A SEAMLESS HYBRID RANS/LES MODEL WITH DYNAMIC REYNOLDS-STRESS CORRECTION FOR HIGH REYNOLDS NUMBER FLOWS ON COARSE GRIDS

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Abstract

To enable economic hybrid RANS/LES of wall-bounded flows at very high Reynolds numbers, two separate RANS and LES velocity and temperature fields are computed in parallel and are dynamically coupled. Both simulations are performed on proprietary grids, each with an optimal, model-adapted topology: a quasi-homogeneous grid for the LES, and a wall-refined high aspect-ratio grid for the RANS simulation. The model automatically governs the blending of RANS and LES stresses on the subgrid-scale level. First, a blending function, based on comparing LES grid quality with integral length scale, automatically determines the mutual correction of RANS and LES solutions. Then, the additional near-wall Reynolds stress correction improves the RANS/LES transitioning behaviour in the viscous and buffer layer. The proposed model ensures consistency, frame invariance and shall increase numerical stability. Overall, this method incorporates ideas from three strands of hybrid modelling, covers heat transfer modelling, also works with second moment closures, is applicable to unstructured grids, requires little user input, and can also be used in an embedded LES mode with even coarser LES grids as outside the local zooming down region only the RANS model drives all the fields.

1 Introduction

Hybrid RANS/LES schemes make use of both statistical and turbulence-resolving methods and are commonly used to overcome the high grid requirements of turbulence-resolving computations of wall-bounded high-Reynolds number flows. The “Two Velocity Fields” model (Uribe et al., 2010) couples RANS and LES on the subgrid-scale stress (SGS) level, where a RANS turbulence model accounts for the mean SGS turbulence, and an LES SGS model contributes the fluctuating, isotropic part. The term “Two Velocity Fields” refers to differentiating between the instantaneous and time-averaged flow field, and to their different characteristics. This concept enables turbulent structures to transition between RANS- and LES-dominant regions without being damped by a high RANS viscosity, hence avoiding the need of additional synthetic turbulence. This model has originally been demonstrated on wall-refined grids with low wall-parallel grid resolution, but also on its capability to return to a RANS mode, when the grid is intentionally coarsened in flow regions of less interest. Furthermore, as one of a few hybrid models, it features a hybrid scalar flux model to account for heat transfer (Rolfo, Uribe, and Billard, 2011). Towards a hybrid model, which is also capable of handling grids with low wall-normal resolution, the framework has recently been extended to a dual-grid model (Xiao and Jenny, 2012), where RANS and LES are computed on separate grids, each optimised for the respective scheme (Nguyen et al., 2018). A homogeneous grid is beneficial for the turbulence isotropy assumption of linear SGS eddy viscosity models. In contrast, the RANS model can be economically computed on a supplemental high-aspect ratio grid.

In the present work, we add a dynamic Reynolds stress correction near walls, where the SGS stresses are continuously compared with RANS-modelled stresses, similar to the model of Chen et al., (2012). The purpose is to assess its capability to further lower the model’s sensitivity to its parameters, especially when the LES grid is extremely coarse in wall-normal direction.

2 Methodology

In LES, the governing continuity, momentum transport and passive scalar transport equations for an incompressible flow are solved for filtered variables \( \overline{\Phi} \), i.e. \( \Phi = \overline{\Phi} + \phi' \), where \( \overline{\cdot} \) denotes filtered and \( \cdot' \) denotes the residual variable. Using the shorthand notation \( \partial_i \) for \( \partial/\partial x_i \), the equations are written as

\[
\partial_i \overline{U}_i = 0,
\]
\[ \partial_t \vec{U}_i + \vec{U}_j \partial_j \vec{U}_i = -\frac{1}{\rho} \partial_t \vec{p} + \nu \nabla^2 \vec{U}_i - \partial_j \tau_{ij}^{SGS}, \]  
(2)

and

\[ \partial_t \vec{U} + \vec{U}_j \partial_j \vec{U} = \Gamma \nabla^2 \vec{U} - \partial_j \tau_{ij}^{SGS}, \]

respectively. \( \vec{U} \) and \( \vec{p} \) are the filtered velocity and static pressure, \( \rho \), \( \nu \) and \( \Gamma \) are density, kinematic viscosity, and scalar diffusivity, \( \tau_{ij}^{SGS} \) is the SGS Reynolds stress tensor

\[ \tau_{ij}^{SGS} = \frac{\partial \vec{U}_i}{\partial x_j} \frac{\partial \vec{U}_j}{\partial x_i} \]  
(4)

and \( \tau_{ij}^{RANS} \) is the SGS scalar flux

\[ \tau_{ij}^{RANS} = \frac{\partial \vec{U}_i}{\partial x_j} \frac{\partial \vec{U}_j}{\partial x_i}, \]

(5)

**Baseline Reynolds stress blending model**

The hybrid model for the deviatoric part of \( \tau_{ij}^{SGS} \),

\[ \tau_{ij}^{hyb} \equiv \tau_{ij}^{SGS} - 1/3 \delta_{ij} \tau_{kk}, \]

is based on a decomposing formulation (Schumann, 1975), in which the Reynolds stresses are separately modelled with a time-averaged, and a fluctuating part. Uribe et al., (2010) modified the model to a blending hybrid Reynolds stress tensor as

\[ \tau_{ij}^{hyb} = -2 f_b \nu_t^{LES} s_{ij}^{\prime \prime} - 2 (1 - f_b) \nu_t^{RANS} \langle \overline{S}_{ij} \rangle. \]

(7)

\( \overline{S}_{ij} = 0.5(\partial_i \overline{U}_j + \partial_j \overline{U}_i) \) is the filtered strain-rate tensor, \( s_{ij}^{\prime \prime} = \overline{S}_{ij} - \langle \overline{S}_{ij} \rangle \) is its fluctuating part, and \( \langle \cdot \rangle \) denotes the time-averaged (or space-averaged in homogeneous directions) variable. The eddy viscosities \( \nu_t^{LES} \) and \( \nu_t^{RANS} \) are provided by LES SGS and RANS eddy viscosity models. \( f_b \) is a blending function,

\[ f_b = \tanh \left[ C_l \left( \frac{L}{\Delta} \right)^n \right], \]

with a length scale \( L = \rho \kappa^{1/2} / c \), the characteristic cell size \( \Delta = 2 \sqrt{\text{Vol}} \) (with Vol being the cell volume), and model constants \( C_l \) and \( n \). The hyperbolic tangent expression is chosen to provide a seamless transition between RANS and LES mode. Depending on the ratio of cell size to length scale, the model can thus return to a RANS mode away from walls, if for example the grid is massively coarsened outside the region of primary interest as in embedded LES mode.

The analogous formulation for a hybrid scalar flux

\[ \tau_{ij}^{SGS} = -f_b \frac{\nu_t^{LES}}{P_{\nu_t^{LES}}} \partial_j (\theta - \overline{\theta}) \]

\[ - (1 - f_b) \frac{\nu_t^{RANS}}{P_{\nu_t^{RANS}}} \partial_j (\theta - \overline{\theta}). \]

(9)

is taken over from Rolfo, Uribe, and Billard, (2011) and is currently left unmodified. \( P_{\nu_t^{LES}} \) and \( P_{\nu_t^{RANS}} \) are the turbulent Prandtl numbers related to the LES and RANS contributions, respectively.

**Near-wall Reynolds-stress correction**

Attempting to alleviate the model sensitivity to the model parameters \( C_l \) and \( n \), and primarily \( P_{\nu_t}^{RANS} \), we here assess a correcting term to the mean Reynolds stresses, \( \langle \tau_{ij}^{hyb} \rangle \), while the fluctuating stresses remain unchanged. The correction tensor \( \tau_{ij}^{RANS} \), similarly to the constraint model of Chen et al., (2012). The correction tensor can be interpreted as an explicit forcing term in the momentum equation, as proposed by Xiao, Wang, and Jenny, (2017). We limit this to a layer near walls, \( \tau_{ij}^{RANS} \) represents the deviation of the sum of mean resolved Reynolds stresses,

\[ \tau_{ij}^{RANS} = \langle \overline{U}_i \overline{U}_j \rangle - \langle \overline{U}_i \rangle \langle \overline{U}_j \rangle, \]

(10)

and the mean hybrid SGS Reynolds stress model,

\[ \langle \tau_{ij}^{hyb} \rangle = -2 (1 - f_b) \nu_t^{RANS} \langle \overline{S}_{ij} \rangle, \]

(11)

from the modelled or given RANS Reynolds stresses \( \tau_{ij}^{RANS} \) as

\[ \tau_{ij}^{RANS} \equiv \tau_{ij}^{RANS} - 2 (1 - f_b) \nu_t^{RANS} \langle \overline{S}_{ij} \rangle. \]

(12)

This correction is limited to the near-wall region, and the SGS Reynolds stress tensor shall return to the original hybrid SGS model formulation. Hence, \( \tau_{ij}^{RANS} \) is blended with \( \langle \tau_{ij}^{hyb} \rangle \) as

\[ \langle \tau_{ij}^{hyb} \rangle = -2 (1 - f_b) \nu_t^{RANS} \langle \overline{S}_{ij} \rangle. \]

(13)

The complete SGS Reynolds stress tensor then is

\[ \tau_{ij}^{RANS} = -2 f_b \nu_t^{LES} s_{ij}^{\prime \prime} - 2 (1 - f_b) \nu_t^{RANS} \langle \overline{S}_{ij} \rangle \]

\[ + (1 - f_b) \tau_{ij}^{RANS}. \]

(14)

\( f_b = \alpha^q \), with \( \alpha \) being the elliptic blending wall proximity parameter and \( q = 5 \), is prescribed to transition from zero close to walls to unity with increasing wall distance. The limits of Equation 14 are therefore given with Equation 7 and

\[ \tau_{ij}^{RANS} \equiv \tau_{ij}^{RANS} - 2 f_b \nu_t^{LES} s_{ij}^{\prime \prime} + \tau_{ij}^{RANS} \]

(15)

for \( f_b = 0 \), with the fluctuating part remaining unchanged.

**LES eddy viscosity model.** In the LES SGS eddy viscosity, only the fluctuating strain-rate is considered (Moin and Kim, 1982), as

\[ \nu_t^{LES} = (C_S \Delta)^2 \sqrt{2 \nu_I^{\prime \prime} s_{ij}^{\prime \prime}}. \]

(16)

The Smagorinsky constant is here \( C_S = 0.065 \), as the common value for channel flows. In contrast to the Smagorinsky model based on the full instantaneous strain-rate tensor \( \overline{S}_{ij} \), it is beneficial that the SGS
model is only based on the fluctuating strain-rate tensor. Van Driest damping close to walls is not used here.

**RANS eddy viscosity model.** Furthermore, we apply an elliptic blending $k-\varepsilon-\omega-\alpha$ model (Billard and Laurence, 2012), which demonstrates an improved prediction of the near-wall turbulence over simpler $k-\varepsilon$ models. In addition to transport equations for $k$ and $\varepsilon$, model equations for $\varphi = (u'^2)/k$, and an elliptic equation for the wall proximity parameter $\alpha$ are solved. The latter is an elementary part of the elliptic blending concept (Manceau and Hanjalic, 2002) where an elliptic equation for $\alpha$ is solved. $\alpha$ transitions from unity to zero as walls are approached, and smoothes the transition between near-wall and quasi-homogeneous models for various turbulence variables. The eddy viscosity is

$$ u'_i^{\text{RANS}} = C_\mu \varphi k \min (T, T_{\text{lim}}), $$

with time scales $T$ and $T_{\text{lim}}$. Additionally, we explore the model’s general applicability using a Reynolds Stress Model with elliptic blending (Manceau and Hanjalic, 2002) as an alternative to the eddy viscosity model. The transport equations for the Reynolds stress components deliver direct solution of the components of $\tau^{\text{RANS}}_{ij}$.  

**Dual-grid parallel simulation**

To enable prediction of the RANS eddy viscosity and Reynolds stresses

$$ \tau^{\text{RANS}}_{ij} \equiv -2 \nu^{\text{RANS}} (\tilde{S}_{ij}), $$

the RANS equations are computed on a secondary wall-refined, high aspect-ratio grid, similar to the Dual Mesh approach (Xiao and Jenny, 2012), on which a blended RANS- and LES-averaged velocity field is computed as

$$ \bar{U}_i^n = (f_a)^n (\overline{U}_i)^n + (1 - (f_a)^n) \tilde{U}_i^{n-1}. $$

$n$ is the current time step. $f_a$ is here used, as only the wall distance should be affecting the transition of $\bar{U}_i$. It is noted that this blended velocity $\bar{U}_i$ is only used for the computation of the RANS turbulence model and stresses, and does not enter the LES momentum equation explicitly. In contrast, Xiao, Wang, and Jenny, (2017) enforce consistency between RANS and LES solutions with explicit source terms in the transport equations.

We use the EDF open-source finite volume solver *Code_Saturne* version 5.0.4 (Fournier et al., 2011) to implement the presented hybrid model. To account for the computation of RANS and LES on two grids, simulations are run in parallel and are coupled via a coupling interface available in *Code_Saturne*, which enables an interpolation of variables between RANS and LES grid.

3 Results

The presented model is validated on a fully developed turbulent plane channel flow. The domain size is $L_x \times L_y \times L_z = 6.4\delta \times 2\delta \times 3.2\delta$ in streamwise, wall-normal and spanwise direction, respectively, $\delta$ is the half-channel height. The hybrid simulation is carried out on a constant-spaced grid with $N_{xyz} = 40 \times 41 \times 32$ grid points. The wall-normal distance of the first grid point off the wall, $y^+_1 \equiv y_1 u_\tau/\nu$, is deliberately chosen to be higher than the suggested limit of $y^+_1 \equiv 1$ for conventional LES and many other hybrid RANS/LES models. For the four different friction Reynolds numbers considered, including $Re_f = u_\tau \delta/\nu = 395$, 1020, 2017, and 4088, $y^+_1$ ranges from 10 to 102. Also, the grid spacing in the wall-normal directions of the first grid point off the wall, $\Delta x^+ = 654$ and $\Delta z^+ = 327$ at $Re_f = 4088$, which is considerably larger than in common practice. In contrast, the RANS simulation is computed on a wall-refined, high-aspect ratio RANS grid, which covers the identical domain as the LES grid, with $y^+_1 \leq 1$ and 128 grid points in wall-normal direction. The computations for the plane channel flow are run on seven CPUs, while the parallel RANS simulation is run on one CPU.

To isolate the beneficial effects of the hybrid modelling, performance is compared against a baseline LES with a wall-function on the same grid, and DNS data (Abe, Kawamura, and Matsuo, 2004; Bernardini, Pirozzoli, and Orlandi, 2014). For the plane channel flow, the hybrid model generally exhibits a good agreement with reference data. The positive effect of the hybrid SGS Reynolds stress tensor modelling appears visible in the velocity prediction (Figure 1) for the considered range of Reynolds numbers, when compared to the standard LES and DNS. The secondary logarithmic layer, predicted by the LES, is avoided, what is in line with the original Two Velocity Fields model performance.

To underline the grid impact, in Figure 2, both the Dual-Grid model with and without the proposed near-wall Reynolds stress correction, are opposed for the aforementioned constant-spaced grid, and for a wall-refined grid (with all other grid parameters kept identical). Both models overlap on the wall-refined grid, while they deviate on the other grid.

The temperature profile (Figure 3) is also improved against the baseline LES, but similar sensitivity to $Pr^{\text{RANS}}_{ij}$, as reported by Nguyen et al., (2018) is observed and is currently subject to further modelling work.

The mean Reynolds shear stress $\tau_{12}$ in Figure 4 for LES shows high peaks for the first grid points off the wall, with increasing Reynolds number. The decomposition into resolved and SGS contributions highlights that the cause for this observation is the SGS model, which is not performing adequately at such low grid resolutions. In contrast, the hybrid model returns an improved SGS part, with benefits for the total Reynolds shear stresses. Also, the turbulence intensi-
The velocity field predictions reflect in the skin friction coefficient \( C_f \), which is here set into comparison to the empirical correlation \( C_{f,0} = 0.073Re^{-0.25} \) by Dean, and to DNS data, Figure 6. The standard LES underpredicts the skin friction in a range of around 6\% to 24\% to the correlation, while the hybrid model is much closer to the empirical value with deviation between 3\% to 14\%.

The seamless transition between the Reynolds stress correction near walls, governed by \( f_\alpha \), and the original blending function \( f_b \) is depicted in Figure 7. With the first grid point at high \( y^+ \) values, both blending functions do not reach very low values any more.

**Pericodic Hill**

The proposed model is assessed on a periodic hill flow (Almeida, Durão, and Heitor, 1993) at \( Re = 10,595 \), based on the hill crest height and bulk velocity. The RANS simulation is carried out on a \( 60 \times 120 \times 1 \) grid with \( y^+ \leq 1 \), while the LES is run on a \( 100 \times 80 \times 40 \) grid with maximum first cell wall distance of \( y^+ \approx 27 \) (see Figure 8). Reference fine LES data is given by Fröhlich et al., (2005). Overall, the hybrid model at the present state does not predict the flow well. The conventional LES predicts a too short recirculation bubble aft of the hill, which is evident in the mean axial velocity profile and skin friction coefficient \( C_f \) (Figures 9, 10). The hybrid model returns an even shorter separation area, and the \( C_f \) profile does not follow the reference data well. It is here concluded that the Reynolds stress prediction with RANS and LES at this grid quality level, especially at critical areas such as the hill crest, but also the current method of feeding back the mean filtered velocity field to the RANS simulation, requires additional modelling work.

### 4 Conclusions

A modification to a dual-grid hybrid RANS/LES model, based on Reynolds stress blending and dynamic correction near walls, is assessed. With the extension of the original Reynolds stress blending model to non-wall-refined grids, and now a near-wall correction to reduce model sensitivity, three different ideas of hybrid modelling are combined. The model allows instantaneous flow simulation even on very low grid resolutions, but also accounts for heat transfer in a hybrid context. The Reynolds stress correction performs well on the plane channel flow, for wall-refined and homogeneous grids. The fluctuating nature of LES is generally not lost, as only the mean stresses are corrected. This is in line with the approaches of Xiao and Jenny, (2012), Chen et al., (2012), and Uribe et al., (2010). The model returns similar results when applying Reynolds stress models instead of an eddy viscosity model on the RANS side.

The low add-on in computational cost of running dual simulations, in comparison to an equivalent LES with wall functions has to be underlined. The computational cost consists of the LES and RANS simulations, the additional transfer of variables across the simulations. The RANS simulation is run on a wall-refined grid, but with high wall-normal expansion ratios, as the dual simulation is relying on the LES part away from walls. For geometries with homogeneous directions, the grid can be simplified in these directions, reducing the computational power enormously,
Figure 4: Mean Reynolds shear stress for $Re_\tau = 2022$.

Figure 5: Mean turbulence intensities for $Re_\tau = 2022$.

Figure 6: Skin-friction coefficient $C_f$ over bulk Reynolds number $Re_b$ for hybrid RANS/LES and LES, compared with DNS and Dean’s correlation.

Figure 7: Blending functions $f_b$ (clear markers) and $1 - f_\alpha$ (solid markers) over wall distance for all Reynolds numbers.

Figure 8: Periodic hill grids: 2D RANS grid (top), 3D LES grid (bottom).

Figure 9: Skin friction coefficient $C_f$ along the bottom wall.

Figure 10: Mean streamwise velocity for RANS, coarse LES, hybrid RANS/LES and reference LES.
as shown for the plane channel and periodic hill flow. Currently, the coupling is performed at each time step. However, as only averaged values are transferred, the RANS-LES coupling can be reduced to every $n$-th time step.

Future work focuses on exploring the Reynolds stress correction further on separating flows, but also on extending the concept to the hybrid scalar flux modelling, where a high sensitivity of the predicted temperature field to the model parameters, especially the turbulent Prandtl numbers of RANS and LES contribution, is currently observed.

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References


