Multi-objective optimization approaches to efficiency assessment and target setting for bank branches

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Abstract

This thesis focuses on combining data envelopment analysis (DEA) and multi-objective linear programming (MOLP) methods to set targets by referencing peers’ performances and decision-makers’ (DMs) preferences. A large number of past papers have proven the importance of a company having a target; however, obtaining a feasible but challenging target has always been a difficult topic for companies. Since DEA was proposed in 1978, it has become one of the most popular performance assessment tools. The performance possibility set and efficient frontier established by DEA provide solid and scientific reference information for managers to evaluate an individual’s efficiency. Based on the successful experience of DEA in performance assessment, many scholars have mentioned that DEA can be used to set appropriate targets as well; however, traditional DEA models do not include DMs’ preferences – information that is crucial to a target-setting process. Therefore, several MOLP methods have been introduced to include DMs’ preferences in the target-setting process based on the DEA efficient frontier and performance possibility set. The trade-off-based method is one of the most popular interactive methods that have been incorporated with DEA. However, there are several gaps in the current research: (1) the trade-off-based method could take so many interactions that no DMs could finish the interactive process; (2) DMs might find it very difficult to provide the preference information required by MOLP models; and (3) DMs cannot have an intuitive view in terms of the efficient frontier.

Regarding the gaps above, this thesis proposes three new trade-off-based interactive target-setting models based on the DEA performance possibility set and efficient frontier to improve DMs’ experience when setting targets. The three models can work independently or can be combined during the decision-making process. The “piecewise
linear model” uses a piecewise linear assumption to simulate DMs’ real utility function. It gradually narrows down the region that could contain DMs’ most-preferred solution (MPS) until it reaches an acceptable range. This model could help DMs who have limited time for interaction but want to have a global view of the entire efficient frontier. This model has also been proven very helpful when DMs are not sensitive to close efficient solutions. The “prioritized trade-off model” provides a new way for a DM to know about the efficient frontier, which allows the DM to explore the efficient frontier following the preferred direction with a series of trade-off tables and trade-off figures as visual aids. The “stepwise trade-off model” focuses on situations where the number of objectives (outputs/inputs for the DEA model) is quite large and DMs cannot provide all indifference trade-offs between all the objectives simultaneously. To release the DMs’ burden, the stepwise model starts from two objectives and gradually includes new objectives in the decision-making process, with the assumption that the indifference trade-offs between previous objectives are fixed, until all objectives are included.

All three models have been validated through numerical examples and case studies of a Chinese state-owned bank to help DMs to explore their MPS in the DEA production possibility set.
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1. Introduction

1.1 Research rationale

The rationale of this thesis developed from an internship in 2008. At that time, I was an intern at a state-owned bank in China, known as one of the “big four” in China. In a south China city, the investigated bank possessed more than 100 secondary branches. The resources consumed by the branches differ, including employees, office space, location, rent, number of ATMs, overheads, etc. The products of each branch can generally be classified into five categories: personal deposits, personal loans, corporate deposits, corporate loans, and intermediary business.

Like most banks in the world, the investigated bank adopts a classical multi-hierarchy structure. In the investigated city, all secondary branches are managed by a three-level hierarchy: city branch, primary branches, and secondary branches. At the beginning of a year, each secondary branch will establish initial targets for each product. At the end of the year, branches’ performances will be measured by the degree to which they accomplished their initial targets.

However, this performance assessment method has received a lot of complaints from the secondary branches regarding two aspects: 1) there is not an objective and scientific way to guide the target-setting process, so a number of secondary branch managers have complained that their initial targets are not feasible; and 2) the final performance assessment score is obtained by the weighted sum of the completed rates of all productions; however, there is no scientific and objective way to decide an appropriate weight for each objective.
The main purpose of this thesis is to provide an objective and scientific way to help the branches to set feasible but advanced targets, through data envelopment analysis (DEA) and multi-objective linear programming (MOLP).

DEA is a non-parametric technique to measure an individual’s efficiency by referencing their peers’ performances. The benefit of DEA is that it does not require prior knowledge of the weight of each criterion when assessing an individual’s efficiency. By referencing an individual’s efficiency, DEA can build up a performance possibility set (PPS) or feasible region. All performances inside the PPS are regarded as achievable. The set of best performances of the PPS is called an efficient frontier. All performances on the efficient frontier are efficient and could be selected as targets for inefficient branches.

Although DEA, the PPS, and the efficient frontier build a way for decision-makers (DMs) to set targets, DMs still need to identify their most-preferred solution (MPS) among all efficient solutions. The classical DEA models do not include DMs’ preference information, so it is necessary to incorporate MOLP models with DEA to include DMs’ preferences in the target-setting process.

A single-objective optimization model minimizes/maximizes one objective function though optimizing a set of decision variables. Practically, DMs may want to maximize/minimize more than one objective at the same time, which is defined as a multi-objective optimization (MOO) problem. Different from single-objective optimization problems, the solution of MOO problems is normally an efficient solution set instead of one dominant solution. Extra techniques are required to collect DMs’ preference information and further select the MPS or the best balance among all efficient solutions. Different MOO techniques have been developed to find the best balance between all the objectives. DEA is actually a special case of MOO with only linear objective functions and constraints, which is also named MOLP. The efficient frontier identified by DEA models
is actually the PPS of the corresponding MOLP problem. Therefore, a number of MOLP models could be utilized to include DMs’ preferences and identify the best balance of the DEA efficient frontier.

After reviewing the existing MOLP models (Section 3.3), trade-off-based interactive methods are selected to be included in the target-setting process. However, there are still gaps between the existing trade-off-based methods and the practical requirements. (1) Although the convergence of most trade-off models can be proven, the interaction times could be so large that no DMs could actually finish the interactive process. (2) DMs might find it very hard to provide the marginal substitutions of all objectives simultaneously, especially when the number of objectives is quite large. (3) The classical trade-off-based models do not provide DMs with enough information to get an image of the real shape of the efficient frontier, which might have a negative effect on the accuracy of DMs’ feedback.

In order to solve these research gaps, this paper proposes three independent models to help DMs to explore the MPS, based on the DEA efficient frontier.

1.2 Research questions

The research questions of this paper can be summarized as follows:

1) Is it possible to assess secondary branches’ performances scientifically and objectively?

2) Is it possible to include DMs’ preference information in the target-setting process, based on the DEA PPS and the DEA efficient frontier?

The target-setting process should satisfy the following requirements. Firstly, the target-setting process should be able to help DMs to identify their MPS among all efficient solutions. Secondly, the interaction should not be too time-consuming, and DMs should be
able to provide the preference information required by the target-setting process. Finally, DMs should be able to check whether their MPS has been achieved or not.

1.3 Contributions of the thesis

The contributions of each model are listed as follows:

- **Contribution 1:** Although DEA possibility set provides a solid reference method for DMs to plan the target for the future, it does not give DMs enough space to express their own preference towards the efficient frontier and explore their most preferred target. The proposed piecewise linear model (Chapter 4) provides a new way for the DMs to explore the efficient frontier and locate the most preferred target within feasible interaction times between model and DMs. Besides, the model also improves the discrimination of close solutions so that DMs can express their preferences more precisely.

- **Contribution 2:** The prioritized trade-off model (Chapter 5) provides a new way for DMs to explore the efficient frontier in the direction they prefer, with a two-dimensional graph-based visual aid to help them to observe the changes between objectives.

- **Contribution 3:** The stepwise model (Chapter 6) releases the DMs’ burden when the number of objectives is quite large. DMs do not need to clarify their trade-off information of all objectives simultaneously. Instead, starting from two objectives, DMs only need to clarify the indifference trade-off between new objectives and the previous ones. In this way, the DMs’ burden of providing preference information can be released.

- **Contribution 4:** All three models have been applied to specific cases, to help a Chinese state-owned bank to assess its branches’ performances and to explore
the MPS by involving DMs’ preference information (Chapters 4, 5, and 6).

1.4 Structure of the thesis

The structure of this thesis is as follows: Chapter 1 (Introduction) briefly introduces this thesis’s rationale, research questions, and contributions. Chapter 2 explains the research methodology adopted by this thesis. Chapter 3 reviews previous literature on DEA and MOLP and clarifies the research gaps between existing models and the practical requirements. Chapters 4, 5, and 6 introduce three trade-off-based interactive MOLP models for target setting, filling the gaps identified in Chapter 3. Finally, Chapter 7 concludes the thesis and provides potential future research directions.
2. Research methodology

2.1 Type of research

This research study followed an inductive process. It was observed from the bank that there was a performance assessment and performance-planning requirement from a state-owned bank’s branches. The researcher collected data from the bank and evaluated the efficiency of each branch. After comparing all existing performance assessment methods, the researcher chose DEA as the performance assessment technique. Although DEA has been proven an effective performance assessment tool, DEA itself cannot satisfy the requirement for performance planning. Most of the DEA models make a strong prior assumption in terms of DMs’ preference structure (for example CCR output oriented model assume that DM wish to increase all outputs with the same proportion), or assume that DMs can specifically express their preference in a certain type of numeric way. However, it is not necessarily true in some real cases. It is sometimes essential for a DM to explore the feasible targets and make comparisons before he/she can make the final decision about the future target. Therefore, the technique of MOLP is used to help to plan an advanced but challenging target in the PPS of DEA. However, during the application process, gaps are identified between existing MOLP models and the practical requirements, which make it difficult for the existing methods to satisfy DMs’ requirements. Therefore, this thesis proposes three models to help DMs to plan targets. These three models can either work independently or cooperate with each other to improve DMs’ experience when setting a target. All three models can be generalized and applied in target-planning cases for all businesses with a branch system.

This research was quantitative and was based on the performance data collected from the investigated bank. The specific data collection methods are described in the following
section and all data are cross-sectional. The analysis technique used quantitative methods as well, including DEA, MOLP, statistical approaches, and other quantitative approaches.

2.2 Data collection

The data collection of this thesis consisted of two parts. The first part was the performance data of the bank’s branches, which were collected from the accounting department of the investigated bank, using unobtrusive methods. The second part was the preference information data, collected from different levels of DMs.

2.2.1 Unobtrusive methods using existing data

Branches’ performance data were collected from the accounting department of a state-owned bank’s central database, established in 2006. Before 2006, city head office knew very little about the detailed performances of its branches. Over 100 types of financial products or services were provided by branches, but at the end of every season, branches only reported to head office several summary indices, such as profits and costs. Because there was no database to store data, some branches did not have the electronic version of their detailed performance data. Insufficient performance information made it very difficult for head office to supervise its branches. Corresponding with this requirement, at the beginning of 2006, the bank established its first information platform. Every operation terminal is connected to this information platform via the bank’s local area network. The system then uploads every transaction’s data to the central database, which is supervised by head office’s accounting department.

Generally, four types of production data are stored by head office’s accounting department. They are personal deposits, personal loans, corporate deposits, and corporate loans. For example, the bank set up the criterion “personal deposit” to represent branches’
deposit volume and financial products with similar characteristics for personal customers. At the end of 2013, the bank created the new criterion “intermediary business volume” to present the bank’s non-deposit and non-loan business, like financial products or services, including investment and financing products, electronic banking business, etc.

On the resource side, the data are collected from the network construction department. When a branch decides to open a new secondary branch, it needs to get approval from the network construction department. Therefore, the network construction department stores detailed information, like staff numbers and overheads.

According to the cooperation agreement with the bank, the researcher had access to all productivity data in terms of personal deposits, personal loans, corporate deposits, corporate loans, and intermediary businesses for all secondary branches in 2015. Besides, the researcher was also able to collect resource data regarding staff numbers and overheads.

2.2.2 Survey of DMs

The second type of data was DMs’ preference information, in terms of target setting, which was collected using face-to-face interviews with the banks’ DMs. DMs from four levels were interviewed, including the vice president of a city branch, the managers of the accounting departments, the managers of the primary branches, and the managers of the secondary branches.
3. Literature review

3.1 Performance assessment and target setting in banks

3.1.1 State-owned banks in China and performance assessment reformation

It is widely accepted that low productivity and low efficiency are common characteristics of Chinese state-owned banks (Yao et al., 2007, Zhang et al., 2009, Xia and Tian, 2012, Matthews and Zhang, 2010, Leung and Young, 2005, Foo and Witkowska, 2014). For example, Yao et al. (2007) compared 22 banks in China, using panel data from 1995 to 2001. The results show that the efficiency of state-owned banks was 8–18% lower than that of other types of banks in China. The most efficient bank during this period was Minsheng Bank, whose average technical efficiency was 89%. However, the most efficient state-owned bank (Bank of China) only had an average efficiency of 58.03%. Similar results were obtained by Leigh (2006). Therefore, how to assess state-owned banks’ performances and help them to improve is the key to them earning back market share.

3.1.2 Branch-based system

In order to improve branches’ efficiency, it is crucial to understand banks’ branch systems and performance assessment systems. It is common for a commercial bank to adopt a multi-hierarchy branch system. A typical four-level hierarchy branch system is head office, district branch, primary branch, and secondary branch. Figure 3.1 displays their relationships.

Secondary branches, also known as “lattice points”, are the main body of state-owned banks. They are the only hierarchy level that operates business with customers face to face. It is reported by the bank that about 80% of bank employees work in secondary branches.
They directly or indirectly contribute more than 75% of the total profits and account for over 90% of the total operational costs every year. To some extent, the efficiency of secondary branches determines a bank’s overall performance. Paradi et al. (2011) also mentioned that compared to the bank level, branch-level efficiency analysis is more meaningful for a bank. Bank branches’ business can be classified into five main categories: personal deposits, corporate deposits, personal loans, corporate loans, and intermediary business. Each category is supervised by one or several business departments of the primary branches.
Figure 3.1 Structure of a commercial bank based on four-level hierarchy branch system
3.1.3 Performance assessment system

The examined Chinese state-owned bank’s existing performance assessment system for evaluating the productivity and efficiency of secondary branches can be summarized as follows:

Step 1: Business line weights allocation

Firstly, district branches deliver the strategy information from the head office to the primary branches. It includes a summary of previous year’s performance, new policies, development advice, and budgets (resources allocated and targets to achieve).

A typical Chinese commercial bank majorly includes 3 business lines: Personal business, cooperate business, and intermediary business. Personal business includes the businesses that aims to serve the personal customers, for example personal deposit and personal loan. Cooperate business includes the businesses that aims to serve the company customers or organization customers, for example organization deposit and organization loan. Intermediary business is a new emerging type of business. It mainly includes the services charges happens along with different types of financial activities. For example, the administration fee of selling financial products.

Based on this information, primary branches allocate a certain weight to each business line, which represents the relative importance of each line (Figure 3.2).
The challenge of this step is to consider both head office’s and business departments’ opinions. However, it is very common for DMs from different levels to have conflicting opinions. The argument on weights among business departments could last for weeks, until the manager of the primary branches makes the final decision.

**Step 2: Criteria selection**

After being allocated a certain weight, each business department chooses a series of criteria to assess secondary branches’ performance. For example, personal business departments utilize criteria like personal deposit, personal loan, and VIP customer increases. After determining the criteria, business departments distribute information on the weights that are allocated to these specific criteria.

**Step 3: Budgeting**

Budgeting is another important process for performance assessment. Primary branches hope to use different budgets to control secondary branches’ resources and to set achievable but challenging targets. For example, large branches with more-advanced equipment are expected to achieve higher targets than small, poorly equipped branches are.
However, most of these budgets are decided subjectively. They depend on the discussion between secondary branches’ managers and primary branches’ managers. Secondary managers want relatively easy targets and sufficient resources, while primary managers want secondary branches to sell the maximum number of products with the minimum amount of resources consumed. The conflicting orientations make the budgeting process time-consuming and controversial.

**Step 4: Marking**

The results of the performance assessment are calculated by the completion rates of targets, as shown in Equation 3.1.3.

\[
\text{Performance score} = \frac{\text{Completed quantity}}{\text{Target (Budget)}} \times \text{Weight} \quad (3.1.3)
\]

The performance scores of secondary branches are collected by different business departments and then reported to primary branches. Primary branches then use these results to make a performance assessment table (Table 3.1). Primary branches’ accounting departments calculate the weighted sum of all performance scores and obtain the overall performance score.

The disadvantages of the method above cannot be ignored. If a branch has extremely good performance in one or a few criteria, the extreme values will dominate the other performance values. The weights of criteria are decided by the discussion between primary branch managers and secondary branch managers at the beginning of every financial year. During the discussion process, there is neither reference data nor solid reason to support them in making decisions.
Table 3.1 Performance assessment table (example)

<table>
<thead>
<tr>
<th>Business Departments</th>
<th>Weights for business line</th>
<th>Weights for criteria</th>
<th>Criteria</th>
<th>Performance Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary branch</td>
<td>W1</td>
<td>W11</td>
<td>Profit</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>W12</td>
<td>Intermediary business</td>
<td></td>
</tr>
<tr>
<td>Personal business department</td>
<td>W2</td>
<td>W21</td>
<td>Personal deposit</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>W22</td>
<td>Personal loan</td>
<td></td>
</tr>
<tr>
<td>Cooperate business department</td>
<td>W3</td>
<td>W31</td>
<td>Cooperate deposit</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>W32</td>
<td>Cooperate loan</td>
<td></td>
</tr>
<tr>
<td>International business department</td>
<td>W4</td>
<td>W41</td>
<td>Cross-border remittances</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>W42</td>
<td>Foreign exchange</td>
<td></td>
</tr>
</tbody>
</table>

Overall score

3.1.4 Summary of challenges

The challenges and problems can be summarized as follows:

1) Weight setting
   The weights of criteria are assigned subjectively. They depend on the discussion between primary branch managers and secondary branch managers. During the discussion process, there is no reference data or solid reasons to support them in making decisions.

2) Dominating effect
   In the existing performance assessment method (Table 3.1), all secondary branches share the same criteria and weights. As a result, extreme performances in some criteria could dominate the overall performance.

3) Budgeting
   The performance assessment results depend heavily on the budgeting results. However, budgeting is a complicated and controversial process. Most performance values are calculated by the accomplishment rates of targets. If the targets are biased, the performance assessment results will be biased too.

4) Insufficient managerial information
The results of performance assessments are only overall scores and ranks, from which branches cannot know what the specific sources of their inefficiency are. Also, they do not have any chance to compare their performance with that of other branches in detail.

5) **Managers’ conflict opinions**

During the budgeting process, primary branch managers and secondary branch managers often have different opinions. For primary branch managers, secondary branches’ target setting should be based on primary targets and overall profit. However, secondary branches prefer easy targets, and they hope that the target allocation can take their strengths and weaknesses into consideration.

<table>
<thead>
<tr>
<th>Research target</th>
<th>Research questions</th>
</tr>
</thead>
</table>
| 1. Identify inefficient branches | 1. How can a series of input and output criteria be chosen that represent the operational processes of bank branches?  
2. How can we build a model to identify inefficient branches based on the selected input and output criteria? |
| 2. Collect data on managers’ preferences and aggregate them | 1. How can we build a model to capture managers’ preferences?  
2. How can we aggregate managers’ preferences? |
| 3. Include managers’ preferences in the secondary branches’ target-setting process | 3. How can we combine efficiency assessment results and managers’ preferences to make feasible targets for secondary branches? |
3.1.5 Past research on performance assessment

Among the past research about performance assessment, efficiency is always the key concept. The definition of efficiency is as follows:

*A DMU is to be rated as fully (100%) efficient on the basis of available evidence if and only if the performances of other DMUs does not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs* (Cooper et al., 2011).

*For a characterized PPS, the set of efficient performances are defined as efficient frontier.*

*In economic field, if there is a unique output, efficient frontier is also called production function.*

DEA plays a dominant role among all existing performance assessment models by referencing peers’ performances. Among the over 10,000 papers about performance assessment published in recent years, 87% utilized DEA as their main approaches. The success of DEA is because of its three significant advantages. Firstly, DEA is a non-parametric method that does not need to set the arbitrary weights for criteria. Secondly, DEA performance assessment is based on referencing peers’ performances, which
guarantees the fairness of the performance assessment. Thirdly, DEA is a linear model and is more likely to be accepted by DMs because of its simplicity.

Considering all the advantages mentioned above, this thesis adopts DEA as the main methodology to assess the performances of bank branches.

3.2 DEA and DEA in the banking industry

3.2.1 DEA and efficiency evaluation

DEA, as proposed by Charnes et al. (1978), is a non-parametric efficiency evaluation method. One significant advantage of DEA is that no prior knowledge is required in terms of the weights of input and output criteria. The core idea of DEA is to formulate a PPS via referencing the existing decision making units’ (DMUs’) performances. Suppose there are \( n \) DMUs and they all consume \( m \) inputs \( X = (x_1, x_2, ..., x_m) \) and produce \( s \) outputs \( Y = (y_1, y_2, ..., y_s) \). The PPS is defined by:

\[
PPS = \{(X,Y)|X \text{ can produce } Y\} \tag{3.1}
\]

In a DEA model, the shape of the PPS is identified by the return to scale (RTS), which is a concept adopted from the field of economics (Yang et al., 2016). DEA models under a constant-return-to-scale (CRS) assumption, also known as CCR models, accept that outputs will change by the same proportion as the inputs. The corresponding PPS is defined by Charnes et al. (1978) as:

\[
PPS(X,Y) = \{(X,Y)\big|X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, j = 1, ..., n\} \tag{3.2}
\]

Banker et al. (1984) proposed a DEA model under the assumption of a variable return to scale (VRS), known as the BCC model. Under VRS, the PPS is defined as:

\[
PPS(X,Y) = \{(X,Y)\big|X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, ..., n\} \tag{3.2.1c}
\]

A DMU is rated fully (100%) efficient on the basis of available evidence if and only if the performances of other DMUs do not show that some of its inputs or outputs can be
improved without worsening some of its other inputs or outputs (Cooper et al., 2011). For a characterized PPS, the set of efficient performances are defined as the efficient frontier. In the economic field, if there is a unique output, the efficient frontier is also called the production function.

Whether under a CRS or VRS assumption, a DEA model constructs a piecewise linear efficient frontier. A DMU’s efficiency is evaluated by measuring the distance between its position and the corresponding efficient frontier. If the assessed DMU is on the efficient frontier, it is efficient. Otherwise, it is inefficient. Suppose we investigate the minimum inputs without sacrificing any outputs using radial measurement: the DEA input-oriented model under CRS is as follows (Charnes et al., 1978):

$$\begin{align*}
\min_{\lambda_j} \theta_{jo} \\
\text{s. t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^+ = \theta_{jo} x_{ijo} \quad i = 1, \ldots, m ; \\
\quad y_{rjo} - \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^- = 0 \quad r = 1, \ldots, s \\
\quad \lambda_j \geq 0 \text{ for all } j.
\end{align*}$$ (3.3)

Suppose we accept the VRS assumption: the DEA BCC model is as follows (Banker et al., 1984):

$$\begin{align*}
\min_{\lambda_j} \theta_{jo} \\
\text{s. t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^+ = \theta_{jo} x_{ijo} \quad i = 1, \ldots, m ; \\
\quad y_{rjo} - \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^- = 0 \quad r = 1, \ldots, s
\end{align*}$$ (3.4)
\[ \sum_{j=1}^{n} \lambda_j = 1 \]

\[ \lambda_j \geq 0 \text{ for all } j. \]

where \( y_{rj} \) denotes the amount of “\( r \)th output” produced by \( DMU_j \) and \( x_{ij} \) represents the amount of “\( i \)th input” consumed by \( DMU_j \). We define \( \lambda_j \) as the composite variable and \( \theta_{j_0} \) as the efficiency score, with a measurement scale from 0 to 1. If \( \theta_{j_0} \) is equal to 1 and both \( s^+ \) and \( s^- \) are equal to 1, the assessed DMU is efficient. Otherwise, it is inefficient. It should be noted that the only difference between the CCR and BCC models is that the BCC model has an extra constraint: \( \sum_{j=1}^{n} \lambda_j = 1 \).

The idea of DEA can also be illustrated graphically (Figure 3.4). Suppose there are six DMUs: A, B, C, D, E, and F. They all consume two inputs and produce one output. Efficient DMUs A, B, C, D, and E together formulate a piecewise linear efficient frontier. As F is an inefficient DMU, its efficiency score can be obtained by calculating \( OG'/OG \).

![Figure 3.4 Illustration of DEA concept](image-url)
Summary statistics of DEA publications are provided by Yang and Emrouznejad (2018), which is illustrated in Figure 3.5. Since 1978, the number of DEA publications has been increased exponentially, which indicates that DEA is becoming a more and more popular technique.

### 3.2.2 DEA and bank

Since Sherman and Gold (1985b) published the first DEA paper about banks’ performance assessment. After Sherman and Gold’s paper, the development of DEA in banks went through an exploring and learning process. The number of publications in this period was not very large. However, many papers pointed out the future directions of DEA in banks’ performance assessment. Dyson and Thanassoulis (1988) attempted to identify DMs’ preferences and combine these preferences with DEA performance assessment to generate feasible and DM-preferred targets. In addition, this was the first weight-restricted DEA model in the banking field. Elyasiani and Mehdian (1990) and Berg et al. (1992) considered efficiency changes over time and discussed the utilization of Malmquist indices in banks’ efficiency assessment.
The period from 1995 to 1996 was a turning point for DEA research in banks. Thanassouli et al. (1996) and Athanassopoulos and Curram (1996) compared DEA with other popular methodologies in that period, including financial ratios and neural networks. According to their comparison results, they point out that DEA can provide important information from different angles in banks’ performance assessment. In the same year, Miller and Noulas (1996) utilized DEA in banks’ performance assessment with a large sample. They compared the technical efficiency of 201 large banks.

From 1996 to 2003, more researchers started to realize the potential capability of DEA. During this period, there were more than 120 DEA papers published every year (Figure 3.6). Ten per cent of these DEA papers were about banks. One important development in this period was the utilization of multi-stage DEA models (Favero and Papi, 1995). The other new research area was assurance region DEA (AR-DEA) models in banks’ performance assessment. Thompson et al. (1997) utilized the AR-DEA model to assess the efficiencies of 100 US banks, through considering their profit ratios. In this period, some successful large-scale applications with DEA in banking industry has appeared, for example Seiford and Zhu (1999) measured and compared the profitability and marketability of the top 55 US commercial banks. Bhattacharyya et al. (1997) examines the productive...
efficiency of 70 Indian commercial banks during the early stages (1986-1991) of the ongoing period of liberalization. DEA is used to calculate radial technical efficiency scores.

From 2003 to 2010, DEA went through an exponential growth period. The number of publications increased from 13 (2003) to 81 (2010). The new research topics included interactive DEA (Wong et al., 2009, Lozano and Villa, 2009), DEA and linear programming (Van Der Meer et al., 2005), cross-efficiency (Wu et al., 2009, Liang et al., 2008), DEA neural networks (Wu et al., 2006), and so on.

Since 2010, the development of DEA has been fast and stable. Every year, about 80 papers are published about DEA in banks’ performance assessment. How to use DEA efficiency assessment results to set feasible but preferred targets has become a popular topic (Ebrahimnejad and Hosseinzadeh Lotfi, 2012, Yang and Xu, 2014).

3.2.3 DEA at branch level

For banks’ performance assessment, the research entities in DEA papers can be classified into two categories: bank-level research and branch-level research. The advantage of bank-level research is that the data are relatively easy to obtain. However, the disadvantage is that the obtained data are too general to provide specific information to the managers. Branch-level researchers can use more-specific data, including the staff numbers of each branch, the number of ATMs, and the value of deposits. These data are usually internal data, mostly confidential data. Compared to bank-level research, branch-level research is able to provide more-specific guidance and recommendations DMs. Correspondingly, the disadvantages are also obvious; for example, data are not easy to collect, and some data cannot be published. Figure 3.7 illustrates the number of published papers based on branch-level research.
Although with the difficulty of data collection, branch level efficiency assessment can provide more specific and detail managerial information to the bank (Paradi and Schaffnit, 2004). The first DEA paper about banking industry focus on branch level information (Sherman and Gold, 1985a).

### 3.3 Multi-objective methods and target setting

Setting an appropriate target plays an essential role in organizational performance analysis and improvement (Amirteimoori and Kordrostami, 2005, Bi et al., 2011, Lovell and Pastor, 1997). Some researchers even claim that a business can never succeed unless its DMs have a clear picture of the future direction of the business (Atrill and McLaney, 2009). The previous section (3.2) proved that DEA is a valid method to build up a reference set or PPS to help DMs to assess DMUs’ performance. However, classical DEA models do not consider DMs’ preferences, which is a limitation when using DEA in the target-setting process. Therefore, it is necessary to utilize MOO techniques. This section (3.3) aims to review the existing MOLP models and further investigates how MOLP techniques can be used to set targets in the DEA efficient frontier.

As section 3.2 introduced, the beauty of DEA is that it can provide a PPS or possibility region based on existing peers’ performances. However, the efficient frontier formulated by DEA consists of numerous non-dominant efficient solutions, every one of which could
act as an efficient target for the assessed DMU. Therefore, it is necessary to include higher-level information to identify the preferred solution or the best balance among all the efficient solutions.

 Actually, most classical DEA models can be understood as DEA target-setting models without any DMs’ preferences but with different sorts of assumptions about DMs’ preference structures. For example, DEA CCR and BCC models assume that the improvement of DMUs should be a ratio improvement. Similarly, the basic DEA addictive model (ADD) (Charnes et al., 1985), the basic DEA SBM model (Tone, 2001), and the basic DEA Russell model (Färe et al., 1985), all make different kinds of assumptions about DMs’ preference structures. However, in most practical cases, it is rather difficult to validate these assumptions. In order to release these a priori assumptions in terms of DMs’ utility structure, a number of past papers have combined different kinds of MOLP techniques with the DEA PPS to set targets.

 This section (3.3) reviews the existing MOLP methods, trying to identify one that can help to find the MPS in a DEA efficient frontier. Following the knowledge structure proposed by Miettinen et al. (2008), this section illustrates the advantages, disadvantages, and suitable situations of each type of MOLP technique. The research gaps are presented at the end of this section.
Figure 3.8 Knowledge structure of MOLP methods
3.3.1 MOLP without DMs’ preference information

3.3.1.1 Global criterion method

The global criterion method or compromise programming was proposed by Yu (1973) and Zeleny (1973). In a global criterion method, a default reference point is set first. Normally, the ideal point is infeasible. Then, an assumption is made that DMs want to minimize the distance between the reference point and the feasible solutions. The formula of the global criterion method can be displayed as follows:

$$
\text{Min } \left( \sum_{i=1}^{n} |f_i(x) - f_i^*|^{\frac{1}{p}} \right)^{\frac{1}{p}} \\
\text{s.t. } x \in \Omega,
$$

where $f_i^*$ is the value for the $i$th objective of the ideal point $f^*$. It should be noted that when $p \to \infty$, the global criterion method can be rewritten as:

$$
\text{Min } \max_{i=1,\ldots,n} |f_i(x) - f_i^*| \\
\text{s.t. } x \in \Omega,
$$

which is a special case of the following compromise programming model (Zeleny, 1973):

$$
\text{Min } \max_{i=1,\ldots,n} w_i |f_i(x) - f_i^*| \\
\text{s.t. } x \in \Omega,
$$

Yang et al. (2009b) proved that the CCR model and the BCC model are actually special cases of the compromise programming model above, with $w_i$ and $f_i^*$ fixed as follows:

$$
w_i = \frac{1}{y_{ij_0}} \\
f_i^* = F^{max}
$$

where $y_{ij_0}$ is the $i$th outputs for the assessed DMU, and $F^{max}$ is the super-ideal point.
3.3.1.2 Neutral compromise model

Another category of DEA target-setting model without DMs’ preferences is the neutral compromise model. The main idea is to find a target somewhere in the middle of the efficient frontier. The original neutral compromise model was proposed by Wierzbicki (1999) as follows:

\[
\text{Min } \max_{i=1,...,n} w_i \left| \frac{f_i(x) - \left( (f_i^* + f_i^-)/2 \right)}{f_i^- + f_i^+} \right|
\]

s.t. \( x \in \Omega \),

where \( f^* \) and \( f^- \) are the ideal and nadir objective vectors, respectively. The idea of this model is to use the average of the ideal and nadir points as the reference point in the global criterion model and then got a neutral compromise solution in terms of the efficient frontier. The DEA SBM model and some Russell models adopt a similar idea. For example, the SBM model proposed by Tone (2001) tries to find a neutral compromise, with the assumption that set targets should be based on the scale of the assessed DMU’s inputs and outputs.

3.3.1.3 Summary of MOLP models without DMs’ preference information

DEA target-setting models without DMs’ preference information do not need to collect DMs’ preference information during the entire process. However, this type of model makes a very strong assumption that DMs’ preferences or utility structures follow the default utility assumptions. For example, the CCR and BCC models make a ratio improvement assumption, and the SBM model makes a neutral compromise assumption. These assumptions might not stand in practical cases. Therefore, models that include DMs’ preferences are necessary for the target-setting process.
3.3.2 Target-setting models with DMs’ preference information

When solving MOO problems, it is common to assume that DMs have a utility function over the entire efficient frontier, which makes the efficient solutions not equivalent to each other. Unlike non-preference methods with a fixed and universal utility function for all DMs, MOLP models with DMs’ preference information assume that DMs’ utility function is implicit. A procedure that collects DMs’ preferences, formulates DMs’ preferences, and incorporates DMs’ preferences into the efficient frontier to identify the MPS is necessary.

According to when DMs’ preferences are collected (before, after, or in the middle of the target-setting process), the multiple-objective methods can be divided into a priori methods, a posteriori methods, and interactive methods.

3.3.2.1 A priori methods

For a priori methods, the preference information from DMs is collected before generating a solution. The assumption is that DMs can clearly express their views in terms of their preference structures without knowing the specific structure of the efficient frontier. The advantage of a priori methods is that when DMs express their a priori preference information, the target-setting process can be very easy and effective. However, in most cases, it is not necessary for DMs to have a priori knowledge about their utility function. In these cases, DMs may find that the generated solutions are far from what they expected.

A typical example of an a priori method is the value function method. The value function method was proposed by Keeney and Raiffa (1993). The general form is as follows:

\[ \max u(f(x)) \]
\[ s.t. x \in \Omega, \]

where \( u(x) \) is the mathematical form of the DMs’ utility function. For all a priori methods, DMs are assumed to be able to provide the specific equation of \( u(x) \). Unfortunately, most DMs will find it difficult or even impossible to provide this type of information. Some
researchers have even proven that single utility functions cannot capture DMs’ preferences through experiments (deNeufville and McCORD, 1984).

Despite the limitations mentioned above, a priori methods are still some of the most popular methods for MOO problems. One typical example is the weighted sum model:

$$\text{Max } \sum_{i=1}^{n} w_i f_i(x)$$

$$s.t. x \in \Omega,$$

The weighted sum model assumes that DMs’ overall utility function can be calculated by the weighted sum of all objectives. DMs are expected to provide the corresponding weight for each objective. The corresponding DEA target-setting models based on the weighted sum method are the DEA ADD models (Cooper et al., 2007):

$$\text{Max } z = \sum_{i=1}^{n} w^- s_i^- + w^+ s_r^+$$

$$s.t. \sum_{j=1}^{n} x_{ij} \lambda_j + s_i^- = x_{ij0}$$

$$\sum_{j=1}^{n} y_{rj} \lambda_j - s_r^+ = y_{rj0}$$

$$\lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0$$

The DEA ADD model is a non-oriented model, which enables DMs to identify the inefficiency from the input and output sides simultaneously. DMs are expected to provide the weights for each output slack and input surplus, and DMs’ overall utility is assumed to be negatively correlated to the weighted sum of output slacks and input surplus.

Because the nature of DEA ADD models is a weighted sum model of MOO problems, DEA ADD models share the same limitations as weighted sum models. Firstly, it is normally difficult or even impossible for most DMs to express a set of weights that can
express DMs’ preferences globally. Secondly, like all the other weighted sum models, the DEA ADD models can only identify the boundary solutions between two adjacent efficient facets, which will lead to the loss of many potential targets.

### 3.3.2.2 A posteriori methods

Different from a priori methods, a posteriori methods tend to collect DMs’ preference information after generating solutions. A posteriori methods are also called “methods for generating the Pareto efficient frontier”. A common idea for a posteriori methods is to generate a representative sample set of the efficient frontier and to show it to DMs. Then, the DMs’ preferences can be collected by making a selection or comparison among all the generated solutions.

A classic a posteriori method is the method of weighted metrics. In compromise programming (Zeleny, 1973), such a method is as follows:

\[
\min \max_{i=1,\ldots,n} w_i |f_i(x) - f_i^*|
\]

\[s.t. \ x \in \Omega_i\]

When setting \(f_i^*\) as an ideal point, for any positive set of \(w_i\), the compromise programming as above can generate an efficient solution. If a weighted metric containing sufficient and representative weights is input into the compromise programming model, the efficient frontier can be sampled by a set of efficient solutions. Theoretically, a compromise programming model is able to generate any efficient solution on the efficient frontier. Then, DMs can make a comparison or selection among all the generated solutions and identify their MPS.

The advantage of a posteriori methods is that they enable DMs to know about the efficient frontier from the generated sample set or visual aids like graphs and figures. However, its limitations cannot be ignored. Firstly, when the problem is relatively complex, for example the number of inputs and outputs is quite large, it will be computationally expensive to effectively sample the efficient frontier. Secondly, on the one hand, if the
sample set is relatively large, DMs may find it difficult to select among the generated samples. On the other, if the sample set is not large enough, the samples cannot effectively represent the overview of the efficient frontier, and DMs might lose their MPS because of insufficient samples.

3.3.2.3 Interactive methods

As people gradually realized that single interaction is insufficient for DMs to express correct preference information and to learn about the efficient frontier, researchers started to develop more-complex models that enable DMs to have multiple chances to express their preferences in terms of different efficient solutions. These types of models are called “interactive models” or “interactive methods”. The idea is to provide DMs with multiple chances to learn about the efficient frontier and to express or correct their preference information, through several iterations. The procedure of interactive methods can generally be summarized in the following steps:

1. Generate an initial efficient solution through non-preference models, for example the global criterion method.
2. Show DMs the initial solution and collect DMs’ preference information.
3. According to DMs’ feedback, generate new efficient solutions that are preferred by the DMs.
4. Judge whether the MPS has been achieved. If the MPS has been achieved, stop the interaction process. Otherwise, set the new solution as the initial solution and go back to Step 2.

Compared to a priori methods, interactive methods do not require DMs to provide a priori global preference structures. On the one hand, by getting access to different efficient solutions, DMs can gradually learn about the efficient frontier. On the other hand, by collecting more and more preference information, the interactive model can gradually approach the MPS. All in all, the interactive method is a learning process for both DMs and the model itself.
Compared to a posteriori methods, the most significant advantage of interactive methods is the interaction efficiency. For interactive models, only part of the efficient frontier is displayed to DMs for each iteration. After collecting DMs’ preference information, the new part of the efficient frontier will be generated accordingly. In other words, the sample of the efficient frontier is guided by the DMs’ preferences, instead of generating the entire efficient frontier randomly like a posteriori methods do.

Many interactive methods have been proposed according to the specific requirements of different cases. According to the research aims, the existing MOLP models can be classified into two types. One type helps DMs to know about the efficient frontier, with less emphasis on locating the final MPS. A typical example of this type of method is the “Pareto race” method (Korhonen and Wallenius, 1988). The other type puts more emphasis on finding a final MPS. A typical example is the gradient projection method (Yang, 1999). It has been proven in many practical cases that both types of methods are necessary for the target-setting process.

A more popular way to classify the interactive methods is according to the types of information required from DMs. Trade-off-based methods require DMs to provide marginal substitutions, which are used to find a solution preferred by DMs. Reference-point-based methods collect DMs’ preference information in the form of ideal points or “goals”. New solutions can be obtained through adjusting the position of the reference point or the distance measurement. Classification methods allow DMs to determine which objectives should be improved, which can be sacrificed, and which should remain the same. This section (3.3.2.3) specifically illustrates the general ideas and typical models of each type of interactive model, with the advantages and disadvantages.

**Trade-off-based methods**

Intuitively speaking, a trade-off is a type of exchange where a sacrifice must be made to get some benefits. In MOLP problems, making a trade-off means to improve one or some
objectives by sacrificing other objectives. The mathematical form of the trade-off in MOLP can be expressed as follows (Miettinen et al., 2008):

For two solutions \( x_1 \) and \( x_2 \), the corresponding objective vectors are \( f(x_1) \) and \( f(x_2) \). The exchange rate between \( f_i \) and \( f_j \) is \( T_{ij}(x_1, x_2) \), where:

\[
T_{ij}(x_1, x_2) = \frac{f_i(x_1) - f_i(x_2)}{f_j(x_1) - f_j(x_2)}
\]

\( T_{ij}(x_1, x_2) \) is said to be a partial trade-off involving \( f_i \) and \( f_j \) between \( x_1 \) and \( x_2 \) if \( f_l(x_1) = f_l(x_2) \) for all \( l = 1, ..., k \), \( l \neq i, j \). If, on the other hand, there exists an index \( l \in \{1, ..., k\}\backslash\{i, j\} \) such that \( f_l(x_1) \neq f_l(x_2) \), then \( T_{ij}(x_1, x_2) \) is called the total trade-off involving \( f_i \) and \( f_j \) between \( x_1 \) and \( x_2 \).

When moving from one efficient solution to another, a trade-off can show DMs the exchange or change rates between objectives.

Trade-offs can also help to structure DMs’ partial utility function. When collecting DMs’ preference information using interactive methods, a DM can be asked questions like “in order to improve one unit of objective one, how many units of the other objectives would you like to sacrifice?” The answer to this question is called the “indifference trade-off”, and its mathematical form can be illustrated as follows (Miettinen et al., 2008):

For two solutions \( x_1 \) and \( x_2 \), if the corresponding objective vectors \( f(x_1) \) and \( f(x_2) \) are on the same indifference curve or they have the same utility value for DMs, the corresponding trade-off \( T_{ij}(x_1, x_2) \) (whether total or partial) is defined as the indifference trade-off involving \( f_i \) and \( f_j \) between \( x_1 \) and \( x_2 \).

Several indifference-trade-off-based interactive methods (also known as trade-off-based methods) have been proposed over the years. Typical trade-off-based methods include Geoffrion methods (Geoffrion et al., 1972), the ZW method (Zionts and Wallenius, 1976), and the GRIST method (Yang, 1999).
There are several advantages of trade-off-based methods. Firstly, no strong assumptions have to be made in terms of DMs’ utility function. Secondly, the mathematical convergence of most interactive methods can be proven. Some trade-off-based methods, for example the ZW method, can be proven to converge in a finite number of interactions. Finally, DMs can always check whether the MPS has been achieved by checking the optimal condition.

Despite the significant advantages outlined above, the limitations of trade-off-based methods cannot be ignored. (1) Given an initial solution, the indifference trade-off can show the direction to improve DMs’ utility. However, it is very difficult to find an appropriate trade-off step size (Korhonen, 2005). An inappropriate trade-off step will lead to a dramatic increase in interaction times. (2) Although mathematical convergence can be proven for most interactive methods, the convergence speed could be infeasible for real applications (Korhonen, 2005, Hwang and Masud, 1979, Yang, 2002). When the number of objectives is quite large, the interaction times required could be so high that no DMs could actually finish the entire process and find out the MPS. (3) Some DMs may find it very difficult to provide the precise indifference trade-offs between objectives, especially when DMs are insensitive to close solutions (Miettinen et al., 2008). (4) The aim of trade-off-based methods is to identify the MPS. If DMs want to have a general view of the efficient frontier, trade-off-based methods might not be a good choice.

Goal-based methods

Goal-based methods are another popular type of interactive method with a long history. The efficient solutions of goal-based methods are generated by minimizing the distance between a “goal” or “target” (normally infeasible) and the PPS. Then, different interactive approaches are designed to help DMs to adjust freely the goal or the distance measurement so that different efficient solutions will be generated and the DMs can explore the efficient frontier according to their preferences. The procedure of goal-based methods can generally be summarized as follows:

1. DMs provide an initial reference point.
2. The goal-based approach responds to DMs with a corresponding efficient solution, by minimizing or maximizing an achievement function.

3. DMs are free to adjust the goal according to their preferences, with different efficient solutions generated. During the process, DMs can gradually learn about the efficient frontier.

A typical example of a goal-based method is the weighted Tchebycheff method (Steuer, 1986). The classical Tchebycheff method without considering DMs’ preferences can be expressed mathematically as follows:

\[
\min \theta \\
\text{s.t. } \theta \geq f_i^* - f_i(x) \\
x \in \Omega
\]

In order to include DMs’ preferences in the Tchebycheff model, we can add a weight to each objective constraint, where the weights should be determined by the interactive process with DMs. The weighted Tchebycheff model can be expressed as follows:

\[
\min \theta \\
\text{s.t. } \theta \geq w_i(f_i^* - f_i(x)) \\
x \in \Omega
\]

It has been proven that given a positive weight set \(w_i\), the weighted Tchebycheff method can always generate an efficient solution. In addition, for every efficient solution \(f\), we can always find a corresponding weight set for the weighted Tchebycheff model. In this way, DMs can explore the efficient frontier by adjusting the weights in the Tchebycheff model.

The advantages of goal-based methods are significant. If the DMs have clear goals or targets, goal-based interactive methods can help them to explore the closest efficient solutions to their goals effectively. However, for most real cases, DMs do not have clear goals before knowing the efficient frontier and the PPS well. Besides, the stopping criterion
of goal-based methods is only based on DMs’ subjective judgement. There is no objective reference index or criterion to help DMs to judge whether the MPS has been achieved or not.

Classification-based methods

Because all solutions on the efficient frontier are non-dominant solutions, improving some objectives means that other objectives must be sacrificed. The original idea of classification-based methods can be summarized as follows. Given an initial target, which objectives need to be improved and which objectives can be sacrificed should be determined by DMs. Unlike trade-off-based methods, classification-based methods offer a more intuitive way for DMs to search the efficient frontier according to their preferred trade-off direction. The stopping criterion is the same as in goal-based methods: based on DMs’ subjective judgement. Several studies have proven that the classification-based method is a valid way for DMs to explore and search the efficient frontier (Thanassoulis and Dyson, 1992a, Zhu, 1996, Uemura, 1999, Wong et al., 2009).

A typical example of a classification-based method is the step method (STEM) (Benayoun et al., 1971). STEM was proposed to solve MOLP problems, but its extensions have proven to be effective in solving non-linear problems (Sawaragi et al., 1985, Vanderpooten and Vincke, 1989, Eschenauer et al., 1990). Given an initial efficient solution \( f_i(x_0) \), DMs are asked to classify all objectives into two classes: objectives that the DMs want to improve \( (I^+) \) and objectives that could be sacrificed \( (I^-) \), where \( I^+ \cup I^- = I \). DMs are also asked to provide an acceptable boundary \( (\epsilon_i) \) for each objective that could be sacrificed \( (I^-) \). Then, the new formulation is:

\[
\begin{align*}
\text{Min } & \max_{i=1,\ldots,n} w_i |f_i(x) - f_i^*| \\
\text{s.t. } & f_i(x) \geq \epsilon_i \quad \text{for all } \; i \in I^- \\
& f_i(x) \geq f_i(x_0) \quad \text{for all } \; i \in I^+ \\
& x \in \Omega,
\end{align*}
\]
where \( w_i \) is a set of fixed weights. Other MOLP methods without DMs’ preferences could be utilized here to settle \( w_i \).

A similar classification method is called the NIMBUS method (Miettinen and Mäkelä, 1999). Based on STEM, the NIMBUS method further classifies objectives into five classes: objectives that could be decreased \( (I^<) \), objectives that could be decreased until a certain level \( (I^\leq) \), objectives that are currently satisfied \( (I^=) \), objectives that need to be increased \( (I^>) \), objectives that need to be increased until a certain level \( (I^\geq) \), and objectives that could be changed freely, where \( I^< \cup I^\leq \cup I^= \cup I^> \cup I^\geq = I \). The difference between \( I^> \) and \( I^\geq \) is that the former objectives should be improved as much as possible but the latter objectives only need to be increased to a certain level.

As a more intuitive type of method, classification-based methods enable DMs to control changes in objectives. However, some limitations make the classification-based methods sometimes inappropriate for target setting. Firstly, the convergence of the model cannot be guaranteed. After several interactions, DMs may find that the new solutions are no more preferred than the initial ones. Secondly, there is no objective stopping criterion to tell DMs when to stop the interactions. There might be situations where DMs stop the interactions before the MPS has been achieved or situations where DMs missed the MPS during the previous interactions.

3.3.3 Summary of existing MOLP approaches and research gaps

The previous section (3.3.2) reviewed existing multi-objective methods that include DMs’ preferences and compared them with each other and with typical models, defining the advantages and disadvantages.

When using the models that consider DMs’ preferences, the following requirements should be considered. Firstly, the primary aim of the adopted model is to locate a specific solution or target among the efficient frontier. Besides, a model that can let DMs intuitively explore and learn about the efficient frontier will also be very helpful. Secondly, because
DMs are involved in the decision-making process, the interactive process must be convergent in finite and feasible interaction times, with a clear and objective stopping criterion. Thirdly, DMs must feel comfortable in providing their preferences in the form required by the selected model. Finally, the interactive process should not be too complex for DMs to understand.

Based on the requirements listed above, the trade-off-based model is more suitable than classification-based methods and goal-based methods. Compared to the other two types of interactive methods, the trade-off-based model is the only type of method that focuses on locating a specific solution among the efficient frontier. Besides, trade-off-based methods can provide DMs with a clear and objective stopping criterion or optimal conditions that could help DMs to judge when to stop the interactive process. Most importantly, the convergence of trade-off-based methods can be proven, which is crucial in the target-setting process.

However, existing trade-off-based methods also have several limitations, which make some interactive processes infeasible in real cases.

1) **Infeasible interaction times:** A very common problem of trade-off-based models is the interaction times. For MOO problems, more objectives mean a more complex efficient frontier, which will cause DMs to engage in many more interactions to find the MPS. Although the convergence of most trade-off models can be proven, it could take so many interactions that no DMs could finish the interactive process. Some researchers even claim that most DMs actually stop the interactive process before the MPS has been achieved (Belton and Stewart, 2002).

2) **Abstract results being presented to the DMs:** When the number of objectives is more than three, trade-off results can only be presented to DMs in the form of tables. It will thus be quite difficult for DMs to imagine the real shape of the efficient frontier, which might have a negative effect on the accuracy of DMs’ feedback. It is very helpful if there is some visual support when DMs are exploring the efficient frontier.
3) **Complex indifference trade-offs**: DMs might find it very hard to provide the indifference trade-off information of all objectives simultaneously. When DMs try to provide their preference information as indifference trade-offs, they are actually trying to quantify their preference structures in pricewise ratios. However, when the number of objectives is more than two, consistency between pricewise ratios cannot be guaranteed.

### 3.4 Research question development and research design

In order to fill these gaps and further improve the feasibility of trade-off-based methods, this thesis proposes three independent trade-off-based models.

Regarding the limitation of infeasible interaction times, the reason why it takes DMs so many interactions to locate the MPS is because the efficient frontier is a continuous surface with infinite alternatives, which makes it difficult or even impossible for DMs to make a comparison between all the potential alternatives. The key to solving this challenge is to transform the infinite alternative solutions into a limited and controllable solutions set.

Corresponding with the above, this thesis proposes a piecewise linear model. The general idea is to divide the continuous efficient frontier into finite and controllable decision-making regions instead of specific efficient solutions. The DMs can make a comparison between the regions and choose the one they prefer the most, by collecting trade-off information. After identifying DMs’ preferred region, the range of this region can be further narrowed down by dividing the preferred solution into several decision-making sub-regions. By repeating this procedure, the range can be gradually narrowed down.

The difficulty of visualizing the efficient frontier is another challenge for target-setting models based on the DEA PPS. When there are fewer than two outputs or fewer than two inputs, a two-dimensional figure showing the efficient frontier can be displayed to DMs. In most cases, DMs find it very helpful when setting a target. However, when the number of both inputs and outputs is more than three, it is impossible to depict the efficient frontier
on a graph. DMs may easily get lost. Therefore, a visual aid can be helpful to DMs when the number of both inputs and outputs is more than three, and it is also very helpful if the approach can enable DMs to search the efficient frontier in the direction they prefer.

Corresponding with the above, this thesis proposes a prioritized model. The prioritized model firstly collects DMs’ indifference information. Based on this information, only the part of the efficient frontier that follows the DMs’ preferred trade-off direction is sampled using a new sampling method. A series of trade-off figures and a trade-off table can then be presented to the DMs to help them to visualize the part of the efficient frontier that they prefer. The figures and table provide intuitive support to show DMs the substitution rates between different objectives.

The difficulty of providing indifference trade-offs when the number of objectives is quite large is another problem for DMs when setting a target. In such a case, it is difficult for DMs to quantify the trade-off relationships between objectives. This raises another issue: how to release DMs’ burden when the number of objectives is large. The key to solving this is to decrease the number of objectives that DMs face simultaneously during each interaction. In other words, during each interaction, a method should be utilized to help DMs to provide trade-off information in terms of a controllable number of objectives. Therefore, this thesis proposes a stepwise model. The stepwise model enables DMs to start from only two objectives. DMs only need to identify the MPS in a two-dimensional space. After that, the other objectives are gradually included in the target-planning process one by one, assuming that the indifference trade-offs between all previous objectives are fixed, until all objectives have been considered. In this way, DMs do not have to provide the indifference trade-offs of more than two objectives, which makes the target-setting process much easier for DMs.

The following chapters are organized as follows. Chapter 4 describes a new piecewise linear interactive target-setting model, which aims to overcome the first limitation (infeasible interaction times). Chapter 5 proposes a prioritized model to overcome the
second limitation (abstract results). Chapter 6 presents a stepwise trade-off model, aiming to address the third limitation (complex indifference trade-offs).
4. A piecewise linear interactive model

In this chapter, local indifference trade-off information is explored for performance planning using DEA. This investigation is based on an optimal condition established in the literature for the termination of an interactive MOO process by testing whether an MPS has been achieved in the sense that the DMs’ implicit utility function is maximized locally. The focus is on developing a new interactive procedure for supporting the convergence of the interaction towards MPS. This model largely alleviates the inconsistency problem of DMs’ preference information through substituting specific solutions with decision sub-regions (DSRs) as the basic unit to collect the DMs’ preference information. In this process, DMs are able to control the number of interactions. The process continues until the optimal condition is satisfied. A numerical example is provided to demonstrate the applicability and effectiveness of the model, and this model is also applied to a real target-setting case for a secondary branch of a Chinese state-owned bank through interactions with its managers.

4.1 Introduction

Setting an appropriate target plays an essential role in organizational performance analysis and improvement (Amirteimoori and Kordrostami, 2005, Bi et al., 2011, Lovell and Pastor, 1997). Some researchers even claim that a business can never succeed unless its DMs have a clear picture about the future direction of the business (Atrill and McLaney, 2009). To obtain appropriate targets, many target-setting models have been proposed. However, guaranteeing that the target is feasible but advanced is always a challenge for DMs. A common method is to reference existing peers’ performances and select the best performances as the targets.

A number of DEA target-setting models have been designed to help DMs to set appropriate targets. The DEA SBM model (Tone, 2001) and its developed models are one of the most popular target-setting models, but one limitation of SBM-based models is that
only corner points can be found. Other DEA target-setting methods, such as goal-based methods (Golany, 1988a, Thanassoulis and Dyson, 1992a, Athanassopoulos, 1995, Stewart, 2010), weight-restricted models (Wong and Beasley, 1990, Halme and Korhonen, 2000, Korhonen et al., 2001), assurance region models (Allen et al., 1997, Thompson et al., 1990), and restricted composites (Athanassopoulos et al., 1999), enable DMs to know about the DEA PPS, but they fail to provide a complete decision-making process to help DMs to decide specific targets, and they fail to provide an optimal condition to help DMs to judge whether the MPS has been achieved or not. Yang et al. (2009b) proved the equivalent relationship between DEA and MOLP, so a variety of MOLP models can be utilized in the DEA PPS to assist DMs to find specific targets (Amirteimoori and Kordrostami, 2005).

MOLP methods can generally be classified into three types. For a priori methods like the value function approach (Steuer et al., 1977), the lexicographic approach (Fishburn, 1974), and the goal programming approach (Charnes et al., 1955), DMs specify their preference information before the solution process. However, a priori methods require DMs to provide an entire preference structure, which is normally not available in practical decision-making processes. A posteriori methods, like weighted metrics (Zeleny, 1973), the achievement scalarizing function (Wierzbicki, 1982), and evolutionary MOO (EMO) methods, are methods for generating efficient solutions instead of DMs searching for a specific MPS. Compared with a priori methods and a posteriori methods, interactive methods do not require DMs to have a global preference structure. Instead, they can gradually learn about the efficient frontier during the interactive process. Therefore, an increasing number of interactive methods have been applied in target-setting processes based on the DEA PPS.

According to the types of preference information that DMs provide, interactive methods can generally be classified into three categories. Trade-off-based methods require DMs to provide marginal substitutions, which are used to find an MPS. Typical trade-off-based methods include the ZW method (Zionts and Wallenius, 1976), Interactive Surrogate Worth
Trade-Off methods (ISWT) (Haimes and Hall, 1974), GDF methods (Geoffrion et al., 1972), Sequential Proxy Optimization Technique (SPOT) methods (Sakawa, 1982), and GRIST methods (Yang, 1999). Reference-point-based-methods collect DMs’ preference information in the form of ideal points or “goals”. New solutions can be obtained through adjusting the position of the reference point and the distance measurement. Classical reference point methods include the Tchebycheff (Steuer, 1986), Pareto race (Korhonen and Laakso, 1986), and REF-LEX (Miettinen and Kirilov, 2005) methods. Classification methods allow DMs to determine which objectives should be improved, which can be sacrificed, and which should remain the same, for example the Step Method (STEM) (Benayoun et al., 1971), the Satisficing TradeOff Method (STOM) (Nakayama, 1995), and the NIMBUS method (Miettinen, 2012).

Applying traditional interactive methods directly in DEA target setting without any adjustments may lead to several problems in practical cases. The DEA efficient frontier is a continuous space that includes infinite potential targets. Given an initial target, interactive methods can gradually find more-preferred solutions and finally achieve the MPS. However, the interactive procedure could require so many interactions that no DMs could actually finish it (Korhonen, 2005). An effective interactive method that can make the best use of limited interaction times and generate an approximate target range is practically preferred. Besides, DMs tend to have various sensitivities for targets in different cases. For example, selling one more car might increase a lot of challenges for a car shop, while increasing or decreasing £1,000 deposit accounts might make little difference to a bank branch. Asking a DM to provide feedbacks recursively in their insensitive region may not only make the interaction procedure inefficient but also cause inconsistent feedback.

Based on the research gaps raised above, this chapter proposes a new piecewise linear interactive procedure to set targets by referencing the DEA efficient frontier, based on collecting DMs’ preference information in the unit of DSRs. Instead of generating an initial target like other traditional interactive methods, the piecewise linear model proposes firstly
to divide the original DEA PPS into several DSRs. Different division methods will be discussed in detail later. Then, DSRs are used as the basic unit to collect DMs’ preference information. The collected indifference trade-offs can be used to build a series of preference relationships between adjacent DSRs. By combining the preference relationships, we can build up a preference digraph, which can eliminate the DSRs that do not contain the MPS and narrow down to the regions that potentially contain the MPS. This process continues until the region is narrowed to a range acceptable to DMs. Although the new piecewise linear model is designed for DEA target setting, it can be applied to any MOLP problem.

Comparing to existing DEA target-setting and interactive models, the new proposed model has several advantages. Firstly, this piecewise linear model to a large extent alleviates the inconsistency problem of DMs’ preference information through substituting specific solutions with DSRs as the basic unit to collect DMs’ preference information. Secondly, the number of interactions can be controlled so that the interaction process can be practical and effective. Thirdly, if the PPS is divided into multiple sub-regions, more than one local MPS may be obtained. In this case, the DMs can make a judgement between them and locate the one they prefer.

The remainder of this chapter is organized as follows. Section 4.2.1 briefly discusses the equivalent relationships between DEA and MOLP, as proven by Yang et al. (2009b). After that, the procedures of the proposed piecewise linear model, including DSR division, efficient DSR identification, seeking the MPS for each DSR, and setting up the preference relationships between DSRs, are discussed in detail. A numerical example is provided to illustrate its applicability and effectiveness in Section 4.3. This model is applied to a real case of a Chinese state-owned bank in Section 4.4. Finally, conclusions are given in Section 4.5.
4.2 Methodology

4.2.1 The equivalent relationship between MOLP and DEA

Suppose an MOO problem has $s$ objectives defined in general as follows (Yang et al., 2009):

\[
\text{Max } f(\lambda) = [f_1(\lambda), ..., f_r(\lambda), ..., f_s(\lambda)] \\
\text{s. t. } \lambda \in \Omega = \{\lambda | g_i(\lambda) \leq 0, h_l(\lambda) = 0; i = 1, ..., k_1, l = 1, ..., k_2 \},
\]

where $\Omega$ is the feasible decision space, $\lambda = \{\lambda_j; j = 1, ..., n\}$ is the independent composite variable, $f_r(\lambda)$ ($r = 1, ..., s$) is the continuously differentiable objective function, and $g_i(\lambda)$ ($i = 1, ..., k_1$) and $h_l(\lambda)$ ($l = 1, ..., k_2$) are the continuously differentiable inequality and equality constraint functions, respectively. In an MOO problem, objectives are generally in conflict with each other and therefore no dominant solution can be found that maximizes all objectives. Instead, we are satisfied with efficient solution $\lambda^*$ in the sense that there is no other feasible solution that is better than $\lambda^*$ on at least one objective and as good as $\lambda^*$ on all other objectives. The set of all efficient solutions is referred to as the efficient frontier.

Yang et al. (2009b) proved the equivalent relationship between DEA and MOLP. When the restriction of radial improvement is relaxed, the conventional DEA dual model can be generalized into the following target-planning models:

Output-oriented DEA target-planning model:

\[
\text{Max } f(\lambda) = [f_1(\lambda), ..., f_r(\lambda), ..., f_s(\lambda)] \\
\text{s. t. } \lambda \in \Omega = \{\lambda | \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij}, i = 1, ..., m ; \lambda_j \geq 0 \text{ for all } j \}
\]

where $f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj}$ is the composite output for the “$r$th output”. This model can be applied when DMs want to investigate the maximum outputs with no extra inputs consumed.
Chapter 4. A piecewise linear interactive model

For example, when opening a new branch with no more than a certain amount of investment, DMs want to know the most-preferred product outcome.

Input-oriented DEA target-planning model for inputs:

\[
\text{Min } f(\lambda) = [f_1(\lambda), ..., f_i(\lambda), ..., f_m(\lambda)]
\]

\[
\text{s.t. } \lambda \in \Omega = \{ \lambda | y_{rj} - \sum_{j=1}^{n} \lambda_j y_{rj} \leq 0, j = 1, ..., n ; \lambda_j \geq 0 \text{ for all } j \}
\]

where \( f_i(\lambda) = \sum_{j=1}^{n} \lambda_j x_{ij} \) is the composite input for the “ith input”. This model can be applied when DMs want to minimize inputs without sacrificing existing outputs. For example, an inefficient DMU may require a benchmark to help it to cut down its redundant resource usage.

There might be situations where DMs want to adjust both inputs and outputs at the same time. Thus, the following combined-oriented model can be applied.

\[
\text{Max } f(\lambda) = [F^{c}_{\text{outputs}}, -F^{c}_{\text{inputs}}]
\]

\[
\text{s.t. } \lambda \in \Omega = \{ \lambda | F^{N}_{\text{outputs}} \geq y_0^N, F^{N}_{\text{inputs}} \leq x_0^N \}
\]

where \( F^{c}_{\text{outputs}} \) and \( F^{c}_{\text{inputs}} \) refer to the set of discretionary composite outputs and discretionary composite inputs, respectively, and \( F^{N}_{\text{outputs}} \) and \( F^{N}_{\text{inputs}} \) refer to the set of non-discretionary composite outputs and non-discretionary inputs, respectively. For illustration purposes, the rest of this chapter will use the output target-planning model to illustrate the following procedure. The same procedure applies to the other two oriented models as well.

All situations mentioned above are actually special cases of 4.1 with linear constraints and objective functions. In the following sections, this chapter proposes a piecewise linear interactive MOO process to support DEA target planning with DMs’ preferences considered progressively.
4.2.2 Decision Sub-Region

Although a common assumption of interactive methods is that there exists an implicit continuous non-linear utility function, DMs’ sensitivity to solutions varies widely. In some rare cases, DMs can tell their preference differences in terms of solutions even when they are relatively similar to each other. However, more generally, DMs can only give different feedback when the solutions are significantly different. For traditional interactive DEA methods, the searching direction is mostly based on DMs’ local preference information.Insensitive feedback can seriously influence the next step of searching. At the same time, the efficient frontier contains an infinite number of efficient solutions. Even though an interactive model has a convergent process, interactions could be so time-consuming that they may cause DMs to lose their patience and end the interactive process without the MPS, or the feedback they provide may be inconsistent.

Compared to using specific solutions, using regions as basic units to collect DMs’ preference information can be more practical. The PPS described by DEA can generally be divided into several regions, denoted as DSRs, in which a common fixed linear utility function can be assumed to approximate DMs’ implicit utility function. Instead of providing opinions on individual solutions, DMs are encouraged to use one common feedback to present their preferences towards all solutions in a DSR. This preference collection method can significantly improve DMs’ discrimination towards solutions, especially when DMs are not sensitive to similar solutions.

DSR division starts from identifying an inclusive feasible region. For an output-oriented DEA target-planning model as described in 4.2, the maximum feasible value for $f_r(\lambda)$ can be calculated by solving a series of single-objective optimization problems (Yang et al., 2009b):

$$\text{Max } f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} \quad r = 1, \ldots, s$$  \hspace{1cm} (4.5)
Chapter 4. A piecewise linear interactive model

\[ s.t. \lambda_j \in \Omega. \]

The solutions are denoted as \( f^*_r(\lambda) \). The performance \( f^*(\lambda) = [f^*_1(\lambda), \ldots, f^*_r(\lambda), \ldots, f^*_s(\lambda)]^T \) is denoted as the positive ideal point. Normally, \( f^*(\lambda) \) is infeasible. On the contrary, if we change 4.5 to a minimization problem with all equations and variables remaining the same, the solutions are denoted as \( f^-_r(\lambda) \). The performance \( f^-(\lambda) = [f^-_1(\lambda), \ldots, f^-_r(\lambda), \ldots, f^-_s(\lambda)]^T \) is denoted as the negative ideal point. Normally, the negative ideal point is infeasible. The positive and negative ideal points can describe an inclusive region \( R^A \) that contains all efficient solutions:

\[ R^A = [R^A_1, \ldots, R^A_r, \ldots, R^A_s] = \{f(\lambda) | f^-_r(\lambda) \leq f_r(\lambda) \leq f^*_r(\lambda), r = 1, \ldots, s\}. \]

There can be two ways to identify the boundaries of DSRs. Suppose DMs are able to distinguish the boundaries of DSRs, which is relatively rare in practical cases. According to the degree of preference, DMs are able to divide the feasible interval \( R^A_r \) into \( B_r (B_r \geq 1) \) intervals. The principle is that DMs do not have significant preference differences in terms of \( f_r(\lambda) \) inside the same interval, but the difference is significant between intervals. Ranking by the upper boundary of the interval, the \( d_r \)th \((d_r = 1, \ldots, B_r)\) interval for \( f_r(\lambda) \) is denoted as \( I^d_r = \{f_r(\lambda) | f^d_{r,r-1}(\lambda) \leq f_r(\lambda) \leq f^d_{r,r}(\lambda)\} \), where \( f^d_{r,r-1}(\lambda) \) and \( f^d_{r,r}(\lambda) \) are the lower bound and upper bound of the \( d_r \)th interval, respectively. \( f^d_0(\lambda) \) is equal to \( f^-_r(\lambda) \), and \( f^d_{r,r}(\lambda) \) is equal to \( f^*_r(\lambda) \). By utilizing the full combinations of \( I^d_r \), \( R^A \) can be divided into several DSRs. A DSR is defined as:

\[ R_D = \{f(\lambda) | f^d_{r,r-1}(\lambda) \leq f_r(\lambda) \leq f^d_{r,r}(\lambda), r = 1, \ldots, s\} \]

\[ D = [d_1, \ldots, d_r, \ldots, d_s], \]

where \( D \) is a unique identification matrix for a DSR.

In a broader picture, when DMs cannot tell the boundaries of DSRs, the idea of dichotomy can be adopted to divide DSRs. Under this situation, \( R^A_r \) is equally divided into two sub-intervals: \( I^U_r = \{f_r(\lambda) | f^-_r(\lambda) \leq f_r(\lambda) \leq (f^*_r(\lambda) + f^-_r(\lambda))/2\} \) and \( I^L_r = \{f_r(\lambda) | (f^*_r(\lambda) + f^-_r(\lambda))/2 \leq f_r(\lambda) \leq f^*_r(\lambda)\} \). The full combinations of \( I^U_r \) and \( I^L_r \) divide \( R^A \) into \( 2^{s-1} \) DSRs. Besides dichotomy, techniques like trichotomy and the Fibonacci
method could also be used here to identify the boundaries of DSRs. The increasing number of DSRs can decrease the possibility of missing the MPSs in local regions, but it requires more feedback at each interaction.

Figure 4.1 graphically illustrates the concept of DSRs via the objective space of a DEA target-planning model with two outputs. A, B, C, D, and E are five observation units. Observation units A, C, D, and E together formulate the efficient frontier, and \( f_1^*(\lambda) \) and \( f_2^*(\lambda) \) are the optimal feasible values for objective 1 and objective 2, respectively. According to DMs’ opinions, the feasible range of objective 1 is divided into four intervals, and the feasible range of objective 2 is divided into three intervals. The boundaries of these intervals divide \( R^4 \) into several DSRs. When collecting DMs’ preference information, these obtained DSRs will act as the basic units.

Using DSRs as the basic unit to collect DMs’ preference information to a large extent increases the DMs’ discrimination of solutions, alleviating the inconsistency problem of interactive methods. Besides, DSR-based preference collection significantly decreases the
number of interactions and makes it practical for DMs to express their opinions on the entire efficient frontier.

### 4.2.3 Efficient DSR identification

As $R^A$ is a much broader space, only some of the DSRs contain part of the efficient frontier. These DSRs are defined as efficient decision sub-regions (EDSRs). For example, in Figure 4.1, only the DSRs characterized by bold dashed lines are EDSRs. It is not necessary to collect DMs’ preference information for the DSRs without efficient solutions. Therefore, (4.7) is proposed to screen out the DSRs without efficient solutions. For a DEA target-planning problem for outputs as defined in 4.2 and a DSR as defined in 4.6, the screening model is as follows:

![Mathematical expression](image)

where $\lambda'$ is the test independent variable, $s^+_r$ is the auxiliary variable, and $\varepsilon$ is a very small positive real number. The definitions of other variables are the same as in 4.2.

**Theorem:** There exist efficient solutions in $R_D$ if and only if there exist feasible solutions in (4.7) and $\sum_{r=1}^s s^+_r$ is equal to 0.

**Proof:** Equation (4.7) can be decomposed into a two-stage screening process. Stage 1 can be characterized as follows:

![Mathematical expression](image)
which is a weighted sum model of 4.1. The feasible objective space is further narrowed down to a DSR by containing the extra constraints \( f(\lambda) \in R_D \). Its aim is to find a feasible and optimal solution in the region of \( R_D \). If there is no feasible solution in \( R_D \), the extra constraint will be conflicting against the original feasible space \( \lambda \in \Omega \), which will cause no solution to be found in (4.8). If \( R_D \) contains efficient solutions, (4.8) will generate at least one of them. If \( R_D \) contains only inefficient solutions, the optimal solution of (4.8) is still inefficient, which will be further eliminated by stage 2:

\[
\begin{align*}
\max_{\lambda'} & \sum_{r=1}^{s} s_{r}^{+} \\
\text{s. t.} & \quad f_r(\lambda') \geq f_r(\lambda) + s_{r}^{+}, \quad (r = 1, \ldots, s) \\
& \quad s_{r}^{+} \geq 0 \\
& \quad \lambda' \in \Omega
\end{align*}
\]

where \( f(\lambda) = [f_1(\lambda), \ldots, f_r(\lambda), \ldots, f_s(\lambda)] \) is the optimal solution obtained from 4.5. The nature of (4.9) is a DEA addictive model (Cooper et al., 2007). \( f(\lambda) \) is efficient if and only if \( \sum_{r=1}^{s} s_{r}^{+} \) is equal to 0. The aim of (4.9) is to test the efficiency of \( f(\lambda) \). If \( f(\lambda) \) is inefficient, there is no efficient solution in \( R_D \), because \( f(\lambda) \) dominate all the other performances in \( R_D \). Otherwise, \( R_D \) contains efficient solutions. Equation (4.7) is actually a combination of (4.8) and (4.9).

The filtering process of (4.7) dramatically decreases the number of interactions and relieves DMs’ burdens. Besides, the identified EDSRs can provide a general view of the efficient frontier to DMs by showing them the boundaries of EDSRs.

**4.2.4 Indifference trade-off collection in terms of each DSR**

The remaining EDSRs will be presented to DMs to collect their preference information. Different feedback forms can be adopted here. In this chapter, DMs are asked to provide the indifference trade-offs between objectives, as defined by Yang et al. (2009b). The first objective \( f_1 \) is set as the reference objective. Then, the indifference trade-off \( M_{1r}^D \) between the first and the \( r \)th objective and the marginal rate of substitution \( M^D \) in \( R_D \) are given by:
where $df_1^{D}$ is a change in $f_1$ that is assumed to be exactly offset by a change $df_r^{D}$ in $R_D$, keeping DMs’ overall utility constant. This marginal rate of substitution is assumed to be fixed inside one DSR but might vary across different DSRs.

### 4.2.5 Setting up preference relationships between adjacent DSRs

Suppose DMs have an underlying utility function: the reason for searching for the MPS for each DSR is actually to find out the solution with the highest utility value. Making the assumption of a linear and fixed utility function inside each DSR, DMs’ utility function in each DSR can be represented by:

$$u(f(\lambda)) = \sum_{r=1}^{s} \frac{f_r(\lambda)}{M_{1r}^{D}} + c$$

where $c$ is an unknown constant. The MPS for each DSR can be obtained by solving the following single-objective optimization problem:

$$\max_{\lambda} u(f(\lambda)) = \sum_{r=1}^{s} \frac{f_r(\lambda)}{M_{1r}^{D}}$$

$$s. t. \quad f(\lambda) \in R_D \quad \lambda \in \Omega$$

The optimal solution is defined as $\lambda^*_D$, and its corresponding objective image is defined as $f(\lambda^*_D)$. $f(\lambda^*_D)$ is preferred over all the other objective images in $R_D$ for DMs. Yang et al. (2009b) have proven that for a certain efficient solution $f(\lambda)$, the projection of the DMs’ utility gradient onto the tangent plane of the efficient frontier, represented by $\Delta \overline{u}^{D}$, along which DMs’ utility of $f(\lambda^*_D)$ can be improved, can be calculated by:

$$\Delta \overline{u}^{D} = [\Delta f_1^{D}, ..., \Delta f_r^{D}, ..., \Delta f_s^{D}]^T = -M^{D} + \frac{(M^{D})^T N^{D}}{(N^{D})^T N^{D}} N^{D}$$

$N^{D}$ is the normal vector for the efficient frontier on $f(\lambda^*_D)$, and $N^{D}$ can be obtained by:
Chapter 4. A piecewise linear interactive model

\[ N^D = [w_1^D \beta_1^D, \ldots, w_r^D \beta_r^D, \ldots, w_s^D \beta_s^D] \]

where:

\[ w_r^D = \frac{f_1(\lambda_r^D) - f_1^*}{f_r(\lambda_r^D) - f_r^*} \quad r = 1, \ldots, s. \]

(4.13)

\( \beta_r^D \) is the Lagrange multiplier for the objective constraints of the following single-objective optimization problem:

\[ \min_{\lambda_j} \theta \]

(4.14)

\[ \text{s. t. } w_r^D \left( f_r^* - \sum_{j=1}^{n} \lambda_j y_{rj} \right) \leq \theta \]

\[ \lambda \in \Omega. \]

If \( \Delta \bar{u}^D = 0 \), the \( M^D \) is parallel to the normal vector on \( f(\lambda_D^0) \). This means that the MPS has been achieved (Yang et al., 2009b). If \( \Delta \bar{u}^D \neq 0 \), we can always find a more-preferred solution than \( f(\lambda_D^0) \) if moving a very small step along \( \Delta \bar{u}^D \). The more-preferred solution can be obtained by solving (4.14) and substituting \( w_r^D \) with \( w_r^{D'} \), where:

\[ w_r^{D'} = \frac{f_1(\lambda_r^D) + \varepsilon \Delta f_1^D - f_1^*}{f_r(\lambda_r^D) + \varepsilon \Delta f_r^D - f_r^*}, \quad r = 1, \ldots, s. \]

(4.15)

where \( \varepsilon \) is a very small positive real number. The obtained solution is denoted as \( f(\lambda_D^+). \) Because \( f(\lambda_D^0) \) is preferred over the other solutions in \( R_D \), unless \( \Delta \bar{u}^D = 0 \), \( f(\lambda_D^+) \) will be located in an adjacent EDSR (denoted as \( R_{D+} \)). Note that there is a more-preferred solution in \( R_{D+} \) than \( f(\lambda_D^0) \) and all other efficient solutions in \( R_D \). This preference relationship is defined as:

\[ R_D \rightarrow R_{D+}, \text{ or } R_{D+} \text{ is preferred over } R_D. \]

1 graphically explains this preference relationship. When solving (4.10) for \( R_{D1}, M_{D1} \) in Figure 4.2 shows that the MPS of \( R_{D1} \) is on the boundary of \( R_{D1} \) and \( R_{D2} \). Following the
projection of $M_{D1}$ on the efficient frontier will lead to a more-preferred solution, which is located in $R_{D2}$. Therefore, a preference relationship is built:

$$R_{D1} \rightarrow R_{D2}, \text{ or } R_{D2} \text{ is preferred over } R_{D1}.$$ 

![Figure 4.2 Preference links between DSRs](image)

### 4.2.6 Preference structure analysis through directed graphs

After obtaining the preference relationships between adjacent EDSRs, a directed graph can be constructed to analyse DMs’ overall preference structure in terms of the efficient frontier. If denoting each EDSR as a vertex $v_u$, denoting each preference relationship as an edge with the direction $r_p$, DMs’ overall preference structure can be formulated as a directed graph, defined as a preference digraph. We define an edge as from its tail to its head.
For a DEA target-setting problem defined in 4.2, a preference digraph $G$ is a directed graph consisting of a vertex set $V = \{v_{it} | v_{it} \text{ is an EDSR identified by (4.7)}\}$ and a directed edge set:

$$E = \{e_{ut} | e_{ut} \text{ is a directed edge from one vertex } v_{it} \text{ to another vertex } v_{jt}; u \text{ could be equal to } t\}.$$  

For example, DMs’ preference structure in Figure 4.2 can be represented by a preference digraph:

![Diagram of preference digraph]

It should be noted that the preference digraph does not necessarily need to be a connected graph because there might be more than one local MPS identified by the piecewise linear model. In this way, the problem of finding the EDSR containing a local MPS (convergent center) is actually transferred to a problem of finding the vertex acting only as a head. It should be noticed that there might be situations where the MPS lies on the boundary between two or more DSRs. For example:

![Diagram of boundary between D1 and D2]

The MPS lies on the boundary between $D_1$ and $D_2$. In this case, $D_1$ and $D_2$ should be combined as a new vertex $D_0$.

### 4.2.7 Summary

The piecewise linear model can be summarized into the following five steps.

1. **Calculate the inclusive region from the original DEA target-setting model**

   Calculate the maximum feasible value for each output using Equation 4.5 and identify the inclusive region $R^A$. 

65
2. Divide $R^A$ into DSRs

Divide the inclusive region into several DSRs equally or according to DMs’ opinions.

3. Identify the EDSRs

Screen out the DSRs with only inefficient or infeasible solutions using (4.7) and identify the EDSRs.

4. Set up the preference relationships between adjacent DSRs

For each DSR, collect the DMs’ indifference trade-offs and calculate the MPS using Equation (4.10). Construct the directed links between adjacent DSRs through (4.10) to (4.15).

5. Identify the DSR(s) containing the MPS(s)

Use a preference digraph or an adjacency matrix to analyse DMs’ preference structure and narrow down the location of the global MPS into the convergent centre(s). Then ask the DMs to select the one(s) containing the MPS(s).

Repeat Step 1 to Step 5 only inside the selected region until it is narrowed down to a region that is satisfactory to DMs.

4.3 A numerical example of commercial banks in the UK

4.3.1 Problem description and efficiency assessment using the DEA CCR model

A numerical example was carried out to illustrate how the piecewise linear model can be applied to assist DMs to find the MPS. The demonstration data are from Yang et al. (2009b), Yang et al. (2012b), and Yang and Xu (2014). In this case, NatWest, which is identified as an inefficient bank by the DEA CCR model, finds its MPS via referencing six competitors using the proposed piecewise linear interactive model.

The initial DEA CCR model includes four inputs and two outputs. The collected data are listed in Table 4.1:
Taking the CRS assumption, Table 4.2 displays the efficiency assessment results for the investigated seven banks.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Inputs (x₁, x₂, x₃, x₄)</th>
<th>Outputs (y₁, y₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abbey National</td>
<td>0.77, 2.18, 2.35, 2.96</td>
<td>6.79, 10.57</td>
</tr>
<tr>
<td>Barclays</td>
<td>1.95, 3.19, 8.43, 3.53</td>
<td>2.55, 13.25</td>
</tr>
<tr>
<td>Halifax</td>
<td>0.80, 2.10, 3.21, 2.41</td>
<td>9.17, 8.14</td>
</tr>
<tr>
<td>HSBC</td>
<td>1.75, 4.00, 13.30, 4.85</td>
<td>5.82, 23.67</td>
</tr>
<tr>
<td>Lloyds</td>
<td>2.50, 4.30, 9.27, 2.40</td>
<td>6.57, 14.01</td>
</tr>
<tr>
<td>NatWest</td>
<td>1.75, 5.30, 7.70, 3.09</td>
<td>4.80, 12.04</td>
</tr>
<tr>
<td>RBS</td>
<td>0.65, 1.73, 2.67, 1.34</td>
<td>7.28, 7.36</td>
</tr>
</tbody>
</table>

Barclays and NatWest are identified as inefficient banks. Two outputs for Barclays could both be improved by 28.37% without consuming extra inputs, which would achieve 7.49 and 17.14, respectively. y₁ and y₂ for NatWest could both be improved by 33.51% without consuming extra inputs, which would achieve 10.53 and 16.07, respectively. Except for Barclays and NatWest, the other branches are all identified as efficient banks, no output of which could be improved without worsening the other outputs.

When applying DEA to assess DMUs’ performances, a radial improvement solution is used as the default reference performance, without including DMs’ preference structures. Actually, a radial improvement benchmark may not be the most-preferred benchmark for DMs over other efficient solutions. Using NatWest as an example, the piecewise linear
model is explored to show how to take DMs’ preference structures into account in an interactive target-planning process.

4.3.2 Searching for the MPS for NatWest using the piecewise linear interactive model

In the proposed interactive model, the very first step is to find out the feasible range for each composite output by solving 4.5, in which the inclusive region $R^A$ can be defined. The results show that, when consuming no more inputs, the maximum and minimum feasible values for the first composite output are 0 and 14.11, respectively, while the maximum and minimum feasible values for the second composite output are 0 and 16.07, respectively. The inclusive region $R^A$ for NatWest is given by:

$$R^A = \{ f(\lambda) | 0 \leq f_1(\lambda) \leq 14.11, 0 \leq f_2(\lambda) \leq 16.07 \}$$

For illustration purposes, the trichotomy method is used to divide the DSRs. The feasible ranges of the first and second outputs are divided into three sub-intervals of equal length. Table 4.3 shows the division boundaries.

<table>
<thead>
<tr>
<th>Table 4.3 DSR boundaries</th>
<th>$f_1^1(\lambda)$/$f_1^2(\lambda)$</th>
<th>$f_2^1(\lambda)$</th>
<th>$f_2^2(\lambda)$</th>
<th>$f_2^3(\lambda)$/$f_2^4(\lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>0</td>
<td>4.70</td>
<td>9.41</td>
<td>14.11</td>
</tr>
<tr>
<td>$y_2$</td>
<td>0</td>
<td>5.36</td>
<td>10.71</td>
<td>16.07</td>
</tr>
</tbody>
</table>

The full combination of these intervals divides the entire inclusive feasibility set into nine ($3^3$) DSRs, listed in Table 4.4 with their identification matrices. The filtering model (4.7) is then applied to eliminate the DSRs without efficient solutions. A DSR contains efficient solutions if a feasible optimal solution can be found in (4.7) with no slack. Table 4.4 illustrates the screening results. Only $R_{[3,3]}$ is identified as an EDSR, in which feasible solutions can be found with 0 slacks. No feasible solution can be found in $R_{[3,1]}$. Except for $R_{[3,3]}$ and $R_{[3,1]}$, slacks can be found for all the other DSRs’ optimal solutions, which indicates that the remaining DSRs only contain inefficient solutions.
As only one DSR contains efficient solutions, there is no need to collect DMs’ indifference trade-offs to determine the DSR with the MPS, so the searching region narrows down from $R^4$ to:

$$R_{[3,3]} = \{ f(\lambda) \mid 9.41 \leq f_1(\lambda) \leq 14.11, 10.71 \leq f_2(\lambda) \leq 16.07 \}$$

This region is further divided in order to obtain the more-precise region for the MPS. The ranges of the first and second outputs are divided into three sub-intervals of equal length. Table 4.5 shows the division boundaries for the second interaction.

The full combinations of these intervals divide the convergent region obtained in the first interaction into 16 DSRs. Four DSRs ($R_{[1,3]}$, $R_{[2,3]}$, $R_{[3,2]}$, and $R_{[3,3]}$) are proven to include efficient solutions, and their boundaries are displayed in Table 4.4. DMs are then asked to provide their indifference trade-offs in terms of these four DSRs. Suppose the DMs agree that:

$$\forall f(\lambda) \in R_{[1,3]} = \{ f(\lambda) \mid 9.97 \leq f_1(\lambda) \leq 11.35, 14.28 \leq f_2(\lambda) \leq 16.07 \}$$
The corresponding marginal substitution is $M^{[1,3]} = [1, 2.5]$. The remaining marginal substitutions are displayed in Table 4.7. The MPS for each DSR can then be calculated using (4.10). For example, the MPS for $R_{[1,3]}$ is $f(\lambda^*_1, 1) = (11.35, 15.57)$, whose first output lies on the upper boundary of $R_{[1,3]}$. The normal vector on $f(\lambda^*_1, 1)$ can be calculated through (4.12) to (4.14), resulting in $N^{[1,3]} = [1, 1, 6]^T$. The marginal direction that improves DMs’ overall utility can be obtained by taking $M^{[1,3]}$ and $N^{[1,3]}$ into (4.11). The obtained result is $\Delta \bar{u}^{[1,3]} = [0.86, -0.52]$, which indicates that DMs’ preference in terms of this $R_{[1,3]}$ is to increase the first objective and to sacrifice the second one. Improving $f(\lambda^*_1, 1)$ for a very small step can guarantee finding a more-preferred solution and will lead to an adjacent DSR($R_{[2,3]}$), so a directed link $R_{[1,3]} \rightarrow R_{[2,3]}$ can be established. By applying the same process, each EDSR is linked to an adjacent DSR, except for $R_{[3,2]}$. The MPS for $R_{[3,2]}$ is $f(\lambda^*_3, 1) = (13.89, 14.04)$, whose normal vector $N^{[3,2]} = [1, 0.42 - 1.6]^T$ is parallel to the corresponding marginal rate of substitution $M^{[3,2]} = [1, 1.25]^T$. According to the optimal condition (Yang et al., 2009b), $f(\lambda^*_3, 1) = (13.89, 14.04)$ is a local MPS, which makes $R_{[3,2]}$ a convergent centre as well. DMs’ preference structure in terms of these four EDSRs can be displayed as follows:

$$R_{[1,3]} \rightarrow R_{[2,3]} \rightarrow R_{[3,3]} \rightarrow R_{[3,2]}.$$
Through the second interaction, the region that contains the MPS narrows down to the DSR:

\[
R_{[3,2]} = \left\{ f(\lambda) \mid 12.73 \leq f_1(\lambda) \leq 14.11, 12.50 \leq f_2(\lambda) \leq 14.28 \right\}
\]

and one local optimal solution is found to be \( f(\lambda_{[3,2]}') = (13.89, 14.04) \). If the DMs are satisfied with the narrowed range and can accept \( f(\lambda_{[3,2]}') \) as the final target, the interaction will be terminated. If the DMs still think that the range of \( R_{[3,2]} \) is too broad and they would like to further narrow down the region, the third interaction should be carried out.

Table 4.7 DSR boundaries of the third interaction (trichotomy division)

<table>
<thead>
<tr>
<th>( y_1 )</th>
<th>( f_1^0(\lambda) )</th>
<th>( f_1^1(\lambda) )</th>
<th>( f_1^2(\lambda) )</th>
<th>( f_1^3(\lambda) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_2 )</td>
<td>12.73</td>
<td>13.19</td>
<td>13.65</td>
<td>14.11</td>
</tr>
<tr>
<td>12.50</td>
<td>13.09</td>
<td>13.69</td>
<td>14.28</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.8 Third interaction convergent region

<table>
<thead>
<tr>
<th>EDSR ID</th>
<th>( y_1 ) From To</th>
<th>( y_2 ) From To</th>
</tr>
</thead>
</table>

Table 4.7 and Table 4.8 display the interaction results. The region that contains the MPS is finally narrowed down to:

\[
R_{[3,3]} = \left\{ f(\lambda) \mid 13.65 \leq f_1(\lambda) \leq 14.11, 13.69 \leq f_2(\lambda) \leq 14.28 \right\}
\]

The obtained MPS is still \( (\lambda_{[3,3]}') = (13.89, 14.04) \). If the DMs agree that the obtained region is narrow enough to terminate the interactive process with the final target of \( f(\lambda_{[3,3]}') = (13.89, 14.04) \), or a region of \( y_1 \) between 13.65 and 14.11 and \( y_2 \) between 13.69 and 14.28, the final target for NatWest is to achieve 13.89 for \( y_1 \) and 14.04 for \( y_2 \) without consuming extra inputs.
4.4 A case study of a branch benchmark search for a state-owned bank in China

4.4.1 Background

The investigated bank is a Chinese state-owned bank with a history of more than 60 years, and the aim for this study is to set feasible but challenging benchmarks for one of the bank’s secondary branches (Branch 79) in a city in south China. The bank adopts a four-level hierarchy management system, including the head office in Beijing, city head offices, primary branches, and secondary branches. Secondary branches, as the main body of the bank, are the operational units in the system. The primary branches or head offices for the higher levels are all supervision organizations. The bank serves more than 100 types of customers. According to the served customer type and the service type, the bank generally classifies its business into five categories: corporate deposits, corporate loans, personal deposits, personal loans, and intermediary business. The resources that secondary branches consume are staff and overheads. The bank uses these five outcome indices and two resource indices to assess a branch’s performance.

Among the over 100 secondary branches of the investigated bank, 103 branches are selected as the reference set to set target for branch 79. The following table illustrates the descriptive statistics for the 103 branches’ performances in 2015:

<table>
<thead>
<tr>
<th></th>
<th>Personal deposit ($y_1$)</th>
<th>Personal loan ($y_2$)</th>
<th>Company deposit ($y_3$)</th>
<th>Company loan ($y_4$)</th>
<th>Intermediary business ($y_5$)</th>
<th>Employees ($x_1$)</th>
<th>Overhead per person ($x_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>881.29</td>
<td>460.43</td>
<td>452.19</td>
<td>461.63</td>
<td>9.74</td>
<td>20.07</td>
<td>54.10</td>
</tr>
<tr>
<td>Median</td>
<td>807.97</td>
<td>397.98</td>
<td>343.41</td>
<td>206.56</td>
<td>7.88</td>
<td>19.00</td>
<td>49.80</td>
</tr>
<tr>
<td>STDEV</td>
<td>388.82</td>
<td>360.11</td>
<td>478.15</td>
<td>638.92</td>
<td>6.96</td>
<td>4.82</td>
<td>17.22</td>
</tr>
<tr>
<td>Max</td>
<td>1876.20</td>
<td>2771.60</td>
<td>3137.47</td>
<td>4245.35</td>
<td>55.72</td>
<td>40.00</td>
<td>130.56</td>
</tr>
<tr>
<td>Min</td>
<td>102.09</td>
<td>52.69</td>
<td>16.96</td>
<td>0.00</td>
<td>0.88</td>
<td>12.00</td>
<td>26.94</td>
</tr>
<tr>
<td>Branch 79</td>
<td>1131.58</td>
<td>536.81</td>
<td>98.83</td>
<td>9.00</td>
<td>15.89</td>
<td>20</td>
<td>75.31</td>
</tr>
</tbody>
</table>
The data are collected in 2015.

Branch 79 is a secondary branch located in a densely populated region. Its performance indices are also displayed in Table 4.9. The DEA CCR model shows that the efficiency score for Branch 79 is 0.926, which indicates that it is an inefficient branch.

In order to guide the development of secondary branches effectively, the bank decides to set a benchmark for each branch. The benchmarks are expected to not only motivate branches but also help branches to explore their suitable short-term and long-term development directions. A suitable benchmark should be neither too difficult nor too easy. An infeasible benchmark is likely to discourage branches rather than motivate them, but an easy benchmark may slacken a branch. Therefore, a method to help branches to explore a feasible but challenging target is required.

The advantages of the DEA target-setting approach proposed in this chapter are significant in this case. On the one hand, the benchmark identified using the approach is achieved on the basis of existing performances or the linear combinations of existing performances, which guarantees the feasibility of the benchmark. On the other hand, the identified benchmark sets the MPS as a reference for the branches, which guarantees that the benchmark is challenging. Normally, the cost of the branches is relatively stable. The bank does not tend to lay off many staff or reduce its branches’ budgets. Therefore, the focus of the benchmark is on improving outcomes, rather than reducing resource consumption. An output-oriented DEA model is built to set targets for the bank, with five performance indices as outputs and two resource indices as inputs.

4.4.2 Identification of the MPS using the piecewise linear model

Because secondary branch managers do not have a priori knowledge about their benchmarks, an interactive procedure is required to collect secondary branch managers’
preference information. An appropriate interactive method for this case should satisfy the following requirements or restrictions. Firstly, the interaction times should be feasible or relatively short. Otherwise, the branch managers may lose patience and end the procedure without identifying an MPS. Secondly, although the performance data are all continuous data, branch managers are not sensitive to the differences in the data within a certain range. For example, the business volume of every corporate customer is relatively large on average, so benchmarks with 100 million RMB difference in terms of corporate deposits may not be significant for branch managers.

The proposed piecewise linear model is suitable for this case. On the one hand, branch managers can avoid unnecessary interactions. This is because branch managers’ preference information is collected in the form of marginal rates of substitution. Through controlling the width of DSR boundaries, the number of marginal rates of substitution can be controlled. On the other hand, using DSRs can help branch managers to discriminate among different solutions when searching for targets.

In this section, Branch 79 is taken as an example to illustrate how the proposed approach is applied to this case study. The first step is to determine the original feasible range for Branch 79 using Equation 4.5. The maximum feasible business volumes obtained for outputs are 137,621.40 (personal deposits), 294,238.60 (personal loans), 137,903.80 (corporate deposits), 244,885.40 (corporate loans), and 1,748.08 (intermediary business). Therefore, the original feasible space is from no outputs at all to the maximum feasible business volumes. The managers of Branch 79 believe that the interval division should build on the branch’s existing performance. For each objective, the range from 90% to 110% of the branch’s existing performance is regarded as the “normal range”, which means that the branch can achieve this range without exerting extra effort. The range above that is the “challenging range”, which means that the branch needs to exert extra effort or sacrifice other outputs to achieve this range. The range below the “normal range” is defined as the “easy range”, which means that with the existing resources, the branch can easily achieve
this range or a better range. Based on the branch managers’ opinions, we divide the original feasible output ranges into three intervals (Table 4.10):

Table 4.10 DSR boundaries of the first interaction

<table>
<thead>
<tr>
<th>$f^1(x)$</th>
<th>$f^2(x)$</th>
<th>$f^3(x)$</th>
<th>$f^4(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>0</td>
<td>101641.99</td>
<td>124473.54</td>
</tr>
<tr>
<td>$y_2$</td>
<td>0</td>
<td>48312.92</td>
<td>59049.12</td>
</tr>
<tr>
<td>$y_3$</td>
<td>0</td>
<td>8894.51</td>
<td>10671.06</td>
</tr>
<tr>
<td>$y_4$</td>
<td>0</td>
<td>810.00</td>
<td>990.00</td>
</tr>
<tr>
<td>$y_5$</td>
<td>0</td>
<td>1430.23</td>
<td>1748.28</td>
</tr>
</tbody>
</table>

These intervals divide the original feasible region into 243 DSRs, among which the screening process of Equation (4.7) identifies nine DSRs that contain efficient frontiers (Table 4.11). Then, the branch managers’ feedback is collected in the form of marginal rates of substitutions (Table 4.11). Following the procedures of Equations (4.10) – (4.15), we build up a preference relationship digraph to display the preference structure of the branch managers (Figure 4.3)

Table 4.11 EDSRs and indifference trade-offs for the first interaction

<table>
<thead>
<tr>
<th>EDSR ID</th>
<th>Marginal rates of substitution</th>
<th>EDSRs</th>
<th>PERSONAL DEPOSIT ($y_1$)</th>
<th>PERSONAL loan ($y_2$)</th>
<th>COMPANY deposit ($y_3$)</th>
<th>COMPANY loan ($y_4$)</th>
<th>Intermediary business ($y_5$)</th>
<th>Convergent center</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1,3,3,3,3]</td>
<td>1 1 0.6 0.5 1</td>
<td>0.00</td>
<td>108.76</td>
<td>190.40</td>
<td>1370.04</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
<tr>
<td>[2,1,3,3,2]</td>
<td>1 1 0.6 0.5 0.6</td>
<td>905.26</td>
<td>1244.74</td>
<td>0.00</td>
<td>420.45</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
<tr>
<td>[2,2,2,2,2]</td>
<td>1 1 0.6 0.5 0.1</td>
<td>905.26</td>
<td>1244.74</td>
<td>420.45</td>
<td>590.49</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
<tr>
<td>[2,3,3,3,2]</td>
<td>1 1 0.6 0.5 1</td>
<td>905.26</td>
<td>1244.74</td>
<td>590.49</td>
<td>1379.04</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
<tr>
<td>[3,3,3,3,1]</td>
<td>1 1 0.6 0.8 0.6</td>
<td>1244.74</td>
<td>1738.21</td>
<td>0.00</td>
<td>420.45</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
<tr>
<td>[3,1,3,3,2]</td>
<td>1 1 0.6 0.8 0.6</td>
<td>1244.74</td>
<td>1738.21</td>
<td>420.45</td>
<td>590.49</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
<tr>
<td>[3,2,2,2,2]</td>
<td>1 1 0.6 0.5 1</td>
<td>1244.74</td>
<td>1738.21</td>
<td>590.49</td>
<td>1379.04</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
<tr>
<td>[3,3,3,3,1]</td>
<td>1 1 0.6 0.8 1</td>
<td>1244.74</td>
<td>1738.21</td>
<td>590.49</td>
<td>1379.04</td>
<td>108.71</td>
<td>2042.20</td>
<td>9.00</td>
</tr>
</tbody>
</table>
In the preference digraph, five EDSRs act only as heads. They are \( D_{23332} \), \( D_{21333} \), \( D_{31332} \), \( D_{31333} \), and \( D_{23333} \). Among these five identified EDSRs, \( D_{21333} \), \( D_{31332} \), and \( D_{31333} \) all offer “relatively extreme targets”. All of them suggest that Branch 79 should set up a goal for its personal loan business in the range of 0 to 429 million RMB, which indicates that Branch 79 should give up its personal loan business and develop other business. Branch 79’s existing business volume for personal loans is 536.81 million RMB, and the branch managers hold a positive attitude towards the future development of this business area. Therefore, \( D_{21333} \), \( D_{31332} \), and \( D_{31333} \) are excluded from the regions containing the MPS.

Compared to \( D_{23333} \), the branch managers believe that \( D_{23332} \) is more suitable. This is because intermediary business is just at the beginning stage, so a target from 17.48 to 35.18 million RMB seems too ambitious. Consequently, the branch managers decide that \( D_{23332} \) is the one containing the MPS. Then, the region containing the MPS is narrowed down from the original space to \( D_{23332} \). However, the managers still think that the obtained range is too broad, so the second interaction is necessary to narrow down this range further.

Because the branch managers do not provide their preferences in terms of the further division of EDSR boundaries, we adopt the equal division method. Considering the interactions involve the participation of the branch managers, interaction time should not
be too long. We observe that when the branch managers are asked to provide more than 20 groups of marginal rates of substitutions, they start to lose patience. However, if the number of EDSRs is too small, the feasible space cannot be narrowed down efficiently during each interaction. Therefore, we control the number of intervals for outputs so that we can get appropriate EDSR numbers for each interaction. Table 4.12 lists the number of intervals and the corresponding number of EDSRs. For example, in the second interaction, if we adopt the dichotomy method, we obtain six EDSRs, and the branch managers need to provide six groups of marginal rates of substitution. If we adopt the trichotomy method, the branch managers need to provide 20 groups of marginal rates of substitution. Getting more intervals for outputs will lead to an unacceptable number of EDSRs for the branch managers. Therefore, we select the trichotomy method for the second interaction. Following the same rule, we divide outputs into five and four intervals in the third and fourth interactions, respectively.

Table 4.12 Interval numbers and corresponding EDSR numbers

<table>
<thead>
<tr>
<th>Intervals for each output</th>
<th>Interaction 2 (Number of EDSRs)</th>
<th>Interaction 3 (Number of EDSRs)</th>
<th>Interaction 4 (Number of EDSRs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (dichotomy)</td>
<td>6</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3 (trichotomy)</td>
<td>20</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>38</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>59</td>
<td>8</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>83</td>
<td>22</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 4.13 The narrowed feasible space containing the MPS after each interaction

<table>
<thead>
<tr>
<th>Number of EDSR</th>
<th>Personal deposit ($y_1$)</th>
<th>Personal loan ($y_2$)</th>
<th>Company deposit ($y_3$)</th>
<th>Company loan ($y_4$)</th>
<th>Intermediary business ($y_5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original space</td>
<td>9</td>
<td>0</td>
<td>1376.21</td>
<td>0</td>
<td>2042.39</td>
</tr>
<tr>
<td>Interaction 1</td>
<td>20</td>
<td>905.26</td>
<td>1244.74</td>
<td>1379.04</td>
<td>2042.39</td>
</tr>
<tr>
<td>Interaction 2</td>
<td>8</td>
<td>1018.42</td>
<td>1131.58</td>
<td>1379.04</td>
<td>1053.27</td>
</tr>
</tbody>
</table>

Table 4.13 displays the narrowed feasible space after the consecutive interactions. After four interactions, the feasible space containing the MPS has been narrowed down to: personal deposits (from 1,029.74 to 1,035.39), personal loans (from 1,326.47 to 1,339.61),
corporate deposits (from 1,950.6 to 1,997.83), corporate loans (from 1,961.06 to 2,001.71), and intermediary business (from 14.06 to 14.14). The corresponding optimal solution of this region is: personal deposits (1,034.27), personal loans (1,337.47), corporate deposits (1,993.29), corporate loans (1,995.15), and intermediary business (14.13). The branch manager is satisfied with the final region width and accepts the optimal solution as the benchmark.

Figure 4.4 compares the existing performance of Branch 79, the CCR reference point, and the benchmark obtained from the piecewise linear model. The CCR reference point suggests that Branch 79 should slightly improve its personal deposit, personal loan, and intermediary business areas and should dramatically improve its corporate deposit and corporate loan areas. However, the piecewise linear model generates a significantly different benchmark by taking into account the branch managers’ preferences, which suggest that Branch 79 should slightly sacrifice its personal deposit and intermediary business areas and dramatically improve its personal loan, corporate deposit, and corporate loan areas.

The manager of Branch 79 believes that the benchmark obtained from the piecewise linear model is much more suitable for the branch’s future development for the following reasons. It is because Branch 79 has played a leading role in terms of personal business and any further improvement in the personal business area would be very difficult. On the contrary, the personal loan, corporate deposit, and corporate loan areas are the weaknesses of Branch 79, and there is considerable improvement space. Besides, a slight sacrifice in the personal deposit and intermediary business areas would save Branch 79 a large amount of resources to improve its weaknesses.
Chapter 4. A piecewise linear interactive model

Figure 4.4 Existing performance of Branch 79, the CCR reference point, and the benchmark obtained from the piecewise linear model

4.5 Contributions of the piecewise linear model

This chapter proposed a piecewise linear model to incorporate DMs’ preferences into target planning for DEA, where DSRs instead of a specific solution are used as the basic unit to search for the MPS. DSRs are obtained by dividing an inclusive region including all efficient solutions into several sub-regions of equal size. Trade-off analysis is then applied to build up the preference relationships between DSRs. Through identifying the convergent region, the region that contains the MPS is narrowed down from the original inclusive PPS into one or more DSRs. After the same process is repeated, the original feasibility set is gradually narrowed down until DMs have no preferential differences in terms of all solutions inside the set.

The piecewise linear model to a large extent decreases the interaction times of DMs. Besides, the model also improves the discrimination of close solutions so that DMs can express their preferences more precisely.

A numerical example was studied to illustrate how the proposed model can be applied to a business case for helping an inefficient bank to search for its MPS. This model was also applied to a real case of benchmark planning for a secondary branch of a Chinese state-owned bank through interactions with its managers.
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

5. A prioritized trade-off model for bank branch target setting under constraints

Although a number of DEA models have been developed for performance monitoring in the banking industry, using DEA as a management planning tool is a relatively new topic. This chapter proposes a prioritized trade-off model for bank branches’ target setting. The new model aims to help branches to explore benchmarks that are preferred by both head office and branches. Bank head offices’ strategies and regulations are collected and transformed into several constraints. These constraints can identify a narrowed and head-office-preferred feasible space from an existing DEA PPS. Within the narrowed feasible space, the feasible range of each objective is divided into two regions: the dominant region and the trade-off region. Then, a prioritized trade-off model is proposed to help DMs to find the MPS gradually. This model has been applied to the secondary branches of a state-owned bank in China as a case study.

5.1 Introduction

It is important for a business to have a target. A business can never succeed unless its DMs have a clear picture of the future direction of the business (Atrill and McLaney, 2009). Instead of generating one optimal target that optimizes all objectives, DEA efficient frontier can be referenced to provide a set of efficient targets (Liu et al., 2001). DMs’ expectations can be included in a utility function. By simulating an underlying utility function or collecting local indifference trade-offs, DMs can find the best solution or target in the efficient frontier (Halme et al., 1999).

Regression analysis (RA) is the most common target-planning approach. However, its disadvantages cannot be ignored comparing to DEA. DEA has several significant advantages over RA (Thanassoulis, 1993):

1. DEA can cope better with multi-input and multi-output problems.
2. The nature of RTS can be better identified.

3. DEA can identify the source of inefficiency regarding specific resources or low outputs.

4. DEA allows variable marginal values for different input–output mixes.

5. DEA offers more-appropriate individual maximum (minimum) targets where outputs (inputs) cannot vary independently from each other.

There are two core assumptions for multi-objective target setting (including DEA target setting). Firstly, the determined targets must be efficient. Secondly, there must be a stable underlying utility function (unknown) about inputs and outputs, which causes efficient solutions no longer non-inferior for DMs. An MPS is pursued. However, the global utility function is hard to obtain. Some researchers even claim that a complete utility function is immeasurable (Halme et al., 1999, Yang et al., 2009a). Corresponding with this requirement, numerous multi-objective methods have been proposed. Although these approaches differ, their common goal is to help DMs to learn about the efficient frontier and locate the MPS.

When exploring the MPS in the efficient frontier, it is very common for DMs to care about some outputs or inputs more than others. In a multi-objective world, “priority” is a well-known concept when DMs have a dominant preference for some objectives over others. A series of single-objective optimizations is conducted to pursue their optimal values from high priority to low priority sequentially. In order to achieve the optimal value for high-priority objectives, low-priority objectives can be sacrificed as much as required.

However, in reality, DMs are less likely to have such a dominant preference throughout the entire feasible space. In fact, when conducting trade-off analysis, DMs have a dominant preference for high-priority objectives over low-priority objectives only within a specific “dominant region”. Within the dominant region, an objective has a dominant priority to be improved, regardless of sacrificing low-priority objectives. Beyond the dominant region, an objective inside the “trade-off region” or the “non-dominant region”
can be substituted with low-priority objectives at satisfied trade-off rates. This chapter defines this relationship as “semi-dominant priority”.

This semi-dominant relationship is very common in real life. For example, when a company allocates resources to branches, branch managers are very likely to prioritize the importance of resources. However, few branch managers would like to pursue the optimal values of all the important resources, regardless of losing less-important resources. Instead, DMs usually have a series of “bottom lines”. More-important resources’ bottom lines have a higher priority to be satisfied. The extra resources above the bottom lines could be used to trade off with other resources.

A bank branch case can be used here to illustrate the concept of semi-dominant priority. Suppose a bank branch consumes three resources: employees, overheads, and ATM terminals. The branch manager cares about employees the most, then overheads, and ATM terminals the least. The bottom lines for these three resources are 25, 40, and five, respectively. The DM believes that overheads and ATM terminals can be sacrificed as much as required in order to achieve more than 25 employees. If the initial plan already allocated this bank branch more than 25 employees, the extra employees could be used in exchange for more overheads or ATM terminals at desirable trade-off rates. Similarly, after satisfying the bottom line of 25 employees, ATM terminals should be sacrificed as much as required to guarantee that more than 40 units of overheads are allocated to the branch.

By incorporating the gradient interactive method proposed by Yang and Li (2002), this chapter proposes a new prioritized interactive target-setting process in order to solve MOO problems with semi-dominant priorities. In this model, an initial solution is firstly generated by non-interactive methods. After that, a prioritized trade-off procedure will guide the objectives to their non-dominant regions from high semi-dominant priority to low semi-dominant priority sequentially, until all of them are located in their trade-off regions. This model is tailored and applied to a case study of bank branch management in China.
The remainder of this chapter is organized as follows. Section 5.2 briefly introduces the basic idea of the DEA CCR model and the equivalent MOLP model. After that, Section 5.3 explains a three-stage DEA target-setting model. 5.4 proposes a three-stage procedure to help bank branches set target considering both primary and secondary branch managers’ preference. Its application to the secondary branches of a commercial bank in China is displayed in Section 5.5. Conclusions and remarks are provided in the final section (5.6).

5.2 Literature review

5.2.1 Previous research

Yang et al. (2009b) proved the equivalent relationship between MOLP and DEA, so MOLP procedures can be utilized in DEA target setting. Hwang and Masud (1979) classified multi-objective models into three categories: a priori articulation of preferences, a posteriori articulation of preferences, and progressive (interactive) articulation of preferences. Because of the difficulty of obtaining a priori information on utility functions, a priori articulation methods are not easy to realize in reality. Although a posterior articulation can present DMs with a general idea of the efficient frontier (e.g. a multi-objective genetic algorithm), determining the optimal condition is always a challenging problem. Progressive/interactive articulation enables DMs to learn about the efficient frontier more intuitively. Although it is difficult to collect global utility function information, partial preferences collected by stepwise interactions could still lead to an MPS (Yang, 2002). Besides, the optimal condition can always be checked by comparing the normal vector of the efficient frontier and DMs’ indifference trade-offs (Yang and Li, 2002).

Many researchers have made significant contributions in terms of interactive DEA target exploration. Thanassoulis and Dyson (1992b) investigated the nature of the DEA Russell model and developed it into a combined target-setting model. Golany (1988b) proposed the first interactive DEA target-exploring model, in which DMs’ preference
information can be collected by determining several solution options. Stewart (2010) introduced a goal-based target-setting model that is based on the fact that the target-exploring process should consider both present performances and long-term goals, and the weights of these two concerns should be adjustable according to practical cases. Yang et al. (2009a) proposed an integrated model to estimate the MPS on the efficient frontier through collecting the local indifference trade-offs from DMs.

### 5.2.2 DEA and the equivalent MOLP models

DEA is a non-parametric technique used to assess the relative efficiency of comparable organizational units or DMUs. Suppose there are n DMUs \((j = 1, \ldots, n)\). They all produce \(s\) types of outputs, where \(y_{rj}\) denotes the amount of “\(r\)th output” produced by \(DMU_j\). At the same time, they all consume \(m\) types of inputs, where \(x_{ij}\) represents the amount of “\(i\)th input” consumed by \(DMU_j\). We define the inverse of \(h_0\) and \(\theta_{jo}\) as the efficiency score and \(\lambda_j\) as the composite variable. The output-oriented CCR model is given by Cooper et al. (1978):

\[
\begin{align*}
\text{Max} & \quad h_0 = \theta_{jo} \\
\text{s. t.} & \quad \theta_{jo} y_{rjo} - \sum_{j=1}^{n} \lambda_j y_{rj} \leq 0, \quad r = 1, \ldots, s, \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ijo}, \quad i = 1, \ldots, m, \\
& \quad \lambda_j \geq 0, j = 1, \ldots, n \\
\end{align*}
\]

The output-oriented CCR model is equivalent to a minimax ideal point model (Yang et al. 2009):

\[
\begin{align*}
\text{Min} & \quad \theta \\
\end{align*}
\]
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

\[
\begin{align*}
&w_r \left( f_r^* - \sum_{j=1}^{n} \lambda_j y_{rj} \right) \leq \theta \quad r = 1, \ldots, s \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij_0} \quad i = 1, \ldots, m, \\
&\lambda_j \geq 0, \quad j = 1, \ldots, n
\end{align*}
\]  

(5.2)

\(w_r\) is the relative weight for each output. \(f_r^*\) is the optimal feasible value for the \(r\)th composite output. \(f_r^*\) can be obtained by solving the following series of single-objective linear programming problems:

\[
\begin{align*}
&\text{Max} \quad f_r^* = \sum_{j=1}^{n} \lambda_j y_{rj} \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij_0} \quad i = 1, \ldots, m, \\
&\lambda_j \geq 0, \quad j = 1, \ldots, n
\end{align*}
\]  

(5.3)

The definitions of the remaining variables are the same as in the DEA CCR model. Yang et al. (2009a) proved that under the following conditions, (5.3) is identical to the DEA CCR model:

\[
\begin{align*}
w_r &= \frac{1}{y_{rj_0}} \\
f_r^* &= \frac{f_{\text{max}}}{w_r} \\
\theta &= f_{\text{max}} - \theta_{j_0} \\
f_{\text{max}} &= \max_{1 \leq r \leq s} \{w_r f_r^*\}
\end{align*}
\]  

(5.2.2d)

When \(w_r\) is changed as required, Equation (5.2) is able to identify any efficient solution on the efficient frontier within the same objective and decision space as in the DEA CCR model.
5.2.3 Normal vector identification and interactive trade-off analysis using the minimax formulation

Given a positive weight vector $\mathbf{w} = \{w^t_1 \cdots w^t_r \cdots w^t_s\}$, the minimax model (5.2) can generate an efficient solution $\boldsymbol{\lambda} = \{\lambda^t_1 \cdots \lambda^t_r \cdots \lambda^t_s\}$. Suppose $\beta_r$ is the Lagrange multiplier for the objective constraints $w_r(f^*_r - \sum_{j=1}^{n} \lambda_j y_{rj}) \leq \theta$ in the minimax model (5.2). Yang and Li (2002) have proven that the normal vector $\mathbf{N}$ at $f(\boldsymbol{\lambda}^0)$ on the efficient frontier is:

$$\mathbf{N}^t = [w^t_1\beta^t_1 \cdots w^t_r\beta^t_r \cdots w^t_s\beta^t_s]$$  \hspace{1cm} \text{(5.2.3a)}

Although DMs’ utility function cannot be measured in a specific mathematical form, the local utility gradient can be estimated in various ways. One of them is indifference trade-offs or marginal rates of substitution, $M$. The indifference trade-off $M^t_{1r}$ between the first and the $r$th objectives and the marginal rate of substitution $M^t$ at $f(\boldsymbol{\lambda}^t)$ are given by Yang et al. (2009b):

$$M^t_{1r} = \frac{df^t_1}{df^t_r} \text{ and } M^t = [M^t_{12}, \ldots, M^t_{1r}, \ldots, M^t_{1s}]^T$$  \hspace{1cm} \text{(5.2.3b)}

As long as the MPS is not achieved, a new trade-off direction on $f(\boldsymbol{\lambda}^0)$, $\Delta \bar{u}^t$, can be calculated to improve the DMs’ utility, using $M^t$ and normal vector $\mathbf{N}^t$.

$$\Delta \bar{u}^t = [\Delta f^t_1, \ldots, \Delta f^t_r, \ldots, \Delta f^t_s]^T = -M^t + \frac{(M^t)^TN^t}{(N^TN^t)^T}N^t$$  \hspace{1cm} \text{(5.2.3c)}

5.3 Prioritized interactive MOO model

5.3.1 Sampled efficient frontier in the DMs’ preferred trade-off direction

Before introducing the prioritized interactive model, an approach of sampling the efficient frontier towards DMs’ trade-off direction needs to be illustrated. Given an efficient
solution \( f(\lambda^t) \), a trade-off direction \( \Delta \bar{u}^t \) that improves DMs’ utility can be calculated by applying Equations 5.2.3a–5.2.3c (Yang et al., 2009b). This means that following the direction of \( \Delta \bar{u}^t \) and moving a sufficiently small step, the DMs’ utility value can be guaranteed to be improved. However, the challenge is determining the step size. If it is possible to illustrate the real efficient frontier following \( \Delta \bar{u}^t \) to the DMs, it will help them to determine the trade-off steps. A sampling method is utilized here to illustrate the specific part of the efficient frontier that follows the DMs’ local trade-off direction. This approach can be utilized for any concave feasible space. Because the DEA PPS is also a concave set, a DEA example is used here to illustrate this sampling approach.

Suppose the initial efficient solution presented to the DMs is \( f(\lambda^t)_0 \) and their preferred trade-off direction is \( \Delta u^t \). Following this direction, suppose the initial efficient solution moves from \( f(\lambda^t)_0 \) to \( f'(\lambda^t)_n \) (Figure 5.1), which is decided by:

\[
f'(\lambda^t)_n = f(\lambda^t)_0 + \alpha \times \Delta \bar{u}^t
\]

\( \alpha \) is the step size. \( f'(\lambda^t)_n \) can be used as a reference point to locate the corresponding efficient solution on the efficient frontier. If the connection line between the ideal point and \( f'(\lambda^t)_n \) is extended, it will reach the efficient frontier at \( f(\lambda^t)_n \). The corresponding weight \( w^t_{rN} \) at \( f(\lambda^t)_n \) in the minimax ideal point model (5.3) can be calculated by:

\[
w^t_{rN} = \frac{f^t_0(\lambda^t)_n - f^t_1}{f^t_2(\lambda^t)_n - f^t_1}
\]

Putting \( w^t_{rN} \) in Equation (5.3) and solving it, the solution will be \( f(\lambda^t)_n \).
If the initial solution is set as the starting point and $\Delta T$ is the step size, part of the efficient frontier in the DMs’ preferred trade-off direction can be sampled (Figure 5.2). In Figure 5.2, for example, the initial solution $f(\lambda^t)_0$ is the starting point and $\Delta T$ is the step size. Following the trade-off direction $\Delta u^t$, reference points $f'(\lambda^t)_1$, ..., $f'(\lambda^t)_5$ can be sampled, and the corresponding efficient solutions $f(\lambda^t)_1$, ..., $f(\lambda^t)_5$ can also be obtained. $f(\lambda^t)_5$ is the boundary of the efficient frontier because moving further away from $f'(\lambda^t)_5$ will lead to inefficient or weakly efficient solutions. In a concave PPS, this boundary can be identified by checking the normal vectors of the sampled efficient solutions.
Figure 5.2 DMs’ preferred trade-off efficient frontier

It will be more intuitive for the DMs to visualize the efficient frontier in the trade-off direction if the sampled efficient frontier can be illustrated through a payoff figure or a payoff table. Figure 5.2 is a payoff figure that samples an efficient frontier of an MOO problem with five objectives. The initial solution is $f(\lambda^0) = \{f_1(\lambda^0), ..., f_5(\lambda^0)\}$, and the sampled efficient frontier is $f(\lambda^1), ..., f(\lambda^N)$. DMs are free to select any objective as the x-axis, and the other objectives act as the y-axes in their corresponding figures. The advantage of the payoff figure is that the DM is able to observe the change in other objectives when a certain objective is adjusted. They can decide to stop when a certain objective is outside their acceptable range. Similarly, the payoff table (Table 5.1) can provide visual support.
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

5.3.2 An interactive trade-off model under semi-dominant priority

Step 1: Problem definition

Suppose an optimization problem has $s$ objectives; it can generally be described as follows (Yang, 2009):

<table>
<thead>
<tr>
<th>Initial solution</th>
<th>Sampled solution (1)</th>
<th>Sampled solution (2)</th>
<th>...</th>
<th>Sampled solution (A)</th>
<th>...</th>
<th>Sampled solution (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output 1</td>
<td>$f_1(\lambda^0)_0$</td>
<td>$f_1(\lambda^0)_1$</td>
<td>...</td>
<td>$f_1(\lambda^0)_A$</td>
<td>...</td>
<td>$f_1(\lambda^0)_N$</td>
</tr>
<tr>
<td>Output 2</td>
<td>$f_2(\lambda^0)_0$</td>
<td>$f_2(\lambda^0)_1$</td>
<td>...</td>
<td>$f_2(\lambda^0)_A$</td>
<td>...</td>
<td>$f_2(\lambda^0)_N$</td>
</tr>
<tr>
<td>Output 3</td>
<td>$f_3(\lambda^0)_0$</td>
<td>$f_3(\lambda^0)_1$</td>
<td>...</td>
<td>$f_3(\lambda^0)_A$</td>
<td>...</td>
<td>$f_3(\lambda^0)_N$</td>
</tr>
<tr>
<td>Output 4</td>
<td>$f_4(\lambda^0)_0$</td>
<td>$f_4(\lambda^0)_1$</td>
<td>...</td>
<td>$f_4(\lambda^0)_A$</td>
<td>...</td>
<td>$f_4(\lambda^0)_N$</td>
</tr>
<tr>
<td>Output 5</td>
<td>$f_5(\lambda^0)_0$</td>
<td>$f_5(\lambda^0)_1$</td>
<td>...</td>
<td>$f_5(\lambda^0)_A$</td>
<td>...</td>
<td>$f_5(\lambda^0)_N$</td>
</tr>
</tbody>
</table>
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

\[
\begin{align*}
\text{Max } f(\lambda) &= [f_1(\lambda), \ldots, f_r(\lambda), \ldots, f_s(\lambda)] \\
\text{s.t. } X \in \Omega &= \{x | g_j(\lambda) \leq 0, h_l(\lambda) = 0; j = 1, \ldots, k_1, l = 1, \ldots, k_2\}
\end{align*}
\] (5.4)

\(X \in \Omega\) is a feasible set. \(f_r(\lambda)\) \((r=1, \ldots, s)\) is the continuously differentiable objective function. \(g_j(\lambda)\) \((j = 1, \ldots, k_1)\) and \(h_l(\lambda)\) \((l = 1, \ldots, k_2)\) are the continuously differentiable inequality and equality constraint functions, respectively. In DEA target setting, \(f_r(\lambda)\), \(g_j(\lambda)\), and \(h_l(\lambda)\) are all assumed to be linear. However, in general MOO cases, \(f_r(\lambda)\), \(g_j(\lambda)\), and \(h_l(\lambda)\) could be non-linear. When including semi-dominant priority in 5.2.3a, the new multi-objective problem can be redefined as follows. Suppose an optimization problem wants to optimize \(s\) objectives with semi-dominant priorities (from low to high) within the feasible space \(\Omega\):

\[
\begin{align*}
\text{Max } f(\lambda) &= [f_1(\lambda), \ldots, f_r(\lambda), \ldots, f_s(\lambda)] \\
\lambda &\in \Omega
\end{align*}
\] (5.5)

The semi-dominant priority for \(f_1(\lambda), \ldots, f_r(\lambda), \ldots, f_s(\lambda)\) is denoted as:

\[p = p(f_1), \ldots, p(f_r), \ldots, p(f_s) = 1, \ldots, r, \ldots, s\] (5.6)

For any two objectives \(f_a(\lambda)\) and \(f_b(\lambda)\) \((a, b \in 1, \ldots, s\) and \(a \neq b)\), if \(p_a > p_b\), \(f_a(\lambda)\) has a higher semi-dominant priority than \(f_b(\lambda)\). \(f_s(\lambda)\) has the highest semi-dominant priority, and \(f_1(\lambda)\) has the lowest semi-dominant priority. DMs’ initial non-dominant region for \(f(\lambda)\) is:

\[
f_r(\lambda) \geq U^0_r \\
r = 1, \ldots, s
\]

\[U^0 = U^0_1, \ldots, U^0_r, \ldots, U^0_s\]

Within this non-dominant priority region, selected objectives have the superiority to be improved, regardless of sacrificing objectives with lower priority. On the other hand, if an
objective is in the trade-off region, it can be traded off with other low-priority objectives. The rest of the definitions are the same as in (5.2).

Step 2: Produce an initial efficient solution

An initial target \( f(\lambda^0) = [f_1(\lambda^0) \cdots f_r(\lambda^0) \cdots f_s(\lambda^0)] \) is generated, according to branches’ practical situations. If a branch hopes to maintain the existing product structure and improve all outputs in equal proportion, then the reference point of a two-stage CCR model (Cooper et al., 2007) could be used as an initial target. If a branch hopes to find an efficient benchmark that is similar to the current performance, it could choose the minimum distance model as the initial solution (Aparicio et al., 2007). If a branch has long-term goals, then a goal-directed benchmarking model is more suitable to set an initial target (Stewart, 2010).

Step 3: Prioritized trade-off

1. Set \( r = 1 \) and \( t = 1 \).
2. If \( r = s + 1 \), stop the interactive procedure. Otherwise, go to 3.
3. If \( f_r(\lambda^{t-1}) \geq U_r \), set \( r = r + 1 \) and go back to 2. Otherwise, go to 4.
4. Solve the following single-objective optimization problem:

\[
\begin{align*}
\text{Max } & f_r(\lambda^t) \\
\text{s.t. } & \lambda \in \Omega \\
& f_l(\lambda^t) \geq f_l(\lambda^{t-1}), \quad l \geq r
\end{align*}
\]

The obtained solution is denoted as \( \lambda_{max}^t \). If \( f_r(\lambda_{max}^t) \geq U_r \), set \( f(\lambda^{t-1}) = f(\lambda_{max}^t) \), \( r = r + 1 \), and go back to 2. Otherwise, go to 5.

5. Set \( f(\lambda^{t-1}) = f(\lambda_{max}^t) \) and collect the indifference trade-off on \( f(\lambda^{t-1}) \). By using Equations 5.2.3a–5.2.3c, the utility improvement gradient \( \Delta \tilde{u}^t \) can be calculated. Following the sampled efficient frontier approach, a payoff table (Table 5.2) and a payoff figure can be obtained. The table and figure can help DMs to decide the trade-off steps following their preferred direction.
### Table 5.2 Payoff table

<table>
<thead>
<tr>
<th>Objective</th>
<th>Initial solution</th>
<th>Sampled solution (1)</th>
<th>Sampled solution (2)</th>
<th>...</th>
<th>Sampled solution (A)</th>
<th>...</th>
<th>Sampled solution (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f_1(\lambda^{t-1})_0$</td>
<td>$f_1(\lambda^{t-1})_1$</td>
<td>$f_1(\lambda^{t-1})_2$</td>
<td>...</td>
<td>$f_1(\lambda^{t-1})_n$</td>
<td>...</td>
<td>$f_1(\lambda^{t-1})_N$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>r</td>
<td>$f_r(\lambda^{t-1})_0$</td>
<td>$f_r(\lambda^{t-1})_1$</td>
<td>$f_r(\lambda^{t-1})_2$</td>
<td>...</td>
<td>$f_r(\lambda^{t-1})_n$</td>
<td>...</td>
<td>$f_r(\lambda^{t-1})_N$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>s</td>
<td>$f_s(\lambda^{t-1})_0$</td>
<td>$f_s(\lambda^{t-1})_1$</td>
<td>$f_s(\lambda^{t-1})_2$</td>
<td>...</td>
<td>$f_s(\lambda^{t-1})_n$</td>
<td>...</td>
<td>$f_s(\lambda^{t-1})_N$</td>
</tr>
</tbody>
</table>

This trade-off table can illustrate which objectives the DM wants to improve, which objectives the DM wants to sacrifice, and which objectives the DM wants to keep the same.

Suppose objective $r$ is decreasing: that means the DM does not want to sacrifice any other objectives to improve objective $r$, so set $r = r + 1$ and go back to 2. Suppose objective $r$ is increasing: that means the DM would like to sacrifice other objectives with higher priority to improve objective $r$. Then, the DM could choose a new efficient solution $f(\lambda^{t-1})_n$ to replace $f(\lambda^{t-1})$ in the payoff table, with $f_r(\lambda^{t-1})_n \geq f_r(\lambda^{t-1})$ but $f_t(\lambda^{t-1})_n \geq U_t$, $l \geq r$.

This process can keep going until $f_r(\lambda^{t-1})_n \geq U_r$. Then, set $r = r + 1$ and go back to 2.

The procedure of the interactive model is as follows:

1. Produce an initial target.
2. Check the objectives sequentially from high priority to low priority to see whether the assessed objective is inside the trade-off region or the dominant region. If it is inside the trade-off region, check the next objective. Otherwise, follow the next step.
3. Check whether the assessed objective can be improved by sacrificing the objectives with lower priority. If yes, improve the assessed objective to the optimal value, which is located in the trade-off region, only by sacrificing objectives with lower priority, and check the next objective. Otherwise, follow the next step.
4. Collect the indifference trade-offs and calculate the utility improvement gradient on the optimal solution from the last step and create the payoff figure and payoff table.

If the figure and table show that the DM does not hope to improve the assessed
objective by sacrificing objectives with higher priority, keep the assessed objective inside the dominant region and check the next objective. Otherwise, it can be gradually improved until it reaches the trade-off region, and check the next region.

5.4 Bank branches’ prioritized target setting with product constraints

For bank branches’ management, individualization is a general trend. Following this tendency, branches are given more freedom to allocate resources and plan their future targets than before. However, without comparing peers’ performances, branches cannot identify their weaknesses and strengths easily. Moreover, the feasibility of targets is a major concern of branches. As a result, the unilateral targets proposed by branches could be over-optimistic or too pessimistic. On the other hand, as a supervision and support organization, head offices need to guide or restrict the development of branches, as branches’ development should be consistent with banks’ overall strategies. However, because head offices seldom operate business directly, they do not clearly know about the specific situations and environments of branches. Therefore, the targets proposed by bank head offices could be unrealistic.

Considering the different orientations and business views, there is obviously conflicting preference information between the bank head office and branch managers. As a result, this chapter also introduces two types of DEA production constraints in order to include the expectations or regulations from bank head office managers. These constraints will first narrow down the DEA PPS into a head-office-manager-preferred feasible space, and then the prioritized trade-off model can be used to support branch managers to explore their MPS in the efficient frontier.

Several papers have contributed to banks’ target setting using DEA and MOLP models, but none of them has incorporated the concept of semi-dominant priority. Lovell and Pastor
(1997) investigated the target-setting procedure employed by a large financial institution. They first evaluated the targets themselves to eliminate redundant targets. After that, a reduced set of influential targets was utilized to re-evaluate the performances of branch offices. Camanho and Dyson (1999) applied DEA to the performance assessment of a Portuguese bank’s branches to display how DEA could complement the profitability measure currently used at the bank. Two alternative target-setting strategies are explored in this chapter. Yang et al. (2009a) proposed the gradient projection method to identify the MPS for managers. The method depends on the DM providing marginal rates of substitution as trade-off information to estimate the local preference information or the gradient of the utility function at the efficient solution, although this process is supported by the identification of a normal vector on the efficient frontier.

Based on previous research, a three-stage model is proposed to help two different levels of DMs to explore compromise targets on the DEA efficient frontiers. Figure 5.4 illustrates the general structure of the three-stage model. The first stage aims to evaluate DMUs’ efficiency. After that, head office managers’ preference information is collected in the second stage, which narrows the DEA PPS into a preferred PPS. In the third stage, a stepwise interactive model is proposed to help branch managers to explore their preferred targets within the narrowed PPS.

![Three-stage DEA target-setting model](image)

**Figure 5.4 Three-stage DEA target-setting model**

### 5.4.1 Stage 1: Efficiency evaluation and criteria validation

In the first stage, each branch’s efficiency is evaluated using a CCR model. If there is an obvious conflict between the experiences of the branch’s managers and the efficiency
evaluation generated, there should be a discussion on whether there are any missing input or output criteria or whether any redundant criteria have been considered.

**5.4.2 Stage 2: Head office’s preference collection**

If the efficiency evaluation results are consistent with the managers’ experience in general, the analysis goes on to the second stage, which aims to collect preference information from the head office. The head office may require its strategies, guidelines, policies, and regulations to be included in branches’ performance assessment. Regarding DEA, this information can be transformed into different types of production constraints. The PPS describes a feasible region that has been or can be achieved by DMUs. The production constraints could further narrow down the PPS into a space that is preferred by the head office.

Two types of constraints are taken into account in this chapter: absolute value constraints and economic-value-added constraints, although more types of constraints may also be considered.

**5.4.2.1 Absolute value constraints**

Absolute value constraints aim to restrict the absolute values of outputs or inputs. Athanassopoulos et al. (1999) proposed similar ideas using the Russell model. For example, a bank may require that the total deposit value of each branch be no less than £5 million in order to prevent a predictable financial crisis. In this case, a target with less than £5 million of deposits may not be preferred by the managers, but it may be given as an efficient solution by the DEA model. Absolute value constraints can restrict the flexibility of inputs or outputs to make sure that the obtained targets do not exceed the upper or lower boundaries preferred by the DMs. In general, the mathematical form of absolute value constraints is given by Equation (5.7):

\[
O_{rl} \leq \sum_{j=1}^{n} \lambda_j y_{rj} \leq O_{ru} \quad r = 1, ..., s
\]
Some concerns should be noted about the absolute constraints when used in practice. For an output-oriented DEA model with absolute value constraints for outputs, it is possible that no feasible solution can be found. This is because the lower bounds of some outputs are so high that no DMU has ever achieved this lower bound using the assessed DMU’s inputs. In this situation, the DMs will be asked to reconsider the boundaries.

Figure 5.5 graphically illustrates absolute value constraints. Suppose $B$ is the assessed DMU. $B_0$ is the radial improvement solution identified by the CCR model. When $O_{1l}$ (the lower bound for output 1) is lower than $B_0$, $B_0$ is still the solution for the constrained model. However, if $O_{1l}$ is larger than $B_0$, $B_0$ is not the preferred solution. $B_1$ becomes the new solution. If $O_{1l}$ is larger than $A$’s output 1 (the maximum feasible value for output 1), no feasible solution can be found. Similar principles can also be applied to the input-oriented model with absolute value constraints for inputs.

![Figure 5.5 Absolute constraints illustration](image-url)
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It is not suggested to apply absolute value constraints for inputs in output-oriented models. If the upper boundaries are higher than the assessed DMU’s input level, then it will not influence the final result. If the upper boundaries are lower than the assessed DMU’s input level, then the meaning of efficiency may be changed for the output-oriented DEA model.

For non-oriented models or generalized models (e.g. SBM models and Russell models), DMs are free to set constraints on both inputs and outputs in the same model, but it is still possible for no feasible solution to be found under the given constraints. One way to avoid having no feasible solution is to show managers the maximum and minimum feasible values of each output (or the minimum feasible value for each input). The maximum values can be obtained by solving Equation (5.3).

5.4.2.2 Economic-value-added constraints

The concept of “economic value added” (EVA) was proposed in 1993 (Tully, 1993). Since then, it has become a popular performance measurement technique in the banking industry. For bank branches, deposits and loans are the two major production outputs. However, the contributions of deposits and loans to final profits are indirect. In order to calculate the total profits produced by branches, EVA proposes that each type of output be given an EVA “price”. The total EVA can be calculated by multiplying the quantity of outputs by the corresponding prices and summing up:

\[
\text{Total EVA} = \sum_{r=1}^{s} \text{Price}_r \times y_{rj0}
\]

(5.8)

Compared with the quantity of outputs, the total EVA more directly presents the overall profit that is produced by branches. Besides, a bank’s head office can guide its branches’ development through adjusting the prices of outputs and setting targets for the total EVA.
Preference information can also be incorporated into DEA target-setting models in the form of constraints. We denote $Price_r$ as the EVA price for the “$r$th output” in the current year and $Price'_r$ as the EVA prices for the next year. The bank expects the total EVA to improve $\alpha\%$. The rest of the definitions are the same as in Equation (5.2). The EVA constraints can be written as:

$$\sum_{r=1}^{s} \sum_{j=1}^{n} \lambda_{ij} y_{rj} Price'_r \geq (1 + \alpha\%) \sum_{r=1}^{s} y_{rj} Price_r$$

(5.9)

Figure 5.6 illustrates the EVA constraints graphically. Only the feasible solutions in the shadow space have satisfied the total EVA.

5.4.3 Stage 3: Branches’ preference collection

The third stage of the model aims to explore the efficient benchmarks for branches within the narrowed DEA feasible space as preferred by head offices. The basic ideas of the integrated model proposed by Yang et al. (2009a) are adopted and further developed in a stepwise process in this section.
As DMs’ overall utility functions are unknown a priori in general, providing them with the freedom to intuitively learn about the efficient frontier and explore the targets might be a more reasonable method of target setting. The stepwise model proposed below could give DMs the freedom to conduct trade-off analysis between initial targets’ outputs. In addition, it could help DMs to determine the size of trade-off steps. The stepwise model could also show the trade-off relationships through a trade-off figure.

5.5 A case study of a commercial bank in China

5.5.1 Application background

The three-stage model has been applied to performance analysis and target setting for a commercial bank located in southern China, which is referred to as “Bank A”. Bank A adopts a two-level organizational structure. On the one hand, the lower-level hierarchy of Bank A consists of 124 branches that operate directly with customers. On the other hand, the head office of the bank is responsible for supervising and supporting the branches.

Target setting and performance assessment are the two major approaches for the head office to supervise and guide the branches. At the beginning of a year, the branches are given a series of targets by the head office. At the end of the year, the completed proportion of the targets is used as a fundamental performance assessment index for the branches. However, the conflict between the branches and the head office makes target setting very difficult. On the one hand, the head office wants a target to be challenging and to embody the bank’s overall strategy. On the other, the branches care more about the feasibility of a target. Therefore, a technique that could help the head office and the branches to find a compromise target is needed by the bank.

The advantages of utilizing the three-stage DEA target-setting model here are obvious. Firstly, the exploration of targets will be limited inside a DEA PPS, which to some extent guarantees the feasibility of the targets. Secondly, the benchmarks proposed by the three-
stage model are efficient solutions, which can guide the branches to achieve the best performances. Thirdly, the preference information from both the head office and the branches is collected and incorporated into the target-setting model.

5.5.2 Input/output selection and general data statistics

All the branches of Bank A operate in the same business areas, including deposits, loans, and intermediary business. The deposits are further categorized into deposits from corporations and deposits from personal customers. Similarly, loans are further categorized into loans for corporations and loans for personal customers. Like other traditional banks, Bank A hopes to increase its amount of loans and deposits to make more profits from the interest. Extra intermediary business could also generate more service income. The investments of the branches generally include staff and overheads. The output and input criteria are summarized as follows:

1. Outputs:
   a) Personal deposits
   b) Corporate deposits
   c) Personal loans
   d) Corporate loans
   e) Intermediary business

2. Inputs:
   a) Staff number
   b) Overheads

Normally, the cost of branches is relatively stable. Bank A does not tend to lay off many staff or reduce its branches’ budgets. Therefore, an output-oriented model is more suitable for Bank A.

DEA requires the criteria to be independent from each other within inputs or within outputs. The Pearson correlation test is used to determine the correlations among the criteria. The results show that most of the criteria have very strong correlations with each other.
However, the correlation could come from two aspects. Different sizes could be one aspect. Normally, a branch of a larger size tends to have better performance regarding all types of outputs because it is equipped with more resources or inputs. In other words, the high correlation between different outputs could result from the high correlation between outputs and branch size. This type of correlation will not influence evaluation results because all the product outputs will be scaled to the same size.

Another correlation may exist between different outputs themselves. For example, a branch with a remarkable performance in “corporate loans” may have established good relationships with companies. These relationships will help branches to perform well in the “corporate deposits” and “intermediary business” areas. This type of correlation will have an impact on the weights of outputs and will influence the results of efficiency evaluation.

The authors have not found a suitable way to distinguish these two types of correlations. Therefore, this research has to assume that all the correlations between outputs are due to the correlation between outputs and size, and there is no correlation between outputs and inputs themselves.

Among these 124 branches, seven branches are newly established. It is not fair to compare them with the well-established branches. Besides, another 15 branches are equipped with special business advantages, which cannot be reproduced by other branches. For example, one branch is especially responsible for providing financial services to government-owned companies. Although this branch has outstanding performance, it is impossible for other branches to find similar opportunities. 102 secondary branches or DMUs are evaluated using the three-stage DEA model. The performance data of this chapter is collected in the early 2016.

The general statistics are displayed in Table 5.3. Because of confidentiality, the data have been transformed and normalized.
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Table 5.3 General data statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outputs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deposit from individual</td>
<td>88.36</td>
<td>39.20</td>
<td>10.21-187.62</td>
</tr>
<tr>
<td>Deposit from corporation</td>
<td>45.54</td>
<td>48.17</td>
<td>1.70-313.75</td>
</tr>
<tr>
<td>Loan for individual</td>
<td>43.78</td>
<td>28.08</td>
<td>5.27-157.89</td>
</tr>
<tr>
<td>Loan for corporation</td>
<td>46.59</td>
<td>64.38</td>
<td>0.01-424.54</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>9.78</td>
<td>7.01</td>
<td>0.88-55.72</td>
</tr>
<tr>
<td>Inputs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stuff Number</td>
<td>20.08</td>
<td>4.87</td>
<td>12.00-24.00</td>
</tr>
<tr>
<td>Overheads</td>
<td>53.85</td>
<td>17.21</td>
<td>26.94-130.65</td>
</tr>
</tbody>
</table>

5.5.3 Stage 1: Efficiency evaluation using the CCR model

The CCR model (5.1) is first utilized to test the DMUs’ efficiency. The results show that 16 branches are efficient, accounting for 15.69% of all 102 branches. The average efficiency score is 0.76, with a standard deviation of 0.20. Some significantly inefficient branches are identified; for example, the efficiency score of Branch 19 is only 0.01.

Beside the input and output data, the profit data of the bank are also collected. Figure 5.7 plots the relationships between the branches’ profits and their efficiency scores. This graph can be divided into four regions:

1. Very few branches are located in the high-efficiency and high-profit region. These branches are the most-preferred branches by Bank A.
2. A large number of branches are located in the low-efficiency and low-profit zone. These branches should find an approach to improve their efficiency.
3. No branches are located in the low-efficiency and high-profit zone. Given the nature of Bank A, its branches’ low efficiency inevitably leads to low profits.
4. Quite a few branches obtain relatively high efficiency scores with very limited profits. The small interest difference could be one reason to explain this situation.
5.5.4 Stage 2: Head office’s preference collection

In Stage 2, two types of constraint data are collected from the head office: absolute value constraints and EVA constraints.

5.5.4.1 Absolute value constraints

According to its long-term strategies and plans, Bank A has set a minimum overall target for each output. For example, Bank A requires that the total deposit from personal customers be no less than 800. Each branch then sets its personal deposit target to be more than 6.56. Similar regulations also apply to the other outputs, as shown in Table 5.4.

<table>
<thead>
<tr>
<th>Output</th>
<th>Lower bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal deposit</td>
<td>6.56</td>
</tr>
<tr>
<td>Corporate deposit</td>
<td>9.84</td>
</tr>
<tr>
<td>Personal loan</td>
<td>5.49</td>
</tr>
<tr>
<td>Cooperate loan</td>
<td>8.20</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>2.05</td>
</tr>
</tbody>
</table>

Figure 5.7 Scatter graph of profits and efficiency scores
5.5.4.2 EVA constraints

For Bank A, different outputs make different degrees of contributions to profits. From 2005 to 2015, China went through an explosive growth period. During this period, a new company was established every six minutes, and credit demand was extremely high. For Bank A, deposits to a large extent determine its profits. Therefore, Bank A gives much higher EVA prices to deposits than loans. Besides, intermediary business is a growing market in China. Bank A hopes that its branches could possess a larger market share of it. The EVA prices for outputs are thus set as follows:

Table 5.5 EVA prices for Bank A

<table>
<thead>
<tr>
<th>Outputs</th>
<th>EVA prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal deposit</td>
<td>15</td>
</tr>
<tr>
<td>Corporate deposit</td>
<td>15</td>
</tr>
<tr>
<td>Personal loan</td>
<td>3</td>
</tr>
<tr>
<td>Cooperate loan</td>
<td>3</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>14</td>
</tr>
</tbody>
</table>

5.5.4.3 Interactive process with branch managers

The two types of constraints collected above narrow the existing PPS into a head-office-preferred space. Five branch managers are then invited to determine the specific targets preferred by their branches through the stepwise interactive process. This chapter illustrates the interactive process for one of the branches (Branch 79).

In the first step, initial targets are generated by the DEA goal-based target-setting model (Stewart (2010)):

Table 5.6 Initial targets for Branch 79

<table>
<thead>
<tr>
<th></th>
<th>Existing performance</th>
<th>Initial outputs</th>
<th>Normal vector</th>
<th>Initial weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal deposit</td>
<td>113.1577</td>
<td>98.6155</td>
<td>0.0608</td>
<td>3</td>
</tr>
<tr>
<td>Cooperated deposit</td>
<td>9.8827</td>
<td>177.2207</td>
<td>0.3878</td>
<td>1</td>
</tr>
<tr>
<td>Personal loan</td>
<td>53.6810</td>
<td>79.3948</td>
<td>0.285</td>
<td>2</td>
</tr>
<tr>
<td>Corporate loan</td>
<td>0.9000</td>
<td>186.3764</td>
<td>0.1658</td>
<td>2</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>15.8916</td>
<td>22.1755</td>
<td>3.2991</td>
<td>9</td>
</tr>
<tr>
<td>EVA</td>
<td>2231.8341</td>
<td>5245.3135</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
None of the elements of the normal vector is equal to 0, which means that the initial targets generated by the goal-based target-setting model are CCR efficient. All outputs of the initial targets satisfy the absolute value constraints (Table 5.4.1), and the EVA increases by 135%, which suggests that the initial targets qualify as starting solutions to conduct the stepwise interactive process.

The branch managers provide two types of feedback: the priorities of the outputs and comments on each output. The feedback from Branch 79 is summarized in Table 5.7.

According to the branch’s feedback, the following trade-off table can be generated by following the instructions of stepwise model Step 4:

<table>
<thead>
<tr>
<th>Initial target output</th>
<th>Comments</th>
<th>Output priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.6155</td>
<td>High enough</td>
<td>1</td>
</tr>
<tr>
<td>177.2207</td>
<td>Too low</td>
<td>4</td>
</tr>
<tr>
<td>79.3948</td>
<td>High enough</td>
<td>2</td>
</tr>
<tr>
<td>186.3764</td>
<td>High enough</td>
<td>3</td>
</tr>
<tr>
<td>22.1755</td>
<td>Too low</td>
<td>5</td>
</tr>
</tbody>
</table>

According to the branch’s feedback, the following trade-off table can be generated by following the instructions of stepwise model Step 4:

<table>
<thead>
<tr>
<th></th>
<th>SS (1)</th>
<th>SS (2)</th>
<th>SS (3)</th>
<th>SS (4)</th>
<th>SS (5)</th>
<th>SS (6)</th>
<th>SS (7)</th>
<th>SS (8)</th>
<th>SS (9)</th>
<th>SS (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual deposit</td>
<td>98.6155</td>
<td>98.2867</td>
<td>97.6290</td>
<td>96.6420</td>
<td>95.3254</td>
<td>93.6788</td>
<td>91.7020</td>
<td>89.3817</td>
<td>86.7125</td>
<td>83.7059</td>
</tr>
<tr>
<td>Corporate deposit</td>
<td>177.2207</td>
<td>177.2929</td>
<td>177.4373</td>
<td>177.6540</td>
<td>177.9431</td>
<td>178.3046</td>
<td>178.7386</td>
<td>179.2148</td>
<td>179.7254</td>
<td>180.3004</td>
</tr>
<tr>
<td>Individual loan</td>
<td>79.3948</td>
<td>79.3410</td>
<td>78.2333</td>
<td>79.0716</td>
<td>78.8560</td>
<td>78.5863</td>
<td>78.2626</td>
<td>77.8688</td>
<td>77.4003</td>
<td>76.8725</td>
</tr>
<tr>
<td>Corporate loan</td>
<td>186.3764</td>
<td>186.1764</td>
<td>185.7762</td>
<td>185.1757</td>
<td>184.3747</td>
<td>185.3729</td>
<td>182.1701</td>
<td>180.7492</td>
<td>179.1643</td>
<td>177.2315</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>22.1755</td>
<td>22.1878</td>
<td>22.2124</td>
<td>22.2492</td>
<td>22.2984</td>
<td>22.3599</td>
<td>22.4337</td>
<td>22.5365</td>
<td>22.6075</td>
<td>22.7100</td>
</tr>
</tbody>
</table>

SS: Sampled Solution
The managers of Branch 79 identify the personal deposits of Sampled Solution (SS) 4 as their most-preferred target value. They want to set it and conduct the trade-off analysis among other outputs. Therefore, SS4 becomes the new initial solution. The comments from the managers are collected:

Table 5.9 Comments on the new initial targets after the first interaction

<table>
<thead>
<tr>
<th>New initial target output after the first trade-off</th>
<th>Comments</th>
<th>Decision priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.6420</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>177.6540</td>
<td>Too high</td>
<td>4</td>
</tr>
<tr>
<td>79.0716</td>
<td>Too high</td>
<td>2</td>
</tr>
<tr>
<td>185.1757</td>
<td>Too high</td>
<td>3</td>
</tr>
<tr>
<td>22.2492</td>
<td>Too low</td>
<td>5</td>
</tr>
</tbody>
</table>
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

Considering the new comments on the new initial targets, a new trade-off table and new trade-off figures can be generated following the same steps as in the first interaction.

Table 5.10 Second interaction trade-off table

<table>
<thead>
<tr>
<th></th>
<th>SS (1)</th>
<th>SS (2)</th>
<th>SS (3)</th>
<th>SS (4)</th>
<th>SS (5)</th>
<th>SS (6)</th>
<th>SS (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal deposit</td>
<td>96.6420</td>
<td>96.6420</td>
<td><strong>96.6420</strong></td>
<td>96.6420</td>
<td>96.6420</td>
<td>96.6420</td>
<td>96.6420</td>
</tr>
<tr>
<td>Corporate deposit</td>
<td>177.6540</td>
<td>177.6386</td>
<td><strong>177.6077</strong></td>
<td>177.5614</td>
<td>177.4997</td>
<td>177.4225</td>
<td>177.3298</td>
</tr>
<tr>
<td>Personal loan</td>
<td>79.0716</td>
<td>78.8739</td>
<td><strong>78.4785</strong></td>
<td>77.8850</td>
<td>77.0936</td>
<td>76.1039</td>
<td>74.9159</td>
</tr>
<tr>
<td>Corporate loan</td>
<td>185.1757</td>
<td>185.1576</td>
<td><strong>185.1214</strong></td>
<td>185.0670</td>
<td>184.9945</td>
<td>184.9039</td>
<td>184.7951</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>22.2492</td>
<td>22.2690</td>
<td><strong>22.3086</strong></td>
<td>22.3681</td>
<td>22.4474</td>
<td>22.5465</td>
<td>22.6655</td>
</tr>
</tbody>
</table>

Figure 5.9 Second interaction trade-off figures
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

The managers of Branch 79 agree that the value of personal loans of SS3 is their most-preferred target value at this stage. Therefore, SS3 during the third trade-off analysis becomes the new initial solution. It needs to be mentioned that the personal loan value remains the same as during the second trade-off process.

<table>
<thead>
<tr>
<th>New initial target output after the second trade-off</th>
<th>Comments</th>
<th>Decision priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.6420</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>177.6077</td>
<td>Too high</td>
<td>4</td>
</tr>
<tr>
<td>78.4785</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>185.1214</td>
<td>Too low</td>
<td>3</td>
</tr>
<tr>
<td>22.3086</td>
<td>Too low</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5.11 Comments on the new initial targets after the second interaction

A new trade-off table and new trade-off figures can be generated following the same steps as in the first interaction.

<table>
<thead>
<tr>
<th></th>
<th>SS (1)</th>
<th>SS (2)</th>
<th>SS (3)</th>
<th>SS (4)</th>
<th>SS (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal deposit</td>
<td>96.6420</td>
<td>96.6420</td>
<td><strong>96.6420</strong></td>
<td>96.6420</td>
<td>96.6420</td>
</tr>
<tr>
<td>Corporate deposit</td>
<td>177.6077</td>
<td>177.4247</td>
<td><strong>177.0586</strong></td>
<td>176.5091</td>
<td>175.7761</td>
</tr>
<tr>
<td>Personal loan</td>
<td>78.4785</td>
<td>78.4785</td>
<td><strong>78.4785</strong></td>
<td>78.4785</td>
<td>78.4785</td>
</tr>
<tr>
<td>Corporate loan</td>
<td>185.1214</td>
<td>185.1226</td>
<td><strong>185.1250</strong></td>
<td>185.1286</td>
<td>185.1334</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>22.3086</td>
<td>22.3301</td>
<td><strong>22.3730</strong></td>
<td>22.4374</td>
<td>22.5233</td>
</tr>
</tbody>
</table>

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The managers of Branch 79 believe that SS3 is the best solution for corporate loans under the current circumstances. Therefore, it becomes the new initial solution in the fourth interaction. Lastly, the DMs only need to determine the trade-off between corporate deposits and intermediary business.

Table 5.13 Comments on the new initial targets after the third interaction
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

<table>
<thead>
<tr>
<th>New initial target output after the second trade-off</th>
<th>Comments</th>
<th>Decision priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.6420</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>177.0586</td>
<td>Too high</td>
<td>4</td>
</tr>
<tr>
<td>78.4785</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>185.1250</td>
<td>-</td>
<td>3</td>
</tr>
<tr>
<td>22.3730</td>
<td>Too low</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5.14 Fourth interaction trade-off table

<table>
<thead>
<tr>
<th></th>
<th>SS (1)</th>
<th>SS (2)</th>
<th>SS (3)</th>
<th>SS (4)</th>
<th>SS (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal deposit</td>
<td>96.64200</td>
<td>96.64200</td>
<td>96.64200</td>
<td><strong>96.64200</strong></td>
<td>96.64200</td>
</tr>
<tr>
<td>Corporate deposit</td>
<td>177.05859</td>
<td>176.94965</td>
<td>176.73171</td>
<td><strong>176.40472</strong></td>
<td>175.96860</td>
</tr>
<tr>
<td>Personal loan</td>
<td>78.47846</td>
<td>78.47846</td>
<td>78.47846</td>
<td><strong>78.47846</strong></td>
<td>78.47846</td>
</tr>
<tr>
<td>Corporate loan</td>
<td>185.12500</td>
<td>185.12500</td>
<td>185.12500</td>
<td><strong>185.12500</strong></td>
<td>185.12500</td>
</tr>
<tr>
<td>Intermediary business</td>
<td>22.37301</td>
<td>22.38581</td>
<td>22.41143</td>
<td><strong>22.44986</strong></td>
<td>22.50113</td>
</tr>
</tbody>
</table>

Figure 5.11 Fourth interaction trade-off figure

SS4 is determined as the final solution for Branch 79, which consists of 96.64 personal deposits, 176.40 corporate deposits, 78.48 personal loans, 185.13 corporate loans, and 22.45 intermediary business.

5.5.4.4 Validation of the three-stage model results

The results of the three-stage model are displayed and explained to both the head office managers and the branch managers for validation.
Chapter 5. A prioritized trade-off model for bank branch target setting under constraints

For the first stage of the model, both the head office managers and the branch managers agree that the efficiency results for the branches are consistent with their experiences. The head office managers compare the obtained efficiency scores with their current performance assessment method, which is a weighted sum approach. Most of the efficient branches (13 of 16) identified by the DEA CCR model rank top 40 in the current performance assessment framework. Besides, the branch managers agree that the evaluation results reflect the strengths and weaknesses of their branches.

For example, although Branch 39 obtains an efficiency score of 0.97, which is very close to efficient, there are significant slacks in corporate deposits and corporate loans. For Branch 39, this indicates that its personal business is relatively strong, but it should make more efforts to improve its corporate business. The managers totally agree with these results. The reason for its strong personal business but poor corporate business is because the branch is located in an electronic market. Most of its customers are small business owners in the market, which contributes to its outstanding personal business. However, only a few large companies are located near Branch 39, which explains why it has relatively poor corporate business.

Bank A does not have an appropriate approach to set benchmarks that consider the opinions of both head office and the branches. Both head office and the branches agree that the second and third stages of this model could provide some solid information to help them to set future business targets and improve performance.

5.6 Contributions of the prioritized model

This chapter introduced a three-stage DEA target-setting model. In the first stage, the performances of individual DMUs are assessed on a common yet fair basis without any preferences taken into account. Compared to the traditional profit-oriented performance assessment framework, DEA can provide more-insightful information about branch operations in terms of efficiency. The second and third stages of this model propose means
to find a compromise solution that can take into account conflicting opinions from head office managers and branch managers.

The prioritized trade-off model provides a new way for DMs to explore the efficient frontier in the direction they prefer, with a two-dimensional graph-based visual aid to help them to observe the changes in objectives. It enables branch managers to explore interactively the feasible space as set by the head office. During the exploration process, managers can gain better knowledge about their peers’ performances, which could help them to identify their branches’ strengths and weaknesses. The targets generated from this process have a better chance to be satisfactory to both head office and branches, which could effectively guide branches’ future development with better mutual understanding and agreement.
Chapter 6. A stepwise trade-off model for target setting based on the DEA PPS

6. A stepwise trade-off model for target setting based on the DEA PPS

In this chapter, we propose a new interactive MOLP model to help DMs to explore the MPS based on the DEA PPS. This investigation is based on an optimal condition established in the literature for the termination of an interactive MOO process by testing whether an MPS has been achieved in the sense that the DMs’ implicit utility function is maximized locally. Starting from two objectives, the new model gradually includes new objectives to help DMs to explore the MPS through an interactive process, with the assumption that the indifference trade-offs between previous objectives are fixed. The new model to a large extent releases DMs’ burden when setting targets for multiple objectives at the same time. The applicability and effectiveness of this model are further proven by a numerical example and a real case of a secondary branch of a Chinese state-owned bank through interactions with its managers.

6.1 Introduction

Setting an appropriate target plays an essential role in organizational performance analysis and improvement (Amirteimoori and Kordrostami, 2005, Bi et al., 2011, Lovell and Pastor, 1997). Some researchers even claim that a business can never succeed unless its DMs have a clear picture of the future direction of the business (Atrill and McLaney, 2009). However, obtaining a feasible but challenging target is always difficult. A common method is to reference existing peers’ performances and select the best one as the target.

DEA is a well-known performance analysis technique based on peers’ performances (Charnes et al., 1978). By comparing the performances of DMUs, DEA identifies a PPS or possibility region based on the assumption that any linear combination of DMUs is considered possible or that the performances inside the PPS are regarded as achievable. The set of best performances of the PPS is identified as the “efficient frontier”. Setting
targets by referencing the DEA efficient frontier has several advantages. On the one hand, the selected target has been achieved by existing peers, which guarantees the target’s feasibility. On the other, the efficient frontier is the set of best performances, which can guarantee that the target is advanced and challenging.

A number of DEA target-setting models have been designed to help DMs to set appropriate targets. The DEA SBM model (Tone, 2001) and its developed models are popular target-setting models, but one limitation of SBM-based models is that they can only find the extreme solutions located on the boundary of two adjacent efficient facets. Other DEA target-setting methods, such as goal-based methods (Golany, 1988a, Thanassoulis and Dyson, 1992a, Athanassopoulos, 1995, Stewart, 2010), weight-restricted models (Wong and Beasley, 1990, Halme and Korhonen, 2000, Korhonen et al., 2001), assurance region models (Allen et al., 1997, Thompson et al., 1990), and restricted composite models (Athanassopoulos et al., 1999), provide DMs with different insights into the DEA PPS, but they fail to provide a complete decision-making process to help DMs to decide specific targets, and they fail to provide an optimal condition for judging whether DMs’ MPS has been achieved or not. Yang et al. (2009b) proved the equivalent relationship between DEA and MOLP, so a variety of MOLP models can be utilized in the DEA PPS to assist DMs to find specific targets (Amirteimoori and Kordrostami, 2005).

MOLP methods can generally be classified into three types. For a priori methods like the value function approach (Steuer et al., 1977), the lexicographic approach (Fishburn, 1974), and the goal programming approach (Charnes et al., 1955), DMs specify their preference information before the solution process. However, a priori methods require DMs to provide an entire preference structure, which is normally not available in practical decision-making processes. A posteriori methods, like weighted metrics (Zeleny, 1973), the achievement scalarizing function (Wierzbicki, 1982), and evolutionary MOO methods (EMO), are methods for generating efficient solutions instead of searching for a specific MPS. Compared with a priori methods and a posteriori methods, interactive methods do
not require DMs to have a global preference structure. Instead, they can gradually learn about the efficient frontier during the interactive process and approach the MPS gradually. Therefore, an increasing number of interactive methods have been applied in target-setting processes based on the DEA PPS.

According to the types of preference information that DMs provide, interactive methods can generally be classified into three categories. Trade-off-based methods require DMs to provide marginal substitutions, which are used to find an MPS. Typical trade-off-based methods include the ZW method (Zionts and Wallenius, 1976), ISWT methods (Haimes and Hall, 1974), GDF methods (Geoffrion et al., 1972), SPOT methods (Sakawa, 1982), and GRIST methods (Yang et al., 2009). Reference-point-based methods collect DMs’ preference information in the form of ideal points or “goals”. New solutions can be obtained through adjusting the position of the reference point or the distance measurement. Classical reference point methods include the Tchebycheff method (Steuer, 1986), the Pareto race method (Korhonen and Laakso, 1986), and the REF-LEX method (Miettinen and Kirilov, 2005). Classification methods allow DMs to determine which objectives should be improved, which can be sacrificed, and which should remain the same, for example the STEM (Benayoun et al., 1971), the STOM (Nakayama, 1995), and the NIMBUS method (Miettinen, 2012).

However, when applying the existing interactive methods in practice, some DMs may find them too difficult to use or understand, especially when the number of objectives is relatively large. Specifically, DMs may find the interactive process difficult in terms of two aspects. One is that DMs may find it difficult to provide the feedback information that the interactive model requires. For example, indifference-trade-off-based methods require DMs to identify the precise marginal substitutions between all objectives, with their overall utility remaining the same. However, when there is a large number of objectives, it is very hard for DMs to express the indifference trade-offs between all objectives at the same time. For goal-based methods, more objectives will lead to an increasing difficulty for DMs to
express their ideal points. For classification methods, it is more difficult for DMs to identify the step when they want to restrict the PPS further if there are more objectives, and it is more likely for them to miss the MPS by providing the wrong step size. Another difficulty for DMs when the number of objectives is high is the unbearable interaction times. In this case, the objective space will be complex as well, and it requires DMs to engage in many more interactions to locate the MPS. It is reported that most DMs actually stop the interaction process before the MPS has been achieved.

Besides, when setting targets, it is very common for DMs to have priorities in terms of different objectives. Objectives with higher priority will have a strong influence on the overall strategies, and DMs tend to set higher priority objectives before other objectives. On the contrary, less-important objectives usually will not be set until the important objectives have been set. For example, high deposits and high loans are the two most important objectives for a bank. As a result, a bank tends to set the targets for deposits and loans first. Other objectives, like bonds and insurance, will normally be set after those for deposits and loans. Therefore, priority information should be included in the target-setting model. Otherwise, DMs may find that the final targets obtained are quite different from what they expected.

In order to release DMs’ burden and make the target-setting process more feasible when the number of objectives is large, this chapter proposes a new interactive target-setting model based on the DEA PPS. The new proposed model starts from two objectives and gradually includes new objectives when setting a target, with the assumption that the indifference trade-offs between previous objectives are fixed. One significant advantage of the new proposed model is that DMs do not need to provide preference information simultaneously in terms of all objectives. Besides, priority information can be included in the model.

The remainder of the chapter is organized as follows. Section 6.2 illustrates the equivalent relationship between DEA and MOLP. Section 6.3 describes the proposed
Chapter 6. A stepwise trade-off model for target setting based on the DEA PPS

interactive procedure, and a numerical example is used to illustrate the process in Section 6.4. Section 6.5 implements this procedure in a case study of a Chinese state-owned bank. Section 6.6 concludes this chapter and provides directions for future research.

6.2 The equivalent relationship between DEA and the MOLP minimax model

Suppose an MOO problem has $s$ objectives defined in general as follows (Yang et al., 2009):

$$\text{Max } f(\lambda) = [f_1(\lambda), ... , f_r(\lambda), ... , f_s(\lambda)]$$

s.t. $\lambda \in \Omega = \{\lambda | g_i(\lambda) \leq 0, h_l(\lambda) = 0; i = 1, ..., k_1, l = 1, ..., k_2\}$,

where $\Omega$ is the feasible decision space. $\lambda = \{\lambda_j; j = 1, ..., n\}$ is the independent composite variable. $f_r(\lambda) (r = 1, ..., s)$ is the continuously differentiable objective function, and $g_i(\lambda) (i = 1, ..., k_1)$ and $h_l(\lambda) (l = 1, ..., k_2)$ are the continuously differentiable inequality and equality constraint functions, respectively. In an MOO problem, objectives are generally in conflict with each other and therefore no dominant solution can be found that maximizes all objectives. Instead, we are satisfied with efficient solution $\lambda^*$ in the sense that there is no other feasible solution that is better than $\lambda^*$ on at least one objective and as good as $\lambda^*$ on all other objectives. The set of all efficient solutions is referred to as the efficient frontier.

Yang et al. (2009b) proved the equivalent relationship between DEA and MOLP. When the restriction of radial improvement is relaxed, the conventional DEA dual model can be generalized into the following target-planning models:

Output-oriented DEA target-planning model:

$$\text{Max } f(\lambda) = [f_1(\lambda), ... , f_r(\lambda), ... , f_s(\lambda)]$$

s.t. $\lambda \in \Omega = \{\lambda | \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ij}^0, i = 1, ..., m; \lambda_j \geq 0 \text{ for all } j\}$

where $f_r(\lambda) = \sum_{j=1}^n \lambda_j y_{rj}$ is the composite output for the “$r$th output”. This model can be applied when DMs want to investigate the maximum outputs with no extra inputs consumed.
Chapter 6. A stepwise trade-off model for target setting based on the DEA PPS

For example, when opening a new branch with no more than a certain amount of investment, DMs want to know the most-preferred product outcome.

Input-oriented DEA target-planning model for inputs:

\[
\begin{align*}
\text{Min} & \quad f(\lambda) = \{f_1(\lambda), ..., f_i(\lambda), ..., f_m(\lambda)\} \\
\text{s.t.} & \quad \lambda \in \Omega = \{\lambda | y_{rjo} - \sum_{j=1}^{n} \lambda_j y_{rj} \leq 0, j = 1, ..., n; \lambda_j \geq 0 \text{ for all } j\}
\end{align*}
\]

where \( f_i(\lambda) = \sum_{j=1}^{n} \lambda_j x_{ij} \) is the composite input for the “ith input”. This model can be applied when DMs want to minimize inputs without sacrificing existing outputs. For example, an inefficient DMU may require a benchmark to help it to cut down its redundant resource usage.

There might be situations where DMs want to adjust both inputs and outputs at the same time. Thus, the following combined-oriented model can be applied.

\[
\begin{align*}
\text{Max} & \quad f(\lambda) = [F_{outputs}^c, -F_{inputs}^c] \\
\text{s.t.} & \quad \lambda \in \Omega = \{\lambda | F_{outputs}^N \geq y_0^N, F_{inputs}^N \leq x_0^N\}
\end{align*}
\]

where \( F_{outputs}^c \) and \( F_{inputs}^c \) refer to the set of discretionary composite outputs and discretionary composite inputs, respectively, and \( F_{outputs}^N \) and \( F_{inputs}^N \) refer to the set of non-discretionary composite outputs and non-discretionary inputs, respectively. For illustration purposes, the rest of this chapter will use the output target-planning model to illustrate the following procedure. The same procedure applies to the other two oriented models as well.

All situations mentioned above are actually special cases of (6.1) with linear constraints and objective functions. In the following sections, this chapter proposes a stepwise interactive MOO process to support DEA target planning with DMs’ preferences considered progressively.
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6.3 The stepwise trade-off model for target setting

6.3.1 Identifying the initial starting point and indifference trade-off regarding two objectives

In order to release DMs’ burden resulting from the unacceptable interaction times when the number of outputs is large, we propose a stepwise trade-off model. The stepwise trade-off model starts from two objectives and gradually includes new objectives (one at a time), assuming that the indifference trade-offs between set objectives are fixed. One significant advantage of this new model is that DMs do not need to express feedback information in terms of all objectives during each interaction. Instead, DMs only need to decide the trade-off between a new objective and the set objectives. This process can be repeated until all objectives are set. The specific interaction process is as follows.

For an MOO problem as defined in (6.2), the very first step is for DMs to select the two objectives that they care about the most. Suppose DMs select \( f_1(\lambda) \) and \( f_2(\lambda) \) as the initial starting objectives; a two-dimensional graph that illustrates the efficient frontier regarding \( f_1(\lambda) \) and \( f_2(\lambda) \) can be obtained by using the minimax reference point approach proposed by Yang et al. (2012a). The specific procedure can be summarized in the following steps:

1. Choose an observed DMU and define the corresponding MOO problem in the DEA PPS in the form of (6.2).
2. Calculate the scaling factors for all original DMUs and use the obtained scaling factors to scale all DMUs into the PPS of the observed DMU, where the scaling factor can be calculated by:

\[
\hat{\lambda}_j = \min_{1 \leq i \leq m} \frac{x_{ij_0}}{x_{ij}}
\]

The scaled outputs are equal to:

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\[ f_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} = \hat{\lambda}_j y_{rj} \]

3. Draw the scaled DMUs using the scaled outputs and identify the initial efficient frontier.

4. Select two adjacent extreme points on the initial efficient frontier in any order and search whether there is a feasible solution located outside the current efficient frontier. Denote one extreme point as \( A = [A_{11}, A_{12}] \) and the other extreme point as \( B = [B_{21}, B_{22}] \), with \( A_{12} \geq B_{22} \). Define \( g(f_1, f_2) \) as:

\[
g(f_1, f_2) = \begin{cases} 
-(B_2 - A_2)f_1(\lambda) + (B_1 - A_1)f_2(\lambda) & \text{if } \vec{N} \cdot \vec{f}_1 \geq 0 \\
-(A_2 - B_2)f_1(\lambda) + (A_1 - B_1)f_2(\lambda) & \text{if } \vec{N} \cdot \vec{f}_1 < 0
\end{cases}
\]

where \( \vec{N} \) is the outwards normal vector of the line \( \overline{AB} \), and \( \vec{f}_1 \) is the horizontal \( (f_1) \) axis. Then, the searching problem is defined by:

\[
\text{Max } g(f_1(\lambda), f_2(\lambda)) \\
\text{s.t. } \lambda \in \Omega
\]

If the optimal solution is \( f_1^* = A_1 \) and \( f_2^* = A_2 \) or \( f_1^* = B_1 \) and \( f_2^* = B_2 \), with \( g^* = |B_1A_2 - A_1B_2| \), then there will be no feasible solution beyond line \( \overline{AB} \).

5. If any line segments have not been expanded, go to Step 4.

Through the real efficient frontier obtained, DMs can find the MPS on the efficient frontier when only two objectives are considered. There are two potential strategies to help DMs to explore the MPS in two-dimensional graphs. Commonly, most DMs can directly identify the MPS with this visual aid. If DMs are not able to achieve this directly, the DMs can express their minimum acceptable value in terms of one objective, and the corresponding value for the other objective can be obtained by solving the following single-objective linear programming problem:

\[
\text{Max } \sum_{j=1}^{n} \hat{\lambda}_j y_{2j}
\]
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\[ s.t. \quad \sum_{j=1}^{n} \lambda_j y_{1j} = \varepsilon \]

\[ \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij0}, \quad i = 1, ..., m; \]

\[ \lambda_j \geq 0 \text{ for all } j. \]

where \( \varepsilon \) is the minimum acceptable value for DMs in terms of the first objective. Suppose the selected target is \([f_1(\lambda_s), f_2(\lambda_s)]\); the normal vector on the initial solution is defined as \( N^s = [N^s_1, N^s_2] \), which can be calculated using the following equation proposed by (Yang and Li, 2002):

\[ N^s = W^s \ast \beta^s \]

\( \beta^s \) is the Lagrange multiplier for the constraints in the following minimax model:

\[ \min_{\lambda_j} \theta \]

\[ s.t. \quad W^s_r(f^*_r - f_r(\lambda)) \leq \theta \quad \lambda \in \Omega. \]

\( W^s_r \) can be calculated by:

\[ W^s_r = \frac{(f^*_r - f_r(\lambda_s))}{(f^*_r - f_r(\lambda_s))} \]

\( f^*_r \) is the maximum feasible value for objective \( r \), and it can be obtained using the following single-objective optimization models:

Max \( \lambda_j \) \( f^*_r = \sum_{j=1}^{n} \lambda_j y_{rj} \quad r = 1, ..., s \)

\[ s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij0}, \quad i = 1, ..., m; \]

\[ \lambda_j \geq 0 \text{ for all } j. \]
Because the selected solution is the MPS for DMs considering two objectives, the indifference trade-off on this solution is the optimal indifference trade-off for DMs. Therefore, the optimal indifference trade-off considering two objectives is as follows:

\[ M^0 = [1, M^0_{12}]^T = \left[ \frac{N^0_2}{N^0_1} \right] \]

### 6.3.2 Including a new objective in the decision-making process

After DMs have determined their MPS for two objectives, the third objective \( f_3(\lambda) \) is included. Suppose the composite solution for the selected target is \( \lambda_j^s \) (\( j = 1, ..., 3 \)); the initial trade-off starting point considering three objectives is:

\[ f(\lambda)_0 = [f_1(\lambda), f_2(\lambda), f_3(\lambda)] = [f_1(\lambda)_0, f_2(\lambda)_0, f_3(\lambda)_0] \]

which can be obtained using the following equation:

\[ f_r(\lambda)_0 = \sum_{j=1}^{n} \lambda_j^s \ y_{rj} \quad r = 1, ..., 3 \]

We show DMs the initial target considering three objectives. DMs can provide the indifference trade-offs between the new objective and one of the previous objectives, assuming the indifference trade-off between the previous two objectives is fixed. Suppose DMs decide that the indifference trade-off between the third and first objective is \( [1, M_{13}] \); the indifference trade-off between the third objective and the previous two objectives is as follows (Yang et al., 2009b):

\[ M = [1, M^0_{12}, M_{13}]^T \]

(6.10)

Suppose DMs decide that the indifference trade-off between the second and third objectives is \( M_{23} \); the indifference trade-off between the third objective and the previous two objectives is as follows (Yang et al., 2009b):

\[ M = [1, M^0_{12}, M^0_{12} \ast M_{23}]^T \]

(6.11)
If the MPS has not been achieved, $\Delta \bar{u}^t$, a direction that improves DMs’ utility, can be obtained using the following formula:

$$
\Delta \bar{u} = [\Delta f_1, \Delta f_2, \Delta f_3]^T = -M + \frac{(M)^T N}{(N)^T N} N
$$

(6.12)

where the normal vector $N$ can be obtained from (6.6)–(6.9). Suppose the initial efficient solution presented to DMs is $f(\lambda)_0$ and the DMs’ preferred trade-off direction is $\Delta \bar{u}$. Following $\Delta \bar{u}$, a new efficient solution $f'(\lambda)_N$ in the DMs’ preferred trade-off direction can be calculated using the following equation:

$$
f'(\lambda)_N = f(\lambda)_0 + \alpha \Delta \bar{u}
$$

(6.13)

where $\alpha$ is the step size. $f'(\lambda)_N$ can be used as a reference point to locate the corresponding efficient solution on the efficient frontier. If the connection line between the ideal point and $f'(\lambda)_N$ is extended, it will reach the efficient frontier at $(\lambda)_N$ (Yang et al., 2009b). The corresponding weight $w_{rN}$ at $f(\lambda)_N$ in the minimax ideal point model (6.3) can be calculated using the following equation (Yang et al., 2009b):

$$
w_{rN} = \frac{f'_1(\lambda)_N - f'_1^*}{f'_r(\lambda)_N - f'_r^*}
$$

(6.14)

A new efficient solution $f(\lambda)_N$ in the DMs’ preferred trade-off direction can be obtained by putting $w_{rN}$ back into Equation (6.7). If a set of different $\alpha$ with fixed intervals is implemented in (6.13), part of the efficient frontier in the DMs’ preferred trade-off direction can be sampled (Figure 6.1).
The sampled efficient frontier can also be displayed in the following trade-off table (Table 6.1):

<table>
<thead>
<tr>
<th>Output 1</th>
<th>Output 2</th>
<th>Output 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u_1$</td>
<td>$f_1(\lambda)_0$</td>
<td>$f_2(\lambda)_0$</td>
</tr>
<tr>
<td>Initial solution</td>
<td>$f_1(\lambda)$</td>
<td>$f_2(\lambda)$</td>
</tr>
<tr>
<td>Sampled solution (1)</td>
<td>$f_1(\lambda)_1$</td>
<td>$f_2(\lambda)_1$</td>
</tr>
<tr>
<td>Sampled solution (2)</td>
<td>$f_1(\lambda)_2$</td>
<td>$f_2(\lambda)_2$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$f_1(\lambda)_A$</td>
<td>$f_2(\lambda)_A$</td>
</tr>
<tr>
<td>Sampled solution (A)</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>Sampled solution (N)</td>
<td>$f_1(\lambda)_N$</td>
<td>$f_2(\lambda)_N$</td>
</tr>
</tbody>
</table>

By referencing the trade-off table and the trade-off figure, DMs can choose satisfactory trade-off steps, determining how far they would like to go in their preferred trade-off direction. Suppose DMs decide that a satisfactory trade-off step is $\alpha$, and the corresponding new solution is $f_3(\lambda)_A$; the DMs can further decide whether the selected solution is the MPS when considering three objectives. If it is not and DMs decide to continue the trade-off process from $f_3(\lambda)_A$, then $f_3(\lambda)_A$ is set as the new initial point, and the interactive process is repeated until the DMs identify the MPS when considering three objectives. If the DMs identify the MPS, then a new objective can be included. The entire interactive process continues until all objectives are considered.
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6.4 A numerical example

This section illustrates the stepwise trade-off model through a numerical example. The data are from Yang et al. (2009b). In this numerical example, the performances of seven banks are measured by four resources ($x_1, x_2, x_3,$ and $x_4$) and three products ($y_1, y_2,$ and $y_3$). The corresponding values are displayed in Table 6.2. Barclays is selected as the assessed DMU and the stepwise trade-off model is utilized to explore the MPS in the future by referencing the current performances of its competitors.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Bank</th>
<th>Inputs ($x_1$)</th>
<th>Inputs ($x_2$)</th>
<th>Inputs ($x_3$)</th>
<th>Inputs ($x_4$)</th>
<th>Outputs ($y_1$)</th>
<th>Outputs ($y_2$)</th>
<th>Outputs ($y_3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Abbey National</td>
<td>0.77</td>
<td>2.18</td>
<td>2.35</td>
<td>2.96</td>
<td>6.79</td>
<td>10.57</td>
<td>0.96</td>
</tr>
<tr>
<td>2</td>
<td>Barclays</td>
<td>1.95</td>
<td>3.19</td>
<td>8.43</td>
<td>3.53</td>
<td>2.55</td>
<td>13.35</td>
<td>1.14</td>
</tr>
<tr>
<td>3</td>
<td>Halifax</td>
<td>0.30</td>
<td>2.30</td>
<td>3.21</td>
<td>2.41</td>
<td>9.17</td>
<td>8.14</td>
<td>0.78</td>
</tr>
<tr>
<td>4</td>
<td>HSBC</td>
<td>1.75</td>
<td>4.00</td>
<td>13.30</td>
<td>4.85</td>
<td>5.82</td>
<td>23.67</td>
<td>1.73</td>
</tr>
<tr>
<td>5</td>
<td>Lloyds</td>
<td>2.50</td>
<td>4.30</td>
<td>9.27</td>
<td>2.40</td>
<td>6.57</td>
<td>14.01</td>
<td>1.45</td>
</tr>
<tr>
<td>6</td>
<td>NatWest</td>
<td>1.73</td>
<td>3.30</td>
<td>7.70</td>
<td>3.09</td>
<td>4.86</td>
<td>12.04</td>
<td>3.00</td>
</tr>
<tr>
<td>7</td>
<td>RBS</td>
<td>0.65</td>
<td>1.73</td>
<td>2.67</td>
<td>1.34</td>
<td>7.28</td>
<td>7.36</td>
<td>0.52</td>
</tr>
</tbody>
</table>

For illustration purposes, we assume that the managers of Barclays want to set efficient targets for all three outputs without consuming extra resources. The managers care about the objectives $y_1$ and $y_2$ the most and want to determine these first. The performances of all seven branches are firstly scaled according to the inputs of Barclays, and the corresponding outputs are plotted in Figure 6.2. However, the efficient frontier in Figure 6.2 is not the real efficient frontier, and the real efficient frontier can be obtained by the minimax reference point approach Steps 1–5 proposed by Yang et al. (2012a). The real efficient frontier is displayed in Figure 6.2.
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Then, the DM is encouraged to select their MPS when only two objectives are considered. The DM can visually identify this from the real efficient frontier (Figure 6.3). Besides, the DM can express their minimum tolerance level regarding one of the objectives, for example $f_1$, and the value of the other objective can be calculated using (6.5).
Suppose the DM cannot identify their MPS visually and believes that the lowest acceptable value for $y_1$ is 10 and the maximum feasible value for $f_2$ is 15.64, with normal vector $N = [1, 1.653]$. This normal vector can help to check whether the MPS has been achieved or not. Suppose the DM agrees that the MPS has been achieved; the composite variable for selected targets is equal to:

$$\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7] = [0.00, 0.00, 0.00, 0.31, 0.00, 0.00, 1.13]$$

Then, the third objective is involved in the decision-making process. The initial solution can be calculated using the following equation:

$$\sum_{j=1}^{n} \lambda_j y_{rj} = 1, \quad r = 1, 2, 3$$

The weights for the initial solution can be obtained by using (6.7), and the normal vector of the initial solution can be obtained by using Equations (6.6)–(6.9). The results are displayed in Table 6.3.

<table>
<thead>
<tr>
<th>Table 6.3 Solution considering three objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial solution when including a third output</strong></td>
</tr>
<tr>
<td>$f_1$</td>
</tr>
<tr>
<td>$f_2$</td>
</tr>
<tr>
<td>$f_3$</td>
</tr>
</tbody>
</table>

Because $f_1$ and $f_2$ have achieved the most-preferred balance, when considering $f_3$, the trade-off calculation should be conducted between the first two objectives and $f_3$, assuming that the indifference trade-off between the first two objectives is fixed. Based on the initial achievement levels of the three objectives, the DM is asked to provide their indifference trade-offs between $f_1$ and $f_3$, or between $f_2$ and $f_3$, and then the indifference trade-offs among all three objectives can be obtained accordingly. Suppose the DM believes that the indifference trade-off $M = [1, M_{12}, M_{13}] = [1.00, 1.65, 0.02]$. Part of the efficient frontier can be sampled by using Equations (6.12)–(6.14). The corresponding trade-off figure and
trade-off table are shown in Figure 6.4 and Table 6.4, respectively, which could help the DM to decide their most-preferred trade-off step. A table of rate changes could also help the DM to observe the trade-off rate between objectives. Both Figure 6.4 and Table 6.4 indicate that the DM wants to improve $f_3$ by sacrificing $f_1$ and $f_2$ simultaneously. Suppose the DM decides that the trade-off step is:

$$f(\lambda)_s = f(\lambda)_8 = [f_1(\lambda)_8, f_2(\lambda)_8, f_3(\lambda)_8] = [8.91, 15.22, 1.26]$$

with the corresponding normal vector:

$$N_s = [N_1, N_2, N_3] = [1.00, 0.66, 42.95]$$
Both the selected solution and the corresponding normal vector are shown to the DM. If the DM agrees that the MPS has been achieved, then the new objective can be included. Otherwise, the current trade-off step is set as a new initial solution, and the DM can provide a new indifference trade-off based on the current trade-off step. Suppose the DM provides the new indifference trade-off as:

$$M = [1, M_{12}, M_{13}] = [1.00, 1.286, 0.014]$$

A new sample efficient frontier following the DM’s preferred solution can be obtained by following (6.12)–(6.14), producing the corresponding trade-off figure (Figure 6.5) and trade-off table (Table 6.5). Then, the DM can decide the trade-off step. This process can continue until the DM finds the MPS considering three objectives. Then, new objectives are included one at a time until all objectives have been considered.

<table>
<thead>
<tr>
<th>Initial point for the second interaction</th>
<th>Second interaction starting point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>Initial solution</td>
</tr>
<tr>
<td>8.91</td>
<td>1.00</td>
</tr>
<tr>
<td>$f_2$</td>
<td>15.22</td>
</tr>
<tr>
<td>$f_3$</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Figure 6.5 Trade-off figure (second interaction)
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Table 6.6 Trade-off table (second interaction)

<table>
<thead>
<tr>
<th></th>
<th>$f(\lambda^0_0)$</th>
<th>$f(\lambda^0_1)$</th>
<th>$f(\lambda^0_2)$</th>
<th>$f(\lambda^0_3)$</th>
<th>$f(\lambda^0_4)$</th>
<th>$f(\lambda^0_5)$</th>
<th>$f(\lambda^0_6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>8.91</td>
<td>8.88</td>
<td>8.75</td>
<td>8.61</td>
<td>8.47</td>
<td>8.33</td>
<td>8.19</td>
</tr>
<tr>
<td>$f_3$</td>
<td>1.26</td>
<td>1.27</td>
<td>1.27</td>
<td>1.28</td>
<td>1.28</td>
<td>1.29</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Change rate

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>-0.139</th>
<th>-0.139</th>
<th>-0.139</th>
<th>-0.139</th>
<th>-0.139</th>
<th>-0.139</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_2$</td>
<td>-0.178</td>
<td>-0.178</td>
<td>-0.178</td>
<td>-0.178</td>
<td>-0.178</td>
<td>-0.178</td>
<td>-0.178</td>
</tr>
<tr>
<td>$f_3$</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
</tr>
</tbody>
</table>

6.5 A case study of a Chinese state-owned bank

In this section, we select a Chinese state-owned bank as our research target. The selected bank has a history of over 60 years, and its 24,000 branches cover all major cities in China. The bank adopts a strict four-level hierarchy management system, including the head office in Beijing, city head offices, primary branches, and secondary branches.

(a) Beijing head office

The Beijing head office is the top level of the bank’s management hierarchy. Its responsibilities include general strategy (regulation) planning, deliberation and adjustment, city head office supervision and assessment, city branch managers’ personnel assignment, etc.

(b) City head offices

According to the specific market environment, city head offices tailor the strategies planned by the Beijing head office into detailed strategies or regulations for their primary and secondary branches. Besides, city head offices also take responsibility for branch supervision, branch assessment, etc.

(c) Primary branches

Unlike the head office in Beijing and city head offices, primary branches do not have the authority to make or adjust strategies or regulations. The responsibilities for primary
branches are to manage and organize secondary branches’ daily operations under the regulations planned by the upper head offices. Normally, a primary branch supervises 6–12 secondary branches. A primary branch has the authority to open a new secondary branch in a suitable location, as well as to close secondary branches with poor performances.

(d) Secondary branches

Secondary branches are the only level with an operational function in the four-level hierarchy. They are responsible for daily operations such as transactions, deposits, loans, selling financial and insurance products, etc.

According to customer types, the bank classifies its more than 100 types of financial products and services into two major categories: personal business and corporate business. Personal business represents the business area that serves personal customers, for example personal deposits, personal loans (mainly property mortgages), payment settlements, electronic banking, investments, financing, and so on. The corporate business area serves enterprises and institutions and provides services such as cooperate deposits, cooperate loans, business funds, bill payments, surety, and so on. 102 The descriptive statistics are summarized in Table 6.7.
### Table 6.7 Descriptive statistics for the bank case

<table>
<thead>
<tr>
<th>Outputs (thousand RMB)</th>
<th>Priority</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Range</th>
<th>Branch 52</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposit from personal customer ((f_1))</td>
<td>P1</td>
<td>88128.86</td>
<td>39072.61</td>
<td>10209.30 to 187619.76</td>
<td>101907.98</td>
<td>137621.42</td>
</tr>
<tr>
<td>Deposit from corporation ((f_2))</td>
<td>P3</td>
<td>45219.47</td>
<td>48049.09</td>
<td>1696.39 to 313746.76</td>
<td>82657.35</td>
<td>346161.87</td>
</tr>
<tr>
<td>Loan for personal customer ((f_3))</td>
<td>P2</td>
<td>46042.61</td>
<td>36186.93</td>
<td>5268.95 to 277160.49</td>
<td>35865.08</td>
<td>277063.00</td>
</tr>
<tr>
<td>Loan for corporation ((f_4))</td>
<td>P4</td>
<td>46162.69</td>
<td>64204.73</td>
<td>0.00 to 424535.00</td>
<td>13116.23</td>
<td>235180.61</td>
</tr>
<tr>
<td>Intermediary business ((f_5))</td>
<td>P5</td>
<td>974.37</td>
<td>698.97</td>
<td>87.78 to 5572.36</td>
<td>1409.17</td>
<td>3720.27</td>
</tr>
<tr>
<td>Inputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Staff number</td>
<td>-</td>
<td>20.08</td>
<td>4.87</td>
<td>12.00 to 24.00</td>
<td>799.33</td>
<td>-</td>
</tr>
<tr>
<td>Overheads (thousand RMB)</td>
<td>-</td>
<td>53.85</td>
<td>17.21</td>
<td>26.94 to 130.65</td>
<td>19.00</td>
<td>-</td>
</tr>
</tbody>
</table>

For illustration purposes, the decision-making process of Branch 52 is described as follows. Branch 52 is a secondary branch, and the relevant primary branch manager hopes to set an advanced but feasible target for Branch 52 to guide its development, based on its peers’ performances. The performance of Branch 52 is also illustrated in Table 6.7.

At the beginning of the target setting, the DM believes that \(f_1\) (personal deposits) and \(f_3\) (personal loans) are the two most important outputs, and they want to determine these two objectives first. Following the minimax reference point approach Steps 1–2 proposed by Yang et al. (2012a), the performances of all branches are scaled into the same objective space, as shown in Figure 6.6.
In Figure 6.6, data from Branches A, B, and C formulate an efficient frontier in the initial PPS. However, the formulated efficient frontier is not the real efficient frontier. In order to obtain the real efficient frontier, the minimax reference point approach Steps 1–6 proposed by Yang et al. (2012a) is adopted here to extend the initial efficient frontier formed by the scaled branches’ performances to a real efficient frontier formed by the branches’ composite performance. Figure 6.7 illustrates the extended PPS formed by the branches’ composite performance.
Then, the extended efficient frontier is shown to the manager of the primary branch, and they are asked to select the MPS regarding the achievement levels of $f_1$ and $f_3$. During the first interaction, the manager decides that the minimum acceptable value for $f_1$ is 100,000.00. According to this information, the corresponding maximum feasible value for $f_3$ is equal to 97,752.86, using Equation (6.5). The corresponding weight in the minimax model and the normal vector are illustrated in Table 6.8.

<table>
<thead>
<tr>
<th>Two-dimensional selection</th>
<th>Initial solution</th>
<th>Weight in minimax model</th>
<th>Normal vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>100000.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$f_3$</td>
<td>97752.86</td>
<td>0.50</td>
<td>1.33</td>
</tr>
</tbody>
</table>

After showing the DM the obtained results, the DM is still not satisfied and wants to adjust the minimum acceptable value for $f_1$ to 110,000.00. The corresponding maximum feasible value for $f_3$ is equal to 90,228.03, with the corresponding weight and normal vector shown in Table 6.9. However, the feedback target is still not satisfactory to the primary branch manager. After another three interactions (Figures 6.10–6.14 and Tables 6.10–6.14), the manager finally decides that the MPS values are 117,500.00 and 68,278.21. The DM
agrees that the optimal trade-off on this solution is parallel to the normal vector $N = [1, 0.19]$. 

![Figure 6.8 Second selection](image)

Table 6.9 Second selection

<table>
<thead>
<tr>
<th>Two-dimensional selection</th>
<th>Initial solution</th>
<th>Weight in minimax model</th>
<th>Normal vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1</td>
<td>110000.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>f3</td>
<td>90228.03</td>
<td>0.26</td>
<td>1.33</td>
</tr>
</tbody>
</table>
Chapter 6. A stepwise trade-off model for target setting based on the DEA PPS

Figure 6.9 Third selection

Table 6.10 Third selection

<table>
<thead>
<tr>
<th>Initial solution</th>
<th>Weight in minimax model</th>
<th>Normal vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>115000.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$f_3$</td>
<td>77872.20</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Figure 6.10 Fourth selection

Table 6.11 Fourth selection

<table>
<thead>
<tr>
<th>Initial solution</th>
<th>Weight in minimax model</th>
<th>Normal vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>117500.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$f_3$</td>
<td>68278.21</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Chapter 6. A stepwise trade-off model for target setting based on the DEA PPS

Figure 6.11 Fifth selection

Table 6.12 Fifth selection

<table>
<thead>
<tr>
<th>Two-dimensional selection</th>
<th>Initial solution</th>
<th>Weight in minimax model</th>
<th>Normal vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>120000.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$f_3$</td>
<td>55910.44</td>
<td>0.05</td>
<td>0.19</td>
</tr>
</tbody>
</table>

After determining the MPS considering only two objectives, $f_1$ and $f_3$, the manager believes that the next most important objective is $f_2$ (corporate deposits) and wants to determine it next. The initial solution considering three objectives is 120,000.00, 55,910.44, and 17,138.30. Assuming that the optimal indifference trade-off between $f_1$ and $f_3$ is fixed, the DM decides that the indifference trade-off between $f_1$ and $f_2$ $M = [1, 0.02]$. Therefore, the DM’s indifference trade-off for all three objectives is:

$$M = [1, M_{12}, M_{13}] = [1.00, 0.19, 0.02]$$

<table>
<thead>
<tr>
<th>Initial solution when including a third output</th>
<th>Initial solution</th>
<th>Weight in minimax model</th>
<th>Normal vector</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>120000.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$f_3$</td>
<td>55910.44</td>
<td>0.05</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>$f_2$</td>
<td>17138.30</td>
<td>0.03</td>
<td>0.05</td>
<td>0.02</td>
</tr>
</tbody>
</table>

According to the DM’s indifference trade-off information, the part of the efficient frontier that follows the DM’s preferred trade-off direction can be sampled by using (7.0)–
(6.14). The obtained three-dimensional trade-off figure and trade-off table are displayed in Figure 6.12 and Table 6.13.

![Figure 6.12 Trade-off figure considering three objectives](image)

<table>
<thead>
<tr>
<th>Table 6.13 Trade-off table considering three objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trade-off table for three outputs</strong></td>
</tr>
<tr>
<td>$f(\lambda)_0$</td>
</tr>
<tr>
<td>$f_1$ 120000.00</td>
</tr>
<tr>
<td>$f_3$ 55910.44</td>
</tr>
<tr>
<td>$f_2$ 17138.30</td>
</tr>
<tr>
<td><strong>Change rate</strong></td>
</tr>
<tr>
<td>$r_1$ -</td>
</tr>
<tr>
<td>$r_2$ -</td>
</tr>
<tr>
<td>$r_3$ -</td>
</tr>
<tr>
<td>$r_4$ -</td>
</tr>
<tr>
<td>$r_5$ -</td>
</tr>
</tbody>
</table>

Both the trade-off figure and the trade-off table indicate that the DM wishes to improve $f_2$ by sacrificing $f_1$ and $f_3$ simultaneously. Table 6.14 also shows the DM the increasing rate of $f_2$ and the corresponding decreasing rates of $f_1$ and $f_3$, which could help them to determine the trade-off step. The primary branch manager decides that the preferred trade-off step is:

$$f(\lambda)_5 = f(\lambda)_2 = [f_1(\lambda)_2, f_2(\lambda)_2, f_3(\lambda)_2] = [117677.51, 34197.13, 45008.81]$$
with the normal vector equal to $N = [1, 0.38, 0.38]$. The DM agrees that this is the optimal indifference trade-off on $f(\lambda)_s$.

Then, the fourth objective $f_4$ (corporate loans) is included. The initial starting point for the trade-off is:

$$f(\lambda)_0 = [f_1(\lambda)_0, f_2(\lambda)_0, f_3(\lambda)_0, f_4(\lambda)_0] = [117677.51, 34197.13, 45008.81, 7268.77]$$

<table>
<thead>
<tr>
<th>Initial solution when including a fourth output</th>
<th>Initial solution</th>
<th>Weight in minimax model</th>
<th>Normal vector</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>117677.51</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$f_2$</td>
<td>45008.81</td>
<td>0.070</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>$f_3$</td>
<td>34197.13</td>
<td>0.048</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>$f_4$</td>
<td>7268.77</td>
<td>0.060</td>
<td>0.16</td>
<td>0.02</td>
</tr>
</tbody>
</table>

After being shown the initial solution, the DM decides that the indifference trade-off between $f_1$ and $f_4$ is equal to $M_{14} = [1, 0.02]$, which gives the indifference trade-off of $f_1$, $f_2$, $f_3$, and $f_4$ as:

$$M = [1, M_{12}, M_{13}, M_{14}] = [1.00, 0.38, 0.38, 0.02]$$

According to the indifference trade-off provided by the DM, the part of the efficient frontier that follows the DM’s preferred trade-off direction can be sampled. The corresponding trade-off table is displayed in Table 6.15. By comparing the trade-off table and the corresponding change rate, the DM decides that the preferred trade-off step is:

$$f(\lambda)_s = f(\lambda)_1 = [f_1(\lambda)_1, f_2(\lambda)_1, f_3(\lambda)_1, f_4(\lambda)_1] = [117677.51, 34197.13, 45008.81, 7268.77]$$

which indicates that the DM wishes to improve $f_4$ by sacrificing $f_1$, $f_2$, and $f_3$ simultaneously. The DM agrees that the normal vector on $f(\lambda)_s$ is parallel to the optimal indifference trade-off on this solution.
Then, $f_5$ is involved in the decision-making process, assuming the indifference trade-offs among the previous four objectives are fixed. The initial starting point for the trade-off is:

$$f(\lambda)_0 = [f_1(\lambda)_0, f_2(\lambda)_0, f_3(\lambda)_0, f_4(\lambda)_0, f_5(\lambda)_0] = [117677.51, 34197.13, 45008.81, 7268.77, 871.98]$$

The manager of the primary branch decides that they would not like to make a trade-off between $f_5$ and the rest of the objectives and accepts that the corresponding normal vector on $f(\lambda)_0$ is parallel to the optimal indifference trade-off on $f(\lambda)_0$.

So far, all objectives have been included, and the DM has found the MPS in terms of five outputs by referencing all its peers. Table 6.17 compares Branch 52’s current performance with the targets set. The results indicate that the DM believes that in the future, Branch 52’s personal deposits should be significantly improved, with personal loans remaining at the same level. At the same time, the DM holds a negative view regarding the future of corporate business and intermediary business because of negative market expectations. All in all, the DM believes that Branch 52 should focus on developing the personal business, especially the personal deposit business. The primary branch manager
believes that the obtained targets fit with Branch 52’s practical situation and market expectations.

<table>
<thead>
<tr>
<th>Outputs (thousand RMB)</th>
<th>Branch 52</th>
<th>Target</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposit from personal customers (f1)</td>
<td>101907.98</td>
<td>117677.51</td>
<td>137621.42</td>
</tr>
<tr>
<td>Deposit from corporation (f2)</td>
<td>82657.35</td>
<td>45008.81</td>
<td>346161.87</td>
</tr>
<tr>
<td>Loan for personal customer (f3)</td>
<td>35865.08</td>
<td>34197.14</td>
<td>277063.00</td>
</tr>
<tr>
<td>Loan for corporation (f4)</td>
<td>13116.23</td>
<td>7268.77</td>
<td>235180.61</td>
</tr>
<tr>
<td>Intermediary business (f5)</td>
<td>1409.17</td>
<td>871.98</td>
<td>3720.27</td>
</tr>
<tr>
<td>Inputs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Staff number</td>
<td>799.33</td>
<td>799.33</td>
<td>-</td>
</tr>
<tr>
<td>Overheads (thousand RMB)</td>
<td>19.00</td>
<td>19.00</td>
<td>-</td>
</tr>
</tbody>
</table>

**6.6 Contributions of the stepwise model**

This chapter proposed a new trade-off-based interactive trade-off model to help DMs to explore the MPS by referencing peers’ performances. Firstly, the peer-performance-based technique DEA is utilized to build up a PPS or feasible space, where the best performances are called the efficient frontier. Setting a target based on the DEA efficient frontier can help DMs to find an advanced but feasible target. Then, a new stepwise interactive MOO model is proposed to help a DM to explore the MPS among all efficient solutions.

The new proposed model to a large extent releases the burden of DMs when exploring the MPS when solving MOO problems, especially when the number of objectives is quite large. The feasibility of this model was further explained through a numerical example and a real case of a Chinese state-owned bank.
7. Conclusions and future work

7.1 Conclusions

This thesis firstly observed a requirement of banks in terms of performance assessment and target planning. After reviewing past research on performance assessment, the technique of DEA was selected as the technique to help a bank to evaluate the performances of its branches. Then, a series of input and output criteria were selected to build up a PPS or feasible space. The set of best performances of the PPS is called the efficient frontier, which can be used as a reference to assess branches’ performances. Then, the DEA model was applied to a case study of bank branch performance assessment.

After the successful implementation of the DEA model in the performance assessment field, questions were raised, including how to use the DEA PPS to plan the future targets of branches. Traditional DEA models cannot achieve this because they do not include DMs’ preference information. Therefore, this thesis combined MOLP techniques and DEA models to include DMs’ preference in the decision-making process. After reviewing the existing MOLP models, this thesis identified three gaps that could make the target-planning process difficult for DMs. Firstly, although most MOLP methods have been proven convergent, the interaction times could be so large that no DMs could finish the entire process. Secondly, because the efficient frontier is a continuous set, most DMs may have a sensitive range. DMs might be insensitive to close solutions. Thirdly, when the number of objectives is quite large, it is very difficult for DMs to provide the trade-off information of all objectives simultaneously. Fourthly, some visual aids can help DMs a lot in understanding the shape the efficient frontier. However, when the number of objectives is more than three, it is very hard for DMs to understand the entire efficient frontier.

To fill the research gaps above and to make the target-planning process easier for DMs, this thesis proposed three MOLP models. According to the situation, DMs can choose one
of these or can combine two or three of them to set targets for branches based on the DEA PPS. Based on this thesis, some efforts could be made to improve DMs’ experiences when setting targets based on the DEA PPS.

### 7.2 Limitations

Although the three models proposed by this thesis to a large extent release DMs’ burden when setting targets based on the DEA PPS and the efficient frontier, there are some limitations.

For the piecewise linear model, one limitation is how to identify the suitable boundaries for the DSRs. In the example provided, we used the trichotomy method to identify the boundaries of the DSRs. However, this might not be the most efficient way because DMs’ utility function may be a non-linear and irregular function. A fixed-interval cut may miss some important reverse or changing point in DMs’ utility function. In this situation, a lot of indifference trade-offs or interactions may be wasted in the non-preferred regions. Therefore, a more-systematic procedure that could help DMs to identify the boundaries of the DSRs could improve the interaction efficiency.

For the prioritized model, the convergence of the exploration process cannot be guaranteed because it can only show DMs the preferred trade-off direction: the trade-off steps need to be determined by DMs by referencing the trade-off table and the trade-off figures. Even with these visual aids, it is still possible for DMs to be unable to identify the trade-off direction precisely, causing inefficient interactions. However, this limitation could be solved by combining the piecewise linear model and the prioritized model. The piecewise linear model could firstly be used to narrow down the entire efficient frontier into a relatively small region. Then, the prioritized model could be used to help the DMs to explore the MPS in the narrowed region.
7.3 Future research

In future research, a machine-learning model could be designed to learn DMs’ preferences based on the preference information collected from the interactive process. The learning process could be in a variety of forms, such as an evidence-based machine-learning process.

DMs’ feedback collected in different interactions could be inconsistent. Therefore, it could be very helpful to create a consistency index to check the consistency of feedback information.
Reference


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