Asset Pricing with Time Varying Pessimism and Rare Disasters

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Abstract

We incorporate time-varying consumption volatility in the representative-agent asset pricing model with generalized recursive smooth ambiguity preferences developed by Ju and Miao (2012). We calibrate the model to data on consumption and asset returns since the Great Depression period. Uncertainty aversion amplifies the perceived probability of the disastrous state coupled with high consumption volatility. We find that the model with time-varying volatility generates a high equity risk premium. When we impose the condition that no consumption disasters ever realized in simulated samples, the model with time-varying volatility can reproduce predictability of returns and non-predictability of consumption growth simultaneously, which are consistent with empirical findings.

JEL Classification: D81; G11; G12.

Keywords: Equity premium, rare disaster, uncertainty aversion, volatility risk.

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1 Introduction

Historical equity premium of the U.S. stock market has been one of the major puzzling stylized facts in financial economics. Mehra and Prescott (1985) show that a standard rational representative-agent model has great difficulty in reconciling low consumption risk and high equity premium, both observed in the data, assuming plausible levels of risk aversion. An important literature pioneered by Rietz (1988) relies on the risk of rare disasters. Investors require sufficiently high risk premium to compensate for the adverse impact of rare disasters on fundamentals. By calibrating disaster probabilities to international data on severe historical contractions, Barro (2006) finds that disasters are infrequent by nature, and in particular, the disaster probability is about 1.7 percent per year on average. Barro, Nakamura, Steinsson, and Ursua (2013) use a long data set on consumption for both OECD and non-OECD countries and find that the average disaster probability is over 3 percent per year from a Bayesian perspective. Barro (2006) and Barro et al. (2013) show that the model with disaster risk can explain high equity premium and low risk-free rate in the data. More recently, Wachter (2013) and Gourio (2012) extend the model of Barro (2006) by allowing for time-varying disaster probabilities and recursive preferences (Epstein and Zin (1989)). Their calibrated models can further account for high return volatility and predictability of excess returns.

The literature on consumption disasters commonly assumes that volatility of consumption growth remains constant across disaster and non-disaster states. However, a number of papers find that consumption volatility risk is important for asset pricing, see Bansal and Yaron (2004), Bollerslev, Tauchen, and Zhou (2009), Bansal, Kiku, Shaliastovich, and Yaron (2014) among others. In this paper, we incorporate volatility risk in a rare disaster model and examine asset pricing implications. Our model builds on the consumption-based model with Markov-switching mean consumption growth and generalized recursive smooth ambiguity preferences, developed by Ju and Miao (2012) (hereafter JM) and Hayashi and Miao (2011). Smooth ambiguity preferences were first proposed by Klibanoff, Marinacci, and Mukerji (2005, 2009). JM’s model can explain salient features of asset returns including a high equity premium, a low risk-free rate, countercyclical variations in both equity premium and volatility of returns. In their calibration to a century long sample, JM find that one state of mean consumption growth corresponds to a severe contraction.

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Gourio (2012) examines rare disasters on productivity growth and the impacts on business cycles and asset prices.
state and thus is of the rare disaster type. The representative agent cannot observe the mean growth state but has to learn about it. The agent is not only averse to consumption risk but also averse to uncertainty (ambiguity) captured by fluctuating beliefs about the unobserved state. The generalized recursive smooth ambiguity utility function also allows for separation between risk aversion and the elasticity of intertemporal substitution (EIS) and thus nests recursive utility as a special case when the agent is ambiguity neutral.

Our paper is different from JM’s model in important aspects. JM assume constant volatility in the two-state Markov chain whereas we extend the model by incorporating volatility risk. We postulate that the volatility of innovations shocks to consumption growth is also regime-switching and depends on the state that determines the mean growth regime. This modeling choice allows for ambiguity about consumption volatility, which however is usually ignored in the literature on ambiguity and asset pricing. Empirical estimation of the Markov-switching model with both regime-dependent mean and volatility identifies two distinct regimes for consumption growth dynamics, one normal regime with high mean growth and low volatility and the other regime with severely low mean growth and high volatility. Model comparison based on the likelihood function value suggests that the model with time-varying volatility is a better description of the data than the constant volatility model. The estimated transition probabilities imply that 1) the normal regime is very persistent, and the conditional probability of the economy falling into the disastrous state is only 1 percent on an annual basis, and 2) the disastrous regime becomes more persistent once regime-dependent volatility is allowed in the estimation.

In contrast to JM’s empirical analysis, we calibrate the model to consumption data since the Great Depression period. In a replication exercise assuming a modest degree of ambiguity aversion, we find that JM’s model, when calibrated to the annual sample 1930—2015, produces equity premium close to zero. However, ceteris paribus, the model with time-varying volatility can match equity premium in the data closely. Another important finding is that while increasing the degree of ambiguity aversion in JM’s model can significantly increase equity premium, the model still cannot reproduce predictability of returns in the data. Nevertheless, the model with time-varying

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2 Suzuki (2016) studies major asset pricing puzzles in an endowment economy with heterogenous beliefs and recursive utility.

3 Exceptions include Epstein and Ji (2013) and Branger, Schlag, and Thimme (2016).
volatility implies that excess returns are predictable by the price-dividend ratio at different horizons. In addition, we also consider the “peso” situation à la Veronesi (2004), i.e., the disastrous state never realized in simulations but the agent is concerned about its possible occurrence. We find that in simulated samples with the peso property, the model can further account for the absence of predictability of consumption growth.

The key mechanism driving these results is that once time-varying volatility is controlled for, the increase in the estimated persistence of the disastrous regime greatly magnifies the impact of uncertainty aversion on the agent’s fluctuating beliefs about the state of the economy. Thus, time-varying pessimism and its effects on equilibrium asset prices are strengthened in the model. Without consumption volatility risk, the distribution of the continuation value conditional on the state of the Markov chain is not dispersed sufficiently, and thus a very modest degree of uncertainty aversion is unable to generate noticeable time-varying pessimism. On the other hand, the model with time-varying consumption volatility implies strong co-movement between the stochastic discount factor and the price-dividend ratio in opposite directions. This modeling feature is important for generating equity risk premium and return predictability.

This paper is closely related to a number of studies addressing the impact of rare disasters or ambiguity on asset prices. Wacther (2013) studies time-varying disaster probabilities in the recursive utility model. The main finding is that time variation in disaster risk is important to generate excess volatility. Indeed, this paper shows how the magnitude and time variation in disaster probabilities assumed by Wacther (2013) can arise endogenously from uncertainty aversion and learning about the unobservable state. Gabaix (2012) examines time-varying dividends shocks in response to disasters in a constant relative risk aversion setting. Using the technique of linearity-generating processes, Gabaix obtains closed-form solutions, and the calibrated model can explain a rich set of empirical regularities of stock and bond returns. Barro (2009) proposes a model with a constant disaster probability and recursive preferences. In a subsequent work, Barro et al. (2013) estimate a model with disasters followed by recoveries and study the impact on equity premium. But these two models lack a mechanism to generate excess volatility due to absence of learning. Veronesi (2004) uses exponential utility without uncertainty aversion. Veronesi shows that learning

4 Many papers examine ambiguity and asset pricing, see, for example, Chen and Epstein (2002), Leippold, Trojani, and Vanini (2008), Collard et al. (2018) and Jahan-Parvar and Liu (2014).
about the disastrous state can generate excess volatility and asymmetric volatility reaction to good and bad news. However, the implied equity premium is still rather low compared to the data.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 calibrates the model and discusses simulation results. Section 4 concludes. The numerical algorithm is similar to that used by JM.

2 The Model

We assume that the growth rate of aggregate consumption follows the Markov-switching process

$$\Delta c_t \equiv \ln \left( \frac{C_t}{C_{t-1}} \right) = \mu(s_t) + \sigma(s_t) \epsilon_{c,t},$$

(1)

where $\epsilon_{c,t}$ is an i.i.d. standard normal random variable, and state $s_t$ follows a two-state Markov chain. Suppose that “1” and “2” indicate two distinct states for mean $\mu(s_t)$ and volatility $\sigma(s_t)$ of consumption growth. The transition probabilities are given by

$$\Pr(s_t = 1|s_{t-1} = 1) = p_{11}, \quad \Pr(s_t = 2|s_{t-1} = 2) = p_{22}$$

Empirical estimation of the model suggests that states 1 and 2 are characterized by “high mean, low volatility” ($\mu_h, \sigma_l$) and “low mean, high volatility” ($\mu_l, \sigma_h$) respectively.

Because empirical evidence shows that aggregate dividends are more volatile than aggregate consumption, it is common in the literature to model dividends and consumption separately, see Abel (1999) and Bansal and Yaron (2004). The dividend growth process is given by

$$\Delta d_t \equiv \ln \left( \frac{D_t}{D_{t-1}} \right) = \lambda \Delta c_t + g_d + \tilde{\sigma}_d \epsilon_{d,t},$$

(2)

where $\epsilon_{d,t}$ is a standard normal i.i.d. shock independent of $\epsilon_{c,t}$, and $\lambda$ is interpreted as the leverage parameter. Given estimates of parameters in the Markov-switching model (1), we can recover values of $g_d$ and $\tilde{\sigma}_d$ from the estimates of unconditional mean and volatility of dividend growth.

We assume that the representative agent cannot observe state $s_t$ in each period but has to form Bayesian beliefs about the state. The observable signals include the history of growth.
rates of consumption and dividends. To keep the model parsimonious, we assume that the agent has complete knowledge in the parameters in the consumption and dividend processes, namely, \(\{\mu_l, \mu_h, \sigma_l, \sigma_h, p_{11}, p_{22}, \lambda, g_d, \tilde{\sigma}_d\}\).

Suppose that the agent’s belief is \(\pi_t = \Pr(s_{t+1} = 1|I_t)\) where \(I_t\) denotes information available at time \(t\). With respect to learning about the unobservable state, dividends do not contain additional information compared to consumption. As a result, given the prior belief \(\pi_0\) and full information, the agent updates his beliefs according to Bayes’ rule:

\[
\pi_{t+1} = \frac{p_{11} f(\Delta c_t|s_{t+1} = 1) \pi_t + (1 - p_{22}) f(\Delta c_t|s_{t+1} = 2)(1 - \pi_t)}{f(\Delta c_t|s_{t+1} = 1) \pi_t + f(\Delta c_t|s_{t+1} = 2)(1 - \pi_t)}
\]

where \(f(\Delta c_t|s_t)\) is conditional density with mean \(\mu(s_t)\) and variance \(\sigma^2(s_t)\):

\[
f(\Delta c_t|s_t) \propto \exp\left[-\frac{(\Delta c_t - \mu(s_t))^2}{2\sigma^2(s_t)}\right].
\]

The agent’s preferences are represented by the generalized recursive smooth ambiguity utility function. Given consumption plans \(C = (C_t)_{t \geq 0}\) the value function \(V_t = V(C; \pi_t)\) is given by

\[
V_t(C; \pi_t) = \left[(1 - \beta)C_t^{1-1/\psi} + \beta \{R_t(V_{t+1}(C; \pi_{t+1}))\}^{1-1/\psi}\right]^{1-1/\psi},
\]

where \(\beta \in (0, 1)\) is the subjective discount factor, \(\psi\) is the elasticity of intertemporal substitution (EIS) parameter, and \(\gamma\) is the risk aversion parameter. The certainty equivalent \(R_t(V(C_{t+1}; \pi_{t+1}))\) is given by

\[
R_t(V_{t+1}(C; \pi_{t+1})) = \left(\mathbb{E}_{\pi_t}\left[\mathbb{E}_{(s_{t+1}, t)}\left[V_{t+1}(C; \pi_{t+1})^{1-\gamma}\right]^{1-\frac{\gamma}{1-\psi}}\right]\right)^{\frac{1}{1-\gamma}}.
\]

Note that the certainty equivalent in period \(t\) involves the expectation operator \(\mathbb{E}_{s_{t+1}, t}[\cdot]\), which is taken with respect to the distribution of consumption growth conditional on the next period’s state \(s_{t+1}\) and all other information available in period \(t\). This characterization corresponds to the conventional notion of risk, i.e., the future continuation value is risky given future state \(s_{t+1}\). Importantly, the expectation operator \(\mathbb{E}_{\pi_t}\) is taken with respect to the posterior belief about the unobservable state, and the integrand is the conditional certainty equivalent given future state.
This feature captures the notion of ambiguity, arising from the hidden nature of state $s_t$. The economic interpretation is that the agent is averse to a mean-preserving spread in conditional certainty equivalent of future utility. The agent displays ambiguity aversion when $\eta > \gamma$, where $\eta$ is the ambiguity aversion parameter. By setting $\eta = \gamma$, we obtain recursive utility under ambiguity neutrality. Uncertainty aversion prescribed by $\eta > \gamma$ precludes the compound reduction between the agent’s subjective beliefs, which are generated from Bayesian learning, and the two distributions of consumption growth conditional on regimes of the economy. Thus, smooth ambiguity utility implies that the agent displays different attitudes toward subjective uncertainty and consumption risk.

The stochastic discount factor (SDF) is given by (see Ju and Miao (2012))

$$M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left( \frac{V_{t+1}}{R_t(V_{t+1})} \right)^{1/\psi - \gamma} \left( \frac{\mathbb{E}_{st_{t+1,t}}[V_{t+1}^{1-\gamma}]}{\mathbb{R}_t(V_{t+1})} \right)^{1/\gamma}^{-(\eta-\gamma)}.$$ 

The risk-free rate, $R_f^t$, is the reciprocal of the conditional expectation of the SDF, that is

$$R_f^t = \frac{1}{\mathbb{E}_t[M_{t,t+1}]}.$$ 

The Euler equation is given by

$$\mathbb{E}_t[M_{t,t+1}R_{t+1}] = 1$$

where $R_{t+1}$ is an asset return. We solve for the price-dividend ratio from the Euler equation

$$\frac{P_t}{D_t} = \mathbb{E}_t \left[ M_{t,t+1} \left( 1 + \frac{P_{t+1}}{D_{t+1}} \right) \frac{D_{t+1}}{D_t} \right].$$

Since $\frac{P_t}{D_t}$ is a functional of the state variable $\pi_t$, $\frac{P_t}{D_t} = \Phi(\pi_t)$, the Euler equation becomes

$$\Phi(\pi_t) = \mathbb{E}_t \left[ M_{t,t+1} (1 + \Phi(\pi_{t+1})) \exp(\Delta d_{t+1}) \right].$$
The Euler equation can be rewritten as

$$0 = \tilde{\pi}_t E_{1,t} \left[ M_{t,t+1}^{EZ} \left( R_{t+1} - R_t \right) \right] + (1 - \tilde{\pi}_t) E_{2,t} \left[ M_{t,t+1}^{EZ} \left( R_{t+1} - R_t \right) \right],$$

where $E_{i,t} [\cdot], i = 1, 2$ denotes $E_{\sigma_{t+1},t} [\cdot]$. The term $M_{t,t+1}^{EZ}$ can be interpreted as the SDF under recursive utility with ambiguity neutrality:

$$M_{\tilde{\pi}_{t+1},t+1}^{EZ} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\gamma}} \left( \frac{V_{t+1}}{R_t (V_{t+1})} \right)^{\frac{1}{\gamma} - \gamma}.$$

With the formulation presented above, we can interpret $\tilde{\pi}_t$ as the ambiguity-distorted belief and write is as:

$$\tilde{\pi}_t = \frac{\pi_t \left( E_{1,t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta - \gamma}{\gamma - \gamma}}}{\pi_t \left( E_{1,t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta - \gamma}{\gamma - \gamma}} + (1 - \pi_t) \left( E_{2,t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{-\frac{\eta - \gamma}{\gamma - \gamma}}}.$$ (5)

For $\eta > \gamma$, the distorted beliefs are not equivalent to Bayesian beliefs. The distortion driven by uncertainty aversion is an equilibrium outcome and implies time-varying pessimistic beliefs. Recall that state 2 represents the disastrous state. We can show by quantitative analysis that

$$\left( E_{1,t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} > \left( E_{2,t} \left[ V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}$$

and thus $\tilde{\pi}_t < \pi_t$. That is, uncertainty aversion effectively increases the posterior probability of the disastrous state.

Variance (risk) premium is defined as the difference between the risk-neutral and objective expectations of stock return variance for a given horizon, that is

$$VRP_t = E_t^Q \left[ \sigma_{r,t+1}^2 \right] - E_t \left[ \sigma_{r,t+1}^2 \right]$$

where the horizon is often assumed to be one month and $\sigma_{r,t+1}^2$ is variance of log returns. The Chicago Board Options Exchange (CBOE) uses a “model-free” approach to calculate the volatility index (VIX) as the risk-neutral expectation of the stock market variance. Since the VIX data are reported in annualized “vol” terms, it is often convenient to convert the index to the monthly
quantity \( VIX^2/12 \). The physical expectation of variance can be calculated based on measures of realized variance for a given month. Drechsler (2013) uses high frequency returns data to construct realized variance measures, and a projection method to estimate conditional variance forecasts that serve as an empirical proxy for the physical expectation of variance. We follow this approach to estimate the VIX and VRP for the sample period 1996—2015.

In the model, variance risk premium is defined as
\[
VRP_t = \mathbb{E}_t \left[ \frac{M_{t,t+1}}{E_t(M_{t,t+1})} \sigma^2_{r,t+1} \right] - \mathbb{E}_t \left[ \sigma^2_{r,t+1} \right]
\]

where \( \frac{M_{t,t+1}}{E_t(M_{t,t+1})} \) characterizes the risk-neutral measure transformation, and \( \sigma^2_{r,t} \) is conditional variance of log returns, i.e., \( \sigma^2_{r,t} \equiv \text{Var}_t(\ln R_{t+1}) \).

3 Calibration

We calibrate the model to assess quantitative impacts of uncertainty aversion and time-varying volatility on asset returns. Due to nonlinearities, the model does not admit an approximate analytical solution. We follow JM and use the projection method to solve the model and then perform Monte Carlo simulations to compute moments of asset returns and run predictive regressions. The quantitative results presented below are based on 20,000 simulations. Gauss-Hermite quadrature is used to compute expectation over a normal distribution.

We estimate the Markov-switching model (1) on the annual sample of aggregate consumption growth rates for 1930—2015. We also re-estimate the Markov-switching model with constant volatility adopted by JM. We obtain real per capita consumption data from National Income Product Accounts (NIPA) and calculate growth rates. Real per capita consumption is measured by the sum of real nondurable and services consumption (items 16 and 17 in the NIPA Table 7.1 “Selected Per Capita Product and Income Series in Current and Chained Dollars”). The NIPA real per capita consumption data are reported in chained 2009 U.S. Dollars and calculated using mid-year population data.

Maximum likelihood estimates of parameters in the constant volatility model are reported in Panel A, Table 1. It is not surprising to see that our estimation results are different from those
reported by JM. We show that these different estimates can significantly affect JM’s calibration results. JM’s analysis builds on empirical estimates of Cecchetti, Lam, and Mark (2000), based on a century long sample from 1890 to 1994, but our sample for estimation includes more recent data. While the estimated transition probabilities reported in Panel A of Table 1 are similar to those in Cecchetti et al. (2000), our estimate of the mean growth rate in the bad state is -0.045, which is moderately higher than that (-0.068) reported by Cecchetti et al. (2000). Additionally, our estimate of constant volatility of innovation shocks is 0.016, lower than that (0.032) reported by Cecchetti et al. (2000). The estimate of the transition probability $p_{11}$ suggests that conditional on a normal economic regime the probability of the economy falling into the disastrous state is about 0.03 per year, similar to empirical evidence provided in the literature on disaster risk. The likelihood function value for this model is 293.78.

Estimation results for the Markov-switching model with regime-dependent volatility are summarized in Panel B, Table 1. These results indicate that time-varying volatility is present in consumption data. The two economic regimes consist of 1) state 1 with high mean growth and low volatility, and 2) state 2 with low mean growth and high volatility (disastrous state). The estimated transition probabilities for this model imply that both regimes are more persistent than in the constant volatility model. Conditional on a normal regime, the probability of the economy experiencing a disaster per year is about 0.01. The likelihood function value for this model is 308.17. In terms of the likelihood function value, the model with regime-dependent volatility characterizes empirical dynamics of consumption growth better than the constant volatility model does.

Parameters in the dividend growth process are calibrated as follows. The leverage parameter $\lambda$ is set at 2.75. The unconditional volatility of dividend growth is set at $\sigma_d = 0.12$, its unconditional mean is set equal to the unconditional mean of consumption growth. These values are in line with the literature, see Drechsler (2013), Wachter (2013), and Bansal and Yaron (2004), among others.

The coefficient of relative risk aversion is set at a low value, $\gamma = 2$. Following the long-run risk literature, we assume that the EIS parameter is greater than 1, and $\psi = 1.5$. As shown by Bansal and Yaron (2004), $\psi < 1$ leads to the counterfactual implication that the price-dividend ratio increases in face of higher uncertainty. Low values of $\psi$ may imply excessive volatility in the risk-free rate. In addition, empirical estimates also suggest that $\psi > 1$, see, for example, Bansal
et al. (2007) and Bansal, Kiku, and Yaron (2016) and Schorfheide, Song, and Yaron (2018). Finally, the subjective discount factor is set at $\beta = 0.975$ to match the mean risk-free rate in the data.

We use data on asset returns for the period 1941—2015 in our calibration analysis. These financial returns data are drawn from Wharton Research Data Services (WRDS). The nominal one-year Treasury Bill rates and annual inflation rates are taken from the WRDS Treasury and Inflation database. The nominal yields are then deflated using inflation rates to deliver the proxy for the risk-free rate. For real returns on stocks, we take annual value-weighted returns including dividends (VWRETD) on the stock portfolio of the NYSE/AMEX/NASDAQ from the Center for Research in Security Prices (CRSP) and deflate nominal returns using inflation rates. Data on the price-dividend ratio are obtained from value-weighted returns including and excluding dividends (VWRETD and VWRETX).

Additionally, we use index options data for the period 1996—2015 to estimate conditional variance under the risk-neutral measure and then compute the variance risk premium. We calibrate models at an annual frequency. We present empirical results for two scenarios. The first scenario concerns the full sample 1941—2015, during which the second World War and the recent financial crisis have been widely viewed as disasters. The second scenario corresponds to the subsample period 1949—2007, during which no disasters ever realized. This case provides a natural benchmark for calibrating the peso version of asset pricing models.

We examine several calibrated models. The first model is the model with ambiguity, learning and Markov-switching mean and volatility of consumption growth, abbreviated as “SV”. We calibrate this model to generate a sufficiently high equity premium, which pins down the ambiguity aversion parameter $\eta$ at $\eta = 4.5$. This parameter value is only half of that adopted by JM and thus implies very modest degree of uncertainty aversion. The second model is the peso version of SV that assumes no disaster realizations in model simulation. This model is abbreviated as $\hat{SV}$. For comparison, we suppress volatility risk but keep everything else the same as in model SV. That is, we examine JM’s model with our estimates of parameters in the Markov-switching model presented in Panel A of Table 1. We name this model “JM1”. A comparison of calibration results between JM1 and SV models is helpful to isolate the impact of time-varying volatility on equilibrium asset returns. Another calibrated model is “JM2”, which is otherwise model JM1 except that the
ambiguity aversion parameter is set at \( \eta = 10 \). Similarly, the peso version of JM2 is denoted as \( \hat{JM}2 \).

[Insert Table 1 about here]

**Quantitative results**

Table 2 reports calibration results for different models discussed above. We consider several unconditional moments of asset returns in our analysis. These moments are 1) mean and volatility of risk-free rates, 2) mean and volatility of excess returns, 3) mean and volatility of VRP, and 4) the market price of risk, defined as the volatility of the SDF scaled by its mean. Important stylized facts observed in the data include 1) high equity premium and excess volatility of returns, 2) low and smooth risk-free rates, and 3) high and volatile VRP. These features of data are evident in Table 2.

We compute unconditional moments from conditional moments simulated under each model. Table 2 shows that model SV generates low and smooth risk-free rates but high and volatile equity premium. The mean equity premium for this model is 6.23 percent per year, and equity volatility is 18.47 percent per year, both of which match the data closely. The model also implies a high market price of risk, \( \sigma(M)/E(M) = 1.01 \). Nevertheless, mean and volatility of VRP implied by the model are still not comparable with the data. The success of the model in reproducing important characteristics of asset returns in the data stems from uncertainty aversion and more importantly, the increased persistence of the disastrous regime due to inclusion of time-varying volatility in the empirical estimation. Both features increases the volatility of the SDF and thus the market price of risk. Assuming that the disastrous state never realized in simulations, model \( \hat{SV} \) implies slightly higher means of equity premium and the risk-free rate. Moments of returns for model \( \hat{SV} \) are also reasonably close to those for the subsample 1949—2007, in which the economy does not experience severe shocks.

The channel of time-varying consumption volatility is shut down in model JM1. Without volatility risk, empirical estimation of the Markov-switching model yields moderate persistence of the disastrous regime. As a result, model JM1 produces mean equity premium close to 0 as well as significantly lower volatility of excess returns compared to that in the SV model. The market price
of risk in this model is also very small. A comparison between JM1 and SV models suggests that accounting for time-varying volatility in consumption dynamics is important for studying asset prices in models with ambiguity. Incorporating volatility risk in empirical estimation of consumption dynamics generates important asset pricing implications. The model-implied pessimism greatly increases responses of equilibrium asset prices and the SDF to changes in the state of the economy. This key mechanism generates high equity risk premium and excess volatility of returns. Assuming $\eta = 10$, the calibrated model JM2 can better match mean equity premium and volatility of returns. The implied market price of risk is close to that in the SV model. Model $\hat{\text{JM2}}$, which is the peso version of model JM2, has similar results. Compared to the SV model, the JM2 model produces higher mean VRP. This result suggests that uncertainty aversion may be important to explain high VRP in the data. Nevertheless, all these models cannot explain high volatility of VRP observed in the data.

[Insert Table 2 about here]

We run predictive regressions to examine predictability of returns and consumption growth. We follow the literature in using the price-dividend ratio as the predictor. In the asset pricing literature, it is challenging for the long-run risks model to generate predictability of returns without generating predictability of consumption growth (see Beeler and Campbell (2012)). For each model of our interest, we simulate 20,000 samples of data and run predictive regressions with different horizons of returns and consumption growth rates. Tables 3 and 4 report results for different models from averaging over simulated samples.

Table 3 presents results for the subsample 1949—2007 and accordingly for the two models, $\hat{\text{SV}}$ and $\hat{\text{JM2}}$, in which disaster realizations are suppressed. As a result, the model environment is consistent with the chosen subsample period. Panel A of Tables 3 shows the empirical evidence that returns are predictable by the $P/D$ ratio but consumption growth rates are not. Results for the $\hat{\text{SV}}$ and $\hat{\text{JM2}}$ models are summarized, respectively, in Panels B and C. The $\hat{\text{SV}}$ model can successfully reproduce predictability of returns and non-predictability of consumption growth. The finding that both equity returns and excess returns are predictable by the $P/D$ ratio indicates that the predictability does not come from variations in the risk-free rate. Consistent with the empirical evidence, the current $P/D$ ratio negatively forecasts future returns. The average $R^2$'s
over simulations for the $\hat{SV}$ model are sufficiently high to imply predictability of returns at different horizons. However, the simulated $R^2$'s for the $\hat{JM2}$ model are too low to deliver significant predictability of returns. The success of the SV model in producing predictability results stems from the model implication that persistent economic regimes in the SV model boosts persistence of the $P/D$. Thus, a significant fraction of future returns can be explained by the current $P/D$ ratio. Remarkably, in predictive regressions for consumption growth, the $\hat{SV}$ model generates $R^2$'s close to 0, a result consistent with the empirical evidence. The result for the $\hat{JM2}$ model is similar. The absence of predictability of consumption growth in the peso environment is mainly because variations in consumption growth are driven by i.i.d. innovation shocks.

[Insert Table 3 about here]

Predictability results for the sample period 1941—2015 are summarized in Table 4. Simulation results of models SV and JM2 are also presented in the table. Empirical results indicate that returns are still predictable for this period, and moreover, long horizon consumption growth rates exhibit certain predictability. In particular, high $P/D$ ratios forecast low future consumption growth at long horizons. Model simulation results suggest that without controlling for the peso property of models, disaster realizations dampen predictability of returns substantially. In predictive regressions for returns and at different horizons of returns, the averages of simulated $R^2$'s are small for both SV and JM2 models. The sign of slope coefficients in predictive regressions with equity returns being the dependent variable is even positive and contradictory to the empirical evidence. Another counterfactual implied by both models is that consumption growth is significantly predictable by the $P/D$ ratio. Simulations from the SV model generate average $R^2$'s of over 0.5 in predictive regressions for consumption growth rates at different horizons, whereas the average $R^2$'s obtained for the JM2 model are much lower. Furthermore, both models imply that the current $P/D$ ratio positively forecasts future consumption growth, which is inconsistent with the empirical finding.

[Insert Table 4 about here]

**Impulse responses and conditional moments**

To better understand the mechanism of the SV model, we do impulse responses analyses for the
SV, JM1 and JM2 models. We investigate impacts of changes in beliefs on the SDF, $P/D$ ratio, conditional equity premium and volatility. We assume that the economy initially remains in the normal regime without innovation shocks. In the third period, consumption growth falls in the disastrous state, and following the regime change, consumption growth evolves according to the respective Markov-switching process specified in each model. Given simulated consumption growth rates and regimes, we compute Bayesian beliefs according to the belief updating equation (3) and distorted beliefs as specified in (5). We then compute responses of variables of interest to changes in simulated states and beliefs. The average responses across simulations for different models are plotted in Figure 1.

When the economy experiences a regime change, the Bayesian belief deteriorates by a large amount in the SV model while only by a small amount in the JM1 and JM2 models without volatility risk. In addition, the belief dynamics after the regime change exhibits much more persistence in the SV model than in the other two models. This high degree of persistence in the belief changes shares important characteristics with the long-run risk component in long-run risk models. The plot illustrates remarkable effects of time-varying volatility on the posterior beliefs. The plot of distorted beliefs further shows that uncertainty aversion, coupled with volatility risk, imputes time-varying pessimism to a great extent by “distorting” the Bayesian belief toward the disastrous state. Because of a higher degree of uncertainty aversion assumed in the calibration, the JM2 model implies a more pessimistically distorted belief than JM1 does.

The remaining plots in Figure 1 indicate significant responses of financial variables in the SV model. Due to time-varying pessimism, the SDF rises sharply in response to the regime change while the $P/D$ ratio displays persistent movements after the drop upon the regime shift. Both conditional equity premium and volatility increase as a consequence of the co-movement between the SDF and $P/D$ ratio in opposite directions. Among the three models examined in the exercise, the JM1 model leads to insignificant responses of key variables of interest because of modest degree of uncertainty aversion and the absence of time-varying volatility.

[Insert Figure 1 about here]

Figure 2 plots conditional risk-free rate, market price of risk, equity premium and equity volatility as functions of the state variable $\pi_t$ for the SV, JM1 and JM2 models. Panel A shows that
compared with the other two models, the SV model implies lower risk-free rates. This is because the concern about time-varying volatility and high persistence of the disastrous regime raise the value of the one-year bond and thus reduces its rate of return. It is interesting to note that conditional risk-free rates display strikingly skewed U-shape. Among the three models, the SV model generates the highest conditional market price of risk, equity premium and equity volatility for all possible values of the state variable $\pi_t$, suggesting the importance of time-varying consumption volatility. Similar to JM's results, these conditional moments display a humped shape. The maxima of conditional market price of risk and equity premium are obtained for values of $\pi_t$ close to its steady-state level. The intuition for these findings is as follows. Suppose that the economy is experiencing a severe shock after having the good regime for certain periods. The posterior belief $\pi_t$ will fall and move away from its steady-state level. The strengthened state uncertainty decreases the risk-free rate but increases the price of risk and thus equity risk premium because uncertainty strongly affects the agent's perception about future consumption growth and cash flow. This effect is indeed amplified in the model with time-varying consumption volatility. Thus, similar to JM's results, the SV model also implies countercyclical price of risk, equity premium and equity volatility.

[Insert Figure 2 about here]

Figure 3 plots Bayesian and distorted beliefs, conditional equity premium and the SDF from a real-time simulation exercise. We only consider the SV model in this exercise. The grey bars in the plots indicate NBER recession periods. We observe that time-varying pessimism caused by uncertainty aversion has led several falls in the distorted belief, i.e., the agent's subjective belief about the disastrous state increases in bad times. These occasions correspond to a few historical contractions in the U.S. The most significant increase in the perceived probability of the disastrous state has occurred in the 2008 crisis. The perceived probability of the disastrous state, after accounting for time-varying pessimism, has increased to almost 0.6. The enhanced state uncertainty generates remarkable spikes in the SDF and conditional equity premium, as shown in Panels B and C, Figure 3.

[Insert Figure 3 about here]
4 Conclusion

We have examined a consumption-based representative-agent asset pricing model with generalized recursive smooth ambiguity preferences and time-varying volatility. The model is an extension of Ju and Miao (2012) by introducing volatility risk. We estimate the Markov-switching model with regime-dependent volatility using consumption data since the Great Depression period. Empirical results suggest two distinct economic regimes, a normal regime and a disastrous regime. Unlike the Markov-switching model with constant volatility, the model with time-varying volatility implies high persistence of the disastrous regime and leads to important asset pricing implications. We calibrate the asset pricing model with volatility risk to data on asset returns and compare its performance with the alternative model assuming constant volatility. The model with time-varying volatility generates a large equity risk premium. When we impose the peso condition that no consumption disasters ever realized in simulated samples, the model with time-varying volatility can reproduce predictability of returns and non-predictability of consumption growth simultaneously, consistent with empirical findings.
Table 1: **Parameter Estimates of Markov-switching Models**

Panel A: Constant volatility

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_l$</td>
<td>-0.045</td>
<td>0.007</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>0.025</td>
<td>0.002</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.016</td>
<td>0.002</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>0.59</td>
<td>0.210</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.97</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Likelihood function value: 293.78

Panel B: Regime-switching volatility

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_l$</td>
<td>-0.045</td>
<td>0.011</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>0.021</td>
<td>0.002</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>0.015</td>
<td>0.002</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>0.021</td>
<td>0.011</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>0.943</td>
<td>0.107</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.989</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Likelihood function value: 308.17

This table reports the maximum likelihood estimates of parameters in Markov-switching models with and without regime-switching volatility. Data for estimation are annual per capita consumption growth rates from 1930 to 2015. Panel A presents parameter estimates of a two-regime Markov-switching model with constant volatility. Panel B presents parameter estimates of a model with regime-switching volatility. Standard errors are reported in parentheses. The values of the likelihood functions for both models are also reported.
This table reports calibration results of several models. All models are simulated at an annual frequency. The moments $\text{E}(r_f)$, $\sigma(r_f)$, $\text{E}(r - r_f)$ and $\sigma(r - r_f)$ are expressed in percentage terms. $r$ is the log return and $r_f$ is the log risk-free rate. The mean and standard deviation of variance risk premium, $\text{E}(VRP)$ and $\sigma(\text{VRP})$, are rescaled to a monthly frequency by multiplying $10^4/12$.

<table>
<thead>
<tr>
<th></th>
<th>U.S. data 1941-2015</th>
<th>U.S. data 1949–2007</th>
<th>$\hat{SV}$</th>
<th>SV</th>
<th>JM1</th>
<th>$\hat{JM2}$</th>
<th>JM2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{E}(r_f)$ (%)</td>
<td>1.41</td>
<td>2.36</td>
<td>1.69</td>
<td>1.22</td>
<td>3.14</td>
<td>2.23</td>
<td>2.01</td>
</tr>
<tr>
<td>$\sigma(r_f)$ (%)</td>
<td>2.82</td>
<td>2.11</td>
<td>0.88</td>
<td>1.39</td>
<td>0.81</td>
<td>0.30</td>
<td>0.93</td>
</tr>
<tr>
<td>$\text{E}(r - r_f)$ (%)</td>
<td>5.32</td>
<td>5.18</td>
<td>6.54</td>
<td>6.23</td>
<td>0.81</td>
<td>5.26</td>
<td>5.87</td>
</tr>
<tr>
<td>$\sigma(r - r_f)$ (%)</td>
<td>17.77</td>
<td>16.86</td>
<td>17.17</td>
<td>18.47</td>
<td>14.87</td>
<td>15.04</td>
<td>16.28</td>
</tr>
<tr>
<td>$\text{E}(VRP)$</td>
<td>11.07</td>
<td>11.13</td>
<td>3.93</td>
<td>2.89</td>
<td>1.16</td>
<td>6.91</td>
<td>7.45</td>
</tr>
<tr>
<td>$\sigma(\text{VRP})$</td>
<td>24.94</td>
<td>12.75</td>
<td>0.92</td>
<td>2.72</td>
<td>1.08</td>
<td>1.28</td>
<td>2.50</td>
</tr>
<tr>
<td>$\sigma(M)/\text{E}(M)$</td>
<td>n.a.</td>
<td>n.a.</td>
<td>1.09</td>
<td>1.01</td>
<td>0.18</td>
<td>0.89</td>
<td>0.89</td>
</tr>
</tbody>
</table>
Table 3: Predictability results: models $\hat{SV}$ and $\hat{JM2}$

<table>
<thead>
<tr>
<th>Horizon (yr)</th>
<th>Excess return</th>
<th>Equity return</th>
<th>Consumption growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope $R^2$</td>
<td>Slope $R^2$</td>
<td>Slope $R^2$</td>
</tr>
<tr>
<td>Panel A: Data (1949-2007)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.1076 0.0730</td>
<td>-0.1054 0.0724</td>
<td>0.0011 0.0018</td>
</tr>
<tr>
<td>2</td>
<td>-0.1841 0.1068</td>
<td>-0.1824 0.1080</td>
<td>0.0001 0.0000</td>
</tr>
<tr>
<td>3</td>
<td>-0.2427 0.1409</td>
<td>-0.2466 0.1475</td>
<td>0.0001 0.0000</td>
</tr>
<tr>
<td>4</td>
<td>-0.3303 0.2026</td>
<td>-0.3412 0.2151</td>
<td>0.0012 0.0004</td>
</tr>
</tbody>
</table>

Panel B: $\hat{SV}$

| 1            | -1.0859 0.1341 | -0.9137 0.1016 | -0.0031 0.0024      |
| 2            | -1.2041 0.1020 | -1.0114 0.0758 | -0.0033 0.0025      |
| 3            | -1.2131 0.0770 | -1.0185 0.0568 | -0.0051 0.0026      |
| 4            | -1.2124 0.0618 | -1.0181 0.0456 | -0.0030 0.0025      |

Panel C: $\hat{JM2}$

| 1            | -1.1611 0.0313 | -0.9975 0.0244 | -0.0031 0.0024      |
| 2            | -1.2037 0.0189 | -1.0370 0.0149 | -0.0032 0.0025      |
| 3            | -1.2176 0.0143 | -1.0513 0.0114 | -0.0052 0.0026      |
| 4            | -1.2223 0.0114 | -1.0566 0.0092 | -0.0035 0.0026      |

This table reports predictive regression results for the subsample 1949—2007 and two models, $\hat{SV}$ and $\hat{JM2}$. The dependent variables include the excess return, equity return and consumption growth rate, all in log terms. The explanatory variable is the log price-dividend ratio. The results are based on averaging over 20,000 simulated samples for each model.
Table 4: Predictability results: models SV and JM2

<table>
<thead>
<tr>
<th>Horizon (yr)</th>
<th>Excess return</th>
<th>Equity return</th>
<th>Consumption growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope</td>
<td>$R^2$</td>
<td>Slope</td>
</tr>
<tr>
<td>Panel A: Data (1941-2015)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.1008</td>
<td>0.0775</td>
<td>-0.0914</td>
</tr>
<tr>
<td>2</td>
<td>-0.1739</td>
<td>0.1192</td>
<td>-0.1610</td>
</tr>
<tr>
<td>3</td>
<td>-0.2276</td>
<td>0.1604</td>
<td>-0.2159</td>
</tr>
<tr>
<td>4</td>
<td>-0.2771</td>
<td>0.1913</td>
<td>-0.2682</td>
</tr>
<tr>
<td>Panel B: SV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.0314</td>
<td>0.0105</td>
<td>0.0078</td>
</tr>
<tr>
<td>2</td>
<td>-0.0321</td>
<td>0.0160</td>
<td>0.0392</td>
</tr>
<tr>
<td>3</td>
<td>-0.0292</td>
<td>0.0207</td>
<td>0.0710</td>
</tr>
<tr>
<td>4</td>
<td>-0.0251</td>
<td>0.0248</td>
<td>0.1014</td>
</tr>
<tr>
<td>Panel C: JM2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.3563</td>
<td>0.0361</td>
<td>-0.2456</td>
</tr>
<tr>
<td>2</td>
<td>-0.5378</td>
<td>0.0459</td>
<td>-0.3704</td>
</tr>
<tr>
<td>3</td>
<td>-0.6349</td>
<td>0.0469</td>
<td>-0.4377</td>
</tr>
<tr>
<td>4</td>
<td>-0.6846</td>
<td>0.0442</td>
<td>-0.4720</td>
</tr>
</tbody>
</table>

This table reports predictive regression results for the subsample 1949—2007 and two models, SV and JM2. The dependent variables include the excess return, equity return and consumption growth rate, all in log terms. The explanatory variable is the log price-dividend ratio. The results are based on averaging over 20,000 simulated samples for each model.
This figure plots the mean impulse response functions for the SV, JM1 and JM2 models when the state of the economy shifts from the normal regime to the disastrous regime.
Figure 2: Conditional moments

This figure plots conditional moments for the SV, JM1 and JM2 models.
Figure 3: Belief, equity premium and SDF

Panel A: Bayesian and Distorted beliefs

Panel B: Conditional equity premium

Panel C: Stochastic discount factor

This figure plots the Bayesian and distorted beliefs, conditional equity premium and SDF for the SV model, simulated based on the historical consumption data in the U.S.
References


