Essays on Effects of Uncertainty on Competition among Firms and Political Parties

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Humanities

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CONTENTS

ABSTRACT

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This thesis investigates different aspects of competition under uncertainty using the tools of game theory. In Chapter 1, I consider a quantity oligopoly game. One of the firms is presented with an opportunity to commit to some output before the demand becomes known, but may add to it afterwards, then moving simultaneously with the rivals. I show that the more cost-efficient firm is more likely to behave like a Stackelberg leader, i.e. to produce the optimal Stackelberg leader quantity ex-ante and refrain from adding to it later, letting the rivals respond to its ex-ante output in the manner of Stackelberg followers.

In Chapter 2, I study a model of an electoral contest. Two symmetric parties allocate their endowments to building platforms on various issues before the start of a campaign. Next, one of the issues becomes decisive in the course of the campaign with a commonly known probability. The outcome of the election depends on the difference in competence in this issue. I show that if the payoff functions are convex in this difference—the case of 'increasing returns to power'—parties differentiate each other by selecting different campaign issues. On the contrary, when the payoff functions are concave in this difference—the case of 'decreasing returns to power'—parties mimic each other by investing the same amounts into the same issues. Thus, incentives for selecting campaign issues depend critically on the shape of the payoff functions, which might be determined by (1) a non-linear technology transforming parties' investment in various topics into voters' perception of their competence, (2) or parties' inherent motivation for winning by a big margin due to parties' ideological convictions or rent-seeking, (3) or an electoral system giving winners or big parties a disproportionate advantage in the assigned number of seats, (4) or a relatively high extent of power given to the winning party once in office.

Keywords: (Chapter 1) quantity competition, endogenous timing, asymmetric costs, uncertain demand, (Chapter 2) elections, campaign, contest, salience, competence, issue convergence, issue divergence.
JEL Classification: (Chapter 1) C72, D43, L13, (Chapter 2) D72, C70.

DECLARATION

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Moim rodzicom
Marii i Andrzejowi
za nieustające wsparcie
i bezwarunkową miłość

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1

COST-EFFICIENCY AND ENDOGENOUS STACKELBERG LEADERSHIP UNDER UNCERTAIN DEMAND

1.1 INTRODUCTION

Modern highly innovative industries, such as consumer electronics or automotive industry, share a number of similarities. First, firms in these sectors tend to pay a lot of attention to launching new products. As their life cycle is relatively short, a successful launch is typically preceded by heavy advertising. For instance, Apple sold more than 13 million new iPhone 6s and 6s Plus models within three days after launch[1]. This business model requires the firms to make strategic decisions well in advance of the sales and hence it is consistent with the assumptions of 'one-shot' competition models.

Second however, those decisions are usually taken at different moments by different firms, even though their products are substitutes. Product release dates vary between competitors, e.g. in recent years, Apple's iPhones and Samsung's Galaxy S series smartphones were launched annually in September and May, respectively.

Third, estimating demand for a new product presents a challenge for the producer, even if data on the historical sales of the previous versions are available (Rob, 1991). That uncertainty vanishes gradually with time as firms collect relevant information and conduct market research. For instance, despite being the first foreign brand to have penetrated the Chinese smartphone market, Apple openly admitted to having underestimated the local demand. By the time it made the decision to expand its production capacity there, it was faced with competition from other brands.[2]

1 http://www.apple.com/uk/pr/library/2015/09/28Apple-Announces-Record-iPhone-6s-iPhone-6s-Plus-Sales.html
2 Source: www.thenextweb.com/apple.

Accordingly, market leaders face a trade-off when deciding when and how much to produce. By opting to move early and committing to deliver some output before any followers entered the market, the leader is able to exercise its market power and crowd out the followers' output at the risk of misjudging the market size. By producing later, simultaneously with the followers, leader loses strategic advantage, but is able to acquire more information about demand.

In this chapter, I consider such a strategic trade-off by generalising the standard Stackelberg model to allow the leader which has underestimated the size of the market to add to its initial quantity upon realising the mistake. The aim here is to understand the output and timing decisions of early innovators operating in oligopolistic markets. To this end, I analyse the extent to which these decisions can be explained by differences in the firms' production technologies.

Even in homogeneous product markets, firms' manufacturing costs are likely to differ, making some of the firms more cost-efficient than others. For instance, the marginal cost of electricity generation varies from almost zero for nuclear power to very high values for the peaking gas turbines power (Green and Newbery, 1992). In the short run, those differences are virtually exogenous from producers' perspective as they arise due to various historical, geographical, or regulatory factors.

Formally, I consider a quantity competition game between firms producing homogeneous product. There are two periods. In the first one, one of the firms is exogenously given an opportunity to lead, i.e. to choose its output before the competitors. At this moment however, the exact size of demand is unknown. It is revealed in period two, when leader's choice from the initial period is announced and all the firms, including the leader, choose their outputs simultaneously, as in Saloner (1987). The market clears once at the end of the game.

The setting presented here corresponds to the market situations in which there exists an exogenous barrier to entry: the followers have to wait for the leader's choice and have no means of committing to a future action. This might be due to a variety of circumstances. For instance, one of the firms might have succeeded in gaining access to a new technology earlier than competitors. The barrier might also result from some informational or psychological asymmetry, or, more plausibly, from historical circumstances (Dowrick, 1986) such as the sunk cost of capacity (Spence, 1977), or a threat of dominance by one firm perceived to be in a position to dictate the play. Regardless of the underlying causes, in all these situations the leader knows that the rivals observe its action and is able to exploit that.

Surprisingly, the intuitive conclusion that more cost-efficient firms have a stronger incentive to fully commit has received relatively little support in the literature. A notable exception is the work of van Damme and Hurkens (1999), who used a variant of the action commitment game (as introduced by Hamilton and Slutsky, 1990 and complemented by Amir (1995)) with linear demand and asymmetric constant marginal costs. Although the game exhibits two subgame-perfect Nash equilibria (SPNE), each with a different firm as the leader, a risk-dominance criterion is used to select the equilibrium in which the low-cost firm moves first.

I compare two alternative scenarios in which either the low or the high cost firm is presented with an opportunity to commit to some output early on. I show that cost-efficient firms are more likely to produce the optimal Stackelberg leader quantity straight away and refrain from adding to it later. In a similar situation less cost-efficient firms would prefer to produce a smaller quantity, but add to this initial output ex-post when having underestimated the demand to a sufficiently high extent. Thus, I conclude that low-cost firms are particularly inclined and predisposed to assuming a leadership position.

This chapter is organised as follows. In Section 1.2, I survey the related literature. I introduce the model in Section 1.3 and analyse it in Sections 1.4 and 1.5. I conclude in Section 1.6.

1.2 RELATED LITERATURE

In this section, I review the existing literature to give background for my agenda. First, I introduce the Stackelberg model and highlight its problems. Next, I discuss the attempts to resolve them, also within the empirical literature.

1.2.1 *Stackelberg model and its shortcomings*

In response to the classic (Cournot, 1838) model of a duopoly with firms setting their output levels simultaneously, von Stackelberg (1934) advocated a sequential-move approach as a more realistic alternative. In this prototypical model, he considers two firms competing in a homogeneous product market. One of them—the leader—sets the level of its output first. The other firm—the follower—observes the leader's choice and then selects its own output. The order of movement is hence exogenously (and somewhat arbitrarily) given. Stackelberg's prediction for this model with profit-maximising firms coincides with what later became known as the perfect Nash equilibrium. Indeed, both Cournot and Stackelberg problems are games of perfect information solved using the same equilibrium concept.

Stackelberg model captures the strategic interactions in quantity games with two natural stages of play. This approach has been employed to entry deterrence models with an incumbent firm as the leader and to several public policy problems involving one or more economic agents as the followers and the government as the leader. Note that one may also view the sequential quantity game differently: although both firms deliver the product to the market at the same moment, one of them is able to credibly commit to a chosen output in full view of the counterparts. Conversely, the follower is unable to make empty threats, such as setting the price to zero whenever the rival produces anything. Thus, the follower perceives the opponent's action as credible and reacts accordingly. Clearly, the action is credible if it is irreversible. That is because, in general, the leading firm is not at its reaction curve *ex post*, but has an incentive to reduce its output. Then however, the other firm would anticipate this move and react by increasing its own production, thereby decreasing the leader's profits. In this sense, flexibility would be harmful to the leading firm. As Tirole (1988) notices, this leads to at least three questions: Why does quantity have a commitment value? What does it mean to compete in quantities? Why does one of the firms have an opportunity to move first?

One way to answer these questions, and thus make Stackelberg's story consistent, was proposed by Spence (1977, 1979) and Dixit (1979, 1980) who interpret the quantity variable as a capacity. This solves the issue of credibility, as building capacity requires incurring sunk costs. Moreover, competition in quantities acquires meaning: given the capacity levels, the profit functions represent reduced-form profit functions obtained from solving for short-run product-market competition. Finally, the advantage of moving first may result from getting earlier access to the technology or acting quicker. In this chapter, I show that uncertainty may encourage the less cost-efficient firms to wait, were they given the first-mover advantage. In contrast, the low-cost firms would prefer to commit early when given the same opportunity. provide some insight into the reasons why some firms take up the opportunity of moving early, while others prefer to wait.

The reason why some firms are not able to challenge the market leaders early enough is the presence of various barriers to entry. The Stackelberg model offers no explanation as to why these barriers limit the actions of only selected firms. Stackelberg (1952, pp. 194–95) thought that his solution, which he termed the asymmetrical duopoly, 'is unstable, for the passive seller can take up the struggle at any time. . . It is possible, of course, that the duopolists may attempt to supplant each one another in the market so that "cut-throat" competition breaks out' (quoted after Vives, 2001). This type of outcome is typically referred to as the Stackelberg warfare or the Stackelberg disequilibrium, in which both duopolists try to lead in a quantity setting game contrary to the opponent's expectations. Stackelberg (1952, section 4.3.2) insisted that duopoly was an unstable regime and equilibrium would be restored only with 'collective monopoly or State regulation'. Similar issues were also raised already by Edgeworth (1925) who wondered which of the two identical firms is to be anointed as the leader and which supplanted as the follower.

These considerations lead to the question of what would happen in the absence of any exogenous barriers. It seems plausible that in particular circumstances firms may choose to produce first or wait for the rival voluntarily. More specifically, a firm may be sometimes inclined to commit to some action early to deter the rival, or a firm may prefer to wait, in order to take advantage of the information conveyed in the rival's decision; hence the need for an integrated model allowing firms to make the timing choices.

Robson (1990a) provides a simple example of an ex-ante symmetric duopoly game with an asymmetric equilibrium outcome of the Stackelberg type. In the example, firms face a linear demand and choose their quantities from a finite set at one of the two periods; precommitting to a given level of output in period 1, however, involves a small positive cost. There are only two pure strategy subgame-perfect equilibria of this game—each of Stackelberg type. Since the first-mover's equilibrium payoff is higher, firms prefer to assume the leadership role to waiting. Nonetheless, The incentive to act early is not robust to game specification. For instance, firms prefer to follow rather than lead when the follower is able to undercut the price of the incumbent or when the follower can overbid the leader and collect a patent in a research and development game (Reinganum, 1985). In spite of these problems, traditional duopoly literature (e.g. Fellner, 1965) treated the timing feature as exogenously given and was limited to comparing outcomes of Cournot and Stackelberg games.

Intuitively, an exogenous difference between the firms could induce them to acquire the roles of the leader and the follower. The main motive to produce first is to influence the competitors, while the main motive to wait is to be able to gather information from observing the leader's choices, and to respond optimally. Depending on the firm's characteristics, it may find one of the incentives involved in this trade-off more appealing and, in this sense, those characteristics may predispose firms to be natural leaders.

Specifically, I focus on the production technology (the level of costs) and access to the information about demand. As Rob (1991) remarks, when a new market opens (for instance, as a result of a product being newly invented) or when an existing market starts to expand, uncertainty with respect to its size is likely to prevail. As a result of this uncertainty, the entry of firms into such markets will typically occur in waves: some firms will enter early, while others will wait to see the consequences of that initial entry. For those reasons, allowing for uncertain demand is essential in studying endogenous entry or timing decisions.

A fixed order of decision-taking is a realistic assumption in the case of: industries with established dominant firms or where entry at an earlier stage was not possible for technological or legal reasons, liberalised markets that were once considered natural monopolies, or those where intellectual property rights play an important role.

Clearly, the leading firm's commitment to a strategy may not be credible in the long run; nonetheless, it might create a credible advantage in markets with a short horizon or when strategies are costly to change. In patent races, for instance, a preliminary investment in research and development represents a solid commitment to an innovation strategy. In seasonal markets, firms choose their production level at the beginning of the season and it is hard to change such a strategic choice afterwards. Prices might be sticky in the short run because the information for reoptimising it is costly or because a price change can induce adverse reputational effects on the consumers' perception. Hence, being the first mover in the price choice provides the leader with a credible commitment in the short run. Indeed, firms in the consumer electronics industry usually announce the price of their product at its launch and tend to adhere to it.

1.2.2 *Comparing payoffs of the first- and second-mover*

The problems with the Stackelberg model outlined in the previous section have spurred growth of the literature examining them. In the first strand of this research, the payoffs of the first- and second-movers in duopoly games are compared. In a short note, Gal-Or (1985) studies two identical firms with strictly monotonic profit functions in their opponent's strategies and globally concave in their own ones. She demonstrates that when players move sequentially in a game, the player that moves first earns lower (higher) profits in the subgame-perfect Nash equilibrium than the player that moves second if the reaction functions of the players are upwards (downwards) sloping.

Dowrick (1986) investigates the firms competing in normal goods which are substitutes, without assuming the global concavity of payoffs and players being identical. He shows first that the duopolists disagree over the choice of roles if they have downward-sloping reaction functions: each prefers to take leadership so long as there is no uncertainty about demand or the rival's output—a restriction I relax. Second, if both firms have upward-sloping reaction functions and one prefers to be the leader, the other must prefer to be the follower. Third, firms also disagree if their reaction functions are upward-sloping and the cost and demand structures are similar. That is because each prefers being the Stackelberg follower. The firm may prefer to follow if its best response function has a sufficiently positive slope to act as an effective deterrent against a price-cutting opponent[3]. This motivates my focus on the cost asymmetry.

Although Boyer and Moreaux (1987) make different assumptions than other contributions in this strand as they allow firms to choose both prices and quantities, they also analyse first and second-mover advantages in a homogeneous good market. Employing a proportional rationing rule, they show first that the non-trivial Nash-Bertrand-Edgeworth equilibrium does not exist. Then they demonstrate that for similar costs both firms prefer the role of the follower; for significant cost differences the equilibrium may be only of two types: either the less efficient firm acts as the leader, selling a limited quantity at a low price, and the more efficient firm as the follower, serving the residual demand at a higher price, or the more efficient firm drives the other out of the market acting as the leader that uses a limit pricing strategy. Studying the price-quantity strategy space might be an interesting extension to my agenda.

Similarly, Deneckere and Kovenock (1992) analyse Bertrand-Edgeworth duopolies which set prices in the presence of capacity constraints. They construct a game-theoretic model of the dominant-firm price leadership by showing that the small firm, being indifferent between leading and playing simultaneously, strictly prefers to be the follower. On the basis of this result, they discuss the timing games with ex-post inflexible prices in which the large firm endogenously becomes the price leader. These results are in line with Furth and Kovenock (1993).

3 For a simplified exposition of Dowrick (1986), see Varian (1992), pp. 295–300.

1.2.3 *Endogenising timing in oligopoly games*

The second strand of research deals directly with the endogenous timing. It attempts to explain why firms may want to act simultaneously or sequentially and which role they might be inclined to adopt. Put differently, instead of imposing some exogenous timing structure, the order of play in a given two-player game should reflect the players' own intrinsic incentives and result from their own decisions. This kind of game was suggested already by both Gal-Or (1985) and Dowrick (1986); the latter compares firms' payoffs in Nash simultaneous (follower-follower) games, Stackelberg leadership (leader-follower and follower-leader), and Stackelberg warfare (leader-leader). This, however, does not address the issue of endogenous timing directly.

In their seminal paper, Hamilton and Slutsky (1990) (henceforth HS) offer a way to model the firms' timing decisions. They assume quasiconcave profit functions in the player's own action to ensure that reaction functions are continuous and a Nash equilibrium in pure strategies exists for the simultaneous game. Consequently, the existence of a pure strategy equilibrium is also guaranteed in the sequential game and each of the basic games has a unique and different equilibrium. In order to explain the firms' timing decisions, they introduce two different extended games. In the first one, called the *extended game with observable delay* (EGOD), a preplay stage in period 0 is added to the basic quantity competition game. In that stage, the firms decide simultaneously whether to deliver their quantity in period 1 or 2. Then, quantity competition unfolds according to these timing decisions: simultaneous play occurs if both players decide to move at the same time (whether in period 1 or 2), sequential play under perfect information occurs otherwise (with the appropriate order of moves as announced by the players). Thus, the subgame-perfect equilibria of the game are extended to include the preplay stage. Given the equilibrium uniqueness in every subgame, the game reduces to a 2 by 2 normal form. Then:

(i) if each firm is better off in the simultaneous move equilibrium than when it is the Stackelberg follower, in the EGOD's equilibrium both firms act in period 1 and get simultaneous move payoffs;

(ii) conversely, if each firm is better off being the Stackelberg follower than being in the simultaneous move equilibrium, both sequential play subgames are Nash equilibria of the EGOD and there is a mixed strategy equilibrium in which firms randomise over their timing strategies;

(iii) sequential play is the unique SPNE of the EGOD if the leader prefers leading to simultaneous play and the latter to following and follower prefers following to simultaneous play.

Subsequently, they contrast their results with Gal-Or (1985) and Dowrick (1986):

(i) if both reaction functions have slopes of the same sign, then either *(a)* none of the reaction functions crosses the Pareto improvement set relative to the simultaneous move equilibrium, so the unique equilibrium of this EGOD is the simultaneous move equilibrium or *(b)* each reaction function enters the Pareto improvement set, so the EGOD has multiple equilibria.

(ii) if the reaction functions have slopes opposite in sign, then only one of them enters the set of Pareto improved outcomes relative to the simultaneous move outcome, so the unique equilibrium outcome in this EGOD is that the player whose reaction function enters the Pareto set moves second and the rival plays first.

However, Amir (1995) finds a counterexample demonstrating that part *(i)* of the above result is not valid. The presupposition that each firm's indifference curves are single-valued functions is not true under the assumptions of HS. He shows that the additional requirement needed here is the monotonicity of each payoff in the opponent's actions.

Moreover, HS define a game as 'qualitatively symmetric' if (1) the firms' reaction functions are either both increasing or both decreasing in the rival's action; (2) the same property applies to the firms' profit functions. If such a game has a unique equilibrium in the interior of the action space, then when the reaction functions slope down, neither crosses the set of outcomes Pareto-improving the simultaneous move outcome, and when reaction functions slope up, both cross this Pareto set.

In the second type of extended game considered by HS, there is no pre-play stage and hence no way to announce the firm's intentions as in the EGOD. This game is called the *extended game with action commitment* (EGAC). Here, a firm can choose to deliver some quantity in period 1 only by committing to some specified action and, at that point, such a firm does not know the decision of the rival. The EGAC is similar in spirit to Dowrick's 1986 study. The difference is his assumption that when both firms try to lead, Stackelberg warfare is an outcome. HS note that this can only occur through error. Otherwise, Stackelberg leader's action is only chosen if the player expects the opponent to wait or select follower's action.

HS prove that there exist only three pure strategy equilibria in the EGAC: either with both firms playing their equilibrium actions from the simultaneous basic game in period 1, or with each playing its Stackelberg leader choice in period 1 and the other waiting until period 2. Further, both Stackelberg equilibria are the only pure strategy equilibria in undominated strategies. Playing the simultaneous move equilibrium strategy is weakly dominated by waiting to play after one's opponent. For all actions of the rival, the firm does either better by waiting and observing the rival's action to play according to its reaction function or is indifferent if the rival plays the simultaneous equilibrium strategy.

The two extended games (EGOD and EGAC) yield different results. As long as the simultaneous- and sequential-move equilibria differ, the properties of the basic duopoly game are not relevant in the game with action commitment. The same multiple equilibria exist in all cases. This is not true with observable delay. The Pareto dominance of the simultaneous move equilibrium depends on the slopes of reaction functions.

In order to choose one of the two Stackelberg equilibria resulting from the EGAC, van Damme and Hurkens (1999) use risk considerations. They consider a quantity-setting game with linear demand and a constant marginal cost. Commiting early involves the risk that the opponent chooses to act early as well hence leading to the Stackelberg warfare outcome. To deal with the equilibrium selection problem, HS use risk dominance. Committing is more risky for the high cost firm. Thus, only the low cost firm chooses to commit and thus emerges as the endogenous Stackelberg leader. This result follows from the fact that the reaction function of the firm with a higher marginal cost is below the reaction function of the rival. In consequence, Stackelberg and Nash quantities of the former are closer to each other, so the firm gains less from committing than the opponent. Conversely, the lower marginal cost firm suffers greater losses if both firms commit themselves and hence it enjoys more bargaining power.

This exercise is repeated for a price setting duopoly game with differentiated products in van Damme and Hurkens (2004). Although the role of the follower is more attractive for both firms, they demonstrate that waiting is more risky for the low cost firm. Again, risk dominance considerations allow to conclude that only the high cost player chooses to wait.

All the papers mentioned in this subsection so far explicitly compare the follower's payoff to the one she would get as a leader or in the simultaneous play. As von Stengel (2010) observes, if the game is symmetric and certain standard assumptions hold, the follower gets either less than in the simultaneous game, or more than the leader. It means that the seemingly natural case when both players profit from sequential play as compared to simultaneous play, but the leader more so than if he was a follower, can only happen in non-symmetric games. This remark motivates my investigation of cost asymmetries.

Amir and Grilo (1999) revisit Stackelberg-Cournot debate in the EGOD framework. In their main results, they show first that the log-concavity of the (inverse) demand function leads to the simultaneous play as the endogenous timing outcome in a quantity setting duopoly regardless of the cost function. In their setting, all the reaction correspondences are downward-sloping. Second, both sequential play outcomes are equilibria if the demand function is log-convex and production is costless (reaction correspondences are downward-sloping in this case). Third, the sequential play with a specific assignment of roles prevails if one firm has a constant marginal cost c, the other is costless, and the demand function $P(\cdot)$ is log-convex while $P(\cdot) - c$ is log-concave. Then, the costless firm has an increasing (and the rival a decreasing) best response. Consequently, the firm with the positive marginal cost emerges as the leader. Although these results generally confirm that the simultaneous play is more common, they also provide justification for the sequential play and suggest it is natural under the conditions explained above. Further, they demonstrate that the cost structures play an essential role in determining the timing incentives.

Tasnádi (2003) studies the timing of moves in an asymmetric price-setting duopoly in the framework of a Bertrand-Edgeworth game with efficient rationing rule and the production-to-order version of the game in which production takes place after the firms have already announced their prices. He finds out that, with sufficiently asymmetric and strictly convex cost functions, the less efficient firm moves first while the more efficient one prefers to move second and sets a higher price than the less efficient firm. This confirms the findings of Boyer and Moreaux (1987); Deneckere and Kovenock (1992), as well as Canoy (1996) under different assumptions. The reason behind it might be that firms compete in prices, i.e. the strategic complementarity. One might reasonably expect this result to reverse under quantity competition, i.e. the cost efficient firm would be eager to lead.

Many other contributions consider the timing problem in the price-setting environment. Tasnádi (2016) extends the contribution by Deneckere and Kovenock (1992) from duopolies to triopolies. He shows for the non-trivial case (in which the Bertrand-Edgeworth triopoly has only an equilibrium in non-degenerated mixed-strategies) that the largest capacity firm sets its price first, while the two other firms wait and set their prices later.

An extended version of a quantity leadership oligopoly is studied by Robson (1990a) and Matsumura (1999). The former employs a generalised Stackelberg model with multiple firms as an empirically testable description of oligopoly. He first considers firms with identical U-shaped average cost functions and, second, with always decreasing average costs as in a natural monopoly. The firms announce their quantities in an exogenously defined sequence. Robson shows that in the first case the SPNE of this game converge to the competetive equilibrium as the number of firms increases to infinity, while in the second case only the first firm remains active. Matsumura analyses a model similar to the EGAC. There are n firms and m periods. In the pre-play stage each firm selects the time t at which it will produce. Contrary to the EGOD, this choice is revealed to the opponents only after t. Market is cleared at the very end after period m. He finds that at least $n-1$ firms play simultaneously in the first period in every pure strategy equilibrium. Thus, the generalised outcome in the spirit of the Stackelberg duopoly does not extend beyond the case of two firms. This discussion constitutes a point of departure for the extension of my approach to the case of the oligopoly with the leader.

Dastidar and Furth (2005) examine endogenous price leadership in the framework of a homogeneous product Bertrand duopoly model in which the firms have different, strictly convex cost functions. Using the continuous version of the Robson (1990b) timing game, they show that, surprisingly, in most cases the endogenous leader is the firm with the highest threshold price and not the more efficient firm.

The potential of the HS approach is demonstrated by the variety of applications of their framework. Pal (1998) studies how the duopoly outcome could change if one firm was private-owned and the other public, the latter maximising social welfare instead of its own profit (mixed duopoly). Baik and Shogren (1992), Leininger (1993) and Hoffmann and Rota-Graziosi (2012) examine the simultaneous versus sequential choice of effort in two-player contests. Kempf and Rota-Graziosi (2010) extend the standard approach of horizontal tax competition by endogenising the timing of decisions made by the competing jurisdictions.

Leadership games are studied by von Stengel and Zamir (2010). They analyse subgame-perfect Nash equilibria of a game with the leader committing to an action to which the followers respond playing among themselves their simultaneous Nash equilibrium strategies. However, the main results concern finite two-player games (the leader and one follower) with commitment to mixed strategies and, as they show, do not extend to the case of multiple followers. The games are studied via their mixed extension, where every mixed-strategy simplex is a set of new pure strategies. Thus, the authors are able to analyse convex and compact strategy sets and, using further standard assumptions, apply Kakutani's fixed point theorem to prove the existence of the equilibrium. They also study incentives for leading. They prove that the leader's equilibrium payoffs are at least as high as its Nash and correlated equilibrium payoffs in the simultaneous game. As such, these games are not directly applicable to the oligopoly games of my interest.

1.2.4 *Allowing firms to act in both periods*

The contribution here fits most closely within the third strand of literature which generalises the framework of the endogeous timing by allowing the firms to act in more than one period. This approach captures the strategic interactions between the firms that are not facing any barriers preventing them from making commitments early. A two-period model seems more plausible than other versions outlined above as the real-world firms usually have more flexibility in their actions. In all the aforementioned studies, a firm is not allowed to split production between two periods. In other words, players need to decide in which period they want to produce. In this strand, firms can commit to producing some quantity in both periods, just as the leader in my approach. That allows to capture strategic interaction between firms that are present in the market at the same time and are able to adjust their quantities in the late period knowing the decision of the rival from the initial period. Such a setting seems natural in situations when producers have an opportunity to increase their capacity before they deliver their product to the market. The incentive for a correction after the initial period might stem from the new information available to firms: whether concerning the demand or the action of the rival from the previous stage. I plan to investigate this possibility too and survey related literature in the subsection 1.2.5.

This strand was initiated by Saloner (1987). The basic structure of the game presented in this chapter follows his contribution. In the short note, he considers a Cournot game with two production periods before the market clears (once). In the first period, both firms choose quantities simultaneously and after decisions have been made, they become common knowledge. In the second period, firms again choose how much more to produce before the market clears. Saloner claims that in this game any outcome on the outer envelope of the best-response functions between (and including) firms' smallest Stackelberg outcomes is attainable with a subgame-perfect Nash equilibrium. It holds even when all the production is in fact carried out solely in the first period. As Ellingsen (1995) points out however, not all of these points are equilibria due to a missing assumption. If the profit function is non-monotonic in the firm's own actions along the set of the claimed equilibria, it may pay one of the firms to reduce production. Nonetheless, even with the use of this additional assumption, there exists a continuum of equilibria. In the setting analysed here, only one of the firms is given the option to commit to producing some quantity in both periods and hence equilibrium multiplicity does not occur.

Pal (1991) generalises Saloner's approach by allowing symmetric firms' costs to change in period 2. He shows that the continuum of equilibria vanishes for any cost differential and, depending on its magnitude, three types of SPNE in pure strategies are possible. When production is cheaper in period 1, there exists a unique SPNE in which both players produce their Cournot-Nash quantities in this period only. Furthermore, if the marginal costs are slightly higher in this period, there exist two SPNE of the Stackelberg type with both firms restricting all of their respective outputs to different periods. For sufficiently high costs in period 1, there exists a unique SPNE with the firms making commitments in period 2 only.

In the second case above, the presence of two Stackelberg equilibria generates a selection problem. In a perfectly symmetric setting, there is no reason to think any of the firms will be willing to coordinate on the less preferred outcome. Pal (1996) addresses this issue by studying the symmetric mixed strategy equilibrium of this game. He shows that the ex-post Cournot competition occurs with a positive probability. This observation may explain some of the experiments reported in subsection 1.2.6. Nonetheless, the use of the mixed strategies raises the question of how one should interpret the randomness in the firms' decisions, which is left unanswered in Pal's paper.

Ellingsen (1995) analyses a duopoly in which both firms can invest in stage 1, but one of them is more flexible in the sense that it is also able to commit to some investment in stage 2. The firms' payoff functions are strictly concave in their own actions and decreasing in the actions of the rival. Hence, the firms' reaction functions slope downwards, i.e their decisions are strategic substitutes. When the investments can be costlessly reversed in period 2, i.e. one of the firms can select a negative action, any SPNE is of the Stackelberg type with the flexible firm in the role of the follower. This reiterates the long-standing observation that the capital investment has to be somewhat difficult to reverse in order to have a commitment value. However, Ellingsen shows that even if the investment cannot be reversed, the flexible firm is still disadvantaged: although there exist multiple SPNE, the only undominated strategy for the flexible firm is to refrain from investing in stage 1, while the rival's best strategy is the Stackelberg leadership investment. The flexibility is a curse in the sense that it gives the opponent the equivalent of the first-mover advantage. This result suggests that it might be fruitful to investigate further implications of asymmetries in Saloner's framework, e.g. ones arising from cost or information differentials between firms.

1.2.5 *Asymmetrically informed firms*

Mailath (1993) is driven by the latter concern. He notes that in the model with symmetrically informed firms, the Stackelberg leader prefers its role to being the Cournot duopolist. Nevertheless, if the leader has more information about the demand, its ex ante profits may be lower than under simultaneous competition. The reason is that actions convey information: an informed agent has an incentive to delay her choice to prevent the rival from free-riding on the signal. In the paper, the firm with superior information is given the opportunity to delay its quantity commitment until the decision period of the uninformed firm, thus, forcing the rival to play simultaneously. This setting is motivated by a scenario in which a firm chooses a new capacity level just before the expiration of a patent. Somewhat surprisingly, the informed firm moves first in the unique stable outcome irrespective of its private information, thus conveying the information to the free-riding follower.

Similarly, in order to demonstrate the reduced advantage of the first-mover in a stochastic environment with private information, Gal-Or (1987) considers a leader-follower quantity game. Once again in the equilibrium, the leader reveals to the follower the information about the demand. Even though the leader reduces its output in an attempt to signal a low demand, the follower always infers the signal correctly, provided that the leader's information is not infinitely noisy. The follower is better off than the leader over a wide range of parameter values.

In comparison to the above contributions, the setting presented in this chapter reverses the information asymmetry. It is the first-mover which is initially uninformed, but can defer its decision to the second period in which the demand size is revealed.

1.2.6 *Experimental evidence on endogenous timing*

Huck et al. (2002) study the EGAC in an experiment. Randomly matched subjects play a quantity competition game with two symmetric firms. The results do not confirm the prevalence of endogenous Stackelberg leadership suggested by HS, Ellingsen (1995), and Robson (1990a) for symmetric games and by Mailath (1993) and Normann (1997) for the asymmetric information case. Endogenous Cournot play prevails in most cases and sometimes collusive play is observed, but Stackelberg outcomes are extremely rare. This might be due to the fact that there exist two Stackelberg equilibria and either firm may take up the leadership, hence severe coordination problems arise. The second reason, strengthening the first one, is that endogenous Stackelberg followers learn over time to reward cooperation and punish exploitation. From the behavioural viewpoint, it is unclear why players would coordinate on an asymmetric equilibrium with large payoff differences given that both firms are symmetric. The theoretical results based on the iterated elimination of weakly dominated strategies have been shown to not hold in the laboratory (see Kübler and Weizsäcker, 2004).

Similar results are reported by Müller (2006) from an experiment with fixed-matching, in which subjects play two types of a duopoly game: the single-period Cournot and the two-period Saloner. In both settings, symmetric outcomes turn out to be the most common. After a short learning phase, the average industry output is the same in both markets and lower than predicted by the classic one-period Cournot duopoly. Moreover, he observes diverse types of behaviour in two-period markets ranging from Cournot-Nash competition to pure collusion. On average however, the vast majority of the total production takes place in the first period.

The above results are supported by Huck et al. (2001) who analyse the Stackelberg duopoly with an exogenously assigned leader in the same experimental setting. They find that the followers frequently punish the Stackelberg leaders who try to exploit their advantage. Hence, the latter are much better off producing less than than prescribed by the sequential SPNE.

To fill the gap between the theory and the experimental evidence, Santos-Pinto (2008) generalises the EGAC by assuming that the players are averse to payoff inequality, i.e. they dislike both the disadvantageous (envy) and advantageous (compassion) inequity. He shows that the relatively high levels of inequity aversion rule out the asymmetric equilibria in the EGAC. Furthermore, the game generates a continuum of symmetric equilibria with both collusive and competitive outcomes.

Thus, to the best of my knowledge, there is no experimental evidence supporting the pure Stackelberg outcomes. Again, I conjecture that it might be due to the game participants being symmetric.

1.3 THE MODEL

Consider a homogeneous good oligopoly, in which firms, indexed by $i \in I = \{1, \ldots, n\}$, choose quantities $q_i \geq 0$. The firms are ordered by their constant marginal cost, $0 \leq c_1 < \ldots < c_n$, which is also assumed common knowledge. The aggregate output Q is sold at the corresponding demand price, given by linear inverse demand function $P(Q) = \alpha - Q$. Thus, profits for firm i are:

$$\pi_i(q_i, Q_{-i}, \alpha) = (\alpha - q_i - Q_{-i} - c_i)q_i.$$

Before the game, firm $j \in I$, henceforth 'leader', is allocated (by nature) with a first-mover advantage exogenously. Then, the timing of moves is as follows:

- In period 1, the demand intercept α is unknown and leader chooses a quantity of output q_j^1, while all other firms are restricted to $q_k^1 = 0$, $k \neq j$.

- In period 2, the realised demand intercept denoted by α_r is revealed to all firms along with leader's quantity q_j^1; then, every firm chooses its output q_i^2 simultaneously with the rivals.

The first-mover knows that the second-movers maximise their profits. Period 2 of the game is therefore equivalent to a Cournot (sub)game in which leader's quantity q_j is bounded from below by q_j^1 and strategies are $q_1, \ldots, q_n$. Thus, I assume that in period 1 firm j expects the unique Cournot-Nash equilibrium (CNE) to follow in period 2, as per the chosen q_j^1 and realised demand intercept α_r. I model the period 1 expectations of firm j with respect to this intercept in two ways.

(M1) In section 1.4, I consider the case in which the first-mover has *naïve* beliefs. This means that the leader has estimate α_e of eventual demand intercept α_r, which it considers as the most likely market size. However, it is ignorant of the statistical properties of the estimate, and sets q_j^1 as if $\alpha_r = \alpha_e$ was *certain to occur*. Of course, the actual value of α_r may turn out to be different, and I am particularly interested in what happens in period 2 when firm j has under- or overestimated the demand, as it happened to Apple in China. Note that I present this approach as a precursor to the more realistic case discussed subsequently. It allows me to illustrate and highlight the factors driving the main result.

(M2) In section 1.5, I extend the results to the case in which the first-mover has probabilistic beliefs about α, given by a continuous probability distribution with finite support $[\alpha^-, \alpha^+]$. I consider a duopoly and I assume that potential leader j is risk-neutral, i.e. it sets q_j^1 to maximise the expected profit with respect to the distribution of α. Further, I assume that:

$$2\alpha^- - \alpha^+ > 3c_2 - 2c_1. \tag{A}$$

This requirement guarantees that in period 2 the follower selects a positive quantity $q_{-j} > 0$ with certainty regardless of which firm is given the first-mover advantage and of the eventual value of the demand intercept α_r. More specifically, consider the following extreme scenario. Firm 1 believes that the distribution of α is concentrated near the upper bound of its support (α^+) and thus produces a large quantity, but the demand intercept is later revealed as the smallest possible and equal to α^-. Assumption A ensures that less cost-efficient firm 2 still decides to produce a positive output.

While (M1) may be considered a model in its own right, it primarily serves an illustrative purpose. It provides an intermediate and intuitive result, which is then readily extended to the more realistic setting (M2). In particular, instead of investigating the likelihood of sequential vs simultaneous play as in the endogenous timing literature started by HS, I attempt to answer the question of whether the cost-efficient firm is more likely to become the Stackelberg leader. The firm is such a leader in the sense that *(i)* in period 1, it chooses to produce the same output as in the classic Stackelberg model; *(ii)* in period 2, it does not add to its period 1 output, letting the rivals produce their optimal response to this output in the manner of Stackelberg followers.

1.4 NAÏVE BELIEFS (M1)

I first analyse a duopoly. The results I obtain here are later generalised to an n-firm oligopoly in subsection 1.4.2.

1.4.1 *Duopoly*

Since firm i's profits are strictly concave in its own strategy, it is sufficient to differentiate i's profits with respect to its quantity q_i and equate the derivative to zero in order to find i's optimal strategy for any quantity selected by rival $-i$:

$$r_i(q_{-i}, \alpha) = \max\{0, (\alpha - c_i - q_{-i})/2\}.$$

Note that it is a stepwise function as the quantity cannot be negative.

If firm j is given the first-mover advantage, it also able to infer the reaction of second-mover $-j$. Thus, j maximises $\pi_j(q_j, r_{-j}(q_j, \alpha), \alpha)$. Then:

- If $q_j > \alpha - c_{-j}$, the reaction of $-j$ is equal to zero. In this case, j operates as a monopolist and produces $(\alpha - c_j)/2$, provided that $\alpha \geq c_j$ and $\alpha \leq 2c_{-j} - c_j$ (to ensure $q_{-j} = 0$).
- If however $q_j < \alpha - c_{-j}$, the reaction of $-j$ is to produce a positive quantity. The maximand of j is strictly concave in q_j. Hence, it is again sufficient to differentiate the maximand with respect to q_j and equate the resulting derivative to zero in order to find j's optimal quantity. This yields $\frac{1}{2}(\alpha + c_{-j}) - c_j$, which is positive for $\alpha > 2c_j - c_{-j}$. To ensure $q_{-j} > 0$, it is also necessary that $\alpha > 3c_{-j} - 2c_j$.
- Otherwise, j prefers to select $q_j = \alpha - c_{-j}$, a limit-pricing strategy which is the exact amount discouraging $-j$ from producing anything. This occurs when α is too large to prevent $-j$ from entering in case j produces monopolist's quantity, $\alpha > 2c_{-j} - c_j$, but small enough to enable j to use the limit-pricing strategy, $\alpha \leq 3c_{-j} - 2c_j$.

To summarise, if j believes that the demand intercept will be equal to α with certainty, its optimal period 1 quantity is:

$$q_j^*(\alpha) = \begin{cases} 0 & \text{if} \quad 0 \leq \alpha \leq \max\{c_j, 2c_j - c_{-j}\}, \\ (\alpha - c_j)/2 & \text{if} \quad c_j < \alpha \leq 2c_{-j} - c_j, \\ \alpha - c_{-j} & \text{if} \quad 2c_{-j} - c_j < \alpha \leq 3c_{-j} - 2c_j, \\ (\alpha - 2c_j + c_{-j})/2 & \text{if} \quad \alpha > \max\{3c_{-j} - 2c_j, 2c_j - c_{-j}\}. \end{cases} \tag{1}$$

Observe that the monopolist's and the limit-pricing strategies can be selected only if $j = 1$ and $c_1 < c_2$, i.e. firm 1 is given the first-mover advantage and its marginal cost is strictly lower than its rival's.

Hence, under model specification (M1), firm j will find it optimal to set $q_j^1 = q_j^*(\alpha_e)$ in period 1. Indeed, the optimal Stackelberg leader quantity yields the highest profit for the first-mover given the rival's optimal reaction, and as such it is better than the unconstrained CNE. However, that is not to say that j would not wish to set $q_j^2 > 0$ if α_r turns out to be sufficiently higher than α_e.

Definition 1. Let $\tau_j : \mathbb{R}_+ \to \mathbb{R}_+$ denote the function mapping α_e expected by first-mover j with the threshold for eventually realised demand intercept α_r, such that first-mover j selects $q_j^2 = 0$ in period 2 CNE if $\alpha_r \leq \tau_j(\alpha_e)$; otherwise, if $\alpha_r > \tau_j(\alpha_e)$, firm j selects $q_j^2 > 0$ and the unconstrained CNE follows.

Thus, if the realised demand intercept is below the threshold defined above, $\alpha_r \leq \tau_j(\alpha_e)$, firm j ends up as the Stackelberg leader, in the sense that *(i)* in period 1, it produces the Stackelberg leader quantity consistent with expected demand intercept α_e; *(ii)* it does not add to this quantity in period 2, letting the rival firm react optimally to its quantity committed in period 1. In order to find threshold τ_j for any given α_e analytically, I solve for α_r such that j's optimal period 1 quantity q_j^* induced by α_e is equal to j's CNE output induced by α_r. The next proposition determines the relationship between both firms' thresholds τ_1 and τ_2. Note that when the expected demand intercept is sufficiently high, $\alpha_e > 2c_2 - c_1$, then regardless of which firm is given the first-mover advantage, it selects a positive output $q_j^*(\alpha_e)$.

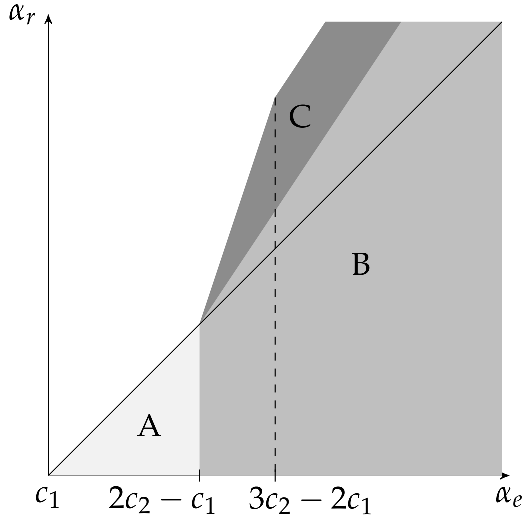
Proposition 1. *In a duopoly ($n = 2$) with firms having naïve beliefs (M1), inequality $\tau_1(\alpha_e) > \tau_2(\alpha_e)$ holds for $\alpha_e > 2c_2 - c_1$.*

Proof. See the proof of Proposition 2. □

Thus to compare both firms as potential first-movers, consider any belief $\alpha_e > 2c_2 - c_1$ shared by them about the demand intercept. Then, by Proposition 1, there exist values of the intercept $\alpha_r \in (\tau_2(\alpha_e), \tau_1(\alpha_e)]$ realised in period 2 inducing firm 2 to add quantity $q_2^2 > 0$ to its initial output, but insufficient to do the same to firm 1, which prefers to select $q_1^2 = 0$ in the same circumstances. The combinations of α_e and α_r for which this occurs are illustrated in Figure 1 by area C, which is bounded from below by $\tau_2(\alpha_e)$ and from above by $\tau_1(\alpha_e)$. For any pair α_e and α_r within area B however, both firm 1 and 2 refrain from adding any output in period 2.

Clearly, the first-mover does not want to add any output in period 2 if the eventual demand is lower than expected, below the solid 45° line from the origin. Areas B and C, however, extend above this line, because moving first allows the leader to commit to an output higher than the one dictated by the CNE, thus also crowding out the output of the follower.

Figure 1.: Expected vs realised demand intercept and the leader's incentive to increase quantity in period 2.



Note: Pairs of (α_e, α_r) for which the leader does not add output in period 2 are in area B for $j = 2$ and in areas B and C for $j = 1$.

Moreover, note that $\tau_1(\alpha_e)$, the upper bound on C, is a piecewise function; a feature inherited from $q_j^*(\cdot)$. Recall that, given the first-mover advantage, firm 1 selects the exact quantity forcing firm 2 out for $\alpha_e \in (2c_2 - c_1, 3c_2 - 2c_1]$; if it expects α_e from above that region, it is not able to force firm 2 to shut its production.

When presented with a first-move opportunity, the low-cost firm produces a relatively large initial amount before reaching the point at which the marginal (Stackelberg) profit from any further output increase is zero. As period 2 opens, this marginal profit is decreased, because increasing one's own output no longer induces the competitor to reduce theirs, making the associated reduction in demand price larger. Since the increased price reduction is applied to a larger period 1 output than in the case of the high-cost firm, the marginal profit from increasing this output in period 2 is more significantly reduced. Hence, it takes a larger increase in the market size (above period one expectations) to make it positive and induce an unconstrained CNE in period 2.

In other words, the low-cost firm gains more than the high-cost one from being the leader and being able to affect the counterpart's output, as the benefits are applied to a larger quantity of its own. Hence, it produces more in period 1 relative to what it would want to produce when moving simultaneously with the rival, and is less likely to increase the initial output even having underestimated the market size.

The fact that the threshold τ_1 is greater than τ_2 has an important implication. Suppose both firms are not just equally likely to be presented with a first-move opportunity, but their expectations of the eventual market size do not systematically differ either. That is to say nature independently draws the value of j (both $j = 1$ and $j = 2$ with equal probability) and a point (α_e, α_r). Then, the low-cost firm is more likely to take on the role of the leader, producing its total output in period 1, and leaving the rival to optimally respond to this in period 2.

This leads to a hypothesis that the reason why Apple's leadership position was challenged by Samsung in China is not only that it underestimated the demand. In addition, it might have had a cost disadvantage which led to a more symmetric equilibrium of a Cournot type. Had Apple been more cost-efficient, perhaps it would not have been forced to increase its output simultaneously with the follower.

I extend Proposition 1 to the case of any finite number of firms in the next result. As before, I examine the case when both potential leaders share the expectations about demand and they believe it will be sufficient for each of them to produce positive quantity in period 1 when given the opportunity to move first.

1.4.2 *Oligopoly*

Unconstrained CNE

When firm i faces multiple rivals, its optimal reaction to aggregate output of its rivals Q_{-i} is $r_i(Q_{-i}, \alpha) = \max\{0, (\alpha - Q_{-i} - c_i)/2\}$. Recall that in the unconstrained CNE, first-mover j adds quantity in period 2 to its commitment from period 1. This can be expressed as $q_j^E \geq q_j^1$, where q_i^E denotes i's quantity in this CNE. By the definition of Nash equilibrium, $q_i^E = r_i\left(Q_{-i}^E, \alpha\right)$ for all $i \in I$. Moreover, all n firms produce positive CNE quantities in period 2 if the demand intercept is sufficiently high. Under this assumption, solving the system of n equations with n unknowns yields:

$$q_{i,n}^E = \frac{1}{n+1}\left(\alpha - nc_i + \sum_{k=1,k\neq i}^{n} c_k\right) \text{ if } \alpha > nc_n - \sum_{k=1}^{n-1} c_k.$$

The above right-hand side inequality ensures that the least cost-efficient firm n produces a positive quantity. More generally, firm i shuts down its production if $\alpha \leq \hat{\alpha}_i = ic_i - c_1 - \ldots - c_{i-1}$. Hence, i's quantity in the period 2 unconstrained CNE is:

$$q_i^E = \begin{cases} 0 & \text{if} \quad \alpha \leq \hat{\alpha}_i, \\ q_{i,i}^E & \text{if} \quad \hat{\alpha}_i < \alpha \leq \hat{\alpha}_{i+1}, \\ q_{i,i+1}^E & \text{if} \quad \hat{\alpha}_{i+1} < \alpha \leq \hat{\alpha}_{i+2}, \\ \vdots & \\ q_{i,n}^E & \text{if} \quad \hat{\alpha}_n < \alpha, \end{cases}$$

The quantity above is a piecewise function[4] of demand intercept α because higher values of the latter encourage more firms to enter with a positive output. Thus, the production of the less cost-efficient firms is more sensitive to the eventual value of the demand intercept.

Constrained CNE

Consider now the case in which leader j does not add any output in period 2. Then, the leader's period 1 quantity exceeds its optimal reaction in period 2, i.e. $q_j^E < q_j^1$. Denote by $q_i^S\left(q_j^1\right)$ the CNE output of follower i, which is induced by first-mover's period 1 quantity q_j^1. By the definition of Nash equilibrium, follower $q_i^S\left(q_j^1\right) = r_i\left(q_j^1 + \sum_{k \in I \setminus \{i,j\}} q_k^S, \alpha\right)$ for all $i \in I \setminus \{j\}$. As it will be

4 I skip its argument for notational brevity.

shown, once again the firms with the higher marginal cost are more likely to shut down their production even for relatively high values of α or low values of q_j^1. Denote by k the firm with the highest marginal cost to produce a positive quantity in the period 2 CNE, i.e. $k = \max\{i \in I \setminus \{j\} : q_k^S(q_j^1) > 0\}$. Let:

$$\mathbb{1}_j^k = \begin{cases} 1 & \text{if } k \leq j, \\ 0 & \text{otherwise.} \end{cases}$$

Assume that α and q_j^1 are such that firm k produces a positive quantity and $k+1$ refrains from delivering any output in the period 2 CNE. Then, the system of $k - \mathbb{1}_j^k$ equations with $k - \mathbb{1}_j^k$ unknowns can be solved to obtain the constrained CNE quantity of every follower $i \in \{1, \ldots, k\} \setminus \{j\}$:

$$q_{i,k}^S\left(q_j^1\right) = \frac{1}{k + \mathbb{1}_j^k}\left(\alpha - \left(k - 1 + \mathbb{1}_j^k\right) c_i + \sum_{l=1, l \neq i,j}^{k} c_l - q_j^1\right).$$

Hence, $q_{k,k}^S\left(q_j^1\right)$ is positive iff:

$$q_j^1 < \hat{q}_j^k = \alpha - \left(k - 1 + \mathbb{1}_j^k\right) c_k + \sum_{i=1, i \neq j}^{k-1} c_i \tag{2}$$

and $q^S_{i,k+1}\left(q^1_j\right)$ is equal to zero iff $q^1_j \geq \hat{q}^{k+1}_j$. Hence, follower i's quantity in the period 2 constrained equilibrium is:

$$q^S_i\left(q^1_j\right) = \begin{cases} 0 & \text{if} \quad \hat{q}^i_j \leq q^1_j, \\ q^S_{i,i}\left(q^1_j\right) & \text{if} \quad \hat{q}^{i+1}_j \leq q^1_j < \hat{q}^i_j, \\ q^S_{i,i+1}\left(q^1_j\right) & \text{if} \quad \hat{q}^{i+2}_j \leq q^1_j < \hat{q}^{i+1}_j, \\ \vdots & \\ q^S_{i,j-1}\left(q^1_j\right) & \text{if} \quad \hat{q}^{j+1}_j \leq q^1_j < \hat{q}^{j-1}_j, \\ q^S_{i,j+1}\left(q^1_j\right) & \text{if} \quad \hat{q}^{j+2}_j \leq q^1_j < \hat{q}^{j+1}_j, \\ \vdots & \\ q^S_{i,n}\left(q^1_j\right) & \text{if} \quad q^1_j < \hat{q}^n_j. \end{cases}$$

Follower i's quantity is a piecewise function of j's period 1 output q^1_j and demand intercept α because their values determine the incentive for the firms to enter the market with a positive output, thereby influencing i's optimal decision.

If k is the least cost-efficient firm producing a positive quantity in the period 2 CNE (i.e. if $\hat{q}^{k+1}_j \leq q^1_j < \hat{q}^k_j$), the aggregate output of all followers equals:

$$Q^S_{-j,k}\left(q^1_j\right) = \sum_{i=1, i\neq j}^{k} q^S_{i,k}\left(q^1_j\right) = \frac{k + \mathbb{1}^k_j - 1}{k + \mathbb{1}^k_j}\left(\alpha - q^1_j\right) - \frac{1}{k + \mathbb{1}^k_j}\sum_{i=1, i\neq j}^{k} c_i.$$

Thus, the followers' aggregate output as a function of q_j^1 is:

$$Q_{-j}^S\left(q_j^1\right) = \begin{cases} 0 & \text{if} \quad \hat{q}_j^1(\alpha_e) \le q_j^1, \\ Q_{-j,1}^S\left(q_j^1\right) & \text{if} \quad \hat{q}_j^2(\alpha_e) \le q_j^1 < \hat{q}_j^1(\alpha_e), \\ Q_{-j,2}^S\left(q_j^1\right) & \text{if} \quad \hat{q}_j^3(\alpha_e) \le q_j^1 < \hat{q}_j^2(\alpha_e), \\ \vdots & \\ Q_{-j,j-1}^S\left(q_j^1\right) & \text{if} \quad \hat{q}_j^{j+1}(\alpha_e) \le q_j^1 < \hat{q}_j^{j-1}(\alpha_e), \\ Q_{-j,j+1}^S\left(q_j^1\right) & \text{if} \quad \hat{q}_j^{j+2}(\alpha_e) \le q_j^1 < \hat{q}_j^{j+1}(\alpha_e), \\ \vdots & \\ Q_{-j,n}^S\left(q_j^1\right) & \text{if} \quad 0 \le q_j^1 < \hat{q}_j^n(\alpha_e). \end{cases} \tag{3}$$

First-mover's optimal quantity

The advantage of moving first allows j to set its quantity at the level which maximises its profits given the followers' reaction in period 2. Formally, j maximises $\pi_j\left(q_j, Q_{-j,}^S(q_j^1), \alpha\right)$. The maximand is a piecewise function as increasing j's quantity forces consecutive firms to shut down their production. Moreover, the maximand is strictly concave in q_j in a trivial way on any interval $(\hat{q}_j^{k+1}, \hat{q}_j^k)$, with $k \in \{1, \dots, n-1\} \setminus \{j\}$, $[0, \hat{q}_j^n)$, and $[\hat{q}_j^n, \infty)$. In order to verify that the maximand is globally strictly concave, I show that the profit's left-hand derivative is greater than the right-hand derivative at the joints of the consecutive intervals:

$$\frac{1}{k+1+\mathbb{1}_j^{k+1}}\left(\alpha - 2q_j + \sum_{i=1, i\neq j}^{k+1} c_i\right) - c_j \ge \frac{1}{k+\mathbb{1}_j^k}\left(\alpha - 2q_j + \sum_{i=1, i\neq j}^{k} c_i\right) - c_j$$

at $q_j = \hat{q}_j^{k+1}$, which can be rearranged to[5]:

$$\alpha + \sum_{i=1, i\neq j}^{k} c_i + \left(k + \mathbb{1}_j^k\right) c_{k+1} - 2\left(k + \mathbb{1}_j^k\right) c_k \ge 0.$$

5 I assume here for the sake of brevity that $\mathbb{1}_j^{k+1} = \mathbb{1}_j^k$. It is easy, albeit somewhat tedious, to verify that the inequality holds in the general case as well.

The above inequality is implied by $\hat{q}_j^{k+1} \geq 0$ and, thus, the leader's profit function is strictly globally concave in its own output.

Hence, the period 1 quantity maximising the expected profits and denoted by q_j^* is unique for any α_e. Observe that the expected profit function has the shape of an inverted hump with kinks at $\hat{q}_j^k(\alpha)$, $k \in I \setminus \{j\}$. Thus, the maximum may lie either at one of the kinks or at the intervals between them. Consider the latter case and assume again that k denotes the firm with the highest marginal cost which produces a positive quantity in the period 2 CNE induced by the leader's period 1 output. Denote this solution by $q_{j,k}^*$. In order to find the explicit expression for the maximiser of this type, differentiate j's profits $\pi_j \left(q_j, Q_{-j,k}^S(q_j), \alpha\right)$ with respect to q_j and compare the resulting derivative to zero. This yields:

$$q_{j,k}^*(\alpha) = \frac{1}{2}\left(\alpha + \sum_{i=1, i\neq j}^{k} c_i - (k + \mathbb{1}_j^k)c_j\right). \tag{4}$$

Recall that, in order to be feasible[6], $q_{j,k}^*(\alpha)$ needs to lie between kinks $\hat{q}_j^{k+1}$ and $\hat{q}_j^k$. The condition $q_{j,k}^*(\alpha) \leq \hat{q}_j^k$ is equivalent to

$$\alpha \geq \alpha_j^k = \left(2k - 1 + 2 \cdot \mathbb{1}_j^k\right) c_k - \left(k + \mathbb{1}_j^k\right) c_j - \sum_{i=1, i\neq j}^{k-1} c_i,$$

while $q_{j,k}^*(\alpha) > \hat{q}_j^{k+1}$ is equivalent to

$$\alpha < \hat{\alpha}_j^{k+1} = 2\left(k + \mathbb{1}_j^{k+1}\right) c_{k+1} - \left(k + \mathbb{1}_j^k\right) c_j - \sum_{i=1, i\neq j}^{k} c_i.$$

Therefore, j selects maximiser $q_{j,k}^*(\alpha)$ between the kinks of the profits function if and only if $\alpha_j^k \leq \alpha < \hat{\alpha}_j^{k+1}$. Since $\alpha_j^{k+1} > \hat{\alpha}_j^{k+1}$, j optimally selects maximiser $\hat{q}_j^{k+1}$ at one of the kinks of the profits function if and only if $\hat{\alpha}_j^{k+1} \leq \alpha < \alpha_j^{k+1}$. Note that the maximising function q_j^* mapping the expected demand intercept

6 Unless $k + 1 = j$, when interval $\left[\hat{q}_j^{j+1}, \hat{q}_j^{j-1}\right]$ should be considered. I continue with this slight abuse of notation without any bearing on the general results.

α_e to j's optimal decision in period 1 is continuous as $q^*_{j,k} = \hat{q}^{k+1}_j$ for $\alpha = \hat{\alpha}^{k+1}_j$. It remains to verify the range of the feasible values of k, i.e. the number of rivals j may face in period 2. Consider α' such that $\hat{q}^{j-1}_j = q^*_{j,j-1}$. Then, $\alpha' = (2j-1)c_{j-1} - jc_j - \sum_{i=1}^{j-2} c_i$ and the resulting optimal quantity in this case is $q^*_{j,j-1}(\alpha') = j(c_{j-1} - c_j)$, which is negative and hence a contradiction. Thus, whenever expected α is sufficiently high to encourage j to produce a positive quantity, the leader expects to face at least all of the more cost-efficient firms. Further, observe that j has the incentive to produce a positive quantity $q^*_{j,j-1}(\alpha) > 0$ if and only if $\alpha > \alpha^j_j$. Finally, the leader's optimal period 1 output can be summarised as follows:

$$q^*_j(\alpha_e) = \begin{cases} 0 & \text{if} \quad \alpha_e < \alpha^j_j, \\ q^*_{j,j-1}(\alpha_e) & \text{if } \alpha^j_j \leq \alpha_e < \hat{\alpha}^{j+1}_j, \\ \hat{q}^{j+1}_j(\alpha_e) & \text{if } \hat{\alpha}^{j+1}_j \leq \alpha_e < \alpha^{j+1}_j, \\ q^*_{j,j+1}(\alpha_e) & \text{if } \alpha^{j+1}_j \leq \alpha_e < \hat{\alpha}^{j+2}_j, \\ \hat{q}^{j+2}_j(\alpha_e) & \text{if } \hat{\alpha}^{j+2}_j \leq \alpha_e < \alpha^{j+2}_j, \\ \vdots & \\ \hat{q}^n_j(\alpha_e) & \text{if } \hat{\alpha}^n_j \leq \alpha_e < \alpha^n_j, \\ q^*_{j,n}(\alpha_e) & \text{if } \alpha^n_j \leq \alpha_e. \end{cases} \tag{5}$$

Observe that the leader's output strictly increases with the demand intercept and the followers' marginal costs, while it decreases with the leader's marginal cost. The sensitivity of the leader's output to all these parameters is higher when the maximiser lies at one of the kinks of the profit function.

Proposition 2. *Under model specification M1 with $n \geq 2$, consider any $i, j \in I$ as potential leaders, $i < j$. Then, $\tau_i(\alpha_e) > \tau_j(\alpha_e)$ for any $\alpha_e > \max\{\alpha^i_i, \alpha^2_2\}$.*

Proof. See the APPENDIX. □

Proposition 2 extends the result from Proposition 1 to the case of more than two potential competitors, $n \geq 2$. Two arbitrarily chosen firms i and j, among which one is more cost-efficient $c_i < c_j$, are compared as potential first-movers. Consider any belief α_e about the demand intercept which ensures that *(i)* at least the efficient firm produces a positive quantity in period 1 $\alpha_e > \underline{\alpha}_i$ and *(ii)* in case $i = 1$, firm 1 expects to compete with at least one follower, $\alpha_e > 2c_2 - c_1$. Then by Proposition 2, there exist values of intercept α_r which lie between $\tau_j(\alpha_e)$ and $\tau_i(\alpha_e)$ which in period 2 induce j to add quantity $q_j^2 > 0$ to its period 1 output, but are insufficient to encourage firm i to do the same and hence it prefers to select $q_1^2 = 0$ in the same circumstances.

1.5 PROBABILISTIC BELIEFS (M2)

I now extend my results to specification M2 in a duopoly setting. Here, leader j has *probabilistic beliefs* about demand intercept $\alpha \in [\alpha^-, \alpha^+]$. Recall that Assumption A requires the following. If firm 1 is the first-mover and it expects the highest possible demand intercept α^+, then even if the eventual intercept is the lowest possible, equal to α^-, firm 2 would still opt to produce a positive output, i.e. $r_2(q_1^*(\alpha^+), \alpha_-) > 0$, which is equivalent to $2\alpha^- - \alpha^+ > 3c_2 - 2c_1$. Note that I assume common knowledge: first-mover j believes that the follower (*i*) knows α^+, (*ii*) knows that firm j expects α^+, (*iii*) knows that firm j knows that the follower knows α^+, etc. Hence, the first-mover expects the follower to best respond to any quantity it chooses, consistently with α^+. In the next result, I compare two alternative scenarios, in which either one of the firms assumes leadership.

Proposition 3. *Consider any probability distribution with support on $[\alpha^-, \alpha^+]$ and satisfying Assumption A. Then:*

(i) outcomes $q_2^2 > 0$ and $q_1^2 = 0$ occur with positive probability, i.e. there exist α_r such that firm 2 as the leader would produce additional output, while firm 1 as the leader would not produce any additional output in period 2;

(ii) outcomes $q_2^2 = 0$ and $q_1^2 > 0$ cannot occur with positive density; i.e. there is no α_r inducing this behaviour.

Proof. See the APPENDIX. □

The intuition behind the proof of Proposition 3 is that even if the firms had identical prior beliefs about the market size, their expected CNE profit-maximising period 1 quantities would be 'as if' they each expected a different market size to occur with certainty. Specifically, in the context of setting (M1), it is as if the value of α_e associated with $j = 1$ was greater than or equal to the one associated with $j = 2$. Loosely speaking, even though firms are risk-neutral expected profit maximisers with the same prior beliefs, the low-cost firm is effectively more optimistic / less risk-averse when moving first, in the sense that its 'certainty-equivalent' α_e is larger. Thus, the low-cost firm may produce even more in period 1 than what is implicit in Proposition (2); and is even less likely to add to this later.

In particular, it is possible to have a prior distribution of α and its realisation α_r such that, in those circumstances, firm 1, but not firm 2, would produce its entire output in period 1 if granted an opportunity to move first. However, there exists no prior distribution of α and its realisation such that the opposite occurs. Consequently, the conclusions from setting M1 continue to hold: so long as firms do not differ in their beliefs about the prospective market size, the low-cost firm is more likely to produce its total output in period 1, leaving the rival to optimally respond to this in period 2. Furthermore, imagine that nature allocates the first-move advantage randomly with equal probabilities to one of the firms. The firm makes its choice and next the demand intercept is drawn. Then, the low-cost firm becomes the leader more often than the rival.

Another matter is under what conditions firm j would choose to produce the same quantity in period 1 as it would produce in the absence of the opportunity to increase its quantity in period 2. In other words, the question is whether the firm's choice of q_j^1 would coincide with the optimal quantity of a (risk-neutral) Stackelberg leader with the same prior beliefs about the prospective market size.

Clearly, this would occur so long as the period 2 CNE has $q_j^2 = 0$ for $\alpha_r = \alpha^+$ and $q_j^1 = q_j^*(\alpha^-)$, which implies it must happen for all $q_j^1 \geq q_j^*(\alpha^-)$ and $\alpha_r \in [\alpha^-, \alpha^+]$. Indeed, j would want to commit at least to the Stackelberg leader quantity associated with the smallest possible market size. Hence, the requirement means that j does not expect to add to any quantity it may wish to select, and so will choose q_j^1 as if not being able to add to it later. However, as the next proposition shows, it is sufficient (and in fact necessary) that this is true for $q_j^1 = q_j^*(E(\alpha)) > q_j^*(\alpha^-)$.

Proposition 4. *The optimal q_j^1 is the same as if a restriction $q_j^2 = 0$ was imposed, i.e. as in the Stackelberg game with demand uncertainty, if and only if:*

$$\alpha^+ \leq [3E(\alpha) - 2c_j + c_{-j}]/2 \tag{6}$$

Additionally, when the above holds, the optimal q_j^1 equals $q_j^*(E(\alpha))$, i.e. the Stackelberg leader quantity given a market size equal with certainty to the expected value of α.

Proof. See the Appendix. □

Observe that condition (6) amounts to condition (3) of Proposition (2) when $\alpha_e = E(\alpha)$ and $\alpha_r = \alpha^+$. This is because $E(\alpha) < \alpha^+$ and (A) implies $2E(\alpha) - \alpha^+ > 3c_2 - 2c_1$, so that:

$$[3E(\alpha) - 2c_j + c_{-j}]/2 < 3E(\alpha) + 2c_j - 4c_{-j}.$$

Thus, not altogether surprisingly, whenever the optimal q_j^1 is the same as if a restriction $q_j^2 = 0$ was imposed, $q_j^2 = 0$ will actually occur in the period 2 CNE, whatever the value of α_r. Recall the main objective of the chapter: to find out which of the firms is more likely to become a Stackelberg leader, in the sense that: a) in period 1, it chooses to produce the same output it would produce if it did not expect to be able to add to it in period 2; b) in period 2 CNE, it does not add to its period 1 output, letting the rival produce its optimal response to this output in the manner of a Stackelberg follower. As established by Proposition 3, the low-cost firm is more likely to satisfy requirement b), i.e. to refrain from adding to whatever output it finds optimal to produce in period 1. Here, I find that a) is an even stronger requirement, in that it implies b) is satisfied for all α_r. However, it is apparent that condition (6) is again more easily satisfied for $j = 1$ than for $j = 2$. Hence, it can be concluded that the low-cost firm is more likely to meet requirement a) as well.

As a final note, observe that condition (6) is more readily fulfilled when $E(\alpha)$ is large relative to α^+. That is to say, requirement a) is more likely to hold (for any firm) when the distribution of α is left skewed, inducing firm j to produce enough output in period 1 to avoid the risk of having underestimated the eventual market size to a large extent.

1.6 CONCLUSIONS

In this chapter, I considered a situation in which one of the oligopolists is presented with an opportunity to move in the first period of play, before being able to add to this initial quantity in period 2, then moving simultaneously with the counterparts. Producing more output straight away induces the rivals to reduce their own quantity, but at the risk of being committed to an excessive output in the event of demand being lower than expected.

I considered two settings: one in which the first-mover has an estimate of the eventual market size and acts as if this was certain to occur, and one in which it has probabilistic beliefs about the market size and seeks to maximise its expected profits. In both cases, I find the results to be similar. Specifically, the more cost-efficient of the firms are more likely to: a) produce the Stackelberg leader quantity consistent with the expected market size in period 1; b) refrain from adding to its initial quantity in period 2, letting the rival respond like a Stackelberg follower.

The reason for this is that the low-cost firm gains more than the less cost-efficient rivals from being able to bring about a reduction in the counterparts' output, as the benefits are applied to a larger quantity of its own. It is also effectively more optimistic, or risk-loving, in deciding to produce more than the less cost-efficient rivals relative to what it would produce under certainty. Thus, it wants to produce more in period 1 than the counterparts relative to what it wants to produce ex-post, when both uncertainty and the potential to strategically influence the rivals are absent. By producing relatively more in period 1, it is less likely to be inclined to add to this later, even having underestimated the demand size.

The novelty of the analysis here is that even though only one firm at a time can move first, and both are equally likely to be presented with such an opportunity, it is still possible to endogenise the timing of moves by showing that the low cost firm is more likely to emerge as the Stackelberg leader. In particular, this is achieved without referring to risk-dominance equilibrium selection criteria as in van Damme and Hurkens (1999).

2

SETTING AGENDA FOR POLITICAL CAMPAIGN WITH AN UNCERTAIN HOT ISSUE: CONVERGENCE OR DIVERGENCE?

2.1 INTRODUCTION

A message is a crucial element of the modern political campaign. It is a concise summary of candidates' plans once in office, not only emphasising their professional skills, but also personal qualities such as integrity, family values, etc. As the attention span of the voters can be narrow, the message is typically short, which makes it easy to repeat and build a lasting impression with the targeted audience. To make the message appealing, candidates need to identify issues of the highest concern to the public. These concerns may not be stable: campaign's media coverage and various events—like economic crises, international conflicts or corruption scandals—can shift voters' attention (Page and Shapiro, 1992; Smith, 1985), giving an advantage to the candidate deemed more suitable for handling the emerging problem.

This problem is well recognised in the press commentary on the 2008 U.S. presidential election quoted by Colomer and Llavador (2012): '"If in October we're talking about Russia and national defense and who can manage America in a difficult world, John McCain will be president", predict[ed] Thomas Rath, the leading Republican strategist in the swing state of New Hampshire. "If we're talking largely about domestic issues and health care, Barack Obama probably will be president". Events can affect that conversation. If Russia invades another country on Oct. 20 or Iran detonates a nuclear weapon, advantage McCain; if there's another Bear Stearns meltdown, or a stock market crash, put a few points on the Obama side.' (Albert R. Hunt, 'Letter from Washington', *New York Times*, 8 Sep 2008).

This raises the question of how such uncertainty affects parties when they select issues for their campaigns. In this chapter, I characterise circumstances leading political rivals to either imitate or differentiate each other's campaigns when the decisive issue for the election result is uncertain.

In the model, two parties decide how to allocate their endowments between multiple issues, e.g. the economy, foreign policy, education, etc. The endowments can be thought to represent either time or funds available for the campaign. The candidates use them to improve their appeal on a given issue either by developing concrete policy proposals or investing in political advertisements; however, I make no such distinction in the model. The key feature of my analysis here is that, just before the voters go to the polls, one of the issues becomes hot with a probability known to both parties. Subsequently, this topic is decisive: it becomes the primary concern of the voters. The election outcome is determined solely by the difference in investment into the hot issue between the parties.

What circumstances lead parties to raise different or the same issues in their campaigns? I distinguish two situations: *increasing* and *decreasing returns to power*. If an advantage over the rival in the hot campaign issue translates into a more than proportionate reward from being in office, returns to power are increasing; conversely, if this advantage results in a less than proportionate reward, returns to power are decreasing. In the model, these features are captured by convex and concave payoff functions, respectively.

I treat the determinants of such an environment as exogenous; they may include various, mutually non-exclusive underlying causes. First, an investment advantage over the rival on a given issue may translate into an advantage in voters' perceptions in a non-linear way. Second, ideology and rent-seeking may lead candidates to risk-seeking or risk-averse behaviour: either preferring victories by a big margin or hedging against the risk of political irrelevance. Third, an electoral system giving the winner a disproportionate advantage or disadvantage in the assigned number of seats in parliament might change parties' rhetoric in the campaign. Fourth, strong and/or decentralised institutions constraining the executive power of elected officials limit potential rents for the winner, while weak institutions present more opportunities for using power to the winner's advantage, such as passing intended reforms or extracting political rents easily. However, only the aggregate effect of all these potential factors matters for parties' behaviour in the model.

I show that parties imitate each other's campaigns—'issue convergence' occurs (Amorós and Puy, 2011)—when returns to power are decreasing. In this case, the payoffs are concave and hence rivals do not have the incentive to gamble their resources; instead, they prefer to play safe by imitating their counterparts. This result is reversed when returns to power are increasing. In this case, parties differentiate themselves from each other by picking different campaign issues—'issue divergence' occurs. In both situations, parties allocate all their endowments to the issues which are the most likely to become hot.

The chapter is organised as follows. In Section 2.2, I present a discussion of the related literature. I introduce the model in Section 2.3 and analyse it in Section 2.4 by comparing environments with decreasing, increasing, and constant returns to power.

2.2 RELATED LITERATURE

In this section, I present a broad review of the economic literature on political competition and emphasise how the current chapter fits into it.

Spatial competition The seminal work by Downs (1957) is one of the first major attempts at integrating a government with private decision-makers in a single general equilibrium theory. The goal of this approach is to explain the rational course of action for governors guided by a particular set of incentives. In the framework based on Hotelling's (1929) model of spatial competition, two parties propose very similar or even identical policies, because they want to win the support of the median voter. This result gave rise to a large body of literature, see e.g. Budge and Farlie (1983) and Budge (1993).

Valence theory The Downsian (or spatial) approach is criticised by Stokes (1963), who emphasises the multi-dimensional aspect of political competition. In his model, parties build their campaign platforms on various independent issues. The resulting valence (or performance) theory asserts that voters support the candidates believed to be most competent to manage the important issues and, importantly, these are issues over which there is virtually no disagreement. Classic examples of valence issues characterised by broad consensus are the economy, safety, quality of public services in education, health, transport, and environmental protection. In all these cases, people are interested in competing parties' ability to deliver the best possible policy outcomes. I follow this approach in the analysis presented here.

The empirical evidence from Britain and other mature democracies (Clarke, 2004) suggests that valence issues typically dominate the political agendas: the majority of the electorate focuses their attention on how rival candidates can deliver on valence issues. Clarke (2009) suggests that contemporary British politics revolves around a cluster of concerns about crime, immigration and terrorism mixed with perennial economic and public service issues. A large share of the electorate agrees on how to resolve these issues and, consequently, the debate focuses on who can do the best job.

Issue selection The seminal work by Riker (1993) (see also Riker, 1996) analyses the ratification of the U.S. constitution in 1787–88. He formulates two principles for the choice of rhetorical effort in a political campaign. The *dominance principle* posits that when one side wins an argument on one dimension, the other side abandons it, while the winner continues to exploit it. This corresponds to the case of increasing returns to power presented here, in which parties avoid competing on the same issues and prefer to find their own niche. The *dispersion principle* posits that when neither of the sides wins on an issue, both sides abandon it and attempt to find another topic. Taken together, these principles predict that parties in any political campaign will choose to emphasise their strengths, while also attacking opponent's weakness.

Petrocik (1996) complements Riker's theory with a notion of *issue ownership*, which suggests that one side may dominate on an issue because it is perceived by the voters to be more competent to deal with it. This perception may stem from its technical expertise or an ideological conviction ensuring its commitment to solving the problem at hand. Similarly to Riker, this theory predicts that each rival will focus its campaign on the issue that it owns, while avoiding the issues owned by the opponent. Again, this approach is encompassed by the case of increasing returns to power analysed here.

Aragonès et al. (2015) investigate the conditions for issue ownership and show when candidates may wish to appropriate the issues previously owned by their opponents. In their model, two office-motivated parties compete for votes. Each side has an established past reputation in handling different issues. Before the election, both parties invest resources to produce policy innovations that increase quality of their proposals on each issue. Next, parties allocate their communication time to shift voters' attention to the issues of their choosing. Thus, parties are able to *prime* voters, i.e. influence their sense of prorities. As a result, (1) the core issue chosen by the party need not coincide with the one that voters find the most important. For this reason, the model presented here abstracts from voters' underlying preferences. (2) Parties' agendas address fewer issues than those the electorate truly cares about. Hence, I focus on the extreme case in which only one issue matters for the election outcome, but parties do not know which it is going to be when starting their campaigns. (3) The better parties become at priming voters, the more homogeneous the electorate becomes. Consequently the competition gets tougher and *issue stealing* may occur in equilibrium, i.e. parties may focus on the issues traditionally perceived as their opponents' strengths.

Amorós and Puy (2011) study a contest where two parties allocate their campaign resources between two salient issues. They distinguish two types of advantage a party may have on an issue: absolute and comparative. The former occurs when a majority of voters prefer its stance on that issue to that of its rival. The latter occurs when the percentage of votes that it would obtain if voters cared only about that issue is larger than those that it would obtain if voters cared only about the other issue. In this model, parties tend to emphasise different issues, unless one of the parties has an absolute advantage on both issues, but its comparative advantage is small. In this case, parties focus on the same issues in the campaign. In my approach, issue convergence or divergence occurs endogenously without assuming that voters are ideologically biased.

Colomer and Llavador (2012) consider an agenda-setting model of electoral competition. In order to make an issue salient in the eyes of the voters, parties need to propose an innovative policy proposal challenging the status quo. The trade-off arises between issues with high salience and those with broad consensus on an alternative policy proposal. In this setting, the issues which are considered the most important by a majority of electorate may not be given salience in the campaign. As a result, an incumbent government may win in spite of its poor policy performance if there is not a sufficiently broad agreement on a new policy. In my model, the issues which are most likely to become decisive are usually emphasised by the candidates. However, as the hot campaign issue is selected by nature—as an outcome of international events, media coverage, etc.—one cannot identify if the omitted issues are perceived as important by the voters and hence I abstract from any welfare judgements.

Krasa and Polborn (2010) study electoral competition as a contest between candidates endowed with different abilities in two distinct policy areas. Candidates are uncertain about voters' preferences when they propose how much money or effort they would allocate to each area. These amounts, along with the winner's level of competence, determine the quality of the public goods provided to the voters. Voters differ in their perceptions of how important each of the provided goods is. This model predicts that candidates' platform policies usually diverge in equilibrium and display a strong rigidity when issues' salience in voters' perception changes. This result prevails because candidates are unable to succesfully imitate their opponents and are subsequently forced to stick to their strengths, even if voters' attention changes focus. In the model presented in this chapter, the issue divergence emerges even though the political rivals are symmetric. Issue rigidity is also present under increasing returns to power in the sense that the equilibrium is insensitive to a range of changes in the probability of different issues becoming hot.

Signalling competence Another contribution to understanding the selection of issues in political campaigns and their dynamics is by Egorov (2015). Two parties may signal their competence on only one of two issues. Voters are more likely to discover candidates' competence when both campaign on the same issue rather than different ones. In this setting, the voters' welfare is a non-monotone function of the informativeness of different-issue campaigns, but the voters benefit if parties are free to pick an issue rather than if the agenda is set exogenously or by voters. If the first-mover is able to change her initial choice, then in the case of the follower choosing a different issue, the politicians highly competent in both issues switch.

Spending and incumbent advantage Meirowitz (2008) attempts to explain empirical spending patterns in political campaigns and sources of incumbency advantage. In his model of electoral contest, candidates exert effort to convince voters of their ability in one dimension only. There are two sources of asymmetry between candidates: first, voters are biased in favour of the incumbent, second, candidates can differ in how effectively they can collect funds for their campaign and how efficiently they can use this money to influence voters' perceptions. Meirowitz shows that only the latter asymmetry can explain the incumbency advantage, because the former cannot reproduce incumbents' outspending behaviour.

Erikson and Palfrey (2000) is similar to the analysis presented here in assuming that campaign spending choices are related to payoffs in a smooth manner. The authors show that close races generate higher spending levels. This explains the simultaneity bias arising in the empirical literature estimating the effect of campaign spending by both incumbents and challengers. Whilst their focus is different from the one presented in this chapter, it demonstrates potential usefulness of this approach in empirical applications.[1]

2.3 THE MODEL

Two symmetric parties, indexed by $p \in \{a, b\}$, compete for votes in an election. There are $n \geq 2$ possible campaign *issues* $i \in I = \{1, \ldots, n\}$, e.g. education, foreign policy, etc. Each party is endowed with a unit budget, or unit amount of time, which can be spent on campaigning on these issues, $x_p^i \in X = [0, 1]$ and $\sum_i x_p^i \leq 1$. When choosing how to allocate their budgets, parties know that issue i will become hot later in the campaign with probability α_i, such

1 See also other contributions in this area: Wittman (1983), Soubeyran (2009), Krasa and Polborn (2014), Page and Shapiro (1992), Bélanger and Meguid (2008), and Demange and Van der Straeten (2009).

that $\alpha \in \Delta$ with $\Delta = \{\alpha \in [0,1]^n | \alpha_1 + \ldots + \alpha_n = 1\}$. The issues are ranked by that probability: $1 \geq \alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n \geq 0$. The hot issue, denoted by h, is revealed in campaign after parties have chosen their platforms $x_p \in X^n$. On the election day, voters compare the parties' proposals on the hot issue and then cast their ballots. Finally, parties are allocated with seats in a parliament and obtain their payoffs. Party p's payoff is a function $\pi : [-1,1] \to \mathbb{R}$ of the difference between its own and its rival's investment into the hot topic $x_p^h - x_{-p}^h$. I focus on the pure strategy Nash equilibrium of the game.

Note that the processes of allocating seats and sharing power are embedded in the payoff functions. One could separate them, for instance, by devising share of seats in parliament function $s : [-1,1] \to [0,1]$ and rents from power function $r : [0,1] \to \mathbb{R}$, such that $\pi(\cdot) = r(s(\cdot))$. If consecutive and equally-sized increases in the hot-issue advantage result in higher and higher increases in the share of seats for the winner, then s is convex; the converse would hold for concave s. Observe that convex s in conjunction with a linear r would result in convex π; moreover, π would be convex even if r was concave to a sufficiently small extent. Other factors may play a role in shaping the parties' payoff functions too. The shape of s could be influenced by the type of electoral system: winner-takes-it-all voting mechanisms traditionally believed to favour large parties would also lead to convex s. Similarly, candidates' strong ideological convictions may be reflected in convex rents from power function r, while a rigid institutional system limiting the extent of the executive power would make r concave.

Nonetheless, the payoff functions analysed in this chapter are in a reduced form. This allows me to focus on explaining how their curvature influences parties' incentives for issue selection. Hence, the example separation above is beyond the scope of the analysis presented here.

I restrict the payoffs by the following assumptions.

Assumption 1. *Parties are risk neutral.*

Therefore, party p's problem is:

$$\max_{x_p \in X^n} \sum_{i \in I} \alpha_i \pi(x_p^i - x_{-p}^i), \text{ subject to } x_p^1 + \ldots + x_p^n \leq 1.$$

The maximised expression is a sum of realised payoffs when either of the topics is hot, weighted by corresponding probabilities.

Assumption 2. *π is of class $\mathcal{C}^2$.*

Assuming continuity for the payoff function and its first two derivatives ensures that results are tractable.

Assumption 3. *π is bounded.*

Bounded payoff function means that parties' perceived gains or losses are limited even in the best or worst case scenario.

Assumption 4. *π is increasing.*

More campaigning on the hot issue translates into higher chances of winning, securing more seats in parliament, higher political rents, or obtaining stronger mandate for implementing intended reforms.

Assuming the continuity of the payoff functions is restrictive considering that the parties' positions on the hot issue are the sole determinant of the election's outcome. The party with a stronger position in this issue would be likely to win by a landslide leading to a discontinuity in the the payoff function. That is not the case here. The reason for this could be that various voters are biased towards one of the parties to a different degree as in the models of probabilistic voting, e.g. see Egorov (2015). Thus, the advantage in the hot issue perceived by the voter needs to be sufficiently high to convince her to switch her political allegiance. In the environment with a continuum of voters whose biases are random, the payoff function becomes smooth.

2.4 RESULTS

In the definitions below, I distinguish two major types of equilibria depending on whether parties select identical or different issues. Let $e_i \in X^n$ be a vector whose i-th element is 1 and all other elements are 0.

Definition 2. *Issue convergence* occurs when both parties select the same strategies $x_a = x_b \in X^n$ in equilibrium. Issue convergence with *specialisation on issue $i \in I$* occurs when parties focus on this issue only: $x_a = x_b = e_i$.

Definition 3. *Issue divergence* occurs when parties campaign on different issues in equilibrium: $x_a = e_i$ and $x_b = e_j$, $i \neq j$ with $i, j \in I$.

2.4.1 *Increasing returns to power*

Recall that increasing returns to power occur when any difference in investment into the hot issue between the parties always translates into a proportionately larger difference in payoffs. This is captured by the payoff functions which are convex in the difference in investments into the hot issue. For this case, Proposition 5 identifies conditions leading parties to choose in equilibrium either the same or different issues for their campaigns. First, each party goes 'all in' by specialising in a single issue. Second, parties select the same, most likely to become hot issue only if the difference in probability between the top and the second issue is sufficiently large. Otherwise, if this difference is relatively small, parties select different issues from among the two most likely ones. This holds for *any* payoff function displaying increasing returns to power and satisfying Assumptions 1–4.

Proposition 5 (Increasing Returns to Power)**.** *If π is strictly convex, satisfies Assumptions 1–4, and*

(i) $\alpha_1 = \alpha_2 = \ldots = \alpha_m$, $2 \leq m \leq n$, *then issue divergence equilibria* (e_i, e_j) *exist for any* $i \neq j$, *where* $i, j \in \{1, \ldots, m\}$;

(ii) $\alpha_1 > \alpha_2 = \alpha_3 = \ldots = \alpha_m$, $2 \leq m \leq n$, *then there exists unique* $\alpha' \in (0, \min\{\frac{1}{2}, \alpha_1\})$, *such that if*

- *a)* $\alpha_2 > \alpha'$, *for any* $j \in \{2, \ldots, m\}$, *issue divergence equilibria* (e_1, e_j) *and* (e_j, e_1) *exist,*
- *b)* $\alpha_2 < \alpha'$, *an issue convergence equilibrium* (e_1, e_1) *with specialisation on issue 1 exists,*
- *c)* $\alpha_2 = \alpha'$, *both (a) and (b) equilibria exist.*

There are no other equilibria in pure strategies.

Proof. Party p's expected payoff Π_p is strictly convex and bounded in x_p^i for all $i \in I$ as a convex combination of strictly convex and bounded functions $\pi(x_p^i - x_{-p}^i)$. Hence, Π_p is continuous in x_p^i, for any $x_{-p} \in X$ and $\alpha \in \Delta$ its maximum $\hat{x}_p(x_{-p}, \alpha)$ exists, and lies on the boundary of X^n. Clearly, $\Pi_p(e_1, e_i) \geq \Pi_p(e_j, e_i)$ for any $i, j \in I \setminus \{1\}$ with strict inequality for $\alpha_1 > \alpha_j$. Similarly, $\Pi_p(e_2, e_1) \geq \Pi_p(e_i, e_1)$ for any $i \in \{3, 4, \ldots, n\}$ with strict inequality for $\alpha_2 > \alpha_i$.

If $x_{-p} = e_1$, party p strictly prefers selecting e_1 to e_j when $\Pi_p(e_1, e_1) > \Pi_p(e_j, e_1)$, equivalently

$$\alpha_1[\pi(0) - \pi(-1)] > \alpha_j[\pi(1) - \pi(0)], \tag{7}$$

which is satisfied for α_j approaching 0, but fails for $\alpha_j = \alpha_1$. Hence, as the right-hand side of (7) is linear in α_j, for any $\alpha_1 \in (0,1)$ there exists unique $\alpha' \in (0, \min\{1/2, \alpha_1\})$, such that if $\alpha_j = \alpha'$, then the lhs is equal to the rhs. Thus, party p's best response to $x_{-p} = e_1$ when $\alpha = (\alpha_1, \ldots, \alpha_n)$ is

$$x_p^*(e_1, \alpha) = \begin{cases} e_1 & \text{for} \quad \alpha_2 \in [0, \alpha'), \\ \{e_1, e_2\} & \text{for} \quad \alpha_2 = \alpha' > \alpha_3, \\ e_2 & \text{for} \quad \alpha_2 > \alpha' \text{ and } \alpha_2 > \alpha_3, \\ \{e_2, e_3\} & \text{for} \quad \alpha_2 = \alpha_3 > \alpha'(\alpha_1) \text{ and } \alpha_3 > \alpha_4, \\ \{e_2, \ldots, e_m\} & \text{for} \quad \alpha_2 = \ldots = \alpha_m > \alpha' \text{ and } \alpha_m > \alpha_{m+1}, \\ \{e_1, e_2, \ldots, e_m\} & \text{for} \quad \alpha_1 = \alpha_2 = \ldots = \alpha_m = \alpha' \text{ and } \alpha_m > \alpha_{m+1}, \end{cases}$$

where $2 \leq m \leq n$. As $x_1^*(\cdot) = x_2^*(\cdot)$, this completes the proof. □

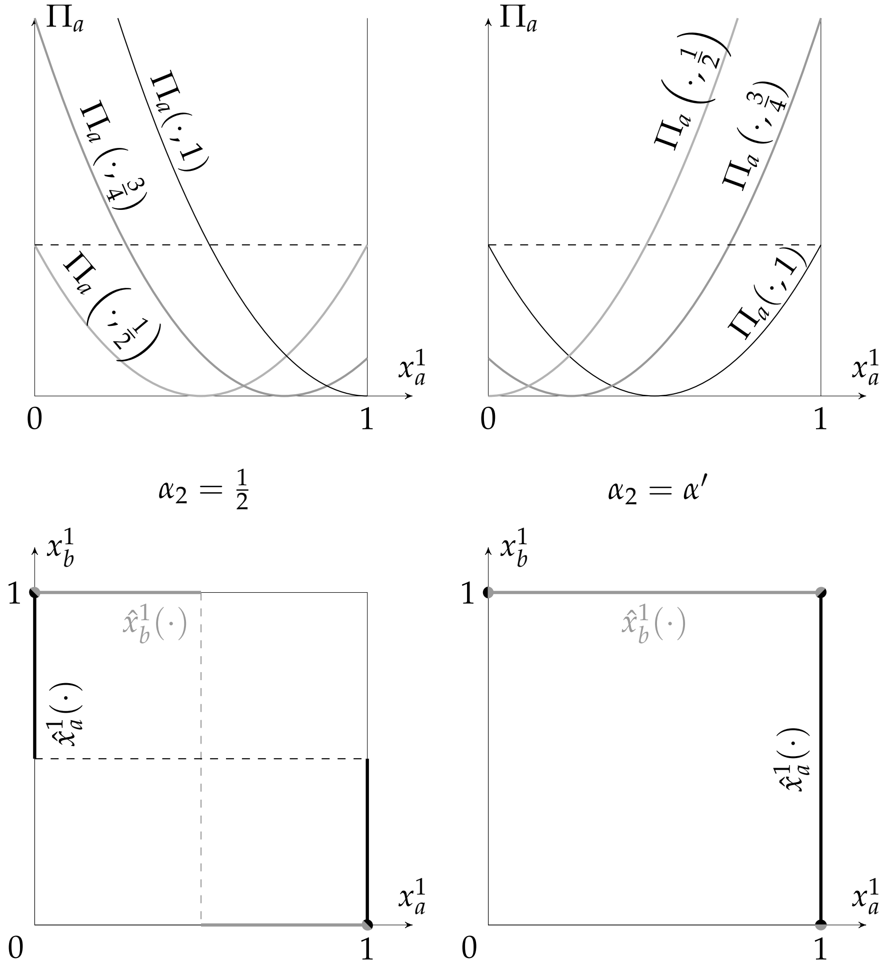
To see the intuition behind this result, consider a case in which there are only two possible campaign issues, thus $\alpha_2 = 1 - \alpha_1$. Since the level of expenditures does not influence the payoff directly, party a uses all its resources in its maximisation problem, $x_a^2 = 1 - x_a^1$. Hence, the problem of party a reduces to a single dimension: selecting the optimal level of investment into issue 1. Thus, the expected payoff function simplifies to $\alpha_1 \pi(x_a^1 - x_b^1) + \alpha_2 \pi(x_b^1 - x_a^1)$. Note that it is strictly convex in x_a^1 as a weighted sum of the final payoffs, which are strictly convex in x_a^1 too. Further, $\pi(x_a^1 - x_b^1)$ and $\pi(x_b^1 - x_a^1)$ are symmetric around x_b^1; while the former increases in x_a^1, the latter decreases.

Assume that both issues become hot with equal probability, as illustrated in the left column of Figure 2. As a result, the expected payoff function has the shape of an inverted hump with a minimum equal to opponent's action, $x_a^1 = x_b^1$, and two local maxima at the boundaries: 0 and 1. That is why the optimal strategy is to go all in by allocating *all* endowment into only one of the issues, while also choosing the strategy furthest away from the one selected by the rival. Thus, two issue divergence equilibria obtain: $(1,0)$ and $(0,1)$. Because both issues become hot equally likely, both parties earn equal expected payoffs in both equilibria.

Consider next that the probability that issue 1 will be hot increases from 1/2 by a sufficiently small amount, such that both $(1,0)$ and $(0,1)$ remain equilibria. Then, the payoff-equivalence breaks down: campaigning on issue 1 in equilibrium yields a higher expected payoff. Increasing further the probability that issue 1 will be hot leads to an even higher payoff difference and, eventually, to the point at which, in equilibrium, one firm specialises in issue 1 and the other is indifferent between specialising in any of the two issues. This occurs when this probability reaches threshold value $\alpha_2 = \alpha'$ (and hence $\alpha_1 = 1 - \alpha'$) described in Proposition 5, which demonstrates that such a threshold exists for any payoff function satisfying the specified assumptions. This case is illustrated in the right column of Figure 2, when party b invests all its endowment into issue 1, party a is indifferent between allocating all its endowment into issue 1 and issue 2. Issue convergence equilibrium $(1,1)$ emerges along the issue divergence equilibria $(1,0)$ and $(0,1)$.

Finally, as issue 1 becomes even more likely to become hot, $\alpha_2 < \alpha'$, the issue divergence equilibria vanish: campaigning on issue 2 yields a too low expected payoff. An analogous argument applies in the opposite direction—when the probability of issue 2 becoming hot increases—leading to the result from Proposition 5.

Figure 2.: Payoffs and best responses of party a to different strategies of party b under increasing returns to power with two possible campaign issues.



Note: Overlaps of best response correspondenes are marked with dots.

2.4.2 *Decreasing returns to power*

How do the incentives for selecting campaign issues change with the circumstances? In this subsection, I consider concave payoff functions. In this case, any difference in investment into the hot issue between the parties always translates into a proportionately smaller difference in payoffs. Proposition 6 identifies conditions leading parties to choose in equilibrium either the same or different issues for their campaigns. First, if there exists a single issue which is the most likely to become hot, then both parties coordinate their campaigns by investing all their endowments into this issue: only the issue convergence equilibrium with specialisation exists. Otherwise, if there are multiple issues which are most likely to become hot with the same probability, parties allocate all their resources between these issues and the campaign resembles a coordination game, i.e. parties mimic each others' strategies. This holds for *any* payoff function displaying decreasing returns to power and satisfying Assumptions 1–4.

Proposition 6 (Decreasing Returns to Power)**.** *For any strictly concave π satisfying Assumptions 1–4,*

(i) if $\alpha_1 = \alpha_2 = \ldots = \alpha_m > \alpha_{m+1}$, $2 \leq m \leq n$, then a continuum of issue convergence equilibria exists: (x, x), where $x_1 + \ldots + x_m = 1$, $x_i \in X$ for $i \leq m$, and $x^i = 0$ for $i > m$, $i \in I$;

(ii) if $\alpha_1 > \alpha_2$, then only an issue convergence equilibrium (e_1, e_1) with specialisation on issue 1 exists.

There are no other equilibria in pure strategies.

Proof. Party p's expected payoff Π_p is strictly concave and bounded in x_p^i for all $i \in I$ as a convex combination of strictly concave and bounded functions $\pi(x_p^i - x_{-p}^i)$. Hence, Π_p is continuous in x_p^i and its maximum $\hat{x}_p(x_{-p}, \alpha)$ exists and is unique for any $x_{-p} \in X^n$ and $\alpha \in \Delta$. In equilibrium $x_p = \hat{x}_p(\hat{x}_{-p}, \alpha)$ and $x_{-p} = \hat{x}_{-p}(\hat{x}_p, \alpha)$.

The Lagrange function for party p's problem can be written as

$$\sum_{i \in I} \Big(\alpha_i \pi(x_p^i - x_{-p}^i) - \lambda_i(-x_p^i) - \lambda_i'(x_p^i - 1) \Big) - \lambda_{n+1} \Big(\sum_{i \in I} x_p^i - 1 \Big).$$

Allocation $\hat{x}_p \in X$ is a constrained maximiser of party p's problem iff:

$$\alpha_i \pi_i'(\hat{x}_p^i - x_{-p}^i) + \lambda_i - \lambda_i' - \lambda_{n+1} = 0, \quad (8)$$
$$\hat{x}_p^i \geq 0,$$
$$\lambda_i \hat{x}_p^i = 0, \; \lambda_i \geq 0,$$
$$\hat{x}_p^i \leq 1,$$
$$\lambda_i'(1 - \hat{x}_p^i) = 0, \; \lambda_i' \geq 0,$$
$$\text{for } i \in I, \text{ and}$$
$$\sum_{i \in I} \hat{x}_p^i \leq 1, \quad (9)$$
$$\lambda_{n+1}\left(\sum_{i \in I} \hat{x}_p^i - 1\right) = 0, \; \lambda_{n+1} \geq 0.$$

Consider $i < j$ with $i, j \in I$. The remainder of the proof proceeds in the steps below.

Step 1. Condition (9) holds with equality, i.e. $\sum_{i \in I} \hat{x}_p^i = 1$; otherwise there exists $\varepsilon = 1 - \sum_{i \in I} \hat{x}_p^i > 0$ such that $\Pi_p(\hat{x}_p^1 + \varepsilon, \hat{x}_p^2, \ldots, \hat{x}_p^n, x_{-p}) > \Pi_p(\hat{x}_p, x_{-p})$.

Step 2. If $\hat{x}_p^i \in (0,1)$ and $\hat{x}_p^j \in (0,1)$, then $\lambda_i = \lambda_j = \lambda_i' = \lambda_j' = 0$.

If $\alpha_i = \alpha_j$, then (8) implies $\pi'(\hat{x}_p^i - x_{-p}^i) = \pi'(\hat{x}_p^j - x_{-p}^j)$, and since π' is one-to-one, $\hat{x}_p^i - x_{-p}^i = \hat{x}_p^j - x_{-p}^j$.

Otherwise, if $\alpha_i > \alpha_j$, then $\alpha_i \pi'(\hat{x}_p^i - x_{-p}^i) = \alpha_j \pi'(\hat{x}_p^j - x_{-p}^j)$. Hence $\pi'(\hat{x}_p^i - x_{-p}^i) < \pi'(\hat{x}_p^j - x_{-p}^j)$, implying $\hat{x}_p^i - x_{-p}^i > \hat{x}_p^j - x_{-p}^j$, as π' is strictly decreasing. Party $-p$'s symmetric problem yields a contradiction: $\hat{x}_{-p}^i - x_p^i > \hat{x}_{-p}^j - x_p^j$.

Step 3. If $\hat{x}_p^i = 0$ and $\hat{x}_p^j = 1$, then $\lambda_i' = \lambda_j = 0$. Consequently, (8) implies $\alpha_i \pi'(-x_{-p}^i) + \lambda_i = \alpha_j \pi'(1 - x_{-p}^j) - \lambda_j'$. As λ_i and λ_j' are non-negative,

$$\alpha_i \pi'(-x_{-p}^i) \leq \alpha_j \pi'(1 - x_{-p}^j). \quad (10)$$

If $\alpha_i = \alpha_j$, condition (10) simplifies to $\pi'(-x^i_{-p}) \leq \pi'(1 - x^j_{-p})$, or equivalently $-x^i_{-p} \geq 1 - x^j_{-p}$, which can hold iff $x^i_{-p} = 0$, $x^j_{-p} = 1$, and $\lambda_i = \lambda'_j = 0$.

If $\alpha_i > \alpha_j$, condition (10) implies $\pi'(-x^i_{-p}) < \pi'(1 - x^j_{-p})$, equivalent to $-x^i_{-p} > 1 - x^j_{-p}$, which is a contradiction.

Step 4. If $\hat{x}^i_p = 1$ and $\hat{x}^j_p = 0$, then $\lambda_i = \lambda'_j = 0$, and consequently (8) implies $\alpha_i \pi'(1 - x^i_{-p}) - \lambda'_i = \alpha_j \pi'(-x^j_{-p}) + \lambda_j$. As λ'_i and λ_j are non-negative,

$$\alpha_i \pi'(1 - x^i_{-p}) \geq \alpha_j \pi'(-x^j_{-p}). \tag{11}$$

If $\alpha_i = \alpha_j$, condition (11) simplifies to $\pi'(1 - x^i_{-p}) \geq \pi'(-x^j_{-p})$, or equivalently $1 - x^i_{-p} \leq -x^j_{-p}$, which can hold iff $x^i_{-p} = 1$, $x^j_{-p} = 0$, and $\lambda'_i = \lambda_j = 0$.

If $\alpha_i > \alpha_j$, condition (11) is non-binding.

Step 5. If $\hat{x}^i_p = 0$ and $\hat{x}^j_p \in (0, 1)$, then $\lambda'_i = \lambda_j = \lambda'_j = 0$. Consequently (8) implies $\alpha_i \pi'(-x^i_{-p}) + \lambda_i = \alpha_j \pi'(\hat{x}^j_p - x^j_{-p})$. As λ_i is non-negative,

$$\alpha_i \pi'(-x^i_{-p}) \leq \alpha_j \pi'(\hat{x}^j_p - x^j_{-p}). \tag{12}$$

If $\alpha_i = \alpha_j$, condition (12) simplifies to $\pi'(-x^i_{-p}) \leq \pi'(\hat{x}^j_p - x^j_{-p})$, or equivalently $x^j_{-p} - x^i_{-p} \geq \hat{x}^j_p$.

If $\alpha_i > \alpha_j$, condition (12) implies $\pi'(-x^i_{-p}) < \pi'(\hat{x}^j_p - x^j_{-p})$, equivalent to $x^j_{-p} - x^i_{-p} > \hat{x}^j_p$, which can be only satisfied if $x^j_{-p} > 0$. In equilibrium, $\hat{x}^i_{-p} \in (0, 1)$ is contradicted by Step 2 and $\hat{x}^i_{-p} = 0$ by a symmetric requirement: $\hat{x}^j_{-p} < x^j_p$.

Step 6. If $\hat{x}^i_p \in (0,1)$ and $\hat{x}^j_p = 0$, then $\lambda_i = \lambda'_i = \lambda'_j = 0$. Consequently (8) implies $\alpha_i \pi'(\hat{x}^i_p - x^i_{-p}) = \alpha_j \pi'(-x^j_{-p}) + \lambda_j$. As λ_j is non-negative,

$$\alpha_i \pi'(\hat{x}^i_p - x^i_{-p}) \geq \alpha_j \pi'(-x^j_{-p}). \tag{13}$$

If $\alpha_i = \alpha_j$, condition (13) simplifies to $\pi'(-x^i_{-p}) \leq \pi'(\hat{x}^j_p - x^j_{-p})$, or equivalently $x^j_{-p} - x^i_{-p} \geq \hat{x}^j_p$. If $\alpha_i > \alpha_j$, condition (13) implies $\pi'(-x^i_{-p}) < \pi'(\hat{x}^j_p - x^j_{-p})$, equivalent to $x^j_{-p} - x^i_{-p} > \hat{x}^j_p$.

Step 7. If $\hat{x}^i_p = \hat{x}^j_p = 0$, then $\lambda'_i = \lambda'_j = 0$. Consequently (8) implies $\alpha_i \pi'(\hat{x}^i_p - x^i_{-p}) + \lambda_i = \alpha_j \pi'(-x^j_{-p}) + \lambda_j$, thus the constraint is non-binding.

Step 8. Consider $j \in I$, such that $\alpha_j < \alpha_1$. Then, Steps 2, 3, and 5 imply that in equilibrium $x^j_a = x^j_b = 0$.

Step 9. Let $D = \{i \in I : \alpha_i = \alpha_1\}$. Steps 1 and 7 imply $\sum_{i \in D} \hat{x}^i_p = 1$. Let $D_+ = \{i \in D : \hat{x}^i_p > 0\}$ and $n' = |D_+|$. Then, Step 2 implies:

$$\begin{aligned} \hat{x}^1_p - \hat{x}^2_p &= x^1_{-p} - x^2_{-p}, \\ \hat{x}^2_p - \hat{x}^3_p &= x^2_{-p} - x^3_{-p}, \\ &\vdots \\ x^{n'-1}_p - x^{n'}_p &= x^{n'-1}_{-p} - x^{n'}_{-p}. \end{aligned} \tag{14}$$

Since $\hat{x}^{n'}_p = 1 - \sum_{i=1}^{n'-1} \hat{x}^i_p$, the last line in (14) is equivalent to $\sum_{i=1}^{n'-2} x^i_p + 2x^{n'-1}_p = \sum_{i=1}^{n'-2} x^i_{-p} + 2x^{n'-1}_{-p}$. In a matrix notation, (14) can be written as:

$$\begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1 \\ 1 & 1 & 1 & \dots & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}^1_p \\ \hat{x}^2_p \\ \vdots \\ \hat{x}^{n'-1}_p \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1 \\ 1 & 1 & 1 & \dots & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x^1_{-p} \\ x^2_{-p} \\ \vdots \\ x^{n'-1}_{-p} \end{bmatrix}. \tag{15}$$

The square matrices in (15) on the lhs and the rhs are equal with linearly independent rows, hence the unique solution to (15) is $\hat{x}^i_p = x^i_{-p}$ for all $i \in D_+$. □

In order to understand what drives this result, consider again the case of two potential campaign issues. As before, $\alpha_2 = 1 - \alpha_1$ and $x^2_a = 1 - x^1_a$; hence the expected payoff function can be written as $\alpha_1 \pi(x^1_a - x^1_b) + \alpha_2 \pi(x^1_b - x^1_a)$. It is strictly concave as a convex combination of strictly concave final payoff functions. Recall that $\pi(x^1_a - x^1_b)$ and $\pi(x^1_b - x^1_a)$ are symmetric around x^1_b; while the former increases in x^1_a, the latter decreases.

Assume that both issues become hot with equal probability, as illustrated in the left column of Figure 3. Then, the expected payoff function has the shape of a hump with a maximum at $x^1_a = x^1_b$. Hence, mimicking the rival is the optimal strategy for both parties, which gives rise to a continuum of issue convergence equilibria.

Next, consider that the probability that issue 1 becomes hot is greater than 1/2. Observe that for all $x^1_a, x^1_b \in X$

$$\frac{\partial^2 \Pi_a}{\partial (x^1_a)^2} = \alpha_1 \pi''(x^1_a - x^1_b) + \alpha_2 \pi''(x^1_b - x^1_a) < 0$$

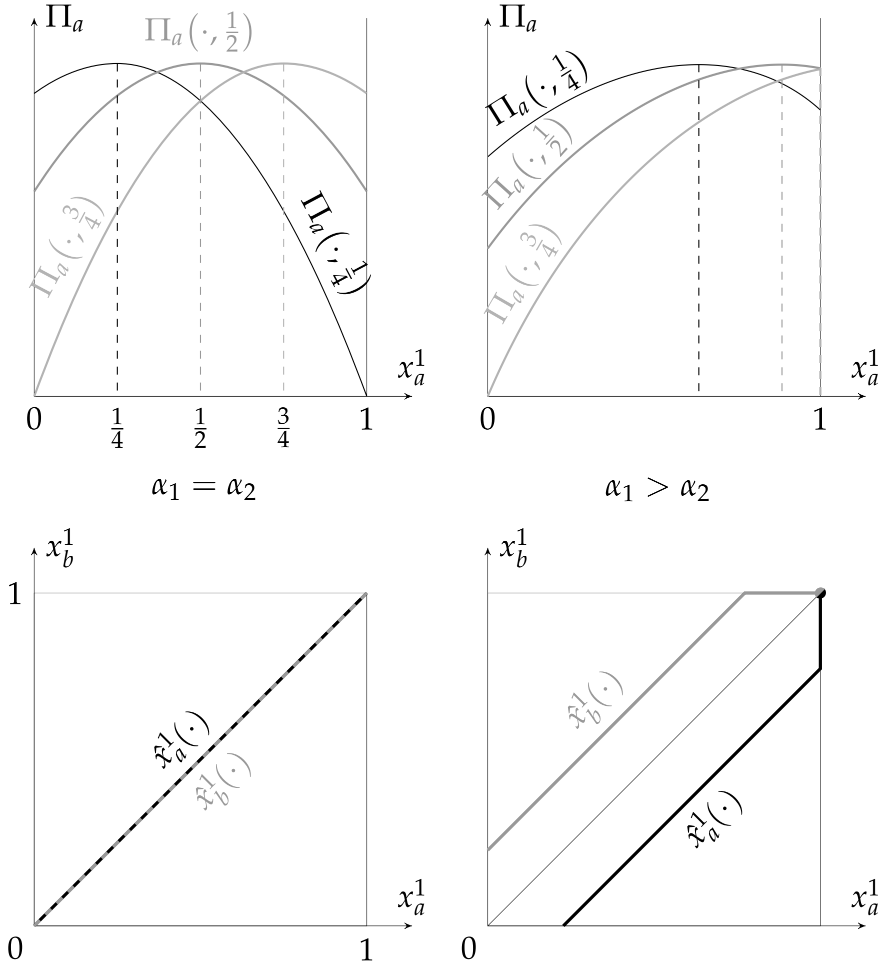
and

$$\frac{\partial^2 \Pi_a}{\partial x^1_a \partial \alpha_1} = \pi'(x^1_a - x^1_b) + \pi'(x^1_b - x^1_a) < 0.$$

Hence, $\frac{\partial \hat{x}^1_a}{\partial \alpha_1} > 0$ for all $x^1_b \in X$, implying that party a's best response function, $\hat{x}^1_a$, is greater than x^1_b for any $\alpha_1 > 0.5$. Thus, parties have an incentive to 'overbid' the rival in the issue which is more likely to become hot. This gives rise to an issue convergence equilibrium with specialisation in this issue.

While the intuition for Proposition 6 presented above is quite formal, it is difficult to extend it to the case of n issues, as it would require investigating properties of an n by n matrix. Thus, the proof presented here is substantially longer and more complex than in the case of the increasing returns to power.

Figure 3.: Payoffs and best responses of party a to different strategies of party b under decreasing returns to power with two possible campaign issues.



Note: Equilibria are marked with dots, in which the best response correspondenes of both parties overlap.

2.4.3 *Linear returns to power*

In order to provide a point of reference to the previous results, I also consider constant returns to power, when the payoff functions are linear and there are two potential campaign issues, $n = 2$.

Definition 4. The payoff function is *linear* if $\pi(z) = \beta z$ for some $\beta > 0$.

Proposition 7 (constant returns to power)**.** *For any linear π satisfying Assumption 1:*

(i) if the issues are equally likely to be decisive, any pair of strategies $(x_1, x_2) \in X^2$ is an equilibrium;

(ii) otherwise, there exists a unique issue convergence equilibrium with specialisation in the issue that is more likely to be decisive,

Proof. Party p's expected payoff is $\beta(2\alpha - 1)(x_p - x_{-p})$, which is strictly monotonic in x_p: increasing for $\alpha > 1/2$, decreasing for $\alpha < 1/2$, constant for $\alpha = 1/2$. □

2.5 CONCLUSIONS

In this chapter, I have identified a link between the curvature of the payoff functions and the incentives for issue selection in the political campaigns, in which the public perceptions are swayed by random events. As the payoffs have been considered in a reduced form, a number of potential factors may determine their shape. Thus, the results presented here provide a broad framework for evaluating the influence of, e.g., reforms of the election system or the executive power, on the campaign rhetoric. Moreover, the issue convergence and divergence results have been obtained without assuming that voters have ideological positions.

A

APPENDIX

Proof of Proposition 2. Two auxiliary lemmas are helpful for the proof.

Lemma 1. For any $\alpha_e > \alpha_j^j$, j's optimal period 1 output is greater than $j+1$'s, $q_j^*(\alpha_e) > q_{j+1}^*(\alpha_e)$.

Proof. Assume otherwise: there exists $\alpha' > \alpha_j^j$ for which $q_j^*(\alpha') \leq q_{j+1}^*(\alpha')$. Since both q_j^* and q_{j+1}^* are continuous, it implies that there also exists $\alpha'' > \alpha_j^j$ for which $q_j^*(\alpha'') = q_{j+1}^*(\alpha'')$.

Recall from equation (5) that $q_{j+1}^*(\alpha'') = q_{j+1,n}^*(\alpha'')$ for $\alpha'' \geq \alpha_{j+1}^n$. Note that equation 4 implies $q_{j+1,n}^*(\alpha) < q_{j,n}^*(\alpha)$ for any $\alpha \in \mathbb{R}$ as these functions are parallel and have the same slope equal to 1/2. Since $q_j^*(\alpha) = q_{j,n}^*(\alpha)$ for $\alpha \geq \alpha_j^n$, functions q_j^* and q_{j+1}^* cannot intersect in the interval in which both take the values of their respective ultimate sub-functions, i.e. for $\alpha'' \geq \alpha_j^n > \alpha_{j+1}^n$.

Further, observe that equation (2) implies $\hat{q}_{j+1}^n < \hat{q}_j^n$ for any given $\alpha \in \mathbb{R}$ as these functions are parallel and have the same slope equal to 1. Recall that these penultimate sub-functions of q_{j+1}^* and q_j^* intersect with the ultimate sub-functions $q_{j+1,n}^*$ and $q_{j,n}^*$, respectively. Thus, functions q_j^* and q_{j+1}^* cannot intersect in the interval in which both take the values of their respective penultimate or ultimate sub-functions, i.e. for $\alpha'' > \max\{\hat{\alpha}_j^n, \hat{\alpha}_{j+1}^n\}$.

Generally, equation (4) implies $q_{j+1,k}^*(\alpha) < q_{j,k}^*(\alpha)$ and equation (2) implies $\hat{q}_{j+1}^k < \hat{q}_j^k$ for any $k \in \{j+2, \ldots, n\}$ and any $\alpha \in \mathbb{R}$. Consider now the antepenultimate sub-functions of q_{j+1}^* and q_j^* which are $q_{j+1,n-1}^*$ and $q_{j,n-1}^*$, respectively. Since $q_{j+1,n-1}^*(\alpha) < q_{j,n-1}^*(\alpha)$ for any $\alpha \in \mathbb{R}$ and both functions intersect with their penultimate counterparts, functions q_j^* and q_{j+1}^* cannot intersect in the interval in which both take the values of their respective antepenultimate, penultimate, or ultimate sub-functions, i.e. for $\alpha'' > \max\{\alpha_j^{n-1}, \alpha_{j+1}^{n-1}\}$. This procedure can be repeated backwards with consecutive sub-functions of q_j^* and q_{j+1}^*, until $\hat{q}_{j+1}^{j+2}$ and $\hat{q}_j^{j+2}$. Hence, functions q_j^* and q_{j+1}^* cannot intersect at α''

for which q^*_{j+1} takes the values of $\hat{q}^{j+2}_{j+1}$, $q^*_{j+1,j+2}$, $\dots$, $\hat{q}^n_{j+1}$, $q^*_{j+1,n}$ and q^*_j takes the values of $\hat{q}^{j+2}_j$, $q^*_{j,j+2}$, $\dots$, $\hat{q}^n_j$, $q^*_{j,n}$, i.e. for $\alpha'' > \max\{\alpha^{j+2}_j, \alpha^{j+2}_{j+1}\}$. In particular, note that $\hat{q}^{j+2}_{j+1}$ in its domain $[\hat{\alpha}^{j+2}_{j+1}, \alpha^{j+2}_{j+1})$ cannot intersect with q^*_j as in this case it would imply that $q_{j+1} \geq q_j$, which is a contradiction. Finally, q^*_{j+1} is equal to its first sub-function $q^*_{j+1,j}$ with the slope equal to 1/2 for $\alpha \in [\alpha^{j+2}_{j+1}, \hat{\alpha}_{j+1,j+2})$. Since $\alpha^j_j < \alpha^{j+1}_{j+1}$ and consequently $q^*_j\left(\alpha^{j+1}_{j+1}\right) > 0 = q^*_{j+1}\left(\alpha^{j+1}_{j+1}\right)$, the supposed α'' cannot belong to this interval, $\alpha'' \notin [\alpha^{j+2}_{j+1}, \hat{\alpha}_{j+1,j+2})$. Hence, $\alpha \in \emptyset$.

□

Lemma 2. Condition $q^*_j(\alpha_e) + c_j \geq q^*_{j+1}(\alpha_e) + c_{j+1}$ holds for any $\alpha_e > \alpha^{j+1}_{j+1}$. In addition, equality $q^*_j(\alpha_e) + c_j = q^*_{j+1}(\alpha_e) + c_{j+1}$ holds only for $\alpha_e \in \left[\hat{\alpha}^k_j, \alpha^k_{j+1}\right]$, where $k = j+2, \dots, n$.

Proof. First observe that for every $k \in \{j+2, \dots, n\}$ inequalities $q^*_{j,j+1} + c_j > q^*_{j+1,j} + c_{j+1}$ and $q^*_{j,k} + c_j > q^*_{j+1,k} + c_{j+1}$ are implied by equation (4) and equalities $\hat{q}^k_j + c_j = \hat{q}^k_{j+1} + c_{j+1}$ are implied by equation (2). Thus every consecutive sub-function of q^*_{j+1} has a counterpart sub-function of q^*_j which is either greater or equal to it for any given α (similarly to the proof of Lemma 1). Since both q^*_j and q^*_{j+1} are continuous, q^*_{j+1} cannot exceed q^*_j and hence $q^*_j + c_j \geq q^*_{j+1} + c_{j+1}$ for $\alpha_e > \alpha^{j+1}_{j+1}$. The equality between the two holds only where $\hat{q}^k_j$ and $\hat{q}^k_{j+1}$ overlap which completes the second part of the proof. □

Now, I proceed to proving Proposition 2. Follower i produces a positive output in period 2 CNE if the realised demand intercept is sufficiently high and condition (2) can be rewritten:

$$\alpha_r > P^i_j = (i-1)c_i - \sum_{k=1,k\neq j}^{i-1} c_k + q^1_j,$$

which is equivalent to $r_i\left(q^1_j + R^{i-1}_{-j}, \alpha_r\right) > 0$, i.e. that the i's best response to the aggregate output in the CNE with $i-1$ firms producing positive outputs is positive. The aggregate followers' output in the period 2 CNE from expression (3) can thus be rewritten as a piecewise function of the demand intercept α_r:

$$Q^S_{-j}\left(q^1_j,\alpha_r\right) = \begin{cases} 0 & \text{if} \quad P^1_j \geq \alpha_r, \\ Q^S_{-j,1}\left(q^1_j,\alpha_r\right) & \text{if} \quad P^2_j \geq \alpha_r > P^1_j, \\ Q^S_{-j,2}\left(q^1_j,\alpha_r\right) & \text{if} \quad P^3_j \geq \alpha_r > P^2_j, \\ \vdots & \\ Q^S_{-j,j-1}\left(q^1_j,\alpha_r\right) & \text{if} \quad P^{j-1}_j \geq \alpha_r > P^{j-2}_j, \\ Q^S_{-j,j+1}\left(q^1_j,\alpha_r\right) & \text{if} \quad P^{j+1}_j \geq \alpha_r > P^{j-1}_j, \\ \vdots & \\ Q^S_{-j,n}\left(q^1_j,\alpha_r\right) & \text{if} \quad \alpha_r > P^n_j. \end{cases}$$

Since the leader's profit is continuous and strictly concave, leader j chooses optimally not to add any output in period 2 CNE ($q^2_j = 0$), if the right-hand derivative of its profit function w.r.t. to q_j is non-positive at $q^*_j(\alpha_e)$:

$$\alpha_r - 2q^*_j(\alpha_e) - Q^S_{-j}\left(q^*_j(\alpha_e),\alpha_r\right) - c_j \leq 0. \tag{16}$$

Note that the followers' aggregate period 2 quantity Q^S_{-j} is a piecewise function of α_r. Similarly, it is also a piecewise function of α_e, a property inherited from q^*_j. If α_r is such that k firms produce positive outputs in period 2 CNE, restriction (16) simplifies to $\alpha_r \leq \tau^k_j(\alpha_e)$, where

$$\tau^k_j(\alpha_e) = (k+1)(q^*_j(\alpha_e) + c_j) - \sum_{i=1}^{k} c_i.$$

Thus, any $\alpha_e \geq \alpha^j_j$ (so that j's initial output is positive) can be mapped to maximal α_r such that j does not have the incentive to add any output once α_r is revealed. Denote this function as $\tau_j(\alpha_e)$.

1. Note that if not all of the firms which are more cost-efficient than the leader produce positive outputs in the CNE, $K < j$, restriction $\alpha_r \leq \tau^K_j$

is slack, as $\tau_j^K \geq P_j^{K+1,r}$ reduces to $c_{K+1} \leq q_j^1 + c_j$, which is true for any $q_j^1 \geq 0$ only when $K < j$.

2. Thus if $c_j < c_{j+1}$ (and $c_j < c_{j+p}, p = 1, \dots, n-j$), there exists $\alpha_e' > \alpha_j^j$ such that $\tau_j^j(\alpha_e') < P_j^{j+1,r}$ (and $\tau_j^{j+p-1}(\alpha_e') < P_j^{j+p,r}$), i.e. $\tau_j^j(\alpha_e')$ (and $\tau_j^{j+p-1}(\alpha_e')$) is binding: at the highest realised demand where the leader does not add to its initial output only $1, \dots, j-1$ (and $1, \dots, j+p-1$) followers produce positive outputs in the second period CNE.

3. Threshold τ_j^{j+1} (and τ_j^{j+p}) is binding when $P_j^{j+1,r} \leq \tau_j^{j+1} \leq P_j^{j+2,r}$ (and $P_j^{j+p,r} \leq \tau_j^{j+p} \leq P_j^{j+p+1,r}$). These requirements simplify to $c_{j+1} \leq q_j + c_j \leq c_{j+2}$ ($c_{j+p} \leq q_j + c_j \leq c_{j+p+1}$).

4. The reasoning from the steps 1–3 can be repeated for firms $j+2, \dots, n$ (and $j+p+2, \dots, n$) to get

$$\tau_j(\alpha_e) = \begin{cases} (j+1)(q_j^*(\alpha_e) + c_j) - \sum_{k=1}^{j} c_k & \text{if } c_j \leq q_j^*(\alpha_e) + c_j < c_{j+1}, \\ (j+2)(q_j^*(\alpha_e) + c_j) - \sum_{k=1}^{j+1} c_k & \text{if } c_{j+1} \leq q_j^*(\alpha_e) + c_j < c_{j+2}, \\ (j+3)(q_j^*(\alpha_e) + c_j) - \sum_{k=1}^{j+2} c_k & \text{if } c_{j+2} \leq q_j^*(\alpha_e) + c_j < c_{j+3}, \\ \vdots & \\ (n+1)(q_j^*(\alpha_e) + c_j) - \sum_{k=1}^{n} c_k & \text{if } c_n \leq q_j^*(\alpha_e) + c_j, \\ 0 & \text{otherwise.} \end{cases}$$

Now, I proceed to show that for any given firms j and k, where $c_j < c_k$, I have $\tau_j(\alpha_e) \geq \tau_k(\alpha_e)$ for any $\alpha_e \geq 0$ and there exists α_e' such that $\tau_j(\alpha_e') > \tau_k(\alpha_e')$. I prove this claim for j and $k = j+1$, as by transitivity of set inclusion, it holds true for any $m > j$.

By lemma 2, I have $q_j^* + c_j \geq q_{j+1}^* + c_{j+1}$ for any $\alpha_e > \underline{\alpha}_{j+1}$. Let $\Theta_j : [c_j, \infty) \to \mathbb{R}_+$ be a function such that $\tau_j(\alpha_e) = \Theta_j\left(q_j^*(\alpha_e) + c_j\right)$. It can be written as

$$\Theta_j(x) = \begin{cases} 0 & \text{if } x \quad = c_j, \\ (j+1)x - \sum_{k=1}^{j} c_k & \text{if } c_j \quad < x \leq c_{j+1}, \\ (j+2)x - \sum_{k=1}^{j+1} c_k & \text{if } c_{j+1} \quad < x \leq c_{j+2}, \\ \vdots & \\ (n+1)x - \sum_{k=1}^{n} c_k & \text{if } c_n \quad < x. \end{cases}$$

It is straightforward to see that Θ_j strictly increases with its argument. Moreover, $\Theta_j = \Theta_{j+1}$ for $x \notin (c_j, c_{j+1}]$ and $\Theta_j > \Theta_{j+1}$ for $x \in (c_j, c_{j+1}]$. Thus, for any $\alpha_e > \alpha_j^j$ I have $\tau_j(\alpha_e) = \Theta_j\left(q_j^*(\alpha_e) + c_j\right) \geq \Theta_j\left(q_{j+1}^*(\alpha_e) + c_{j+1}\right) \geq \Theta_{j+1}\left(q_{j+1}^*(\alpha_e) + c_{j+1}\right) = \tau_{j+1}(\alpha_e)$. □

Proof of Proposition (3). Denote the density function of the firms' belief about the demand intercept by f. Denote the profit expected by firm $j \in \{1, 2\}$ in period 1 conditional on that belief by $\Pi_j|f : [0, \infty) \to \mathbb{R}$ and let $\mathfrak{q}_j|f = \{q_j : q_j = \arg\max \Pi_j|f\}$.

Recall that (A) implies that j expects $r_{-j}^2\left(q_j, \alpha_r\right) > 0$ for any $\alpha_r \in (\alpha^-, \alpha^+]$ and any $q_j \in [q_j^-, q_j^+)$, where $q_j^- = r_j^1\left(\alpha^-\right)$ and $q_j^+ = r_j^1\left(\alpha^+\right)$. Hence, $\mathfrak{q}_j|f$ is bounded for any f satisfying A and $q_j^- < q_j^* < q_j^+$ for any $q_j^* \in \mathfrak{q}_j|f$. Thus in period 1, leader j expects either (*i*) Stackelberg outcome where $q_j^2 = 0$ for $\alpha \leq \tau_j(q_j^*)$ or (*ii*) Cournot outcome where $q_j^2 > 0$ for $\alpha > \tau_j(q_j^*)$. Let $\hat{\alpha}_j(q_j) = \min\{\tau_j(q_j), \alpha^+\}$. The expected profit of leader j then equals

$$\Pi_j(q)|f = \int_{\alpha^-}^{\hat{\alpha}_j(q)} \frac{1}{2} q(\alpha - 2c_j + c_{-j} - q) f(\alpha) d\alpha + \int_{\hat{\alpha}_j(q)}^{\alpha^+} \frac{1}{9} (\alpha - 2c_j + c_{-j})^2 f(\alpha) d\alpha$$

for $q \geq 0$. The integrals on the left and the right in $\Pi_j(q)|f$ represent j's expected profit for the outcomes of type (*i*) and type (*ii*) respectively. Differentiating this w.r.t. q by applying Leibniz's rule yields:

$$\Pi_j'(q)|f = \frac{1}{2} E\left(\alpha | \alpha < \hat{\alpha}_j(q)\right) + \frac{1}{2}\left(c_{-j} - 2c_j - 2q\right) F\left(\hat{\alpha}_j(q)\right).$$

Note that Π is differentiable. In particular, it is differentiable for q'' such that $\tau_j(q'') = \alpha^+$. Due to Assumption A, equality $\tau_j(q) = 3q + 2c_j - c_{-j}$ holds for $q \geq \max\{0, c_{-j} - c_j\}$ and $j \in \{1,2\}$. Observe that equality $\tau_1(q + c_2 - c_1) = \tau_2(q)$ holds for $q \in (0, \infty)$ and hence, by the strict monotonicity of τ_j, there exists unique $\tilde{q}_2$ such that $\tau_1(\tilde{q}_2 + c_2 - c_1) = \tau_2(\tilde{q}_2) = \alpha^+$ and it is $\tilde{q}_2 = \frac{1}{3}(\alpha^+ - 2c_2 + c_1)$. Thus, we obtain

$$\Pi_1'(q + c_2 - c_1)|f - \Pi_2'(q)|f = \frac{1}{2}(c_2 - c_1)F\left(\tau_2(q)\right) > 0 \text{ for } q > q_2^- > 0. \quad (17)$$

Note that due to the continuity and differentiability of f we have $\Pi_j'(q_j^-)|f > 0$ and $\Pi_j'(q_j^+)|f < 0$; hence, $\mathfrak{q}_j|f$ is nonempty for $j \in \{1,2\}$. Moreover, due to (17) and because $\Pi_2(\min \mathfrak{q}_2)|f = \Pi_2(\max \mathfrak{q}_2)|f$ we get $\Pi_1(\min \mathfrak{q}_2 + c_2 - c_1)|f < \Pi_1(\max \mathfrak{q}_2 + c_2 - c_1)|f$ and $\Pi_1'(\max \mathfrak{q}_2 + c_2 - c_1)|f > 0$. Thus, $\max\{\mathfrak{q}_2|f\} + c_2 - c_1 < \min\{\mathfrak{q}_1|f\}$ holds and in consequence $\tau_2(q_2^*) < \tau_1(q_1^*)$ for any $q_j^* \in \mathfrak{q}_j|f$, $j \in \{1,2\}$ and any f satisfying A.

Note that restriction $\tau_2(\max \mathfrak{q}_2|f) < \alpha^+$ is equivalent to $\max \mathfrak{q}_2|f < \tilde{q}_2$. The sufficient condition for this is $\Pi_2'(\tilde{q}) < 0$ and this in turn is equivalent to $E\alpha < \frac{1}{3}(2\alpha^+ - c_1 + 2c_2)$. □

Proof of Proposition 4. The marginal expected profit of firm j is given by:

$$\int_{\alpha^-}^{\hat{\alpha}} \frac{1}{2}(\alpha - 2c_j + c_{-j} - 2q_j^1)f(\alpha)d\alpha, \quad (18)$$

where $\hat{\alpha} = \min\{\alpha^+, 2c_j - c_{-j} + 3q_j^1\}$. In contrast, the marginal expected profit of the Stackelberg leader faced with demand uncertainty is given by:

$$\int_{\alpha^-}^{\alpha^+} \frac{1}{2}(\alpha - 2c_j + c_{-j} - 2q_j^1)f(\alpha)d\alpha. \quad (19)$$

In both cases, the expected profit function is continuous in q_j^1 and the optimal q_j^1 must satisfy $q_j^1 > 0$. Thus, for both functions to be maximised at the same q_j^1, both expression (18) and expression (19) need to be equal to 0. However, $\alpha - 2c_j + c_{-j} - 2q_j^1 > 0$ for $\alpha > 2c_j - c_{-j} + 3q_j^1$. Hence, expression (18) is smaller than expression (19) for $\hat{\alpha} < \alpha^+$ which can be rearranged to $q_j^1 <$

$(\alpha^+ - 2c_j + c_{-j})/3$. Thus, the optimal q_j^1 has to satisfy $q_j^1 \geq (\alpha^+ - 2c_j + c_{-j})/3$ and:

$$\int_{\alpha^-}^{\alpha^+} \frac{1}{2}(\alpha - 2c_j + c_{-j} - 2q_j^1) f(\alpha) d\alpha = \frac{1}{2}[E(\alpha) - 2c_j + c_{-j} - 2q_j^1] = 0,$$

which after rearranging yields:

$$q_j^1 = [E(\alpha) - 2c_j + c_{-j}]/2 = q_j^*(E(\alpha)),$$

which is implied by the fact that Assumption A guarantees an interior Stackelberg solution. Combining these two requirements results in:

$$[E(\alpha) - 2c_j + c_{-j}]/2 \geq (\alpha^+ - 2c_j + c_{-j})/3 \Leftrightarrow E(\alpha) \geq [2(\alpha^+ + c_j) - c_{-j}]/3.$$

□

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