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Fractional quantum Hall effect in strained graphene: stability of Laughlin states in disordered (pseudo)magnetic fields

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We address the question of the stability of the (fractional) quantum Hall effect (QHE) in presence of pseudomagnetic disorder generated by mechanical deformations of a graphene sheet. Neglecting the potential disorder and taking into account only strain-induced random pseudomagnetic fields, it is possible to write down a Laughlin-like trial ground-state wave function explicitly. Exploiting the Laughlin plasma analogy, we demonstrate that in the case of fluctuating pseudomagnetic fluxes of relatively small amplitude both the integer and fractional quantum Hall effects are always stable upon the deformations. By contrast, in the case of bubble-induced pseudomagnetic fields in graphene on a substrate (a small number of large fluxes) the disorder can be strong enough to cause a glass transition in the corresponding classical Coulomb plasma, resulting in the destruction of fractional quantum Hall regime and in a quantum phase transition to a non-ergodic state of the lowest Landau level.

INTRODUCTION

Massless Dirac fermions were discovered [1, 2] in graphene via the observation of an unusual (“half-integer”) quantum Hall effect (QHE) [1–7] which is a manifestation of the existence of a topologically protected zero-energy Landau level [1, 4, 7]. This means that this level is not broaden by any inhomogeneity of the magnetic field. It was realized very soon after this discovery [8] that inhomogeneities of the effective magnetic field are unavoidable in graphene, due to the effect of pseudomagnetic fields induced by strain (for a review, see [7, 9, 10]). In earlier works [8, 11] random pseudomagnetic fields created by defects (such as intrinsic and extrinsic ripples) were considered. Later it was theoretically predicted [12, 13] and experimentally confirmed [14] that (pseudo) Landau level quantization and valley quantum Hall effect can be created in graphene by external smooth deformation with a trigonal symmetry, and that effective fields as high as hundreds of Tesla may be easily reached in this way (an order of magnitude stronger than what may be observed in conventional high-field magnetic laboratories).

The fractional quantum Hall effect (FQHE) has been experimentally discovered [15, 16] in graphene via the observation of an unusual (“half-integer”) quantum Hall effect (QHE) [1–7] which is a manifestation of the existence of a topologically protected zero-energy Landau level [1, 4, 7]. This means that this level is not broaden by any inhomogeneity of the magnetic field. It was realized very soon after this discovery [8] that inhomogeneities of the effective magnetic field are unavoidable in graphene, due to the effect of pseudomagnetic fields induced by strain (for a review, see [7, 9, 10]). In earlier works [8, 11] random pseudomagnetic fields created by defects (such as intrinsic and extrinsic ripples) were considered. Later it was theoretically predicted [12, 13] and experimentally confirmed [14] that (pseudo) Landau level quantization and valley quantum Hall effect can be created in graphene by external smooth deformation with a trigonal symmetry, and that effective fields as high as hundreds of Tesla may be easily reached in this way (an order of magnitude stronger than what may be observed in conventional high-field magnetic laboratories).

The fractional quantum Hall effect (FQHE) has been experimentally discovered in graphene and its observation was reported in [15, 16]. A very natural and interesting question is whether this state is also protected, to some extent, with respect to inhomogeneities of the (pseudo)magnetic field or not. Here we answer this question within a framework of a model with random pseudomagnetic fields but assuming the absence of potential disorder. In terms of deformations this means strong shear deformations and no dilatation [7]. It was shown recently [17] that at least in some graphene samples random strain-induced pseudomagnetic fields is indeed the main source of electron scattering and therefore the model may be quite realistic.

THE MODEL OF PSEUDOMAGNETIC DISORDER

It was shown experimentally [17] that in many cases the sources of pseudomagnetic field can be considered as randomly distributed centers of deformations. The typical size of these centers is much smaller than the distances between them, so we can effectively treat them as point-like objects. The quenched pseudomagnetic disorder therefore emerges in a form of a set of highly localized fluxes, and our problem reduces to the study of properties of a correlated electron gas in presence of a random flux distribution coexisting with a homogeneous background magnetic field.

Disorder of this geometrical structure appears in suspended graphene due to ripples [11]. Another system where it can be observed is a graphene sheet put on a substrate, e.g. platinum surface [14]. When epitaxial graphene is grown on such a substrate, bubbles of characteristic width about 10nm and height 2nm tend to form. In this case, the number of fluxes is much smaller and the value of each flux is much larger than in the case of rippled graphene.

In what follows we neglect intervalley scattering and model graphene as two independent massless-Dirac-fermion systems, associated to the valleys $K$ and $K'$ at the corners of its hexagonal Brillouin zone. The two valleys differ by the sign of the strain-induced pseudomagnetic field they experience. A trial wave function for the zero-energy Landau level of a system of massless Dirac fermions in the presence of localized magnetic fluxes can be constructed using the Aharonov-Casher solution [18]. We briefly recall this construction in [19]. Following this prescription we derive a wave function that can be viewed as a square root of partition function of 2D Coulomb plasma evolving in a background of randomly distributed (quenched) point charges, i.e.
\[ Z = \int d\tilde{z}|\psi(\tilde{z})|^2 = \int d\tilde{z}e^{-\mathcal{H}/m}, \]
where
\[ \mathcal{H} = -2m^2 \sum_{k<l}^N \ln |z_k - z_l| + \frac{m}{2} \sum_{n}^N |\dot{z}_n|^2 \]
\[ + 2m \sum_{i}^N \sum_{j}^{N_0} \Phi_j \ln |z_i - \tilde{z}_j| \]

This classical Hamiltonian is the central object of our study. For the sake of simplicity we assume that all pseudomagnetic fluxes have the same value \( \Phi_j = \Phi \).

A few comments are now in order. While the two aforementioned physical realizations of disorder can be described by the same formal model, they are pretty different on a phenomenological level. First of all, inhomogeneities of pseudomagnetic field in rippled graphene are normally not very strong, with deviations \( |\delta B| \approx 1T \) on a length scale of \( l \approx 1nm \) [8], resulting in very moderate flux values \( \Phi \approx 10^{-3} \Phi_0 \). In contrast, nanobubbles observed in graphene on a metal substrate lead to very strong pseudomagnetic fields, of the order of \( |\delta B| \approx 300T \), localized within regions of characteristic size \( \sim (5-10)nm \) [14]. Hence, the corresponding fluxes are of the order of \( \Phi \approx 10^{-15} \Phi_0 \). Another fundamental difference is that out-of-plane deformations of freely suspended graphene (ripples) can occur in both direction, hence the signs of \( \Phi_j \) fluxes are randomly distributed, and each of them acts as local repulsive or attractive potential on the particles of the associated classical plasma.

On the other hand, the fluxes due to nanobubbles are all of the same sign, making the potential landscape for the Laughlin plasma either purely repulsive or purely attractive (depending on the valley). In what follows we mainly perform computations for the second case and demonstrate that a transition of the liquid plasma to a glass state is possible. The first case turns out to be more trivial due to the weakness of the disorder, and it will be clear that quantum Hall effect is insensitive to deformations of this kind.

**DISCUSSION OF THE MODEL**

Since pseudomagnetic fields preserve the time-reversal symmetry [7], the same bubble deformation of the graphene sheet induces a flux co-directed with the background magnetic field in one valley (which result in an attractive potential for the classical plasma), and oppositely directed in the other valley. In the latter case it enters as a quenched repulsive potential in the action of the classical plasma. Great care should be taken here: since we allow for strong variations of (pseudo)magnetic field, to remain within the regime of validity of the model we need to make sure that these inhomogeneities never lead to mixing between Landau levels (LLs). In the valley where pseudomagnetic fluxes are co-directed with the background magnetic field the problem is not expected to occur: while the lowest Landau level (LLL) is protected upon variations of the magnetic field, the gap between it and the next LL is bounded from below by the corresponding homogeneous value. In the other valley the situation is potentially more dangerous, as the presence of oppositely directed large magnetic fluxes may imply the existence of zero-magnetic field lines in the sample and, possibly, regions where LL merge. However, since the corresponding regions act on classical particles in the Laughlin plasma as strong repulsive potentials, on the quantum level we expect the system to avoid occupying states within these domains, and the effect of level mixing should be mild.

Another potentially problematic aspect related to the existence of \( B = 0 \) lines is the possibility of percolation through the bulk of the sample that destroys the quantization of the Hall conductivity [22]. But since we consider only the case of low density of fluxes, and the distance between any two fluxes is much bigger than their characteristic size, such a line is an isolated loop circumventing a flux, and there is no chance of percolation.

**PLASMA STATIC STRUCTURE FACTOR**

Having defined the model, we can numerically calculate the static structure factor of the Coulomb plasma in the presence of a number of static (disorder) charges by means of the replica Ornstein-Zernike equations derived for a partly quenched two-component fluid in [23]. Referring the reader to the original paper for a detailed discussion, hereafter we just briefly quote the idea. In this language, the potential landscape provided by pseudomagnetic fluxes can be implemented as a frozen “liquid” whose direct \( c_{00} \) and full \( h_{00} \) pair correlation functions (and thus the static structure factor \( S_{00} = 1 + \rho_0 h_{00} \)) are fixed. The correlation functions \( h_{ij} \), \( c_{ij} \) of the annealed
component can be obtained by solving the replicated system of equations

\begin{align}
    h_{01} &= c_{01} + \rho_0 c_{00} \otimes h_{01} + \rho_1 c_{01} \otimes (h_{11} - h_{12}), \\
    h_{11} &= c_{11} + \rho_0 c_{01} \otimes h_{01} + \rho_1 c_{11} \otimes h_{11} - \rho_1 c_{12} \otimes h_{12}, \\
    h_{12} &= c_{12} + \rho_0 c_{01} \otimes h_{01} + \rho_1 c_{11} \otimes h_{12} + \\
    &\quad \rho_1 c_{12} \otimes h_{11} - 2\rho_1 c_{12} \otimes h_{12},
\end{align}

where the \( \otimes \) symbol stands for convolution \( f \otimes g = \int f(r-r') g(r') dr' \). Here the index 0 denotes the quenched component (i.e. the magnetic fluxes), while 1 and 2 refer to two replicas of the annealed one (the electron liquid). These equations are to be supplemented by the closure relations. We use the hypernetted chain closure [20, 24], which has been proven to give very accurate results for quantum Hall plasmas

\[
h_{ij}(r) = \exp(h_{ij}(r) - c_{ij}(r) - \beta v_{ij}(r)) - 1,
\]

where \( \beta v_{ij}(r) \) are radially symmetric interaction potentials (\( \beta = m \) is the inverse temperature of the classical plasma), and replicas are required not to interact with each other directly, i.e. \( v_{12}(r) = 0 \). Here \( \rho_0 \) is the density of pseudomagnetic fluxes which can be estimated as follows. The typical background magnetic field which is required to develop a \( \nu = \frac{1}{2} \) FQHE state is \( B_0 \approx 15 \text{T} \) [16], hence \( l_B \approx 6 \text{nm} \). In experiments on graphene on a platinum substrate, a density of nanobubbles of about 5 per 2500 nm\(^2\) [14] was observed, which in rescaled units would correspond to \( \rho_0 \approx 0.07 l_B^{-2} \). The particle density of itinerant electrons is instead fixed to \( \rho_1 = 1/(2\pi l_B^2 m) \) in a Laughlin state with the corresponding filling factor \( \nu = 1/m \). Hereafter we set the magnetic length \( l_B = 1 \) for convenience.

Before proceeding with solving (2) we have to fix the static structure factor \( S_{00}(q) \) of the pseudomagnetic disorder. While the latter is not explicitly known at all momentum scales, there are two qualitatively distinct cases that correspond to different behavior at small wave vectors. Inhomogeneities of the pseudomagnetic field can be long- or short-range correlated. In the former case the correlator of pseudomagnetic vector potential \( \langle |A_q|^2 \rangle \) behaves as \( 1/q^2 \) for \( q \to 0 \) [11, 17], resulting in the correlator of pseudomagnetic fields \( S_{00}(q) \propto q^2 \langle |A_q|^2 \rangle \) to approach a non-vanishing constant at \( q = 0 \). This kind of pseudomagnetic disorder is relevant for ripple-scattering dominated electronic transport [7, 11, 17]. At the same time, short-range disorder with \( \langle |A_q|^2 \rangle \) approaching a constant at \( q \to 0 \) should always exist but does not lead to any appreciable contribution to the electron mobility at zero magnetic field. However, as we will see in what follows, it can substantially effect the FQHE. In this case \( S_{00}(q) \propto q^2 \) at \( q \to 0 \).

A natural way to generate such model structure factors is to imagine for a second that, prior to being quenched, the fluxes themselves were released to anneal as a liquid. If we assume that they interact with each other via a two-dimensional Coulomb potential, we will end up with a structure factor corresponding to short-range disorder (represented by blue curve in Fig. 1). If the “annealing” potential is instead taken to be of the hard-sphere type, we obtain a model of long-range disorder (red curve in Fig. 1). Since real correlation functions of pseudomagnetic fields are unknown (and may be very different for different samples and substrates) we use those obtained from these two models. This is enough to demonstrate a qualitative difference between short-range and long-range correlated disorder.

The solutions of the replica Ornstein-Zernike equations in the two aforementioned cases are shown in Figs. 2 and 3, respectively. We can see that the two types of pseudomagnetic disorder lead to very different physical effects. The structure factor \( S_{11}(q) \) of a quantum Hall plasma in the presence of a short-range disorder remains vanishing at \( q \to 0 \) whatever the strength of pseudomagnetic fluxes is. We can also check that the incompressibility sum rule is always satisfied: \( \frac{1}{2\pi} \int (g(r) - 1) d^2 r = -1 \) [21]. On the other hand, long-range correlations in the pseudomagnetic disorder already at small strength \( \Phi \) lead to a change in the small-\( q \) behavior of the QHE plasma structure factor, making it compressible and thus destroying the quantum Hall effect.

The high peak in \( S_{11}(q) \) for a strong enough magnetic disorder is a precursor of a glass phase transition of the Laughlin plasma. The latter can be interpreted as a breakdown of ergodicity of the corresponding quantum ground state. To show this, we use the mode coupling theory [25, 26]. Details of this approach are given in

\[
\int f(r) g(r') dr' = \int f(r) g(r') dr' = \int f(r) g(r') dr'.
\]
DISCUSSION OF THE RESULTS

We have considered two models of pseudomagnetic disorder, namely with long- and short-range correlations. Long-range correlated disorder occurs in the presence of ripples, and is characterized by structure factor which does not vanish in the limit \( q \to 0 \) [7]. Its effect on the fractional quantum Hall state turns out to be dramatic. Even for small concentrations of not very strong disorder, the structure factor of the annealed component (i.e. the electron liquid) does not vanish in the limit of \( q \to 0 \). This in turn implies that the liquid becomes compressible and the fractional quantum Hall state is completely destroyed [20].

On the other hand, the short-range disorder, whose quenched structure factor resembles that of a normal liquid, has a more subtle effect on the fractionalized state, which needs to be discussed in more detail. It is well known that inhomogeneous ground states can be realized in fractional quantum Hall systems at filling factors between those corresponding to incompressible states [33–40]. Striped or bubble phases have been predicted to have energies of the order of the homogenous (liquid) ground state, and to appear at intermediate filling factors whenever long-range interactions are present [33, 38, 39]. Striped phases are especially favoured by non-isotropic or dipolar-like interactions (like, e.g., those resulting from the screening of the Coulomb potential by nearby metal plates) [33]. A characteristic hallmark of such phases is, for example, a non-homogeneous Hall conductivity [41–45]. In clean systems and at low temperature such phases usually exhibit a long-range order.

An incompressible state in the presence of quenched magnetic disorder will exhibit a somewhat similar phenomenology. Local fluctuations of the filling factor will result in the formation of chaotic patterns. Although the pattern looks at a first sight completely random, a careful study can reveal the hidden typical length scale, corresponding to a sharp maximum of the structure factor. Such patterns can, in general, be thought of as stripe or bubble glass phases [46, 47]. Their origin is due to the fact that the system finds itself frustrated by the presence of disorder, and wants to modulate with a period corresponding to the typical length scale but in all possible directions at the same time [46, 47].

We predict a phase transition as a function of the strength of the magnetic disorder. At all strengths the structure factor will exhibit a liquid-like behavior at small momentum (i.e. \( S(q) \to q^2 \) in the limit of \( q \to 0 \)). A sharp peak develops at finite wavevectors, and grows with the strength of the quenched magnetic disorder \( \Phi / \Phi_0 \). This structure of \( S(q) \) is crucial and leads to the emergence of a glassy behavior. The latter is revealed by the presence of non-decaying density fluctuations, encoded in the finiteness of the long-time part of the dynamical structure factor. Below the critical value of the strength of the quenched magnetic disorder, \( S(q,t) \to 0 \) at large time. However, above this critical value of \( \Phi / \Phi_0 \) it remains finite, signaling that density fluctuations do not decay over time. This behavior is typical for frozen systems.
From the behavior of the classical plasma associated to the deformed Laughlin wavefunction in the presence of quenched magnetic disorder, we infer that there is a phase transition analogous to the formation of a Wigner crystal at small filling fraction. In our case the transition is driven not by the small density but by the presence of magnetic disorder and, depending on its the strength, can occur at all filling fractions.

Whether the quantum electron liquid itself behaves non-ergodically can be tested by means of scanning single-electron transistor experiments in combination with thermal cycling. The chaotic charge texture we predict is similar to electron-hole puddles observed in graphene in presence of potential disorder and can be read off with the same experimental techniques [48]. However, in the case of inhomogeneous pseudomagnetic field we do not expect the texture to reproduce the geometric pattern of the underlying quenched disorder matrix. That is, if we heat up a sample and cool it down again for a number of times, the texture should be unique at every iteration. If this prediction is confirmed, it would mean that the quantum liquid indeed exhibits aging [49]. In this case the excited thermal states over such a vacuum can be thought of as an example of many-body localization [50].

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[19] See Section 1 of Supplemental Material at [URL will be inserted by publisher] for the details of the derivation.
[27] See Section 2 of Supplemental Material at [URL will be inserted by publisher] for a brief introduction into the mode-coupling theory.
magnetic field, Phys. Rev. B 45, 11354-11357.