Fluid-Structure Interaction for a Cantilever Rod in Axial Flow: An Experimental Study

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<th>Description</th>
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<tbody>
<tr>
<td>BWR</td>
<td>Boiling Water Reactor</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FIV</td>
<td>Flow-Induced Vibration</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid-Structure Interaction</td>
</tr>
<tr>
<td>GTRF</td>
<td>Grid-To-Rod Fretting</td>
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<tr>
<td>PSD</td>
<td>Power Spectrum Density</td>
</tr>
<tr>
<td>PWR</td>
<td>Pressurized Water Reactor</td>
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<td>RMS</td>
<td>Root-Mean-Squared</td>
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<tr>
<td>$A$</td>
<td>rod cross-sectional area</td>
</tr>
<tr>
<td>$C_m$</td>
<td>added mass coefficient</td>
</tr>
<tr>
<td>$D$</td>
<td>rod outer diameter</td>
</tr>
<tr>
<td>$D_h$</td>
<td>hydraulic diameter</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$f$</td>
<td>ordinary frequency</td>
</tr>
<tr>
<td>$f_D$</td>
<td>Darcy friction factor</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$I$</td>
<td>second moment of area</td>
</tr>
<tr>
<td>$L$</td>
<td>rod length</td>
</tr>
<tr>
<td>$m$</td>
<td>rod mass per unit length</td>
</tr>
<tr>
<td>$M$</td>
<td>added mass per unit length</td>
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<tr>
<td>$\Delta P$</td>
<td>pressure loss</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$T$</td>
<td>axial tension</td>
</tr>
<tr>
<td>$U$</td>
<td>flow velocity</td>
</tr>
<tr>
<td>$w$</td>
<td>angular frequency</td>
</tr>
<tr>
<td>$y$</td>
<td>rod displacement</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>damping ratio</td>
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\( \mu \) fluid dynamic viscosity
\( \nu \) fluid kinematic viscosity
\( \rho \) fluid density
\( \rho_r \) rod density
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Abstract

The phenomenon of fluid-structure interaction is present in many industrial applications, such as bridge cables, transmission wires, drilling risers in petroleum production, biomedical engineering, etc. In the nuclear industry, fluid-structure interactions have also been identified, one form of which is Flow-Induced Vibrations (FIVs) of the fuel rods in a typical pressurized nuclear water reactor core. As a result, the vibrating fuel rods may have contact with the neighbouring structures, such as spacer grids which are technically a structure for preventing the fuel rods from excessive movement, and concurrently initializing fretting on the fuel rod surfaces, called grid-to-rod fretting. Thus, for safety concerns, the characteristics of the flow-induced rod vibrations in such a system are required to be understood, and accordingly monitoring the wear-through failure using a prediction tool.

In the present study, an experimental approach is proposed to advance our understanding of these phenomena and extend the range of available correlations. In the present experiment, a flow-induced structural vibration system has been designed, in which the geometry is prototypical of pressurized water nuclear reactor core and the flow parameters replicate the flow conditions during its full power operation. Following a validation of the methodology, a series of tests on a cantilever rod in pipe flows directed from the rod free end towards the fixed end has been carried out, in which the rod features either a blunt or a tapered free-end shape, and has been filled internally with either air or lead (for mimicking the fuel pellets in a real nuclear fuel rod). Through
analysis of the resulting data, it has been found that the vibrating amplitude of a cantilever rod is more sensitive to the free-end shape of the rod, while the vibrating frequency is mostly influenced by the internal loading material. These findings give an insight into the future design of relevant structures in nuclear reactor cores, the fluid-structure interaction community will also benefit from the availability of such data to fine-tune relevant numerical codes.
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Chapter 1 Fluid-structure interaction in industrial applications

Interactions between fluids and structures are involved in many industrial applications, such as bridge cables, transmission wires, drilling risers in petroleum production, biomedical engineering, etc. One frequently encountered phenomenon in such a Fluid-Structure Interaction (FSI) system is flow-induced vibration (FIV), which has attracted great attention for both safety and economic concerns. A notable failure due to FIV is the collapse of the Tacoma Narrows Bridge in 1940, posing a potential threat to the public safety and an economic loss. Therefore, it is of importance to investigate the FIV mechanisms and accordingly propose counter-measures to mitigate or even avoid potential losses.

In this Chapter, a general description of the issues encountered in FSI systems, particularly in nuclear water reactor cores, and mechanisms of the FIVs in a system with certain flow configurations is briefly given. Conventional approaches to estimating such system behaviours are also discussed. Section 1.1 introduces the accidents/incidents identified in many industrial applications such as bridge cables subjected to wind and drilling risers in petroleum production, as a result of FSIs. Section 1.2 highlights structural failures occurring in nuclear reactor cores in particular pressurized water nuclear reactor cores, as a result of FSIs. Section 1.3 describes the objectives of this PhD study, and Section 1.4 lists the structures of this thesis.
1.1 Fluid-structure interaction: in general

This section introduces the phenomena observed in industrial applications such as bridge cables in a wind and drilling risers in petroleum production, as a result of fluid-structure interactions.

Matsumoto et al. [1] reported that an inclined cable of 187 m in length mounted onto a cable-stayed bridge, during the passing of a typhoon, showed a violent vibration with an amplitude being estimated to be more than 1.5 m. As a result, a part of the edge fairing installed at the bridge girder edges and cable surface were severely damaged, as shown in Figure 1.1. The wind velocity during the passing of the typhoon was estimated to be 18 m/s and the wind blew with certain yawing angle to the bridge axis which was based upon the data measured at a meteorological observatory located at approximately 1 km upstream away from the bridge. It was also observed that rain had already stopped while wind was still present when the cable vibration started, which might be described as a dry-state galloping. Wind FIV is also suspected to be the culprit of damage of an oil damper installed to the stay cables of a bridge in Japan, as shown in Figure 1.2 [2].

![Figure 1.1. Cable vibration observed at a cable-stayed bridge in Japan [1]](image)
Another engineering failure due to wind flow induced vibrations is the breaking of the tension bars of a tower crane in Germany. The tower crane, shown in Figure 1.3, was revealed to have a maximum outreach of 50 m, a load capacity of 20.5 t, and a maximum load capacity of 30 t. The top of the tower was manufactured to be connected to the counterweight jib via tension bars. In March 1995, 100 days after assembly of the new crane, tension bar No. 3 broke as depicted in Figure 1.4. As a result, the counterweights of 6 steel slabs with a total mass of 85 t crashed down from 170 m height through the supporting steel structures, causing severe damage to the concrete structure of the supporting building as in Figure 1.5. Witnesses reported horizontal bending vibrations of the counterweight tension bars when the crane was left out of operation and was
exposed to a high speed wind. Thus, it was proposed that wind flow induced vibrations of the counterweight tension bar may have developed during downtime. The oscillating bending stresses might have initiated a fatigue crack on the tension bar. After the final fracture of the tension bar, the counterweight jib tilted and the supported counterweights started to fall [3]. Several wind flow induced vibrations have also been observed in many other bridges, interested readers may refer to the papers [4]–[7].

Figure 1.3. Tower crane with counterweight jib with tension bars (1;2;3) to the top [3]

Figure 1.4. Broken parts of the right and left tension bars No. 3 as in Figure 1.3 [3]
Wind-induced or Aeolian vibrations of overhead electricity transmission lines are another manifestation of FSI. These vibrations are typically induced by winds ranging from 1 and 8 m/s in velocity and are characterized to occur at frequencies between 5 and 100 Hz, leading frequently to bending fatigue failures of the transmission lines. For controlling the vibrations, the Stockbridge-type damper has been ubiquitously introduced into the lines. A Stockbridge-type damper (as in Figure 1.6) generally consists of two weights rigidly attached to the ends of a steel cable with either 7 or 19 strands, known as a messenger wire, and a clamp for mounting the damper to the transmission line. The damper’s rationale of controlling the unexpected vibrations is illustrated as follows. Due to the relatively high masses of the damper weights and the low stiffness of the supporting damper cable, the damper weights cannot accurately track the motions of the damper.
clamp when the transmitting conductor vibrates, such as in a cross-sectional direction. This differential in motion between the damper clamp and the damper weights results in a bending of the damper cable. This bending is extremely large in amplitude when the damper responds in one of its own natural frequencies. During the process of the bending, each inter-strand of the damper cable relatively slides on its neighbouring strand's surface, thus dissipating unexpected energy through the inter-strand friction. In practical applications, many factors such as the installation location, conductor’s tension, terrain, wind orientation and conductor span length, all play a vital role in the damper’s effectiveness, thus requiring great attention in particular in its design stage [8], [9].

Figure 1.6. Stockbridge-type damper with cross-sectional view [9]

Wind-induced vibration under precipitation generally called rain-wind induced vibration, has also been observed at many cable-stayed bridges all around the globe. One important feature of the rain-wind induced vibration is the occurrence of a larger vibrating amplitude, compared with the wind-alone induced vibration. Such a phenomenon was initially observed during the construction period of the three-span Meikonishi Bridge of 176.5-405-176.5 m in length, which is located at the inlet of Nagoya Harbor in Japan. During the construction, the observed vibration of a cable reached a peak-to-peak
amplitude of 55 cm under 14 m/s wind. Hence, to monitor this bridge’s health and to find causes of the vibrations, a full scale field measurement was carried out for a five-month period. The measurement involved 24 cables of the south-side plane extending from the east-side pylon. These cable lengths varied from 65 to 200 m, and each cable was made up of bundles of parallel wires using a polyethylene pipe and cement grout for corrosion protection. The typical outer diameter of these cables was 140 mm, and the mass per unit length was 37 kg/m before grouting and 51 kg/m after grouting. Figure 1.7 shows a 10-hour record of the vibration amplitude and the weather conditions including wind speed and directions. Notably, although the wind speed and direction were almost the same, the vibrations only occurred during the precipitation periods. Thus, it can be implied that the vibrations were caused by the combined influence of wind and rain [10]. A further study on the rain-wind induced vibrations of this bridge found that the instabilities are characterized by a lower frequency and by a higher amplitude than the vortex-induced vibrations. Many studies of the rain-wind induced vibrations have also been carried out through field measurements, interested readers may refer to the literature [9]–[12].
FSIs have also been witnessed in many deepwater applications. For oil and natural gas exploration systems deployed in deep waters, the use of steel catenary risers becomes a potential solution based on the fact that it is technically feasible and cost effective. However, as ocean water flows by, the risers tend to vibrate in the in-line and cross-flow directions, called vortex induced vibrations. The vibrations were found to not only contribute to the riser fatigue damage [15], but also cause extensive damage to the riser touchdown zone due to the riser-soil interactions [16]. Wang et al. [17] studied the fatigue damage of a steel catenary riser immersed in still water, as the riser’s top end experienced a two-dimensional in-plane motion. The prototype steel catenary riser featured 4100 m in length, 0.4 m in outer diameter, and operating at a water depth of
1500 m. As limitations of the test facility, the prototype riser was truncated down based on the Froude similarity law, where the riser total length was scaled down to 23.7 m and the water depth was 9 m, as shown in Figure 1.8. This truncated model was pinned at the bottom end and was connected to a motion controlled track at the top end. The motion apparatus was installed with a force transducer for monitoring the motions in the vertical and horizontal directions. For measuring the vibration responses, 25 evenly distributed locations along the riser span were installed with strain sensors. At each measuring location, four sensors at 90° interval were deployed around the circumference of the riser cross section, as in Figure 1.8, two of which were for the in-plane motion measurement, and the other two of which were for the out-of-plane motion measurement. When the force transducer exerts a force on the riser, the whole span of the riser is induced to vibrate which is being monitored by the strain sensors. Figure 1.9 shows the fatigue damage distribution with 3 force periods when the motion amplitude is 0.105 m, in which the horizontal axis is the measuring location number from the bottom to the top, and the vertical axis is the fatigue damage in units of 1/year. The experimental test shows that the maximum fatigue damage of the riser occurs near its touchdown point (enclosed in the blue rectangle). Thus, a countermeasure against the fatigue damage near the fragile touchdown point of a steel catenary riser is required during its design stage. A similar study to the response of a curved circular cylinder immersed in a water flow has also been conducted by Assi et al. [18], where the cylinder was in either a concave or a convex configuration.
In rotating machines, cracks due to FSI, known as sub-synchronous vibration (or named as “1/2X vibration”) at a frequency at just below one-half of the rotational speed, have also been identified. One mechanism of such vibrations is the occurrence of temperature gradients on the shaft’s surface, which results from different viscous shear stress in the
bearing lubricant. Occasionally, the sub-synchronous vibration can be a critical issue as the induced vibrating amplitude may go beyond a danger level, damaging the machine. Yu [19] studied the sub-synchronous vibration of a rotating machine experimentally. The test facility consists of a turbine, a generator, and four connecting journal bearings, as shown in Figure 1.10. At each bearing, a pair of non-intrusive probes in X and Y directions has been mounted to monitor the motions of the turbine and the generator. In the tests, the connecting shaft was powered to reach a velocity of 3600 revolutions per minute (rpm) at time 16:27:09. During the initial 20 min after reaching 3600 rpm, the vibration levels at all the bearings were found to be below 51 μm. At time 16:46:39, about 20 min after reaching 3600 rpm, the shaft centerline had a rise of about 51 – 76 μm at Bearing 3. Then the sub-synchronous vibration started to develop, as shown in Figure 1.11 (a)-(b). The developing vibration amplitude reached the trip level of 305 μm at time 16:52:59, triggering the machine to slow down. Surprisingly, the sub-synchronous vibration amplitude kept increasing, and reached the highest level of 328 μm at 16:53:14. As speed dropped further, the sub-synchronous vibration amplitude started to decrease. An inspection for abnormality and damage was conducted on Bearing 3 after the machine shutdown. It was found that the bearing diametrical clearance was larger than its specification, and in particular a crack on the bearing surface was observed as in Figure 1.12.

This section reviewed the problems encountered in many industrial applications in which fluid-structure interactions are involved, such as bridge cables in wind, drilling risers in petroleum production, and rotating machines.
Rotating machine train diagram and monitoring probes in X and Y directions [19]

Orbital plots at different stages at Bearing 3 as in Figure 1.10 [19]

(a) Normal orbit without $\frac{1}{2}X$ vibration at 16:46:34
(b) Starting $\frac{1}{2}X$ vibration at 16:46:39
(c) Full-blown $\frac{1}{2}X$ vibration at 16:52:59
(d) Maintaining $\frac{1}{2}X$ vibration at 3466 rpm at 16:53:14

Orbital plots at different stages at Bearing 3 as in Figure 1.10 [19]
Figure 1.12. Identified damage on Bearing 3 surface after inspection of a rotating machine [19]
1.2 Fluid-structure interaction: in nuclear reactors

This section gives a general introduction to the issues encountered due to fluid-structure interactions in nuclear reactors, particularly in Pressurized Water Reactors (PWRs) as a widely used type of in-service reactor.

A nuclear reactor is a device used to control a sustained nuclear fission chain reaction, from which heat is generated. In the world, there exist a few types of in-service nuclear reactors: Pressurized Water Reactor (PWR), Boiling Water Reactor (BWR), Pressurized Heavy Water Reactor (PHWR), Advanced Gas-cooled Reactor (AGR), Molten Salt Reactor (MSR), etc. A typical pressurized light water nuclear reactor as in Figure 1.13, consists of a steel pressure vessel within which nuclear fuel elements and control rods are mounted, a pressurizer to control the internal pressure level, a steam generator for exchanging heat, a pump for accelerating water, connecting pipes, etc. Within the space enclosed by these structures, water is utilized for exchanging the energy generated: high-speed cold water is initially accelerated by the pump, flows into the pressure vessel through its annular periphery downwards, then axially travels through the fuel elements during which the cold water is heated up by released energy from fuel fission. The hot water then flows out of the pressure vessel at its upper plenum, into the steam generator in which the temperature of the hot water reduces as heat is exchanged with another water flow at a lower temperature, the cold water then runs back to the pump. Many types of nuclear reactors such as BWRs and PHWRs, have a similar flow configuration as in the pressurized light water nuclear reactor, in particular the fluid flow inside the pressure vessel.
Clearly, for a PWR, FSIs can be observed at many locations along the water flowing path, such as fuel elements, transporting pipes, annulus periphery of the pressure vessel, U-shaped tubes of the steam generator. As a consequence of such interactions, structural vibrations are generally induced under high-speed fluid flows, called Flow-Induced Vibrations. Of the possible occurring locations, FIVs of the fuel elements have attracted more attention as they play an important role in the integrity and safety of a PWR. Kim [21] reported a systematic fuel failure occurring at 16x16 Korean Optimized Fuel Assemblies (KOFAs), as in Figure 1.14. The first failure signal was detected at a twice-burned fuel assembly (FA) at around 90 days after startup. It has been found that the primary cause of the failure was a fuel assembly vibration with a peak amplitude of about 200 μm with a frequency of between 1 and 200 Hz. As the vibrating fuel rods had contact with the spacer grids (SGs), depicted as in Figure 1.15, which are a type of structure.
mounted for preventing the fuel rods from excessive movement and generally comprise of springs and dimples, fretting on the fuel rod surfaces were initialized, this is called grid-to-rod fretting (GTRF). As time accumulated, the fretting eroded further, eventually leading to a wear-through. Such a wear-through phenomenon has been identified on some Korean fuel rods presented in Figure 1.14, where the brighter spots denote the perforated locations. Kim [22] numerically studied the effect of fuel rod supporting conditions on GTRF wear, where the fuel rod was divided into 9 equally spaced segments by 8 spacer grids. Comparing with a grid-to-rod axial fretting wear profile observed in a Korean PWR in Figure 1.16, it was concluded that the 1st mode shape dominates the vibrating pattern of the rod, as the fretting wear depth (denoted as black spots in the plot) is the largest at the 4th mid-grid position, the second largest at the 3rd mid-grid, the third largest at the 5th mid-grid, the fourth largest at the 6th mid-grid, and no wear identified at the 1st, 2nd, 7th and 8th grids. In this structural configuration, the outer surface of a fuel rod near the mid-span was found to be most vulnerable to GTRF and hence requiring more attention during the design stage. Kim [23] carried out out-of-pile fretting wear tests on a Zr-Nb alloy fuel rod for 20 days, where the rod was in an adiabatic environment at a temperature of 200 °C. Figure 1.17 presents the GTRF wear progress as a function of time in 20 days, where the spring and dimple wear marks on the outer surfaces can be clearly identified. The wear is shown to be initiated in the point-contact regions, and its area and depth increases as time accumulates. Through this study, it has also been found that the fretting wear rate is strongly dependent on the spacer grid designs such as grid-to-rod gap size and pre-oxidation level of the rod outer surface. GTRF may also be determined by a power uprate in PWRs due to increased water flow rates [24].
Hence, it can be seen that in PWRs, GTRF as a result of fluid-induced fuel rod vibrations, plays a crucial role in deteriorating the integrity of the fuel rods. Occasionally, GTRF wear may erode further resulting in perforated wear, and accordingly radioactive materials will be released into the coolant water and potentially into the public environment. According to Kim [25], GTRF induced failure accounted for around 70% of the worldwide nuclear fuel rod failures in the year 2011. Therefore, the mechanism of GTRF in nuclear reactor cores is necessary to be understood in detail, and based on that, estimating the life span of fuel rods at a level which fulfills the state-of-the-art criteria such as safety constrains during its design stage.

Figure 1.14. Perforated 16×16 KOFA fuel due to grid-to-rod fretting [21]
Figure 1.15. Structure of the rod rod bundle with spacer grid inside a PWR core [26]

Figure 1.16. Grid-to-rod axial fretting wear profile in a Korean PWR, where the spacer grids (SGs) are ordered from the upstream to the downstream of water flow [22]
Fuel rod failures have also been observed in some CANadia Deuterium Uranium (CANDU) reactors. A fuel bundle in the Darlington CANDU-type nuclear power plant was found to be damaged in the year 1990, as a result of extensive vibration of the fuel rods. A subsequent inspection reported severe wear marks in many fuel channels in Units 1 and 2 [27]. Dennier et al. [28] carried out a series of tests at Darlington Unit 3 to investigate the vibration mechanism of the fuel rods. It was found that the severe wear marks were caused by the bearing pad interactions with the outer fuel elements, as the fuel rods vibrated partially due to turbulence-induced excitation by turbulent water flows. Fretting phenomena due to flow-induced structural vibrations have also been identified in many other types of nuclear reactors, including lead/lead alloy cooled fast reactors, accelerator driven systems [29]–[31], and boiling water nuclear reactors [24].

Therefore, for safety concerns, the behaviour of the fuel rods immersed in fluid flows in an operating nuclear reactor is required to be monitored. There are generally two approaches to achieving this goal: one is a conventional way through in situ surveillance, the other is by life-span estimation at its design stage. In situ surveillance, as many vendors have been employing via estimating the number of the failed fuel rods, refers to measuring the activity ratios of Xe-133 to Xe-135 or I-131 to I-133 inside the fluid flows, or
conducting an inspection on the fuel rods during the refueling of the reactor. This approach is clearly not economically sound as the damaged rods may be replaced ahead of the planned time. If a wear scar due to GTRF goes unnoticed during inspection, then the fuel rod may fail to function as a barrier against radioactive material releasing during operation. On certain occasions, this fuel load failure may cause shut-down of the nuclear reactor for replacement. For commercial nuclear reactors in operation, the shut-down criteria are determined by the degree of the failure which is defined by each nuclear reactor vendor, and by the specific regulations issued by local nuclear regulatory bodies. Life-span estimation prior fuel rod loading, as an alternative approach which has attracted great attention from nuclear reactor owners, can be realized by high fidelity models/tools developed by the FSI community, which take relevant factors into consideration such as the spacer grid configurations. However, due to confidentiality concerns by nuclear reactor vendors, no such a model has been made public and available to academics [32]. Under the circumstances, in order to derive such models, a good characterisation of flow-induced rod vibration is necessary to be examined.

This section discussed one serious phenomenon for safety concerns, namely the grid-to-rod fretting as a result of FIV encountered in nuclear reactor cores, particularly in pressurized water nuclear reactors. Meanwhile, to accurately predict the mechanical behaviours of fuel rods, an evaluation system is required to be well established. One step towards achieving this goal is characterizing the rod vibrations induced by axial fluid flows.
1.3 Motivation for this PhD study

FSI has been found to be present in many industrial applications, particularly in certain type nuclear reactor cores in the form of FIVs. Such vibrations can pose a great threat to the safety operation of the nuclear reactors, as a consequence of interaction of the fuel rods and neighbouring spacer grids. The aim of this PhD study was to investigating the structural dynamics of such a fuel rod subjected to a water flow, in particular the vibrating amplitude and frequency, such that making it possible to initiate a sound solution to mitigate the effects of the flow-induced rod vibrations on the rod’s failures. This work includes a literature review of FIV occurring in particular in the nuclear reactor cores of interest, by summarizing and describing its mechanisms, prediction tools of relevance and the limitations of the tools. Based on the findings from this literature review, an experimental solution to pushing forward the research on potentially improving the performance of the prediction tools in use and mitigating flow-induced rod failures, was raised and examined.

1.4 Thesis structure

The structure of this thesis is organised as follows. Chapter 2 undertakes a literature review of FIV, including the mechanisms of FIV in both axial and cross directions, prediction tools in use such as empirical correlations and a numerical approach, and an evaluation of the behaviour of the prediction tools. Based on the findings of this literature
review, an experimental approach to improving the performance of the prediction tools was established. The description of the test facility for realizing this goal in an experimental approach is made in Chapter 3, including its test configurations and measuring techniques such as measurement of water velocity in a confining tube and rod movement measurement in a non-invasive manner.

In Chapter 4, test results on a nuclear fuel-like rod subjected to an axial flow are presented and discussed. It starts with a preliminary test for validating the capabilities of the test facility in capturing the structural dynamics of the rod of interest, and then follows this by a series of tests on a rod with varying free-tip shapes and internal-loading materials, from which the effects of the varying parameters on the structural dynamics of the rod in particular vibrating amplitude and frequency are unveiled.

Finally, based on the knowledge accumulated, Chapter 5 gives conclusions on the present study and suggestions for future work. The Appendix section in the present thesis includes an uncertainty analysis of the parameters encountered in the experimental tests.
Chapter 2  Literature review of flow-induced vibration

As one aspect of this study is experimentally investigating the flow-induced vibrations in pressurized water nuclear cores, attention is directed to a flexible clamped-clamped cylindrical structure subjected to turbulent water flow in the axial direction, since this is the basic configuration encountered in the nuclear reactor cores of interest.

In this chapter, a literature survey of flow-induced rod vibrations in particular an axial flow, is conducted, in which prediction tools for estimating the structural dynamics of the rod are enumerated, the accuracy and limitations of the prediction tools are also compared and discussed.

The remaining part of this Chapter is organised as follows: Section 2.1 introduces mechanisms of FIVs, in which Section 2.1.1 gives a description of the mechanisms of axial FIVs, while Section 2.1.2 introduces the mechanisms of cross FIVs.

Section 2.2 lists some prediction tools either in use or in development: Section 2.2.1 gives a mathematical description of the structural movement of a rod subjected to a flowing fluid, Section 2.2.2 reviews correlations for estimating the vibrating amplitude (Section 2.2.2.1) and frequency (Section 2.2.2.2), Section 2.2.3 describes a promising methodology, namely numerical calculations, to understanding such a fluid-structure system, from which both the fluid and structural domains can be described. This is followed by Section 2.2.4 in which an examination of the behaviour of the correlations enumerated in Section
2.2.2 is performed. Specifically, a collection of experimental tests in a similar test configuration of interest from the literature is presented in Section 2.2.4.1, based on which the performance of the correlations in estimating the structural dynamics of the rod in particular the vibrating amplitude, is discussed in Section 2.2.4.2.

Based on the accumulated knowledge, Section 2.3 makes a conclusion on the literature review. Based on the finding from this conclusion, an experimental work to be conducted is defined in Section 2.4.

2.1 Flow-induced vibration mechanism

In pressurized water nuclear reactor cores, axial water flow dominates the flow pattern, and there also exists a small portion of water flow traveling in the cross direction due to the influence of neighbouring spacer grids. This section reviews the mechanisms of flow-induced vibrations of a flexible clamped-clamped cylindrical structure subjected to external axial/cross flows.

2.1.1 Axial FIV mechanism

This Section examines the axial flow-induced vibration mechanisms for a solitary fuel rod and/or a fuel rod array, in which some theories as to the fluid-structure interactions such as the effects of flow on the fuel rod dynamics, are enumerated.
For a solitary fuel rod immersed in an axial turbulent water flow, Blevins [33] classified the FIV phenomena observed as being turbulence-induced vibration and fluid-elastic instability. When a turbulent flow travels by a fuel rod, the turbulent energy in the flow leads to random fluctuating pressures across the surface of the fuel rod, thus causing the fuel rod to vibrate, this is called turbulence-induced vibration. The fluctuating pressures have some random characterisations: they oscillate over a broad band of frequencies, they change in amplitude over the surface of the fuel rod, and they are coherent over only small areas. It was believed that turbulence-induced vibration is dependent on eddies which exist in flows regardless of the presence or absence of fuel rod vibration, thus time dependent, and features a gradual amplitude increase of the fuel rod movement with increasing flow velocity when less than a critical velocity, which is a function of the dynamic properties of the fuel rod and the aerodynamic coefficients supplying the fluid force on the fuel rod. Once this critical velocity has been exceeded, fluid-elastic instability will generally develop in a form such as flutter characterizing a large amplitude vibration compared with that caused by turbulence-induced vibration, and is primarily determined by the position and velocity of the fuel rod relative to the fluid flow. For a closely space array of fuel rods, fluid-elastic instability can be described as whirling, in which the fuel rod movement is influenced by both its adjacent fuel rods and the neighbouring fluid. To analytically predict the amplitude of these vibrations, movement of the fuel rod and the fluid force exerted on the rod are assumed to be in a harmonic motion such as a sinusoidal oscillation. Based on this hypothesis, the vibration amplitude of the fuel rod can be estimated whether in an increasing trend or not, by comparing the energy input to the fuel rod from the turbulent flow per cycle of vibration and the energy dissipated by the fuel rod via internal damping.
Paidoussis [34], [35] proposed four different responses of a flexible fuel rod subjected to an axial turbulent water flow, as shown in Figure 2.1, in which the relationship between vibration amplitude and water flow velocity is plotted:

i) Forced vibration or buffeting of fuel rods

It has been widely accepted that the random pressure fluctuations on the surface of a fuel rod constitute the excitation force field for buffeting vibration at all flow velocities. The pressure field exciting the vibration has near-field and far-field components. The near-field includes the local pressure fluctuations associated with the boundary layer and the non-propagating part of disturbances associated with singularities such as valves, bends, protuberances, supports. The far-field component comprises propagating disturbances in a form of acoustic waves. The resulting vibration amplitude due to buffeting is typically less than 1% of the fuel rod diameter, rarely larger than 10% (see Figure 2.2). As the flow velocity increases, the vibration amplitude also gradually increases in a power range from 1 to 2 (see the region with a lower slope in Figure 2.1).

ii) Hydrodynamic coupling

For an array of fuel rods immersed in a fluid, hydrodynamic coupling is another influential factor for the vibration characteristics. Taking a system of two clamped-clamped fuel rods in a still water fluid (contained by a circular channel) as an example for a detailed illustration (see Figure 2.3). One rod is given an initial displacement with its first mode shape, then the acceleration of the fluid produced by the subsequent free motion of the rod induces a pressure field on the neighbouring rod which causes it to oscillate. The induced oscillation motion is not simply harmonic. Also, due to hydrodynamic coupling, instead of having an independent set of flexural modes for each rod, the system has a set
of coupled flexural modes. For a system of N rods, there are 2N modes in first mode axial shape, 2N modes with second mode axial shape. It has also been found that the amplitude of the FIV for hydrodynamically coupled rods is qualitatively larger than that for a solitary one. This may be explained by the fact that a single rod inside an array is viewed as a shaped band-pass filter (see Figure 2.4), this band-pass “window” is much broader in the system of hydrodynamic coupling than that of weak coupling; thus, the frequency band over which energy may be absorbed by the rods, becomes wider.

iii) Vibrations due to flow periodicity

The mean flow velocity passing an array of rods may be harmonically perturbed, so that the flow velocity $U$ is expressed as:

$$U = U_0 \left(1 + \mu \cos \omega t\right)$$

(2.1)

where $U_0$ is the undisturbed flow velocity, $\mu$ is a small parameter, $\omega$ is the angular frequency of the perturbation. As observations in the literature suggest, vibrations due to the flow periodicity may develop around $\omega / \omega_n = 2k, k = 1, 2, 3...$ where $\omega_n$ is the natural frequency of the rod. The instability of vibration due to flow periodicity is characterized, as shown in Figure 2.1, as an abrupt peak in vibration amplitude. Figure 2.5 presents the instabilities for a fixed-fixed rod subjected to a harmonically perturbed flow, in which instabilities occur around $\omega / \omega_1 = 1$ and $\omega / \omega_2 = 1$ [36] (To clarify, the frequencies in the plot have been normalized by dividing by the first-mode frequency in still fluid, making it difficult to observe).

iv) Fluid-elastic instabilities
A rod in an axial flow of sufficiently high flow velocity will be subject to fluid-elastic instabilities, generally buckling (divergence) which is a non-oscillatory instability, followed by flutter at higher flow velocities which features a large amplitude of vibration. The mechanism of the instability relates to the reduction of the effective flexural rigidity of the rod through a centrifugal load on the instantaneously bent rod, where this force is proportional to the flow velocity squared. Fluid-elastic instabilities may also be developed in rod arrays, but at lower flow velocity due to the effect of hydrodynamic coupling; And the tighter the array, the lower is the critical flow velocity. Fortunately, the critical flow velocity lies far beyond the operating range of typical engineering systems, including rod arrays.

Figure 2.1. Generic idealized vibration amplitude of a fuel rod exposed to an axial turbulent flow as a function of the flow velocity [37]
Figure 2.2. Relationship between measured and predicted relative amplitudes of vibration (using an empirical correlation) of a fixed-fixed rod subjected to an axial flow, in which most data is less than 1% [38]

Figure 2.3. Response of a system of two parallel-aligned fixed-fixed cylinders in a still-water confining tube, in which cylinder 1 was given an initial displacement in the y-direction (upper two plots), and in the z-direction (lower two plots) [39]
Figure 2.4. Response of an array of four rods in unconfined still fluid when rod 1 was initialized to vibrate sinusoidally, (a) for a loosely spaced array, (b) for a relatively tightly spaced array [40]

Figure 2.5. Regions of parametric instabilities (enclosed by the curves) for a fixed-fixed rod, as a result of flow periodicity, in which x-axis is the amplitude parameter $\mu$ in Equation (2.1), y-axis is the ratio of perturbation frequency to the first mode frequency of the rod in still water, and $u_0$ is the undisturbed flow velocity [36]
Hence, it can be concluded from the two theories that the random pressure fluctuations around a fuel rod are a primary cause of rod vibration, featuring a gradual vibration amplitude increase as flow velocity increases. Above a critical flow velocity, fluid-elastic instabilities are observed to initiate and start developing, which characterize an abrupt jump in vibration amplitude and a large amplitude at high flow velocities. Fortunately, fluid-elastic instabilities can hardly develop in the current in-service pressurized water nuclear reactors, as it runs at flow velocities which are far less than the critical flow velocity. Meanwhile, some other factors which may have an impact on the rod response, such as harmonically perturbed flow, have been recognized and investigated. Similar research into axial FIV mechanisms has also been made by other researchers, interested readers may refer to references [41]–[43].

This section reviewed the axial flow-induced vibration mechanisms for a solitary fuel rod and/or a fuel rod array immersed in turbulent water flows.

2.1.2 Cross FIV mechanism

Cross flow is one common flow pattern in certain type of heat exchangers’ U-bend regions in the nuclear industry (see Figure 2.6), in which water flows across rod arrays at a certain attack angle relative to its axis which absorb the heat generated by nuclear fuel. Also, minor cross flow is identified in the nuclear reactor core as a result of interaction between the axial water flow and the spacer grids. In this section, a review of the cross-flow-induced rod vibration mechanism is provided. This is followed by a brief discussion on the design guidelines available for alleviating the influences caused by such a vibration.
Païdoussis [34] classified cross-flow-induced response of a rod into two phenomena: one is forced vibration response, the other is fluid-elastic instability. Generally, the forced vibration consists of a non-resonant and a resonant component, as shown in Figure 2.1. The non-resonant component results from energy extraction by the rod from the fluctuating pressure field within a frequency band around its natural frequency. The amplitude levels remain small at all flow velocities, and increase with the flow velocity. However, for a rod array, the fluctuating pressure spectrum may contain one or more peaks of discrete periodicity, or there may not be discrete periodicity but simply a quasi-periodic peak in the turbulent energy spectrum. In either case, the vibrating amplitude of the rods will show a resonance peak. Hence, designers may keep the rod natural frequencies away from the resonance flow conditions, for minimizing its vibratory
response. In addition to forced vibration, rod arrays may be subject to fluid-elastic instability which is believed to be a self-excited vibration phenomenon, when the flow velocity value is above a critical level. The fluid-elastic instability characterizes a much larger amplitude than that of force-induced vibration. This critical flow condition is generally represented in a dimensionless form:

\[
\frac{U_{cr}}{f_n D} = K \sqrt{\frac{m \zeta}{\rho D^2}}
\]

(2.2)

where \( U_{cr} \) is the critical flow velocity, \( f_n \) is the rod natural frequency, \( D \) is the characteristic length of the system (herein the rod outer diameter), \( m \) is the rod mass per unit length, \( \zeta \) is the damping ratio either in vacuo or in still water, \( \rho \) is the water density, and \( K \) is an empirical factor which can be obtained from fitting experimental data. Connors [45] set \( K \) to be 9.9 for a single row of rods, while Pettigrew & Taylor [46] proposed it to be 7.5 after compiling extensive experimental data, as shown in Figure 2.7. For multi-row rods, experimental points available to the authors [47]–[51] yield an ultra-conservative value of \( K \) being 0.8. However, this conservative value makes designers face with a difficult task in that the use of 0.8 in \( K \) may lead to excessively conservative designs. Thus, a new expression was suggested to use as a design guideline:

\[
\frac{U_{cr}}{f_n D} = C \left( \frac{m}{\rho D^2} \right)^{0.4} \zeta^{0.4} \left( \frac{p}{D} - 1 \right)^{0.5}
\]

(2.3)

where \( C \) equals to either 2.3 or 5.8, \( p \) is the center-to-center spacing between rods.

In rare occasions, acoustical vibrations can also develop in which an acoustical natural frequency coincides with the frequency of a flow periodicity, leading to intense sound amplification and large fluctuations of pressure. For pressurized water nuclear reactors,
such vibrations are very difficult to achieve due to the presence of a high acoustical natural frequency and of a flow periodicity of low frequency.

Figure 2.7. Summary of fluid-elastic instability experimental data for single-phase cross flow [46]

Païdoussis [35] made an extensive study of cross FIVs which occur in reactor components, for both a solitary rod and rod arrays. For a solitary rod, two phenomena of interest are generally found: vibration induced by vortex shedding and vibration induced by turbulent buffeting. The vortex shedding which is similarly being experienced by a fast swimmer on the arms, can generate forces on the rod's surface in both transverse and streamwise directions. In the subcritical and supercritical flow regimes featuring $40 \leq \text{Re} \leq 2*10^5$ and $\text{Re} \geq 3.5*10^6$ respectively, periodic shedding of vortices which correspond to Strouhal numbers (denoted as $St = \frac{f_{vs} D}{U}$, where $f_{vs}$ is the vortex shedding frequency, $D$ is a characteristic length, and $U$ is the flow velocity) of 0.2 and 0.3 respectively, may result in

60
large amplitude oscillations when the system damping is small. If the vibration amplitude
is not negligibly small, i.e. greater than 0.01 \( D \) to 0.02 \( D \), and if the parameter \( m\zeta / \rho D^2 \)
is not too large, then a lock-in phenomenon may arise where the vortex shedding
frequency \( f_{vs} \) is entrained to follow the rod natural frequency \( f_n \), over a range of flow
velocities whose width has been found to be determined by the vibration amplitude. In
this lock-in range, oscillations in the streamwise direction at double the shedding
frequency have been identified to dominate the rod dynamics, contrary to most cases in
which oscillations in the transverse direction dominate. In the past, many expressions for
predicting the lock-in vortex-induced vibration amplitude have been proposed as a
function of damping [52]–[55]. For avoiding lock-in occurring in such structures, designers
usually change the rod natural frequency and keep it \( \pm 40\% \) away from the vortex
shedding frequency. Other effective measures include suppressing the vortex shedding or
reduce its strength and increasing the damping to reduce the vibration amplitude. In
addition to forces due to vortex shedding, a rod immersed in cross flow is also subjected
to turbulent buffeting due to the random pressure perturbations of the fluid stream on
the rod’s surface. During this process, the rod behaves as a band-absorption filter which
exchanges energy with the neighbouring flow field in a narrow window about the natural
frequency of the rod (mostly around the lowest natural frequency). The resulting vibration
is generally featured by a small amplitude across all flow velocities, which is similar to that
excited by turbulence in axial flow conditions. However, it has been found that if the
intensity of incident turbulence exceeds a certain level, the distinction between vortex
shedding and turbulent buffeting is quite arbitrary. At high turbulence, lock-in
phenomena were found to entirely disappear [56].
For a rod array in a repeated geometric pattern, Paidoussis [35] summarized the cross FIV mechanisms as being turbulent buffeting, vortex shedding and fluid-elastic instability. The turbulent buffeting recognized in a rod array was thought to be similar to the case of a solitary rod, but also differentiated in a sense that the incident turbulent field in the rod array is generated mostly by the upstream rows of rods, instead of by free-stream turbulence. Similarly, vibration induced by turbulent buffeting occurs at all flow velocities and its amplitude is rather small. The vibration induced by vortex shedding or sometimes called Strouhal periodicity featuring an amplified vibration amplitude over and above the buffeting background, was found to commonly occur within the first few rows of rods, unless the upstream turbulence level is sufficiently high to suppress it. Deep into the rod array, Strouhal periodicity may or may not survive, depending on some factors such as Reynolds number, rod layout geometry, fluid-to-rod mass ratio and rod vibration amplitude. When the shedding frequency coincides with the rod’s natural frequency, Strouhal resonance is observed to occur in the rods in one of the following geometrical layouts:

1) Tightly spaced arrays in gaseous flows, in the first couple of rows provided that upstream turbulence is kept quite low

2) Tightly spaced arrays in liquid flows of low turbulent level, in the first few rows

3) Widely spaced arrays, throughout the whole array

In design guidelines, a rod’s natural frequency is generally kept to be ± 40% or ± 50% away from the vortex shedding frequency, as a conservative criterion to avoid loss due to the amplified vibration.
Fluid-elastic instability in rod arrays, on the other hand, was described as self-excited oscillations and was believed to dominate the vibration amplitude if the flow velocity exceeds a critical level. As flow velocity exceeds the critical level, rods within the rod array move in synchronism with the neighbouring rod motions. During the process, energy is extracted by the rods from the flow; if this energy gain is sufficiently larger than offset dissipation, the motion is amplified to a self-sustaining limit cycle. In the past, such a critical value has been obtained experimentally from various sources in a similar form as in Equation (2.2), as shown in Figure 2.8. It shows that at low $m\zeta/\rho D^2$, the slope is almost horizontal; for $0.2 < m\zeta/\rho D^2 < 20$, the slope is around $1/3$ and gradually increases with increasing $m\zeta/\rho D^2$. Theses semi-empirical findings are of great importance in relevant structural designs.

Figure 2.8. Experimental values of the critical flow velocity of fluid-elastic instability for rod arrays immersed in cross flow [35]
State-of-the-art examination of the mechanisms of cross FIV was carried out by Kaneko et al. [57]. For a solid rod immersed in a cross flow, the vibration phenomena observed are caused by steady flows and in particular by unsteady (oscillating) flows as in the case of marine structures. The vibration caused by steady flow is further classified into four categories, namely, forced vibration by Kármán vortex shedding, synchronization accompanied by alternating Kármán or symmetric vortex street, turbulence-induced vibration and vibration induced by tip-vortices in a high flow velocity regime.

i) Forced vibration by Kármán vortex shedding

As a flow travels around a bluff body (or a rod in the current case of interest), a vortex street in the wake region, herein being the Kármán vortex street, is commonly generated. When these vortices periodically shed from the surface of the body, periodic pressure fluctuations on the structure surface are generated. As a result, it is possible to induce vibrations in both transverse and streamwise directions. In the transverse direction, the excitation force features a dominant frequency called the Kármán vortex shedding frequency. While in the streamwise direction, the dominant frequency is at twice the Kármán vortex shedding frequency.

ii) Synchronization accompanied by alternating Kármán or symmetric vortex street

If the vortex shedding frequency in either transverse direction or streamwise direction, is close to the natural frequency of the rod, synchronization is generally introduced, this is called lock-in. Such synchronization is characterized as an abrupt increase in vibration amplitude, as Figure 2.9 presenting the synchronizations accompanied by a symmetric vortex street in the streamwise direction, by a Kármán vortex street in the streamwise direction, and by a Kármán vortex street in the transverse direction as an increase in flow...
velocity. From engineering experiences, the critical velocity for this symmetric vortex shedding synchronization is lower than that for the Kármán type of synchronization.

iii) Turbulence-induced vibration

Outside the synchronization region, a rod is also excited to vibrate by vortex-induced forces. The excitation force has not only a periodic component at a dominant frequency, but a component over a wide frequency band. If this dominant vortex shedding frequency is far separated from the rod natural frequency, the rod will be excited by the wide-band component closest to the natural frequency. The resulting excitation is called turbulence-induced vibration. Turbulence-induced vibration can be generally observed in the cases which upstream turbulence is high enough to induce vibration.

iv) Vibration induced by tip-vortices in high flow velocity regime

At high flow velocities, large amplitude vibrations (comparing with synchronization induced vibrations) may be excited beyond the Kármán shedding lock-in region. This phenomenon is induced by vortices generated at the rod extremity as in Figure 2.10. These vortices are characterized to shed at a frequency about one-third of the Kármán shedding frequency. Hence, the critical flow velocity for lock-in by tip-vortices is roughly three times the critical velocity for the Kármán shedding.

Hence, it can be summarized that for a solitary rod immersed in cross flow, there exist two classes of response: non-synchronization response by forces due to the Kármán vortex shedding and turbulence, and synchronization response consisting of synchronization accompanied by alternating Kármán or symmetric vortex street and synchronization induced by tip-vortices at high flow velocity.
This section outlined the development of cross flow-induced vibration mechanisms for a solitary rod/rod arrays.

Figure 2.9. Vortex-induced synchronization as a function of flow velocity

Figure 2.10. Tip-vortex shedding as fluid flows by a rod
2.2 Approach to predicting FIV

It is of importance to predict the flow-induced vibration system, in particular the structural dynamics such as vibrating amplitude and frequency in the industrial applications of relevance. For a pressurized water nuclear reactor, the accuracy level at which a prediction tool can estimate the rod dynamics can have a great impact on economic and safety concerns. In this section, approaches to predicting the axial flow-induced rod vibrations which occur in pressurized water nuclear reactor cores are reviewed, including a mathematical description of the system, empirical correlations for estimating the vibrating amplitude and frequency, and a novel methodology via numerical calculations. In this context, performance of the empirical correlations is examined by comparing with experimental results in the literature. The limitations of these methods are also discussed.

2.2.1 Mathematical description

In this Section, the equations of motion for a flexible clamped-clamped rod subjected to axial flow are briefly introduced. Also, theories to solving the equations are enumerated, followed by a discussion on the features and limitations of the theories.

The system of interest in the current research consists of a rod, centrally located in a rigid channel within which a fluid flows parallel to the channel centerline. Assuming the rod can be viewed as an Euler-Bernoulli beam, the rod’s equations of motion can thus be obtained either by Newton’s second law of motion following the Galerkin’s technique [58], or by
applying Hamilton’s principle concerning energy conservation [59]. Generally, the obtained equations of motion are expressed in the following form:

\[ M \ddot{y} + C \dot{y} + K y = F \]  

(2.4)

where \( y \) is the lateral displacement in generalized coordinates as a function of time; \( M \), \( C \) and \( K \) are the mass, damping and stiffness coefficients, respectively; \( F \) is the fluid force on the rod, which consists of a time-dependent force term, and a force term as a function of the rod displacement, velocity and acceleration, proposed by Kaneko [57].

Païdoussis [58] split the fluid force into an inviscid, a pressure and a viscous contribution in the linear models. The inviscid term was obtained by Lighthill [60], as a result of the acceleration of the added mass which is the virtual mass of the fluid and equal to the fluid mass per unit length replaced by the rod. The pressure term comes from the buoyancy force on the rod. The viscous contribution is based on an empirical description by Taylor [61], in which the viscous force is expressed as a function of the fluid density, fluid bulk velocity, rod diameter and coefficients associated with form and friction drag. Nonlinear models have also been derived and studied in Refs. [59], [62], in which the system’s response predicted by the nonlinear models was found to be in qualitative agreement with experimental observation: the system loses stability and develops divergence (static buckling), and the new equilibrium becomes unstable at higher flow velocities and this leads to flutter (oscillating). Herein, the onsets of divergence and flutter are determined mathematically by the eigenvalues of the system of study: if all the complex eigenvalues have negative real parts, then the system is stable; when one of the complex eigenvalues becomes zero, then static buckling occurs; if a pair of complex conjugate eigenvalues crosses the imaginary axis, flutter starts to develop. As a general observation, the linear
analysis yields the natural frequency, the vibration mode, the amplitude growth rate, the
frequency response spectra and the transient response, while the stability boundary, the
post-instability limit cycle amplitude and the time-history response can only be obtained
from the nonlinear analysis to such a clamped-clamped flexible rod as in nuclear reactor
cores [57].

All the theoretical methods or analyses, however, are empirical/semi-empirical, in that
relevant fluid force information such as force coefficients and fluid pressure spectra, must
be known in advance, and this information is generally obtained from experimental tests
on specific configurations [63], [64].

This section introduced the mathematical equations of motion for a flexible clamped-
clamped rod subjected to axial turbulent flow, and theories available to solving the
equations.

2.2.2 Correlations

In this Section, empirical/semi-empirical correlations for predicting the deflection
amplitude of a flexible clamped-clamped rod subject to axial flow, as an alternative
approach being extensively applied in nuclear industry, are provided and discussed. As
many of the correlations are a function of the natural frequency of the rod in air or still
water, a review to obtaining the natural frequency of a rod is also made. The correlations
discussed are then implemented into a newly generated Matlab modelling suite and their
prediction accuracy is successively evaluated by comparing with a large experimental databank collected from the literature.

### 2.2.2.1 Amplitude

This section collects the correlations for estimating the amplitude, in most cases the maximum amplitude, of a flexible clamped-clamped rod subject to axial flow. As the first mode generally dominates the oscillation pattern in this field of study, the maximum amplitude of the rod is believed to occur at the mid-span. In this context, deflection amplitude of the rod at the mid-span is solely studied through the empirical correlations.

Burgreen et al. [65] were one of the earliest researchers to experimentally study axial flow-induced rod vibration. In the tests, rods made of either brass or aluminum were under fixed and pin-ended support conditions. The rods were manufactured to be either hollow inside in the form of a cylindrical shell or solid. The rods were then mounted in triangular lattices with equivalent hydraulic diameters of 2.16, 6.03 and 14.33 cm, as in Figure 2.11. The water velocity varied from 2 to 6 m/s, at ambient temperature. Based on the obtained vibrating amplitude, a simple correlation for peak to peak amplitude estimation was proposed, consisting of three dimensionless parameters:

\[
\left( \frac{y_{\max}}{D_h} \right)^{1.3} = 0.83 \times 10^{-10} k \left( \frac{\rho U^2 L^4}{EI} \right)^{0.5} \frac{\rho U^2}{\mu f_0}
\]

(2.5)

here \( k \) is the end fixity factor which equals to 5 for hinged condition and to 2.92 for clamped condition; \( y_{\max} \) is the peak to peak amplitude, \( D_h \) is the hydraulic diameter, \( \rho \)
is the fluid density, \( U \) is the fluid bulk velocity in axial direction, \( L \) is the rod length, \( E \) is the Young’s modulus, \( I \) is the second moment of area of the rod, \( \mu \) is the fluid dynamic viscosity, \( f_0 \) is the rod fundamental frequency in air (these parameters also apply to the following correlations). Herein, the two dimensionless parameters \( \frac{\rho U^2 L^4}{EI} \) and \( \frac{\rho U^2}{\mu f_0} \) represent the ratio of the hydrodynamic force of the water to the elastic force in the rod and the ratio of the hydrodynamic force of the water to damping force on the rod, respectively.

Basile et al. [66] experimentally studied axial FIVs in different types of fuel rods. The test section, as in Figure 2.12, mainly consisted of two grids at the inlet and outlet regions for tuning the eccentricity of the rod within the channel, an aluminum-made clamped-clamped rod of 1 m in length, strain gauges, and a blocking system against rotation. By fitting the derived amplitude data, a correlation for estimating the half-peak to peak amplitude known as the EUR-formula was proposed, as expressed in Equation (2.6). In the fitting process, only 5 of the total 60 experimental data lay outside the \( \pm 50\% \) range around the values given by the formula, which is deemed to be in satisfactory agreement with the experimental results.

\[
\frac{y_{1/2\text{max}}}{D} = 10^{-9} \frac{U}{f_0 D} \left( \frac{\rho U D_{b}}{\mu} \right)^{0.5} \left( \frac{L}{D} \right)^{1.5} \left( \frac{f_0}{f_{w0}} \right)^{0.5} \left( \frac{\rho}{\rho_f} \right)^{0.25}
\]

(2.6)

where \( y_{1/2\text{max}} \) is the half-peak to peak amplitude, \( D \) is the rod diameter, \( \mu \) is the fluid dynamic viscosity, \( f_{w0} \) is the rod fundamental frequency in still water, the ratio of \( \frac{f_0}{f_{w0}} \) is called the coefficient of added mass and equals to \( \left( 1 + K \frac{M}{m} \right)^{0.5} \) where \( K \) is a geometry
dependent factor varying from 1 to 3 and $M$, $m$ being the added mass per unit length and rod mass per unit length, respectively. The rod density is denoted as $\rho_r$.

Figure 2.11. Arrangements of rods in three lattices in the vibration studies (reproduced from [65])
Figure 2.12. Schematic of test section by Basile et al. [66]
Reavis [41] developed an amplitude-to-velocity mathematical relation, known as the WV-1 correlation, based on the turbulent pressure field within the boundary-layer generated as a fluid axially flows by a rod. Prior to deriving this formula, some assumptions were made which mainly include: the rod is treated as a pinned beam on motionless supports, the rod motion is assumed to have no effect on the pressure field, the lateral shear stress due to cross-flow fluctuations is discounted as a significant source of excitation, the first bending-mode response dominants all the possible modes and so on. The obtained maximum amplitude is expressed as:

\[
y_{\text{max}} = C \eta_D \eta_{D_h} \eta_{\nu} \frac{Dv^{0.5} N^{0.5}}{n \eta_0^{1.5} \zeta^{0.5}} \mathcal{U}
\]

(2.7)

where \( C \) is a factor manually introduced to better fit experimental data after comparing the results obtained from this expression with those from experiments in the literature, and is a function of ratio of hydraulic diameter to rod length, shown in Figure 2.13; \( \eta_D \), \( \eta_{D_h} \) and \( \eta_D \) are three dimensionless scale factors as functions of rod fundamental frequency, characteristic length (e.g. rod diameter, hydraulic diameter, or rod length), and fluid flow velocity; \( \nu \) is the fluid kinematic viscosity; \( N \) is the number of rods in the array studied, implying the capabilities of applying to not only a solitary rod but also rod arrays; \( \zeta \) is the rod critical damping ratio.
Figure 2.13. Average disparity in amplitude between theoretical results and those obtained from experiments by Reavis, being a function of hydraulic diameter to rod length ratio [41]

Paidoussis [67] also gave an expression for the ratio of the vibrating amplitude \( y \) to rod diameter \( D \) for a flexible clamped-clamped rod in an axial flow, from fitting with experimental data:

\[
\frac{y}{D} = 5 \times 10^{-4} K \alpha_1^{-4} \frac{u^{16} \varepsilon^{1.8} \text{Re}^{0.25}}{1 + u^2} \left( \frac{D_h}{D} \right)^{0.4} \frac{\beta^{2/3}}{1 + 4 \beta}
\]  

(2.8)

where \( K \) equals to 1 for very “quite” circulating systems (e.g. low turbulence wind or water tunnels) and equals to 5 for industrial environments; \( \alpha_1 \) is the dimensionless first-
mode eigenvalue of the rod, which is equal to $\pi$ for pinned and simply supported rods, and equal to 4.73 for a rod with clamped-clamped ends; $u$ is a dimensionless fluid flow velocity defined as $\left(\frac{\rho A}{EI}\right)^{0.5}UL$, here $A$ is the cross-sectional area of the rod; $\varepsilon$ is the slenderness ratio being rod length to diameter ratio; $Re$ is the Reynolds number; $\beta$ is the mass ratio and is defined as $\rho A/(\rho A + m)$.

Chen [68] proposed a correlation for the relative vibrating amplitude for a flexible rod in axial flow:

$$\frac{y}{D_h} = \left(1 - \left(\frac{U}{U_c}\right)^2\right)^{-1} \left(\frac{\beta U}{U_c}\right)^2$$  \hspace{1cm} (2.9)

where $U_c = \left(\frac{\pi^2 EI}{L^2} \right)^{0.5}$, $c_L$ is the skin friction coefficient of the axial flow on the rod surface; $\beta$ equals to 0.5, 1 and 2 for “quiet”, “average” and “noisy” flow conditions, respectively.

Based on the random vibration theory, Wambsganss and Chen [69] obtained a Root-Mean-Square (RMS) vibrating amplitude $y_{rms}$ as a function of distance away from the clamped end $x$, for a solitary rod centrally located in a circular channel, written as:

$$y_{rms}(x) = \frac{0.018 K D^{1.5} D_h^{1.5} U^2 \phi(x) g}{L^{0.5} f_1^{1.5} (M + m) \left(\zeta_0 + a_1 U + a_2 U^2\right)^{0.5} \left(1 - \frac{\beta_M U^2 L^2}{EI + \beta_ITL^2}\right)^{0.75}}$$  \hspace{1cm} (2.10)
where $K$ is equal to 0.7995, $a_1$ and $a_2$ equal to $8.0052 \times 10^{-4}$ and $3.7028 \times 10^{-5}$, respectively. $\phi_1$ is the rod eigenfunction and can be expressed as $\sqrt{2} \sin\left(\pi x / L\right)$ for a pinned-pinned rod, $g$ is the gravitational acceleration. $M$ is the added mass which is expressed as $C_m \rho \left(\pi D^2 / 4\right)$, where $C_m$ is the added mass coefficient which is a function of the ratio of the inner diameter of the annular flow channel to the rod diameter, given in Figure 2.14. $\zeta_0$ is the effective viscous damping factor in still fluid which includes internal damping and external damping due to friction at the supports; $\beta_1$ is equal to 0.101 for a pinned-pinned rod and 0.0246 for a clamped-clamped rod; $T$ is the axial tension. Also, this correlation has been validated for the following values of parameters:

$$\frac{f_i D_b}{U} < 2.5, \quad 0.032 < \frac{2\pi f_i D}{U} < 1.76 \quad \text{and} \quad 8 < \frac{2\pi f_i L}{U} < 800.$$ 

For all the correlations enumerated in this section, a parametric study of the variables involved can be presented as in Table 2.1. This parametric study shows that in many correlations, the vibrating amplitude of a flexible rod subjected to an axial turbulent flow is commonly dependent on flow velocity, fluid density, fluid viscosity and rod mass per unit length. Of the correlations, most are in the form of a ratio of the maximum amplitude to the characterizing length (normally the rod outer diameter), except the correlation proposed by Wambsganss & Chen [69] in which the obtained amplitude is an RMS result. As the relationship between the maximum and RMS values in such a system follows a non-linear trend this can only be derived from experiments [41], no factors will be used, as in subsequent sections, to transfer the resulting RMS values to the maximum ones for the correlation by Wambsganss and Chen.
However, Table 2.1 also shows that the vibrating amplitude dependencies on the influential parameters have been experimentally proved to be inconsistent with each other. This implies that at present there is no consensus on what the key influencing parameters are and their respective effects. In some cases, the inconsistence can reach up to a very large level, e.g. vibrating amplitude is flow velocity to the powers of 3 and 1 respectively, for the correlations proposed by Burgreen et al. and Reavis, even though data obtained from experimental tests under almost identical conditions were utilized for fitting and calibrating the two correlations. A recent study made by Kang et al. [70] even challenged the constant parametric power-factor at varying flow conditions: in this study, an FIV model derived from the Lagrange’s method was supported by springs at both ends,
where the spring constant varied from 50, 400 kN/m, to infinity (called simple-supported).

It has been shown that the vibrating amplitude was proportional to the flow velocity to a power of 1.348 when the span length was 0.62 m, and to a power of 1.305 when the span length reduced to 0.52 m. One could easily identify the differences in the power factor of flow velocity: in the correlations enumerated as in Table 2.1, the power factors of flow velocity are 1, 1.5, 1.85, 2 and 3, thus implying a large deviation with the experimental results if applied. Meanwhile, this factor changing with flow velocity from 1.348 to 1.305 may suggest that flow velocity has more impact on the vibrating amplitude at longer spans, thus implying that both flow velocity and rod span length may have an effect on the vibrating amplitude. This result contradicts that claimed in the correlations presented as in Table 2.1. Therefore, it clearly highlights the need for more fundamental experimental studies.

This section reviewed the correlations for estimating the vibrating amplitude of a flexible clamped-clamped rod subjected to an axial turbulent water flow. These correlations available in the literature feature a capability for predicting the maximum amplitude of the rod which generally occurs at the mid-span, assuming that the first mode dominates the motion shape. The properties of the correlations were also discussed.
Table 2.1. Parametric relations in the correlations examined

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<td>U</td>
<td>L</td>
<td>D</td>
<td>EI</td>
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<td>0</td>
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<td>-1.5</td>
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<td>0.2</td>
<td>-1.6</td>
<td>-0.8</td>
<td>-0.8</td>
<td>0</td>
<td>1 -- 12 0.5 -- 0.9 16 -- 19 52 -- 0.1 -- 0.6</td>
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<td>0</td>
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<td>1.5</td>
<td>-0.5</td>
<td>-1.5</td>
<td>0</td>
<td>-1</td>
<td>-0.5</td>
<td>See Page 77</td>
</tr>
<tr>
<td>Chen</td>
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<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>-2</td>
<td>0</td>
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</tbody>
</table>
2.2.2.2 Frequency

It can be seen from Section 2.2.2.1 that the fundamental frequency of a rod in still water/air is a key parameter that plays a decisive role in the resulting vibrating amplitude from the correlations. Also, the fundamental frequency of a rod in still water has been found to be close to the vibrating frequency of the rod in an axial turbulent flow, and in particular the vibrating frequency in axial flows has been regarded as an important parameter to describe the structural dynamics and vibration-induced damage such as fretting wear [25], [71], [72]. Hence, formulae available in the literature for mathematically obtaining the fundamental frequency of a clamped-clamped rod are highlighted in this section. To clarify, symbols appearing in this section have an identical meaning to those in the previous section, unless otherwise specified.

In the 1950s, Burgreen et al. [65] experimentally studied the natural frequency of a simply supported rod, and accordingly proposed a very simple correlation to estimate the natural frequency of the rod being immersed in air:

\[ f_0 = \frac{1}{L^2} \sqrt{\frac{EI}{\rho A}} \]  \hspace{1cm} (2.11)

Wambgsanss and Chen [69] solved the equation of a flexible fixed-fixed rod in axial flow, and expressed the fundamental frequency of a rod in a still fluid, in which the mass term includes the rod mass and the added mass contributions:

\[ f_{w0} = \frac{\lambda^2}{2\pi L^2} \sqrt{\frac{EI}{M + m}} \]  \hspace{1cm} (2.12)
here $\lambda$ is equal to $\pi$ for simply-supported end conditions and 4.73 for clamped-clamped end conditions.

When a rod is immersed in an axial flowing fluid, the oscillating frequency of the rod can be developed from the fundamental frequency in still fluid as in Equation (2.12):

$$f_w(U) = f_{w0} \sqrt{1 - \frac{\lambda M U^2 L^2}{EI + \lambda T L^2}}$$  \hspace{1cm} (2.13)

where $f_w$ is the oscillating frequency as a function of flow velocity, $\lambda$ is equal to 0.101 for simply-supported end conditions and 0.0246 for clamped-clamped end conditions.

Similarly, Basile et al. [66] experimentally studied the vibrations of different types of fixed-fixed rods subjected to still air, in which the rod was excited to vibrate by deflecting from its equilibrium position and releasing it suddenly. By comparing with the test results, a correlation for predicting the fundamental frequency of a rod immersed in air has been validated:

$$f_0 = \frac{a^2}{2\pi L^2} \sqrt{\frac{EI}{m}}$$  \hspace{1cm} (2.14)

here $a$ is a function of the type of fixation and the harmonic form of the rod.

Wachel et al. [73] also gave an expression to the natural frequencies of a rod immersed in air, assuming the rod to be an Euler-Bernoulli beam in the absence of a transverse load:

$$f_0 = \frac{\beta}{2\pi L^2} \sqrt{\frac{EI}{m}}$$  \hspace{1cm} (2.15)

here $\beta$ is a dimensionless frequency factor, equals to 22.4 for clamped-clamped end conditions.
A recent study on the oscillating frequency of a rod was made by Someya et al [74], who proposed a correlation with a similar form featuring being able to be applied to a rod immersed in still water and flowing water:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{M + m}}$$  \hspace{1cm} (2.16)

where \( f \) is an unified symbol for the frequency in either still water or flowing water, \( k \) is the spring constant (unfortunately not defined in the paper).

The frequency expressions for a flexible clamped-clamped rod immersed in air/water have been reviewed in this section. It can be seen that these expressions have very similar structures which are in a form of a factor multiplied by the square root of the ratio of flexural rigidity to mass. Meanwhile, the correlation proposed by Wambsganss and Chen [69] also features the capability for predicting the oscillating frequency under different flow velocities and varying axial tensions. In this paper, the correlation proposed by Wachel et al. [73] for estimating the fundamental frequency of a rod immersed in air, which has been widely and successfully applied in engineering applications, is utilized for making subsequent calculations.

2.2.3 Numerical calculation

In this Section, a brief insight into a novel and promising methodology for understanding the axial flow-induced vibrations particularly in nuclear cores, namely numerical calculation, is made and discussed.
In the past, numerical calculations of fluid dynamics in many industrial applications have achieved great success in describing the flows involved, including off-shore flows, aerodynamic jets, drug delivery and many other fields. In most cases, the fluid of interest is spatially discretized into multiple volumes called control volumes, within which conservation equations of mass, momentum and energy are solved given relevant boundary conditions. Hence, this approach via numerical calculation is capable of describing a steady-state system, can also be extended to solving a system under transient states in which a well-defined temporal discretization is required. For an axial FIV system such as pressurized water nuclear reactor cores, however, such a conventional numerical simulation approach has been challenged by the system’s nature that the system of interest involves two different physics: structural mechanics and fluid dynamics. In such a system, the fluid motion induces the structure to move, and concurrently the structural motion exerts a force on the fluid in return, thus the two components are coupled in spatial and temporal scales. Blom [75] discussed a staggered approach and a monolithical approach for describing fluid and structure in an FSI system. The staggered approach preserves the fluid and structure solvers as separate solvers, both of which are alternately integrated in space and time. Firstly state of the structure at the end of each time step is predicted based on either the information in the previous time step or boundary conditions as specified. This step is followed by integrating the fluid to the next time step using the predicted state of the structure. In the end, proceeding the calculation of the structure to the next time step using the fluid pressures on the fluid-structure interface. This approach features the presence of a time lag between the integration of the fluid and structure, which can be neglected if the time steps are small. In the monolithic approach, however, the fluid is spatially discretized by finite volume method
(FVM) and the structure is discretized by finite element method (FEM); Time dependent information transfer between the fluid and the structure is made on the fluid-structure interface. In particular, the fluid pressure is transferred to the structure as an external force, and on the fluid-structure interface, the velocity of the fluid mesh is equal to the velocity of the structural mesh. Thus it differs from the staggered approach that this information transfer is not made synchronously. In this approach, both FVM and FEM are a computational method to study a system of field equations which is mathematically described by partial differential equations. For the FEM, the system of study is initially discretized into small but finite-sized elements. For each element, the fields of interest are then approximated based on the partial differential equations by a function such as a linear or quadratic polynomial, thus making a local description to the system. Assembling the descriptions on all elements of the system gives a sparse matrix equation system which can thus be numerically solved. While the FVM is very similar to the FEM in that the system is also discretized into small but finite-sized elements. As the system of study follows conservation laws such as mass, momentum and energy, for each element in such a system, a mathematical description based on flux conservation equations is formulated in a form of a matrix equation. Hence the mathematical description to the system can be realized by solving such a matrix equation.

Liu et al. [63] numerically investigated an FSI system in which two simple fuel rods in parallel have been immersed in an axial water flow. The two rods are considered to be Euler-Bernoulli beams and free to move in any transverse directions. The solvers for the system were CFD software Ansys Fluent as the fluid solver and an in-house beam code as the structural solver. In the simulations, the explicit partitioned scheme was used in which the fluid and structural solvers exchange data within one time step. The results derived by
the numerical calculation show that the critical dimensionless flow velocity for buckling instability is around 25% in difference with experimental data; flutter instability observed in experiments cannot be realized in the simulations, which may be attributed to severe mesh distortion at high flow velocities leading to a crash of the calculation scheme. The obtained results also show the turbulence induced force fluctuation is not strongly dependent on the dimensionless flow velocity, even if the bucking instability arises. However, according to a study by Paidoussis [58], turbulence induced force plays an important role in the buckling instability and is proportional to the second derivative of displacement with respect to the axial coordinate. Thus, the finding from the numerical study by Liu et al. contradicts the experimental data, requiring an improvement to the simulation schemes and models. Okui et al. [76] studied flow-induced GTRF wear experimentally and numerically. In the experiments, a full-scale 17x17 PWR fuel mockup manufactured by Nuclear Fuel Industries, Japan was picked as the structure for testing flow-induced structural vibrations and vibration induced wear. The fuel assembly in this study consists of 9 spacer grids without protective grid, and end plugs of each fuel rod have tight contact with the bottom nozzle surface for reducing the pathway that debris may pass through. During the tests, the fuel assembly was immersed in water flows of bundle-averaged velocity of 5.0 m/s for 1000 hours, from which the effects of grid-to-rod gap on fuel rod acceleration, wear volume and fretting wear work rate were experimentally investigated. Meanwhile, the fretting wear of the fuel mockup was also numerically calculated using a Computational Fluid Dynamics (CFD) code and VITRAN (Vibration Transient Analysis Non-linear) which is a code developed by Westinghouse to specifically simulate flow induced fuel rod vibration and GTRF. The working principle of this numerical calculation can be described as follows: firstly, the steady-state flow
distribution within the fuel assembly is derived; the second step involves the calculation to the average axial and cross flow velocity distributions near each rod; in the last step, excitation force acting on each rod is obtained using an empirical correlation which is a function of average velocity. Then, the derived excitation force data is transferred into the VITRAN code for simulating rod vibration and in particular predicting fretting wear. Comparing the results from the experiments and the simulations, it has been found that the RMS fuel rod acceleration measured from the tests is ± 30 % in relative discrepancy with that obtained from the numerical calculations. As a direct result of such discrepancies, probability distribution of the wear volume predicted by the numerical method may be overestimated. Such a discrepancy in describing the structural dynamics of the rod in the FSI system may be attributed to the applicability of the numerical approach in use such as a staggered approach or a monolithical approach, to describing such a system. Also, selection of the available models for numerically solving the governing equations of the FSI system, in particular for describing the turbulence terms in the Navier-Stokes equations, can have an effect on the estimation accuracy. Thus it requires a more extensive investigation on the performance of such FSI codes through comparing its results with high accuracy experimental data for a system in a nuclear core-like configuration, and potentially calibrating and fine-tuning relevant models for a prediction with higher accuracy and confidence.

In this section, numerical approach to understanding FIV systems was briefly introduced. This numerical method features coupling calculations of structural and fluid fields. As an evidence from large deviations between the numerical results and experimental data, providing experimental data for fine-tuning relevant FSI codes, notably for fretting wear studies in the nuclear industry, becomes a necessary and important work.
2.2.4 Performance study of the correlations

Following the correlations discussed in Section 2.2.2, performance evaluation of the correlations is conducted in this section, through extensive comparisons with the experimental data in the literature. Section 2.2.4.1 gives a general introduction to the experimental work that has been done in the past, including a databank of the experimental configurations involved in the studies. Section 2.2.4.2 makes a detailed evaluation to the performance of the correlations in predicting the vibrating amplitude by comparing with the experimental data in the databank.

2.2.4.1 Experimental tests review

This section endeavors to provide a review of the experimental tests on a flexible clamped-clamped rod subject to an axial flow, including features of interest such as flow configuration, materials, etc.

Pavlica and Marshall [77] experimentally studied the vibrations of a rod assembly immersed in an axial flow. The test rod assembly, as in Figure 2.15, is 1.8 m long with 16 rods in a 4 by 4 square pitch array, and has been surrounded by a Plexiglas shroud. The geometric parameters of the assembly of interest are: outer diameter of the stainless steel rod is 11.11 mm, 14.99 mm in pitch (distance between the centers of two adjacent rods), 0.89 kg/m in mass per unit length for each rod. Also, each rod has been loaded with lead for mimicking the fuel pellet in real nuclear reactors and both ends of each rod have
been secured tightly to end plates to closely approximate the fixed ends of a fuel rod in nuclear reactor cores. In this experiment setup, the spacer grids (having a determined influence on stiffness) are of the spring clip type, whose numbers are either 1, 2 or 4 in the tests. Within the space formed by the separate rods and the Plexiglas shroud, an axial water flow with a temperature of either 20 °C or 60 °C has been accelerated to travel through from the bottom to the top as in the top figure of Figure 2.15, and the FIV amplitude has been measured by an externally mounted pickup. The obtained results showed that the vibrating amplitude increases with increasing flow velocity, but decreases with the fluid viscosity, which are in qualitative agreement with that predicted by the Paidoussis and Burgreen correlations [65], [67]. However, if a quantitative comparison with the correlations of estimating the vibrating amplitude as in Section 2.2.2 is desired to be made, a precaution needs to be taken about how to calculate the parameters involved, since most of the correlations are only applicable to a solitary rod instead of a rod assembly. In this context, Pavlica and Marshall held the following suggestions: values of the assembly stiffness should be obtained from in situ measurements instead of being replaced by the stiffness value of a single rod from calculations; The virtual added mass should originate from a single rod instead of the rod assembly; All the remaining parameters should be kept as in their original context.

It has been widely accepted that the primary excitation mechanism is the randomly fluctuating pressure on the surface of a fuel rod when being subjected to an axial turbulent flow. To study the mechanical response of a rod in such a system, Choi et al. [78] experimentally investigated the structural dynamics of a rod which was subject to a varying force for mimicking the fluctuating force due to turbulent flows. In the study, the rod is 9.5 mm in external diameter, 0.64 mm in thickness and 2189 mm in length. Inside
the rod, lead pellets with density of 11.4 g/cm³ have been filled for mimicking the uranium pellets in real nuclear reactors. The whole setup of the testing apparatus including the installation positions of the measuring sensors which consist of accelerometers and a laser vibrometer, is shown in Figure 2.16. In this apparatus, the fuel rod has been split into 4 spans of interest by spacer grids of either Optimized H Type (OHT) or New Doublet (ND) type, hence lateral movement of the rod segments near the spacer grids is prohibited. At the three-quarters position of the second span, a shaker has been installed to exert a force on the rod under the control of a monitoring system. It also has a laser displacement sensor at the half position of the second span, which has been installed in order to obtain the deflection of the span when the shaker is activated. In the experimental tests, displacement of the fuel rod under different excitation forces of 0.25, 0.50, 0.75, 1.00 and 1.25 N was measured, where the system had been immersed in either air or water. It has been found that under identical excitation levels, the deflection amplitude for the Optimized H Type spacer grid is larger than that of the New Doublet spacer grid, and the oscillating frequency for the Optimized H Type is smaller. These phenomena can be explained by the fact that if the same energy inputs are supplemented into the same structure, it is obvious that the system with a larger deflection amplitude has a lower oscillating frequency, to keep the energy balanced. Meanwhile, it has also been revealed that for both the Optimized H Type and the New Doublet spacer grids, the oscillating frequencies show a decreasing trend with increasing force levels. This trend is a typical behaviour of a structure having nonlinear characteristics, and should be paid great attention to in the design of relevant structures.
Figure 2.15. Schematic of the test section by Pavlica and Marshall [77]

Figure 2.16. Experimental setup including installation positions of sensors by Choi et al. [78]
De Pauw et al. [79] experimentally studied the FIV of a fuel rod designed for a lead-bismuth eutectic cooled reactor which is subjected to similar flow conditions as in most PWR cores. A schematic viewing of the test facility in this test can be shown as in Figure 2.17, which mainly consists of a pump, a drain valve, a throughput valve, a fuel rod, two laser Doppler velocimetry systems for monitoring the vibrations, and connecting pipes. The fuel rod is made of stainless steel and has a diameter of 6 mm and a length of 1400 mm (see Figure 2.18 and Figure 2.17). The fuel rod had been filled with a piece of solid lead-bismuth supported by hollow spacers (#3 in Figure 2.18). Inside the fuel rod, a spring (#6) has also been inserted to simulate pressure applied to the lead-bismuth pellet. As the fuel rod in the reactor of investigation experiences a nominal flow of approximately 2 m/s of lead-bismuth eutectic at 300 °C in the axial direction, in order to yield the same flow conditions (such as a same turbulent flowing regime) for water being the fluid instead of lead-bismuth, a criterion has been applied in which the flow speed and temperature of water have been calculated accordingly. Under this criterion, it has been found that the fuel rod excitation conditions resulting from a lead-bismuth eutectic flow of 2 m/s at 300 °C are similar to those generated by a water flow of 5.2 m/s at 25 °C. The obtained results from the experimental tests showed that in the range studied, the oscillating amplitude is proportional to the water velocity to the power 1.5, which is in good agreement with that in the literature. This phenomenon can be explained as a result of a turbulence change: as water velocity increases, the main excitation mechanism namely turbulence intensity rises up accordingly. Through a novel operational modal analysis as a tool for retrieving fundamental frequency, De Pauw et al. [79] found that the fundamental frequencies for the low mode increase with flow velocity, which clearly contradicts against the trend declared as in Equation (2.13). Hence, the operational modal analysis
methodology and the extremity fixation conditions (highly suspicious by De Pauw et al.) may need to be carefully re-examined to be capable of better characterizing such systems.

Figure 2.17. Schematic of the test facility by De Pauw et al. [80]

Figure 2.18. Design of the fuel rod by De Pauw et al. [80]

Meanwhile, numerous efforts via experimental approach have been made to characterize the FIVs in nuclear core-like systems. Hence, in the remaining part of this section, experimental tests for studying FIV systems are summarized and discussed, featuring flow geometry and rod parameters. For simplicity, the tests conducted by Burgreen et al. [65] is represented as number 1, Pavlica and Marshall [77] as 2, Basile et al. [66] as 3, Ohlmer et al. [43] as 4, Kang et al. [70] as 5, Choi et al. [78] as 6, Elmahdi et al. [81] as 7, Bakosi et al. [82] as 8, and De Pauw et al. [80] as 9. As such, Figure 2.19 and Table 2.2 show a comparison of the parameters in the FIV systems of study. It shows that in most studies, water flow velocity is around 5 m/s and the water has been kept at room temperature.
which makes the tests considerably simple as the temperature influence on the rod response is considered to be small and negligible in first approximation; Rod diameter varies from 10 mm to 30 mm in which a diameter of about 10 mm dominates in all the studies; Rod span length varies from 1 m to 4 m; Rod mass per unit length is around 0.5 kg/m; Rod flexural rigidity varies from 10 N*m² to 1000 N*m² (except the cases of flexural rigidity being 0 and infinity which are for a simulation study only); Hence, Reynolds number can be accordingly obtained as in the order of 10⁵. These parameters have been designed in such a way that mimics the structures and flow configurations in a real pressurized water nuclear reactor core, in which water flows at velocity around 5 m/s, the rod has an outer diameter of about 10 mm and has a length of 3.6 m, mass per unit length of the rod is around 0.4 kg/m, flexural rigidity of the rod is about 40 N*m². However, values of the rod flexural rigidity in an operating nuclear reactor core may be subject to change, as the interface bonding conditions of the rod are altered with burnup (involving changes in the mechanical properties resulting from neutron irradiation). In the present study, such influential factors on the mechanical response of a fuel rod are not considered as in a reduced scale test rig without irradiation.

In this section, a collection of data on the mechanical response of a rod subjected to axial turbulent flows has been created, based on the experimental tests available in the literature. After comparing the parameters of interest, it has been found that the test conditions in these experimental tests are very close to those in real nuclear reactors, thus making it capable of testing the accuracy of the empirical correlations in Section 2.2.3 which are claimed to behave well in nuclear core-like systems. This databank will also be of particular importance in guiding our subsequent plans of investigating an FIV system through an experimental approach.
(c)

Rod diameter (mm)

Data set label

(d)

Rod length (m)

Data set label
Figure 2.19. Comparison of key parameters in the FIV studies: (a) water temperature, (b) water velocity, (c) rod diameter, (d) rod length, (e) rod mass per unit length, (f) rod rigidity
Table 2.2. Parameters of interest in the FIV studies except those in Figure 2.19

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<td>958</td>
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2.2.4.2 Accuracy comparison

In this Section, a performance study of the accuracy of the correlations discussed in Section 2.2.3 is conducted, based on the databank created in Section 2.2.4.1.

In the present study, experimental tests made by Burgreen et al. [65], Pavlica & Marshall [77], Basile et al. [66] and De Pauw et al. [80] become the candidates for verifying the accuracy of the correlations collected in Section 2.2.3, due to their completeness features of the data set. Hence, comparisons in amplitude are made through Matlab between the experimental results and those predicted by the correlations including Burgreen et al. [65], Basile et al. [66], Reavis [41], Paidoussis [67], Wambsganss & Chen [69] and Chen [68]. Figure 2.20 presents such comparisons in a sequential order in which the experimental data are extracted from (a) Burgreen et al. [65], (b) Pavlica & Marshall [77], (c) Basile et al. [66] and (d) De Pauw et al. [80]. It can be seen from all the comparisons that many amplitude values predicted by the correlations, including the correlations by Wambsganss & Chen [69], Burgreen et al. [65] and Reavis [41], deviate more than ± 50% from the experimental results, even deviate up to 2 to 3 orders of magnitude. Interestingly, the correlation proposed by Wambsganss & Chen [69] tends to underestimate the vibrating amplitude while the correlations by Burgreen et al. [65] and Reavis [41] tend to overestimate. The large deviations from the correlation by Wambsganss & Chen [69] may be partially explained by the fact that the resulting amplitude is an RMS value which is clearly less than the maximum amplitude by a certain factor. However, there has not been a consensus on defining the value of this factor [41], thus only the RMS amplitude results obtained from the correlation by Wambsganss & Chen [69] have been presented and
compared with the values of maximum amplitude recorded from the experiment tests. The large deviations from the correlation proposed by Burgreen et al. [65] are partially attributed to the unsatisfactory performance of the correlation in fitting to its original experimental data, as it is evidently presented in Figure 2.20 (a) (see marks in round black) that one order of magnitude of deviation is generally resulted: most amplitude values estimated by the correlation by Burgreen et al. [65] are larger than those obtained from the experimental tests upon which the correlation was based. For the correlation proposed by Reavis [41], its relatively large discrepancies in estimating the vibrating amplitude can be partially explained by the fact that during the process of obtaining the correlation through fitting the experimental data, deviations up to 2 to 3 orders of magnitude have been observed for the test cases with large hydraulic diameters. To reach a good agreement with the experimental data, a test based empirical factor as a function of the ratio of hydraulic diameter to rod length has been preliminarily introduced. However, the applicability of this factor has never been extensively verified in other experimental tests, hence having a potential effect on the performance of the correlation in estimating the amplitude. Unfortunately, most experimental tests discussed in the present section fall into this category of having large hydraulic diameters, hence the performance in predicting the vibrating amplitude by the correlation by Reavis [41] is inevitably influenced.

In comparison, the correlations obtained by Basile et al. [66], Paidoussis [67] and Chen [68] have a better performance, in which many predicted amplitude values are generally having a discrepancy of around ± 50% with those from the experimental tests. Of these three correlations, Chen’s correlation [68] seemingly behaves the best by fitting with the experimental results of study, in which many data fall within the ± 50% boundary limits.
The deviations may also come from the application range on which the performance of the empirical correlations is highly dependent (see Table 2.1), and the correlations generally perform well in the tests with similar application ranges upon which the empirical correlations have been fitted. Taking the comparisons with the experimental tests by Basile et al. [66] (see Figure 2.20 (c)) as an example, it shows that the results obtained from the correlation by Paidoussis [67] are in good agreement with the experimental data, since the application range of the correlation by Paidoussis [67] is very similar to that in the experimental tests.

However, even with correlations with the most accuracy in predicting amplitude, structural dynamics of a rod in a pressurized water nuclear reactor core is still far from being satisfactory described, as the nuclear core is such a subtle system that the vibrating amplitude level plays an important role in the severity of vibration-induced fretting wear [21][22], and subsequently in the safety of the nuclear plant. These findings clearly highlight the need to develop a more accurate and cost-effective tool for practical design applications and potentially in-operation maintenance.

In this section, a databank of experimental tests for studying the FIVs in nuclear core-like systems have been assembled and discussed. Using this databank, the empirical correlations discussed in Section 2.2.3 have been evaluated in the performance of predicting the structural dynamics of interest. This performance suggests a need to develop a more sound and accurate tool for design applications in nuclear reactor cores.
Figure 2.20. Comparison in vibrating amplitude between values obtained from the correlations and experimental data which originate from (a) Burgreen et al. [65], (b) Pavlica & Marshall [77], (c) Basile et al. [66], (d) De Pauw et al. [80]
2.3 Conclusions on the literature review

This chapter presented a literature review of cross and in particular axial flow-induced vibrations, which have been identified as a culprit for fretting wear leading to fuel rod failures in PWR cores. It started with a review on the mechanisms of cross and axial FIVs, and was followed by a description of the system in mathematical form. A further study on the methodologies of solving the mathematical formulae found that many formulae require an input of the force field exerted on the rod, which generally comes from experimental tests and varies with experimental conditions. Meanwhile, in the recent few decades, numerical approaches to describing the FIV systems have attracted great attention, in which the fluid and structure are spatially discretized by the finite volume method and finite element method, respectively. For solving such a system, the fluid in the system can be described by the Navier-Stokes equations, while the structure can be regarded as an Euler-Bernoulli beam and free to move in any transverse direction. An interface between the structural and fluid domains is commonly introduced to exchange the shared information such as force. However, the results obtained from the numerical calculations are not satisfactory as large discrepancies have been observed. As indicated in the study with a numerical approach, it has been further suggested that high accuracy experimental data for a system in a nuclear core-like configuration is necessary for fine-tuning relevant CFD models.

In this Chapter, correlations for estimating the maximum vibrating amplitude of a flexible clamped-clamped rod subjected to axial turbulent flows have also been enumerate and discussed, assuming the first mode dominates the motion pattern, hence the maximum
vibrating amplitude occurs at the mid-span of the rod. This has been followed by a section on evaluating the behaviour of the correlations via comparing with an archival database of vibrating amplitude obtained from experimental tests in the literature. The evaluation results show that the correlations behave differently in predicting the vibrating amplitude, but the overall performance is still far from being satisfactory as the accuracy level achieved so far is not well matching the requirements of nuclear industry applications particularly in face of state-of-the-art nuclear concepts. This performance is partially attributed to the very limited range of relevant parameters being validated for the correlations. Thus it further proves a necessity to conduct more experimental tests on a flexible rod immersed in axial water flows.

### 2.4 Experimental work definition

In this chapter, it has been evidently proven that it is of importance and necessity to generate more benchmark data with high accuracy on axial turbulent flow induced vibration of a single rod as being used in the nuclear industry. Specifically, for this PhD study, the issue of the flow-induced structural vibrations of a system which has a configuration close to a typical pressurized water nuclear reactor core, is investigated experimentally. In this system, the geometry is prototypical of a commercial light water nuclear reactor (including PWRs and BWRs), and the flow parameters are replicating the flow conditions during its full power operation of the reactor. For measuring the rod vibration and potentially the flow field in particular the field around the rod, a transparent test rig is used for optical access hence water temperature is kept at ambient
temperature. This rig features the first experimental facility in development where both the rod vibration and the fluid motion could be potentially simultaneously recorded. Hence, the obtained data from this rig will allow a better trouble-free operation of nuclear fuel elements and heat-exchanger tubes [83], and also allow the fine tuning of the numerical codes for simulating the response of FSI systems.
Chapter 3  Test facility

The previous Chapter led to the clear conclusion that more benchmark data on axial flow-induced vibration of a single rod, in particular its structural dynamics such as vibrating amplitude and frequency, are necessary to be generated. In the present study, such data have been obtained from a test facility. In this Chapter, the test facility employed in this study is introduced in detail, including the test apparatus such as the structural geometry and materials, and the measuring techniques.

In the rest of this Chapter, Section 3.1 gives a detailed description of the test facility, including its mounting configurations and in particular the testing rod such as its geometry and binding conditions. Section 3.2 introduces the measuring techniques of relevance applied in the present test, such as bulk water velocity measurement and rod movement tracking.

3.1  Test apparatus

In this Section, the test section of the test apparatus is firstly introduced, including the geometry and materials. This is followed by a general description of other key features of the apparatus, such as the water flow monitoring system.

The test section of the test apparatus employed in this study, as shown in Figure 3.1 (a), consists of a rod, a Plexiglas confining pipe, two stainless steel confining tubes, a cross
tube fitting, a flow straightener and some connecting appliances. The stainless steel (316L) rod is 1.05 m in length, 8.8/10 mm in I.D./O.D. and has been attached with an end-piece of either a blunt (actual shapes used in many commercial nuclear power plants) or a tapered shape at its free-end. Inside the rod, either lead (mimicking fuel pellets in nuclear fuel rods) or ambient air has been loaded. On the outer surface of the rod near its free-end, several axially aligned and roughly round white ink marks (two are sufficient for relevant measurement usage) have been made. The waterproof ink is manufactured by Valspar. The selection of this type of ink marks generates a large contrast in brightness between the ink marks and the background (see Figure 3.1 (b) and Figure 3.1 (c)), in particular the neighbouring rod surface which has not been covered by any ink marks and has been painted black: when a beam of incident lights travels at a small angle relative to the normal direction to the rod surface, the amount of light reflected by the ink marks per unit surface is larger than that by the rod surface per unit surface. This makes it feasible to track the movement of the marks, thus the movement of the rod, through observing the marks’ movement as the presence of a large contrast in brightness between pixels, using an optical fast-imaging camera. In the present study, the camera is a Panasonic Lumix DMC-FZ200 camera which is equipped with a 1/2.3” MOS image sensor, and has a high framing frequency of 200 Hz and a resolution of 480x640 pixels. This camera has been installed 15 cm in a radial direction away from the rod centerline, maximising its performance in tracking the rod’s movement. As the first mode dominates the motion pattern for a nuclear fuel rod in vibration, the optical tracking position on the rod in the present study has been chosen as to be close to its free-end, where the first-mode has the maximum amplitude, thus resulting in more accurate data.
Outside the rod, clamps have been mounted at the top of the vertically standing rod, onto a confining tube. The confining tube is comprised of a Plexiglas tube of 0.655 m in length starting from 4.5 cm below the cross section of the rod free-end till the mid-span of the rod, and two stainless steel tubes of which one, of 0.4 m in length, has been mounted onto the top of the Plexiglas tube and the other, of 0.955 m in length, has been mounted onto the bottom. The sealings between the Plexiglas tube and the two stainless steel confining tubes, both of which have the same inner and outer diameters, have been achieved through flexible rubber tubes and iron wires. At each joint region, a rubber tube, which has the same inner diameter in value as the outer diameter of the confining tubes, has been pushed over the ends of the two confining tubes, secured with iron wires at its top and bottom sections respectively, such that no leakage out of the tubes may occur when the confining tubes are filled with fluid. The Plexiglas window allows for optical access for viewing the ink marks and thus recording the rod movement with a high speed camera which is set to be at the same height as the ink marks, when the rod is subjected to a piping flow. Also, a flow straightener comprising of 8 tubes in a form of a tube bundle, has been installed within the confining tube at the entrance to the test section, for eliminating the secondary flow when a fluid axially flows into the test section. Downstream of this flow straightener, until the rod free-end tip, there exists a section of confining tube of 0.955 m in length, which is completely filled with flowing fluid. This design is motivated by a concept called the entrance length which is defined as the distance a fluid flows after entering a pipe before the flow becomes fully developed. In most engineering applications, this entry length can be generally approximated by ten times the pipe diameter for a turbulent flow (encountered in the present study), equal to 0.21 m in the present case. Thus the setup of the pipe with 0.955 m in length is sufficient
for a flow to develop. A SWAGELOK cross tube fitting has been mounted on the top of the binding clamps for guiding water flows out of the test section.

The water circulation system, through the apparatus is presented as in Figure 3.2. Water in the holding tank is circulated by the pump (which is a Clarke pump of model number CBM 240, rotating at 2,800 revolutions per minute, maximum head is 48 m, and maximum output is 101 L/min), transported to the test section (flowing from the bottom to the top of the rod as in Figure 3.1 (a)), and travels through the annulus formed between the confining Plexiglas pipe and the testing rod, finally running back to the water tank. At the pump outlet, two ball valves have been mounted to control the flow rate in the test section. A digital thermometer has also been installed for measuring the water temperature within the water tank. A pressure gauge has been installed onto a section of the straight Plexiglas tube between the test section and the water tank, from which the water velocity within the annulus of the test section can be monitored. To get a fully developed water flow for the pressure gauge, a section of straight Plexiglas tube of 1.2 m in length, viewing upstream from the pressure gauging position, has been reserved. The working principle of this pressure gauge will be discussed in the subsequent section.
Figure 3.1. (a) Schematic view of the test section of the experimental rig, including rod free-end shapes, (b) ink marks alignment on the rod surface (only two of the five marks are chosen for relevant calculation use), (c) optical travelling pathway of the lights for tracking the movement of the rod by a camera.
In this study, the selection of stainless steel as test material is mainly due to the fact that its key mechanical properties are similar to those of the materials used in industrial applications, e.g. the 316L stainless steel’s density is very close to that of Zircaloy as the cladding material of a fuel rod in light water reactors. The lead loading inside the rod of study is for mimicking the fuel pellets in nuclear fuel rods as both materials having a close density value, while the air loading inside the rod under study is for calibration and comparison use. In some types of pressurized water nuclear reactors, the rod at its inlet region displays the most frequent interactions with its surrounding structures, since the turbulent pressure acting on the rod surface is relatively larger at its inlet section [76]. Thus, free end configuration has been selected as the boundary condition in the present study. Meanwhile, water flows in the present study are set to be at ambient temperature instead of at around 300 °C for a typical nuclear reactor, which allows for optical access through the transparent Plexiglas tube thus making it feasible to simultaneously measure
the rod movement and fluid flow velocity field, the former of which is achieved through an optical camera, the latter is achieved by the particle image velocimetry technique. Under these configurations, the Reynolds number that can be achieved is in the range of $10^4$, which is one order of magnitude less than that in a typical nuclear reactor core being $5 \times 10^5$. This difference is mainly attributed to the difference in fluid viscosity as this is a function of temperature. In the present study, the ambient temperature instead of 300 °C for a typical nuclear reactor core has also an effect on influencing the density and Young’s modulus of the rod, hence on its mechanical movements when the rod is subjected to axial water flows. Except fluid temperature, loading configurations inside the fuel rods are another aspect that differentiates the present system of test with commercial nuclear reactor cores. For a single fuel rod in a commercial nuclear reactor core, a narrow annulus gap between its Zircoloy cladding and fuel pellets is reserved for keeping the radioactive fission gases from releasing out of the fuel rod, when the fuel rod is initially loaded into the reactor core. During the process of fission reactions, this gap may shrink in size even completely disappear as the fuel pellets may swell or/and crack in such a radioactive and high-temperature environment, from which a fully contact configuration between the fuel pellets and cladding could be developed. Hence, as the increase of burnup, the fuel rod changes from a two-body system at its start-up state to a one-body system after reaching a certain burnup level. As a result, the structural dynamics of the rod changes as the increase of burnup, even the system is under the identical flow conditions. While the system of investigation in the present study for both the air-loading and lead-loading rods only mimics a one-body system, limiting its applicability to study the behaviours of high burnup fuel rods in which the fuel pellets and cladding are in contact with each other.
Meanwhile, surface roughness of the nuclear fuel rods is subject to change during its operation, as the corrosion of the internal structures causes the release of ions and these ions precipitate on the rods’ surface to form particulate material. As a consequence of such rough surfaces, the fluid’s mean velocity and shear stress close to the rod surface may be influenced hence witnessing a change in momentum transport. Such influences depend on several factors such as topography and geometry of the roughness elements. When the roughness height is small such that the roughness elements are confined in the viscous sub-layer, the flow characteristics are not affected by the presence of such roughness elements. However, if the roughness height is relatively large such that the roughness elements protrude through the viscous sub-layer into the turbulent boundary layer even protrude through the turbulent boundary layer, the flow is inevitably influenced in particular the near wall axial velocity diminishes, and the thickness of the turbulent boundary layer may become larger. However, as the effect of rod’s surface roughness on the fluid flow adjacent to the rod wall is a very complicated phenomenon, this effect is not investigated in the present study.

To the author’s best knowledge, the structural dynamics of a cantilever rod subjected to an axial flow directed from the rod free end towards the fixed end has received very little attention in the literature, most of which use very flexible rubber rods. Hence, the present test matrix with a stainless steel rod is very novel for investigating FIV at a fundamental level. The present test rod has been designed specifically to mimic the fuel rods in light water nuclear reactors, the data generated will contribute to calibrate fluid structure interaction computer codes applied in the nuclear community. Also, the novel design of the present test rig allows for simultaneous measurements on rod vibration and flow field, both of which use a non-invasive tool. These aspects highlight the significance of the
present experimental approach on studying the FIV phenomena encountered in many industrial applications, in particular in the nuclear applications.

This Section has provided a detailed description of the test apparatus in the present study, including the structural dimensions, materials, mounting conditions, etc.

3.2 Measuring techniques

In this Section, measuring techniques applied in the current study are introduced and discussed in detail, including rod movement measurement via a fast camera and water flow velocity by means of a pressure gauge.

3.2.1 Bulk water velocity measurement

In the present study, the measurement of the bulk water velocity is indirectly realized by a pressure measurement. The pressure measurement is made by recording the water head difference between two capillaries (see Figure 3.3) which are located on a section of the Plexiglas tube between the test section and the water tank (see Section 3.1). Each of the two capillaries have been kept open to atmosphere at one end and perpendicularly connected to the main transporting pipe at the other end. The Darcy–Weisbach equation, see Equation (3.1), states that for a cylindrical pipe of inner diameter $D$, the pressure loss $\Delta P$ due to viscous effects is proportional to length $L$:
\[
\frac{\Delta P}{L} = f_D \frac{\rho \overline{U}^2}{2D_h}
\]  \hfill (3.1)

where \(f_D\) is the Darcy friction factor, \(\rho\) is the density of the fluid, \(D_h\) is the hydraulic diameter of the pipe, and \(\overline{U}\) is the mean flow velocity. For a fully turbulent flow in a smooth conduit, Blasius [84] estimated the Darcy friction factor using the correlation:

\[
f_D = 0.316 \text{Re}^{-0.25}
\]  \hfill (3.2)

where Reynolds number \(\text{Re} = \frac{\rho D_h \overline{U}}{\mu}\), \(\mu\) is the fluid dynamic viscosity. In the present test setup, the water flow is a fully developed turbulent flow as the reservation of a section of straight pipe with a sufficient length upstream the measuring location, the Plexiglas employed has a smooth inner surface, and in the present study range of Reynolds number, the effect of the relative pipe roughness (about 1.0*10^{-4} relative to its inner diameter) parameter on the Darcy factor is negligible, thus this correlation is applicable to be employed to calculate the Darcy factor. As the pressure drop can be measured through the pressure gauge, the bulk water velocity in the tests can be obtained as this is the only unknown parameter. In this series of tests, the length \(L\) was 500.0 mm, and the hydraulic diameter \(D_h\) of the pipe, equaling to its inner diameter, was 21.0 mm.
3.2.2 Rod movement measurement

In this study, the movement of the rod is captured by a high speed non-invasive camera via observing the ink marks axially aligned on the rod (see Figure 3.1). The captured images from the camera employed in the present study are stored as an RGB video format. For one frame in the video data, the true colour RGB image has been firstly converted to a greyscale intensity image. This is followed by comparing the brightness of a region defined by one pixel with those of its surroundings. Since there is a large difference in brightness between the ink marks and the rod surface (see Figure 3.4), the locations of the pixels representing an ink mark can thus be identified, and each of the pixels is defined by a
binary number. As the camera has been fixed during the test, these pixels can be described by coordinates in a pre-defined coordinate system, in which the zero/equilibrium positions of the rod have been pre-identified. In the present study, the position of each ink mark is set to be located at its centre of mass, which can be described by a block of pixels. Therefore, the movement distance of each ink mark can be depicted as the pixel coordinates difference when the rod oscillates. The factors that may have an effect on identifying the true coordinate and shape of the ink marks, such as the lens distortion of the camera, the refraction of transmitting light between air, water and Plexiglas, have been investigated. One of the most important parameters that can evaluate the accuracy level of such an identification is the total area of ink marks: when an ink mark attached on the rod surface oscillates, some light reflected from the ink mark surface may be distorted as the existence of refraction phenomenon, thus the location of this block of ink mark may be wrongly recorded, the total area of this ink mark obtained by the camera accordingly changes. In the present study, the total area of an ink mark has been found to fluctuate with a standard deviation of 2.3 % relative to its average value. If the shape of the ink mark can be assumed to be a circle and the circle has the same area, then the diameter of the circle has a standard deviation of 1.1 % relative to its mean value. Therefore, the present measuring technique and setup is accurate enough to locate the ink marks’ position in the pixel coordinate system. To retrieve the real movement amplitude of the ink marks, the length on the rod occupying one pixel is required to be computed. This has been realised by calculating the ratio between the distance between two ink marks on the rod surface which has been measured by a ruler and the number of pixels between the two ink marks in the frame. Thus, the movement amplitudes of the ink marks, and also of the rod, can be obtained. The time interval between any two
successive frames is determined by the framing rate of the camera, which is 0.005 seconds. As the natural frequency of the rod employed in the present study is less than 10 Hz, the camera has sufficient temporal resolution to track the movement of the rod. As the rod and flow have been designed to be cylindrically symmetric in the test system, the rod oscillation is assumed to display no preferential direction, the circumferential angle of the camera needs not to be specified. Therefore, the time series of displacement of the vibrating rod can be retrieved. This digital image post-processing has been carried out using a Matlab script.

Through analysis of the resulting vibrating displacement, the structural dynamics of the rod subjected to axial flows, such as the maximum amplitude and frequency, can be obtained for characterisation. For example the vibrating frequency can be retrieved by performing a Fourier transform on the time series of movement. Any periodic discrete time-domain signal, such as the time series of movement in the present study, can be approximated by a sum of several sinusoids with an appropriate amplitude $A$, frequency $\omega$ and phase $\psi$, and expressed as:

$$x(t) = \sum_{i}^{N} A_i \sin(\omega_i t + \psi_i)$$

(3.3)

where the signal $x(t)$ comprises of $N$ sinusoid components. To extract the amplitude and phase of each sinusoid function, a discrete Fourier transform (DFT) is generally applied to transform the finite sequence of equally-spaced time domain signal into a sequence of equally-spaced frequency domain spectrum, which is a complex-valued function of frequency. For a sequence of $N$ real numbers $x_0, x_1, \ldots, x_{N-1}$, another
sequence of complex numbers $X_0, X_1, \ldots, X_{N-1}$, which is a function of frequency, can be resulted from the DFT with the definition:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N}$$  \hspace{1cm} (3.4)$$

where $k$ is $0, 1, \ldots, N-1$. The equally-spaced frequency sequence equals to the multiplication of the number sequence $k$ and the frequency interval, defined as the ratio of the reciprocal of time interval of the time domain signal and the sequence total number. When the oscillation frequency of factor $e^{-i2\pi kn/N}$ well matches that of a harmonic sinusoid function, each element of the multiplication $x_n e^{-i2\pi kn/N}$ will always have the same positive/negative sign, thus the sum of this multiplication is a function of the amplitude of this sinusoid function; when the oscillation frequency of factor $e^{-i2\pi kn/N}$ does not match that of a sinusoid function, the sum of the multiplication $x_n e^{-i2\pi kn/N}$ would be close to zero for a sufficiently large number of discrete data in the time domain, since the multiplication terms have equally numbered positive and negative components, and these components cancel out. Hence, for each resulted complex number, its amplitude implies the amplitude of the corresponding harmonic function, while the arctangent of the imaginary to real components ratio represents the phase. Such a transform generally requires a time in the scale of $N^2$. For efficiently computing the DFT of a finite discrete signal, many algorithms have been proposed to reduce its calculation cost, and this is called Fast Fourier Transform (FFT). In the present study, an FFT on a discrete signal has been realised using a function implemented in Matlab [85].

In this section, the measuring techniques were introduced, including rod movement measurement via a fast camera and water flow velocity by means of a pressure gauge.
3.3 Summary

In this chapter, the test facility employed in the present study was introduced, including the geometry, material, model, and measuring techniques such as measurements on rod movement amplitude and bulk water velocity. The test facility features the first facility available in the literature in which a stainless steel cantilever rod in an axial flow directed from the rod free end towards the fixed end is investigated, the measurements of the rod vibration and flow field can be made simultaneously and both use non-invasive tools. In the present study, an emphasis is placed on investigating the structural dynamics of the rod.

Figure 3.4. A simplified frame recorded by the camera, in which the ink marks and rod surface are distinguish by brightness
Chapter 4 Results and discussions

In this Chapter, experimental results of a flexible cantilever rod subjected to an external water flow, featuring its dynamic characteristics, are presented and discussed. This mainly includes a parametric study of the effects of free-end shape of the rod in either a blunt or a tapered shape, mass per unit length of the rod in which has been loaded with either air or lead for mimicking fuel pellets in nuclear fuel rods, and the water flow velocity.

Through the tests, high accuracy experimental data for a system in a nuclear core-like configuration are derived, which are very necessary for fine-tuning relevant CFD models. Meanwhile, the influential factors of the structural dynamics of the rod in the present configuration in particular the vibrating amplitude and frequency, are unveiled from analysing the obtained data. This is very helpful for structural designs of relevance.

The structure of this chapter is organised as follows. Prior to conducting the tests of interest, consensus between the data obtained from the present test system and the results available in the literature are indispensable to be checked. Section 4.1 depicts the correlations for retrieving the fundamental frequency (Section 4.1.1) and damping ratio (Section 4.1.2) of a flexible cantilever rod in a decay movement.

Section 4.2 introduces a series of preliminary tests in which a cantilever rod is immersed in a still fluid, for validating the measuring techniques. Specifically, Section 4.2.1 describes tests on a rod in open air and Section 4.2.2 describes tests on a rod in still water, in which the resulting vibrating frequency and damping ratio values are compared with the formula discussed in Section 4.1.
Following the validation of the measuring techniques applied in the present test facility, tests on a cantilever rod in pipe flows are conducted in Section 4.3: Section 4.3.1 describes tests on an air-loaded rod with a blunt free-end shape, Section 4.3.2 introduces tests on an air-loaded rod with a tapered free-end shape, Section 4.3.3 describes tests on a lead-loaded rod with a blunt free-end shape, and Section 4.3.4 introduces tests on a lead-loaded rod with a tapered free-end shape. Based on the obtained experimental data, a summary to the main findings is given in Section 4.3.5.

4.1 Parameter calculations

This Section reviews some basic formulae for correlations describing the characteristics of a flexible cantilever rod immersed in either free air or still water in a confining tube, where the rod in air is initialized by striking at its free end with a small hammer for a decay movement, the rod in still water is initialized by deflecting a certain distance away from its equilibrium position at its free end using a wire which is accessed from outside the confining tube through a drilled hole on the flexible rubber tube nearby. In the current study, such characteristics mainly include frequency and damping ratio, which are used in subsequence sections.
4.1.1 Oscillation frequency

This Section reviews analytical results and correlations for obtaining the fundamental frequencies of an Euler-Bernoulli beam immersed in a still fluid.

For a dynamic homogeneous Euler-Bernoulli beam in the absence of a transverse load, the equation of motion can be expressed as [86]:

\[ EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = 0 \]  \hspace{1cm} (4.1)

where \( EI \) is the flexural rigidity, \( y(x,t) \) is the beam displacement as a function of distance \( x \) along axial direction and time \( t \); The beam mass per unit length is denoted as \( m \).

Assuming that the beam’s displacement is the sum of harmonic vibrations, the displacement \( y(x,t) \) can be expressed in a form \( y(x,t) = \text{Re}\left[ \hat{y}(x)e^{-2\pi if t} \right] \), where \( \hat{y}(x) \) is a new form of displacement as a function of distance \( x \), \( f \) is the frequency of vibration in Hz. Then Equation (4.1) can be transformed into:

\[ EI \frac{d^4 \hat{y}}{dx^4} - m(2\pi f)^2 \hat{y} = 0 \]  \hspace{1cm} (4.2)

By applying relevant boundary conditions, the first natural frequency of the beam can be obtained [73]:

\[ f = 0.56 \sqrt{\frac{EI}{mL^4}} \]  \hspace{1cm} (4.3)

Where \( L \) is the beam total length.
If the beam is immersed in a fluid, the natural frequency can also be obtained from Equation (4.3) except mass $m$ being comprised of the beam mass per unit length and an additional term called added mass [87]:

\[ m = m_b + M \]  

(4.4)

\[ M = C_m m_f \]  

(4.5)

\[ C_M = \frac{(D_e / D)^2 + 1}{(D_e / D)^2 - 1} \]  

(4.6)

where $m_b$ is the beam mass per unit length, $M$ is the added mass which is a function of the fluid replaced by beam $m_f = \rho \frac{\pi}{4} D^2$, $\rho$ is the fluid density and $D$ is the beam outer diameter, and $C_m$ is the added mass coefficient which depends on the equivalent diameter of the surrounding tubes $D_e$ and $D$. If the beam is confined by a concentric tube, then $D_e$ is the inner diameter of the tube.

In the present study, as the rod moves with a small lateral displacement, the rod immersed in either air or water can be considered as an Euler-Bernoulli beam. Therefore, Equations (4.3)-(4.6) can be utilized to obtain the natural frequency of the rod as a reference for comparisons with subsequent experimental results.
4.1.2 Damping ratio

Damping ratio is a term which quantifies how well a mechanical system dissipates vibrational kinetic energy. In this section, measuring methods and correlations of obtaining such a damping ratio are discussed.

In engineering applications, the damping ratio is a dimensionless term which describes how oscillations in a system decay after a disturbance. Taking a spring-mass system as an example, if the mass in such a system slowly returns back to its equilibrium position without overshooting, the state of the system is called overdamped. If the mass overshoots its equilibrium position and returns, the state of the system is called underdamped. Between the overdamped and underdamped states, there is a level of damping at which the system returns to its equilibrium position in the minimum amount of time. The equation of motion for a damped harmonic oscillation system with mass \( m \), damping coefficient \( c \) and spring constant \( k \), can be mathematically expressed as:

\[
m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky = 0 \tag{4.7}
\]

where \( y \) is the displacement away from its equilibrium position as a function of time \( t \).

For such a system, the damping ratio \( \zeta \) is a function of mass \( m \), damping coefficient \( c \) and spring constant \( k \), and is defined as:

\[
\zeta = \frac{c}{2\sqrt{km}} \tag{4.8}
\]
In many practical applications, the values of the damping coefficient \( c \) and the damping ratio \( \zeta \) are not known in advance, and must be obtained from experimental measurement.

In the literature, the empirical formula to calculate the damping ratio of a rod in a confining tube filled with water can be expressed as [87], [88]:

\[
\zeta_T = \zeta_v + \zeta_s = \frac{\pi}{2\sqrt{2}} \left( \frac{\rho D^2}{m} \right)^{0.5} \left( \frac{2\nu}{\pi fD^2} \right)^{0.5} \left[ \frac{1}{1-\left( \frac{D}{D_o} \right)^2} \right] + \zeta_s
\]  

(4.9)

Here \( \zeta_v \) and \( \zeta_s \) are the viscous and structural damping, respectively. \( \rho, D, m \) and \( f \) are the water density, rod outer diameter, mass per unit length including the added mass and vibrating frequency of the rod, respectively. The water kinematic viscosity is denoted by \( \nu \), and \( D_o \) is the inner diameter of the confining tube.

The empirical formula for the viscous damping in Equation (4.9) is only valid for \( \pi fD^2/2\nu > 3300 \) and \( D/D_o < 0.5 \) [87], [89], [90], however these restrictions have not been mentioned in some publications [88]. Unfortunately, the two conditions above are not fully satisfied in most of the current tests. Since Equation (4.9) is the equation in the literature for conditions closest to those of the current tests, it is consequently employed here to provide values for some preliminary comparisons with the current data.

The structural damping ratio \( \zeta_s \) as mentioned in Equation (4.9) cannot be deduced deterministically, since it depends on several parameters: number of structural supports, length of each support, mounting conditions, etc. In practice, the structural damping ratio
can only be estimated from experimental tests on a free decaying rod by the following
two approaches:

a) Logarithmic decrement: the damping ratio $\zeta_s$ can be calculated as follows

$$\zeta_s = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}}$$  \hspace{1cm} (4.10)

where $\delta = \frac{1}{n} \ln \frac{y(t)}{y(t + nT)}$ is called the successive peak ratio, here $T$ is the oscillation
period, and $n$ is an integer number of successive positive peaks. $y(t)$ and $y(t + nT)$ are
the peak deflections at time $t$ and time $t + nT$, respectively.

In real experimental tests, to avoid a zero offset influence on the result, the following
formula for obtaining the successive peak ratio is utilized [91]:

$$\delta = \ln \frac{y(1) - y(2)}{y(3) - y(4)}$$  \hspace{1cm} (4.11)

where $y(1)$, $y(2)$, $y(3)$ and $y(4)$ are the displacement values at peaks 1, 2, 3 and 4 as
schematically shown in Figure 4.1, respectively.

b) Amplitude peak fitting function [91]

Free vibration decay of an ideal viscously damped system is restricted by an envelope
developed as $y(t) = \pm A \exp(-\zeta_s \omega t)$, where $y(t)$ is the deflection as function of time $t$,
$\zeta_s$ is the damping ratio, and $\omega$ is the deflecting angular velocity in $\text{rad/s}$. 
In the following sections, the two methods for estimating damping ratio will be simultaneously applied and compared: for the first method, the successive peak ratio expressed in Equation (4.1) will be applied instead of being originally defined, for avoiding a zero offset influence especially on cases where no bias eliminations are made to the raw data.

This section reviewed some basic formulae for obtaining the vibrating frequency and damping ratio of a rod in either air or water, which will be employed to provide values in subsequent sections for some comparisons.
4.2 Preliminary tests on a cantilever rod immersed in a still fluid

In this Section, preliminary tests on a flexible cantilever rod which is immersed in a still fluid and has a blunt free-end shape (see Figure 3.1) are performed, and the obtained results are compared with the correlations listed in Section 4.1, for checking the capability of the measuring techniques to capture the main structural dynamics of interest, and more importantly, to validate the test rig and experimental procedures.

4.2.1 In open air

In the present study, an air-filled flexible cantilever rod (see Figure 3.1) in open air has been initialized by striking at its free end with a small hammer, hence performing a free decay motion. To characterize such a motion, a high speed camera has been setup to capture its time history deflection via the axially aligned ink marks. For more details about the test apparatus and measurement uncertainties involved, readers may refer to Chapter 3 and the Appendix Section, respectively. As the initial conditions have an influence on the final results of interest, and a changing hammer force can pose a difference on the initial deflection, two tests in which the rod has been struck by two people have been conducted, represented as s1 and s2.
The complete time series up to 60 seconds of the two tests’ rod deflections of 25 mm axially away from the free end, and a detailed view ranging from 4 to 5 seconds and from 50 to 51 seconds are shown as in Figure 4.2.

By means of an FFT of the rod deflections at the free end in the two tests by Matlab, see Figure 4.3, the vibrating frequency can be identified as being the frequency corresponding to the highest peak. Due to the small lateral displacement of the rod, the rod can be considered as an Euler-Bernoulli beam. Thus the theoretical natural frequency can be obtained from Equations (4.3)-(4.6) in which Young’s modulus $E$ is $1.93 \cdot 10^{11}$ Pa, and density $\rho$ is $7990$ kg/m$^3$. Comparisons between the experimental data (the error accounts for human reading of the frequency plot) and the theoretical results are shown in Table 4.1, in which the relative deviation computed refers to: (theoretical value – measurement value) / measurement value * 100.

From Table 4.1, the oscillating frequency obtained from the two tests shows a relative difference of less than 0.2%, compared with the results calculated by the theoretical formulae as in Equations (4.3)-(4.6). It can thus be concluded that the test rig and data acquisition system are capable of capturing the oscillating frequency of the rod which is one key feature of interest in the present study. Meanwhile, one can also observe a peak from the frequency plot (see Figure 4.3) near 52.8 Hz in frequency, which is expected to be the 2$^{nd}$ frequency of the rod, showing a good agreement between the experimental data and the results obtained by the theoretical formulae. Other peaks at frequencies except the 1$^{st}$ and 2$^{nd}$ frequencies that have been identified are believed to correspond to the harmonic frequencies as the frequencies have integer multiples of the 1$^{st}$ frequency value.
Figure 4.2. Time histories of deflections in two tests represented as s1 and s2, where for each test, rod is immersed in free air: figures i(a) and ii(a) present complete rod deflections up to 60 seconds at free end, figures i(b) and ii(b) present deflections from 4 to 5 seconds, and figures i(c) and ii(c) illustrate deflections from 50 to 51 seconds after each hammer strike.
Table 4.1. Frequency comparison between experimental results and theoretical value for a cantilever rod in free air

<table>
<thead>
<tr>
<th></th>
<th>s1</th>
<th>s2</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>8.3(±0.1)</td>
<td>8.3(±0.1)</td>
<td>8.312</td>
</tr>
<tr>
<td>Rel. deviation (%)</td>
<td>0.14(±1.2)</td>
<td>0.14(±1.2)</td>
<td>-</td>
</tr>
</tbody>
</table>

To further check the performance of the measuring techniques used in the current study, another parameter namely the damping ratio is herein selected for a validation. One method of obtaining the damping ratio is through the logarithmic decrement as discussed in Section 4.1.2. Using this method, the change in damping ratio values with the number of peaks (defined as \( n \) in Equation (4.10)): 1, 2, 3, 4, 5 and 10 can be obtained as in Figure 4.4(a). In addition, to eliminate the possible influence of the deflection range on retrieving damping ratio, another two series of deflection data with an almost identical deflection range (from 3.95mm decaying to 2.80mm) have been selected from the two tests, represented as s1.2 and s2.2, respectively. A comparison of the damping ratio between the two tests s1.2 and s2.2, and a comparison of damping ratios between tests s1 and s1.2 are made as depicted in Figure 4.4(b) and Figure 4.4(c), respectively. During calculation of the damping ratio for each test, more than one damping ratio value can be obtained as over 1000 peaks have been recorded in each test. Thus in the present study, the damping ratio is represented as a mean and a standard deviation by an error bar as shown in the figures. It is apparent that the damping ratio reaches an asymptotic value of around 0.05%, as the peak difference number increases. It also implies that the estimated damping ratios are a function of the oscillation deflections, and vary slightly from test to test as the tests cannot be replicated under identical conditions. These phenomena have also been observed by Zuo et al. [92], who experimentally studied the galloping of a slender tower in a wind flow.
Figure 4.3. Fast Fourier Transforms of rod deflections at free ends in two tests s1 and s2, where for each test, rod is immersed in free air.
Figure 4.4. Damping ratio by logarithmic decrement as a function of peak differences number from: (a) complete datasets, (b) partial datasets of two tests with deflection values ranging from 3.95mm to 2.80mm, and (c) complete and partial datasets in test s1
Another method of estimating the damping ratio is through exponential fitting (see Section 4.1.2). In the current study, a Matlab exponential fitting function has been applied, and the oscillation frequency has been set to be 8.3 Hz. The results are presented as in Table 4.2:

Table 4.2. Damping ratio in two tests by exponential fitting, where for each test, rod is immersed in open air

<table>
<thead>
<tr>
<th></th>
<th>s1</th>
<th>s2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio</td>
<td>0.0435%</td>
<td>0.0396%</td>
</tr>
</tbody>
</table>

It can been seen from Table 4.2 that the estimated damping ratio of each of the two tests reaches a value around 0.04%, which is within one standard deviation away from the mean estimated damping ratio by the logarithmic decrement method. The differences in the damping ratios estimated by the two methods can be attributed to the measurement accuracy of the oscillation frequency. Hence, it can be summarized that the two methods reach a good agreement in estimating the damping ratios.

In the literature, the structural damping ratio of slender steel structures is around 0.1% to 0.2% [93], and the viscous damping ratio, according to Equation (4.9), equals to 0.01%. Thus the total damping ratio ranges from 0.11% to 0.21%. This is in the same range as that calculated by the logarithmic decrement approach at peak difference of 1, but relatively larger deviations are observed by amplitude peak fitting and by logarithmic decrement at larger peak difference numbers such as 10. The difference can be explained by the fact that the structural damping is subject to change due to various factors such as mounting conditions, support length, etc. In particular, the criteria required in Equation (4.9) are not fully satisfied as some researchers have discussed [87], [89], [90]: \( \pi fD^2 / 2 \nu \) has a value
of around 100 if relevant parameters in the current study are applied, which is clearly out of the range required (larger than 3300), this may be another influential factor to the deviations in the tests.

This section has presented decay vibration tests using a cantilever rod in air, from which the vibration frequency and the damping ratio can be obtained. From comparisons between the experimental data and the results obtained by the calculation formulae, it has been found that the test rig and data acquisition system are capable to capture the features of interest. Meanwhile, it was also found that for a free vibrating rod in air, the damping ratio value obtained by logarithmic decrement is subjected to change as changing the peak difference number, and the damping ratio value approaches to an asymptotic value at a large peak difference number.

**4.2.2 In a water-confining tube**

In this section, an air-filled flexible cantilever rod subjected to external still water in a coaxial confining tube is studied. The configurations of the current tests as shown in Figure 3.1, are identical with the tests in Section 4.2.1 where the blunt free-end rod vibration has been initialized by deflecting a certain distance away from its equilibrium position at its free end using a wire which is accessed from outside the confining tube through a drilling hole on the flexible rubber tube nearby, except for adding a coaxial confining tube made of Plexiglas of 21 mm and 25 mm in inner and outer diameters respectively, and the tube has been filled with either air or water.
Two sets of tests have been conducted in air, and seven tests in water, represented by symbol # followed with an algebraic number. Taking one typical test in each fluid as an example, the complete and the partial time histories (two partial time histories: one in the beginning period, the other in the ending period) of the deflections of 25 mm axially away from the free end are shown in Figure 4.5.

It shows that though the initial deflections of the two tests are in a similar range, the rod vibrating in the confining tube filled with water has a faster decay rate than that in air: the deflection value gets halved after about 5 seconds in air but only about half a second in water. This is attributed to the different dissipation rates in different fluids: energy is dissipated with the surrounding fluid when a rod moves, and this energy dissipation rate in water is higher than in air.

To characterize the rod dynamics in the tests, an FFT function, included in Matlab, is applied to the time history deflection data, from which the vibrating frequency of the rod can be identified from the obtained PSD plot as being the frequency having the largest peak power. Figure 4.6 shows the PSDs of deflections at the free end of the cantilever rod in a confining tube filled with air or water. The vibrating frequency in air is clearly larger than that in still water, which can be explained by the effect of added mass: for a rod immersed in a fluid, the movement of the rod must push some volume of its surrounding fluid, as if an added mass was applied to the rod. Also, either through a half-power bandwidth calculation or by simply observation, it can also be seen from Figure 4.6 that the spectrum around the fundamental frequency peak gets broadened more in water than that in air, the spectra in the low frequency range, e.g. < 10 Hz, is increased compared with that in air. This phenomenon can be explained by that due to the
existence of the surrounding water, the energy dissipation of the rod gets influenced by fluid damping, and has a larger frequency range.

Comparing the obtained frequency values from the present tests with that calculated by Equations (4.3)-(4.6): Table 4.3 shows the frequency comparisons of two tests for a cantilever rod in a confining tube filled with air, and Table 4.4 shows the frequency comparisons of two tests for a cantilever rod in a confining tube filled with still water, in which the relative deviation computed refers to: (theoretical value – measurement value) / measurement value * 100, the error accounts for human reading in every frequency plot. It shows that for both experimental configurations, the relative deviations are less than 1.3 % and 15 %, respectively. This behaviour further proves the capabilities of the present measuring techniques in capture the vibrating frequency of the rod of study.

Another key parameter in the present study, damping ratio of the cantilever rod in a confining tube filled with air can be obtained by the logarithmic decrement method and the exponential fitting method. Figure 4.7 shows the damping ratio change with the peak difference number ranging from 1 to 10. It shows that the damping ratio of the rod confined by a tube filled with air has an asymptotic value of around 0.18 %. Table 4.5 shows a summary of the damping ratios from the present tests and those calculated by Equation (4.9). The two methods are in good agreement in estimating the damping ratio of the rod which is in the range of 0.12 % to 0.18 %. Meanwhile, the damping ratio of the rod in the current configuration is larger than that of an identical rod but without a confining tube being 0.05 %. This can be explained by the fact that because the surrounding fluid is confined, this affects the motion and energy dissipation of the rod in vibration.
Figure 4.5. Time history of deflection at rod free end after releasing from a deflection state: figures i(a) and ii(a) present complete rod deflections for rods in air and in water, figures i(b) and ii(b) present deflections in the early periods after releasing for rods in air and in water, and figures i(c) and ii(c) illustrate deflections in the ending periods for rods in air and in water respectively.
Figure 4.6. Power spectral densities of deflections at the free end of a cantilever rod in a confining tube filled with air or water
Table 4.3. Frequency comparison between experimental results and theoretical values for a cantilever rod in a confining tube filled with air

<table>
<thead>
<tr>
<th></th>
<th>#1</th>
<th>#2</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>$8.2(\pm 0.1)$</td>
<td>$8.2(\pm 0.1)$</td>
<td>$8.31$</td>
</tr>
<tr>
<td>Rel. deviation (%)</td>
<td>$-1.3(\pm 1.2)$</td>
<td>$-1.3(\pm 1.2)$</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.4. Frequency comparison between experimental results and theoretical value for a cantilever rod in a confining tube filled with water

<table>
<thead>
<tr>
<th></th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>$5.4(\pm 0.1)$</td>
<td>$5.3(\pm 0.1)$</td>
<td>$5.3(\pm 0.1)$</td>
<td>$5.4(\pm 0.1)$</td>
<td>$5.3(\pm 0.1)$</td>
<td>$5.4(\pm 0.1)$</td>
<td>$5.3(\pm 0.1)$</td>
<td>$6.063$</td>
</tr>
<tr>
<td>Rel. deviation (%)</td>
<td>$12.3(\pm 1.9)$</td>
<td>$14.4(\pm 1.9)$</td>
<td>$14.4(\pm 1.9)$</td>
<td>$12.3(\pm 1.9)$</td>
<td>$14.4(\pm 1.9)$</td>
<td>$12.3(\pm 1.9)$</td>
<td>$14.4(\pm 1.9)$</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4.7. Damping ratios of a cantilever rod in a confining tube filled with air, as function of peak difference number ranges from 1 till 10
Table 4.5. Damping ratio comparison between experimental results and calculation values by empirical formula for a cantilever rod in a confining tube filled with air

<table>
<thead>
<tr>
<th></th>
<th>Logarithmic decrement</th>
<th>Exponential fitting</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio</td>
<td>0.18% (± 0.10%)</td>
<td>0.122%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.02%</td>
</tr>
</tbody>
</table>

Due to relatively strong damping of a rod immersed in water confined by a tube, the rod deflection decays very fast, thus relatively small deflections are recorded at the very end of the test. In the present study the damping ratio of the rod immersed in still water in a confining tube is investigated, only the early period of about 6 to 10 consecutive cycles after the rod being released from its initial deflection state, has been chosen for relevant damping calculations. Table 4.6 shows a summary of the damping results for such a rod in a water-filled tube, by the logarithmic decrement method, the exponential fitting method and Equation (4.9). Again, the two methods show good agreement in estimating the damping ratio of the rod.

Table 4.6. Damping ratio comparison between experimental results and calculation value by empirical formula for a cantilever rod in a confining tube filled with water

<table>
<thead>
<tr>
<th></th>
<th>Logarithmic decrement</th>
<th>Exponential fitting</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio</td>
<td>3.6% (± 1.0%)</td>
<td>4.0%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.5%</td>
</tr>
</tbody>
</table>

The damping ratio value changes from 0.05% to 0.16% and 3.8% for the case of a rod in open environment to that of a rod confined first in an air-filled tube and then in a water-filled tube, respectively. The damping increase can be explained as follows: when a rod vibrates in an air-filled tube, the displaced air will exert a force on the rod in a direction which is opposite to that of the movement direction, thus partial kinetic energy of the rod
will transform into heat. However, the force for a rod in an open environment without a confining tube is almost negligible as the surrounding displaced air has nearly no “feedback” to the oscillating rod. The damping difference in water-confined and air-confined tubes lies in that water has a larger viscosity than air, meaning larger resistance to deformation, thus more kinetic energy will be transformed into heat within a cycle.

This section presented tests of a rod immersed in either air or water. The obtained oscillating frequency in both air and water shows good agreement with the results by theoretical formulae. It has also been found that the damping ratio value of a rod confined in a water-filled tube is larger than that of a rod in open environment, which can be explained as a result of more energy dissipation in a water-filled tube. These findings also prove the capability of the measuring techniques used in the present test facility to capture the main structural dynamics of interest, thus is suitable to be applied in subsequent studies.

4.3 Tests on a cantilever rod in pipe flows

In this section, the structural dynamics of a cantilever rod with varying loading materials and free-end shapes in an axial water flow are investigated. In the present study, the loading materials include air and lead, the free-end shapes of the rod include a blunt-end and a tapered-end shape.
4.3.1  Tests on an air-loaded rod of blunt-end shape

It has been found that the free-end shape of a cantilever rod plays an important role in the structural dynamics subjected to an axial flow [83]. Therefore, to investigate the influences of free-end shape on the structural dynamics, two series of tests have been designed and conducted: one was with a blunt-end shape (the actual shape of many nuclear fuel rods), the other was with a tapered-end shape. In the tests, the water velocity value being achieved was up to 3.5 m/s limited by the pump capability. As the maximum water velocity that can be achieved is reasonably close to the 4.0 m/s typical water velocity of PWR nuclear reactors, it is consequently utilized here to mimic the real water flows in nuclear water reactors.

The first series of tests featured a blunt shape free end, its mounting configuration shown in Figure 3.1 and Figure 3.2, where water with velocities ranging from 1.5 m/s to 3.5 m/s was pumped from the free-end to the clamped-end of the rod. A transparent confining tube made of plexiglas has been setup, through which the rod vibrations can be recorded by a non-invasive camera (200 Hz in frequency as previously utilized). In the present tests, the rod displacement was measured at locations very close to the rod free end: 12, 22, 32 and 42 mm axially away from the free end. In the present section, the rod displacement at a location 32 mm axially away from the free end is presented. Figure 4.8 shows the time trace of displacement at a flow velocity of 1.49 m/s and temperature of 25 °C, Figure 4.9 shows the time trace of displacement at a flow velocity of 2.33 m/s and temperature of 23.4 °C, and Figure 4.10 shows the time trace of displacement at a flow velocity of 3.41 m/s and water temperature of 20.9 °C, respectively:
Figure 4.8. Time trace of displacement at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water at a velocity of 1.49 m/s and temperature of 25 °C (a) complete (b) 10 to 20 seconds
Figure 4.9. Time trace of displacement at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water at a velocity of 2.33 m/s and temperature of 23.4 °C (a) complete (b) 20 to 30 seconds
Figure 4.10. Time trace of displacement at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water at a velocity of 3.41 m/s and temperature of 20.9 °C (a) complete (b) 10 to 20 seconds
From the time traces of displacement in the tests, the RMS and maximum displacements under different water velocities ranging from 1.5 to 3.4 m/s (Reynolds number from 18,144 to 38,022 when the hydraulic diameter of the annulus is chosen as the characteristic length), and ratio of the maximum to the RMS values can be obtained as in Figure 4.11. It clearly shows that the vibrating displacement increases with velocity, which is physically consistent: at the velocities studied, the vibrating mechanism is mostly turbulence driven, where the pressure field around the rod fluctuates due to turbulence, thus a larger Reynolds number corresponds to a larger displacement. The observed phenomenon is also in qualitative agreement with that observed by Rinaldi & Païdoussis [94], [95], in which a flexible rubber-made cantilever rod subjected to an axial air flow directed from the rod free end towards the fixed end, was studied experimentally. Since it is the only experiment in the literature for conditions close to the current tests, it is consequently employed here to provide some preliminary comparisons with the current results.

Furthermore, the maximum to RMS displacement ratio is around 2.3 (taking the uncertainties into account), showing a good consensus with that observed in an experimental test conducted by Rinaldi & Païdoussis, in which a value of 2.5 was obtained [94]. For more details about the uncertainties, reader may refer to the uncertainty analysis in the Appendix Section of this report.

For comparison purposes, water velocity is routinely expressed in a dimensionless way as [95]:

\[ u = \left( \frac{\rho A}{EI} \right)^{1/2} UL \]  

(4.12)
where \( \rho, U, A, EI \) and \( L \) are the water density, uniform axial velocity, cross-sectional area, flexural rigidity, and length of the flexible cylinder, respectively.

If the above parameters are applied, then the dimensionless water velocities covered in the study range from 0.072 to 0.17. In this range, it has been found that the displacements of both maximum and RMS are in quantitative agreement with that obtained by Rinaldi & Paidoussis [94].

![Graph showing RMS and Maximum displacements at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity values ranging from 1.5 m/s to 3.5 m/s, and ratio of Maximum to RMS values (error bars represent precision limit of maximum displacement; precision limits of RMS are 0.0001 mm)](image)

Figure 4.11. RMS and Maximum displacements at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity values ranging from 1.5 m/s to 3.5 m/s, and ratio of Maximum to RMS values (error bars represent precision limit of maximum displacement; precision limits of RMS are 0.0001 mm).

Power spectrum density (PSD) of the deflection can be obtained via an FFT. These are presented for a water velocity of 1.49 m/s and temperature of 25 °C in Figure 4.12 i(a)-(d), for a water velocity of 2.33 m/s and temperature of 23.4 °C in Figure 4.12 ii(a)-(d), and for
a water velocity of 3.41 m/s and temperature of 20.9 °C in Figure 4.12 iii(a)-(d), respectively (to more conveniently identify peaks of interest, each test case is plotted in four scales: linear-logarithmic with 0 to 100 Hz of frequency range, logarithmic-logarithmic with 0 to 10 Hz of frequency range, linear-logarithmic with 0 to 10 Hz of frequency range, and linear-linear with 0 to 10 Hz of frequency range).

As shown in Figure 4.12, one frequency can be explicitly identified as dominating the rod movement under each flow condition, which corresponds to the broad peak in the power spectrum, this is the fundamental vibrating frequency. Thus, the relationship between the water velocity and vibrating frequency can be presented as in Figure 4.13 (Reynolds number ranges from 18,144 to 38,022 when the hydraulic diameter of the annulus is chosen as the characteristic length, and dimensionless water velocity from 0.072 to 0.17 as in Equation (4.12)).

It has been found that the vibrating frequency shows a small decrease as velocity goes up, e.g. 5.7 Hz to 5.0 Hz, which is in the same range as that in still water, 5.3 Hz in the previous section. This indicates that the axial flow velocity has a minor effect on the oscillating frequency, at least in the present range studied. The decreasing trend has also been witnessed by Rinaldi & Païdoussis [95]. In the tests, they found that the oscillating frequency as a function of velocity follows a paraboloidal relationship, which drops from around 1.1 to 0.2 Hz at dimensionless air velocities of 0.4 and 1.6. As the dimensionless velocity range in the present study is much smaller than that in the tests by Rinaldi & Païdoussis [95], this paraboloidal changing trend is too early to be easily observed. Generally, the decrease of vibrating frequency as a function of flow velocity may be attributed to the increase of damping from fluid. When the flow velocity increases thus
the flow becomes more turbulent, not only the component of the fluid pressure on the rod close to its natural frequency which results in a larger displacement of vibration, becomes larger in magnitude, but other components of the fluid pressure concurrently increase in magnitude, hence causing larger damping on the vibrating rod. This phenomenon is similar to that observed in a typical spring-mass system in which a mass is attached to a spring and a damper: the vibrating frequency decreases with the increase of damping.
Figure 4.12. PSDs of displacement at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water: i(a), i(b) for water velocity 1.49 m/s and temperature of 25 °C, ii(a), ii(b) for 2.33 m/s and 23.4 °C, and iii(a), iii(b) for 3.41 m/s and 20.9 °C in range of 0 – 100 Hz in linear-log and log-log scales, respectively. While i(c), i(d), ii(c), ii(d), iii(c) and iii(d) present PSDs of the three tests in frequency range of 0 – 10 Hz in linear-log and linear-linear scales, respectively.
Figure 4.13. Vibrating frequency of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity values ranging from 1.5 m/s to 3.5 m/s, through two different scales on y-axis: (a) from 0 to 6 Hz showing its overall range, (b) from 4 to 6 Hz showing its changing trend.
The PSD comparisons of the rod in air, still water and flowing water of 3.41 m/s are shown in Figure 4.14. For the PSD of flowing water of 3.41 m/s, to remove the spikes in particular those near the fundamental vibrating frequency, a Savitzky-Golay filter has been applied to smooth the data. A Savitzky-Golay filter is a digital filter for smoothing a set of digital data points which is achieved by fitting successive data points with a low-degree polynomial by the method of linear least squares. Figure 4.14 (a) shows the PSDs of deflections for a water flow of 3.41 m/s: the raw data shown in blue, and the filtered data is in red through a Savitzky-Golay filter whose filter coefficients are derived by performing an unweighted linear least-squares fit to 21 data points using a polynomial of degree 3. After applying this filter, the PSD plot gets smoothed over the whole frequency range, thus making it more convenient to identify the peaks. In the present section, this smoothing has a negligible effect on the identification of the fundamental frequencies, thus it is not presented here in detail, but is applied in subsequent sections.

From Figure 4.14 (b), it has also been found that the spectrum around the fundamental frequency peak in flowing water becomes more broadened, compared with those in still water. This can be partially attributed to the introduction of turbulence from the water flow, under which energy is dissipated over a larger frequency range. As discussed in Section 3.2.2, the PSD value at a certain frequency in a Fourier transformed plot is a function of the corresponding oscillation amplitude; for a free decay motion, this PSD value relates with the RMS value of the oscillation amplitude. Hence, the PSDs near the fundamental frequency peaks in still water and air are very close, and both are larger than that in flowing water of 3.41 m/s in velocity, as the RMS oscillation amplitude values for the tests in still water, air and flowing water of 3.41 m/s in velocity are 0.93, 0.72 and 0.22mm respectively.
Figure 4.14. (a) PSDs of displacement at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity 3.41 m/s: raw data is in blue and smoothed data is in red colour from 6 to 10 Hz. (b) PSDs of displacement at the free end of a cantilever rod in air (blue colour), still water in a confining tube (green colour) and flowing water of velocity 3.41 m/s (red colour).
In order to check the effectiveness of low-pass filters in removing unwanted signals in the retrieved data, a Butterworth low-pass filter was designed and applied to the raw data. A Butterworth filter is a type of signal processing filter which is designed to reject unwanted signals, and features a flat frequency response in the passband. In the current tests, a Butterworth low-pass filter of cutoff frequency 40 Hz and order ranging from 1 to 5, is applied to raw time series data from 10 s to 10.2 s and to PSDs of time series at a water velocity of 3.41 m/s. The results are shown in Figure 4.15. In influencing the PSDs, the applied filtering shows a faster attenuation as in higher filter order. In influencing the time series data, the filtered data show a smoother trend, even more explicit at large filter orders where high frequency sections are greatly attenuated. To investigate the features of the removed signals, the signal difference of the raw data and the filtered data was examined by a Jarque-Bera test, which is in statistics a goodness-of-fit test of whether sample data characteristics are matching a normal distribution. The Jarque-Bera test shows that it cannot reject the null hypothesis of data coming from a normal distribution at the 5% significance level. Thus, the removed signals can be deemed to fit a normal distribution. As noise signals are recognized as in normal distribution, it is possible to regard the removed signals as noise. This result will be of great importance for future use in more complex gauging systems.

In conclusion, this section showed that the maximum and the RMS displacement of a fixed-free flexible rod under pipe flow directed from the rod free end towards the fixed end concurrently increase with water velocity, as this is mainly turbulence driven in the velocity range studied. Meanwhile, it has been shown that the dominating vibratory frequency declines with water velocity, which is in quantitative agreement with that described in the literature.
Figure 4.15. Effectiveness of applying Butterworth low-pass filter of cutoff frequency 40 Hz, order ranges from 1 to 5 on (a) time trace of displacement from 10 s to 10.2 s, (b) PSDs of displacement at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity 3.41 m/s.
In this Section, a cantilever air-loaded rod with a tapered free-end shape, roughly semi-spherical, is studied in a pipe water flow with velocity ranging from 1.5 to 3.5 m/s, where water flows from its free end towards its fixed end. Some of the key structural dynamics in each test such as vibrating amplitude and frequency, are obtained as a function of water velocity and compared with those of the rod with a blunt free end shape in Section 4.3.1.

In the present test, the experimental setup can be depicted as in Figure 3.1 and Figure 3.2, in which a rod with a tapered free end shown in Figure 3.1 (a) has been utilized. During the test, the deflection of the rod was measured at locations very close to the rod free end tip: 10, 20, 30, 40, 50 and 60 mm axially away from the free end tip. In this section, the deflection characteristics at the location 30 mm axially way from the free end tip are presented. In the test, water velocity between the two cylindrical shells ranged from 1.50 to 3.52 m/s, and its temperature ranged from 19.5 to 25.6 °C. The time traces of vibrating displacement with water flows of 1.50, 2.69 and 3.52 m/s in velocity are shown in Figure 4.16, Figure 4.17 and Figure 4.18, respectively:
Figure 4.16. Time trace of displacement at the free end of a cantilever rod with a tapered free-end shape in a confining tube filled with water at a velocity 1.50 m/s and temperature 22.2 °C: (a) complete, (b) 40 to 50 seconds
Figure 4.17. Time trace of displacement at the free end of a cantilever rod with a tapered free-end shape in a confining tube filled with water at a velocity 2.69 m/s and temperature 24.0 °C: (a) complete, (b) 30 to 40 seconds
Figure 4.18. Time trace of displacement at the free end of a cantilever rod with a tapered free-end shape in a confining tube filled with water at a velocity 3.52 m/s and temperature 19.5 °C: (a) complete, (b) 50 to 60 seconds
By computing the DFT of the signals measured using an FFT algorithm, the fundamental vibrating frequency can be identified through the obtained PSD plots, taking two tests as an example: one has a water velocity of 2.69 m/s and temperature of 24.0 °C, the other one has a water velocity of 3.52 m/s and temperature of 19.5 °C. In the PSD plots in Figure 4.19, around 0.1 to 4.5% of total data (corresponding to 35 and 1621 data points, respectively) in the frequency domain have been selected and smoothed by a Savitzky-Golay filter which is almost identical with the one applied in Section 4.3.1. The relationship between the water velocity (Reynolds number from 17,050 to 41800 when the hydraulic diameter of the annulus is chosen as the characteristic length) and the corresponding fundamental vibrating frequency is presented in Figure 4.20 (based on the raw data and smoothed data).

From Figure 4.20, it can be seen that in the studied range, the fundamental vibrating frequency obtained from the raw data fluctuates slightly, but the one obtained from the smoothed data shows a decreasing trend. This difference can be explained by the fact that in the unsmoothed plot the fundamental vibrating frequency is identified as the frequency corresponding to the highest peak, in the presence of some spikes nearby. In the smoothed plot, there spikes have been removed. The changing trend observed from the smoothed data is found to be in good agreement with that observed by Rinaldi and Paidoussis [94], who made an experiment on a very flexible rubber-made cantilever rod in axial air flow directed from the rod free end towards the fixed end. This also indicates the necessity of applying a filter to such PSD data, whose results are used for comparisons in subsequent sections.
Figure 4.19. PSDs of displacement at the free end of a cantilevered rod with a tapered free-end shape in a confining tube filled with water, of which 0.1%, 1.2%, 2.3%, 3.4% and 4.5% of total data sets are chosen as the span length for smoothing by a Savitzky-Golay filter (a) water velocity of 2.69 m/s and temperature of 24.0 °C (b) water velocity of 3.52 m/s and temperature of 19.5 °C
Figure 4.20. Frequency change as water velocity, for a cantilever rod with a tapered free-end shape in a confining tube filled with water, through two different scales on y-axis: (a) from 0 to 7 Hz showing its overall range, (b) from 4 to 7 Hz showing its changing trend.
The RMS and maximum displacement with water velocity ranges from 1.5 to 3.5 m/s, and ratio of the maximum to the RMS values are shown in Figure 4.21. It clearly shows that the vibrating displacement increases with velocity, this phenomenon is in qualitatively agreement with that observed in experiments by Rinaldi and Paidoussis [94], [95], in which a very flexible cantilever rod subjected to an axial air flow travelling from the free end towards the fixed end was studied experimentally. The ratio of the maximum to the RMS displacement has an average value of around 2.2 (taking the uncertainties into account), showing a good agreement with that observed in an experimental test conducted by Rinaldi and Paidoussis, in which a value of 2.5 was obtained [94]. For more details about the uncertainties, reader may refer to the uncertainty analysis in the Appendix Section of this report.

Figure 4.21. RMS, Maximum displacements at the free end of a cantilever rod with a tapered free-end shape in a confining tube filled with water, and Maximum to RMS ratio as a function of water velocity (error bars represent precision limit; precision limits of RMS are 0.00006 mm)
Comparison of the obtained vibrating characteristics such as fundamental frequency (smoothed using a Savitzky-Golay filter), maximum and RMS displacements from the blunt-end rod discussed in Section 4.3.1 and those from the tapered-end rod in the present section, are shown in Figure 4.22, Figure 4.23 and Figure 4.24 respectively. The movement signals in both tests are obtained at an identical position 1018 mm axially away from the rod fixed end, thus the time series of movement for the tapered-end rod is obtained by linearly interpolating the measured movement values at the nearest neighbouring positions. It is interesting that for the tapered-end rod, its fundamental frequency is higher and both the maximum and the RMS displacements are lower in the range studied (Reynolds number ranges from 15,000 to 40,000) than that of the blunt-end rod. The different behaviours partially result from water flow streamline changes caused by the rod-end shape, notably its influence on the entry region. As the rod is tapered at the entry region in a roughly semi-spherical shape, flowing water is better ‘guided’ than with the blunt-end rod, thus resulting in less turbulence into the system after the entry region.

The data presented in this section is that for a tapered end shape rod subject to a pipe flow directed from the rod free end towards the fixed end, the maximum and the RMS displacements increase with water velocity, as this is mainly turbulence driven in the velocity range studied. It also showed that the dominating vibratory frequency declines with water velocity. Furthermore, for a tapered end shape rod subject to an external water flow, the oscillating amplitude is smaller but the oscillating frequency is larger compared with those with a blunt shape. These findings will allow fine tuning of the numerical codes for simulating the response of FSI systems, and be beneficial to the design of relevant structures in the nuclear industry.
Figure 4.22. Comparison of vibrating frequency change as Reynolds number, for cantilever rods with blunt free-end and tapered free-end shapes in an identical confining tube filled with flowing water, with two y-axis ranges: (a) from 0 to 7 Hz, (b) from 4 to 7 Hz
Figure 4.23. Comparison of RMS displacement change at the free ends of cantilever rods with blunt free-end and tapered free-end shapes in an identical confining tube filled with flowing water, as a function of Reynolds number.

Figure 4.24. Comparison of maximum displacement change at the free ends of cantilever rods with blunt free-end and tapered free-end shapes in an identical confining tube filled with water, as a function of Reynolds number (error bars represent precision limits).
4.3.3 Tests on a lead-loaded rod of blunt-end shape

In Sections 4.3.1 and 4.3.2, the effect of the free end shape of a cantilever rod subject to an axial water flow on its structural dynamics has been studied. While in this and subsequent sections, the effect of the loading material of a cantilever rod immersed in an axial water flow on its structural dynamics such as vibrating frequency and amplitude is investigated. In the present study, the loading material inside the cantilever rod refers to either air (as in Sections 4.3.1 and 4.3.2) or lead.

In this Section, a cantilever rod with a blunt free-end and with a lead loading internally (see Figure 3.1) is firstly studied in open air and a still water confining tube for verifying the performance of the designed system, and then in an axial water flow (see Figure 3.2 for more details). Specifically, the addition of the lead spheres makes mass per unit length of the rod be $0.586 \pm 0.001 \text{ kg/m}$, and the rod displacement was measured at locations very close to the rod free end: 39, 48, 58 and 68 mm axially away from the free end. In this section, the vibrating characteristics of the rod at the location 48 mm axially away from the free end tip are presented.

For verifying the performance of the designed system and in particular of the measuring system, the vibrating frequency of the rod in a free decaying motion (being initially deflected at a certain distance away from its equilibrium position at the free end, and released as has been done in Section 4.2) is firstly obtained via making an FFT of the time series of displacement, and the resulting frequency value is then compared with that computed by Equations (4.3)-(4.6) which are used to retrieve the frequency of an Euler-Bernoulli beam. Such comparisons for the rod immersed in air and still water are depicted.
in Table 4.7. It shows that the vibrating frequencies in air and still water are 4.1 and 3.7 Hz respectively, less than 8.3 and 5.4 Hz the corresponding vibrating frequencies of the rod in an almost identical condition except for the internal loading material being air (see Section 4.2). These differences in frequency can be explained as the difference in the mass value as presented in Equations (4.3)-(4.6): a larger mass per unit length results in a lower vibrating frequency, while the mass value depends on both the rod’s mass and the added mass. Meanwhile, the relative deviations of the vibrating frequency under both conditions are around 0.2 % and 0.3 % respectively. This further proves the capabilities of the present measuring techniques in capture the vibrating features of the rod of study, thus the present system is applicable to be used in subsequent tests.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Test</th>
<th>Theoretical</th>
<th>Rel. deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>4.1</td>
<td>4.09</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>4.2</td>
<td></td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>4.1</td>
<td></td>
<td>-0.2</td>
</tr>
<tr>
<td>Still water</td>
<td>3.7</td>
<td>3.71</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>3.7</td>
<td>3.7</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>3.7</td>
<td>3.7</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>3.7</td>
<td>3.7</td>
<td>0.3</td>
</tr>
</tbody>
</table>
After verifying the present system in use, another series of tests has been designed and conducted in which the cantilever rod with lead loading internally is subjected to an axial water flow with velocity ranging from 1.01 to 3.46 m/s and with temperature ranging from 21.3 to 28.1 °C. As the maximum water velocity that can be achieved is reasonably close to the 4.0 m/s typical water velocity of PWR nuclear reactors, it is consequently utilized here to mimic the real water flows in nuclear water reactors. Figure 4.25 shows the time trace of displacement at a flow velocity of 1.01 m/s and temperature of 26.2 °C, Figure 4.26 shows the time trace of displacement at a flow velocity of 2.23 m/s and temperature of 23.8 °C, and Figure 4.27 shows the time trace of displacement at a flow velocity of 3.46 m/s and temperature of 21.3 °C.
Figure 4.25. Time trace of displacement at the free end of a cantilever rod with lead loaded internally and with a blunt free-end shape in a confining tube filled with flowing water at a velocity of 1.01 m/s and temperature of 26.2 °C (a) complete (b) 40 to 50 seconds.
Figure 4.26. Time trace of displacement at the free end of a cantilever rod with lead loaded internally and with a blunt free-end shape in a confining tube filled with flowing water at a velocity of 2.23 m/s and temperature of 23.8 °C (a) complete (b) 20 to 30 seconds
Figure 4.27. Time trace of displacement at the free end of a cantilever rod with lead loaded internally and with a blunt free-end shape in a confining tube filled with flowing water at a velocity of 3.46 m/s and temperature of 21.3 °C (a) complete (b) 40 to 50 seconds

From the time traces of displacement, the RMS and the maximum displacements under different water velocities ranging from 1.01 to 3.46 m/s (Reynolds number from 12,700 to 43,200 when the hydraulic diameter of the annulus is chosen as the characteristic length), and ratio of the maximum to the RMS values in each test can be obtained (see Figure 4.28). It shows the vibrating displacement increases with water velocity, which follows the same trend as that of an air loaded rod in Sections 4.3.1 and 4.3.2. This behaviour is also in qualitative agreement with that observed by Rinaldi and Paidoussis [94], who conducted an experimental study on a very flexible rubber-made cantilever rod subjected to an axial air flow directed from its free end towards the fixed end. Again, this phenomenon can be well explained as that in the velocity range of study, the vibration is
mostly turbulence induced, thus a large Reynolds number corresponds to a large displacement. Rewriting water flow velocity in a dimensionless form as in Equation (4.12), it can be found that the displacements of both maximum and RMS are also in quantitative agreement with that by the researchers [94]. Meanwhile, the maximum to RMS ratio value is between 2 and 2.7 (taking the uncertainties into consideration), which is reasonably close to 2.5 which was found by Rinaldi and Paidoussis [94]. For more details about the uncertainties, reader may refer to the uncertainty analysis in the Appendix Section of this report.

![Figure 4.28. RMS and Maximum displacements at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity values ranging from 1.0 m/s to 3.5 m/s, and ratio of Maximum to RMS values (error bars represent precision limit of maximum displacement; precision limits of RMS are 0.0001 mm)](image)

Figure 4.28. RMS and Maximum displacements at the free end of a cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity values ranging from 1.0 m/s to 3.5 m/s, and ratio of Maximum to RMS values (error bars represent precision limit of maximum displacement; precision limits of RMS are 0.0001 mm)
Meanwhile, another key parameter in the present study namely the fundamental vibrating frequency can be retrieved by computing the PSD of the measuring signals using an FFT algorithm. Figure 4.29, Figure 4.30 and Figure 4.31 show the power spectrum densities of displacement at the free end of the lead-loaded cantilever rod with a blunt free-end shape in a confining tube filled with water: of 1.01 m/s in velocity, of 2.23 m/s in velocity and of 3.46 m/s in velocity, respectively. To remove the spikes in the original PSD plots, a Savitzky-Golay filter has been applied as was done in Sections 4.3.1 and 4.3.2, in which the filter coefficients are derived by performing an unweighted linear least-squares fit to 5% of the complete data points using a polynomial of degree 21. Also, for reducing the noise level in the signals, the Welch’s method has also been used as a reference. Welch’s method technically breaks the time series of a signal into multiple segments, calculates a modified periodogram spectrum estimate for each segment and in the end computes the average of these estimates. In the present study, the “pwelch” function embedded in Matlab, as a Welch’s power spectral density estimate, has been performed, in which a Hanning window of length 1000 is selected. The resulting spectrum is plotted with ten times the logarithm of its PSD estimate with base ten. The two methods Savitzky-Golay filtering and Welch’s method are marked as “Savitzky-Golay” and “pwelch” respectively in Figure 4.29, Figure 4.30 and Figure 4.31.

In each test, one frequency can be explicitly identified as dominating the rod movement under each flow condition, this corresponds to the fundamental vibrating frequency. Hence, the relationship between the water velocity ranging from 1.01 to 3.46 m/s (Correspondingly Reynolds number covers roughly from 12,700 to 43,200 when the hydraulic diameter of the annulus in the present test facility is chosen as the characteristic length) and vibrating frequency can be obtained (see Figure 4.32), in which the frequency
is derived from the Savitzky-Golay filtered signal. As the outcome differential in frequency between the Savitzky-Golay filtering and the Welch’s method discussed in the previous paragraph is generally less than 1% over the water velocity of study, the frequency values obtained by the Welch’s method are not presented here.

It shows that as water velocity increases from 1.01 to 3.46 m/s, the vibrating frequency of the rod slightly drops from 3.7 to 3.4 Hz. Comparing with 3.7 Hz the vibrating frequency of the identical rod immersed in a still water, it indicates that in the present water range of study, the axial flowing water has a minor effect on the oscillating frequency. This trend has also been observed in a similar experimental test except for air flows instead of water flows [94]. This decreasing trend in vibrating frequency as a function of water velocity may be caused by the increase of damping from fluid: a more turbulent flow results in not only an increase in the component of fluid pressure on the rod that drives the rod to vibrate, but also an increase in other components of the fluid pressure that cause a larger damping. Figure 4.33 depicts the PSDs of the rod’s displacement immersed in open air, still water confined by a tube, and flowing water of 3.09 m/s in velocity, from which the vibrating frequency change in different fluids can be identified. Also, it shows that the spectrum around the vibrating frequency peak for the rod in the flowing water becomes more broadened than that in air, which is partially due to the introduction of turbulence into the system of study. The PSD amplitude of the rod in flowing water with 3.09 in velocity at its natural frequency is one order of magnitude lower than those in still water and air, which is attributed to the fact that the RMS vibrating amplitude of the rod in flowing water is lower than those in still water and air.
Figure 4.29. Three PSDs of displacement at the free end of a lead-loaded cantilever rod with a blunt free-end shape in a confining tube filled with water of 1.01 m/s in velocity and of 26.2 °C in temperature: original PSD (left y-axis), smoothed PSD by a Savitzky-Golay filter (left y-axis), and PSD estimating using Welch’s overlapped segment averaging estimator (plotted with ten times the logarithm of its PSD estimate with base ten on the right y-axis)
Figure 4.30. Three PSDs of displacement at the free end of a lead-loaded cantilever rod with a blunt free-end shape in a confining tube filled with water of 2.23 m/s in velocity and of 23.8 °C in temperature: original PSD (left y-axis), smoothed PSD by a Savitzky-Golay filter (left y-axis), and PSD estimating using Welch’s overlapped segment averaging estimator (plotted with ten times the logarithm of its PSD estimate with base ten on the right y-axis)
Figure 4.31. Three PSDs of displacement at the free end of a lead-loaded cantilever rod with a blunt free-end shape in a confining tube filled with water of 3.46 m/s in velocity and of 21.3 °C in temperature: original PSD (left y-axis), smoothed PSD by a Savitzky-Golay filter (left y-axis), and PSD estimating using Welch’s overlapped segment averaging estimator (plotted with ten times the logarithm of its PSD estimate with base ten on the right y-axis)
Figure 4.32. Vibrating frequency of a lead-loaded cantilever rod with a blunt free-end shape in a confining tube filled with flowing water of velocity ranging from 1.01 m/s to 3.46 m/s, with two y-axis ranges: (a) from 0 to 4 Hz, (b) from 3 to 4 Hz
Figure 4.33. PSDs of displacement at the free end of a lead-loaded cantilever rod in air (blue colour), still water in a confining tube (red colour) and flowing water of velocity 3.09 m/s (green colour)
This section firstly presented the tests on a lead-loaded cantilever rod with a blunt free-end shape which is being immersed in either open air or still water confined by a tube. Through comparing the test results with those obtained by relevant correlations in the literature, it has again been proved that the measuring techniques used in the present test facility are capable of capturing the main structural dynamics of interest, thus are applicable to be applied in subsequent studies. Following these tests in still fluids, the structural dynamics of a cantilever rod with a blunt free-end and an internal lead loading is studied, when being subjected to a water pipe flow directed from the rod free end towards the fixed end. Of the parameters of interest, the vibrating amplitudes of both the maximum and RMS have been found to increase with water velocity, which can be explained by the fact that the rod movement in the present study is mainly turbulence driven, thus a larger water velocity (also a larger Reynolds number) corresponds to a larger amplitude of displacement. Meanwhile, the dominating vibrating frequency of the present system has been found to decrease with water velocity, which is in quantitative agreement with the findings from an experimental test with a similar mounting configuration in the literature.

4.3.4 Tests on a lead-loaded rod of tapered-end shape

In this Section, a cantilever rod with a tapered free-end and with a lead loading internally (see Figure 3.1) is firstly studied in open air and still water confined by a tube, for verifying the performance of the designed system, and then in an axial water flow (see Figure 3.2 for more details). Specifically, the additions of the lead spheres and tapered free-end
make mass per unit length of the rod be $0.589 \pm 0.001 \text{ kg/m}$, and the rod displacement was measured at locations very close to the rod free end: 19, 29, 39, 48, 58 and 68 mm axially away from the free end. In this section, the vibrating characteristics of the rod at the location 48 mm axially away from the free end tip are presented.

For verifying the performance of the designed system in the present test facility, the vibrating frequency of the rod in a free decaying motion (being initially deflected at a certain distance away from its equilibrium position at the free end, and released as was done in Section 4.2) is firstly obtained via making an FFT of the time series of displacement, and the resulting frequency value is then compared with that computed by Equations (4.3)-(4.6) which are applied for obtaining the frequency of an Euler-Bernoulli beam. Such comparisons for the rod immersed in air and still water are depicted in Table 4.8. The vibrating frequencies in air and still water are shown to be 4.3 Hz and 3.8 Hz respectively, less than the 8.3 Hz and 5.4 Hz of the corresponding vibrating frequencies of the rod under almost identical conditions except for the internal loading material being air and the free-end shape being blunt (see Section 4.2). These differences in frequency can be mainly explained as the difference in the mass term as presented in Equations (4.3)-(4.6): a larger mass per unit length corresponds to a larger vibrating frequency, in which the mass term has both the rod’s mass and added mass contributions. The rod free-end shape has also a minor effect on frequency: the rod with a tapered free end shape has a frequency 4.6 % in relative difference larger than that with a blunt free end shape in Section 4.3.3. Meanwhile, the relative deviations of the vibrating frequencies under both conditions are less than 5.2 % and 5.0 % respectively, this further proves that the present measuring techniques is capable of capturing the vibrating features of the rod of study, thus the present system is applicable to be used in subsequent tests. In comparison, the
relative deviations of the vibrating frequency of around 5% in the present section are
larger than those of less than 0.3 % in Section 4.3.3 in which an almost identical rod has
been tested except the free-end shape being blunt. This phenomenon can be explained by
the fact that Equations (4.3)-(4.6) are only applicable to compute the frequency of a rod
with a blunt free-end, as the effect of the free-end shape on frequency has not been
taken into account during the development of these equations. This results in a larger
discrepancy for the rod with a tapered free-end.

Table 4.8. Frequency comparison between test results and theoretical values for a lead-loaded cantilever rod with a tapered free end, when immersed in either air or still water confined by a tube

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Test</th>
<th>Theoretical</th>
<th>Rel. deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>4.3</td>
<td>4.077</td>
<td>-5.2</td>
</tr>
<tr>
<td>Air</td>
<td>4.3</td>
<td>4.077</td>
<td>-5.2</td>
</tr>
<tr>
<td>Still water</td>
<td>3.9</td>
<td>3.704</td>
<td>-5.0</td>
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<tr>
<td>Still water</td>
<td>3.8</td>
<td>3.704</td>
<td>-2.5</td>
</tr>
</tbody>
</table>

In the remaining part of this section, structural dynamics of the lead-loaded cantilever rod
immersed in an axial water flow with velocity ranging from 1.07 to 3.43 m/s is
experimentally studied. Figure 4.34 shows the time trace of displacement at a flow
velocity of 1.19 m/s and temperature of 26.4 °C, Figure 4.35 shows the time trace of
displacement at a flow velocity of 2.41 m/s and temperature of 24.2 °C, and Figure 4.36
shows the time trace of displacement at a flow velocity of 3.43 m/s and temperature of
22.1 °C.
Figure 4.34. Time trace of displacement at the free end of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with a flowing water at a velocity of 1.19 m/s and temperature of 26.4 °C (a) complete (b) 10 to 20 seconds
Figure 4.35. Time trace of displacement at the free end of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with a flowing water at a velocity of 2.41 m/s and temperature of 24.2 °C (a) complete (b) 20 to 30 seconds
Figure 4.36. Time trace of displacement at the free end of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with a flowing water of 3.43 m/s in velocity and 22.1 °C in temperature (a) complete (b) 40 to 50 seconds
From the time series of displacement displayed in this section, the RMS and maximum displacements under different water velocities ranging from 1.19 to 3.43 m/s (correspondingly, Reynolds number covering 13,600 to 45,400 when the hydraulic diameter of the annulus is chosen as the characteristic length), and ratio of the maximum to RMS values in each test can be derived (see Figure 4.37). The vibrating displacement is shown to increase with water velocity, which shows the same trend as that of an air-loaded rod in Sections 4.3.1 and 4.3.2. This finding is also in qualitative agreement with that observed on a flexible rubber-made cantilever rod in axial air flows by Rinaldi and Paidoussis [94]. This phenomenon is attributed to the fact that in the current range of study, the rod’s movement is mainly turbulence driven, hence a larger Reynolds number corresponds to a larger vibrating amplitude of displacement. Quantitatively, the vibrating displacement in the present test is also in good agreement with that found in the literature [94]. Furthermore, the maximum to RMS ratio value falls between 1.6 and 3 (taking the uncertainties into account), which is reasonably close to 2.5 a value which was concluded from another experimental test with a similar configuration [94]. For more details about the uncertainties, reader may refer to the uncertainty analysis section in the Appendix of this report.
Figure 4.37. RMS and Maximum displacements at the free end of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with flowing water of velocity values ranging from 1.19 m/s to 3.43 m/s, and ratio of Maximum to RMS values (error bars represent precision limit of maximum displacement; precision limits of RMS are 0.00006 mm thus not plotted herein)

In the present study, another key feature of interest, the fundamental vibrating frequency, can be derived from the PSD plot of the measuring signals. Herein, this has been realized by applying an FFT algorithm, Figure 4.38, Figure 4.39 and Figure 4.40 illustrate the PSDs of displacement at the free-end of the lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with water: of 1.19 m/s in velocity, of 2.41 m/s in velocity and of 3.43 m/s in velocity, respectively. To mitigate the influence of the spikes appearing in the PSD plots upon identifying the dominating contributions in frequency, a Savitzky-Golay filter has been used in which the filter coefficients are obtained by performing an unweighted linear least-squares fit to 5% of the complete data points using a polynomial of degree 21. Meanwhile, the Welch’s method has also been applied as a reference due to
its good performance in reducing the noise level in the signals. Technically, the Welch’s method breaks the time series of a signal into multiple segments, following by computing a modified periodogram spectrum estimate for each segment and then outputs the average of these estimates. In the present study, the “pwelch” function embedded in Matlab, as a Welch’s power spectral density estimate, has been applied, in which a Hanning windows of length 1000 is selected. The obtained spectrum is presented as ten times the logarithm of its PSD estimate with base ten. In the plots, the two methods Savitzky-Golay filtering and Welch’s method are marked as “Savitzky-Golay” and “pwelch” respectively in Figure 4.38, Figure 4.39 and Figure 4.40.

In each test, the fundamental vibrating frequency is believed to correspond to the highest peak, the vibration with which dominates the rod movement. Thus, the relationship between water velocity ranging from 1.19 to 3.43 m/s (Reynolds number correspondingly ranging from 13,600 to 45,400 if the hydraulic diameter of the annulus in the test facility is chosen as the characteristic length) and fundamental vibrating frequency can be derived (see Figure 4.41). In this section, the fundamental frequency is obtained from the Savitzky-Golay filtered signals, due to its better performance in presenting the changing trend than the original signals from the past experience. Meanwhile, as the outcome differential in frequency between the Savitzky-Golay filtering and the Welch’s method is generally less than 1.5 % over the water velocity range of study, can be negligible, the frequency values obtained by the Welch’s method are not presented.
Figure 4.38. Three PSDs of displacement at the free end of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with water of 1.2 m/s in velocity and of 26.4 °C in temperature: original PSD (left y-axis), smoothed PSD by a Savitzky-Golay filter (left y-axis), and PSD estimating using Welch’s overlapped segment averaging estimator (plotted with ten times the logarithm of its PSD estimate with base ten on the right y-axis)
Figure 4.39. Three PSDs of displacement at the free end of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with water of 2.4 m/s in velocity and of 24.2 °C in temperature: original PSD (left y-axis), smoothed PSD by a Savitzky-Golay filter (left y-axis), and PSD estimating using Welch’s overlapped segment averaging estimator (plotted with ten times the logarithm of its PSD estimate with base ten on the right y-axis)
Figure 4.40. Three PSDs of displacement at the free end of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with water of 3.4 m/s in velocity and of 22.1 °C in temperature: original PSD (left y-axis), smoothed PSD by a Savitzky-Golay filter (left y-axis), and PSD estimating using Welch’s overlapped segment averaging estimator (plotted with ten times the logarithm of its PSD estimate with base ten on the right y-axis)
Figure 4.41. Vibrating frequency change of a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with flowing water of velocity ranging from 1.1 m/s to 3.4 m/s, with two y-axis ranges: (a) from 0 to 4 Hz, (b) from 3 to 4 Hz
As water velocity increases from 1.19 to 3.43 m/s, the fundamental vibrating frequency of the rod is shown to drop from 3.8 to 3.65 Hz. This changing trend has also been observed in another experimental test in which a flexible cantilever tube is subjected to an axial air flow directed from the rod free end towards the fixed end [94]. Comparing with 3.8 Hz the vibrating frequency of the rod immersed in still water, it suggests that in the current range of study, the axial flowing water has a minor but non-negligible effect on the vibrating frequency.

In this Section, a preliminary test on a lead-loaded cantilever rod immersed in a still fluid such as air and tube-confining water has been initially presented. The results in measuring the vibrating frequency of the rod showed a good agreement with that obtained by a correlation for an Euler-Bernoulli beam. This verified the capabilities of the measuring techniques applied in the present test facility in capturing the main structural dynamics of interest. Following these tests in still fluids, a test on the identical rod immersed in a water flow has been conducted. Through the test, it has been found that the vibrating amplitude of displacement increases, but the vibrating frequency slightly drops with water velocity.

### 4.3.5 Summary

In this Section, structural dynamics of the rods encountered in Sections 4.3.1 till 4.3.4, featuring either a blunt (the actual shape of many nuclear fuel rods) or a tapered free end shape and either an air or a lead loading internally, is compared and discussed.
Prior to conducting tests on a cantilever rod immersed in water flows, a series of tests on the identical rod immersed in still fluids has been initially performed. Through comparing the structural dynamics in particular the vibrating frequency of the rod during the test with relevant correlations, it has been explicitly suggested that the present test facility including the measuring techniques is capable of capturing the main features of interest, thus can be applied in subsequent tests. Reader may refer to Sections 4.3.1 till 4.3.4 for more details.

In the remaining part of this section, an emphasis is placed on the behaviour of the rod immersed in water flows, as this is of interest to the nuclear communities. Figure 4.42, Figure 4.43 and Figure 4.44 present the comparisons of the fundamental vibrating frequency, the maximum amplitude of displacement and the RMS amplitude of displacement of the rod, respectively, as a function of Reynolds number. In this section, the characteristics of the rod movement at an identical position 1011 mm axially away from the rod fixed end are studied, thus the movement signals for both air-filled rods are obtained by linearly interpolating the measured movement values at the nearest neighbouring positions.

In the plots, the test on an Air-loaded rod with a blunt free-end shape is denoted as “Air blunt”. Generally, the vibrating amplitude in each test increases with Reynolds number, which the fundamental vibrating frequency decreases. This changing trend in vibrating amplitude can be explained by the fact that in the present range of study, the rod’s movement is mainly driven by turbulence, hence a large Reynolds number causes a large vibrating amplitude in both the maximum and RMS displacements. While the changing trend in vibrating frequency can be attributed to the specific mass change as in Equations
(4.3)-(4.6): the mass term comprises of two contributions, one is the rod mass itself, the other one is the added mass which is introduced if the test condition such as the surrounding fluid, is changed. Hence, a higher mass corresponds to a lower frequency. These two trends have also been well observed in the literature [94], [95]. Meanwhile, for the rod with a blunt free-end shape, the maximum and RMS amplitudes of displacement of the rod loaded with air are in the same range as those loaded with lead. For the rod with a tapered free-end shape, the maximum and RMS amplitudes of displacement of the rod loaded with air are also in the same range as those loaded with lead, when Reynolds number is less than 35,000, but seemingly increases at a larger slope when Reynolds number is beyond 35,000 till 45,000. This suggests that in the aspect of vibrating amplitude, a rod with a tapered free-end shape is more sensitive to the loading material than that with a blunt free-end shape. Also, the vibrating amplitude of the rod with a blunt free-end shape is larger than that with a tapered free-end shape, which can be explained by the fact that when a water flows by the rod with a tapered free-end shape, roughly semi-spherical, the water follows better streamed streamlines at the entry region than that with a blunt shape, thus less turbulence is generated and entrained with flow.

In the vibrating frequency, the rod loaded with air internally has a higher value than that loaded with lead by about 50 %. This could be explained as a result of a differential mass per unit length (see Equations (4.3)-(4.6)). The vibrating frequency is also shown to be partially determined by the free-end shape: the rod with a tapered free-end has a higher frequency than that with a blunt free-end, by about 4 %. In the present study, however, the loading material is found to play a more significant role in the vibrating frequency than the free-end shape.
This section summarized the structural dynamics of the rods encountered in Sections 4.3.1 till 4.3.4 such as vibrating amplitude and frequency, and based on that made a deep discussion on the influential factors to the parameters of interest. These findings will give an insight into the future design of relevant structures in nuclear reactor cores, the FSI community will also benefit from the availability of such validation data.

Figure 4.42. Vibrating frequency as a function of Reynolds number for the rod with different loadings and free-end shapes, with two y-axis ranges: (a) from 0 to 6 Hz, (b) from 3 to 6 Hz
Figure 4.43. Maximum amplitude of displacement as a function of Reynolds number for the rod with different loadings and free-end shapes

Figure 4.44. RMS amplitude of displacement as a function of Reynolds number for the rod with different loadings and free-end shapes
Chapter 5  Conclusions and outlook

In this research, the issue of flow-induced vibrations in particular those occurring in nuclear reactor cores has been discussed, including its mechanism, prediction tools and the performance of the tools. Empirical correlations, one of the prediction tools, were particularly evaluated through a large databank.

Meanwhile, a test facility for the investigation of flow-induced structural vibrations has been established. The cantilever rod of test is made of stainless steel and features a geometry which is prototypical of a commercial light water nuclear reactor core, and is subjected to an axial water flow directed from the rod free end towards the fixed end in which the water flow parameters replicate the flow conditions during its full power operation. In the literature, an experimental investigation on such a configuration has attracted very little attention, and most experimental tests are conducted with very flexible rubber tubes. This highlights the novel characteristics of the present test facility, in particular in the context of using a stainless steel rod as the test piece for studying flow-induced vibration at a fundamental level, and allowing the simultaneous measurement of rod vibration and flow field both of which use a non-invasive tool, which is of the first kind in the literature. The design of the facility and measuring techniques have been proved to be capable of capturing the structural dynamics of the rod of interest. Through a series of tests on the cantilever rod in pipe flows, more benchmark data with high accuracy on axial turbulent flow induced vibration of a single rod as being used in the nuclear industry, have been generated. These data will give an insight into fine-tuning relevant numerical codes
for the FSI community. By conducting analyses on the data, the present understanding of
the flow-induced vibration phenomena has been advanced. For a cantilever rod in axial
flow directed from the rod free end towards the fixed end, the vibrating amplitude
increases with Reynolds number as its motion is mainly turbulent driven; the vibrating
amplitude of the rod is more sensitive to the free-end shape of the rod, the rod with a
well-streamed free end results in a lower vibration in amplitude than that with a blunt one;
while the vibrating frequency is mostly influenced by the internal loading materials, the
rod with a higher mass per unit length has a lower vibrating frequency.

The place where future effort can be made to further develop the established test facility,
would be introducing flow visualisation via particle image velocimetry. It was shown that
some of the available prediction tools in describing the structural dynamics of a rod
subjected to an axial flow, in particular numerical calculations, require the surrounding
flow field of the rod as an input. A better characterisation of the flow field would generate
a more accurate result in estimating the rod’s behaviours. Hence, the work such as
choosing appropriate seeding particles, particle density in the mixture and a light sheet,
would be necessary to be carefully examined.

Further developments of the present test facility could also be introducing a different
mounting configuration of the rod. This configuration in change could be a fixed-fixed
mounting condition for a single rod subjected to an axial water flow confined by a coaxial
rigid tube, or a fuel bundle which is the mounting condition of nuclear fuel rods in
pressurized water nuclear reactors, such as a 3-by-3 layout. Particular attention should be
paid to the behaviours of the rod in a fuel bundle as it would perform differently to a
single rod. A further development could also be investigating the effect of the gap
between the fuel pellets and cladding on the structural dynamics of the rod into account, such as designing a nuclear fuel-like two-body system, to have a broader understanding to the mechanical vibrations of the fuel rods as a function of burnup.

In addition, fluids of different properties could also be introduced into the test rig, in place of the water at an ambient pressure and room temperature as being applied in the present study. Such a change may be redefining the present water to be at a pressure such that a viscosity level similar to that in nuclear reactors can be achieved, and/or introducing a two-phase flow for mimicking the flow patterns in BWRs, such as water-vapor mixtures. Also, the temperature of the testing rod may be redefined as its effect on mechanical movements of the rod when the rod is subjected to forces and torques. As the rod’s temperature increases, the atomic thermal movements of the rod increase, resulting in the changes of lattice potential energy and accordingly a change on the Young’s modulus. Concurrently, the rod will expand in volume as the increase of temperature, and this causes a decrease in density.
Appendix

Uncertainty analysis

In the experimental tests, the 95% confidence precision limits of the variables are: 0.5°C for temperature as limited by the measuring thermometer’s capability, 0.05 mm for rod diameter, 0.21 mm for inner diameter of Plexiglas pipe, 0.5 mm for length between two measuring points for obtaining pressure drop as in Equation (3.1), 0.01 mm for rod displacement, 3% for relative pressure difference as in Equation (3.1).

In Section 4.3, the 95% confidence precision limit $P_{Re}$ of Reynolds number can be expressed as:

$$
\left( \frac{P_{Re}}{Re} \right)^2 = \left( \frac{1}{Re} \frac{\partial Re}{\partial U} P_U \right)^2 + \left( \frac{1}{Re} \frac{\partial Re}{\partial D_h} P_{D_h} \right)^2 + \left( \frac{1}{Re} \frac{\partial Re}{\partial \rho} P_{\rho} \right)^2 + \left( \frac{1}{Re} \frac{\partial Re}{\partial \mu} P_{\mu} \right)^2
$$

where $Re$ is Reynolds number, $U$ is water velocity within the test section annulus, $D_h$ is hydraulic diameter of the annulus, $\rho$ is water density, $\mu$ is dynamic viscosity of water, and $P_U$, $P_{D_h}$, $P_{\rho}$ and $P_{\mu}$ are the 95% confidence precision limits of water velocity, hydraulic diameter, water density and water viscosity, respectively.

In Equation A(1), the precision limits of water density and viscosity which are temperature dependent can be deduced from relevant relationship curves, thus the remaining unknown terms are the precision limits of water velocity within the annulus and hydraulic
diameter of the annulus. In the present tests, the value of hydraulic diameter of the annulus equals to the value difference of the inner diameter $D_c$ of the confining tube in the test section and the rod’s outer diameter $D$. Therefore, the 95% confidence precision limit $P_{D_h}$ of hydraulic diameter can be written as:

$$
\left( \frac{P_{D_h}}{D_h} \right)^2 = \left( \frac{1}{D_h} \frac{\delta D_h}{\delta D_c} P_{D_c} \right)^2 + \left( \frac{1}{D_h} \frac{\delta D_h}{\delta D} P_D \right)^2
$$

where $P_{D_c}$ and $P_D$ are the 95% confidence precision limits of the confining tube’s inner diameter and the rod’s outer diameter, respectively.

In the present tests, the values of the water velocity within the annulus have been obtained from:

$$
U \frac{\pi}{4} (D_c^2 - D^2) = V \frac{\pi}{4} D_i^2
$$

where $V$ is water velocity inside the measuring pipe which is a section of straight pipe after the test section, $D_i$ is inner diameter of the measuring pipe.

Thus, the precision limit $P_U$ of water velocity within the annulus can be expressed as:

$$
\left( \frac{P_U}{U} \right)^2 = \left( \frac{1}{U} \frac{\delta U}{\delta V} P_V \right)^2 + \left( \frac{1}{U} \frac{\delta U}{\delta D_i} P_{D_i} \right)^2 + \left( \frac{1}{U} \frac{\delta U}{\delta D_c} P_{D_c} \right)^2 + \left( \frac{1}{U} \frac{\delta U}{\delta D} P_D \right)^2
$$

where $P_V$, $P_{D_i}$, $P_{D_c}$ and $P_D$ are the 95% confidence precision limits of water velocity inside the measuring pipe, inner diameter of the measuring pipe, confining tube’s inner diameter in the test section and rod’s outer diameter, respectively.
According to Equation (2.11), the 95% confidence precision limit $P_v$ of water velocity inside the measuring pipe can be expressed as:

\[
\left( \frac{P_v}{V} \right)^2 = \left( \frac{1}{V} \frac{\delta V}{\delta \Delta p} P_{\Delta p} \right)^2 + \left( \frac{1}{V} \frac{\delta V}{\delta P} P_P \right)^2 + \left( \frac{1}{V} \frac{\delta V}{\delta D_1} P_{D_1} \right)^2 + \left( \frac{1}{V} \frac{\delta V}{\delta L_1} P_{L_1} \right)^2
\]

where $\Delta p$ is pressure difference between two points which were axially aligned along the measuring pipe, the distance of the two points is denoted as $L_1$, $P_{\Delta p}$, $P_{D_1}$ and $P_{L_1}$ are precision limits of the pressure difference $\Delta p$, inner diameter of the measuring pipe, and distance $L_1$ of the two measuring points, respectively.

According to Equations A(2)-A(5), the 95% confidence precision limit $P_{Re}$ of Reynolds number in each test can be obtained: for the two rods with either air or lead loading internally, of which one rod features with a blunt free-end shape and the other rod features with a tapered free-end shape, the precision limits of Reynolds number in the tests are 4.3%.

In this report, RMS amplitude is obtained from a data reduction equation as in Equation A(6), whose 95% confidence precision limit can be calculated as in Equation A(7):

\[
y_{rms} = \sqrt{\frac{1}{n} \left( y_1^2 + y_2^2 + ... + y_n^2 \right)}
\]

\[
\left( \frac{P_{y_{rms}}}{y_{rms}} \right)^2 = \left( \frac{1}{y_{rms}} \frac{\delta y_{rms}}{\delta y_1} P_1 \right)^2 + \left( \frac{1}{y_{rms}} \frac{\delta y_{rms}}{\delta y_2} P_2 \right)^2 + ... + \left( \frac{1}{y_{rms}} \frac{\delta y_{rms}}{\delta y_n} P_n \right)^2
\]

where RMS amplitude $y_{rms}$ is a function of $n$ displacement values $(y_1, y_2, ..., y_n)$, $P_{y_{rms}}$ is the 95% confidence precision limit of RMS amplitude, and 95% confidence precision limits for the displacement values are denoted as $P_1, P_2, ..., P_n$. 

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For the tests in Section 4.3.1 which feature measuring time duration of 1 minute, the 95% confidence precision limit of RMS amplitude is around 0.0001 mm. For the tests in Section 4.3.2 which feature measuring time duration of 3 minutes, the 95% confidence precision limit of RMS amplitude is 0.00006 mm. For the tests in Sections 4.3.3 and 4.3.4, the 95% confidence precision limits of RMS amplitude are 0.00006 mm, respectively.

For the tests in Sections 4.3.1 till 4.3.4, the 95% confidence precision limits $P_{y_{\text{max}}}$ of the maximum amplitude $y_{\text{max}}$ are 0.01 mm. Therefore, the precision limits $P_r$ of ratios of maximum to RMS amplitude values can be expressed as:

$$
\left( \frac{P_r}{r} \right)^2 = \left( \frac{1}{r} \frac{\delta r}{\delta y_{\text{max}}} P_{y_{\text{max}}} \right)^2 + \left( \frac{1}{r} \frac{\delta r}{\delta y_{\text{rms}}} P_{y_{\text{rms}}} \right)^2
$$

where $r$ is ratio of maximum to RMS amplitude values.

For the tests on an air-loaded rod with a blunt free-end shape in Section 4.3.1, the precision limits of ratios of maximum to RMS amplitude values are shown in Figure A1.

For the tests on an air-loaded rod with a tapered free-end shape in Section 4.3.2, the precision limits of ratios of maximum to RMS amplitude values are shown in Figure A2.

For the tests on a lead-loaded rod with a blunt free-end shape in Section 4.3.3, the precision limits of ratios of maximum to RMS amplitude values are shown in Figure A3.

For the tests on a lead-loaded rod with a tapered free-end shape in Section 4.3.4, the precision limits of ratios of maximum to RMS amplitude values are shown in Figure A4.

Thus, the precision limits of ratios between maximum to RMS amplitude values, discussed in Section 4.3, may indicate the very small scatter with 2.5 a typical value extensively concluded in the literature.
Figure A1. Ratio of maximum to RMS displacement for a cantilever air-loaded rod with a blunt free-end shape in a confining tube filled with flowing water, as function of water velocity (error bars represent precision limits).

Figure A2. Ratio of maximum to RMS displacement for a cantilever air-loaded rod with a tapered free-end shape in a confining tube filled with flowing water, as function of water velocity (error bars represent precision limits).
Figure A3. Ratio of maximum to RMS displacement for a cantilever lead-loaded rod with a blunt free-end shape in a confining tube filled with flowing water, as function of water velocity (error bars represent precision limits)

Figure A4. Ratio of maximum to RMS displacement for a lead-loaded cantilever rod with a tapered free-end shape in a confining tube filled with flowing water, as function of water velocity (error bars represent precision limits)
Furthermore, the 95% confidence precision limit $P_\zeta$ of damping ratio $\zeta$ obtained by logarithmic decrement as in Equation (4.10) and (4.11) can be expressed as:

$$\left(\frac{P_\zeta}{\zeta}\right)^2 = \left(\frac{1}{\zeta} \frac{\delta\zeta}{\delta y(1)} P_{y(1)}\right)^2 + \left(\frac{1}{\zeta} \frac{\delta\zeta}{\delta y(2)} P_{y(2)}\right)^2 + \left(\frac{1}{\zeta} \frac{\delta\zeta}{\delta y(3)} P_{y(3)}\right)^2 + \left(\frac{1}{\zeta} \frac{\delta\zeta}{\delta y(4)} P_{y(4)}\right)^2$$

where $P_{y(1)}$, $P_{y(2)}$, $P_{y(3)}$ and $P_{y(4)}$ are the precision limits of measured displacement $y(1)$, $y(2)$, $y(3)$ and $y(4)$, respectively, the values equal to 0.1 mm.

In Section 4.2, the precision limits of the damping ratios are less than 0.05 %, including those obtained from various numbers of peak difference.

In summary, the 95% confidence precision limits of the primary measured parameters and of the calculated parameters are tabulated in Table A1, where the precision limit of Reynolds number is dependent on the values of and on the precision limits of temperature, rod outer diameter, confining tube inner diameter, inner diameter of the pipe for pressure measurement, and relative pressure difference; the precision limits of the maximum and RMS displacements, ratio of the maximum to RMS displacement values, and damping ratio are dependent on the value of and on the precision limit of rod displacement.
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<th>Calculated parameters and the corresponding precision limits</th>
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<td>Rod displacement (mm)</td>
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<td>RMS displacement $y_{rms}$ (mm)</td>
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<td>Ratio of Maximum to RMS displacement</td>
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<tr>
<td>Damping ratio $\zeta$</td>
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References


