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On the steady-state drop size distribution in stirred vessels. Part II: effect of continuous phase viscosity.

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Abstract

In Part I¹ we used silicon oils with viscosities across six orders of magnitude to investigate the effect of the dispersed phase viscosity on the DSD of dilute emulsions. In this study we extended Part I by using three glucose aqueous solutions to thicken the continuous phases approximately an order of magnitude while keeping the Power number constant. It was found that increasing the continuous phase viscosity decreases the maximum drop size despite having drops well above the Kolmogorov length-scale. Our results are in disagreement with the mechanistic models for the turbulent inertia regime. The results were explained using the full turbulent energy spectrum proposed by Pope² instead of the Kolmogorov -5/3 spectrum. Our analysis revealed that most of the steady-state drop sizes do not fall in the isotropic turbulence size range.

Keywords:

Emulsification / Viscosity / Droplet size distribution / Stirred vessel / Drop breakup

Introduction

In this study we varied the viscosity of the continuous phase systematically in a turbulent stirred vessel. Mechanistic models consider that the

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continuous phase should not influence the drop size or drop size distribution provided that the drops are larger than the Kolmogorov length-scale η and that the vessel is in the turbulent regime. However the mechanistic models consider that the viscosity of the continuous phase μ_c becomes relevant if the drops are smaller than η as discussed in reference³.

Due to the predictions of the models, the effect of μ_c in turbulent vessels has not been thoroughly investigated and there are only a few publications, such as the ones by Stamatoudis and Tavlarides⁴, by Boxall et al.⁶ and by Vankova et al.⁵, where the combined effect of μ_d and μ_c is considered. Furthermore, the experiments done by Stamatoudis and Tavlarides⁴ and by Boxall et al.⁶ were not carried out at constant Power numbers Po .

Stamatoudis and Tavlarides⁴ dispersed kerosene ($\mu_d \approx 1.5 \times 10^{-3}$ Pa s) and mineral oil ($\mu_d \approx 2.6 \times 10^{-2}$ Pa s) in aqueous several glycerol solutions (3.5×10^{-3} Pa s $< \mu_c < 2.2 \times 10^{-1}$ Pa s) in the transitional and turbulent regimes ($3.6 \times 10^2 < Re < 1.7 \times 10^4$) using a Rushton turbine and no surfactant. The $\bar{d}_{32}$ dependency on μ_d for kerosene showed two patterns depending on N : (1) an increase followed by a decrease for low N and (2) a decrease followed by a plateau for high N . For the mineral oil only pattern (2) was observed but the decrease was more pronounced. They attributed the decrease in $\bar{d}_{32}$ to a decline in the coalescence rate.

Boxall et al.⁶ emulsified deionized water in oils with viscosities in the 3.1×10^{-3} Pa s to 10^{-1} Pa s using a stirred vessel with a 6-blade Rushton turbine with a diameter D of 0.051 m at different stirring speeds. They used the mean energy dissipation rate per unit mass $\bar{\epsilon}$ to calculate η and were able to observe a transition from turbulent inertia to sub-Kolmogorov shear stress regimes. Their results were well correlated by the models of Hinze⁷ and Shinnar and Church³. Nevertheless Boxall et al.⁶ worked with very high μ_c and their experiments were not carried out at a constant Power number Po , i.e. the Reynolds number (Re) of their systems ranged from 27 to 3.4×10^3 , therefore some of their systems were done in the laminar and transition regimes.

Vankova et al.⁵ used a narrow-gap homogenizer to emulsify a 9.5×10^{-2} Pa s SiOil in different glycerol aqueous solutions varying μ_c from 1×10^{-3} Pa s to 1.8×10^{-2} Pa s. They found that even the drops emulsified in 3×10^{-3} Pa s glycerol solution produced smaller drops than when the 1×10^{-3} Pa s one was used. The decrease in droplet size was attributed to a change in regime from inertia to viscous inertia as μ_c increased.

Theory

Hinze⁷ theorized that drops larger than η are broken up by their interaction with a single eddie, and that such eddies are of a comparable size to the diameter of the droplet. Larger eddies are only responsible for transporting the droplets while smaller eddies do not have enough energy to make the drops burst⁸⁻¹⁰. Under the assumption of isotropic turbulence, he used the Kolmogorov -5/3 energy spectrum and determined that the stresses experienced by the largest droplet of diameter d_{max} are

$$\rho_c \overline{v'^2} = C \rho_c (\epsilon d_{max})^{\frac{2}{3}} \quad (1)$$

Where ρ_c is the density of the continuous phase, $\overline{v'^2}$ is the average of the square of velocity differences, ϵ is the energy dissipation rate per unit mass and C is the Kolmogorov constant². The Kolmogorov length-scale η is given by

$$\eta = \left(\frac{\nu_c^3}{\epsilon} \right)^{\frac{1}{4}} \quad (2)$$

Where ν_c is the kinematic viscosity of the continuous phase.

Turbulence in stirred vessels is not homogeneous across the whole vessel. According to Davies¹¹ the steady-state DSD will be determined by the maximum energy dissipation rate ϵ_{max} which is located in the vicinity of the impeller^{12,13}. Determining ϵ_{max} is difficult, however it is proportional to the mean energy dissipation rate per unit mass $\bar{\epsilon}$. Several $\epsilon_{max}/\bar{\epsilon}$ have been reported in literature for different impeller geometries and have been proven to be independent of N using particle image velocimetry¹³.

The $\bar{\epsilon}$ can be obtained through power draw P measurements

$$\bar{\epsilon} = \frac{P}{\rho_c V} = \frac{\text{Po} N^3 D^5}{V} \quad (3)$$

Where V is the volume of the vessel, Po is the dimensionless Power number ($\text{Po} = P \rho_c^{-1} N^{-3} D^{-5}$), N is the impeller speed and D is the diameter of the impeller. Therefore two geometrically identical systems with the same Po and D , N share the same $\bar{\epsilon}$; furthermore, if these two systems have the same ρ_c , these should have the same disruptive forces (see Eq. 1).

For $d_{max} < \eta$, Shinnar and Church³ determined through dimensional

analysis that the stresses due to k are given by

$$\rho_c \overline{v'^2} \propto \frac{\epsilon}{\nu_c} d_{max}^2 \quad (4)$$

This model includes an effect of ν_c on the disruptive forces, however it has been criticized arguing that the disruptive forces for $d_{max} < \eta$ should be mainly viscous¹⁴. Shinnar and Church³ proposed another model viscous stresses being the disruptive force for $d_{max} \ll \eta$; in their model the viscous shear stresses over a distance d are

$$\mu_c \left[\left(\frac{\partial v'}{\partial d} \right)^2 \right]^{\frac{1}{2}} \propto \mu_c \left(\frac{\epsilon}{\nu_c} \right)^{\frac{1}{2}} = \rho_c (\nu_c \epsilon)^{\frac{1}{2}} \quad (5)$$

Where μ_c is the dynamic viscosity of the continuous phase.

Equations 1, 4 and 5 are the stresses of the commonly known inertia subrange ($d_{max} \gg \eta$), sub-Kolmogorov inertia stress ($d_{max} < \eta$) and sub-Kolmogorov-viscous stress models ($d_{max} \ll \eta$). The former models consider that the interfacial forces are the only cohesive forces balancing the disruptive stresses. However, as the viscosity of the dispersed phase μ_d increases, the viscous internal resistance becomes important^{1,7,15,16}. Dimensionless numbers have been proposed to relate the viscous forces to the inertial and the interfacial ones such as the Viscosity group Vi_H (also known as Ohnesorge number) by Hinze⁷ ($Vi_H = \mu_d (\rho_d \sigma d)^{-1/2}$) and the one by Calabrese et al.¹⁶ ($Vi_c = \mu_d N D \sigma^{-1} (\rho_c / \rho_d)^{1/2}$). Both of the mechanistic models proposed by these authors are implicit functions.

For very high μ_d , the droplet's viscous forces become the dominant cohesive force and the interfacial ones become negligible^{1,10,15-20}. As shown by Calabrese et al.¹⁶ and by Padron²¹ the mechanistic models become explicit functions for very large μ_d . Padron²¹ made a comprehensive review on the mechanistic models and their derivation. Table 1 shows the models for constant Po for the two limiting cases $\mu_d \rightarrow 0$ and $\mu_d \rightarrow \infty$. The dimensionless numbers used in this table are the Weber number of the stirrer ($We_s = \rho_c N^2 D^3 \sigma^{-1}$) and its Reynolds number ($Re = N D^2 \nu_c^{-1}$); where D is the diameter of the impeller, N its stirring speed and σ the interfacial tension.

Many authors^{8,10,16,17,22,23} have reported that as viscosity increases, drops elongate forming two spherical ends connected by a filament which entails

Table 1: Summary of mechanistic models for constant Po in its dimensionless form. The four columns at the right are the exponential dependencies for each parameter. From Padron²¹.

Range	Model	$\frac{d_{max}}{D} \propto$	Dependency			
			μ_d	μ_c	ρ_c	N
$d_{max} > \eta$	$\mu_d \rightarrow 0$	$We_S^{-\frac{3}{5}}$	0	0	-3/5	-6/5
	$\mu_d \rightarrow \infty$	$\left(\frac{\mu_d}{\mu_c}\right)^{\frac{3}{4}} \left(\frac{\rho_c}{\rho_d}\right)^{\frac{3}{8}} Re^{-\frac{3}{4}}$	3/4	0	-3/8	-3/4
$d_{max} < \eta$	$\mu_d \rightarrow 0$	$(We_S Re)^{-\frac{1}{3}}$	0	1/3	-2/3	-1
	$\mu_d \rightarrow \infty$	$\left(\frac{\mu_d}{\mu_c}\right)^{\frac{1}{2}} \left(\frac{\rho_c}{\rho_d}\right)^{\frac{1}{4}} Re^{-\frac{3}{4}}$	1/2	1/4	-1/2	-3/4
$d_{max} \ll \eta$	$\mu_d \rightarrow 0$	$We_S^{-1} Re^{\frac{1}{2}}$	0	-1/2	-1/2	-3/2
	$\mu_d \rightarrow \infty$	$\left(\frac{\mu_d}{\mu_c}\right) \left(\frac{\rho_c}{\rho_d}\right)^{\frac{1}{2}} Re^{-\frac{3}{4}}$	1	-1/4	-1/4	-3/4

multiple fragmentation. Solsvik et al.²⁴ found that for a given parent daughter drop size the number of daughter drops also depends on the ϵ , a greater amount of daughter drops is produced in the vicinity of the impeller. Maaß et al.²⁵ and Solsvik and Jakobsen²⁶ found that as the size of the parent drop decreases, the number of daughter droplets also decrease. Maaß et al.²⁵ reported that for 2 mm petroleum drops, ternary breakup was the most probable, however they reported that a maximum of 97 drops per breakup event; for 0.50 mm drops they found that binary breakup was the most likely. In Carrillo De Hert and Rodgers¹, the increase in the amount of small daughter drops was attributed this breakup mechanism.

Eastwood et al.⁸ found that the sizes of the deformed drops were comparable to the integral length scale and that a single eddie comparable to the size to the initial drop diameter could not responsible for the drop breakup as its turnover frequency was faster than the breakup frequency. Andersson and Andersson²³ found that large drops of order of the integral length scale the length of the filament could be up to 20 times larger than the initial drop size and that these winded around a rotating vortex, from their observations they hypothesized that viscous shear could contribute to the drop breakup.

Materials and equipment and methods

Materials

Eight of the Fluid XX Silicon Oils (SiOil, dimethyl siloxane, Dow Corning, Michigan, U.S.A.) used in Part I¹ with nominal dynamic viscosity in the 10 to 100 000 cSt range and blends were emulsified for this investigation. Their ρ_d and μ_d can be consulted in Table 2.

Table 2: Density ρ_d and viscosity μ_d for the SiOils used at 25 °C.

SiOil [cSt]	ρ_d [kg m ⁻³]	μ_d [Pa s]
10	934	9.58×10^{-3}
50	960	4.91×10^{-2}
350	970	3.28×10^{-1}
1000	970	9.47×10^{-1}
*2800	970	2.75×10^0
*4500	970	4.34×10^0
*7600	970	7.36×10^0
10 000	970	1.05×10^1
12 500	970	1.33×10^1
30 000	970	2.93×10^1
[†] 50 000	970	4.88×10^1
100 000	970	1.06×10^2

*1000 and 10 000 cSt SiOil blend.

[†]30 000 and 100 000 cSt SiOil blend.

The continuous phases studied were three Sweet Glucose Syrup M01632 (Brenntag, Mülheim, Germany) aqueous solutions of concentration C_g 1.50 mol L⁻¹, 2.50 mol L⁻¹ and 2.90 mol L⁻¹. All the continuous phases contained 27.85 g of Texapon N701 (Cognis, Hertfordshire, UK), which yields 1 wt.% of Sodium Laureth Sulphate (SLES) for the systems emulsified in water ($C_g = 0$). Naturally the wt.% was different depending of the continuous phase used as their ρ_c are different; the SLES concentrations ranged from 0.86% for $C_g = 2.90$ mol L⁻¹ to 1.0% for $C_g = 0$ mol L⁻¹.

The relevant properties of the continuous phases are shown in Table 3. The ρ_c were obtained using a pycnometer and the μ_c was obtained using the same methods used to determine the properties of the SiOils in Part I¹.

Table 3: Density ρ_c and viscosity μ_c for the continuous phases used at 25 °C.

C_g [mol L ⁻¹]	ρ_c [kg m ⁻³]	μ_c [Pa s]
0	997	1.04×10^{-3}
1.50	1098	2.65×10^{-3}
2.50	1166	5.73×10^{-3}
2.90	1194	9.79×10^{-3}

The measured σ for the 10 and 350 cSt SiOils and the continuous phases are shown in Table 4. These were obtained by the methodology described in Part I¹. For a given C_g the interfacial tension for the 10 and 350 cSt SiOils is very similar, therefore σ for the SiOils with μ_d greater than 350 cSt was assumed constant as done by other authors^{1,16,18}. The results show that σ decreases with C_g .

Table 4: Interfacial tensions σ of the SiOils and the continuous phases. The percentages represent one standard deviation.

SiOil [cSt]	σ [mN m ⁻¹]			
	$C_g = 0$ mol L ⁻¹	1.50 mol L ⁻¹	$C_g = 2.50$ mol L ⁻¹	$C_g = 2.93$ mol L ⁻¹
10	9.25 $\pm$ 3.0%	7.56 $\pm$ 1.5%	6.52 $\pm$ 0.8%	5.57 $\pm$ 1.2%
350	9.43 $\pm$ 3.9%	7.57 $\pm$ 2.2%	6.46 $\pm$ 1.3%	5.54 $\pm$ 2.2%

Equipment and methods

Emulsification

The impeller used was a 6-blade-45° pitched blade turbine with $D = 4.83 \times 10^{-2}$ m. The vessel and impeller used for this study were the same used in Carrillo De Hert and Rodgers¹. The diameter of the vessel T was 1.37×10^{-1} m and had four baffles with dimensions $T/10$. The clearance and height of the liquid were $T/3$ and T respectively. The volume of the system was 1.97 L.

The emulsions were prepared by pouring 1.95 L of continuous phases into the vessel, N was fixed using a CT6 Hand-Held Tachometer (Compact Instruments, Lancashire, UK) and injecting 20 mL of SiOil in the impeller vicinity using a syringe. The concentration of the SiOil was 1% by volume for all systems.

As done in Part I¹, the systems were stirred for 24 h and steady-state was verified by analysing the DSD at different times. All the SiOils in the 10 to 30 000 cSt viscosity range were emulsified in the three different glucose solutions. Additionally, the 50 000 and 100 000 cSt SiOils were emulsified in the 2.90 mol L⁻¹ solution. All combinations were studied for two N : 10.0 s⁻¹ and 13.3 s⁻¹. The total number of emulsions prepared for Part II was 62, however the data obtained in Part I¹ was also used in the analysis.

The drop size analysis was also carried out using the Mastersizer 3000 (Malvern Instruments, Malvern, UK) using the same operation procedure described in Carrillo De Hert and Rodgers¹.

Power draw and rate of energy dissipation

The Po was estimated following the procedure described in James et al.²⁷, the torque M was measured using a TorqSense RWT321 1 N m torque sensor (Sensor Technology, Oxfordshire, UK). The power draw P can be estimated using $P = 2\pi NM$. Seven materials were used: (1) water, (2) a 2.9 mol L⁻¹ glucose in water solution, (3-7) five different SiOils. The torque was measured for different N and the real torque M was obtained by subtracting the mechanical losses estimated by calculating the y -intercept through regression of measured M vs. N and measured M vs. N^2 for the laminar and turbulent regimes respectively.

Results and discussion

Power draw

The “power curve” we obtained is shown in Figure 1. The laminar power constant K_P ($K_P = \text{RePo}$) and the Po plateau in the turbulent regime were 64.1 and 1.33 respectively. This curve shows that all our experiments were carried out in the turbulent regime at a constant Po.

Phenomenology, drop size and drop size distribution

The effect of μ_d for the systems emulsified in water/SLES ($C_g = 0$) for the five different N investigated were presented in Part I¹. The systems with $C_g > 0$ showed a similar trend: the maximum drop size increases with μ_d , the 10 and 50 cSt SiOils yielded monomodal DSD, whereas thicker oils yielded bimodal distributions, the maximum drop size reached a maximum for the 10 000 cSt SiOil before plummeting. This behaviour has been previously discussed in Part I¹.

Figure 1: Power curve.

Figure 2 show the DSD we obtained for the 50 cSt, 1000 cSt, 10 000 cSt and 100 000 cSt SiOil for $N = 13.3 \text{ s}^{-1}$ for the continuous phases used. This Figure shows that the droplet size decreases with μ_c (or C_g). Stamatoudis and Tavlarides⁴ also obtained a decrease in drop size within the same μ_c range at high stirring speeds (such as the ones in this investigation). It can further be noticed in Figures 2b and 2c that increasing C_g reduces the size of the satellite droplets and increases their amount. Interestingly 2c suggests that increasing C_g gives birth to a third merged distributions, which droplet size is smaller than the ones obtained with low μ_c . This is reasserted by Figure 2d where two distinct peaks with a third distribution in their valley was obtained; as demonstrated in our previous work using rotor-stators^{20,28}.

The emulsions produced in this study were surfactant-laden systems, with the anionic surfactant concentration 120 times the critical micelle concentration. Interfacial phenomena such as Marangoni stresses or interfacial elasticity were not considered in this work as their influence has not been thoroughly studied for turbulent systems and these are surfactant-dependent; Padron²¹ showed that interfacial effects could be important depending on μ_d , N and the type of surfactant used. Surfactant reduce the interfacial tension which leads to slower breakup kinetics²⁹. The necking prior to breakup can give rise to Marangoni stresses if the surfactant molecules are not readily adsorbed either because there is not enough surfactant in the bulk or because of slow transfer to the interface, the latter having a more pronounce effect in highly

Figure 2: DSD obtained for $N = 10 \text{ s}^{-1}$ for different continuous phases. The shadowed areas represent one standard deviation.

viscous systems^{21,29}. The breakup time scale is mainly affected by the stirring speed (or energy dissipation rate) and by the μ_d ; while the adsorption rate is affected by the diffusion, quality of mixing and micelle dissociation kinetics²¹. Padron²¹ found that for concentrations above the critical micelle concentration, drops of low viscosity tend to decrease for low stirring speeds and remain constant for high stirring speed as concentration is increased; for thicker drops he found a small decrease as concentration was increased when using non-ionic surfactants, the behaviour was attributed to the slower diffusion from the micelles to the interface. He found that for SiOils of 1000

cSt, the drop sizes became independent of stirring speed, we found a similar behaviour for higher viscosities. However, Skartlien et al.³⁰ did Lattice Boltzmann simulations and found that at high turbulence levels Marangoni effects can be discarded.

It is worth mentioning that according to the mechanistic models presented in Table 1 for $\mu_d \rightarrow \infty$ a decrease in drop size is expected as incrementing C_g leads to an increase in ρ_c . The smaller drop sizes obtained as C_g increases could not be justified only by the change in ρ_c ($\approx 20\%$ variation, see Tab. 3).

Figure 3 shows the $\bar{d}_{32}$ obtained as a function of μ_d for $N = 13.3 \text{ s}^{-1}$ for the four continuous phases used in a log-log scale. It is evident that $\bar{d}_{32}$ decreases with C_g . For μ_d at the left of the vertical dotted line d_{max} increases with μ_d ; and at the right of the same line d_{max} was independent of μ_d . This behaviour has been previously discussed in Part I¹; the decrease in $\bar{d}_{32}$ can be noticed for $\mu_d < 10 \text{ Pa s}$ and is due to the increase in amount of satellite drops. It is interesting to notice that this behaviour is observed for all the continuous phases at the same μ_d despite the DSDs having a different shape. As done in Part I¹ we will restrict our analysis to the emulsions to the left of the dashed line.

Figure 3: Sauter mean diameter $\bar{d}_{32}$ as a function of μ_d for $N = 13.3 \text{ s}^{-1}$ with C_g as parameter.

Figure 4 shows the evolution of the $\bar{d}_{32}$ to d_{max} ratio as a function

of μ_d for different C_g . The plot shows that for the 50 cSt SiOil, which yielded a monomodal distribution, $\bar{d}_{32}/d_{max}$ is independent of μ_c . In Part I¹, the monomodal distributions (0.65, 10 and 50 cSt SiOils) yielded the same $\bar{d}_{32}/d_{max}$ and the ratio would decrease as the amount of satellite drops increased. An increase in μ_c favoured the appearance of satellite drops, therefore the $\bar{d}_{32}/d_{max}$ is expected to be lower as μ_c increases. The inclined-dashed lines, their indexes and R^2 displayed in Figure 4 were obtained using the method of the least squares for the 3.28×10^{-1} -7.36 Pa s viscosity range; thicker oils displayed a chaotic trend and were not fitted.

Figure 4: $\bar{d}_{32}$ to d_{max} ratio as a function of viscosity for the different continuous phases used.

As done in Part I¹, the mode of the daughter droplets Mo_d was used in our analysis. Figure 5 shows the mode of the daughter droplets Mo_d as a function of μ_d for $N = 13.3 \text{ s}^{-1}$. Most of the results obtained for the 10 000 cSt SiOil are not shown in this plot as the daughter droplets do not exhibit a clear Mo_d (see Fig. 2c). The comparison in-between these plots show that $\bar{d}_{32}$ is not the best measure of central tendency to elucidate the drop break-up mechanism when the DSD are multimodal.

The first part of our analysis of variance contemplated μ_d , ρ_d , μ_c , ρ_c , σ and N as variables. The results of the analysis of variance with all the aforementioned variable is shown in the Supplementary material for

$$Mo_d = A_1 \mu_d^{A_2} \mu_c^{A_3} \rho_d^{A_4} \rho_c^{A_5} \sigma^{A_6} N^{A_7} \quad (6)$$

Figure 5: Effect of μ_d on the Mo_d for $N = 13.3 \text{ s}^{-1}$ with C_g as parameters.

The mean and 95% confidence interval (%CI) obtained for A_7 was $-1.21 \pm 16.4\%$, which is in agreement with the Kolmogorov-Hinze⁷ theory for the turbulent inertia regime. An analysis on the effects of μ_c , ρ_c and σ is difficult because the variables are linked (see Tab. 3). Provided that N is in agreement with the mechanistic models and that according to Wang and Calabrese¹⁷ interfacial cohesive forces are negligible for high μ_d , different models were proposed by fixing $Mo_d \propto N^{-6/5}$ and $Mo_d \propto (\text{We}_s D^{-3})^{-3/5}$. The mean of the fitting coefficients, p -value and 95%CI, as well as the coefficient of determination R^2 and mean absolute error MAE of the fit can be consulted in the Supplementary material attached to this paper. The models using $Mo_d \propto N^{-6/5}$ instead of $Mo_d \propto (\text{We}_s D^{-3})^{-3/5}$ provided better fits. In the same Table it can be seen that if the kinematic viscosity ($\nu = \mu/\rho$) is used, the power dependency on ν_d and ν_c is statistically similar (based on the 95%CI). Therefore the model which yielded the smallest MAE and has the least fitting parameters was

$$Mo_d = A_1 \left(\frac{\nu_d}{\nu_c} \right)^{A_2} N^{-\frac{6}{5}} \quad (7)$$

Where the coefficients and their 95%CI are $A_1 = 3.15 \times 10^2 \pm 8.05\%$ and $A_2 = 0.403 \pm 3.98\%$. The coefficient of determination R^2 and the MAE of the fit were 0.954 and 12.4% respectively. The goodness of the fit using the

mean coefficient value can be assessed in Figure 6.

Figure 6: Experimental vs Modelled Mo_d using Equation 7.

If the results presented in Figure 3 are standardized by $\nu_c^{-2/5} N^{-6/5}$ (see Fig. 7), the $\bar{d}_{32}$ collapses into a Master curve relatively well in the $9.58 \times 10^{-3} \text{ Pa s} > \mu_d > 4.34 \text{ Pa s}$ interval. The scattered results for the 10 cSt are due to the presence of a small tail in the distribution for the emulsion using the two most viscous continuous phases; to our knowledge this has not been reported in literature and the reasons are unknown *a priori*. It is unclear if the scattered results for the high-viscosity end is due to a different $\bar{d}_{32}$ dependency on ν_d/ν_c or to experimental error. The DSD analysis of these emulsions is difficult due to sampling as a single large drop can alter the shape of the whole distribution.

Figure 7: Collapsed $\bar{d}_{32} \nu_c^{2/5} N^{6/5}$ as a function of μ_d . The shadowed area and the percentages represent one standar deviation from the mean.

The DSD were normalized by Mo_d to show the changes in the shape of the DSD. Figure 8 shows the effects of μ_c on the shape of the DSD for the 1000 cSt and the 4500 cSt SiOils. The proportionality $Mo_d = 1.54d_{95}$ found in Part I¹ was also obtained; from these Figures, and the results obtained in Part I¹ we concluded that the size of the large daughter drops is independent of μ_c , μ_d , σ and N .

Figure 8: Mo_d -standardised DSD $f_v(d_i/Mo_d)$ with C_g as parameter for (a) 1000 cSt SiOil for $N = 13.3 \text{ s}^{-1}$ and (b) 4500 cSt SiOil for $N = 10.0 \text{ s}^{-1}$.

The effect of μ_d for the four continuous phases used is shown in Figure 9. For the distribution where the amount of daughter drops were not enough to obtain the Mo_d directly, these were obtained by extrapolating Equation 7. These curves show that all DSD for the 10 and 50 cSt SiOils are monomodal and that bimodality arises somewhere in-between the 50 and 350 cSt SiOils despite μ_c changing an order of magnitude. In our previous work on the effect of μ_d on the DSD in rotor-stators²⁰, we also found that bimodality occurred in the same μ_d range.

The degree of polydispersity increases as the normalized minimum droplet diameter also decreases with μ_d . The shape of the DSD for the 7600 and 10 000 cSt SiOils in Figures 9b-9d suggest that at least three distribution are merged together. To our knowledge, the only study reporting trimodal DSDs has been the one of Nishikawa et al.³¹. A possible explanation is that some of the daughter drops produced from the stretching breakup mechanism suffer further breakup in the viscous turbulent regime.

Figure 9: Mo_d -standardized DSDs obtained for $N = 10 \text{ s}^{-1}$ for all dispersed SiOils in (a) Water/SLES and in (b) 1.5 mol L^{-1} (c) 2.5 mol L^{-1} and (d) 2.9 mol L^{-1} glucose aqueous solutions.

Figure 10 shows an estimate of the volume fraction of the daughter droplets ϕ_d , the points were obtained by fitting a log-normal distribution to the right end of the DSDs in Figure 9. Fitting of the whole DSDs was impractical due to the appearance of a third merged distribution. Nevertheless a clear trend can be observed in Figure 10, which is in agreement with the DSDs presented in Figure 8a: increasing μ_c favours the stretching breakup mechanism resulting in DSDs with a larger amount of satellite drops. The dashed lines in this Figure were obtained by $\phi_m = \exp[-(\mu_d/\beta)^\alpha]$ which corresponds to subtracting a Weibull distribution to 100%. Even though the

parametrization of α and β was not possible, it was found that a Weibull distribution follows the trend adequately while requiring a small number of fitting parameters.

Figure 10: Volume fraction of daughter drops ϕ_d as a function of μ_d with C_g as parameter. Fits were obtained using $\alpha = 0.68$ and $\beta = 5.3$ for emulsions in water and $\alpha = 0.76$ and $\beta = 3.7$ for emulsions in the 2.9 mol L^{-1} glucose solution.

Turbulence and emulsification sub-regime

The mechanistic models presented in Table 1 predict different dependencies on the variables depending on the size-scale of d_{max} in relation with D and η . Nevertheless we found that μ_c had a large effect on the drop size and on the DSD, despite all our experiments being at constant Po; furthermore, the d_{max} obtained varied across two orders of magnitude (see Fig. 5 and $d_{max} = 1.54 Mo_d$) making it unlikely that emulsification occurs in the same emulsification regime in all our systems studied. In this section a theoretical analysis on the relevant scales under the assumption of isotropic turbulence was performed to offer an explanation on the effect of μ_c despite d_{max} being larger than η .

The theory postulated by Hinze⁷ assumes isotropic turbulence occurs at scales much smaller than the integral length-scale and that Kolmogorov -5/3 energy spectrum³² for drops larger than η . However, Kolmogorov's hypothesis is only valid for infinite Re and industrial processes occur at finite Re. The turbulent inertia sub-range for finite Re is narrower and in some cases non-existent³³. Solsvik and Jakobsen³³ recently reviewed other models

of energy spectra for isotropic turbulence, using the one proposed by Pope² as benchmark. Pope's energy spectrum describes the energy-containing range, the inertial sub-range and the dissipation range.

$$E(\kappa) = C\epsilon^{\frac{2}{3}}\kappa^{-\frac{5}{3}}f_L(\kappa\Lambda)f_\eta(\eta\kappa) \quad (8)$$

where κ is the wave number of the eddies (inversely proportional to the eddie size); $f_L(\kappa\Lambda)$ and $f_\eta(\eta\kappa)$ are non-dimensional functions. The first dimensionless function determines the shape of the energy containing subrange and tends to unity for large $\kappa\Lambda$ and the second determines the shape of the dissipation subrange and tends to unity for small $\kappa\eta$. Their equations read

$$f_L(\kappa\Lambda) = \left(\frac{\kappa\Lambda}{[(\kappa\Lambda)^2 + c_\Lambda]^{\frac{1}{2}}} \right)^{\frac{5}{3}+p_0} \quad (9)$$

$$f_\eta(\eta\kappa) = \exp \left\{ -\beta \left\{ [(\kappa\eta)^4 + c_\eta^4]^{\frac{1}{4}} - c_\eta \right\} \right\} \quad (10)$$

Where β , p_0 , c_Λ and c_η are constants. According to Pope² $\beta = 5.2$ and $p_0 = 2$; the values of the constants c_Λ and c_η have to be determined so that

$$k(\kappa) = \int_0^\infty E(\kappa)d\kappa \quad (11)$$

$$\epsilon(\kappa) = \int_0^\infty 2\nu\kappa^2 E(\kappa)d\kappa \quad (12)$$

The energy spectrum model can normalized (as a function of the dummy variable $\kappa' = \kappa\Lambda$) so that it is uniquely a function of the integral Reynolds number (Re_Λ)³³

$$\frac{\eta}{\Lambda} = \text{Re}_\Lambda^{-\frac{3}{4}} \quad (13)$$

Where Λ is the integral length-scale and is considered to be of the size of the trailing vortices $\Lambda = D/10$ ¹², however values ranging from $D/12.5$ to $D/6$ have been adopted¹³. The derivation of the normalized spectrum can be found in Solsvik and Jakobsen³³.

The determine η in the proximity of the impeller (from Eq. 2), ϵ_{max} needs to be known. We estimated ϵ_{max} by calculating $\bar{\epsilon}$ from our power measurements and Equation 3 and using $\epsilon_{max}/\bar{\epsilon}$ reported in literature. Gabriele

et al.¹³ summarized some of the $\epsilon_{max}/\bar{\epsilon}$ obtained for pitched-blade turbines by several authors (Table 1 and Table 3 in Ref.¹³). The values ranged from 3.27 to 78 for 45°-pitched blade turbines depending on the number of blades, the geometrical ratios of the vessel and the methodology used to determine ϵ_{max} . The authors suggest that the Smagorinsky closure method provides more realistic results than the direct and dimensional analysis methods (see discussion in Ref.¹³). Using this method Khan³⁴ determined that $\epsilon_{max}/\bar{\epsilon} = 78$ for a 6-PBT-D with geometric ratios $C = T/3$ and $D = T/3$; which are very similar to the ones in this study. Despite the discrepancy on the reported $\epsilon_{max}/\bar{\epsilon}$, using the value reported by Khan³⁴ will provide the most “optimistic” scenario (widest turbulent inertia regime). The calculated $\bar{\epsilon}$ and ϵ_{max} using the $\epsilon_{max}/\bar{\epsilon}$ reported by Khan³⁴ can be consulted in Table 5.

Table 5: Mean energy dissipation rate $\bar{\epsilon}$ and maximum local energy dissipation rate ϵ_{max} for different N .

N [s ⁻¹]	$\bar{\epsilon}$ [m ² s ⁻³]	ϵ_{max} [m ² s ⁻³]
8.33	0.100	7.82
10.0	0.173	13.5
11.7	0.278	21.7
13.3	0.408	31.8
15.0	0.585	45.7

Figure 11 shows the resulting Pope’s normalized energy spectra for the four studied C_g for $N = 13.3 \text{ s}^{-1}$. The dashed vertical line at 2π (left) shows the location of Λ . The solid-decaying straight line shows the Kolmogorov -5/3 spectrum. In Figure 11a the dashed vertical lines indicate the location of estimated η for the studied continuous phases and dotted lines represent the spectrum proposed by Shinnar and Church³ for $\kappa > \eta$ (from Eq. 4). The dashed lines in Figure 11b show the $\kappa\Lambda$ at which a 20% (to represent a good approximation) deviation from the Kolmogorov -5/3 spectra was found. It can be noted that the width of turbulent inertia sub-range shortens with μ_c . The calculated η , Re_Λ and turbulent inertia sub-range size-range are listed in Table 6 for all different N and C_g studied.

In Figure 12 the experimental d_{95} ($d_{95} = 1.54M_o_d$) for $N = 13.3 \text{ s}^{-1}$ were plotted as a function of μ_d . The limits of the turbulent sub-regimes shown in Figure 11 were redrawn as dashed lines for the TI regime and the η s are denoted with dotted lines. This Figure shows that most of our resulting d_{95}

Figure 11: Pope's normalized Energy Spectrum for the systems studied for $N = 13.3 \text{ s}^{-1}$. (a) the vertical dashed line at the left show Λ (at 2π), the ones at the right demarcate $\eta(\mu_c)$ and the dotted lines shows the sub-Kolmogorov spectrum theorized by Shinnar and Church³. (b) show the turbulent inertia range if a 20% variation in-between the energy spectra of Kolmogorov³² and Pope² is considered.

Table 6: Kolmogorov scale η and integral turbulent Reynolds number Re_Λ , and turbulent inertia (TI) range for the experimental C_g and N .

C_g [mol L ⁻¹]	N [s ⁻¹]	η [μm]	$\text{Re}_\Lambda/10^2$ [-]	TI range [μm]
0	8.33	20.0	15.1	361-4830
	10.0	17.4	18.1	321-4830
	11.7	15.5	21.1	287-4830
	13.3	14.0	24.1	255-4830
	15.0	12.9	27.1	241-4830
1.50	10.0	32.8	7.76	509-4830
	13.3	26.5	10.3	454-4830
2.50	10.0	55.9	3.82	906-4830
	13.3	45.1	5.09	762-4830
2.90	10.0	82.0	2.29	1436-4830
	13.3	66.1	3.05	1208-4830

are not in the turbulent inertia regime; as a reminder the $\epsilon_{max}/\bar{\epsilon}$ used was the largest reported and lower dissipation ratios would reduce the width of the TI regime even further.

Figure 12: Maximum drop diameter d_{max} as a function of μ_d for $N = 13.3 \text{ s}^{-1}$ with C_g as parameter. The different length-scales are shown as vertical lines: The highest one shows the integral length-scale $\Lambda = D/10$, the dashed lines show the lower boundary of the turbulent inertia regime (TI) and the dotted lines shows the location of η for the different C_g .

Most of our steady-state experimental data are above η but below the region where the Kolmogorov -5/3 energy distribution law is valid. Therefore a combination of viscous and inertia stresses would be expected, which might explain why μ_c had an effect on the DSD. As shown in Figure 2 some of our SiOils yielded broad DSD with drop sizes extending over three orders of magnitude; therefore some of the drops might be in the turbulent inertia regime and in the dissipation range above and below η .

According to high-speed image processing experiments^{8,10,23} the parent drops giving rise to the steady-state DSD should have stretched considerably before breakup. Eastwood et al.⁸ suggested that the relevant length-scale causing drop breakup is of the order of Λ as only the largest eddies have enough energy and life-span to breakup such viscous structures. On the other hand, Andersson and Andersson²³ argued that existing models assume negligible viscous drag, as is expected from gas-liquid systems; but that these

effect should be considered when the viscosity of the phases is not negligible as the momentum transfer becomes more efficient and that it would be reasonable to consider that shear contributed to the breakup of the elongated structures. To our knowledge there are no reported studies where viscous drag has been considered.

Another cause explaining the effect of μ_c on the drop size would be if the thickness of the liquid thread formed prior to breakup is considered as the relevant length-scale. If the large daughter droplets are the ones resulting from the separation of the spherical ends of the elongated structures, it would be reasonable to assume that the thickness of the liquid thread is in the viscous regime and therefore shear stresses are important.

It is interesting to notice that our results correlated with $d_{max} \propto N^{-6/5}$, in agreement with the Kolmogorov-Hinze theory for the TI regime despite most of our data not falling in the size range where the Kolmogorov -5/3 spectrum is valid even under the most optimistic considerations, which is contradictory and the reasons are unknown.

Conclusions

In Part I we focused on the effect of μ_d and N on the DSD; for most of our emulsion the interfacial cohesive forces could be neglected as explained by Wang and Calabrese¹⁷. The μ_d dependency $d_{max} \propto \mu_d^{0.365}$ was in disagreement with the mechanistic models found in literature; however it was on agreement with the one found in our previous work using in-line rotor-stators²⁰ despite the obvious geometrical differences. Furthermore, we proposed using two log-normal distribution to describe the bimodal DSD.

In Part II we expanded our previous study by emulsifying the SiOils used in Part I in other continuous phases for the same Po. The mechanistic models presented in Table 1 are based on the work of Hinze⁷, whose hypothesis that the disruptive energy causing drop breakup is the turbulent kinetic energy k , and therefore μ_c should not play a role. Pope's energy spectrum revealed that most of the drop sizes were in the size range in which the Kolmogorov -5/3 spectrum is no longer valid and therefore shear stresses are expected; despite the drops being larger than η . The commonly accepted assumption that emulsification in turbulent stirred vessels occur in the inertia regime is likely to be false.

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Notation

Latin symbols

$95\%CI$	95% confidence interval [-]
ℓ	size of the eddie [m]
$\bar{d}_{32}$	Sauter mean diameter [m]
A_i	fit constants [-]
C	Kolmogorov constant [-]
C_g	glucose concentration [mol l ⁻¹]
D	diameter of the stirrer [m]
d_i	diameter of the drop class i [m]
d_{95}	95th percentile [m]
d_{max}	maximum drop diameter [m]
$E(\kappa)$	turbulent kinetic energy spectrum [J m ⁻¹]
$f_\eta(\kappa\eta)$	specified dimensionless function [-]
$f_\Lambda(\kappa\Lambda)$	specified dimensionless function [-]
$f_v(d_i)$	frequency by volume of the drop class i [%]
k	kinetic energy [m ² s ⁻²]
K_P	laminar power number constant [-]
M	torque [N m]
Mo_d	mode of the daughter droplets [m]
N	stirring speed [s ⁻¹]
n_{ri}	refractive index [-]
P	power [W]
R^2	coefficient of determination [-]
T	diameter of the tank [m]
V	volume of the vessel [m ³]

Greek letters

ϵ	energy dissipation rate per unit mass [$\text{J kg}^{-1} \text{s}^{-1}$]
ϵ_{max}	maximum energy dissipation rate per unit mass [$\text{J kg}^{-1} \text{s}^{-1}$]
η	Kolmogorov length-scale [m]
κ	eddy wavenumber [m^{-1}]
κ'	dimensionless eddy wavenumber [-]
Λ	Integral length-scale [m]
μ_c	dynamic viscosity of the continuous phase [Pa s]
μ_d	dynamic viscosity of the dispersed phase [Pa s]
ν	kinematic viscosity [$\text{m}^2 \text{s}^{-1}$]
ν_c	kinematic viscosity of the continuous phase [$\text{m}^2 \text{s}^{-1}$]
ν_d	kinematic viscosity of the dispersed phase [$\text{m}^2 \text{s}^{-1}$]
$\bar{\epsilon}$	mean energy dissipation rate per unit mass [$\text{J kg}^{-1} \text{s}^{-1}$]
ρ_c	density of the continuous phase [kg m^3]
ρ_d	density of the dispersed phase [kg m^3]
σ	intefacial tension [N m^{-1}]

Dimensionless numbers

Po	Power number $\text{Po} = P\rho^{-1}N^{-2}D^{-5}$
Re	characteristic Reynolds number of the stirred vessel $\text{Re} = ND^2\nu^{-1}$
Re_Λ	integral Reynolds number $\text{Re}_\Lambda = v'\Lambda\nu_c^{-1}$
We_S	Weber number of the stirrer $\text{We}_S = \rho_c N^2 D^3 \sigma^{-1}$

Abbreviations

DSD	droplet size distribution
SiOil	silicon oil
SLES	sodium laureth sulphate
TI	turbulent inertia sub-range

Literature cited

1. Carrillo De Hert, S., Rodgers, T.L.. On the steady-state drop size distribution in stirred vessels. part i: Effect of dispersed phase viscosity. *AIChE Journal* 2018;doi:10.1002/aic.16171.
2. Pope, S.B.. Turbulent Flows. Cambridge University Pr.; 2000. ISBN 0521598869.
3. Shinnar, R., Church, J.M.. Statistical theories of turbulence in predicting particle size in agitated dispersions. *Industrial & Engineering Chemistry* 1960;52(3):253–256. doi:10.1021/ie50603a036.

4. Stamatoudis, M., Tavlarides, L.L.. Effect of continuous-phase viscosity on the drop sizes of liquid-liquid dispersions in agitated vessels. *Industrial & Engineering Chemistry Process Design and Development* 1985;24(4):1175–1181. doi:10.1021/i200031a047.
5. Vankova, N., Tcholakova, S., Denkov, N.D., Ivanov, I.B., Vulchev, V.D., Danner, T.. Emulsification in turbulent flow 1. mean and maximum drop diameters in inertia and viscous regimes. *Journal of Colloid and Interface Science* 2007;312(2):363–380. doi:10.1016/j.jcis.2007.03.059.
6. Boxall, J.A., Koh, C.A., Sloan, E.D., Sum, A.K., Wu, D.T.. Drop size scaling of water-in-oil emulsions under trubulent flow. *Langmuir* 2012;28(1):104–110. doi:10.1021/la202293t.
7. Hinze, J.O.. Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. *AIChE Journal* 1955;1(3):289–295. doi:10.1002/aic.690010303.
8. Eastwood, C.D., Armi, L., Lasheras, J.C.. The breakup of immiscible fluids in turbulent flows. *Journal of Fluid Mechanics* 2004;502:309–333. doi:https://doi.org/10.1017/S0022112003007730.
9. Kresta, S.M., Brodkey, R.S.. Turbulence in mixing applications. In: Edward L. Paul and Victor A. Atiemo-Obeng and Suzanne M. Kresta, , ed. *Handbook of Industrial Mixing: Science and Practice*; chap. 2. John Wiley and Sons; 2003:19–87. doi:10.1002/0471451452.ch2.
10. Nachtigall, S., Zedel, D., Kraume, M.. Analysis of drop deformation dynamics in turbulent flow. *Chinese Journal of Chemical Engineering* 2016;24(2):264–277. doi:10.1016/j.cjche.2015.06.003.
11. Davies, J.T.. Drop sizes of emulsions related to turbulent energy-dissipation rates. *Chemical Engineering Science* 1985;40(5):839–842. doi:10.1016/0009-2509(85)85036-3.
12. Kresta, S.. Turbulence in stirred tanks: Anisotropic, approximate, and applied. *The Canadian Journal of Chemical Engineering* 1998;76(3):563–576. doi:10.1002/cjce.5450760329.
13. Gabriele, A., Nienow, A., Simmons, M.. Use of angle resolved piv to estimate local specific energy dissipation rates for up- and down-pumping pitched blade agitators in a stirred tank. *Chemical Engineering Science* 2009;64(1):126–143. doi:10.1016/j.ces.2008.09.018.
14. Bourne, J., Baldyga, J.. Drop breakup in the viscous subrange: a source of possible confusion. *Chemical Engineering Science* 1994;49(7):1077 – 1078. doi:http://dx.doi.org/10.1016/0009-2509(94)80013-8. URL

- <http://www.sciencedirect.com/science/article/pii/S0009250994800138>.
15. Arai, K., Konno, M., Matunaga, Y., Saito, S.. Effect of dispersed-phase viscosity on the maximum stable drop size for breakup in turbulent flow. *Journal of Chemical Engineering of Japan* 1977;10(4):325–330. doi:<http://doi.org/10.1252/jcej.10.325>.
 16. Calabrese, R.V., Chang, T.P.K., Dang, P.T.. Drop breakup in turbulent stirred-tank contactors. part i: Effect of dispersed-phase viscosity. *AIChE Journal* 1986;32(4):657–666. doi:10.1002/aic.690320416.
 17. Wang, C.Y., Calabrese, R.V.. Drop breakup in turbulent stirred-tank contactors. part ii: Relative influence of viscosity and interfacial tension. *AIChE Journal* 1986;32(4):667–676. doi:10.1002/aic.690320417.
 18. Hall, S., Cooke, M., El-Hamouz, A., Kowalski, A.. Droplet break-up by in-line silverson rotor-stator mixer. *Chemical Engineering Science* 2011;66(10):2068–2079. doi:doi:10.1016/j.ces.2011.01.054.
 19. Chen, H.T., Middleman, S.. Drop size distribution in agitated liquid-liquid systems. *AIChE Journal* 1967;13(5):989–995. doi:10.1002/aic.690130529.
 20. Carrillo De Hert, S., Rodgers, T.L.. On the effect of dispersed phase viscosity and mean residence time on the droplet size distribution for high-shear mixers. *Chemical Engineering Science* 2017;172:423 – 433. doi:<https://doi.org/10.1016/j.ces.2017.07.002>. URL <http://www.sciencedirect.com/science/article/pii/S0009250917304426>.
 21. Padron, G.A.. Effect of surfactants on drop size distribution in a batch, rotor-stator mixer. Ph.D. thesis; University of Maryland; 2005.
 22. Tcholakova, S., Vankova, N., Denkov, N.D., Danner, T.. Emulsification in turbulent flow:. *Journal of Colloid and Interface Science* 2007;310(2):570–589. doi:10.1016/j.jcis.2007.01.097.
 23. Andersson, R., Andersson, B.. On the breakup of fluid particles in turbulent flows. *AIChE Journal* 2006;52(6):2020–2030. doi:10.1002/aic.10831.
 24. Solsvik, J., Becker, P.J., Sheibat-Othman, N., Mohallick, I., Farzad, R., Jakobsen, H.A.. Viscous drop breakage in liquid–liquid stirred dispersions: Population balance modeling. *Journal of Dispersion Science and Technology* 2014;36(4):577–594. doi:10.1080/01932691.2014.910471.
 25. Maaß, S., Gbler, A., Zacccone, A., Paschedag, A., Kraume, M.. Experimental investigations and modelling of breakage phenomena in stirred liquid/liquid systems. *Chemical Engineering Research and Design* 2007;85(5):703–709. doi:10.1205/cherd06187.

26. Solsvik, J., Jakobsen, H.A.. Single drop breakup experiments in stirred liquid–liquid tank. *Chemical Engineering Science* 2015;131:219–234. doi: 10.1016/j.ces.2015.03.059.
27. James, J., Cooke, M., Trinh, L., Hou, R., Martin, P., Kowalski, A., Rodgers, T.. Scale-up of batch rotorstator mixers. part 1power constants. *Chemical Engineering Research and Design* 2017;124:313 – 320. doi:http://dx.doi.org/10.1016/j.cherd.2017.06.020. URL <http://www.sciencedirect.com/science/article/pii/S0263876217303490>.
28. Carrillo De Hert, S., Rodgers, T.L.. Continuous, re-cycle and batch emulsification kinetics using a high-shear mixer. *Chemical Engineering Science* 2017;167:265 – 277. doi:https://doi.org/10.1016/j.ces.2017.04.020. URL <http://www.sciencedirect.com/science/article/pii/S0009250917302671>.
29. Kovalchuk, N.M., Nowak, E., Simmons, M.J.. Kinetics of liquid bridges and formation of satellite droplets: Difference between micellar and bi-layer forming solutions. *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 2017;521:193–203. doi: 10.1016/j.colsurfa.2016.08.060.
30. Skartlien, R., Sollum, E., Schumann, H.. Droplet size distributions in turbulent emulsions: Breakup criteria and surfactant effects from direct numerical simulations. *The Journal of Chemical Physics* 2013;139(17):174901. doi:10.1063/1.4827025.
31. Nishikawa, M., Mori, F., Kayama, T., Nishioka, S.. Drop size distribution in a liquid-liquid phase mixing vessel. *Journal of Chemical Engineering of Japan* 1991;24(1):88–94. doi:10.1252/jcej.24.88.
32. Kolmogorov, A.. The local structure of turbulence in incompressible viscous fluid for very large reynolds numbers. *Comptes Rendus De L Academie Des Sciences De L URSS* 1941;30:301–305.
33. Solsvik, J., Jakobsen, H.A.. A review of the statistical turbulence theory required extending the population balance closure models to the entire spectrum of turbulence. *AIChE Journal* 2016;62(5):1795–1820. doi:10.1002/aic.15128.
34. Khan, F.R.. Investigation of turbulent flows and instabilities in a stirred vessel using particle image velocimetry. Ph.D. thesis; Loughborough University; 2005.