Improved CMB anisotropy constraints on primordial magnetic fields from the post-recombination ionization history

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ABSTRACT

We investigate the impact of a stochastic background of Primordial Magnetic Fields (PMF) generated before recombination on the ionization history of the Universe and on the Cosmic Microwave Background radiation (CMB). Pre-recombination PMFs are dissipated during recombination and reionization via decaying MHD turbulence and ambipolar diffusion. This modifies the local matter and electron temperatures and thus affects the ionization history and Thomson visibility function. We use this effect to constrain PMFs described by a spectrum of power-law type, extending our previous study (based on a scale-invariant spectrum) to arbitrary spectral index. We derive upper bounds on the integrated amplitude of PMFs due to the separate effect of ambipolar diffusion and MHD decaying turbulence and their combination. We show that ambipolar diffusion is relevant for $n_B > 0$ whereas for $n_B < 0$ MHD turbulence is more important. The bound marginalized over the spectral index on the integrated amplitude of PMFs with a sharp cut-off is $\sqrt{\langle B^2 \rangle} < 0.83$ nG. We discuss the quantitative relevance of the assumptions on the damping mechanism and the comparison with previous bounds.

Key words: Cosmology: CMB – theory – observations

1 INTRODUCTION

Primordial magnetic fields (PMFs) generated prior to cosmological recombination provide an interesting window on the physics of the Early Universe and could have seeded the astrophysical large scale magnetic fields we observe in clusters and voids. These PMFs leave imprints on the Cosmic Microwave Background (CMB) through different mechanisms. PMF gravitate at the level of cosmological perturbations and source magnetically-induced perturbations. The comparison of theoretical predictions with different combinations of CMB data has been presented in several works (Paoletti & Finelli 2011, Shaw & Lewis 2012, Paoletti & Finelli 2013, Planck Collaboration XVI 2014, Planck Collaboration XIX 2016, Zucca et al. 2016), leading to constraints on the amplitude of PMFs smoothed at 1 Mpc of the order of few nG. PMFs also induce a Faraday rotation of CMB polarization, mixing E- and B-modes with an angle inversely proportional to the square of the frequency (Kosowsky & Loeb 1996, Kahniashvili et al. 2009, Pogosian et al. 2011). At present, Faraday rotation leads to constraints which are weaker than those obtained by considering the gravitational effect, but represents a good target for the future low-frequency polarization experiments (Kahniashvili et al. 2009, Pogosian et al. 2011, Planck Collaboration XIX 2016).

PMFs in the cosmological plasma prior to the recombination may also have an impact on the thermal and ionization history (Sethi & Subramanian 2005, Seshadri & Subramanian 2005, Sethi & Subramanian 2009). Around recombination the reduced ionization fraction induces an ambipolar diffusion effect, whereas the drop in radiation viscosity after recombination allows for the development of magneto-hydrodynamic (MHD) turbulence. Both these effects depend on the integrated magnetic energy and dissipate it into the plasma, producing spectral distortions in the CMB absolute spectrum (Jedamzik et al. 2000) and altering the ionization history. This latter effect modifies the CMB anisotropies angular power spectra in both temperature and polarization (Sethi & Subramanian 2005, Seshadri & Subramanian 2005). Recent works (Kunze & Komatsu 2014, 2015, Chluba et al. 2015, Planck Collaboration XIX 2016) have reconsidered these effects and derived an upper limit on the PMFs integrated amplitude for a nearly scale-invariant and negative indices (Kunze & Komatsu 2015) stochastic background at the nG level, tighter than those derived on the basis of the gravitational effects only. However, as stressed previously (Chluba et al. 2015, Planck Collaboration XIX 2016), significant uncertainties exist in the description of the heating rates and consequently the derived constraints.

The scope of this paper is to derive the CMB constraints on a stochastic background of PMFs by their impact on the modified ionization history and anisotropies angular power spectra beyond
the nearly-scale invariant case previously reported (e.g., Planck Collaboration XIX 2016 Chluba et al. 2015). Constraints for PMF spectral indices $n_B = -1.5$ and $-2.5$ were already obtained by Kunze & Komatsu (2015). Here we extend the analysis to arbitrary spectral index and improve the treatment including subtle effects. We improved the numerical accuracy of the recombination code Recfast++ (Chluba & Thomas 2011), which includes the heating effect of PMFs by means of two different methods dedicated specifically to MHD turbulence and to ambipolar diffusion. In order to maximize the numerical stability of CAMB, following Hart & et al. (2018), we also enhanced the time-step settings during recombination which hampered the precision of the obtained CMB power spectra at large scales, leading to a slower convergence of MCMC chains.

The paper is organized as follows. In section 2 we describe the details of a stochastic background of PMFs and of the induced modified ionization history. In section 3 we describe the impact of the MHD decaying turbulence and of the ambipolar diffusion on the CMB power spectra. We present the constraints from PLANCK 2015 data in section 4. In section 5 we discuss our results and we draw our conclusions in section 6. In appendix A we describe the implications of our results on the commonly adopted amplitude of PMF smoothed at 1 Mpc scale.

## 2 IMPACT OF PRIMORDIAL MAGNETIC FIELDS ON THE POST-RECOMBINATION IONIZATION HISTORY

We consider a fully inhomogeneous stochastic background of non-helical PMFs scaling as $B(x, t)\sim B_0 e^{\alpha(t)}$ described by:

$$\langle B_i(k)B_j(k')\rangle = (2\pi)^3\delta(k-k')(\delta_{ij}-k_i k_j)\frac{P_B(k)}{2}$$

where the magnetic power spectrum is $P_B(k)=A_B k^n$. Radiation viscosity damps PMFs at a damping scale $k_D$ (Fedamzik et al. 1998; Subramanian & Barrow 1998):

$$k_D = \left(\frac{5.5 \times 10^4 (2\pi)^{\frac{6-n}{2}}}{\sqrt{(B^2)/nG}}\right) \sqrt{\frac{\Omega_b h^2}{0.022}} \cdot (2)$$

In this paper, we choose to model this damping by imposing a sharp cut-off at the scale $k_D$ to regularize ultraviolet divergencies in integrated quantities, as done in the study of the PMFs gravitational effects. We therefore define the root mean square as:

$$\langle B_i^2 \rangle = \frac{A_B}{2\pi^2} \int_0^\infty d k k^2 nG = \frac{A_B}{2\pi^2(n_B + 3)} \frac{\Omega_b h^2}{0.022} \cdot (3)$$

Note that in our previous paper (Chluba et al. 2015) we considered a Gaussian smoothing as in Kunze & Komatsu (2015) to regularize the integrated amplitude of the stochastic background. According to Sethi & Subramanian (2005), the heating due to PMFs to the electron temperature equation is modelled as:

$$\frac{d T_e}{d r} = -2HT_e + \frac{8\pi \gamma N_e n_b^2 (T_g - T_e)}{3mc\nu_N} + \frac{\Gamma}{(3/2)k}\sqrt{\sum_{n=1}^{\infty} (4 \pi)^{n+3}}$$

where $H(z)$ denotes Hubble rate, $N_{n} = N_{\text{tot}}(1 + f_{\text{He}} + X_{\text{He}})$ the number density of all ordinary matter particles that share the thermal energy, beginning tightly coupled by Coulomb interactions; $N_{\text{He}}$ is the number density of hydrogen nuclei, $f_{\text{He}} \approx Y_{\text{He}}/(1-Y_{\text{He}}) \approx 0.079$ for helium mass fraction $Y_{\text{He}} = 0.24$; $X_{\text{He}} = N_{\text{He}}/N_{\text{H}}$ denotes the free electron fraction and $\rho_\gamma = n_{\gamma} T_{\gamma}^4 \approx 0.26 \epsilon (1+z)^4$ the CMB energy density. The first term in Eq. (4) describes the adiabatic cooling of matter due to the Hubble expansion, while the second term is caused by Compton cooling and heating. The last term accounts for the PMF heating due to the sum of the decaying magnetic turbulence ($\Gamma_{\text{mHD}}$) and ambipolar ($\Gamma_{\text{amb}}$), respectively.

We review in the following the approach of the aforementioned heating terms and describe the regularization and numerical improvements we provide with respect to previous treatments (Kunze & Komatsu 2015 Chluba et al. 2015).

### 2.1 Decaying MHD turbulence

On scales smaller than the magnetic Jeans scale, PMFs may be subject to non-linear effects and develop MHD turbulence. Before recombination the radiation viscosity over-damps the velocity fluctuations maintaining the Reynolds number small. After recombina-
tion, the sudden drop of radiation viscosity allows for the development of large Reynolds number and for the transfer of energy from large towards smaller scales, dissipating energy. The dissipation of the fields injects energy into the plasma, with a rate that can be approximated as (Sethi & Subramanian 2009):

$$\Gamma_{\text{amb}} = \frac{3m}{2} \left[ \ln \left( \frac{1 + \frac{\gamma}{m} \ln \rho_{\text{b}}(z) \right) \right]^{8/3} \frac{H(z)\rho_{\text{b}}(z)}{\ln \rho_{\text{b}}(z)}, \quad (5)$$

with the parameters $m = 2(m_{\text{b}} + 3)/m_{\text{b}} + 5$, $t_{\text{is}} \approx 14.8(\langle B^2 \rangle^{1/3}/nG)^{1}(k_{\text{b}}/M_{\text{pc}})^{1}/s^{-1}$, and magnetic field energy density $\rho_{\text{b}}(z) = \langle B^2 \rangle(1 + z)^{3}/(8\pi) \approx 9.5 \times 10^{-8}((\langle B^2 \rangle/nG)^{3}m_{\text{b}}(z)$.

### 2.1.1 Regularizing around recombination

Following previous approaches (Planck Collaboration XIX 2016, Chluba et al. 2015) the heating term due to decaying magnetic turbulence in Eq. (5) switches on abruptly at $z_{c} \sim 1088$. Although the rate is a continuous function, the cusp at $z_{c} = 1088$, shown in Fig. 1, creates numerical issues for the derivatives within the modified recombination code we have developed to include PMFs. The decaying magnetic turbulent rate in Eq. (5) is weakly coupled to the rate is a continuous function, the cusp at $z_{c} = 1088$, shown in Fig. 1, creates numerical issues for the derivatives within the modified recombination code we have developed to include PMFs. The decaying magnetic turbulent rate in Eq. (5) is weakly coupled to the time evolution of the electron temperature in Eq. (4) for $\frac{\rho_{\text{b}}(z)}{\rho_{\text{hi}}}$, and make it zero at $1 - \frac{\rho_{\text{b}}(z)}{\rho_{\text{hi}}}$. In order to capture the effect of heating by ambipolar diffusion we use the approximation (Sethi & Subramanian 2005, Schleicher et al. 2008):

$$\Gamma_{\text{amb}} \approx \frac{(1 - \rho_{\text{b}})}{\gamma_{\text{b}}\rho_{\text{b}}^{2}} \langle B^2 \rangle \quad (6)$$

where $\langle B^2 \rangle = \langle |\nabla \times B| \rangle = \langle B^2 \rangle/2nG \approx \langle B^2 \rangle/2nG$ denotes the average square of the Lorentz-force $\rho_{\text{b}} = m_{\text{b}}n_{\text{b}}$ the baryon mass density with baryon number density $N_{\text{b}}$, and $X_{\text{b}} = N_{\text{b}}/N_{\text{hi}}$ the coupling between the ionized and neutral component. The coupling coefficient is given by $y = (\sigma v)_{\text{mH}}/2n_{\text{b}}$ with $(\sigma v)_{\text{mH}} = 6.49 \times 10^{-10}(T/K)^{0.375}cm^3 s^{-1}$. For $-2.9 < n_{\text{b}} < 2$, the integral for the Lorenz force according to a sharp cut-off prescription is:

$$\langle |\nabla \times B| \rangle = 16\pi n_{\text{b}}^{2}g_{\text{L}}(z)g_{\text{L}}(n_{\text{b}} + 3) \quad (7)$$

$$g_{\text{L}}(z) = 0.6615[1 - 0.1367z + 0.007574z^2]^{0.8874} \quad (8)$$

with $h = a/k_{\text{b}}$. Note that the Lorentz force is computed in this paper for a sharp cut-off, consistently with the rms amplitude of the stochastic background in Eq. (5), whereas in our previous paper (Chluba et al. 2015) we instead adopted a Gaussian smoothing to compare with the results in Kunze & Komatsu (2014).

In order to solve the numerical issues with the ambipolar diffusion effect for PMFs with positive spectral indices we also improved the numerical integration of RectFast++ (Chluba & Thomas 2011), adding an explicit solve of the linear algebra problem appearing at each time-step in the ordinary differential equation problem. This improved the numerical stability at the onset of ambipolar diffusion around redshift $z \approx 100 - 200$.

### 3 CMB ANGULAR POWER SPECTRA

We now briefly present the impact of the PMF dissipation on the CMB angular power spectra in temperature and polarization. These are very similar to previous computations; however, the numerical noise which was present at large angular scales is eliminated thanks to the improved time-sampling inside CAMB.

#### 3.1 The impact of MHD decaying turbulence

We start by describing the MHD decaying turbulence effect. In the left column of Fig. 3, we illustrate the effect on the temperature and E-mode polarization angular power spectra for $\sqrt{\langle B^2 \rangle} = 4$ nG, note that for this specific figure we have increased the amplitude of the fields with respect to the others of this section in order to visually enhance the effect. In the right column of Fig. 3, we present the relative differences of the angular power spectra which include and do not include the MHD turbulence effect, note that for these figures the amplitude of the fields is $\sqrt{\langle B^2 \rangle} = 0.4$ nG, which is closer to the value obtained in the data analysis. We note in particular a strong effect on the E-mode polarization at intermediate and small angular scales and a sub-percent effect in temperature on small angular scales. In contrast to previous computations (e.g. Kunze & Komatsu 2014, Planck Collaboration XIX 2016), the effect at large angular scales is less pronounced. This is because following Hart & et al. (2018), we significantly increased the time-sampling in CAMB (≈ 100 times) to better resolve the onset of heating around $z \approx 1088$. This improvement eliminates the dependence of the angular power spectrum on large scales on the accuracy parameters $2$.

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making the Boltzmann code very stable as can be seen in Fig. 3, where large scales do not show any feature.

We have described the regularization function we apply in order to solve numerical issues of the MHD turbulence treatment for positive spectral indices (Sect. 2.1.1). In Fig. 4, we show the relative differences of the cases with and without MHD decaying turbulence effect, $\sqrt{\langle B^2 \rangle} = 0.4 \text{ nG}$. The effect of our regularization remains at the sub-percent level in all considered cases, with the largest effect seen for $n_B = 2$. Please note that for $n_B = 2$ an amplitude $\sqrt{\langle B^2 \rangle} \approx 0.4$ for the root mean square of the PMFs is already ruled out by data. For indices smaller or equal zero the angular power spectra do not show any significant dependence on the chosen regularization scheme. We can therefore conclude that for the amplitudes we are able to constrain with this methodology the application of the regularization of the rate does not affect the results of the analysis.

### 3.2 Ambipolar diffusion

We now proceed by illustrating the effect of the ambipolar diffusion on the CMB angular power spectra. In the left column of Fig. 5, we show the angular power spectrum in temperature and E-mode polarization with the effect of ambipolar diffusion compared with the case without PMFs. We considered different spectral indices and PMFs with an amplitude of $\sqrt{\langle B^2 \rangle} = 0.4 \text{ nG}$ as in the previous case. For more clarity, in the right column of Fig. 5, we show the relative difference between the ambipolar diffusion case and the case without PMF contribution. The main effect of ambipolar diffusion heating is a reduction of the overall amplitude of the $TT$ power spectra at intermediate and small scales ($\ell \gtrsim 10$). In contrast, for...
the EE power spectra, the effect is more pronounced at large angular scales around the reionization bump which for very blue indices of the order of $n_\beta = 1 - 2$ is strongly suppressed (cf., Fig. 5). This illustrates that the main effect of ambipolar diffusion heating is an increase of the total Thomson optical depth to last scattering. The overall features are consistent with previous studies (e.g., Kunze & Komatsu 2014).

### 3.3 Combining both effects

Having discussed the two dissipative effects separately we now analyse the combined effect of PMF heating on the CMB angular power spectra. In Fig. 6 we again show the $TT$ and $EE$ angular power spectra and their relative difference with respect to the case without PMFs, for fields of $\sqrt{\langle B^2 \rangle} = 0.4$ nG and different spectral indices. We note how the combination of the two effects results in an impact of both temperature and polarization both on small and large angular scales, with the effect increasing for positive spectral indices. In the next section we will derive the constraints with current CMB data, which are foreground and cosmic-variance limited in temperature, but strongly affected by systematics in polarization. Future CMB polarization dedicated observations will be therefore crucial to fully exploiting the potential of the impact of ambipolar diffusion on the E-mode polarization.

### 4 CMB CONSTRAINTS ON THE AMPLITUDE OF PMFS

In this section, we derive the constraints with the CMB anisotropy data from Planck 2015 release. We use the extension of the CosmoRec and Refeast++ codes developed in our previous work (Chluba et al. 2015) with the regularization of the MHD rate and the improved numerical treatment for the ambipolar diffusion discussed in the previous sections. We use the CosmoMC (Lewis & Bridle 2002) code with the inclusion of the modified recombination codes in order to compute the Bayesian probability distribution of cosmological and magnetic parameters. We vary the baryon density $\Omega_b = 0.022$, the cold dark matter density $\Omega_c = 0.27$ (with $h$ being $H_0/100$ km s$^{-1}$Mpc$^{-1}$), the reionization optical depth $\tau$ with a Gaussian prior, the ratio of the sound horizon to the angular diameter distance at decoupling $\theta$, $\ln(10^{10} A_s)$, $n_s$ and the magnetic parameter $\sqrt{\langle B^2 \rangle}$. We either fix $n_\beta$ to the values $-2.9$, $-2$, $-1$, $0$, $1$, $2$ or we allow $n_\beta$ to vary in the range $[-2.9, 2]$.

Together with cosmological and magnetic parameters we vary the parameters associated to calibration and beam uncertainties, astrophysical residuals, which are included in the Planck public likelihood (Planck Collaboration XI 2016). We assume a flat universe, a CMB temperature $T_{CMB} = 2.725$ K and a pivot scale $k_0 = 0.05$ Mpc$^{-1}$. We sample the posterior using the Metropolis-Hastings algorithm (Hastings 1970) generating eight parallel chains.
and imposing a conservative Gelman-Rubin convergence criterion \cite{Gelman1992} of $R < 1 < 0.02$.

We use public Planck high-$\ell$ likelihood temperature likelihood (Planck Collaboration XI 2016) combined with the Planck lensing likelihood (Planck Collaboration XV 2016). We use a conservative Gaussian prior for the optical depth \(\tau\) and imposing a conservative Gelman-Rubin convergence criterion of $R < 1 < 0.02$.

Note that the likelihood code for the more recent analysis of large angular scales HFI polarisation data (Planck Collaboration Int. XLVI 2016; Planck Collaboration Int. XLVII 2016) has not been released and we therefore make use only of Planck 2015 data.

4.1 Constraints with MHD decaying turbulence

We first present the constraints on the amplitude of PMFs obtained by considering only the heating due to the MHD decaying turbulence term with the use of the regularized rate.

In Fig. 7 we plot the one-dimensional marginalized posterior probabilities for \(\langle B^2 \rangle^{1/2}\) at different fixed values of the spectral index \(n_B\). We also plot the same quantity obtained when \(n_B\) is allowed to vary. In the first column of Table 1 we report the 95% CL constraints on \(\langle B^2 \rangle^{1/2}\) for all the cases considered. The constraints are

<table>
<thead>
<tr>
<th>(n_B)</th>
<th>(\sqrt{\langle B^2 \rangle} ) (nG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>&lt; 0.25</td>
</tr>
<tr>
<td>1</td>
<td>&lt; 0.37</td>
</tr>
<tr>
<td>0</td>
<td>&lt; 0.58</td>
</tr>
<tr>
<td>-1</td>
<td>&lt; 0.90</td>
</tr>
<tr>
<td>-2</td>
<td>&lt; 0.93</td>
</tr>
<tr>
<td>-2.9</td>
<td>&lt; 1.04</td>
</tr>
<tr>
<td>[-2.9,2]</td>
<td>&lt; 0.87</td>
</tr>
</tbody>
</table>

Table 1. Comparison of the constraints from the separate effects and their combination.

at the nano-Gauss level with tighter constraints for positive spectral indices (reduced \(\approx 3 - 4\) times for \(n_B \approx 2\) with respect to the quasi-scale invariant case).
In this subsection we present the constraints on the amplitude of PMFs considering both the heating effects for fixed values of the spectral index \( n_B \) compared with its corresponding value marginalized on \( n_B \) allowed to vary in the range \([-2.9, 2]\).

4.2 Constraints with the ambipolar diffusion

In this subsection we present the constraints on the amplitude of PMFs considering only the heating due to the ambipolar diffusion. In Fig. 8, we plot the one-dimensional marginalized posterior probabilities for \( \langle B^2 \rangle^{1/2} \) at different fixed values of the spectral index \( n_B \). We also plot the same quantity obtained when \( n_B \) is allowed to vary. In the second column of Table 3, we report the 95% CL constraints on \( \langle B^2 \rangle^{1/2} \) for all the cases considered. We note how the ambipolar diffusion gives stronger constraints for growing spectral indices as it is expected from its impact on the CMB angular power spectra. The improvement of the constraint for \( n_B = 2 \) with respect to the quasi-scale invariant case is dramatic, reaching a factor \( \approx 100 \). This implies that a combination of turbulent MHD and ambipolar diffusion heating is expected to improve the constraints in particular for very blue spectra, as we will see below.

4.3 Constraints including both heating terms

In this subsection we present the constraints on the amplitude of PMFs considering both the effects of the ambipolar diffusion and MHD decaying turbulence. In Fig. 9, we plot the one-dimensional marginalized posterior probabilities for \( \langle B^2 \rangle^{1/2} \) at different fixed values of the spectral index \( n_B \). We also plot the same quantity obtained when \( n_B \) is allowed to vary. In the third column of Table 3, we report the 95% CL constraints on \( \langle B^2 \rangle^{1/2} \) for all the cases considered. For \( n_B \leq -1 \), MHD turbulent heating drives the constraint, while for \( n_B \geq -1 \), ambipolar diffusion become most relevant.

In Fig. 10, we present the comparison of the amplitude constraints marginalized over the spectral index. We note how the MHD turbulence has a much sharper posterior distribution compared with the long tail at high amplitudes of the ambipolar diffusion. This effect is mainly due to the strong dependence of the constraints of the ambipolar diffusion with the spectral index. While the MHD turbulence has similar constraining power for all the indices, the ambipolar diffusion is weaker for negative ones resulting in a longer tail. The combination of the two gives a sharp constraint as shown in Fig. 10, the lower amplitude part of the distribution is dominated by the ambipolar diffusion whereas the higher amplitude side is dominated by the MHD decaying turbulence.

Finally, in Fig. 11, we present the two-dimensional posteriors of the amplitude of PMFs with the other cosmological parameters. We note the presence of a slight degeneracy with the angular diameter distance \( \theta \) especially for the varying spectral index case, this is expected considering the effect of the heating on the recombination.
5 DISCUSSIONS

We now discuss the dependence of the results presented in Table 1 on the physics at the damping scale. This is tricky and several approaches have been considered in the past. There is indeed a dependence of both the MHD decaying turbulence and ambipolar rates on $k_0$ and a dependence on the damping profile in the Lorentz force (compare Eq. (7) with Eqs. (A3-A4) of Appendix A of Chluba et al. (2015)). We therefore compare the results of Table 1 with the ones obtained by adopting an exponential damping profile as in Chluba et al. (2015) and Kunze & Komatsu (2015), with the following damping scale:

$$\bar{k}_D = \frac{299.66}{(B_0/1 \text{ nG})},$$

where $B_0$ denotes the integrated amplitude of the stochastic background of PMFs for this second approach to the damping. Note that $\bar{k}_D$ does not depend on the spectral index as the one in Eq. (2) adopted in the previous discussion and has been also used in our previous work Chluba et al. (2015) for the nearly scale-invariant case. See Fig. [12] for a difference between these two damping scales. We mention that in recent numerical simulations Trivedi et al. (2018) a significantly larger damping scale ($\text{smaller } k_0$) is found, but leave a more detailed discussion to future work.

We have repeated the previous analysis for this alternative model of damping. The qualitative aspects remain similar to the case discussed in Section 2: the MHD term is relevant for negative spectral indices, whereas the ambipolar term is for positive ones. Note however that whereas the MHD term leads to constraints similar in the two approaches because of the mild dependence on $k_0$ of the rate in Eq. (5), the ambipolar term leads to much looser constraints when this alternative modelling of the damping scale is adopted. The constraint with the ambipolar term are indeed of the same order of magnitude of the ones obtained with the MHD term by using this alternative damping envelope. In Table 2 we show the results when both the MHD and ambipolar terms are considered: for all values of $n_B$, the combined constraints are at the $\text{nG}$ level.

Our analysis improves in several ways on Kunze & Komatsu (2015): i) the methodology as described in Section 2, ii) the range of considered PMF spectral indices, which in Kunze & Komatsu (2015) was limited to $n_B = -2.9, -2.5, -1.5$, iii) and the data combination: here we consider the most recent Planck 2015 data, whereas Kunze & Komatsu (2015) used Planck 2013 data. The numerical stability we have achieved removes the large scale instability which could have biased the results especially concerning the indices with a stronger heating. With these new settings, in contrast to Kunze & Komatsu (2015), there is almost no variation with the spectral index of the constraints and therefore we do not find tighter bounds for $n_B > -2.9$ as Kunze & Komatsu (2015) do and our 95 %CL constraint $B_0 < 1.1 \text{ nG}$ for $n_B = -2.9$ is more conservative than their corresponding bound: $B_0 < 0.63 \text{ nG}$. Note that for positive spec-

![Figure 10. Comparison of the constraints marginalized over the spectral index for the three heating cases.](image)

![Figure 11. Two dimensional posteriors for the amplitude of the fields with the other cosmological parameters. The results are shown for three spectral indices, in blue $n_B = 2$, in red varying $n_B$ in grey is the almost scale invariant $n_B = -2.9$.](image)
construal indices the constraints from this alternative model of damping are relaxed by a factor 5-20 with respect to the model described in Section 2. The reason for different results in the two approaches is due to the ambipolar term. As already said, the differences could be traced to the different Lorentz force obtained by a different damping envelope or a different damping scale. In order to understand what is the most relevant difference, we have substituted the damping scale in Eq. (9) in the sharp-cut off profile for the damping discussed in Section 2 for \( n_B = 2 \). We obtain \( \sqrt{B^2} < 1.0 \, nG \) at 95 \% CL for the combined case, a very similar result to Table 2. This means that the most relevant difference is due to choice of \( k_B \) for the two models of damping discussed here.

It is now interesting to assess the the implications of the constraints derived in this paper on the amplitude of the stochastic background of PMF smoothed at 1 Mpc, which is commonly adopted in the literature. Since the damping scale enters in the magnetic field amplitude smoothed \( B_1 \) as function of the integrated amplitude (see Appendix A), \( B_1 \) can be different for the two dissipation scales in Eq. (2) [Jedamzik et al. 1998; Subramanian & Barrow 1998] and in Eq. (5) [Chiubra et al. 2015; Kunze & Komatsu 2015], in particular for positive spectral indices, even with equal integrated amplitudes. Table A1 shows that for \( n_B = -2.9 \) the constraints on \( B_1 \) from the two different damping envelopes are similar and of the same order of magnitude of the constraints on the integrated amplitude. This can be understood by realizing that for quasi-scale independent power spectrum the increase of \( \langle B^2 \rangle \) (which simply is a proxy for the total PMF energy density) caused by small scales is logarithmic, and hence \( B_1 \equiv \sqrt{\langle B^2 \rangle} \).

For \( n_B = 2 \) instead, the energy density is dominated by modes around the damping scales. In this case, we see from Table A1 that the constraint on \( B_1 \) with the damping scale in Eq. (2) is tighter than the one obtained with the alternative damping by several orders of magnitude. To a large extend this is due to the large disparity of the damping scale \( \lambda_D = 1 - 10 \, kpc \) and the smoothing scale \( \lambda = 1 \, Mpc \), as can be seen from Eq. (A3). In the most conservative case, the window for PMF between the CMB bounds and the lower limit due to the interpretation of non-observation of GeV gamma-ray emission in intergalactic medium is severely squeezed for \( n_B = 2 \). The tightest constraint obtained with Eq. (2) would instead completely rule out the causal case \( n_B = 2 \) in combination with the lower limit derived from high-energy observations in the intergalactic medium.

6 CONCLUSIONS

We have obtained the constraints on the integrated amplitude of PMFs due to their dissipation around and after recombination caused by the MHD decaying turbulence and the ambipolar diffusion. We have improved our previous treatment by including a regularization of the heating rate due to the MHD decaying turbulence which is particularly important for stochastic background of PMFs with a positive spectral index. At the same time, we have also improved the numerical treatment of the ambipolar diffusion allowing for the stability of the numerical code, again for stochastic background of PMFs with positive spectral indices. These improvements have allowed to constrain the integrated amplitude of PMFs for different spectral indices, extending our previous studies restricted to the nearly scale-invariant case [Kunze & Komatsu 2015; Planck Collaboration XIX 2016; Chiubra et al. 2015].

The results of the three analysis which considered separately the heating by MHD decaying turbulence and ambipolar diffusion and their combination are summarized in Table 2 for a regularization of the integrated amplitude by a sharp cut-off. Our results show that both MHD decaying turbulent and ambipolar effects need to be taken into account, the first one being important for negative spectral index and the second for positive spectral index. For a sharp cut-off the combined constraint from MHD and ambipolar is of the order of \( nG \) for the scale-invariant case as in [Planck Collaboration XIX 2016], and becomes tighter with a larger spectral index reaching \( \sqrt{B^2} < 0.06 \, nG \) (95 \% CL) for \( n_B = 2 \). These constraints on PMFs from the ionization history are the tightest ones for any single spectral index. Thanks to our numerical improvements we have also been able to derive the constraints on the integrated amplitude when the spectral index is allowed to vary, obtaining \( \sqrt{B^2} < 0.83 \, nG \) (95 \% CL) [see Fig. 10].

We have also investigated how the PMFs heating effects are sensitive to the physics at the damping scale. We have shown how two proposed damping scales, Eq. (2) and Eq. (9), usually adopted in the literature, lead to a different magnitude of the effect induced by the ambipolar term on the CMB anisotropy power spectra, in particular for positive spectral indices. As a consequence, the constraints obtained on the integrated amplitude of PMFs, and even more on the smoothed amplitude on 1 Mpc, depend on the physics at the damping scale, which deserve further investigation. In the future, some of these aspects can be clarified with detailed numerical MHD simulation that track the evolution of the PMF across the recombination era [Trivedi et al. 2018].

We also note that although recently refined computations of the magnetic heating rates due to MHD turbulence have become available [Trivedi et al. 2018], here we improved the treatment remaining within the framework first introduced by Sethi & Subramanian 2005. However, the improved heating rate computations show a direct dependence of the onset of heating on the magnetic field amplitude and spectral index. We anticipate this to affect the

<table>
<thead>
<tr>
<th>( n_B )</th>
<th>2</th>
<th>-2.9</th>
<th>[-2.9,2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_0 ) (nG) [( k_B ])</td>
<td>&lt; 0.95</td>
<td>&lt; 1.10</td>
<td>&lt; 0.91</td>
</tr>
</tbody>
</table>

Table 2. Constraints from the combined effects for the alternative model of the damping profile, \( B_0 \).
overall constraints, but a more detailed study is left to future work. Finally, it will also be important to repeat the analysis with the next release of Planck data, expected later this year. In particular, improvements of the polarization power spectra are expected to further tighten the constraints.

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APPENDIX A: CONSTRAINTS ON SMOOTHED MAGNETIC FIELD AMPLITUDE

In most of the literature, constraints on a stochastic background of PMFs are reported on the amplitude smoothed at 1 Mpc scale, which is a quantity closer to astrophysical observations of large scale magnetic fields. It is therefore interesting to understand our results for the integrated amplitude in terms of the smoothed amplitude $B_1$, which is defined as:

$$B_1^2 = \int_0^\infty \frac{dk}{2\pi^2} \frac{k^2}{e^{-Dk^2}} P_B(k).$$  \hspace{1cm} (A1)$$

The smoothed amplitude $B_1$ is related to the integrated amplitude by

$$\langle B^2 \rangle = \frac{2}{D} \frac{\Gamma(nG + 3)}{\Gamma(nG + 3/2)} \lambda^2 B_1^2 ,$$  \hspace{1cm} (A2)$$

for the first damping envelope and by

$$B_1^2 = \frac{2}{D} \frac{\Gamma(nG + 3/2)}{\Gamma(nG + 3)} \lambda^2 B_1^2 ,$$  \hspace{1cm} (A3)$$

for the second damping envelope.

In Table A1 we report the implications for $B_1$ from our results on the integrated amplitude. A cautionary note must be considered when discussing these results. The derived constraints on the smoothed amplitude seems very sensitive to the model of damping, in particular for positive spectral index. Nevertheless, the resulting constraints are extremely tight for positive $n_B$ compared to those obtained with the gravitational contribution only. As a comparison, we remind that the 95 % CL Planck 2015 upper bound on the smoothed amplitude is $B_1 < 0.011 \text{nG}$ for $n_B = 2$ derived from gravitational effects [Planck Collaboration XIX 2016].

<table>
<thead>
<tr>
<th>$n_B$</th>
<th>$B_1^{1\text{Mpc}} (\text{nG})$</th>
<th>$B_1^{1\text{Mpc}} (\text{nG}) [\kappa_2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$&lt; 5.22 \times 10^{-16}$</td>
<td>$&lt; 1.13 \times 10^{-6}$</td>
</tr>
<tr>
<td>-2.9</td>
<td>$&lt; 0.76$</td>
<td>$&lt; 0.84$</td>
</tr>
</tbody>
</table>

Table A1. Constraints from the combined effect for different spectral indices with the $B_1^{1\text{Mpc}}$ parametrization.