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Generalised Control-Oriented Modelling Framework for Multi-Energy Systems

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Abstract—A requirement for new sources of flexibility in meeting energy demand and the growing interest in the interaction between energy sectors have drawn attention to multi-energy systems, which can offer numerous benefits, e.g. providing flexibility to counteract the intermittency of renewable generation and increase energy efficiency. In order to maximise these benefits, advanced control methods are required. The model predictive control methodology is a promising candidate for such applications as it is capable of incorporating economic and operational objectives whilst respecting various technical, regulatory and environmental constraints. In order to implement such a control strategy effectively, it is necessary to develop appropriate system models. This paper presents a novel generalised modelling framework for multi-energy systems that is particularly well suited, though not limited to, predictive control applications. The proposed approach is capable of representing energy converter arrangements of arbitrary complexity containing multiple energy vectors, as well as multi-directional energy flow, multi-generation and multi-mode devices, a wide range of controllable producers/consumers, energy storage and flexible loads. The effectiveness of the approach is demonstrated using a representative case study based on three buildings at the University of Manchester where the developed model is incorporated into a model predictive control scheme. The simulation results show how the controller minimises the cost of purchasing energy whilst satisfying operational constraints, and managing various types of flexible demand.

Index Terms—Model Predictive Control, control-oriented modelling, multi-energy systems, demand-side management

NOMENCLATURE

k	Sampling period
h	Scheduling period
n_i, n_j	Node indices
$\eta_{(n_i \rightarrow n_j)}$	Conversion factor from node n_i to n_j
$P_{(n_i \rightarrow n_j)}$	Power flow from node n_i to node n_j
P_{n_i}	Power flow in/out of node n_i
$\mathcal{I}_{\rightarrow n_l}$	Index set of source nodes that connect to n_l
$\mathcal{I}_{n_l \rightarrow}$	Index set of sink nodes that n_l connects to
$\delta_{(n_i \rightarrow n_j)}$	Binary decision variable associated with arc $n_i \rightarrow n_j$
$\mathbf{x}$	State vector
$\mathbf{u}$	Input vector
$\mathbf{A}$	State transition matrix
$\mathbf{B}$	Input matrix
$\mathbf{C}$	Output matrix
$\mathbf{D}$	Feed-forward matrix

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v	Storage model charging/discharging input
$\mathbf{w}$	Storage model input disturbance vector
y	Storage model output state
p_{j,n_i}	Prosumer j connected to node n_i
$P_{p_{j,n_i}}$	Net power demand/consumption of prosumer p_{j,n_i}
G_m	m^{th} prosumer generating component
L_n	n^{th} prosumer load component
$N_{G,p_{j,n_i}}$	Number of generating components connected to prosumer p_{j,n_i}
$N_{L,p_{j,n_i}}$	Number of load components connected to prosumer p_{j,n_i}
N_H	Scheduling/control horizon
N_n^i	Number of energy segments for demand n
$l_{n,i}$	Actual energy consumption of i^{th} segment of demand n
$\tilde{l}_{n,i}$	Energy consumption based on the nominal energy requirement for the i^{th} segment of demand n
$\Delta l_{n,i}^+$	Slack variable relating to the amount of scheduled energy in excess of the nominal demand for the i^{th} segment of demand n
$\Delta l_{n,i}^-$	Slack variable relating to the amount of curtailed energy from the nominal demand for the i^{th} segment of demand n
$E_{n,i}$	Total nominal energy requirement of the i^{th} segment of demand n
$\delta_{n,i}^p$	Binary variable to indicate whether or not (1 or 0) the i^{th} segment of demand n is <i>processing</i>
$\delta_{n,i}^c$	Binary variable to indicate whether or not (1 or 0) the i^{th} segment of demand n is <i>complete</i>
$\delta_{n,i}^w$	Binary variable to indicate whether or not (1 or 0) the i^{th} segment of demand n is <i>waiting</i>
L_n	Resulting energy consumption of demand n accounting for contributions from all energy segments
$\mathbf{T}_n^p$	Energy user's time-preference vector
S_{n_i}	Standby efficiency of storage device connected to node n_i
Q_{n_i}	Charging/discharging power of storage device connected to node n_i
R_{n_i}	Energy stored in device
λ_A^+, λ_A^-	Control incentive/penalty for exceeding/not meeting the nominal energy requirement of adjustable portion of demand.

$\lambda_{AP}, \lambda_{SI}$	Respective control penalties to ensure smoothest possible scheduling profile is chosen amongst multiple optima for adjustable-pliable and shiftable-interruptible demands
π_e^{buy}, π_g^{buy}	Per unit purchasing price of electricity and gas respectively
π_e^{sell}	Per unit revenue received for selling electrical energy to grid
$J(\cdot)$	Objective function
$\underline{z}$	Lower bound on variable z
$\bar{z}$	Upper bound on variable z

I. INTRODUCTION

To support progression toward a low-carbon future and away from dependency on non-renewable energy resources, electrification of heat and transport sectors as well as increased penetration of low-carbon technology is expected to continue at ever increasing pace. This also means an increase in so-called prosumers, as distributed generation technologies provide the means for energy users to produce energy locally along with consuming energy provided by the wider infrastructure. While this presents great challenges for distribution network operators, it is also an opportunity for novel energy management methodologies to be developed and applied. This is due to the increase in the number of flexibility sources made available through the integration of low-carbon technologies, which in many cases represent controllable system components. Electrification of heat and transport entails new loads, such as heat pumps and electric vehicles, which are associated with inherent storage capabilities (e.g. thermal storage, thermal load inertia, plug-in electric vehicle batteries) and can be harnessed to further increase system flexibility. In particular, by employing programs designed to influence desirable changes in consumer's energy use, i.e. demand-side management (DSM) [1]. By considering the inherently multi-variable nature of energy systems and exploiting the interaction between different energy vectors, a variety of benefits in power system operation can be realised such as increased flexibility in load balancing, frequency response and reserve [2]. These implications have influenced significant interest in the concept of multi-energy systems (MES). Correctly managing the newly available flexibility sources has the potential to significantly reduce the requirement for electricity network re-enforcement that would otherwise be required as they can assist an economically and/or environmentally optimal dispatch of generation/load, thus accommodating more renewable energy sources (RES) and maximising capacity utilisation of the existing infrastructure. A promising control methodology to accommodate these requirements is model predictive control (MPC). MPC is suited to this type of application since it incorporates multi-step optimisation which accounts for system changes over time. In addition, limiting/inequality constraints are explicitly handled allowing the inclusion of both hard (e.g. system's physical and regulatory limits) and soft (e.g. user's comfort thresholds) constraints [3]. As its name suggests, MPC is critically reliant on the availability of a sufficiently accurate

system model that allows it to forecast and, therefore, optimise future system behaviour. In particular, the model should suitably balance the trade-off between accuracy/detail and complexity/tractability. The modelling framework proposed in this paper enables and facilitates the development of advanced control algorithms, such as MPC, to more efficiently manage MES and the flexibility sources they can provide.

It has been proven that flexibility optimisation strategies, which also account for synergies between electricity and heat energy systems, in combination with demand response services and energy storage technologies, have the potential to offer numerous benefits, e.g., providing flexibility to counteract the intermittency of RES. [4]. Currently, to the best of the author's knowledge, no modelling approach that captures all these aspects in a unified framework exists. This need is addressed in this paper which presents a novel, generalised approach to system level modelling of MES for optimisation based energy management problems. The proposed modelling approach is particularly well suited, though not limited, to predictive control applications.

A. Related Literature

The energy hub has been a prevalent concept for the modelling of MES over the last decade. Energy hubs are described as units with inputs and outputs in which the storage and conversion of different energy carriers take place [5]. In the original formulation, a hub is comprised of converter and storage elements. Converter elements represent how efficiently energy carriers are converted from one form to another, whilst storage elements represent energy storage devices. An excellent resource, containing software tools for energy hub modelling can be found in [6]. The energy hub modelling approach has been applied to a variety operational challenges associated with MES such as expansion planning [7] and [8], optimal design [9] and [10], resource optimisation [11] and [12] and reliability assessment [13]. The energy hub is capable of representing MES flexibility provided by energy storage and the increased degrees of freedom in the energy resource dispatch. However, as highlighted and addressed in [14], existing energy hub approaches do not allow the modelling of arbitrarily complex converter topologies (e.g. including bi-directional power flow and splitting of energy flow at any point in the converter arrangement). Recent contributions based on the energy hub that also aim to address some of these issues are [15] and [16]. However, while both address the issue of arbitrarily complex uni-directional flow paths across an energy hub, neither address the possibility of bi-directional power flow through, or within a hub as is the case with the approach proposed herein.

Alternative approaches to MES modelling are included in [17], [18], [19], [20] and [21]. The models proposed in [17] and [18] are not presented as a generalised approaches for MES, focusing on specific technologies instead, and do not including detailed models of flexible energy demand/production. Generalised MES model is presented in [19] that incorporates spatio-temporal dependencies in a multi-objective optimisation for planning, design and operation of MES. However, the

emphasis on planning and design as well as the lack of detailed flexible prosumer models and multi-mode devices make it unsuitable for the control-oriented applications considered in this paper. The model presented in [20], while presenting a more general framework based on game theory, is focused on system design with an emphasis on stakeholder benefit over the lifetime of a MES project and is not concerned with day-to-day or intra-day operation as is typically the case with control focused applications. Also, similarly to [17], [18] and [19], flexible energy demand/production models are not included. A more general approach to MES modelling, considering aspects of control is proposed in [21] where energy systems are described using three fundamental components that exchange both energy and information. However, while the so-called *client modules* have the potential to model some demand flexibility, they are not able to capture some of the more detailed flexibility characteristics e.g. to shift or interrupt a particular demand profile.

Due to the lack or insufficiency of flexible energy demand/production models, the aforementioned approaches do not capture the full range of flexibility benefits that result from considering MES. In particular, existing methods do not capture the full range of flexibility offered through integrated demand side management (IDSM) which extends the domain of DSM to MES, where in addition to shifting or modulating their energy use, which is realised by DSM, energy users are able to switch the primary source of the consumed energy [22]. Contributions focusing on IDSM include [23], [24] and [25]. However, these are not fully comprehensive approaches because they either focus on demand response (DR) in the cases of [23] and [24], which is a sub-set of DSM/IDSM and relates to the DSM programs implemented to alter energy use in response to particular events such as periods of poor system reliability or high wholesale energy cost, or they focus on specific energy vectors as is the case in [25].

The application of optimisation-based DSM techniques in electrical power systems has been widely researched topic, with models relating to electrically powered resources being recently proposed in [26], [27] and [28]. However, since these approaches focus on electrical energy systems, they do not include multi-energy technologies that unlock additional flexibility and enable IDSM, such as co-generation. It is worth mentioning that the DSM model in [27] covers a range of flexible demand *characteristics* captured by the model presented in this paper but is only focused on a single primary energy vector.

IDSM and the energy hub modelling approach have been combined in [29], [30] and [31]. These references all include flexible demand models of varying detail, but none offer a fully comprehensive approach to modelling all the possible flexibility characteristics in MES. Therefore, these approaches cannot realise the opportunities from fully exploiting all the possible flexibility characteristics that arise in controllable energy demand/production. In [29], the DR model allows modulation of the demand profile but does not consider the case where the demand profile (or at least a portion of it) is fixed and can only be shifted in time. The DR model in [30] only considers shiftable and curtailable demands while the

model proposed in [31] cannot model either fully adjustable or interruptible demands.

Applications of IDSM within MES are considered for the specific case of microgrid operation with DR capabilities in [32], [33] and [34]. However, these contributions only consider curtailable and/or shiftable demands.

Most of the references listed thus far do not address the control problem of automatically operating MES in the presence of uncertain disturbances. The exceptions are [33] and [34], both of which include operational considerations. Also, MPC applications to MES have been reported in [35], [36], [37] and [38]. However, these approaches either do not include IDSM (which is the case in [35]) or the proposed models omit at least some of the flexibility characteristics included in the framework proposed in this paper, i.e. [36] and [38] do not include interruptible demands and [37] only considers adjustable demands based on thermal dynamics associated with buildings.

B. Summary of Contribution

To the best of the authors' knowledge, no formalised approach to MES modelling for control applications has yet been proposed that incorporates the full range of flexibility sources available in MES. In order to address this, an innovative generalised approach to modelling MES for predictive control design is presented in this paper. The contributions of the paper can be summarised as follows:

- Provision of an innovative energy conversion model that improves on existing approaches (e.g. the energy hub) to facilitate the representation of arbitrarily complex energy converter topologies, i.e. arrangements that can include any number of energy vectors, inputs/outputs, splits/combinations of energy flow, multi-mode devices and/or co-generation devices and bi-directional power flow.
- Proposal of a novel flexible energy demand/production model that, combined with the aforementioned energy conversion model, can capture the full range of flexibility benefits made available through IDSM. While alternative IDSM models have been proposed in the literature, they focus only on a single energy vector and/or none include the full range of flexibility characteristics that can be modelled using the approach proposed herein. Furthermore, the proposed flexible demand/production model can readily incorporate aspects linked to energy-user comfort.
- Unification of MES modelling and IDSM into a new framework that is not based on or restricted to any particular technologies or energy vectors. The modular nature of the framework removes need for spending excessive time developing bespoke mathematical models for particular systems/scenarios as pre-defined components can be connected together to form relatively complex compound models. This aspect makes the approach particularly amenable to software implementation.
- A mechanism for the updating of sequential dispatch schedules and inter-temporal constraints relating to physical dynamic relationships (e.g. storage, thermal loads)

in order to facilitate on-line application rather than static 'look-ahead' optimisation.

While the proposed modelling approach has been developed primarily for the deployment within predictive control formulations, it can be readily applied to alternative optimisation based energy management applications. For example, developed causal models of MES can be used to evaluate the impact of various 'what-if' scenarios for the purposes of risk analysis, planning, process design (e.g. determining the optimal size of storage or energy conversion device), economic dispatch and unit commitment. Hence, the proposed framework should be of interest to the wider multi-physics and engineering community. Additionally, the proposed modelling approach places no restriction on the particular objective function employed in the optimisation, allowing functions other than that employed for the case study in Section VI-B to be used in other applications (e.g. flexibility optimisation, environmental concerns, ancillary service provision etc.). Alternative functions can also be combined for the purposes of multi-objective optimisation. Finally, the proposed modelling framework can be incorporated into stochastic and/or robust optimisation formulations that rigorously address inevitable uncertainties, such as fixed demand profiles, system model parameters and energy prices.

Further discussion highlighting the superiority of the proposed approach over existing methods is provided in Section VI.

C. Outline of the Paper

The remainder of this paper is organised as follows. In Section II an overview of the control oriented modelling tool is provided and the key components of the proposed MES model are introduced. The first of these components, namely the energy conversion model, is described in detail in Section III. In Section IV the storage model component is described. A prosumer model component is described in Section V along with a detailed description of the flexible demand modelling approach and its extension to energy producers. In Section VI, a particular case-study is introduced that illustrates capabilities of the proposed modelling approach when integrated within MPC control framework. The simulated results relating to the case study are presented in Section VII before the paper is concluded and discussion of future work is provided in Section VIII.

II. CONTROL ORIENTED MODELLING FRAMEWORK FOR MULTI-ENERGY SYSTEMS

Figure 1: General Model of Multi-EEZ MES

The MES modelling approach proposed in this paper assumes that for certain sections of MES it is reasonable to represent the transportation networks of the various energy carriers within the section as non-dynamic 'lossy' connections. Such sections are termed energy exchange zones (EEZ) and extend the well known energy hub concept [39]. As with the energy hub, the spatial boundaries of a particular EEZ depend upon a given application (e.g. a building, district, city etc.) and are essentially defined by the validity of assuming non-dynamic and linear relationships in the transportation networks. An EEZ contains three types of components, which are the *energy conversion model* (ECM), the *storage model* (SM) and *prosumer model* (PM). The flow, conversion and splitting of the energy carriers are modelled by means of the ECM. Description of various energy storage devices that may exist within an EEZ is provided using the SM. Finally, different types of demand and/or production that may exist within an EEZ are modelled using PM. In a given EEZ, only one ECM component may exist. This is in contrast to SM and PM components, which are not limited by number of occurrences within a single EEZ.

A schematic of a single generic EEZ that exists within a larger multi-zone MES is depicted in Figure 1. The ECM component provides an interface between SM and PM sub-components in a given EEZ and a wider multi-energy network. Separation of energy conversion, storage dynamics and various prosumer characteristics, as shown in Figure 1, facilitates modular approach to the modelling of MES. Also, the proposed approach for the ECM allows configurable bi-directional converter topologies to be described in a compact manner whilst maintaining a reasonable representation of the underlying physical system. In the following sections, modelling of these three types of EEZ components will be described in greater detail. Whilst the remainder of the paper considers a single EEZ, the proposed modelling framework can be readily applied to general MES composed of multiple EEZs interconnected with transportation networks as indicated in Figure 1.

While EEZ components can be modelled, to some extent, using the existing approaches found in the literature, to the best of the authors' knowledge, detailed models of each have not been combined, formally, into a single general

modelling framework. Furthermore, the proposed approach includes innovative development of ECM and PM models that provides them with flexibility necessary for the integration within generalised modelling framework. The generality of the propose approach allows it, under certain assumptions, to model any MES that may be comprised by the full set or a subset of the components shown in Figure 1.

III. ENERGY CONVERSION MODEL

The ECM, preliminarily introduced in [14], represents a novel extension of the original energy hub concept used as a paradigm for representing energy conversion within MES. In particular, the ability of the ECM modelling approach to represent energy conversion topologies of arbitrary complexity represents one of the key contributions of this paper. No restrictions are imposed on the direction of power flow through a particular arrangement of converters nor the number and/or position of energy carrier splits within. This is achieved whilst still maintaining a reasonable representation of physical conversion devices, with particular reference to multi-output converters such as combined heat and power (CHP) devices. Furthermore, the proposed methodology is well suited to software implementation thus expediting wider model development.

The ECM representation is based on graph theory and is fully characterised by the set of nodes it contains and the associated arcs that provide mutual interconnections between the nodes. Each node of the graph has an energy carrier associated with it. The arcs of the graph represent directional paths through the ECM along which the power can flow and are denoted as $(n_i \rightarrow n_j)$ where n_i and n_j correspond to an arc's source node and sink node, respectively. Each arc has an associated conversion factor, denoted as $\eta_{(n_i \rightarrow n_j)}$. The power that flows to n_j node is then given by $P_{(n_i \rightarrow n_j)}(k)\eta_{(n_i \rightarrow n_j)}$, where $P_{(n_i \rightarrow n_j)}(k)$ is the power flowing from node n_i during sampling period k . Also, two index sets are associated with each node. These are $\mathcal{I}_{\rightarrow n_l}$, which denotes the index set of source nodes that connect to n_l , and $\mathcal{I}_{n_l \rightarrow}$, which denotes the index set of sink nodes that n_l connects to.

Each node in the ECM belongs to one of the following four categories: terminal nodes, sum nodes, switch nodes and transmitter nodes. Terminal nodes represent connections of the ECM to externalities that include prosumers, storage elements and the wider energy network. Terminal nodes can be further sub-divided into input, output and input/output nodes depending on whether power flows in or out of the ECM. Input nodes have a single outgoing arc and facilitate import of energy to the ECM. Output nodes, conversely, have a single incoming arc and facilitate export of energy out of the ECM. Lastly, input/output nodes have both a single incoming and single outgoing arc connected to the same adjacent node. Input/output nodes allow interface of the ECM with other entities within MES that both import and export energy, e.g. prosumers and storages. The power flowing into or out of the ECM's terminal node n_l , denoted as P_{n_l} , is related to

the incoming and/or outgoing arc powers according to the following equation:

$$P_{n_l}(k) = \sum_{i \in \mathcal{I}_{\rightarrow n_l}} P_{(n_i \rightarrow n_l)}(k)\eta_{(n_i \rightarrow n_l)} - \sum_{j \in \mathcal{I}_{n_l \rightarrow}} P_{(n_l \rightarrow n_j)}(k) \quad (1)$$

Furthermore, non-negativity constraints are imposed on all ECM arc flow variables which ensures that (1) respects a sign convention such that $P_{n_l} \geq 0$ for input nodes and $P_{n_l} \leq 0$ for output nodes. In addition, upper bounds on arc decision variables can be employed to limit capacity of particular conversion devices.

Sum nodes describe the combining and splitting of a particular energy carrier within the ECM. For each sum node the algebraic sum of incoming and outgoing power flows is equal to zero:

$$\sum_{i \in \mathcal{I}_{\rightarrow n_l}} P_{(n_i \rightarrow n_l)}(k)\eta_{(n_i \rightarrow n_l)} - \sum_{j \in \mathcal{I}_{n_l \rightarrow}} P_{(n_l \rightarrow n_j)}(k) = 0 \quad (2)$$

Sum nodes represent junctions of energy flow and can be used to describe particular parts of a multi-energy system, such as an electrical busbar or pipework manifold.

Switch nodes, similarly to sum nodes, obey the relationship between their incoming and outgoing arcs given in (2). However, additional binary decision variables, denoted as δ , are associated with each outgoing arc of a switch node to obey the condition (3a) and the constraint (3b) which ensure that only a single outgoing arc is active at any instant in time.

$$P_{(n_l \rightarrow n_j)}(k) > 0 \iff \delta_{(n_l \rightarrow n_j)}(k) = 1 \quad \forall j \in \mathcal{I}_{n_l \rightarrow} \quad (3a)$$

$$\sum_{j \in \mathcal{I}_{n_l \rightarrow}} \delta_{(n_l \rightarrow n_j)}(k) \leq 1 \quad (3b)$$

Therefore, switch nodes facilitate modelling of multi-mode devices, such as a heat pump that can operate in both heating and cooling modes.

Transmitter nodes are used to model the conversion of energy through a co-generation plant, such as CHP, by requiring that each of their outgoing arc flow values are equal to the sum of the incoming arc flow values, as described in (4). For each transmitter node there are as many equations as there are outgoing arcs.

$$P_{(n_l \rightarrow n_j)}(k) = \sum_{i \in \mathcal{I}_{\rightarrow n_l}} P_{(n_i \rightarrow n_l)}(k)\eta_{(n_i \rightarrow n_l)} \quad \forall j \in \mathcal{I}_{n_l \rightarrow} \quad (4)$$

To prevent the unrealistic case of simultaneous power flow in two parallel but directionally opposite arcs between two nodes, binary decision variables are associated with the particular arcs that obey an equivalent condition to (3a) and the following mutual exclusivity constraint:

$$\delta_{(n_i \rightarrow n_j)}(k) + \delta_{(n_j \rightarrow n_i)}(k) \leq 1 \quad (5)$$

IV. STORAGE MODEL

The modular nature of the proposed approach allows for the storage dynamics to be represented by a general n-th order discrete-time state space model, i.e. $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) +$

$\mathbf{B}\mathbf{u}(k)$; $\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k)$. Note that the generality of state-space models allows multi-input, multi-output SM to be considered. Also, note that the sampling time for a particular application must depend on the dominant time constant of the storage dynamics and be sufficiently small to capture any transient effects. However, this choice must be made with respect to the speed of the dynamics that are neglected in the wider framework i.e. if the sampling time is too short, then the steady-state assumptions applied to the rest of the model no longer hold.

Inputs to a SM generally consist of charging/discharging power, denoted as $v(k)$, as well as N_w other disturbance inputs, denoted as $\mathbf{w}(k) \in \mathbb{R}^{N_w}$, that affect storage, i.e. $\mathbf{u}(k) = [v(k) \ \mathbf{w}(k)^T]^T$. The output of SM, denoted as $y(k)$, represents the stored energy at time k . Storage models are interfaced with the rest of the MES via ECM's terminal input/output nodes. Equation (6) indicates how the interface between a SM and ECM is accomplished by setting SM input $v(k)$ equal to the negated node power of the associated terminal node of the ECM, in order to maintain the adopted sign convention. Thus, the power associated with the terminal input/output node of ECM to which a SM is connected is positive during discharging and negative during charging. Also, as SMs are connected to terminal nodes, (5) ensures that simultaneous charging and discharging does not occur.

$$v(k) = -P_{n_i}(k) \quad (6)$$

Charging/discharging capacities can be limited by implementing upper bounds on the relevant arc decision variables as described in Section III while finite energy capacity is limited by imposing bounds on the stored energy output state of the model, i.e.

$$\underline{y}(k) \leq y(k) \leq \bar{y}(k) \quad (7)$$

where $\underline{y}$ and $\bar{y}$ are the respective lower and upper bounds.

V. PROSUMER MODEL

In the wider context of energy systems and the 'smart grid', the prosumer is defined as an energy user that both produces and consumes energy, i.e. producer-consumer. As such, within the context of the proposed modelling framework, the prosumer model (PM) represents a terminus of the MES where energy can be consumed and/or produced. Crucially, the model incorporates additional degrees of freedom allowing it to model dispatchable generation and/or IDSM.

The PM is represented by a set of components. Each component models a particular type of energy demand or production (e.g. local generator or RES). A particular PM is denoted, p_{j,n_i} , where j is an index corresponding to the PM when more than one is connected to a terminal ECM node n_i . The net energy demand of a particular prosumer at sampling instant k , denoted as $P_{p_{j,n_i}}(k)$ is given by (8).

$$P_{p_{j,n_i}}(k) = \sum_{m=1}^{N_{G,p_{j,n_i}}} G_m(k) - \sum_{n=1}^{N_{L,p_{j,n_i}}} L_n(k) \quad (8)$$

Where $G_m(k) \geq 0$ and $L_n(k) \geq 0$ represent prosumer's m^{th} generating component and n^{th} load component, respectively.

Also, $N_{G,p_{j,n_i}}$ is the total number of energy generators within the PM and $N_{L,p_{j,n_i}}$ is the total number of energy consumers. The sign of $P_{p_{j,n_i}}(k)$ indicates whether it is producing or consuming, i.e. positive $P_{p_{j,n_i}}(k)$ represents production and negative $P_{p_{j,n_i}}(k)$ represents consumption. As such, end users that are either sole consumers or producers can be represented by (8) with $G_m(k) = 0$ or $L_n(k) = 0$ respectively.

As discussed in the following sub-sections prosumer components can be one of two types: fixed or flexible. In the remainder of this section only consumer type components of the PM will be considered since the potential flexibility of generation type components can be described by using the same approach.

A. Fixed Demand

Fixed demand components represent energy consumption that cannot be modified either in the rate or time of usage and, therefore, must be met completely and instantaneously. Examples include electrical demand for lighting and television as well as 'on-demand' heating to produce hot water drawn from a tap. Within the proposed framework decision variables are included to represent the consumption with their values fixed at each sampling instant by equality constraints.

B. Flexible Demand

Each flexible demand component (FDC) is characterised by a particular schedule of its associated decision variables over a future scheduling horizon with length equal to $N_H \in \mathbb{I}$ and divided into individual scheduling periods, denoted as $h \in \{0, 1, \dots, N_H - 1\}$. Also, it is assumed that the values of the decision variables are constant during each scheduling period. FDC are classified using four different flexibility characteristics:

- **Shiftable:** Commencement of the demand is not fixed and can be moved within the scheduling horizon but no interruptions are allowed.
- **Adjustable:** Baseline energy demand can be exceeded or curtailed at particular sampling instants in the scheduling horizon.
- **Pliable:** The energy demand profile can be manipulated through the scheduling horizon, provided the total energy meets its requirement.
- **Interruptible:** Energy demand, once commenced at some fixed scheduling period, can be paused between particular sampling instants in the scheduling horizon.

Figure 2 shows how the four flexibility characteristics can be combined to represent different types of demand. In this paper the model representing the combination of shiftable adjustable pliable interruptible (SAPI) demand and shown in the centre of Figure 2 will be described in detail. The other combinations in Figure 2 are special cases of the SAPI demand that result in simplified versions of the model. The model is based on [40], with novel extensions and developments including the adoption of slack decision variables and soft constraints to quantify mismatches to the baseline energy requirements for loads. In particular, the slack variables allow for an explicit

Figure 2: Classification of Flexible Demand Types

link to comfort based adjustable loads that require additional dynamic models for their inclusion. Due to space limitations, this feature of the modelling framework is excluded from this paper. However, an example can be found in the associated technical report [41].

In the proposed modelling framework the time-profile of the energy consumption for a particular demand n is split into N_n segments that, once started, cannot be interrupted. Energy consumption of the i^{th} segment of demand n during time period h is denoted as $l_{n,i}(h)$ and is given by:

$$l_{n,i}(h) = \tilde{l}_{n,i}(h) + \Delta l_{n,i}^+(h) - \Delta l_{n,i}^-(h) \quad \forall n, i, h \quad (9)$$

where $\tilde{l}_{n,i}(h)$ is the energy consumption related to the baseline energy requirement for the i^{th} segment of demand n during time period h , and $\Delta l_{n,i}^+(h)$ and $\Delta l_{n,i}^-(h)$ are *adjustable flexibility slack variables* that quantify the amount of energy for the demand n that has been curtailed or exceeded, respectively, for each segment i during each time period h effectively relaxing the requirement to meet the entire baseline energy demand at all times as required by the inequalities in (10). The baseline energy requirement represents a *preferred* amount of energy to be consumed for each segment if adjustable flexibility is not required. The cumulative sum over the scheduling horizon of the baseline energy consumption for the i^{th} segment of demand n must remain within the baseline lower and upper bounds of the segment's total energy requirement, denoted $\underline{E}_{n,i}$ and $\overline{E}_{n,i}$ respectively:

$$\underline{E}_{n,i} \leq \sum_{h=0}^{N_H-1} \tilde{l}_{n,i}(h) \leq \overline{E}_{n,i} \quad \forall n, i \quad (10)$$

In the cases of shiftable and/or interruptible and/or adjustable demand type the baseline segment time-profile is fixed and is not a degree of freedom. For these cases $\underline{E}_{n,i} = \overline{E}_{n,i} = E_{n,i}$ and (10) collapses to an equality constraint. Conversely, in the case of any demand that includes pliable type flexibility, the baseline time-profile is not pre-determined and represents an additional degree of freedom that is determined whilst respecting the constraint imposed on the baseline total energy requirement specified in (10). Furthermore, since the total energy requirement is fixed in the cases of shiftable and/or interruptible and/or pliable demand types, its curtailment/increase is not allowed and hence $\Delta l_{n,i}^+(h) = 0 \quad \forall i, h$ and $\Delta l_{n,i}^-(h) = 0 \quad \forall i, h$. However, in the case of a flexible demand

that combines adjustable flexibility with any of the other three types, $\Delta l_{n,i}^+$ and $\Delta l_{n,i}^-$ represent degrees of freedom. Hence, in order to be able to represent any combination of the four demand types, degrees of freedom include the baseline energy consumption time-profile, given by $\tilde{l}_{n,i}(h)$, as well as both curtailment and increase in energy demand, given by $\Delta l_{n,i}^-(h)$ and $\Delta l_{n,i}^+(h)$ respectively. In the case of adjustable flexibility characteristic both demand curtailment ($\Delta l_{n,i}^-(h)$) and increase ($\Delta l_{n,i}^+(h)$) are manipulated through appropriate cost-function selection, i.e. equally large penalties must be imposed on each variable under 'normal' operation, when neither demand curtailment or increase is required, and only one the responses (curtailment or increase) be incentivised at any given h .

The resulting energy consumption of a particular demand n at a given time h is represented by $L_n(h)$ and is related to the actual energy consumption of its energy segments in the following way:

$$L_n(h) = \sum_{i=1}^{N_n} l_{n,i}(h) \quad \forall n \quad (11)$$

An additional three sets of binary decision variables are utilised to commence or to halt individual demand segments. The first set, δ_n^p , contains *segment processing* variables which determine whether or not an energy segment i is to be processed during each time step h . In particular, $\delta_{n,i}^p(h) = 1$ if and only if energy segment i is in process at time h . The second set, δ_n^c , is comprised of *segment complete* variables which indicate whether or not a particular energy segment has been completed. More specifically, $\delta_{n,i}^c(h) = 1$ if and only if segment i has been completed before time step h . The third set, δ_n^w , consists of *segment waiting* variables which indicate whether or not a demand has been halted between completed and yet-to-be-completed segments i.e. $\delta_{n,i}^w(h) = 1$ if and only if energy segment $i-1$ has been completed and segment i is waiting to begin.

The seven sets of decision variables that are associated with the general SAPI FDC are listed in (12).

The inequalities relating to the auxiliary variables are provided in (13) where (13a) and (13b) determine the (hard) limits on the size of the energy requirement deviation variables $\Delta l_{n,i}^+$ and $\Delta l_{n,i}^-$, with $\overline{\Delta l}_{n,i}^+$ and $\overline{\Delta l}_{n,i}^-$ representing the corresponding upper bounds. Similarly, (13c) determine the minimum $\underline{l}_{n,u}$ and maximum $\overline{l}_{n,u}$ amount of energy that can be scheduled for any particular demand segment at a particular instant. In (13a)-(13c), the multiplication of the segment *processing* binary decision variables by the lower and/or upper bounds ensures that the corresponding continuous decision variable cannot be non-zero unless the particular segment i is currently under way. The length of a particular demand segment, given in terms of a number of scheduling periods, is constrained from below and above by $\underline{N}_{n,i}^p$ and

$$\mathbf{l}_n = [l_{n,1}(0), \dots, l_{n,1}(N_H - 1)]^T, \dots, [l_{n,N_n}(0), \dots, l_{n,N_n}(N_H - 1)]^T \in \mathbb{R}_{\geq 0}^{N_n \cdot N_H} \quad (12a)$$

$$\delta_n^p = [\delta_{n,1}^p(0), \dots, \delta_{n,1}^p(N_H - 1)]^T, \dots, [\delta_{n,N_n}^p(0), \dots, \delta_{n,N_n}^p(N_H - 1)]^T \in \mathbb{I}_{\{0:1\}}^{N_n \cdot N_H} \quad (12b)$$

$$\delta_n^c = [\delta_{n,1}^c(0), \dots, \delta_{n,1}^c(N_H - 1)]^T, \dots, [\delta_{n,N_n}^c(0), \dots, \delta_{n,N_n}^c(N_H - 1)]^T \in \mathbb{I}_{\{0:1\}}^{N_n \cdot N_H} \quad (12c)$$

$$\delta_n^w = [\delta_{n,2}^w(0), \dots, \delta_{n,2}^w(N_H - 1)]^T, \dots, [\delta_{n,N_n}^w(0), \dots, \delta_{n,N_n}^w(N_H - 1)]^T \in \mathbb{I}_{\{0:1\}}^{(N_n-1) \cdot N_H} \quad (12d)$$

$$\tilde{\mathbf{l}}_n = [\tilde{l}_{n,1}(0), \dots, \tilde{l}_{n,1}(N_H - 1)]^T, \dots, [\tilde{l}_{n,N_n-1}(0), \dots, \tilde{l}_{n,N_n-1}(N_H - 1)]^T \in \mathbb{R}_{\geq 0}^{N_n \cdot N_H} \quad (12e)$$

$$\Delta \mathbf{l}_n^+ = [\Delta l_{n,1}^+(0), \dots, \Delta l_{n,1}^+(N_H - 1)]^T, \dots, [\Delta l_{n,N_n-1}^+(0), \dots, \Delta l_{n,N_n-1}^+(N_H - 1)]^T \in \mathbb{R}_{\geq 0}^{N_n \cdot N_H} \quad (12f)$$

$$\Delta \mathbf{l}_n^- = [\Delta l_{n,1}^-(0), \dots, \Delta l_{n,1}^-(N_H - 1)]^T, \dots, [\Delta l_{n,N_n-1}^-(0), \dots, \Delta l_{n,N_n-1}^-(N_H - 1)]^T \in \mathbb{R}_{\geq 0}^{N_n \cdot N_H} \quad (12g)$$

$$\mathbf{L}_n = [L_n(0), \dots, L_n(N_H - 1)]^T \in \mathbb{R}_{\geq 0}^{N_H} \quad (12h)$$

$\overline{N}_{n,i}^p$ respectively.

$$0 \leq \Delta l_{n,i}^-(h) \leq \overline{\Delta l}_{n,i}^-(h) \delta_{n,i}^p(h) \quad \forall n, i, h \quad (13a)$$

$$0 \leq \Delta l_{n,i}^+(h) \leq \overline{\Delta l}_{n,i}^+(h) \delta_{n,i}^p(h) \quad \forall n, i, h \quad (13b)$$

$$l_{n,i}(h) \delta_{n,i}^p(h) \leq l_{n,i}(h) \leq \tilde{l}_{n,i}(h) \delta_{n,i}^p(h) \quad \forall n, i, h \quad (13c)$$

$$\underline{N}_{n,i}^p \leq \sum_{h=0}^{N_H-1} \delta_{n,i}^p(h) \leq \overline{N}_{n,i}^p \quad \forall n, i \quad (13d)$$

The following inequalities imposed on *segment processing* and *segment complete* binary decision variables ensure that once an energy segment has commenced, it is not interrupted and must run to completion:

$$\delta_{n,i}^p(h) + \delta_{n,i}^c(h) \leq 1 \quad \forall n, i, h \quad (14a)$$

$$\delta_{n,i}^p(h-1) - \delta_{n,i}^p(h) \leq \delta_{n,i}^c(h) \quad \forall n, i \quad \forall h \in \{1 : N_H - 1\} \quad (14b)$$

$$\delta_{n,i}^c(h-1) \leq \delta_{n,i}^c(h) \quad \forall n, i \quad \forall h \in \{1 : N_H - 1\} \quad (14c)$$

The correct sequencing of individual segments is ensured by the following inequality constraint imposed on *segment processing* and *segment complete* binary decision variables:

$$\delta_{n,i}^p(h) \leq \delta_{n,i-1}^c(h) \quad \forall n, h \quad \forall i \in \{2 : N_n\} \quad (15)$$

The *segment waiting* decision variables allow halting of the corresponding energy segment after its predecessor has been completed:

$$\delta_{n,i}^w(h) = \delta_{n,i-1}^c(h) - (\delta_{n,i}^p(h) + \delta_{n,i}^c(h)) \quad \forall n, h \quad \forall i \in \{2 : N_n\} \quad (16)$$

The halting period is bounded from below and above by $\underline{N}_{n,i}^w$ and $\overline{N}_{n,i}^w$:

$$\underline{N}_{n,i}^w \leq \sum_{h=0}^{N_H-1} \delta_{n,i}^w(h) \leq \overline{N}_{n,i}^w \quad \forall n, h \quad \forall i \in \{2 : N_n\} \quad (17)$$

Finally, (18) specifies the equality constraint which ensures that no segment of demand n can run at particular scheduling instants listed in vector $\mathbf{T}_n^p$, which correspond to the user time preferences for the demand:

$$\delta_{n,i}^p(h) = 0 \quad \forall n, i \quad \forall h \in \mathbf{T}_n^p \quad (18)$$

As stated, the other categories of flexible demand shown in Figure 2 are special cases of the SAPI. In particular, the constraints described above take a particular form and not all the decision variable vectors in (12a)-(12h) are required, depending on the classification. For example, S, P and I demands do not require the slack variables $\Delta \mathbf{l}^+$ and $\Delta \mathbf{l}^-$. Due to space limitations, the mathematical description of the fundamental demand types S, A, P and I are not covered in this paper. However, the interested reader is referred to the technical report, which includes more detailed descriptions and illustrations associated with each of the different fundamental demand types [41].

If the above constraints, which model the allowable behaviour of a FDC, are part of an optimisation problem solved repeatedly by a predictive controller (i.e. in an MPC scheme), it is then necessary to account for the effect of the control actions determined by the solution of the previous optimisation (at $k-1$) on the constraints of the current optimisation (k), a detailed description of which is also provided in [41].

VI. CASE STUDY

The MES used to demonstrate the modelling framework presented in Sections II-V is schematically shown in Figure 3. It represents energy infrastructure for three buildings located at the University of Manchester (UoM) North Campus, namely Barnes Wallis Building (BWB), Sackville Street Building (SSB) and Ferranti Building (FB). The transportation networks

interconnecting the energy infrastructure of the three buildings is assumed to be non-dynamic and, therefore, is considered as a single EEZ introduced in Section II. It can be seen that multiple facets of MES are included within the EEZ: multiple energy vectors (electricity, gas, heat and cooling), multi-generation (CHP), multi-mode operation (heat pump), energy storage (battery and thermal store), bi-directional flow (grid connection and heat network between FB and BWB), prosumer (FB both consumes electricity and produces it from a PV installation) and IDSM with a shiftable-interruptible (SI) electricity demand in SSB, and an adjustable-pliable (AP) heat demand in FB. Not all of the technology shown in Figure 3 currently exists within the actual buildings. In particular, the CHP, heat pump, heat storage, chiller and FDC have all been added to the simulation for demonstration purposes.

The particular MES considered in this case study has been chosen as it demonstrates most features of the modelling framework as well as highlighting the superiority of the proposed approach over existing methods. In particular, it is not possible to represent the energy conversion topology, shown in Figure 3, as a single hub using existing energy hub approaches, even using the recent modifications in [15] and [16], due to the presence of bi-directional power flow within the arrangement. The non-energy hub approach reported in [21] requires the user to distinguish between inputs and outputs referred to sources and clients in [21] when modelling bi-directional power flow in which case, as described in Section III, it is entirely possible for a given interface to act both as an input *and* an output. The comprehensive MES approach in [19] does facilitate the modelling of bi-directional power flow but, similarly to that reported in [21], it provides no explicit option for multi-mode devices. Furthermore, none of these approaches are capable of modelling the FDCs of the types shown in Figure 3 (or, more generally, fully flexible prosumers). As mentioned in Section I-A, none of the other contributions that include more detailed flexible demand models for MES (e.g. [29] and [31]), are able to capture *all* the flexibility characteristics modelled in this paper (i.e. shiftable, adjustable, pliable and/or interruptible flexibility), nor can they be mutually combined (e.g. an AP demand can be modelled by the approach in [38] but a SI demand cannot). Another novel aspect of the proposed approach, and one that is not considered already published methods is the procedure of updating receding horizon constraint updates as mentioned in Section V).

The graph diagram that represents the ECM is shown in Figure 4. With reference to Section III, the terminal nodes are represented as squares, sum nodes as circles, transmitter nodes as triangles and switch nodes as irregular pentagons. The alphabetic characters in the node identifiers seen in Figure 4 relate nodes to specific energy carriers (i.e. *e* is electricity, *g* is gas, *c* is cooling and *h* is heat) which, combined with the numeric characters, generate unique indices for each node. For example, employing the node identifiers in Figure 4, the arc power flow variable from node *g2* to *c1* is denoted $P_{(g2 \rightarrow c1)}$ and the associated arc weighting is denoted $\eta_{(g2 \rightarrow c1)}$. In Figure 4, where conversion devices exist in the ECM and consequentially the arc weightings are not equal to 1, the η

Figure 3: North Campus Buildings Energy System Schematic

symbols associated with particular devices have been labelled. For example, $\eta_{(g2 \rightarrow c1)} = \eta_{gc}^C$ represents the gas-to-cooling conversion efficiency of the chiller. The values associated with these conversion factors used for the simulations can be seen in Table I. The full ECM requires a single equation for each non-transmitter node and multiple equations for each transmitter node corresponding to the number of its outgoing arcs. Therefore, 21 (19+2) equations represent the ECM for this case study.

The nodes *h5* and *e4* in Figure 4 are the dedicated terminal nodes for the battery and thermal storage devices with the weightings of their incoming and outgoing arcs representing the respective charging and discharging efficiencies of the particular devices. For the case study, simple single state models have been adopted to represent the storage devices. Hence, the *A, B, C* and *D* state-space matrices introduced in Section IV become scalars and equal to S_{n_i} , 1, 1 and 0 respectively. For each storage $S_{n_i} < 1$ represents the standby storage efficiency of the device connected to node n_i . Also, $u(k) = Q_{n_i}(k)$ is a scalar quantity representing the charging/discharging power for each storage device and $y(k) = R_{n_i}(k)$ is equal to the stored energy in each device. The particular parameter values for the case study are displayed in Table II.

A. Simulation Data

The UoM Estates department operates a building management system (BMS) that, among other functions, logs energy usage data at half-hourly intervals. Data relating to a particular energy vector for a particular building may be logged using a variety of physical meters (e.g. an electricity meter might be present at each switchboard) but the data logging software includes *virtual meters* for each building that logs total energy consumption for each separate energy vector. Therefore, data was collected for the total electricity and heat demands of both buildings, in addition to the amount of energy produced by the PV installation on FB. Because the heat pump and chiller were simulated additions to the case study, it was necessary to synthesise a data set for the cooling demand. Also, to match the frequency of the data logging, the duration of each

Table I: Conversion Efficiencies Related to Physical Devices where Applicable

ECM Arc Factors	Parameter Label	Device	Conversion	Value
$\eta_{(e1 \rightarrow e2)}, \eta_{(e2 \rightarrow e1)}$	η_{ee}^T	Transformer	electricity	0.98
$\eta_{(g2 \rightarrow h2)}, \eta_{(g2 \rightarrow h3)}, \eta_{(g2 \rightarrow h4)}$	η_{gh}^B	Boiler	gas to heat	0.698
$\eta_{(g3 \rightarrow h1)}$	η_{gh}^{CHP}	CHP	gas to heat	0.485
$\eta_{(g3 \rightarrow e2)}$	η_{ge}^{CHP}	CHP	gas to electricity	0.294
$\eta_{(g2 \rightarrow c1)}$	η_{gc}^C	Chiller	gas to cooling	1.1
$\eta_{(e3 \rightarrow h4)}$	η_{eh}^{HP}	Heat pump	electricity to heat (summer)	3.2
			(autumn)	2.8
$\eta_{(e3 \rightarrow c1)}$	η_{ec}^{HP}	Heat pump	electricity to cooling (summer)	2.5
			(autumn)	2.1
$\eta_{(h3 \rightarrow h5)}$	η_{hh}^+	Heat store	charging efficiency	0.95
$\eta_{(h5 \rightarrow h3)}$	η_{hh}^-		discharging efficiency	1/0.95
$\eta_{(e2 \rightarrow e4)}$	η_{ee}^+	Battery	charging efficiency	0.9
$\eta_{(e4 \rightarrow e2)}$	η_{ee}^-		discharging efficiency	1/0.9

Table II: Additional Simulation Parameters

Parameter	Description	Value	Parameter	Description	Value
$P_{(g2 \rightarrow g3)}$	CHP input capacity	340 kW	$P_{(e2 \rightarrow e3)}$	HP input capacity	20 kW
$\underline{R}_{h5}/\overline{R}_{h5}$	Min./max. heat energy storage	30/150 kWh	$\overline{Q}_{h5}/\underline{Q}_{h5}$	Max. heat store charge/discharge	100 kW
$\underline{R}_{e4}/\overline{R}_{e4}$	Min./max. battery storage	27/180 kWh	$\overline{Q}_{e4}/\underline{Q}_{e4}$	Max. battery charge/discharge	225 kW
S_{h5}	Heat storage standby loss factor	0.95	S_{e4}	Battery standby loss factor	0.96

scheduling period h is set equal to each sampling period k to result in a half-hourly sampling frequency.

For the purposes of the case study, it was assumed that 60% of heat energy consumed in the FB was fixed with the remaining 40% assumed to be adjustable and pliable (AP). A set of energy demand segments was then computed using the 40% of AP demand by summing the energy consumption over every 2hr period. Hence the total *baseline* amount of heat energy delivered to the building remains equal to the original demand profile but the controller has freedom to manipulate that portion of the energy consumption profile according to its objective. This configuration could represent a situation where thermal modelling of a building has previously been conducted and, using historic data, it produced a predicted *baseline* energy consumption profile over bi-hourly periods. It is also assumed that the AP heat demand, denoted L_{AP} , is a continuously running demand. Therefore, the minimum and maximum number of scheduling periods during which each energy segments must be processed are both equal to 4 (2hr time interval) i.e. $\underline{N}_{AP,i}^p = \overline{N}_{AP,i}^p = 4 \forall i$. The maximum amount of energy the AP demand can consume over one sampling period is set to 300kWh (i.e. $\overline{U}_{AP,i}(k+h) = 300 \forall i, k, h$). Also, the maximum deviation from the baseline requirement during a sampling period is ± 20 kWh.

The shiftable and interruptible (SI) electricity demand, denoted L_{SI} , represents the energy consumption of a machine located within SSB that has a fixed demand profile each time it runs. The example profile used for the case study is equal to $\mathbf{D}_{SI} = [E_{SI,1}, E_{SI,2}, E_{SI,3}, E_{SI,4}, E_{SI,5}, E_{SI,6}, E_{SI,7}]^T = [10, 10, 15, 10, 15, 15, 10]^T$. Each element of $\mathbf{D}_{SI}$ has kWh units. Also, $\underline{N}_{SI,i}^p = \overline{N}_{SI,i}^p = 1 \forall i$. Since the demand is interruptible, between any energy segment in the profile the controller is able to delay the commencement of the next segment by up to maximum of 3 scheduling periods (i.e.

$\overline{N}_{SI,i}^w = 3 \forall i$). Further, as the demand is shiftable, the particular period during which the machine commences its operation can be determined by the controller.

Figure 4: North Campus Buildings Energy Conversion Model

Other parameter values related to the case study are presented in Table II along with their units where applicable.

B. Control Scheme

In order to exemplify the control-oriented modelling framework presented in this paper, a certainty equivalent model predictive controller (CEMPC), where future uncertainties in the optimisation are assumed deterministic based on predicted values [42], is applied to the case study introduced earlier in this section. The CEMPC is introduced in this section whilst the results of its application are provided in Section VII.

The CEMPC solves the deterministic multi-period optimisation problem given in (19) during each sampling period k . The problem (19) is defined over a user-specified future control horizon, which is without loss of generality set equal to the scheduling horizon, N_H , introduced in Section V-B. For the purposes of the case study a 48 period (24hr) control horizon is employed, i.e. $N_H = 48$. The objective function $J(k)$ is predominantly economic, aiming to minimise the cost of purchasing energy from external entities, e.g. the power grid. Additional terms are included in the objective function that relate to control performance as well as incentives and/or penalties imposed by external entities. The solution of the optimisation problem produces a schedule defined over the length of the control horizon for the system's manipulating variables, which are namely the amounts of electrical energy and gas purchased from the external entities, the dispatch of energy to various conversion devices, as well as the schedule of the flexible demand characterised by the various binary decision variables introduced in Section V over the length of the horizon. The control actions determined by the schedule for the current sampling period are applied to the system which then evolves based on its inherent dynamics and any associated disturbances before the state of the system is observed again at the next sampling instant and the optimisation problem is resolved.

Since the decision variables specified in (12) are calculated repeatedly during each sampling period with the scheduling horizon remaining constant and defined over the next/future N_H scheduling periods, the component elements of the decision variable vectors in (12) during online control implementation are time-indexed with $k + h$ instead of h .

Furthermore, if the constraints introduced in the previous section, which model the allowable behaviour of a FDC, are part of an optimisation problem solved repeatedly by a predictive controller (i.e. in an MPC scheme), it is then necessary to account for the effect of the control actions determined by the solution of the previous optimisation (at time $k - 1$) on the constraints of the current optimisation (at time k). For example, future energy requirements of particular segments reduce, based on how much energy they are scheduled at a given control instant. An interested reader can find a detailed description of the constraint updates in the associated technical report. [41].

Following standard receding horizon implementation, prediction horizon length N_H remains constant throughout the simulation.

$$\begin{aligned} \min J(k) = & \\ & \sum_{h=0}^{N_H-1} [V_{cost}(k+h) + V_A(k+h) + V_{smooth}(k+h)] \\ & s.t. \\ & (1) - (11), (13) - (18) \end{aligned} \quad (19)$$

where:

$$\begin{aligned} V_{cost}(k+h) = & [P_{(e1 \rightarrow e2)}(k+h)\pi_e^{buy}(k+h) \\ & - P_{(e2 \rightarrow e1)}(k+h)\eta_{(e2 \rightarrow e1)}\pi_e^{sell}(k+h) \\ & + P_{g1}(k+h)\pi_g^{buy}(k+h)] \end{aligned} \quad (20a)$$

$$\begin{aligned} V_A(k+h) = & \\ & \sum_{i=1}^{N_n} [\Delta l_{AP,i}^+(k+h)\lambda_A^+(k+h) + \Delta l_{AP,i}^-(k+h)\lambda_A^-(k+h)] \end{aligned} \quad (20b)$$

$$\begin{aligned} V_{smooth}(k+h) = & \\ & [L_{SI}(k+h+1) - L_{SI}(k+h)]^2 \lambda_{SI}^\Delta(k+h) \\ & + [L_{AP}(k+h+1) - L_{AP}(k+h)]^2 \lambda_{AP}^\Delta(k+h) \end{aligned} \quad (20c)$$

In (19), V_{cost} is the economic term of the objective function with π_e^{buy} and π_e^{sell} representing the purchase price and received revenue for selling electrical energy, respectively. Similarly, π_g^{buy} represents the cost of purchasing gas.

The term V_A relates to the adjustable portion of demand in the system. Variables $\Delta l_{AP,i}^+$ and $\Delta l_{AP,i}^-$ are the slack variables (introduced in Section V-B) for the AP FDC. Also, λ_A^+ and λ_A^- represent a penalty or incentive for respectively exceeding ($\Delta l_{AP,i}^+ > 0$) or not meeting ($\Delta l_{AP,i}^- > 0$) the apparent energy requirement of the AP energy segment profile. These parameters can be used to implement the provision of services to system operators e.g. frequency response, demand response etc. During 'normal' operation, where no additional service is being provided, λ_A^+ and λ_A^- take positive values, greater than the maximum π_e^{buy} or π_g^{buy} . Hence, unless it is necessary to meet an energy balancing constraint imposed on the entire EEZ, the deviations will remain equal to zero. If, however, it becomes desirable to deviate from the baseline energy requirement during some period(s) in the control horizon then the corresponding $\lambda_A^+(k+h)$ or $\lambda_A^-(k+h)$ will become negative, thus influencing the optimisation to maximise the corresponding deviation variable during the period i.e. if curtailment is required then the corresponding λ_A^- will be negative and if an increase in demand is required the corresponding λ_A^+ will be negative. Also, it is possible to incentivise the operation of a particular energy converter (e.g. the heat pump) by changing the cost coefficient, related to its corresponding variable, into a negative value when, at the same time, the demand response is required.

Finally, the V_{smooth} term imposes a control performance penalty on the difference in energy consumption of the FDC between consecutive periods in the control horizon. This ensures that in the presence of multiple optimal scheduling solutions of the FDC, a unique solution that produces the smoothest possible energy consumption profile is selected. The control performance penalty is included based on the assumption that it is typically desirable to avoid large ramp rates applied to manipulating variables. To ensure that the control performance penalty does not significantly influence the major economic objective of the controller the corre-

sponding objective function coefficients are set such that $\lambda_{SI}^{\Delta}, \lambda_{AP}^{\Delta} < \pi_e^{buy}, \pi_e^{buy}$.

The certainty equivalence of the CEMPC means that future values of uncertain variables, namely *fixed* energy demand and PV production, are treated as deterministic values in the corresponding MPC optimisation problem. These future values are obtained by utilising an adaptation of a Self Organising Map (SOM) neural network architecture [43]. Note that exploring forecasting techniques is beyond the scope of this paper and the SOM was chosen due to its simplicity in implementation. The SOMs are trained offline using historical data and then, during online implementation, they utilise at each time k the previous $N_H - 1$ observations, along with the observation at k , to predict the future $N_H - 1$ values of each uncertain variable. These future values of the uncertain variables are then used to solve the deterministic MPC optimisation problem in order to obtain the optimal schedule of control inputs. For the particular case study introduced in Section VI, collected data spanning the period March 2016 - April 2017 was used to train the SOMs to predict future values of energy demand and PV production.

Due to the presence of the binary decision variables in the EEZ model as well as the quadratic term (20c), the corresponding MPC optimisation problem is a mixed integer quadratic program (MIQP) which is solved using the commercial solver CPLEX at each sampling period. For the case study in this paper the resulting optimisation problem is defined with 9790 decision variables of which 3312 are binaries. The average computation time to solve the optimisation problem during each control period is 32 seconds using a general purpose laptop computer.

VII. RESULTS

Simulations were run, using data from a random week in the autumn of 2017. In order to highlight particular controller responses, an artificial time of use tariff was used for the cost of purchasing electricity from the grid. Fixed prices were used for the purchasing of gas and the export of electricity. The daily price profiles are shown in Figure 5a. The SI demand is

Figure 5: Energy Price Profile for One Day

shown in Figure 5b. It can be seen that the controller schedules the SI demand to run during the period when electricity is

least expensive (between 23:00 and 05:30). Also, as there is a slight price increase between 02:30 and 03:30, the controller interrupts the demand for 1.5hrs to avoid buying electricity at the slightly elevated price. Figures 6a and 6b show the battery charging/discharging profile and the stored energy in the battery respectively. The battery is charged through the periods of relatively inexpensive electricity before being discharged once the price is increased at 03:00, 05:30 and 16:30. Figure 7 shows the heat demand of each building

Figure 6: SSB Controllable Demand and Battery Operation

(7a) along with the CHP operation (7b) during an Autumn weekday. As the CHP is sized for base load it runs at capacity throughout the day. For most of the day, the heat produced by the CHP is diverted to SSB as does it not benefit from a heat interconnection with either the heat pump or energy storage. Figure 8 shows the heat dispatch for the BWB (8a) along with the heat energy storage profile (8b). The BWB heat dispatch

Figure 7: Building Heat Demands and CHP Dispatch

plot shows the HP being operated at maximum capacity until 07:30. It also shows that an excess in heat production is sent to the FB building where the heat store is situated. This is the case except between 05:00 - 05:30 when, coinciding with the peak in heat demand in BWB, heat flow between BWB and FB is reversed and heat storage discharge is sent from FB to BWB which is combined with the HP heat production to meet the increase in heat demand. Between 07:30 and 12:00 the HP operation is reduced in accordance with the reduction in FB heat demand as, combined with the heat store discharge, it is

sufficient to meet the demands of both FB and BWB. The HP is again operated at maximum capacity from 12:00 to 16:30 and heat is largely exported to FB charge the store, except at 12:30 when another relative peak in BWB heat demand causes heat storage discharge and heat to flow from FB to BWB. At 16:30 the electricity price reaches its premium and, after discharging the heat store, it is more economic to run the boiler in BWB. This also coincides, as shown in Figure 7b, with some of the CHP heat output being diverted from SSB to FB, which avoids the use of FB boiler.

Figure 8: BWB Heat Dispatch and Ferranti Heat Store Operation

In order to demonstrate the features of the AP heat demand in the FB two separate simulations were run. In the first, no incentive was provided at any point for violating the baseline energy requirement. In the second, an incentive was provided to increase the electricity demand of the heat pump between 17:30 and 19:30. Figure 9a shows the fixed heat demand for the FB along with the AP demand profile for both cases (with and without the incentive). Figure 9b shows the BWB heat dispatch for the case when the incentive is applied (the equivalent plot for the case without the incentive is shown in the upper plot of Figure 8a). Also, Figure 9a shows the value of the exceeded energy requirement deviation variable $\Delta I_{AP,1}^+$. It can be seen that, due to the incentive, additional energy consumption is scheduled during the incentivised period. In particular, as the maximum allowable deviation from the baseline energy requirement over any sampling period is ± 20 kWh, the controller schedules a maximum increase of 80 kWh over the corresponding period. This additional consumption can also be seen in Figure 9a and 9b where the HP is operated during the incentivised period and heat is exported to FB. This is in contrast to the ex-incentive case where only the boiler is operated during this period (see Figure 8a). Simulations were also run for the scenario where the CHP plan was not present in the system for both Summer and Autumn days. The average cost savings for the two days were approximately 50%.

VIII. CONCLUSION

This paper has presented a new general, comprehensive and formalised modelling framework for the representation of multi-energy systems that facilitates the optimisation of a

Figure 9: Ferranti AP Heat Demand with Barnes Wallis Heat Dispatch

broad range of flexibility benefits. The framework is particularly suited for, but not restricted to, the design of predictive control strategies. The approach is capable of representing converter topologies of arbitrary complexity that can include any number of energy vectors, inputs/outputs, splits/combinations of energy flow, multi-mode devices and/or co-generation devices and bi-directional power flow.

The modelling tool was used to develop the optimal on-line energy management for a group of university buildings within a simulation environment. Where possible, actual energy consumption data was used to ensure a representative case-study. The simulation results demonstrated benefits of utilising predictive control to economically optimise operation of a relatively complex multi-energy system.

Future work will focus on utilising the proposed modelling framework to formulate a control scheme with incorporated robustness against various uncertainties, such as fixed demand profiles, system model parameters and energy prices.

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