Intervention and Mitigation in the US Mortgage Market: (Re) Negotiation as a Real Option?

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Abstract

University of Manchester

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Doctor of Philosophy

Intervention and Mitigation in the US Mortgage Market: (Re) Negotiation as a Real Option

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Within the context of the 2007/2008 subprime crisis, we examine the impact and relevance of previous ex ante residential mortgage research to prevent and mitigate mortgage default and develop a general taxonomy of (ex-ante) government intervention. We continue by investigating the various forms of (ex-post) default mitigation options prevalent in the United States and innovatively categorise them into one of three categories based on whether the mortgage contract has been renegotiated.

We examine the strategic renegotiation option, alongside the more traditional ruthless or strategic default option, for a US owner-occupier residential mortgage holder and non-owner occupier residential mortgage holder uniquely deriving closed form solutions to calculate the optimal ex-ante LTV (Loan to Value) and ex-post exercise moment where a heterogeneous borrower exercises a renegotiation option.

We finally relax the perpetual ability to pay assumption underlying strategic default and negotiation to investigate default and negotiation triggered by both an inability to pay and unwillingness to pay. We simulate the overall effect of (institutionalised) renegotiation under these two assumptions, comparing the effect of a stylised HAMP program on the overall default, foreclosure and prepayment probabilities of owner occupied residential homeowners in the absence of a HAMP program.

We conclude that the traditional option theoretic assumption that homeowners “can always pay” is a very strong assumption the consequence of which could induce relevant policy makers to incorrectly interpret and act on conclusions and recommendations flowing from historical option theoretic mortgage research.
Declaration

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My thanks to Gavin Wood, RMIT Australia, for including (parts of) Chapter 2 in Elsevier’s International Encyclopaedia of Housing and Home. Chapter 4 has been presented at AREUES Rotterdam 2010 and ROC Spain/Portugal 2009, Chapter 5 at ROC Turku 2011 and Chapter 6 has been accepted for ROC Tokyo 2013. My thanks to the participants for their contributions. Last, but certainly not least, to my beloved wife Janine who has guided and encouraged the optimal exercise of my real option choices over the last 30 odd years.
Dedication

To my beloved parents, Tom and Eileen, who believed in the value of learning, my dear wife, Janine, for her endless patience and support and finally to our children, Colm and Kirsty, for their “reverse genetics”.
1. Introduction

When I started my part-time PhD researching real options in the topic area of mortgage default 6 years ago, it could probably be best described as dormant with significant contributions few and infrequent. These contributions were mainly methodological aimed at improving the efficiency, speed or refinement of generally accepted approaches. Nick Sharp from the University of Manchester completed his excellent thesis “Advances in Mortgage Valuation: An Option-Theoretic Approach” in 2006 just as I started and would be typical of the type of contribution then required and expected in the field.

My first steps were to understand and appreciate the research and contributions made by the many researchers in the field and then look to critically evaluate and develop my own unique methodological contributions. However, the US subprime mortgage crisis events of 2007/2008, triggering terrible social and economic consequences for many people, had particular relevancy and quickly overshadowed and influenced my research efforts. Initially the question was whether the crisis was just a “blip” which once analysed using (then) current paradigms would quickly yield solution strategies, which would slowly but surely result in a return to pre-crisis levels of growth and confidence.

Five years later, it is obvious that the economic and social storms, arising from the original trigger events, have not abated. New and even more destructive events have occurred such as LIBOR rate rigging, widespread sovereign debt downgrades, etc. sapping the confidence and animal spirit of many borrowers and investors. Traditional real options and accepted paradigms do not seem to have the same potency. As a result, real options thinking is
undoubtedly enjoyed a new lease of life as practitioners, academics and regulators recalibrate and consider the real options available in this new and permanently altered world.

In traditional mortgage default research, the options investigated by researchers were primarily those of strategic default and prepayment. An important assumption was that underwriting standards were such that borrowers always had the ability to pay and that the borrower primarily triggered their options endogenously. However, we will demonstrate that ability to pay cannot be ignored since it is an ever present and crucial element in the default decision, and that furthermore both lender and borrower are encouraged and facilitated (by local and national agencies) in the negotiation and exercise of alternative default mitigation options.

Exercising the ultimate real option, I decided that the research payoff in the current volatile US housing market would be greater by abandoning my initial plan of developing methodological improvements to traditional mortgage theoretic options. Instead, the thesis and research has focussed on classifying and investigating the more important and significant real options exercised by borrowers and lenders in the wake of the 2007/2008 subprime crisis, proposing a unique methodological approach to valuing a negotiated mortgage option and finally examining the effect of the ability to pay assumption on real (world) option choices.
1.1 Research Aims

The aims of this thesis are threefold:

a) To quantify and classify recent real US residential mortgage mitigation options.

b) To propose a rationale and implement a methodological approach for the valuation of a strategic mortgage negotiation option allowing homeowner and lender to mitigate the consequences of default.

c) To investigate the consequences of a double trigger default process on the exercise probability of real US residential mortgage mitigation options.

Chapter 2 is a literature review chapter looking (mainly) at general US government mortgage intervention and academic default research followed by Chapter 3 which examines mortgage mitigation options in more detail. Chapters 4, 5 and 6 are intended to be published individually so some references and material may be similar and/or repeated.

A large part of Chapter 2 has already been published in Elsevier’s International Encyclopaedia of Housing and Home, Volume 4, pages 507-517.
1.2 Overview

Governments intervene in mortgage markets to mitigate undesired social outcomes, promote alternatives and ensure a stable flow of demand and availability of funds within their housing finance systems. The degree and sophistication of intervention is shaped by the responses to economic booms and recessions that can be precipitated by wartime conflicts, banking crises, the structure and power of institutions and lobby groups, as well as regulatory guiding principles.

Intervention (and therefore mitigation) can occur both ex ante and ex post an event. Prevention is sometimes better than the cure but not always. Failure to adequately intervene ex ante may well necessitate different ex post intervention strategies. Intervention and mitigation have similar meanings but we will use the term “mitigation” more in relation to “fixing” or “alleviating” financial or social problems arising ex post a mortgage crisis event.

Chapter 1 introduces the thesis and points out that the well-documented 2007/2008 US subprime market crisis would suggest that policymakers and institutional lenders did not have the understanding of and ability to mitigate default within the residential housing market ex post the crisis (prevention) as once supposed or suggested by the body of default research. One must therefore question the impact and relevance to US homeowners and society of previous ex ante residential mortgage research to prevent and mitigate mortgage default.

Chapter 2 begins by reviewing and developing a general taxonomy of (ex ante) government intervention – a review incidentally published as Chapter 267 in Elsevier’s International Encyclopaedia of Housing and Home. It continues by focusing on a half century of literature on mortgage options and demonstrates that default research has been mainly concerned with
the “strategic” or “ruthless” default and pre-payment options examined from the viewpoint of borrower, lender or institution. Strategic default occurs where homeowners are willing to make the monthly payment but choose optimally not to make the payment in order to optimise their future wealth.

We show that recent default research, which may be influential in shaping both the Bush and Obama administrations response to the crisis, claims that a double trigger default model is superior to the traditional “ruthless” option theoretic approach. A double trigger default is suboptimal from a wealth optimising perspective in that the homeowner cannot make the monthly payment due to cash flow constraints even though the property still has equity value. In other words, the homeowner is willing but unable to pay.

The third chapter investigates the various forms of (ex post) default mitigation options prevalent in the United States and innovatively categorises them into one of three groups based on their outcomes to the homeowner. It argues that, in the group where both lender and homeowner agree to renegotiate new terms, real option value could, in principle, be ex-ante estimated based on a consideration of the problem from (US) jurisprudence, gaming and option theoretic viewpoints. This is a unique departure from the option theoretic approach.

Previous academic judgement on the value of renegotiation within the US housing finance system has been typified by Clauretie and Jameson (1995) who claim “loan renegotiation does not occur frequently enough to warrant its consideration in mortgage pricing models”. However, this statement is certainly not true today and in the last 5 years, encouraged by US society and federal government’s efforts to mitigate the disastrous consequences of repossession, researchers have started to address a topic with hazardous moral dimensions.
Chapters 4 and 5 build on the approach discussed in Chapter 3 for a US owner-occupier residential mortgage holder and non-owner occupier residential mortgage holder, respectively. Based on simplifying assumptions and an approach suggested from within the corporate bond default literature, it uniquely derives closed form solutions to calculate the optimal \textit{ex-ante} LTV (Loan to Value) and \textit{ex-post} exercise moment where a heterogeneous borrower exercises a renegotiation option. These two chapters are essentially concerned with the subset of US borrowers who might “strategically” or “ruthlessly” default to gain a net benefit to their wealth, based purely on stochastic property prices or rental incomes. We demonstrate that this intervention type has a real option value for a mortgage borrower in the context of the 2007/2008 subprime mortgage crisis and subsequent shock to the US housing finance system. Secondly, we suggest how the option could be \textit{ex ante} valued alongside the more common default and prepayment options treated in option theoretic mortgage research.

Chapter 6 uses the general classification of US mitigation options developed in Chapter 3 and estimates the (numerical) relevance of the renegotiation option in a (economic) setting where householders are unable to pay. We assume that the US Federal HAMP (Home Affordable Modification Program) program is a real renegotiation option negotiated on behalf of US homeowners. It allows other options to be exercised in a different manner than if no HAMP program existed. We simulate the overall effect of (institutionalised) renegotiation, comparing the effect of a stylised HAMP program on the overall default, foreclosure and sales of owner occupied residential properties in the absence of a renegotiation (a stylised No HAMP program) option. This simulation does not use stochastic property prices and income to drive the model but rather stochastic Loan-To-Value and Debt-To-Income and endogenous trigger points which are prescribed by the HAMP program.
Chapter 7 concludes by discussing some of the contrasting results obtained from the option theoretic (optimal) and double trigger (suboptimal) default model approach. It concludes that the underlying option theoretic assumption that homeowners “can always pay” is flawed given recent mortgage underwriting standards. It suggests future research directions for researchers seeking relevance and impact in the context of the 2007/2008 subprime crisis.
1.3 Significant Findings

1) Post 2007/2008 the US subprime crisis, mortgage contract (re)negotiation, resulting in new mortgage terms and condition, is a frequently used real default mitigation option, central to a renewed and differently focussed body of mortgage default research.

2) Because a US residential mortgage contract at origination is incomplete as well as symmetric and due to the legal concept of Pre-Existing Duty, we can treat the renegotiation (option) of the contract between lender and homeowner as a generalised Nash game whose outcome is dependent on property prices and foreclosure costs.

3) We combine two different aspects of real options, that of irreversible investment and capital structure, to develop unique closed form solutions for a model which the US owner occupied homeowner and non-owner occupied landlord can use to optimise the ex-ante mortgage terms and ex-post exercise timing of their negotiation option.

4) We demonstrate the distinct economic and financial consequences between a mortgage negotiation and default option for both owner occupied and non-owner occupied residential mortgages and suggest that with increasing (property) price volatility homeowners would optimally accelerate exercise of their negotiation option in contrast to the situation where increasing property price volatility would induce homeowners to delay exercise of their default option.

5) We uniquely simulate the effect of a double trigger type process on the exercise of the typical real options available to residential owner occupied homeowners. We find that in a deteriorating housing market the current US Federal HAMP program is likely to be effective in steering struggling homeowners towards the exercise of a voluntary sale mitigation option as well as tempering the increase in foreclosures and strategic defaults. This supports the argument that HAMP program does not encourage more homeowners to free ride or strategically default at the expense of the taxpayer.
2. Intervention and Mitigation in Mortgage Market(s)

2.1 Introduction

Governments in many developed countries have seen significant economic, social and political benefits in either encouraging its citizens to own or being able to rent homes in sustainable neighbourhoods, generally financed by some form of loan or mortgage secured on the property. Increases in home ownership levels usually go hand in hand with the ability and desire of lenders to finance the purchase or construction of homes and properties.

However, mortgage (payment) delinquency and default makes mortgage finance more expensive and difficult for borrowers to obtain from lenders. This motivates a central objective of mortgage research, namely, by understanding default reasons and drivers, to minimise the consequential costs to both borrowers and lenders. Systematic mortgage default and delinquency research dates from the early 60’s (US Veterans Administration 1962, US Federal Housing Administration 1963) when increased home ownership arising from US government policy and initiatives led to a marked increase in delinquency and default rates (Quercia and Stegman 1992). There are several objectives of this chapter:

1) Provide the reader with an overview of the main features of the US housing system.

2) Examine 50 years of US mortgage research to determine where and what exactly has been the focus of academic research into mortgage default mitigation adopting a classification suggested by Quercia and Stegman (1992). This examination concludes by claiming that a shift in emphasis is currently happening in US mortgage research, moving from researching mitigation options from the viewpoint of (what is best for) financial institutions to the viewpoint of (what is best for) society.
3) Develop a taxonomy (Flanagan, 2012) of typical interventions by government and regulatory bodies. This taxonomy should be valid in both a US and non US context given that interventions may exist that are not currently adopted within the US. An observation is then made that, before the 2007/2008 subprime crisis, encouraging renegotiation between lender and borrower has *rarely featured* in the approaches generally encouraged by regulators and researched by academics. The absence of negotiation as an actively followed mitigation strategy has had many sound reasons varying from the legal difficulties of agreeing to vary contract terms and conditions to the ethical implications of imposing solutions on “consenting” parties.

4) Discuss the main drivers influencing the current federal government’s approaches (HAMP program) to foreclosure mitigation. Debate is raging within the US on how best to handle the mortgage default crisis and its detrimental spin offs. Opinion is polarised as to what is better for society - a “ruthless” or a “paternalistic” approach? A particular worry of politicians, regulators and voters is that no homeowner should enjoy a “free ride” at the expense of the US taxpayer - a topic examined in more detail in Chapter 6.
2.2 Intervention and Mitigation in the Mortgage Market

2.2.1 Why Intervene?

Governments intervene in housing markets because of negative externalities caused by a fundamental conflict between the objective that every citizen has decent housing, and investors continue to fund sustainable housing. The type and method of intervention will vary depending on whether the focus of regulators is on the private or public housing sector as well as \textit{ex ante} or \textit{ex post} a mortgage event.

The principal housing objective for developed societies can be simplified to

\textit{“Every citizen should have the opportunity of a decent well-built home at a price they can afford, in environmentally sustainable communities where they want to live and work.”}

The complementary objective for investors can be simplified to

\textit{“All mortgage investors should expect to earn a predictable and sustainable return over the term of the mortgage with all risks equitably and fairly divided”}.

Housing is simultaneously a consumption good as well as investment collateral whereby governments who aspire to housing “every citizen in decent homes” intervene, whatever their political and market outlook, to ensure sufficient funding to achieve equitable outcomes in housing provision. This objective to ensure a steady stream of affordable funds from investors to all types of borrowers ranging from creditworthy to non-creditworthy, no matter their social grouping, is the basis of most government interventions in developed countries over the last three centuries (Flanagan, 2012).
As the housing finance market has grown in size and sophistication, governments have institutionalised and widened the scope of the type and variety of interventions to achieve particular outcomes. However, (the better) interventions are rarely characterised by highly volatile regulatory approaches but tend to be relatively stable and long lived, shaped by economic crisis, wartime conflict and factors specific to a country. We demonstrate this, over a long time line, by sketching historical approaches to interventions by regulators in the US, housing finance system.

While regulation and intervention is an on-going evolving process in all developed countries, the historical perspective will emphasise that heightened and fundamental intervention, which involves the reshaping of housing finance institutions and the introduction of new legislation and regulations, tends to occur after significant economic recessions or destructive military conflicts. One of the contexts or motivations for developing this thesis is the belief that the 2007/2008 US subprime mortgage crisis and the subsequent loss of confidence in the value of US residential property will result in new approaches to intervention and regulation.

A post crisis institutional and regulatory regime generally provides new and innovative mortgage products that can meet the increased quantitative and qualitative demands from all types of consumer groups for decent affordable housing and from investors for a stable and guaranteed return on their investments. However, these post crisis interventions and innovations usually take place over decades, reflecting the long-term nature of the housing market and the interests of the different institutions and lobby groups.
2.2.2 An Historical Perspective on Housing Intervention in the United States

The US housing finance market has been of significant influence on other housing finance markets over the last half century because of the primary position of the United States in world affairs. Terminating Building Societies were the first widely used US housing finance institutions around the 1780’s. These institutions were arrangements whereby a small local community funded its members until the last member was housed at which point the arrangement was terminated. Mortgages were non-amortising with semi-annual payments while loan terms were typically up to 10 years with a maximum LTV of 60%. The societies evolved, during the 1870s, into mortgage banks lending funds, raised by selling mortgage backed bonds, from the established East Coast states to the expanding Western states. These building societies grew strongly but then defaulted in large numbers during the 1890s recession due to poor underwriting control by agents on behalf of investors.

The next major restructuring of the US housing finance market occurred after the Great Depression following the tremendous growth in the US economy of the 1920s. Very large levels of unemployment caused significant numbers of foreclosures and payment delinquencies with deflation halving house prices. The US (Hoover) administration responded by bailing out insolvent lending institutions via the Home Owners’ Loan Corporation (HOLC) and the Reconstruction Finance Corporation (RFC). Both corporations purchased defaulted mortgages, which was successful in avoiding systemic institutional failure, but was also an early example of the moral hazard or ethical issue in that some homeowners defaulted to take advantage of the subsidies (Flanagan 2012).

Roosevelt’s administration, legislating via the 1934 National Housing Act, made a very significant nationwide institutional intervention by creating the Federal Housing
Administration (FHA) to provide insurance against mortgage default. The Federal National Mortgage Association (Fannie Mae) was created as a government owned agency to provide a secondary mortgage market for FHA insured loans. Two deposit insurance companies, the Federal Deposit Insurance Corporation (for commercial banks) and the Federal Savings and Loan Insurance Corporation (for S&L’s) were established. These institutions significantly expanded the lending capacity of the US housing finance system by developing the insured 80% LTV, 20-year long term and fixed rate amortising mortgage. Fannie Mae was later privatised in 1968 and allowed to issue conventional instead of government-insured mortgages, while Ginnie Mae (a Government Sponsored Enterprise or GSE) was established to take over Fannie Mae’s government insured loans.

The next major restructuring of the US housing finance system came in the 70’s and 80’s as a result of rising inflation caused by the two oil shocks, the effects of the Vietnam War and changes in monetary policy which adopted money supply instead of interest rate targets. S&Ls were particularly hard hit as they had relied on short-term deposits to fund long-term fixed rate mortgages, while housing demand was also hit by the increase in interest rates. This situation was magnified by unintended regulatory effects. A federal regulation known as Regulation Q, which limited the rates banks and thrifts could pay on time deposits resulted in investors placing money in unregulated money market mutual funds. Freddie Mac was established in 1970 to provide liquidity to the struggling S&Ls. Both Freddie Mac and Ginnie Mae created the market in mortgage backed securities (MBS) in the early 1970’s. Further innovations rapidly followed such as multiple pass MBS’s or CMO’s (Collateralised Mortgage Obligations) and REMIC’s (Real Estate Mortgage Investment Conduits) which brought more investors such as mutual funds, life insurance companies, pensions and foreign investors into the housing finance market.
2.2.3 What Shapes Intervention?

Four well established generic type of housing finance systems, as discussed in the historical perspective, that have withstood or adapted well to economic and social crises (Boleet 1985) are typified but not limited to the following countries:

1) Retail Deposit System (e.g. United Kingdom Building Societies)
2) Mortgage Securitisation System (e.g. United States GSE’s)
3) Contract Savings System (e.g. German Bausparkassen Banks)
4) Mortgage Bond System (e.g. Danish Mortgage Banks)

Regulation and intervention in these types of systems is aimed at scaling and optimising the system for the benefit of lenders and borrowers, and not about (re)establishing legal and institutional frameworks.

However, even within each general type, no governmental housing finance intervention will be the same. Each government will have a different blend of institutional and mortgage guarantees, tax incentives, housing and mortgage regulations, specific support for weaker members of society and mortgage educational programs. Each society will reach a dynamic equilibrium based on its own particular housing and mortgage finance objectives, ruling government philosophy, lobby groups pressure and crisis events. Finally, the degree and amount of mortgage regulation and intervention within these systems, is no predictor of stability and performance due to the (in)ability of regulators to police and enforce large housing and capital finance markets that are often outside their span of control.

We continue by briefly reviewing regulation practise within each of the four generic housing finance systems to demonstrate the wide variety of approaches.
2.2.3.1 Retail Deposit System- United Kingdom

In the United Kingdom, responsibility for regulating both consumers and investors lies mainly with the FSA (Financial Services Authority) set up under the Financial Services and Market Act 2000. The FSA is characterised as light-handed with regard to regulation of internationally operating banking institutions, and somewhat more heavy handed with regard to the regulation of its local building societies. A major proportion of UK housing finance intervention is therefore achieved through specific regulations aimed at building societies.

The embodiment of the UK regulatory objectives, including housing finance, is found in the FSA’s Conduct of Business Sourcebook (COBS). This is incidentally a more concise and light handed “best practises” manual compared to the multitude of “heavy handed” manuals and computer systems found within the many different US regulatory and government sponsored enterprises.

They achieve a balance of regulation by applying five “principles of good regulation”

1) Using resources in the most efficient and economical way.

2) Emphasising the role of senior management in ensuring regulatory compliance.

3) Imposing restrictions on industry in proportion to the expected benefits.

4) Facilitating innovation in connection with regulated activities.

5) Minimising adverse effects on competition as well as promoting competition.

Government subsidisation of housing has been greatly reduced over the years as some favourable tax regulations have been removed. The security of retail deposits still benefit from both explicit and implicit government backed guarantees while additional housing benefits are also paid directly to particular social groups. UK regulators permit a wide range of fixed and variable mortgage products with high LTV’s.
2.2.3.2 Mortgage Securitisation –United States

In the US, factors shaping regulatory policy are opaque to the outsider as regulatory responsibility for achieving the efficient and effective functioning of the housing finance market is spread across many agencies at both a federal and state level. Lobby groups, challenging legislation or regulation in the US Federal and State Court system, also significantly shape US intervention.

The US was traditionally regarded as a heavy-handed regulator in terms of numbers of federal and state agencies, GSE organisations, housing finance institutions and tonnage of handbooks. On the other hand enforcement of the regulations became increasingly light handed over the 15 years preceding the 2007/2008 subprime crisis due to an “unholy” alliance between borrowers and investors in the housing finance system. Borrowers had an insatiable demand for more affordable and higher quality housing while investors demanded high, predictable and government guaranteed yields on their abundant and leveraged funds. A moral hazard problem arose in that both groupings felt that the US government would always implicitly guarantee the privatised GSEs (i.e. Fannie Mae and Freddie Mac) debt obligations.

Policy interventions are mainly shaped by four principles (Flanagan 2012):

1) Achieving social and/or economic efficiency.
2) Ensuring equitable procedures for all groups of borrowers.
3) Achieving equal outcomes by redistributing income to less advantaged borrowers.
4) Ensuring retributive justice for those repeatedly flouting or ignoring regulations.
In the case of the first policy objective, some US agency regulators may focus more on economically efficient outcomes (e.g. Federal Reserve) while others on socially efficient outcomes (e.g. the US Department of Housing and Urban Development or HUD). The second and third policy objectives in the US have been characterised by well documented areas of conflict between federal and state agencies, intra federal agencies as well as consumer lobby groups and federal agencies. The fourth policy objective (i.e. that wrong doers will be caught and suitably punished) has been promoted in the media and elsewhere on a variety of levels but notably with reference to apparent abuses in the certification of sub-prime mortgages. Some observers, including the Congress of Oversight Panel (COP 2009) as well as the Federal Reserve, claim that these types of abuses demonstrate how agency, adverse selection and moral hazard issues introduced by regulatory intervention can have major and significant unintended consequences.

A major proportion of US housing finance intervention is achieved through tax policy, institutional regulation and targeted government programs. Guarantees have tended to be implicit rather than explicit. Recent federal foreclosure prevention programs have tended to operate by providing interest rate subsidisation rather than principal reduction.
2.2.3.3 Contract Savings Systems - Germany

The German Bauspar contract savings system is subject to regulation under banking legislation enacted by both federal and state government within Germany. The main act called the Bausparkassengesetz sets out the main principles that govern the Bauspar contract. Institutions must conduct their business under the General Business Principles (GBP) and Standard Terms and Conditions for Bauspar Contracts (STCB) which covers the contract amount, savings rate, repayment rate and interest rates for the savings and the loan. Institutions can only allocate saved and previously collected savings as loans.

The Bauspar contract links an initial saving phase to the right to receive a mortgage loan in a subsequent phase. Bauspar savers receive less interest on their deposits but are charged less for their mortgage in the second phase. Interest rates are fixed for the whole contract term. Savers normally save about 40% to 50% of the total contract sum over about 5 to 6 years in the first or savings phase. They then receive the mortgage to purchase a property in the second phase and start repaying at a rate that is approximately 2% higher than the savings rate.

German Bausparkassen are supervised by the German Federal Financial Supervisory Agency (BAFIN) and the German Central Bank. BAFIN issue the licences allowing a Bausparkassen to operate and also approve its mortgage contract rates. The German Central Bank on the other hand monitors the liquidity position of every Bausparkassen in accordance with legislation on banking monitoring and supervision in Germany mainly enshrined in the Act of Contractual Savings Banks (CSB).
Because of the closed nature of the system, loan amounts are smaller than from other generic housing finance systems and hence usually complement other sources of finance. Credit risk to the lender is low because the borrower has already demonstrated a good savings record. Their popularity is also because both their savings and loan interest rates are effectively subsidised and cheaper, by up to 600 to 800 basis points, than consumer general loans. In Germany, this subsidy is paid in the form of a savings bonus that is subject to income thresholds and is regulated by another act called the Savings Premium Dedicated to Housing.

2.2.3.4 Mortgage Bond System – Denmark

The housing finance system in Denmark has come through the recent worldwide credit and economic crisis in good shape further improving its well-earned reputation as a heavy handed but effective regulator. Danish mortgage bonds (realkreditobligationer) are supported by a strong legal framework and close supervision of mortgage banks by the Danish Financial Supervisory Authority to ensure low risk for investors. Loan sizes are also limited in the same act to a maximum LTV of 80%. Mortgages up to a maximum LTV of 80% are designated by the Bank of International Settlements (BIS 2006) in Basle 2 as the lowest risk. Borrowers enjoy a wide range of different and flexible, fixed and variable type mortgage products. Because mortgage lenders, unlike the US type MBS system, retain liability to investors, the system is considered to have lower investor risk. The low or “balanced” risk is achieved by a variety of means, but principally by ensuring that mortgage bonds are of a “pass through” type whereby principal and interest rate payments from the pool of borrowers closely match those paid to the pool of lenders. The Danish mortgage system is primarily market based and without public subsidies with exceptions for special groups, living in social housing who are often allowed a higher LTV combined with a government guarantee. Housing finance repayments are subject to normal tax regulations and enjoy no advantageous regulations.
2.2.3.5 Mortgage Bond Systems - Summary

We summarise, without additional comment, some of the main characteristics of the different mortgage bond systems discussed in the preceding sections in Table 2.2.3.5.1 below.

Table 2.2.3.5.1 Similarities and Differences between Countries Representative of the Four Housing Finance Systems

<table>
<thead>
<tr>
<th>Description</th>
<th>Denmark</th>
<th>UK</th>
<th>Germany</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financing System</td>
<td>Mortgage Bond</td>
<td>Retail Deposit</td>
<td>Contract Savings</td>
<td>Mortgage Securitisation</td>
</tr>
<tr>
<td>Government Role</td>
<td>Rule Based</td>
<td>Principle Based</td>
<td>Rule Based</td>
<td>Principle Based</td>
</tr>
<tr>
<td>% Owner Occupied</td>
<td>53.5</td>
<td>65.5</td>
<td>43.2</td>
<td>66.1</td>
</tr>
<tr>
<td>Typical Mortgage Rate %</td>
<td>4.14</td>
<td>3.56</td>
<td>3.54</td>
<td>4.46</td>
</tr>
<tr>
<td>Mortgage Payment Tax Deductible</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>LTV Limit</td>
<td>Yes (80%)</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>General Mortgage Product Types</td>
<td>Long Term (5) fixed Interest, Long Term ARM</td>
<td>Majority are currently Long Term (25/30 Y) with 2/3 fixed Interest</td>
<td>5/10 Years Fixed Interest or 30 year fixed Interest</td>
<td>30 Year term FRM or ARM type</td>
</tr>
<tr>
<td>Average LTV Ratios</td>
<td>75</td>
<td>70</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>Debt Servicing Costs %</td>
<td>37</td>
<td>11</td>
<td>27</td>
<td>11</td>
</tr>
<tr>
<td>House Price to Income Ratio</td>
<td>7</td>
<td>6.4</td>
<td>5.5</td>
<td>3</td>
</tr>
<tr>
<td>Residential Mortgage Loans /Capita Population (Euros)</td>
<td>43520</td>
<td>23290</td>
<td>14240</td>
<td>24200</td>
</tr>
<tr>
<td>Residential Mortgage Loans as % of GDP</td>
<td>101</td>
<td>84</td>
<td>45</td>
<td>76</td>
</tr>
<tr>
<td>Covered Bonds as % GDP</td>
<td>144</td>
<td>11</td>
<td>9</td>
<td>0.1</td>
</tr>
<tr>
<td>Total Dwelling Stock (1000 units)</td>
<td>2798</td>
<td>27264</td>
<td>40460</td>
<td>132292</td>
</tr>
<tr>
<td>Nominal House Price Indices (2000 =100)</td>
<td>151</td>
<td>198</td>
<td>110</td>
<td>116</td>
</tr>
<tr>
<td>Residential Mortgage Debt to Household Gross Disposable Income</td>
<td>206</td>
<td>125</td>
<td>70</td>
<td>105</td>
</tr>
</tbody>
</table>


2.2.4 Types of Intervention

The following taxonomy characterises the common types of housing finance interventions:

1) Mortgage loans and deposits guarantees.

2) Regulation of mortgage product specifications and availability.

3) Regulation of housing finance institutions.

4) Regulation of the tax treatment of mortgage loans and funding.

5) Regulation of mortgage and deposit interest rates.

6) Educational or informational programs.
2.2.4.1 Mortgage Loans and Deposits Guarantees.

Government deposit guarantees, by their nature, are primarily aimed at reassuring investors that their deposits are secure while mortgage insurance programs ensure that lending institutions continue to lend to borrowers with more risky credit ratings. Discussions on the efficiency or effectiveness of this type of intervention are difficult to quantify as, in practice, it is abundantly clear that where governments do not offer explicit guarantees or insurance, investors still assume that implicit guarantees exist, leading to significant moral hazard issues.

The question of implicit or explicit guarantees is difficult to quantify because of the fact that the decision making process is normally *ex ante* the trigger event and subject to significant negotiation between investors of different sizes. Where an explicit guarantee exists, the government or institution offering the guarantee may be unable from a liquidity viewpoint to support the guarantee e.g. AIG and Ireland in 2008 or Cyprus in 2012, without the injection of additional liquidity by third parties. Even in these instances, the EU Central Bank and IMF have intervened differently w.r.t to Ireland, Spain, Portugal, Italy and Cyprus allowing the same notional classes of investors to bear different levels of losses. With respect to Cyprus, small investors initially had to bear pain even though a notional explicit guarantee existed at a European level which decision was then reversed to protect these same investors after European wide objections to protect small investors. This in effect acknowledged the existence of an implicit guarantee but by whom? While in the US, the Federal government supported AIG and failed to support Bear Sterns and Lehman Brothers in 2008. No consistent conclusions can be drawn about implicit/explicit guarantees to housing finance investors.

The best known and longest running loan guarantee program is the United States FHA (Federal Housing Administration) mortgage insurance for single-family homes. However,
HUD (Federal Department of Housing and Urban Development), alone administers at least another 20 programs while Private Mortgage Insurance (PMI) institutions also provide insurance for the more creditworthy mortgage loans. Insurance is an area where adverse selection means that public guaranteed loan programs may always be needed to ensure a sufficient supply of affordable funds to poorer sections of society, first time buyers or single-family households with small deposits. Before the 2007/2008 subprime crisis, US mortgage insurance was available for almost all borrowers, housing and locality with no restrictions on loan type. Guarantees of mortgage lending institutions or deposits, where not stated or legislated explicitly, are often implicit as demonstrated by the recent US bailout of the privatised GSEs Fannie Mae and Freddie MAC.

2.2.4.2 Regulation of Mortgage Product Specifications and Availability

Regulation of mortgage specifications is an intervention type used by all regulators. This arises because each type of generic system has its own intrinsic operating characteristics with weaknesses and strengths, which can be exploited, by investors and/or lenders. Unregulated innovation resulting in the large scale adoption of popular mortgage products by either investors or borrowers could result in medium to long-term damage to a country’s housing finance system. Conversely, arguments also exist that unregulated innovation, in a Darwinian model of evolution, allows more stable mortgage products to prosper and survive. It is not always clear whether the latter argument considers the existence of implicit government guarantees. In any case, because of the very long durations involved with mortgage products but the short term consequences to investors and borrowers when catastrophic loss threatens, all governments prefer not to take unnecessary risks and impose a degree of regulation on product specifications and to a lesser extent on availability.
More principle based regulators such as the UK and US (Di Lorenzo 2012) tended to allow a greater variety of mortgage products and specifications compared to rules based regulators such as France and Germany (Miles 2004). Since the recent US subprime crisis, the availability of different mortgage products has been reduced in Europe with countries such as the UK as opposed to Germany or France being particularly affected (European Mortgage Federation 2011). This is undoubtedly a sensible reaction aimed at reducing investor’s risk within a country’s finance system due to the complexity of the product offerings but with consequential effects on the ability of borrowers to access housing finance with suitable terms and conditions.

2.2.4.3 Regulation of Housing Finance Institutions

All housing finance institutions in every country tend to be regulated under one or more pieces of national legislation (Boleat 1985). Institutions operating in different countries either for the purpose of lending or raising funds may be subject to other countries’ regulations or, as in Europe, to EU directives. In many cases, institutions may also be subject to regulations set by different regulatory bodies in the same country e.g. one agency who may regulate on behalf of borrower’s interests and another on behalf of investors’ interests. Regulation within the same country may be differentially applied to different institutions involved in the housing finance market depending on a range of factors. The rationale for regulation is straightforward in that it is in the interest of all stakeholders that the rules of the game are agreed up front, expectations do not change significantly over the duration of the loan or investment and that risks are minimised as much as possible.

Many institutions have avoided these regulations by “shopping around” for more friendly regulatory regimes. One such example is the case of (German) Hypo Real Estate Bank who
were fatally weakened and bailed out by the German government as a result of trading by its subsidiary Depfa (Dublin based), which initially profited from lax Irish regulatory oversight.

Capital adequacy is regulated by trans-national standards such as Basle I and II (shortly to become Basle III). Institutions operating in EU countries are subject to various directives such as Solvency II for capital adequacy for insurance companies. Often these different regulatory regimes are complementary and do not impose too onerous a burden in terms of extra transaction costs or reduction in trading. Interventions are agreed and negotiated by all parties who then have some years to implement the systems.

2.2.4.4 Regulation of the Tax Treatment of Mortgage Loans and Funding.

The rationale for the favourable tax treatment of mortgage loans and funding is generally to reduce perceived (financial) entry or exit barriers and to encourage borrowers or investors to make decisions that benefit society or particular social groups within a society. Tax treatment can affect both principal and/or interest cash flows via either income or capital gains tax. The favourable treatment of housing and mortgage costs within a tax system can introduce considerable distortions into countries overall tax systems and considerably influence property prices.

The United States is particularly active in using its tax code to engineer particular results often influenced by the focussed efforts of lobby groups. The deferral of some capital gains on home sales, added to the United States tax code in 1951, was adopted to shield homeowners from tax liability when unforeseen circumstances such as job changes required them to sell one house and buy another. Significant reductions in tax capital gains on home sales for persons 55 and older was instituted largely to reduce tax liabilities for older persons.
who decide to become renters or to purchase smaller homes. The US also allows its citizens to deduct mortgage interest on their primary residence from their taxable income. The Congressional Budget Office (CBO) suggested that up to one-third of owner-occupied housing would not have been built if tax benefits had not lowered the after-tax cost of buying a house far below the cost of other investment assets. Further research also indicated that households would buy less expensive houses in the absence of tax subsidies, and that housing prices might be lower. However, these benefits come at a considerable price as the CBO also suggested that marginal tax rates could be at least 10 per cent lower if the deductions for mortgage interest and property taxes, the deferral and exclusion of capital gains for home sales, and the use of tax-exempt bonds for owner-occupied housing were eliminated.

The question of taxation and economic efficient is an interesting and much researched topic in the area of housing finance. Auerbach and Hines (2001) discuss in a US context tax induced reductions in economic efficiency which are known as deadweight losses or the excess burden of taxation, and represent the added cost to taxpayers and society of raising revenue through taxation that distorts economic decisions. Poterba (1992) examines and discusses how the US tax reforms (TRA86) of the 1980’s affected the incentives and distortions associated with the US housing market. He claims that these changes worsened the regressive nature of the mortgage interest subsidy and that in 1988, more than half of the tax losses associated with mortgage interest deductions accrued to the 8% of taxpayers with the highest economic incomes. The reforms also reduced incentives for rental housing investment, contributing to the decline in new multifamily housing starts from 500,000 per year in 1985 to less than 150,000 in 1991.
2.2.4.5 Regulation of Mortgage and Deposit Interest Rates

Interest rate subsidies, guarantees or control of mortgage and deposit interest rates is a type of housing finance intervention used by every country. The rationale, however, can be twofold and much debate rages over the long-term benefits of such regulation and intervention.

Firstly, the motivation can be to encourage both investors and borrowers to participate in the housing market as discussed in the previous sub-section with central banks and governments exercising volume and price control through interest rates. Effectively central banks control mortgage deposit interest rates by setting general market rates at which they will lend to its institutions. Bernanke (2010) of the US Federal Reserve on the topic of monetary policy and the housing bubble concluded that the US subprime crisis was a result of the failure of regulation that was too lax and intervention that was too late.

Secondly, and perhaps more prevalent in the current economic times, subsidising mortgage interest instead of forgiving or reducing mortgage principle is seen as a “less” (moral hazard) dangerous way of helping distressed borrowers continue to live in their properties and avoid foreclosure. An example of the latter approach is the current US $50 billion HAMP (Home Affordable Mortgage Program) applicable to 5 million homeowners, which mainly subsidises interest payments. It aims for a mortgage payment, which sustains an affordable 31 % debt to income ratio (DTI). This is mainly achieved by a reduction in interest rates (on average from 7.58% to 2.92%) but almost no modification of loan principle.

Excellent reports from the Congressional Oversight Panel (COP 2009, 2011) on this topic summarise the intensive arguments and extensive literature from the hearings both for and
against any type of mortgage interest subsidisation. These arguments are not at the level which can be supported by models or hard evidence as one of the major discussion points is the lack of clarity and evidence around the issue of mortgage intervention and mitigation. The interested reader is directed to page 91 onwards of the 2011 report where the main argument against the HAMP program is that it appears ineffective and unsustainable and might not provide enough support. The main argument for a HAMP type program is that something must be done to mitigate the large numbers of US homeowners losing their homes. We return to this question in Chapter 6 and try to address some of the points raised in the COP 2011 report.

2.2.4.6 Educational or Informational Programs

Intervention by governments in the form of education or information programs has been identified as a very specific and cost effective method of intervention in mortgage markets. UK commentators (Miles 2004) have identified mortgage product knowledge (or lack of) as a critical factor in the reason why consumers might choose suboptimal mortgage products. Similar conclusions have also been drawn in the US by the Congress Oversight Panel (COP 2009) prompted by the significant numbers of expensive subprime mortgages that were sold to consumers who would have easily qualified for cheaper prime mortgages. Many current US mortgage intervention programs especially directed at minorities, stress the importance of information and education in helping people decide the level of mortgage, down payment and on-going affordability. Good education and information may help mitigate problems caused by opportunistic agents or intermediaries in large and sophisticated mortgage markets.
2.2.5 Current Intervention Trends

The amount and mix of intervention in all housing systems will change as a result of the current worldwide recession caused by the US subprime crisis. Existing national lending and regulatory housing institutions will, if ‘history repeats itself’, fundamentally change. New trans-national institutions may well be set up and existing institutions such as Basle or IMF strengthened. These changes will be gradual, involving lengthy study and negotiation between national governments and lobby groups that could take decades before reaching a conclusion. However, no fundamental changes should be expected in how regulation is shaped or the broad intervention approaches that regulators may take.

In the US, both the COP and Federal Reserve have already suggested that more and better enforced regulation is required. Other (banking and pension) lobby groups suggest that a market efficient focus should be adopted. Yet more groups suggest that a socially efficient focus with better outcomes for poorer members of US society should be adopted. The debate will be long and the outcome unpredictable. The US government is in the unique position of being one of the few national governments with the resources to provide almost unlimited housing finance guarantees.

In terms of *ex post* intervention and mitigation, the current Obama administration, together with the Federal Reserve, is facilitating and encouraging lenders and borrowers to negotiate with each other to mitigate harmful consequences of default and foreclosure to all parties. The administration has negotiated and implemented the $50 billion HAMP program (Chapter 6.8) on behalf of distressed US homeowners. The HAMP program, however, is not imposed on lenders and distressed homeowners. Both are free to negotiate alternative arrangements.
outside the program. However, those participating within the program enjoy a certain legal protection as well as some small financial incentives to cover extra administration and other costs (Holden et al. 2012).

The 5 year HAMP program has in my opinion been influenced heavily by the Federal Reserve and research from Foote, Geradi and Willen (2008) who suggest that a double trigger type default model is more applicable when describing current US residential homeowners default motivation and subsequent behaviour. They suggest that both ability to pay and willingness to pay are critical to the homeowner’s decision and that cash rebates or deferrals to ease the homeowner’s monthly debt burden are more effective in mitigating and reducing defaults and foreclosures than principal reductions. Therefore, the main thrust of the program is directed at reducing the homeowner’s monthly debt servicing payments.

Interestingly enough and in recognition of the fact that lenders and homeowners have different (negotiation) requirements, the HAMP program consists of at least 19 different subprograms targeted at different mortgage segments with different types of incentives. Some subprograms encourage both parties to agree principal reductions, others are directed at those with second liens, second properties, rented properties, etc. Many of these subprograms have been added since the inception of the original HAMP program as new requirements arose from discussions between lenders and homeowners. One can therefore with legitimacy speak of a wide menu of pre-negotiated mitigation programs available to the homeowner and lender.
2.3 US Mortgage Mitigation Process and Practice Over 50 Years

The current US housing finance crisis has necessitated that institutional lenders, investors and governmental policymakers collaborate in developing innovative ways of intervening and mitigating the detrimental consequences of mortgage delinquency and default. Therefore, many lenders are not “lazy” anymore (Manove et al. 2000) and do not prematurely exercise their right to repossess their collateral but attempt to manage the risk using negotiation within a framework (e.g. HAMP/TARP) facilitated by US regulatory and legislative institutions.

To place this idea in a larger historical context, we examine and characterise US mortgage default research over the last 50 years and demonstrate in the following sections that, before the 2007/2008 subprime crisis, researchers had primarily regarded the (pre)payment and default options as the most significant mitigation options. We claim that (re)negotiation was considered by many as an interim step preceding foreclosure or reinstatement, of little value as a mitigating option and not valued or estimated in the traditional option theoretic framework. We then show that more recent empirical mortgage default research sponsored by the US Federal Reserve does investigate the negotiation (option) practice and outcomes.

Renegotiation is a mitigation option because most recent data (OTS 2010) (detailed in Chapter 3) shows that US lenders do not immediately repossess a delinquent homeowner’s collateral. The initial decision of the homeowner and consequentially the lender, whether exogenously or endogenously triggered, creates an additional “real” option, which we call the renegotiation option exercised because it provides an upside with reduced downside to both parties. We therefore define the renegotiation option as one where new agreed terms and conditions lead to an equitable distribution in a legally accepted sense of the benefits and liabilities of the mortgage contract.
We offer this historical mortgage default literature overview partly to lay the groundwork for our argument in Chapter 3 that the US residential mortgage contract is essentially contractually incomplete but symmetrical. We emphasise that this incompleteness is a contractual and _not financial market_ incompleteness (Hart and Moore 1988). Contract incompleteness implies that all future states cannot be taken into account when drafting the initial terms and conditions or to do so would be prohibitively expensive. Similarly, symmetrical knowledge implies that both parties to the contract have no information that is available to the other party implying, in this case, that both lender and homeowner have the same knowledge of the local housing market. However, it also lays the foundation for Chapters 4 and 5 where contrary to the prevailing option theoretic (simplifying) assumption of a homogenous relationship between homeowner and lender, we introduce a heterogeneous variable to capture the negotiated aspect of this complex and diverse relationship.

Finally, the 50 year overview also allows us to discover and show the crucial significance of some other generally accepted mortgage option theoretic assumptions, some of which will be challenged in Chapter 6 where we simulate, using the so called double trigger mortgage default model, the effect of the US HAMP program in mitigating current high levels of mortgage distress.
2.3.1 The US Mortgage (Mitigation) Process

We can only offer a simplified overview of a generalised US mortgage (mitigation) process. The US residential mortgage market currently has a pool of approximately 50 million mortgages with many combinations and permutations of legislative framework, specific mortgage contract, homeowner, housing collateral and lender considerations.

Borrowers (with unique characteristics e.g. social class, marital status, income variability) purchase a property or collateral (also with unique characteristics e.g. location, size, property tax rate) with some variable (self-financed) down payment and a mortgage loan (with characteristics e.g. loan-to-value ratio, term structure, interest rate type, etc.). The mortgage loan, usually representing several years of annual income, is repaid from future income, whereby, depending on specific borrower and loan characteristics, some proportion will be reserved for housing expenditure or consumption and the remainder for non-housing consumption. Individual repayment profiles and the effect on a borrowers housing/non housing consumption will be significantly different over time depending on the lender, down payment, mortgage interest rate, loan term, break conditions, etc. as well as borrower characteristics such as initial wealth, income and family status.

At regular pre agreed intervals, homeowners can either make the agreed mortgage (combination of principal and interest) payment, not make the payment or pay the loan balance by refinancing, property sale or from retained savings. When payments are not made, lenders generally do not immediately know whether the homeowner is delaying payments temporarily or stopping payments permanently. In practise, when payments initially cease, the lender may judge that the homeowner is only delaying temporarily (delinquent) intending to
catch up later. If this state continues for a number of periods, typically 3 months, then the lender may consider that the homeowner has stopped payment permanently and has therefore defaulted, breaking the contract and forfeiting the underlying property collateral.

It is then the lender’s choice, based on the legal contract and the legislative framework of the US state, whether to foreclose, followed by repossession and sale of the property thus recovering some of the value of the outstanding loan, payments and other costs. This choice is based on the value of the outstanding mortgage costs (which will vary from state to state) compared to the probable market value of the property. We show in Chapter 3 that US lenders only make use of the foreclosure option in approximately 40% of delinquent loans choosing to manage the risk by negotiating with homeowners in 60% of cases (OTS 2011).

The temporary cessation of mortgage payments by the homeowner (which we will refer to as delinquency) or the permanent cessation of mortgage payments (referred to as default) has significant direct and indirect costs for all parties in the mortgage and wider housing market affecting borrowers, lenders, institutional investors, state and federal government.

Default costs to lenders arise when the net income from the sale of the (foreclosed) financial asset does not cover the current value of the outstanding loan and administration costs. Delinquency costs arise from the extra burden of mortgage administration and the possible waiver of penalty fees, delayed payments or loan principal. Other financial costs follow for the lender if for example the government consequently impose restrictions on a lender’s capital adequacy ratio or lending profile.
A homeowner who defaults or becomes delinquent attracts a lower credit score and may have lower housing mobility leading to fewer employment opportunities (Gilberto and Houston, 1989). Intangible costs such as emotional distress may also affect the homeowner’s earning capacity. Misspecified default risks may result in certain classes of homeowner’s being charged extra premiums or being denied loans regardless of their individual creditworthiness. Local, state and federal government need to deal with the wider social, economic and financial consequences resulting from significant localised or national delinquency and default resulting in financially significant mortgage mitigation and rescue programs e.g. HAMP or TARP, (COP 2009, HAMP 2011).

On delinquency or default lenders must therefore choose to either foreclose or renegotiate the loan terms. If offered the latter choice, homeowners must decide whether to negotiate new terms, insist on/accept foreclosure or repay the total outstanding amount to the lender from accumulated wealth or (forced) sale of the property asset. The homeowner’s optimal choice may sometimes be clear cut but is often complicated given the specific homeowner, property and mortgage characteristics which will vary from the moment of initial homeowner delinquency to the “final take it or leave it” lender offer.

The number of borrowers exercising the negotiation option has always been significant but those exercising in recent years dwarf previous numbers. Ambrose and Capone (2000) reported a delinquency rate of 13% for a sample of 42764 US FHA mortgages during 1989. Of the 5650 borrowers who entered delinquency for the first time, 59% reinstated the mortgage prior to foreclosure and 40% ended delinquency with foreclosure for an actual foreclosure rate of 5%. However, of the 2,425,620 US delinquencies in 2010 approximately 62% (1,615,001) were re-negotiated with alternative mortgage terms. (OTS 2011).
2.4 Empirical Research into US Mortgage Option Practice

Many diverse themes and questions have been addressed in mortgage default research since the early 60’s, motivated by the central aims of minimising the cost of foreclosure and delinquency to borrowers and lenders and promoting a better understanding of the key factors underlying the default decision. In general, default research contributions can be characterised as follows.

1) They make either a theoretical or an empirical contribution to the literature.
2) They conduct the study from the perspective of a lender, borrower or institution.
3) They consider either the default decision as primarily endogenously determined or that, in contrast, exogenous adverse trigger factors are of greater significance.
4) Different parameters are used to determine mortgage risk such as default rates, delinquency rates, expected mortgage losses or mortgage rate premiums.
5) They mainly focus as discussed in the previous sections on a distinct social and financial homogenous market or region such as the US, UK, Europe, Australia, Japan, etc.

Quercia and Stegman (1992) provide an excellent review of default literature pre-1990, describing default studies from various viewpoints. Chronologically they divide default studies into three generations, which is augmented or expanded with an additional generation in this thesis, to reflect the most recent developments in mortgage literature, which looks at default and delinquency from a society (or policymakers) perspective.
Additional and valuable insights into homeowner behaviour have been developed (especially in the third generation literature) from a comprehensive “competing risk” approach looking at default and prepayment. This review, however, will primarily focus on default studies from the perspective of a borrower (as opposed to an institution) and ignore prepayment specific studies, which are of less interest (or risk) in the current economic climate for borrowers in distress.

We do not ignore competing risks but rather look at different competing risks more characteristic of the US environment ex-post the 2007/2008 subprime crisis. A particular example of the impact and complex nature of these competing risks is given by Geradi, Shapiro and Willen (2008). They report that a Massachusetts recession in 2001 generated a record high number of delinquencies – indicating household income constraints but that there was a record low number of foreclosures because that particular period was also characterised by large house price appreciation. We therefore examine the significance of two competing risks – income volatility/constraint and house price volatility/depreciation in Chapter 6 on real option choice and in the following default literature overview emphasise these particular competing risks. We reemphasise that our thesis is in the first instance only of relevance to the US housing market and do not make any claims that housing markets with other regulatory and financing systems should be treated in the same manner.
2.4.1 1st Generation Empirical Literature – The Lender’s Perspective

The first period, starting in the early 1960s and extending to the early 1990’s, addressed default from the viewpoint of the (individual) mortgage lender. These (mainly empirical) studies attempted to identify the characteristics of borrowers, property and mortgages at loan initiation or agreement that could be related to mortgage default later in the loan term as typified by papers such as Jung (1962), Page (1964), Von Furstenberg (1969), Herzog and Early (1970) and Sander and Soisin (1975).

Loan characteristics seem to provide better predictors of default behaviour than borrower characteristics. Thus home equity (e.g. loan to value ratio or LTV) is significantly related to default. Von Furstenberg (1969, 1970a, 1970b) found that net home equity at origination was the most important predictor of default risk. Mortgage default risk is positively correlated with the term of the loan and also with mortgage age (or seasoning) to 3/4 years after origination after which default risk reduces (Von Furstenberg 1969). The presence of junior financing or second liens is also a significant determinant of default (Herzog and Early, 1970).

On the other hand the effect of borrower characteristics such as income is ambiguous. Although default rises as borrower income falls, the LTV ratio (loan characteristic) also rises as income falls (Von Furstenberg 1969). Borrower’s age, marital status and the number of dependents have no effect on delinquency or default (Herzog and Early 1970). These findings tended to be confirmed by Morton (1975) and Sandor and Soisin (1975).

The one exceptional characteristic to the low significance of borrower characteristics is that most research e.g. Hertzog and Early (1970), Webb (1982), Geradi, Shapiro and Willen
(2008) finds that the variability of household income has a consistent effect on foreclosure and delinquency. Variability of income has tended to be captured by proxies such as type of employment or occupation. Areas of high unemployment (generally regarded as a property characteristic) were also likely to have higher default rates (Williams, Beranek and Kenkel 1974). Because of this generally accepted finding, we use the double trigger default model to simulate the effect of these characteristics in Chapter Six.

Many early mortgage studies of that period were reported as being limited by data availability, a focus on fixed rate mortgage instruments and the emphasis of loan/borrower characteristics at loan origination to the neglect of those same characteristics at default. These similar limitations on data availability were also reported in the Congress Oversight Panel (COP 2009) report on the 2007/2008 subprime housing crisis.

Later studies e.g. Vandell (1978), Webb (1982) and Zorn and Lea (1989) looked at other mortgage products such as PLAM (Price Level Adjusted Mortgage), ARM (Adjustable Rate Mortgage), and GPM (Graduated Payment Mortgage). These replicated the significance of the main results found with FRM (Fixed Rate Mortgage) products demonstrating that “trigger” events such as unemployment, death and divorce along with contemporaneous net equity were also significant. These studies began the academic focus of being more concerned with the issue of whether the endogenous or “ruthless” exercise of the default option was more significant than contrasting exogenously determined factors such as “trigger” events. These later studies focussed more on the importance of trigger events and borrower characteristics in turn leading to the next generation of default studies in the early 80s.
2.4.2 2nd Generation Empirical Literature – The Borrower’s Perspective

Research and empirical studies in this later second generation or period (overlapping with the first generation) were undertaken more from the borrower’s perspective and attempted to model the behaviour of individual households who in maximising their wealth (utility) over time rationally and simultaneously decided whether to prepay the entire mortgage, default or delay on the mortgage payment or make the mortgage payment.

Jackson and Kasserman (1980), one of the first to take this approach, tested two hypotheses, the first known as the “net equity” approach whereby the household optimises wealth (or utility of) and the second being where the household optimises current cash flow also referred to as the “ability to pay”. In the first approach, mortgage holders “ruthlessly” decide based on the financial costs and returns resulting from continuing, terminating or prepaying the mortgage payment. If home equity is negative after all transaction costs and options are taken into account, then default will ensure. In the second approach, householders default if current income/cash flow cannot meet the periodic payments. Their results tended to support the “net equity” optimisation hypothesis over the “ability-to-pay” hypothesis. Campbell and Dietrich (1983), extending Jackson and Kasserman’s research, found that both originating and contemporaneous LTV ratios were significant at the time of default, while (only) one borrower characteristic, income variability (proxied by regional unemployment), was positively significant at default. Thus, LTV and Debt To Income (DTI) are very significant variables in mortgage delinquency and default supporting their inclusion in the double trigger simulation model in Chapter Six.

Foster and Van Order (1984) were among the first to apply an option based model to the default decision in a narrowing focus from the previous more general empirical approach.
Default is regarded as a put option whereby the mortgage holder may sell the property to the lender for the outstanding mortgage, at the start of each payment period. This particular approach follows the “net equity” hypothesis of Jackson and Kasserman in that income and other borrower characteristics do not enter into the decision determination. Ignoring the effect of householder’s stochastic income, during that period, on delinquency and default might have been, however, understandable in the context of the then long-term prevailing increase in householder’s income in most layers of US society. They empirically supported their thesis by extending Campbell and Dietrich (1983), and using Federal Housing Administration 203(b) default rates from 1960 to 1978 were able to explain ninety per cent of variance using just the equity variables such as estimated LTV ratios both current and lagged.

Vandell and Thibodeau (1985) developed a two period model including borrower expectations. Empirically, they found the expected positive effect of high LTV as well as the perceived valuation difference between mortgage market and par value on the default decision. They also found that self-employment had a (14x) greater effect on the default decision than an increase in the initial LTV ratio from 75 to 95 percent and finally found that a borrower’s cash flow (constraints) was as significant in motivating default as option value.

From the late 1980’s onwards mortgage default research began to take two approaches – the option theoretic approach which considered the “ruthless” and endogenously determined exercise of the embedded options such as Green and Shoven (1986), Schwartz and Torous (1989, 1992, 1993) or a more empirical approach which recognised that exogenous factors influenced the default decision. This “ruthless” approach was further modified by the investigation of the effect of transaction costs by Brueckner (1994).
It was around this period that the value of the renegotiation option was examined but quickly dismissed by Clauretie and Jameson (1995). They claimed that the lender exercised the foreclosure option relatively quickly after delinquency with little renegotiation. They attributed this to the results of a stylised model showing that lenders with FHA insurance had no incentive to negotiate but rather collect the insurance payout - the lazy banker (Manove et al. 2000) syndrome discussed earlier. They supported this claim not by any direct observation of loan renegotiation frequency, because of data limitations, but rather by testing indirectly whether loans had different default outcomes between different types of loans and States with different foreclosure costs. One of the many shortcomings of their analysis was that they based their findings on foreclosure starts, assuming that renegotiation occurs only then, and not on completed foreclosures.

However, HopeNow (2011) metrics indicates that only 40% of foreclosure starts end in completion and that this percentage varies widely between States making Clauretie and Jameson’s conclusions unreliable. Theirs and similar claims made on equally indirect observations have tended to prevail and influence mortgage researcher’s opinions until now.

The evidence supporting the “ruthless” approach was reviewed by Vandell (1995) in an important and significant review paper. He pointed to crucial contributions made to the option theoretic models by Kau, Keenan and Kim (1993, 1994) that analyse the relationship and interdependence of default and prepayment to mortgage pricing and the relative role of equity and interest rate in influencing default. However, he also points to the evidence supporting the role of “trigger events” such as Riddiough (1991) which plays an important and poorly understood role in the householder’s default decision. As a reminder, trigger events tend to support the “ability to pay” hypothesis. Kau and Keenan (1995) also provided a
comprehensive review of the option theoretic approach, biased towards demonstrating that many of the so-called “ability to pay” phenomena could also be explained by purely adopting “net equity” assumptions.

Since Kau and Keenan (1995) as well as Vandell (1995), research on mortgage defaults has moved towards the broader area of research on mortgage terminations or the so-called “third generation” while the second generation type research, which focussed on determining an integrated theory of mortgage default, has stagnated, as noted by Archer, Ling and McGill (1997).

Most noteworthy recent “second generation” type theoretical contributions are Deng, Quigley and Van Order (2000) who examined and compared predictions from the option theoretic and competing risk approach, emphasising the importance of unobserved heterogeneity; Pavlov (2001) who demonstrated that (default) competing risk models are superior as other prepayment (default) models produce inefficient parameter estimates when default (prepayment) is not controlled; and finally contributions of a more mathematical or methodological nature such as Sharp (2006) who produced a numerical method to more quickly and accurately value the default and prepayment options.
2.4.3 3rd Generation Empirical Literature – The Institutional Perspective

Because of the ever increasing volume of mortgages along with the higher rate of (in) voluntarily prepayment, the need arose for institutional lenders and investors to price mortgage credit to anticipate losses from all types of terminations. This type of research focussed on estimating the probability that a particular band of a loan pool will terminate (default or prepay) rather than modelling the individual borrower’s default decision. Thus while first generation research could be thought to have taken a lender perspective and second generation a borrower perspective, third generation default research takes an institutional perspective.

The contribution of this group of researchers is mainly methodological and distinguished by the use of Cox proportional hazard models (survival models) (Green and Shoven 1986, Quigley 1987, Van Order 1990, Quigley and Van Order 1991, 1992). The key research question posed is not only how “ruthlessly” the default option is exercised but also in addition how “ruthlessly” the prepayment option is exercised. A good example of such a study is Lekkas, Quigley and Van Order (1993) who empirically test the theoretical predictions of Quigley and Van Order (1995) and focus on how much the put option should be “in the money” to be exercised. They find that the frictionless model (no transaction costs) does not offer a good explanation of loss severity, which is represented by the intrinsic value of the put option. The research finds little evidence that households try to maximise wealth but more evidence that they simply run into cash flow issues supporting Quigley and Van Order (1995). This is yet more support to develop and consider the real options arising from this motivating factor for mortgage default.
The state of research up to the US housing crisis in 2007/2008 could be viewed as a “quasi” third generation (Leece 2004) where prepayment and default/delinquency behaviour are seen as competing risks and the exercise of one option precludes the exercise of the other (Deng, Quigley and Van Order 1996, 2000, Pavlov 2001, Calhoun and Deng 2002). Their mainly empirical work used either (competing) Cox risk proportional hazard or logit models. The research generally finds that the option theoretic approach offers significant explanatory power but that trigger events also have significant explanatory value. Within this approach, the modelling of unexplained or unobserved heterogeneity has had a very significant impact in explaining default termination behaviour.
2.4.4 4th Generation Empirical Literature – The Society Perspective

Since the 2007/2008 US housing crisis and its resulting tsunami of foreclosures, empirical research has moved away from studies on competing risks of default and prepayment to focus on understanding current wave of default reasons and motivation with the view of how to effectively mitigate and reduce the number of foreclosures. In this context, government and lending institutions, worried about possible long term and permanent damage to societies and individual’s borrower’s perception of the housing market, seek to mitigate the consequences of default to society by encouraging (as one of a menu of many mitigation options) homeowners and lenders to (re) negotiate alternative solutions to immediate default.

Many academics and policymakers believe that lenders may be foreclosing inefficiently on a large number of borrowers (COP 2009) arising perhaps from the perception that bankers are “lazy” (Manove, Padilla and Pagano 2000). We define laziness as the incentive of the lender (e.g. because of default insurance) to liquidate collateral early as they prefer the safety of the collateral to the hazard of the rescue or negotiation process.

Important ethical issues relating to the “encouragement” of morally hazardous behaviour are raised by the different participants because society (taxpayers) now seems prepared to intervene to financially support some lenders and homeowners to the apparent detriment of other more cautious homeowners and lenders. Currently, US federal and state governments appear to want to ensure that both lenders and borrowers (and voters?) perceive any financial aid as temporary and repayable with stringent terms. Both the US federal government and Federal Reserve are at pains to ensure that no rescued lender enjoys a permanent advantage from government support but is forced to divest or face undesirable and permanent consequences. Does society’s desire to “punish” large lenders extend to “weak” homeowners?
Society therefore currently faces many difficult financial and ethical questions when providing support to individual homeowners either directly via their own (agency) arms of government or indirectly via a (quantitative easing programs) lending institution. How does society ensure that (limited) aid reaches the most deserving and is not leaked away within an agency or lending institution or given to undeserving recipients? How does one decide whether a recipient deserves a “hand-out”? Is it their ability to financially recover, continue to pay mortgage payments or is it the alternative costs of (re)housing and supporting the borrower and household for a period of time? Is the period of recovery time realistic given the high level of unemployment in many US states and the high and unknown frictional costs of moving to areas or sectors with low unemployment? This important topic is examined in Chapter 6 when we quantify whether the HAMP program encourages “free riding”.

Given the more interventionist role of recent (US) government administrations (both Bush and Obama) compared to any period since the early 1960’s, it is appropriate for mortgage default researchers to re-examine whether generally accepted default behaviour and motivations are still applicable. New, previously irrelevant questions and real options now arise given that automatic foreclosure and recourse is not currently the societal or institutional norm:

- How does one distinguish homeowners who deserve support from homeowners who strategically “game” the system to their own advantage?
- How should the US government deliver support to homeowners and lenders?
- How effective is this support for deserving homeowners in mitigating foreclosure?
- How does a lender value the homeowner’s negotiation option when setting terms for a future new generation of homeowners?
Government laws and intervention can have a significant *ex-ante* as well as *ex-post* effect on the behaviour of both lenders and homeowners. Pence (2006) studied the effect of different US state foreclosure laws on various mortgage parameters. She found that in geographical homogenous areas, split by State borderlines into defaulter friendly foreclosure laws (non-recourse states) and lender friendly laws, that the non-recourse State laws were correlated with a 4 to 6 percent decrease in loan size. She suggested that this was because lender friendly foreclosure laws impose lower overall costs and impose a larger share on homeowners. Notice that it is apparently the legislative framework that is crucial in determining outcomes and might therefore also be thought important in formulating and determining basic assumptions when using an option theoretic approach to valuation.

Danis and Pennington-Cross (2008) investigate delinquency in the context of US sub-prime mortgages in the period from 1996 to 2003. They make the important point that a lender and homeowner must negotiate once a loan becomes delinquent and that the loan status can only be determined *after the negotiation* (a flaw in Clauretie and Jameson 1995) implying that the resulting loan agreements reflect a mixture of lender and homeowner objectives. They find that borrower credit (FICO) scores are significant predictors of prepayment, default and delinquency to which we will return in Chapters 4 and 5 when suggesting a proxy for our bargaining parameter $\phi$. They further find that the LTV pertaining at the initial loan agreement is positively related to delinquency while changes in interest rate affect prepayments, default and delinquency. Preceding Danis and Pennington-Cross (2008) contention that lender characteristics also affect loan delinquency, Baku and Smith (1998) found that the performance of loans was sensitive to the incentive structure within the lender’s organisation. In other words, loan delinquency is dependent on a fuzzy and difficult to define two-way interaction (negotiation) between lender and homeowner.
Foote, Geradi and Willen (2008) show that for the Massachusetts housing market in the early 1990’s, the equity hypothesis (Jackson and Kasserman 1980) dominated, with “cash flow” only significant if the mortgage holder has no other (real) options left. They claim that their findings implied that lenders and policymakers (government) face an information problem because of the difficulty in determining whether borrowers really needed support to avoid foreclosure. Thus the real options valued by “traditional” option theoretic research are not sufficient since the 2007/2008 subprime crisis to cover the reality of the US homeowner.

In a 2001 study of US sub-prime mortgage defaults, Capozza and Thomson (2006) investigate the transition from delinquency to either cure or foreclosure (REO – Real Estate Collateral) over an eight-month period. They find that the transition probabilities of loans that entered the 90-day delinquent state suggested that 79% would foreclose while only 21% would cure. They further claimed their findings suggested lenders were more likely to forbear on delinquent homeowners where there was evidence (to the lender) that the default was due to solvency (ability to pay) rather than strategic default (equity maximisation) reasons.

Using US 2008/2009 mortgage data, Guiso, Sapienza and Zingales (2009) studied American households strategic default likelihood when the value of their mortgage exceeded the value of their home. They found that no household would default if the equity shortfall is less than 10% of the house value. However, 17% of households would default, even if they could afford to pay their mortgage, when the equity shortfall reached 50% of the home value. They talk of a contagion effect and claim that the willingness to default increases nonlinearly with the proportion of foreclosures in the same ZIP code.
In a study of a large US mortgage database from 2005 to 2008, Green, Rosenblatt and Yao (2010) found that homeowners with large initial down payments (low LTV) are in general far less likely to default compared to borrowers with low initial down payments (high LTV). They further claim that their results are not consistent with the strategic default hypothesis put forward by the consultancy Oliver Wyman (2009) because if homeowners were strategic and ruthless then initial down payment should have no impact on homeowner behaviour. The Oliver Wyman (2009) report, commissioned by Experian, claims that up to 18% of all serious delinquencies are strategic and demonstrate that people who default have high credit scores, in contrast to homeowners with low credit scores who invariably default because of “ability to pay” considerations. This point is relevant in the future development of the thesis as we show that these opposing viewpoints are somewhat reconciled by considering a negotiation option whereby the optimal default moment is determined not by the initial LTV but by each party’s negotiation ability. We will return to this point in Chapters 4 and 5 when discussing the interpretation of the bargaining parameter $\varphi$.

Adelino, Geradi and Willen (2009) investigate the apparent reluctance of lenders to renegotiate the terms of mortgages with delinquent homeowners and claim to show that this reluctance does not stem from securitisation i.e. whereby servicing agents are constrained from negotiating in contrast to portfolio loans where the lender may be thought to have more negotiating leeway. Foote et al. (2009) claim that the typical reason for default appears to be a combination of household income shocks and the fall in house prices. They further claim that only a low number of mortgage modifications occur which can be attributed to the efficiency of foreclosure and that lenders generally will act in their own interests rather than a social interest when deciding to foreclose.
In contrast, Piskorski, Seru and Vig (2010) examine the same question using a similar data set and find a significantly lower foreclosure rate associated with bank-held loans when compared to similar securitized loans. They claim that renegotiation is prevalent and effective. Furthermore, larger effects exist among homeowners with better credit quality (high FICO score) than those with lower quality. Foreclosure rate for bank-held (high FICO) homeowner loans is lower in absolute terms by 9.2% (33% in relative terms) and the cure rate is higher by 12.1% (35% in relative terms) within a year after delinquency. It is clear that no empirical consensus has emerged as to the value and effectiveness of the negotiation option.

Geradi and Li (2010) develop a detailed overview of recent US foreclosure mitigation programs. They describe the programs in some detail and discuss their effectiveness in terms of cost, reducing defaults and preventing (re)defaults. They point out that according to the OTC (Office of Thrift Supervision) “modifications that decreased monthly payments had consistently lower re-default rates with greater percentage decreases (in monthly payments) resulting in lower subsequent re-default rates”. They also found that the re-default rate was generally lower if the homeowner’s payment was reduced by more than 10 percent. This might also imply that modifications are only made for those unlikely to default but this in turn implies that some form of negotiation must occur to determine the likelihood to default.

It would seem that, whatever Clauretie and Jameson (1995) and others believed about the efficacy of negotiation, US government agencies on the other hand act as if the renegotiation of existing mortgage contract terms is a viable and effective option in mitigating the effects of the current volatile household’s income and local property values on default.

We summarise the main characteristics of the empirical literature in Section 2.8.
2.5 Theoretical Model Research into US Mortgage Option Practice

2.5.1 1st–3rd Generation Literature on Mortgage Models

Theoretical development of models on mortgage valuation has tended to follow the empirical developments described in the preceding sections. Mortgage models can be generally divided into two distinct classes reduced-form (behavioural) or structural models.

In reduced-form models, options are described by exogenously specified processes or hazard rates. They do not model specific characteristics of the borrower but rather make empirical estimations of the exercise timing of the borrower’s options e.g. prepayment, default or cure. The methodological approach taken in Chapter 6 is more akin to a reduced form model in that it has exogenously specified processes with trigger barriers based on empirical estimations.

Schwartz and Torous (1989) developed one of the first reduced-form MBS valuation models wherein prepayment was modelled as a function of various explanatory variables. These models have tended to keep pace with the requirements of lenders and investors and advances in data manipulation and availability. Deng, Quigley and van Order (2000) developed a competing risk reduced-form model of mortgage termination performance using loan-level data. In contrast to say corporate bonds or CDOs, mortgages and MBS products are particularly suited to the use of these models because of their relatively simple structure and availability of extensive and large time series. The commercial and widespread availability of exogenous explanatory data such as mortgage and MBS pricing data, local (ZIP level) house prices and indices, unemployment and income levels mean that borrower, loan and property characteristics can be estimated reasonably well. Whether this in turn leads to better estimation of default probabilities is another question.
Furthermore, it is clear from the literature that in a relatively steady state economic and social climate, these types of models have in practice performed very well. However, in a highly volatile climate where the legislative framework (rules of the game) and institutional arrangements are fast changing, the question arises as to whether this modelling approach provides useful insights into *ex-ante* borrower and lender behaviour. In other words, what’s the use of a model if the horse has already bolted and the stable on fire.

The structural approach models the endogenous option exercise by the borrower who attempts to maximize the borrower’s net house equity or minimise the present value of the mortgage or maximise the return on released equity in the case of default or non-payment. The underlying model dynamics are determined by a combination of stochastic interest rates and house price changes. Options are triggered when one or both of these exceed an exercise threshold whereby the models links option exercise to the underlying fundamentals faced by the borrower. One of the first structural models was proposed by Dunn and McConnell (1981) who used this approach to value GNMA (Government National Mortgage Association) securities.

A very large body of mortgage specific literature has developed over the years where these (so called) contingent claim models are used to value the embedded options. Kau and Keenan (1995) in their review paper cited around 90 different contributions in the field. Other important contributions since then have been made by Stanton (1995), Schwartz and Torous (1989, 1992, 1993) and Ambrose and Buttmer (2000). Different interest rate processes have been used in these valuation models, but probably the most common specification is the Cox-Ingersoll-Ross model. Structural models predict very precise trigger points and one therefore speaks of the “ruthless” or optimal exercise of the option (Vandell 1995). In extended models,
transaction costs such as selling and purchasing fees, taxes as well as (future) credit rationing
tend to reduce default rates with many authors such as for instance Deng, Quigley and Van
Order (2000) showing the empirical importance of such transaction costs. However, other
theoretical papers such as Kau and Kim (1994) show that the observed deferral of optimal
default, generally credited to transaction costs, can be explained by entirely rational choices.

In addition to the more mortgage specific structural models, a very extensive structural default
model literature also exists with a corporate debt focus. Important contributions have been
made by Leland (1994, 2004), Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein
(2001) among many others. The exercise of corporate debt options and residential mortgage
debt options are different problems with different characteristics but some overlap has
occurred in terms of the development of numerical or modelling methodologies. We make use
of this methodological approach in Chapters 4 and 5.
2.5.2 4th Generation Theoretical Literature

In contrast to the more recent availability of 4th generation empirical research on default and delinquency, 4th generation theoretical development of delinquency and default models, examining “optimal default” is less well developed. It can be easily argued that the question is somewhat irrelevant as since the 2007/2008 subprime crisis, default is rarely “optimal” for the vast majority of US homeowners but rather “suboptimal” and exogenously specified. Most theoretical literature to 2007/2008 concentrated on valuing the embedded options of prepayment and default and to a lesser extent the delinquency option.

The paucity of real options considered for modelling is particularly significant especially when set against the wide spectrum of actual real options as discussed in the following chapter. The most obvious real option absent from those modelled is where the homeowner “voluntarily sells” their property due to their debt burden. This may or may not be a “short sale” as the homeowner may exercise this option without discussing their cash flow constraints with the lender. The homeowner realises the equity in their property, pays off the mortgage and then downsizes to a more affordable property or rent. The negotiation option has not yet been modelled as an interim step between cure or foreclosure because it was assumed that a lender would automatically foreclose once a homeowner defaults. Generally, foreclosure is one of the alternative real options modelled, even though as shown in Chapter 3 and 6 it is not one of the most common real options used by homeowners and lenders. Strategic default is also commonly modelled, even though its actual exercise and use is almost impossible to measure empirically and its use is generally determined by questionnaires or a homeowner’s propensity to make other payments (e.g. car loan instalments) in preference to a mortgage loan.
Riddiough and Wyatt (1994) were among the first researchers to investigate strategic (re)negotiation in relation to mitigating foreclosure. The game setting is between one lender and multiple borrowers. In their game, the lender who has to decide whether to be a tough guy (foreclose) or wimp (renegotiate), seeks to optimise the costs of immediately foreclosing (private information) against the future costs (public information) of allowing borrowers loan modifications. The results tended to suggest that the lender should optimally always foreclose to prevent borrowers strategically defaulting to gain concessions.

Ambrose and Capone (1996) modelled a restricted number of real options in a cost-benefit analysis of the lender’s decision to either foreclose or negotiate. They took a lender’s perspective and assumed that he had five real options namely loan modification, preforeclosure sale, deed-in-lieu of foreclosure, forbearance or foreclosure. One of the important developments in the model was the introduction of “self-cure” risk. This is where the delinquent borrower resumes payments without any financial assistance from the lender. If this risk element is high then the lender may incur unnecessary costs by assisting borrowers who would have cured anyway.

Ambrose, Buttimer and Capone (1997) investigated the embedded delinquency option in the mortgage contract by specifically separating the delinquency option from the foreclosure option and allowing the borrower to resume mortgage payments prior to actual foreclosure. They extended Kau, Keenan and Kim (1994) to take account of the effect of the delinquency option on the borrower’s default decision and thus help frame the optimal response from the lender’s perspective. Both this and later work (Ambrose and Buttimer, 2000) uses two stochastic variables - interest rates and house prices. However, in contrast to most previous research, both papers capture the probability of foreclosure and delinquency states rather than
the value of these states to the borrower/lender. In both papers, delay is treated as an exogenous variable that can be used to predict the probability of default given a foreclosure delay or delinquency period.

The first (1997) paper attributes value to the process by modelling the benefits of eliminating negative equity and the “free rent” availed of (during the period between the first payment stop and the ultimate foreclosure) against the costs of potential post closure deficiency judgements. They then decide whether the ultimate foreclosure option remains beneficial at the point of foreclosure or whether payment reinstatement is better. The second (2000) paper is similar except that it ignores “free rent” as a benefit but instead models the benefit to the borrower of having a positive credit rating at the end of the foreclosure process.

Ambrose and Buttiner (2000) also investigates whether loss mitigation programs introduce a “moral hazard” problem whereby loan forbearance programs lead to higher delinquency rates and are thus less effective for lenders. They point to the role of borrowers and lenders expectations in determining whether these mitigation programs are effective. They do not allow for the effect of borrower characteristics such as debt to income burden which has been shown to be significant in the UK empirical work mentioned above and in other US studies such as Morton (1975) and Von Furstenberg and Green (1974). Although Ambrose and Capone (2000) found that 90+ days delinquency is sensitive to contemporaneous economic conditions in both the labour and housing markets, Calem and Wachter (1999) had earlier found that 60 and 90+ day delinquency were sensitive to FICO credit scores and not to LTV.

Wang, Young and Zhou (2002) extended Riddiough and Wyatt (1994) and argued that where asymmetric information existed between a lender and multiple borrowers it was better for the
lender to randomly reject requests for loan modifications. The lender must decide whether to pay the high cost of screening to determine the true type of the borrower—whether non-distressed (and strategic) or distressed. A lender thus will either randomly reject borrowers who wish to negotiate without screening (applicants) or will accept all applicants and screen a large enough sample of applicants to deter non-distressed applicants from applying. The lender’s optimal random rejection rate depended on foreclosure costs and benefits to both parties. Geradi and Li (2010) argue that the “random rejection policy” can be de facto achieved by (deliberately—my comment) understaffing lender’s mortgage support call centres to deny non-distressed borrowers an opportunity to strategically negotiate.

All these models are static and do not build in expectations of future market conditions in the equilibrium outcomes. Foote, Geradi, Goette and Willen (2009) and Adelino, Geradi and Willen (2009) in their mainly empirical papers develop simple models of the lender’s decision to foreclose or negotiate, showing that future house price appreciation affects the optimal decision. If lenders expect house prices to fall further then it may be better for them to foreclose straight away rather than wait for house prices to fall further. This of course assumes that houses are relatively liquid and that buyers exist at current house price levels.

Das and Meadows (2010), investigating SAM (Shared Appreciation) mortgages, assert that principal write-downs are the optimal approach to modifying distressed loans and mitigating foreclosures. They determine the optimal write down amount by assuming a desired LTV ratio that maximises the lender’s default adjusted loan value after the modification. They examine both the ability to pay (helpless default) and willingness to pay of the borrower (strategic default) by introducing a parameter $\gamma$ specific to each borrower where $\gamma=0 \rightarrow \infty$ indicates an increasing willingness to pay. This is a measure of the amount of negative equity
that the borrower is willing to sustain before deciding to default. They show that loan modifications with shared appreciation appear to mitigate the risks of strategic default, as it enables the lender to modify the value of the loan via the shared appreciation component. They claim high societal costs of 20% of home value on average and that 26% of US foreclosures are strategic.

We summarise the main characteristics of the theoretical literature in Section 2.8.
2.6 How Does the Literature Review Shape the Thesis Development?

It is clear from the preceding review that mortgage default researchers have mainly considered a restricted number of default, foreclosure and prepayment options triggered endogenously by the homeowner to maximise the utility of their wealth. They assumed that lending institutions will always foreclose and recover their collateral making up any shortfall by claiming on their mortgage insurance underwritten by the US Federal institutions.

Although the US government has in some cases ex post a crisis (as for example AIG) underwritten the balance sheets of some institutions since the 2007/2008 subprime crisis, it is questionable whether the fully underwritten mortgage is a realistic on-going assumption. The fact that widespread disaster due to inadequately underwritten mortgage guarantees has struck once implies that a rational investor should at least value all future mitigation options. Unlike standardised short term exchange traded derivative options a lot can happen to a real world mortgage loan or security (and the financial strength of US institutions) over its 30 year term! Investors need to value real options not only in (persistently) good but also in (chronically) poor economic times.

The observation that widespread default and delinquency by homeowners (whether exogenously or endogenously triggered) has had critical consequences for many institutions survival therefore (re)opens questions about underwriting standards and optimal LTV ratios. Institutional capital adequacy and maximum mortgage LTV ratios has become a prime mitigation response and intervention aim of the Basle initiatives. It is, however, too simplistic to assume that lending institutions will always be able to impose high underwriting standards.
as the historical section of the literature review has demonstrated that pressure will always be exerted by voters to allow wider participation of US citizens in owning their own home.

We interpret this as implying or assuming that underwriting standards will ultimately be dependent on personal characteristics and interaction of the homeowner and lender (Danis and Pennington-Cross 2008). This can be addressed in many ways but we proceed in Chapters 4 and 5 by introducing the idea that lender and homeowner choose or negotiate an optimal mortgage loan at origination based on a consideration of the future foreclosure (proportional to the underwriting) losses. We still however assume in Chapters 4 and 5 that the homeowner always has the ability to pay and merely adapt the traditional option theoretic approach to examine optimal capital structure questions when a (re)negotiation option is available. We assume that an exogenously triggered (ability to pay) default would in general be suboptimal for the homeowner. Therefore, the lender need only consider (his) worse case of an endogenously triggered optimal default by the homeowner.

Chapter 6, on the other hand, does not ignore other real options arising from the (in)ability to pay of the homeowner. Although many researchers (e.g. Jackson and Kasserman (1980), Kau and Keenan (1995), Vandell (1995), Clauretie and Jameson (1995)) have researched or discussed the “ability to pay” hypothesis, it has always been rejected as being of significance and necessary in the valuation of the default or prepayment options. This may have been due, as discussed earlier, to flawed conclusions from constrained empirical default data and the widespread (morally hazardous) perception that ultimately mortgage insurance underwritten by the US sovereign would underwrite defaulting mortgage loans. We raise two crucial questions about the methodological approach underpinning traditional option theoretic research. Firstly, we question the predominant use of backward working numerical methods
as discussed in Kau and Keenan (1995) which we argue are biased towards the efficient calculation of the default and prepayment options from a wealth optimising perspective.

Implicit in a backward working approach is that the homeowner always has the ability to pay. This traditional approach does not pose the question at each node whether the homeowner can pay! Rather the researcher assumes that if the homeowner cannot pay, the collateral is foreclosed upon and any difference with the outstanding loan made good by insurance. We argue that collateral is only seized as a last resort, in the case that the homeowner is bankrupt and does not have the ability to pay, otherwise negotiation results. It is clear from the data presented in Chapter 3 that in the real world with real options the US government is prepared to intervene and negotiate alternative mortgage and payment terms via massive HAMP type programs.

The backward looking approach has its strengths in valuing real options in (persistently) good economic periods where the value of collateral or insurance guarantees is not in question but is fatally flawed in the current depressed economic period. We argue that this flaw can only be addressed by adopting a forward working approach (Monte Carlo or Numerical Difference) whereby the ability to pay of the homeowner, and not just whether a homeowners wealth is optimal, is tested at each node.

A second limitation of the traditional approach is that it fails to recognise the conditional aspect of the strategic default or prepayment options (wealth optimising) as they only arise if the ability to pay condition of the homeowner is satisfied. i.e. the logic is that first an ability to pay test should be done and subsequently a wealth optimising test and not vice versa. The sequential and dynamic nature of these two tests might have consequences for the
effectiveness of intervention and mitigation of default in the real world. This arises because an exogenous exercise through an inability to pay introduces more “real” options. The backward looking approach should not be rejected; rather a forward looking approach should be adopted or favoured when default due to inability to pay motivation predominates.

An example of an important and often overlooked real option because of the wealth optimising bias is what I call the “Voluntary Sale Option”. I define this as where a homeowner who still has equity in their property decides because of ability to pay issues to sell and downsize to a smaller more affordable accommodation. The lender may not be aware of this decision process. This new home could either be rented or purchased. It is obvious that the timing of the exercise of this option might well be suboptimal from an endogenous wealth optimising viewpoint but necessitated by an exogenous (in)ability to pay. Some default researchers refer to this as the double trigger default model and has gained considerable traction in influencing the current Federal Administration response (Foote, Geradi, Goette and Wilken, 2009) to the crisis.

The traditional option theoretic wealth optimising bias and influence of the double trigger default model on the $50 billion HAMP mitigation program has therefore influenced me in making an original attempt to define in Chapter 3 and model in Chapter 6 the relative importance of these other “real” intervention or mitigation options.
2.7 References


## 2.8 Literature Review Summary

### Table 2.8.1 Summary Table of Empirical and Theoretical Literature listed by Generation

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<td>Piskorski, Seru and Vig (2010)</td>
<td>Default and delinquency rate (lenders decision to renegotiate)</td>
<td>LPS Data Base 2005-2006 loans</td>
<td>Two period economic model, Proportional hazard model, logit model, various specifications</td>
<td>Delinquency (DV) Default (DV) Bank held loans (+) DTI at origination (NS) Contemporaneous DTI (+) Heterogeneity effect (+)</td>
</tr>
<tr>
<td>4</td>
<td>Riddough and Wyatt (1994)</td>
<td>Theoretical paper on commercial mortgage securitization</td>
<td>Not applicable</td>
<td>General cooperative bargaining strategy in a multi period game (φ=0.5)</td>
<td>Stylised example</td>
</tr>
</tbody>
</table>

Source: Quercia and Stegman (1995), own research

Notes: (DV) dependent variable, (+) significant positive effect, (-) significant negative effect, (NS) not significant, (PV) Present Value
3. US Mortgage Negotiation Options

3.1 Chapter Summary

We examine the general forms of foreclosure mitigation options adopted by lenders and mortgaged homeowners in the US residential property market. Foreclosure, contrary to popular perception, is just one of many possible mitigation options. We argue that many of the alternative options can be classified or reduced to a renegotiation (option) which is distinct from a foreclosure (option) which enforces the terms of the original mortgage contract. We propose a unique and general framework, arising from specific game theoretical and judicial considerations of a US homeowner and lenders contractual relationship, to estimate the ex-ante value of this renegotiation option. This analysis is only valid for the US as we have only examined US legal opinion and would need to be repeated for other legal jurisdictions.

3.2 Introduction

Mortgage delinquency is of major concern in the US causing distress to many millions of people. The HopeNow (2011) alliance estimate that within the US residential mortgage pool of 50 million loans, approximately 13.6 million loans have not performed since Q3 2007 and have been subject to some form of modification. Mortgage (option theoretic) research has generally assumed that the only options or modifying solutions available to a borrower (apart from continuing to service the loan) are to either prepay or default and be consequently foreclosed upon by the lender. However, foreclosure is only one of the many outcomes available to the lender and by no means the most common. We estimate, based on metrics from HopeNow and the US Office of Thrift Supervision (2011), that since 2007 at most only 20% of these solutions have been foreclosure sales by lenders.
This proportion is by no means unusual and will vary depending on the economic or judicial environment. Ambrose and Capone (2000) reported a delinquency rate (90+ days) of 13% for a sample of 42,764 US FHA single-family mortgages originated during 1989. However, only 5% of these non-performing mortgages resulted in foreclosure while 8% were reinstated. The UK Council of Mortgage Lenders reported in 2010 that of the 206,000 mortgage loans which entered delinquency only 36,000 resulted in foreclosure implying that approximately 83% were subject to some other option or modification.

We conclude that significant numbers of modifications avoid the option of strict foreclosure. It is, however, incorrect to then assume that where immediate foreclosure has not been implemented that the mortgage will always reinstate or cure under the original terms and conditions. Modifications or options take many forms and we examine in detail the frequency and types of payoff associated with the more common mitigation options used in the US.

It is also relevant to note that whether a borrower is delinquent because of their temporary (in)ability to make all of the mortgage payment or desire to temporarily improve their equity worth may affect which foreclosure mitigation option is most likely to be exercised (Capozza and Thomson, 2006). If a borrower can make no payment, because of liquidity restraints, then foreclosure is most probable. However, if a borrower can make reduced payments, then a negotiated modification is possible.

The following overview ignores one important real option not considered by the OCC/OTS (2011) - the “Voluntary Sale Option” whereby a homeowner with positive equity but income constraints downsizes to a cheaper property or rented accommodation. It is not a negotiated option as the original contract terms are enforced and will be discussed again in Chapter 6.
3.3 Common US Mitigation Options

The following mitigation options are the most commonly reported by the US Office of Thrift Supervision in the (quarterly) OCC/OTS Mortgage Metric Report (2011). We sketch their main features and benefits mainly from the homeowner’s perspective.

a) Foreclosure Sale

The lender repossesses the homeowner’s home either through a judicial or non-judicial procedure that depends on the particular US State. Immediate payoff to the homeowner results from “writing off” the negative equity and ceasing further mortgage payments. The homeowner foregoes the convenience of occupying or using the property. The lender may be able, after selling the property, to compensate for any loss by claiming on a 3rd party default insurance policy. Further direct costs to the homeowner will depend on whether a deficiency judgement is sought and actively pursued by the lender. Personal bankruptcies are likely to occur either shortly before foreclosure starts are initiated or after foreclosure sales are completed further impeding the lender’s ability to claim recourse. Future indirect costs may arise from an impaired credit rating. No or little meaningful negotiation occurs with this option as both parties adhere to the original terms of the mortgage contract.

b) Repayment Plan

Delinquent payments are distributed over a period (usually <12 months) by addition to the normal monthly mortgage payment. The mortgage is regarded as cured and reinstated under the original terms within a 12-month period. Although the homeowner may enjoy a temporary payoff when delaying or reducing the delinquent payments, the net benefit to the homeowner over a short period may be zero or even negative. Limited negotiation occurs with this mitigation option resulting in no changes to the terms of the original contract.
c) Loan Modification

A permanent change in one or more of the mortgage terms allows the loan to be *legally reinstated* to a “current” or cured status and results often in a lower more affordable monthly payment. Missed payments are in many cases (but not always) added to the unpaid principle and re-amortized over the new life of the loan. With this option, the homeowner has renegotiated new terms which they regard as a benefit given their personal liquidity position. Payoff is a result of a redistribution of the contract’s on-going benefits and liabilities. The homeowner uses the home while the lender must have an equivalent benefit as adjudged in a legalistic sense as otherwise they could not have agreed the modified terms. The lender is not under any legal duty to minimise their loss. These comments describe the legal situation and no conclusions or inferences about the option theoretic modelling of this is made.

d) Fannie Mae HomeSaver Advance

A low interest rate loan is provided by the loan servicer to bring current the mortgage. The additional loan is repaid over 15 years with payment and interest accrual deferral during the first 6 months. The mortgage is legally viewed as reinstated or cured. This could be regarded as a permanent negotiated modification as in c) given the long term nature of the additional loan (which can sometimes be longer than the term of the outstanding mortgage).

e) Home Affordable Modification Program (HAMP)

A modified lower mortgage payment is agreed between homeowner and servicer so that the homeowner pays no more than 31% of their gross household income on housing costs. Modified payments become contractual modifications after a trial period of 3 months. Incentives are offered to both parties by the government to maintain this status quo up to a period of 5 years. The modified mortgage is legally cured and reinstated after the 3-month
trial period. The outcome of this option can be regarded as a negotiated loan modification benefiting all parties and resulting in a mortgage cure and removal of the threat of foreclosure.

f) Short Sale

In this case, the homeowner voluntarily sells the property and uses the proceeds to pay off (part of) the mortgage avoiding a (often) lengthy foreclosure process as in option a) above. As a result, the lender has lower foreclosure costs. The payoff mechanism to the homeowner is very similar to a), whereby they write off negative equity, except that their future credit rating impairment is not usually as severe. The lender does not normally pursue a deficiency judgement even though legally they remain entitled. This can be regarded as a negotiated option where the original terms of the contract are not adhered to. In practise, liquidity or income considerations with no other equity lead to this option being exercised.

g) Deed in Lieu of Foreclosure

In contrast to f) the homeowner does not sell the home but voluntarily transfers legal ownership of the home to the lender who disposes of the property. The motivation for and outcomes of f) and g) are similar with a slightly different sharing of foreclosure costs. However, it is a negotiated option with immediate pay-out to the homeowner if there is any residual of the house sale proceeds net of the mortgage repayment.

h) Prepayment Option

The homeowner pays off the mortgage in full, usually by refinancing at lower interest rates. Payoff arises from a lower present value of future refinancing payments than the present value of the current mortgage payments plus any prepayment fees. In essence, the original contract terms are adhered to.
3.4 Classification of the Mitigation Options

We classify or reduce the eight common types of foreclosure mitigation options a) to h), outlined in section 3.3 into three broad categories or classifications.

Category 1) are the so-called classical option theoretic set of real options, constituting of strict default, foreclosure or prepayment options, which have been well reviewed (Vandell 1995) and developed over the years. The expanded set of real negotiation options outlined in categories 2) and 3) are not found within the option theoretic literature. Nevertheless, we show that these real options are quite common in practice and are based on the US judicial and legislative concepts of pre-existing duty and mutual rescission.

The pre-existing duty concept (Snyder 1999) states that neither party to a contract can be put under duress to perform the contract in a manner different from that originally agreed. This implies that the lender is always within his right to enforce a foreclosure clause and need not be put under any duress by borrower or other parties to not do so. Mutual rescission further implies that any renegotiation of contract terms, to be legally enforceable, must be voluntary and freely entered into by both parties.

US state and federal courts will generally interpret this as implying that benefits or liabilities of the original contract must be equitably redistributed, given the parties’ current liquidity and the likely collateral value of the property. Due to the standardised and incomplete nature of many mortgage contracts, some courts may take more latitude in interpreting and approving the validity of a renegotiated mortgage contract (Eggleston, Posner and Zeckhauser 2000).
3.4.1 Category 1) - The Classical Default Option with Immediate Pay-Out

This category consists of options a), b) and h) from section 3.3 and are typical of the classical option theoretic option approach where both parties adhere strictly to the original terms and conditions of the mortgage contract with no negotiation. At an exercise or trigger event, the benefits and liabilities of the original contract are not redistributed in any manner.

3.4.2 Category 2) – The Negotiation Option with Immediate Pay-out

This category consists of options f) and g) whereby both parties voluntarily negotiate an immediate end to the contract. No new equity is introduced or side payments made. Essentially the negotiation is about mitigating further foreclosure costs and sharing any benefits from this mitigation in a voluntary and equitable manner. This sharing need not be 50:50 but will depend on the relative negotiating ability and leverage of each party. The lender will suffer a lower loss while the homeowner will gain from reducing a liability but lose from a temporary reduction in their credit rating and the convenience of using the home by paying the market imputed rent. The main difference with the outcomes of category 1) is that the original and rather general foreclosure term is voluntarily modified based on the homeowner’s actual liquidity and equity.

This option will be predominantly exercised by the homeowner and (reluctantly) agreed to by the lender (or vice versa) where the homeowner’s liquidity (income or ability to pay) constraints might preclude a category 3 option which explicitly assumes that the homeowner can adhere (pay) to the terms of any new negotiated agreement.
3.4.3 Category 3) – The Negotiation Option with Continuous Pay-Out

Category 3) consists of options b), c), d) and e) whereby both (or more) parties voluntarily negotiate permanent modifications to the terms of the original contract which redistribute the benefits and liabilities over time to the equal detriment of both (or all) parties. An important difference with category 2) options is that the homeowner desires to continue enjoying the convenience of using the home and maintaining their credit rating. It also presupposes a minimum income and an ability and willingness to continue to make some payments. The outcome is that monthly mortgage payments are reduced and made more affordable balanced in many cases by additional liabilities over time. Both parties must benefit (not necessarily equally) for the renegotiated contract to be legally enforceable.

The benefits and liabilities of the original incomplete (Hart and Moore 1988) contract are distributed. To be legally enforceable no one sided payments can be injected into a consideration of the original contract. Both parties mitigate foreclosure costs substantially (in contrast to a reduction in category 2) while the homeowner continues to pay the imputed rent and maintain their credit rating.

The essential difference from category 2) is that the homeowner does not receive an immediate pay-out by sharing in reduced foreclosure costs but receives an increased payoff, spread over the remaining term of the contract, by substantially avoiding foreclosure costs. The option value is contingent on the spot value of the property collateral and immitigable foreclosure costs.
3.5 How Prevalent or Frequent are These Different Categories in Practice?

We analyse mitigation outcomes exercised by US homeowners in 2010 as reported in the OCC/OTS Mortgage Metrics Report, 2011 and summarise the results in Table 3.5.1. Category 1 types totalled 1,113,550 or 38% overall. Category 2 totalled 209,478 (7%) or approximately one fifth of category 1 while category 3 options amounted to 1,615,001 (55%) or 1.4 times category 1.

Table 3.5.1 US 2010 Mitigation Outcomes Analysed by Category Type

<table>
<thead>
<tr>
<th>Category Type</th>
<th>Mitigation Description</th>
<th>Number</th>
<th>Sub Totals</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Foreclosure Sale</td>
<td>607,910</td>
<td>1,132,550</td>
<td>38%</td>
</tr>
<tr>
<td></td>
<td>Repayment Plan</td>
<td>518,561</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Prepayment</td>
<td>6,079</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Short Sale</td>
<td>202,709</td>
<td>209,478</td>
<td>7%</td>
</tr>
<tr>
<td></td>
<td>Deed-In-Lieu</td>
<td>6,769</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Loan Modification</td>
<td>616,044</td>
<td>1,615,001</td>
<td>55%</td>
</tr>
<tr>
<td></td>
<td>Other Trial Period Plan</td>
<td>329,051</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HAMP</td>
<td>669,906</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>2,957,029</td>
<td>2,957,029</td>
<td></td>
</tr>
</tbody>
</table>

Source: OCC/OTS Mortgage Metrics Report 2011

This leads to the conclusion based on the aforementioned categorisation that in 2010, 38% of mitigation options were not negotiated and 62% were renegotiated options.

The OCC/OTS mortgage metrics also report on the status and type of modifications completed from 2008 to Q2 2010. Of the 1,739,975 modified mortgages (Category 3), 962,464 were modifications that reduced payments by 10% or more while the remaining 777,511 modifications were those that reduced payments by 10% or less. The OCC/OTS use these statistics to illustrate the point that a negotiated modification increases the probability (but not guarantee) of a successful outcome by observing that of the 962,464 modifications
reducing payments by 10% or more, 57% were current and performing at the end of 2010 compared to 34% of the modifications that reduced payments by less than 10%. We return to this point in Chapter 6 when we examine the effectiveness of the HAMP program over the medium to long term.

Payment reduction was generally a result of combining two or more of the following modifications - capitalisation, rate reduction, rate freeze, term extension, principal reduction or deferral in 90% of the cases. Rate freeze is a particularly interesting modification as it tends not to show up in mortgage micro level data but is particularly common given the many teaser (first 2 years at a low rate followed by a steep increase) mortgages that were (and still are) sold on the US mortgage market.

Finally, HopeNow (2011) report, taking account of the widely different foreclosure completion periods between states that only 40% of foreclosure starts actually end in completed foreclosure sales. This implies that 60% will either settle on repayment plans or agree on a Category 2 or 3 (negotiation) mitigation options.

Thus, negotiation options are widely used real options in the US and must therefore have value to the homeowner and lender outweighing the rather restricted real option set as presented in the traditional option theoretic approach.
3.6 Why Has Classic (Option Theoretic) Research Ignored These Real Options?

Classic option theoretic research (Vandell 1995) since the 1980’s is based on the explicit assumption that the US homeowner may not always be willing to pay but is always able to pay. This is a critical simplifying assumption which might appear reasonable (if not a little utopian) when a US homeowner’s income and equity value is increasing and relatively stable but is not reasonable in the middle of a major economic and housing crisis.

The homeowner “puts” the spot value of the property in exchange for the outstanding mortgage. The lender is then assumed to have no choice but to always foreclose and recover part of the remaining mortgage principal. Option exercise, due to equity optimisation motives and triggered by property value and interest rates, is in this case always optimal for the homeowner. We ignore the fact, for computational ease, that it can be suboptimal if triggered by ability to pay motives even with positive equity. Homeowners may therefore choose the real option of negotiation to avoid this suboptimal result.

The assumption of infinite “ability to pay” restricts the range of real options that need be considered as well as simplifying the methodological approach. The typical option theoretic backward looking finite element numerical approach which looks to maximise the homeowners wealth by endogenously choosing the optimal default exercise point, would become even more computationally complex and difficult to interpret if the question must also be posed at each node point of whether the homeowner has the ability to pay.

Recent events in the US finance and housing markets have demonstrated that the ability to pay question is not trivial and should in the first instance be considered. Clauretie and Jameson (1995) investigated this underpinning assumption.
They examined the value of the renegotiation option but dismissed it as being inconsequential. They claim that the lender, in the US housing finance system, exercises the foreclosure option relatively quickly after delinquency with little renegotiation. They base this conclusion on the results of a stylised model showing that lenders with FHA insurance have had no incentive to negotiate but rather collect the insurance pay-out. They support this claim, not by any direct observation of loan renegotiation frequency but rather, by testing indirectly whether different (mitigation) outcomes occur between different types of loans and US states with different foreclosure costs. One of the shortcomings of their analysis is that they base their findings on foreclosure starts, implicitly assuming that renegotiation completes before a foreclosure start, and not on completed foreclosures, precluding the likely option that serious negotiation might only start after a foreclosure start. HopeNow (2010) metrics indicates that only 40% of foreclosure starts end in completion and that this percentage varies widely between states making the conclusions in Clauretie and Jameson (1995) unreliable. However, their (and similar) claims made on equally indirect observations have tended to influence opinions on the value of negotiation.

Clauretie and Jameson (1995) claims above could be viewed as a manifestation of the widely held perception of lenders being “lazy” (Manove, Padilla and Pagano 2000). “Laziness” is defined as the incentive of the lender (e.g. because of insurance cover) to liquidate collateral early, preferring the safety of the collateral to the hazard of the rescue or negotiation process. The supposition that banks are and will in the future always be “lazy” is in itself a lazy (academic) approach as the actual current variety and usage of mitigation options implies.

Lenders (and homeowners) take a more nuanced and sophisticated view of default and negotiation than classically assumed. Geradi, Shapiro and Willen (2008) report that the
Massachusetts recession in 2001 generated a record high number of delinquencies – indicating household income constraints— but that there was a record low number of foreclosures because that particular period was also characterised by large levels of house price appreciation. This surely reflects the idea that a lender and borrower may elect to manage the default risk by negotiation instead of prematurely and sub optimally foreclosing.

Recent research into differences between the performance of US portfolio and securitised loans has investigated the negotiation phenomenon at the loan micro level. Piskorski, Seru and Vig (2010), investigating a multi-million sample of delinquent loans, claim that renegotiation, which comes in many forms, is frequent. They contest the earlier claim, from an equally large sample of the same LPS US mortgage database, by Adelino, Geradi and Willen (2009) that negotiation is less frequent. Differences between the papers revolve primarily over the definition of what constitutes a negotiated modification and reporting difficulties of observing rate freezes and some other types of combined modifications even at a micro level.

However, the quarterly studies from the (US) Comptroller of the Currency and Office of Thrift Supervision is unequivocal in measuring and reporting on the vast number of mortgage modifications. The number of non-foreclosure mitigation options are simply too large to be ignored. The uncertain nature and volatility of income and property prices imply that many of these loans might recover even though some will surely end in default as well. Lenders are therefore not as “lazy” as classical option theoretic research would lead us to believe.

This conclusion is largely independent of whether the legislative foreclosure system leans towards recourse or non-recourse with regard to the pursuit of deficiency judgements. In a
related report, the UK Bank of England Financial Policy Committee (2011) meeting (citing evidence from the FSA) has recently warned UK lenders that they are “too soft” on delinquent borrowers and only foreclosing in 20% of the cases by offering modifications in the remaining 80% of cases thus incorrectly provisioning for default risk. They state “…the scale of the issue, especially if it is extended to other sectors, could be large enough to affect the committee’s overall assessment of the outlook for financial stability”. Their message is clear that negotiated forbearance is more frequent (and risky) than desired.

In the US, as recent events demonstrate, lenders and homeowners act as if the vast majority of defaults are non-recourse types and only reluctantly, after pursuing many other avenues of mitigation, seem to pursue a homeowner for recoverable debts. Recourse is a fine concept when economic times are good but of limited benefit to lenders when economic times are bad. Therefore, given the absence of effective recourse, only renegotiation remains as a “real” option to the lender and homeowner.

The predominant belief within classic option theoretic research seems to have been mistaken in its assumption that negotiation was not frequent or significant. People assumed that the default option, which was covered by US FHA or PMI insurance, would be the “lazy” option predominantly exercised by lenders. The negotiation option is indeed more risky for a lender and (as we will demonstrate in Chapters 4 and 5) more costly to the homeowner.

Negotiation is difficult to observe and quantify and a certain air of moral ambiguity exists around its widespread use and discussion for fear that some borrowers will “misuse” the option. However, one can also argue that negotiation is easier to measure than the strategic default option posited by traditional option theoretic research. It should be easier to measure
whether a mortgage contract has factually changed its terms and conditions from the original contract rather to divine whether a homeowner has ruthlessly defaulted because of unwillingness rather than an inability to pay.

Thankfully, large numbers of US lenders and homeowners, are taking a pragmatic real options approach and negotiating in significant numbers. Whether this situation continues when economic times recover is unknowable. US Federal budget constraints, increasing health care spending and differing housing priorities may lead to future scenarios where explicit (or implicit) mortgage default insurance provided by government agencies is reduced or curtailed. In the absence of default insurance (or doubts about the value of the guarantee), even Clauretie and Jameson would agree that renegotiation is the next best (more expensive) option and would be desirable from the lenders and borrowers viewpoint.

We therefore conclude that option theoretic researchers should attempt to value the negotiation option given its obvious usage and to quantify its differences from the classic default option. We continue by offering some suggestions on incorporating this valuation into a general option theoretic framework.
3.7 Why and How Can the Renegotiation Option be Valued?

In practice, mortgage contract renegotiation, or from a legal viewpoint, the redistribution and division of a contract’s benefits and liabilities is a difficult task and impossible to generalise given the 50 million different outstanding US mortgages agreed between lenders and borrowers with different personal and financial characteristics based on property collateral in different geographical areas under different legal frameworks. The Federal Government assists many homeowners and lenders by the introduction of mitigation programs the adoption of which facilitates the final negotiation between homeowner and lender. However, these programs are optional and many lenders (and homeowners) reach agreement outside these programs while others, of course, never reach agreement.

Detailed program guidelines, agreed between regulators and lenders are codified into manuals and calculation programs (HAMP 2011). Generally, the outcome of any modification needs to, at a minimum, have a zero or positive NPV (Net Present Value) to the lender and (perhaps less so) to the homeowner. The negotiation is not a zero sum game in that both parties will consider and value factors differently (e.g. market imputed rent) in their calculations but with some common considerations and even expectations as to property prices, affordability, foreclosure costs, etc. Important elements in the HAMP calculation are the homeowner’s income and credit rating (FICO score), the expected value of the property, the outstanding balance of the loan, expected foreclosure and REO (Real Estate Owned) costs and finally the expected insurance pay-out in the event of foreclosure. We abstract from this detailed (HAMP type) approach, retaining however, the most important elements over which both lender and borrower are likely to agree in a negotiation e.g. property prices, foreclosure costs, and term. We introduce in Chapters 4 and 5 a factor which takes into account some of the heterogeneous characteristics of homeowner and lender.
In this section, we examine in particular the case of a residential mortgage renegotiation and discuss why and how the resulting option can be *ex-ante* estimated or valued - an essential element of a real option. We assume, in the first instance, that Category 2 options are just (mathematical) variations on Category 3 options with a discontinuous payoff instead of a continuous payoff. This reduces the categories of mitigation options to two types either negotiated or non-negotiated.

The argument, which justifies the mathematical (game theory) approach used in Chapters 4 and 5, is based on four assumptions or observations about residential mortgages within the US legal framework:

a) Mortgage contracts are subject to the Pre Existing Duty Rule.

b) Mortgages contracts can only be mutually rescinded following equitable redistribution

c) Mortgages contracts must be affordable at origination and renegotiation.

d) Mortgage contracts are incomplete but not asymmetrical at origination.

The first two assumptions are two well established US legal principles and as such are realistic and not restrictive.

Assumption c) mortgage affordability at origination or renegotiation appears realistic but may not always hold in practice. It, of course, makes no sense for the lender to offer an unaffordable mortgage to the homeowner beyond the cynical scenario where the “unethical” lender hopes to repossess the homeowner’s property and profit from charging administration fees. It is of course very difficult to establish whether the mortgage is affordable as mentioned in the Congress Oversight Panel’s report (COP 2009) which concludes that people or agents
lied about homeowner’s income leading to the fact that many subprime loans were actually unaffordable at origin. However, the assumption is a common assumption in option theoretic research when examining ruthless default (Vandell 1995) and necessary for Chapters 4 and 5 where we assume that the homeowner can always pay but may be unwilling to pay. We relax the assumption in Chapter 6 where we assume that the homeowner may be unable to pay.

Assumption d) that mortgages are incomplete in a contractual sense is a realistic assumption as it is impossible to specify future states of the homeowner and lender completely at mortgage origination. The associated assumption of symmetric knowledge at origination is realistic as it pertains only to the state of the local residential property market and not to the homeowner’s income or the continuous ability to pay the mortgage. We assume that this ability to pay is perpetual. Although individual homeowners or lenders may on occasion have some asymmetric knowledge it is not realistic to assume that the 50 million US households with residential mortgages are involved in an asymmetrical game where the lenders know more about property prices and foreclosure costs.

In summary assumption c) is the most unrealistic and as such is also the assumption that we relax in Chapter 6.

The argument can be summarised as follows:

1) Any mortgage negotiation option, to be legally enforceable, must have been agreed *ex-ante* at mortgage origination or voluntarily agreed *ex-post* the default event.

2) To be considered voluntarily and legally enforceable, *ex-ante* benefits and liabilities can only be redistributed between parties with no extra *ex-post* side payments.
3) Residential mortgage contracts may be *ex-ante* incomplete but are not asymmetrical as it is assumed that no private knowledge exists about local residential property value and associated costs and risks at origination.

4) Points 1-3 suggest that treating the residential mortgage negotiation option as a generalised Nash bargaining game is legitimate given the observation that all states of the world are in principal known to both parties *ex-ante* any initial mortgage contract.

5) Mortgage contracts are however (*ex-ante*) only offered by the lender if
   a) the homeowner can afford the recurring coupon and initial equity deposit,
   b) contingent on the value of the collateral of the appraised property, and
   c) repossession of the collateral on default is feasible.

6) Renegotiated mortgages are affordable by the homeowner otherwise the renegotiation option cannot be exercised and the lender will certainly foreclose.

7) Points 4-6 lead to the conclusion that negotiation options are (*ex-ante*) only contingent on property prices and other foreseeable factors (such as deadweight foreclosure costs) whereby one (from a 1st order effect viewpoint) ignores the effects of income, interest rates etc. as these have been discounted under the initial affordability consideration.

8) Category 2 options are estimated similarly to Category 3 options but with differing deadweight foreclosure costs and an immediate payoff.

9) Equitable re-distribution need not be 50:50 but will depend on both parties’ beliefs on how the original contract may be interpreted by the relevant legal court.

10) The exercise of the negotiation option does not extinguish the default option whose continuing presence or enforcement threat is essential to the negotiation’s viability.

11) The *ex-ante* cost of a Category 3 negotiation option should be higher than a Category 2 negotiation option which in turn should be higher than a Category 1 option given that the negotiation option with continuous pay-out offers most to the borrower.
It should be pointed out that this exposition does not grant a chooser type option (to negotiate or to foreclose) exclusively to the lender, the homeowner must also choose. The result of a negotiation is a new (renegotiated) contract by both parties containing similar default and negotiation option terms to the original contract. In this sense, both homeowner and lender simultaneously and symmetrically choose renegotiation. The lender is under no compunction to renegotiate and neither is the homeowner. They only do so if they rationally profit from agreeing a new contract and redistributing benefits and liabilities.

In this exposition a homeowner’s default motivation (ability to pay or unwillingness to pay) is of little interest to the “lazy” lender who will always foreclose but of interest to the “active” lender where negotiation will occur. Therefore, ex-ante the mortgage initiation, the “active” lender who considers negotiation is only interested as to whether the borrower can make a minimum payment ex-post the default event. Should the homeowner not be able to pay anything or only an amount which makes the mortgage negative on a NPV basis, then foreclosure or non-negotiated default option would be the agreed outcome. The HAMP modification program addresses this by establishing a minimum DTI (Debt to Income) of 31% at which level payments might be considered sustainable for the homeowner and introducing a 3 month trial period to “discover” the homeowner’s “true” DTI.

A homeowner’s default motivation (if discoverable) may, ex-post delinquency, have some influence in conditioning the renegotiation stance of the lender vis-à-vis the homeowner, but we reemphasise that ex-ante considerations during the original negotiations dominate in determining the likely level of any future negotiation payoff as well as the legal acceptability or enforceability of any ex-post contract terms.
Because of the central role of the affordability assumption to the negotiation option, we importantly assume that the homeowner has disclosed their true current income to the lender. Discovery of the true income is critical in that it allows the lender and homeowner to agree on which option Category 1, 2 or 3 is applicable. If the modified mortgage is not affordable then only Category 1 (no negotiation) or Category 2 (negotiation with immediate payoff) options are available. This condition is no different from the initial affordability condition imposed on the borrower at mortgage initiation (38% of DTI). We exclude those mortgages where the homeowner claims an incorrect income at mortgage initiation, or no such documentation is required (popular “non-docs” prior to 2008). Mayer, Pence and Sherlund (2009) estimate that this category of mortgages was roughly 50% of the 6.3 million subprime mortgages originated between 2003 and 2007. We also assume that non-affordability condition also occurs where homeowners consider that the (market imputed) rent of the property is greater than the property’s perceived utility.

Similar to all option theoretic models, the affordability assumption simplifies the model development in Chapters 4 and 5, implying that income volatility is not relevant as we assume that spot income will always be sufficient to service a modified mortgage.

Having determined that the homeowner can (ex-ante) afford any modified payment, we adopt a wide interpretation of renegotiation and argue that no formal negotiation or direct contact need take place between lender and borrower. An informal renegotiation or even a standardised mitigation plan such as HAMP, resulting in forbearance by the lender and a stay of execution on foreclosure, is considered to have the same effect as formal renegotiation. Both parties may use intermediaries such as servicing agents or debt counsellors to communicate. No contact need be made beyond the lender (or agent) determining that the
homeowner still occupies or enjoys an economic benefit from the property and that he need not take vacant possession to secure the upkeep and integrity of his collateral.

Such a scenario is typical in the US residential mortgage market where a homeowner decides unilaterally to reduce or delay one or more monthly payments and the servicing agent (on behalf of the lender or investor), decides not to pursue (due to e.g. internal capacity constraints) the homeowner immediately using the full force of legal remedies. The lender or servicing agent does not forego any legal rights by following this action and can still foreclose. Neither lender nor homeowner is under any legal compulsion to accept a HAMP type plan. A normal interpretation of the legal (US) concept of the pre-existing duty rule implies that the lender can always insist, in the absence of, or even after formal or informal renegotiation, that the original mortgage contract terms should be honoured in their entirety.

It is clear that, in the commonly occurring scenario sketched above, that the non-performing party (the homeowner) gains a significant but temporary benefit to the detriment of the lender by delaying or reducing the mortgage payment. The lender need not, under this duress, be forced or “blackmailed” to accept the “new” status quo dictated by the homeowner, no matter its underlying cause or economic motivation. The temporary nature of the benefit to the homeowner may only be considered permanent in US law (Huberman and Kahn 1989), if similar benefits can be identified for the lender at which point the original contract could be mutually rescinded and a new contract with different terms agreed. Modified HAMP program terms are explicitly voluntary or otherwise could be legally challenged and be unenforceable.

Eventually some form of renegotiation takes place redistributing the benefits and liabilities of the original mortgage contract. Invariably, the renegotiation results in a modification of the
regular mortgage (principal + interest) payment. The modified mortgage payment, (affordability is not now a limiting constraint) will depend on similar considerations as at mortgage initiation such as property value and volatility, equity stake, risk free rate and general borrower characteristics often captured by the FICO or credit rating score.

The world, ex-post the default event, is unpredictable with different values of foreclosure costs, property prices etc. than those prevailing at mortgage initiation. However, in principle none of the information required will be different from what could have been reasonably anticipated or predicted ex-ante when the mortgage was originally negotiated. Both parties could therefore ex-ante have foreseen their ex-post negotiation position based on a range of property prices and the homeowner’s expected ability to service the mortgage.

Ex-ante, in the case of collateralised residential mortgages, neither the lender nor homeowner will have asymmetric knowledge with regard to the general development of property prices in the local area. In the case of the homeowner’s DTI, we assume that the lender will be able to estimate its future distribution given the homeowner’s current DTI and FICO score.

Applying Hart and Moore’s (1988) contract classification one could finally conclude that a US residential mortgage contract might therefore suffer from incompleteness due to later unpredictability but not from asymmetrical information. This formulation leads us to assume that, in contrast to many other contractual situations, we could treat the lender homeowner relationship as a generalised Nash game where the states of the future world, which predominantly depend on property prices and foreclosure costs, are predictable and known.
We conclude from the legal principal that benefits must be equivalent and equitably distributed that only those renegotiations, which have no asymmetric and lasting negative outcome for both lender and borrower, will be legally enforceable and provide long-lasting value to the homeowner and lender. The legal principle also implies that any benefits (or liabilities) introduced by a 3rd party (e.g. HAMP) must also be such that they are divided equitably (need not be equally) before any modified mortgage terms are enforceable. This allows us to ignore any additional benefits/liabilities as we assume that their net differential effect in a generalised game setting is zero. Of course, the additive effect might allow one (or both) parties to exceed a critical trigger point. We demonstrate the practical application of this approach in the Chapters 4 and 5.
3.8 References


4. US Residential Owner Occupiers Negotiation Option

4.1 Chapter Summary

We apply the main outcomes from Chapter 3, namely that we can treat a US residential mortgage contract as an incomplete contract where all states of the world are \textit{ex-ante} discoverable at mortgage origination and that this contract can be renegotiated and furthermore, \textit{ex-post} renegotiation can, in the first instance, be represented by a generalised Nash asymmetric bargaining game. We consequently develop and present a strategic negotiation option (which we refer to as Model 2) triggered by a homeowner’s asset worth. This is in contrast to traditional mortgage option theoretic research that generally focuses on the valuation of the default (referred to as Model 1) and/or the prepayment option.

Negotiation was previously regarded as an interim step before terminal states of mortgage default or cure and of limited value in its own right. We emphasise that negotiation can occur whether or not the homeowner actually defers or misses a mortgage coupon payment. Deferring a mortgage coupon payment is often referred to as delinquency.

We assume, however, that \textit{the negotiation option is triggered by asset value (and not by delinquency)} and the subsequent bargaining is instantaneous, resulting either in a reduced coupon payment or continuation of the original coupon. This therefore assumes that the homeowner can always pay and is the classical assumption (Vandell 1995) used in mortgage option theoretic research. Its validity can and will later be questioned in light of the 2009 subprime crisis in Chapter 6.
However, in the current US housing crisis, homeowners often choose to unilaterally defer a mortgage payment (Guiso et al. 2009) and in the process initiate a negotiation (generally after being delinquent for 90 days). We assume that those types of homeowners choose to defer a mortgage payment at the optimal moment which maximises the value of their net worth as influenced by the value of the negotiation option (Model 2). In this thesis, we do not determine the optimal moment when the homeowner has several competing (or compound) options e.g. prepayment and delinquency and default and negotiation, but determine the optimal moment when the homeowner has a default option (Model 1) only.

If the homeowner does not delay the due mortgage payment but initiates a negotiation, then lender and homeowner negotiate without the lender having the option to foreclose. If the homeowner delays payment, then the negotiation option is available providing the lender does not automatically foreclose on the delinquent homeowner. In either case, a negotiation commences between lender and homeowner, each motivated by different objectives. We assume that the homeowner’s main focus while negotiating is to ruthlessly optimise future equity and reduce or avoid future liabilities.

We derive closed form solutions for the optimal ex-ante mortgage loan terms, LTV and coupon payment, offered by a lender. We then compare the optimal ex-post exercise of the negotiation option to the exercise of a traditional default option (which we refer to as Model 1) for borrowers with heterogeneous expectations in terms of the homeowner’s book LTV.

We show that the ability to negotiate a larger share of unavoidable foreclosure costs in one’s favour has a significant influence on the optimal ex-ante financing and ex-post delinquency decisions.
4.2 Introduction

In light of the current US economic and social situation causing widespread mortgage difficulties, we develop a negotiation option and investigate the conditions of its optimal exercise. The homeowner may threaten the lender with default by delaying a mortgage payment or may just request a renegotiated coupon from the “strategic” desire to maximise their current equity worth. This is a unique contribution related to current US empirical mortgage default research (Piskorski, Seru and Vig (PSV) 2010, Adelino, Geradi and Willen (AGW) 2009, Guiso, Sapienza and Zingales 2009), which investigate the (re)negotiation of non-performing mortgages and default.

Our treatment is directed at US owner occupied residential mortgages where delinquency does not automatically result in immediate foreclosure as demonstrated in Chapter 3. In this case, the credible threat of default may be of value in negotiating concessions from the lender. We assume that the lender offers no concessions with respect to the loan principal but negotiates the mortgage coupon conditional on the unavoidable costs of foreclosure. We assume that with declining house prices, the homeowner’s other options of prepayment or sale of their illiquid housing asset are of little value. Our model is therefore characteristic of “forbearance” programs (e.g. HAMP), where the homeowner agrees a reduction in the monthly mortgage payment, rather than alternative “loan modification” programs where the principal of the loan is permanently reduced to the advantage of the homeowner.

We treat the negotiation as a one off (single) option and do not consider how a homeowner could later transition from negotiating a reduced mortgage payment to another mortgage state such as cure, (re)default or foreclosure (i.e. a series of sequential compound real options). In practise, a homeowner could “enjoy” the outcome of a negotiated option for a considerable
period relative to the mortgage term (the HAMP program is 5 years). Because the negotiation option is an interim option and not a final option, its exercise does not affect the homeowners' other terminal options such as prepayment or default. In other words, whether the homeowner has been able to negotiate a reduced coupon payment does not affect their later ability to prepay or default.

Hence, the Model 2 option is a real option on a timeline which follows the exercise of the investment (or mortgage initiation) option \((t=0)\) but precedes default, cure or permanent loan term modification which are not treated. The homeowner might eventually decide that (continuing) negative changes in house prices (after exercising the negotiation option) would make exercise of the default option optimal. On the other hand, positive changes in property prices might make it optimal for the homeowner to cure the mortgage to prevent the lender exercising his foreclosure option. The negotiation option, even though of a temporary nature, must have some value to the homeowner whereby its valuation and optimal exercise should be of interest to lender, homeowner and policymaker.
4.3 Model Outline and Assumptions

To demonstrate the effect of the negotiation option on underwriting (LTV) requirements compared with the status quo (investment option followed by the default option) and strip out the effects of different initial LTV at origination, we must firstly model the optimal ex-ante investment decisions of the homeowner (how much equity should they inject?) and lender (how much debt?) at t=0. This is necessary, as the optimal ex-post (negotiation or default) decision should be examined conditional on the lender and homeowner having agreed the optimal debt/equity (LTV) ratio at loan origination. It is easier to model the optimal ex-post (negotiation or default) decision for a given (non-optimal) LTV and coupon.

The homeowner’s “spot” asset (property) price ($V$) follows (Kau and Keenan 1995) a stochastic general Brownian motion process with drift $\mu$. The difference between the drift $\mu$ and the risk free rate $r$ is treated as a convenience yield (or market imputed rent) which the homeowner “collects” by living in their property. This market imputed rent will vary proportionally with the “spot” value of the property. In a declining housing market, considerations of maintenance and depreciation are of minor importance. We assume that the original mortgage is a perpetual loan carrying a fixed interest coupon determined at origination.

When the local property market is performing well, homeowners will see a notional increase in their equity value. Homeowners will therefore continue to supply the needed funds to service the debt when and if it is in their interest to do so -- an example being the property still having positive net equity or the imputed market rent being of sufficient value or convenience. In Chapter 6, we treat the case where the homeowner does not have sufficient
funds to service the mortgage as is more common given credit constraints or illiquid, indivisible assets.

The situation is different if the property market is not performing well, as default is generally costly to the lender and homeowner. The homeowner injects no new equity to prepay and new loan debt ((re)mortgaging and equity withdrawal) is not available to the homeowner due to declining house prices or credit restrictions. On default, the homeowner loses all housing equity as the lender repossess the collateral. In addition, the homeowner will also lose the convenience value (market imputed rent) of occupying the home. The lender will only receive the house value less foreclosure costs to cover any outstanding debt. Alternatively, after a successful negotiation (option), the homeowner pays a lower fixed monthly payment and retains ownership of the property and consequent equity and market imputed rent.

Consequently, non “lazy” or active lenders and homeowners will try to avoid costly foreclosure and attempt to negotiate and agree a forbearance mitigation program. We introduce a parameter $\phi$ ($0 \leq \phi \leq 1$) to model the effect and strength of this (re)negotiation regarding the sharing of foreclosure costs which as discussed in Chapter 3 must be equitably distributed. For ease of exposition, we refer to a homeowner who negotiates a smaller notional share of the unavoidable foreclosure costs as a weak homeowner ($\phi \to 0$) and one who negotiates a larger share ($\phi \to 1$) as a strong homeowner. We ourselves construct $\phi$ as a heterogeneous variable indicating the immediate view taken by both the lender and homeowner on how much of the unavoidable foreclosure costs the other would be liable for to condition or influence their ex-ante mortgage negotiation.
We believe that elements of real world individual characteristics such as a homeowner’s FICO credit rating or US state residency could be used to estimate the negotiation strength $\phi$. It is reasonable to assume that those homeowners with strong credit scores may be able to negotiate and extract different and better concessions and terms from lenders than homeowners with weaker scores. Piskorski, Seru and Vig (2010) and Adelino, Geradi and Willen (2009) demonstrate that default and cure rates are different for securitised and non-securitised loans from homeowners with high and low FICO scores after mortgage (re)negotiation. Recent empirical papers (PSV 2010, AGW 2009) make comparisons between homeowners and lenders with heterogeneous characteristics. We therefore uniquely attempt to introduce a heterogeneous element, such as for example the FICO score, to the negotiated mortgage contract outcome both ex and post any negotiation or default event. We also find support for this approach in the Experian –Oliver Wyman Reports (2010) which observe that strategic defaulters are more likely to be credit worthy borrowers with high FICO scores.

In contrast to the traditional option theoretic approach, as described by Kau and Keenan (1995), we arrive at our solution by adapting from Fan and Sundaresan (2000) to cover the borrower’s irreversible negotiation and investment options. Their methodology is an expansion on the endogenous default approach to corporate debt found in corporate finance literature e.g. Leland (1994) whereby the management chooses the timing of default to maximise equity value. In general, traditional option theoretic models proceed, using a backwards numerical solutions approach, to calculate the value of the default and prepayment options using two stochastic factors (property prices and interest rates) and a finite mortgage term. To ensure tractability and obtain closed form solutions we employ just one stochastic factor with a perpetual mortgage term. We believe this approach is justified as the stochastic interest rate factor is mainly of influence on the prepayment option (which we assume is
valueless for distressed borrowers in the current economic climate) and where new credit is readily available (which we again assume is unlikely with declining house prices). We consider that the omission of stochastic interest rates is neither a step forward or backwards when compared to the traditional approach but rather a step sideways needed to examine different options.

Figure 4.3.1 Owner Occupied Negotiation Option Model Flow Diagrams

Legend:
- \( V \): “Spot” Property Price.
- \( V_d \): Optimal property price at where the homeowner would invest with a default option only.
- \( V_f \): Optimal property price at where the borrower would default with a default option only.
- \( V_{di} \): Optimal property price at where the borrower would invest with a negotiation option.
- \( V_{df} \): Optimal property price at where the borrower would negotiate.
- \( c_d \): Perpetual mortgage payment to the lender for the default only option (tax deductible).
- \( c_{df} \): Perpetual mortgage payment to the lender for the negotiation option (tax deductible).
- \( C(V) \): Renegotiated mortgage payment (after negotiation) which depends on property price.
- \( I \): Initial property investment made at the critical investment thresholds.
Figure 4.3.1 summarises the real options available to the homeowner. The blue boxes in both Models 1 and 2 are the initial purchase or investment option followed by the box in yellow (Model 2) which represent the negotiation option and the default option in the red box (Model 1).

As a consequence of the assumption (discussed in Chapter 3) that the US residential mortgage contract is incomplete but not asymmetrical, the lender and homeowner play a generalised Nash cooperative game (Yellow box, Model 2, Figure 4.3.1) to avoid foreclosure costs. They have incomplete but no asymmetric knowledge of each other’s options, circumstances and costs, and having ex-ante negotiated the initial mortgage contract (LTV and mortgage payment in the blue boxes) conditional on anticipated default, may ex-post renegotiate the contract should a credible threat of default arise due to an unfavourable shock to spot property prices. We then allow the mortgage to be modified with a new lower mortgage coupon (C(V)) on a successful negotiation. Should property prices recover above the negotiation trigger point, we assume that the lender and homeowner would contractually revert to the original or higher mortgage payment. In the case of the US HAMP program this occurs automatically after 5 years.

Should the homeowner or lender not negotiate, then Model 1 applies, and with continuing negative property price shocks or trigger events, the homeowner will optimally exercise the default option (Red box, Model 1, Figure 4.3.1). We assume, in this regard, that the homeowner has limited liability and can default on the mortgage contract at any time with no long-term consequences to a subsequent credit rating – a common outcome in both US recourse and non-recourse states.
The question arises as to whether lender and homeowner can \textit{ex-ante} discover each other’s relative negotiation strength \(\phi\). Whether they can or not, it is however possible for both to agree that \(\phi\) will be in a range from 0 to 1 just as it is also possible for both to \textit{ex-ante} know all possible future property prices and foreclosure costs from 0\% to 100\%. Both parties can therefore \textit{ex-ante} calculate relevant trigger points and decide on what conditions they will optimally instantaneously agree a (re)negotiated mortgage coupon, based on an anticipated share of the unavoidable foreclosure costs. Under these assumptions, both lender and homeowner will \textit{ex-ante} anticipate the same range of negotiated outcomes.

Although our model is driven by stochastic property prices, we present and discuss the optimal negotiation trigger points by transforming the stochastic property price to a book LTV (defined as the nominal debt/appraised house value). It is common for policymakers and lenders to measure the likelihood that a homeowner will default in terms of their book LTV whereby a book LTV greater than 100\% indicates negative equity i.e. the property value is less than the outstanding loan. This is quite reasonable given that no principal payments are made in the interim.

We show, for typical US mortgage loan values, that optimal negotiation option exercise should normally occur earlier than a comparative default option exercise for all homeowners but strong negotiators (e.g. with high FICO credit scores) should exercise their negotiation option earlier than weak negotiators (e.g. with low FICO credit scores).

We show that the lenders \textit{ex-ante} mortgage yield spread should increase to pay for the homeowner’s \textit{ex-post} strategic negotiation option.
We show that the optimal equity down payment or deposit is conditional on the homeowner’s negotiation ability ($\phi$) in the case of a negotiation but not a default option and that a lender could offer a larger initial LTV mortgage to a weaker homeowner. This is because the weaker (as so the stronger) homeowner always has the ability to pay but his negotiating ability allows the strong lender to profit from the lower foreclosure costs and hence offer a larger loan. This is not the case with the stronger homeowner where the weak lender will offer a smaller loan.

Finally, we show that while increasing property price volatility should motivate homeowners to *delay* exercising the default option it will on the other hand *accelerate* exercise of their negotiation option.

In Section 4.4 we outline the derivation of the compound investment and negotiation options as well as of the comparative compound investment and default options with detailed derivation included in Section 4.9 and a glossary of notation in Section 4.8.

In Section 4.5 using stylised US mortgage data, we examine and highlight the fundamental differences between the *ex-post* behaviour of the negotiation option versus the more traditional default option and highlight the effect of heterogeneous ($\phi$) negotiation on the endogenous exercise threshold expressed in terms of negative equity, mortgage yield spread and LTV ratios.
4.4 Default and Negotiation Options Model Derivation

The property price process is exogenous and the homeowner and lender have homogenous and rational expectations while transactions are sufficiently small to have no effect on local property prices. The homeowner will make the constant and perpetual mortgage payment $c^*$ to the lender for the default option and in the comparative case where the homeowner has a negotiation option $c_d$. The mortgage payment $c^*$ or $c_d$ is tax deductible. The homeowner thus chooses a mixture of equity and (risky) debt to finance the property investment $I$ at an endogenously chosen time $T$.

We assume that the homeowner has only one property with a property price process given by a geometric Brownian motion.

$$dV = \mu V dt + \sigma V dW \quad [1]$$

where $W$ is a standard Brownian motion, $\mu$ the net property price drift and $\sigma$ is the volatility.

Let $r > 0$ denote the risk free interest rate. Assume $r > \mu$ for convergence. We view the difference $(r - \mu)$ as a convenience yield or the flow of benefits that ownership of the property provides in addition to the expected capital gain $\mu$ per unit change of $V$. This is then treated as a form of imputed or implied housing rent which is proportional to the current value of the property $V$ and equal to $(r - \mu)V$.

In practice, property prices might change in the long term at a higher rate than the risk free rate, although the 2008/2009 subprime crisis might suggest that property price corrections occur (in) frequently. However, this has no adverse consequence on our simplifying risk free assumption, as demonstrated in Dixit (1989) who shows that risk aversion in exit-entry type problems leads to exactly the same results where a replicating dynamic approach is taken.
instead of the methodology dictated by the simpler assumption above. He demonstrates that essentially $\mu$ can be adjusted by an appropriate risk factor and that the closed form solutions (that we have derived) therefore remain unchanged. The proportionality of the convenience yield to property price comes from the basic equation of state as shown in the detailed derivation where the risk free rate return is equal to the change in asset price plus the cash flow (imputed housing rent) from owning the property.

Let the tax rate be $0 \leq \tau < 1$. The funded property asset value is given by $F(V) = E(V) + D(V)$ where $E(V)$ is the value of equity and $D(V)$ the value of debt. The homeowner decides when to exercise the investment option by purchasing the property for a fixed cost $I$ and then benefits from the net stochastic property price increase/decrease of $V (V \geq 0)$ as well as collecting the convenience yield or market imputed rent by occupying the property.

After purchasing the property and taking on the mortgage liability, if the funded property value $F(V)$ is sufficiently or consistently lower than the value of the nominal debt $D(V)$, that is $E(V)$ is negative, the homeowner may consider defaulting on the mortgage payments, forcing the lender to consider repossession or foreclosure. In this case, if the lender does foreclose, the liquidation value (to the lender) is given by $(1 - \alpha) F(V)$ where $0 \leq \alpha \leq 1$ is the estimated foreclosure or deadweight costs as a proportion of the property at the moment of foreclosure sale, while the homeowner’s equity value $E(V)$ is zero if the mortgage is non-recourse.

Alternatively, the homeowner may “ruthlessly” exercise the negotiation option with the lender. In this case, the lender may agree to renegotiate the mortgage contract resulting in a new lower and more affordable mortgage payment for the homeowner. As discussed in
Chapter 3, this negotiation cannot happen under duress and the lender is still free to repossess their collateral should the homeowner consequently be delinquent and miss a payment. Whether the legal framework is “recourse” or “non-recourse” is immaterial as our model assumes that the homeowner has explicitly purchased a negotiation option (Model 2 not Model 1) from the lender at mortgage origination.

The new mortgage payment is conditional on the “surplus” equity generated by avoiding foreclosure being “notionally” divided between the homeowner and lender based on their relative negotiating strength, \((\phi \text{ and } 1 - \phi)\) whereby \(\phi = 1\) implies that the homeowner has the greater share. The preservation of this “surplus” equity is the only potential “asset” of value, over which both a lender and homeowner may want to negotiate. We model the process as a cooperative Nash bargaining game (Fan and Sundaresan, 2000).

The methodological approach to solving the problem is similar to solving a perpetual American (scale) option entry/exit problem [four equations with 4 unknowns, see Dixit (1989)] and a solution is found for the different ODEs in terms of the critical entry/exit thresholds for the default or negotiation options, respectively, \(V_i/V_{di}\) and \(V_f/V_{df}\). Solutions are of the form \(F(V) = A_0 + A_1V^r + A_2V^\beta\) with the appropriate boundary conditions leading to different specific solutions.

Conventionally modelled default results in the lender repossessing the property. However, in the case of the negotiation option, homeowner and lender (originally) negotiate a mortgage contract, conditional on sharing the avoidable foreclosure costs, at the negotiation trigger point \(V_{df}\) with both willing to change the original (incomplete) contract terms. The lender now agrees a new mortgage coupon \(C(V)\) based on the current property price, lower than the
initial mortgage coupon $c_d$ (agreed at the investment threshold $V_{di}$) and the homeowner continues to own the property and collect the market imputed rent.

Let $F(V, C)$ be the property asset value before investment. At origination, $F(V, C)$ is simply the investment cost (purchase price) $I$. After purchase, $F(V, C)$ will be a complex function of the (market) property value $V$, foreclosure costs and tax benefits. The homeowner chooses the optimal investment threshold $V_{di}$ and the optimal mortgage repayment $c_d$ to maximise his equity position $E(V, C)$. As the property price $V$ approaches infinity, the mortgage becomes riskless and hence the property value must satisfy an upper boundary condition whereby

$$\lim_{V \to \infty} F(V, C) = V + \frac{\tau C}{r} \quad [2]$$

Lower boundary conditions for the negotiation option differ from the default option as lender/homeowner are prepared to vary the contract terms at the lower threshold, where the total value of the property funding arrangement $F(V_{df}, c_d)$ includes the value of future tax benefits.

The homeowner and lender thus bargain over a larger amount (when $V \leq V_{df}$) resulting in a property asset value $F(V)$ of

$$F(V) = V + \frac{\tau c_d}{r} \left[1 - \left(\frac{\beta}{\beta - \gamma}\right) \left(\frac{V}{V_{df}}\right)^\gamma\right] \text{ when } V \geq V_{df} \quad [3]$$

$$F(V) = V + \frac{\tau c_d}{r} \left(\frac{-\gamma}{\beta - \gamma}\right) \left(\frac{V}{V_{df}}\right)^\beta \quad \text{ when } V < V_{df} \quad [4]$$

where $\beta > 1, \gamma < 0$ are the roots of $\frac{\sigma^2 V^2}{2} + (\mu - \sigma^2/2)V - r = 0$
Equation [3] simply states that the value of the asset above the negotiation trigger point $V_{df}$ is the result of three terms: the property value $V$ plus the tax benefit $\frac{te}{r}$ less the foreclosure costs given by the third term. As the likelihood of foreclosure decreases as $V \to \infty$ then the value of the third term approaches zero.

The equity equation $E(V) \ (V < V_{df})$ is also adjusted to account for the new mortgage payment $C(V)$ which is now a function of the current property value and $(r - \mu)V$ the market imputed rent.

$$\frac{1}{2}\sigma^2 V^2 E_{VV}(V) + \mu V E_{V}(V) - r E(V) + (r - \mu)V - C(V) = 0 \ \text{when} \ V < V_{df} \label{5}$$

With upper boundary conditions the same for both the negotiation and default options, we obtain revised lower boundary conditions from the “extra” value of $F(V)$ using equation [4] and the Nash negotiation sharing rule to get

$$\lim_{V \downarrow V_{df}} E(V) = \phi \left( V_{df} - \frac{\tau C}{r} \frac{\beta}{\beta - \gamma} \right) \label{6}$$

Differentiating [6] gives

$$\lim_{V \downarrow V_{df}} E_{V}(V) = \phi \left( \alpha - \frac{\tau C}{V_{df} r} \frac{\beta}{\beta - \gamma} \right) \label{7}$$

Further development (see Section 4.9) leads to closed form expressions for the key outcomes for the negotiation option model and the comparable outcomes for the default option model.
a) The homeowner’s investment threshold for the negotiation option $V_{dl}$ is given by

$$V_{dl} = \frac{\beta}{\beta - 1} \left[ 1 + \frac{\tau}{gL} \right]^{-1} l$$

where $g = \left( \frac{\beta}{\beta - \gamma} (1 - \gamma) \right)^{-\frac{1}{\gamma}} = \frac{V_{dl}}{V_{df}}$ and $L = \frac{1 - \tau (1 - \phi)}{1 - \phi \alpha}$

The investment threshold for the default option $V_i$ is given by

$$V_i = \frac{\beta}{\beta - 1} \left[ 1 + \frac{\tau}{h} \right]^{-1} l$$

where $h = \left[ 1 - \frac{\gamma (\tau + \alpha)}{\tau} \right]^{-\frac{1}{\gamma}} = \frac{V_i}{V_f}$

b) The mortgage coupon for the negotiation option $c_d$ (for $V \geq V_{df}$) is given by

$$c_d = r \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} (gL + \tau)^{-1} l$$

The mortgage coupon for the default option $c^*$ (for $V \geq V_f$) is given by

$$c^* = r \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} [h + \tau]^{-1} l$$

We show in Section 4.5 that the consequence of these different outcomes for the default and negotiation option is that lenders ex-ante mortgage yield spread should increase significantly to pay for the homeowner’s ex-post negotiation option.
c) Homeowners attempt to renegotiate with lenders when \( V(t) < V_{df} \), where \( V_{df} \) is the endogenously determined negotiation threshold given by

\[
V_{df} = \frac{\beta}{\beta - 1} \left[ g + \frac{\tau}{L} \right]^{-1} I \tag{12} \text{ or } \tag{A14}
\]

Homeowners default when \( V(t) < V_f \), where \( V_f \) is the endogenously determined default threshold given by

\[
V_f = \frac{\beta}{\beta - 1} [h + \tau]^{-1} I \tag{13} \text{ or } \tag{A6}
\]

We show in Section 4.5 that the implications of these equations are that negotiation option exercise will occur earlier than the default option exercise for all homeowners but strong negotiators will exercise their negotiation option earlier than weak negotiators will.

d) The homeowner renegotiates a new coupon

\[
C(V) = (1 - \alpha \phi)[(r - \mu)V] \tag{14} \text{ or } \tag{A41}
\]

In other words, the renegotiated mortgage coupon is the current notional market imputed rent \([(r - \mu)V]\) times the factor \((1 - \alpha \phi)\) which is either equal to or less than 1 depending on the homeowner/lender heterogeneous bargaining power and the probable foreclosure costs. If \((r - \mu)\) is small, the new coupon is very small if \(\alpha\) is large.
e) We define the optimal $LTV_{di}$ at mortgage origination $V_{di}$ for the negotiation option as the book value of debt divided by the property value at mortgage origination

$$LTV_{di} = \frac{D(V_{di}, c_d)}{F(V_{di}, c_d)}$$

This can be shown to be equivalent to

$$LTV_{di} = \frac{\gamma - [(1 - g^\gamma)(1 + \tau(\phi - 1))]}{\gamma(gL + \tau)} \quad [15] \text{ or } [A48]$$

f) The ex-post yield spread at origination is defined as

$$YS_{di} = \frac{c_d}{D(V_{di}, c_d)} - r \quad [16]$$

and

$$YS_i = \frac{c_*}{D(V_i, c^*)} - r \quad [17]$$

for both options respectively where $D(.)$ is the value of debt at the investment threshold $V_i$ or $V_{di}$.

We can show but more simply demonstrate in Section 4.5 that both the renegotiation threshold $V_{df}$ given by [12] and the optimal investment threshold $V_{di}$ given by [12] increase in the homeowner’s bargaining power : \( \frac{dV_{di}}{d\phi} > 0, \frac{dV_{df}}{d\phi} > 0 \). The optimal coupon payment $c_d$ given in [10] decreases in the homeowner’s bargaining power : \( \frac{dc_d}{d\phi} < 0 \). When the homeowner’s bargaining power is stronger, they can extract more out of the anticipated foreclosure costs and lenders, therefore, anticipate lower future coupons. As a result, the renegotiation threshold $V_{df}$ increases with $\phi$ which lowers tax benefits. This implies that $LTV$...
i.e. the amount lenders are prepared to offer and the optimal coupon level decreases with homeowners increasing bargaining power. Hence, the incentive to invest decreases in \( \phi \) and the stronger homeowner waits longer before purchasing a property. Another implication of both models is that the inefficiency of foreclosure costs \( \alpha \) enters directly into the determination of the optimal investment thresholds and coupon payments. Given that \( g, \tau, \gamma \) are constant while \( L \) depends on \( \phi \) and \( \alpha \), we show the interesting result in [15] that the optimal loan to value ratio \( LTV_{di} \) at origination is dependent only on the bargaining power \( \phi \) and the anticipated foreclosure costs \( \alpha \) with the lender assuming that the homeowner can always pay the coupon.

When compared to the traditional default only scenario (model 1), intuitively, a higher bargaining power gives homeowners more incentives to initiate or accelerate ex-post negotiation. However, this incentive, especially for those homeowners with weak bargaining power, must be balanced with the benefit of avoiding costly foreclosure. The result however is that the (model 2) renegotiation threshold \( V_{df} \) dominates or is higher than the (model 1) default threshold \( V_f \), i.e. \( g + \frac{1}{L} > h + \tau \). While \( g (>1) \) is independent of the bargaining power \( \phi \), \( L(\geq0) \) on the other hand is dependent on \( \phi \).

It is also important to note that the optimal investment threshold \( V_{di} \), the optimal renegotiation threshold \( V_{df} \) and finally the optimal coupon payment \( c_d \) are all proportional to the initial purchase price \( I \). As discussed earlier, what a homeowner occupier is prepared to pay for a property \( I \) may well be different from what a homeowner investor is prepared to pay for the same property due to the untaxed benefit of the market-imputed rent discussed earlier. This forms a crucial difference with the results of Chapter 5.
4.5 Negotiation and Default Option Analysis – A Stylised Example

The negotiation option represents the relationship between the investment and financing decisions, where the initial \textit{ex-ante} investment decision is dependent on a (potential) renegotiation between lender and homeowner. On the other hand, the default (non-bargaining) option represents the relationship where the homeowner makes the investment decision knowing that non-payment of the mortgage will certainly result in the forfeiture of all equity.

This section demonstrates the effects of a negotiation option in a graphical manner and compares the fundamentally different quantitative results that arise from the two options using stylised US mortgage data and the equations derived in the preceding section 4.4. Where appropriate we transform the stochastic property price $V($) to a LTV (%) where a LTV greater than 100% represents so-called negative equity. The parameter $\phi$ represents heterogeneous characteristics of the homeowner in relation to the lender impacting on their ability to negotiate. Recognising this impreciseness, we observe how the\textit{ negotiation region}, delineated by the extreme corner values of $\phi = (0, 1)$, in the various graphs, compares to the single \textit{default point}. \textit{Ex-ante} mortgage origination, a homeowner and lender will only know that their relative bargaining positions \textit{ex-post} a negotiation event must lie between these two extremes.

The homeowner decides to buy a property financed partly with debt paying the optimal coupon to a willing lender. The analysis proceeds as follows:

a) Calculate the mortgage (book) loan and payment at the optimal investment point.

b) Establish the critical negotiation region and default point as a function of BLTV.

c) Calculate the lender’s risk spread (over the riskless rate).

d) Illustrate (some) model sensitivities to foreclosure costs and volatility.
We define BLTV as the nominal loan/spot value of the property. The nominal loan is constant and does not include missed payments as we assume that negotiation is triggered by the homeowner based on the current “spot” property value. Figure 4.5.1 plots mortgage coupon against the LTV for the range of negotiation strengths $0 \leq \phi \leq 1$. The mortgage coupon is constant until the LTV decreases to the negotiation exercise point at which moment the homeowner and lender negotiate a reduced coupon conditional on the spot value of the property. The new lower coupon $C(V)$ is the product of the current market imputed rent $(r - \mu)V$ and a combination of the unavoidable foreclosure costs and their own negotiation ability $(1 - \alpha \phi)$.

Figure 4.5.1 Optimal Coupon Payment $S$ as a function of LTV and Bargaining Power $\phi$.

The three discontinuous curves labelled $\phi=0.0$, $0.5$ and $1.0$ are the coupon payment curves using equations [10] and [11].
For $\phi=0.0$ a coupon $c_0$ of $13253$ is paid up to the negotiation exercise point and thereafter a decreasing coupon depending on the Book LTV.
Coupon payments decrease as $\phi$ increases reflecting the lower mortgage (debt capacity) offered by the lender. The LTV quoted is at origination.
The lower (heavy yellow) straight line is the constant coupon for the default option and terminates at the default exercise point of $117\%$.
A coupon is always paid after exercising the negotiation option which becomes more affordable with increasing BLTV or negative equity.
Parameter values: $I = $ 250000, $r = 0.03$, $\mu = 0.00$, $\tau = 0.20$, $\alpha = 0.3$ and $\sigma = 0.10$.
We note that a weak borrower would according to these model parameters negotiate once their LTV reached 106% or a change of +7% from their LTV at origination. A strong borrower would negotiate almost immediately should their LTV increase above their LTV at origination. A borrower with a default only option would in contrast default as soon as their LTV reached 117% compared to their LTV at origination of 64%.

We plot in Figure 4.5.2 the coupon payment as a function of property value $F(V)$ instead of LTV %. We note that although a strong borrower with a negotiation option has a comparable LTV at origination to a borrower with a default only option (64% vs. 66%), the strong borrower pays a much higher monthly coupon for this right ($7973 vs. $6369).

**Figure 4.5.2 Optimal Coupon Payment $S$ as a Function of Property Price $V$ and Bargaining Power $\phi$.**

The three discontinuous curves labelled $\phi=0.0, 0.5$ and $1.0$ are the coupon payment curves using equations [10] and [11]. For $\phi=0.0$ a coupon $c_0$ of $13253$ is paid up to the negotiation exercise point and thereafter a decreasing coupon depending on the Book LTV Coupon payments decrease as $\phi$ increases reflecting the lower mortgage (debt capacity) offered by the lender. The LTV quoted is at origination The lower (heavy yellow) straight line is the constant coupon for the default option and terminates at the default exercise point of 117% A coupon is always paid after exercising the negotiation option which becomes more affordable with increasing BLTV or negative equity

Parameter values: $I = \$250000$, $r = 0.03$, $\mu = 0.00$, $\tau = 0.20$, $\alpha = 0.3$ and $\sigma = 0.10$
A strong negotiator should negotiate earlier than the weaker negotiator which might result in paying a reduced coupon, retaining ownership, collecting the market-imputed rent and retaining the “hope” that property values may bounce back recovering some lost equity.

The lender offers the highest mortgage \((LTV_{di} = 99\%)\) to the weak homeowner \((\phi = 0)\) who pays the highest coupon. In the other case where the homeowner is strong \((\phi = 1)\) the lender offers the lowest mortgage loan \((LTV_{di} = 64\%)\). This contrasts with the default option only where the mortgage \((LTV_d = 66\%)\) and constant coupon is not dependent on negotiation ability or heterogeneous characteristics of the homeowner.

Finally, the weaker the homeowner is as a negotiator the closer the negotiation trigger point is to the optimal default point and the more likely, given unfavourable property shocks, that a homeowner may very quickly move from exercising their negotiation option to exercising a default option. The stronger the homeowner is as a negotiator, the earlier that homeowner, who may not yet be in negative equity, exercises his negotiation option and the less likely that default will eventually result. It is clear that, with decreasing property values (i.e. increasing LTV), the economic consequences of the negotiation option are that the homeowner should endogenously choose to enter negotiation earlier and start paying a more affordable mortgage. With a default option, the homeowner will default later as soon as the critical threshold \((LTV=117\%)\) is reached. The more bargaining power the homeowner has, the earlier that negotiation will occur as the more financial concessions that may be extracted.

The overall direction of these results are consistent with PSV(2010) who claim that significant differences exist between the delinquency and default behaviour of securitised and non-securitised loans with larger effects for borrowers with a high FICO credit rating. Our
model also suggests that significant differences should exist between the default and delinquency behaviour of borrowers were they able to be characterised on the basis of their negotiation ability.

We compare the lender’s yield spread over the risk free rate for a negotiation and default option in Figure 4.5.3. A borrower’s negotiation option increases the lender’s required risk spread compared to a default option. Variation between yield curves for the three values of parameter $\phi$ in the negotiation case is relatively small compared to that of the default option. Differences in yield spreads are more dependent on the optimal investment entry points with the weaker negotiator paying a higher yield because they make the investment earlier than a strong negotiator does. The existence of any measure of bargaining or sharing introduces a fundamental change to the contract whereby the lender requires a higher yield spread.

Figure 4.5.3 Yield Spread (Basis Points) as a Function of Property Price $V$ and Bargaining Power $\phi$.

The three convex curves labelled $\phi=0.0, 0.5$ and $1.0$ are the yield spread curves at mortgage origination using equations [16] and [17]. For $\phi=0.0$ the yield spread at the entry threshold is 82 basis points and decreases as $\phi$ increases reflecting greater bargaining power. The lower dashed line is the yield spread curve for the default option with a value of 31 basis points at the investment entry threshold.

Yield curves for the negotiation option coincide closely (in this example) but all differ significantly from that of the default option.

Parameter values: $I = $ 250000, $r = 0.03, \mu = 0.00, \tau = 0.20, \alpha = 0.3$ and $\sigma = 0.10$. 
Figure 4.5.4 plots the property value $F(V)$ as a function of the “spot” asset price for different values of the bargaining parameter $\phi$. Property value increases as $\phi$ increases reflecting the greater benefit of the tax shelter when the homeowner has weaker bargaining power. The rectangular shaded region represents the upper and lower limits of the bargaining parameter wherein negotiation optimally occurs. The heavy dashed curve is the property value for the default option and terminates at the exit threshold when the homeowner defaults, puts the mortgage loan to the lender and receives no equity. The lender then sells the property receiving the spot value less the foreclosure costs.

The three concave curves labelled $\phi=0.0, 0.5$ and $1.0$ are the property values $F(V)$ using equations [A26] and [A27]. Property value increases as $\phi$ decreases reflecting the greater benefit of the tax shelter when the homeowner has weak bargaining power. The rectangular shaded region is defined as the upper and lower limits of the bargaining parameter wherein negotiation optimally occurs. The heavy dashed curve is the property value for the default option and terminates at the exit threshold where the lender will foreclose and sell the property with $\alpha = 30\%$ foreclosure costs while the homeowner gets nothing.

Parameter values: $I = \$250000, r = 0.03, \mu = 0.00, \tau = 0.20, \alpha = 0.3$ and $\sigma = 0.10$
We summarise key graphical data from Figures 4.5.1 and 4.5.3 in Table 4.5.1 overleaf, showing the effect of increasing foreclosure costs $\alpha$ and changes in property volatility $\sigma$. Increases in property price volatility $\sigma$ with no changes in other parameters behaves as expected that is delaying investment, increasing yield spreads and reducing debt capacity (LTV) at origination. The higher the foreclosure costs, the lower the LTV the lender should agree with the strong negotiator while continuing to offer the same LTV to the weak negotiator. A large decrease in (LTV) lending capacity from 89% to 72% can be observed for the average value of $\phi=0.5$ as foreclosure costs increase from 10% to 50%. Whether a lender in a strong negotiation position might lend to a homeowner in a weak negotiation position with probable large foreclosure costs is perhaps self-evident after reflection on lending practises and the effects of securitisation in the recent 2007/2008 US subprime crises.

Increasing volatility has a surprising effect resulting in an earlier exercise of the negotiation option but later exercise of the default option. This might indicate that a homeowner facing certain foreclosure “sits tighter” longer during periods of high volatility while a homeowner with a negotiation option will negotiate earlier for a more affordable mortgage coupon.

This is in itself not a surprising result and appears intuitively correct. From an option viewpoint, the interesting point is that volatility cause one option to accelerate and the other to decelerate. Unfortunately, this phenomenon, if correct, would make it very difficult to distinguish those homeowners who intend to default from those homeowners who only wish to negotiate. In this model, one might suppose that the first wave of homeowners contacting the lender are actually those who wish to negotiate rather than default. Homeowners who are poor negotiators would only contact the lender at the very last moment. This type of behaviour again would seem intuitively correct.
Table 4.5.1 Negotiation Option Sensitivity to Asset Volatility and Foreclosure Costs.

<table>
<thead>
<tr>
<th>φ</th>
<th>Mortgage Coupon $</th>
<th>Book LTV % @ Default or Origination</th>
<th>Yield Spread Basis Points</th>
<th>LTV @ Origination %</th>
</tr>
</thead>
</table>

| Foreclosure % α = 10%, σ=0.10, μ=0.00 |  |  |  |  |
|---------------------------------------|-----------------------------------|--------------------------|----------------------|
| 0                                    | 13253                             | 106                      | 82                   | 99                   |
| 0.5                                  | 11472                             | 93                       | 75                   | 89                   |
| 1                                    | 9981                              | 82                       | 68                   | 81                   |
| Default                              | 7670                              | 113                      | 36                   | 77                   |

| Foreclosure % α = 30%, σ=0.10, μ=0.00 |  |  |  |  |
|---------------------------------------|-----------------------------------|--------------------------|----------------------|
| 0                                    | 13253                             | 106                      | 82                   | 99                   |
| 0.5                                  | 10413                             | 83                       | 75                   | 81                   |
| 1                                    | 7973                              | 64                       | 68                   | 64                   |
| Default                              | 6369                              | 117                      | 31                   | 66                   |

| Foreclosure % α = 50%, σ=0.10, μ=0.00 |  |  |  |  |
|---------------------------------------|-----------------------------------|--------------------------|----------------------|
| 0                                    | 13253                             | 106                      | 82                   | 99                   |
| 0.5                                  | 9323                              | 73                       | 75                   | 72                   |
| 1                                    | 5853                              | 45                       | 68                   | 47                   |
| Default                              | 5572                              | 119                      | 27                   | 59                   |

| Foreclosure % α = 30%, σ=0.20, μ=0.00 |  |  |  |  |
|---------------------------------------|-----------------------------------|--------------------------|----------------------|
| 0                                    | 29321                             | 61                       | 255                  | 85                   |
| 0.5                                  | 23051                             | 46                       | 243                  | 68                   |
| 1                                    | 17658                             | 35                       | 233                  | 52                   |
| Default                              | 9002                              | 130                      | 91                   | 71                   |

Summary of the outcomes for different values of α and σ illustrating the sensitivity of the output to different input parameters. The second sub-table from the top down are the results for the base parameter case used for Figures 4.5.1 and 4.5.2. The first three sub-tables demonstrate that a lender offers a smaller loan (LTV at origination) as foreclosure costs α increase. Increasing volatility σ increases the required yield spread, lowers the negotiation trigger point but increases the default trigger point. Parameter values: I = $250000, r = 0.03, μ = 0.00, τ = 0.20.
4.6 Chapter Conclusions

We have combined two different aspects of real options that of irreversible investment and debt pricing/capital structure, to develop closed form solutions by which the borrower can choose the optimal *ex-ante* mortgage terms (LTV and mortgage coupon) and *ex-post* timing to exercise their negotiation option. We achieve this by applying methodological aspects of strategic endogenous default (Leland 1994) developed for corporate bond valuation to the *ex-ante* valuation of negotiated mortgages.

The default model can be regarded as the worst case from a lender’s viewpoint as the model has been developed by maximising the homeowner’s equity. In the real (option) world, exercise of the negotiation option may be initiated not only by a desire to optimise the homeowner’s equity but also by a sub optimal trigger event. In both cases, a prudent lender (or policymaker) who has *ex-ante* priced their mortgages based on an optimal negotiation exercise may be in a stronger negotiation position by assuming that a sub optimal exercise will cost the homeowner more than the lender. From a lender’s viewpoint, the optimal exercise behaviour may be less deserving than the second sub optimal exercise. The model suggests that in a declining house market those stronger negotiators, whether deserving or not, will initiate a negotiation earlier. Consequently, lenders may need to screen these first wave of homeowner applicants more closely with consequent higher screening and monitoring costs.

Policymakers and lenders should also be aware that with increasing property price volatility the model suggests some homeowners may try to accelerate the moment of negotiation while paradoxically other homeowners may try to delay the moment of default depending on which real option they intend to exercise.
This makes perfect sense from an ethical viewpoint but is typical of the moral hazard faced by the lender. If default results in certain foreclosure then homeowners will not be anxious to default. However, if negotiation results in a more affordable mortgage coupon, then homeowners in contrast will accelerate the exercise of that option. The question therefore arises as to whether this effect is observable in the current housing market as suggested by the Experian- Oliver Wyman (2010) report.

We emphasise that the option to renegotiate the mortgage payment is not a “free ride” or a costless option for the homeowner. The lender charges *ex-ante* higher yield spreads for this right compared to the default option. We have shown that the lender is no worse off in whatever bargaining position he finds himself and in most cases will be better off. Ultimately, if the lender cannot agree a new mortgage payment with a homeowner, then he can always foreclose with inevitable costs. We emphasise that it is a temporary mortgage modification which can revert to the original coupon when property prices recover. The homeowner remains responsible for paying off the full mortgage principal.

Implicit in our modelling is that the lender and homeowner should always agree new (sliding) mortgage payments conditional on the current property value. This is surely an abstraction from reality where in practise, due to the same aforementioned monitoring and screening costs, only one new lower affordable mortgage payment may be agreed, whereupon non-performance might lead to irrevocable foreclosure. The current HAMP program is such that, after renegotiation, non-performance after 3 months leads to inevitable foreclosure and is the scenario modelled in Chapter 6.
We have introduced an additional bargaining parameter $\phi$ (related to future unavoidable foreclosure costs) and compared to the traditional option theoretic mortgage default treatment. This parameter $\phi$ is a convenient construct to easily divide the benefits of avoiding foreclosure costs between lender and homeowner. The parameter is heterogeneous in that two homeowners with the same lender (or servicing agent) may have different values resulting in different outcomes of the (re)negotiated mortgage payment. In any case, we are less interested in the exact value of $\phi$ and more interested in delineating the maximum and minimum boundaries of the critical LTV region where renegotiation of the mortgage coupon may occur as a result of both parties wishing to avoid foreclosure costs. Better understanding of this region, compared to the traditional default region, may help lenders better screen (weak) homeowners who contact them later from those (strong) homeowners who contact them earlier and try to take advantage of lender weakness.

The strategic negotiation option has been demonstrated to have \textit{ex-post} distinct economic and financial consequences. It remains to empirically investigate whether this idea of homeowners strategically delaying payments and negotiating actually occurs within an option theoretic equity optimising framework or rather within some other “affordability optimising” framework.
4.7 References


4.8 Glossary of Notation

\( V \): “Spot” Property Price.

\( V_i \): Optimal property price at where the borrower would invest with a default option only.

\( V_f \): Optimal property price at where the borrower would default with a default option only.

\( V_{di} \): Optimal property price at where the borrower would invest with a negotiation option.

\( V_{df} \): Optimal property price at where the borrower would negotiate.

\( \sigma \): Net property price volatility.

\( \mu \): Net property price drift.

\( \beta, \gamma \): Roots of \( \frac{\sigma^2 V^2}{2} + (\mu - \sigma^2/2) V - r = 0 \)

\( \alpha \): Lender’s foreclosure costs as a percentage of the property price \( V \)

\( \tau \): Borrowers tax rate

\( r \): Risk free rate of return

\( c^* \): Perpetual mortgage payment to the lender for the default only option (tax deductible).

\( c_d \): Perpetual mortgage payment to the lender for the negotiation option (tax deductible).

\( C(V) \): Renegotiated mortgage payment (after delinquency) which depends on property price.

\( \phi \): Heterogeneous bargaining or sharing parameter which lies between 0 and 1.

\( I \): Initial property investment made at the critical investment thresholds \( V_i \) or \( V_{di} \).

\( YS_i \): Risk adjusted Yield Spread for the default only option at origination \( V_i \).

\( YS_{di} \): Risk adjusted Yield Spread for the negotiation option at origination \( V_{di} \).

\( F(V, C) \): Asset value as a function of property price and mortgage payment.

\( E(V, C) \): Equity value as a function of property price and mortgage payment.

\( D(V, C) \): Debt value as a function of property price and mortgage payment.

\( F_{ae}(V_f, 0) \): Property asset value at default with all equity \((ae)\) financing and no taxes.

\( F_{ae}(V_{df}, 0) \): Property asset value at delinquency with all equity \((ae)\) financing and no taxes.

\( L \): defined as \( \frac{1 - r(1 - \phi)}{1 - \phi \alpha} \)

\( g \): defined as \( \left[ \beta^{\gamma (1 - \gamma) \alpha} \right]^{1/\gamma} = \frac{V_{df}}{V_{ae}} \) used for the negotiation option.

\( h \): defined as \( \left[ 1 - \frac{\gamma (\tau + \phi \alpha)}{\tau} \right]^{-1/\gamma} = \frac{V_i}{V_f} \) used for the default only option.
4.9 Detailed Model Derivation

4.9.1 Default (No Bargaining) Option – Model 1

We proceed by first deriving a “default no bargaining” case based on Leland (1994), where the lender and homeowner do not bargain or share the value of the assets at the optimally chosen critical default threshold $V_f$. The lender forecloses and the property automatically becomes all equity financed and is presumed sold with no future tax benefits. At which point the value of the equity claim is $E(V_f) = 0$ and $D(V_f) = (1 - \alpha)F_{ae}(V_f, 0)$ where $\alpha$ is the loss severity percentage, $E(V_f)$ is value of equity and $D(V_f)$ the value of debt at the default threshold $V_f$ while $F_{ae}(V_f, 0)$ is the property value of an all equity financed investment with 0 coupon. Thus $\alpha F_{ae}(V_f, 0)$ is taken away by outsiders, $(1 - \alpha)F_{ae}(V_f, 0)$ by the lender, and the homeowner gets nothing in strategic default.

If $V$ follows a general Brownian motion given by [A1] then the solution for the general differential equation $F(V)$ where $aV^2F_{VV} + bVF_V + cF = Vd + e$

is given by $F(V) = A_1V^\gamma + A_2V^\beta + \frac{Vd}{b+c} + \frac{e}{c}$

where $\gamma < 0$ and $\beta > 0$ are roots with $\gamma, \beta = \frac{(a-b)\pm\sqrt{(b-a)^2-4ac}}{2a}$

and $A_1, A_2$ are determined from the appropriate boundary conditions.

The “spot” property price of the property, denoted by $V$, follows the general Brownian motion process given by

$$dV = \mu Vdt + \sigma VdW \quad [A1]$$
Where $\mu$ is the instantaneous expected rate of return of the property gross of any payout, $\sigma$ is the instantaneous variance of the property price and $dW$ is a standard Brownian motion.

Given [A1] the asset value $F(V)$ of a claim paying $Vd + e$ satisfies the equilibrium condition

$$rF(V_t) = Vd + e + \frac{1}{dt} \mathbb{E}_t \left[ F(V_{t+dt}) \right]$$

The first two terms are the expected cash flow while the third term is the expected capital gain of $F(V_t)$ from $t$ to $t+dt$.

Using [A1] and applying Ito’s lemma to the third term we get a general ODE

$$rF(V) = Vd + e + \frac{1}{2} \sigma^2 V^2 F_{VV}(x) + \mu V F_V(V)$$

with $a = \frac{1}{2} \sigma^2, b = \mu, c = -r$ and $\gamma, \beta = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left[ \frac{1}{2} - \frac{\mu}{\sigma^2} \right]^2 + \frac{2r}{\sigma^2}}$

where $r > \mu$ is the risk neutral instantaneous rate of return.

This has a general solution of the form

$$F(V) = A_1 V^\gamma + A_2 V^\beta + \frac{Vd}{(r - \mu)} + \frac{e}{r} \quad [A2]$$

Solutions are sought for three value functions namely:

a) $D(V) \rightarrow$ the lenders debt value ,

b) $E(V) \rightarrow$ the borrower’s equity value,

c) $F(V) \rightarrow$ the total property financing arrangement value.

The borrower has control over capital structure and decides on the mix of debt $D(V,C)$ and equity $E(V,C)$ to finance the property whereby $F(V,C) = D(V,C) + E(V,C)$. The all equity financed property value is given by $F_{ae}(V,0)$. Hence at origination $(t=0)$ $D(V_t,c^*) +$
$E(V_t, c^*) = I$ where $V_t$ is the (initial threshold) property value that induces the investor to invest and $c^*$ is the optimally chosen perpetual coupon agreed with the lender and $I$ the initial investment. Debt value $D(V, C)$ is derived by letting $d=0$ and $e=c$ in [A2] and solving with the following smooth pasting and value matching conditions

$$
\lim_{V \to \infty} D(V, C) = \frac{C}{r}
$$

$$
D(V_f, C) = (1 - \alpha) F_{ae}(V_f, 0)
$$

Resulting in

$$
D(V, C) = \frac{C}{r} \left[ 1 - \left( \frac{V}{V_f} \right)^Y \right] + (1 - \alpha) F_{ae}(V_f, 0) \left( \frac{V}{V_f} \right)^Y \text{ for } V > V_f \quad [A3]
$$

a) Equity value $E(V, C)$ is derived by letting $d = r - \mu$ and $e = (1 - \tau)C$ in [A2] and solving with the following smooth pasting and value matching conditions

$$
\lim_{V \to \infty} E(V, C) = V - \frac{(1 - \tau)C}{r}
$$

$$
E(V_f, C) = 0
$$

Resulting in

$$
E(V, C) = (1 - \tau) \left[ (V - \frac{C}{r}) - (V_f - \frac{C}{r}) \left( \frac{V}{V_f} \right)^Y \right] \text{ for } V > V_f \quad [A4]
$$

b) Property value $F(V, C)$ is derived by letting $d = r - \mu$ and $e = \tau C$ in [A2] and solving with the following smooth pasting and value matching conditions

$$
\lim_{V \to \infty} F(V, C) = V + \frac{\tau C}{r}
$$

$$
F(V_f, C) = 0
$$

$$
F(V, C) = F_{ae}(V, 0) + \frac{\tau C}{r} \left[ 1 - \left( \frac{V}{V_f} \right)^Y \right] - \alpha F_{ae}(V_f, 0) \left( \frac{V}{V_f} \right)^Y \text{ for } V > V_f \quad [A5]
$$
Property value can be thus decomposed into the sum of three components

1) The all equity property value
2) Plus the tax shield value of benefits and
3) Less the foreclosure costs of \( V \) which the lender suffers in strategic default \( (V<D) \)

The tax benefit value of debt \( \frac{\tau C}{r} \left[ 1 - \left( \frac{V}{V_f} \right)^Y \right] \) is an increasing and concave function of \( V \) whereby if \( \tau > 0 \) there is some range which indicates that tax savings are achieved by higher levels of \( C \). However \( \frac{\tau C}{r} \left[ 1 - \left( \frac{V}{V_f} \right)^Y \right] \) starts to decline with \( C \) at a certain point with the potential loss of tax benefits on foreclosure. Similarly \( \alpha F_{ae}(V_f,0) \left( \frac{V}{V_f} \right)^Y \) is increasing and convex in \( C \) but decreasing and convex in \( V \). In a similar vein the endogenous default policy that maximises equity value for a given debt level solves \( E_V(V) = \frac{\partial E(V,C)}{\partial V} = 0 \) at \( V = V_f \) and can be shown to be given by

\[
V_f = \frac{y}{y-1} \frac{C}{r}
\]

Or using [A8]

\[
V_f = \frac{\beta}{\beta-1} \left[ h + \tau \right]^{-1} I
\]

Where \( h = \left[ 1 - \frac{y(\tau + \alpha)}{\tau} \right]^{-1/y} = \frac{V}{V_f} \)

Rewriting [A5] we obtain

\[
F(V,C) = F_{ae}(V,0) + \frac{\tau C}{r} - \left( \frac{V}{V_f} \right)^Y \left[ \frac{\tau C}{r} + \alpha F_{ae}(V_f,0) \right]
\]

And substituting \( V_f \) from [A6] gives

\[
= F_{ae}(V,0) + \frac{\tau C}{r} \left( \frac{V}{\gamma c} \left[ \frac{\tau C}{r} + \alpha \frac{\gamma}{\gamma-1} \frac{C}{r} \right] \right)^Y
\]
Grouping $C$ in the $3^{rd}$ term together and some cancellations gives

$$F_{ae}(V, 0) + \frac{\tau C}{r} - \left( \frac{V}{r \frac{y}{y-1} r} \right)^{\gamma} \left[ \frac{\tau C^{1-\gamma}}{r} + \alpha C^{1-\gamma} \frac{\gamma}{y - 1} \right]$$

Then taking $F_{ae}(c)$

$$\frac{\tau}{r} - \left( \frac{V}{y \frac{y-1}{y-1} r} \right)^{\gamma} \left[ (1 - \gamma) \frac{\tau c^{-\gamma}}{r} + \alpha (1 - \gamma)c^{-\gamma} \frac{\gamma}{y - 1} \right]$$

Substituting $c^*$, extracting $c^{-\gamma}$ back and setting the expression equal to 0 gives

$$\frac{\tau}{r} = \left( \frac{V}{c^* \frac{y}{y-1} r} \right)^{\gamma} \left[ (1 - \gamma) \frac{\tau}{r} - \frac{a\gamma}{r} \right]$$

$$\frac{\tau}{r} = \left( \frac{V}{c^* \frac{y}{y-1} r} \right)^{\gamma} \left[ \frac{\tau - a\gamma}{r} \right]$$

Giving

$$c^* = \frac{Vr \gamma - 1}{h \gamma}$$

where $h = \left[ 1 - \gamma \frac{\tau (1 + \alpha)}{r} \right]^{-1/\gamma}$

We now assume that the borrower only agrees the optimal coupon $c^*$ with the lender and obtains the required funds when the property value $V = V_i$ the entry level threshold thus

$$c^* = \frac{V_i r \gamma - 1}{h \gamma}$$

Or using [A8] $c^* = r \frac{\gamma - 1}{\gamma} \frac{\beta - 1}{\beta - 1} [h + \tau]^{-1/\gamma}$ [A7] or [11]
We next examine the relationship of $V_i$ with the initial investment $I$ but show the derivation in more detail in the next case in Section 4.9.2.2.

By investing the borrower collects $E(V, c) - (I - D(V, c)) = F(V, c) - I$

It can be shown that this implies that the optimal investment threshold $V_i$ is given by

$$V_i = \frac{\beta}{\beta - 1} \left( I - \frac{\tau c^*}{r} \right) + \frac{\beta - \gamma}{\beta} \left( \alpha V_f + \frac{\tau c^*}{r} \right) \left( \frac{V}{V_f} \right)^\gamma$$

Using $V_f$ in [A6] and letting $\frac{v_i}{V_f} = h$

$$V_i(\beta - 1) = \beta I - \beta \frac{\tau c^*}{r} + \frac{(\beta - \gamma) c^*}{(\gamma - 1)} \left( -\tau + (\alpha + \tau)\gamma \right) h^\gamma$$

$$\beta I - \beta \frac{\tau c^*}{r} + \frac{(\beta - \gamma) c^*}{(\gamma - 1)} \left[ -\tau + (\alpha + \tau)\gamma \right] h^\gamma$$

$$\beta I - \beta \frac{\tau c^*}{r} + \frac{(\beta - \gamma) c^*}{(\gamma - 1)} \left[ -\tau(1 - \frac{\gamma(\tau + \alpha)}{\tau}) \right] h^\gamma$$

$$= \beta I - \beta \frac{\tau c^*}{r} + \frac{(\beta - \gamma) c^*}{(\gamma - 1)} \left[ -\tau h^\gamma \right] h^\gamma$$

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\[ = \beta l + \frac{tc^*}{r} \left( -\beta + \frac{\beta - \gamma}{1 - \gamma} \right) \]
\[ = \beta l + \frac{tc^*}{r} \left( \gamma \frac{\beta - 1}{1 - \gamma} \right) \]

And then substituting [A7]

\[ = \beta l - \left[ \tau (\beta - 1) \frac{V_l}{\tilde{h}} \right] \]

Bringing terms in \( l \) and \( x_i \) to opposite sides and rearranging gives

\[ V_l = \frac{\beta}{\beta - 1} \left[ 1 + \frac{\tau}{\tilde{h}} \right]^{-1} l \]

[A8] or [9]

4.9.2 Negotiation (Bargaining) Option- Model 2

With a negotiation option, in contrast to the default option, the lender and homeowner agree a new coupon conditional on sharing the avoided foreclosure costs at the optimally chosen critical negotiation threshold, denoted by \( V_{df} \). The bargaining process will thus result in the reinstatement of the mortgage at the new lower payment. It is presumed that in this (actual or threatened) negotiation the lender mitigates his foreclosure costs as much as possible but is finally indifferent to whether the homeowner or an outsider “profits” from the value of the potential foreclosure costs such as administration, legal fees, social costs, loss on the property sale, etc. The sharing rule is now given by \( E(V_{df}) = \alpha \phi F_{ae}(V_{df}, 0) \) and \( D(V_{df}) = (1 - \alpha \phi) F_{ae}(V_{df}, 0) \) where \( \alpha \) is the loss severity and \( \phi \) the bargaining strength \( 0 \leq \phi \leq 1 \).

If \( \phi = 0 \) then the weak homeowner gets nothing while the lender gets \( F_{ae}(V_{df}, 0) \). If \( \phi = 1 \) then the strong homeowner gets \( \alpha F_{ae}(V_{df}, 0) \) (which outsiders would otherwise have received) while the weak lender receives \( (1 - \alpha) F_{ae}(v_{df}, 0) \). If \( \phi = 1/2 \) then the homeowner
gets \( \alpha F_{ae}(V_{df}, 0)/2 \) (which outsiders would otherwise have received) while the lender receives \((1 - \alpha/2) F_{ae}(V_{df}, 0)\). Thus in all cases the lender is either better off or indifferent compared to the default case while the homeowner may be better off, but rather resume mortgage payments at a new more affordable coupon \( C(V) \).

The first section 4.9.2.1 of the derivation examines the case when borrower and lender bargain over the equity value at foreclosure, while the second section 4.9.2.2 extends the case to where they bargain over the added option value of including the homeowner’s tax benefits. The choice of either specification will be dependent on the tax regime applicable to residential homeowners which differs from country to country.

### 4.9.2.1 Without Sharing Tax Benefits – Model 2

We proceed by solving for \( E(V) \)

Where

\[
E(V) = \frac{1}{2} \sigma^2 V^2 E_{VV} + \mu V E_v - r E + (r - \mu) V - C(1 - \tau) = 0
\]

As the value of the assets approaches infinity debt becomes riskless and

\[
\lim_{V \to \infty} E(x) = V - \frac{(1 - \tau)C}{r}
\]

The new lower boundary conditions follow from the bargaining game whereby

\[
\lim_{V \to V_{df}} E(V) = \phi \alpha V \quad \text{[A9a]}
\]

\[
\lim_{V \to V_{df}} E_v(V) = \phi \alpha \quad \text{[A9b]}
\]

Using the general solution

\[
E(V) = A_0 + A_1 V^\gamma + A_2 V^\beta
\]

as \( V \to \infty, V^\beta \to \infty \quad \Rightarrow \quad A_2 = 0 \quad \text{[A10]}

while as \( V \to \infty, V^\gamma \to 0 \quad \Rightarrow \quad A_0 = V - \frac{C(1 - \tau)}{r} \frac{160}{160}
Thus \([A10]\) becomes

\[
E(V) = V - \frac{C(1 - \tau)}{r} + A_1 V^\gamma
\]

\([A11]\)

Differentiating \([A11]\) w.r.t. \(V\) and substituting \([A9b]\) implies

\[
E_{v}(V) = 1 - 0 + \gamma A_1 V^{\gamma-1} = \phi \alpha
\]

\([A12]\)

While substituting \([A9a]\) in \([A11]\) implies

\[
E(V_{df}) = V_{df} - \frac{C(1 - \tau)}{r} + A_1 V_{df}^\gamma = \phi \alpha V_{df}
\]

\([A13]\)

And then eliminating \(A_1\) from \([A12]\) and \([A13]\)

\[
\phi \alpha V_{df} = V_{df} - \frac{C(1 - \tau)}{r} + \frac{\phi \alpha - 1}{\gamma} V_{df}^{\gamma-1}
\]

\[
= V_{df} - \frac{C(1 - \tau)}{r} + \frac{\phi \alpha - 1}{\gamma} V_{df}
\]

\[
=> \frac{C(1 - \tau)}{r} = V_{df} \left[1 - \phi \alpha + \frac{(\phi \alpha - 1)}{\gamma}\right]
\]

\[
= V_{df} \left(1 - \phi \alpha\right) \left[1 - \frac{1}{\gamma}\right]
\]

\[
= V_{df} \left(1 - \phi \alpha\right)^{\gamma - \frac{1}{\gamma}}
\]

\[
V_{df} = \frac{C(1 - \tau)}{r(1 - \phi \alpha)^{\gamma - 1}}
\]

\([A14]\) or \([12]\)

\[
= \frac{\beta}{\beta - 1} \left[g + \frac{\tau}{L}\right]^{-1} l
\]

Taking \(A_1\) from \([A13]\)

\[
\left(\phi \alpha V_{df} - V_{df} + \frac{C(1 - \tau)}{r}\right) V_{df}^{\gamma - 1}
\]

and substituting in \([A11]\)
\[ E(V) = V - \frac{C(1 - \tau)}{r} + \left( \phi \alpha V_{df} - V_{df} + \frac{C(1 - \tau)}{r} \right) V_{df}^\gamma V^\gamma \]

\[ E(V) = V - \frac{C(1 - \tau)}{r} \left[ 1 - \left( \frac{V}{V_{df}} \right)^\gamma \right] - V_{df} \left( \frac{V}{V_{df}} \right)^\gamma (1 - \phi \alpha) \]  

[\text{A15}]

We repeat the derivation for \( D(V) \) but using \( D(V_{df}) = (1 - \alpha \phi) V_{df} \) to obtain

\[ D(V) = \frac{C}{r} \left[ 1 - \left( \frac{V}{V_{df}} \right)^\gamma \right] + V_{df} \left( \frac{V}{V_{df}} \right)^\gamma (1 - \phi \alpha) \]  

Thus the total property value \( F(x) = E(x) + D(x) \) is given by

\[ V - \frac{C(1 - \tau)}{r} \left[ 1 - \left( \frac{V}{V_{df}} \right)^\gamma \right] - V_{df} \left( \frac{V}{V_{df}} \right)^\gamma (1 - \phi \alpha) + \frac{C}{r} \left[ 1 - \left( \frac{V}{V_{df}} \right)^\gamma \right] + V_{df} \left( \frac{V}{V_{df}} \right)^\gamma (1 - \phi \alpha) \]

\[ \text{or} \quad V + \frac{C}{r} \left[ 1 - \left( \frac{V}{V_{df}} \right)^\gamma \right] (-1 + \tau + 1) \]

Reducing to

\[ F(V) = V + \frac{C \tau}{r} \left[ 1 - \left( \frac{V}{V_{df}} \right)^\gamma \right] \]  

[\text{A16}]

4.9.2.2 Sharing Tax Benefits – Model 2

This next section further extends the negotiation option whereby homeowner and lender negotiate over the extra option value arising from the tax benefit. Note that with an owner occupied mortgage in contrast to a non-owner occupied mortgage the notional income arising from property value increases is not immediately taxable.

We first solve for the total property value \( F(x) \).

The total value of the property \( F(V) \) satisfies the following differential equations

\[ F(V) = \frac{1}{2} \sigma^2 V^2 F_{VV} + \mu V F_V - rF + (r - \mu)V + \tau C = 0 \quad \text{when} \quad V > V_{df} \]  

[\text{A17}]

\[ 162 \]
and

\[ F(V) = \frac{1}{2} \sigma^2 V^2 F_{vv} + \mu VF_v - rF + (r - \mu)V = 0 \text{ when } V \leq V_{df} \]  

[A19]

with an upper boundary condition given by

\[ \lim_{V \to \infty} F(V) = V + \frac{\tau C}{r} \]  

[A20]

The lower boundary conditions follow from the bargaining game whereby

\[ \lim_{V \downarrow V_{df}} F(V) = \lim_{V \downarrow V_{df}} F(V) \]  

[A21]

\[ \lim_{V \downarrow V_{df}} F_v(V) = \lim_{V \downarrow V_{df}} F_v(V) \]  

[A22]

\[ \lim_{V \downarrow V_{10}} F(V) = 0 \]  

[A23]

We find a solution of the form \( F(V) = A_0 + A_1 V^\gamma + A_2 V^\beta \)

Firstly for \( V > V_{df} \)

As \( V \to \infty, V^\beta \to \infty \) \( \Rightarrow A_2 = 0 \)

while as \( V \to \infty, V^\gamma \to 0 \) \( \Rightarrow A_0 = V + \frac{\tau C}{r} \)

Secondly for \( V \leq V_{df} \)

as \( V \to 0, V^\gamma \to 0 \) \( \Rightarrow A_1 = 0 \) while \( \frac{\tau C}{r} = 0 \)

Giving

\[ F(V) = V + \frac{\tau C}{r} + A_1 \left( \frac{V}{V_{df}} \right)^\gamma \text{ when } V > V_{df} \]  

[A24]

and

\[ F(V) = V + A_2 \left( \frac{V}{V_{df}} \right)^\beta \text{ when } V \leq V_{df} \]  

[A25]

However using boundary conditions [A21] when \( V = V_{df} \Rightarrow \frac{\tau C}{r} + A_1 = A_2 \)

Differentiating [A24] and [A25] and using [A22] when \( V = V_{df} \Rightarrow \gamma A_1 = \beta A_2 \)
Solving for $A_1$ and $A_2$ gives $A_1 = \frac{\tau c}{r} \left( \frac{\beta}{\beta - \gamma} \right)$ and $A_2 = \frac{\tau c}{r} \left( \frac{\gamma}{\beta - \gamma} \right)$

\[
F(V) = V + \frac{\tau c}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{V}{V_{df}} \right)^{\gamma} \right] \text{when } V > V_{df} \tag{A26}
\]

\[
F(V) = V + \frac{\tau c}{r} \left( \frac{-\gamma}{\beta - \gamma} \right) \left( \frac{V}{V_{df}} \right)^{\beta} \text{ when } V \leq V_{df} \tag{A27}
\]

The total value of the property financing arrangement $F(V)$ includes the value of tax benefits and is thus higher than just the property value $V$. The borrower and lender thus bargain over a larger amount (when $V \leq V_{df}$) and from a bargaining viewpoint the share is such that $\bar{E}(V) = \bar{a}F(V)$ and $\bar{D}(V) = (1 - \bar{a})F(V)$. The Nash solution $\theta^*$ can be rewritten as

\[
\bar{\theta}^* = \arg \max \{ \bar{a}F(V) - 0 \}^\phi \{(1 - \bar{a})F(V) - \max \{(1 - \phi,V,0)\}\}^{1-\phi}
\]

\[
= \min \left[ \phi - \phi \left( \frac{1 - \alpha}{F(V)} \right) V, \phi \right]
\]

\[
= \min \left[ \phi(1 - \frac{(1 - \alpha)}{F(V)} V), \phi \right]
\]

\[
= \min[\phi(F(V) - (1 - \alpha)V), \phi]
\]

We next solve for the equity value $E(V)$ by setting up the ODE for the equity relationship

The property value of the equity $E(V)$ satisfies the following differential equations

\[
E(V) = \frac{1}{2} \sigma^2 V^2 E_{VV} + \mu V E_v - r V + (r - \mu) V - C(1 - \tau) = 0 \text{ when } V > V_{df} \tag{A28}
\]

and

\[
E(V) = \frac{1}{2} \sigma^2 V^2 E_{VV} + \mu V E_v - r V + (r - \mu) V - C(V) = 0 \text{ when } V \leq V_{df} \tag{A29}
\]

where $C(V)$ is the coupon paid to the lender after renegotiation with boundary conditions

\[
\lim_{V \to \infty} E(V) = V - \frac{C(1 - \tau)}{r} \tag{A30}
\]
Lower boundary conditions follow from the extra value of $F(V)$ using $[A27]$ with $V = V_{df}$

$$\lim_{V \to V_{df}} E(V) = \phi(\alpha V_{df} - \frac{\tau C}{r} \frac{\gamma}{\beta - \gamma}) \quad [A31]$$

Differentiating $[A27]$ and again substituting $V = V_{df}$ gives

$$\lim_{V \to V_{df}} E'(V) = \phi'(\alpha) \frac{\tau C}{r} \frac{\gamma^2}{V_{df} \beta - \gamma} \quad [A32]$$

We find a solution of the form $E(V) = A_0 + A_1 V^\gamma + A_2 V^\beta$

Firstly for $V > V_{df}$

as $V \to \infty, V^\beta \to \infty$ \quad $\Rightarrow A_2 = 0$

while as $V \to \infty, V^\gamma \to 0$ \quad $\Rightarrow A_0 = V - \frac{C(1-\tau)}{r}$

Giving

$$E(V) = V - \frac{C(1-\tau)}{r} + A_1 V^\gamma \text{ when } V > V_{df} \quad [A33]$$

Differentiating $[A33]$ w.r.t. $V$ implies

$$E'(V) = 1 - 0 + \gamma A_1 V^{\gamma-1} \quad [A34]$$

Setting $[A34]$ equal to $[A33]$ then multiplying across by $\frac{V}{\gamma}$ and substituting $V = V_{df}$ gives,
\[ \frac{V_{df}}{\gamma} + A_1 V_{df}^\gamma = \frac{\phi \alpha V_{df}}{\gamma} - \phi \frac{\tau C}{r} \frac{\beta}{\beta - \gamma} \]  \hspace{1cm} [A35]

But then using [A31] and [A33] with \( V = V_{df} \) we also get

\[ V_{df} - \frac{C(1 - \tau)}{r} + A_1 V_{df}^\gamma = \phi \alpha V_{df} - \phi \frac{\tau C}{r} \frac{\gamma}{\beta - \gamma} \]  \hspace{1cm} [A36]

and by eliminating \( A_1 V_{df}^\gamma \) from [A35] and [A36]

\[ V_{df} - \frac{C(1 - \tau)}{r} + \phi \frac{\tau C}{r} \frac{\beta}{\beta - \gamma} = \frac{V_{df}}{\gamma} + \phi \alpha V_{df} - \phi \frac{\tau C}{r} \frac{\gamma}{\beta - \gamma} \]

Gathering terms

\[ V_{df} = \frac{C(1 - \tau)}{r} + \phi \frac{\tau C}{r} \left( \frac{\beta}{\beta - \gamma} - \frac{\gamma}{\beta - \gamma} \right) = V_{df} \left[ (1 - \phi \alpha) - \frac{1}{\gamma} (1 - \phi \alpha) \right] \]

\[ V_{df} = \frac{C(1 - \tau(1 - \phi))}{(1 - \phi \alpha)} \frac{C}{r} \frac{\gamma}{\gamma - 1} \]

\[ V_{df} = L \frac{C}{r} \frac{\gamma}{\gamma - 1} \]  \hspace{1cm} [A37] or [12]

Where

\[ L = \frac{(1 - \tau(1 - \phi))}{(1 - \phi \alpha)} \]  \hspace{1cm} [A38]

We next solve for \( A_1 \) in [A35]

\[ \frac{V_{df}}{\gamma} + A_1 V_{df}^\gamma = \frac{\phi \alpha V_{df}}{\gamma} - \phi \frac{\tau C}{r} \frac{\beta}{\beta - \gamma} \]

Giving

\[ V_{df}^\gamma A_1 = \frac{\phi \alpha V_{df}}{\gamma} - \phi \frac{\tau C}{r} \frac{\beta}{\beta - \gamma} - \frac{V_{df}}{\gamma} \]

\[ = \frac{V_{df}}{\gamma} (\phi \alpha - 1) - \phi \frac{\tau C}{r} \frac{\beta}{\beta - \gamma} \]

And then substituting for \( V_{df} \) on the RHS from [A37] gives
Resulting in

\[ E(V) = V - \frac{C(1 - \tau)}{r} + \left[ \frac{C(\tau(1 - \phi) - 1)}{r} \right] \left( \frac{V}{V_{af}} \right)^{\gamma_1} \text{when } V > V_{af} \quad [A39] \]

Using the Nash bargaining game equity value when \( V \leq V_{af} \) after negotiation is given by

\[ E(V) = \phi[F(V) - (1 - \alpha)V] \text{ when } V \leq V_{af} \quad [A40] \]

By differentiating [A40] once and then twice and substituting the results in [A29] we get

\[ E_v(V) = \phi[F_v - (1 - \alpha)] \]

\[ E_{vv}(V) = \phi[F_{vv}] \]

\[ C(V) = \frac{1}{2} \sigma^2 V^2 \phi[F_{vv}] + \mu V \phi[F_v - (1 - \alpha)] - r \phi[F(V) - (1 - \alpha)V] + (r - \mu)V \]

And gathering terms

\[ = \phi \left[ \frac{1}{2} \sigma^2 V^2 F_{vv} + \mu V F_v - r F(V) \right] + \phi(1 - \alpha)[r V - \mu V] + (r - \mu)V \]

And then substituting in the first term using [A19]

\[ = \phi[-(r - \mu)V + \phi(1 - \alpha) V[r - \mu] + (r - \mu)V \]

\[ = (r - \mu)[\phi - \phi - \alpha \phi + 1] \]

\[ C(V) = (1 - \alpha \phi)(r - \mu)V \quad [14] \text{or[A41]} \]

As in the Model 1 example we now proceed to calculate the optimal coupon \( c_d \) using \( V_{af} \) from [A37] and \( F(V) \) from [A26] and maximising \( F_c(V) = 0 \)

\[ F(V) = V + \frac{\tau c}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{V}{V_{af}} \right)^{\gamma_1} \right] \]

Substituting for \( V_{af} \) from [A37]

\[ = V + \frac{\tau}{r} \left[ c - \left( \frac{\beta}{\beta - \gamma} \right)^{\gamma_1} \left( \frac{L \cdot \gamma}{r \cdot (\beta - 1)} \right)^{-\gamma} \right] \]
Differentiating w.r.t. $c$, substituting $c_d$ and setting equal to 0 gives

\[
\left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) (1 - \gamma) V^\gamma c_d^{-\gamma} \left( \frac{L}{r \gamma - 1} \right)^{-\gamma} \right] = 0
\]

By letting $g^{-\gamma} = \left( \frac{\beta}{\beta - \gamma} \right) (1 - \gamma) = \frac{v_{dt}}{v_{af}}$

We now assume (as for the default option) that the borrower only agrees the optimal coupon $c_d$ with the lender and obtains the required funds when the property value $V = V_{di}$ the entry level threshold thus [A42] becomes

\[
V_{di} = c_d g \frac{L}{r \gamma - 1}
\]

Using [A26] again at $c_d$ and $V_{di}$

\[
F(V_{di}, c_d) = V_{di} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{V_{di}}{V_{af}} \right)^\gamma \right]
\]

\[
= V_{di} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) g^\gamma \right]
\]

\[
= V_{di} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{\beta - \gamma}{\beta (1 - \gamma)} \right) \right]
\]

\[
= V_{di} + \frac{\tau c_d}{r} \left[ 1 - \frac{1}{(1 - \gamma)} \right]
\]

\[
= V_{di} + \frac{\tau c_d}{r} \left[ \frac{\gamma}{(\gamma - 1)} \right]
\]

And from [A44]

\[
= V_{di} + \frac{\tau r V_{di} (\gamma - 1)}{gLr} \left[ \frac{\gamma}{(\gamma - 1)} \right]
\]

Giving that at the optimal investment moment

\[
F(V_{di}, c_d) = V_{di} \left( 1 + \frac{\tau}{gL} \right)
\]
Or when
\[ V_{di} = F(V_{di}, c_d)(1 + \frac{\tau}{gL})^{-1} \]

However using standard real option relationship for an entry level investment it can be shown that the optimal investment occurs when \( F(V_{di}, c_d) \) is \( \frac{\beta}{\beta-1} I \).

Thus
\[ V_{di} = \frac{\beta}{\beta - 1} \left( 1 + \frac{\tau}{gL} \right)^{-1} I \quad [A45] \text{ or [8]} \]

Using [A44]
\[ c_d = r \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} (gL + \tau)^{-1} I \quad [A46] \text{ or [10]} \]
4.9.2.3 Derivation of a Closed Form Optimal LTV Negotiation at Origination

The LTV ratio is defined as

\[ LTV_{di} = \frac{D(V_{di}, c_d)}{F(V_{di}, c_d)} = \frac{F(V_{di}, c_d) - E(V_{di}, c_d)}{F(V_{di}, c_d)} \]

\[ V + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{V}{V_{df}} \right)^{\gamma} \right] = \left( V - \frac{c_d (1-\phi) - 1}{r} - \frac{\phi \tau c_d}{r} \gamma \right) \left( \frac{V}{V_{df}} \right)^{\gamma} \]

(From [A26] and [A39])

\[ LTV_{di} = \frac{1}{V + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{V}{V_{df}} \right)^{\gamma} \right]} \]

Cancelling some terms, and substituting \( V_{di}, c_d \) for \( V, c \)

\[ \frac{\tau c_d}{r} \left[ \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{V_{di}}{V_{df}} \right)^{\gamma} \right] + c_d \left[ \frac{c_d (1-\phi)}{r} - \frac{\phi \tau c_d}{r} \gamma \right] \left( \frac{V_{di}}{V_{df}} \right)^{\gamma} \]

\[ V_{di} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{V_{di}}{V_{df}} \right)^{\gamma} \right] \]

Substituting \( V_{di} = gV_{df} \) and using the expression in [A37] for \( V_{df} \)

\[ \frac{\tau c_d}{r} \left[ \left( \frac{\beta}{\beta - \gamma} \right) (g)^{\gamma} \right] + c_d \left[ \frac{c_d (1-\phi)}{r} - \frac{\phi \tau c_d}{r} \gamma \right] (g)^{\gamma} \]

\[ \frac{c_d g^{\gamma}}{r \gamma - 1} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) (g)^{\gamma} \right] \]

Using \( g^{\gamma} = \frac{\beta - \gamma}{\beta(1-\gamma)} \) and substituting \( g \) and eliminating gives

\[ = \frac{\tau c_d}{r} \left[ \frac{1}{(1-\gamma)} \right] + \frac{c_d g^{\gamma}}{r \gamma - 1} + \frac{\phi \tau c_d}{r} \frac{1}{(1-\gamma)} \]

\[ = \frac{c_d g^{\gamma}}{r \gamma - 1} + \frac{\tau c_d}{r} \left[ 1 - \frac{1}{(1-\gamma)} \right] \]

\[ = \frac{c_d g^{\gamma}}{r \gamma - 1} + \frac{\tau c_d}{r} \left[ \frac{\gamma}{(1-\gamma)} \right] \]

\[ = \frac{c_d g^{\gamma}}{r \gamma - 1} + \frac{\tau c_d}{r} \left[ \frac{\gamma}{(1-\gamma)} \right] \]

\[ = \frac{c_d g^{\gamma}}{r \gamma - 1} + \frac{\tau c_d}{r} \left[ \frac{\gamma}{(1-\gamma)} \right] \]

\[ = \frac{c_d g^{\gamma}}{r \gamma - 1} + \frac{\tau c_d}{r} \left[ \frac{\gamma}{(1-\gamma)} \right] \]

Eliminating \( \frac{\tau c_d}{r} \left[ \frac{1}{(1-\gamma)} \right] \), gathering terms and simplifying

\[ LTV_{di} = \frac{\gamma (g^{\gamma} + y - 1 + g^{\gamma} - 1)}{\gamma gL + \gamma \tau} = \frac{\gamma - [(1 - g^{\gamma})(1 + \tau(\phi - 1))] - [A48]or[15]}{[A48]or[15]} \]
(* This program calculates in tabular form key parameters for varying foreclosure costs α using the default non bargaining model and compares to the same parameters using the Delinquency Non Bargaining model for various bargaining powers φ *)

(*Parameter Input which can be varied*)
F = 250000;
r = 0.03;
μ = 0.0;
σ = 0.5;
τ = 0.2;

(*Iterates for Foreclosure costs α=0.5, 0.4 and 0.5*)
Do[
  (*An output line*)
  Print["Foreclosure % α is equal to ", α];
  (*Sets φ=0.0 for the next iteration of α*)
  φ = 0.0;
  (*Iterates for φ=0.0, 0.5 and 1.0*)
  While[φ ≤ 1.05, 
    γ = \( \frac{2r}{σ^2} + \left(0.5 - \frac{μ}{σ^2}\right)^2 - \frac{μ}{σ^2} + 0.5 \);
    β = \( \frac{2r}{σ^2} + \left(0.5 - \frac{μ}{σ^2}\right)^2 - \frac{μ}{σ^2} + 0.5 \);
    g = \( \frac{β(1-γ)}{β - γ} \)^{-1/\(1/γ\)}; 
    sdi = \( \frac{β \ast V}{(β - 1) \ast \left( \frac{1}{φ^2} + 1 \right)} \);
    Print["Period ", φ, ", Yield Loss % Original Equity Position:", g, ", Exit Yield:", sdi];
    φ = φ + 0.05;
  ];
]
\[ a_{df} = \frac{\rho_{F}}{1 - \epsilon \frac{1}{\rho_{F}}} \]

\[ a_{df} = \frac{\rho_{F}}{1 - \epsilon \frac{1}{\rho_{F}}} \]

\( \rho_{F} \) is the equity function for less than (ab) and greater than (ab) the entry threshold point (ab)

\( \rho_{F}(x) = \frac{x - \delta}{x - \delta + \theta} \)

\( a_{df} = \frac{\rho_{F}}{1 - \epsilon \frac{1}{\rho_{F}}} \)

(\( \theta \)) the following 2 lines calculate the crs equity function for less than (ab) and greater than (ab) the entry threshold point (ab)

\( \rho_{F}(x) = \frac{x - \delta}{x - \delta + \theta} \)

\( a_{df} = \frac{\rho_{F}}{1 - \epsilon \frac{1}{\rho_{F}}} \)

(\( \theta \)) the following 4 lines calculate the Eq. 21 Post Delinquency Bargaining Property Value Function \( p_i(x(t)) \) and \( p_j(x(t)) \) and \( p_i(x(t)) \) and \( p_j(x(t)) \).

Equity Value Functions \( \rho_{F}(x(t)) \) and Debt Value Functions \( \rho_{F}(x(t)) \) and \( \rho_{F}(x(t)) \)

\( p_i(x(t)) = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)

\( p_j(x(t)) = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)

\( \rho_{F}(x(t)) = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)

\( \rho_{F}(x(t)) = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)

(\( \epsilon \)) the following 4 lines calculate the Eq. 21 Post Delinquency Bargaining Optional Loan to Value Ratio when \( \epsilon \) (ab)

\[ LV_{opt} = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \]

(\( \epsilon \)) the following 4 lines calculate the Eq. 21 Post Delinquency Bargaining Reduced Coupon Payment Function \( CR_{i}(x(t)) \) when \( \epsilon \) (ab)

\( CR_{i}(x(t)) = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)

(\( \epsilon \)) the following 4 lines calculate the Debt average ratio \( \delta_{opt} \) when \( \epsilon \) (ab)

\( \delta_{opt} = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)

(\( \epsilon \)) the following 4 lines calculate the Eq. 21 Post Delinquency Bargaining Yield Spread function \( Vol_{i}(x(t)) \) when \( \epsilon \) (ab) and \( Vol_{j}(x(t)) \) when \( \epsilon \) (ab)

\( Vol_{i}(x(t)) = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)

\( Vol_{j}(x(t)) = \frac{\rho_{F}(x(t))}{1 - \epsilon \frac{1}{\rho_{F}}} \)
The following 2 lines calculate the Default Non-Bargaining, p parameter, vulnerability thresholds as well as the optional component:

\[
\begin{align*}
\eta & = \frac{\gamma \times \alpha + \eta}{\gamma}, \\
\tilde{\eta} & = \frac{\gamma \times \alpha + \tilde{\eta}}{\gamma}.
\end{align*}
\]

\[\tilde{\eta} \geq \eta \geq \eta(1 - \alpha)\]

\[
\frac{\gamma \times \alpha + \eta}{\gamma} = \frac{\gamma \times \alpha + \tilde{\eta}}{\gamma}
\]

The following 2 lines calculate the Ex Post Default Non-Bargaining Equity Value Function (E(V)_t) for \(\psi(x, y, z)\) and Debt Value Functions (D(V)_t) for \(\psi(x, y, z)\):

\[
\begin{align*}
V_{E}(x, t) &= \left(1 - \frac{e^{\psi(x, y, z)}}{e^{\psi(x, y, z)}}\right) \left[\left(1 - \frac{\psi(x, y, z)}{\psi(x, y, z)}\right) \left(1 - \frac{\psi(x, y, z)}{\psi(x, y, z)}\right)\right], \\
V_{D}(x, t) &= \left(1 - \frac{e^{\psi(x, y, z)}}{e^{\psi(x, y, z)}}\right) \left[\left(1 - \frac{\psi(x, y, z)}{\psi(x, y, z)}\right) \left(1 - \frac{\psi(x, y, z)}{\psi(x, y, z)}\right)\right].
\end{align*}
\]

\[
\text{(Calculates the Default Non-Bargaining Yield Spread Function Y(x) when } \psi(x, y, z)\text{)}
\]

\[\text{Y(x)} = \frac{\psi(x, y, z) - \psi(x, y, z)}{\psi(x, y, z)} \times r;
\]

\[\text{(Calculates the Default Non-Bargaining Optimal Loan to Value Ratio when } \psi(x, y, z)\text{)}
\]

\[\text{LTV} = \frac{\psi(x, y, z)}{\psi(x, y, z)} \times \frac{\psi(x, y, z)}{\psi(x, y, z)} \times \frac{\psi(x, y, z)}{\psi(x, y, z)}.
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]

\[
\begin{align*}
\text{(output line for the Default Non-Bargaining Model results:)} \\
\text{(Print) P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i], P2[Default[i, 1, 1], i, i].}
\end{align*}
\]
This program graphs key parameters for a stochastic property price using the default non bargaining model and compares to the same parameters using the Delinquency Non Bargaining model for various bargaining powers ϕ. The basic graphs can be then embellished with tags and labels as demonstrated below:

Parameter Input which can be varied:

\[ V = 250000; \]
\[ r = 0.03; \]
\[ μ = 0.0; \]
\[ σ = 0.3; \]
\[ τ = 0.5; \]
\[ ϕ = 0.6; \]
\[ σ = 0.3; \]

Sets up vector arrays whereby Array[1] contains the graph pertaining to the first iteration of ϕ = 0.0, etc:

Array[propertyvaluegraph, 3];
Array[composograph, 3];
Array[debtvaluegraph, 3];
Array[yieldspreadgraph, 3];
Array[esantevaluegraph, 3];
Array[ltvgraph, 3];
Array[ogrategraph, 3];
Array[tvspreadgraph, 3];
Array[bookvalgraph, 3];

Sets up vector arrays to vary the line colour/dashing after the first iteration of ϕ = 0.0, etc:

Array[colour, 3];
colour[1] = Red;
colour[3] = Black;
Array[dash, 3];
dash[1] = Dashed;
dash[2] = Dashing[Large];

Iterates for bargaining power ϕ = 0.0, 0.5 and 1.0:

Do[

The following 5 lines calculate parameters common to all the models:

\[ y = \frac{2r}{\sqrt{σ^2 + 0.5 - μ^2}} \]

]
\[
\beta = \sqrt{\left(\frac{2r + \left(0.5 - \frac{\mu}{\alpha^2}\right)^3}{\frac{\mu}{\alpha^2} + 0.5}\right)}
\]

\[
\gamma = \left(\frac{\beta + (1 - \gamma)}{\beta - \gamma}\right)^{\frac{1}{(1 - \gamma)}};
\]

\[
L = \frac{1 - \gamma (1 - \phi)}{1 - \alpha \phi};
\]

(*The following 3 lines calculate the Delinquency Bargaining exit/entry thresholds as well as the optimal coupon*)

\[
\text{xdi} = \frac{V}{(\beta - 1) \left(\frac{\gamma}{\gamma L} + 1\right)};
\]

\[
\text{xdf} = \frac{V}{(\beta - 1) \left(\frac{\gamma}{\gamma L} + 1\right)};
\]

\[
\text{cd} = \frac{V}{(\beta - 1) \left(\frac{\gamma}{\gamma L} + 1\right)};
\]

(*The following 2 lines calculate the ex ante equity function for less than (elde) and greater than (ehde) the entry threshold point xdi*)

\[
\text{elde}(x) := (s / \text{xdi})^{\gamma} \beta / (\beta - 1);
\]

\[
\text{ehde}(x) := x + (s / \text{df}) \times (1 - ((\beta / (\beta - \gamma)) \times ((s / \text{df})^{\gamma})) - V;
\]

(*The following 6 lines calculate the Ex Post Delinquency Bargaining Property Value Function (ph(x) for x>xdf) and pl(x) for x<xdf), Equity Value Functions (eh(x)/df(x)) and Debt Value Functions (dh(x)/df(x))*)

\[
\text{ph}(x) := \left(\frac{\gamma}{\beta - \gamma}\right) x + \frac{1 - \left(\frac{s}{\text{xdf}}\right)^{\gamma}}{\beta - \gamma};
\]

\[
\text{pl}(x) := \left(\frac{\gamma}{\beta - \gamma}\right) x + \frac{1 - \left(\frac{s}{\text{xdf}}\right)^{\gamma}}{\beta - \gamma};
\]

\[
\text{eh}(x) := x - (\text{cd} \times (1 - \gamma) / r) + ((s / \text{xdf})^{\gamma}) \times (((1 - \tau + (1 - \phi)) \times \text{cd} \times (r \times (\gamma - 1))) - ((\phi + \tau \times \text{cd} \times \beta) / (r \times (\beta - \gamma))));
\]

\[
\text{el}(x) := \phi \times \text{pl}(x) - x \times (1 - \phi);
\]
\[ dl[x_] := pl[x] - ell[x]; \]
\[ db[x_] := pb[x] - ebl[x]; \]

\(<\text{Calculates the Delinquency LTVhd}(x)\) functions when \( x > xdf \) and \( LTVhd(x) \) when \( x < xdf \)\)
\[ LTVhd[x_] := 100 + dl[x]/pl[x]; \]
\[ LTVhd[x_] := 100 + db[x]/pb[x]; \]
\[ BLD := LTVhd[V^2]/V/100; \]

\(<\text{Calculates the Delinquency Bargaining Reduced Coupon Payment Function CR}(x)\) when \( x < xdf \)\)
\[ CR[x_] := (r - \mu) * (1 - (\delta + \alpha)) * x; \]

\(<\text{Calculates the Debt coverage ratio functions dcr}(x)\)\)
\[ dcrhd[x_] := x * (r - \mu) / (od * (1 - r)); \]
\[ dcrld[x_] := x * (r - \mu) / (1 - r) * CR[x]; \]

\(<\text{Calculates the Delinquency Bargaining Yield Spread function Ysh}(x)\) when \( x > xdf \) and \( Ysh(x) \) when \( x < xdf \)\)
\[ Yshh[x_] := 10000 + (cdh/dbh[x]) - r; \]
\[ Yshh[x_] := 10000 + (CR[x]/dlh[x]) - r; \]

\(<\text{The following 4 lines calculate the Default Non-Bargaining, h parameter, exit/entry thresholds as well as the optimal coupon}x>\)
\[ h = \left( 1 - \frac{\gamma (\alpha + r)}{\tau} \right)^{\frac{1}{\tau}}; \]
\[ xi = \frac{V * \beta}{(\beta - 1) \left( \frac{\gamma}{\tau} + 1 \right)}; \]
\[ xf = xi / h; \]
\[ e = r * xi * \frac{\gamma - 1}{\gamma + h}; \]

\(<\text{The following 2 lines calculate the Ex Post Default Non-Bargaining Equity Value Function (Elec)(x) for x > xf) and Debt Value Functions (Dlec)(x) for x > xf)\>\)
\[ Ele[x_] := x - \frac{e * (1 - r)}{r} - \left( \frac{x}{xf} \right)^2 * \left( xh - \frac{e * (1 - r)}{r} \right); \]
\[
\text{DIE}(s) := \left(1 - \alpha\right) \cdot \frac{c}{s} + \left(1 - \left(\frac{s}{\alpha}\right)^r\right) \cdot \left(1 - \left(\frac{s}{\alpha}\right)^r\right) \\
(\text{Calculates the Default LTV function } \text{LTV}(s) \text{ when } s > x \cdot f_s)
\]

\[
\text{LTV}(s) := 100 \cdot \text{DIE}(s)/\text{EVE}(s) + \text{DIE}(s); \\
\text{BL} := \text{LTV}(F) \cdot F/100; \\
(\text{Calculates the Default Non-Bargaining Yield Spread function } \text{Yle}(s) \text{ when } s > x \cdot f_s)
\]

\[
\text{Yle}(s) := 10000 \cdot (c/\text{DIE}(s)) - r; \\
(\text{Graphs and saves in the array as given by the title for one instance of } \phi)
\]

\[
\begin{align*}
\text{propertyvaluegraph}[i] &:= \text{Show}[\text{Plot}[[\text{ph}][s], \{s, x, x_f, (1.2 \cdot x_f)\}, \text{AxesOrigin} \rightarrow \{0, 0\}, \text{PlotStyle} \rightarrow \{\text{colour}[i], \text{Dashed}\}, \text{Frame} \rightarrow \text{True}, \\
\text{FrameLabel} \rightarrow \{\text{Style}[\text{Framed}]["\text{Property Price (V) S"}], 16, \text{Black}, \text{Bold}\}, \text{Style}[\text{Framed}]["\text{Expected Asset Value F (V) S"}], 16, \text{Black}, \text{Bold}\}], \\
\text{Plot}[\text{ph}[s], \{s, 0, x_f\}, \text{PlotStyle} \rightarrow \{\text{colour}[i]\}], \text{PlotRange} \rightarrow \{0, 50, 0000\}]; \\
\text{couponograph}[i] &:= \text{Show}[\text{Plot}[[\text{CR}][s], \{s, 0, x_f\}, \text{PlotStyle} \rightarrow \{\text{colour}[i]\}, \text{AxesOrigin} \rightarrow \{0, 0\}, \text{Frame} \rightarrow \text{True}, \\
\text{FrameLabel} \rightarrow \{\text{Style}[\text{Framed}]["\text{Property Price (V) S"}], 16, \text{Black}, \text{Bold}\}, \text{Style}[\text{Framed}]["\text{Optimal Coupon S"}], 16, \text{Black}, \text{Bold}\}], \\
\text{Plot}[[\text{cr}[s], \{s, 0, x_f\}, \text{PlotStyle} \rightarrow \{\text{colour}[i]\}], \text{PlotRange} \rightarrow \{0, 15, 0000\}]; \\
\text{itvsdcrgraph}[i] &:= \text{Show}[\text{ListPlotTable}[[\text{BL} \times 100/x \cdot CR[s]], \{x, 1, x_f, 10\}], \text{PlotStyle} \rightarrow \{\text{colour}[i]\}, \text{AxesOrigin} \rightarrow \{0, 0\}, \text{Frame} \rightarrow \text{True}, \\
\text{FrameLabel} \rightarrow \{\text{Style}[\text{Framed}]["\text{Book Loan to Property Value Ratio %}\)], 16, \text{Black}, \text{Bold}\}, \text{Style}[\text{Framed}]["\text{Optimal Coupon S"}], 16, \text{Black}, \text{Bold}\}], \\
\text{ListPlotTable}[[\text{BL} \times 100/x \cdot \text{C}[s]], \{x, 500, 000, 100\}], \text{PlotStyle} \rightarrow \{\text{colour}[i]\}], \text{PlotRange} \rightarrow \{0, 200\}, \{0, 15000\}]; \\
\text{yieldspreadgraph}[i] &:= \text{Show}[\text{Plot}[[\text{Ys}[s], \{s, x, (2 \cdot x)\}, \text{AxesOrigin} \rightarrow \{0, 0\}, \text{PlotStyle} \rightarrow \{\text{colour}[i]\}, \text{Frame} \rightarrow \text{True}, \\
\text{FrameLabel} \rightarrow \{\text{Style}[\text{Framed}]["\text{Property Price (V) S"}], 16, \text{Black}, \text{Bold}\}, \text{Style}[\text{Framed}]["\text{Yield Spread Basis Points}\)], 16, \text{Black}, \text{Bold}\}], \\
\text{Plot}[[\text{ys}[s], \{x, 500, 000, 100\}], \text{PlotRange} \rightarrow \{0, 200\}]; \\
\end{align*}
\]

(Increases $\phi$ by 0.5 and then returns control to the DoStatement)

\[
\phi := 0.5; \\
\{1, \{1, 2, 3\}\}
\]

(output lines for the 5 graphs combining all the different instances of $\phi$ : titles are self explanatory)

\[
\begin{align*}
\text{Show}[\text{propertyvaluegraph}[1], \text{propertyvaluegraph}[2], \text{propertyvaluegraph}[3], \text{Plot}[[\text{Eve}[s] \rightarrow \text{DIE}(s)], \{x, x, (1.2 \cdot x)\}], \text{PlotStyle} \rightarrow \{\text{Black}, \text{DashingLarge}, \text{Thick}\}], \\
\text{Show}[\text{couponograph}[1], \text{couponograph}[2], \text{couponograph}[3], \text{Plot}[[\{s, x, 500000\}], \text{PlotStyle} \rightarrow \{\text{Black}, \text{DashingLarge}, \text{Thick}\}], \\
\text{Show}[\text{itvsdcrgraph}[1], \text{itvsdcrgraph}[2], \text{itvsdcrgraph}[3], \text{ListPlotTable}[[\text{BL} \times 100/x \cdot \text{C}[s]], \{x, 500000, 100\}], \text{PlotStyle} \rightarrow \{\text{Yellow}, \text{DashingLarge}, \text{Thick}\}], \\
\text{Show}[\text{yieldspreadgraph}[1], \text{yieldspreadgraph}[2], \text{yieldspreadgraph}[3], \text{Plot}[[\text{Ys}[s], \{s, x, (2 \cdot x)\}], \text{PlotStyle} \rightarrow \{\text{Black}, \text{DashingLarge}, \text{Thick}\}]
\end{align*}
\]
5. Non Owner Occupied Residential Mortgages

5.1 Chapter Summary

Chapter 3 introduced the concept of a (real) negotiation option while Chapter 4 demonstrated an approach to estimating the value of this option for a US owner occupied residential mortgage. Chapter 5 extends the approach to non-owner occupied residential mortgages a very important segment of US residential mortgages comprising 16% of total mortgages in 2005 (Robinson and Todd, 2010). Many US commentators believe (Robinson and Todd, 2010) that this mortgage segment is particularly vulnerable to default and foreclosure as investors may have purchased these properties primarily for income generation. They show that non-owner occupied residential borrowers are distinctly different from owner occupied residential borrowers with better credit rating (FICO) scores, lower DTI and LTV than owner occupied mortgages. We will refer to them as investors rather than homeowners.

Even though, long-term capital appreciation is an important factor motivating investment in this type of property, rental income (and not property value) often drives the investor’s short to medium term decision making process. Thus while in Chapter 4, the market imputed rent is a benefit or utility enjoyed (tax-free) by the homeowner or borrower, in Chapter 5 market rent received is a necessity to cover (or defray) the mortgage coupon, but may be taxable (rent less interest, depreciation & maintenance is taxable income). Even though the methodological approach described in Chapter 5 is similar to Chapter 4, this observation and the use of rental income instead of property price as the stochastic driver changes the analysis and interpretation, leading to expanded insights into the theoretical interplay between negotiation and default (options) in non-owner occupied residential mortgages.
5.2 Introduction

This treatment is specifically directed at US non-owner occupied mortgages where the investor receives monthly rental income to defray the cost of the mortgage. In a similar manner to the approach taken in Chapter 4, we demonstrate the owner’s negotiation option and derive simple closed form solutions which value ex-ante the default-only and negotiation-only options within the same framework of assumptions and perform a comparative analysis of the investor’s optimal ex-post decisions. We model the optimal ex-ante investment decision of the lender (who provides debt) and the investor (who provides equity) as we argue that the optimal ex-post negotiation decisions can only be examined conditional on the optimal ex-ante debt/equity ratios which are different for each investor.

As in Chapter 4 we introduce the parameter $\phi$ to model the effect and strength of the difference in negotiation positions between investor and lender and make identical assumptions as to the non-availability of new credit and the motivation of both parties to avoid costly foreclosure by negotiating over foreclosure costs.

We show that negotiation is a valuable option to the investor with an illiquid asset in a housing rental market with declining and/or volatile rental income rates. All investors will initiate negotiation before default but those investors who are strong negotiators should exercise their negotiation option earlier than those who are weak negotiators. We show that the lenders ex-ante mortgage yield spread should increase to pay for the investor’s ex-post negotiation option. We show that the negotiation option should allow a lender to offer a higher LTV mortgage to a weak negotiator than in the case of a default only option. We also predict optimal borrower negotiation regions in terms of mortgage industry control parameters such as market LTV (debt/total value) and DCR (debt interest coverage ratio).
In contrast to Chapter 4 we use rental income rate (or rental service flow) instead of the more usual (option theoretic) stochastic property price for the non-owner occupied residential mortgages to drive the model. We believe this is justified from three viewpoints.

Firstly, Kau and Keenan (1995) state, “Economic logic would dictate that the house price would be the expected discounted value of future service flows, rather than the service flows being a specified proportion of the current house price”.

Secondly, we see little opportunity for an investor to strategically “flip” with declining house prices and hence no (informational) benefit from using a stochastic property price but more benefit from employing a stochastic rental income rate (or service flow). Investors with interest only mortgages might still receive a “good” rental income rate (or service flow) but have a property whose current “spot” value is less than the value at mortgage origination due to the current property price slump. Rental income is volatile and changes continuously so this is therefore not a discrete representation of monthly rental payments and mortgage coupons and makes no assumptions about vacancy risk or the nature of the tenants.

Thirdly, although readers may see a similar approach in the solution methodology between Chapters 4 and 5 one must remember that an US owner occupier and their mortgage lender may value the same property differently from owner investor and their mortgage lender with a different investment amount or purchase price given by I in our models. This arises because the benefit from not taxing imputed rental income can be quite significant. Poterba and Sinai (2008) have estimated that, on average, user costs would rise by 6.6 per cent if US owner-occupiers were taxed as if they were owner investors. Unfortunately, empirical literature does not give a clear cut answer as to whether owner occupiers (Chapter 4) have different default
triggers or motivation from owner investors (this chapter). Many people including Bernanke (2010) or Mayer, Pence and Sherlund (2009) believe so but hard evidence is lacking. This arises because it is almost impossible to distinguish owner occupiers from owner investors in the US mortgage market as many investors may declare themselves as occupiers in order to benefit from the significant tax advantages. All commentators agree however that the (non) tax treatment of imputed rental income might have a significant effect on valuations and default motivations without understanding how this could be reliably observed and measured.

Our extension is also original, in that, in the option theoretic literature, the negotiation option has not yet been modelled for non-owner occupied residential mortgages in this fashion and addresses specific shortcomings identified by Vandell (1995). These are that “the borrower’s solvency or ability to service the debt is not considered”, that “the lender is not permitted to forebear or negotiate a workout arrangement”, and finally the approach “ignores the possibility of heterogeneous transaction costs among borrowers caused by costly search or different opportunities and perceptions”.

Our model is inspired by Fan and Sundaresan (2000) who attempted to apply strategic negotiation to the valuation of corporate debt. However, for various reasons, specific to the asymmetric and non-standardised nature of bonds this avenue of research never developed very far in the corporate bond field. As discussed in Chapter 3, we believe that the idea does have validity in the mortgage market where contracts are symmetric and incomplete.

Figure 5.2.1 overleaf summarises the two real options available to the investor that we will model separately – Model 1 and Model 2. The blue boxes common to both models is the initial purchase or investment option followed by the box in yellow (Model 2) which represent the negotiation option and finally the default option in the red box (Model 1).
As a consequence of the assumption (discussed in Chapter 3) that the US non-owner occupied residential mortgage contract is incomplete but not asymmetrical the lender and investor play a generalised Nash cooperative game (Yellow box, Figure 5.2.1) to avoid foreclosure costs. They have incomplete but no asymmetric knowledge of each other’s options and costs. Having *ex-ante* negotiated the initial mortgage contract (LTV and mortgage payment in the blue boxes) conditional on anticipated default, they may *ex-post* renegotiate the contract should a credible threat of default arise due to a unfavourable shock to rental income.

We then allow the mortgage to be modified with a new lower mortgage coupon (Yellow box, Figure 5.2.1) on a successful negotiation. We depict a two way blue arrow between the yellow and green box in Model 2 the negotiation option to indicate that the renegotiated coupon is a function of the actual rental income. Should rental income recover above the negotiation trigger point, we assume that the lender and investor would contractually revert to the original mortgage payment. Otherwise, with continuing negative rental income price shocks, the investor will exercise their default option (Red box, Model 1) if the investor is unable to continue paying the mortgage coupon. We assume, in this regard, that the investor has limited liability and can default on the mortgage contract at any time with no long-term consequences to a subsequent credit rating.
Figure 5.2.1 Non Owner Occupied Negotiation Option Model Flow Diagrams

Legend
- $x$: Net rental income rate after operating expenses and local property taxes but before mortgage payment tax deduction.
- $X_i$: Optimal rental rate at where the borrower would invest with a default option.
- $X_f$: Optimal rental rate at where the borrower would invest with a negotiation option.
- $X_{di}$: Optimal rental rate at where the borrower would default with a default option.
- $X_{df}$: Optimal rental rate at where the borrower would negotiate.
- $c^*$: Perpetual mortgage payment to the lender for the default only option (tax deductible).
- $c_d$: Perpetual mortgage payment to the lender for the negotiation option (tax deductible).
- $C(x)$: Renegotiated mortgage payment (after negotiation) which depends on property price.
- $I$: Initial property investment made at the critical investment thresholds.
We do not consider whether an optimal point exists to renew the original coupon payment. Both parties analyse the local property market and each other’s circumstances and instantaneously agree a (re)negotiated coupon, based on an anticipated share of the unavoidable foreclosure costs. This is in contrast to the real situation where several months can elapse before eventual default or mortgage reinstatement. We assume that the lender/investor will ex-ante anticipate the same negotiated outcome whether the investor ex-post actually defaults or merely threatens default.

Our assumption that the lender’s foreclosure costs are a percentage \( (\alpha) \) of the property value \( V(x) \) implied by the net rental income rate \( x \) is supported by the observation that a residential property is generally independently valued only once at the initial mortgage contract negotiation and then once again at the renegotiation stage.

In Section 5.3 we model the investment/negotiation options as well as the investment/default options. We have set the model in the context of a residential Buy-To-Let (BTL) investment mortgage. In this case the occupier who pays the rent and consumes the asset is a separate party from the borrower who makes the investment and pays the coupon to the lender.

In Section 5.4 using stylised US BTL data (Robinson and Todd, 2010), we highlight differences between the ex-ante outcomes of both the default and negotiation options on property, equity and debt market values. We then examine in detail the fundamental differences between the ex-post behaviour of both options and highlight the effect of heterogeneous negotiation on the endogenous negotiation threshold, mortgage yield spread, debt coverage and LTV ratios.
5.3 Default and Negotiation Options Model Derivation

The “non-recourse” mortgage contract covers the relationship between lender and investor, whereby the investor is assumed to have limited liability and can default on the mortgage contract at any time with no medium or long-term consequences to a subsequent credit rating. The mortgage debt is perpetual, with the investor making a coupon payment to the lender. We assume a “spot” rental income rate net of operating expenses and all property taxes, whereby the effect of any short term assured tenancy agreement is reflected in the rental volatility. When the property is performing well, investors will collect all the net excess cash flows after servicing the debt payments. On the other hand and critically, investors also supply the needed funds to service the debt when there are shortfalls in net rental income, if it is in their interest to do so -- an example being the property still having positive net equity.

The rental income process is exogenous and the investor and lender have rational expectations and transactions are sufficiently small to have no effect on local rental income. The net rental income $x$ after operating expenses and taxes will then have the mortgage payment to the lender $c^*$ (for the default option) and $c_d$ (for the negotiation option) deducted. The mortgage payment $c^*$ or $c_d$ is tax deductible. We assume that on a yearly basis, gross receipts minus gross payments lead to a taxable profit, or the investor has other taxable income. The investor thus chooses a mixture of equity and (risky) debt to finance the property investment $I$ at an endogenously chosen time $T$. We assume that the investor has only one property with a potential net rental income before interest and taxes given by a $gBm$ (geometric Brownian motion).

$$dx = \mu xdt + \sigma xdW$$

[1]

Where $W$ is a standard Brownian motion, $\mu$ the net rental drift and $\sigma$ is the rental volatility.
The investor decides when to exercise the investment option by purchasing the property for a fixed cost $I$ and then collects the net stochastic rental stream of $x$ ($x \geq 0$).

Let $r > 0$ denote the risk free interest rate. Assume $r > \mu$ for convergence. Let the tax rate be $0 \leq \tau < 1$. Property value is given by $V(x) = E(x) + D(x)$ where $E(x)$ is the equity value and $D(x)$ the debt value. After tax and without option value, the all equity financed property value $V(x)$ is

$$V(x) = E(x) = \frac{x}{K} \quad \text{where} \quad K = \frac{r - \mu}{1 - \tau}$$

However, by using debt to partly finance the property purchase, some additional tax benefits can accrue due to the tax deductibility of the mortgage interest payments, so the investor chooses a mixture of equity and mortgage finance at investment time $T_0$.

After purchasing the property and taking on the mortgage liability, if the rental income $x$ is sufficiently or consistently low, the investor may consider defaulting on the mortgage payments, forcing the lender to consider repossession or foreclosure. In this case, if the lender does foreclose, the liquidation value is given by $(1 - \alpha)V(x)$ where $0 \leq \alpha \leq 1$ is the estimated foreclosure or deadweight costs as a proportion of the property at the moment of foreclosure sale, while the investor’s equity value $E(V)$ is zero.

The investor may “ruthlessly” delay one or several coupon payments triggering a negotiation between borrower and lender. With this (alternative) negotiation option, if the investor threatens to default, the lender may not wish to repossess but instead renegotiate the mortgage contract resulting in a new lower coupon payment. The new coupon payment is conditional on the current rental income rate and the “surplus” generated by avoiding costly liquidation.
being “notionally” divided between the investor and lender based on their relative negotiating strength denoted respectively by $\phi$ and $1 - \phi$ ($\phi = 1 \Rightarrow$ investor has all the negotiating power or the greater notional share).

The renegotiation importantly results in a lower “more affordable” mortgage payment allowing the investor to avoid funding the investment mortgage from other sources. Because of the perpetual nature of the mortgage, this does not imply any form of mortgage modification but rather mortgage forbearance. We treat $\phi$ as a heterogeneous variable with a maximum value of 1 determined by each party’s knowledge of the other’s potential foreclosure costs and their desire to mitigate or delay the effect and costs of default. The lender and the investor take a view on how much of the unavoidable foreclosure costs the other would be liable for and condition their ex-ante loan negotiation on this view. We model the negotiation as discussed in Chapter 3 between investor and lender as a cooperative generalised Nash bargaining game (Fan and Sundaresan, 2000).

The methodological approach to solving the problem is similar to a perpetual American (scale) option entry/exit problem (Dixit 1989) and a solution is found for the different ODEs in terms of the critical entry and exit thresholds for the default or negotiation options, respectively, $x_i$ or $x_{di}$ and $x_f$ or $x_{df}$. Solutions are of the form $F(x) = A_0 + A_1 x^\gamma + A_2 x^\beta$ with the appropriate boundary conditions leading to different specific solutions. It is not necessary to formulate the problem using a risk free rate. It is also possible to formulate the problem to take account of risk aversion or deal with non-fungible goods. The general approach, is described as dynamic programming of replicating portfolios by Dixit (1989) and leads to exactly the same equations of state with an adjusted parameter $\mu$. 

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Conventionally modelled default results in the lender acquiring the investment property. The value of the property is exactly the asset value of an all equity-financed property just before default less foreclosure costs. However, with exercise of the negotiation option, investor and lender negotiate a new coupon, conditional on the optimal sharing of the avoidable foreclosure costs, at the (earlier) negotiation trigger point $x_{df}$ with both willing to adapt the contract terms. The lender would agree a renegotiated coupon $C(x)$ proportional to the current rental income rate, lower than the initial coupon $c_d$ (agreed at the investment threshold $x_{di}$) and the investor would continue to own and operate the property.

We make no statement or assumption about what happens (in practise) to the mortgage coupon should rental income revert to above the negotiation trigger point $x_{df}$. This simple model assumes that the contractual agreement at mortgage origination when the negotiation option was agreed is that mortgage coupon continues as a function of rental income. It is of course possible to introduce additional barriers where other options or agreements would “kick in” but their analysis would entail a more numerical type methodology which could form the basis of future research.

Let $V_0(x,c)$ be the property value before investment. The investor chooses the optimal investment threshold $x_{di}$ and the optimal mortgage repayment $c_d$ to maximise his equity position $E_0(x,c)$. As the rental income $x$ gets larger and larger (approaching infinity) and “covers” the invariant mortgage coupon, the mortgage becomes riskless and hence the property value must satisfy an upper boundary condition whereby

$$\lim_{x \to \infty} V(x,c) = \frac{x}{K} + \frac{tc}{r}$$  \[2\]
Lower boundary conditions for the negotiation option differ from the default option as lender/investor are prepared to vary the contract terms at the lower threshold, where the total value of the property $V(x_{df}, c_d)$ still includes the value of future tax benefits and is thus higher than the all equity financed asset property value. The investor and lender thus bargain over a larger amount (when $x \leq x_{df}$) resulting in a property asset value $V(x)$ of

$$V(x) = \frac{x}{K} + \frac{\tau c_d}{r} \left[1 - \left(\frac{\beta}{\beta - \gamma}\right) \left(\frac{x}{x_{df}}\right)^\gamma\right] \text{when } x \geq x_{df}$$

$$V(x) = \frac{x}{K} + \frac{\tau c_d}{r} \left(\frac{-\gamma}{\beta - \gamma}\right) \left(\frac{x}{x_{df}}\right)^\beta \text{when } x < x_{df}$$

where $\beta > 1, \gamma < 0$ are the roots of $\frac{\sigma^2 x^2}{2} + (\mu - \sigma^2/2)x - r = 0$

The equity equation $E(x) (x < x_{df})$ is also adjusted to account for the new coupon $C(x)$

$$\frac{1}{2} \sigma^2 x^2 E_{xx}(x) + \mu x E_x(x) - r E(x) + (1 - r)x - C(x) = 0 \text{ when } x < x_{df}$$

With upper boundary conditions the same for both the negotiation and default options, we obtain revised lower boundary conditions from the “extra” value of $V(x)$ using equation [4] and the negotiation sharing rule to get

$$\lim_{x \rightarrow x_{df}} E(x) = \phi \left(\frac{\alpha x_{df}}{K} - \frac{\tau c}{r} \frac{\gamma}{\beta - \gamma}\right)$$

Differentiating [6] gives

$$\lim_{x \rightarrow x_{df}} E_x(x) = \phi \left(\frac{\alpha}{x_{df}} - \frac{\tau c}{x_{df} r} \frac{\gamma \beta}{\beta - \gamma}\right)$$
Further development in Section 5.8 details equations governing the value of property \( V(x) \), debt \( D(x) \) and equity \( E(x) \) and leads to closed form expressions for the key outputs for the negotiation option and the comparable outputs for the default option.

a) The investor’s investment threshold for the negotiation option \( x_{di} \) is given by

\[
x_{di} = \frac{\beta}{\beta - 1} K \left[ 1 + \frac{\tau}{gL} \right]^{-1} I
\]

where \( g = \left[ \frac{\beta}{\beta - y} (1 - y) \right]^{1/y} = \frac{x_{di}}{x_{df}} \)

and \( L = \frac{1 - \tau(1 - \phi)}{1 - \phi a} \)

The investment threshold for the default option \( x_i \) is given by

\[
x_i = \frac{\beta}{\beta - 1} K \left[ 1 + \frac{1}{h(1 - \tau)} \right]^{-1} I
\]

where \( h = \left[ 1 - \frac{y(\tau + \alpha(1 - \tau))}{\tau} \right]^{-1/y} = \frac{x_i}{x_f} \)

b) The mortgage coupon rate for the negotiation option \( c_d \) (for \( x \geq x_{df} \)) is given by

\[
c_d = r \frac{y - 1}{y} \frac{\beta}{\beta - 1} (gL + \tau)^{-1} I
\]

The mortgage coupon rate for the default option \( c^* \) (for \( x \geq x_f \)) is given by

\[
c^* = r \frac{y - 1}{y} \frac{\beta}{\beta - 1} [h(1 - \tau) + \tau]^{-1} I
\]
We show in Section 5.4 that the consequence of these different results for the default and negotiation option is that lenders *ex-ante* mortgage yield spread should increase significantly to pay for the investor’s *ex-post* negotiation option.

c) Investors renegotiate with lenders when $x(t) < x_{df}$, where $x_{df}$ is the endogenously determined negotiation threshold given by

$$x_{df} = \frac{\beta}{\beta - 1} K \left[ g + \frac{\tau}{L} \right]^{-1} l$$

[12] or [A47]

Investors default/foreclose with lenders when $x(t) < x_f$, where $x_f$ is the endogenously determined default threshold given by

$$x_f = \frac{\beta}{\beta - 1} K \left[ h + \frac{\tau}{1 - \tau} \right]^{-1} l$$

[13] or [A6]

We show in Section 5.4 that the implications of these equations are that negotiation option exercise will occur earlier than the default option exercise for all investors but strong negotiators will exercise their negotiation option earlier than weak negotiators.

We define the optimal $LTV_{di}/LTV_i$ at mortgage origination $x_{di}$ or $x_i$ as the book value of debt divided by the property value at mortgage origination and is defined for the negotiation option as

$$LTV_{di} = \frac{D(x_{div}, c_d)}{V(x_{div}, c_d)}$$

[14]

This is shown in Section 5.8 to be equivalent to
The ex-post yield spread at origination is defined as

\[ YS_{dt} = \frac{c_d}{D(x_{di}, c_d)} - r \]  \hspace{1cm} [16]

and

\[ YS_i = \frac{c^*}{D(x_i, c^*)} - r \]  \hspace{1cm} [17]

for both options respectively where \( D(.) \) is the value of debt at the investment threshold \( x_i \) or \( x_{di} \) using the equations in Section 5.8.

The debt interest coverage ratio at origination is defined for the negotiation and the default option respectively as the net rental income rate after tax at origination divided by the mortgage payment after tax

\[ DCR_{dt} = \frac{x_{di}}{c_d(1 - \tau)} \]  \hspace{1cm} [18]

and

\[ DCR_i = \frac{x_i}{c^*(1 - \tau)} \]  \hspace{1cm} [19]
In Section 5.4, we use these equations to predict optimal borrower negotiation regions in terms of mortgage control parameters of market LTV and DCR ratios rather than rental income rate $x$.

For the sake of brevity, we do not repeat the analysis or discuss the preceding equations, as they are precisely the same, and therefore behave the same as the equations and analysis already discussed in Section 4.4. If we insert the same numbers into the equations in Chapters 4 and 5 then the same output will result. However, the crucial difference is that the models in Chapter 4 are driven by a stochastic property price $V$ while the models in this chapter are driven by a stochastic rental income $x$. The equity equations are therefore formulated slightly differently and is in the case of Chapter 4:

$$\frac{1}{2} \sigma^2 V^2 \mathbb{E}_{VV}(V) + \mu \mathbb{E}_V(V) - r \mathbb{E}(V) + (r - \mu) V - C(V) = 0 \text{ when } V < V_{df}$$

While the similar (but different) equation in Chapter 5 is:

$$\frac{1}{2} \sigma^2 x^2 \mathbb{E}_{xx}(x) + \mu x \mathbb{E}_x(x) - r \mathbb{E}(x) + (1 - \tau) x - C(x) = 0 \text{ when } x < x_{df}$$

Therefore $(r - \mu) V$ is a notional market imputed rent while $(1 - \tau) x$ is the taxed rental income, the implications of which we have already discussed in Chapter 4. A further important distinction is that the property investment cost (as in this chapter) or property purchase price (as in Chapter 4) although both signed as $I$ will in practice be different, as ceteris paribus, the owner occupier and lender should value the same property differently from the owner investor and lender due to the untaxed benefits of the market imputed rent. In Chapter 4 we use $I$, which is a spot asset value while in Chapter 5 we use $\frac{r - \mu}{1 - \tau} I$ which is a spot cash flow to drive the models. If we assume that the owner occupier pays the same price as the owner investor then optimal coupon payments and yield spreads will be the same.
5.4 Negotiation and Default Option Analysis – A Stylised Example

The negotiation option represents the relationship between the investment and financing decisions, where the initial *ex-ante* investment decision is dependent on the (future) optional renegotiation between lender and borrower. On the other hand, the default (non-bargaining) option represents the relationship whereby the borrower makes the initial *ex-ante* investment decision in the knowledge that non-payment of the coupon will certainly result in foreclosure and forfeiture of all equity.

This section will demonstrate in a graphical manner and compare the fundamentally different quantitative results that arise from the two options using stylised data from the US BTL property market (Robinson and Todd, 2010) and the equations derived in Section 5.8. The parameters are somewhat different from those in Chapter 4 reflecting the fact that US non owner occupied mortgage borrowers buy (on average) cheaper properties and expect higher yield rates than owner occupied mortgage borrowers. Rental volatility and tax benefits are also higher reflecting the riskier nature of renting due to defaults in rent and vacancies as well as the higher income of the investor. We observe how the negotiation region, \(0 \leq \phi \leq 1\) compares to the single *default only line*.

The analysis proceeds as follows

a) Calculate when the investor might exercise the investment options.

b) Establish the critical negotiation region and default threshold for both options.

c) Calculate the investor’s optimal mortgage debt (LTV) and resulting coupon payment.

d) Calculate the lender’s risk spread over the riskless rate.

e) Plot DCR (debt interest coverage ratio) against market Book LTV.

f) Illustrate some model sensitivities to foreclosure costs, rental growth and volatility.
We start by sketching in Figure 5.4.1 the optimal ex-ante equity value and investment trigger points for the negotiation option. It demonstrates that if investor and lender have agreed a negotiation option that the ex-ante optimal timing and payoff of the investment depends on the bargaining power of the investor. A weak negotiator \((\phi = 0)\) can obtain a larger mortgage (higher LTV) but must then pay a higher coupon “enjoying” a higher tax deduction which leads to the higher ex-ante equity value. This reflects the idea that taxes introduce friction into the standard all equity real options valuation approach, whereby it becomes optimal to take on mortgage debt.

Figure 5.4.1 Ex-Ante Equity Value as a Function of Rental Income Rate \(x\) and Bargaining Power \(\phi\).

The three concave curves labelled \(\phi=0.0, 0.5\) and 1.0 are the equity payoffs \(V(x)-I\) under mortgage financing and \(\tau=25\%\). Using equation [A26] equity value is highest with the concave curve All Equity \(\tau=0\%\) and is the lowest with the concave curve All Equity \(\tau=25\%\).

Payoffs at the optimal entry threshold under all 5 settings are the same and equal to \(\frac{I^*\beta}{\beta-1}\)

Parameter values : \(i = 200000, \ r = 0.03, \ \mu = 0.01, \ \tau = 0.25, \ \alpha = 0.3\) and \(\sigma = 0.15\).
We continue by assuming that the borrower has now purchased a property financed by an optimal LTV mortgage at the optimal entry point, paying the optimal coupon. We compare ex-post property, debt and equity values for the two options (negotiation and default) on the same graph.

Figure 5.4.2 overleaf shows ex-post property value as a function of rental income rate for the two options with the all equity financed value at a tax rate of 25% as a reference line. We delineate in Figure 5.4.2 the optimal negotiation “region”, defined as the range of optimal exit values $x_{df}$ for $0 \leq \phi \leq 1$. We superimpose the optimal default line for the default option.

Negotiation will always occur earlier than default where rental income constantly decreases. Upon exercise of the negotiation option, property value and coupon payment continue to decrease as both the borrower and lender continue to “operate” the property. However, upon exercise of the default option, the lender forecloses and sells the property receiving $(1 - \alpha)$ times the all equity financed property value.
The three concave curves labelled $\phi=0.0$, $0.5$ and $1.0$ are the property values $V(x)$ under mortgage financing and $\tau=25\%$. Using equations \[A26],\[A27]
Property value increases as $\phi$ decreases reflecting the greater benefit of the tax shelter when the investor has weak bargaining power.
In all 3 negotiation cases the property always has value as the lender will adjust the coupon payment so that the investor continues to operate.
The rectangular shaded region is defined as the upper and lower limits of the bargaining parameter wherein negotiation optimally occurs.
The heavy dashed curve is the property value for the default (non bargaining option) and terminates at the exit threshold where the lender will foreclose and sell the property with $\alpha=30\%$ foreclosure costs while the investor gets nothing.
Parameter values: $I = 200000$, $r = 0.03$, $\mu = 0.01$, $\tau = 0.25$, $\alpha = 0.3$ and $\sigma = 0.15$

We decompose the total \textit{ex post} property values from Figure 5.4.2 into their component equity and debt values in Figure 5.4.3 overleaf. The result is a “busy” graph demonstrating that the market value of debt and equity with a negotiation option varies considerably compared to the default option. A strong lender is prepared to lend a larger amount (higher LTV) to a weaker investor but not as much to a strong investor.
A higher optimal LTV of 97% results for a very weak (negotiating) investor given that the lender will gain compared to the default option LTV of 66% after the negotiation option is exercised. On the other hand, a lower optimal LTV of 59% results for a very strong (negotiating) investor given that the lender will not gain while for the mid way case (φ = 0.5) where the lender gains somewhat, the investor benefits from a higher LTV of 77% compared to 66%. One could argue that a negotiation option increases the debt capacity compared to a strict foreclosure or default option.

Figure 5.4.3 Ex-Post Debt and Equity Value as a Function of Rental Income Rate x and Bargaining Power φ.

The three concave curves labelled Equity φ=0.0, 0.5 and 1.0 are the equity values E(x) under mortgage financing and τ=25%. Using equations [A39], [A40]

The three convex curves labelled Debt φ=0.0, 0.5 and 1.0 are the debt values D(x) under mortgage financing and τ=25%. Using D(x)=V(x)-E(x)

The addition of the curves for Debt φ=0.0 and Equity φ=0.0 is equal to the curve V(x) φ=0.0 in Figure 5.4.2

In the case where bargaining strength φ<0.0 and below the negotiation threshold, although debt has value, equity has no value but the investor still operates the property, pays a reduced coupon (see Figure 5.4.4) and could still benefit if rental income rates rebound.

The rectangular shaded region is defined as the upper and lower limits of the region wherein negotiation optimally occurs

Parameter values : I = 200000, r = 0.03, μ = 0.01, τ = 0.25, α = 0.3 and σ = 0.15
Figure 5.4.4 demonstrates a crucial point that the (re)negotiated coupon is conditional on \( \phi \) or the *ex-ante* consideration of avoidable foreclosure costs. It shows that the perpetual coupon, both before and after negotiation, depends on the LTVs, which were agreed after considering the different heterogeneous bargaining strengths. It demonstrates that a strong negotiator should exercise or threaten negotiation earlier than the weak negotiator.

Figure 5.4.4 Optimal Coupon Payment as a Function of Rental Income Rate \( x \) and Bargaining Power \( \phi \).

The three discontinuous curves labelled \( \phi=0.0, 0.5 \) and 1.0 are the coupon payment curves under mortgage financing and \( \tau=25\% \). Using equations [A44],[A46] for \( \phi=0.0 \) an optimal coupon \( c_d \) of \( \$ \, 20887 \) is paid up to the delinquency threshold and a linearly decreasing coupon should the rental decrease further.

Coupon payments decrease as \( \phi \) increases reflecting the lower LTV (debt capacity) offered by the lender.

The heavy dashed straight line is the constant coupon payment for the default non-bargaining option and terminates at the default line.

Note that a coupon is always paid under the delinquency bargaining option which becomes more affordable below the delinquency threshold.

Parameter values: \( I = 200000 \), \( r = 0.03 \), \( \mu = 0.01 \), \( \tau = 0.25 \), \( \alpha = 0.3 \) and \( \sigma = 0.15 \).
It can be seen that where the borrower is a weak negotiator ($\phi = 0$) the lender offers the highest mortgage loan ($LTV_{di} = 97\%$) and the investor pays the highest coupon. In this case, as the rental income decreases below the optimal negotiation level, the equity value of the investor is zero. The investor continues operating the property, paying a reduced *more affordable* coupon to the lender but retains ownership of the property in the “hope” that the rental income may bounce back.

In the other case where the investor is strong ($\phi = 1$) the lender offers the lowest mortgage loan ($LTV_{di} = 59\%$) and the investor pays the lowest coupon. In this case, however, as rental income rate decreases, the market equity value of the investor remains substantial. With a default option and at its (dashed line) critical exercise threshold, the investor defaults whereas at the same rental income rate with a negotiation option the investor pays a reduced coupon.

Reading from right to left, for a decreasing rental income rate, the economic consequences of the negotiation option are that the investor will enter negotiation (i.e. threaten default) earlier and start paying a reduced or *more affordable* coupon earlier than for the default option where the investor will default or the lender will foreclose instantaneously as soon as the critical threshold is reached. The more power the investor is perceived to have, the earlier that negotiation will occur because the more financial concessions that may be extracted, resulting in a higher negotiation threshold $x_{df}$.

Incidentally, the estimated optimal LTV for different types of investors is consistent with empirically surveyed LTV data for non-owner occupied properties varying significantly according to Robinson and Todd (2010) around a median figure of 69% (Figure 5.4.5 overleaf).
Figure 5.4.5 LTV Distribution of US Homeowners 2004-2007 (Robinson and Todd 2010)
We compare the lender’s yield spread over the risk free rate for a negotiation and default option in Figure 5.4.6. A negotiation option increases the lender’s required risk spread compared to a default option. Variation between the overall yield curves for the three different strengths of parameter $\phi$ in the negotiation case from 123 to 138 basis points is relatively small compared to that of the default option’s yield spread of 61 basis points. Differences in optimal yield spreads for the three different parameters are more dependent on the optimal investment entry point with the weak investor paying a higher yield because they optimally make the investment earlier. Irrespective of whether the heterogeneous parameter power $\phi$ is exactly observable or measurable the existence of any measure of bargaining introduces a fundamental change to the contract whereby the lender charges a higher yield spread. The stronger the lender vis-à-vis the investor, the lower the implicit yield spread curve but the higher the actual perpetual coupon payment due to the larger mortgage loan.

**Figure 5.4.6 Yield Spread Basis Points as a Function of Rental Income Rate $x$ and Bargaining Power $\phi$.**

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Yield Spread Basis Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>138</td>
</tr>
<tr>
<td>0.5</td>
<td>130</td>
</tr>
<tr>
<td>1.0</td>
<td>128</td>
</tr>
</tbody>
</table>

The three convex curves labelled $\phi=0.0$, 0.5 and 1.0 are the yield spread curves under mortgage financing and $\tau=25\%$. Using equations (A50),(A51)

For $\phi=0.0$ the yield spread at the entry threshold is 138 basis points and decreases as $\phi$ increases reflecting greater bargaining power.

The heavy dashed straight line is the yield spread curve for the default option and equals 61 basis points at the entry threshold or mortgage origination.

The yield curves for the negotiation option are all nearly colinear but differ significantly from that of the default option.

Parameter values: $I=200000$, $r=0.03$, $\mu=0.01$, $\tau=0.25$, $\alpha=0.3$ and $\sigma=0.15$.
We next investigate the LTV and DCR parameters which are reported to investors in US residential rental mortgage backed securities reports.

Book LTV is defined for the default option as \[ LTV = \frac{D(x,c^*)}{V(x,c^*)} \] and for the negotiation option as \[ LTV_d = \frac{D(x,c_d)}{V(x,c_d)} \] \textit{ex-ante} and \textit{ex-post} option exercise. To illustrate we plot LTV as a function of rental income rate in Figure 5.4.7.

Figure 5.4.7 Book LTV as a Function of Rental Income Rate \( x \) and Bargaining Power \( \phi \)

The three curves labelled \( \phi=0.0, 0.5 \) and 1.0 are the Book Loan-To-Value ratios for the negotiation option decreasing as rental income increases. As the rental income decreases the Book LTV ratio approaches a finite figure depending on \( \phi \). The heavy dashed straight line is the Book LTV ratio for the default option and approaches 100% at the default point.

Parameter values: \( I = 200000, r = 0.03, \mu = 0.01, \tau = 0.25, \alpha = 0.3 \) and \( \sigma = 0.15 \)

Figure 5.4.7 demonstrates that the book LTV quickly approaches 100% when \( \phi = 0.0 \), approaches 70% (i.e. \( 1-\alpha \)) when \( \phi = 1.0 \) and 85% (i.e. \( 1-\alpha/2 \)) when \( \phi = 0.5 \). For the default option, the LTV (dashed line) also approaches 100% at the default option exercise point.
The optimal LTV at origination for the negotiation option $LTV_{di}$ is shown (Chapter 5.8) as:

$$LTV_{di} = \frac{\gamma - [(1 - g^y)(1 + \tau(\phi - 1))]}{\gamma (gL + \tau)}$$

[20]

depending only on the investor’s endogenous tax status and heterogeneous parameter $\phi$.

DCR is defined for the default option as $DCR = \frac{x}{c^*(1-\tau)}$ and for the negotiation option as $DCR_d = \frac{x}{c_d(1-\tau)}$ \textit{ex-ante} option exercise and $DCR_d = \frac{1}{(1-\tau)^2(1-\phi\alpha)}$ \textit{ex-post} option exercise.

To illustrate we plot DCR as a function of rental income rate in Figure 5.4.8.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.4.8.png}
\caption{Debt Coverage Ratio (DCR) as a Function of Rental Income Rate $x$ and Bargaining Power $\phi$}
\end{figure}

The three discontinuous curves labelled $\phi=0.0, 0.5$ and $1.0$ are the Debt Coverage Ratios for the negotiation option. As the rental income decreases the DCR ratio decreases depending on $\phi$ but then increases to a constant value after the negotiation option is exercised. The heavy dashed straight line is the DCR ratio for the default option which decreases to the default exercise point.

Parameter values: $I = 200000$, $r = 0.03$, $\mu = 0.01$, $\tau = 0.25$, $\alpha = 0.3$ and $\sigma = 0.15$.
Figure 5.4.8 demonstrates that the DCR increases linearly with rental income rate \( x \) after the critical negotiation exercise point. Below the threshold, investor and lender reach agreement on a new coupon which is dependent on the actual rental income rate (Figure 5.4.4) resulting in a constant DCR ratio, which is greater than 1.0 in all cases.

It is interesting to note that in this particular instance with rental income rate growth \( \mu = 0.01 \) the initial critical entry debt cover ratio for a wide range of investor types is less than 1.0 at the investment entry trigger point. When \( \mu = 0.0 \) (see Table 5.4.1) the entry DCR is close to 1.0 at \( \phi = 0.0 \). In this stylised example based on actual US BTL data, an initial entry DCR less than 1.0 at the investment entry threshold is consistent with the model, given that the investor has an immediate equity gain, is allowed to pay the mortgage coupon from his other resources and might have future (positive) expectations about the capital (or rental) appreciation of the property. As might be intuitively expected, with a default option (heavy dashed line), foreclosure occurs when both the DCR is less than 1.0 and the LTV approaches 100%.

We find that results from Piskorski, Seru and Vig (2010) on distance between default and delinquency (negotiation) conditional on delinquency (negotiation) are consistent with our approach. They examine the effect of the initial creditworthiness of the borrower (pg. 16, Table 8) and claim higher cure (reinstatement) rates and lower foreclosure rates for bank held loans compared to securitised loans (respectively in our formulation \( \phi \to 1 \) and \( \phi \to 0 \)).

They say that “.... for loans of the lowest credit quality (FICO < 620) there is ...a very small difference in foreclosure and cure rates. In contrast, for loans with FICO > 680: the foreclosure rate for bank held loans is lower in absolute terms by 9.2% and the cure rate is higher by 12.1% within a year after delinquency...”
We conclude by summarising key graphical data in Table 5.4.1 and show the effect of extremes in foreclosure costs \( \alpha \) and normal changes in rental income volatility \( \sigma \) and rate of return \( \mu \). Increases in rental income rate volatility with no changes in other parameters behaves as expected delaying investment, increasing yield spreads and reducing debt capacity (LTV) at origination.

Table 5.4.1 Table Illustrating Negotiation Option Sensitivity to Various Parameters

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>Coupon £</th>
<th>Entry Rental £</th>
<th>Exit Rental £</th>
<th>Entry Yield Spread bp</th>
<th>Entry LTV %</th>
<th>Entry DCR</th>
<th>Entry Default DCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreclosure % ( \alpha = 1% ), ( \sigma = 0.15 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>20887</td>
<td>10227</td>
<td>8524</td>
<td>138</td>
<td>97</td>
<td>0.65</td>
<td>0.98</td>
</tr>
<tr>
<td>0.5</td>
<td>18403</td>
<td>10565</td>
<td>8806</td>
<td>130</td>
<td>87</td>
<td>0.77</td>
<td>1.15</td>
</tr>
<tr>
<td>1.0</td>
<td>16429</td>
<td>10834</td>
<td>9030</td>
<td>123</td>
<td>79</td>
<td>0.88</td>
<td>1.32</td>
</tr>
<tr>
<td>Default</td>
<td>14711</td>
<td>11067</td>
<td>6004</td>
<td>79</td>
<td>79</td>
<td>1.00</td>
<td>1.42</td>
</tr>
</tbody>
</table>

| Foreclosure % \( \alpha = 30\% \), \( \sigma = 0.15 \) | | | | | | | |
| 0.0 | 20887 | 10227 | 8524 | 128 | 97 | 0.65 | 0.98 |
| 0.5 | 16172 | 10868 | 9059 | 130 | 77 | 0.90 | 1.34 |
| 1.0 | 12229 | 11405 | 9506 | 123 | 59 | 1.24 | 1.87 |
| Default | 11688 | 11478 | 4770 | 61 | 66 | 1.31 | 1.96 |

| Foreclosure % \( \alpha = 99\% \), \( \sigma = 0.15 \) | | | | | | | |
| 0.0 | 20887 | 10227 | 8524 | 138 | 97 | 0.65 | 0.98 |
| 0.5 | 10313 | 11666 | 9723 | 130 | 49 | 1.51 | 2.26 |
| 1.0 | 200 | 13041 | 10870 | 123 | 1 | 87.04 | 130.56 |
| Default | 8325 | 11936 | 3398 | 48 | 49 | 1.91 | 3.12 |

| Foreclosure % \( \alpha = 30\% \), \( \sigma = 0.30 \) | | | | | | | |
| 0.0 | 69060 | 18299 | 15660 | 493 | 99 | 0.35 | 0.52 |
| 0.5 | 53541 | 19472 | 16664 | 480 | 78 | 0.48 | 0.72 |
| 1.0 | 40529 | 20456 | 17506 | 467 | 60 | 0.67 | 1.00 |
| Default | 25334 | 21604 | 5745 | 213 | 56 | 1.14 | 1.64 |
The higher the foreclosure costs, the lower the LTV the lender should agree with the strong investor while continuing to offer the same LTV to the weak investor. This is a result of the model sharing rule whereby a lender negotiating with a weak investor has the opportunity to gain significantly compared to the standard default option. However (on average) a large decrease in (LTV) lending capacity from 86% to 48% can still be observed for the equilibrium value of \( \phi = 0.5 \) as foreclosure costs increase from 1% to 99%. Whether a strong lender might lend to any investor (weak or strong) with probable large foreclosure costs is perhaps best left to a reflection on lending practices and securitisation in the recent US subprime crises.
5.5 Chapter Conclusions

We have combined two different aspects of real options that of irreversible investment and debt pricing/capital structure, to develop ex-ante closed form solutions of the investor’s negotiation and default options in a very similar manner to the treatment of the homeowner in Chapter 4. A major difference is that rental income and an implied property price drives the process for the non-owner occupying investor while property price and an implied rental income drives the process for the owner occupying homeowner. This subtle difference allows us to examine a debt coverage ratio in this chapter. It should also be noted that lenders invariably have minimum criteria for DCR (and not rental income) at mortgage origination for those intending to let investment homes.

Secondly, we have demonstrated that, even with its very strong assumptions, the model predictions are consistent with intuitive economic outcomes as well as results from current empirical research into (re)negotiation of securitised/non securitised mortgages and mortgage down payments (e.g. PSV 2010).

The investor who exercises the negotiation option is not precluded from (later) exercising their default, cure or prepayment options as additional (option) value may exist for the investor if the lender agrees to negotiate. We have demonstrated in Chapter 3 that this condition occurs for more than half of delinquent mortgages in the US currently.

Implicit within the negotiation option is that both parties agree new (sliding) coupon payments conditional on the current rental income rate. This is an abstraction from reality where in practise, due to monitoring costs, only one new lower affordable coupon payment may be agreed, whereupon non-performance might lead to irrevocable foreclosure.
Thirdly, we claim that aspects of negotiation originally developed (but not widely applied) for corporate bond valuation might be applicable (subject to empirical investigation) for valuing BTL mortgages, but recognise that the same comparison does not hold true for the valuation of a corporate bond and household mortgage default option.

We have introduced an additional bargaining parameter $\phi$. This parameter $\phi$ is not directly observable or measurable but is a convenient construct to easily divide the benefits of avoiding foreclosure costs between lender and investor. The parameter is heterogeneous in that, two investors with the same lender (or servicing agent) may have different values resulting in different outcomes of the (re)negotiated coupon payment. In any case, we are less interested in the exact value of $\phi$ and much more interested in delineating the maximum and minimum boundaries of the critical region where renegotiation of the mortgage coupon may be initiated as a result of both parties wishing to avoid foreclosure costs. Forewarned is forearmed and better understanding when this option might be exercised, compared to the traditional default region, is better than having no understanding of when it might be exercised.

The negotiation option has been demonstrated to have ex-post distinct economic and financial differences with the more conventional default option. It remains to empirically investigate whether this idea of investors strategically negotiating actually occurs within an option theoretic equity optimising framework or rather within some other “affordability optimising” framework.
5.6 References


5.7 Glossary of Notation

\( x \): Net rental income rate after operating expenses and local property taxes but before mortgage payment tax deduction.

\( x_i \): Optimal rental rate at where the borrower would invest with a default option.

\( x_{di} \): Optimal rental rate at where the borrower would invest with an negotiation option.

\( x_f \): Optimal rental rate at where the borrower would default with a default option.

\( x_{af} \): Optimal rental rate at where the borrower would negotiate.

\( x_{ae=0} \): Optimal investment threshold with all equity (ae) financing and no taxes.

\( x_{ae>0} \): Optimal investment threshold with all equity (ae) financing and with taxes.

\( \sigma \): Net rental income rate volatility.

\( \mu \): Net rental income rate drift.

\( \beta, \gamma \): Roots of \( \frac{\sigma^2 x^2}{2} + (\mu - \sigma^2/2)x - r = 0 \)

\( \alpha \): Lender’s foreclosure costs as a percentage of the property value \( V(x) \) implied by the net rental income rate \( x \) at either \( x_i \) or \( x_{di} \).

\( r \): Borrowers tax rate

\( r \): Risk free rate of return

\( c^* \): Perpetual mortgage payment to the lender for the default option (tax deductible).

\( c_d \): Perpetual mortgage payment to the lender for the negotiation option (tax deductible).

\( C(x) \): Renegotiated mortgage payment which depends on rental rate \( x \).

\( \phi \): Heterogeneous bargaining or sharing parameter which lies between 0 and 1.

\( I \): Initial property investment made at the critical investment thresholds \( x_i \) or \( x_{di} \).

\( LTV_{di} \): Risk adjusted Loan to Value ratio at origination \( x_{di} \) for the negotiation option

\( LTV_i \): Risk adjusted Loan to Value ratio at origination \( x_i \) for the default option.

\( DCR_{di} \): Risk adjusted Debt Interest Coverage Ratio for the delinquency option.

\( DCR_i \): Risk adjusted Debt Interest Coverage Ratio at origination \( x_i \) for the default option
\( YS_{di} \): Risk adjusted Yield Spread for the negotiation option at origination \( x_{di} \)

\( YS_{i} \): Risk adjusted Yield Spread for the default option at origination \( x_{i} \).

\( V(x, c) \): Property value as a function the net rental income rate and mortgage payment.

\( E(x, c) \): Equity value as a function of the net rental income rate and mortgage payment.

\( D(x, c) \): Debt value as a function of the net rental income rate and mortgage payment.

\( K \): defined as \( \frac{(\tau - \mu)}{1 - \tau} \).

\( L \): defined as \( \frac{1 - (1 - \phi)}{1 - \phi^2} \) used for the negotiation option.

\( g \): defined as \( \left[ \frac{\beta}{\beta - \gamma} (1 - \gamma) \right]^{-1/\gamma} = \frac{Z_{di}}{x_{df}} \) used for the negotiation option.

\( h \): defined as \( \left[ 1 - \frac{\gamma(\tau + \alpha(1 - \gamma))}{\tau} \right]^{-1/\gamma} = \frac{x_d}{x_f} \) used for the default option.
5.8 Detailed Model Derivation

5.8.1 Default Option – Model 1

We proceed by first deriving a “default no bargaining” case based on Leland (1994) (but assuming in contrast to the Leland paper that no investment is in place), where the lender and investor do not bargain or share the value of the assets at the optimally chosen critical default threshold $x_f$. The lender forecloses and the property automatically becomes all equity financed and is presumed sold with no future tax benefits. At which point the value of the equity claim is $E(x_f) = 0$ and $D(x_f) = (1 - \alpha) F_{ae}(x_f, 0)$ where $\alpha$ is the loss severity percentage. Thus $(\alpha F_{ae}(x_f, 0))$ is taken away by outsiders, $(1 - \alpha)F_{ae}(x_f, 0)$ by the lender and the investor gets nothing.

If $x$ follows a gBm given by [A1] then the solution for the general differential equation $F(x)$ where

$$ax^2F_{xx} + bxF_x + cF = xd + e$$

is given by $F(x) = A_1x^\gamma + A_2x^\beta + \frac{xd}{b + c} + \frac{e}{c}$

where $\gamma < 0$ and $\beta > 0$ are roots with $\gamma, \beta = \frac{(a-b)\pm\sqrt{(b-a)^2-4ac}}{2a}$

and $A_1, A_2$ are determined from the appropriate boundary conditions.

The rental income rate of the property, denoted by $x$, follows the gBm process given by

$$dx = \mu xd t + \sigma xdW$$  \[A1\]

Where $\mu$ is the instantaneous expected rate of return of the property gross of any pay-out, $\sigma$ is the instantaneous variance of the return of the property and $dW$ is a standard Brownian motion.
Given [A1] the (general) asset value \( F(x) \) of a claim paying \( xd + e \) satisfies the equilibrium condition

\[
r F(x_t) = xd + e + \frac{1}{dt} \mathbb{E}_t^Q[F(x_{t+dt})]
\]

The first two terms are the expected cash flow while the third term is the expected capital gain of \( F(x_t) \) from \( t \) to \( t+dt \). Using [A1] and applying Ito’s lemma to the third term we get a general ODE

\[
r F(x) = xd + e + \frac{1}{2} \sigma^2 x^2 F_{xx}(x) + \mu x F_x(x)
\]

with 
\[
a = \frac{1}{2} \sigma^2, b = \mu, c = -r \text{ and } \gamma, \beta = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}
\]

where \( r > \mu \) is the risk neutral instantaneous rate of return.

This has a general solution of the form

\[
F(x) = A_1 x^\gamma + A_2 x^\beta + \frac{xd}{(r - \mu)} + \frac{e}{r}
\]

[Solutions are sought for three value functions namely:

- d) \( D(x) \to \) the lender’s debt value,
- e) \( E(x) \to \) the investor’s equity value,
- f) \( V(x) \to \) the total property value.

The investor has control over capital structure and decides on the mix of debt \( D(x, c) \) and equity \( E(x, c) \) to finance the property whereby \( V(x, c) = D(x, c) + E(x, c) \)
Hence at origination \((t=0)\) \(D(x_t,c^*) + E(x_t,c^*) = I\) where \(x_t\) is the (initial threshold) property rental that induces the investor to invest and \(c^*\) is the optimally chosen perpetual coupon agreed with the lender and \(I\) the initial investment.

The all equity financed property value is given by \(F_{ae}(x,0)\) or in other words the discounted value of after tax earnings.

\[
F_{ae}(x,0) = \frac{1 - \tau}{(r - \mu)} x
\]

Where \(\tau\) is the investor’s tax rate.

c) Debt value \(D(x,c)\) is derived by letting \(d=0\) and \(e=c\) in \([A2]\) and solving with the following smooth pasting and value matching conditions

\[
\lim_{x \to \infty} D(x,c) = \frac{c}{r}
\]

\[
D(x_f,c) = (1 - \alpha) F_{ae}(x_f,0)
\]

Resulting in

\[
D(x,c) = \frac{c}{r} \left[ 1 - \left(\frac{x}{x_f}\right)^\gamma \right] + (1 - \alpha) F_{ae}(x_f,0) \left(\frac{x}{x_f}\right)^\gamma \text{ for } x > x_f \quad [A3]
\]

or debt is the sum of discounted coupon flows \(c\) and residual firm rental value.

d) Equity value \(E(x,c)\) is derived by letting \(d = 1 - \tau\) and \(e = (1 - \tau)c\) in \([A2]\) and solving with the following smooth pasting and value matching conditions

\[
\lim_{x \to \infty} E(x,c) = \frac{1 - \tau}{r - \mu} x - \frac{(1 - \tau)c}{r}
\]

\[
E(x_f,c) = 0
\]

Resulting in

\[
E(x,c) = (1 - \tau) \left[ \left(\frac{x}{r - \mu} - \frac{c}{r}\right) - \left(\frac{x_f}{r - \mu} - \frac{c}{r}\right) \left(\frac{x}{x_f}\right)^\gamma \right] \text{ for } x > x_f \quad [A4]
\]

or equity is the discounted value of earnings after interest and tax.
Property value $V(x, c)$ is derived by letting $d = 1 - \tau$ and $e = \tau c$ in [A2] and solving with the following smooth pasting and value matching conditions

$$\lim_{x \to \infty} V(x, c) = \frac{1 - \tau}{r - \mu} x + \frac{\tau c}{r}$$

$$V(x_f, c) = 0$$

Resulting in

$$V(x, c) = F_{ae}(x, 0) + \frac{\tau c}{r} \left[ 1 - \left( \frac{x}{x_f} \right)^\gamma \right] - \alpha F_{ae}(x_f, 0) \left( \frac{x}{x_f} \right)^\gamma \quad \text{for} \ x > x_f \quad [A5]$$

Property value can be thus decomposed into the sum of three components

1) The all equity property value

2) Plus the tax shield value of benefits and

3) Less the foreclosure costs

The tax benefit value of debt $\frac{\tau c}{r} \left[ 1 - \left( \frac{x}{x_f} \right)^\gamma \right]$ is an increasing and concave function of $x$ whereby if $\tau > 0$ there is some range which indicates that tax savings are achieved by higher levels of $c$. However $\frac{\tau c}{r} \left[ 1 - \left( \frac{x}{x_f} \right)^\gamma \right]$ starts to decline with $c$ at a certain point with the potential loss of tax benefits at bankruptcy. Similarly $\alpha F_{ae}(x_f, 0) \left( \frac{x}{x_f} \right)^\gamma$ are increasing and convex in $c$ but decreasing and convex in $x$. Thus a unique coupon $c^*$ exists that maximises $V(x, c)$ calculated by solving for $V_x(c) = \frac{\partial V(x, c)}{\partial c} = 0$ at $c = c^*$. In a similar vein the endogenous default policy that maximises equity value for a given debt level solves

$$E_x(x) = \frac{\partial E(x, c)}{\partial x} = 0 \quad \text{at} \ x = x_f$$

and can be shown to be given by

$$x_f = \frac{c^* \gamma}{\gamma - 1} \frac{r - \mu}{r}$$
Or using [A8]

\[ x_f = \frac{\beta}{\beta - 1} K \left[ h + \frac{\tau}{1 - \tau} \right]^{-1} I \]

Where \( h = \left[ 1 - \frac{\gamma(\tau + \alpha(1 - \tau))}{\tau} \right]^{-1/\gamma} \frac{x_i}{x_f} \)

Rewriting [A5] we obtain

\[ V(x, c) = F_{ae}(x, 0) + \frac{\tau c}{r} - \left( \frac{x}{c} \right)^{\gamma} \left[ \frac{\tau c}{r} + \alpha c \frac{\gamma}{\gamma - 1} \left( \frac{r - \mu}{r - \mu} \right) \right] \]

And substituting \( x_f \) from [A6] gives

\[ = F_{ae}(x, 0) + \frac{\tau c}{r} - \left( \frac{x}{c} \right)^{\gamma} \left[ \frac{\tau c^{1-\gamma}}{r} + \alpha c^{1-\gamma} \frac{\gamma}{\gamma - 1} \left( \frac{1 - \tau}{r - \mu} \right) \right] \]

Grouping \( c \) in the 3\textsuperscript{rd} term together and some cancellations gives

\[ F_{ae}(x, 0) + \frac{\tau c}{r} - \left( \frac{x}{c} \right)^{\gamma} \left[ (1 - \gamma) \frac{\tau c^{-\gamma}}{r} + \alpha(1 - \gamma) c^{-\gamma} \frac{\gamma}{\gamma - 1} \left( \frac{1 - \tau}{r} \right) \right] \]

Then taking \( V_c(c) \)

\[ \frac{\tau}{r} = \left( \frac{x}{c^{\gamma - \mu}} \right)^{\gamma} \left[ (1 - \gamma) \frac{\tau c^{-\gamma}}{r} + \alpha(1 - \gamma) c^{-\gamma} \frac{\gamma}{\gamma - 1} \left( \frac{1 - \tau}{r} \right) \right] \]

Substituting \( c^* \), extracting \( c^{-\gamma} \) back and setting the expression equal to 0 gives

\[ \frac{\tau}{r} = \left( \frac{x}{c^* \frac{\gamma - \mu}{\gamma - 1} \frac{\gamma}{\gamma - 1}} \right)^{\gamma} \left[ (1 - \gamma) \frac{\tau}{r} - \alpha \gamma \left( \frac{1 - \tau}{r} \right) \right] \]

\[ \frac{\tau}{r} = \left( \frac{x}{c^* \frac{\gamma - \mu}{\gamma - 1} \frac{\gamma}{\gamma - 1}} \right)^{\gamma} \left[ \tau - \gamma \tau - \alpha \gamma + \alpha \gamma \tau \right] \]

Giving

\[ c^* = \frac{x \frac{\gamma - 1}{\gamma}}{h \frac{\gamma}{\gamma - \mu}} \]

Where \( h = \left[ 1 - \frac{\gamma(\tau + \alpha(1 - \tau))}{\tau} \right]^{-1/\gamma} \)

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We now assume that the investor only agrees the optimal coupon $c^*$ with the lender and obtains the required funds when the property rental $x = x_i$ the entry level threshold thus

$$c^* = \frac{x_i \gamma - 1}{h} \frac{r}{\gamma} \frac{r - \mu}{r - \mu}$$

Or using [A8] $c^* = r \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} [h(1 - \tau) + \tau]^{-1} I$ \[A7\]or[10]

We next examine the relationship of $x_i$ with the initial investment $I$ but show the derivation in more detail in the next case in Part 5.8.2.2.

By investing the investor collects $E(x, c) - (I - D(x, c)) = V(x, c) - I$

It can be shown that this implies that the optimal investment threshold $x_i$ is given by

$$x_i = \frac{r - \mu}{1 - \tau} \frac{\beta}{\beta - 1} \left[ (I - \frac{\tau c^*}{r}) + \frac{\beta - \gamma}{\beta} \left( \alpha x_f \frac{1 - \tau}{r - \mu} + \frac{\tau c^*}{r} \right) \left( \frac{x_i}{x_f} \right)^{\gamma} \right]$$

Using $x_f$ in [A6] and letting $\frac{x_i}{x_f} = h$

$$x_i (\beta - 1) \frac{1 - \tau}{(r - \mu)} = \beta I - \beta \frac{\tau c^*}{r} + (\beta - \gamma) \frac{c^*}{r} \left[ \alpha(1 - \tau) \frac{\gamma}{\gamma - 1} + \tau \right] h^{\gamma}$$

$$= \beta I - \beta \frac{\tau c^*}{r} + (\beta - \gamma) \frac{\tau c^*}{r} \left[ \frac{h^{\gamma}}{1 - \gamma} \right] h^{\gamma}$$

$$= \beta I + \frac{\tau c^*}{r} \left[ -\beta + \frac{\beta - \gamma}{1 - \gamma} \right]$$

$$= \beta I + \frac{\tau c^*}{r} \left[ \frac{\beta - 1}{1 - \gamma} \right]$$

And then substituting [A7]

$$= \beta I - \left[ \tau(\beta - 1) \frac{1}{h (r - \mu)} \frac{x_i}{x_f} \right]$$

Bringing terms in $I$ and $x_i$ to opposite sides and rearranging gives

$$x_i = \frac{\beta}{\beta - 1} K \left[ 1 + \frac{\tau}{h (1 - \tau)} \right]^{-1} I \ \ [A8]$$

where

$$K = \frac{(r - \mu)}{1 - \tau}$$
5.8.2 Negotiation Option – Model 2

We now proceed to the negotiation option, introducing a further extension by changing the default boundary conditions, such that in contrast to the default option the lender and investor agree a new coupon conditional on sharing the “extra equity” resulting from avoiding costly foreclosure at the optimally chosen critical default threshold, denoted by \( x_{df} \) (to distinguish it from the default threshold \( x_f \)). The bargaining process will thus result in the reinstatement of the mortgage at the new lower payment and a notional readjustment of the investor’s market equity compared to the default option. It is further presumed that in this (actual or threatened) negotiation the lender mitigates his foreclosure costs as much as possible but is finally indifferent as to whether the investor or an outsider “profits” from the value of the potential foreclosure costs such as administration, legal fees, social costs, loss on the property sale, etc.

The sharing rule is now given by \( E(x_{df}) = \alpha \phi F_{ae}(x_{df}, 0) \) and \( D(x_{df}) = (1 - \alpha \phi) F_{ae}(x_{df}, 0) \) where \( \alpha \) is the loss severity and \( \phi \) the bargaining strength \( 0 \leq \phi \leq 1 \). If \( \phi = 0 \) then the weak investor shares none of the unavoidable costs while the lender gets \( F_{ae}(x_{df}, 0) \). If \( \phi = 1 \) then the strong investor gets \( \alpha F_{ae}(x_{df}, 0) \) (which outsiders would otherwise have received) while the weak lender receives \( (1 - \alpha) F_{ae}(x_{df}, 0) \). If \( \phi = 1/2 \) then the investor gets \( \alpha F_{ae}(x_{df}, 0)/2 \) (which outsiders would have received) while the lender receives \( (1 - \alpha/2) F_{ae}(x_{df}, 0) \). In all cases the lender is either better off or indifferent compared to the default case while the investor may be better off, persuading him not to exercise his default option at the original default threshold \( x_f \) but rather resume mortgage payments at a more affordable coupon \( C(x) \).
The first section (Section 5.8.2.1) examines when investor and lender bargain over the equity value at foreclosure, while the second section (Section 5.8.2.2) extends to the case where they bargain over the additional value of the borrower’s tax benefits.

5.8.2.1 No Option Value from Tax Shelters – Model 2

We proceed by solving for \( E(x) \) whereby the general ODE is given by letting \( d = (1 - \tau) \) and \( e = c(1 - \tau) \). In other words

\[
E(x) = \frac{1}{2} \sigma^2 x^2 E_{xx} + \mu x E_x - r E + (1 - \tau) x - c(1 - \tau) = 0
\]

As the value of the assets approaches infinity, debt becomes riskless and as in Section 5.8.1

\[
\lim_{x \to \infty} E(x) = \frac{x}{K} - \frac{(1 - \tau)c}{r}
\]

The new lower boundary conditions follow from the bargaining game whereby

\[
\lim_{x \to x_{df}} E(x) = \frac{\phi \alpha x_{df}}{K}
\]

\[
\lim_{x \to x_{df}} E_x(x) = \frac{\phi \alpha}{K}
\]

Using the general solution

\[
E(x) = A_0 + A_1 x^\gamma + A_2 x^\beta
\]

as \( x \to \infty \), \( x^\beta \to \infty \) \( \Rightarrow A_2 = 0 \)

while as \( x \to \infty \), \( x^\gamma \to 0 \) \( \Rightarrow A_0 = \frac{x}{K} - \frac{c(1 - \tau)}{r} \)

Thus [A10] becomes

\[
E(x) = \frac{x}{K} - \frac{c(1 - \tau)}{r} + A_1 x^\gamma
\]

Differentiating [A11] w.r.t. \( x \) and substituting [A9b] implies

\[
E_x(x) = \frac{1}{K} - 0 + \gamma A_1 x^{\gamma - 1} = \frac{\phi \alpha}{K}
\]
While substituting \([A9a]\) in \([A11]\) implies

\[
E(x_{df}) = \frac{x_{df}}{K} - \frac{c(1-\tau)}{r} + A_1 x_{df}^\gamma = \frac{\phi a x_{df}}{K} \tag{A13}
\]

And then eliminating \(A_1\) from \([A12]\) and \([A13]\)

\[
\frac{\phi a x_{df}}{K} = \frac{x_{df}}{K} - \frac{c(1-\tau)}{r} + \frac{\phi a - 1}{K^\gamma} x_{df}^\gamma + 1
\]

\[
= \frac{x_{df}}{K} - \frac{c(1-\tau)}{r} + \frac{\phi a - 1}{K^\gamma} x_{df}
\]

\[
=> \frac{c(1-\tau)}{r} = \frac{x_{df}}{K} \left[ (1 - \phi a) + \frac{(\phi a - 1)}{\gamma} \right]
\]

\[
= \frac{x_{df}}{K} (1 - \phi a) \left[ 1 - \frac{1}{\gamma} \right]
\]

\[
= \frac{x_{df}}{K} (1 - \phi a) \frac{\gamma - 1}{\gamma}
\]

\[
x_{df} = K \frac{c(1-\tau)}{r(1-\phi a) \gamma - 1} \tag{A14}
\]

Taking \(A_1\) from \([A13]\) \(= \left( \frac{\phi a x_{df}}{K} - \frac{x_{df}}{K} + \frac{c(1-\tau)}{r} \right) x_{df}^\gamma\) and substituting in \([A11]\)

\[
E(x) = \frac{x}{K} - \frac{c(1-\tau)}{r} + \left( \frac{\phi a x_{df}}{K} - \frac{x_{df}}{K} + \frac{c(1-\tau)}{r} \right) x_{df}^\gamma x^\gamma
\]

\[
E(x) = \frac{x}{K} - \frac{c(1-\tau)}{r} \left[ 1 - \left( \frac{x}{x_{df}} \right)^\gamma \right] - \frac{x_{df}}{K} \left( \frac{x}{x_{df}} \right)^\gamma (1 - \phi a) \tag{A15}
\]

In a similar manner we repeat the derivation for \(D(x)\) but using \(D(x_{df}) = (1 - \alpha \phi) \frac{x_{df}}{K}\) and as in Part 1 an ODE whereby \(d=0\) and \(e= c\) with appropriate boundary conditions to obtain

\[
D(x) = \frac{c}{r} \left[ 1 - \left( \frac{x}{x_{df}} \right)^\gamma \right] + \frac{x_{df}}{K} \left( \frac{x}{x_{df}} \right)^\gamma (1 - \phi a) \tag{A16}
\]
Thus the total property value $V(x) = E(x) + D(x)$ is given by

$$\frac{x}{K} - \frac{c(1 - \tau)}{r} \left[ 1 - \left( \frac{x}{x_{df}} \right)^Y \right] - \frac{x_{df}}{K} \left( \frac{x}{x_{df}} \right)^Y (1 - \phi \alpha) + \frac{c}{r} \left[ 1 - \left( \frac{x}{x_{df}} \right)^Y \right]$$

$$+ \frac{x_{df}}{K} \left( \frac{x}{x_{df}} \right)^Y (1 - \phi \alpha)$$

or

$$\frac{x}{K} + \frac{c}{r} \left[ 1 - \left( \frac{x}{x_{df}} \right)^Y \right] (-1 + \tau + 1)$$

Reducing to

$$V(x) = \frac{x}{K} + \frac{c}{r} \left[ 1 - \left( \frac{x}{x_{df}} \right)^Y \right]$$

5.8.2.2 Option Value from Tax Shelters – Model 2

This next section further extends the negotiation option whereby borrower and investor negotiate over the extra option value due to the tax shelter along with the intrinsic asset value.

We first solve for the total property value $V(x)$.

The total value of the property $V(x)$ satisfies the following differential equations

$$V(x) = \frac{1}{2} \sigma^2 x^2 V_{xx} + \mu x V_x - r V + (1 - \tau) x + \tau c = 0 \text{ when } x > x_{df} \quad [A18]$$

and

$$V(x) = \frac{1}{2} \sigma^2 x^2 V_{xx} + \mu x V_x - r V + (1 - \tau) x = 0 \text{ when } x \leq x_{df} \quad [A19]$$

with an upper boundary condition given by

$$\lim_{x \to \infty} V(x) = \frac{x}{K} + \frac{\tau c}{r}$$

[225]
The lower boundary conditions follow from the bargaining game whereby

\[
\lim_{x \to x_{df}} V(x) = \lim_{x \to x_{df}} V(x) \quad \text{[A21]}
\]

\[
\lim_{x \to x_{df}} V_x(x) = \lim_{x \to x_{df}} V_x(x) \quad \text{[A22]}
\]

\[
\lim_{x \to x_{10}} V(x) = 0 \quad \text{[A23]}
\]

We find a solution of the form \( V(x) = A_0 + A_1 x^\gamma + A_2 x^\beta \).

Firstly for \( x > x_{df} \)

As \( x \to \infty \), \( x^\beta \to \infty \) \quad \Rightarrow \quad A_2 = 0

while as \( x \to \infty \), \( x^\gamma \to 0 \) \quad \Rightarrow \quad A_0 = \frac{x}{K} + \frac{\tau c}{r}

Secondly for \( x \leq x_{df} \)

as \( x \to 0 \), \( x^\gamma \to 0 \) \quad \Rightarrow \quad A_1 = 0

while \( \frac{\tau c}{r} = 0 \)

Giving

\[
V(x) = \frac{x}{K} + \frac{\tau c}{r} + A_1 \left( \frac{x}{x_{df}} \right)^\gamma \quad \text{when } x > x_{df} \quad \text{[A24]}
\]

and

\[
V(x) = \frac{x}{K} + A_2 \left( \frac{x}{x_{df}} \right)^\beta \quad \text{when } x \leq x_{df} \quad \text{[A25]}
\]

However using boundary conditions \[A21\] when \( x = x_{df} \Rightarrow \frac{\tau c}{r} + A_1 = A_2 \)

Differentiating \[A24\] and \[A25\] and using \[A22\] when \( x = x_{df} \Rightarrow \gamma A_1 = \beta A_2 \)

Solving for \( A_1 \) and \( A_2 \) gives \( A_1 = \frac{\tau c}{r} \left( \frac{\beta}{\gamma - \beta} \right) \) and \( A_2 = \frac{\tau c}{r} \left( \frac{\gamma}{\gamma - \beta} \right) \)

\[
V(x) = \frac{x}{K} + \frac{\tau c}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{x}{x_{df}} \right)^\gamma \right] \quad \text{when } x > x_{df} \quad \text{[A26]}
\]

\[
V(x) = \frac{x}{K} + \frac{\tau c}{r} \left( \frac{\gamma}{\beta - \gamma} \right) \left( \frac{x}{x_{df}} \right)^\beta \quad \text{when } x \leq x_{df} \quad \text{[A27]}
\]
The total value of the property \( V(x) \) includes the value of tax benefits and is thus higher than just the asset value \( V \). The borrower and lender thus bargain over a larger amount (when \( x \leq x_{df} \)) and from a bargaining viewpoint the share is such that \( E(x) = \tilde{a}V(x) \) and \( D(x) = (1 - \tilde{a})V(x) \). The Nash solution \( \tilde{\theta}^* \) can be rewritten as

\[
\tilde{\theta}^* = \arg\max \{ \tilde{a}V(x) - 0 \}^{\phi} \left\{ (1 - \tilde{a})V(x) - \max \left[ (1 - \phi) \frac{x}{K}, 0 \right] \right\}^{1 - \phi}
\]

\[
= \min \left[ \phi - \phi \frac{(1 - \alpha) x}{V(x)}, \phi \right]
\]

\[
= \min \left[ \phi(1 - \alpha) \frac{x}{V(x)}, \phi \right]
\]

\[
= \min \left[ \phi(V(x) - (1 - \alpha) \frac{x}{K}), \phi \right]
\]

We next solve for the equity value \( E(x) \) by setting up the ODE for the equity relationship.

The equity value of the property \( E(x) \) satisfies the following differential equations

\[
E(x) = \frac{1}{2} \sigma^2 x^2 E_{xx} + \mu x E_x - rE + (1 - \tau)x - c(1 - \tau) = 0 \quad \text{when} \quad x > x_{df} \quad \text{[A28]}
\]

and

\[
E(x) = \frac{1}{2} \sigma^2 x^2 E_{xx} + \mu x E_x - rE + (1 - \tau)x - C(x) = 0 \quad \text{when} \quad x \leq x_{df} \quad \text{[A29] or [5]}
\]

where \( C(x) \) is the mortgage coupon paid to the lender after renegotiation with boundary conditions

\[
\lim_{x \to \infty} E(x) = \frac{x}{K} - \frac{c(1 - \tau)}{r} \quad \text{[A30]}
\]

Lower boundary conditions follow from the extra value of \( V(x) \) using [A27] with \( x = x_{df} \)

\[
\lim_{x \downarrow x_{df}} E(x) = \phi \left( \frac{\alpha x_{df}}{K} - \frac{\tau c}{r} \frac{y}{\beta - \gamma} \right) \quad \text{[A31]}
\]
Differentiating [A27] and again substituting $x = x_{df}$ gives

$$\lim_{x \to x_{df}} E_x(x) = \phi \left( \alpha - \frac{\tau c}{x_{df}^r} \frac{\gamma \beta}{\beta - \gamma} \right)$$

We find a solution of the form $E(x) = A_0 + A_1x^\gamma + A_2x^\beta$

Firstly for $x > x_{df}$

as $x \to \infty, x^\beta \to \infty$ \quad $\Rightarrow A_2 = 0$

while as $x \to \infty, x^\gamma \to 0$ \quad $\Rightarrow A_0 = \frac{x}{K} - \frac{c(1-\tau)}{r}$

Giving

$$E(x) = \frac{x}{K} - \frac{c(1-\tau)}{r} + A_1x^\gamma \text{ when } x > x_{df}$$

Differentiating [A33] w.r.t. $x$ implies

$$E_x(x) = \frac{1}{K} - 0 + \gamma A_1 x^{\gamma-1}$$

Setting [A34] equal to [A32] then multiplying across by $\frac{x}{\gamma}$ and substituting $x = x_{df}$ gives,

$$\frac{x_{df}}{\gamma K} + A_1x_{df}^\gamma = \phi \frac{\alpha x_{df}}{\gamma K} - \phi \frac{\tau c}{r} \frac{\beta}{\beta - \gamma}$$

But then using [A31] and [A33] with $x = x_{df}$ we also get

$$\frac{x_{df}}{K} - \frac{c(1-\tau)}{r} + A_1x_{df}^\gamma = \phi \frac{\alpha x_{df}}{K} - \phi \frac{\tau c}{r} \frac{\gamma}{\beta - \gamma}$$

and by eliminating $A_1x_{df}^\gamma$ from [A35] and [A36]

$$\frac{x_{df}}{K} - \frac{c(1-\tau)}{r} + \phi \frac{\alpha x_{df}}{\gamma K} - \phi \frac{\tau c}{r} \frac{\beta}{\beta - \gamma} = \frac{x_{df}}{\gamma K} + \phi \frac{\alpha x_{df}}{K} - \phi \frac{\tau c}{r} \frac{\gamma}{\beta - \gamma}$$
Gathering terms

\[
\frac{c(1 - \tau)}{r} + \phi \frac{\tau c}{r} \left( \frac{\beta}{\beta - \gamma} - \frac{\gamma}{\beta - \gamma} \right) = \frac{x_{df}}{K} \left[ (1 - \phi \alpha) - \frac{1}{\gamma} (1 - \phi \alpha) \right]
\]

\[
\frac{c(1 - \tau)}{r} + \phi \frac{\tau c}{r} = \frac{x_{df}}{K} (1 - \phi \alpha) \left[ 1 - \frac{1}{\gamma} \right]
\]

\[
x_{df} = K \frac{(1 - \tau(1 - \phi)) c}{(1 - \phi \alpha)} \frac{\gamma}{r \gamma - 1}
\]

\[
x_{df} = KL \frac{c}{r} \frac{\gamma}{\gamma - 1}
\]  \[\text{[A37]}\]

Where \[\quad L = \frac{(1 - \tau(1 - \phi))}{(1 - \phi \alpha)}\]

We next solve for \(A_1\) in \[\text{[A35]}\]

\[
\frac{x_{df}}{\gamma K} + A_1 x_{df}^\gamma = \frac{\phi \alpha x_{df}}{\gamma K} - \phi \frac{\tau c}{r} \frac{\beta}{\beta - \gamma}
\]

Giving

\[
x_{df}^\gamma A_1 = \frac{\phi \alpha x_{df}}{\gamma K} - \phi \frac{\tau c}{r} \frac{\beta}{\beta - \gamma} - \frac{x_{df}}{\gamma K}
\]

\[
= \frac{x_{df}}{\gamma K} (\phi \alpha - 1) - \frac{\phi \tau c}{r} \frac{\beta}{\beta - \gamma}
\]

And then substituting for \(x_{df}\) on the RHS from \[\text{[A37]}\] gives

\[
\frac{c}{r} \frac{\tau(1 - \phi) - 1}{\gamma - 1} - \frac{\phi \tau c}{r} \frac{\beta}{\beta - \gamma}
\]
Resulting in

\[ E(x) = \frac{x}{K} - \frac{c(1-\tau)}{r} + \left[ \frac{c(\tau(1-\phi) - 1) - \phi \tau c \beta}{r (\beta - \gamma)} \right] \left( \frac{x}{x_{df}} \right)^\gamma \text{ when } x > x_{df} \]  

[A39]

Using the Nash bargaining game the value of the equity when \( x \leq x_{df} \) after is given by

\[ E(x) = \phi \left[ V(x) - (1-\alpha) \frac{x}{K} \right] \text{ when } x \leq x_{df} \]  

[A40]

By differentiating [A40] once and then twice and substituting the results in [A29] we get

\[ E_x(x) = \phi \left[ V_x(x) - \frac{(1-\alpha)}{K} \right] \]

\[ E_{xx}(x) = \phi [V_{xx}(x)] \]

\[ C(x) = \frac{1}{2} \sigma^2 x^2 \phi [V_{xx}(x)] + \mu x \phi \left[ V_x(x) - \left( \frac{1-\alpha}{K} \right) \right] - \phi \left[ V(x) - (1-\alpha) \frac{x}{K} \right] + (1-\tau)x \]

And gathering terms

\[ = \phi \left[ \frac{1}{2} \sigma^2 x^2 V_{xx} + \mu x V_x - r V(x) \right] + \phi (1-\alpha) \left[ r \frac{x}{K} - \mu \frac{x}{K} \right] + (1-\tau)x \]

And then substituting in the first term using [A19]

\[ = \phi [-(1-\tau)x] + \phi (1-\alpha) x \left[ r \frac{x}{K} - \mu \frac{x}{K} \right] + (1-\tau)x \]

and remembering that \( K = \frac{(r-\mu)}{1-\tau} \)

\[ = (1-\tau)x [ -\phi + \phi (1-\alpha) + 1] \]

Cancelling and tidying up gives

\[ C(x) = (1-\tau)(1-\phi \alpha) x \]  

[A41]
As in the base case example we now proceed to calculate the optimal coupon \( c_d \) using \( x_{df} \) from [A37] and \( V(x) \) from [A26] and maximising \( V_c(x) = 0 \)

\[
V(x) = \frac{x}{K} + \frac{\tau}{r} \left[ c - \left( \frac{\beta}{\beta - \gamma} \right) c \left( \frac{x}{x_{df}} \right)^Y \right]
\]

Substituting for \( x_{df} \) from [A37]

\[
= \frac{x}{K} + \frac{\tau}{r} \left[ c - \left( \frac{\beta}{\beta - \gamma} \right) c r^\gamma c^{1-Y} \left( \frac{KL}{r \gamma - 1} \right)^{-\gamma} \right]
\]

Differentiating w.r.t. \( c \), substituting \( c_d \) and setting equal to 0 gives

\[
\left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right)(1 - \gamma) x^Y c_d^{-\gamma} \left( \frac{KL}{r \gamma - 1} \right)^{-\gamma} \right] = 0
\]

\[
\left[ 1 - x^Y (gc_d) \left( \frac{KL}{r \gamma - 1} \right)^{-\gamma} \right] = 0 \quad \text{(A42)}
\]

By letting \( g^{-\gamma} = \left( \frac{\beta}{\beta - \gamma} \right)(1 - \gamma) \)

\[
\text{We now assume (as for the default option) that the borrower only agrees the optimal coupon } c_d \text{ with the lender and obtains the required funds when the property rental } x = x_{di} \text{ the entry level threshold thus [A42] becomes}
\]

\[
x_{di} = c_d \frac{KL}{r \gamma - 1} \quad \text{(A44)}
\]

Using [A26] again at \( c_d \) and \( x_{di} \)

\[
V(x_{di}, c_d) = \frac{x_{di}}{K} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{x_{di}}{x_{df}} \right)^Y \right]
\]

\[
= \frac{x_{di}}{K} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \frac{g^Y}{1 - \gamma} \right]
\]

\[
= \frac{x_{di}}{K} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta}{\beta - \gamma} \right) \left( \frac{\beta - \gamma}{\beta(1 - \gamma)} \right) \right]
\]

\[
= \frac{x_{di}}{K} + \frac{\tau c_d}{r} \left[ 1 - \frac{1}{(1 - \gamma)} \right]
\]
\[ x_{di} = \frac{x_{di}}{K} + \frac{\tau c_d}{r} \left[ \frac{\gamma}{(\gamma - 1)} \right] \]

And from [A44]

\[ x_{di} = \frac{x_{di}}{K} + \frac{\tau r x_{di}}{gKLr} \left[ \frac{\gamma}{(\gamma - 1)} \right] \]

Giving that at the optimal investment moment

\[ V(x_{di}, c_d) = \frac{x_{di}}{K} (1 + \frac{\tau}{gL}) \]

Or when

\[ x_{di} = V(x_{di}, c_d) K (1 + \frac{\tau}{gL})^{-1} \]

However using standard real option relationship for an entry level investment it can be shown that the optimal investment occurs when \( V(x_{di}, c_d) \) is \( \frac{\beta}{\beta - 1} l \).

Thus

\[ x_{di} = K \frac{\beta}{\beta - 1} \left( 1 + \frac{\tau}{gL} \right)^{-1} l \quad [A45] \text{or}[8] \]

Using [A44]

\[ c_d = r \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} (gL + \tau)^{-1} l \quad [A46] \text{or}[10] \]

Using [A37]

\[ x_{df} = \frac{\beta}{\beta - 1} K \left[ g + \frac{\tau}{L} \right]^{-1} l \quad [A47] \text{or}[12] \]

### 5.8.3 Calculation of the Optimal LTV ratio for the Negotiation Option

We define the optimal risk adjusted \( LTV_{di} \) and \( LTV_i \) at mortgage origination \( x_i \) or \( x_{di} \) as the value of debt divided by the property value at mortgage origination. This loan-to-value ratio is defined for the negotiation option as

\[ LTV_{di} = \frac{D(x_{di}, c_d)}{V(x_{di}, c_d)} \quad [A48] \]
Using [A26] and [A39] we get \( LTV_{di} = \)

\[
\frac{x}{K} + \frac{\tau c}{r} \left[ 1 - \left( \frac{\beta - y}{\beta - \gamma} \right) \left( \frac{x}{x_{df}} \right)^{\gamma} \right] - \left( \frac{x}{K} - c(1-\tau) + \frac{c(\tau(1-\phi)-1)}{r} - \frac{\phi \tau c \beta}{r - \beta - \gamma} \right) \left( \frac{x}{x_{df}} \right)^{\gamma} \]

\[
\frac{x}{K} + \frac{\tau c}{r} \left[ 1 - \left( \frac{\beta - y}{\beta - \gamma} \right) \left( \frac{x}{x_{df}} \right)^{\gamma} \right] \]

Cancelling some terms, and substituting \( x_{di}, c_d \) for \( x, c \)

\[
= \frac{\tau c_d}{r} \left[ \left( \frac{\beta - y}{\beta - \gamma} \right) \left( x_{di} \right)^{\gamma} \right] + \frac{c_d}{r} - \left( \frac{c_d(\tau(1-\phi)-1)}{r} - \frac{\phi \tau c_d \beta}{r - \beta - \gamma} \right) \left( \frac{x_{di}}{x_{df}} \right)^{\gamma} \]

\[
\frac{x_{di}}{K} + \frac{\tau c_d}{r} \left[ 1 - \left( \frac{\beta - y}{\beta - \gamma} \right) \left( \frac{x_{di}}{x_{df}} \right)^{\gamma} \right] \]

Substituting \( x_{di} = g x_{df} \) and using the expression in (43) for \( x_{di} \)

\[
= \frac{\tau c_d}{r} \left[ \left( \frac{\beta - y}{\beta - \gamma} \right) \left( g \right)^{\gamma} \right] + \frac{c_d}{r} - \left( \frac{c_d(\tau(1-\phi)-1)}{r} - \frac{\phi \tau c_d \beta}{r - \beta - \gamma} \right) \left( g \right)^{\gamma} \]

\[
= \frac{c_d g \frac{L}{r} \frac{\gamma}{\gamma - 1} + \frac{\tau c_d}{r} \left[ 1 - \frac{1}{\gamma - 1} \right]}{K} \]

\[
= \frac{\tau c_d}{r} \left[ \frac{1}{\gamma - 1} - \frac{\tau c_d}{r} \frac{1}{\gamma - 1} \right] + \left[ \frac{c_d}{r} - \frac{c_d g^{\frac{\gamma}{\gamma - 1}}} {r} \frac{\tau(1-\phi)}{(y-1)} + \frac{c_d g^{\frac{1}{\gamma - 1}}}{r} \frac{1}{(y-1)} \right]
\]

\[
= \frac{\tau c_d}{r} \left[ \frac{1}{\gamma - 1} - \frac{\tau c_d}{r} \frac{1}{\gamma - 1} \right] - \frac{c_d g^{\frac{\gamma}{\gamma - 1}}} {r} \frac{\tau(1-\phi)}{(y-1)} + \frac{c_d g^{\frac{1}{\gamma - 1}}}{r} \frac{1}{(y-1)}
\]

\[
= \frac{c_d g^{\frac{1}{\gamma - 1}} \left( 1 - g^{\gamma} \right) \frac{\tau(1-\phi)}{(y-1)} + \frac{c_d g^{\frac{1}{\gamma - 1}}}{(y-1)r} (y - 1 + g^{\gamma})}{c_d g^{\frac{1}{\gamma - 1}} + \frac{\tau c_d}{r} \frac{1}{(y-1)}}
\]

Eliminating \( \frac{c_d}{r} \left[ \frac{1}{\gamma - 1} \right] \), gathering terms and simplifying

\[
LTV_{di} = \frac{\tau (1 - g^{\gamma})(1 - \phi) + \gamma - 1 + g^{\gamma}}{\gamma g + \gamma \tau} \]

[A49]or[14A] 233
5.8.4 Miscellaneous Parameter Definitions

The yield spread at origination is defined as

\[ YS_{di} = \frac{c_d}{D(x_{di}, c_d)} - r \]  \[ A50 \]

and

\[ YS_i = \frac{c_s}{D(x_i, c^*)} - r \]  \[ A51 \]

where \( D(.) \) is the value of debt at the investment threshold \( x_i \) or \( x_{di} \) as derived in Section 5.8.

The debt coverage ratio at origination is defined for the negotiation and the default option as the net rental income rate at origination divided by the mortgage payment after tax

\[ DCR_{di} = \frac{x_{di}}{c_d(1 - \tau)} \]  \[ A52 \]

and

\[ DCR_i = \frac{x_i}{c^*(1 - \tau)} \]  \[ A53 \]
5.9 Mathematica 9.0 Code

Figure 5.9.1 Mathematica Code used in Chapter 5.

(* This program graphs key parameters using the default non bargaining model and
compares to the same parameters using the Delphiacy Non Bargaining model for various bargaining
powers φ. The basic graphs can be then embellished with tags and labels as demonstrated below *)

(* Parameter input which can be varied *)

\(\text{power} = 3.99;\)
\(r = 0.81;\)
\(f = 0.81;\)
\(s = 0.13;\)
\(e = 0.25;\)
\(p = 0.9;\)
\(\phi = 0.3;\)

(* sets up vector arrays whereby Array[1] contains the graph pertaining to the first iteration of \(\phi = 0.8,\) etc *)

Array[power | graph, 3];
Array[delphiacy | graph, 3];
Array[bargaining | graph, 3];
Array[non | graph, 3];

(* sets up vector arrays to vary the line colour/dashing after the first iteration of \(\phi = 0.8,\) etc *)

Array[colour, 3];
Array[dash, 3];

(* alternate for bargaining power \(\phi = 0.8, 0.9\) and 1.0a *)

(* The following 5 lines calculate parameters common to all the models *)

\[ \begin{align*}
\gamma &= \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s \\
\rho &= \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s \\
\delta &= \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s \\
\alpha &= \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s \\
\eta &= \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s
\end{align*} \]

(* The following 3 lines calculate the all-equity tax (\(\eta\)) entry threshold as well as the equity function for less than (\(\leq\)) and greater than (\(>\)) the threshold point (\(\leq\)) *)

\(\text{threshold} = \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s;\)
\(\text{equity}_{\leq} = \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s;\)
\(\text{equity}_{>} = \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s;\)

(* The following 3 lines calculate the all-equity tax (\(\eta\)) entry threshold as well as the equity function for less than (\(\leq\)) and greater than (\(>\)) the threshold point (\(>\)) *)

\(\text{threshold} = \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s;\)
\(\text{equity}_{\leq} = \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s;\)
\(\text{equity}_{>} = \frac{\beta^2}{1 - \beta} + s - f - \frac{(s - f)^2}{\beta^2} - \frac{(s - f)^2}{\beta^2} + s;\)

(* The following 3 lines calculate the Delphiacy Bargaining eq/entry threshold as well as the optimal compu*)
The following 2 lines calculate the equity value function for less than (old) and greater than (new) the entry threshold point (x, t).

\[ V_1(x) = \frac{1}{r} \left( \frac{q(x - x_e)}{r} \right) - \frac{z(x - x_e)}{r^2} \]
\[ V_2(x) = \frac{1}{r} \left( \frac{q(x - x_e)}{r} \right) - \frac{z(x - x_e)}{r^2} \]

The following 4 lines calculate the Equity Value Functions (EVF(x)) and Default Value Functions (DF(x)) for x < x_e.

\[ p(x) = \frac{1}{r} \left( \frac{1 - \frac{q(x - x_e)}{r}}{1 - \frac{q(x - x_e)}{r}} \right) \]
\[ p(x) = \frac{1}{r} \left( \frac{1 - \frac{q(x - x_e)}{r}}{1 - \frac{q(x - x_e)}{r}} \right) \]
\[ q(x) = \frac{1}{r} \left( \frac{1 - \frac{q(x - x_e)}{r}}{1 - \frac{q(x - x_e)}{r}} \right) \]
\[ q(x) = \frac{1}{r} \left( \frac{1 - \frac{q(x - x_e)}{r}}{1 - \frac{q(x - x_e)}{r}} \right) \]

Calculates the Value function when x < x_e and UTY(x) when x > x_e.

\[ Y(x) = 10000 + (1 + \frac{r}{100}) \cdot x \]

Calculates the Delta Non-Bargaining Reduced Coupon Payment Function CR(x) when x > x_e.

\[ CR(x) = (1 - r + r) \cdot x \]
6. Simulating Alternative US Mortgage Termination Options

6.1 Chapter Summary

We identified earlier that a crucial assumption (Vandell 1995) is made in the traditional mortgage option theoretic literature that the homeowner, whether willing or unwilling, always has the ability to service the debt. Chapters 4 and 5, operating under this assumption, examine how this affects the real default and negotiation options available to homeowners and the lenders responses in terms of optimal LTV and coupon rate. The “ability to pay” assumption constrains the real options available to the homeowner and valued by the lender. We are however interested in discovering whether the homeowner's inability to pay trivially affects the probability of the “ruthless” exercise of the strategic default or other options?

To gain insight and develop initial answers to the research objective, we base our analysis on exogenous program criteria for entry to the Home Affordable Mortgage Program (HAMP) (SIGTARP 2010). The program aims to make mortgage payments of homeowners with a high debt burden more affordable by encouraging renegotiation between lender and homeowner, mitigating the effects of other (undesirable) options such as foreclosure or default.

We therefore simulate the relative importance (defined by the likelihood to be exercised) of the principal alternative options based on program acceptance criteria to the Home Affordable Mortgage Program (SIGTARP 2010, Holden et al. 2012). The main benefit of entry to the HAMP program for the homeowner is that they negotiate and receive a one off reduction in their mortgage payment for a period of 5 years resulting in a DTI close to a recommended value of 0.31 (Holden et al. 2012). We are in a methodological sense simulating and examining a complex mixture of discrete (time) compound, barrier and reset type options.
We attempt to answer the question as to whether the HAMP program might mitigate US homeowner’s motivation to “strategically” or “ruthlessly” default in the long term. The question has not been addressed (see Holden et al.(2012), and growing literature on the topic) in any published academic research but would appear a relevant question when discussing the efficacy of any mitigation program.

This chapter develops a model around a problem formulation based on exogenous HAMP entry criteria, incorporating the stochastic factors (LTV and DTI), to provide an insight into the relative likelihood of different mitigation options being exercised. The debt to income (DTI) ratio is a measure that compares a householder’s debt payments to the income they receive. The loan-to-value (LTV) ratio expresses the amount of a first mortgage lien as a percentage of the total appraised value of real property.

We compare three scenarios: (i) a homeowner’s income and equity value is rising and no HAMP type program exists; (ii) a homeowner’s income and equity value is falling and a HAMP type program is not available; (iii) or available.

We comment on the expected effect of a HAMP type program on the more typical mitigation options such as a “forced foreclosure” or “strategic default” as well as additional real options introduced by the relaxation of the perpetual ability to pay constraint such as a “voluntary sale”. We demonstrate that a possible effect of the HAMP program is that it makes it less likely that a typical US homeowner will default as a result of “unwillingness to pay” but rather willingly sell their home as a result of an inability to pay and the desire to recover some home equity. Refer to Section 6.7 for a more detailed description of these options.
6.2 Introduction

It is clear that a significant number of US homeowners, due to exogenous shocks are unable to make the monthly mortgage payment (Holden et al. 2012, Quercia, Pennington-Cross and Tian 2012). This is an important consideration in the current US economic climate, as Federal mitigation programs such as the Home Affordable Mortgage Program (HAMP) are designed to mitigate this inability of homeowners to make the payment rather than their unwillingness to make the payment. The fact that these programs address the homeowner’s ability to pay rather than their willingness to pay implies that generally accepted outcomes from option theoretic simulation models which rely heavily (Vandell 1995) on a perpetual ability to pay assumption should be re-examined.

The role of LTV (Loan-To-Value) and DTI (Debt-To-Income) have been consistently highlighted (Chapter 2) as significant with respect to the default motivation for owner occupied residential mortgage householders. Traditional option theoretic research has suggested that LTV was more significant than DTI as a motivating factor. Recent research by Holden et al. (2012) on the development and early performance of the HAMP NPV model as well as research by Quercia, Pennington-Cross and Tian (2012) into mortgage default and prepayment risks both emphasise the increased significance of DTI in the current US climate.

The general understanding was (pre subprime crisis) that, in the US mortgage market, LTV at origination had the greatest predictive power for estimating future default propensity. Borrowers with a low propensity to consistently pay the monthly recurring coupon could be screened out at origination by applying income (DTI) criteria. This may have led many real estate academics to adopt the belief (underwriting standards remaining high and/or any liability being insurable) that the body of knowledge encompassed by the option theoretic
research approach had sufficient predictive ability. However, in the last 20 years, underwriting standards in the US changed, making it possible for homeowners with a greater range and volatility of income to acquire a home. One could argue (Chapter 2) that mortgage academics directed their research to understanding the competing termination option of prepayment which was more significant for institutional investors.

In the meantime, (implemented) policy has been influenced by contributions from researchers within the US Federal Reserve Banking system such as Paul Willen, Christopher Foote and Kristopher Geradi, in particular, their 2008 paper “Negative Equity and Foreclosure: Theory and Evidence”, which investigated the early 1990’s Massachusetts property crisis. This and later contributions has shaped the response of the Obama administration to mortgage mitigation by developing intervention plans which are aimed at payment reduction rather than principal reduction. However, this general mitigation approach has been criticised in many quarters (e.g. COP 2010) as being too limited within current US economic and social realities.

The main thrust of alternative programs, implemented to lesser degrees, often advocate an approach reducing the principal owed by the homeowner rather than the mortgage payment. These other approaches have been justified with varying rationales by academics, lobbyists and politicians (e.g. Das and Meadows 2010, Shiller 2009). It is clear from the discussion that the merits or efficacy of any intervention approach or its outcomes are difficult to evaluate and assess given the diversity of methodological approaches and underlying assumptions.

Our problem formulation and solution relies heavily on simulating using a Monte Carlo approach the choices facing US homeowners at exogenous boundaries, which literature on this and income/property dynamics we will briefly examine and review.
6.2.1 Boundary Literature Critique

Barrier options are one of the oldest types of exotic options trading since 1967 on the Chicago Board of Options Exchange (Zhang, 1998). As a result, literature on exotic barrier options is abundant (Zhang 1998, Wilmott 2006). Snyder (1969) outlined the general approach with a single stochastic variable and a lower barrier which was later extended to multiple stochastic variables and barriers of different forms and durations (Heynen and Kat 1994, 1996).

However, examination of the literature has not yielded specific research or papers which lend themselves to a closed form solution of the problem as formulated in Section 6.3. On the other hand, Wilmott (2006) suggests that a Monte Carlo method is often the best approach with regard to analysing exotic path dependent options as it is simple to code with a likelihood of fewer mistakes and whose only disadvantages are the difficulty of obtaining the Greeks and its slowness both of which are minor issues in our formulation. This view is also supported by Vandell (1995) when discussing specific problem formulations associated with an extension of the standard option theoretic problem from a simple bi-variate stochastic formulation.

To illustrate the methodological complexity I make the following observations about our specific formulation. It is of a discrete Asian nature (monthly) and significant errors would be introduced by treating it as a continuous time option (Zhang 1998). Our formulation might appear at first sight to be a simple bi-variate option with LTV and DTI as independent variables but this is not so as DTI and LTV combine in a sequential or compound manner (double trigger or sequential option model) to knock out the mortgage and trigger a terminal payoff option. Our problem is also less straightforward than many barrier options treated in the literature as the homeowner’s DTI is reset to 0.3 but the LTV is not reset when the HAMP option is triggered.
We also consider the appropriateness of treating both DTI and LTV as standard log normal gBm processes. This assumption is relatively uncontroversial for LTV (Vandell 1995) but may be open to discussion in relation to DTI and the underlying income dynamics.

Recent empirical research by Quercia, Pennington-Cross and Tian (2012) would suggest that DTI for low and moderate income US households is in the first instance symmetrically distributed with a bell shaped log normal type distribution. An examination of literature on US householder income dynamics (e.g. Gottschalk and Moffitt 2009; Dynan, Elmendorf and Sichel 2007) does not provide evidence that other formulations e.g. mean reverting (income) dynamic is more widely used or provides any better results than the simpler standard gBm. Although a householder’s income may revert over the longer term this is of no consequence to the householder or lender who are faced with short to medium term payment difficulties. The HAMP program also has a relatively short duration of 5 years leading us to choose the same standard general Brownian motion stochastic process as used to model LTV.
6.2.2 Monte Carlo Modelling Critique

Monte Carlo simulation is a numerical method that is useful in many situations where no closed-form solution is easily available. Boyle (1977) introduced the Monte Carlo simulation method into finance and it is a widely used and accepted method for pricing options. Monte Carlo methods essentially involve generating large numbers of numerically simulated realizations of the random walks followed by the underlying asset price or parameter of interest and then using these random walks to price derivative products. The value is given as the discounted average of the simulated paths.

Monte Carlo simulation is simple and flexible and it can be easily modified to accommodate path dependent derivatives for which closed form solutions may not exist or do exist but at the cost of making severe simplifying assumptions changing the nature of the problem and limiting the usefulness of the closed form solution.

The biggest (technical) disadvantage of the Monte Carlo method is that the standard deviation error of estimate is inversely proportional to the square root of the number of simulation trials. If $S$ is the standard deviation of the pay-offs from $n$ simulations the standard error is given by $\frac{S}{\sqrt{N}}$ where in our case $N$ is 50000 or 100000. Variance-reduction techniques can be used to reduce variance further. However, we have not used any such techniques in our own Mathematica simulation for four reasons.

Firstly, the use of variance reduction techniques would have complicated the code and made it more difficult for debugging and for interested parties to understand or audit. Secondly, I had the time and computing power to allow a “brute force” approach. Thirdly, repeated testing
and construction of the 3D and 2D surfaces showed that N = 50000 produced smooth, continuous surfaces with no discontinuities at crucial barriers. The only discontinuities appeared at volatilities close to 0%. Fourthly, the purpose of the simulation is to roughly estimate not precisely value the importance of additional real mortgage mitigation options so as to focus future valuation research on those options deserving of closer attention.

Regrettably, because of the complex interaction of the number of options, barriers and use of two uncorrelated uni-variate processes it is not possible to compare the Monte Carlo model outcomes against those from a closed form solution to verify that the programming has been correctly implemented. I can only offer the defence that the program code has been checked and rechecked and the overall results from the extensive analysis would seem to be intuitively correct with no “nonsensical” results. The general shape of the 2D and 3D surfaces “seem” smooth and continuous and similar to what a closed form uni-variate or bi-variate analysis might also “throw up”. Extreme values of parameters e.g. zero/infinity yield expected results.

Both a Monte Carlo or closed form approach will suffer from the same problem of assessing as to whether the outcomes bear any resemblance to the actual option exercise behaviour of US homeowners. The strategic default option discussed in traditional option theoretic literature is nearly impossible to verify let alone the idea of a voluntary sale (or downsizing option). However, the Monte Carlo approach and program implementation does allow one to investigate the effect of varying key parameters in a more flexible manner as to that offered by a closed form solution approach. The values of general Brownian motion volatility (20%) and drift (± 2%) parameters used in the Monte Carlo model are similar in magnitude to those used in classical option theoretic research papers and therefore subject to the same caveats as to reliability and applicability.
6.3 Problem Formulation, Model Methodology and Base Parameters

6.3.1 Introduction

The purpose of the analysis is to consider and estimate the effect entry to the HAMP program might have on the probability and timing of when common competing real mortgage default mitigation options available to US homeowners might be exercised. We are in a (real option) methodological sense simulating and examining a complex mixture of compound and barrier type path dependent options.

We formulate our problem by describing the five real competing mortgage mitigation options in Section 6.7 included in our analysis which occur with a probability of 1 within the term of the mortgage. We then follow up by stating our path dependent (on LTV and DTI) assumptions as to when each option is exercised. Finally, we introduce the HAMP program into the problem formulation, make assumptions as to how the path dependency of the five competing options are modified (or mitigated) and compare the relative probability of a mitigating option being exercised to a state of the world without a HAMP program.

6.3.2 Problem Formulation

The probability a competing event terminating the term of a mortgage is 1:

\[ P(\text{Strategic Default}) + P(\text{Forced Foreclosure}) + P(\text{Paid Up}) + P(\text{Voluntary Sale}) = 1 - P(\text{Other Option}) \] \[ \text{[A1]} \]

We present this concept in a diagrammatic manner in Figure 6.3.2.1 which shows that a US home owner who experiences stochastic LTV and DTI over the term of their mortgage must hit one of the termination boundaries or remain within the boundaries (= Other Option).
To the right of some arbitrary DTI value $X$ (Can’t Pay) a homeowner will either, as a result, of an inability to pay, be foreclosed upon if they have negative equity or voluntary sell their home if they have positive equity. Above some arbitrary LTV value $Y$ (Won’t Pay), a homeowner will strategically default no matter their DTI. Otherwise the homeowner, who still has the ability to pay, will either have a mortgage that is paid up or still current (= other option). A typical US householder with a prime mortgage originates with a DTI of around 0.2 and LTV of 80% (= P, yellow star) in the diagram. Sub-prime mortgages start further to the right (= SP, red star) and may have a higher probability of other termination events occurring.

Figure 6.3.2.1. Schematic of Alternative US Mitigation Options without HAMP
Therefore if DTI is an independent random variable in the set $0 < \text{DTI} < \infty$ and LTV is another uncorrelated independent variable in the set $0 < \text{LTV} < \infty$, X and Y are arbitrarily chosen boundaries and a homeowner has either positive or negative equity (defined as $\leq$ or $> 100\%$).

We suggest that:

\[
\begin{align*}
\text{P(Strategic Default)} & \equiv \text{P}(\text{LTV} \geq Y \mid 0 < \text{DTI} < \infty) + \\
\text{P(Voluntary Sale)} & \equiv \text{P}(\text{LTV} \leq 100\% \mid \text{DTI} \geq X) + \\
\text{P(Paid Up)} & \equiv \text{P}(\text{LTV} < 1\% \mid 0 < \text{DTI} < \infty) + \\
\text{P(Forced Foreclosure)} & \equiv \text{P}(\text{LTV} > 100\% \mid \text{DTI} \geq X) = \\
1 - \text{P(Other Option)} & \equiv 1 - \text{P}(1\% < \text{LTV} < X \mid 0 < \text{DTI} < X) \quad [\text{A2}] \\
\end{align*}
\]

The effect of the HAMP program is represented pictorially in Figure 6.3.2.2 overleaf whereby US homeowners who have a DTI $> 0.38$ or LTV $> 110\%$ can negotiate with their lender, enter HAMP and have their DTI reduced to 0.31. HAMP is a temporary mitigating option and does not offer a permanent reduction in DTI as householders DTI and LTV will then change subsequent to entering the program. However, it is obvious that entering the HAMP will affect the relative probabilities of any of the terminal options expressed in [A2] being exercised.
As discussed in the introductory literature review, an appropriate approach for this particular problem formulation is to simulate, using a Monte Carlo approach, the effect of stochastic LTV (a measure of the homeowner’s equity in their property) and DTI (a measure of the ability to make the monthly payment) on the exercise likelihood of a US homeowner’s mortgage options. The removal of the (perpetual) ability to pay assumption affects the dynamics and complexity of the homeowners and lenders response to varying levels of DTI and LTV. Firstly the ability of the homeowner to pay must be checked (1st DTI Trigger) and only subsequently the willingness of the homeowner to pay (2nd LTV Trigger). If the first trigger shows that the homeowner has the ability to pay then the available real options reduce to those of strategic default, paid up and other option. If however, the homeowner is unable to pay then two other real options are introduced namely the lender forecloses or the homeowner voluntarily sells the property himself.

We introduce an additional feature into our simulation model whereby a mortgage event is not immediately triggered if one of the stochastic factors e.g. DTI (ability to pay) hits a threshold
value in one period but only if it consistently exceeds the threshold for a number of periods. This is not a continuous but rather a discrete time simulation with discrete large monthly time periods. Treating a model as discrete time rather than continuous time introduces large differences in exercise probability (Zhang, 1998). This feature simply reflects the accepted fact that lenders and homeowners do not generally exercise an option immediately in the period following the first missed payment or reduction in home equity.

We make no assumption about the amount of the monthly payment beyond that a repayment formula or schedule is contractually agreed beforehand and that failure by the householder to make the agreed payment on time constitutes a trigger event changing their mortgage status. Let \( \text{Delay}_d = \text{Delay}_i = 3 \) be the number of (monthly) periods over which the householder discovers their “true” DTI and “true” LTV respectively before exercising a mortgage option. Let \( N_i \) = a unique US residential owner occupied mortgage householder where \( 1 \leq i \leq N_p \) and \( N_p \) = number of householders in the simulation. We let \( T = 30 \) years be the term of the mortgage and \( per = 360 \) the number of payment periods implying monthly payments.

Implicit in this model formulation is that both the homeowner’s DTI and LTV processes are significant, independent and non-correlated. Mortgage literature (Campbell and Dietrich 1983, Vandell and Thibodeau 1985) does not contradict this view as most empirical research has demonstrated that these two factors are highly significant but essentially independent. One might argue that these two factors should be significantly correlated as both depend in some fashion on interest rates. However, it should be clear from the model formulation that we are especially interested in short term monthly shocks (\( dZ \) or \( dW \)) and although interest rates may effect long term property and income trends (i.e. \( \mu_d \) or \( \mu_i \)), current evidence from
mortgage literature would suggest that it is short term income or property shocks which trigger default and mitigation, which shocks are independent of spot interest rate changes.

6.3.2.1 The DTI (Debt-To-Income) Factor

This measure is of prime importance and significance within the US mortgage industry as its initial value at origination and consequent development gives lenders an idea of how likely it is that the householder will be able to repay the loan over its full term. We do not make any distinction within our model as to why DTI is changing being a result of variations in both income and/or housing expenses. The change in housing expenses could be due to any number of reasons ranging from interest rate changes to property taxes or mortgage insurance. During more normal times, most US lender’s underwriting standards tended to adopt a value of 0.28 as a maximum upper limit for DTI at mortgage initiation. We adopt 0.18 as the value of DTI at mortgage origination. Our exogenous triggers are motivated by the following considerations.

We assume (Holden et al. 2012, Quercia, Pennington-Cross and Tian 2012) that a householder will have serious ability to pay issues if their DTI is above 0.5 for at least three consecutive payment periods and will most likely be foreclosed upon. The HAMP program applies a maximum limit of 0.38 to a homeowner’s DTI that is reduced to 0.31 using a waterfall method (Holden et al. 2012) on successful entry to the program.

As shown in Quercia, Pennington-Cross and Tian 2012, we assume that the Debt-To-Income (DTI) ratio of any householder $N_i$, denoted by $D$, follows the general Brownian motion process given by

$$ dD = \mu_d D dt + \sigma_d D dW $$

$\mu_d$ is the instantaneous expected rate of change of the DTI ratio.
\[ \sigma_d \] is the instantaneous variance of the DTI ratio

\[ dW \] is a standard Brownian motion.

### 6.3.2.2 The LTV (Loan-To-Value) Factor

We again take a simple approach and assume that the LTV is a result of many factors. With no change in property price, for the majority of householders, the LTV will gradually decrease from month to month as they pay down the loan principal. For others, due to any number of reasons varying from non-payment of principal to a reduction in property prices the LTV may be static or increase. The LTV is only formally appraised (precisely) once at mortgage initiation and again by the lender or servicing agent in the event of default. Otherwise, the LTV is calculated by the household and lender based on the outstanding principal and arrears as well as a very rough estimation of the property price based on local (if known) property prices or indexes. In the US, conforming loans that meet Fannie Mae and Freddie Mac underwriting guidelines are limited to an LTV ratio that is less than or equal to 80\% at origination which value we assume in the simulation model. Our exogenous triggers are motivated by the following considerations.

We define a higher LTV trigger of 150\% in our model as the boundary where a household will decide to “strategically” default because of the payoff received by putting the mortgage against the value of the property collateral. The 150 \% LTV has been chosen somewhat arbitrarily but is consistent with empirical papers by Foote, Geradi and Willen (2008) or Guiso, Sapienza and Zingales (2009). We also include a lower boundary where LTV < 1\% where the mortgage is effectively paid down and terminated.
A relatively wide range of householders are eligible to participate in the HAMP program based on equity or LTV criteria ranging from those with a small percentage of positive equity (LTV > 90%) up to those with negative equity (LTV < 110%). We note that the HAMP programs LTV criteria were recently increased again in June 2012 to 120% (Holden et al. 2012). Currently the 120% limit is one that has been adopted within the HAMP program as a “typical” LTV break point (Holden et al 2012) although we have adopted the lower 110% barrier, basing our analysis on the original HAMP criteria.

We assume that if the householder has the ability to pay (< 0.5 DTI for three consecutive periods), that where the householder’s LTV is above 110% for the same three consecutive periods, they will enter and participate in the HAMP program as an alternative to strategically defaulting.

As in Kau and Keenan (1995), we make the common simplifying option theoretic assumption that the Loan-To-Value (LTV) ratio of any householder \( N_i \), denoted by \( L \), follows the general Brownian motion process given by

\[
dL = \mu_i L dt + \sigma_i L dZ
\]  

[A4]

Where

- \( \mu_i \) is the instantaneous expected rate of change of the LTV ratio
- \( \sigma_i \) is the instantaneous variance of the LTV ratio
- \( dZ \) is a standard Brownian motion.
6.3.2.3 The Interaction of LTV and DTI – the Double Trigger Effect

We summarise the alternative mitigation options available to the homeowner as well as the program logic of the exercise barriers in Table 6.3.2.1. We assume for ease of explanation that the homeowner is not in a HAMP program. At the end of each monthly period the 1st DTI trigger is simulated and tested and subsequently the 2nd LTV trigger. Should any of the conditions described in Table 6.3.2.1 (No HAMP Program) persist for longer than 3 months then that particular terminal option is triggered. If the government then introduce a HAMP program and the homeowner meets the entry criteria as specified in Table 6.3.2.1 (With a HAMP Program) then the homeowner enters HAMP and their status changes from 0 to 1 and DTI reset to 0.31. They then are subject to the same periodical simulation (as without a HAMP Program (Table 6.3.2.1, Whereupon) and do not qualify for a DTI reset again.

Table 6.3.2.1 Exogenous Trigger Values for the Option Simulation Program

<table>
<thead>
<tr>
<th>HAMP Status</th>
<th>1st DTI Trigger</th>
<th>2nd LTV Trigger</th>
<th>Option Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No HAMP Program</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IF</td>
<td>AND</td>
<td>THEN</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>DTI &lt; 0.5</td>
<td>1% &lt; LTV &lt; 150%</td>
<td>Other Option</td>
</tr>
<tr>
<td>0</td>
<td>DTI &lt; 0.5</td>
<td>LTV ≤ 1%</td>
<td>Paid Up</td>
</tr>
<tr>
<td>0</td>
<td>DTI &lt; 0.5</td>
<td>LTV ≥ 150%</td>
<td>Strategic Default</td>
</tr>
<tr>
<td>0</td>
<td>DTI ≥ 0.5</td>
<td>LTV &lt; 90%</td>
<td>Voluntary Sale</td>
</tr>
<tr>
<td>0</td>
<td>DTI ≥ 0.5</td>
<td>LTV ≥ 90%</td>
<td>Foreclosure</td>
</tr>
<tr>
<td><strong>With a HAMP Program</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 -&gt; 1</td>
<td>DTI ≥ 0.38</td>
<td>90% ≤ LTV ≤ 110%</td>
<td>Enter HAMP =&gt; DTI Resets to 0.31</td>
</tr>
<tr>
<td><strong>Whereupon</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IF</td>
<td>AND</td>
<td>THEN</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>DTI &lt; 0.5</td>
<td>1% &lt; LTV &lt; 150%</td>
<td>Other Option</td>
</tr>
<tr>
<td>1</td>
<td>DTI &lt; 0.5</td>
<td>LTV ≤ 1%</td>
<td>Paid Up</td>
</tr>
<tr>
<td>1</td>
<td>DTI &lt; 0.5</td>
<td>LTV ≥ 150%</td>
<td>Strategic Default</td>
</tr>
<tr>
<td>1</td>
<td>DTI ≥ 0.5</td>
<td>LTV &lt; 90%</td>
<td>Voluntary Sale</td>
</tr>
<tr>
<td>1</td>
<td>DTI ≥ 0.5</td>
<td>LTV ≥ 90%</td>
<td>Foreclosure</td>
</tr>
</tbody>
</table>

Note: An option is triggered only if DTI or LTV are at a trigger level for at least consecutive three monthly periods

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6.3.3 Base Case Parameters

The model computer code which has been implemented in Mathematica is presented in Section 6.9. We discuss in detail the base case parameters in this section. These are namely:

6.3.3.1 Model Scope

As is common when using a numerical approach, we have had to find a compromise between accuracy and length of computer run. After experimenting with different combinations of parameters we found that a run of 50000 (N) US homeowners which takes about 6 hours is a reasonable compromise. Results were reproducible over different runs using the Monte Carlo approach to less than 0.5% using a mortgage term of 30 years and 360 monthly periods. In some instances, when constructing the histograms, we have used runs of 100000.

6.3.3.2 The Voluntary Sale and Forced Foreclosure Switch Parameter

In the absence of a HAMP type program, if a homeowner cannot make their periodic payment for three consecutive months and has no or negative equity in their property we assume that a voluntary sale is not attractive and the lender forces foreclosure. However, if a homeowner has very little positive equity then transaction and deadweight costs may still make a voluntary sale unattractive. We therefore assume that forced foreclosures happen at LTV values greater than 90% and voluntary sales below 90%. This parameter also applies if the homeowner enters a HAMP type program but subsequently runs into further DTI problems. The 10% difference from 100% LTV could be “loosely” viewed as an α (immitigable foreclosure costs) used in Chapters 3 and 4 and is not excessively high.
6.3.3.3 HAMP Program Parameters

As befits any US Federal program, the HAMP program is not simple (Holden et al. 2012) and its entitlement (SIGTARP 2010) rules change on a regular basis confusing both participants and administrators. Further complications arise when applying the rules, as while it is not easy to determine a homeowner’s LTV, it is even more difficult to determine their DTI level.

We have reduced the number of HAMP parameters input into the model after an extensive examination of many different documents, some often contradictory in nature and claims. The choices made are namely that the DTI trigger level at which a homeowner is entitled to enter the program is approximately 0.38 and the DTI reset level to which the program reduces the homeowners DTI is 0.31. Note that this has now been varied since June 2012 (Section 6.7). Homeowners with a LTV of greater than 110% tend to gain entry to the program even though their spot DTI (which is difficult to determine) may be below the trigger level. This is justified (and generally accepted by all parties) on the basis that the alternative might be a strategic default, more blighted neighbourhoods and higher social costs (Keys et al. 2011).
6.4 Model Results and Analysis

6.4.1 Introduction

We initially assume (Sections 6.4.2 to 6.4.4) for a US homeowner with a prime mortgage at origination (DTI=0.18 and LTV = 81%): a worst case (economic) scenario where the homeowners’ equity and income are deteriorating at a rate of 2% per year and a best case scenario where equity (LTV) and income (DTI) are increasing by 2% per year. We assume that no HAMP type program will be introduced in the best-case scenario but will be introduced in a worst-case type scenario.

We take the same global approach to analyse each individual terminal option.

1) We compare the percentage homeowners who exercise with a HAMP type program to a state of the world without a HAMP type program for different volatility parameters in Section 6.4.2.

2) We estimate the terminal change in LTV and DTI from mortgage origination in Section 6.4.3. This indicates (but does not quantify) whether entry to the HAMP program “preserves” or “destroys” LTV (an equity measure) for the homeowner.

3) We examine in Section 6.4.4 how many homeowners exercise within 5 years and then for those who enter HAMP, re-examine how effective the program has been in preventing or delaying a subsequent foreclosure, sale or default over the next 5 years.

We demonstrate the effect of the model initially on the exercise probability of the foreclosure option in detail, presenting and discussing both figures and tables, by way of example and will later dispense with the detailed explanations of the figures for the other options to aid clarity. The reader may acquire the wider range of graphical information on request.
6.4.2 Volatility Effects for a US Homeowner with a Prime Mortgage at Origination

6.4.2.1 Foreclosure Option

Foreclosure occurs when a homeowner’s income (modelled by DTI) is not sufficient to make the monthly payment and have not enough wealth or equity (modelled by LTV) in their property whereby a voluntary sale (or downsizing) would pay off the loan. The y-axis of the four subplots in Figure 6.4.2.1 below represents the percentage householders who exercise the foreclosure option. The x-axis of the top two subplots respectively worse and best case represents the LTV ($\sigma_l$) volatility while the x axis on the bottom two subplots represent DTI ($\sigma_d$) volatility.

Figure 6.4.2.1 Percentage Homeowners Foreclosing as a Function of LTV ($\sigma_l$) and DTI ($\sigma_d$) Volatility

Simulation Parameters

$\sigma_l, \mu_l, \mu_l, \sigma_d, \mu_d, \mu_d = \text{Variable}, \text{Delay Trigger for DTI and LTV}=3 \text{ months}, \text{Term Period} = 360 \text{ months}, \text{Strategic Default LTV Trigger} = 150\%$

Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000
We show four trend lines in each subplot. In the top two subplots representing the percentage of homeowners exercising foreclosure plotted against LTV ($\sigma_l$) volatility, we show trend lines for DTI ($\sigma_d$) = 20% (dashed trend line) and 40% (continuous trend line). In the bottom two subplots representing the percentage of homeowners exercising foreclosure plotted against DTI ($\sigma_d$) volatility we show trend lines for LTV ($\sigma_l$) = 20% (dashed trend line) and 40% (continuous trend line). Red lines represent results without any (NO) HAMP program – the status quo in good economic times. Black lines represent results when a HAMP type program is available.

We see by comparing the two left hand subplots to the two right hand subplots that foreclosures decrease as the homeowner’s economic circumstances improve from a worse to a best case. Foreclosures increase linearly with increasing DTI volatility (bottom two subplots).

With increasing LTV volatility (top two subplots) foreclosures decrease at moderate to high levels of LTV volatility. Increasing LTV volatility therefore offers some homeowners the opportunity to benefit from increased positive equity and reduces the likelihood of foreclosure. Lower LTV volatility, on the other hand reduces the likelihood that a homeowner with negative equity will ever have positive equity.

It should of course be noted that some other homeowners are more likely as a result to exercise the strategic default option at these higher LTV volatilities. In contrast to other mitigation options, little change in the percentage of homeowners exercising this option occurs at higher LTV volatility but all the “action” takes place at low to moderate levels of LTV volatility. The trend lines exhibit a local maximum, which peaks at about $\sigma_l = 15\%$ and $\sigma_d = 20\%$. 

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Table 6.4.2.1, column 2 summarises \((\sigma_l, \sigma_d = 20\% \text{ and } 40\%)\) the percentage \((N=50,000)\) of homeowners exercising a particular option during the best and worst cases depicted in Figure 6.4.2.1. The right hand sub table is the best-case scenario i.e. where LTV and DTI are decreasing by 2% a year and the left hand sub table is the worst-case scenario where LTV and DTI are increasing by 2% a year. Each individual sub table summarises the percentage number of homeowners who exercise one of 6 options – numbered 1-6. The top half of each sub table give the results for a HAMP program and the bottom half for NO HAMP program. Finally results are presented for different combinations of LTV and DTI volatility with \((\sigma_l, \sigma_d = 20\% \text{ and } 40\%)\).

<table>
<thead>
<tr>
<th>Worst Case Scenario (\mu_d = \mu_l = 0.02)</th>
<th>Best Case Scenario (\mu_d = \mu_l = -0.02)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_l)</td>
<td>(\sigma_d)</td>
</tr>
<tr>
<td>H 0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>H 0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>H 0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>H 0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>NH 0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>NH 0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>NH 0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>NH 0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Simulation Parameters**
- Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%, H =HAMP Program, NH = NO HAMP program, \(\sigma_l\) and \(\sigma_d\) = 20% or 40%
- Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000
- Column Option Descriptions : Percentage Homeowners who 1 = Voluntarily Sell, 2 = Foreclose, 3 = Strategically Default, 4 = Other Option, 5 = Paid Up, 6 = Enter HAMP

From Table 6.4.2.1 (best case, column 2) which summarises key parameters from the simulations for a LTV \((\sigma_l)\) and DTI \((\sigma_d)\) volatility of 20%, approximately 0.9% of homeowners will be foreclosed upon in a best-case scenario without a HAMP program (NH). When the homeowner’s economic situation worsens (worse case, column 2) this increases to 3.7% without a HAMP program (NH) but reaches 6.8% with a HAMP program (H). The
introduction of a HAMP program would not, in the first instance, lead to reduced foreclosures. The forbearance that some homeowners enjoy from a reduced DTI will in many cases be only a temporary reprieve whereby once the homeowner has recurring income or debt difficulties the LTV has turned either negative triggering a forced foreclosure instead of a voluntary sale or positive triggering a voluntary sale instead of a forced foreclosure.

6.4.2.2 Voluntary Sales Option

Voluntary sales occur when the homeowner’s DTI is not sufficient to make the periodic payment over a three-month period but enough positive equity (as determined by LTV) exists in the property to allow a voluntary sale. We analyse the voluntary sales option in a similar manner to that described for the foreclosure option. In the interest of brevity, we will not repeat the analytical procedure or present the same graphical figures but rather use the summary results in (option) column 1 of the best and worst-case sub tables in Table 6.4.2.1.

Overall homeowners exercise the voluntary sales option more often as the economy deteriorates. From Table 6.4.2.1 (table best-case scenario, column 1) for a LTV ($\sigma_l$) and DTI ($\sigma_d$) volatility of 20% approximately 7.6% of homeowners will voluntarily sell in a best-case scenario when NO HAMP program is available. When the homeowner’s economic circumstances deteriorate (table worst-case scenario, column 1) this increases to 17.3% without the presence of a HAMP program but reaches 28.8% with a HAMP program. HAMP induces a large increase in voluntary sales by homeowners- a desirable outcome from the viewpoint of lenders and regulators, because the forbearance effect created by lowering DTI is such that the increase in some homeowner’s LTV leads to more voluntarily sales.
6.4.2.3 Strategic Default Option

The strategic default option is exercised no matter the DTI if the negative equity of the homeowner as determined by the LTV reaches 150%.

As the economy deteriorates the number of homeowners exercising the strategic default option increases significantly.

However, examining the values in Table 6.4.2.1, (table worst-case scenario, column 3) the introduction of the HAMP program might see strategic default increasing from 21.6% to 39.4% as compared to an increase from 21.6% to 45.4% with NO HAMP program - a 7% reduction. A HAMP type program might induce roughly 15% fewer people to strategically default which might be seen as a desirable outcome by lenders and regulators.

6.4.2.4 Paid Up Option

This option is exercised if the LTV of the homeowner is below 1% over a three-month period. Effectively, the mortgage is repaid and it is extremely unlikely that any other option will be exercised.

As might be expected, when the economy deteriorates the number of homeowners exercising the paid up option decreases. The option is insensitive to whether or not a HAMP type program is available with the same percentage of homeowners exercising the option regardless (0.20%, table worst-case scenario, column 5). Thus, one could conclude that the availability of a HAMP type program should have little effect on the rate at which some homeowners pay off their mortgages.
6.4.2.5 Other Option

Homeowners who are still current and have never entered a HAMP type program (if available) within the three month period at the end of the computer simulation are deemed to have exercised the Other Option.

With deteriorating economic conditions, the percentage homeowners exercising the other option decreases from 68.2% to 33.4% with NO HAMP program (table worst case scenario, column 6 and (with a HAMP program) further reduces to 16.6% because of homeowners exercising other options and because around 25.0% of homeowners remain in the HAMP program at the end of the term.
6.4.3 How Much Does LTV and DTI Change Conditional on the Option Exercised?

We are particularly interested in discovering what effect the introduction of a HAMP program has on the average change in LTV and DTI from mortgage origination compared to a NO HAMP scenario. This might help increase our understanding of whether homeowners who enter HAMP experience beneficial changes in LTV from a scenario where NO HAMP program is available? Strictly speaking rational homeowners should only be motivated by future expectations but it is not unreasonable to assume, that notwithstanding HAMP benefits and lender forbearance, that the average US homeowner may compare terminal LTV to LTV at origination. Similarly, rational lenders should only be interested in NPV (Holden et al. 2012) but they are also concerned with managing their loan losses and interested in the net change in LTV of their mortgage loans and customer’s DTI from origination.

We assume that the difference in LTV between mortgage origination and the exercise of a terminal option such as strategic default, forced foreclosure or voluntary sale gives a measure of the change in equity to the homeowner (and therefore lender). The difference, depending on the type of option, can be negative and positive as demonstrated in the histogram of the change in LTV from mortgage origination in Figure 6.4.3.1 for US homeowners.

Figure 6.4.3.1 LTV Change from Mortgage Origination for Homeowners Exercising the Voluntary Sale Option

Simulation Parameters
Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%
Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000
Worse Case = \( \mu_d = \mu_l = 0.02 \), H =HAMP Program, \( \sigma_l = \sigma_d = 20\% \), LTV0 = 81\%, DTI0 = 0.18
We construct the voluntary sales histogram by selecting those homeowners from the 100000 simulated who exercise the option (at the reference volatilities) which in this case numbers 29418. We divide these homeowners into LTV “buckets” (change in LTV) across a range from -100% to +20%. We finally numerically integrate the area to calculate the mean change in LTV from origination for those homeowners. We report the mean change in LTV and DTI in Table 6.4.3.1 for the voluntary sale, forced foreclosure and strategic default options. The “discontinuity” in the voluntary sale histogram at approximately +10% is explained by the observation that more homeowners who had an initial LTV of approximately 80% may decide to foreclose above the 90% LTV trigger point (as discussed in Section 6.3.3.2).

Table 6.4.3.1 Homeowners Option Exercise Frequency and Average Change in LTV and DTI from $t_0$.

<table>
<thead>
<tr>
<th></th>
<th>% Homeowners Exercising</th>
<th>Change in LTV from $t_0$</th>
<th>Change in DTI from $t_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td><strong>Best Case</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NH $\mu=0.2$, $\sigma=0.2$</td>
<td>7.5</td>
<td>0.8</td>
<td>21.5</td>
</tr>
<tr>
<td>$\sigma=0.4$</td>
<td>6.1</td>
<td>0.4</td>
<td>34.7</td>
</tr>
<tr>
<td><strong>Worse Case</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NH $\mu=0.2$, $\sigma=0.2$</td>
<td>17.1</td>
<td>3.8</td>
<td>45.7</td>
</tr>
<tr>
<td>$\sigma=0.4$</td>
<td>16.6</td>
<td>1.2</td>
<td>45.3</td>
</tr>
<tr>
<td><strong>Worse H</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu=0.2$, $\mu=0.02$, $\sigma=0.02$, $\alpha_5=20%$ or $40%$, $\sigma=0.02$, $\sigma=0.02$</td>
<td>29.3</td>
<td>6.8</td>
<td>39.0</td>
</tr>
<tr>
<td>$\sigma=0.4$</td>
<td>28.2</td>
<td>2.0</td>
<td>41.4</td>
</tr>
</tbody>
</table>

**Simulation Parameters**
Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%, H = HAMP Program, NH = NO HAMP program
Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000
Worse Case $\mu=\mu=0.02$, Best Case $\mu=\mu=0.02$, $\alpha_5=20\%$ or $40\%$, $\sigma=0.02$, $\sigma=0.02$

**Column Option Descriptions**: Percentage Homeowners who 1 = Voluntarily Sell, 2 = Forceclose, 3 = Strategically Default
6.4.3.1 Voluntarily Sale Option

For $\sigma_i = \sigma_d = 20\%$, we calculate (table 6.4.3.1 sub-table 2 column 1) a mean change in LTV of -33.8% in the best case No HAMP situation, -23.6% in the worst case NO HAMP situation and -20.4% in the worst case HAMP situation. Homeowners, exercising the voluntary sale option after entering a HAMP type program have a LTV that is on average 3.2% higher at option exercise compared to when NO HAMP is available.

Although LTV and DTI is increasing for both the HAMP and NO HAMP cases in the worst case scenario, because the homeowner “hangs in” longer by entering the HAMP program, gaining the DTI reset benefit, some may also “lose” more equity due to a higher LTV at termination. This would appear on average advantageous to lenders depending on how the DTI reset benefit is financed.

6.4.3.2 Forced Foreclosure Option

For $\sigma_i = \sigma_d = 20\%$, we calculate (table 6.4.3.1 sub-table column 2) a change in LTV of 30% in the best case No HAMP situation, 32.7% in the worst case NO HAMP situation and 34.6% in the worst case HAMP situation.

Homeowners, being foreclosed upon after entering a HAMP type program, have a LTV that is on average 1.9% higher compared to NO HAMP which appears beneficial to homeowners but (perhaps) at the expense of lenders.

6.4.3.3 Strategic Default Option

We can conclude (table 6.4.3.1 sub-table 2, column 3) that the change in homeowners’ LTV from mortgage origination is almost identical where a HAMP program exists than where NO
HAMP program exists. We calculate a change in LTV of 75.3% in the best case NO HAMP situation, 77.1% in the worst case NO HAMP and 77.1% in the worst case HAMP situation.

In other words, homeowners, defaulting even with a HAMP type program, have a change in equity that is the same compared to a NO HAMP type situation. Lenders are no worse off with respect to defaults in terms of the losses they might suffer on loans. However, they (might) still gain from the much greater number of homeowners who voluntarily sell. Homeowners who enter HAMP have of course benefitted from the DTI reset.
### 6.4.4 How is the seasoning of the option affected by the HAMP program?

Our third analytical question examines the effect of HAMP on the duration or “seasoning” of the voluntary sale, forced foreclosure and strategic default options. We have picked five years because the HAMP program terminates after this period though homeowners who have already entered continue to benefit for an additional 5 years. We are interested in discovering:

a) How many homeowners who (eventually) exercise a particular option enter the HAMP program within the first five years?

b) How many of these homeowners in a) who enter the HAMP program eventually exercise the same option in the following 5 years?

The simulation program notes the period when the homeowner enters the HAMP program and then the period when the homeowner subsequently exercises one of the terminal options – strategic default, forced foreclosure or a voluntary sale. We plot histograms of duration effects for the different options and then integrate and summarise key data from the histograms for three types of options - strategic default, voluntary sales and forced foreclosures, in Table 6.4.4.1 overleaf. By way of example only, the left hand subplot in Figure 6.4.4.1 below helps answer question a) while the right hand subplot answers question b) for voluntary sales.

![Figure 6.4.4.1 Histograms of Homeowners who Voluntary Sell as a Function of Entry Time and Program Status](image_url)

**Simulation Parameters**

- Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%
- Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000
- Worse Case = μ_d = μ_l = 0.02, H =HAMP Program, σ_d = σ_l = 20%, LTV_0= 80%, DTI_0 = 0.21
Table 6.4.3.1 LTV Change, Durations and Exercise Frequency for Prime Homeowners.

<table>
<thead>
<tr>
<th>Option Description</th>
<th>Strategic Default</th>
<th>Forced Foreclosure</th>
<th>Voluntary Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic Case</td>
<td>Best NH</td>
<td>Worse NH</td>
<td>H</td>
</tr>
<tr>
<td>Number of homeowners who exercise over 30 years</td>
<td>18787</td>
<td>43355</td>
<td>37767</td>
</tr>
<tr>
<td>Percentage of those who exercise within 5 years of t₀</td>
<td>23%</td>
<td>22%</td>
<td>25%</td>
</tr>
<tr>
<td>Number of homeowners entering HAMP</td>
<td>1647</td>
<td>5942</td>
<td>8735</td>
</tr>
<tr>
<td>Percentage of those entering HAMP within 5 years of t₀</td>
<td>51%</td>
<td>58%</td>
<td>61%</td>
</tr>
<tr>
<td>Percentage of those who enter and exercise in + 5 years</td>
<td>55%</td>
<td>69%</td>
<td>36%</td>
</tr>
<tr>
<td>Mean Change in LTV from t₀</td>
<td>85%</td>
<td>85%</td>
<td>85%</td>
</tr>
</tbody>
</table>

Simulation Parameters
Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%, H = HAMP Program, NH = NO HAMP program
Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000
Worse Case = μd = μl = 0.02, Best Case = μd = μl = -0.02, σl = σd = 20%, LTV₀ = 80%, DTI₀ = 0.21

6.4.4.1 Voluntary Sale Option

We calculate (row 1, voluntary sale column) for a simulation of 100000 runs that the number of homeowners who exercise the voluntary sale option over the 30 years term as 8883 (best case/NO HAMP), 21595 (worse case/NO HAMP) and 33622 (worse case/HAMP). i.e. HAMP increases voluntary sales absolutely.

Row 2 in the table and the left hand sub plot of Figure 6.4.4.1 shows that of the 33622 who enter HAMP, 23% (or 7733) exercise the voluntary sale option within the first 5 years of their 30-year mortgage term which answers a) above. In contrast, with NO HAMP program although 21595 homeowners eventually voluntary sell only 9% (1943) of these do so within the first five years of their 30 year mortgage term. HAMP “encourages” voluntary sales.

Under HAMP, 8735 (row 3) of those voluntarily selling (33622) will at some time (not necessarily in the first 5 years) enter the program whereby 61% of these 8735 (i.e. 5328 of the 7733) will have entered within the first 5 years of their 30-year mortgage term. Of those 8735 that enter, 36% will nevertheless voluntarily sell within 5 years of entering HAMP which answers part b) above.
We conclude that the HAMP program is particularly effective when compared in the next two sub sections to the foreclosure and strategic default options in “steering” prime mortgage holders towards a voluntary sale within a short time period of initiating their mortgage but not necessarily in delaying the exercise of the voluntary sale option for a third of this group.

The mean decrease in LTV of the 8735 HAMP homeowners voluntarily selling is 10% compared to a mean decrease of 21% for the total 33622 homeowners who voluntarily sell over the 30-year term (row 4). HAMP homeowners are therefore in contrast to those other (voluntary sales) homeowners who did not enter the HAMP program significantly worse off when they eventually sell their homes, which reflects the fact that LTV is increasing over the period. They will of course have benefitted from the DTI reset.

6.4.4.2 Forced Foreclosure Option

We see in row 1 of Table 6.4.4.1 that foreclosures increase absolutely under a HAMP program from 4440 to 6437 homeowners although the percentage homeowners exercising within the first 5 years of their term is very similar at 14% and 16% respectively (row 2).

Of the total 6437 homeowners selling over the 30-year term, 5942 (or 58%) would have entered the HAMP program within the first 5 years. Of these 58%, 69% would foreclose within 5 years of entering the HAMP program.

The HAMP program has little effect on “steering” more homeowners towards a foreclosure within a very short time period of initiating their mortgage as compared to the voluntary sale or strategic default options and of those that do enter the HAMP program many will most likely still foreclose within its lifetime.
Examining the change in LTV since origination, we note that the mean change in LTV is 43%, which is 2% higher than the worst-case scenario without a HAMP program. In other words, on average, individual prime mortgage holders who foreclose appear to be slightly “better off” in terms of their equity position if a HAMP program is available in that they can “put” a lower value property/higher loan to the lender. In addition, they have benefitted from the DTI reset. Lenders are therefore “losing” on average.

6.4.4.3 Strategic Default Option

The number of homeowners who exercise the strategic default option over the 30 years term is 43355 (worse case/NO HAMP) compared to 37767 (worse case/HAMP). i.e. in contrast to the other two options HAMP reduces strategic defaults absolutely.

When a HAMP program is available 1647 of those defaulting will enter HAMP at some time within their 30 year mortgage term of which 51% will be within the first 5 years. Of those 1647 that enter, 55% will nevertheless strategically default within 5 years of entering HAMP.

We make the important conclusion that the HAMP program does not necessarily “steer” prime mortgage holders towards a strategic default within a very short time period of initiating their mortgage and that the majority will nevertheless still strategically default within a 5 year period.

Examining the change in LTV since origination, we note that the mean change in LTV is 85% which is the same as in a worst case scenario without a HAMP program. Those subprime mortgage homeowners who enter the HAMP program on average have a change of LTV of 86% and are therefore only slightly “better off” when they put their property to the lender but will have enjoyed some benefit from the DTI reset.
6.4.5 Comparison of the HAMP effect on Prime and Sub Prime US Homeowners

The detailed demonstration in Sections 6.4.2 to 6.4.4 is directed at prime mortgages reflecting their preponderance in the US mortgage market. In contrast, the following analysis gives more insight into how a HAMP type program might differentially affect the so-called sub-prime segment - those homeowners who we define as having a DTI of 0.31 and a LTV of 95% at origination.

Table 6.4.5.1 summarises the results for six options. So for instance, the column for Voluntary Sales is divided into sub columns for subprime and prime. We then compare in descending order the best case NO HAMP, to the worst-case NO HAMP and the worst case HAMP.

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td><strong>Voluntary Sale</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>Best Case</strong></td>
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</tr>
<tr>
<td>NH 0.2 0.2</td>
<td>19.7</td>
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<td>NH 0.4 0.2</td>
<td>28.9</td>
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<td>NH 0.4 0.4</td>
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<td>14.1</td>
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<tr>
<td><strong>Worse Case</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NH 0.2 0.2</td>
<td>29.6</td>
<td>17.3</td>
</tr>
<tr>
<td>NH 0.4 0.2</td>
<td>30.7</td>
<td>16.6</td>
</tr>
</tbody>
</table>

**Simulation Parameters**

Delay Trigger for DTI and LTV=3 months, Term Period = 360 months, Strategic Default LTV Trigger = 150%, H =HAMP Program, NH = NO HAMP program

Voluntary Sales DTI Trigger = 0.5, HAMP DTI Trigger = 0.38, HAMP DTI Reset = 0.31, HAMP LTV Trigger = 110%, Number of Householders = 100,000

Worse Case = $\mu_d = \mu_l = 0.02$, Best Case = $\mu_d = \mu_l = -0.02$, $\sigma_d = \sigma_l = 20\%$ or $40\%$

SP = Subprime Mortgage where $LTV_0 = 95\%$, DTI= 0.31, P = Prime Mortgage where $LTV_0 = 81\%$, DTI= 0.18
6.4.5.1 Forced Foreclosure Option

The percentage of homeowners facing foreclosures, decrease in the best case NO HAMP scenario (Table 6.4.5.1, foreclosure Column 2) as the quality of mortgages at origination increase from subprime to prime (SP->P). This decrease is also mirrored in the worst-case NO HAMP scenario as mortgage quality improves but is significantly muted when a HAMP type program is introduced as would be expected after a DTI reset.

Examining subprime mortgages, in particular at our reference volatility values of 20%, one can see that the percentage of foreclosures increases from 4.2% in a best-case scenario and NO HAMP program to 11.8% in a worst case NO HAMP scenario and then decreases to 6.6% in a worst case HAMP scenario. In contrast, for prime mortgages, foreclosures increase from 0.7% to 4.3% and then further increase to 6.7% when a HAMP program is introduced.

In other words, a HAMP type program (as modelled here) produces contrasting results depending on the initial characteristics of the borrower and collateral at mortgage initiation. Subprime mortgages will see a marked decrease in foreclosures, due to the reduction in DTI, while prime mortgages will see small increases in foreclosure levels. This is because, with the extra delay induced by the HAMP program and volatile LTV, some subprime homeowners (at origination) move into that equity zone where a voluntary sale is feasible while some (but not many) prime homeowners due to the higher LTV volatility move into the foreclosure region. This is an important and unique observation.

6.4.5.2 Voluntary Sale Option

In contrast to the foreclosure option, the percentage of homeowners selling voluntarily (Table 6.4.5.1, Voluntary Sales Column 1) decreases rapidly both for the worse and best-case scenarios as the quality of mortgages at origination increases.
Examining subprime mortgages, in particular at our reference volatility values of 20%, one can see that voluntary sales increase from 19.7% in a best-case scenario and NO HAMP program to 29.6% in a worst case NO HAMP scenario but increases further to 46.1% in a worst case HAMP scenario. In a similar manner, for prime mortgages, voluntary sales increase from 8.7% to 21.7% and then increase further to 33.3% when a HAMP program is introduced. In contrast to the forced foreclosure option, a HAMP type program does not produce contrasting results with respect to the voluntary sale option.

Homeowners entering HAMP, enjoying a reduction in their mortgage payments that eases their debt burden, will gain some temporary relief but will inevitably have to (voluntarily) sell their property. However, from the viewpoint of lenders and regulators this may be seen as a positive outcome as it may be preferable to induce subprime mortgage holders to sell their property rather than to foreclose on their property.

6.4.5.3 Strategic Default Option

In a manner similar to the voluntary sale option, the percentage of homeowners strategically defaulting (Table 6.4.5.1, Strategic Default Column 3) decreases both for the worse and best-case scenarios as the quality of mortgages at origination increase.

Examining subprime mortgages, in particular at our reference volatility values of 20%, one can see that strategic default increases from 25.4% in a best-case scenario and NO HAMP program to 43.2% in a worst case NO HAMP scenario and decreases to 37.0% in a worst case HAMP scenario. In a similar manner, for prime mortgages, strategic default increases from 18.9% to 43.1% and then decreases to 37.6% with a HAMP program. This reemphasises the earlier point that HAMP mitigates against strategic default. A HAMP type program, reducing payment burden, should have a measureable effect on reducing strategic default.
In a bizarre sort of way, regulators need not worry too much about “free riders” with a HAMP type program, as its main effect is to increase the probability of exercising the voluntary sales option with a mitigating effect on the exercise of the strategic default option.

6.4.5.4 Other Option

As would be expected the number of homeowners who don’t run into trouble decreases rapidly as the quality of the mortgage goes to subprime: in a best case NH scenario from 68.2% to 50.6% compared to 33.4% to 15.3%. This reduces further to 16.6% and 3.8% in the case of a HAMP program the difference from the NO HAMP percentages reflecting the fact that more homeowners make use of the HAMP program.

6.4.5.5 Paid Up Option

The percentage of homeowners paying off their mortgage (Table 6.4.5.1, Paid Up, Column 5) increases both for the worse and best-case scenarios as the quality of mortgages at origination increase. As might be expected, more homeowners exercise this option when LTV volatility is high. (\(\sigma_l=40\%\)) and rarely at our reference LTV volatility \((\sigma_l)\) and DTI volatility \((\sigma_d)\) of 20%.

6.4.5.6 HAMP Option

The percentage of homeowners (Table 6.4.5.1, HAMP Program, Column 6) that enter and remain in the HAMP program is quite constant across the different mortgage segments at around 24% implying that the program is not particularly biased towards any particular group. This would seem intuitively correct as entry to the HAMP program is mainly based on DTI criteria and not LTV criteria.
6.4.6 The Strategic Default and the Voluntary Sales Trigger

We investigate the exogenous triggers at which homeowners finally succumb to their debts and are forced to sell their homes either strategically or voluntarily. We vary the base case DTI trigger value of 0.5 downwards to 0.4 and upwards 0.6 to investigate the effects of homeowners respectively deciding to exercise an option earlier due to debt burdens or trying to “stick it out” a little longer.

Table 6.4.6.1 illustrates that the higher the DTI trigger at which homeowners finally succumb to their debt burden the lower the number of voluntary sales and foreclosures that are exercised and the greater the number of strategic defaults.

If strategic defaults are regarded as morally hazardous and expensive to lenders and society then in some ways, the earlier that homeowners decide to sell or foreclose the better. Homeowners who game the system and “free ride” by staying as long as possible in a home and then strategically defaulting will invariably have a very high DTI. Our chosen DTI trigger level of 0.5 based on industry commentary would seem in this regard to be reasonable.

We also vary the LTV strategic default trigger from the base case value of 150% to 130% and 170% to investigate the effects of homeowners respectively deciding to exercise the default option earlier or trying to stick it out a little longer perhaps due to future expectations of property price increases.

Table 6.4.6.2 illustrates that the higher the LTV trigger at which homeowners finally default the lower the number of strategic defaults that are exercised and the greater the number of voluntary sales and foreclosures.
### Table 6.4.6.1: Homeowner Option Exercise Frequencies for Varying (DTI) Voluntary Sales Trigger Values

<table>
<thead>
<tr>
<th></th>
<th>DTI T</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td><strong>H</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>31.4</td>
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<td>16.4</td>
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</tr>
<tr>
<td>0.5</td>
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<td>6.8</td>
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<td>16.6</td>
<td>0.2</td>
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<td>27.7</td>
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<td>41.8</td>
<td>16.4</td>
<td>0.2</td>
<td>22.3</td>
<td></td>
</tr>
<tr>
<td><strong>NH</strong></td>
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<td></td>
<td></td>
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<td></td>
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<td></td>
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**Simulation Parameters**

- Delay Trigger for DTI and LTV = 3 months
- Term Period = 360 months
- H = HAMP Program
- NH = NO HAMP program
- Strategic Default LTV Trigger = 150%
- HAMP DTI Trigger = 0.38
- HAMP DTI Reset = 0.31
- HAMP LTV Trigger = 110%
- Number of Householders = 50,000
- Voluntary Sale Trigger = DTI T = 0.4, 0.5 or 0.6

| Column Option Descriptions: | Percentage Homeowners who 1 = Voluntarily Sell, 2 = Forceclose, 3 = Strategically Default, 4 = Other Option, 5 = Paid Up, 6 = Enter HAMP |

### Table 6.4.6.2: Homeowner Option Exercise Frequencies for Varying Strategic Default (LTV) Trigger Values

<table>
<thead>
<tr>
<th></th>
<th>LTV T</th>
<th>1</th>
<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
</tr>
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<tbody>
<tr>
<td><strong>H</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>26.6</td>
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<td>48.9</td>
<td>16.5</td>
<td>0.2</td>
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<tr>
<td>150</td>
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<td>0.2</td>
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</tr>
<tr>
<td><strong>NH</strong></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>130</td>
<td>15.8</td>
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<td>27.4</td>
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<td>0.0</td>
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<tr>
<td>150</td>
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<td>45.3</td>
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<tr>
<td>170</td>
<td>18.1</td>
<td>5.1</td>
<td>38.9</td>
<td>37.7</td>
<td>0.2</td>
<td>0.0</td>
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</tbody>
</table>

**Simulation Parameters**

- Delay Trigger for DTI and LTV = 3 months
- Term Period = 360 months
- H = HAMP Program
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- Voluntary Sales DTI Trigger = 0.5
- HAMP DTI Trigger = 0.38
- HAMP DTI Reset = 0.31
- HAMP LTV Trigger = 110%
- Number of Householders = 50,000
- Voluntary Sale Trigger = LTV T = 130%, 150% or 170%

| Column Option Descriptions: | Percentage Homeowners who 1 = Voluntarily Sell, 2 = Forceclose, 3 = Strategically Default, 4 = Other Option, 5 = Paid Up, 6 = Enter HAMP |

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6.5 Chapter Conclusions

We conclude by summing up some of the main findings in Section 6.4 at our reference volatilities of $\sigma_{t}, \sigma_{d} = 20\%$. We examine the simulated effect of a HAMP program on real mortgage mitigation options and by extension answer our research objective of whether ignoring the ability to pay of homeowners is justified by its effect on strategic default and foreclosure. We repeat that the outcomes of the HAMP program are very dependent on the initial LTV and DTI at origination. Therefore, for the typical prime US homeowner:

1) A HAMP type program leads to a significant increase in homeowners exercising the voluntary sales option from 17.3% to 28.8% - a desirable outcome from the viewpoint of homeowners, lenders and regulators due to the (presumed) reduced associated deadweight costs. This occurs because the temporary mitigation effect created by lowering the DTI of homeowners is such that the positive development of some homeowner’s LTV due to higher volatility induces more voluntary sales. This conclusion might however be difficult to verify empirically as it is probably not easy to divine homeowners’ motivations for selling.

2) Forced foreclosures double during the lifetime of a HAMP program from 3.7% to 6.7%. The forbearance that homeowners enjoy from a reduced DTI will in many cases be only a temporary reprieve whereby once the homeowner has recurring income or debt difficulties the LTV has turned either negative forcing a foreclosure instead of a voluntary sale or positive permitting a voluntary sale instead of a foreclosure. This conclusion is more easily verifiable. However, some opponents of the HAMP program, taking a narrow view, may claim that the program has therefore failed.
3) A HAMP type program induces roughly 15% fewer people to strategically default which might also be seen as a desirable outcome by lenders and regulators. In better economic circumstances, the most common terminal option chosen is the strategic default option (21.6%). When economic conditions worsen, although the percentage of homeowners that strategically default increases to 39.4% (with HAMP), this increase is much less than the increase in voluntary sales to 46.1% from 19.7%. The HAMP program is certainly not a “free rider” program. This result would strongly indicate that accepting the ability to pay assumption in deteriorating economic circumstances, as suggested by Vandell (1995) might lead to large over estimations of strategic defaults as many homeowners may be more likely to voluntarily sell.

However, the outcomes of a HAMP program are more nuanced when we compare the subprime and prime type mortgage segments and will depend on the characteristics of the borrower and collateral at mortgage origination. These effects can be difficult to differentiate due to the greater number of prime to subprime mortgages in the US mortgage pool.

4) Subprime mortgages will see a marked decrease in foreclosures from 11.8% to 6.6%, due to the reduction in DTI, while prime mortgages will see an increase in foreclosure levels from 4.3% to 6.7%. This is because, with the extra delay induced by the HAMP program and volatile LTV, that some subprime homeowners (at origination) move into that equity zone where a voluntary sale is feasible while some (but not many) prime homeowners due to the higher LTV volatility move into the foreclosure region.
5) In contrast to the forced foreclosure option, a HAMP type program does not produce contrasting results with respect to the voluntary sale option but reinforces the overall conclusion about the significant effect of a HAMP program on voluntary sales. Subprime mortgage homeowners see an increase from 29.6% to 46.1% and prime mortgage homeowners from 21.7% to 33.3%.

6) Both subprime and prime mortgage holders exercise the strategic default option to a lesser extent: subprime decreasing from 43.2% to 37.0% and prime mortgage homeowners decreasing from 43.1% to 37.6%. This reemphasises the point that the HAMP program is not a “free rider” charter but does actually help reduce strategic default even for those subprime borrowers who imprudently took on an unaffordable loan. Therefore, a HAMP program would seem beneficial to lenders and society alike.

We looked at the effect of changing the DTI trigger level at which homeowners finally succumb to their debt burden.

7) The higher the DTI trigger at which homeowners succumb to their debt burden the lower the number of voluntary sales and foreclosures that are exercised and the greater the number of strategic defaults. Therefore, it probably makes little sense for regulators and lenders to encourage homeowners to “tough it out” and impose harsh income requirements as the value of exercising the strategic default option keeps increasing. It is interesting to note that the recent relaxation in the HAMP eligibility criteria (June 12th, 2012) seems to recognise this aspect as it reduces the criteria from
the current 0.38. If strategic defaults are regarded as morally hazardous and expensive to lenders and society then in many ways, the earlier that homeowners are encouraged to decide to sell or foreclose the better. Homeowners who game the system and “free ride” by staying as long as possible in a home and then strategically default will invariably have a very high DTI.

We examined the effect of varying the strategic default trigger higher and lower.

8) The higher the LTV trigger at which homeowners finally default the lower the number of strategic defaults and the greater the number of voluntary sales and foreclosures.

A unique aspect of this simulation model is the ability to compare the homeowner’s final equity worth as represented by their LTV to that at mortgage origination. Although general LTV drift is positive (i.e. value is destroyed) during an economic crisis we simply pose the question whether the HAMP program is more likely to “create” or “destroy” more or less equity (LTV) value for homeowners or lenders compared to NO HAMP program. In this regard, we presume that strategic defaults are “good” for homeowners and “bad” for lenders while voluntary sales and foreclosures can be either “good” or “bad” depending on the terminal LTV with or without a HAMP program.

9) Homeowners who exercise the voluntary sale option from within a HAMP program have a LTV that is on average 3.2% higher compared to a NO HAMP type situation. Except at very low LTV volatilities, homeowners are worse off in terms of their terminal LTV by waiting longer to sell their property. Homeowners benefit from the reduction in mortgage payments but at the expense of more negative equity when they eventually sell their home. The HAMP program is therefore not a “free ride” charter.
10) Homeowners, being foreclosed upon from after entering a HAMP type program, have a LTV that is on average 1.9% higher compared to a NO HAMP type situation. Lenders are consequently slightly worse off with respect to foreclosures in terms of the losses they might suffer on loans. However, this is compensated by their gain from the much greater number of homeowners who voluntarily sell.

11) With respect to the strategic default option, a homeowners’ average terminal LTV is almost identical after entry to the HAMP program than where NO HAMP program exists. Lenders are no worse off with respect to strategic defaults in terms of the losses they might suffer on loans. However, they still gain from the much greater number of homeowners who voluntarily sell. It is a moot point whether the homeowner will take the presence of a HAMP program into account when considering their strategic default decision. On the other hand, they will gladly make use of any DTI reduction. It remains difficult for regulators or lenders to design a mitigation program where strategic defaulters do not attempt to game to their own advantage.

We conclude by summarising the seasoning or duration effect of a HAMP program on the exercise frequency of prime mortgage homeowners.

12) HAMP seems particularly effective in “steering” prime mortgage holders towards a voluntary sale within a short time of initiating their mortgage. Prime mortgage holders who enter HAMP are significantly worse off in terms of the change in LTV from origination when they eventually sell their homes.
13) The HAMP program does nothing to reduce prime foreclosures within the 5-year lifetime of the HAMP program, in contrast to its effect on the voluntary sale option. In this sense, it might appear justifiable to accept the ability to pay assumption.

14) In contrast to its effect on the voluntary sale option, HAMP does not “steer” more prime mortgage holders towards a foreclosure within a very short time period of initiating their mortgage and those that do enter HAMP will most likely still foreclose.

15) The HAMP program is not effective in “steering” prime mortgage holders away from strategic defaulting. Only a small number of these types of homeowners enter HAMP as would be expected from the HAMP application criteria and most of those entering will still “ruthlessly” default. In this sense also it might appear justifiable to accept the ability to pay assumption.

We summarise the findings to the main questions in this chapter in Table 6.5.1.

Table 6.5.1 Summary of the Simulated HAMP Program Results Compared to a Scenario with No HAMP Program.

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<th>Strategic Default</th>
<th>Voluntary Sale</th>
<th>Forced Foreclosure</th>
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<td>What happens to the option exercise probability?</td>
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<td>Increases</td>
<td>Increases</td>
</tr>
<tr>
<td>Does the change in LTV benefit the homeowner?</td>
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<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>How does the DTI change from origination?</td>
<td>Increases</td>
<td>Decreases</td>
<td>No Change</td>
</tr>
<tr>
<td>Does HAMP have on balance a permanent effect?</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Subprime Homeowners</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What happens to the option exercise probability?</td>
<td>Decreases</td>
<td>Increases</td>
<td>Decreases</td>
</tr>
<tr>
<td>Does the change in LTV benefit the homeowner?</td>
<td>No</td>
<td>No Change</td>
<td>No</td>
</tr>
<tr>
<td>How does the DTI change from origination?</td>
<td>Decreases</td>
<td>Decreases</td>
<td>Decreases</td>
</tr>
<tr>
<td>Does HAMP have on balance a permanent effect?</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Answers pertain to those homeowners who eventually exercise the terminal option

These form the main conclusions from our study of how we might expect terminal options such as voluntary sales, foreclosure and strategic default to behave in the medium to longer...
term given the presence of a HAMP type program. The main effect using this model is that a HAMP program steers more US homeowners towards a voluntary sale and reduces the number of defaults and foreclosures. This outcome, depending on deadweight default costs, is most likely to be advantageous to US homeowners and lenders. Overall, many homeowners who enter HAMP still end up exercising a (generally more favourable) terminal option within the typical 5 years HAMP duration.

Unfortunately, the two main options in percentage terms, affected by the HAMP program, voluntary sales and strategic default, are notoriously difficult to measure empirically. It will remain difficult for opponents and supporters of the HAMP program to conclusively assess its benefit to US society. We have demonstrated that, given the program’s cost ($50 billion), its immediate benefit to distressed US homeowners, the significant increase in voluntary sales and reduction of defaults with the consequent reduction in default deadweight costs, it is probably better to have a HAMP program in place rather than having NO HAMP program.

Finally, we return to our main research question as to whether the ability to pay assumption is valid or not. On balance, we have not demonstrated that ignoring the ability to pay of residential homeowners is invalid when examining the strategic default option only. However, we can argue as a result of our simulation that where the foreclosure option is of interest then one must consider both stochastic DTI and LTV and that the ability to pay assumption is invalid. This is because traditional option theoretic modelling focuses narrowly on the strategic default and foreclosure options to the exclusion of alternative options. In this sense, introducing a stochastic DTI parameter mainly effects the voluntary sale option which is ignored by the traditional approach focussing on LTV. Stochastic DTI has less effect on strategic default which is influenced mainly by LTV but does affect foreclosures.
6.6 References


COP (2009), Foreclosure Crisis: Working Toward a Solution, *Congressional Oversight Panel*, Publication No. 110-343


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HAMP Website (2012), http://www.makinghomeaffordable.gov/Pages/default.aspx


6.7 Option Description

Forced Foreclosure

Forced foreclosure occurs when a homeowner does not have sufficient income, as determined by DTI, to make the monthly payment to the lender. We assume that the homeowner has no savings or other means of paying down the loan. Furthermore, the LTV or homeowner’s equity is negative such that a voluntary sale is of no interest to the homeowner or lender. The option is triggered by the lender if the homeowner is delinquent over a number of monthly periods.

Voluntary Sale

Voluntary sale occurs when a homeowner does not have sufficient income, as determined by DTI, to make the monthly payment to the lender. We assume that the homeowner has no savings or other means of paying down the loan. However, in contrast to the forced foreclosure option, enough positive equity exists in the property to make a voluntary sale attractive to the borrower as the least bad option.

Strategic Default

Strategic default occurs where a homeowner considers that the amount of negative equity in the property as determined by LTV, is such that it makes more sense to permanently default on all future payments and “put” the property to the lender. The homeowner will invariably have enough income (DTI) to make the mortgage payment but their other assets or wealth might be so low that it is of little benefit for the lender to pursue the borrower for the outstanding amount.
**Paid Up**

We assume that this option is exercised if the LTV of the property falls below 1% through either appreciation of the property value or reduction in the outstanding loan as most homeowners whose property has a LTV of less than 1% will not consider exercising other options such as default or prepayment due to their expense and risk.

**Other Option**

Homeowners that have never exercised one of the four previous (terminal) options will therefore have an “Other Option” status. In other words, they continue to make due periodic payments to lenders when the model stops computing after the term period of 30 years. They are thus not necessarily “current” as it is possible that they are into (e.g. first month of) payment difficulties.
6.8 HAMP Eligibility Criteria

This information has been taken from the US Federal Government Home Affordability Modification Program Website (http://www.makinghomeaffordable.gov/Pages/default.aspx). The HAMP eligibility criteria change on a regular basis. The Obama Administration extended the original criteria on June 1, 2012 whereby homeowners that now may be eligible for the Home Affordable Modification Program include homeowners who:

- are applying for a modification on a home that is not their primary residence, but the property is currently rented or the homeowner intends to rent it.
- previously did not qualify for HAMP because their debt-to-income ratio was 31% or lower.
- previously received a HAMP trial period plan, but defaulted in their trial payments.
- previously received a HAMP permanent modification, but defaulted in their payments, therefore losing good standing.

Eligibility Criteria pre 1st June 2012. Homeowners who:

- obtained their mortgage on or before January 1, 2009.
- owe up to $729,750 on their primary residence or single unit rental property
- owe up to $934,200 on a 2-unit rental property; $1,129,250 on a 3-unit rental property; or $1,403,400 on a 4-unit rental property
- have financial hardship and are either delinquent or in danger of falling behind on their mortgage payments (non-owner occupants must be delinquent to qualify).
- have sufficient, documented income to support a modified payment.
- have not been convicted within the last 10 years of felony larceny, theft, fraud or forgery, money laundering or tax evasion, in connection with a mortgage or real estate transaction.

The program enrollment (but not the program) ends on December 31st 2013.
6.9 Mathematica Code

The Mathematica code consists of a thousand lines, most used to produce the graphs, plots and tables used in the data analysis sections 6.4. The core of the program is the subroutine (Figure 6.9.1) which calculate the “behaviour” of a homeowner given the initial parameters.

Figure 6.9.1 Mathematica Code for Calculating the NO HAMP Scenarios

```mathematica
(* Mathematica code for calculating the NO HAMP scenarios *)

(* Define the function to calculate the behaviour of a homeowner *)

(* Define the initial parameters *)

(* Define the main loop to simulate the homeowner's decision process *)

(* Calculate the initial values for the model *)

(* Iterate through the simulation *)

(* Output the results *)

(* End of Mathematica code *

(* Note: The actual Mathematica code snippet is too complex to reproduce here due to its length and detailed nature. It includes various Mathematica commands and functions specific to the model's simulation of homeowner behavior. *)
```
7. Conclusion

We conclude by briefly summarising the unique aspects of this dissertation as well as the key findings before criticising the approach and suggesting avenues for future research.

The main contribution of this dissertation is that it, in the context of the aftermath of the US 2007/2008 subprime housing crisis, categorises real options available to an US homeowner and examines two additional real options that may have been “undervalued” in the past. The first real option is a Negotiation Option, a variant of which is examined in more detail in Chapters 3, 4 and 5 and the second real option is what we define as the Voluntary Sale Option which is examined in Chapter 6. Our variant of the negotiation option is where homeowners and lenders renegotiate the mortgage coupon on a perpetual interest only loan contingent on the property value. Our definition of a voluntary sale option is where a homeowner is unable (but willing) to pay the mortgage coupon and decides voluntarily to sell their property and “downsize” to cheaper property or rented accommodation with a lower overall cost.

We demonstrate the existence of the Negotiation Option in Chapter 3 (as well as our particular variant) by categorising current US mitigation options and develop a unique framework for valuing a “ruthless” variant of the negotiation option. Although we base our methodology on original work by Leland (1994) and Fan and Sundaresan (2000) on corporate bond default, our own development is more appropriate to the mortgage context as we show in Chapter 3 that, in the first instance, the negotiation option can be treated as a generalised symmetrical Nash game due to unique characteristics of the US residential mortgage market. One could argue that this is not the case with respect to corporate bond default where bonds
are non-standardised and there may be asymmetric information. We also contribute by extending the methodology of Fan & Sundaresan (2000) by providing a full and detailed derivation of closed form solutions. Finally, our treatment, uniquely, does not assume as is common in option theoretic modelling as in Vandell (1995) that all homeowners and lenders are homogenous but introduce a heterogeneous negotiation parameter which distinguishes homeowners and lenders.

In our exposition, we demonstrate the distinct economic and financial consequences between a mortgage negotiation and default option. We suggest that with increasing (property) price volatility, homeowners would optimally accelerate exercise of their negotiation option in contrast to the situation with the default option where increasing property price volatility would induce homeowners to delay exercise.

In view of the greater importance placed by regulators on responsible mortgage underwriting, we demonstrate a methodology to calculate optimal LTV ratios at mortgage initiation if homeowners and lenders (ex post) negotiate. We also uniquely derive a closed form solution for the optimal LTV at origination. This is unique in that it demonstrates that whereas the optimal LTV at origination for a default only real option is invariant, in the case of a negotiation option the optimal LTV is dependent on the heterogeneous variable. In other words, it offers a range of LTV’s depending, in this particular model, on a homeowner’s bargaining strength. We further speculate that a credit rating type score (e.g. FICO) might be used to proxy this negotiation strength i.e. homeowners with higher FICO scores are able to negotiate better than homeowners with lower FICO scores.
Chapters 4 and 5 are similar in their methodological approach but do address very different housing finance sectors where default or mitigation behaviour may be fundamentally different. The non-owner occupied segment (Chapter 5) is increasingly important and significant and we believe that it is justifiable to demonstrate both the similarities and differences in Chapters 4 and 5. We also believe that a negotiation option may be more significant in particular housing finance (sub) sectors such as non-owner occupied than say owner occupied.

Empirical verification of any real option model in either (sub) sector is difficult. In fact, we have demonstrated in Chapter 2 that it has proved difficult to even verify or confirm the extent of “ruthless” or strategic default. We do not suggest that it is any easier to verify whether homeowners or landlords strategically or ruthlessly negotiate. However anecdotal comments from a board member of the Manchester Credit Union after a presentation on this topic suggests that some high credit rating borrowers do “try it on” much earlier than lower credit score borrowers. We suggest that the rental market non-owner occupied negotiation option modelled in Chapter 4 might be easier to verify empirically than the owner occupied negotiation option. Considering the growing significance of this sector in social housing policies and mortgage finance, we believe that it may be possible to observe whether and under what conditions “strong” landlords negotiate with their lenders as compared to “weaker” landlords. We believe that recent empirical work as discussed in Chapters 3, 4 and 5 by federal researchers such as Geradi, Shapiro and Willen (2008) demonstrate that in contrast to say 15 years ago, databases and statistical techniques now exist to examine the life cycle of mortgage conditional on lender, borrower and property characteristics.
Chapters 4 and 5 are based on and follow methodological approaches from previous default option and corporate bond default researchers. One could criticise an approach which essentially takes a methodology applied in the valuation of strategic debt negotiation in corporate bonds applied to residential mortgages. However, it is clear from subsequent corporate bond literature that this methodology never found favour due we believe to the limiting constraints in that particular arena. However, we believe that this approach does have merit when applied to large numbers of mortgage debt contracts as our detailed exposition in Chapter 3 demonstrates that a US residential mortgage contract is essentially standardised, symmetrical and incomplete. This insight removes a serious objection to this methodological approach.

Chapter 6 takes an unusual approach to the identification of real mortgage options and attempts to examine and estimate the probability of particular real (world) options occurring should homeowners be unable to pay the mortgage in addition to the classical default research assumption of being unwilling to pay. We believe that the current housing crisis in the US demonstrates that many lenders do not automatically foreclose on delinquent homeowners but negotiate. These negotiations have resulted in various real (world) mitigation options some of which have been examined extensively in the literature, but some such as the voluntary sale option have been generally ignored. Chapter 6 therefore attempts to estimate the significance of real options in mitigating consequences of both homeowner’s willingness and ability to pay.

Because a) a homeowner’s ability to pay cannot be optimised (in terms of utility) in contrast to the classical willingness to pay and b) we did not know a priori the significance of the different real options, we develop a Monte Carlo simulation model (of this double trigger
process) instead of the more traditional closed form or backward looking finite simulation models. The uniqueness and contribution of this is that we may discover the more significant real options to be able to discard the less significant and permit the development of a more focused and relevant closed form methodology in the future.

We uniquely simulate the effect of a double trigger type process on the exercise of the typical real options available to residential owner occupied homeowners. We find that in a deteriorating housing market the current US federal HAMP program is most likely to be cost effective in steering struggling homeowners towards the exercise of a voluntary sale mitigation option as well as tempering the increase in foreclosures and strategic defaults. HAMP should not encourage more homeowners to free ride or strategically default at the expense of the US taxpayer. We therefore propose that the most significant real options may well be strategic default which is the classical option well examined by academic researchers and that of a voluntary sale or “downsizing” – a less well examined option. Previously “downsizing” has been associated (Lehnert 2004) with older home owning people selling their housing assets to realise their capital gains. We suggest that “downsizing” could also, in addition, be currently associated with a wider range of homeowners selling their housing assets to mitigate their financial losses. This points to an interesting research opportunity.

The methodological approach taken in Chapter 6 is less well-grounded on previous research than that taken in Chapters 4 and 5. We have discussed some of the shortcomings in Chapter 6 but contend that we are essentially applying a very simple Monte Carlo option model with in-out barriers. However, due to the complexities of the barriers and the inability to “optimise” the onset of an “ability to pay trigger” a closed form solution is in the first instance beyond the scope of this dissertation. Whether we could or could not derive closed form solutions
would not add or subtract from the outcomes and significant findings. A closed form approach with similar underlying assumptions may not yield any differences in the importance of the (advanced) real options. We believe that our results and ranking of real options are significant enough to allow an empirical study of whether many current US homeowners mitigate their housing finance and consumption problems by (sub optimally) voluntarily selling their housing asset and downsizing.

Empirically, the investigation of whether this real option is exercised in large numbers is no less easy than whether homeowners strategically default. Currently, US Federal housing researchers imply the level of strategic housing default by linking default behaviour on consumption type (credit card) loans. In other words, they consider someone has strategically defaulted on their housing loan if they do not default concurrently on their credit card loans.

To discover the prevalence of the voluntary sale option, a study would be needed to identify homeowners’ subsequent accommodation mode, on voluntarily selling their property. In a similar manner to strategic default, it might be possible to link “new” housing loans or credit card expenditure to discover how many “downsize” to accommodation that is cheaper to operate. Perhaps a qualitative study based on a large sample of questionnaires such as carried out by Guiso, Sapienza and Zingales (2009) on strategic default would be appropriate.

In summary, I believe that this dissertation provides significant methodological contributions to existing option theoretic research in its investigation of a negotiation option as well as the significant contribution based on a model of a stylistic HAMP program that in a world where homeowners are unable to pay their mortgage coupon that a voluntary sale real option could be a significant negotiated mitigation option.
7.1 References


