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OPF-based CVR Operation in PV-Rich MV-LV Distribution Networks

L. Gutierrez-Lagos, *Graduate Student Member, IEEE* and L. F. Ochoa, *Senior Member, IEEE*

Abstract—Conservation Voltage Reduction (CVR) has been traditionally applied adopting moderate settings at primary substations and when distributed generation was uncommon. However, as new infrastructure is deployed across European-style MV and LV networks, driven by increasing photovoltaic (PV) penetration levels, the opportunity arises to develop more advanced CVR schemes. This work proposes a centralized, three-phase AC OPF-based CVR scheme that, using monitoring, on-load tap changers (OLTCs) and capacitors across MV and LV, actively manages voltages to minimize energy consumption, even with high PV penetration, whilst considering MV-LV constraints. To tackle scalability issues brought by discrete variables, a two-stage approach is proposed to solve the OPF as a non-linear programming problem (relaxing integer variables). A process that continuously checks customer voltages is adopted to trigger the optimization only when needed. Moreover, CVR benefits are not only quantified at a network level but also for customers, providing useful insights to policy makers. The proposed control is assessed using a realistic, unbalanced UK residential MV-LV network (2,400+ customers) with high PV penetration, and 1-min resolution time-varying profiles and load models. Results demonstrate that the proposed control effectively coordinates voltage regulation in MV and LV levels throughout the day, minimizing energy imports for all customers.

Index Terms—Conservation voltage reduction, distribution networks, on-load tap changers, optimal power flow, PV systems.

I. INTRODUCTION

CONSERVATION Voltage Reduction (CVR) is a well-known method to reduce energy consumption by reducing the supplied voltage within statutory limits. This method, investigated since the '70s in different countries [1], takes advantage of the positive correlation of voltage and demand featured by most appliances. Given the energy efficiency targets adopted by many countries, CVR has started to be incentivized in recent years to counteract the effects that distribution network operators (DNOs, either solely managing the wires or also being the energy retailer) might have on revenues due to the reduced consumption [2-5]. This is primarily

because, differently from other strategies that also promote energy efficiency (e.g., loss minimization), CVR benefits customers too (reducing energy bills). These incentives, in turn, are making CVR an attractive option for DNOs. Indeed, some have already adopted CVR even at off-peak times [6-8].

Due to the historical lack of observability and limited voltage regulation capabilities closer to low voltage (LV) customers, CVR has been traditionally implemented by adopting modest, fixed voltage reductions at primary substations equipped with on-load tap changers (OLTCs) [1]. Moreover, most studies have been carried out in a context where distributed generation (DG) was not common and, therefore, the corresponding effects on CVR remain largely unexplored. Thus, as customers adopt photovoltaic (PV) systems and new infrastructure (for voltage regulation and monitoring) is deployed across medium voltage (MV) and LV networks, it is necessary to understand the extent to which advanced CVR approaches could bring benefits. Given the intrinsic differences between American and European-style networks, LV regulation devices are more likely to be adopted in the latter.

CVR implementations are overcoming the observability issue thanks to the advent of advanced metering infrastructure. This has allowed the implementation of rule-based closed-loop schemes that can adjust voltages at primary substations throughout the day [1, 7, 9], resulting in even lower energy consumption. Nonetheless, the degree to which voltages can be reduced in these large areas will still be constrained by the compliance of MV customers with statutory limits, a few (critical) LV customers with significant voltage drops, and by the OLTC itself which, depending on the time of the day, might have limited tap positions to reduce voltages [10]. Further voltage reductions, however, could be achieved by also actively controlling voltages closer to end customers and compensating voltage drops in heavily loaded feeders. Although no CVR studies have reported the use of enabling technologies in LV networks (European-style), such as OLTC-fitted transformers and capacitors, their effectiveness in managing customer voltage excursions due to DG has been recently shown in [11-13]. Moreover, the use of OLTCs in LV networks has been also economically justified in [11].

The interactions between CVR and widespread DG have been explored in [14, 15] for American-style networks (typically a few LV customers directly connected to distribution transformers). While both studies show that demand and energy losses can be reduced, the adopted CVR schemes used fixed settings for the MV devices [14] and a rule-based

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closed-loop approach with a single remote monitoring point [15]. This limits further benefits from a much more granular voltage management closer to LV customers. Other studies have also found that adequately located and sized (or controlled) DG installations can boost voltages at critical points, thus allowing further voltage reductions at primary substations [16, 17]. These studies, however, do not explore the effects of widespread non-dispatchable DG installations (such as residential PV systems) on CVR.

Given the inherent limitations of CVR schemes that use fixed settings or rules, the optimal operation of CVR (i.e., having as objective function the minimization of power or energy consumption) has also been explored in the literature [9, 18-29] and recently deployed in a few American-style utilities [6, 30]. Some studies have focused on solution algorithms considering snapshot approaches (e.g., static demand and static load model) [18, 19, 27]. In some others, realistic operational aspects, such as the time-varying nature of the demand, have also been considered [6, 9, 20-26, 28-30]. However, the corresponding time-varying load models, critical to quantify demand reduction, have been largely neglected, with the exception of [28, 29]. Similarly, most works do not consider more detailed network (three-phase) and load modeling so as to incorporate LV customers. From an operational perspective, the time resolution adopted by most of these studies are limited to one hour [20, 21, 26] or at most 15 minutes [6, 23-25], and rely on control cycles equivalent to those periods. This, in practice, neglects the minute-scale changes in demand and generation and, therefore, could result in voltage issues. Consequently, more realistic and granular network and load models are still needed to adequately quantify opportunities for voltage reduction, the corresponding operational aspects, and, ultimately, the resulting CVR benefits.

To help DNOs understanding the extent to which their own assets can be used to exploit CVR even in cases of high PV penetration, while addressing the above challenges, this work proposes a centralized, three-phase AC OPF-based CVR scheme that, using monitoring, OLTCs and staged capacitors banks across unbalanced MV and LV networks, actively manages voltages to minimize energy consumption, even with high PV penetration levels, whilst considering MV-LV constraints. The adopted formulation captures the unbalances and interactions across voltage levels by also considering Delta-wye transformers and voltage-dependent single-phase customer loads. Differently from using fixed control cycles, the concept of ‘problem persistency’ is adopted to trigger the CVR optimization only when needed. This process constantly checks that the lowest customer voltages are within a predefined bandwidth that favors CVR. If not, the optimization is run to determine the most suitable control actions to enhance the CVR performance. To improve scalability issues brought by discrete variables (tap positions and capacitor stages), a two-stage approach is proposed to solve the AC OPF as a non-linear programming problem (relaxing integer variables). This approach delivers close to optimal results within solving times suitable for operation purposes. Finally, CVR benefits are not only quantified at a network level but also for customers,

Fig. 1. Operation overview of the proposed OPF-based CVR scheme.

providing useful insights to policy makers. The proposed CVR approach is demonstrated with a realistic, unbalanced UK residential MV-LV network (2,400+ customers) with 50% of the customers having PV systems, and high resolution (1-min) time-varying profiles and load models. This work has been developed as part of the ‘Smart Street’ project [31] run by Electricity North West Limited (ENWL), the first demonstration of a centralized optimal CVR scheme in MV-LV networks in the UK.

II. OVERVIEW OF THE PROPOSED CVR SCHEME

The proposed CVR scheme takes advantage of advanced metering and communication infrastructures (including customers and controllable devices) expected to be available to most DNOs in the near future. Its operation is shown in Fig. 1.

For a given area (e.g., a MV feeder or network), customer voltages as well as active and reactive powers for demand and PV generation are frequently collected and sent to the control room (SCADA system), for instance, every minute. Based on customer voltages and using the concept of ‘problem persistency’ (i.e., values outside a given bandwidth for longer than a given period), the control room identifies when the CVR operation needs to be optimized (section III-A), triggering the optimization engine. To maximize customer benefits as well as network efficiency, the optimization engine minimizes the overall imported active power (i.e., customers demand plus network losses minus PV generation) whilst considering network constraints (section III-C). Using the corresponding three-phase network model as well as data from customers and controllable elements (OLTCs and capacitors in MV and LV networks), the OPF-based engine, utilizing a proposed two-stage approach, determines the most adequate settings to be implemented immediately after. Also, to improve coordination, tap positions and capacitor stages are directly assigned instead of using voltage targets.

The adopted problem persistency approach is aligned with DNO practices and similar to the use of time delays in the operation of OLTC controllers [32]. Depending on the defined length of the problem (i.e., how persistent it is) as well as the corresponding bandwidth within which the variable of interest is checked against, it can help avoiding unnecessary control actions due to short excursions whilst acting fast enough to tackle the problem. For the proposed CVR scheme, given that

the load mix in residential networks (focus of this work) reduces consumption with voltages [1], the minimization of imported power is expected to be achieved when customer voltages are reduced as much as possible. Consequently, the bandwidth to be adopted should consider a range of voltages close to the minimum statutory limit.

III. PROBLEM FORMULATION

This section first details operational aspects of the CVR scheme as well as load modeling concepts and assumptions. Then, the three-phase AC OPF problem formulation and the proposed two-stage approach are presented.

A. CVR Operation

To harvest the maximum benefits from CVR in residential networks, customer voltages shall be kept as low as possible, i.e., close to the minimum statutory limit but not below. One way to achieve this is to constantly assess customer voltages and check whether the one who creates the bottleneck from a CVR perspective (lowest voltage) is within a predefined bandwidth. This is formulated in (1) at time t , where D is the set of customers (indexed by d) and V_{band}^- and V_{band}^+ correspond to the minimum and maximum voltage values of the bandwidth. V_{band}^- is equal or larger than the minimum statutory limit and V_{band}^+ is slightly larger. This bandwidth allows not only to check whether the bottleneck customer is above the minimum statutory limit but also that is not far from it, ensuring a good CVR performance.

$$V_{band}^- \leq \min_{d \in D} V_d(t) \leq V_{band}^+ \quad (1)$$

In the context of problem persistency, the OPF-based engine will only be triggered if a bottleneck customer persistently experiences voltages outside the bandwidth for as long as the predefined persistency period, pp . This avoids taking actions for short duration excursions outside the bandwidth.

B. Voltage dependency of Loads and PV

The extent to which CVR can save energy depends on two main factors that change in time: the load composition of end customers and their voltages. The former determines the time-varying voltage-demand characteristics of the aggregated loads, whilst the latter constrains the extent to which voltages can be reduced without violating limits.

Most of the electrical loads exhibit a direct relation between voltage and demand. This can be represented with voltage dependent load models for individual appliances or for their aggregation at any level (e.g., households, substations). These are typically classified as steady-state (aka static) or dynamic [33]. Given that the focus of CVR is energy consumption, steady-state load models are adopted. However, these load models will be time-varying when representing aggregated loads as its composition varies in time.

As in [34], the well-known ZIP model is adopted for each aggregated load i (in this case, for each customer i) and time t as shown in (2), where $P_i(t)$ and $P_{i0}(t)$ represent its active power for a given voltage magnitude $V(t)$ and for the nominal voltage magnitude V_0 , respectively. This time-varying ZIP load model consists of constant impedance $\hat{Z}_i^P(t)$, constant

current $\hat{I}_i^P(t)$, and constant power $\hat{P}_i^P(t)$ parameters. Similar equations can be written for the reactive component.

$$P_i(t) = P_{i0}(t) \left[\hat{Z}_i^P(t) \frac{V^2(t)}{V_0^2} + \hat{I}_i^P(t) \frac{V(t)}{V_0} + \hat{P}_i^P(t) \right] \quad (2)$$

Moreover, as the load model represents an aggregation of different loads within the customer premises (e.g., appliances), its ZIP parameters shall be obtained as a weighted sum, as detailed by the authors in [34]. The other most common steady-state model is the exponential, shown in (3) [33]. While the ZIP representation is adopted for simulations due to its higher parameter reliability [35], for simplicity, load models will be presented in the exponential form where $n_i^P(t)$ is obtained using (4) [34].

$$P_i(t) = P_{i0}(t) \left(\frac{V(t)}{V_0} \right)^{n_i^P(t)} \quad (3)$$

$$n_i^P(t) \approx 2\hat{Z}_i^P(t) + \hat{I}_i^P(t) \quad (4)$$

Accurate time-varying customer load models are crucial not only to assess CVR benefits but also to implement active CVR schemes that result in voltages close to the limits, given that a load reduction will also affect voltage drops/rises. Here, it is assumed that online ZIP parameters are available. They can be obtained (and used as inputs for the optimization engine) for each customer using fitting techniques on recently recorded data [33]. Furthermore, the chosen technique should be robust under sudden load composition changes (common at less aggregated load levels), missing data and noise, as the one recently proposed in [36]. Less accurate modeling can also be adopted (e.g., based on historical values).

On the other hand, the steady state response of distribution level PV systems can be approximated as constant power devices, where generation mainly depends on the solar irradiance [33]. This is because PV systems are connected through inverters equipped with a Maximum Power Point Tracker. This device maximizes the generated power for a given solar irradiance regardless the ac voltage [37]. The latter can result in larger percentage CVR benefits for customers with PV, who would further reduce energy imports. However, energy losses can reduce or increase (with large reverse power flows) depending on the level of generation and demand.

C. Three-Phase AC OPF Formulation

A current injection-based three-phase AC OPF formulation is used to realistically represent the main network elements. This features voltage dependent customer loads, PV systems, staged capacitors, lines (considering mutual couplings) and delta-wye transformers (typical connection in primary and secondary substations in the UK) equipped with OLTCs, which are grouped by the sets D, G, C, L and Y , respectively.

The formulation uses the real and imaginary parts of voltages and currents per phase as state variables, where $V_{n,\varphi}^{(re,im)}$ represents the real or imaginary part of the voltage at node $n \in N$ (set of nodes) and phase $\varphi \in \Phi = \{1,2,3\}$ (set of phases). Similar indexes are used for the current components $I_{e,n,\varphi}^{(re,im)}$, where $e \in (D \cup G \cup C \cup L \cup Y)$ maps a network element. It is worth highlighting that since this formulation does not have radiality constraints, it could be applied to meshed networks.

1) Objective Function and Control Variables

To maximize network energy efficiency and customer benefits through CVR, the proposed objective function minimizes the active power flows from the primary substation or the head of a given MV feeder ($e \in Y \cup L$) at node $n \in N$ (5). While this objective function ensures the reduction of overall network energy imports, as it will be demonstrated later in section V, if required, it can be modified as multi-objective to give more prominence to certain components (e.g., losses, customer consumption, etc.). The controllable network variables are OLTC positions and capacitor stages in MV and LV networks.

$$\min \sum_{\varphi \in \Phi} (V_{n,\varphi}^{re} I_{e,n,\varphi}^{re} + V_{n,\varphi}^{im} I_{e,n,\varphi}^{im}) \quad (5)$$

In the case of reverse power flows due to large PV injections, customer voltages will be higher than normal and, therefore, their appliances will be likely to consume more. This affects all customers but mainly those without PV systems (fully reliant on imports). The adopted objective function will still be effective as it will try to make reverse power flows even larger by reducing voltages and, hence, consumption for all customers (PV systems are not affected).

2) Equality Constraints – Network Model

The voltage-dependent consumed power of each single-phase customer $d \in D$, connected at node n and phase φ is represented in pu using ZIP models (6), where $V_{n,\varphi}^{mag^2} = V_{n,\varphi}^{re^2} + V_{n,\varphi}^{im^2}$ (a similar equation is used for its reactive component). Then, the real and imaginary parts of the load current are given by (7) and (8), respectively. Residential PV systems $g \in G$ are modeled as negative constant power loads.

$$P_d = P_{d0} \left(\hat{Z}_d^P V_{n,\varphi}^{mag^2} + \hat{I}_d^P V_{n,\varphi}^{mag} + \hat{P}_d^P \right), \forall d \in D \quad (6)$$

$$I_{d,n,\varphi}^{re} = \frac{P_d V_{n,\varphi}^{re} + Q_d V_{n,\varphi}^{im}}{V_{n,\varphi}^{mag^2}}, \forall d \in D \quad (7)$$

$$I_{d,n,\varphi}^{im} = \frac{P_d V_{n,\varphi}^{im} - Q_d V_{n,\varphi}^{re}}{V_{n,\varphi}^{mag^2}}, \forall d \in D \quad (8)$$

As the active and reactive power measurements for each customer (P_d , Q_d) are already affected by the voltage magnitude, the input parameters P_{d0} and Q_{d0} need to be firstly computed using the last customer ZIP parameters from (6).

A three-phase staged capacitor ($c \in C$) connected at node n injects a total reactive power per phase $Q_{c,n,\varphi}$ that depends on the total rated capacity Q_{c0} , the number of stages st_c^+ and the current stage st_c . Moreover, given their constant impedance nature, $Q_{c,\varphi}$ also varies quadratically with the voltage magnitude (9). Thus, the real and imaginary parts of the capacitor current at phase φ are given by (10) and (11), respectively.

$$Q_{c,n,\varphi} = \frac{Q_{c0} st_c}{3 st_c^+} V_{n,\varphi}^{mag^2}, \forall c \in C, \forall \varphi \in \Phi \quad (9)$$

$$I_{c,n,\varphi}^{re} = \frac{Q_{c,n,\varphi} V_{n,\varphi}^{im}}{V_{n,\varphi}^{mag^2}} = \frac{Q_{c0} st_c}{3 st_c^+} V_{n,\varphi}^{im}, \forall c \in C, \forall \varphi \in \Phi \quad (10)$$

$$I_{c,n,\varphi}^{im} = -\frac{Q_{c,n,\varphi} V_{n,\varphi}^{re}}{V_{n,\varphi}^{mag^2}} = -\frac{Q_{c0} st_c}{3 st_c^+} V_{n,\varphi}^{re}, \forall c \in C, \forall \varphi \in \Phi \quad (11)$$

The phase to ground voltage drop in a three-phase line $l \in L$ with starting and ending nodes $\beta_l^1, \beta_l^2 \in N$ is given by the Kirchhoff's Voltage Law. This is expressed in state variables in (12) and (13), where $R_l^{\varphi,\vartheta}$ and $X_l^{\varphi,\vartheta}$ are the resistance and reactance of the line between phases $\varphi, \vartheta \in \Phi$. Here, given the line lengths in distribution networks, shunt admittances are neglected [38] and therefore, $I_{l,\beta_l^1,\varphi} = I_{l,\beta_l^2,\varphi} = I_{l,\varphi}$.

$$V_{\beta_l^1,\varphi}^{re} - V_{\beta_l^2,\varphi}^{re} = \sum_{\vartheta \in \Phi} (R_l^{\varphi,\vartheta} I_{l,\vartheta}^{re} - X_l^{\varphi,\vartheta} I_{l,\vartheta}^{im}), \forall l \in L, \forall \varphi \in \Phi \quad (12)$$

$$V_{\beta_l^1,\varphi}^{im} - V_{\beta_l^2,\varphi}^{im} = \sum_{\vartheta \in \Phi} (R_l^{\varphi,\vartheta} I_{l,\vartheta}^{im} + X_l^{\varphi,\vartheta} I_{l,\vartheta}^{re}), \forall l \in L, \forall \varphi \in \Phi \quad (13)$$

The equivalent phase to ground complex voltage relation for delta-wye transformers (with β_y^1 and β_y^2 being the delta and wye nodes respectively) is described by (14), while their phase currents (now different at each side) are related by (15) [38].

$$V_{\beta_y^1,\varphi} = -\frac{nr_y \tau_y}{3} \left\{ 2V_{\beta_y^2,\varphi+1} + V_{\beta_y^2,\varphi+2} + Z_y \left(2I_{y,\beta_y^2,\varphi+1} + I_{y,\beta_y^2,\varphi+2} \right) \right\}, \forall y \in Y, \forall \varphi \in \Phi \quad (14)$$

$$I_{y,\beta_y^1,\varphi} = \frac{1}{nr_y \tau_y} \left(I_{y,\beta_y^2,\varphi} - I_{y,\beta_y^2,\varphi+1} \right), \forall y \in Y, \forall \varphi \in \Phi \quad (15)$$

where $\varphi+1$ and $\varphi+2$ represent the two consecutive phases to φ (e.g., for $\varphi=2$, these will be 3 and 1, respectively), Z_y is the complex impedance of the transformer, the parameter nr_y is the transformer nominal ratio (delta line to line voltage over wye line to neutral voltage), which is equal to $\sqrt{3}$ (in pu) if both values coincide with the network nominals, and τ_y is the three-phase transformer tap position in pu ($\tau_y \in [\tau_y^-, \tau_y^+]$). The latter relates to the actual tap position $k_y \in [1, k_y^+]$, as shown in (16). Adopting the industry convention, the smaller tap position corresponds to the largest tap ratio, and therefore, the smallest voltage at the secondary side. Thus, for $k_y = 1$ the tap ratio is τ_y^+ and for $k_y = k_y^+$ the tap ratio is τ_y^- .

$$\tau_y = \frac{\tau_y^+ (k_y^+ - k_y) + \tau_y^- (k_y - 1)}{k_y^+ - 1}, \forall y \in Y \quad (16)$$

The distribution network has at least one point of connection with the upstream network. This corresponds to a subset of nodes $X \subset N$, where one $x_0 \in X$ will represent the source node for which the three-phase voltage is imposed by measurements, becoming a parameter for the AC OPF. Finally, the current balance at each node and phase is given by Kirchhoff's Current Law (17), which is split into real and imaginary parts.

$$I_{x,n,\varphi} + I_{g,n,\varphi} + I_{c,n,\varphi} + \sum_{e \in LUY | \beta_e^2 = n} I_{e,\beta_e^2,\varphi} = I_{d,n,\varphi} + \sum_{e \in LUY | \beta_e^1 = n} I_{e,\beta_e^1,\varphi}, \forall n \in N, \forall \varphi \in \Phi \quad (17)$$

3) Inequality Constraints – Operational Limits

The voltage magnitude $V_{n,\varphi}^{mag}$ at node n and phase φ is constrained by maximum and minimum operational limits $V_n^{(+,-)}$ (18). Given that voltage standards are usually defined at customer connection points, statutory limits are adopted for nodes connecting customers only. Similarly, the magnitude of phase currents in each line l and transformer y are constrained by their capacity I_e^+ (19), with $e \in L \cup Y$.

$$V_n^- \leq V_{n,\varphi}^{mag} \leq V_n^+, \forall n \in N, \forall \varphi \in \Phi \quad (18)$$

$$I_{e,\beta_e,\varphi}^{mag} \leq I_e^+, \forall e \in L \cup Y, \forall \beta_e \in \{\beta_e^1, \beta_e^2\}, \forall \varphi \in \Phi \quad (19)$$

For lines, given that shunt admittances are neglected, it suffices to enforce (19) in one (any) of their nodes.

D. Optimization Engine – Two-Stage Approach

The discrete nature of OLTC positions and capacitor stages makes the AC OPF formulation a mixed integer non-linear program (MINLP), which, given the large number of variables involved, can present scalability issues for real networks [39]. To reduce computational burden, discrete variables are relaxed (treated as continuous), taking the MINLP problem to a non-linear program (NLP) that can be efficiently solved. Note that the AC OPF problem is non-convex and, therefore, there is no guarantee on global optimality.

Here, a NLP two-stage approach is proposed to maximize CVR benefits, providing coordination between controllable elements at different voltage levels and avoiding customer voltage problems. In the first stage, a relaxed AC OPF is run to find locally optimal settings for OLTCs and capacitors (continuous values). This results in customer voltages that are as low as possible, with some at exactly V_n^- , if attainable. The settings found for MV OLTCs and capacitors are then rounded to their closest discrete value, providing the base voltage for all LV networks and reducing voltage drops in feeders. The second stage provides a ‘fine tuning’ of voltage regulation through the determination of the most adequate taps for LV OLTCs. This is achieved by running a second NLP AC OPF that takes as inputs the settings determined in the first stage. Given that the relaxed values for LV OLTC taps can result in voltages equal to V_n^- , this time taps are rounded to their upper discrete value to avoid customer voltages below the limit. In cases where the voltage control at different levels is not attainable (e.g., control of MV OLTCs alone), the resulting relaxed settings are directly rounded up in the first stage.

Although this is not an optimal solution, it provides high-quality results and adequate coordination among all controllable items to deliver CVR. This is demonstrated in the Appendix with comparisons against other rounding alternatives as well as an exhaustive search.

IV. PERFORMANCE METRICS

To adequately quantify the benefits brought by the OPF-based CVR control, as well as the wear and tear of the assets (OLTCs and capacitors), the following metrics are adopted.

- Network Energy Consumption, E_{net} : Daily active energy consumption of the MV-LV network (i.e., including losses). Reverse power flows can lead to negative values.
- Network Energy Reduction, ΔE_{net} : Decrease in E_{net} resulting from the CVR scheme (compared to the case without CVR). This relates to the objective function of the OPF-based CVR scheme and quantifies its energy performance.
- Individual Customer Energy Imports Reduction, $\Delta E_{imp,d}$: Decrease in daily active energy imports of customer d resulting from the CVR scheme (compared to the case without CVR). Imports are zero at periods of PV exports.

- Customer Energy Imports Reduction, ΔE_{imp} : Average of all $\Delta E_{imp,d}$. Min. and max. values are also used.
- Energy Losses, E_{loss} : Daily active energy losses of the MV-LV network.
- Energy Losses Reduction, ΔE_{loss} : Decrease in E_{loss} resulting from the CVR scheme (compared to the case without CVR).
- Voltage Problems: The percentage of customers for whom the applicable voltage standard is not met.
- Tap Changes: Daily number for OLTCs.
- Capacitor Stage Changes: Daily number for capacitors.
- Optimization Triggers: Daily optimization triggering events.

V. CASE STUDY

In this section, the proposed OPF-based CVR control is demonstrated with a realistic UK MV-LV network and considering 50% of PV penetration level, i.e., 50% of residential customers with PV systems. This is a large PV penetration level that does not create thermal nor voltage issues, i.e., reverse power flows can be handled without the need for new assets or operational strategies, such as curtailment. Its performance is assessed with different combinations of controllable elements (or schemes) for three weekday types: winter, summer clear sky, and summer cloudy sky. The first day type allows to assess the scheme with maximum demand and no PV generation. The second corresponds to minimum demand and maximum PV generation. The third, allows to assess the performance of the schemes under rapid voltage changes.

LV customers with voltage problems are defined according to the UK standard BS EN50160 [40] on a daily basis, i.e., 10-min average rms customer voltages must remain between 0.94 and 1.1 pu (base 230 V line to neutral) for at least 95% of the time and never below 0.85 or above 1.1 pu. Here, the problem persistency approach is implemented with a bandwidth defined by $V_{band}^- = 0.94$ and $V_{band}^+ = 0.98$ pu, and a persistency period $pp = 10$ min, consistent with the standard.

It is assumed that average rms voltages, active and reactive power, and ZIP parameters are obtained/calculated every minute using monitoring devices such as smart meters. This is aligned with the 1-min resolution adopted for the simulations (section V-B). From the problem persistency perspective, this 1-min sampling also means that a problem is identified only when the minimum customer voltage presents 10 consecutive measurements outside the bandwidth.

The three-phase MV-LV distribution network is fully modeled in OpenDSS [41], while the SCADA in MATLAB. The optimization engine is implemented in AIMMS with CONOPT 3.14v as solver [42].

A. Real UK MV-LV Network

A realistic residential MV-LV network from the North West of England is used. All MV and LV feeders have been validated in close collaboration with the DNO using monitoring data and according to the criteria in [43]. For clarity, only the most populated MV feeder (6.6 kV) is fully modeled while others are represented with a lumped load and PV generation at the MV busbar as shown in Fig. 2. The fully-modeled MV feeder supplies 2,430 single-phase residential customers through nine

Fig. 2. Realistic UK MV-LV network.

Fig. 3. Topology of two LV feeders (values in meters).

TABLE I
MAIN LV NETWORKS' CHARACTERISTICS

Feature	Min	Average	Max
Customers per LV feeder/network	21/70	58/270	100/553
Feeders per LV network	2	4	7
Feeder main path length (m)	271	400	640
Transformer rating (kVA)	315	608	1000

Delta-wye secondary substations and 36 underground three-phase four-wire 0.4 kV LV feeders (transformer capacities and number of feeders per LV network are also shown in Fig. 2). This results in a network model of 4,580 nodes, suitable for assessing the implementability of the proposed approach. The main characteristics of the LV networks are summarized in Table I. For illustration purposes, the topologies of the LV feeders with the minimum and maximum number of customers are shown in Fig. 3 (a) and (b), respectively; where the secondary transformer and customers per phase can be seen.

The network is supplied through two 33 to 6.6 kV Delta-wye parallel primary transformers of 11.5 MVA each (parallel transformer due to UK reliability requirements). The corresponding OLTCs (MV OLTCs) operate following the same setting. These OLTCs regulate voltages in 17 steps of 1.43% each, providing a +17.16/-5.72% regulation range with respect to the nominal position (tap 5). For simplicity, the high-voltage side is considered balanced and fixed at 1 pu. Secondary transformers have a voltage ratio of 6.6 to 0.433 kV (8.7% boost) and are equipped with no-load tap changers (NLTC) with $\pm 5\%$ range (5 taps, 2.5% steps).

In this study, two LV technologies are considered: OLTC-equipped secondary substations (LV OLTCs) with 9 steps of 2% ($\pm 8\%$) as commercially available [32] and LV staged capacitors. LV OLTCs are connected so that tap 8 delivers 0.433 kV, resulting in a voltage range from 0.953 to 1.109 pu.

Three-phase capacitors are placed and sized in LV feeders with over 80 customers (longest and most populated) using the simple two-thirds rule [44]. This design criteria aims to keep to a minimum reactive flows in a feeder by locating capacitors at 2/3 of its length, which are also sized to provide 2/3 of the feeders' reactive power at peak time, so each third of the feeder carries only 1/3 of the total reactive demand (assuming uniformly distributed loads). After a power flow analysis, the rule resulted in an average three-phase size of 15 kvar. Thus,

Fig. 4. (a) Single household active power profile and model for winter; (b) diversified active power and model for winter and summer; and (c) real summer irradiance profiles for clear and cloudy sky days.

two-stage capacitors of 7.5/15 kvar are considered, where the smaller stage (7.5 kvar) can help in less loaded periods.

B. Time-Varying Load Profiles and Models

Realistic one-minute resolution domestic household profiles are created using a tool developed in [45]. This tool uses UK statistics to determine the status of domestic appliances in a day considering occupancy of the house, type of day and seasonality. Active and reactive profiles, as well as time-varying ZIP models, are then obtained per household by aggregating individual appliances at each minute, as in [34]. In this network, no major constant energy appliances are involved as all households are assumed to use gas for heating purposes.

One thousand (1,000) individual household profiles with models, as the one shown in Fig. 4 (a), are created for winter and summer considering recent UK statistics for the number of occupants per house [46], and randomly assigned among LV customers. This allows to realistically model the time-varying responses of customers to changes in voltages. For the fully-modeled MV feeder (with 2,430 LV customers) this results in a peak demand of approximately 2 MVA.

Similarly, the lumped MV load is characterized considering the corresponding diversified time-varying demand and load models, shown in Fig. 4 (b). To meet the maximum demand of the primary substation (capacity of one primary transformer, 11.5 MVA), the lumped MV load is scaled up accordingly. The same scaling factor is also used for summer days.

The main differences in demand shown in Fig. 4 (b) are due to different sunrise/sunset hours and its effect on lighting. Here, it can be seen that regardless the season, day time presents higher voltage-demand dependency than night, which together with high voltages due to PV can result in larger opportunities to reduce voltages and demand. However, these reductions would be limited in days with little PV generation due to larger voltage drops. Contrarily, the lower voltage-demand dependency at night might be compensated by large opportunities to reduce voltages due to small voltage drops.

C. PV Generation

PV generation is simulated with unity power factor and using a linear relationship between PV system ratings and solar irradiance [33]. In this study, 50% of the LV customers are randomly selected to have PV systems with rated capacities aligned with UK statistics [47] (from 1 to 4 kWp and with an average of 3 kWp). For consistency, the lumped PV generation for the other MV feeders is sized with the same criteria.

One-minute resolution solar irradiance recordings from Manchester, UK [48] are used as shown in Fig. 4 (c) for each of the summer weekdays. For simplicity, and given the rela-

Fig. 5. Profile of the bottleneck voltage for each scheme in: (left) winter, (center) summer cloudy sky and (right) summer clear sky.

Fig. 6. CVR operational behavior of scheme 2 in: (left) winter, (center) summer cloudy sky and (right) summer clear sky.

atively small geographical distance (less than 5 km), it is assumed that all customers get the same solar irradiance. The latter, represents a worst-case scenario for the voltage control (particularly for the cloudy sky day) as, in reality, due to cloud movements, not all PV systems will have the same rate of generation change, resulting in smoother voltage profiles.

D. Controllable Element Combinations (CVR Schemes)

Three OPF-based CVR schemes are compared to assess the benefits brought by different combinations of controllable elements for each of the three day types.

- **Scheme 1:** Only the MV OLTCs are available. Secondary transformers have NLTCs fixed at their minimum position (tap 1, 5% voltage reduction) to compensate for high voltages due to PV in summer (as current DNO practice).
- **Scheme 2:** All secondary substations are equipped with LV OLTCs, adding to the existing MV OLTCs.
- **Scheme 3:** LV capacitors are added to scheme 2 to compensate for voltage drops.

These schemes are compared to the business as usual operation (BAU), which is similar to ‘scheme 1’ but with the primary substation regulating the MV busbar voltage to 1 pu in winter (as done typically) and to 0.98 pu in summer (to avoid customer overvoltages with 50% of PV penetration). Moreover, differently to the three CVR schemes where OLTC positions and capacitor stages are directly assigned, a bandwidth of $\pm 1.5\%$, a persistency problem delay of 120 seconds and 15 seconds between tap changes are used to model the MV OLTCs in the BAU scheme, in line with current UK practices. Note that the bandwidth relative to the voltage target (1.5%) is only slightly larger than the voltage regulation step of the MV OLTC (1.43%). This ensures a good regulation performance.

Finally, to avoid biased results towards minimizing the lumped MV load, the objective function is adapted to minimize active power flows at the head of the fully-modeled MV feeder rather than the whole primary substation. For con-

sistency, E_{net} and E_{loss} are also measured at this point (head of the fully modelled MV feeder).

E. Time-Varying Control Assessment

The ability of the different CVR schemes to exploit the available voltage footroom (and, hence, reduce demand) is illustrated in Fig. 5 for the three day types. The bottleneck voltage profile, i.e., the one triggering the CVR, is shown for each scheme. For the BAU operation (no CVR), the minimum customer voltage across the network at each time step is included for comparison purposes.

In winter, during the night, the BAU is shown to have a significant voltage footroom of approximately 0.08 pu (18.4 V, with respect to $V_{band}^- = 0.94$ pu). A similar footroom appears in summer during daylight due to PV generation, and a still large footroom of 0.06 pu is seen during night. This results in about 18 hours a day with large potential for CVR in summer. As expected, the investigated CVR schemes are able to exploit the time-varying voltage footroom to different extents. For scheme 1 (only MV OLTCs), the minimum tap (position 1) is the optimal setting at these periods of higher voltages. This, however, still leaves a footroom of 0.03-0.04 pu. At times of larger demand (and voltage drops), the scheme results in higher taps (up to 6 in winter) to keep bottleneck voltages within the CVR bandwidth. Note that with only MV OLTCs, the results are more conservative as the taps are rounded up to avoid customer problems. Schemes 2 and 3 are the most effective given the extra flexibility from LV OLTCs and capacitors. They show a very similar performance that leaves almost no voltage footroom throughout the day (see Fig. 5), unlocking most of the CVR potential in the network. Indeed, the average footroom for all the day types was below 0.007 pu. Capacitors (in scheme 3) reduce feeder voltage drops at periods of high demand, allowing, in combination with OLTCs, to reduce voltages even further. This, however, is mainly limited (in this network) to winter peak time. This is shown in Fig. 5 where,

TABLE II
PERFORMANCE OF CVR SCHEMES

Performance Metrics	winter				summer cloudy sky				summer clear sky			
	BAU	Scheme 1	Scheme 2	Scheme 3	BAU	Scheme 1	Scheme 2	Scheme 3	BAU	Scheme 1	Scheme 2	Scheme 3
$E_{net} \Delta E_{net}$ (MWh/%)	22.52	22.16 1.6	21.80 3.2	21.69 3.7	-4.05	-4.39 -8.5	-4.65 -15.0	-4.69 -15.8	-11.31	-11.73 -3.7	-12.00 -6.1	-12.02 -6.2
ΔE_{imp} (min/avg/max)	-	-0.4 1.6 5.6	-1.0 2.9 9.3	-0.8 3.4 9.5	-	0.0 2.4 9.8	0.1 4.0 18.6	0.3 4.2 16.0	-	0.8 3.0 11.4	0.1 4.6 17.9	0.2 4.7 18.7
$E_{loss} \Delta E_{loss}$ (MWh/%)	0.507	0.505 0.4	0.484 4.6	0.482 5.0	0.652	0.695 -6.6	0.693 -6.4	0.680 -4.4	0.859	0.923 -7.5	0.937 -9.2	0.934 -8.8
MV tap chg. (#)	0	21	5	7	1	17	7	9	1	14	11	9
LV tap chg. (min/avg/max)	-	-	13 20 27	13 26 40	-	-	12 16 23	15 18 20	-	-	15 19 23	14 19 26
Cap. chg. (min/avg/max)	-	-	-	4 14 30	-	-	-	2 10 18	-	-	-	2 10 18
Optimization triggers (#)	-	21	23	35	-	16	16	17	-	20	21	24

from 21 to 23 hours, a small footroom otherwise left with scheme 2 (up to 0.02 pu) is exploited by scheme 3.

To illustrate the operation of the proposed CVR approach, customer voltages and control actions of scheme 2 are plotted in Fig. 6 for the three day types. All customer voltages across the network are represented with a grey area to show the spectrum of values. In addition, to demonstrate the effects of MV and LV OLTCs, the voltage profile of the furthest customer in the most populated LV network, V_{fc} (at the end of the longest LV feeder of TR7 in Fig. 2), is included. The control actions correspond to the tap positions of the MV and the TR7 LV OLTCs. Optimization triggering events are shown with dots and bandwidth limits are represented with dashed lines. The results confirm how scheme 2 leads to customer voltages as close as possible to the minimum statutory limit regardless of PV generation conditions. LV OLTCs operate at low positions (1 to 3) most of the time. MV OLTCs, on the other hand, from positions 5 to 8 in winter and from position 3 and 7 in summer. In this way, the scheme not only minimizes LV demand but also losses in the MV feeder (due to higher MV voltages). Moreover, Fig. 5 shows that the bottleneck customer voltage in scheme 2 experiences only short duration excursions outside the bandwidth, with some persistent enough to trigger the optimization (e.g., night ‘spikes’ below V_{band}^- did not trigger it), as seen in Fig. 6. These events (up to 23 in winter), however, tend to be localized, resulting in changes for only some elements (those with problems or voltage footroom).

Finally, Table II summarizes the performance of the investigated schemes using the metrics from section IV (voltage problems were zero in all cases and, therefore, this metric has not been included). It shows how the combined flexibility of MV and LV OLTCs results in fewer MV tap changes (up to 11 for the clear sky day) than with the MV OLTC only. On the other hand, LV OLTCs are now actively used, with an average of up to 20 tap changes a day. This is because the OPF achieves the minimum power flow at the head of the MV feeder by simultaneously reducing LV consumption (lowering LV taps) and MV losses (increasing MV taps). In a year, the tap changes performed by the LV OLTCs would result in only 1% of the maximum number that, according to one manufacturer, they can stand in their lifetime [32].

F. Energy Performance Assessment

Without CVR, the widespread presence of PV systems leads, mainly during summer, to higher voltages across the MV feeder (as shown in Fig. 5). From an energy perspective, this means that appliances within households will consume more during daylight. This has a more evident effect on cus-

tomers without PV systems as they will fully rely on network imports. In fact, for the summer clear sky day, it was found that, in average, customers without PV increased their daily energy consumption by 1.8% (95% of them between 0.1 and 4.7%) when compared to the same day without any PV installation. Customers at the end of long LV feeders are the most affected as they experience the largest voltage increase. This increase also affects customers with PV systems but is largely offset by their corresponding local generation.

Nonetheless, higher voltages also mean larger voltage footroom for CVR purposes. Results in Table II demonstrate how network energy reductions (ΔE_{net}) are improved as flexibility is added to the network. In winter, ΔE_{net} of 1.6, 3.2 and 3.7% are achieved for schemes 1, 2 and 3, respectively. For customers, this translates to energy import reductions (ΔE_{imp}) of 1.6, 2.9 and 3.4%, respectively, evidencing small extra benefits from using LV capacitors.

In summer, due to the reverse power flows through the MV feeder, the reduced consumption is seen in a different way. Without CVR, the daily active energy at the head of the MV feeder is already negative ($E_{net} = -11.31$ MWh for clear sky). After CVR, the OPF-based schemes reduce consumption while keeping PV injections (constant power devices), thus making E_{net} even more negative. Scheme 3, for instance, results in $\Delta E_{net} = 710$ kWh. For customers, the use of LV OLTCs brings, in average, 4.6% less imported energy for the clear sky day.

Energy losses (E_{loss}) range from 0.5 MWh in winter (only 2% of E_{net}) to 0.9 MWh in the clear sky day (8% of a lower E_{net} due to reverse power flows). Depending on the PV generation conditions, however, CVR schemes might reduce or increase losses (according to what is best overall). In winter, schemes with LV OLTCs reduce losses by up to 5%. This is not only because MV losses are reduced (higher voltages) but also because lower LV demand draws less current due to a $n^P > 1$ during most of the day (see Fig. 4 (b)). In summer, however, losses increase by up to 9% due to lower demand resulting in larger reverse power flows.

Results in Table II also confirm that capacitors bring marginal energy benefits compared to scheme 2, especially in the summer days (in average, only 0.2% of extra ΔE_{imp}). PV generation results in voltages that are higher at capacitor locations than at LV busbars, limiting their actions (to compensate for voltage drops) to only a few hours.

G. Computational Performance

Scheme 3, the largest problem (10 OLTCs and 15 capacitors), translates to 81,101 variables and 122,293 constraints.

For this scheme, solving times were, in average, 34 and 12 s for the first and second stages, respectively. This demonstrates the implementability of the approach, reaching a feasible solution in less than a minute (adopted time resolution). This performance corresponds to a Windows OS machine with an Intel Core i7-4770 processor at 3.40 GHz and 16 GB of RAM.

VI. DISCUSSION

This work aims to help DNOs understanding the extent to which their own assets can be used, in a coordinated way, to exploit CVR, even in cases of high PV penetration. Thus, the control of PV systems was outside the scope of this work. Yet, the generic nature of the approach allows for its incorporation. In the context of PV curtailment, the same objective function would result in minimal curtailment as it will be used only when needed (i.e., other flexible elements are not enough, for instance, due to thermal congestion). Similarly, reactive power capabilities of PV inverters could be exploited if available.

Other low carbon technologies (LCTs) such as electric vehicles and storage systems could also be modeled (as shunt elements). These, being DC technologies, are expected to behave as constant power devices (unaffected by voltage). However, storage systems could help reducing peak demand, enabling larger voltage reductions at those times and, therefore, benefiting CVR operation. The adopted objective function would still apply as it delivers energy efficiency and benefits customers regardless the load/generation mix.

The proposed formulation is generic and can be used to pursue other objectives than CVR, for instance, loss minimization. The latter, however, can reduce or increase customers' demand depending on the load composition. In the worst case, when loads are between constant power and constant current, losses can be minimized by increasing voltages as this reduces currents from loads. This, however, increases customers' energy consumption and (most likely) the overall network energy imports. Furthermore, with PV systems (constant power devices), currents from reverse power flows (and losses) can be reduced by increasing voltages. This, however, increases customer demand. Thus, minimizing losses in PV-rich distribution networks would lead to a counterproductive outcome: larger energy consumption for customers and less renewable energy exports to the upstream network.

In this work, all the network impedances are considered to be known a priori. Given that, in reality, these can present variations, a sensitivity analysis could be carried out to assess the robustness of the proposed schemes (e.g., as in [49]). Similarly, the topology of the network is assumed to be known and fixed throughout the analysis. However, the model can be updated (online or offline) to cater for network reconfiguration or expansions, provided the corresponding data is available.

For an exact three-phase line modeling, shunt admittances can be incorporated (if available) by replacing (12) and (13) with the real and imaginary parts of (20), where $[U]$, $[Z_{123}]$ and $[Y_{123}]$ are the identity, series impedance and shunt admittance matrices, respectively [38].

$$\begin{bmatrix} V_{\beta_l^1,1} \\ V_{\beta_l^1,2} \\ V_{\beta_l^1,3} \end{bmatrix} = \left([U] + \frac{1}{2} [Z_{123}] [Y_{123}] \right) \begin{bmatrix} V_{\beta_l^2,1} \\ V_{\beta_l^2,2} \\ V_{\beta_l^2,3} \end{bmatrix} + [Z_{123}] \begin{bmatrix} I_{\beta_l^2,1} \\ I_{\beta_l^2,2} \\ I_{\beta_l^2,3} \end{bmatrix}, \forall l \in L. \quad (20)$$

This paper assumed that customer measurements of powers and voltages can be accurately extracted from smart meters and received at the control room. In reality, the level of accuracy of these measurements depends on the characteristics of the smart meters and the communication infrastructure. In terms of the communication burden between customers and the control room, this could be reduced, for instance, by applying the 'problem persistency' concept at a customer level. Once critical customers flag voltages outside the predefined bandwidth (e.g., as considered in the UK standard [50]), powers, voltages and other necessary parameters from all customers could be sent to the control room by request (only when the optimization engine needs to be run).

VII. CONCLUSIONS

This work proposes a centralized, three-phase AC OPF-based CVR scheme to unlock benefits from actively and coordinately managing DNO-owned infrastructure (OLTCs and staged capacitors) in PV-rich, unbalanced MV-LV networks.

Here, the concept of 'problem persistency' is adopted to trigger the CVR optimization only when needed, keeping customer voltages as low as possible but compliant with statutory limits. To improve scalability, a two-stage approach is proposed to relax integer variables while avoiding undervoltages. The approach is demonstrated in a realistic UK network with 2,400+ residential customers, 50% of PV penetration and 1-min resolution time-varying profiles and load models.

Results demonstrate that the proposed control effectively coordinates voltage regulation in MV and LV levels throughout the day, minimizing energy imports for all customers. Moreover, the two-stage approach delivers close to optimal results within solving times suitable for operation purposes.

In terms of energy, the study first shows that, without CVR, the widespread presence of PV leads, as expected, to higher voltages. This, in turn, results in higher energy imports, particularly for customers without PV systems. With the proposed CVR approach, it is demonstrated that the active, simultaneous control of MV and LV OLTCs can significantly reduce energy imports for all customers in winter and summer, compared to the sole use of MV OLTCs (as typically implemented in practice), while keeping tap changes low. Capacitors, instead, provided limited benefits, particularly in high PV generation days.

VIII. APPENDIX – TWO-STAGE APPROACH ASSESSMENT

The performance of the two-stage approach is compared to two single-stage alternatives considering scheme 3 and the winter day presented in section V. These alternatives are single-stage rounding (SSR) and ceiling (SSC), where integer variables are rounded to their closest or upper integer value, respectively. Results in Table III demonstrate that, as mentioned in section III-D, SSR causes voltage problems and also

TABLE III
PERFORMANCE OF ROUNDING APPROACHES

Performance Metrics	SSR	SSC	Two-stage (from section V-E)
Voltage Problems (%)	33	0	0
E_{net} (MWh)	21.63	22.06	21.69
E_{loss} (MWh)	0.482	0.488	0.482
MV tap chg. (#)	21	7	7
LV tap chg. (min avg max)	11 29 40	7 13 17	13 26 40
Cap. chg. (min avg max)	18 30 48	4 7 14	4 14 30
Optimization triggers (#)	59	12	35

TABLE IV

SNAPSHOT RESULTS: TWO-STAGE APPROACH V/S EXHAUSTIVE SEARCH												
Approach	OF (MW)	Diff (%)	Vmin cust. (pu)	Time (s)	OLTC Tap Position							
					M L1	L2	L3	L4	L5	L6	L7	L8 L9
ES	1.9450	-	0.9406	~1800	8	1	1	1	1	2	3	3 2
TS	1.9501	0.26	0.9429	21.5	7	2	2	2	1	3	4	4 3
SSR	1.9263	-0.96	0.9367	10.9	7	2	2	1	1	3	3	3 2
SSC	1.9606	0.80	0.9443	10.9	7	3	3	2	1	4	4	4 3

more optimization triggers (to try to solve them). SSC results in no voltage issues but in a more conservative CVR operation than the two-stage (TS) approach (1.7% less energy savings).

To assess the quality of the solution delivered by the TS approach, an exhaustive search (ES) is performed and the resulting objective functions (OF) are compared for a snapshot. For completeness, the two single-stage approaches are also evaluated. To make the exhaustive search possible, the search space had to be reduced. Here, capacitors are not available (scheme 2) and 3 possible taps were considered around the solution of the proposed approach (3^{10} combinations, solved in ~30 min). For simplicity, the case considers a maximum diversified demand of winter (0.8 kW without PV) for all loads, together with $n_i^P=1$ and $n_i^Q=2$ (close to the diversified load model at peak time). Results in Table IV show that, while the resulting tap combination can be different, the objective function of the proposed approach was only 0.26% larger than the optimal (in the explored search space). Moreover, it is confirmed that SSR results in customer problems (minimum customer voltage below the limit of 0.94 pu) and that the SSC, being more conservative, results in less energy savings.

The performance of the chosen solver (CONOPT 3.14v) was assessed for the same snapshot against a different NLP solver (IPOPT 3.11) considering two different initialization points (customer voltages at 1 or 0.96 pu). The results were the same for OF and relaxed tap positions, however, while CONOPT took around 10s for either case, IPOPT took 54 and 112s, respectively. This shows that although both solvers deliver the same result in less than a couple of minutes (suitable for the CVR control), CONOPT is faster and less sensitive to initialization inputs, justifying its selection. For completeness, the solving times for the TS, SSR and SSC schemes, using CONOPT as solver, are also shown in Table IV.

Solving the original MINLP problem (same snapshot) was also attempted using the AOA solver in AIMMS [42]. Yet, the process was stopped after 20 hours without converging.

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