is a well established optical design used in many CMB experiments such as QUIJOTE (Génova-Santos et al., 2015). It benefits from minimised cross-polarisation leakage (see Sect. 2.3.5) so that polarised foreground does not contaminate the faint HI signal. The BINGO telescope is currently under construction in Paraiba, north-east of Brazil, which has relatively low radio frequency interference (RFI) contamination. Fig. 2.1 shows the optical design of the BINGO concept where the two mirrors both have a diameter of $\approx 40$ m, corresponding to an angular resolution of $\approx 40$ arcmin ($\theta \approx \frac{\lambda}{D}$), at an observing frequency of 1 GHz. The sky signal will firstly be collected by the primary mirror, reflected to the secondary mirror and finally converged at the focal plane, which consists of $\sim 50$ dual polarisation feed horns and each has a diameter of 1.7 m and length of 4.7 m (Dickinson, 2014; Bigot-Sazy et al., 2015; Battye et al., 2016).

For a single dish experiment, the stability of the receiver system is important for the
successful detection of the signal. We will use a standard correlation receiver system for the BINGO experiment to reduce the $1/f$ noise caused by the gain instability of the receiver system (see Sect. 2.3.3 for a detailed introduction of the $1/f$ noise). For each receiver chain, there will be one main horn collecting the signal from the dish and a cold-load low-noise reference source COLFET for calibration. The output of each receiver chain will be the difference between the main horn and the cold-load, thus subtracting the correlated background instrumental noise but leaving the sky signal (Battye et al., 2013; Dickinson, 2014).

Table 2.1 summarises the BINGO instrumental and observing parameters. The receiver has no cryogenic cooling system in order to reduce the cost and simplify the construction. The system temperature of the BINGO telescope is $\sim 50$ K, which is within expectation for a well-designed system based on room temperature low noise amplifiers (Bigot-Sazy et al., 2015; Battye et al., 2016). The focal plane array of $\sim 50$
2.2. INTENSITY MAPPING EXPERIMENTS

horns has a field of view of $15 \times 15^\circ$. For reasons of stability, simplicity and cost, the telescope is static so relying on the Earth’s rotation, it will perform a drift scan with a $\sim 15^\circ$ stripe of the sky centring on the declination of $-5^\circ$. The choice of the central declination is due to the minimal foreground emission between $\pm 10^\circ$ declination while maximizing the survey area. The sky coverage is $\sim 5000 \text{deg}^2$, which reduces to $\sim 2900 \text{deg}^2$ after masking out the Galactic plane (Olivari et al. 2018). Battye et al. (2013) demonstrated that after $2000 \text{deg}^2$, an increase in survey area will not yield much improvement on the detecting sensitivity of BAOs. This is because after $2000 \text{deg}^2$, the uncertainties from cosmic variance will be within the uncertainties caused by thermal noise so that cosmic variance is no longer the restricting term of a better detection. They also analysed the sensitivity of different frequency ranges at $600 < f < 900 \text{MHz}$ and $960 < f < 1260 \text{MHz}$. The $600 < f < 900 \text{MHz}$ band requires a $\sim 100 \text{m}$ telescope to resolve the smaller BAO scales at higher redshift, which will cost $5 - 10$ times extra but improve little on sensitivity. In contrast, the observing frequency between $960 \text{MHz}$ ($z \approx 0.48$) and $1260 \text{MHz}$ ($z \approx 0.13$) provides information of the late Universe when dark energy dominates and has a higher amplitude of the detected HI signal, with a detecting sensitivity of $\delta k_A/k_A \sim 0.02$ of the acoustic scale (Battye et al., 2013; Bigot-Sazy et al., 2015; Battye et al., 2016). There will be at least 300 frequency channels spreading over the $300 \text{MHz}$ bandwidth, each with a $1 \text{MHz}$ channel width. At this frequency range, Battye et al. (2013) showed that from $\theta_{\text{FWHM}} = 2^\circ$ down to $\theta_{\text{FWHM}} = 40 \text{arcmin}$, there is a significant improvement on detecting sensitivity but little improvement below $40 \text{arcmin}$. This is because there is not much extra information of BAO at smaller scales. They found that the optimal combination of survey parameters is when the thermal noise and cosmic variance are approximately equal. The total proposed on-source integration time will be $\approx 1 \text{year}$, which in practice will take $\sim 2 \text{year}$ to complete due to telescope maintenance, weather condition, RFI, etc (Battye et al., 2013; Dickinson, 2014; Bigot-Sazy et al., 2015; Battye et al., 2016).
While the BINGO telescope is under construction, a few forecasts have been made on the BINGO performance, which have shown BINGO as a promising intensity mapping experiment to detect BAO and constrain dark energy parameter. Bigot-Sazy et al. (2015) shows that BINGO is able to detect the BAO with $\sim 5 - 7 \sigma$ detection and measure the acoustic scale with $\sim 2\%$ accuracy. Using a maximum likelihood method, Olivari et al. (2018) showed that by adding BINGO data to Planck, the current constraint on $h$ can be improved by a factor of $\sim 2$, the constraint on the HI density has the accuracy of $\sim 2\%$, and that of $w$ is $\sim 4\%$. In Chapter 4, we will discuss the further improvement of the constraints by including the redshift-space distortion term into the HI power spectrum calculation, which was not done by Olivari et al. (2018).

### 2.2.2 SKA

The Square Kilometre Array\(^1\) (SKA) is the world largest radio telescope to be built in 2023, with a collecting area of over a square kilometre (e.g., Wilkinson, 1991). The SKA organisation has 10 member countries, with its headquarters at Jodrell Bank Observatory near Manchester in the UK. By the time SKA is operating, it will broaden our current understanding in many fields of astronomy, such as stellar and galactic evolution, cosmology, pulsars, and cosmic dawn.

The majority of the SKA telescope will primarily be hosted in two sites, the Karoo

\(^{1}\)https://skatelescope.org

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<table>
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Table 2.1: Instrumental and observing parameters for BINGO (Bigot-Sazy et al., 2015; Battye et al., 2016; Olivari et al., 2018).
2.2. INTENSITY MAPPING EXPERIMENTS

region in South Africa and Murchison Shire in western Australia. The SKA project will be delivered into two phases. Phase I of SKA (SKA1), due in 2023, will build \( \sim 200 \) dishes including 64 Meerkat Dishes in South Africa and \( \sim 130000 \) individual antennas in Australia. SKA1 has two sub-arrays - SKA1-LOW and SKA1-MID. SKA1-LOW is a low-frequency aperture array, observing at frequencies below 350 MHz. SKA1-MID is a mid-frequency array, consisting of 130 dishes with a diameter of 15 m each, and extending to a 100 km baseline (e.g., Turner et al., 2016). SKA1-MID will have two operation bands, with Band 1 between 350 MHz to 1050 MHz (0.35 < \( z \) < 3), and Band 2 between 950 MHz to 1421 MHz (0 < \( z \) < 0.49) (e.g., Bull et al., 2015). Phase II of SKA (SKA2) will increase the number of SKA1 dishes to \( \sim 2000 \) in South Africa and number of antennas to up to 1,000,000 in Australia. SKA2 aims to improve the sensitivity of SKA1 by a factor of \( \sim 10 \) (Santos et al., 2015; Turner et al., 2016; Bull, 2016).

Bull et al. (2015) assessed the ability of SKA1 for conducting IM survey and suggested SKA1 be operated in the auto-correlation mode, where the total power of all dishes are added together as a single dish. They plot the SKA sensitivity as a function of redshift and scale as shown in Fig. 2.2. They found that at BAO scales, the auto-correlation mode is more sensitive than interferometer mode at lower redshift (\( z \lesssim 1 \)). In a following paper, Bull (2016) showed that if operated at the auto-correlation mode, SKA1-MID Band 2 IM survey gives the best prospect of constraining cosmological parameters, comparable with Euclid, thanks to its high precision at \( z \approx 0 \) where dark energy dominates. By comparison, SKA1-MID Band 1 IM survey is less competitive than HI galaxy surveys at the same redshift range but, however, useful as a part of a future 'multi-tracer' strategy, due to its large survey volume. In contrast, an IM survey with SKA1-LOW operated in the interferometric mode is hardly competitive with optical galaxy surveys in terms of improving parameter constraints. However, SKA1-LOW IM survey will be unique at \( z \gtrsim 3 \) since no optical galaxy survey has yet reached to such high redshift (e.g., Square Kilometre Array Cosmology Science
CHAPTER 2. INTRODUCTION TO HI INTENSITY MAPPING

Santos et al. (2015) also showed that if the foreground and instrumental noise are carefully cleaned from the data, IM survey with SKA1-MID in auto-correlation mode can provide tight constraints on dark energy, curvature of the Universe, and modified gravity models. Therefore, in this thesis, we will only focus on using the SKA1-MID as our SKA IM specifications. The instrumental and observing parameters for both bands of SKA1-MID are summarised in Table 2.2, based on Santos et al. (2015) and Turner et al. (2016).

![Figure 2.2: The sensitivity of SKA as a function of scale and redshift for interferometer (blue) and single dish mode (red) (Bull et al., 2015). The dashed vertical lines highlight the BAO scales so that at higher redshift interferometer is more sensitive than single dish and vice versa. In this case, the dish diameter is $D_{\text{dish}} = 15$ m, survey bandwidth is $\Delta \nu = 600$ MHz, and survey area is $S_{\text{sky}} = 25000$ deg$^2$. The minimum and maximum interferometer baselines are $D_{\text{min}} = 15$ m and $D_{\text{max}} = 1000$ m.](image)

2.2.3 COMAP

The CO Mapping Array Pathfinder (COMAP) is a single dish CO IM experiment led by CALTECH, with collaborations from JPL, Stanford, Maryland, Manchester and Oslo (e.g., Li et al., 2016). The goal of COMAP is to map the CO signal at a frequency of...
### 2.2. INTENSITY MAPPING EXPERIMENTS

#### Table 2.2: Instrumental and observing parameters for SKA1-MID (Santos et al., 2015; Turner et al., 2016; Harper et al., 2018).

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<tr>
<td>Dish diameter, $D_{\text{dish}}$ (m)</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Number of dishes, $n_{\text{dish}}$</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Number of beams, $n_{\text{beam}}$</td>
<td>$1 \times 2$</td>
<td></td>
</tr>
<tr>
<td>Sky coverage area, $A_{\text{sky}}$ (deg$^2$)</td>
<td>20500</td>
<td></td>
</tr>
<tr>
<td>Integration time, $t_{\text{obs}}$ (yr)</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>System temperature, $T_{\text{sys}}$ (K)</td>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td>Frequency range, [$\nu_{\text{min}}, \nu_{\text{max}}$] (MHz)</td>
<td>[350, 1050]</td>
<td>[950, 1410]</td>
</tr>
<tr>
<td>Redshift range, [$z_{\text{min}}, z_{\text{max}}$]</td>
<td>[0.35, 3.0]</td>
<td>[0, 0.49]</td>
</tr>
<tr>
<td>Beam width, $\theta_{\text{FWHM}}$ (arcmin)</td>
<td>122</td>
<td>66</td>
</tr>
</tbody>
</table>

26 – 34 GHz, corresponding to a redshift of $2.4 < z < 3.4$ for the CO $J = 1 \rightarrow 0$ emission line, and $5.8 < z < 6.7$ for the CO $J = 2 \rightarrow 1$ emission line (e.g., Moradinezhad Dizgah et al., 2018). By cross-correlating with galaxy surveys in the same redshift, COMAP will enrich our current knowledge of star formation and gas content in the 'epoch of galaxy assembly' (Chung et al. 2018).

The instrumental and observing parameters of COMAP are summarised in Table 2.3 (Li et al., 2016; Chung et al., 2017). COMAP will be delivered in two phases. Phase I is to use an existing $\sim 10.4$ m dish at Owens Valley Radio Observatory (OVRO), which was previously used for the Millimeter Array CARMA. The focal-plane has 19 pixels with dual-polarisation sensitivity, centred at 30 GHz. Phase I will observe a 2.25 deg$^2$ of the sky, with a total integration time of 6000 hours. The phase II of COMAP will increase the number of feed horns to 5 times that of phase I and the observed sky area will be increased to 25 deg$^2$ in total, with a better sensitivity (e.g., Olivari et al., 2019). Depending on funding, there is also a possibility of adding an antenna at 15 GHz, to allow the detection of higher-redshift signal, targeting the Epoch of Reionization. The analysis performed in Sect. 2.3.5 is based on an initial proposal of COMAP at the early stage of preparing the experiment. The proposed instrumental and observing parameters are listed in Table 2.3. The initial idea was to use an existing 6 m dish to observe 6.8 deg$^2$ of the sky, divided into 4 patches, each with an
area of 1.7 deg$^2$ and an integration time of 1500 hours per patch (e.g., Li et al., 2016).

Li et al. (2016) forecasts the performance of COMAP based on a simulated CO intensity field and detects the CO signal at $\sim 8\sigma$ and $144\sigma$ significance for the pathfinder and fully commissioned experiment respectively. The measured CO power spectrum will reveal the relation between CO luminosity and halo mass, which will place tight constraints on the underlying galaxy population. Moradinezhad Dizgah et al. (2018) used Fisher matrix to forecast the potential of COMAP to constrain non-Gaussianity and found that by using the three-dimensional volume of the COMAP survey Phase II, together with the Planck prior, it is capable of constraining the non-Gaussianity with an uncertainty of $\sigma(f_{NL}^{loc}) = 3.7$ assuming a 2000 deg$^2$ sky coverage.

### 2.2.4 Other experiments

**GBT** Chang et al. (2010) was the first to detect the 21 cm intensity field using the Green Bank Telescope (GBT), which is a 100 m single dish telescope in West Virginia, US. They observed over the redshift range between 0.53 and 1.12, with an angular resolution of 15 arcmin. They found that the detection of HI density field is significantly limited by radio-frequency interference (RFI), Galactic foreground and extragalactic radio sources. They managed to measure the HI brightness temperature of $157 \pm 42 \mu$K at the mean effective redshift of 0.8, through cross-correlating with the DEEP2 optical galaxy survey. From the cross-correlation function, they placed a constraint on the HI
density \( r b_{rel} \Omega_{\text{HI}} = (0.55 \pm 0.15) \times 10^{-3} \), where \( r \) is the correlation coefficient between optical galaxy and HI gas, and \( b_{rel} \) is the relative bias between HI gas and the DEEP2 optical galaxies so that \( b_{rel} = b_{\text{HI}} / b_{\text{opt}} \) with \( b_{\text{HI}} \) (\( b_{\text{opt}} \)) to be the HI(optical) bias.

Masui et al. (2013) extended the work of Chang et al. (2010) by observing a larger sky area (41 deg\(^2\) vs. 2 deg\(^2\)) at redshift between 0.6 and 1, and improved the results in Chang et al. (2010) both in its precision and in the probed range of scales. By cross-correlating their intensity field with WiggleZ optical data, they achieved a constraint on the HI density of \( r b_{\text{HI}} \Omega_{\text{HI}} = [0.43 \pm 0.07 \text{(stat.)} \pm 0.04 \text{(sys.)}] \times 10^{-3} \). Their measurement is limited by both the shot noise from WiggleZ fields and calibration systematic errors from radio observation.

Switzer et al. (2013) used the observed data of Masui et al. (2013) and computed the HI auto-power spectrum. They claimed that apart from RFI and astrophysical foreground, the main challenge of the HI signal detection is the imperfect instrument, such as the bandpass mis-calibration and the polarisation leakage to intensity, which will lose the smooth spectral structure of the astrophysical foreground and thus affect the effective removal of foreground to recover the weak HI signal. They measured \( \Omega_{\text{HI}} b_{\text{HI}} \) from the detected auto-spectrum, and treated it as the upper bound since it was biased by a positive amplitude from residual foreground contamination. They interpreted the measurement of \( \Omega_{\text{HI}} b_{\text{HI}} r \) from Masui et al. (2013) as the lower bound on \( \Omega_{\text{HI}} b_{\text{HI}} \), since \( |r| < 1 \). By combining both the auto-spectrum and the cross-spectrum into a posterior distribution on \( \Omega_{\text{HI}} b_{\text{HI}} \) through Bayes’ theorem, they broke the degeneracy between \( \Omega_{\text{HI}} \) and the correlation coefficient \( r \) and constrained \( \Omega_{\text{HI}} b_{\text{HI}} = 0.62^{+0.23}_{-0.15} \times 10^{-3} \).

CHIME The Canadian Hydrogen Intensity Mapping Experiment is an Canadian-based interferometer HI intensity mapping experiment to detect BAO at the redshift between 0.8 to 2.5 (Bandura et al. 2014). The proposed array consists of four 20 × 100 m cylinders and will map \( \sim 20000 \text{ deg}^2 \) of the sky. A pathfinder version of CHIME, i.e., a 1/10th scale prototype, is commissioned, which has two 37 × 20 m cylinders with a
field of view of $\sim 100$ degrees by $\sim 2$ degrees.

**FAST** The Five-hundred-meter Aperture Spherical radio Telescope (FAST) is the largest single dish radio telescope in the world (Nan et al. 2011). It is based in the southwest of China and has finished its construction in 2017. FAST telescope has an effective aperture of 300 meters covering a large frequency range between 70 MHz and 3 GHz (Nan et al. 2011). Smoot & Debono (2017) and Bigot-Sazy et al. (2016) have discussed the potential of FAST to conduct a 21 cm intensity mapping survey and showed that HI IM with FAST can be competitive to detect BAO and constrain dark energy parameters. Bigot-Sazy et al. (2016) however pointed out that FAST is only slightly better than BINGO at small scales, based on the simulated BAO power spectrum. Given that BINGO is designed specifically for HI IM while FAST is a general-purpose telescope, it is unlikely that FAST will have much dedicated time for intensity mapping as bespoke experiments such as BINGO. Therefore, BINGO is expected to perform better than FAST in terms of conducting HI IM surveys.

**HIRAX** The Hydrogen Intensity and Real-time Analysis eXperiment (HIRAX) is proposed by the University of KwaZulu-Natal in South Africa as an interferometer array to conduct HI IM (Newburgh et al. 2016). It will contain 1000 dishes, each of which has a diameter of $\sim 6$ m. The observing frequency will be around $400 – 800$ MHz, mapping most of the southern sky. HIRAX and CHIME have the same redshift range of $z = 0.8 – 2.5$ but observe in different hemispheres. The first stage of the project is to build 128 dishes in Karoo and eventually extend to 1000 dishes (Newburgh et al. 2016).

**TIANLAI** The TIANLAI (Sound from Heaven in Chinese) project is an interferometric IM experiment based in China (e.g., Chen, 2012). The Phase I of TIANLAI consists of sixteen 5 m dishes and three adjacent cylindrical reflectors, each with 15 m wide and 40 m long, equipped with 96 dual polarization receivers. The Phase II of
TIANLAI project will consist of 8 cylinders, each with 15 m wide and 120 m long, and 2000 dual polarization receivers in total. The observing frequency is between 400–1420 MHz, corresponding to a redshift of between 0 and 2.5 (Chen, 2012; Xu et al., 2015; Huang et al., 2018).

**PAPER** The Precision Array to Probe the Epoch of re-ionization (PAPER) is an interferometric instrument aiming to detect 21 cm signal at EoR time (e.g., Parsons et al., 2010). PAPER consists of 16 antennas on a 300 m diameter circle at the Green Bank Galford Meadow site, 4 antennas with a maximum baseline of 150 m arranged in a trapezoid pattern in Western Australia, and 64 antennas in a square pattern in South Africa. The observing frequency is between 100 and 200 MHz, corresponding to a redshift range of $6 < z < 13$ (Parsons et al., 2010; Ali et al., 2015). Ali et al. (2015) placed a 2σ upper limit of 22.4 mK$^2$ on the HI power spectrum in the range $0.15 < k < 0.5 \ h \ Mpc^{-1}$ at $z = 8.4$, based on a 135 day period observation using the 64 antennas of PAPER.

**LOFAR** The LOw-Frequency ARray (LOFAR) is a general-purpose interferometer primarily based in the North of the Netherlands with stations across the Europe (e.g., van Haarlem et al., 2013). LOFAR covers the low-frequency range between 10 to 240 MHz. The scientific motivation of LOFAR is very broad and the detection of highly redshifted 21 cm signal is one of the most interesting drivers behind it. The high band of LOFAR will be able to detect the EoR signal from $z = 11.4$ (115 MHz) to $z = 6$ (203 MHz) and the the low band is to detect 21 cm signal at around $z \sim 20$ from the Cosmic Dawn (van Haarlem et al., 2013; Patil et al., 2017). Patil et al. (2017) conducted a 13 hours of observation using LOFAR and detected a best 2σ upper limit of 79.6 mK$^2$ of the HI power spectrum at $k = 0.053 \ h \ C\text{Mpc}^{-1}$ at $9.6 < z < 10.6$. They claimed that the main challenges of their detection are from the incompleteness of the sky model used for the calibration, the residual calibration inaccuracy, and the instability of the receiver gains.
The Murchison Widefield Array (MWA) is an interferometer located in Western Australia, the planned SKA1-LOW site (Bowman et al. 2013). The minimum and maximum baseline of MWA are 7.7 and 2864 m respectively, and it operates between 80 and 300 MHz, designed with the goal of detecting 21 cm signal from EoR, spanning the scale of $0.01 \lesssim k \lesssim 1 \text{ c Mpc}^{-1}$. The sensitivity of MWA is able to measure the power spectrum of the 21 cm signal at the redshift between 6 and 10, targeting at relatively high Galactic latitudes, centred at $b = -49^\circ$ and $b = +38^\circ$, respectively (Bowman et al. 2013). Dillon et al. (2014) placed a $2\sigma$ upper limit of 0.3 K at $z = 9.5$ and $k = 0.046 \text{ c Mpc}$, based on a 9 days observation using the 32-tile prototype of MWA.

The Hydrogen Epoch of Reionization Array (HERA) is an interferometer designed to measure the 21 cm power spectrum throughout period of EoR ($5 < z < 27$). The HERA instrument will comprise 350 14-m parabolic dishes in South Africa, 19 of which have been deployed and 18 of which are currently under construction. The operation frequency will be between 50 and 250 MHz (DeBoer et al. 2017).

The Owens Valley Long Wavelength Array (OVRO-LWA) is an interferometer with 288 dual polarization antennas located in California. Of the 288 antennas, 251 are placed within a 200 m diameter core, 32 are outliers to extend the maximum baseline length to $\sim 1.5$ km, and 5 are calibrators. LWA is a low-frequency instrument operating between 27 MHz to 85 MHz, aiming to measure the highly-redshifted 21 cm signal from the Cosmic Dawn at redshift around 20 (Eastwood et al., 2016, 2018).

The HI IM experiments introduced in this section are summarised in Table 2.4. The redshift range of the current proposed experiments expands from low redshift, where the dark energy dominates, to the high redshift at the epoch of reionization. This wide redshift coverage makes HI IM as a competitive technique for studying the broad range of physics of our Universe and a new frontier in modern cosmology. Different IM experiments, e.g., using an interferometer vs. a single dish, at the same redshift range can
2.3 Challenges for intensity mapping

Due to the faintness of the emission line signal, the success of intensity mapping experiment relies primarily on two aspects - (i) the effective removal of foreground contamination, and (ii) the sensible control of the instrumental noise and systematic errors. In this section, we will briefly overview some of the key challenges for intensity mapping experiments.

2.3.1 Foregrounds

The effective removal of the Galactic foregrounds is the key to the success of IM experiments, since they are at least $\sim 10^4$ stronger than the HI or CO signal (e.g., Olivari et al., 2016, 2019). As introduced in Sect. 1.3, the Galactic emission contains four components: the synchrotron emission, free-free emission, thermal dust emission, and spinning dust emission. In addition, the extragalactic radio sources, i.e., bright point sources and faint background sources, also contribute to the foreground contamination. At the HI observing frequency of $\sim 1$ GHz, the most relevant foreground contamination

<table>
<thead>
<tr>
<th>Project</th>
<th>Frequency (MHz)</th>
<th>Redshift</th>
<th>Location</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAST</td>
<td>70 – 3000</td>
<td>0 – 2.5</td>
<td>China</td>
<td>Single Dish</td>
</tr>
<tr>
<td>SKA-MID</td>
<td>350 – 1241</td>
<td>0 – 3</td>
<td>South Africa</td>
<td>Single Dish</td>
</tr>
<tr>
<td>BINGO</td>
<td>960 – 1260</td>
<td>0.1 – 0.5</td>
<td>Brazil</td>
<td>Single Dish</td>
</tr>
<tr>
<td>GBT</td>
<td>660 – 960</td>
<td>0.5 – 1.1</td>
<td>US</td>
<td>Single Dish</td>
</tr>
<tr>
<td>TIANLAI</td>
<td>400 – 1420</td>
<td>0 – 2.5</td>
<td>China</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>CHIME</td>
<td>400 – 800</td>
<td>0.8 – 2.5</td>
<td>Canada</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>HIRAX</td>
<td>400 – 800</td>
<td>0.8 – 2.5</td>
<td>South Africa</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>SKA-LOW</td>
<td>5 – 350</td>
<td>3 – 27</td>
<td>Australia</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>HERA</td>
<td>50 – 250</td>
<td>5 – 27</td>
<td>South Africa</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>MWA</td>
<td>80 – 300</td>
<td>6 – 10</td>
<td>Australia</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>LOFAR</td>
<td>10 – 240</td>
<td>6 – 11.4</td>
<td>Netherlands</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>PAPER</td>
<td>100 – 200</td>
<td>6 – 13</td>
<td>South Africa</td>
<td>Inteferometer</td>
</tr>
<tr>
<td>LWA</td>
<td>27 – 85</td>
<td>16 – 20</td>
<td>US</td>
<td>Inteferometer</td>
</tr>
</tbody>
</table>

Table 2.4: Summary of proposed HI intensity mapping projects.

also provide consistent check with each other, since they have different systematics.
is from the \textit{synchrotron} emission, \textit{free-free} emission and extragalactic point sources. In particular the Galactic \textit{synchrotron} emission, which has a typical brightness temperature of $\sim 10$ K at $\sim 1$ GHz, is at least four orders of magnitude stronger than the HI signal, which is typically $\sim 1$ mK at this frequency (e.g., Battye et al., 2013; Olivari et al., 2016). For CO emission at $\sim 30$ GHz, the spinning dust emission will contribute in addition to the \textit{synchrotron}, \textit{free-free} and point source emission. An effective component separation method is thus essential to extract the desired HI or CO signal from the foreground contaminations.

The main characteristic that enables the separation of foregrounds from the HI (or CO) signal is the smooth frequency spectra of the foregrounds, which can be approximated by power-laws along frequency range (e.g., Ansari et al., 2012; Liu & Tegmark, 2012; Olivari et al., 2016). While the HI (or CO) signal is uncorrelated in frequency domain, it is this unique property of foreground spectra that makes it possible to distinguish the foregrounds from the HI (or CO) signal.

There are many component separation methods in the literature for HI intensity mapping experiments. Some of them require prior knowledge to build a parametric model in order to describe the physical properties of the foregrounds, such as the Karhunen-Loeve Decomposition (Shaw et al. 2014). Others do not use a parametric model prior but only observed data, such as Principal Component Analysis (Masui et al., 2013; Switzer et al., 2013; Alonso et al., 2015; Bigot-Sazy et al., 2015), Independent Component Analysis (Alonso et al., 2015), and FastICA (Wolz et al., 2014).

Of particular notice is the Generalized Needlet Internal Linear Combination (GNILC), which is a non-parametric component separation method that uses not only the frequency property of foregrounds but also the angular and spatial information to recover the HI signal from the observed data. The GNILC method works in two main steps. Firstly, it uses a pre-known HI angular power spectrum as a prior to determine the effective subspaces in space and angular scale where the observed data is comparable to the expected HI power spectrum. Secondly, it perform a multidimensional
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filter within the effective subspaces to reconstruct the HI signal, by assigning a weight to each frequency channel to minimize the total variance of all frequency maps. Olivari et al. (2016) has shown that GNILC is robust to recover the HI signal from their simulated complexity of the foregrounds. At the BINGO redshift of $z \sim 0.25$, it can reconstruct the HI power spectrum with 6% accuracy at the BAO scales. By applying GNILC to the COMAP simulation, Olivari et al. (2019) showed that GNILC can recover the CO signal with a bias relative to the input power spectrum of $-0.76\%$ for COMAP phase I and of $-0.22\%$ for phase II.

2.3.2 Thermal noise

Thermal noise defines the fundamental sensitivity of an instrument. It is the voltages generated by thermal agitations in the resistive components of the receiver. It is also termed as ‘white’ noise since the amount of power per unit bandwidth is independent of frequency, and can be characterised as a Gaussian field. The thermal noise is calculated by the radiometer equation (Wilson et al. 2009)

$$\sigma_T = A \frac{T_{sys}}{\sqrt{t_{pix}\Delta\nu}},$$

(2.2)

where $T_{sys}$ is the entire system temperature detected by the receiver, including noise from the receiver, ground, and the desired signal, etc. $\Delta\nu$ is the frequency bandwidth and $A$ is a constant which has a different value depending on the specific receiver system. For a total-power single-dish system with dual-polarisation sensitivity, we have $A = 1$. The integration time per pixel $t_{pix}$ is given by

$$t_{pix} = t_{obs}n_{beam}n_{dish}\frac{\Omega_{pix}}{\Omega_{sur}},$$

(2.3)

where $t_{obs}$ is the total integration time, $n_{beam}$ is the number of beams, $n_{beam}$ is the number of dishes, $\Omega_{sur}$ is the survey area, and $\Omega_{pix}$ is the beam area, which is approximately the square of the beam resolution so that $\Omega_{pix} \approx \theta^2_{FWHM}$. 
CHAPTER 2. INTRODUCTION TO HI INTENSITY MAPPING

The radiometer equation in Equ. 2.2 is the optimal noise in a given system. Of course, different receiver systems will have a different thermal noise level but no system can avoid the thermal noise. At least, the thermal noise will only increase the overall system noise baseline without disordering the signal in the frequency domain. To make a detection of the real signal, the least requirement is to have a system with its thermal noise level lower than the signal brightness temperature, which can be achieved by, for example, increasing the integration time and frequency bandwidth.

2.3.3 The receiver $1/f$ noise

In a radio telescope, the receiver $1/f$ noise is caused by gain fluctuations, $\Delta G(t)/G$, in the receiver system (e.g., Nyquist, 1928). On long time scales, this gain fluctuations can dominate the Gaussian thermal noise due to its ambient temperature changes, quantum fluctuations in transistors, and variations in power voltage which makes it look like a signal, and can thus mimic or obscure the real signal. When viewed on the observed map, the $1/f$ noise will introduce stripes in the map along the direction of the drift scan (e.g., Bigot-Sazy et al., 2015). Therefore, for the detection of a weak signal such as HI, one should take extra care to mitigate the $1/f$ noise.

Unlike the thermal noise, the $1/f$ noise has more fluctuating power on longer timescales, meaning that its power spectrum increases inversely with frequency, which is why it is termed as $1/f$ noise, also known as 'flicker' or 'pink' noise (e.g., Nyquist, 1928). The $1/f$ noise is independent from the thermal noise so for a receiver system that is contaminated by both, its power spectral density (PSD) is the quadratic addition of these two such that (e.g., Seiffert et al., 2002; Bigot-Sazy et al., 2015; Harper et al., 2018)

$$\text{PSD}(f) = \sigma_t^2 \left[ 1 + \left( \frac{f}{f_{\text{knee}}} \right)^\alpha \right]$$

(2.4)

where the first term in the bracket denotes the contribution from the thermal noise and the second power-law term is from the $1/f$ noise. $f_{\text{knee}}$ is known as the knee
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frequency, at which the $1/f$ noise has the same amplitude as the thermal noise, and
\(\alpha\) is the spectral index of the $1/f$ noise with \(\alpha > 0\). Fig. 2.3 shows the PSD of a
receiver system where there are both thermal noise and $1/f$ noise, with \(f_{\text{knee}} = 0.5\) Hz
and \(\alpha = 1\). It can be seen that while the thermal noise is constant at \(\sigma_t^2\), the $1/f$ noise
drops down along frequency following a power law and crosses the thermal noise at
the knee frequency of \(f_{\text{knee}} = 0.5\) Hz. The total noise of these two is dominated by the
$1/f$ (thermal) noise before (after) the knee frequency.

\[
\text{Harper et al. (2018) introduces an extra term, } H(\omega), \text{ in the } 1/f \text{ noise PSD to take}
\text{into account the correlation of } 1/f \text{ noise along frequencies such that}
\]

\[
\text{PSD}^{\frac{1}{f}}(f) = \left(\frac{f_{\text{knee}}}{f}\right)^{\alpha} \left(\frac{\omega}{\omega_0}\right)^{-\frac{1-\beta}{\beta}}, \quad (2.5)
\]
where $H(\omega)$ adopts a power-law model

$$H(\omega) = \left(\frac{\omega_0}{\omega}\right)^{1-\beta/\beta}, \quad (2.6)$$

where $\omega$ is the Fourier mode of the spectral frequency, i.e., the wavenumber. For a system with $N$ frequency channels and a bandwidth of $\delta \nu$ each such that the total bandwidths is $\Delta \nu = N \delta \nu$, the values of $\omega$ range from the smallest, $\omega_0 = \frac{1}{N \delta \nu} = \frac{1}{\Delta \nu}$, to the largest, $\omega_N = \frac{1}{\delta \nu}$. The correlation parameter $\beta$ describes the correlation of $1/f$ noise in frequency and has a value between 0 and 1. For $\beta = 0$, the $1/f$ noise is perfectly correlated across all frequency channels and for $\beta = 1$, the $1/f$ noise is entirely uncorrelated in frequency.

![Time-frequency plots of $1/f$ noise from Harper et al. (2018) at $\beta = 0.25$ (left) and $\beta = 0.5$ (right) respectively. They other parameters are $f_{knee} = 0.5$ Hz and $\alpha = 1$. The horizontal axis is along time and the vertical axis expands over the 950 – 1410 MHz frequency band for simulated SKA1-MID Band2 IM experiment. The left panel is more stripy than the right panel due to a more correlated $1/f$ along frequencies at a smaller $\beta = 0.25$.](image)

Fig. 2.4 shows an example of correlated $1/f$ noise along frequencies from Equ. 2.5 and Equ. 2.6 for an SKA IM simulation in Harper et al. (2018). The horizontal axis is along time and the vertical axis expands over the 950 – 1410 MHz frequency band for simulated SKA1-MID Band2 IM experiment. Two examples at $\beta = 0.25$ (left)
and $\beta = 0.5$ (right) are plotted respectively, with other parameters set to $f_{\text{knee}} = 1$ Hz and $\alpha = 1$. It can be seen that $\beta = 0.25$ gives a more correlated $1/f$ noise along frequencies, shown as stripes along the vertical direction in the left panel of Fig. 2.4. In contrast, a larger $\beta$ value at $\beta = 0.5$ is less stripy in the right panel, due to a less correlated $1/f$ noise along frequencies. For the benefit of component separation, one would prefer $\beta = 0$ so that the component separation method can use this frequency correlation information to subtract the $1/f$ noise from the signal like for other smooth foregrounds.

2.3.4 Atmospheric noise

Apart from the receiver $1/f$ noise, another type of noise comes from the atmosphere. When a ground-based telescope observes the astronomical signal behind the atmosphere, the atmosphere will affect the observation mainly in three aspects - (i) by adding to the total system temperature, (ii) by absorbing the signal, and (iii) by emitting its own signal. The atmospheric effect can be quantified with the radiative transfer function as

$$I_{\text{obs}} = I_{\text{source}}e^{-\tau} + I_{\text{atm}}(1 - e^{-\tau}),$$

where $I_{\text{obs}}$ is the observed intensity at the antenna, $I_{\text{source}}$ is the actual intensity of the source, $I_{\text{atm}}$ is the emitted intensity from the atmosphere, and $\tau$ is the optical depth, which varies with the local weather and the observing frequency (e.g., Lenoble, 1985). The main source of atmospheric turbulence is from the troposphere, with the oxygen and the precipitable water vapor being the two main contributors to the optical depth at the HI observing frequency of $\sim 1$ GHz (Wilson et al., 2009).

Fig. 2.5 shows the atmospheric optical depth as a function of observing frequency at different humidity conditions - 0% humidity (blue), 50% humidity (green) and 100% humidity (red). The optical depths are computed using the am software (Paine, 2018).
At 0% humidity (blue), the main contributor of the optical depth in the plotted frequency range is Oxygen, which increases optical depth with frequency towards its peak at 60 GHz. With increased humidity, the precipitable water vapor comes into effect, giving a peak at 22 GHz, due to the H$_2$O rotational line. At frequencies below 22 GHz, the optical depth is very low ($\tau \ll 1$) but at higher frequencies, e.g., $\sim 30$ GHz for COMAP, the average optical depth is $\tau \sim 0.03 - 0.04$ but can be much larger depending on local weather (Smith, 1982).

![Figure 2.5: The atmospheric optical depth as a function of observing frequency at 0% humidity (blue), 50% humidity (green) and 100% humidity (red).](image-url)

Apart from increasing the total system noise level, the main challenge from atmosphere for radio observations is that it introduce fluctuations. Because of the inhomogeneous distribution of the water vapor, the fluctuations of the atmospheric noise change within a short timescale and vary spatially. This atmospheric turbulence results in a $1/f$ noise-like component in the time-ordered data and the changeable fluctuation of the water vapor can dominate the HI signal and the thermal noise. Therefore, it is important to quantify the atmospheric turbulence and understand how it will affect the HI signal detection. In Sect. 2.4.2, we will describe the author’s work on quantifying...
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the atmospheric effect on the BINGO experiment.

2.3.5 Polarisation leakage

The intensity of the incident signal can be decomposed into two orthogonal components, e.g., $E_X$ and $E_Y$, and received by two orthogonal feeds at the antenna. Taking the two components $E_X$ and $E_Y$ as an example, the polarisation of the signal in terms of the Stoke parameters is calculated by the combination of the two orthogonal components such that

\begin{align*}
I &= \langle E_X E_X^* \rangle + \langle E_Y E_Y^* \rangle \\
Q &= \langle E_X E_Y^* \rangle - \langle E_Y E_X^* \rangle \\
U &= \langle E_X E_Y^* \rangle + \langle E_Y E_X^* \rangle \\
V &= -i(\langle E_X E_Y^* \rangle - \langle E_Y E_X^* \rangle),
\end{align*}

where $I, Q, U, V$ are the Stokes parameters with $I$ the total intensity, $Q$ and $U$ the perpendicular components of linear polarisation, and $V$ the circular polarisation component (e.g., Hamaker & Bregman, 1996; Hull & Plambeck, 2015). For an ideal receiver system, the orthogonal components, $E_X$ and $E_Y$, of the incident signal should be measured by two separate orthogonal feeds, and are thus independent. However, in reality, due to the imperfections of the receiver system, there will be some leakage between the two feeds, leading to the leakage between the four Stokes parameters.

For the detection of polarised signal, one is usually concerned about the leakage from total intensity into polarised intensity, since the total intensity is mostly much stronger than the polarised intensity (e.g., Planck Collaboration X, 2016). However, for intensity mapping experiment, due to the faintness of the HI signal, the opposite, i.e., the reverse polarisation leakage where the polarised intensity leaks into the total intensity, is more of a problem to contaminate the HI signal. As mentioned in Sect. 2.3.1, the
CHAPTER 2. INTRODUCTION TO HI INTENSITY MAPPING

Galactic foreground, which is at least $10^4$ times stronger than the HI signal, has a typical polarisation fraction of 30% and can be as high as 50% (e.g., Wolleben et al., 2006; Vidal et al., 2015). A small amount of reverse polarisation leakage is thus enough to contaminate the HI signal and affect the accuracy of the measurements. Alonso et al. (2015) used simulation to show that the reverse polarisation leakage can reduce the Galactic foreground smoothness, and thus affect the effectiveness of component separation for foreground removal. Therefore, one must take a careful control of the receiver system to watch for the reverse polarisation leakage when conducting intensity mapping experiment.

2.3.6 Beam imperfection

Antennas collect most power over a limited range of angles, known as the so-called 'primary or main beam' width, which is defined as approximately the full width at half maximum (FWHM) on either side of the straight ahead directions. For an ideal telescope with a parabolic antenna, when the incident radio waves strike the surface, the parabolic shape will ensure the waves are spherical and converging on the feed horns at the prime focus. This assumes that the feed horns collect power uniformly from all points across the parabolic surface and all of the collected power is from the main beam (Popping & Braun, 2007).

However, in reality, the real beam is not only confined to the main beam, but also spread out over the full $4\pi$ rad$^2$ due to diffraction at edges of the telescope. Although the antenna is primarily sensitive to the main beam, it will also get contributions from outside of the main beam, known as “sidelobes” (Hartmann et al., 1996). Sidelobes are caused by, e.g., the irregularities of the parabolic surface, and the blockage due to the focal legs and receiver cabin (Napier, 1999). Sidelobes will weaken the real signal by, e.g., losing the coherence of the signal since the reflecting surface irregularity will introduce phase errors in the incident radio wave front. It will add correlation between spatial and frequency components due to its changing with frequency (e.g., Harper &
2.3. CHALLENGES FOR INTENSITY MAPPING

The ground signal picked up by the edge of the aperture will also contaminate the real signal.

In order to reduce the sidelobes, the feed horn is designed to collect less power from the edge of the aperture, the so-called “taper” (e.g., Wilson et al., 2009). This will reduce the side lobes and unwanted ground pick-up but it will also reduce the aperture efficiency, resulting in a lower resolution.

2.3.7 Mis-calibration

Different telescopes have different responses to the same source and even the measurements from the same telescope continuously vary due to, e.g. gain fluctuations. Therefore, to obtain comparable measurements, a universal system is needed to understand the equivalence between the measurements taken at different telescopes, or one telescope at different time (O’Neil, 2002). This can be achieved by observing a bright source with known flux. The scale measured by the telescope is adjusted to match the known flux of the calibrator. This is the absolute calibration of the receiver system (e.g., Wilson et al., 2009).

There are several other forms of calibration. For example, one has to perform the relative calibration, where the gain differences caused by the imperfect receiver system need to be calibrated. Bandpass calibration is another important step. An ideal bandpass filter is like a top-hat window function which is 1 for the desired band and 0 elsewhere. However, in practice, the bandpass response fluctuates, which will result in, e.g., the foreground components losing their smooth frequency spectrum and thus placing further challenges on component separation techniques (e.g., Burke & Graham-Smith, 2009). One might also need to perform polarisation calibration, where the scales measured by the polarisation feeds need to be adjusted to match the polarised intensity from a known calibrator (e.g., Chen, 2015).

The previously mentioned IM experiment using GBT (Switzer et al., 2013) and
LOFAR (Patil et al., 2017) both claimed that the mis-calibration, e.g., the gain variations and bandpass imperfection, to be one of the main limiting factors for the detection of the HI signal. Therefore, one needs to be very careful in tackling calibration errors for IM experiments.

### 2.3.8 RFI

The radio-frequency interference from active users, such as computers, cell phones, aircraft, satellites, has always been a problem for radio astronomy. To minimise the impact of RFI, radio astronomers tend to build the telescope in remote areas, or observe in a RFI-clean frequency range. However, with the desire to extend science beyond the protected frequency bands, RFI place a new challenge to radio astronomers.

Harper & Dickinson (2018) simulated the effects of global navigation satellite services (GNSS) for future HI intensity mapping surveys and found that emission from GNSS will exceed the HI emission for all angular scales at frequencies above 950 MHz. This will be problematic for, e.g., SKA intensity mapping survey operated in single dish mode. In particular, the integrated GNSS emission is not smooth in frequency space, making it difficult to be removed using component separation methods, which works only for smooth spectra. Therefore, a more careful hardware design might be the solution to mitigate the RFI effects, e.g., by spatially filtering out GNSS emissions which however is very difficult.

Both Chang et al. (2010) and Switzer et al. (2013) found that the HI density field is significantly limited by RFI. Peel et al. (in prep.) measured the RFI at several potential sites at Brazil and Uruguay for the BINGO experiment (see Sect. 2.2.1). The final site will be at Serra do Urubu in the state of Paraiba, a remote part in the North-East of Brazil. The site has no detectable RFI and a radio quiet zone is being defined where the use of electronic devices and industrial activities will be restricted.
2.3.9 Standing waves

For an ideal telescope, when the incident radio waves enter the receiver system, all wave paths should have the equal length. However, due to, e.g., the irregularity of the aperture surface or focal leg blockage, the incident radio waves are usually reflected by the dish and feed structures. It might also get reflected within the cables connecting different receiver components, due to impedance mismatches. This will result in radio waves travelling in paths with different lengths. When entering the receiver system, the delay between one copy of the incident wave and another will imprint sinusoidal oscillations on the gain across the receiver bandpass, the period of which depends on the exact observing frequency (e.g., Briggs et al., 1997; Popping & Braun, 2007; Bigot-Sazy et al., 2015). One issue as a result of having standing waves is the complexity it brings to the telescope’s gain response to the foreground, losing the smoothness of the foreground spectrum and thus affecting the effective component separation.

2.4 Foreground and systematics for BINGO and COMAP experiments

This section describes the author’s work on quantifying, simulating and mitigating some foreground or systematics effects for the BINGO and COMAP experiment. Sect. 2.4.1 is the effort on testing the destriping map-making method for $1/f$ noise removal. Sect. 2.4.2 quantifies the atmospheric noise for BINGO experiment. Sect. 2.4.3 simulates an AME map for the COMAP foreground simulation. Sect. 2.4.4 calculates the required COMAP optical design for mitigating the reverse polarisation leakage.
2.4.1 The Destriping map-making for $1/f$ noise removal

Introduction

As mentioned in Sect. 2.3.3, $1/f$ noise places a challenge for HI intensity mapping experiments. From the observed time-ordered data (TOD) to the map, there are two stages where the $1/f$ noise can potentially be removed. One is through time-filtering during the map-making stage and another one is through component separation along with other foregrounds. Before passing to the second stage, one shall try to reduce it during map-making first. The more components that need to be subtracted by the component separation technique, the less degrees of freedom that are available for precise descriptions of the foregrounds. Therefore, it is worth investigating if we can remove the $1/f$ noise in the map-making stage, to free the burden from the component separation technique. In this section, the effect of the $1/f$ noise on the BINGO experiment is quantified and the destriping map-making algorithm is investigated for the $1/f$ noise removal through the map analysis where maps made from a few different map-making methods are compared.

Map-making algorithm

In our analysis, we tested three map-making algorithms - the maximum-likelihood (ML) algorithm, the naive binning algorithm, and the destriping algorithm. We focus on the destriping algorithm as it can deal with correlated $1/f$ noise compared with the naive binning method, and meanwhile faster and less computing-intensive than the ML method (e.g., Ashdown et al., 2007).

The ML algorithms produce optimal maps where the noise is minimized but the signal is completely preserved, through maximizing the likelihood function (e.g., Tegmark, 1997a; Doré et al., 2001). This algorithm can be used throughout the whole pipeline, from TOD (Ferreira & Jaffe, 2000) to power spectrum (e.g., Tegmark, 1997b). This approach has been proven as a viable map-making option for, e.g., single-detector
Planck data analysis (e.g., Ashdown et al., 2009) and can be extended to include systematic errors (Stompor et al., 2002). One of the main shortcomings of this algorithm is that it is computing-intensive. Therefore, for multi-frequency intensity mapping data, the ML algorithm might not be the optimal choice and a method which can give almost the same performance under certain conditions but a faster speed is thus more suitable (e.g., Ashdown et al., 2007, 2009).

In the naive binning algorithm, the inverse-variance of the data is calculated and the data are simply binned based on the minimal inverse-variance weighted binning. If the data contains only signal and Gaussian noise, e.g. white noise, this method is the same as ML map-making (e.g., Borrill, 1999). However, if the background noise contains a correlated component, such as the $1/f$ noise, simply binning is not able to take the correlation into account and thus can not effectively minimize the $1/f$ noise.

A promising alternative to the ML algorithm is destriping map-making (destriper), where the noise $n$ is modelled into two components (e.g., Keihänen et al., 2005) such that

$$n = Fa + w.$$  \hspace{1cm} (2.9)

The component $w$ represents the uncorrelated white noise. The component $Fa$ is a series of discrete offset functions $F$ with amplitude $a$, modelled from the correlated information of crossing-points, to represent the correlated noise. The offset functions are then subtracted from the TOD to remove the correlated noise (e.g., Delabrouille, 1998; Maino et al., 1999). Therefore, for a dataset where there is $1/f$ noise, destriper can subtract the long baseline noise drifts caused by $1/f$ noise without filtering out the signal part of the TOD. Compared with ML map-making, destriper is much quicker and requires far less computing resources (Sutton et al., 2009, 2010). Simulations have shown that destriper can produce both total intensity and polarised maps with negligible difference compared to the optimal (e.g., Ashdown et al., 2007; Sutton et al., 2009, 2010).
In our analysis, the three map-making algorithms are from a PYTHON code developed by Dr. Stuart Harper and are used to produce maps from simulated BINGO TOD.

**Map analysis**

As mentioned in Sect. 2.3.3, one effect of the $1/f$ noise is that it will introduce stripes in the map along the direction of the drift scan. Therefore, by visually inspecting the map produced from the map-making procedure, one can get the first sight of whether the $1/f$ noise can be removed during the map-making stage. If the map-making algorithm can remove the $1/f$ noise, there should be no stripes left on the destriped map.

The TODs used in our analysis is simulated using a BINGO simulation pipeline developed by Dr. Marie-Anne Bigot-Sazy. The instrumental parameters used in this simulation primarily follow Table 2.1, except for a few extra parameters based on Bigot-Sazy et al. (2015), that are listed in Table 2.5. Here we adopt a frequency channel width of 15 MHz and an integration time of 288 hour. The map-making algorithms are independent of the bandwidth and integration time so a narrower bandwidth and longer integration time is not necessary for this specific analysis. We only include the thermal and $1/f$ noise in our simulation without any signal, since the goal of the destriper is to remove the correlated component of the $1/f$ noise and minimize the background Gaussian noise field, which does not require the existence of a scientific signal. In the focal plane configuration, the receiver horns are aligned in both horizontal and vertical directions. During drift scan, there will be some unobserved gaps in the sky between adjacent horns in the vertical direction. The gaps can be corrected by rotating the horns by a certain angle, e.g. 5° (Bigot-Sazy et al., 2015) as listed in Table 2.5.

For the destriping map-making, the baseline integration time for the map-making is 2000 s. This baseline length is to estimate the offset functions for the correlated component to be subtracted from the TODs. If the baseline integration time is too small,
2.4. **BINGO AND COMAP SYSTEMATICS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel width (MHz)</td>
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</tr>
<tr>
<td>Integration time (hr)</td>
<td>288</td>
</tr>
<tr>
<td>Sample rate (Hz)</td>
<td>0.1</td>
</tr>
<tr>
<td>Knee frequency (Hz)</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Rotation of horns</td>
<td>5°</td>
</tr>
</tbody>
</table>

Table 2.5: Input instrumental parameters for simulating the BINGO TOD data, based on Bigot-Sazy et al. (2015).

It will require more than necessary computing power to estimate all these offsets. If the baseline integration time is too large, the estimated offsets are not accurate enough. We tested the baseline integration time between 500 s to 10000 s, which all give consistent results. We use 2000 s to get enough data without being too computing-intensive. Three maps are generated from the simulated TOD data by the three map-making algorithms and are shown in Fig. 2.6.

All of the three maps show clearly visible stripes along the horizontal direction due to the $1/f$ noise. The standard deviations of the destriped map, naive map and ML map are 2.05, 2.43 and 1.85 respectively. By comparing the standard deviation of the destriped map with that of the naive map, the destriper does not significantly improve the map by removing the correlated $1/f$ noise (15.6% improvement). From the standard deviation, the ML algorithm seems to give the smoothest map although it is not dramatically smoother than the other two maps (9.8% and 23.9% improvement with respect to the destriped map and the naive map). The residual of the destriped map subtracted by the naive map (bottom panel in Fig. 2.6) shows that the destriper does seem to have removed some stripes from the naive map since stripes are present in the residual map.

The reason why the destriper is not effective in removing the $1/f$ noise in this case is due to the transit telescope and the drift-scan strategy of the BINGO experiment, where there is no cross-points, i.e., overlap of observations by adjacent horns, along the latitude direction to be used by the destriper to remove the $1/f$ noise. We also tested the case where the rotation of horn is $0°$ rather than $5°$, to see if the redundancy
between multiple horns, where each pixel along longitude direction will be observed by different horns at different times, will give more correlation information for the destriper to better remove the $1/f$ noise. However, it is found that the destriper is even less effective in this case. The reason might be that the gaps in the observed sky due to the less overlapping of horns give even less correlation information along latitude for destriper to use. For other rotation angles between $0^\circ - 10^\circ$, no significant difference is obtained compared with the $5^\circ$ case. Therefore, a scan strategy with cross-points along both the longitude and latitude direction is important for the efficient performance of the destriper.
2.4. BINGO AND COMAP SYSTEMATICS

Summary

Through the map analysis by comparing the maps made from the ML mapping, naive binning and the destripping map-making, it is found that the destriper cannot effectively remove the 1/f noise for the BINGO experiment, due to the transit telescope and the drift-scan strategy. A possible solution in this case might be to cross-correlate data from adjacent horns observing the same declination, where the cross-correlated HI signal can stand out from the potentially uncorrelated 1/f noise. However, the different beam shapes of the two horns because of imperfection might corrupt the HI signal during cross-correlation. As a future work, one could investigate through simulation to test if cross-correlation between horns can mitigate 1/f noise, given other systematics.

Among all of the three tested algorithms, the naive binning map-making algorithm might be a better choice for BINGO since it is the quickest and simplest map-making method, given that the ML and destripping methods do not make a much difference in terms of dealing with the correlated 1/f noise. The 1/f noise will contaminate the HI signal at the scale of interest for BAO detection (\( \ell \approx 100 \)) and it is crucial that the 1/f noise should be kept to a minimum in the receiver and properly removed using component separation after map-making. In order for the destriper to reduce the correlated 1/f noise from the TODs, a scan strategy, such as a circular scan, providing cross-points along both longitude and latitude direction is needed. Another future work would be to use the 1/f noise correlations between frequency channels, to investigate if map-making can take advantage of these information for 1/f noise removal.

2.4.2 Quantifying the atmosphere effect for BINGO experiment

As introduced in Sect. 2.3.4, the atmospheric noise is one type of the 1/f noise which will affect the HI signal detection. It is thus important to quantify the atmospheric noise and if necessary, mitigate it. In this section, we quantify the fluctuation amplitude of the atmospheric noise for the BINGO experiment.
At sea level, the temperature of the atmosphere is about 285 K and will approximately decrease by 10 K as the altitude increases per 1000 m (e.g., Lowe 2005). Therefore, the atmosphere temperature $T_{\text{atm}}$ is related to the altitude $z$ by

$$T_{\text{atm}}(z) = 285 - \left(\frac{z[\text{m}]}{1000}\right) \times 10[\text{K}], \quad (2.10)$$

where $T_{\text{atm}}$ is in units of K and $z$ is in units of metre.

For a single-dish, Church (1995) gives the antenna atmospheric temperature fluctuation $\Delta T_{\text{atm}}$ at 15 GHz by

$$\bar{\Delta T}_{\text{atm}}^2(z) = 0.3 \sqrt{\frac{\pi}{2} L_0^{5/3}} \int_{z_0}^{z_{\text{at}}} C_\alpha^2(z) T_{\text{atm}}^2(z) \left(1 + \frac{w^2(z)}{2L^2}\right)^{-1} dz, \quad (2.11)$$

where $L_0$ is the outer scale of the turbulence, which has a typical value of 10 m (Church 1995), and $L = 0.3L_0 \approx 3$ m. $C_\alpha^2(z)$ quantifies the amplitude of the atmospheric fluctuation and is given by

$$C_\alpha^2(z) = 2.0 \times 10^{-14} \exp\left(\frac{-z}{z_0}\right), \quad (2.12)$$

with $z_0 = 2000$ m. $w(z)$ is related to the beam waist $w_0$ by

$$w(z) = w_0 \left[1 + \frac{\lambda^2 z^2}{\pi^2 w_0^4}\right]^{1/2} \quad (2.13)$$

where $w_0$ is given by

$$w_0 = \frac{\lambda}{410^2} \sqrt{\frac{2\log 2}{\pi \theta_b}}, \quad (2.14)$$

with $\lambda$ the observation wavelength and $\theta_b$ the resolution of the telescope. For the BINGO experiment, we have $\lambda \approx 0.2$ m and $\theta_b \approx 0.01$ in radian (Table 2.1). Substituting into Equ. 2.14, the value of $w_0$ is approximately 7.5 m. Substituting $w_0 = 7.5$ into Equ. 2.13, the expression of $w(z)^2$ is

$$w(z)^2 = 56 \left(1 + 1.3 \times 10^{-6} z^2\right). \quad (2.15)$$
By substituting Equ. 2.15, Equ. 2.12, and Equ. 2.10 into Equ. 2.11, the antenna atmospheric temperature fluctuation at 15 GHz is then

$$\Delta T_{atm}^2(z) = 3.5 \times 10^{-13} \int_{z_l}^{z_u} \exp\left(-\frac{z}{2000}\right) \times \left(285 - \frac{z}{100}\right)^2 \times \left(4 + 4 \times 10^{-6}z^2\right)^{-1} dz.$$  \hfill (2.16)

By integrating Equ. 2.16 between the sea level at $z_l = 0$ m and the water vapor scale height at $z_u = 2000$ m, the value of $\Delta T_{atm}$ is approximately 2.3 mK at 15 GHz. The background emission of a water vapor spectrum is a blackbody radiation following the Rayleigh-Jeans law so that it increases as the square of the frequency (e.g., Wilson et al., 2009; Smith, 1982). Therefore, at the BINGO observing frequency of 1 GHz, the antenna temperature fluctuation due to atmosphere is

$$\Delta T_{atm}^{1\text{GHz}} = \Delta T_{atm} \times \left(\frac{1}{15}\right)^2 = 0.01 \text{ mK}. \hfill (2.17)$$

Compared with the $\sim 1$ mK instrumental noise level at the timescale of 20 mins for BINGO (Bigot-Sazy et al. 2015), the atmosphere effect should not be a challenge for BINGO. In addition, in Church (1995), the condition of Equ. 2.13 is that the layer height $z$ must be much larger than the dish diameter. Therefore, for the BINGO 40 m telescope (Dickinson 2014), $z_l$ should in fact be at least 500 m. In Equ. 2.16, by setting $z_l$ to be 500 m, the resulting $\Delta T_{atm}^{1\text{GHz}}$ is about 0.007 mK, much lower than the $\sim 1$ mK BINGO instrumental noise level. The $1/f$ noise in the system for the BINGO experiment will thus be dominated by the receiver gain fluctuations, rather than the atmosphere.

However, the above calculations are based on a clear day. A rainy day can significantly change the local fluctuation level of the atmospheric noise. For example, the amplitude of the atmospheric fluctuation $C_2^2$ (Equ. 2.12) varies between $2 \times 10^{-14}$ and $2 \times 10^{-10}$ depending on weather conditions (Thompson et al., 1986). At a very bad day with $C_2^2 = 2 \times 10^{-10}$, the atmospheric fluctuation in antenna can increase by $\sim 2$ orders of magnitude, resulting in $\Delta T_{atm}^{1\text{GHz}}$ being comparable with the BINGO thermal noise.
level at \(\sim 1\) mK. Therefore, the atmosphere will have a negligible impact on BINGO only if the observation is taken on a clear day.

### 2.4.3 The anomalous microwave emission model

As introduced in Sect. 2.2.4, the COMAP experiment maps the CO signals at frequencies between 26 GHz and 34 GHz. In this frequency range, one dominant foreground component is AME (see Sect. 1.3.4). It is therefore crucial to quantify and, if possible, separate the AME component from the COMAP data. Olivari et al. (2019) tested the GNILC method as an efficient component separation candidate for removing foreground contaminations from COMAP simulated data. In this section we describe the author’s work of simulating a Galactic AME map for Olivari et al. (2019), where the author takes the co-authorship.

Since AME is closely correlated with dust in the spatial domain, we use the Planck \(\tau_{353}\) optical depth map as our dust template, which is the dust opacity map produced by the GNILC method at 353 GHz using the Planck data (Planck Collaboration XLVIII 2016). Compared with other dust maps such as the IRAS 100 \(\mu m\) map (Schlegel et al. 1998), the Planck \(\tau_{353}\) map is a good tracer of AME at high Galactic latitude (Planck Collaboration XXV, 2016; Dickinson et al., 2018), with reduced contamination from cosmic infrared background and more accurate estimates of dust temperature and spectral index (Planck Collaboration XLVIII 2016). The Planck \(\tau_{353}\) map has a HEALPIX pixel resolution of \(N_{side} = 2048\) and 5 arcmin beam size.

In order to get the AME brightness temperature, we adopt the factor \(8.3 \times 10^6 \mu K / \tau_{353}\), given by Planck Collaboration XXV (2016), which converts the dust optical depth at 353 GHz to AME brightness temperature at 22.8 GHz, \(T_{22.8}\), in units of \(\mu K\). In order to scale the AME brightness temperature to the desired frequencies of COMAP, we compute the AME emissivities \(E\) as a function of frequency, and the
brightness temperature is related to the emissivity through

\[
\frac{T_{\text{CO}}}{T_{22.8}} = \frac{E_{\text{CO}}}{E_{22.8}} \left( \frac{22.8}{\nu_{\text{CO}}} \right)^2,
\]

where \(T_{\text{CO}}\) and \(E_{\text{CO}}\) are the desired AME brightness temperature and emissivity at the desired COMAP frequency \(\nu_{\text{CO}}\). \(T_{22.8}\) and \(E_{22.8}\) are the AME brightness temperature and emissivity at 22.8 GHz. The last term in the right-hand-side accounts for the \(1/\nu^2\) factor to relate flux density with brightness temperature.

The AME emissivity curve as a function of observing frequency is computed using the \texttt{SPDUST2} code (Ali-Haïmoud et al., 2009; Silsbee et al., 2011), which gives emissivity for various environments of the interstellar medium, such as cold neutral medium (CNM) and warm neutral medium (WNM), given the corresponding gas density \(n_{\text{H}}\), gas temperature \(T\), dust temperature \(T_d\), strength of the interstellar radiation field \(\chi\), and the fraction of molecular hydrogen \(y\), ions of hydrogen \(x_{\text{H}}\), and heavier ions \(x_{\text{M}}\). Fig. 2.7 shows the AME emissivities given by the \texttt{SPDUST2} code under the idealized CNM (\textit{green}) and WNM (\textit{red}) interstellar medium environments for comparison. It can be seen that the two curves have similar shapes but different peak frequencies and magnitudes due to different environments.

We use the CNM environment to compute the AME emissivity for COMAP, since it is a good approximation to the spectrum derived from molecular clouds (e.g., Dickinson et al., 2018). The parameters of the CNM environment are from Draine & Lazarian (1998b) and listed in Table 2.6. We then interpolate the AME emissivities at 22.8 GHz \(E_{22.8}\) and at the COMAP frequency \(E_{\text{CO}}\) respectively from the CNM curve (\textit{green}) in Fig. 2.7, and calculate the AME brightness temperature \(T_{\text{CO}}\) at the desired COMAP frequency from Equ. 2.18.

The AME brightness temperature map produced for COMAP at 33 GHz as an example is shown in Fig. 2.8. It can be seen that the diffused AME distribute at both low latitudes surrounding the Galactic plane and at high latitudes close to the poles. The
Figure 2.7: The AME emissivity curve as a function of frequency, calculated from the SPDUST2 code, for idealized CNM and WNM phases of the interstellar medium.

Table 2.6: The parameters of cold neutral medium environment from Draine & Lazarian (1998b) used in the SPDUST2 code for calculating the AME emissivity as a function of frequency. $n_H$ is the gas density, $T$ is the gas temperature, $T_d$ is the dust temperature, and $\chi$ is the strength of the interstellar radiation field. $y$, $x_H$ and $x_M$ are the fractions of molecular hydrogen, ions of hydrogen and heavier ions respectively.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CNM Phase</th>
</tr>
</thead>
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<tr>
<td>$n_H$ (cm$^{-3}$)</td>
<td>30</td>
</tr>
<tr>
<td>$T$(K)</td>
<td>100</td>
</tr>
<tr>
<td>$T_d$(K)</td>
<td>20</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1</td>
</tr>
<tr>
<td>$y \equiv 2n(H_2)/n_H$</td>
<td>0</td>
</tr>
<tr>
<td>$x_H \equiv n(H^+)/n_H$</td>
<td>0.0012</td>
</tr>
<tr>
<td>$x_M \equiv n(M^+)/n_H$</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

The corresponding angular power spectrum of the map in Fig. 2.8 is shown in Fig. 2.9. The power spectrum is computed using the POLSPICE (Szapudi et al., 2001; Chon et al., 2004; Challinor & Chon, 2005) estimator, which firstly computes the angular correlation function in real space and then transforms to the power spectrum in harmonic space. POLSPICE has the advantages of fast speed, public availability, optional pixelization and beam size correction. The power spectrum is binned with $\Delta \ell = 10$, to

http://www2.iap.fr/users/hivon/software/PolSpice/
smooth out the small scale fluctuations. From Fig. 2.9, the AME power gradually decreases from large scales to small scales and after $\ell \sim 3000$, the power spectrum is dominated by the beam effect due to the 5 arcmin limited resolution of the map.

Figure 2.8: The AME temperature map produced based on the Planck $\tau_{353}$ dust template map (Planck Collaboration XLVIII 2016), scaled to 33 GHz using the SPDUST2 code (Ali-Haïmoud et al., 2009; Silsbee et al., 2011).

Olivari et al. (2019) uses 128 frequency channels with $\Delta \nu = 62.4$ MHz between the frequency of 26 GHz and 34 GHz, as the COMAP instrumental parameters. An AME temperature map is produced for each frequency channel to simulate the observed AME emission. Fig. 2.10 shows the standard deviation of the brightness temperature as a function of the observing frequency of the foreground components for the COMAP phase I setup (Olivari et al., 2019). It can be seen that the AME emission is the second strongest foreground components following the point sources. There is no way to detect the CO signal without an effective removal of the AME emission.

Olivari et al. (2019) applied the GNILC method to the simulated intensity maps to extract CO signals from the instrumental noises and foreground components. It is found that as a competitive component separation candidate, the GNILC method
Figure 2.9: The angular power spectrum of the AME map at 33 GHz (Fig. 2.8). The power spectrum is binned with $\Delta \ell = 10$. The vertical axis is the AME angular power spectrum and the horizontal axis is the multipoles. The AME power decreases with the angular scales and around $\ell \sim 3000$ it begins to be dominated by the beam effect.

Figure 2.10: The standard deviation of the brightness temperature as a function of observing frequency of the foreground components for the COMAP phase I setup (Olivari et al., 2019).

can recover the CO plus instrumental noise signal from the total foreground (free-free + Synchrotron+ Thermal dust + AME + CMB + point sources) with a reasonable
2.4. BINGO AND COMAP SYSTEMATICS

accuracy of a $-0.76\%$ bias for phase I and a $-0.22\%$ bias for phase II. These biases are relatively small compared to the detection. Therefore, the GNILC method is a very promising candidate to be used on the CO intensity maps.

2.4.4 The mitigation of reverse polarisation leakage for COMAP experiment

As mentioned in Sect. 2.3.5, due to the faintness of the CO signal, one challenge of the COMAP experiment is the reverse polarisation leakage. It is essential to quantify the reverse polarisation leakage and ensure that the optical design of the COMAP experiment can meet the requirement where the reverse polarisation leakage can be neglected. One way of quantifying it is by calculating the required polarisation fraction $F_X$ of the receiver system, which quantifies the fractional amount of polarisation that leaks into the system, to make sure the polarisation noise level is smaller than the thermal noise level and can thus be neglected.

In our calculation, we focus on the initially proposed pathfinder version of COMAP since this is the instrument that is currently operating. From Table 2.3 (Li et al. 2016), the system temperature of COMAP is 40 K. The total bandwidth of COMAP is 8 GHz over 26 to 34 GHz. The proposed pathfinder will observe 4 patches of the sky, each of which has an area of $\Omega_{\text{pat}} = 1.7 \text{ deg}^2$ and an integration time of $\tau_{\text{pat}} = 1500 \text{ hr per patch}$. Given the beam width is $\theta = 6' = 0.1^\circ$, the number of independent beams, $N_{\text{beam}}$, within each patch is

$$N_{\text{beam}} \approx \frac{\Omega_{\text{pat}}}{\theta^2} = 170.$$  \hspace{1cm} (2.19)

Therefore, the integration time per beam is

$$\tau_{\text{beam}} = \frac{\tau_{\text{pat}}}{N_{\text{beam}}} \approx 31765 \text{ s}.$$  \hspace{1cm} (2.20)
From the radiometer equation, the thermal noise level per beam is

\[ \Delta T_{\text{ther}} = \frac{\sqrt{2} T_{\text{sys}}}{\sqrt{\Delta v \tau_{\text{beam}}}} \]  

(2.21)

where the factor \( \sqrt{2} \) comes from the design that the antenna is only sensitive to one polarisation. By substituting Equ. 2.20, \( \Delta v = 8 \text{ GHz} \), \( T_{\text{sys}} = 40 \text{ K} \) into Equ. 2.21, the sensitivity per beam is \( \Delta T_{\text{ther}} \approx 4 \mu\text{K} \).

The amount of polarisation \( \Delta T_{\text{pol}} \) leaked into the receiver system can be quantified as

\[ \Delta T_{\text{pol}} = T_{\text{polsky}} \times F_X, \]  

(2.22)

where \( T_{\text{polsky}} \) is the polarised sky signal and \( F_X \) is the fraction of the polarised sky signal that leaks into the receiver system. To make the polarisation leakage negligible, it is required that \( \Delta T_{\text{pol}} \ll \Delta T_{\text{ther}} \). Therefore, from Equ. 2.22 we have

\[ F_X \ll \frac{\Delta T_{\text{ther}}}{T_{\text{polsky}}} \]  

(2.23)

In order to get the polarised sky signal \( T_{\text{polsky}} \), we assume that the polarised sky signal in this case is the polarised synchrotron emission \( T_{\text{polsyn}} \), since it is the dominant foreground in the polarised sky at the COMAP frequency range (e.g., Planck Collaboration X, 2016). The temperature of synchrotron emission \( T_{\text{syn}} \) at a given frequency \( \nu \) is approximately

\[ T_{\text{syn}}(\nu) = T_{408} \left( \frac{\nu}{408 \text{ MHz}} \right)^\beta, \]  

(2.24)

where \( T_{408} \) is the temperature of synchrotron emission at 408 MHz and \( \beta \) is the spectral which has the value of \( \approx -3 \) (Lawson et al., 1987; Kogut et al., 2007; Macellari et al., 2011; Vidal et al., 2015). Fig. 2.11 shows the map of the sky synchrotron emission at 408 MHz (Remazeilles et al. 2015). It can be seen that at higher latitudes beyond the Galactic plane where the COMAP observation will be conducted, the synchrotron emission has the temperature of \( T_{408} \approx 30 \text{ K} \). Substituting \( T_{408} \) into Equ. 2.24,
the synchrotron emission at around 30 GHz in the COMAP experiment has a typical temperature $T_{\text{syn}} \approx 75\,\mu K$. Given that the synchrotron emission has been observed to have a typical polarisation fraction of about 30 percent at high latitude (e.g., Vidal et al. 2015), the temperature of the polarised synchrotron signal is $T_{\text{polsyn}} \approx 22\,\mu K$. By assuming $T_{\text{polsky}} \approx T_{\text{polsyn}}$ and substituting the values of $T_{\text{polsky}}$ and $\Delta T_{\text{ther}}$ into Equ. 2.23, the threshold of fractional amount of polarisation should be

$$F_X \ll \frac{4}{22} \approx -7.4\,\text{dB}.$$  \hspace{1cm} (2.25)

To ensure that the polarisation signal is negligible, we should aim for $F_X \times T_{\text{polsky}} < 0.1\Delta T_{\text{ther}}$, i.e., 10% of the noise level. Therefore, the required optical design should aim to have at least $F_X < -17\,\text{dB}$.

However, the side lobes can also contribute to the polarisation leakage, especially when the side lobes are pointing towards the Galactic plane. In order to quantify this side lobe effect, we assume that all the emissions from the Galactic plane are picked up by the side lobes. We estimate the emission temperature of the Galactic plane to
be $T_{\text{gal}}^{308} \approx 1000 \text{ K}$ (e.g. Haslam et al. 1981). Substituting into Equ. 2.24, the emission temperature of the Galactic plane is $T_{\text{gal}}^{30} \approx 2500 \mu \text{K}$ at 30 GHz. Based on the 30% polarisation fraction assumption, the polarised emission from the Galactic plane is $T_{\text{pgal}}^{30} \approx 800 \mu \text{K}$. Substituting into Equ. 2.23, to ignore the side lobe effect, the side lobe should be tapered to a level of at least $-23 \text{ dB}$. In practice, the side lobe are tapered to be less than $-60 \text{ dB}$. Therefore, the contributions from the side lobes to the reverse polarisation leakage can be neglected.

In summary, the side lobe contributions of the reverse polarisation leakage from the beams to the COMAP experiment are negligible. In order to have a negligible reverse polarisation leakage, the required optical design of COMAP shall aim to have the fractional amount of polarisation leakage to be $F_X < -17 \text{ dB}$. In practice, this standard should not be too hard to achieve and therefore, the polarisation leakage effects could potentially be neglected for the optical design in the COMAP experiment.

### 2.5 Summary

HI intensity mapping is a competitive technique for mapping the large-scale-structures through the detection of an emission line. It does not require a fine resolution telescope to detect a large number of individual galaxies to overcome the statistic limit. Therefore, it is a cost-saving alternative method for galaxy surveys to detect BAO, especially given that it can provide accurate redshift information thanks to its multi-frequency channels.

Many intensity mapping experiments have been proposed but they are almost all at the start-up stage. Due to the faintness of the signal, intensity mapping experiments face many challenges such as the effective removal of the astrophysical foregrounds and a sensible control of the instrumental systematics. In this section, the author investigated a few systematics and foreground issues focused on the BINGO and COMAP experiment.
2.5. SUMMARY

The performance of the destriper for $1/f$ noise removal has been investigated for the BINGO experiment, through both map analysis and power spectrum analysis. It is found that due to the drift scan strategy, the destriper cannot remove $1/f$ noise. Therefore, it is worth testing cross-correlation analysis to remove $1/f$ noise, or one has to rely on the component separation technique for the proper removal of $1/f$ noise, since it will significantly contaminate the HI signal.

The atmospheric effects for BINGO is calculated by using the model of Church (1995). It is found that the atmospheric effect is negligible on a clear day. However, on a rainy day, the atmospheric effect might significantly contaminate the HI signal and regular monitor of the weather is thus required during BINGO observation.

An AME map is simulated for each frequency channel of the COMAP experiment, to simulate the AME contamination at the observing frequency between 26 – 34 GHz. Olivari et al. (2019) shows that the GNILC component separation method can effectively recover the CO signal from AME, along with other foreground and systematics contaminations.

The required optical design where the reverse polarisation leakage can be neglected for COMAP experiment has been investigated. The required receiver system for COMAP shall aim to control the fractional reverse polarisation leakage to be less than $-18\,\text{dB}$, and the contributions from the side lobes are expected to be negligible.
Chapter 3

Impact of SZ Cluster Residuals in CMB Maps and CMB-LSS Cross-Correlations

Residual foreground contamination in cosmic microwave background (CMB) maps, such as the residual contamination from thermal Sunyaev-Zeldovich (SZ) effect in the direction of galaxy clusters, can bias the cross-correlation measurements between CMB and large-scale structure (LSS) optical surveys. It is thus essential to quantify those residuals and, if possible, to null out SZ cluster residuals in CMB maps.

In this chapter we quantify, for the first time, the amount of SZ cluster contamination in the released Planck 2015 CMB maps through (i) the stacking of CMB maps in the direction of the clusters, and (ii) the computation of cross-correlation power spectra between CMB maps and the SDSS-IV large-scale structure data. Sect. 3.1 introduces the astrophysics background. Sect. 3.2 describes the data sets used in our analysis. Sect. 3.3 gives the first sight of the correlation between SZ cluster residuals in CMB maps and galaxies through stacking analysis. Sect. 3.4 presents the results from cross-power spectrum analysis and discusses the impact of the SZ cluster residuals on the ISW detection. Sect. 3.5 summarizes the results and draws the conclusions.

3.1 Introduction

3.1.1 The distortions of the CMB radiation

The interactions between the large-scale structures (LSS) and the cosmic microwave background (CMB) radiation in the Universe lead to gravitational (spatial) and spectral distortions to the CMB radiation. Studying the signature of these CMB distortions enables to probe dark matter and dark energy. These distortions include weak gravitational lensing (WGL) effects on the CMB (Lewis & Challinor, 2006), Sunyaev-Zeldovich (SZ) effect (Sunyaev & Zeldovich, 1972), and the integrated Sachs-Wolfe (ISW) effect (Sachs & Wolfe, 1967; Rees & Sciama, 1968; Hu & Okamoto, 2002).

The weak gravitational lensing. When travelling along the line-of-sight, the CMB photons will encounter under- and over-densities of matter and thus be deflected by the gravitational potential gradients induced by LSS (Lewis & Challinor, 2006). The deflection changes the CMB temperature power spectrum, induce non-Gaussianities, and generate a B-mode polarisation signal. The B-mode polarisation is 45° with respect to the wave vector, in contrast with the E-mode polarisation which is perpendicular or parallel to the wave vector. The lensing signal has the same frequency spectrum so the lensed CMB energy spectrum remains a blackbody. The weak lensing effect can be modelled through the integrated lensing potential along the line-of-sight, which can be re-constructed through quadratic estimators (e.g., Blanchard & Schneider, 1987; Cole & Efstathiou, 1989). Lensing quadratic estimators (e.g., Hu & Okamoto, 2002) perform a weighted convolution of CMB maps in spherical harmonic space in order to
reconstruct the lensing potential $\Phi$:

$$\hat{\Phi}(L) \propto \int \frac{d^2 \ell'}{(2\pi)^2} W(L, \ell) T_{\text{CMB}}(\ell) T_{\text{CMB}}(L - \ell),$$

(3.1)

where $T_{\text{CMB}}(\ell)$ is the spherical harmonic coefficient of the CMB map at multipole $\ell$, and

$$W(L, \ell) = \frac{L \cdot \ell C_\ell + L \cdot (L - \ell) C_{|L - \ell|}}{(C_\ell + N_\ell) (C_{|L - \ell|} + N_{|L - \ell|})},$$

(3.2)

with $C_\ell$ and $N_\ell$ being the CMB and noise angular power spectra.

The ISW effect. The ISW effect (Sachs & Wolfe, 1967; Rees & Sciama, 1968) is a secondary anisotropy in the CMB caused by the interaction of the CMB photons with the evolving gravitational potential wells from LSS across time, due to the accelerated expansion of the Universe. The ISW effect leaves the CMB photons blue-shifted, and is characterised by

$$\Theta = \frac{\Delta T}{T_{\text{CMB}}} = -\frac{2}{3} \int_0^{\chi_{\text{CMB}}} d\chi \frac{\partial \Phi}{\partial \chi},$$

(3.3)

where $\chi$ is the comoving distance. The fractional temperature perturbation $\Theta$ is calculated as the integral over the time-evolving potentials $\Phi$ of LSS, to the surface of last scattering $\chi_{\text{CMB}}$. The ISW effect is mostly significant on the largest angular scales $\gtrsim 10$ degrees, so that the measurement of the ISW effect requires a close-to full-sky CMB map. Crittenden & Turok (1996) discussed the potential of detecting ISW through the cross-correlation between CMB and LSS, with the first detection of ISW made using WMAP data (e.g., Fosalba et al., 2003; Fosalba & Gaztañaga, 2004).

The SZ effect. The SZ effect (Sunyaev & Zeldovich, 1972) is the distortion of the low energy CMB photons through the inverse Compton scattering by high energy electrons residing in the hot gas of galaxy clusters. One type of the SZ effect is the kinetic SZ effect where the CMB photons gain energy from energetic electrons due to their bulk motion, which does not change the CMB spectrum and only accounts for $\sim 10\%$
of the total SZ distortion (Ostriker & Vishniac, 1986). Another type is the thermal SZ effect where the CMB photons gain energy from energetic electrons due to their hot temperature. The thermal SZ effect is quantified by the Compton parameter $y_0$, which is defined as the averaged fractional energy transferred per collision times the averaged number of collisions such that

$$y_0 = \frac{\int \sigma_T n_e k_B T_e}{m_e c^2} dl,$$

(3.4)

where $\sigma_T$ is the Thomson cross-section, $k_B$ the Boltzmann constant, $m_e$ the mass of the electron, $c$ the speed of light, $n_e$ and $T_e$ the number density and temperature of electrons, integrated along the line of sight $dl$ (Sunyaev & Zeldovich, 1972). At a given frequency $\nu$, the thermal SZ intensity in CMB temperature unit is given in the non-relativistic limit by

$$\delta T = y_0 T_{\text{CMB}} \left( x \coth(x/2) - 4 \right),$$

(3.5)

where $x = \frac{h \nu}{k_B T_{\text{CMB}}}$ (e.g., Sunyaev & Zeldovich, 1972). Fig. 3.1 shows the frequency spectrum (spectral energy distribution) of the thermal SZ effect, which has its relative temperature to CMB negative at $\nu < 217$ GHz, zero at $\nu = 217$ GHz, and positive at $\nu > 217$ GHz, ignoring the relativistic effects. Therefore, the SZ signal will show up as negative (positive) spots in the CMB maps observed below (above) 217 GHz and is null at 217 GHz.

The released Planck maps of the CMB lensing field (Planck Collaboration XV, 2016), thermal SZ $y$-Compton parameter (Planck Collaboration XXII, 2016), and ISW effect (Planck Collaboration XXI, 2016), all provide indirect tracers of the matter distribution in the sky through the imprint of the large-scale structures on the CMB. Conversely, optical surveys of large-scale structures, such as BOSS/SDSS-III (Dawson et al., 2013) and future surveys by LSST (LSST Science Collaboration, 2009) and Euclid (Laureijs et al., 2011), offer direct tracers of the matter distribution in the Universe.
3.1. INTRODUCTION

Figure 3.1: The frequency spectrum of the thermal SZ effect. The vertical axis gives the flux density of an SZ signal at \( y_0 = 0.0005 \) as a function of frequency. The SZ signal is negative at \( \nu < 217 \, \text{GHz} \), zero at \( \nu = 217 \, \text{GHz} \), and positive at \( \nu > 217 \, \text{GHz} \).

3.1.2 The contamination of LSS foregrounds in CMB maps

As mentioned in Sect. 1.2.3, there is a growing interest in cross-correlating multiple dark matter tracers from different datasets, such as CMB and LSS surveys. Giusarma et al. 2018 have cross-correlated the Planck CMB lensing map with the SDSS galaxy density map to measure the scale-dependence of the galaxy bias and constrain neutrino masses. Cross-power spectra between Planck CMB lensing and galaxy lensing shear maps have been computed to measure the galaxy lensing shear bias (Liu & Hill, 2015) and the amplitude of the gravitational lensing effect (Singh et al., 2017). The cross-correlations between CMB maps and radio source surveys (e.g., Boughn & Crittenden, 2002, 2004; Giannantonio et al., 2012), infrared surveys (e.g., Afshordi et al., 2004; Giannantonio et al., 2012; Goto et al., 2012; Shajib & Wright, 2016), X-ray background (e.g., Boughn & Crittenden, 2004; Giannantonio et al., 2012; Goto et al., 2012; Shajib & Wright, 2016), and optical surveys (e.g., Padmanabhan et al., 2005; Giannantonio et al., 2006; Planck Collaboration XIX, 2014; Planck Collaboration XXI, 2016), have been used to measure the ISW effect and constrain dark energy. The cross-correlations between CMB maps and
SDSS data (e.g., Peiris & Spergel, 2000; Sherwin et al., 2012), 2MASS data (Afshordi et al., 2004), and *Planck* thermal SZ map (Hill & Spergel, 2014) have been used to constrain cosmological parameters such as the equation of state of dark energy, \( w \), and the r.m.s fluctuations of dark matter, \( \sigma_8 \). Ma et al. (2015) and Hojjati et al. (2015) constrained cosmological parameters, hydrostatic mass bias, and baryon component, by cross-correlating the CFHTLenS mass map with the *Planck* thermal SZ map.

However, CMB-LSS cross-correlations may suffer from another kind of systematic error: the contamination of the CMB products by LSS foregrounds, such as the residual emission from SZ galaxy clusters in the CMB maps. As mentioned in Doux et al. (2018), when performing CMB-LSS cross-correlations, a possible source of systematic error is due to the fact that the residual SZ clusters in the CMB maps must host some of the galaxies in the LSS survey. In Giusarma et al. (2018), the scale-dependent galaxy bias measured through CMB lensing-SDSS cross-correlation was found to be lower than expected, which was attributed to possible thermal SZ contamination in the *Planck* CMB lensing map. Also, Madhavacheril & Hill (2018) have shown that the SZ contamination of the CMB lensing map causes significant bias on the dark matter halo masses when measured from cross-correlations between CMB lensing and galaxy density fields.

LSS residuals in the CMB products can thus no longer be ignored in the context of CMB-LSS cross-correlations for unbiased measurements. Although the contamination by LSS foregrounds has been mitigated in the public CMB maps through component separation algorithms, there is always a non-zero residual of thermal SZ effect from galaxy clusters (see e.g., Planck Collaboration Int. LIII, 2017). Luzzi et al. (2015) have also observed SZ residuals by stacking the *Planck* 2013 SMICA, NILC and SEVEM CMB temperature maps in the direction of galaxy clusters. In particular, SZ residuals in CMB maps may lead to spurious, non-physical, (anti)-correlations with galaxy surveys. Those SZ residuals must also propagate to the CMB lensing potential map in a non-trivial way since it is derived from the CMB map itself by means of quadratic
estimators (e.g., Hu & Okamoto, 2002). Therefore, for unbiased CMB-LSS cross-correlations, it is essential to quantify and minimize the LSS contamination in CMB maps.

CMB foreground residuals can impact both ISW and lensing measurements, and as a consequence might lead to confusion in constraints on dark energy and neutrino masses. Indeed, thermal SZ residuals appear as negative temperature fluctuations (blue) in the CMB maps because most of the component separation weights for CMB reconstruction actually come from the 100–143 GHz frequency channels (Planck HFI Core Team, 2011), where the CMB is dominant but the thermal SZ spectral energy distribution (SED) is negative as shown in Fig. 3.1. Negative SZ residuals in the CMB will thus anti-correlate with galaxy flux/number density from LSS surveys, therefore causing a deficit of power on small scales in the matter cross-power spectrum, which might be confused with effects from massive neutrinos.

3.2 Data description

In this section, we describe the set of CMB temperature maps (Sect. 3.2.1 and 3.2.2) and the LSS optical survey map (Sect. 3.2.3) that we use for the cross-correlation analysis. A thermal SZ map at 143 GHz (Sect. 3.2.4) is also generated from the Planck 2015 SZ catalogue (Planck Collaboration XXVII, 2016) for simulations.

3.2.1 The Planck NILC CMB map

Four CMB maps have been released by Planck (Planck Collaboration IX, 2016). Each of them has been estimated by an independent component separation algorithm: COMMANDER (Eriksen et al., 2008), a Bayesian parametric pixel-by-pixel fitting with MCMC Gibbs sampling; SEVEM (Fernández-Cobos et al., 2012), an internal template fitting in wavelet space; SMICA (Cardoso et al., 2008), a blind power-spectra
fitting approach in harmonic space; NILC (Delabrouille et al., 2009), a minimum-variance internal linear combination in needlet (spherical wavelet) space. The four Planck CMB products have shown good consistency at a level of a few $\mu$K (Planck Collaboration IX, 2016), with minimized residuals from foreground contamination (Galactic foreground emissions, SZ, extragalactic sources). The Planck CMB maps have been mapped on the sphere through a HEALPix pixelization scheme (Górski et al. 2005) on a $N_{\text{side}} = 2048$ grid (pixel size $\approx 1.7$ arcmin). The optical beam resolution of the Planck CMB maps is 5 arcmin.

While we consider the four Planck CMB maps for our stacking analysis in Sect. 3.3.1, we focus on the Planck NILC CMB map for the CMB-LSS cross-power spectrum analysis in Sect. 3.4, as the Planck NILC CMB map is processed in a most similar way with the SZ-free CMB map (see Sect. 3.2.2) that we produced for this work, thus giving a more direct comparison.

The basics of the NILC component separation algorithm can be summarized as follows. In intensity units, the data, $x_\nu$, in each frequency band, $\nu$, and in each pixel are the superposition of the CMB temperature fluctuations, $s \equiv \Delta T/T_{\text{CMB}}$, and the contamination, $n_\nu$, which include foregrounds and noise:

$$x_\nu = a_\nu s + n_\nu,$$  \hspace{1cm} (3.6)

where $a_\nu \equiv dB_\nu(T)/dT|_{T=T_{\text{CMB}}}$ is the derivative of the blackbody spectrum $B_\nu(T)$ with respect to temperature $T$, i.e. the spectral energy distribution (SED) of the CMB temperature anisotropies, $s$, across frequencies. The NILC method then consists of estimating the CMB signal, $\hat{s}$, as a weighted internal linear combination (ILC) of the Planck frequency maps $x_\nu$,

$$\hat{s} = \sum_\nu w_\nu x_\nu,$$  \hspace{1cm} (3.7)
that is constrained to give unit response to the CMB SED, i.e.,

$$\sum_{\nu} w_{\nu} a_{\nu} = 1,$$  \hspace{1cm} (3.8)

and to be of minimum variance, i.e.,

$$\frac{\partial \langle \hat{s}^2 \rangle}{\partial w_{\nu}} = \frac{\partial}{\partial w_{\nu}} \left( \sum_{\nu'} \sum_{\nu''} w_{\nu'} C_{\nu'\nu''} w_{\nu''} \right) = 0,$$  \hspace{1cm} (3.9)

where matrix $C_{\nu\nu'} \equiv \langle x_{\nu}, x_{\nu'} \rangle$ is the frequency-by-frequency covariance matrix of the data ($9 \times 9 \times N$ matrix, where $9$ is the number of Planck frequencies and $N$ is the total number of pixels in Planck maps). The elements of the covariance matrix are computed in each pixel $p$ as

$$C_{\nu\nu'}(p) = \frac{1}{N_{p}} \sum_{p' \in D(p)} x_{\nu}(p)x_{\nu'}(p),$$  \hspace{1cm} (3.10)

where the sum runs over $N_{p}$ pixels $p'$ of a circular domain $D(p)$ surrounding the pixel $p$. The choices for the size and morphology of the pixel domains $D(p)$ have been described in Basak & Delabrouille (2012). The solution for the vector of NILC weights is obtained through a Lagrange multiplier. The method of Lagrange multipliers is a way to find optimization, satisfying given constraints. In our case, the Lagrange function is constructed so that

$$\frac{\partial}{\partial w^t} \left[ (w^t C w) + \lambda (1 - w a) \right] = 0,$$  \hspace{1cm} (3.11)

which minimised the variance given in Equ. 3.9 and meanwhile satisfies Equ. 3.8, by introducing a auxiliary variable $\lambda$, known as the Lagrange multiplier. By solving Equ. 3.11, the solution for the vector of NILC weights, $w = \{w_{\nu}\}$, is thus given by

$$w^t = \frac{a^t C^{-1}}{a^t C^{-1} a},$$  \hspace{1cm} (3.12)
where the superscript $t$ stands for transposition. The NILC estimate is then

$$\hat{s} = s + \sum_\nu w_\nu n_\nu,$$

thus providing an unbiased estimate of the CMB, $s$, thanks to the constraint in Eq. 3.8, while residual foregrounds, $\sum_\nu w_\nu n_\nu$, are minimized thanks to the condition in Eq. 3.9.

The NILC reconstruction of the CMB is performed in needlet (spherical wavelet) space (Narcowich et al., 2006; Guilloux et al., 2009) to allow the ILC weights, $w_\nu$, to vary over the sky, adjusting to the local conditions of contamination both over the sky and angular scales.

### 3.2.2 The 2D-ILC CMB map

The four component separation pipelines applied to the Planck data have been optimized to minimize the global contamination from astrophysical foregrounds and instrumental noise. However, a certain amount of residual foregrounds is inevitably present in those CMB maps, at different levels depending on the area of the sky and the angular scale.

Thermal SZ (tSZ) residuals will be left in the NILC CMB map (Eq. 3.13) through the following expression:

$$\text{tSZ residuals} = \sum_\nu w_\nu b_\nu y,$$

where $w_\nu$ are the NILC weights, $b_\nu$ is the SED of the thermal SZ effect:

$$b_\nu = x \coth(x/2) - 4, \text{ with } x \equiv h\nu/(k_B T_{\text{CMB}}),$$

and $y$ is the tSZ $y$-Compton parameter. For Planck, the signal-to-noise ratio is favourable to the CMB in the 100–143 GHz frequency range. This is because the total
foreground contamination is minimum at $\sim 100$ GHz and the Planck frequency channels have their largest sensitivity at $\sim 100$–143 GHz. Therefore, for the Planck CMB reconstruction, the bulk of the NILC weights is attributed mostly to this frequency range, for which the tSZ SED is negative relative to the CMB. As a consequence, tSZ residuals in Planck CMB maps appear as negative fluctuations in the direction of the galaxy clusters (see Fig. 3.5).

Depending on the scientific objective, in particular cross-correlations between CMB and galaxy surveys, it might be essential to nullify specific LSS foregrounds in the CMB map, such as the thermal SZ contamination from galaxy clusters, rather than minimizing the global contamination.

In this context, we also use in this work the 2D-ILC CMB map (e.g., Planck Collaboration Int. LIII, 2017), a thermal SZ-free CMB map which we have produced by applying the ‘Constrained ILC’ component separation method (Remazeilles et al., 2011) to the Planck 2015 data. The Constrained ILC method is similar to NILC as it is a weighted linear combination of the frequency maps in needlet space that offers unit response to the CMB spectrum, but has an additional constraint of giving zero response to the thermal SZ spectrum. In other words, the vector of weights, $w_\nu$, for the Constrained ILC is constructed to be orthogonal to the thermal SZ SED vector, $b_\nu$, so that Eq. 3.8 is replaced by

$$\sum_\nu w_\nu a_\nu = 1,$$
(3.16a)

$$\sum_\nu w_\nu b_\nu = 0.$$  
(3.16b)

In this way, the thermal SZ contamination Eq. 3.14 is nulled out, thus guaranteeing the total absence of thermal SZ residuals in the resulting 2D-ILC CMB map\(^1\). A unique solution to Eqs. 3.9 and 3.16a-3.16b is derived by using Lagrange multipliers

\(^1\)The name 2D-ILC comes from the two-dimensional (2D) constraint in Eqs. 3.16a-3.16b.
The SZ-free 2D-ILC CMB map is produced specifically for this work by Dr. Mathieu Remazeilles in Manchester (Remazeilles et al., 2011). The map is Mollweide-projected and produced with a HEALPix resolution of $N_{\text{side}} = 2048$, and 5 arcmin beam size, with Galactic plane masked out as grey.

(Remazeilles et al., 2011):

$$w' = \frac{(b'^{-1}C^{-1}b') a'^{-1}C^{-1} - (a'^{-1}C^{-1}b') b'^{-1}C^{-1}}{(a'^{-1}a) (b'^{-1}b) - (a'^{-1}b)^2} \quad (3.17)$$

The SZ-free 2D-ILC CMB map\(^2\) is produced specifically for this work by Dr. Mathieu Remazeilles in Manchester. It has been produced at the same HEALPix $N_{\text{side}} = 2048$ resolution and 5 arcmin beam size than the public Planck CMB maps by assigning to the nine Planck frequency maps the specific weights Eq. 3.17 that fulfill the constraints of Eqs. 3.16a-3.16b. Fig. 3.2 shows the SZ-free 2D-ILC CMB map used in our analysis.

The Constrained ILC method (and 2D-ILC map) is the first solution proposed in the literature (Remazeilles et al., 2011) to null out thermal SZ residuals in CMB maps, and its unique property was used to detect the kinetic SZ effect in the Planck data by Planck Collaboration Int. XIII (2014). An alternative approach based on sparsity

\(^2\)The 2D-ILC used in this work is available upon request to the authors.
(LGMCA) has then been proposed by Bobin et al. (2014, 2016) to achieve the same goal as the 2D-ILC map. The LGMCA CMB map has also been used to measure the kinetic SZ effect by Hill et al. (2016); Ferraro et al. (2016). While the LGMCA CMB map has been produced from the combination of Planck and WMAP data, the 2D-ILC CMB map, like the released Planck 2015 CMB maps, is based on internal Planck data. Therefore for internal consistency with the released Planck 2015 CMB maps we use the 2D-ILC map as SZ-free CMB template in most of our analysis, and we consider the alternative LGMCA map in Sect. 3.4.2 for consistency check of our results.

3.2.3 SDSS catalogue

The LSS dataset used in this analysis is the Main Photometric Galaxy sample from the DR13 release of the SDSS-IV survey (hereafter MphG, Albareti et al., 2017). The MphG catalogue was downloaded from the SDSS DR13 database\(^3\). The total number of galaxies in this release is about 208 millions, covering a sky area of 14555 deg\(^2\) (Albareti et al., 2017).

SDSS probes galaxies within five optical filter bands \(u, g, r, i, z\). Five different measurements of the magnitude, derived using different fitting methods in the SDSS pipeline, are given in each band for each source. The composite model (hereafter cModel) flux is the combined flux of the best-fitting exponential and de Vaucouleurs fluxes in each band, where the fraction of each term is optimised to give the best-fitting to the source profile (Stoughton et al., 2002). The cModel magnitude is used in our analysis since it is an adequate proxy to use as a universal magnitude for all types of objects and a reliable estimate of the galaxy flux under most conditions\(^4\) (Stoughton et al., 2002).

We use the \(r\) band for our analysis because this band has a better sensitivity

---

\(^3\)http://www.sdss.org/dr13/

\(^4\)http://www.sdss.org/dr12/algorithms/magnitudes
(r < 22.2 for 95% completeness for point sources, York et al., 2000) and photometric calibration accuracy (0.8%, Padmanabhan et al., 2008). The r band is also least affected by Galactic dust extinction. We discard faint sources with the r band cModel magnitude below the completeness level r < 22.2 as those faint sources can smear the cross-correlation signal by adding uncorrelated background noise. We also discard the brightest sources (r < 17 amounting to ≈ 1%) to avoid a small number of bright sources dominating the statistical results. After the selection, the number of galaxies in our sub-sample is about 133 million, which is ≈ 64% of the galaxies in the SDSS DR13 MphG sample.

The SDSS MphG density contrast map. The galaxies detected by SDSS have a typical size of a few arcsec (e.g., Albareti et al., 2017; Stoughton et al., 2002), much smaller than the ∼ 1.7 arcmin pixel size of the Planck maps with a HEALPIX N_{side} = 2048. Therefore, each selected SDSS source is assigned into a single HEALPIX N_{side} = 2048 pixel corresponding to its sky coordinates given by the MphG catalogue. A density contrast map is then constructed from the selected sources, where the density contrast is defined as

\[ n = \frac{(N - \bar{N})}{\bar{N}}, \]  

with N being the number of SDSS sources in each pixel and \( \bar{N} \) the average number of sources per pixel. We use density contrast as it is unaffected by astrophysical interactions such as dust extinction, unbiased by the galaxy brightness and also the simplest quantity used, e.g., in Planck Collaboration XXI (2016) and Planck Collaboration XIX (2014), to detect ISW effect through cross-correlation with CMB maps. The density contrast map is then convolved with a 5 arcmin Gaussian beam using the HEALPIX SMOOTHING routine (Górski et al. 2005) in order to have consistent beam resolution with the Planck CMB maps described in Sect. 3.2.1 and 3.2.2. Fig. 3.3 shows the SDSS MphG density contrast map, where the grey area is the SDSS unobserved sky.
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Figure 3.3: The SDSS MphG density contrast map. The sources in the map are selected by their $r$ band magnitudes between 17 and 22.2. The map is Mollweide-projected with a HEALPix resolution of $N_{\text{side}} = 2048$, and smoothed by a 5 arcmin Gaussian beam. Red (resp. blue) colors mean overdensities (resp. underdensities) of galaxies with respect to the average number of galaxies.

The SDSS MphG flux map In addition to the density contrast map, we also generate a flux map from the SDSS MphG catalogue, where instead of using the number counts, the $r$ band cModel flux after dust extinction correction of each source is used for generating the map. The SDSS catalogue gives the observed galaxy flux (before dust extinction correction), $F_b$, in the unit of nanomaggy\(^5\). The Galactic extinction $A_\lambda$ in magnitude for each source at each band is given by the SDSS database:

\[ m_a = m_b - A_\lambda, \]  

(3.19)

where $m_a$ and $m_b$ are the source magnitudes after and before correcting the dust extinction. The flux after dust extinction $F_a$ is calculated by

\[ m_a - m_b = -2.5 \log \left( \frac{F_a}{F_b} \right) = -A_\lambda. \]  

(3.20)

\(^5\)Maggy is a linear flux unit and one nanomaggy is approximately $3.631 \times 10^{-6}$ Jansky (http://www.sdss.org/dr12/algorithms/magnitudes).
Same as the density contrast map, we only include sources with their $r$ band magnitude after dust extinction correction between 17 and 22.2 to discard weak sources below completeness level and top brightest sources, to avoid biasing statistical results. The corrected $r$ band cModel flux of the selected sources is assigned into one HEALPIX pixel with $N_{\text{side}} = 2048$, corresponding to its coordinates. The values of $F_a$ of all galaxies falling in a given pixel are summed. The MphG flux map is smoothed by a 5 arcmin Gaussian beam, to be consistent with the Planck CMB maps described in Sect. 3.2.1 and 3.2.2.

We emphasise that the main results are not strongly dependent on the LSS map unit (density contrast vs. flux), optical bands, and source selection criteria. We will show in Sect. 3.4.3 that all LSS maps give consistent results for the cross-power spectrum analysis for detecting thermal SZ residuals in the Planck CMB maps.

### 3.2.4 SZ catalogue

In order to interpret the structure of the spurious correlation signal between CMB and galaxy surveys due to large-scale structure residuals in CMB maps, we also construct a pure thermal SZ map at 143 GHz (hereafter, catalogue SZ map) from the Planck 2015 SZ catalogue (Planck Collaboration XXVII 2016). The catalogue SZ map is helpful for simulating the anti-correlation between SZ clusters and galaxy clusters in SDSS, since it does not contain instrumental noise or other foreground contamination but bright SZ clusters. The total number of galaxy clusters in the Planck 2015 SZ catalogue is 1653. For each galaxy cluster, the Planck catalogue provides the value of the integrated thermal SZ flux $Y_{5R_{500}}$ within a radius of $5R_{500}$:

$$Y_{5R_{500}} = \hat{y}_0 \int_{\theta < 5 \times \theta_{500}} dr \tau_{\theta_5}(r),$$

where $\hat{y}_0$ is the Comptonization parameter, $\tau_{\theta_5}$ is the thermal SZ pressure profile of the cluster, and $\theta_{500}$ is the radius within which the average density is 500 times the critical
3.2. DATA DESCRIPTION

density of the Universe, and is related to the cluster angular size, \( \theta_s \), by 
\[ \theta_{500} = c_{500} \times \theta_s, \]
with \( c_{500} = 1.177 \) (Planck Collaboration XXVII 2016). The cluster angular size \( \theta_s \) and 
integrated flux \( Y_{SR500} \) are given in the form of a joint probability distribution in the \( (\theta_s, \ Y_{SR500}) \) plane. For each SZ cluster, the integrated thermal SZ flux \( Y_{SR500} \) and the cluster 
angular size \( \theta_s \) are determined from the maximum likelihood point in the \( (\theta_s, Y_{SR500}) \) 
plane. We use a minimal value of 1.7 arcmin for \( \theta_s \), corresponding to the pixel size.
The SZ clusters have a typical angular size of a few arcmin (e.g., Planck Collaboration V, 2013), smaller than the 5 arcmin beam size. Therefore, our cross-power spectrum 
analysis is not sensitive to the exact profile of the SZ clusters. For simplicity, we 
assume each SZ cluster to have a circular top-hat profile with radius \( \theta_s \) and uniform 
brightness. Under this assumption, Eq. 3.21 reduces to

\[
Y_{SR500} = \hat{y}_0 \times \pi \times \theta_s^2,
\]
(3.22)

from which we derive the Compton parameter, \( \hat{y}_0 \), for each cluster.

For cross-power spectrum analysis, one has to be careful that the result is not domi-
nated by a small number of bright SZ clusters in the statistical sample, which can cause 
a bias relative to the average distribution of clusters. We confirmed that this is not the 
case in our analysis by removing the brightest SZ sources and repeating the analysis.
We will show in Sect. 3.4.4 that excluding the brightest 1–10% of clusters made no 
significant difference to the results. When excluding > 10% the cross-correlation sig-
nal begins to reduce due to the lack of SZ signal. The SZ clusters are projected onto a 
HEALPIX map of \( N_{\text{side}} = 2048 \), according to their sky coordinates given by the Planck 
SZ catalogue. The pixels included in the circles of radius \( \theta_s \) for each cluster are equally 
given the value \( \hat{y}_0 \) of that source. In each pixel, the values \( \hat{y}_0 \) of all SZ clusters falling 
in that pixel are summed. The catalogue SZ \( y \)-map is then converted to thermodynamic 
temperature units at 143 GHz through the spectral energy distribution of the thermal 
SZ effect from Equ. 3.5. The choice of 143 GHz is determined by the fact that most of
residual SZ contamination in *Planck* CMB maps comes from this frequency channel, where the *Planck* signal-to-noise is favourable to the CMB. Finally, the catalogue map at 143 GHz is convolved with a 5 arcmin Gaussian beam to be consistent with the resolution of the *Planck* CMB and SDSS MphG maps described earlier. Fig. 3.4 shows the catalogue SZ map at 143 GHz. A few bright clusters such as the Coma and Virgo clusters are clearly visible from the catalogue SZ map. It can be seen that apart from a few big clusters, the majority of the clusters have their sizes around the 5 arcmin beam size.

![Figure 3.4: The catalogue SZ map at 143 GHz, generated from the Planck 2015 SZ catalogue (Planck Collaboration XXVII 2016). The map is Mollweide-projected with a HEALPIX resolution of $N_{\text{side}} = 2048$, and smoothed by a 5 arcmin Gaussian beam.](image)

### 3.3 Map analysis

In this section, we first perform visual inspection of the residual SZ contamination in the *Planck* 2015 CMB maps, and highlight the resulting spurious anti-correlation with large-scale structure data.
3.3. MAP ANALYSIS

Figure 3.5: Stacked *Planck* CMB maps in the directions of the SZ clusters of the *Planck* 2015 SZ catalogue (Planck Collaboration XXVII, 2016) in the Galactic coordinate system with the north Galactic pole upwards. From left to right: SMICA, SEVEM, COMMANDER and NILC CMB maps. Each map covers $3^\circ \times 3^\circ$ and the angular resolution is 5 arcmin. Blue spots in the centre show evidence for negative thermal SZ contamination from galaxy clusters.

3.3.1 Stacking CMB maps in the direction of galaxy clusters

To demonstrate that there are LSS-correlated residuals in the CMB maps, each of the four *Planck* 2015 CMB maps – SMICA, SEVEM, COMMANDER, NILC – is stacked at the locations of the SZ clusters of the *Planck* 2015 SZ catalogue (Planck Collaboration XXVII 2016). Each CMB map is projected onto $3^\circ \times 3^\circ$ patches of the sky centred at each SZ cluster location. The patches are then averaged all together, giving the stacked maps shown in Fig. 3.5. Clearly, the four stacked *Planck* CMB maps show strong negative temperature fluctuations in the centre (blue spot), showing clear residual contamination from thermal SZ effect in the direction of galaxy clusters. The thermal SZ residuals appear as negative temperature fluctuations as the bulk of the weights assigned to the *Planck* frequency maps by component separation algorithms is around 100–143 GHz frequencies, where the CMB emission is dominant over the foreground emission but the thermal SZ SED is negative (Equ. 3.5). While four different component separation techniques have been operated on the *Planck* frequency data to minimize the overall foreground contamination and extract the CMB signal, there is still significant residual thermal SZ emission from galaxy clusters in the *Planck* CMB maps.

Following the same procedure, we show in the right panel of Fig. 3.6 the SDSS
Figure 3.6: Stacked maps in the directions of the SZ clusters of the Planck 2015 SZ catalogue (Planck Collaboration XXVII, 2016) in the Galactic coordinate system with the north Galactic pole upwards: the 2D-ILC CMB map (left), the difference (NILC−2D-ILC) map (middle), and the SDSS MphG galaxy density map (right). Each map covers $3^\circ \times 3^\circ$ and the angular resolution is 5 arcmin.

MphG density contrast map stacked in the directions of the same clusters from the Planck 2015 SZ catalogue. As expected from the agglomeration of galaxies within clusters, the stacked SDSS MphG map shows a strong positive overdensity in the direction of SZ clusters. Our stacking analysis thus provides visual evidence for anti-correlation between SZ cluster residual fluctuations in CMB maps and galaxy overdensities in the SDSS survey. The presence of clusters in the CMB maps may bias any cross-correlation analysis between CMB products and optical galaxy surveys. Therefore, we warn that any statistical interpretation of cross-correlation results (e.g., CMB lensing-galaxy lensing correlations) must bear in mind the amount of spurious correlations from cluster residuals in CMB maps.

Depending on the scientific purpose, for example CMB-LSS cross-correlations, it might be more useful to filter out LSS residuals, such as thermal SZ emission from galaxy clusters, in the CMB maps rather than minimizing the global foreground contamination. In this regard, we propose an SZ-free CMB map, termed as 2D-ILC map, which we have constructed from the Planck 2015 data using the Constrained ILC component separation technique (Remazeilles et al., 2011). The Constrained ILC is specifically designed to null out thermal SZ effects in the reconstructed CMB map (Sect. 3.2.2).
3.3. MAP ANALYSIS

The result of stacking the 2D-ILC CMB map in the direction of the clusters of the Planck SZ catalogue is shown on the left panel of Fig. 3.6. In the case of the 2D-ILC CMB map there is a clear absence of thermal SZ contamination from galaxy clusters, contrasting against the other Planck CMB maps. The cost of this extra filtering constraint is that the 2D-ILC CMB map is slightly noisier (small-scale granularity) than the public Planck CMB maps. In the middle panel of Fig. 3.6, we show the result of stacking the difference map between the NILC and 2D-ILC CMB maps. The difference highlights the negative SZ cluster residuals from the NILC CMB map, dominating over the noise fluctuations from the 2D-ILC map, and anti-correlating with the galaxy density contrast from SDSS (right panel). The absence of LSS residuals (or negligible SZ contamination with respect to noise) in the 2D-ILC map makes it particularly suited for cross-correlations studies with LSS optical surveys.

3.3.2 Projection of individual clusters

The spurious correlation between SZ cluster residuals in CMB maps and SDSS galaxies is also visible from the maps by looking at individual cluster locations in the sky. To highlight thermal SZ residuals in the Planck NILC CMB map, we show in the top 54 panels of Fig. 3.7 the difference (NILC − 2D-ILC) map projected onto the locations of 54 selected clusters of the Planck SZ catalogue (Planck Collaboration XXVII, 2016) sorted by decreasing signal-to-noise ratio. The size of each map (‘stamp’) is 0.5° × 0.5°. Each stamp clearly shows negative (blue) temperature fluctuations at the position of the cluster due to residual thermal SZ emission in the Planck NILC CMB map. Similarly, the bottom 54 stamps of Fig. 3.7 show the SDSS MphG density contrast map projected onto the same cluster locations in the sky. We see in this case an overdensity from SDSS galaxies at the positions of the SZ clusters of the Planck SZ catalogue. These direct projections of the maps at the cluster positions gives a complementary view of the spurious anti-correlations between the SZ cluster residuals in CMB maps and galaxy densities in the SDSS survey.
Figure 3.7: Gnomic projection small maps of 54 individual clusters (sorted by decreasing signal-to-noise ratio given by Planck Collaboration XXVII (2016)) of the difference (NILC−2D-ILC) map (top 54 panels; upper half) and the SDSS MphG density contrast map (bottom 54 panels; lower half). Each map is a $0.5^\circ \times 0.5^\circ$ field-of-view centred at the cluster position in the Galactic coordinate system with the north Galactic pole upwards. Grey areas in some of the stamps are masked regions of the sky.
We recommend the use of the 2D-ILC CMB map for CMB-LSS cross-correlation studies, given that this SZ-free CMB map appears to be safe from residual LSS foreground correlations. A more quantitative analysis on cross-power spectra is developed in the next section in order to corroborate our preliminary findings based on visual inspection of the maps.

3.4 Cross-power spectrum analysis

In this section, we quantify the detection of thermal SZ residuals in the Planck CMB maps by computing the CMB-LSS cross-power spectra and Pearson correlation coefficient across angular scales. We also discuss the impact of the SZ residuals on ISW effect detection.

3.4.1 Power spectrum estimator

We use the POLSPICE (Szapudi et al., 2001; Chon et al., 2004; Challinor & Chon, 2005) estimator to compute the angular cross-power spectra between the Planck CMB and SDSS galaxy survey maps. POLSPICE firstly computes the two-point correlation function of the two maps, and then transform into harmonic space to compute the cross-spectrum. To minimize the presence of residual contamination from Galactic foreground emission in the CMB maps, the Galactic plane is masked. We tested a Galactic mask with $|b| < 10^\circ$, $|b| < 20^\circ$, $|b| < 30^\circ$ and $|b| < 40^\circ$, which all give consistent results. In the end, we mask out all pixels at Galactic latitudes $|b| < 30^\circ$, which mitigates the contamination from Galactic foregrounds while keeping enough signal for the cross-correlation analysis with SDSS. The unobserved sky region of SDSS is also masked out on the HEALPIX map in addition to the CMB mask. Finally, the Planck 143 GHz intensity point source mask (Planck Collaboration XI, 2016) is used to avoid contamination from point sources. The total masked region is shown as the

\[http://www2.iap.fr/users/hivon/software/PolSpice/\]
white area in Fig. 3.8. The mask is apodized by an 80 arcmin beam to avoid spherical harmonic transform artefacts from the sharp boundary of the mask when computing angular power spectra. Afterwards, a threshold of 0.5 is imposed where pixels with a value below this threshold are set to zero. The sky coverage corresponds to 27% of the full sky ($f_{\text{sky}} = 0.27$).

![Figure 3.8: The mask used in the power spectrum analysis. The Galactic plane between the Galactic latitudes $|b| < 30^\circ$ and the unobserved sky region of SDSS (white) is masked out from our analysis. The valid area (grey) outside of the mask corresponds to a fraction of the sky $f_{\text{sky}} = 0.27$.](image)

The maximum distance $\theta_{\text{max}}$ used in the POLSPICE estimator to integrate the correlation functions (e.g., Szapudi et al., 2001) for power spectrum calculation is 40°. This value is chosen to minimise the effects of masking and because the available sky area means that the very largest angular scales ($\ell \lesssim 10$) are not reliably measured. We tested several values of $\theta_{\text{max}}$, from 10° to 80° and found consistent results for $\ell > 100$. The scale factor of the correlation function tapering (e.g., Szapudi et al., 2001) is half of $\theta_{\text{max}}$ (i.e., 20°), in order to have a smooth boundary for correlation function computation. The $f_{\text{sky}} = 0.27$ factor due to the mask, the HEALPIX $N_{\text{side}} = 2048$ pixel window function, and the 5 arcmin beam convolution effect are all corrected for by POLSPICE. Depending on the analysis, the angular power spectrum values are computed into multipole bins of width varying from $\Delta \ell = 100$, since we do not have accurate information
on large scales below $\ell = 100$ due to mask, to $\Delta \ell = 1000$, in order to calculate the averaged cross-correlation signal over a larger bin width.

In order to quantify the (anti-)correlation between SZ cluster residuals in CMB maps and SDSS galaxies, we compute the dimensionless Pearson correlation coefficient over the angular scales as:

$$c_\ell = \frac{C_{\ell}^{\text{CMB} \times \text{SDSS}}}{\sqrt{C_{\ell}^{\text{CMB}} C_{\ell}^{\text{SDSS}}}}.$$  \hfill (3.23)

To understand the Pearson correlation coefficient, two completely correlated maps will have $c_\ell = 1$, since in this case the two maps are identical and thus have their cross-spectrum same as the auto-spectrum. Two completely uncorrelated maps will have $c_\ell = 0$, since the cross-spectrum is zero in this case. For two completely anti-correlated maps, they will have $c_\ell = -1$ since the two maps are identical except their opposite signs, giving a negative cross-spectrum. In Equ. 3.23, 'CMB' stands for either the Planck 2015 NILC CMB map or the SZ-free 2D-ILC CMB map, and 'SDSS' stands for the SDSS MphG galaxy survey map smoothed to the same angular resolution of 5 arcmin as the CMB maps. Here $C_{\ell}^{\text{CMB} \times \text{SDSS}}$ is the cross-power spectrum between the CMB map and the SDSS map, and $C_{\ell}^{\text{CMB}}$ ($C_{\ell}^{\text{SDSS}}$) is the auto-power spectrum of CMB (SDSS) map.

In order to focus on the thermal SZ residuals, we will also consider the correlation coefficient:

$$\tilde{c}_\ell = \frac{C_{\ell}^{\text{Diff} \times \text{SDSS}}}{\sqrt{C_{\ell}^{\text{Diff}} C_{\ell}^{\text{SDSS}}}},$$  \hfill (3.24)

where 'Diff' stands for the difference (NILC–2D-ILC) map. The numerator quantifies the correlated part between two maps and the denominator is the normalisation factor, given by the square root of the auto-correlation spectrum of each map. The dimensionless cross-correlation coefficient quantifies the fractional correlated signal in the maps. A non-zero positive correlation coefficient suggests correlation between two maps while a negative value suggests anti-correlation.
3.4.2 CMB $\times$ SDSS

We first compute the cross-correlation coefficient (Eq. 3.23) between CMB and large-scale structures. Due to the ISW effect, the CMB temperature fluctuations and the distribution of large-scale structures are expected to show a positive correlation on the largest angular scales, corresponding to $\ell \lesssim 200$ (or $> 1^\circ$) (e.g., Planck Collaboration XXI, 2016). However, CMB and large-scale structures must be theoretically uncorrelated on smaller angular scales $\ell > 500$, as long as the CMB map is not contaminated by large-scale structure foregrounds or extragalactic sources, such as SZ clusters.

The scale-dependent cross-correlation coefficient (Eq. 3.23) computed between the SDSS MphG density contrast map and the NILC (resp. the 2D-ILC) CMB map is shown in red (resp. green) in the upper panel of Fig. 3.9. The first bin of 100 multipoles ($\ell = 2$–100) are removed from our analysis because the largest scale modes are significantly affected by the mask and apodization. The cross-correlation is thus divided into 5 bins of multipoles: $100 < \ell < 500$, $500 < \ell < 1500$, $1500 < \ell < 2500$, $2500 < \ell < 3500$, and $3500 < \ell < 4000$. Note that here each bin is independent of each other, since each data point in Fig. 3.9 already has a bin width of $\Delta \ell = 100$, giving independent signal at this scale.

In order to assess the level of sample uncertainty inherent to the cross-correlation coefficient statistics, we perform the following Monte Carlo (MC) simulation. We first simulate 1000 random Gaussian realisations of pure CMB maps from a theoretical CMB power spectrum generated by CAMB (Challinor & Lewis 2011; Howlett et al. 2012), based on the Planck 2015 $\Lambda$CDM model (Planck Collaboration XIII, 2016). We then compute the cross-correlation coefficient (Eq. 3.23) between the 1000 random pure CMB realisations and the SDSS density contrast map, so that the dispersion of the sample provides the $1\sigma$ uncertainty in each $\ell$ bin. The $1\sigma$ sample variance based on the MC simulation is shown as the grey shaded region in Fig. 3.9.

We detect a clear excess of anti-correlation at more than $1\sigma$ between the Planck NILC CMB map and the SDSS galaxy survey (red) on the full range of angular scales...
between $\ell = 500$ and $\ell = 2500$. The observed anti-correlation between the Planck NILC CMB map and SDSS density contrast map is caused by the negative thermal SZ residual contamination from galaxy clusters in the CMB map that we have highlighted in Sect. 3.3. In contrast, the cross-correlation of the 2D-ILC CMB map with LSS from SDSS in Fig. 3.9 (green) shows less correlation on average, and is consistent with zero within $\sim 1\sigma$ sample variance, although for the multipole bin of $1500 < \ell < 2500$ the correlation is slightly larger than $1\sigma$. This $1\sigma$ deviation from zero might be attributed to chance correlations with kinetic SZ residuals that do not average out due to sample selection, mis-calibration of thermal SZ (i.e. relativistic SZ effects which were neglected in the analysis), or other compact foreground residuals in the 2D-ILC map.
The cross-correlation coefficient between the SDSS density contrast map and another SZ-free LGMCA CMB map is also computed for a consistency check as shown in blue in the upper panel of Fig. 3.9. By comparing the blue and green lines, LGMCA gives consistent results with 2D-ILC such that the cross-correlation coefficients in both cases are consistent with zero within $\sim 1\sigma$ sample variance. The level of anti-correlation between the Planck NILC CMB map and the SDSS galaxies due to SZ residuals is not larger than 0.5%, nevertheless is significantly detected over a broad range of multipoles. Below $\ell = 200$, the positive correlation between the Planck NILC CMB map and SDSS is due to ISW effect but still might be underestimated because of large-scale SZ contamination. Fig. 3.9 confirms that the 2D-ILC CMB map contains no thermal SZ effect by construction, and thus LSS-correlated residuals are minimised.

The lower panel of Fig. 3.9 shows the difference between the NILC $\times$ SDSS and 2D-ILC $\times$ SDSS correlation coefficients (green), highlighting the scale-dependence of the anti-correlation between CMB SZ residuals and SDSS galaxies. Similar to the 2D-ILC map, a clear excess of anti-correlation is present in the difference between the NILC $\times$ SDSS and LGMCA $\times$ SDSS correlation coefficients (blue), due to the thermal SZ residuals in the NILC CMB map. The residuals of the thermal SZ impacts a wide range of scales, from the largest angular scales down to $\ell \sim 2500$. For the first bin ($\ell = 100–500$) both CMB maps do not show a significant correlation with LSS and therefore the contamination is probably dominated by Galactic and extragalactic foregrounds. In the range $\ell = 500–2500$ the difference is attributed primarily due to the removal of the thermal SZ effect in the 2D-ILC map. Above $\ell \sim 2500$ there is no significant correlation due to the lack of angular resolution and the presence of noise in the CMB maps. However, we warn that for CMB maps measured at a higher resolution from, e.g., ACT (Fowler et al., 2010) and SPT (Lueker et al., 2010), one might see correlations at smaller scales. We will use the 2D-ILC CMB map for the cross-spectrum calculation in the rest of our analysis to be consistent with the released Planck 2015 CMB maps, since the LGMCA CMB map combines both Planck and
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WMAP data.

![Figure 3.10](image)

**Figure 3.10:** *Upper panel:* Cross-correlation coefficients of the SDSS MphG flux map with the *Planck* 2015 NILC CMB map (*red*) and the SZ-free 2D-ILC CMB map (*green*). The grey shaded area shows the 1σ uncertainty due to sample variance calculated from Monte Carlo simulations of pure CMB realisations. *Lower panel:* the cross-correlation coefficient difference of the (SDSS × NILC) − (SDSS × 2D-ILC). Filled circles connected by dotted lines give the data binned within multipole ranges of ∆ℓ = 100, while thick horizontal bars give the data averaged over larger bin widths.

For comparison, instead of using the SDSS MphG density contrast map, we also use the SDSS MphG flux map to compute the cross-correlation coefficients with CMB maps. The results are shown in Fig. 3.10. The ∼ 1σ sample variance in this case is computed by cross-correlating the SDSS MphG flux map with the simulated 1000 random Gaussian realisations of pure CMB maps. The SDSS MphG flux map (Fig. 3.10) gives consistent results with the density contrast map (Fig. 3.9) such that in both cases, the cross-correlation coefficient from the NILC CMB map shows a negative power excess due to its thermal SZ cluster residuals anti-correlated with LSS while the cross-power spectrum with the SZ-free 2D-ILC CMB map is consistent with zero within ∼ 1σ sample variance. As mentioned before, since the density contrast map is unbiased by the galaxy flux and not affected by other astrophysical interactions such as dust extinction, we will use SDSS MphG density contrast map for the rest of our analysis.
3.4.3 SZ residuals × SDSS

In order to quantify the amount of spurious anti-correlation between SDSS galaxies and SZ cluster residuals in the Planck NILC CMB map, we now consider the difference map between the NILC CMB map and the SZ-free 2D-ILC CMB map. The difference (NILC−2D-ILC) map is dominated by residual thermal SZ emission from galaxy clusters, while the CMB signal has been cancelled out by the difference. This difference map contains also residual foregrounds (Galactic and extragalactic) and noise due to the different processing of the Planck data by the NILC and 2D-ILC algorithms, but at a negligible level compared to SZ residuals, at least outside of the mask (see e.g., middle panel of Fig. 3.6). Following the same process described in Sect. 3.4.2, we now compute the cross-correlation coefficient (Eq. 3.24) between the SDSS MphG density contrast map and the difference map over the angular scales. This can be considered a proxy for the cross-correlation coefficient between SDSS galaxies and SZ cluster residuals in the CMB.

We start with the r band since this is the most sensitive band of SDSS and first test the different source selection criteria by discarding different percentages of the brightest sources from SDSS MphG catalog and then cross-correlating with the difference map. The cross-correlation coefficient in each case is shown in Fig. 3.11. Seven selection criteria have been tested where no source is discarded (no cut), sources below completeness level ($r > 22.2$) are discarded (comp+), sources below completeness magnitude and top $N\%$ brightest are discarded (comp+$N\%$−) with $N\%$ ranges from $[1\% (r < 17.0), 5\% (r < 19.4), 10\% (r < 20.1), 20\% (r < 20.8), 50\% (r < 21.9)]$. A density contrast map is created for each of the source selection criteria and cross-correlated with the difference map. From Fig. 3.11, all selection criteria give cross-correlation coefficients with a consistent trend such that the anti-correlation signals are maximum at large scales below $\ell \approx 500$ and gradually reduce towards small scales until vanish at $\ell \approx 2500$. After removing weaker sources below the completeness level, the anti-correlation signal gradually reduces with the increasing percent (beyond 5\%)
3.4. **CROSS-POWER SPECTRUM ANALYSIS**

![Figure 3.11: The cross-correlation coefficients between the difference map and SDSS MphG maps at different source selection criteria. In all cases, the SDSS MphG map is the density contrast map created using $r$ band magnitude for seven different source selection criteria. *nocut* stands for where no source is discarded; *comp+* stands for where sources below completeness level are discarded; *comp+N%−* stands for where only sources above completeness level and dimmer than the top N% brightest are included, with N% ranges from [1%, 5%, 10%, 20%, 50%]. The cross-correlation coefficients are binned with $\Delta\ell = 100$.](image)

The cross-correlation coefficients are binned with $\Delta\ell = 100$. Of brightest sources discarded, by comparing the cases when discarding 5%, 10%, 20%, 50% of the brightest sources. The reduction of the anti-correlation signal in this case is caused by the lack of correlations due to the cut of galaxy clusters. By discarding sources weaker than the completeness level or brighter than the top 1%, it slightly increases the anti-correlation signal, compared with the case where no source is discarded. This enhancement of the anti-correlation signal might be attributed to the removal of outliers that are weakly detected with low S/N or strong enough to potentially distort the general statistics. Therefore, for conservative reasons, we only include sources above the completeness level and those weaker than the top 1% brightest from the SDSS MphG catalogue as our LSS samples for the rest of our analysis.

We also compute the cross-correlation coefficients between the difference map and...
SDSS MphG density contrast map at different optical bands to see if different optical bands will affect our cross-correlation results. We create 5 SDSS MphG density contrast maps at the 5 SDSS optical bands - $u, g, r, i, z$. In each band, we use the selection criteria set before such that we discard the top 1% brightest sources and those below the completeness level of that band. The magnitudes of the selected sources in each bands are $[18.6 < u < 22.0, 18.4 < g < 22.2, 17.0 < r < 22.2, 17.0 < i < 21.3, 17.0 < z < 20.5]$. The cross-correlation coefficient in each band is shown in Fig. 3.12, where all five bands give consistent results. However, it does show that the $r$ band gives the largest anti-correlation signal, due to its better sensitivity and calibration accuracy.

We then move on to a more detailed inspection using the SDSS MphG $r$ band density contrast map. Figure 3.13 shows the scale-dependent anti-correlation between the SDSS MphG map and the difference map (red). In this case, the sample variance is computed as follows. We randomize the galaxy positions in the selected SDSS MphG
samples and generate 1000 random LSS catalogues with randomly distributed galaxies. A simulated LSS map is created from each of the 1000 random catalogues. The 1000 simulated LSS maps are then cross-correlated with the difference map, giving the $1\sigma$ uncertainty due to sample variance that is shown as the grey shaded area in Fig. 3.13.

Figure 3.13: The cross-correlation coefficients between the SDSS MphG map and the difference (NILC−2D-ILC) map (i.e., the SZ cluster residual map). The grey shaded area shows the $1\sigma$ uncertainty due to sample variance, calculated from Monte Carlo simulations with random positions of SDSS galaxies. Red squares give the data binned within multipole ranges of $\Delta \ell = 100$, while thick horizontal bars give the data averaged over larger bin widths.

Figure 3.13 confirms that the spatial (anti-)correlation with galaxy overdensities in SDSS comes from SZ cluster residuals in the Planck NILC CMB map. This spurious correlation is detected well beyond $1\sigma$ sample variance at the cluster scale, and with larger significance at large angular scales due to the clustering of SZ clusters. Again, the spurious correlation signal is weak in absolute value, with a few % on cluster scales ($\ell = 1500–2500$) and up to $\approx 6\%$ on large scales ($\ell = 100–500$). Nevertheless, the correlation is detected at high significance across a wide range of scales.
We consider this result as our best estimate of the amount of spurious correlation between SZ foregrounds in CMB maps and SDSS galaxies, given that any chance correlation due to CMB or Galactic foregrounds has been eliminated from the difference (NILC−2D-ILC) map. These results are consistent with those of Fig. 3.9, where the cross-correlation was computed between the CMB and SDSS maps. Interestingly, the spurious correlation due to SZ residuals is not dominant at the cluster scale but on large angular scales. We will interpret this overall trend in Sect. 3.4.5.

We quantify the significance of the detection of the anti-correlation between CMB SZ cluster residuals and SDSS galaxies by computing the total signal-to-noise ratio (SNR) in each bin of multipoles in Fig. 3.13 as:

$$\frac{S}{N} = \left[ \sum_{\ell'=\ell-\Delta\ell/2}^{\ell+\Delta\ell/2} \left( \frac{c_{\ell'}}{\sigma_{\ell'}} \right)^2 \right]^{1/2},$$

where $c_{\ell}$ is the value of the cross-correlation coefficient at multipole $\ell$ between SDSS and the SZ residual map, while $\sigma_{\ell}$ is the corresponding 1σ sample variance from the Monte Carlo simulations at the same multipole. The SNR results for different ranges of angular scales and their fractional correlation signal read from Fig. 3.13 are listed in Table 3.1.

<table>
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<tr>
<th>Multipole Range</th>
<th>S/N</th>
<th>Fractional Anti-Correlation</th>
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</thead>
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<tr>
<td>[100, 500]</td>
<td>23.0</td>
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<tr>
<td>[500, 1500]</td>
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</tr>
<tr>
<td>[100, 2500]</td>
<td>54.5</td>
<td>2.60%</td>
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</table>

Table 3.1: Detection significance of the spurious correlation between SZ cluster residuals in the Planck CMB map and SDSS galaxies, along with the fractional anti-correlation signal (Pearson correlation coefficient), over different ranges of angular scales from Fig. 3.13.

At the cluster scale ($1500 < \ell < 2500$), the anti-correlation is detected with $\approx 30\sigma$ significance, while overall the detection significance is $\approx 54\sigma$ over the full range of angular scales from $\ell = 100$ to $\ell = 2500$. However, it should be mentioned that in addition to the contribution from SZ cluster residuals, the correlated signal might also
hide a small contribution from other foreground residuals in the difference map due to inherent differences between the two processed CMB maps. To confirm this, we compute the cross-correlation coefficient between the difference map and the Planck GNILC thermal dust map at 545 GHz and 857 GHz (Planck Collaboration XLVIII, 2016). The SZ clusters from the Planck SZ catalogue are masked out to avoid any correlations caused by SZ cluster residuals in the Planck dust templates. The results are shown in Fig. 3.14. There is a positive correlation at the first bin of $\ell = [100−200]$ and a negative correlation at $100 < \ell < 3000$, suggesting that the difference map has other foreground residuals in addition to thermal SZ clusters, which shows up when cross-correlated with the thermal dust maps. This negative correlation on cluster scales will contribute to the cross-correlation coefficient in Fig. 3.13, resulting in a stronger detection SNR. The positive correlation observed between the difference map and foreground dust maps at the first bin of $\ell = [100−200]$ might explain the positive bias in the first $\ell$-bin ($\ell = 100−500$) of Fig. 3.9, although the positive bias is at the $\sim 1\sigma$ level.
3.4.4 \textit{SZ} \times \textit{SDSS}

To interpret the cross-correlation of CMB tSZ residuals with SDSS, it is instructive to generate a pure SZ cluster map. We use the catalogueSZ map at 143 GHz from the Planck \textit{SZ} catalogue, as described in Sect. 3.2.4. In order to investigate if the brightest SZ clusters in the Planck \textit{SZ} catalogue will distort the results, we first remove the top $N\%$ brightest SZ clusters sorted by the detection S/N given by Planck \textit{SZ} catalogue, with $N\%$ ranges from [0\%, 3\%, 10\%, 50\%]. A catalogue SZ map at 143 GHz is created in each case and cross-correlated with the SDSS MphG map. The results are shown in Fig. 3.15, where the removal of less than top 10\% brightest clusters makes no difference to the results but if increased to 50\%, the anti-correlation signal decreases due to the lack of SZ clusters. Since the brightest SZ clusters do not affect the cross-correlation results, we include all sources from the Planck \textit{SZ} catalogue in the catalogue SZ map at 143 GHz, which is used in all of our analysis in this section.

We then inspect the cross-correlation between the SDSS MphG map and the
143 GHz catalogue SZ map in more detail as shown in red in Fig. 3.16. The grey area shows the $1\sigma$ sample variance, computed by generating 1000 SZ maps obtained by randomizing the locations of the SZ clusters in the Planck 2015 SZ catalogue, while keeping the Compton parameter fluxes from the catalogue. The catalogue SZ map at 143 GHz confirms the overall trend of anti-correlation with SDSS galaxies, as observed from CMB data. Of course, in this case the correlation coefficient is larger because the full thermal SZ signal contributes to the correlation signal, while on the data only a fraction of the thermal SZ emission is present as a residual contamination in the Planck CMB maps.

As further evidence, we also cross-correlate the Planck 2015 NILC thermal SZ y-map (Planck Collaboration XXII, 2016), based on Planck sky observations, with the SDSS MphG map (green curve in Fig. 3.16). The Planck SZ y-map has a HEALPIX
pixel resolution of $N_{\text{side}} = 2048$ but a beam resolution of 10 arcmin. In order to cross-correlate the Planck $y$-map with the SDSS MphG map, we thus smooth the SDSS MphG map down to 10 arcmin. Since the $y$-Compton parameter is positive, the cross-correlation coefficient of the Planck SZ $y$-map with the galaxy overdensities of the SDSS MphG map is also positive. Note that the SZ $y$-map gives a larger fractional correlation than the 143 GHz catalogue SZ map. This is due to other instrument noise, foreground residuals and the diffuse SZ signal between clusters in the SZ $y$-map, that are correlated with the SDSS galaxies. Indeed, the SZ $y$-map looks very noisy when we visually check the map. Therefore, we emphasize that the results obtained from $y$-map shall be interpreted with caution. Nevertheless, we recover the general trend observed in the cross-correlations between the Planck CMB map and the SDSS survey map, which confirms that the observed spurious anti-correlation is due to SZ cluster residuals in the Planck CMB maps.

Both cross-correlation coefficients in Fig. 3.16 show that there is a correlation between SZ clusters and SDSS galaxies both in the typical cluster scale at $\ell \sim 2000$, as expected from the agglomeration of galaxies within clusters, but mostly on large angular scales $\ell < 1500$, that we attribute to the clustering of SZ clusters.

In order to estimate the amount of thermal SZ emission that has been left over in the Planck 2015 NILC CMB map, we calculate the cross-correlation coefficient

$$\delta = \frac{C^{\text{Diff} \times \text{SZ}}}{\sqrt{C^{\text{SZ}} C^{\text{SZ}}}},$$

(3.26)

which gives an estimated percentage of the thermal SZ emission that has been left over in the Planck 2015 NILC CMB map. We calculate $\delta$ using the 143 GHz catalogue SZ map and the Planck $y$-map converted into temperature at 143 GHz respectively. The results are shown in Fig. 3.17. At the typical cluster scale of $\ell \sim 2000$, both maps give an estimated 10% left-over of the thermal SZ emission in the Planck NILC CMB map. On large scales, the estimated left-over based on the catalogue SZ map ($\sim 40\%$) is
higher than that based on the Planck y-map (~15%). The difference between the two maps might be due to the foreground contamination in the Planck y-map, which smears out some of the SZ signal and thus results in a smaller percentage when estimating the left-over of the thermal SZ emission in the Planck NILC CMB map.

The percentage of the left-over thermal SZ emission in the Planck NILC CMB map is further quantified through aperture photometry. At each SZ cluster location given by the Planck SZ catalogue, a circle with a radius of 1.5 times the cluster radius convolved with the 5 arcmin beam is used to calculate the integrated flux density within the circle. Two annuli with respective radii of 1.8 and 2.2 times the cluster radius convolved with the beam are used to subtract the background flux density from the foreground circle, giving the integrated flux density of the cluster in the foreground circle. The scales of the foreground and background radii are chosen such that the foreground circle fully covers the cluster signal, taking into account the smearing effect from the beam at the
edge of the clusters, and the background annuli is large enough to not contain any foreground signal but small enough to contain only the local background noise. Under this criteria, we tried a few other choices of the radii, which all yield consistent results.

The total number of SZ clusters from the Planck SZ catalogue outside of the mask region included in our analysis is 549. The integrated flux density given by aperture photometry of the included SZ clusters are calculated from both the difference map and the catalogue SZ cluster map, the ratio of which gives the percentage of the SZ emission that has been left in the Planck NILC CMB map. Among the 549 included SZ clusters, 97 (18%) give a ratio of either larger than 1 or smaller than 0, due to the noisy background in the difference map which affects the performance of aperture photometry. We further discard these 97 noisy clusters from our statistics and the left-over percentage distribution of the rest 452 (82%) clusters are plotted in Fig. 3.18. Over half (58%) of the clusters give a ratio between 0.2 and 0.6 and overall give an average of 0.44, meaning that 44% of the thermal SZ emission has been left over in the Planck NILC CMB map.

To test the robustness of this result we also split the 452 clusters into 5 groups of decreasing detection S/N ratio and found mean values of 0.488, 0.465, 0.413, 0.394 and 0.410, respectively. This shows that the mean residual flux is robust with a standard deviation of 0.04. This is consistent with the cross-spectrum analysis using the difference map and the catalogue SZ cluster map (red) in Fig. 3.17 which suggests $\sim 30\text{--}60\%$ on scales $\ell = 100\text{--}1000$. Therefore, we consider $44 \pm 4\%$ as our best estimate for the percentage of thermal SZ emission that has been left over in the Planck NILC CMB map.

### 3.4.5 Large-scale anti-correlation due to clustering of SZ clusters

Figures 3.9, 3.13, and 3.16 all show large anti-correlation signals at large angular scales, $100 < \ell < 1500$, which disappears in the Monte Carlo simulations where source
3.4. **CROSS-POWER SPECTRUM ANALYSIS**

locations have been randomized. Therefore, the large-scale anti-correlation signal cannot be attributed to artefacts due to masking and apodization. In order to understand large-scale anti-correlation, we first visually inspect the 143 GHz catalogue SZ map, which indeed have SZ clusters that are clustered together. To quantify the large-scale clustering of SZ clusters, we compute the two-point correlation function of the thermal SZ emission as a function of angular separation over the sky to confirm the detection of large-scale clustering of SZ clusters. We use the TreeCorr package (Jarvis et al. 2004) to compute the two-point correlation function as

$$\xi = \frac{DD - 2DR + RR}{RR},$$

(3.27)

where $DD$ is the counts of pairs of SZ clusters in the Planck 2015 SZ catalogue as a function of separation $r$ for each bin, $RR$ is the counts of pairs in a random catalogue with Poisson distributed sources and $DR$ is that between the SZ catalogue and the
Figure 3.19: Two-point angular correlation function of the Planck 2015 SZ catalogue (red) and a random catalogue with Poisson distributed sources (green). The 1σ sample variance calculated from Monte Carlo simulations in each case is shown as the shaded areas.

random catalogue (Landy & Szalay 1993).

The minimum and maximum separation is chosen to be 1 degree and 10 degrees respectively, split into 10 bins on a logarithmic scale. We create 1000 random catalogues of Poisson distributed sources and compute the two-point correlation (Eq. 3.27) for the SZ catalogue using each random Poisson catalogue, the mean of which is plotted as the red curve in Fig. 3.19, and the dispersion of which provides the 1σ uncertainty shown as the red-shaded area. For comparison, we also compute the two-point correlation among the 1000 random Poisson catalogues shown as the green curve in Fig. 3.19. In each case, the larger uncertainty towards smaller separations is because, for each source, one annulus with a width equal to the bin width is used to compute the correlation function for that bin. Therefore, a larger separation corresponds to a larger area with more samples for calculating the correlation function and thus gives less sample variance. While the two-point correlation function for the random Poisson catalogues is on average, separation is chosen to be 1 degree and 10 degrees respectively, split...
3.4. **CROSS-POWER SPECTRUM ANALYSIS**

into 10 bins on a logarithmic scale. We create 1000 random catalogues of Poisson distributed sources and compute the two-point correlation (Eq. 3.27) for the SZ catalogue using each random Poisson catalogue, the mean of which is plotted as the red curve in Fig. 3.19, and the dispersion of which provides the 1σ uncertainty shown as the red-shaded area. For comparison, we also compute the two-point correlation among the 1000 random Poisson catalogues shown as the green curve in Fig. 3.19. In each case, the larger uncertainty towards smaller separations is because, for each source, one annulus with a width equal to the bin width is used to compute the correlation function for that bin. Therefore, a larger separation corresponds to a larger area with more samples for calculating the correlation function and thus gives less sample variance. While the two-point correlation function for the random Poisson catalogues is on average zero over the full range of angular separations over the sky, the two-point correlation function for the SZ catalogue shows positive peaks of correlation on degree scales. After adding up all 10 bins (similar as in Equ. 3.25), the total detecting S/N of the large-scale clustering of SZ clusters is 3.3σ.

Since the typical angular size of SZ clusters is a few arcmins (e.g., Planck Collaboration V, 2013), corresponding to a multipole range of $\ell \sim [1000 - 3000]$ as $\ell \sim \frac{180}{\theta}$, our result confirms that the anti-correlation signal at degree scales corresponding to $\ell \sim [100 - 1000]$ is mainly caused by the clustering of SZ galaxy cluster residuals in CMB maps on scale of a few degrees.

### 3.4.6 Impact on the measurement of the Integrated Sachs-Wolfe effect

Given that SZ cluster residuals in the Planck CMB map create a spurious anti-correlation signal in the cross-power spectrum between CMB and SDSS maps, this could potentially be an issue for the measurement of the ISW effect by cross-correlation. Indeed the amplitude of the positive correlation signal due to ISW effect might be underestimated because of the competing anti-correlation signal due to SZ
residuals in the CMB map.

In Planck Collaboration XIX (2014) and Planck Collaboration XXI (2016), the cross-power spectrum between the Planck CMB maps and the SDSS MphG density contrast map was computed to measure the ISW effect at large angular scales. In order to quantify the impact of the SZ cluster residuals on the ISW detection under the same conditions as in Planck Collaboration XXI (2016), in Fig. 3.20 we instead compute the cross-power spectrum between the SDSS MphG map and (i) the Planck NILC CMB map (red), (ii) the SZ-free 2D-ILC CMB map (green) on large angular scales. We see that the spectra are almost identical, with up to a \( \approx 40\% \) difference relative to the NILC CMB map in the power on the largest angular scales (\( \ell < 10 \)).

![Figure 3.20: Cross-power spectra between the SDSS MphG density contrast map and the NILC CMB map (red), the 2D-ILC CMB map (green), and an artificial SZ-contaminated CMB map (blue), where the catalogue SZ 143 GHz map has been added in to the SZ-free 2D-ILC CMB map.](image)

To understand whether the gap between the two ISW measurements comes either from SZ cluster residuals in the Planck NILC CMB map or from stronger residual contamination from Galactic foregrounds in the SZ-free 2D-ILC CMB map, we artificially include SZ contamination into the 2D-ILC CMB map, by adding the catalogue SZ 143 GHz map to the SZ-free 2D-ILC CMB map. We calculate the cross-power spectrum between this artificial map and the SDSS MphG map as shown in Fig. 3.20

To understand whether the gap between the two ISW measurements comes either from SZ cluster residuals in the Planck NILC CMB map or from stronger residual contamination from Galactic foregrounds in the SZ-free 2D-ILC CMB map, we artificially include SZ contamination into the 2D-ILC CMB map, by adding the catalogue SZ 143 GHz map to the SZ-free 2D-ILC CMB map. We calculate the cross-power spectrum between this artificial map and the SDSS MphG map as shown in Fig. 3.20.
3.5. CONCLUSIONS AND DISCUSSIONS

(\textit{blue}). Compared with the result from the SZ-free 2D-ILC CMB map (green), the artificial SZ contamination indeed reduces the amplitude of the cross-power spectrum by $\approx 1.7\%$ due to the anti-correlation between the SZ cluster contamination and SDSS galaxies. However, the gap in amplitude between the SZ-free 2D-ILC (green) and NILC (red) maps is a factor of $\approx 17$ larger than this artificially-added anti-correlation. Given that the artificial SZ contamination in this case is the full power of the SZ emission from the \textit{Planck} 2015 SZ catalogue while the actual SZ cluster contamination in the \textit{Planck} NILC CMB map is $\sim 40\%$, the impact on the ISW measurement appears to be insignificant.

This result is somewhat expected from the absolute value of the correlation coefficients between SDSS and CMB SZ residuals that we have measured in Fig. 3.13 to be less than 6\% at large angular scales. This is consistent with Afshordi et al. (2004), where by cross-correlating WMAP with 2MASS, they found that the ISW and SZ components dominate before and after $\ell = 20$ respectively. The removal of SZ cluster residuals in the 2D-ILC CMB map is at the cost of more contamination from Galactic foregrounds. Therefore, the excess of amplitude in the CMB-SDSS cross-power spectrum at large angular scales observed for the 2D-ILC map might result from chance correlation with stronger residual Galactic foreground contamination. Nevertheless, Madhavacheril & Hill (2018) shows that using a 2D-ILC CMB map to construct the CMB lensing potential map will allow for robust SZ-free CMB lensing measurement without much penalty caused by the increased global noise.

3.5 Conclusions and discussions

We have quantified the amount of residual thermal SZ contamination from galaxy clusters in the \textit{Planck} 2015 CMB maps through stacking analysis and cross-correlation with the SDSS survey of galaxies. Based on our cross-power spectrum analysis, the
residual thermal SZ clusters in the Planck NILC CMB map is detected with $\approx 30\sigma$ significance at the cluster scale ($\ell \sim 2000$) and overall $\approx 54\sigma$ significance after including large angular scales. By comparison, the 2D-ILC CMB map that we have produced by using the Constrained ILC component separation technique shows negligible SZ cluster residuals.

The percentage of the thermal SZ emission that has been left over in the Planck 2015 NILC CMB map is estimated to be $44 \pm 4\%$ through aperture photometry. The impact on measurements of the ISW effect of SZ cluster residuals in CMB maps at large angular scales has been calculated to be negligible. However, we warn that one can no longer ignore the SZ cluster residuals in any cross-correlation analysis between CMB and LSS surveys, especially for CMB maps observed with a higher resolution instrument, such as ACT (Fowler et al., 2010), SPT (Lueker et al., 2010) and the upcoming CMB-S4 experiment (CMB-S4 collaboration, 2016), where the thermal SZ cluster residuals will have an effect on even smaller scales ($\ell > 3000$). Therefore, one should take extra care about the thermal SZ cluster residuals for higher resolution CMB maps.

As pointed out in Madhavacheril & Hill (2018), SZ cluster residuals in CMB maps will also affect the CMB lensing potential field reconstruction by propagation through lensing quadratic estimators. In this case, lensing quadratic estimators should be better applied to an SZ-free CMB map, such as the 2D-ILC map, in order to avoid spurious correlations between the CMB lensing potential map and LSS optical survey maps. One potential extension of the current work is to estimate the lensing potential from the NILC and 2D-ILC CMB maps respectively, and then quantify the impact of SZ cluster residuals on CMB lensing-LSS cross-correlation analysis.
Chapter 4

The Auto-Correlation of HI Intensity Mapping and the Impact of $1/f$ Noise

In this chapter, we forecast the constraints on cosmological parameters from IM experiments using a Fisher matrix approach. We focus on the upcoming SKA experiment with comparisons to the BINGO experiment. The novel aspect of our work is that we investigate the impact of $1/f$ noise on parameter constraints following the semi-empirical $1/f$ noise model from Harper et al. (2018).

4.1 Introduction

As introduced in Chapt. 1, we have come to the era of precision cosmology where the CMB measurement from the Planck satellite provides tight constraints on the ΛCDM cosmological parameters (e.g., Planck Collaboration VI, 2018). However, for the study of dark energy which dominates in the late-time Universe, one requires to probe the Universe with a low-redshift survey in order to break degeneracies among cosmological parameters. Therefore, although the Planck CMB measurement is extremely accurate for the study of ΛCDM model, it does not provide much information for dark energy.
As mentioned in Chapt. 2, intensity mapping experiments map the large-scale-structure of the Universe in three dimensions, with accurate redshift information of the detected emission line from the large number of frequency channels. It can provide an independent measurement of cosmological parameters in addition to CMB and optical galaxy surveys. Many IM experiments have been proposed in the recent few years. In this section, we focus on the upcoming SKA Phase I IM experiment. This is because SKA will be the most powerful upcoming radio telescope facility in the world on the timescale of $\sim 10$ years. Before SKA is fully operational, it is useful to forecast its likely performance and in particular, the impact of instrumental effects, such as the $1/f$ noise. Therefore, we extend the work in Harper et al. (2018) to forecast the impact of $1/f$ noise on cosmological parameter constraints from an SKA1-MID Band 2 IM survey. We will also perform a forecast of the BINGO IM survey to compare with SKA, since BINGO is a pathfinder for an SKA IM experiment.

Forecasts have been made on the performance of SKA as an IM facility in terms of cosmological parameter constraints. Santos et al. (2015) showed that by combining the SKA IM with \textit{Planck} CMB, the curvature density parameter can be constrained to $|\Omega_k| < 10^{-3}$. Bull et al. (2015) showed that by combining SKA IM with Stage IV galaxy surveys, the constraints on dark energy is nearly five times tighter than either survey individually. The ‘multi-tracer’ effect has the benefit of cancelling out cosmic variance by measuring the same cosmic density field from several distinct tracers. The wide range of redshifts and large sky coverage of SKA1 IM survey will benefit the detection of e.g., non-Gaussianity and spatial curvature. The Fisher analysis in Raccanelli et al. (2015) gave a 1% constraint on the growth rate $f\sigma_8$ with SKA1-MID Band 2 IM, and 4% with SKA1-MID Band 1 out to $z \approx 2$. Bull (2016) showed that SKA1 IM survey is able to constrain $f\sigma_8$ to 1% accuracy out to $z \approx 1$. SKA1-MID Band 2 IM survey can potentially surpass, e.g. \textit{Euclid}, in terms of constraining modified gravity. Li & Ma (2017) projected a constraint on the local shape of primordial non-Gaussianity
4.1. INTRODUCTION

with $\sigma_{NL} = 0.54$ using SKA1 IM forecast. More recently, Olivari et al. (2018) suggested a possible constraint of $w_0$ with 2% accuracy and an upper limit on the mass of neutrino with $\sum m_\nu < 0.12$, by combining SKA1-MID Band 2 with Planck. Dinda et al. (2018) projected a constraint on $w$ with different parametrization and found that with SKA1-MID IM alone, a constraint of $w_0 = -1.0^{+0.06}_{-0.06}$ and $w_a = -0.01^{+0.19}_{-0.17}$ is obtained within the BA model parametrization given by Barboza & Alcaniz (2008). Note that all forecasts to date have assumed perfect calibration and foreground removal, since they are essential for intensity mapping experiments to be competitive with optical galaxy surveys.

The forecast presented in this chapter differs from previous literature in a number of ways: i) We adopt the 2-D angular power spectrum calculation $C_\ell(z)$ of the HI signal, which is a more directly measurable quantity than the 3-D power spectrum $P(k,z)$ once real data are obtained. Previous forecasts made by Bull et al. (2015) and Bull (2016) used $P(k,z)$. We also use the updated SKA instrumental parameters from the SKA handbook in Turner et al. (2016) and the SKA simulation in Harper et al. (2018). ii) Compared with Olivari et al. (2018), we use the full calculation of HI power spectrum $C_\ell(z)$. In that work, they neglected the redshift-space-distortion contribution and adopted the Limber approximation. iii) The focus of our analysis is to use the semi-empirical $1/f$ noise model given in Harper et al. (2018) and to investigate the impact of this on cosmological parameter constraints, which has not been studied in any of the previous literature.

In this chapter, Sect. 4.2 presents the survey parameters used in the forecast, while sect. 4.3 gives the analytical formulae for power spectrum, noise and Fisher analysis calculations. In sect. 4.4 we calculate the S/N of the HI signal. Sect. 4.5 gives the constraints on cosmological parameters from the IM surveys under consideration. Sect. 4.6 investigates the impact of $1/f$ noise on cosmological parameter constraints. The conclusions and discussions are given in Sect. 4.7.
4.2 The surveys

We focus on the SKA1-MID Band 2 IM survey for our forecast of cosmological parameter constraints, because of its low redshift ($z < 0.5$) where dark energy dominates. We also project parameter constraints using the BINGO experiment for comparison. The instrumental and observing parameters of the two surveys are given in Table 2.2 & 2.1 respectively. For the SKA1-MID Band 2 IM survey, we focus on using the same number of frequency channels ($N_{\text{freq}} = 23$) and bandwidth ($\Delta \nu = 20 \, \text{MHz}$) as in Harper et al. (2018), in order to extend their work into parameter constraints. In Sect. 4.5.3, we will investigate the impact of $N_{\text{freq}}$ on our parameter constraints, where we vary $N_{\text{freq}}$ in the range of [5, 10, 15, 23]. The number of frequency channels and bandwidth used in each case for SKA1-MID Band 2 are listed in Table 4.1. For the BINGO experiment, we use $N_{\text{freq}} = 30$ and $\Delta \nu = 10 \, \text{MHz}$, to be comparable with Olivari et al. (2018).

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<tr>
<td></td>
<td></td>
<td>5</td>
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</tr>
<tr>
<td>BINGO</td>
<td>300</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4.1: The frequency parameters used for SKA1-MID Band 2 and BINGO IM forecasts. The first column gives the survey names with their total bandwidth in the second column. The third column is the number of frequency channels assumed in each case with the corresponding channel width in the fourth column. The row in bold gives the default baseline parameters used for SKA1-MID Band 2.
4.3. The power spectrum calculation

4.3.1 The HI power spectrum

For the auto-correlation of an IM experiment, the angular power spectrum of the HI signal is (e.g., Bonvin & Durrer, 2011; Battye et al., 2013)

\[
C_{\text{HI}}^\ell(z, z') = \left(\frac{2}{\pi}\right) \int dz \left[ W_{\text{HI}}(z) D(z) \right] \int dz' \left[ W_{\text{HI}}(z') D(z') \right] \times \int dk k^2 P_m(k, z = 0) \left[ b_{\text{HI}} j_\ell(k\chi) - f(z) j''_\ell(k\chi) \right] \left[ b_{\text{HI}} j_\ell(k\chi') - f(z') j''_\ell(k\chi') \right]
\]

(4.1)

where \( z \) and \( z' \) denote different redshift bins so that when \( z = z' \), \( C_{\text{HI}}^\ell(z, z) \) is the auto-frequency power spectrum for each channel, and when \( z \neq z' \), \( C_{\text{HI}}^\ell(z, z') \) is the cross-frequency power spectrum between two channels. Note that here one shall differ this cross-frequency spectrum \( C_{\text{HI}}^\ell(z, z') \) with the cross-spectrum to be introduced in Chapt. 5, in which the cross-spectrum means the cross-correlation between two different experiments, instead of the cross-frequency correlation in a single IM experiment, named as the auto-correlation of IM experiment in this chapter. In Equ. 4.1, \( W_{\text{HI}}(z) \) is the window function of each redshift bin at a central redshift \( z_c \) with bin width \( \Delta z \), such that

\[
W_{\text{HI}}(z) = \begin{cases} 
\frac{1}{\Delta z}, & z_c - \frac{\Delta z}{2} \leq z \leq z_c + \frac{\Delta z}{2}, \\
0, & \text{otherwise}.
\end{cases}
\]

(4.2)

\( D(z) \) is the growth factor and \( P_m(k, z = 0) \) is the underlying dark matter power spectrum introduced in Sect 1.1.4. \( b_{\text{HI}} \) is the HI bias that relates the HI density to the matter density. \( j_\ell \) and \( j''_\ell \) are the Bessel function and its 2nd order derivative. \( \chi \) is the comoving distance which is related to redshift and Hubble parameter by

\[
\chi(z) = \int_0^z \frac{dz'}{H(z')},
\]

(4.3)
In Equ. 4.1, the term proportional to $b_{HI}$ gives the contribution of HI density to the power spectrum. The term involving the growth rate, $f(z)$, quantifies the contribution from the redshift-space-distortion (hereafter RSD).

The RSDs come from the peculiar motion of galaxies on top of the Universe expansion (e.g., Agrawal et al., 2017). The Doppler effect from galaxy peculiar velocities will result in distortions in the galaxy distributions in redshift-space. Galaxies that are bound in overdensities such as in a cluster, will move towards the centre of the cluster through gravity. Therefore, galaxies that are closest to us will move towards the centre of the cluster and thus appear more distant than its redshift caused by the Universe expansion. On the other hand, galaxies that are furthest from us will move towards us under the gravity, and thus appear closer than they actually are. At large scales, the radius of the infalling galaxy shell is large compared to the peculiar movement so that the shell is squashed in redshift space, known as the Kaiser effect (Kaiser, 1987). At small scales, due to stronger gravity and smaller infalling shell radius, the collapsing shell appear ‘inside out’ in redshift space where the closest galaxy appear the furthest away and vice versa. The collapsed shells appear elongated along the line-of-sight towards the observer, known as the Fingers of God (Jackson, 1972). In angular power spectrum space, the RSD is encoded into the growth rate term $f(z)$, which is linked to the growth factor $D(z)$ by (e.g., Hamilton, 1998)

$$f(z) = \frac{d \ln D(z)}{d \ln a}.$$ (4.4)

Fig. 4.1 plots the HI power spectrum at $z = 0.1$ and $z = 0.5$, with a frequency bandwidth of $\Delta \nu = 20$ MHz and $\Delta \nu = 200$ MHz respectively. In each case, we plot the total HI power spectrum, the HI density component ($b_{HI}$ term in Equ. 4.1) and the RSD component ($f(z)$ term in Equ. 4.1) respectively. It can be seen that the total HI power decreases with higher redshift and wider bandwidth, consistent with Battye et al. (2013) and Olivari et al. (2018). In each case, the RSD component is much smaller than
the HI density component, especially at a wider bandwidth of, say, $\Delta \nu = 200 \text{ MHz}$, where it becomes almost negligible. The bandwidth-sensitivity of the RSD component is due to the fact that using a wider bandwidth will average out the peculiar motions of galaxies, which have their velocities in all directions. Our HI power spectra with different components are consistent with Fig. 1 in Hall et al. (2013) where their total HI power spectrum is in the same order of magnitude as the one used here, and also has the HI density component dominating over the RSD component.

The RSD component was ignored in both Battye et al. (2013) and Olivari et al. (2018) for simplicity. They also adopted the Limber approximation (Limber, 1953) for the HI power spectrum calculation, which replaces the Bessel function in Equ. 4.1 with a delta-function such that

$$ j_\ell(k \chi) = \sqrt{\frac{\pi}{2(\ell + 1/2)}} \delta_D((\ell + 1/2) - k \chi). \quad (4.5) $$
However, the Limber approximation is only a good approximation for $\ell \gtrsim 50$ (Loverde & Afshordi, 2008), and will overestimate the auto-frequency spectra but underestimate the cross-frequency spectra (Olivari et al., 2018). Therefore, our full calculation of the HI power spectrum following Equ. 4.1 is more accurate with the RSD component and the Bessel function included.

Our HI power spectrum is computed from scratch with a Python code primarily written by Dr. André Costa\textsuperscript{1}, with modifications from the author to suit the analysis performed in this chapter. We make use of the transfer function output from CAMB (Challinor & Lewis, 2011; Howlett et al., 2012) to compute the growth factor $D(z)$ and the underlying matter power spectrum $P_m(k, z = 0)$. The RSD term $f(z)$ is calculated using Equ. 4.4 from $D(z)$. The window function $W(z)$ of each redshift bin is computed and multiplied to $D(z)$. The Bessel function $j_\ell(k\chi')$ and $j_\ell''(k\chi')$ are computed beforehand and written into a table at sampled redshifts $z$ and scales $k$. For each particular set of IM experiment, the corresponding Bessel functions are interpolated from the Bessel function table at the desired redshifts and scales. The HI power spectrum as a function of redshift bins is calculated from the integral of first all scales and then over the redshift of each bin. The author introduces flexibility in optionally including different components (e.g., $b_{\text{HI}}$, $f(z)$ and cross-frequency) of power spectrum to suit the analysis performed in Sect. 4.5.4 and Sect. 4.5.5. The author also speeds up the code by pre-calculating the RSD component $f(z)$ outside of the redshift loop. Samples of redshifts and scales for calculating the Bessel function table are also modified from Dr. Costa’s original table to suit very low redshift for SKA IM.

### 4.3.2 The thermal noise calculation

The thermal noise power spectrum for an IM survey is given by

$$ N_\ell(z) = \left( \frac{4\pi}{N_{\text{pix}}} \right) \sigma_\chi^2, \quad (4.6) $$

\textsuperscript{1}Instituto de Física, Universidade de São Paulo. Visited Manchester during 2016–2017.
4.3. THE POWER SPECTRUM CALCULATION

where \( N_{\text{pix}} \) is the number of pixels in the map, and \( \sigma_T \) is the r.m.s. noise per pixel calculated from the radiometer equation given by Equ. 2.2 and Equ. 2.3, with parameters from Table 2.1. To get the dimensionless noise power spectrum, one normalizes \( N_\ell (z) \) by dividing the 21-cm mean temperature \( \bar{T}^2(z) \) at each frequency channel, given by (Battye et al. 2013)

\[
\bar{T}(z) = 180 \Omega_{HI}(z) b \frac{(1+z)^2}{H(z)/H_0} \text{mK},
\]

where \( \Omega_{HI}(z) \approx 4 \times 10^{-4} (1+z)^{0.6} \) (Crighton et al., 2015). One also needs to take into account the beam effect \( b_\ell(z) \) at each frequency channel such that

\[
b_\ell(z) = e^{\frac{\sigma_B^2}{2}},
\]

where \( \sigma_b = \theta_B(z)/\sqrt{8 \ln 2} \), and

\[
\theta_B(z) = \theta_{\text{FWHM}}(v_0) \frac{v_0}{v}.
\]

On power spectrum, the beam effect is the square of \( b_\ell(z) \) such that

\[
B_\ell(z) = b_\ell^2(z) = e^{\ell^2 \sigma_b^2},
\]

and the product of \( N_\ell(z)B_\ell(z) \) gives the thermal noise with the beam effect applied. Note that we assume thermal noise is uncorrelated between different frequency channels so that the mean noise for cross-frequency bins is zero. Our thermal noise expression and beam effect hereby have only one dimension along redshift. In principle, one could write thermal noise by \( N_\ell(z,z')B_\ell(z,z') \) where \( N_\ell(z,z') = 0 \) for \( z \neq z' \).

The dotted curves in Fig. 4.1 give the expected thermal noise power spectra from SKA for the three cases described in Sect. 4.3.1. In all three cases, we see that the thermal noise increases exponentially after \( \ell > 100 \), due to the beam effect. By comparing the red (\( \Delta v = 20 \text{MHz} \)) curve with the green (\( \Delta v = 200 \text{MHz} \)) curve, a wider bandwidth decreases the thermal noise which can be deduced from the radiometer equation
in Equ. 2.2. The thermal noise is also lower at high redshift as the mean HI brightness temperature from Equ. 4.7 increases towards higher redshift, and thus gives a lower dimensionless thermal noise after normalization.

4.3.3 Fisher matrix analysis

Fisher matrix analysis is widely used in astronomy. It measures the amount of information that an observable $\mathcal{O}$ carries about a parameter $\theta$, which models $\mathcal{O}$ (e.g., Ly et al., 2017). One important use of Fisher matrix in cosmology is to forecast the constraint on parameter $\theta$ given the model and uncertainties of the observable $\mathcal{O}$. Compared to a detailed end-to-end simulation, it provides a quicker and simpler way to predict the performance of an experiment before doing it, and can be used to guide the experimental design. Since the Fisher matrix forecast is purely analytical, it does not take into account practical errors, such as the receiver imperfections or residual foreground contamination that one might be able to introduce in a full end-to-end simulation. In addition, uncertainty in the observable $\mathcal{O}$ is treated as being Gaussian distributed, which might only be an approximation in practice. Therefore, the forecast from Fisher analysis gives the optimal results one could possibly expect from the upcoming experiment. One would not expect the results from a Fisher matrix forecast to be surpassed.

Formally, the Fisher matrix is a way to calculate the expected value of the observed information. Let $f(\mathcal{O}; \theta)$ be the likelihood function for $\theta$, which gives the probability distribution function of the observable $\mathcal{O}$ with an output $o$ at a certain value of $\theta$. The partial derivative $\frac{\partial}{\partial \theta} \log f(\mathcal{O}; \theta)$, of the natural logarithm of $f(\mathcal{O}; \theta)$ with respect to $\theta$, is known as the “score”, which describes the sensitivity of $f(\mathcal{O}; \theta)$ to the changes of $\theta$. The expected value of the score is defined as (e.g., Ly et al., 2017)

$$E \left[ \frac{\partial}{\partial \theta} \log f(\mathcal{O}; \theta) \middle| \theta \right] = \int \left( \frac{\partial}{\partial \theta} \log f(o; \theta) \right) f(o; \theta) do. \quad (4.11)$$
4.3. **THE POWER SPECTRUM CALCULATION**

The Fisher information $\mathcal{I}(\theta)$ is the variance of the score such that

$$
\mathcal{I}(\theta) = \mathbb{E}\left( \left( \frac{\partial}{\partial \theta} \log f(o; \theta) \right)^2 \right) = \int \left( \frac{\partial}{\partial \theta} \log f(o; \theta) \right)^2 f(o; \theta) do.
$$

(4.12)

If $f(\theta'; \theta)$ is twice differentiable with respect to $\theta$, Eq. 4.12 can be written as

$$
\mathcal{I}(\theta) = \int \left( \frac{\partial^2}{\partial \theta^2} \log f(o; \theta) \right) f(o; \theta) do.
$$

(4.13)

In the case of forecasting cosmological parameters from power spectrum measurement, the Fisher information can be written in a matrix format such that (e.g., Asorey et al., 2012; Hall & Challinor, 2012)

$$
M_{i,j} = \sum_{\ell=2}^{\ell_{\text{max}}} \sum_{XX',YY'} \frac{\partial S_{XX'}}{\partial \theta_i} \left[ \text{Cov}(XX',YY') \right]_{\ell} \frac{\partial S_{YY'}}{\partial \theta_j}.
$$

(4.14)

In this case, the power spectrum $S_{\ell}$ is the observable, which can be parametrised by parameter $\theta_{i,j}$. For auto-correlation analyses, e.g., the analysis performed in this chapter, $X$, $X'$, $Y$, and $Y'$ denote redshift bins of the measured power spectrum. Cov$(XX',YY')$ in Equ. 4.14 is the covariance matrix of the measured power spectrum such that

$$
[\text{Cov}(XX',YY')]_{\ell} = \frac{1}{(2\ell+1)f_{\text{sky}}} \left( \hat{C}_{\ell}^{XY} \hat{C}_{\ell}^{XY'} + \hat{C}_{\ell}^{XY'} \hat{C}_{\ell}^{XY} \right),
$$

(4.15)

where $f_{\text{sky}}$ is the fractional sky coverage of the survey and $\hat{C}_{\ell}$ is the measured power spectrum with noise such that

$$
\hat{C}_{\ell}(z,z') = C_{\ell}^{\text{HI}}(z,z') + N_{\ell}(z,z') B_{\ell}(z,z').
$$

(4.16)

For SKA or BINGO IM auto-correlation, $C_{\ell}^{\text{HI}}(z,z')$ and $N_{\ell}(z,z')$ are calculated using Equ. 4.1 & 4.6 respectively. Note that in Equ. 4.16, the beam $B_{\ell}(z,z')$ is applied to noise $N_{\ell}(z,z')$ only but not to the HI signal $C_{\ell}^{\text{HI}}(z,z')$. The reason is that during observation, the HI signal will be reduced because of the limited beam resolution, which however
does not affect instrumental noise. Once observed map (signal+noise) is obtained, one corrects the beam effect in the map for HI signal, but will result in the noise increasing towards higher multipoles, because of the beam correction.

In Eq. 4.14, \( \frac{\partial S_\ell}{\partial \theta_i} \) is the partial derivative of the observable with respect to each parameter \( \theta_i \). In our case, the partial derivative is calculated numerically by varying each parameter with a step of \( \pm \Delta \theta_i \). The value of \( \Delta \theta_i \) should not be too large so that it miscalculates the derivative, and should not be too small so that it introduces numerical noise. The value of \( \Delta \theta_i = 0.5\% \theta_i \) is used such that

\[
\frac{\partial S_\ell}{\partial \theta_i} = \frac{C_{\ell}^{\text{HI}}(\theta_i + 0.5\% \times \theta_i) - C_{\ell}^{\text{HI}}(\theta_i - 0.5\% \times \theta_i)}{1\% \times \theta_i}.
\] (4.17)

We have checked that the numerical derivatives are not affected by numerical noise in the calculations of these derivatives. For auto-correlation, Eq. 4.15 can be simplified to

\[
[Cov(z, z')]_\ell = \frac{2}{(2\ell + 1)f_{\text{sky}}} \left[ C_{\ell}^{\text{HI}}(z, z') + N_\ell(z, z')B_\ell(z, z') \right]^2.
\] (4.18)

The uncertainty on the measured HI power spectrum is thus the square-root of Eq. 4.18 such that

\[
\Delta C_{\ell}^{\text{HI}}(z, z') = \sqrt{\frac{2}{(2\ell + 1)f_{\text{sky}}} \left[ C_{\ell}^{\text{HI}}(z, z') + N_\ell(z, z')B_\ell(z, z') \right]}.
\] (4.19)

For the cross-correlation analysis performed in Chapter 5 where two independent tracers are cross-correlated, \( X \) and \( Y \) in Eq. 4.15 denote redshift bins from the same experiment, while \( X' \) and \( Y' \) denote redshift bins from the other experiment. Therefore, \( \hat{C}_\ell^{XY} \) is the measured power spectrum of the first tracer at the redshift bin \((X, Y)\), and \( \hat{C}_\ell^{X'Y'} \) is the measured power spectrum of the second tracer, at the redshift bin \((X', Y')\). \( \hat{C}_\ell^{XY'} \) and \( \hat{C}_\ell^{X'Y} \) are the measured cross-spectra between the two tracers, at the redshift bin \((X, Y')\) and \((X', Y)\) respectively. For experiments where \( C_\ell(X, Y') = C_\ell(X', Y) \)
with \(N\) redshift bins, such as the SKA IM auto-correlation, the power spectrum covariance matrix \(\text{Cov}(XX', YY')\) has a dimension of \([N(N+1)/2 \times N(N+1)/2]\). For the cross-correlation of two tracers where \(C_\ell(X, Y') \neq C_\ell(X', Y)\), \(\text{Cov}(XX', YY')\) has the dimension of \([N_1N_2 \times N_1N_2]\), where \(N_1\) and \(N_2\) are the number of redshift bins for the two tracers respectively.

In our forecasts of SKA1-MID Band 2 and BINGO IM experiment, we constrain cosmological parameters based on the CPL model introduced in Sect. 1.1.5. Our cosmological parameter set is

\[
\theta_{i,j} = \{\Omega_b h^2, \Omega_c h^2, w_0, w_a, h, n_s, \ln(10^{10} A_s), b_{\text{HI}}\}
\]

The Fisher matrix in Equ. 4.14 sums over all multipoles and redshifts, resulting in an \(8 \times 8\) matrix for this parameter set. The covariance between the cosmological parameters are given by the inverse of the Fisher matrix to get the parameter covariance matrix \(\mathcal{C}_{i,j}\) such that

\[
\mathcal{C}_{i,j} = M^{-1}_{i,j}.
\] (4.20)

Each diagonal element in \(\mathcal{C}_{i,j}\) gives the \(1\sigma\) variance \(\sigma^2_{ii}\) of the corresponding parameter \(\theta_i\), marginalized over the other parameters. The marginalization means that assuming a Gaussian distribution, when calculating the \(1\sigma\) uncertainty \(\sigma_{ii}\) of parameter \(\theta_i\), the other parameters are allowed to vary rather than being fixed, which is equivalent to integrating the probability distribution over all possible values of other parameters on the joint probability distribution plane. Each off-diagonal element in \(\mathcal{C}_{i,j}\) gives the covariance \(\sigma_{ij}\) between the corresponding parameter \(\theta_i\) and \(\theta_j\), and \(\sigma_{ij} = \rho_{\theta_i \theta_j}\) where \(\rho\) is the correlation coefficient between the two parameters (e.g., Coe, 2009).

Therefore, by performing the Fisher analysis, one can obtain the \(1\sigma\) uncertainties of cosmological parameters, along with the correlations among them. We use this Fisher matrix analysis to constrain cosmological parameters for SKA1-MID Band 2
and BINGO IM auto-correlation forecasts in this Chapter, and the multi-tracer cross-correlation forecast in Chapter 5.

### 4.4 The detection S/N

It is useful to quantify the overall detection significance of the IM surveys before constraining parameters. We first calculate the detection S/N of the HI power spectrum from SKA and BINGO at different multipoles, summing over all redshift bins. The S/N at each multipole is calculated by

\[
\frac{S}{N}_\ell = \sqrt{\sum_{z=z_{\text{min}}}^{z=z_{\text{max}}} \left( \frac{C_{\ell}^{HI}(z,z')}{\Delta C_{\ell}^{HI}(z,z')} \right)^2},
\]

where \(C_{\ell}^{HI}(z,z')\) and \(\Delta C_{\ell}^{HI}(z,z')\) are calculated from Equ. 4.1 & 4.19 respectively. Here the S/N at each multipole is calculated from the sum of squares of the S/N from all redshift bins, including only the auto-frequency terms, since the signal in the cross-frequency terms is not the power of the HI signal but the cross-correlated signal between two channels which is not the quantity we use in the S/N calculation. We further bin the data with a width of \(\Delta \ell = 50\). For each multipole bin, the S/N is calculated from the sum of squares of the S/N at each multipole such that

\[
\frac{S}{N}_{\ell, \Delta \ell=50} = \sqrt{\sum_{\ell = \ell_0}^{\ell_0+45} \left( \frac{S}{N}_\ell \right)^2}.
\]

In Fig. 4.2, we plot the uncertainty of the HI power spectra (upper) and the detection S/N (lower) for SKA (left) and BINGO (right). In each case, we show the uncertainty on a particular redshift bin at \(z = 0.2\) as an example in the upper panel. The solid curve is the HI signal (Equ. 4.1) with the uncertainty (Equ. 4.19) shown as the shaded area. The HI signal from BINGO is slightly stronger than that from SKA, due to the narrower bandwidth of BINGO (\(\Delta v = 10\) MHz) compared to SKA.
(Δν =20 MHz). For both SKA and BINGO, the uncertainty is increasing towards the very large scale (ℓ < 5) due to the cosmic variance. For BINGO, the cosmic variance uncertainty is larger since it has a smaller sky coverage area, thus a smaller f_{sky}. The S/N is largest around ℓ ≈ 100 and after ℓ ≈ 150, it starts decreasing due to the limited beam resolution. The increased uncertainty at beam scale particularly affects SKA more than BINGO, since SKA has a larger beam and thus a lower resolution. In the lower panels of Fig. 4.2, the S/N of the two surveys summing over all redshift bins are shown as a function of multipole with Δℓ = 50. Apart from the last two multipole bins which are significantly affected by the beam, the S/N of SKA is in general higher than that of BINGO by a factor of ∼ 2 − 3, thanks to its larger sky coverage, lower system temperature, and larger number of dishes (see Table 2.2).

Finally we calculate the total S/N of the two surveys, summing over all redshift bins and multipoles such that

$$\frac{S}{N} = \frac{\ell = \ell_{\text{max}}}{\ell = \ell_{\text{min}}} \sum_{\ell = \ell_{\text{min}}}^{\ell = \ell_{\text{max}}} \sum_{z = z_{\text{min}}}^{z = z_{\text{max}}} \left[ \frac{C^{\text{HI}}_{\ell}(z, z')}{\Delta C^{\text{HI}}_{\ell}(z, z')} \right]^2.$$  (4.23)
CHAPTER 4. HI IM AUTO-CORRELATION

The total S/N calculated from Equ. 4.23 is $\approx 351$ and $\approx 220$ for SKA1-MID Band 2 and BINGO respectively. Therefore, SKA1-MID Band 2 will give a factor of $\approx 1.6$ better detection than BINGO for conducting IM survey. Our calculation of S/N for the two surveys is comparable with Pourtsidou et al. (2016), where they calculated the S/N summing over all multipoles of MeerKAT-16 at a particular redshift of $z = 0.1$ with $\Delta z = 0.2$, and yielded a S/N of $\sim 100$. The consistency between our S/N and that in Pourtsidou et al. (2016) gives confidence to our calculation.

4.5 Cosmological parameter constraints

In this section we project constraints on cosmological parameters using the power spectrum calculation and Fisher matrix analysis described in Sect. 4.3. We focus on the SKA1-MID Band 2 IM experiment and investigate the impact of different factors, such as the number of frequency channels and Planck prior. We also include the parameter constraints from BINGO and BINGO+Planck for comparison.

4.5.1 The constraints from IM alone

We project constraints on cosmological parameters within the CPL model using the Fisher matrix analysis described in Sect. 4.3. In order to be comparable with previous literature, we use the fiducial values of cosmological parameters from Olivari et al. (2018) where they constrained parameters from BINGO + Planck and SKA + Planck. The parameters and their fiducial values are given in the first column of Table 4.2. We use the SKA1-MID Band 2 instrumental parameters from Table 2.2, with $N_{\text{freq}} = 23$ and $\delta \nu = 20$ MHz from Table 4.1, which are the same as in Harper et al. (2018), in order to obtain comparable results.

The parameter constraints from the SKA Fisher matrix alone are given in the second column of Table 4.2. It can be seen that the projected constraints on the standard $\Lambda$CDM cosmological parameters, such as $\{\Omega_b, \Omega_c, n_s, \ln(10^{10} A_s)\}$, are not competitive.
with those already obtained from the *Planck* CMB measurement (e.g., *Planck Collaboration VI, 2018*) but will provide useful cross-check. This is because the high resolution full sky CMB map of the *Planck* satellite provides independent measurements of a huge number of modes, allowing strong constraints of parameters with $\lesssim 1\%$ accuracy (see Sect. 1.2.1). The redshifted HI power spectrum from SKA IM experiment, in contrast, is less sensitive to these standard $\Lambda$CDM parameters, but breaks the degeneracy inherent in CMB. It also provides important information on dark energy thanks to its low redshift where dark energy dominates over matter.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SKA</th>
<th>Planck</th>
<th>SKA+Planck</th>
</tr>
</thead>
<tbody>
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<td>$\Omega_bh^2$</td>
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<td>±0.00150</td>
<td>±0.00015</td>
</tr>
<tr>
<td>$\Omega_ch^2$</td>
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<td>±0.0037</td>
<td>±0.0016</td>
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<td>$w_0$</td>
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<td>±0.45</td>
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<tr>
<td>$w_a$</td>
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<td>±0.59</td>
<td>...</td>
</tr>
<tr>
<td>$h$</td>
<td>0.6727</td>
<td>±0.0032</td>
<td>±0.1308</td>
</tr>
<tr>
<td>$\ln(10^{10} A_s)$</td>
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<td>±0.217</td>
<td>±0.039</td>
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<tr>
<td>$n_s$</td>
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<td>±0.0046</td>
</tr>
<tr>
<td>$b_{HI}$</td>
<td>1.00</td>
<td>±0.14</td>
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</table>

Table 4.2: The cosmological parameters (1st column) and their projected uncertainties from SKA alone (2nd column), *Planck* likelihood (3rd column) from COSMO-MC, and SKA+*Planck* (last column). The parameters which are significantly improved by SKA+*Planck* are highlighted in bold.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>BINGO</th>
<th>BINGO+Planck</th>
</tr>
</thead>
<tbody>
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<td>$\Omega_bh^2$</td>
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<td>±0.00395</td>
</tr>
<tr>
<td>$\Omega_ch^2$</td>
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<td>±0.0139</td>
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<td>$\ln(10^{10} A_s)$</td>
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<td>±0.350</td>
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<td>$n_s$</td>
<td>0.9641</td>
<td>±0.0383</td>
</tr>
<tr>
<td>$b_{HI}$</td>
<td>1.00</td>
<td>±0.04</td>
</tr>
</tbody>
</table>

Table 4.3: The cosmological parameters and their projected uncertainties from BINGO alone (2nd column), and BINGO+*Planck* (3rd column).

The parameter constraints from the BINGO Fisher matrix alone are shown in the middle column of Table 4.3 to compare with SKA. On average, the constraints are degraded by a factor of $\sim 3$, except the HI bias $b_{HI}$, which is improved by a factor
of 3.5, compared to SKA. The reason for the improved constraint on $b_{\text{HI}}$ is due to the narrower bandwidth assumed for BINGO projection which results in a stronger RSD component (see Fig. 4.1). From Equ. 4.1, the RSD component is the key to break the degeneracy between $b_{\text{HI}}$ and $A_s$. Otherwise, the HI angular power spectrum will be $C_\ell \propto b_{\text{HI}}^2 A_s$. A stronger RSD component in the BINGO case thus helps break the $b_{\text{HI}} - A_s$ degeneracy and obtain a better constraint on $b_{\text{HI}}$.

4.5.2 The Planck prior

Since Planck can provide tighter constraints on $\Lambda$CDM parameters than IM, we add the Planck prior to our IM fisher matrix in order to help constraining dark energy parameters in CPL model at low redshift, where IM has an advantage.

The Planck prior is obtained from Planck 2015 TT+TE+EE likelihood data (Planck Collaboration XI, 2016) using COSMOMC (Lewis & Bridle, 2002), which uses the Markov Chain Monte Carlo technique to estimate the maximum-likelihood value for cosmological parameters and returns, e.g., the mean and standard deviation of different parameters. Here, we take the covariance matrix of cosmological parameters from COSMOMC and add into our IM Fisher matrix as the prior. The Planck prior is produced under the $w_{\text{CDM}}$ model. The output covariance matrix contains 35 parameters where we make use of 6 parameters \{\Omega_b, \Omega_c, n_s, \ln(10^{10} A_s), w_0, H_0\}, and treat others as nuisance parameters.

We follow Coe (2009) to add the Planck prior into our IM Fisher matrix. We first marginalize the nuisance parameters from the Planck covariance matrix by removing the corresponding row and column of each nuisance parameter, and end up with the 6 relevant parameters in the covariance matrix. We then take the inverse of the covariance matrix to get the Planck Fisher matrix. Since the IM Fisher matrix has two extra parameters \{w_a, b_{\text{HI}}\}, a row and a column is appended to the Planck Fisher matrix for $w_a$ and $b_{\text{HI}}$ respectively. Since Planck does not provide information on these two parameters, the appended columns and rows are all zeros. After this, the Planck Fisher
matrix includes \( \{ \Omega_b, \Omega_c, n_s, \ln(10^{10}A_s), w_0, w_a, H_0, b_{HI} \} \). Finally, we transfer \( H_0 \) into \( h \) in order to be consistent with the IM Fisher matrix. The parameter transformation is performed through a transformation matrix \( \mathcal{T} \) such that (Coe, 2009)

\[
[F'] = [\mathcal{T}]^T [F] [\mathcal{T}],
\]

(4.24)

where \([F']\) is the new Fisher matrix after the parameter transformation, with new parameters \( \mathbf{p}' = (p_1', p_2', p_3', \ldots) \), and \([F]\) is the old Fisher matrix with old parameters \( \mathbf{p} = (p_1, p_2, p_3, \ldots) \). The transformation matrix \( \mathcal{T} \) is calculated through the partial derivative of the old parameters to the new parameters that \( \mathcal{T}_{i,j} = \frac{\partial p_i}{\partial p_j'} \), and \( \mathcal{T}^T \) is the transpose of the transformation matrix (Coe, 2009). In this particular case of the transformation from \( H_0 \) to \( h \), the transformation matrix \( \mathcal{T} \) is a diagonal matrix with the element \( \frac{\partial H_0}{\partial h} = 100 \) and other diagonal elements all ones.

The parameter constraints in the Planck prior are listed in the third column of Table 4.2. The constraints on \( \{ \Omega_b h^2, \Omega_c h^2, \ln(10^{10}A_s), n_s \} \) from Planck are on average a factor of \( \sim 5 \) times tighter than that from SKA alone. Notice that the Planck constraint on \( w_0 \) is \( \sim 4 \) times worse than SKA alone, due to the high redshift Planck measurement which has limited information of dark energy that dominates at a much later time. Similarly, Planck can barely constrain the Hubble constant today \( h \) within the \( \omega \)CDM model, because of its high redshift observation, and the degeneracy between \( w_0 \) and \( h \).

The addition of the Planck prior is achieved by adding the Planck Fisher matrix to the IM Fisher matrix (Coe, 2009). One then inverts the combined Fisher matrix to get the covariance matrix, which contains the uncertainties and correlations of parameters with the Planck prior added in. The cosmological parameter forecasts from SKA+Planck are listed in the last column of Table 4.2. Compared to SKA alone, the constraints on \( \{ \Omega_b h^2, \Omega_c h^2, \ln(10^{10}A_s), n_s \} \) from SKA+Planck are \( \sim 6 \) times tighter than that from SKA alone, thanks to the tight constraints from the Planck prior. The
Planck prior also helps constraining other parameters by breaking degeneracies between parameters. The constraint on $w_0$ from SKA+Planck is 4 times tighter than that from SKA alone, with major contributions from the low redshift information provided by SKA. A $\sim 4.5$ times tighter constraint on $w_a$ is obtained compared to SKA alone. There is also an improvement on the constraint of $h$, although not significant compared to SKA alone but however much tighter than Planck alone. The tight constraint on $\ln 10^{10} A_s$ from Planck breaks the degeneracy between $A_s$ and $b_{\text{HI}}$. Therefore, the constraint on $b_{\text{HI}}$ from SKA+Planck is 7 times tighter than that from SKA alone.

For comparison, we also add the Planck Fisher matrix to the BINGO Fisher matrix. The constraints from BINGO+Planck are listed in the last column of Table 4.3. The constraints on $\{\Omega_b h^2, \Omega_c h^2, \ln(10^{10} A_s), n_s\}$ are comparable with those from SKA+Planck, since they are already tightly constrained by Planck. The constraints on $w_0$ and $w_a$ are $\sim 2$ times worse than SKA+Planck while the constraint on $h$ is about $\sim 4$ times worse. This is because the constraints from BINGO alone is $\sim 3$ times worse than that from SKA alone, due to the lower S/N of BINGO. The constraint on $b_{\text{HI}}$ is the same from BINGO+Planck and SKA+Planck, thanks to the narrower bandwidth and thus stronger RSD contribution of BINGO.

Our constraints from BINGO+Planck are comparable with those from Olivari et al. (2018). They however reported a slightly better constraint on $w_0$ with $\Delta w_0 = 0.06$. The main reason is that we parameterise the EoS parameter with $w(a) = w_0 + w_a(1 - a)$ (Equ. 1.28) using the CPL model, while they adopted the $w$CDM model with $w = w_0$ and therefore had one less degree of freedom. This also helped with the $w_0 - h$ degeneracy and in return resulted in a better constraint on $h$ with $\Delta h = 0.009$ in their case. The same reason applies for the SKA+Planck case where they claimed a better constraint with $\Delta w_0 = 0.02$, compared to our $\Delta w_0 = 0.03$. They however reported worse results with $\Delta n_s = 0.0047$, $\Delta \Omega_{\text{HI}} = 0.22(3\%)$, and $\Delta \Omega_c h^2 = 0.0014$. These degraded results are due to the following: i) They did not include RSD in their power spectrum calculation; ii) They used Limber approximation which has no cross-frequency term; iii) They
binned their power spectrum with $\Delta \ell = 12$; iv) They performed maximum-likelihood estimation on simulated maps, which gives a more realistic forecast than Fisher matrix. Nevertheless, our parameter forecasts from BINGO+$Planck$ and SKA+$Planck$ are comparable to Olivari et al. (2018).

4.5.3 The dependency on number of frequency channels

It is expected that by increasing the number of frequency channels, the uncertainties of cosmological parameters will decrease. This is because although the total probed volume is fixed, by increasing the number of frequency channels, one has a finer resolution along the line-of-sight and thus gathers more information in redshift dimension. In addition, with increased channel number and reduced bandwidth, the RSD component becomes more important, which will bring information to the HI signal and help breaking the degeneracy between parameters. However, there should be a turning point, beyond which including more channels will no longer improve results. This is because that the thermal noise will increase with a narrower channel width (see Equ. 2.2), and the shot noise might become important once each narrow frequency channel contains too few galaxies. Therefore, above a certain number of frequency channels, the increased noise will balance out the extra redshift information, and no improvement can be obtained further.

Bearing this expectation in mind, we vary the number of frequency channels in the range of $[5, 10, 15, 23]$ and constrain parameters in each case, in order to investigate the impact of the number of frequency channels on parameter constraints. The number of frequency channels with the corresponding channel widths are given in Table 4.1. In this and follow-up sections, we will focus on SKA1-MID Band 2 as the IM survey, since the impact of different factors we will be discussing is insensitive to the exact experiment but IM survey in general.

Our results are shown in Fig. 4.3. The solid lines in Fig. 4.3 give the parameter constraints as a function of total channel number. The left panel is the constraint from
SKA alone while the right is from SKA+Planck. For SKA alone, we have seen an improvement on the constraints for all parameters with increased number of frequency channels. On average, the constraint with $N_{\text{bin}} = 23$ is $\sim 5.5$ times better than that with $N_{\text{bin}} = 5$.

![Figure 4.3: The fractional uncertainties of cosmological parameters as a function of the number of frequency channels from SKA (left) and SKA+Planck (right). We vary $N_{\text{bin}} = [5, 10, 15, 23]$. The vertical axis gives the fractional uncertainty of each parameter, i.e., the error $\delta A$ divided by its fiducial value $A_{\text{fid}}$ for each parameter $A$, except for $w_a$, where the absolute uncertainty $\delta w_a$ is given since $w_a = 0$. The solid lines in each panel are the constraints obtained using the full HI power spectrum. The dashed lines are obtained by including only the auto-frequency bins in the Fisher matrix.]

In the right panel of Fig. 4.3, the addition of the Planck prior reduces the impact of channel number. This is because apart from $h$, $w_0$ and $w_a$, other cosmological parameters are strongly constrained by the Planck prior without much contributions from SKA (see Table 4.2). On average, after adding the Planck prior, the constraint with $N_{\text{bin}} = 23$ is $\sim 2.3$ times better than that with $N_{\text{bin}} = 5$. Note that although $b_{\text{HI}}$ depends only on SKA, one relies on the Planck prior to break the $b_{\text{HI}} - A_s$ degeneracy for the constraint on $b_{\text{HI}}$. Hence, once the uncertainty of $A_s$ has been decided by the Planck prior, the uncertainty on $b_{\text{HI}}$ is less dependent on the SKA channel number. Therefore, for SKA+Planck, increasing the number of SKA frequency channels has negligible effect on $\{\Omega_b h^2, \Omega_c h^2, \ln(10^{10} A_s), n_s, b_{\text{HI}}\}$ but improves constraints on $w_0$, $w_a$ and $b_{\text{HI}}$. 
4.5. COSMOLOGICAL PARAMETER CONSTRAINTS

The results in Fig. 4.3 are consistent with our expectation such that the uncertainties of cosmological parameters decrease with increased number of frequency channels. In order to find the turning point, one however needs an even larger number of frequency channels than what is used here, which can be a possible future work. In our analysis, we keep using 23 frequency channels rather than including more channels, in order to be consistent with Harper et al. (2018).

4.5.4 The cross-frequency contribution

In order to understand the importance of cross-frequency signal, we repeat the analysis of Sect. 4.5.3 with auto-frequency bins (the auto-correlation of each channel) only, i.e., we include only the on-diagonal terms in our Fisher matrix analysis. It is expected that the cross-frequency bins will not contribute much to the parameter constraints. This is because that the cross-frequency signal has a much (∼2 orders of magnitude) smaller amplitude than the auto-frequency signal. Therefore, including cross-frequency bins should not make a big difference to the Fisher matrix.

The constraints without cross-frequency bins are shown as the dashed lines in Fig. 4.3. By comparing the dashed lines with the solid lines where the full power spectrum is used, it can be seen that the cross-frequency bins bring little improvement to the parameter constraints as expected, so that the gaps between solid and dashed lines are almost negligible compared to the parameter fractional uncertainties themselves. This is particularly the case with a small number of frequency channels, say, \( N_{\text{bin}} = 5 \), for SKA+Planck. This is because the less frequency channels one has, the less cross-frequency bins there are. Since cross-frequency bins already provide limited contribution, a small number of channels will further reduce its contribution, and thus have a even less impact. For SKA+Planck, as mentioned before, those standard \( \Lambda \)CDM parameters are strongly constrained by the Planck prior with the rest significantly improved. Therefore, the cross-frequency bins do not impact much on parameter constraints for SKA+Planck.
Table 4.4 gives the parameter constraints with 23 frequency channels without cross-frequency bins. Compared with Table 4.2, on average, the exclusion of the cross-frequency bins degrades the parameter constraints by a factor of $\approx 1.7$ for SKA, and $\approx 1.2$ for SKA+$Planck$. Given that this is the case for 23 frequency channels, the degradation will be even less for the case with less frequency channels. Therefore, the cross-frequency bins do not contribute much to cosmological parameter constraints, which is consistent with our expectation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SKA diagonal</th>
<th>SKA diagonal +Planck</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$ [0.02224]</td>
<td>$\pm 0.00178$</td>
<td>$\pm 0.00014$</td>
</tr>
<tr>
<td>$\Omega_c h^2$ [0.1198]</td>
<td>$\pm 0.0071$</td>
<td>$\pm 0.0013$</td>
</tr>
<tr>
<td>$w_0$ [-1.00]</td>
<td>$\pm 0.23$</td>
<td>$\pm 0.04$</td>
</tr>
<tr>
<td>$w_a$ [0.00]</td>
<td>$\pm 1.17$</td>
<td>$\pm 0.19$</td>
</tr>
<tr>
<td>$h$ [0.6727]</td>
<td>$\pm 0.0034$</td>
<td>$\pm 0.0029$</td>
</tr>
<tr>
<td>$\ln(10^{10} A_s)$ [3.096]</td>
<td>$\pm 0.473$</td>
<td>$\pm 0.038$</td>
</tr>
<tr>
<td>$n_s$ [0.9641]</td>
<td>$\pm 0.0161$</td>
<td>$\pm 0.0040$</td>
</tr>
<tr>
<td>$b_{HI}$ [1.00]</td>
<td>$\pm 0.29$</td>
<td>$\pm 0.02$</td>
</tr>
</tbody>
</table>

Table 4.4: The cosmological parameters and their uncertainties from SKA alone (2nd column) and Planck+SKA (3rd column) with 23 frequency channels excluding cross-frequency bins.

**4.5.5 The RSD contribution**

As mentioned in Sect. 4.5.1, the RSD component in the HI power spectrum is the key to breaking the $b_{HI} - A_s$ degeneracy. Without it, one is not expected to constrain $A_s$ or $b_{HI}$ at all because of the complete degeneracy. In this section, we test a scenario where RSD is neglected so that the RSD component ($f(z)$ term in Equ. 4.1) is artificially set to zero. The parameter constraints in this case are given in Table 4.5. For SKA alone, no constraint is obtained for $b_{HI}$ or $A_s$ at all, consistent with the expectation. Once the Planck prior is added in, $A_s$ is constrained by the Planck prior, enabling $b_{HI}$ to be measured.

Therefore, Olivari et al. (2018) was able to constrain $b_{HI}$ and $A_s$ only because they have included the Planck prior. Otherwise, they would not have constrained $b_{HI}$ or $A_s$ since they have neglected the RSD component in their power spectrum calculation.
Therefore, we caution that one needs to be careful of neglecting the RSD component in cosmological parameter constraints.

4.6 The impact of $1/f$ noise on parameter constraints

As explained in Sect. 2.3.3, $1/f$ noise is a challenge for IM experiment. Therefore, it is useful to forecast the effect of $1/f$ noise on upcoming IM experiments in order to guide the experimental design. In this section, we will adopt the semi-empirical $1/f$ noise model from Harper et al. (2018) to add into the Fisher analysis and study its impact on cosmological parameter constraints.

4.6.1 The $1/f$ noise calculation

As introduced in Sect. 2.3.3, Harper et al. (2018) came up with a semi-empirical model for $1/f$ noise power spectrum calculation and adopted a power-law to account for the correlation of $1/f$ noise along frequency. Their $1/f$ noise power spectrum density has the form of

$$\text{PSD}^\frac{1}{2} (f) = \left( \frac{f_{\text{knee}}}{f} \right)^{\alpha} H(\omega),$$

(4.25)

where $\alpha$ is the $1/f$ noise spectral index, and $f_{\text{knee}}$ is the knee frequency at which $1/f$ noise has the same amplitude as the thermal noise. $H(\omega)$ describes the correlation of

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SKA−RSD</th>
<th>SKA−RSD+Planck</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$ [0.02224]</td>
<td>±0.00146</td>
<td>±0.00012</td>
</tr>
<tr>
<td>$\Omega_c h^2$ [0.1198]</td>
<td>±0.0033</td>
<td>±0.0011</td>
</tr>
<tr>
<td>$w_0$ [-1.00]</td>
<td>±0.13</td>
<td>±0.04</td>
</tr>
<tr>
<td>$w_a$ [0.00]</td>
<td>±0.65</td>
<td>±0.30</td>
</tr>
<tr>
<td>$h$ [0.6727]</td>
<td>±0.0033</td>
<td>±0.0030</td>
</tr>
<tr>
<td>$\ln(10^{10} A_s)$ [3.096]</td>
<td>...</td>
<td>±0.039</td>
</tr>
<tr>
<td>$n_s$ [0.9641]</td>
<td>±0.0108</td>
<td>±0.0039</td>
</tr>
<tr>
<td>$b_{\text{HI}}$ [1.00]</td>
<td>...</td>
<td>±0.02</td>
</tr>
</tbody>
</table>

Table 4.5: The cosmological parameters and their uncertainties from SKA alone (2nd column), and Planck+SKA (3rd column) with 23 frequency channels excluding the RSD component.
1/f noise in wavenumber space, by the use of a correlation parameter $\beta \in [0, 1]$, where $\beta = 0$ ($\beta = 1$) gives completely correlated (independent) 1/f noise.

The power-law $H(\omega)$ from Harper et al. (2018) is given by Equ. 2.6 but for the convenience of cross-reference, we give Equ. 2.6 here again in Equ. 4.26

$$H(\omega) = \left(\frac{\omega_0}{\omega}\right)^{1-\frac{\beta}{\beta}}. \quad (4.26)$$

The wavenumber $\omega$ is

$$\omega = \left[\frac{1}{N\delta \nu}, \frac{2}{(N)\delta \nu}, \ldots, \frac{N-1}{(N)\delta \nu}, \frac{1}{\delta \nu}\right], \quad (4.27)$$

with the minimum wavenumber of $\omega_0 = \frac{1}{N\delta \nu} = \frac{1}{\Delta \nu}$ (see Sect. 2.3.3 for more details).

The correlation of 1/f noise in frequency space $G(f)$ is given by the inverse discrete Fourier transform of Equ. 4.26 to transform from wavenumber space to frequency space, such that

$$G(f) = \frac{1}{N} \sum_{n=0}^{N-1} H(\omega)e^{2\pi i f \omega}. \quad (4.28)$$

The array $G(f)$ gives the correlation of the first SKA frequency channel with other channels. In order to get the covariance matrix $G(f_i, f_j)$ of 1/f noise, we construct a Toeplitz matrix from $G(f)$, which has constant descending diagonals from left to right (e.g., Golub & van Loan, 1996). The Toeplitz matrix has been used in, e.g., the Planck CMB map-making analysis to describe the 1/f noise covariance matrix (Ashdown et al., 2007). In our case, the Toeplitz matrix $G(f_i, f_j)$ is

$$G(f_i, f_j) = \begin{bmatrix}
G(f_1) & G(f_2) & \ldots & G(f_n) \\
G(f_2) & G(f_1) & \ldots & G(f_{n-1}) \\
\vdots & \vdots & \ddots & \vdots \\
G(f_{n-1}) & G(f_{n-2}) & \ldots & G(f_2) \\
G(f_n) & G(f_{n-1}) & \ldots & G(f_1)
\end{bmatrix}. \quad (4.29)$$
4.6. THE IMPACT OF $1/f$ NOISE

If the $1/f$ noise is purely correlated in frequency space, Equ. 4.29 is a matrix of ones and for completely uncorrelated $1/f$ noise, Equ. 4.29 is an identity matrix.

Harper et al. (2018) calculates the $1/f$ noise power spectrum at a given $\beta$ by

$$ F_\ell(\beta) = d \sin(2\pi \beta) + \beta, \quad (4.30) $$

where $d = -0.16$ and $F_\ell(\beta = 1)$ is the purely uncorrelated $1/f$ noise power spectrum at $\beta = 1$ modelled by

$$ \log_{10} \left( \frac{F_\ell(\beta = 1)}{\mu K^2} \right) = \log_{10} \left( \frac{A}{\mu K^2} \right) + a [\alpha - 1] + b \sqrt{\alpha} \log_{10} \left( \frac{v_t}{\text{deg s}^{-1}} \right) - c \alpha \log_{10}(\ell), \quad (4.31) $$

where $\alpha$ is the $1/f$ spectral index, $v_t$ is the telescope speed, and the best fitted values of $a$, $b$ and $c$ are 1.5, -1.5 and 0.5 respectively (Harper et al. 2018). The amplitude $A$ is parameterised by

$$ A = \left( \frac{T_{\text{sys}}}{21 \text{K}} \right) \left( \frac{f_{\text{knee}}}{1 \text{Hz}} \right)^{\alpha} \left( \frac{\delta v}{20 \text{MHz}} \right)^{-1} \left( \frac{N_t}{200} \right)^{-1} \left( \frac{T_{\text{obs}}}{30 \text{days}} \right)^{-1} \left( \frac{\Omega_s}{20500 \text{deg}^2} \right) 10^2 \mu \text{K}^2, \quad (4.32) $$

where $T_{\text{sys}}$ is the system temperature, $f_{\text{knee}}$ is the knee frequency, $\delta v$ is the frequency channel width, $N_t$ is the number of telescopes, $T_{\text{obs}}$ is the number of observing days, and $\Omega_s$ is the sky coverage area (Harper et al. 2018).

From Equ. 4.29 and Equ. 4.30, the $1/f$ noise for each frequency channel is

$$ F_\ell(f_i, f_j) = F_\ell(\beta) \times G(f_i, f_j). \quad (4.33) $$

Same as the thermal noise, we then normalize $F_\ell(f_i, f_j)$ by the mean brightness temperature of HI signal at each redshift bin given by Equ. 4.7. For the Fisher analysis with $1/f$ noise, we add Equ. 4.33 as an extra component to the measured power spectrum.
in Equ. 4.16 such that

\[
\hat{C}_\ell(z, z') = C^\text{HI}_\ell(z, z') + N_\ell(z, z')B_\ell(z, z') + F_\ell(z, z')B_\ell(z, z'),
\]

(4.34)

where \( B_\ell(z, z') \) takes into account the beam effect on the 1/f noise and is given by Equ. 4.10.

Figure 4.4: The HI power spectrum (blue) at \( z = 0.2 \) along with 1/f noise (green), thermal noise (red), and their combinations. Note that the spectra shown here are the auto-frequency spectra at \( z = 0.2 \). The 1/f noise in this case has \( \beta = 0.5, \alpha = 1, f_{\text{knee}} = 1 \text{Hz}, v_t = 1 \text{deg/s} \). The top left equation is from Equ. 4.35 where the power spectrum degradation factor \( \delta \) after adding 1/f noise is calculated by dividing the yellow curve by the magenta curve.

Fig. 4.4 shows different components in Equ. 4.34, taking the auto-frequency bin at \( z = 0.2 \) as an example. The 1/f noise in this case has \( \beta = 0.5, \alpha = 1, f_{\text{knee}} = 1 \text{Hz}, v_t = 1 \text{deg/s} \). Unless otherwise specified, we will use these 1/f noise parameters as our baseline reference 1/f noise. In Fig. 4.4, the 1/f noise (green) dominates the thermal noise (red) for all scales. This emphasizes the importance of eliminating 1/f noise for IM experiment. The 1/f noise decreases towards small scales until \( \ell \sim 100 \) where the beam effect becomes important and makes both thermal noise and 1/f noise surpass the HI signal (blue). We also show the combination of different components from Equ. 4.34 in Fig. 4.4. For example, the yellow curve shows the measured power
4.6. **THE IMPACT OF 1/f NOISE**

spectrum $\hat{C}_\ell(z,z')$, which closely follows the HI power spectrum (*blue*) until the beam scale. The *cyan* curve of the thermal noise plus 1/f noise further confirms that 1/f noise is dominant over thermal noise, as the *cyan* curve is almost overlapping with the *green* curve that represents the 1/f noise.

### 4.6.2 Power spectrum degradation

In order to understand the degradation on the measured power spectrum after introducing 1/f noise, we calculate the power spectrum degradation factor $\delta$, defined as the measured power spectrum with 1/f noise divided by the measured power spectrum without 1/f noise so that

$$
\delta_\ell(z,z') = \frac{C_{\ell}^{\text{HI}}(z,z') + N_\ell(z,z')B_\ell(z,z') + F_\ell(z,z')B_\ell(z,z')}{C_{\ell}^{\text{HI}}(z,z') + N_\ell(z,z')B_\ell(z,z')}.
$$

Equ. 4.35 quantifies the amount of extra power brought by 1/f noise relative to the measured power spectrum without 1/f noise, and thus gives the degradation factor of the power spectrum.

Fig. 4.5 plots $\delta$ as a function of redshift and multipole for the 23 SKA auto-frequency bins. The 1/f noise in this case has $[\beta = 0.5, \alpha = 1, f_{\text{knee}} = 1\text{ Hz}, v_t = 1\text{ deg/s}]$. Vertically inspecting Fig. 4.5, a factor of $\delta \gtrsim 3$ degradation is seen at $\ell \gtrsim 150$, for all redshift bins. This is because after $\ell \sim 150$, both the HI signal and the thermal noise start to be dominated by 1/f noise, as can be seen in Fig. 4.4. The degradation factor peaks at $\ell \sim 200$, where the 1/f noise has already started its exponential increase due to the beam while the thermal noise has not caught up due to its relatively low amplitude compared to the 1/f noise. After $\ell \sim 300$, the thermal noise catches up with 1/f noise so that the degradation factor flattens afterwards. This can be confirmed in Fig. 4.4, where the gap between the *yellow* and *magenta* curves does not vary much above $\ell \sim 300$. In Fig. 4.5, below $\ell \sim 150$, the S/N peaks around $\ell \sim 30$. This is because from $\ell \sim 2$ to $\ell \sim 50$, the 1/f noise keeps decreasing as can be seen in
Fig. 4.4. At $\ell \lesssim 100$, a degradation of $\delta \sim 1.5$ can be read from Fig. 4.5 for all redshift, indicating that $1/f$ noise does not affect the measured power spectrum much at these scales. This is because $1/f$ noise is dominated by HI signal at $\ell \lesssim 100$, as can be seen from Fig. 4.4, where the yellow curve is only slightly above the magenta curve.

Horizontally inspecting Fig. 4.5, higher redshift with $z \gtrsim 0.2$ gives a higher degradation factor in general, compared with lower redshift. This is because the HI signal at lower redshift is stronger than that at higher redshift (see Fig. 4.1), and thus less affected by $1/f$ noise, which is redshift-independent given a constant $T_{\text{sys}}$ under our assumption.

Since the degradation factor varies a lot in redshift and multipole, we divide $\delta$ into two redshift blocks separated by $z = 0.25$, and two multipole blocks separated by $\ell = 150$, to calculate the averaged $\delta$ in each block. The mean degradation factor and its standard deviation of each block are listed in Table 4.6. One can see that $\delta$ indeed varies significantly along redshift and multipole. Lower redshift and smaller multipoles give a smaller degradation which is consistent with Fig. 4.5. Due to the
large variation of the degradation factor in redshift and multipole, the values of \( \delta \) is widely distributed and spread out, giving a large standard deviation. On average, the degradation factor \( \delta \) has a mean value of 5.7 with a standard deviation of \( \pm 3.4 \), meaning that the 1/f noise at \( \beta = 0.5 \) will increase the measured power spectrum by a factor of \( \sim 2 - 9 \).

<table>
<thead>
<tr>
<th>Redshift range</th>
<th>( \ell = [2 - 150] )</th>
<th>( \ell = [150 - 400] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z = [0 - 0.25] )</td>
<td>1.9 ( \pm ) 1.5</td>
<td>6.4 ( \pm ) 1.1</td>
</tr>
<tr>
<td>( z = [0.25 - 0.5] )</td>
<td>2.3 ( \pm ) 1.8</td>
<td>9.6 ( \pm ) 1.8</td>
</tr>
<tr>
<td>Total</td>
<td>5.7 ( \pm ) 3.4</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6: The mean and standard deviation of the power spectrum degradation factor (Equ. 4.35) at different redshift and multipole ranges. The last row gives the averaged mean and s.t.d over all redshift and multipoles. The 1/f noise in this case has \( \beta = 0.5, \alpha = 1, f_{\text{knee}} = 1 \text{Hz}, v_i = 1 \text{deg/s} \).

4.6.3 Base parameters

In order to understand the impact of 1/f noise on cosmological parameters, we vary 1/f noise and calculate the fractional uncertainties of cosmological parameters in each case. The fractional uncertainty is defined by \( \frac{\delta A}{\delta A(\beta = 0)} \), where \( A \) is a cosmological parameter with its uncertainty \( \delta A \) with 1/f noise, and \( \delta A(\beta = 0) \) without 1/f noise, giving the fractional degradation caused by 1/f noise.

Fig. 4.6 gives the fractional uncertainties of cosmological parameters at various \( \beta \) values in the case of \( \alpha = 1 \) (upper) and \( \alpha = 2 \) (lower), from SKA alone (left) and SKA + Planck (right). All panels observe an increased fractional uncertainties of cosmological parameters towards a higher value of \( \beta \). This is expected from Harper et al. (2018) where a purely uncorrelated 1/f noise with \( \beta = 1 \) was shown to reduce the detection \( S/N \) of the HI signal. By vertically comparing the upper and lower panels of Fig. 4.6, the spectral index \( \alpha \) affects the results significantly such that the fractional uncertainty in the case of \( \alpha = 2 \) is about \( \sim 3 \) times bigger than that of \( \alpha = 1 \) for SKA alone, and \( \sim 1.5 \) times for SKA+Planck on average. This is consistent with Harper et al. (2018) who showed that \( \alpha \) can have a major impact on detecting HI signal and
Figure 4.6: The fractional cosmological parameter constraints as a function of the $1/f$ noise correlation parameter $\beta$. In each panel, the x-axis gives the $\beta$ values varying in the range of $\beta = [0, 0.25, 0.5, 0.75, 1]$. The y-axis gives the ratio of the parameter uncertainties obtained at the corresponding $\beta$ values, divided by the case without $1/f$ noise ($\beta = 0$). The two left panels are obtained from the SKA alone while the right panels are from SKA+Planck. The other parameters are set to $[v_t = 1\, \text{deg/s}, f_{\text{knee}} = 1\, \text{Hz}, \alpha = 1]$ (upper panels) and $[v_t = 1\, \text{deg/s}, f_{\text{knee}} = 1\, \text{Hz}, \alpha = 2]$ (lower panels).

should be minimised if possible. By horizontally comparing the left and right panels in Fig. 4.6, the Planck prior helps reduce the fractional uncertainty by constraining $\Omega_b h^2$, $\Omega_c h^2, n_s, \ln(10^{10} A_s)$ and $b_{\text{HI}}$, leaving $h, w_0$ and $w_a$ the most affected parameters since they cannot be constrained by Planck. On average, the fractional uncertainty from SKA alone is $\sim 1.5$ times that from SKA+Planck for $\alpha = 1$, and $\sim 3$ times for $\alpha = 2$. Note that in the upper left panel of Fig. 4.6, the averaged fractional uncertainty over all cosmological parameters at $\beta = 0.5$ is $\approx 1.8$, comparable to the measured power spectrum degradation given in Sect. 4.6.2, where we have $\delta = 5.7 \pm 3.4$ at $\beta = 0.5$. 


4.6. THE IMPACT OF 1/F NOISE

We then fix $\beta$ at $\beta = 0.5$, and study the impact of the knee frequency $f_{\text{knee}}$ and the slew speed of telescope $v_t$ on parameter constraints, shown in Fig. 4.7 and Fig. 4.8 respectively. From Fig. 4.7, a smaller knee frequency than 0.1 Hz is desirable to have a negligible degradation on parameter constraint, where on average a 14% and 8% degradation is reported for SKA alone and SKA+Planck respectively. Our results are consistent with Harper et al. (2018) where they suggested that ideally the knee frequency should be $f_{\text{knee}} < 10$ mHz.

![Figure 4.7: The fractional cosmological parameter constraints as a function of the knee frequency $f_{\text{knee}}$, varying in the range of $f_{\text{knee}} = [0.01, 0.1, 1, 10]$ Hz. The other parameters are set to $[\beta = 0.5, v_t = 1 \text{deg/s}, \alpha = 1]$. The constraints in the left panel are obtained from SKA alone and the right panel are from SKA+Planck.](image)

From Fig. 4.8, a higher slew speed at 2 deg/s is desired which gives an averaged fractional uncertainty of $\sim 1.4$ ($\sim 1.8$ for $v_t = 1 \text{deg/s}$) and $\sim 1.2$ ($\sim 1.4$ for $v_t = 1 \text{deg/s}$) for SKA alone and SKA+Planck respectively. This is consistent with Harper et al. (2018) who reported a desire for higher slew speed. The fractional uncertainty from SKA alone is in general $\sim 1.4$ times that of SKA+Planck, which is consistent with Fig. 4.6 and Fig. 4.7.

In summary, a small $\alpha$ is crucial to reduce the degradation on parameter constraints due to 1/f noise. A smaller $\beta$, a smaller $f_{\text{knee}}$, and a larger $v_t$ are desired to minimize the degradation. By adding the Planck prior, it can reduce the degradation by a factor of $\sim 1.5$. It is worth noting that, $h$, $w_0$ and $w_a$ are always the most affected parameters
Figure 4.8: The fractional cosmological parameter constraints as a function of the telescope slew speed $v_t$, varying in the range of $v_t = [0.5, 1, 1.5, 2]$ deg/s. The other parameters are set to $[\beta = 0.5, f_{\text{knee}} = 1 \text{Hz}, \alpha = 1]$. The constraints in the left panel are obtained from SKA alone and the right panel are from SKA+Planck.

4.6.4 Dark energy parameters

Since IM has the advantage of constraining late-time parameters, in this section we focus on the dark energy equation of state parameter $[w_0, w_a]$, and study the effect of $1/f$ noise on the $w_0 - w_a$ confidence ellipse. The confidence ellipses are plotted by following the method from Coe (2009) based on the Fisher analysis results. The confidence ellipse of two parameters contains information of each parameter along with the correlation between them. The projection of the ellipse on each axis gives the possible value of each parameter, and the angle of the ellipse represents the correlation between the two parameters.

The $1\sigma$ (solid) and $2\sigma$ (dashed) ellipses without the $1/f$ noise are shown in red in Fig. 4.9 from SKA alone (left) and SKA+Planck (right). It can be seen that $w_0$ and $w_a$ have a negative correlation such that a larger $w_0$ tends to have a smaller $w_a$. SKA alone

by $1/f$ noise in all cases. This emphasizes the importance of controlling $1/f$ noise for IM experiments, since the degraded uncertainties of these parameters cannot be mitigated by Planck prior and the effect of $1/f$ noise on IM will be propagated into the uncertainties of these parameters.
obtains a $\sim 10\%$ uncertainty on $w_0$ and $\sim 50\%$ uncertainty on $w_a$. SKA+Planck improves the constraint on $w_0$ to $\sim 2\%$ and $w_a$ to $\sim 9\%$. The $1\sigma$ ellipse from SKA+Planck is consistent with that given in Bull (2016), who forecast the performance of SKA IM on constraining general relativity, including a constraint on $w_0 - w_a$. However, our ellipse is slightly smaller than theirs which can be attributed to: i) Bull (2016) constrained different cosmological parameter set with 11 cosmological parameters while we constrain 8 parameters so their Fisher matrix gave larger uncertainties; ii) We adopt the latest SKA configuration with 200 dishes and dual polarisation beams while they used 130 dishes with single polarisation; iii) We have more frequency channels than them which brings finer information along line-of-sight. Nevertheless, our ellipse without $1/f$ noise is consistent with Bull (2016).

We move on to add two sets of $1/f$ noise to our measured HI power spectrum and re-plot the ellipses. In the first case, we have $\beta = 0.5$ and in the second case we have $\beta = 1$. The other parameters are set to the baseline values with $[v_t = 1 \text{deg/s}, f_{\text{knee}} = 1 \text{Hz}, \alpha = 1]$. The ellipses are shown in green and cyan respectively in each panel of Fig. 4.9. In the left panel, the $1/f$ noise with $\beta = 0.5$ degrades the uncertainty of $w_0$ to $\sim 15\%$ and $w_a$ to $\sim 80\%$. For $\beta = 1$, the uncertainty of $w_0$ will be further degraded to $\sim 30\%$ and $w_a$ to any possible values between $\sim -1.2$ to $\sim 1.2$. Compared to SKA...
alone without $1/f$ noise, $\beta = 0.5$ will degrade the uncertainties of $w_0$ and $w_a$ by a factor of 1.5. At $\beta = 1$, this is a factor of $\sim 3$ degradation. For SKA+Planck in the right panel, $1/f$ noise with $\beta = 0.5$ degrades the uncertainty of $w_0$ to $\sim 3\%$ and $w_a$ to $\sim 11\%$. For $\beta = 1$, the uncertainty of $w_0$ is degraded to $\sim 4\%$ and $w_a$ to $\sim 16\%$. This is a factor of $\sim 1.5$ degradation at $\beta = 0.5$ and a factor of $\sim 2$ degradation at $\beta = 1$, compared to SKA+Planck without $1/f$ noise.

It is interesting to note in Fig. 4.9 that $1/f$ noise weakens the correlation between $w_0$ and $w_a$ for SKA+Planck, which is not visible in the case of SKA alone. This is because SKA IM is more capable to constrain $w_0$ than $w_a$, making $w_0$ to be the more IM-sensitive parameter. Therefore, while the Planck prior generally helps breaking the $w_0 - w_a$ degeneracy, it improves the constraint on $w_0$ and $w_a$ at different levels. As a result, once SKA IM is affected by $1/f$ noise, it will affect $w_0$ more than $w_a$ and thus stretches the ellipse to a circle.

### 4.6.5 The Hubble rate $H(z)$

To study the impact of $1/f$ noise on the expansion rate of the Universe, we constrain the Hubble parameter $H(z)$ in this section through parameter transformation of the 8 base parameters. The parameter transformation is performed through the transformation matrix $\mathcal{T}$ introduced in Equ. 4.24, which contains the partial derivatives of old parameters to new parameters. The new parameter set is

$$\theta_i' = \{H(z_0), \Omega_c h^2, H(z_1), H(z_2), h, n_s, \ln(10^{10} A_s), b_{HI}\},$$

where $\Omega_b h^2$, $w_0$ and $w_a$ are translated into the Hubble rate at three redshift bins $H(z_0)$, $H(z_1)$ and $H(z_2)$. The reason that we did not transform other parameters is that $\Omega_c h^2$ is completely degenerate with $\Omega_b h^2$ under the parametrisation of $H(z)$ that

$$H(a) = 100 \sqrt{(\Omega_b h^2 + \Omega_c h^2)a^{-3} + (h^2 - \Omega_b h^2 - \Omega_c h^2)\exp[3w_a(a - 1)]a^{3(1+w_0+w_a)}}. \quad (4.36)$$
4.6. **THE IMPACT OF 1/F NOISE**

In(10^{10}A_s), n_s and b_{HI} are independent of \(H(z)\) from Equ.4.36. \(h\) is the Hubble rate at \(z = 0\). Therefore, our constraints on the Hubble rate is limited to three redshift bins through the parameter transformation method. The criteria of choosing the three redshift bins is that they should properly sample the partial derivative of old parameters to \(H(z)\) so that they can best represent the shape and amplitude of \(\frac{\partial \theta_i}{\partial H(z)}\), where \(\theta_i\) are the old parameters to be transformed into \(H(z)\). We choose \(z = [0.05, 0.2, 0.4]\) and also test a few other redshift sets under our criteria, which give consistent results.

The partial derivatives of the transformed old parameters with respect to \(H(z)\) in the transformation matrix are calculated by

\[
\frac{\partial \Omega_b h^2}{\partial H(z)} = \left[ \frac{\partial H(z)}{\partial \Omega_b h^2} \right]^{-1} = \frac{100 \times \frac{1}{2} \left( a^{-3} - \exp[3w_a(a-1)] \right)}{\sqrt{(\Omega_b h^2 + \Omega_c h^2) a^{-3} + (h^2 - \Omega_b h^2 - \Omega_c h^2) \exp[3w_a(a-1)] a^{3(1+w_0+w_a)}}}^{-1},
\]

\[
\frac{\partial w_0}{\partial H(z)} = \left[ \frac{\partial H(z)}{\partial w_0} \right]^{-1} = \frac{100 \times \frac{-3}{2} \left( h^2 - \Omega_b h^2 - \Omega_c h^2 \right) \exp[3w_a(a-1)] \log a}{\sqrt{(\Omega_b h^2 + \Omega_c h^2) a^{-3} + (h^2 - \Omega_b h^2 - \Omega_c h^2) \exp[3w_a(a-1)] a^{3(1+w_0+w_a)}}}^{-1},
\]

and

\[
\frac{\partial w_a}{\partial H(z)} = \left[ \frac{\partial H(z)}{\partial w_a} \right]^{-1} = \frac{100 \times \frac{-3}{2} \left( h^2 - \Omega_b h^2 - \Omega_c h^2 \right) \exp[3w_a(a-1)] (\log a + (1-a))}{\sqrt{(\Omega_b h^2 + \Omega_c h^2) a^{-3} + (h^2 - \Omega_b h^2 - \Omega_c h^2) \exp[3w_a(a-1)] a^{3(1+w_0+w_a)}}}^{-1}.
\]

The constraints on the Hubble rate at the three selected redshift bins after the parameter transformation are shown in Fig.4.10. The upper panels give the fractional uncertainty of the Hubble rate \(\frac{\sigma(H)}{H}\), defined by the uncertainty divided by the fiducial value of \(H(z)\) at each redshift bin. The constraints are obtained from SKA alone (left) and SKA+Planck (right). In each case, three scenarios are considered - i) no 1/f noise (green); ii) \(\beta = 0.5\) (red); iii) \(\beta = 1\) (cyan). Other 1/f noise parameters are set to the baseline values with \(v_i = 1\ \text{deg/s}, f_{\text{knee}} = 1\ \text{Hz}, \alpha = 1\). The lower panels give the ratio of the fractional uncertainty with \(\beta = 0.5\ (\beta = 1)\) over that with no 1/f noise.

From Fig.4.10, the uncertainties on the Hubble rate increase with redshift. This is
Figure 4.10: The fractional uncertainty of $H(z)$ against redshift (upper panels) obtained from SKA alone (left) and SKA+Planck (right). The $1/f$ noise correlation parameter is set to $\beta = 0$ (no $1/f$ noise; green), $\beta = 0.5$ (red), and $\beta = 1$ (cyan) respectively. The other parameters are set to $[v_t = 1\,\text{deg/s}, f_{knee} = 1\,\text{Hz}, \alpha = 1]$. The lower subplot in each panel gives the ratio of the $\beta = 0.5$ and $\beta = 1$ curves over the $\beta = 0$ curve from the upper subplot.

because dark energy dominates at late time and thus gives a better constraint on $H(z)$ at low redshift. The decreased uncertainty towards low redshift in Fig. 4.10 is consistent with our constraint on $h$ in Table 4.2, where at the fractional uncertainty is $\sim 0.5\%$ for SKA alone and $\sim 0.4\%$ for SKA+Planck. By comparing the left and right panel, the Planck prior improves the fractional constraints $\frac{\sigma(H)}{H}$ by a factor of $\sim 3$. For each panel, the $1/f$ noise at $\beta = 0.5$ degrades the constraints by a factor of $\sim 1.5$ averaging all redshift bins. The $1/f$ noise at $\beta = 1$ on average degrades the constraints by a factor of $\sim 2.5$ for SKA alone and $\sim 2.2$ for SKA+Planck. These degradation factors are consistent with the degradation on $w_0$ and $w_a$ in Sect. 4.6.4.

### 4.6.6 The angular diameter distance

To investigate the impact of $1/f$ noise on the angular diameter distance $D_A(z)$, same as in Sect. 4.6.5, we transform $\Omega_b h^2$, $w_0$ and $w_a$ into three redshift bins of $D_A(z)$ at $z = [0.05, 0.2, 0.4]$. The formula to calculate the angular diameter distance is given in Equ. 1.34. The partial derivatives of the transformed parameters with respect to $D_A(z)$
in the transformation matrix are

\[
\frac{\partial \Omega_b h^2}{\partial D_A(z)} = \left[ \frac{\partial D_A(z)}{\partial \Omega_b h^2} \right]^{-1} = \left[ \frac{-c}{200(1+z)} \int_0^z \frac{a^{-3} - \frac{\exp[3w_a(a-1)]}{a^{3(1+w_0+w_a)}}}{(\Omega_b h^2 + \Omega_c h^2)a^{-3} + (\Omega_b h^2 - \Omega_c h^2)\frac{\exp[3w_a(a-1)]}{a^{3(1+w_0+w_a)}}} dz' \right]^{-1},
\]

(4.40)

\[
\frac{\partial w_0}{\partial D_A(z)} = \left[ \frac{\partial D_A(z)}{\partial w_0} \right]^{-1} = \left[ \frac{3c}{200(1+z)} \int_0^z \frac{(h^2 - \Omega_b h^2 - \Omega_c h^2)\frac{\exp[3w_a(a-1)]}{a^{3(1+w_0+w_a)}} \log a}{(\Omega_b h^2 + \Omega_c h^2)a^{-3} + (\Omega_b h^2 - \Omega_c h^2)\frac{\exp[3w_a(a-1)]}{a^{3(1+w_0+w_a)}}} dz' \right]^{-1},
\]

(4.41)

and

\[
\frac{\partial w_a}{\partial D_A(z)} = \left[ \frac{\partial D_A(z)}{\partial w_a} \right]^{-1} = \left[ \frac{3c}{200(1+z)} \int_0^z \frac{(h^2 - \Omega_b h^2 - \Omega_c h^2)\frac{\exp[3w_a(a-1)]}{a^{3(1+w_0+w_a)}} (\log a + (1-a))}{(\Omega_b h^2 + \Omega_c h^2)a^{-3} + (\Omega_b h^2 - \Omega_c h^2)\frac{\exp[3w_a(a-1)]}{a^{3(1+w_0+w_a)}}} dz' \right]^{-1}.
\]

(4.42)

The fractional uncertainties of \( D_A(z) \) from SKA alone (left) and SKA+Planck (right) are shown in Fig. 4.11. We consider the same three scenarios as in Sect. 4.6.5 with \( \beta = [0, 0.5, 1] \). Similar to \( H(z) \), the fractional uncertainty of \( D_A(z) \) decreases towards low redshift, which is attributed to the extra information brought by dark energy dominating in lower redshift. Comparing the left and right panels in Fig. 4.11, the Planck prior improves the uncertainties by a factor of \( \sim 4 \) on average. For SKA alone, the \( 1/f \) noise at \( \beta = 0.5 \) and \( \beta = 1 \) will degrade the constraints by a factor of \( \sim 1.5 \) and \( \sim 2.5 \) respectively, averaging over all redshift bins. For SKA+Planck, the
1/f noise at $\beta = 0.5$ will degrade the constraints by a factor of $\sim 1.5$ and a factor of $\sim 2.1$ at $\beta = 1$. These degradation levels are consistent with the degradation on $w_0$ and $w_a$ in Sect. 4.6.4 and $H(z)$ in Sect. 4.6.5, and comparable to the power spectrum degradation in Sect. 4.6.2.

### 4.6.7 The growth rate

Another quantify to reveal the Universe expansion history is the growth rate (Equ. 4.4). In this section we investigate the effect of 1/f noise on the growth rate $f \sigma_8(z)$. For the Fisher analysis of the growth rate, we calculate the analytical derivative of the HI power spectrum (Equ. 4.1) with respective to $f \sigma_8(z) = f(z)D(z)\sigma_8$ so that

\[
\frac{\partial C^\text{HI}_\ell(z, z')}{\partial f \sigma_8(z)} = \frac{2}{\pi} \int dk k^2 \frac{P_m(k, z = 0)}{\sigma_8} \times \\
\left\{ \\
\int dz' W^\text{HI}(z') D(z') \left[ b_{\text{HI}} j_{\ell}(k\chi') - f(z') j''_{\ell}(k\chi') \right] f dz W^\text{HI}(z) (-j''_{\ell}(k\chi)) \quad z \neq z' \\
2 \int dz W^\text{HI}(z) D(z) \left[ b_{\text{HI}} j_{\ell}(k\chi) - f(z) j''_{\ell}(k\chi) \right] f dz W^\text{HI}(z) (-j''_{\ell}(k\chi)) \quad z = z' \\
\right\}
\tag{4.43}
\]
The constraints on $f\sigma_8(z)$ are obtained from the Fisher matrix in Equ. 4.14 with the derivative of $f\sigma_8(z)$ calculated from Equ. 4.43. We use 10 frequency bins equally spaced within the SKA frequency band. The $f\sigma_8(z)$ in each frequency bin is treated as a free parameter and marginalised over, with all other base parameters fixed. The parameter set in this case is thus $\{f\sigma_8(z_i)\}$ with $z_i$ corresponding to $f_i = [990, 1030, 1070, 1110, 1150, 1190, 1230, 1270, 1310, 1350]$ MHz.

Figure 4.12: The fractional uncertainty of $f\sigma_8(z)$ against redshift obtained from SKA alone. The $1/f$ noise correlation parameter is set to $\beta = 0$ (no $1/f$ noise; green), $\beta = 0.5$ (red), and $\beta = 1$ (cyan) respectively. The other parameters are set to $v_t = 1$ deg/s, $f_{knee} = 1$ Hz, $\alpha = 1$. The lower subplot in each panel gives the ratio of the $\beta = 0.5$ and $\beta = 1$ curves over the $\beta = 0$ curve from the upper subplot.

The fractional uncertainty of $f\sigma_8(z)$ is shown in Fig. 4.12. The constraints are from SKA alone as *Planck* does not have prior information on the growth rate. In Fig. 4.12, the uncertainty decreases towards lower redshift, consistent with $H(z)$ and $D_A(z)$ in Fig. 4.10 and Fig. 4.11. After adding $1/f$ noise, at $\beta = 0.5$, the constraint will be degraded by a factor of $\sim 1.2$ and at $\beta = 1$, the degradation is about a factor of $\sim 1.5$. Compared to the degradation factor on $H(z)$ and $D_A(z)$, the degradation on $f\sigma_8(z)$ is smaller but still comparable.
4.7 Conclusions and discussions

In this chapter, we forecast the constraints on cosmological parameters from upcoming IM surveys using the Fisher matrix method. We focus on SKA-MID Band 2, and study the impact of $1/f$ noise on parameter constraints.

Our power spectrum calculation is an improvement over other treatments in terms of: i) we calculate the full power spectrum instead of using the Limber approximation; ii) we include RSD component, which was often neglected in literature; iii) we calculate the 2-D angular power spectrum, which is a more direct measurable once real data is obtained, instead of 3-D power spectrum $P(k)$ in the wavenumber space.

We first constrain cosmological parameters within the CPL model without $1/f$ noise. We report constraints of $w_0 = -1.0 \pm 0.03$, $w_a = 0.0 \pm 0.13$, and $b_{HI} = 1.0 \pm 0.02$ for SKA+Planck, compared to $w_0 = -1.0 \pm 0.06$, $w_a = 0.0 \pm 0.25$ and $b_{HI} = 1.0 \pm 0.04$ for BINGO+Planck. Our constraints are consistent with previous literatures (e.g., Bull, 2016; Olivari et al., 2018), where forecasts have been made from similar analysis.

We study the dependency of parameter constraints on number of frequency channels, cross-frequency contributions, and RSD component. It is found that the parameter uncertainties will decrease with increased number of frequency channels. Due to the limited number of frequency channels analysed in our case, we did not reach the turning point where an increase in frequency channels no longer improves the results, leaving as a future work to do. The cross-frequency channels are found to have negligible contributions to constraining cosmological parameters. We caution that one needs to be careful of neglecting the RSD component, since it is the key to breaking the $b_{HI} - A_s$ degeneracy.

We study the impact of $1/f$ noise on cosmological parameters using the semi-empirical $1/f$ noise model from Harper et al. (2018). A representative $1/f$ noise model degrades the measured HI power spectrum by a factor of $\delta = 5.7 \pm 3.4$ on average, with large variations in redshift and angular scales. Consequently, the constraints on cosmological parameters are degraded by $\sim 50\%$. In order to mitigate this, one requires
4.7. CONCLUSIONS AND DISCUSSIONS

A minimised $1/f$ noise spectral slope, a low knee frequency ($f_{\text{knee}} < 0.1$ Hz) and a large telescope slew speed ($v_t \rightarrow 2$ deg/s). A correlation in frequency is also preferred ($\beta \rightarrow 0$).

Given the $\sim 50\%$ degradation on parameter constraints, $1/f$ noise does not seem to be too problematic for cosmologists. However, this degradation can be serious for IM to be competitive with optical galaxy surveys for precision cosmology. Besides, one has to keep in mind that $1/f$ noise can increase the absolute noise level by $\sim 2$ orders of magnitude at certain angular scales (see Fig. 4.4). This can impact, e.g., the test of GR theory, where one compares the differences between two models and will thus be sensitive to the absolute noise level. Due to the large variations of $1/f$ noise in redshift and angular scales, IM experiments at a higher redshift with lower beam resolution will be more affected by $1/f$ noise. Also, instrumental designs without a proper consideration of $1/f$ noise can result in a much larger degradation factor than the case analysed here. We therefore emphases the importance of mitigating $1/f$ noise for IM experiments.

Possible extensions of our work include: i) Adopt a more complicated parametrisation of $w$, or calculate analytical derivatives of HI power spectrum with respect to $H(z)$, $D_A(z)$ and $f(z)$, in order to include more redshift bins for these time-dependent parameters; ii) Use finer steps and more combinations of $1/f$ noise parameters to investigate their impact on cosmological parameters; iii) Investigate the impact of $1/f$ noise on testing GR theory.
Chapter 5

The Cross-Correlation between Galaxy Lensing and HI Intensity Maps

In this chapter, we make forecasts of the cross-correlation between a 21 cm intensity map and a galaxy lensing map, in terms of detection S/N and constraining cosmological parameters. We adopt the Fisher analysis method as in Chapt. 4, start with two upcoming experiments, study the impact of survey parameters on cross-correlation, and finally present results from two perfect surveys.

5.1 Introduction

As mentioned in Sect. 1.2.3, the cross-correlation between multi-tracers benefits from a zero noise bias on the measured cross-correlation signal, mitigated systematics and calibrated mass bias. Cross-correlating IM with redshifted galaxy surveys has been important since the first measurement of 21 cm intensity map was made by cross-correlating 21 cm map from GBT observation with WiggleZ data (Chang et al. 2010, see Sect. 2.2.4). Since then, forecasts have been made on cross-correlating upcoming HI IM experiments with optical galaxy surveys.
Fonseca et al. (2015) forecast the cross-correlation between SKA1 IM and Euclid-like photometric surveys. They found that one could potentially constrain the primordial non-Gaussianity to $\sigma(f_{\text{NL}}) \simeq 1.4 - 0.5$. Pourtsidou et al. (2016) calculated the detection S/N of IM×optical surveys and found that 21 cm lensing×galaxy density field can considerably improve the 21 cm lensing detection. The same applies to 21 cm lensing×galaxy lensing. The cross-correlation between 21 cm IM and galaxy density fields can bring information about the HI-galaxy correlation coefficient. In a following paper, Pourtsidou et al. (2017) forecast that the cross-correlation between HI IM and galaxy density fields could obtain competitive constraints on the HI bias parameter and growth rate. Fonseca et al. (2017) forecast that the cross-correlation between intensity maps from MeerKAT with DES can deliver an up to 3 times better measurement of $f_{\text{NL}}$ than Planck. Anderson et al. (2018) cross-correlated 21 cm intensity maps from the Parkes telescope with 2dFGS and detected the cross-correlation signal with 5.18$\sigma$ significance. Witzemann et al. (2018) simulated a full sky 21 cm intensity map and a galaxy map with foregrounds, and measured the bias ratio of two tracers. Their multi-tracer analysis showed that cross-correlation is immune to foreground contamination, and can improve cosmic-variance contaminated measurement by a factor of 2−4.

Our analysis in this chapter focuses on the cross-correlation between a 21 cm intensity map and a galaxy lensing map (hereafter $\delta_{\text{IM}} \times \kappa_g$), which has not been studied in detail in previous literature. The $\delta_{\text{IM}} \times \kappa_g$ is free from galaxy bias (see Sect. 5.3), and thus does not have a $b_{\text{HI}} - b_g$ degeneracy as in HI IM × galaxy density field cross-correlation. We adopt the Fisher analysis introduced in Chapt. 4, to forecast cosmological parameter constraints from $\delta_{\text{IM}} \times \kappa_g$. Sect. 5.2 introduces the surveys used in our analysis. Sect. 5.3 gives the analytical formulae for power spectrum calculation. Sect. 5.4 presents the results from BINGO×DES. Sect. 5.5 investigates the impact of survey parameters on cross-correlation and Sect. 5.6 gives the results for two ideal surveys. Sect. 5.7 draws conclusions.
5.2 The surveys

As introduced in Sect. 1.2.3, to obtain a cross-correlation signal between two tracers, it is required that the two tracers have overlapping sky areas and redshift coverage. It is also preferred that the two surveys will have data available at approximately the same time for cross-correlation.

We start with BINGO as the IM survey for our cross-correlation analysis. The BINGO instrumental parameters are given in Table 2.1 and we assume 30 frequency channels with a channel width of $\delta \nu = 10 \text{ MHz}$ as in Chapt. 4. In order to find the appropriate optical survey for cross-correlation, we plot the BINGO sky coverage along with a few major redshift galaxy surveys as shown in Fig. 5.1. It can be seen that those galaxy surveys which have a decent overlapping sky coverage with BINGO are 6dFGS, 2dFGS, BOSS and DES. However, both 6dFGS ($0 < z < 0.15$; Jones et al. 2004) and 2dFGS ($0 < z < 0.3$; Colless et al. 2001) have a limited redshift coverage with BINGO. We use DES as the redshift galaxy survey since it has a reasonable sky and redshift coverage, and is currently collecting data.

The DES experiment is an ongoing stage-III photometric redshift galaxy survey and one aim of DES is to measure BAO. It is mapping a $\sim 5000 \text{ deg}^2$ of the sky using the Dark Energy Camera on the Blanco 4 m telescope in Chile. Table 5.1 gives the observing parameters of DES that are relevant to our cross-correlation analysis. DES will observe galaxies in 5 optical bands [$g, r, i, z, Y$], at the redshift range of $0 < z < 2$ with a medium redshift of $z_0 \approx 0.7$ (e.g., Hoyle et al., 2018). The average number density of galaxies is estimated to be $n_g = 10 \text{ arcmin}^{-2}$, which is adopted from the assumption in Pourtsidou et al. (2016) for consistency. One might note that the latest DES data release gives a lower $n_g \sim 4 \text{ arcmin}^{-2}$ (e.g., Zuntz et al., 2018), which however is from the first data release, with possible improvement from upcoming data release. Nevertheless, later in Sect. 5.6 where we cross-correlate two ideal surveys with no noise, we will show that our main conclusion will not be affected by the exact value of galaxy density.
Figure 5.1: The sky coverage of BINGO and a few major galaxy surveys in Celestial coordinate. The graticule and numbers give the right ascension and declination. The sky coverage of each experiment is indexed with a specific color. The Galactic plane is plotted in temperature contour curves.

number density assumed for the redshift galaxy survey.

<table>
<thead>
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<th>DES parameters</th>
<th></th>
</tr>
</thead>
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<tr>
<td>Sky coverage area, $A_{\text{DES}}$ (deg$^2$)</td>
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</tr>
<tr>
<td>Galaxy number density, $n_g$ (arcmin$^{-2}$)</td>
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</tr>
<tr>
<td>Redshift coverage</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Cross-correlation parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Redshift overlapping</td>
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</tr>
<tr>
<td>Sky overlapping area, $A_{\text{cross}}$ (deg$^2$)</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 5.1: The BINGO×DES cross-correlation parameters. For the DES parameters, only those that are used in the cross-correlation analysis are listed here.

For BINGO×DES cross-correlation, the overlapping sky area is $\sim 800$ deg$^2$ from Fig. 5.1. The overlapping redshift coverage is the same as the BINGO redshift range, since DES covers the whole BINGO observing redshift band. These used cross-correlation parameters are given in Table 5.1.
5.3 Analytical formula

5.3.1 HI power spectrum

The HI angular power spectrum is calculated using Eq. 4.1 as in Chapt. 4. Fig. 5.2 shows the HI power spectra at the lowest (\( z = 0.13 \), green) and highest (\( z = 0.48 \), blue) BINGO redshift, with a channel width of \( \delta \nu = 10 \) MHz (solid) and \( \delta \nu = 100 \) MHz (dashed). As discussed in Sect. 4.3.1, a higher redshift and wider channel width will decrease the HI signal.

![HI power spectra figure](image)

Figure 5.2: The HI power spectra at the lowest (\( z = 0.13 \), green) and highest (\( z = 0.48 \), blue) BINGO redshift. Also shown is the thermal noise at \( z = 0.13 \) (red) and \( z = 0.48 \) (magenta). We consider two channel widths, \( \delta \nu = 10 \) MHz (solid) and \( \delta \nu = 100 \) MHz (dashed).

The BINGO thermal noise is calculated using Eq. 4.6 with the BINGO instrumental parameters in Table 2.1. The thermal noise has the beam applied (Eq. 4.10). The thermal noise at \( z = 0.13 \) (red) and \( z = 0.48 \) (magenta) are shown in Fig. 5.2. The thermal noise decreases with a wider bandwidth, and due to being normalised by the HI brightness temperature (Eq. 4.7), it decreases with a higher redshift. In each case, the thermal noise is much lower than the HI signal until \( \ell \approx 200 \), after which, the thermal noise will surpass the HI signal due to the beam.
5.3.2 Galaxy lensing power spectrum

Similar to the CMB weak lensing as introduced in Sect. 3.1.1, the galaxy weak lensing effect is due to the bending of light from distant sources when passing through LSS. One requires accurate measurement of the shapes of a large number of small and faint galaxies to observe the galaxy weak lensing. The strength of lensing for a given configuration of source, lens and observer is quantified by the lensing potential. Two quantities derived from the lensing potential are the shear field, \( \gamma \), and the convergence field, \( \kappa \). Shear measures the distortions of the source galaxy shapes, and convergence quantifies the magnification of the lensed image, which can be reconstructed from the shear.

The galaxy lensing power spectrum is calculated by

\[
C_\ell^\kappa = \int_0^{\chi_H} d\chi \left( \frac{W^\kappa(\chi)}{\chi} \right)^2 P_m \left( \frac{\ell + 1/2}{\chi}, z \right),
\]

where \( W^\kappa(\chi) \) is the kernel of lensing as a function of the comoving distance \( \chi \), defined by (e.g., Kirk et al., 2016)

\[
W^\kappa(\chi) = \frac{3}{2} \Omega_m \left( \frac{H_0}{c} \right)^2 g(\chi) \frac{\chi}{a},
\]

and

\[
g(\chi) = \int_\chi^{\chi_H} d\chi' p_S(\chi') \left( \frac{\chi' - \chi}{\chi'} \right),
\]

with \( \chi_H \) denoting the comoving distance to the last scattering surface. For DES, the empirical source function \( p_S(\chi') \) takes the form of

\[
p_S(z) \propto z^\alpha \exp\left[-\left(\frac{z}{z_0}\right)^\eta\right],
\]

with \( \alpha = 2 \), \( \eta = 3/2 \) and the median redshift \( z_0 = 0.7 \) (LSST Science Collaboration,
The source function is then normalised so that
\[
\int d\chi p_S(\chi) = 1.
\] (5.5)

Fig. 5.3 shows the source function \( p_S(z) \) (Equ. 5.4) and \( g(z) \) by replacing the independent variable from \( \chi \) to \( z \) in Equ. 5.3. In each panel, we plot \( p_S(z) \) and \( g(z) \) at different medium redshift values \( z_0 \), to study how \( p_S(z) \) and \( g(z) \) change with the medium redshift of the galaxy survey. In the left panel, the source function has its peak shifted with the corresponding medium redshift \( z_0 \), and due to the normalisation in Equ. 5.5, \( p_S(z) \) with a lower \( z_0 \) is narrower but taller than that with a higher \( z_0 \), giving the same area of coverage under each curve. In the right panel, the \( g(z) \) curves all start from 1 at \( z = 0 \), since Equ. 5.3 will become Equ. 5.5 with \( \chi = 0 \) at \( z = 0 \). A higher medium redshift \( z_0 \) gives a wider coverage of \( g(z) \), due to a wider \( p_S(z) \).

![Figure 5.3: The DES galaxy source function \( p_S(z) \) (Equ. 5.4, left) and \( g(z) \) (Equ. 5.3, right) as a function of redshift at different survey medium redshift \( z_0 \). In the right panel, the independent variable \( \chi \) in Equ. 5.3 has been substituted by \( z \) for a more direct understanding.](image)

We plot the kernel of lensing \( W_\kappa \) (Equ. 5.2) in Fig. 5.4 and replace the independent variable from \( \chi \) to \( z \). Due to a wider \( g(z) \) at a higher medium redshift \( z_0 \), the kernel of lensing is both taller and wider towards a larger \( z_0 \). Therefore, from Fig. 5.4, a higher medium redshift will give a lensing kernel that not only covers a wider range of redshift but is also higher in amplitude. One must note that here we keep using the empirical DES galaxy source function given in Equ. 5.4 for lensing kernel calculation at different
survey medium redshift $z_0$. However, in reality, the galaxy source function might take a different form, or at least a different value of $\alpha$ and $\eta$ in Equ. 5.4, when shifting to a lower survey medium redshift. Therefore, a higher medium redshift in reality does not necessarily return a wider and higher lensing kernel as shown in Fig. 5.4.

Figure 5.4: The lensing kernel (Equ. 5.2) as a function of redshift at different survey medium redshift $z_0$. The independent variable $\chi$ in Equ. 5.2 has been substituted by $z$.

The lensing power spectrum (Equ. 5.1) is shown in Fig. 5.5. Here we plot the lensing power spectra at the DES medium redshift $z_0 = 0.7$, and a lower medium redshift $z_0 = 0.3$, as two examples for comparison. As expected from the lensing kernel shown in Fig. 5.4, a higher medium redshift returns a higher lensing power spectrum signal in Fig. 5.5. We compare our lensing power spectrum with Sato & Nishimichi (2013) who plotted a theoretical lensing power spectrum at $z_0 = 1$, and we obtain consistent spectrum once shifted to $z_0 = 1$.

The advantage of the galaxy lensing power spectrum is that unlike in the galaxy density power spectrum, it does not have galaxy bias. Therefore, during cross-correlation, it will not introduce a galaxy bias parameter that is degenerate with other mass biases.

The shot noise of the measured DES lensing power spectrum is the reciprocal of
5.3. ANALYTICAL FORMULA

The galaxy number density per steradians such that

\[ N^\kappa = \frac{1}{n_g} \approx 8.46 \times 10^{-9}. \] (5.6)

Fig. 5.5 plots the shot noise in red. It can be seen that due to the shot noise, the detection of the DES lensing signal is significantly limited especially at small scales.

5.3.3 The cross-power spectrum

The cross-power spectrum of HI IM and galaxy lensing is calculated by

\[
C_{\ell}^{\text{HI} \times \kappa}(z) = \frac{2}{\pi} \int dz W_{\text{HI}}(z) D(z) \int_0^\infty dz' \left( \frac{c}{H(z')} \right) W^\kappa(z') D(z') \\
\times \int dk k^2 \left[ b_{\text{HI}} j_\ell(k\chi) - f(z) j''_\ell(k\chi) \right] j_\ell(k\chi') P_m(k, z = 0). 
\] (5.7)

From Equ. 5.7, one can see that the cross-correlation spectrum only includes the HI bias \( b_{\text{HI}} \) but no galaxy bias, which is the benefit of our \( \delta_{\text{IM}} \times \kappa_g \) analysis.

Fig. 5.6 plots the cross-spectrum of BINGO×DES with the DES medium redshift
$z_0 = 0.7$ at the $z = 0.13$ and $z = 0.48$ redshift bin. The solid curves have a channel width of $\delta \nu = 10 \text{ MHz}$ and the dashed curves have $\delta \nu = 100 \text{ MHz}$. It can be seen that the cross-spectrum does not depend much on the channel width, which is because the lensing part is bandwidth-independent by integrating over all redshift. The $z = 0.13$ redshift bin gives a stronger signal than $z = 0.48$, because of a stronger HI signal at a lower redshift. However, from Fig. 5.4, at the DES medium redshift of $z_0 = 0.7$, a higher redshift bin at $z = 0.48$ should return a stronger lensing kernel than $z = 0.13$. This effect compensates some of the decrease on the cross-spectrum caused by a decreased HI signal towards higher redshift, such that the two redshift bins in the cross-spectrum of Fig. 5.6 are comparable to each other after a certain scale ($\ell \gtrsim 80$).

![Figure 5.6](image.png)

**Figure 5.6:** The cross-spectrum of HI IM and galaxy lensing. The solid curves give the cross-spectra at the redshift bin $z = 0.13$ (green) and $z = 0.48$ (blue) with the DES medium redshift of $z = 0.7$. The solid curves have a frequency channel width of $\delta \nu = 10 \text{ MHz}$ while the dashed curves have $\delta \nu = 100 \text{ MHz}$.

Since BINGO and DES have completely different systematics and uncorrelated noise, it is assumed that the mean noise level of the cross-spectrum is $N^{\text{HI} \times \kappa} = 0$. Therefore, unlike auto-correlation where the measured auto-spectrum of each survey is the signal plus its noise, the measured cross-spectrum has no noise bias thanks to uncorrelated noise of two surveys, which is the advantage of cross-correlation analysis.
5.4. THE CROSS-CORRELATION OF BINGO×DES

5.3.4 The Fisher matrix

We use the Fisher analysis introduced in Sect. 4.3.3 for the forecast of BINGO×DES. The Fisher matrix calculation is given in Eq. 4.14 with the covariance matrix of the cross-spectrum given in Eq. 4.15. In this case, in the covariance matrix, \( X \) (\( Y \)) loops over the 30 frequency bins of BINGO, and \( X' \) (\( Y' \)) denotes the lensing signal which only has one bin integrated over all redshift, so that Eq. 4.15 becomes

$$\Delta C^{\text{HI} \times \kappa}(z_i, z_j) = \sqrt{\frac{1}{(2\ell + 1)f_{\text{sky}^\ell}} \left( C^{\text{HI}}_\ell(z_i, z_j) C^{\kappa}_\ell + C^{\text{HI} \times \kappa}_\ell(z_i) C^{\text{HI} \times \kappa}_\ell(z_j) \right)}.$$  (5.8)

The covariance matrix of the cross-spectrum has a dimension of \([30 \times 30]\). The measured power spectra in the covariance matrix for the cross-correlation analysis are

$$C^{\text{HI}}_\ell(z_i, z_j) = C^{\text{HI}}_\ell(z_i, z_j) + N^{\text{HI}}_\ell(z_i, z_j) B_\ell(z_i, z_j)$$

$$\hat{C}^{\kappa}_\ell = C^{\kappa}_\ell + N^{\kappa}_\ell$$

$$\hat{C}^{\text{HI} \times \kappa}_\ell(z) = C^{\text{HI} \times \kappa}_\ell(z).$$

Instead of using the CPL model adopted in Chapt.4, the set of cosmological parameters in the BINGO×DES analysis is within the \( w \)CDM model with \( w = w_0 \), for simplicity. The cosmological parameter set in this case is

$$\theta_{i, j} = \{\Omega_b h^2, \Omega_c h^2, w_0, h, n_s, \ln(10^{10} A_s), b_{\text{HI}}\}.$$  

The Fisher matrix (Eq. 4.14) in this case is thus a \([7 \times 7]\) matrix, containing information of the uncertainties and correlations of cosmological parameters.

5.4 The cross-correlation of BINGO×DES

In this section, we present results from BINGO×DES cross-correlation based on our Fisher analysis. We calculate the detection S/N, constrain cosmological parameters,
and investigate the improvement after adding in the Planck prior or fixing known parameters.

### 5.4.1 The detection S/N

We follow the procedure described in Sect. 4.4 to calculate the detection S/N of BINGO×DES. The S/N per multipole is calculated from \( C_{\ell}^{\text{HI} \times \kappa} (z) \) in Eq. 5.7 and \( \Delta C_{\ell}^{\text{HI} \times \kappa} (z, z) \) in Eq. 5.8 by

\[
\frac{S}{N}_\ell = \sqrt{\sum_{z=z_{\text{min}}}^{z=z_{\text{max}}} \left( \frac{C_{\ell}^{\text{HI} \times \kappa} (z)}{\Delta C_{\ell}^{\text{HI} \times \kappa} (z, z)} \right)^2}.
\]

We then bin the power spectrum with a bin width of \( \Delta \ell = 50 \) and calculate the S/N for each multipole bin.

![Figure 5.7](image_url)

**Figure 5.7:** The S/N of the cross-spectrum between HI and galaxy lensing as a function of multipole. The upper panel shows the cross-spectrum of BINGO and DES at one redshift bin of \( z = 0.13 \) and \( \Delta \nu = 10 \text{ MHz} \). The multipoles are binned with \( \Delta \ell = 50 \). The shaded area shows the uncertainty of the cross-spectrum of that particular redshift bin. The lower panel gives the total S/N as a function of multipole after integrating over the full redshift range \( 0.13 < z < 0.48 \).

The detection S/N is shown in Fig. 5.7. In the upper panel, we show the cross-spectrum at \( C_{\ell}^{\text{HI} \times \kappa} (z = 0.13) \) as an example. The shaded area is the \( 1 \sigma \) uncertainty.
calculated from Equ. 5.8. It can be seen that the uncertainty is larger at $\ell \lesssim 50$ due to cosmic variance and at $\ell \gtrsim 150$ due to limited beam resolution. In the lower panel, the S/N integrated over all redshift bins is plotted as a function of multipole, with a bin width of $\Delta \ell = 50$. A maximum S/N of $\sim 6.8$ is obtained at $50 < \ell < 100$ and a minimum of $\sim 1.9$ is obtained at $300 < \ell < 350$. After integrating over all multipole bins (Equ. 4.23), we obtain an overall $\approx 13\sigma$ detection. This is consistent with Pourtsidou et al. (2016), who calculated the predicted detection S/N of $\delta_{\text{IM}} \times \kappa_g$ using MeerKAT and DES, and obtained a $\sim 11\sigma$ detection.

5.4.2 The cosmological parameter constraints

We constrain the seven cosmological parameters given in Sect. 5.3.4 within the $w$CDM model, using BINGO×DES cross-correlation. The fiducial value of each parameter is the same as in Chapt. 4, taken from Olivari et al. (2018) for consistency. The parameter constraints are shown in the second column of Table 5.2. One might immediately notice that $A_s$ and $b_{\text{HI}}$ are not constrained at all. This is due to the $A_s - b_{\text{HI}}$ degeneracy mentioned in Sect. 4.5.5. Although here we include the RSD component in our power spectrum calculation in Equ. 5.7, because it is much smaller than the $b_{\text{HI}}$ density component, the inclusion of RSD in the cross-spectrum is not enough to break the $A_s - b_{\text{HI}}$ degeneracy. We also test this by artificially removing the RSD component from the cross-spectrum, which results in no constraints on either $A_s$ or $b_{\text{HI}}$ (uncertainties equal to infinity).

In order to break the $A_s - b_{\text{HI}}$ degeneracy, we assume that we have a perfect knowledge of $A_s$ such that we fix $A_s$ by removing the corresponding row and column of $A_s$ from the Fisher matrix and re-calculate the covariance matrix to obtain the constraints afterwards (Coe, 2009). The results in this case are shown in the third column of Table 5.2. It can be seen that the constraint on $b_{\text{HI}}$ is indeed improved but is hardly competitive with, e.g., BINGO and SKA auto-correlations. By comparing Table 5.2 with those obtained from SKA and BINGO auto-correlation in Table 4.2 and 4.3, the
BINGO×DES cross-correlation is significantly surpassed by the auto-correlation in terms of constraining cosmological parameters. This is consistent with the calculated detection S/N where we only obtain an overall $\approx 13\sigma$ detection of the cross-spectrum, but a $\approx 351\sigma$ and $\approx 220\sigma$ detection of the auto-spectrum with SKA and BINGO respectively (see Sect. 4.4).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BINGO×DES</th>
<th>Fix $\ln(10^{10}A_s)$</th>
<th>Add Planck</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>[0.02224]</td>
<td>±0.06804</td>
<td>±0.06525</td>
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<tr>
<td>$\Omega_c h^2$</td>
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<td>$h$</td>
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</tr>
<tr>
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<td>...</td>
</tr>
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<td>±0.3850</td>
</tr>
<tr>
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<td>3.39</td>
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<tr>
<td>$b_{HI}$</td>
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<td>±55.57</td>
<td>±1.7</td>
</tr>
</tbody>
</table>

Table 5.2: The parameter constraints from BINGO×DES cross-correlation. From left to right, the first column gives the parameters and their fiducial values from Olivari et al. (2018). The second column gives the uncertainty of each parameter from the BINGO×DES Fisher matrix. The third column gives the uncertainties after fixing $\ln(10^{10}A_s)$. The last column gives the uncertainties after adding the Planck prior to the BINGO×DES Fisher matrix.

We then add the same Planck $w$CDM prior used in Chapt. 4 obtained from CosmoMC into the BINGO×DES Fisher matrix (without fixing $A_s$), following the procedure described in Sect. 4.5.2. The results in this case are shown in the last column of Table 5.2. The Planck prior constrains $\{\Omega_b h^2, \Omega_c h^2, \ln(10^{10}A_s), n_s\}$ and improves the constraints on $\{h, w_0, b_{HI}\}$, which, nevertheless, are still hardly competitive compared to those obtained from IM auto-correlation (see Table 4.2 & 4.3).

Finally, given the limited detection S/N, we fix all other parameters except $b_{HI}$ or $w_0$, and constrain each of them separately. This is to see if the BINGO×DES cross-correlation is useful in constraining dark energy or mass bias, in a scenario that all other parameters are fixed by other dataset, e.g. Planck in the $\Lambda$CDM model. In this case, we obtain a constraint on $b_{HI}$ with $b_{HI} = 1.00 \pm 0.08$, and a constraint on $w$ with $w_0 = -1.00 \pm 0.15$ respectively. These constraints are still worse than those from SKA+Planck or BINGO+Planck (Table 4.2 & 4.3), even though one has fixed all other parameters, which is the best possible scenario.
Based on our analysis, the cross-correlation between HI IM and galaxy lensing field using BINGO and DES does not give competitive results in terms of constraining cosmological parameters, due to a limited \(\approx 13\sigma\) detection of the cross-spectrum. Nevertheless, the BINGO \(\times\) DES cross-correlation might be used to constrain \(b_{\text{HI}}\), given all rest parameters fixed somehow. We will investigate in Sect. 5.5 the possible factors leading to the limited S/N and parameter constraints. Therefore, for the particular survey of BINGO and DES, one might prefer using the auto-correlation of each survey instead of cross-correlation.

5.5 The impact of survey parameters on \(\delta_{\text{IM}} \times \kappa_g\) cross-correlation

In this section, we investigate the effects of several survey parameters, such as \(f_{\text{sky}}\) and noise level, on the detection S/N and parameter constraints for the \(\delta_{\text{IM}} \times \kappa_g\) cross-correlation.

5.5.1 The impact of number of frequency channels

We investigate the impact of number of frequency channels on the \(\delta_{\text{IM}} \times \kappa_g\) cross-correlation by varying the number of frequency channels in the range of \([3, 10, 20, 30]\), and calculating the cosmological parameter constraints in each case with the Planck prior. As explained in Sect. 4.5.3, the parameter uncertainties are expected to decrease with increased number of frequency channels, due to a finer resolution in redshift space, and flatten after a turning point where increased thermal noise and shot noise due to narrow channel width surpass the signal.

Our results are shown in Fig. 5.8, where the fractional uncertainty of each parameter is plotted as a function of number of frequency channels. The fractional uncertainty is defined as the uncertainty \(\delta A\) of cosmological parameter \(A\) calculated with a certain number of frequency channels, divided by the corresponding uncertainty.
\( \delta_{\text{BINGO} \times \text{DES}} \) from BINGO \times \text{DES} with the fiducial \( N_{\text{bin}} = 30 \).

From Fig. 5.8, the constraints on \( \{h, w_0, b_{\text{HI}}\} \) are improved with more frequency bins as expected, although not significantly. From \( N_{\text{bin}} = 3 \) to \( N_{\text{bin}} = 30 \), one only obtains a \( \sim 9\% \), \( \sim 8\% \) and \( \sim 6\% \) improvement on \( h \), \( w_0 \), and \( b_{\text{HI}} \) respectively. This is because that the lensing part of the cross-spectrum (see Equ. 5.7) is integrated over all redshifts, independent of frequency channel width, so that the cross-spectrum is less sensitive to channel width (see Fig. 5.6). In order to reach the turning point, one however needs to further increase the number of frequency channels than what is used here. From Fig. 5.8, the constraints on \( \{\Omega_b h^2, \Omega_c h^2, \ln(10^{10} A_s), n_s\} \) are independent of \( N_{\text{bin}} \). This is because they are almost completely constrained by the Planck prior as discussed in Sect. 5.4.2. In summary, the number of frequency channels does not have a big impact on the parameter constraints for \( \delta_{\text{IM}} \times \kappa_R \).
5.5. IMPACT OF SURVEY PARAMETERS

5.5.2 The impact of noise

We investigate the impact of survey noise on the $\delta_{\ell M} \times \kappa_g$ cross-correlation by reducing BINGO and DES noise respectively, and re-calculating the detection S/N and cosmological parameter constraints in each case. It is expected that the uncertainties of parameters will decrease with decreased survey noise, due to a higher detection S/N.

We first reduce the BINGO noise to [80%, 60%, 40%, 20%, 0%] of its original level, while keeping DES noise unchanged. Note that we also keep all other observing parameters unchanged, i.e., $N_{\text{bin}} = 30$, $A_{\text{cross}} = 800 \text{deg}^2$ and $z_0 = 0.7$. The total detection S/N as a function of the BINGO noise factor is shown in the left panel of Fig. 5.9, where the noise factor is multiplied to the original BINGO noise power spectrum $N^\text{HI}$ (Equ. 4.6) to reduce the amplitude. The S/N indeed improves with decreased BINGO noise as expected. However, it is worth noting that even with no BINGO noise, the total detection S/N is merely improved to $\approx 16.7\sigma$, compared to $\approx 13\sigma$ in the original BINGO×DES cross-correlation. This is just a $\sim 20\%$ improvement on the total detection S/N.

![Figure 5.9: The total S/N (left) and fractional cosmological parameter constraints (right) from BINGO×DES Fisher matrix with the Planck prior, as a function of the input BINGO noise level. For both panels, the x-axis gives the BINGO noise factor such that the input BINGO noise is given by the original BINGO noise multiplied by the noise factor. The y-axis in the left panel gives the total S/N summing over all frequency channels and multipoles. The y-axis in the right panel gives the fractional uncertainty of each cosmological parameter relative to the original input BINGO noise case.](image-url)
We calculate the fractional uncertainty of each cosmological parameter as a function of the BINGO noise factor in the right panel of Fig. 5.9. The fractional uncertainty in this case is defined as the uncertainty $\delta A$ of parameter $A$ at a given noise factor, divided by the uncertainty $\delta A_{\text{BINGO} \times \text{DES}}$ at the original BINGO noise level. All constraints have the Planck prior added in. As expected, uncertainties on $\{h, w_0, b_{\text{HI}}\}$ decrease with decreased BINGO noise but however are limited to a less than 20% improvement, when completely switching off the BINGO noise. The improvement on these parameter constraints is consistent with the $\sim 20\%$ improvement on the total detection S/N. The constraints on $\{\Omega_b h^2, \Omega_c h^2, \ln(10^{10} A_s), n_s\}$ are independent of the BINGO noise factor since they are completely constrained by the Planck prior. Based on our analysis, a reduced IM noise does not improve the $\delta IM \times \kappa_g$ cross-correlation significantly, in terms of the detection S/N and cosmological parameter constraints. The reason is that as can be seen in Fig. 5.2, the BINGO thermal noise is already $\sim 2-3$ orders of magnitude weaker than the HI signal, leaving limited space of improvement with a reduced thermal noise.

![Figure 5.10: The total S/N (left) and fractional cosmological parameter constraints (right) from BINGO $\times$ DES Fisher matrix with the Planck prior, as a function of the input DES lensing shot noise level. For both panels, the x-axis gives the DES lensing shot noise factor such that the input DES shot noise is given by the original shot noise multiplied by the noise factor. The y-axis in the left panel gives the total S/N summing over all frequency channels and multipoles. The y-axis in the right panel gives the fractional uncertainty of each cosmological parameter relative to the original input DES noise case.](image)

We repeat the same analysis to study the impact of DES noise on the total detection
5.5. IMPACT OF SURVEY PARAMETERS

S/N and cosmological parameter constraints, while keeping the BINGO noise to its original level. The results are shown in Fig. 5.10. As expected, both the detection S/N and parameter uncertainties are improved with decreased DES noise. From the left panel, the total detection S/N is improved by a factor of 2 when completely switching off the DES shot noise. From the right panel, an up to \( \sim 35\% \) improvement is achieved on the constraints of \( \{ h, w_0, b_{\text{HI}} \} \) without the DES shot noise. Compared with the IM noise, the reduction of DES shot noise is more important to obtain a better detection S/N of the \( \delta M \times \kappa_g \) cross-spectrum and a tighter cosmological parameter constraint. The reason is that as seen in Fig. 5.5, the DES shot noise significantly restricts the detection S/N of the lensing signal. As a result, the cross-correlation signal will be more sensitive to the reduction of DES noise than BINGO noise.

5.5.3 The impact of sky coverage

In order to study the impact of sky coverage on the \( \delta M \times \kappa_g \) cross-correlation, we artificially change the \( f_{\text{sky}} \) factor in the Fisher matrix (Equ. 4.14) while keeping all other observing parameters unchanged as the original BINGO and DES setup. Note that in reality, when the observed sky area is increased, the noise level might be increased due to a reduced integration time per pixel given a fixed total observing time. In our analysis, we however keep the noise level unchanged while varying \( f_{\text{sky}} \), in order to isolate the effect of \( f_{\text{sky}} \) from other factors.

The original BINGO and DES setup has an overlapping sky coverage of \( \approx 800 \text{deg}^2 \), corresponding to \( f_{\text{sky}} \approx 2\% \). In our analysis, we vary \( f_{\text{sky}} \) among \([0.02, 0.2, 0.4, 0.6, 0.8, 1] \), where \( f_{\text{sky}} = 1 \) corresponds to two full-sky surveys. It is expected that a larger overlapping sky area will improve the detection S/N and parameter constraints due to reduced cosmic variance.

The total detection S/N as a function of \( f_{\text{sky}} \) is shown in the left panel of Fig. 5.11. As expected, a full sky coverage can improve the S/N by a factor of \( \sim 9 \), indicating that \( f_{\text{sky}} \) is crucial to improving the cross-spectrum detection.
In the right panel of Fig. 5.11, the fractional uncertainty of cosmological parameters are plotted against $f_{\text{sky}}$. The fractional uncertainty in this case is defined as the uncertainty $\delta A$ of parameter $A$ at a given $f_{\text{sky}}$ value, divided by the uncertainty $\delta A_{\text{BINGO} \times \text{DES}}$ at the original $f_{\text{sky}} \approx 2\%$. From $f_{\text{sky}} = 2\%$ to $f_{\text{sky}} = 1$, an $\sim 80\%$, $\sim 70\%$ and $\sim 40\%$ improvement is achieved on the constraint of $h$, $w_0$, and $b_{\text{HI}}$ respectively, with other parameters barely improved due to being strongly constrained by the Planck prior.

Based on Fig. 5.11, the overlapping sky coverage area of the two surveys is crucial to a decent detection S/N of the cross-spectrum, and thus key to a better cosmological parameter constraint. Compared with the limited improvement from a decreased survey noise given in Sect. 5.5.2, seeking for two surveys that have a larger overlapping sky coverage is a more immediate approach to a better cross-correlation result.

### 5.5.4 The impact of redshift coverage

We vary the DES medium redshift $z_0$ in Eq. 5.1 and Eq. 5.7, to study the impact of two surveys’ redshift coverage on the cross-spectrum detection and cosmological parameter constraints. The values of $z_0$ are varied in the range of [0.1, 0.3, 0.5, 0.7]
while all other observing parameters are kept to the original BINGO and DES setup. It is expected that a medium redshift at $z_0 = 0.3$ will give the maximum S/N detection and the optimal parameter constraints, since this is where the two surveys have the same medium redshift and thus the maximum overlap in redshift.

Our results are shown in Fig. 5.12. In the left panel, the total detection S/N increases with $z_0$ so that the original DES medium redshift $z_0 = 0.7$ gives the largest detection S/N. This is inconsistent with our expectation where a peak of S/N at $z_0 = 3$ is expected. Our results however can be understood from Fig. 5.4 where a larger $z_0$ gives a higher and wider lensing kernel, leading to a stronger lensing signal to be correlated with HI signal. This larger lensing window at a higher redshift, say, $z_0 = 0.7$, actually gives a stronger cross-correlation signal with a higher detection S/N.

![Figure 5.12: The total S/N and parameter constraints from BINGO×DES Fisher matrix with the Planck prior, as a function of the DES median redshift. For both panels, the x-axis gives the DES median redshift $z_0$. The y-axis in the left panel gives the total S/N summing over all frequency channels and multipoles. The y-axis in the right panel gives the fractional uncertainty of each cosmological parameter relative to that from the original DES medium redshift $z_0 = 0.7$.](image)

In the right panel of Fig. 5.12, the fractional uncertainty of cosmological parameters are plotted against $z_0$. The fractional uncertainty in this case is defined as the uncertainty $\delta A$ of parameter $A$ at a given $z_0$ value, divided by the uncertainty $\delta A_{\text{BINGO×DES}}$ at the original DES medium redshift $z_0 = 0.7$. It can be seen that $z_0 = 0.7$ gives the tightest parameter constraints, consistent with the detection S/N in the left panel.

However, we must warn that as mentioned in Sect. 5.3.2, the way we shift the
galaxy survey medium redshift \( z_0 \) might not be realistic, where we keep using exactly the same galaxy source function as in Equ. 5.4, except a different \( z_0 \) each time. The continuous improvement on S/N and parameter uncertainties towards a higher \( z_0 \) might be due to the lensing kernel not being appropriate for other redshifts than \( z_0 = 0.7 \). The expected detection S/N and cosmological parameter constraints should peak at \( z_0 = 0.3 \), where both surveys have the same medium redshift and a maximum overlap in redshift space.

5.6 The very ideal case

In this section, we consider the very ideal case of \( \delta_{\text{IM}} \times \kappa_g \), where the two surveys have no noise and a full-sky coverage. We use a medium redshift of \( z_0 = 0.7 \) as it gives the highest S/N in our analysis. We use \( N_{\text{bin}} = 30 \) since we have shown in Sect. 5.5.1 that the parameter constraints do not improve much with increased number of frequency channels. We calculate the total detection S/N and constrain cosmological parameters in this very ideal case, in order to understand the optimal results one can possibly achieve from \( \delta_{\text{IM}} \times \kappa_g \) cross-correlation.

5.6.1 The detection S/N

We repeat the analysis described in Sect. 5.4.1 to compute the detection S/N in the ideal case. The results are shown in Fig. 5.13. The upper panel shows the cross-spectrum at \( z = 0.13 \) as an example. The spectrum is binned with \( \Delta \ell = 50 \). The shaded area shows the 1\( \sigma \) uncertainty of the cross-spectrum calculated from Equ. 5.8. The lower panel gives the S/N of each multipole bin, summing up all redshift bins. It can be seen that large scales at \( \ell \lesssim 50 \) give a larger uncertainty and thus a smaller S/N, due to cosmic variance. Unlike in Fig. 5.7 where the S/N decreases at beam scale, since there is no noise in the two ideal surveys, the S/N keeps increasing towards small scales.

The total detection S/N after adding up all multipole and redshift bins is \( \approx 308 \sigma \).
5.6. THE VERY IDEAL CASE

Figure 5.13: The S/N of the cross-spectrum between HI IM and galaxy lensing as a function of multipole in an ideal case, with no survey noise and $f_{\text{sky}} = 1$. The upper panel shows the cross-spectrum at one redshift bin of $z = 0.13$, $\Delta \nu = 10 \text{ MHz}$ and $z_0 = 0.7$. The multipoles are binned with $\Delta \ell = 50$. The shaded area shows the uncertainty of the cross-spectrum of that particular redshift bin. The lower panel gives the total S/N as a function of multipole by adding up all redshift bins.

Compared with the $\approx 13\sigma$ detection for the BINGO×DES cross-correlation, the ideal case improves the detection S/N by a factor of $\sim 24$. However, this $\approx 308\sigma$ detection is still less than the $\approx 351\sigma$ detection from SKA1-MID Band 2 auto-correlation, and is only slightly higher than the $\approx 220\sigma$ detection from BINGO auto-correlation. Given that the two surveys used in this cross-correlation are optimal with no noise and a full-sky coverage, which are unlikely to be realistic, the cross-correlation of HI intensity with galaxy lensing field does not seem to be a wise choice in terms of signal detection.

5.6.2 The cosmological parameter constraints

The cosmological parameter constraints from the ideal cross-correlation are shown in the second column of Table 5.3. It can be seen that one cannot obtain constraints on $A_s$ and $b_\text{HI}$ due to their degeneracy, as discussed in Sect. 5.4.2.

We then fix $A_s$, assuming a perfect knowledge of it. The constraints in this case are
shown in the third column of Table 5.3. Compared with BINGO×DES (Table 5.2), the parameter constraints are improved by a factor of \(\sim 27\) on average, consistent with the factor of \(\sim 24\) improvement on the total detection S/N.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HI×κ</th>
<th>Fix (\ln(10^{10}A_s))</th>
<th>Add Planck</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Omega_b h^2) [0.02224]</td>
<td>±0.00337</td>
<td>±0.00324</td>
<td>±0.00013</td>
</tr>
<tr>
<td>(\Omega_c h^2) [0.1198]</td>
<td>±0.0050</td>
<td>±0.0050</td>
<td>±0.0011</td>
</tr>
<tr>
<td>(h) [0.6727]</td>
<td>±0.0232</td>
<td>±0.0227</td>
<td>±0.0086</td>
</tr>
<tr>
<td>(\ln(10^{10}A_s)) [3.096]</td>
<td>±3.302</td>
<td>—</td>
<td>±0.039</td>
</tr>
<tr>
<td>(n_s) [0.9641]</td>
<td>±0.0131</td>
<td>±0.0131</td>
<td>±0.0035</td>
</tr>
<tr>
<td>(w_0) [-1.00]</td>
<td>±0.13</td>
<td>0.13</td>
<td>±0.06</td>
</tr>
<tr>
<td>(b_{HI}) [1.00]</td>
<td>±3.20</td>
<td>±0.06</td>
<td>±0.05</td>
</tr>
</tbody>
</table>

Table 5.3: The parameter constraints from an ideal HI×κ cross-correlation. From left to right, the first column gives the parameters and their fiducial values from Olivari et al. (2018). The second column gives the uncertainty of each parameter from the ideal HI×κ Fisher matrix. The third column gives the uncertainties after fixing \(\ln(10^{10}A_s)\). The last column gives the uncertainties after adding the Planck prior.

We add the Planck prior to the ideal cross-correlation Fisher matrix and the results are shown in the last column of Table 5.3. Same as in the BINGO×DES case, \(\{\Omega_b h^2, \Omega_c h^2, \ln(10^{10}A_s), n_s\}\) are completely constrained by the Planck prior. The constraints on \(\{h, w_0, b_{HI}\}\) are improved by a factor of \(\sim 6\) on average, compared to BINGO×DES. However, compared with the constraints from SKA+Planck (Table 4.2), the constraints from \(\delta_{IM} \times \kappa_g\) cross-correlation with two ideal surveys are less competitive, but comparable with the constraints from BINGO+Planck (Table 4.3). Given that SKA and BINGO are much more realistic than the two ideal surveys assumed in this section, it is better to use HI IM auto-correlation than \(\delta_{IM} \times \kappa_g\) cross-correlation to constrain cosmological parameters.

Finally, we fix all other parameters but \(b_{HI}\) and \(w_0\) respectively, to consider the scenario where all other parameters are constrained by other dataset, and see how much \(\delta_{IM} \times \kappa_g\) cross-correlation can contribute to the constraints on dark energy or mass bias. In this case, the constraints on \(b_{HI}\) and \(w_0\) are \(b_{HI} = 1.000 \pm 0.004\) and \(w_0 = -1.000 \pm 0.007\) respectively. However, we have to emphasize that these tight constraints are unlikely to be achieved in reality, since it requires two ideal surveys.
5.7. CONCLUSIONS AND DISCUSSIONS

for cross-correlation, and all other parameter fixed by other dataset. Nevertheless, the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation might be useful when combined with other dataset to help constraining dark energy and mass bias.

5.7 Conclusions and discussions

In this chapter, we cross-correlate HI intensity maps with galaxy lensing field to study its ability of detecting cross-correlation signal, and constraining cosmological parameters. We first forecast the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation using BINGO and DES setup, and then study the impact of survey parameters on cross-correlation results. Finally, we forecast the performance of the optimal $\delta_{\text{IM}} \times \kappa_g$ cross-correlation with two ideal surveys.

The BINGO×DES cross-correlation gives a $\approx 13\sigma$ detection of the cross-spectrum, and does not give competitive $w_{\text{CDM}}$ parameter constraints, even with the Planck prior, compared to SKA1-MID Band 2 or BINGO auto-correlation. With only $\approx 13\sigma$ detection, it is able to constrain merely one cosmological parameter each time. We managed to obtain $b_{\text{HI}} = 1 \pm 0.08$ and $w_0 = -1 \pm 0.15$ respectively, by fixing all other parameters, which, nevertheless, are still worse than the constraints from BINGO auto-correlation + Planck.

We investigate the impact of number of frequency channels, survey noise, sky coverage, and redshift coverage on the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation. It is found that the cross-correlation results are not sensitive to number of frequency channels, due to the lensing part integrating over all redshifts. The noise of two surveys should be reduced if possible, in order to improve the cross-correlation results. A large overlapping sky coverage between the two surveys is critical to obtain a decent detection of the cross-correlation signal and reasonable constraints of cosmological parameters. From the galaxy source function we adopted from DES observation (LSST Science Collaboration, 2009), a higher galaxy survey medium redshift $z_0$ is favored, regardless of the IM
survey medium redshift, due to a larger lensing kernel at a larger $z_0$, which increases
the cross-correlation signal as a result. However, we must warn that in reality, the exact
galaxy source function will change with survey medium redshift, so that the lensing
kernel is unlikely to keep increasing continuously with an increased $z_0$. One might
expect the strongest cross-correlation signal when the IM survey and the galaxy sur-
vey have the same medium redshift and thus the maximum overlap in redshift space.
As a future work, one should adopt an appropriate galaxy source function to properly
calculate the lensing kernel at different redshifts.

We consider the optimal $\delta_{\text{IM}} \times \kappa_g$ cross-correlation with two ideal surveys that have
no noise and a full-sky coverage, at a medium galaxy survey redshift $z_0 = 0.7$. The
ideal case detects the cross-spectrum with a total S/N of $\approx 308\sigma$, $\sim 24$ times higher
than that from BINGO×DES cross-correlation. With the Planck prior, the constraints
on $\{h, w_0, b_{\text{HI}}\}$ are improved by a factor of $\sim 6$ on average compared to BINGO×DES.
Nevertheless, the constraints from the ideal $\delta_{\text{IM}} \times \kappa_g$ cross-correlation are still less
competitive than SKA+Planck, and are comparable to BINGO+Planck. Given that the
two ideal surveys assumed in our optimal $\delta_{\text{IM}} \times \kappa_g$ cross-correlation are unrealistic
while BINGO and SKA are proposed real experiments under construction, the cross-
correlation of HI intensity maps with galaxy lensing field is not useful in terms of
signal detection and cosmological parameter constraints.

The reason that $\delta_{\text{IM}} \times \kappa_g$ cross-correlation is less competitive compared to IM auto-
correlation is due to the low S/N of galaxy lensing, which will increase the cross-
spectrum uncertainty in contrast to IM auto-spectrum. Besides, the galaxy lensing
field loses its redshift resolution by integrating over all redshifts, which reduces the
cross-correlation coefficient between HI IM and galaxy lensing. In comparison, galaxy
density field has a stronger S/N and better redshift resolution than lensing field. Pourt-
sidou et al. (2016) cross-correlated MeerKAT IM and DES density field to calculate a
detection S/N of $\sim 100\sigma$, compared to a S/N of $\sim 10\sigma$ with DES lensing field. Their
results together with ours indicate that the detection S/N and redshift information of
5.7. CONCLUSIONS AND DISCUSSIONS

galaxy surveys are important in the cross-correlation with HI IM. However, as mentioned before, one should adopt a proper galaxy source function in the future, in order to obtain more accurate forecasts on the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation, given a maximum redshift coverage of two ideal surveys.

Nevertheless, in combination with other dataset, the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation might be useful to help constraining dark energy and mass bias, given other parameters fixed or tightly constrained. The optimal $\delta_{\text{IM}} \times \kappa_g$ cross-correlation gives a constraint of $b_{\text{HI}} = 1.000 \pm 0.004$ and $w = -1.000 \pm 0.007$ respectively, by fixing all other parameters. The $\delta_{\text{IM}} \times \kappa_g$ cross-correlation might also be useful to be combined with, e.g., HI IM and galaxy density field cross-correlation, to separate the HI bias and galaxy bias.
Chapter 6

Conclusions and Future Work

In this chapter, we summarise the author’s work in this thesis and discuss the potential future work.

6.1 Conclusions

We have come to the era of precision cosmology where the CMB measurement from the Planck satellite gives incredibly tight constraints on ΛCDM cosmological parameters. However, due to the high redshift of CMB emission, it does not provide much information on dark energy, which dominates at a much lower redshift. Therefore, it is useful to probe the Universe with lower redshift surveys, to study the dark energy component. In addition, low redshift surveys provide a cross-check to the CMB measurement as an independent probe, and the complementary information brought by it will break degeneracies inherent in the CMB measurement.

HI intensity mapping is a relatively cheap and efficient technique to measure the large-scale-structures, such as BAOs, through the detection of the redshifted HI line. It provides 3-D information of aggregated emission from multiple galaxies without detecting individual galaxies. The 3-D mapping of large-scale-structures can reveal the expansion history of the Universe, and provide information on constraining dark energy. Many IM experiments have been proposed in the past few years, such as
BINGO and SKA, with the first detection made by the GBT team Chang et al. (2010).

However, due to the faintness of HI signal, HI IM has to face lots of challenges. Two of the major challenges are the effective removal of Galactic foregrounds, and the mitigation of systematics. The former requires component separation to recover HI signal from foregrounds, and the latter requires a careful design of instrumentation, calibration and effective algorithm to minimise its effect.

One way to mitigate systematics is by cross-correlating two tracers, where the two surveys trace the same underlying density field but have uncorrelated instrumental noise and systematics. During cross-correlation, the cross-correlated signal will stand out while the uncorrelated noise or systematics will cancel out, leaving a non-biased measurement of the cross-spectrum. In terms of astrophysics and cosmology, cross-correlation can also bring information about the calibrated mass bias.

Through the cross-correlation of Planck 2015 NILC CMB maps and SDSS galaxy samples, we detect the thermal SZ cluster residuals in the NILC CMB map with $\approx 30\sigma$ significance at cluster scale ($\ell \sim 2000$), and overall $\approx 51\sigma$ significance including large scales. The thermal SZ cluster residuals are visually detected on the stacked NILC CMB map, and at locations of individual clusters. The percentage of thermal SZ emission that has been left over in the Planck CMB map is quantified to be $44\pm4\%$ through aperture photometry. Although the impact of thermal SZ cluster residuals on the ISW measurement is proved to be negligible, we must warn that one can no longer ignore the SZ cluster residuals in CMB maps, especially for upcoming CMB experiments with a higher resolution. In those cases, the thermal SZ cluster residuals will impact at even smaller scales ($\ell > 3000$), which can contribute a negative anti-correlation signal during CMB and LSS cross-correlation analysis. In contrast, we provide an alternative CMB map, produced from 2D-ILC component separation technique, that has been shown to be free from thermal SZ contamination.

One systematic error for IM experiment is $1/f$ noise. In order to understand the impact of $1/f$ noise on cosmological parameter constrains in IM experiments, we forecast
the performance of SKA1-MID Band 2 as an IM experiment, to constrain cosmological parameters, with and without $1/f$ noise. Using Fisher analysis, we obtain constraints of $w_0 = -1 \pm 0.06$, $w_a = 0 \pm 0.13$ and $b_{\text{HI}} = 1 \pm 0.02$, for SKA+Planck without $1/f$ noise.

Using the semi-empirical $1/f$ noise model from Harper et al. (2018), we study the impact of $1/f$ noise on cosmological parameters. With a representative baseline $1/f$ noise, the constraints of cosmological parameters are degraded on average by $\sim 50\%$, compared to no $1/f$ noise. It is found that the $1/f$ noise spectral index $\alpha$ has a major impact on parameter constraints. Therefore, in order to reduce the impact of $1/f$ noise, $\alpha$ should be reduced to the minimum if possible. In addition, a smaller knee frequency $f_{\text{knee}} < 0.1$ Hz, and a larger telescope velocity $v_t \rightarrow 2$ deg/s are desired. A more correlated $1/f$ noise along frequency ($\beta \rightarrow 0$) is also favoured to minimise the $1/f$ noise impact.

Given this $\sim 50\%$ degradation on parameter constraints, $1/f$ noise does not seem to be too problematic for cosmologists. However, one must bare in mind that $1/f$ noise can increase the absolute noise level by $\sim 2$ orders of magnitude, and have a larger impact on higher redshift and larger angular scales. In practice, an instrumental design can potentially have a worse $1/f$ noise than the default model analysed in this work, which will further degrade the results. One therefore cannot neglect the impact of $1/f$ noise on IM experiments, and should seek for potential mitigation.

The cross-correlation between HI intensity map and galaxy lensing field is shown to be less competitive than auto-correlation of IM, in terms of signal detection and cosmological parameter constraints. With BINGO and DES setup, the total detection S/N is calculated to be merely $\approx 13\sigma$. An increased overlapping sky coverage is more efficient in improving the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation, compared to a decreased noise level of individual surveys. However, even with two ideal surveys of no noise and a full-sky coverage, the total detection S/N and parameter constraints from $\delta_{\text{IM}} \times \kappa_g$ are still surpassed by those from SKA IM auto-correlation + Planck. The reason might be
attributed to the loss of redshift information of the galaxy lensing field, which reduces the cross-correlation coefficient. Therefore, one might prefer using IM auto-correlation than $\delta_{\text{IM}} \times \kappa_{g}$ for signal detection and parameter constraints. Alternatively, the cross-correlation between HI IM maps with galaxy density field, rather than lensing field, might be a better option for cross-correlation (e.g., Pourtsidou et al., 2017). Nevertheless, the advantage that $\delta_{\text{IM}} \times \kappa_{g}$ does not introduce galaxy bias might be useful when combined with other dataset, for a calibrated mass bias measurement.

6.2 Future work

In the near future, the author plans to extend the CMB $\times$ LSS cross-correlation work to the CMB lensing potential field. In this case, the lensing potential from the NILC CMB map and 2D-ILC map will be constructed and cross-correlated with LSS optical survey respectively. One can then quantify the amount of SZ cluster residuals left in the contaminated CMB lensing potential through cross-correlation, and study its effects.

In Chapt. 4, we quantify the impact of $1/f$ noise on cosmological parameter constraints through Fisher matrix. As mentioned in Sect. 4.7, the main defect of our analysis is that our approach of constraining $H(z)$ and $D_A(z)$ are limited to three redshift bins only. To improve this, one can: i) Adopt a more complicated parametrisation of $w(a)$ which allows more redshift bins during parameter transformation; ii) Instead of using parameter transformation, directly constrain $H(z)$ and $D_A(z)$ by analytically calculate the partial derivatives of HI power spectrum with respect to $H(z)$ and $D_A(z)$. This approach, however, is not obviously feasible, since it involves partial derivatives of Bessel functions; iii) Instead of using 2-D angular power spectrum $C_{\ell}^{\text{HI}}(z_i, z_j)$ and $F_{\ell}$, one can use 3-D power spectrum $P_{\text{HI}}(k)$ and $P^{1/f}(k)$. This will avoid the partial derivative of Bessel function addressed in point ii).

It is worth extending the work to end-to-end simulations for a more realistic forecast. One can use the Manchester intensity mapping pipeline to simulate HI maps with
and without $1/f$ noise, calculate the power spectra from simulated maps and constrain cosmological parameters from there. With the end-to-end simulation, one can also test the ability of component separation, such as the GNILC method, on removing $1/f$ noise and thus minimising its impact.

It is interesting to measure $1/f$ noise in laboratory, in order to investigate if $1/f$ noise is indeed correlated between frequency channels, i.e., to measure the $1/f$ noise correlation parameter $\beta$. In order to mitigate $1/f$ noise, one can test receiver components in the lab, and seek for mitigation methods on the technical side.

In terms of cosmology, the test of General Relativity (GR) model requires a precise measurement of power spectrum so that one can compare the measured power spectrum to the predictions from GR theory or other models for comparison. However, since $1/f$ noise can potentially increase the absolute noise level by a factor of $\sim 100$, it could result in large inaccuracies for GR testing, where one cares about differences between models. Therefore, one potential future work is to forecast the impact of $1/f$ noise on GR testing.

As mentioned in Sect. 5.7, our cross-correlation analysis of $\delta_{\text{IM}} \times \kappa_g$ does not take into account the modification of the galaxy source function, after shifting the galaxy survey medium redshift from $z_0 = 0.7$ to lower redshift. We obtain a continuously increasing cross-correlation signal with increased $z_0$, which is unlikely to be the case in reality. Given this result, it appears that our galaxy source function might be inappropriate for low redshift. Therefore, in the future, it is worth adopting a proper galaxy source function for low redshift, in order to obtain a more accurate forecast and investigate the potential improvement on the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation by increasing the overlapping redshift coverage of the two surveys.

One could potentially extend the $\delta_{\text{IM}} \times \kappa_g$ cross-correlation work into simulation, where a simulated HI map and a galaxy lensing map is generated respectively based on the same underlying dark matter distribution, and are cross-correlated with each other. With the simulation, we can testify the conclusion from the Fisher matrix analysis and
the possible usage of $\delta_{IM} \times \kappa_{g}$ cross-correlation. It will also be useful to simulate the cross-correlation between HI maps and galaxy density field. From there, one can add constraints obtained from $\delta_{IM} \times \kappa_{g}$ to break the $b_{HI} - b_{g}$ degeneracy, in the HI IM×galaxy density cross-correlation.
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