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Crashworthiness optimisation of a composite energy-absorbing structure for subway vehicles based on hybrid particle swarm optimisation

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Abstract: To improve the crashworthiness of subway vehicles, a composite energy-absorbing structure (EAS) is designed by coupling a thin-walled metal tube and aluminium honeycomb structures. On this basis, the effectiveness of the composite structure and aluminium honeycomb finite element model (FEM) is validated by conducting trolley impact tests and quasi-static compression experiments. Based on the verified FEM, the surrogate models, including polynomial response surface (PRS), Kriging, radial basis function (RBF) and supported vector regression (SVR), are established to get the relationship between crashworthiness indexes and design variables. Then, the most accurate model is employed for crashworthiness optimisation through comparing the accuracies of these four models. Owing to the mutually effects among components of the composite EAS and the relatively complex mathematical formulae acquired by the high-precision surrogate model, a hybrid particle swarm optimisation (HPSO) algorithm is put forward. The performance of the HPSO is tested by a typical engineering optimization case. The results indicate that the HPSO algorithm presents various advantages (e.g. strong global search ability, high optimisation accuracy, etc.). By optimising the mathematical model of the composite EAS, the optimal configurations of the structure are obtained, which confirms that the HPSO algorithm has favourable applicability and performance in the crashworthiness optimisation of the EASs for subway vehicles.

Keywords: *Composite energy-absorbing structure; Impact test; Surrogate model; Hybrid particle swarm optimisation; Global search*

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1 Introduction

To improve the safety of passengers during the subway crash accidents, the vehicles have to be designed with good crashworthiness. Otherwise, the casualties and loss of property resulting from collision accidents are unbearable. For example, on 22 November, 2012, a subway in Pusan, South Korea broke down and then the following subway train crashed into it at high speed, causing the occurrence of a rear-end collision with over 100 casualties. To reduce the loss caused by such collisions, it is necessary to install energy-absorbing structures (EASs) on the front end of subway vehicles.

Subway vehicles running on the fixed tracks are generally marshalled as a six-car train, which are high quality, high-energy level, high-capacity, self-guided vehicles which can generate a significant amount of initial kinetic energy compared with that of automobiles. Therefore, the EAS on the subway vehicles are required to show high collision resistances to meet passive safety protection requirements. Many investigations have concerned themselves with the structural crashworthiness design of subway vehicles. Xie and Zhou (2014) proposed the design concept of a crashworthy subway vehicle based on the characteristics of subway vehicles and divided energy-absorbing process of the head car (HC) impact into four steps. Moreover, they designed the energy-absorption structures on the basis of the impact analysis of a square thin-walled structure. To improve the impact performance of honeycomb-filled structures under eccentric loading for subway vehicles, Yao et al. (2018) conducted an optimisation that the cross-sectional shape of the guide was changed based on the bending strength and found that no instability phenomenon arose in the improved structure. Xie and Zhou (2014) designed a composite EAS for a certain subway vehicle to meet the requirements for passive safety protection during railway vehicle collisions. Peng et al. (2015) presented a composite EAS mainly composed of a square tube and aluminium honeycomb structure for use in subway vehicles and discussed its collision performance: they found that a structural form that can induce more regular folding buckling will promote the energy absorption and crashworthiness of a structure.

To date, many scholars have carried out testing, simulation, and theoretical research on different EASs composed of thin-walled tubes or aluminium honeycombs (Li et al. 2018; Peng et al. 2017; Sun et al. 2017; Xie and Zhou 2015; Zeng and Duddeck 2017; Duddeck et al. 2016). A

favourable energy absorbing structure is able to absorb, or dissipate, all initial kinetic energy in a controllable manner. Thin-walled metal structures were extensively investigated owing to their low cost and advantageous energy-absorbing efficiency (Alavi Nia et al. 2012; Chen et al. 2017; Chen et al. 2018; Kohar et al. 2015; Alavi Nia and Parsapour 2013). In addition, aluminium honeycomb with its high strength-weight ratio and high energy absorption efficiency has been widely applied in EASs (Zhu et al. 2016; Sun et al. 2016; Wang et al. 2014; Yamashita and Gotoh 2005). Researchers throughout the world have studied the energy-absorbing characteristics of combining thin-walled metal structures with cellular honeycomb structures, because the composite EASs are capable of absorbing adequate energy during the collision process. Wang et al. (2016) investigated matching relationships between the inside honeycomb core and outside metallic thin-walled structures and carried out parametric simulations to understand further the matching effect. They found that deformation mode evolution of the honeycomb-filled thin-walled square tube (HFST) structures is closely related to the structural geometric configuration. Zhou et al. (2017) designed the energy-absorbing structure of a crashworthy railway vehicle by combining the characteristics of thin-walled metal structures and aluminium honeycomb structures, and they discussed the safe velocity standard for rail vehicle collisions, which was improved from 25 km/h to 45 km/h by adopting a combined energy-absorbing structure. Hussein et al. (2017) conducted experimental investigations aimed to studying the axial crushing behaviour of square aluminium tubes with different configurations. The results show square aluminium tubes filled with both polyurethane foam and aluminium honeycomb is the most crashworthy combination.

Basically, the main challenge in the crashworthiness design and development of subway vehicles is how to find the optimal design of an EAS that can satisfy best the crashworthiness requirements. Optimisation of structural crashworthiness using surrogate models and intelligent algorithms is considered as a research focus in vehicle engineering (Fazilati and Alisadeghi 2016; Sun et al. 2017; Yin et al. 2013; Forsberg and Nilsson 2005; Yin et al. 2014). In aims to address the energy absorption behaviour and crashworthiness optimisation of short length circular tubes under quasi-static lateral loading, Baroutaji et al. (2015) adopted the response surface methodology (RSM) and multi-objective optimisation design (MOD) to find the optimal configuration of a range of proposed energy absorption structures. Song et al. (2013) performed a comparative study of four surrogate models including PRS, Kriging, SVR, and RBF and used

sequential quadratic programming (SQP) and particle swarm optimisation (PSO) for these models to find the optimal results, respectively. Xie et al. (2018) demonstrated the crashworthiness optimisation of a composite EAS for railway vehicles based on the polynomial response surface (PRS) method and PSO. Yang and Qi (2013) employed a multi-objective particle swarm optimisation (MOPSO) algorithm to generate the Pareto fronts for different MOD problems under oblique impact on the basis of Kriging meta-models constructed using finite element analysis (FEA) results. To design lightweight automobile parts and decrease their fuel consumption, Gao et al. (2018) proposed a cylindrical negative Poisson's ratio (NPR) structure based on the double-V unit cell. Then, they established a Kriging approximation model and operated the MOPSO algorithm to achieve an optimal cylindrical NPR structure. To improve the crashworthiness of a gradual EAS for subway vehicles, Xu et al. (2016) addressed a new cut-out groove design and used optimisation technology with the response surface (RS) method and adaptive simulated annealing (ASA) algorithm to find optimal structure.

Many scholars have mainly focused their investigations on optimising thin-walled metal tubes or foam-filled thin-walled metal tubes installed on railway vehicles or automobiles; however, the crashworthiness optimisation, in most of the literature, is implemented by employing a single surrogate model and existing heuristic algorithms, which may not guarantee that the chosen surrogate model was necessarily capable of leading to the best fitness and the relatively complex expressions obtained on the basis of surrogate methods may cause local optimum when using certain algorithms. The optimisation of the EASs coupling thin-walled metal tubes and aluminium honeycomb structures using the improved PSO has rarely been investigated. The current project presents a composite EAS used for subway vehicle and studies the crash performance. Based on the validated FEM of the composite EAS, crashworthiness optimisation was conducted by the high-precision surrogate model and a hybrid particle swarm optimisation (HPSO) algorithm, in an attempt to provide a theoretical basis for the optimisation of the crashworthiness of composite EASs for subway vehicles.

2 Numerical analyses

2.1 Optimisation index for crashworthiness

According to the literature (Yin et al. 2015; Djamaluddin et al. 2015), the crashworthiness behaviour of a structure was evaluated based on various indexes including specific energy absorption (SEA), peak crushing force (F_p), average crushing force (F_{avg}), and crash force efficiency (CFE). SEA is an important index used for measuring the energy absorption capacity of a structure, which is equal to the energy absorbed of structure per unit mass during the collision process, as shown in Formula (1):

$$SEA = \frac{EA}{m} = \frac{\int_0^s F(x)dx}{m} \quad (1)$$

where, F , s , and m refer to the crash force, the compression displacement, and the total mass of a structure, respectively.

F_{avg} is used to measure the capacity of buffering deformation for EASs, which can be acquired by employing energy absorption (EA) divided by the compression displacement s , as shown in Formula (2):

$$F_{avg} = \frac{EA(s)}{s} \quad (2)$$

CFE , also called the utilisation rate of load, is the ratio of F_{avg} of a structure to F_p . The larger the CFE , the more steady the crash force of the structure, as shown in Formula (3):

$$CFE = \frac{F_{avg}}{F_p} \quad (3)$$

Additionally, the initial peak crushing force (F_{ip}) needs to be considered. If F_{ip} exceeds a certain threshold, a large initial deceleration will be induced to threaten the safety of passengers. Therefore, it is necessary to reduce F_{ip} as far as possible.

2.2 Numerical simulation

2.2.1 Design of the composite EAS

The composite EAS designed in the study is illustrated in Fig. 1. The EAS is installed at two sides of front end of the underframe of subway vehicles, which is mainly composed of a

quadrate-tapered thin-walled tube, aluminium honeycomb, diaphragm for isolation from the honeycombs, front-and back-end plates and guide crossing the centre of the diaphragms and connecting with the front-end plate. Fig. 1 shows the compositions and sizes of components of the structure: the length and thickness of quadrate-tapered thin-walled tube are 787mm and 2.5 mm, respectively, whose two sides stand the front-and back-end plates with rectangular sections. The cross-sectional areas of the two plates are 280mm×188mm and 280mm×232mm, respectively. An inducement groove with 40 mm radius is located near the front-end plate to prevent from excessive initial peak crushing force. The internal part of the quadrate-tapered thin-walled tube contains two types of aluminium honeycombs which are separated by the diaphragms, and these honeycombs are called type A and B type with the same cross-sectional dimensions of 150 mm × 90 mm and the different axial dimensions of 97 mm and 62 mm, respectively. The configuration of the aluminium honeycombs can prevent the structure from instability under axial loads. There are nine diaphragms, each of 3 mm, in thickness in the composite EAS and each diaphragm is welded to the quadrate-tapered thin-walled tube.

Fig. 1 The composite EAS and its components (1-front-end plate ; 2-guide; 3-A type aluminium honeycomb; 4-quadrate-tapered thin-walled tube; 5-diaphragm; 6-B type aluminium honeycomb; 7-back-end plate)

2.2.2 Definition of the FEM

The collision analysis model used for crashworthiness optimisation of the composite EAS was established in non-linear finite element software ANSYS/LS-DYNA, as shown in Fig. 2. The quadrate-tapered thin-walled tube is discretised by employing element type Shell 163 while Solid 164 is used to discretise the front-and back-end plates and the guide. The welding of the quadrate-tapered thin-walled tube, the diaphragms and front-and back-end plates is simulated by coupling nodes. To complete, effectively and accurately, the crashworthiness optimisation of the structure, the honeycomb core is defined by using the Type 126 honeycomb equivalent material with anisotropy (mechanical property) and its discretisation with element type, Solid 164 (Yao et al. 2018).

During the analysis process, one end of the composite EAS is fixed while the other end is compressed for 35 ms by using an ideal rigid wall at a uniform speed. The wall thickness t_1 of the quadrate-tapered thin-walled tube, the side length l and thickness t_2 of unit cell of honeycomb structure are used as optimisation parameters. The crashworthiness behaviour of the EAS is evaluated by using various indexes including SEA , F_{ip} , F_{avg} , and CFE .

Fig. 2 The finite element model analysis conditions for optimisation

2.2.3 Material model of the structure

Considering the strain rate effect for a low strain rate condition, Cowper-Symonds models were chosen to model the steel structure (Hallquist 2006; Zhong 1993). The constitutive equation of the material can be described by Eq. (4):

$$\sigma_y = \left[1 + \left(\frac{\dot{\epsilon}}{C} \right)^{\frac{1}{p}} \right] (\sigma_0 + \beta E_p \epsilon_{eff}^p) \quad (4)$$

where σ_y is the yield stress; C and P are the strain rate parameters for the Cowper-Symonds model; $\dot{\epsilon}$ is the strain rate; σ_0 represents the initial stress; ϵ_{eff}^p is the equivalent plastic strain; E_p is the plastic hardening modulus, $E_p = E_{tan}E/(E - E_{tan})$; E is the Young's modulus; E_{tan} is the tangent modulus of plastic deformation; β denotes the hardening parameters that varies between 0 and 1. According to the literature (Yao et al. 2018; Xie et al. 2018), on the basis of the material of energy-absorbing structure, the parameters in Table 1 are taken as the material parameters for the Q235 steel structure in this numerical crash analysis.

Table 1 Material parameters for the steel structure.

Material parameters	ρ /(kg m ⁻³)	ν	E /GPa	β	σ_0 /MPa	E_{tan} /MPa	C /s ⁻¹	P
Numerical value	7.85×10^3	0.30	210	1	235	763	40	5

Fig. 3 shows a typical honeycomb core structure with its double thickness cell walls. Based on the literature (Wang et al. 2014; Xie and Zhou 2014; Yao et al. 2018; Xie et al. 2018), the thickness, width, and length of the honeycomb core are represented by T , W , and L ; t_2 is the thickness of the aluminium foil; l and h are side lengths of the unit cell; θ is the cell wall angle of unit cell; and when the cells are regular, $l = h$ and $\theta = 30^\circ$. The size $t_2 \times l \times h$ of unit cell of aluminium honeycomb in the composite EAS is $0.06 \text{ mm} \times 2 \text{ mm} \times 2 \text{ mm}$.

Fig. 3 Specification of honeycomb core of the composite EAS

In addition, the material model of the aluminium honeycomb of the composite EAS has been validated elsewhere (Xie and Zhou 2014; Yao et al. 2018; Xie et al. 2018) by the authors.

2.3 Validation of the numerical model

To analyse the crash performance of the composite EAS and validate the accuracy of the FEM, the modelling results of the structure under dynamic loads were compared with the experimental data. The impact experiment was carried out on the impact experimental bench of Key Laboratory of Traffic Safety on Track (Central South University), Ministry of Education. The impact experimental bench was consisted of the rigid wall installing force sensors with high sensitivity and precision, the crash test trolley, standard tracks, the driving and braking systems, the control system, and the experimental data acquisition and test systems. The experimental and simulation model consisted of trolley and EAS as shown in Fig. 4, respectively. During the process of simulation, the boundary conditions were fully consistent with those applied to the experiment, and the same sampling frequency and filtering methods were used throughout.

(a)

(b)

Fig. 4 (a) The experimental model of the trolley and EAS (b) FEM of the trolley and EAS

Many impact experiments and simulations have been carried out (Yao et al. 2018), which were not deemed to have been a priority in this research. Therefore, the deformation of the

structures was only presented when the initial impact velocity of the trolley was $v_0 = 19.5 \text{ m}\cdot\text{s}^{-1}$ in this work. Fig. 5 shows the final deformation of the composite EAS from both the impact test and simulation, it can be seen that the simulation results showed similar trends to those found experimentally. Because the experimental conditions are more complex than those of the numerical simulation, the force-time curve, and the energy-displacement curve, in the experiment are not in good agreement with those of the simulation, as shown in Fig. 6; nevertheless, the trends therein are essentially consistent. All comparisons indicate that the simulation model can be applied to the next optimisation based on the verification thereof.

Fig. 5 Final deformation of the EAS through experiment and simulation: (a) Experimental result; (b) Simulation result; (c) The deformation of the inner honeycomb of the composite EAS for simulation

(a)

(b)

Fig. 6 Comparison of experimental and simulation results: (a) force-time curve; (b) energy-displacement curve

3 Optimisation with different surrogate models and HPSO

3.1 Optimisation formulation

During the optimisation design of the composite EASs, it is expected that *SEA* of the structure is as large as possible within the effective compressive distance. This study focuses

much more on the *SEA* of the composite EAS than on other crashworthiness indexes, so the *CFE*, F_{avg} and F_{ip} will be taken as extra constraints, and maximising the *SEA* is taken as our major research objective. The wall thickness t_1 of the quadrate-tapered tube, the wall thickness t_2 of the unit cell of the honeycomb foil and the side length l of the unit cell are taken as design variables, with the data ranges as follows: $1.5\text{mm} \leq t_1 \leq 3.5\text{mm}$, $1.0\text{mm} \leq l \leq 5.0\text{mm}$ and $0.04\text{mm} \leq t_2 \leq 0.16\text{mm}$, respectively.

According to the literature (Peng et al. 2015; Yao et al. 2018) and the requirement of GM/RT2100, the constraint of the optimisation is that the initial peak crushing force induced by collision is not more than 1,500 kN and the deformation displacement should be less than 600 mm considering the load borne by the passengers in such subway vehicles. Additionally, the crashworthiness indexes including *CFE*, F_{ip} and F_{avg} are defined as the constraints to protect the passengers from severe damage during the collision process. To account for these design criteria, the optimisation problem can be formulated as a single-objective mathematical model as follows:

$$\left\{ \begin{array}{l} \text{Max} \quad SEA(t_1, l, t_2) \\ \text{s.t.} \quad \left\{ \begin{array}{l} S \leq 600\text{mm} \\ CFE(t_1, l, t_2) \geq 0.30 \\ F_{ip}(t_1, l, t_2) \leq 1500\text{kN} \\ F_{avg}(t_1, l, t_2) \geq 360\text{kN} \\ 1.5\text{ mm} \leq t_1 \leq 3.5\text{mm} \\ 1.0\text{mm} \leq l \leq 5.0\text{mm} \\ 0.04\text{mm} \leq t_2 \leq 0.16\text{mm} \end{array} \right. \end{array} \right. \quad (5)$$

3.2 Design of experiment (DOE)

Orthogonal experimental design (OED) can search optimal allocation and analysis information reflecting the comprehensive experimental conditions by conducting fewer experiments. Therefore, the study selected 25 training samples based on three-factor and five-level OED to fit (or interpolate) different surrogate models and chose 20 testing samples to validate the accuracy of each surrogate model. The spatial distributions of 25 training samples and 20 testing samples are displayed in Fig. 7.

Fig. 7 Spatial distributions of training and testing samples

3.3 The establishment of surrogate models

To reduce the time required for finite element simulation and analysis, the surrogate model was employed in optimising the crashworthiness of the composite EAS. It is unclear which surrogate model is the most accurate one for predicting of crashworthiness indexes before the optimisation design. Therefore, a comparative investigation of different surrogate models to choose the most suitable one for subsequent optimization becomes even more necessary. In this study, the approximate functions of crashworthiness indexes on the design variables are obtained by using four surrogate models involving the polynomial response surface (PRS) model Xie and Zhou (2014), radial basis function (RBF) model (Sun et al. 2011; Zhao and Xue 2010; Sun et al. 2010; Esfahlani et al. 2013), Kriging model (Simpson TW 2015, Xie and Zhou 2014) and supported vector regression (SVR) model (Clarke et al. 2005, Zhu et al. 2012). The approximate functions and relevant parameters of the four models are shown in Table 2.

Table 2 Approximate and basic (correlation and kernel) functions of four types of surrogate models.

Surrogate model	Approximate function	Basic(correlation, kernel) function
PRS-3	$a_0 + \sum_{i=1}^n a_i x_i + \sum_{i=1}^n \sum_{j=1}^n b_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n b_{ijk} x_i x_j x_k$	<i>Cubic polynomial functions</i>
RBF	$\sum_{i=1}^n r_i \varphi(\ \mathbf{x} - \mathbf{x}_i\)$	$\varphi(\ \mathbf{x} - \mathbf{x}_i\) = \sqrt{\ \mathbf{x} - \mathbf{x}_i\ ^2 + c^2}$ $c = 1.5$

Kriging	$\mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}) + \mathbf{f}(\mathbf{x})^T \hat{\boldsymbol{\beta}}$	$R(x_i, x_j) = \prod_{k=1}^{n_R} \exp(-\theta_k x_k^i - x_k^j ^2)$ $0.1 \leq \theta_k \leq 30$
SVR	$\sum_{i=1}^n (a_i - a_i^*) k(\mathbf{x}_i \cdot \mathbf{x}) + b$	$k(\mathbf{x}, \mathbf{x}') = \exp(-\frac{\ \mathbf{x} - \mathbf{x}'\ ^2}{2\sigma^2})$ $\sigma = 2$

3.4 Error evaluation criterion

To evaluate the accuracy of the surrogate models, 20 testing samples are selected from the sample space, as shown in Fig. 7. The mean relative error (MRE) is used for measuring the local accuracy of the surrogate model while the root-mean-square error (RMSE) is used to measure its global precision. The lower the MRE and RMSE, the higher the accuracy of the models. RMSE and MRE between the FEA result y_i and the approximation result based on the corresponding predicted $\hat{y}_i$ are calculated using Eqns (6), and (7), respectively:

$$RMSE = \sqrt{\frac{\sum_{i=1}^M (y_i - \hat{y}_i)^2}{M}} \times 100\% \quad (6)$$

$$MRE = \frac{\sum_{i=1}^M |(\hat{y}_i - y_i) / y_i|}{M} \times 100\% \quad (7)$$

R^2 , the coefficient of determination, is considered as another important index when evaluating the accuracy of models and it can be calculated as follows:

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^M (y_i - \hat{y}_i)^2}{\sum_{i=1}^M (y_i - \bar{y})^2} \quad (8)$$

where, M and $\bar{y}$ represent the number ($M = 20$) of the testing samples and the mean value for the simulation analysis, respectively.

3.5 HPSO

3.5.1 The principle of the optimisation algorithm

The linear decreasing inertia weight PSO (LDIW-PSO) with proper parameter setting (Arasomwan and Adewumi 2013) is introduced as a major part of hybrid particle swarm

optimisation (HPSO) algorithm. With increasing iterations through the LDIW-PSO algorithm, the deep-search capacity of the algorithm gradually decreases. On the premise of failing to impair cooperation between particles, the explosion operation of the fireworks algorithm (FWA) (Tan and Zhu 2010) is introduced in which the individual extremum updated by the LDIW-PSO algorithm is taken as the explosion firework to strengthen the local search capacity and diversity of the algorithm. The explosion number and radius of each firework in the firework explosion operation depend on the values of fitness functions for all individual extremums. To adjust, self-adaptively, the explosion radius of the firework to adapt different decision variables and increase disturbance accuracy, the explosion update of the individual extremums and the number of fireworks generated by the explosion update are as displayed in Formulae (9) to (12). Additionally, to avoid premature convergence of the algorithm, the global extremum undergoes logistic mapping treatment (Yong et al. 2015) to help the HPSO algorithm jump over local optima.

$$A^{l,t} = \hat{A} \cdot \frac{F(pbest^{l,t}) - F(pbest^t)^{\min} + \varepsilon}{\sum_{l=1}^{Sizepop} (F(pbest^{l,t}) - F(pbest^t)^{\min}) + \varepsilon} \quad (9)$$

$$S^{l,t} = M \cdot \frac{F(pbest^t)^{\max} - F(pbest^{l,t}) + \varepsilon}{\sum_{l=1}^{Sizepop} (F(pbest^t)^{\max} - F(pbest^{l,t})) + \varepsilon} \quad (10)$$

$$S^{l,t} = \begin{cases} S^{\min}, & \text{if } S^{l,t} < S^{\min} \\ S^{\max}, & \text{if } S^{l,t} > S^{\max} \\ \text{round}(S^{l,t}), & \text{otherwise} \end{cases} \quad S^{\min} = a \cdot M, S^{\max} = b \cdot M \quad (11)$$

$$pbest_i^{l,t,\alpha} = pbest_i^{l,t} + \text{round}(\text{rand}(0,1)) \cdot \omega^t \cdot A^{l,t} \cdot \text{rands}(-1,1) \cdot (xs_i^{\max} - xs_i^{\min}), \alpha \in \{1, 2, S^{l,t}\} \quad (12)$$

In the t th iteration, $pbest_i^{l,t,\alpha}$, $pbest_i^{l,t}$, and ω^t refer to the i th-dimensional location of the individual extremum of the particle l in the α th explosion, the i th-dimensional location of the individual extremum of the particle l , and inertia weight, respectively. Moreover, $A^{l,t}$, $xs_i^{\max}$ and $xs_i^{\min}$ represent the explosion radius of the individual extremum of the particle l , the maximum and minimum locations of the particle in the i th-dimension, respectively. $S^{l,t}$, $Sizepop$ and $\hat{A}$ refer to the explosion number of individual extremum of particle l , population size, and the constant of explosion radius of fireworks, respectively. M , $F(pbest^t)^{\max}$ and $F(pbest^t)^{\min}$ represent the constant of explosion number, and the maximum and minimum fitness values of

individual extremums, respectively. Additionally, $\text{round}(\cdot)$ means rounding and $\text{rands}(-1,1)$ refers to a random value on $[-1,1]$ while ε represents an infinitesimal real number. $S^{\max}$ and $S^{\min}$ refer to upper-and lower-limit parameters for controlling the number of explosion; a and b denote the constant parameters controlling fireworks' bounds.

3.5.2 Implementation of the proposed algorithm

Step 1: Initialisation;

1.1 Setting parameters of HPSO algorithm;

1.2 The speed and location of the particle are randomly initialised to acquire xs^t and vs^t

1.3 The fitness value $F(xs^t)$ of the particle is calculated and the individual extremum $pbest^t = xs^t$. Afterwards, the global extremum $gbest^t$ is updated based on $pbest^t$.

Step 2: Set $t = t + 1$, the speed vs^t and location xs^t of the particles are updated, as shown in Formulae (13), (14), (15), and (16).

Step 3: Whether, or not, the individual extremum is updated is judged by comparing the fitness function values of $pbest^{t-1}$ and updated particle xs^t .

Step 4: According to Formulae (9), (10), (11), and (12), the $pbest^t$ is taken as a firework to undergo explosion updating. On this basis, the fitness value is calculated to update $pbest^t$.

Step 5: By comparing the fitness values of $gbest^{t-1}$ and $pbest^t$ updated in Step 4, $gbest^t$ can be updated.

Step 6: According to Formulae (17) and (18), the fitness value of the $gbest^t$ in Step 5 is subjected to chaotic mapping treatment, and then update $gbest^t$ again.

Step 7: If $t > T$, the HPSO algorithm ends to output the globally optimal solution. Otherwise, return to Step 2.

$$\begin{cases} vs_i^{l,t} = \omega^{t-1} \cdot vs_i^{l,t-1} + c_1 \cdot \text{rand} \cdot (pbest_i^{l,t-1} - xs_i^{l,t-1}) + c_2 \cdot \text{rand} \cdot (gbest_i^{t-1} - xs_i^{l,t-1}) \\ xs_i^{l,t} = xs_i^{l,t-1} + vs_i^{l,t} \end{cases} \quad (13)$$

$$\omega^{t-1} = \omega^{\max} - \frac{(\omega^{\max} - \omega^{\min}) \cdot (t-1)}{T} \quad (14)$$

$$vs_i^{l,t} = \begin{cases} vs_i^{\max}, & \text{if } vs_i^{l,t} > vs_i^{\max} \\ vs_i^{\min}, & \text{if } vs_i^{l,t} < vs_i^{\min} \end{cases} \quad (15)$$

$$xs_i^{l,t} = \begin{cases} xs_i^{\max}, & \text{if } xs_i^{l,t} > xs_i^{\max} \\ xs_i^{\min}, & \text{if } xs_i^{l,t} < xs_i^{\min} \end{cases} \quad (16)$$

$$xc_i^{k+1} = \mu \cdot xc_i^k \cdot (1 - xc_i^k), k = \{0, 1, 2, \dots, N-1\} \quad (17)$$

$$xc_i^k = \frac{gbest_i^t - xs_i^{\min}}{xs_i^{\max} - xs_i^{\min}} \quad (18)$$

In the t th iteration, $vs_i^{l,t}$ and $xs_i^{l,t}$ denote the speed and location of particle l in the i th-dimension, respectively; c_1 and c_2 mean acceleration coefficients; T , $\omega^{\max}$, and $\omega^{\min}$ represent the maximum number of iterations, and maximum and minimum inertia coefficients, respectively, and $gbest_i^t$ denotes the location of the global extremum in the i th dimension. Moreover, $vs_i^{\max}$ and $vs_i^{\min}$ separately refer to the maximum and minimum speeds of the particle in the i th dimension. Moreover, μ , xc_i^k , and N represent the control parameter, the chaotic variable in the i th dimension acquired in the k th mapping and the maximum number of chaotic mapping operation, respectively.

Fig. 8 shows the detailed optimisation process by employing HPSO for the composite EAS.

Fig.8 The optimisation process of HPSO algorithm for the composite EAS

3.5.3 Performance testing

The performance of the HPSO algorithm was tested by a typical engineering optimisation problem, and then the optimised results were compared with those of the algorithms in the existing literature. This problem involves three variables (including mean coil diameter x_1 , wire diameter x_2 , and the number of active coils x_3) and four non-linear constraints, as shown in Fig. 9.

Fig. 9 Optimised design variables concerning tension/compression spring

The mathematical model of the problem is given by Formula (19):

$$\left\{ \begin{array}{l} \min \quad f(x) = (x_3 + 2)x_2x_1^2 \\ s.t. \quad \left\{ \begin{array}{l} F_1(x) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leq 0 \\ F_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0 \\ F_3(x) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0 \\ F_4(x) = \frac{x_2 + x_1}{1.5} - 1 \leq 0 \end{array} \right. \end{array} \right. \quad (19)$$

The data ranges of design variables of the model are displayed as follows: $0.05 \leq x_1 \leq 2$, $0.25 \leq x_2 \leq 1.3$ and $2 \leq x_3 \leq 15$. The parameters of the HPSO algorithm are set as follows: $sizepop = 60$, $T = 500$, $c_1 = c_2 = 1.9$; $\hat{A} = 1$, $M = 100$, $\varepsilon = 0.0000001$, $N = 10$ and $\mu = 4$. The algorithm independently works 30 times. To compare the performance between the HPSO and other algorithms in a fair manner, the efficiency of the algorithm is reflected by the number of function evaluations Yildiz (2009), as shown in Table 3. By employing the HPSO algorithm, the optimal solution can be found after conducting 30,000 function evaluations. It can be seen from the evaluation indexes of optimisation results that the HPSO is superior to other algorithms. By checking the results of each evaluation, it can be speculated that the algorithm presents stable optimisation performance due to the very low standard deviation (1.397E-6). Therefore, it is feasible to apply the HPSO algorithm into subsequent crashworthiness optimisation of the composite EAS.

Table 3 The optimised results of the tension/compression spring based on seven algorithms.

Different Algorithms	Best	Mean	Worst	Standard deviation	Function evaluations
HPSO	0.01266523	0.012666	0.012671	1.397E-6	30,000
Yong et al. (2015)	0.01266602	0.012763	0.012856	3.52 E-5	35,000

Arasomwan and Adewumi (2013)	0.01266526	0.012708	0.012797	7.07 E -6	35,000
Yildiz(2009)	0.01266527	0.012673	0.012708	6.24 E-6	30,000
Mahdavi et al. (2007)	0.0126706	0.012735	0.012832	3.05 E-5	30,000
Huang et al. (2007)	0.0126747	0.012730	0.012924	5.20 E -5	204,800
Coello and Montes (2002)	0.01268100	0.012742	0.012973	5.90 E -5	80,000

Table 4 A comparison of the optimised results for the tension/compression spring by using seven algorithms.

Constrain ed variables	HPSO	Yong et al. (2015)	Arasomwan and Adewumi (2013)	Yildiz(2009)	Mahdavi et al. (2007)	Huang et al. (2007)	Coello and Montes (2002)
$x_1(d)$	0.05167724	0.05148182	0.051725645	0.0516904023	0.05115438	0.051728	0.051989
$x_2(D)$	0.3564334	0.35175268	0.3575984	0.3567500	0.34987116	0.357644	0.363965
$x_3(p)$	11.3056574	11.58610148	11.2375167	11.2871200	12.0764321	11.244543	10.890522
$F_1(x)$	-5.19E-8	-4.43E-8	-9.11E-13	-0.0000058368	0.000000	-0.000845	-0.000013
$F_2(x)$	-3.73E-10	-4.65E-8	-3.16E-13	-0.0000010045	-0.000007	-1.26E-5	-0.000021
$F_3(x)$	-4.053223373	-4.0438661	-4.055521597	-4.0537944322	-4.0278401	-4.051300	-4.061338
$F_4(x)$	-0.727926260	-0.73117699	-0.727117248	-0.7277134115	-0.7365723	-0.727090	-0.722698
$f(x)$	0.01266523	0.01266602	0.01266526	0.01266527	0.0126706	0.0126747	0.0126810

4 Results and discussion

4.1 Error analysis of surrogate models

To compare the accuracy of four surrogate models (PSR, RBF, Kriging, and SVR), 20 testing samples were randomly selected from the sample space, as illustrated in Fig. 7. According to Formulae (6) to (8) for error evaluation in Section 3.4, the accuracy evaluation results of four models are obtained, as shown in Fig. 10 and Table 5.

Fig. 10 A comparison of errors of four surrogate models

Table 5 Evaluation results of errors of four surrogate models.

Objective	Error index	PRS-3	RBF	Kriging	SVR
SEA	RMSE	0.6436	0.6375	0.3488	0.3310
	MRE	0.0681	0.0496	0.0288	0.0331
	R^2	0.9905	0.9907	0.9972	0.9975
F_{ip}	RMSE	5.4832	3.4881	2.6978	2.4938
	MRE	0.0295	0.0100	0.0107	0.0150
	R^2	0.9980	0.9992	0.9995	0.9996
F_{avg}	RMSE	9.4244	7.9828	8.1916	7.6176
	MRE	0.0693	0.0517	0.0409	0.0320
	R^2	0.9941	0.9947	0.9987	0.9989
CFE	RMSE	0.0571	0.0276	0.0498	0.0256

MRE	0.1239	0.0552	0.0290	0.0296
R^2	0.9466	0.9593	0.9675	0.9893

As shown in Table 5 and Fig. 10, the $RMSE$ and MRE of the PRS-3 model for predicting crashworthiness indexes are all larger than those of the other three models. The values of R^2 of RBF and Kriging models for predicting CFE are relatively low. However, the crashworthiness indexes of the composite EAS predicted by the SVR model are both relatively optimal.

Fig. 11 shows the different four-dimensional response model of the SEA on design variables under constraints. It can be seen from the figure that, except for the PSR-3 model, the other three response models are similar, showing approximately the same monotonicity: the objective function monotonically increases with growing thicknesses of quadrate-tapered tube and unit cell of aluminium honeycomb while monotonically decreases with increasing side length of the unit cell of the aluminium honeycomb. The overall consistency of the four models basically conforms to R^2 values of the SEA in Table 5. Additionally, it can be concluded that compared with the thickness t_2 of the unit cell of the aluminium honeycomb, the side length l of the unit cell of the aluminium honeycomb and the thickness t_1 of the quadrate-tapered tube have a more significant effect on the SEA of the composite EAS.

(c) **Kriging** model

(d) **SVR** model

Fig. 11 Four response models of *SEA* under the effect of influence factors: (a) PRS-3 model (b) RBF model (c) Kriging model (d) SVR model

According to the above analysis, the four surrogate models show approximately similar trends in predicting *SEA*. However, there is the highest accuracy in predicting four crashworthiness indexes of the composite EAS by using the SVR model, therefore the SVR model is employed as the surrogate mode for subsequent optimisation. The expressions for crashworthiness indexes were acquired by applying the support vector regression method, as shown in Formulae 20 to 23 in the Appendix.

4.2 Optimisation and validation

The HPSO algorithm is proposed based on the LDIW-PSO by integrating the explosion operator of the FWA. To acquire the optimal crashworthiness design of the composite EAS for subway vehicles, the single-objective mathematical model in Section 3.1 is optimised by using FWA, LDIW-PSO and the HPSO algorithms proposed in Section 3.5, respectively. The algorithms are separately run 30 times independently, and the parameters used by the algorithms for the optimisation are as follows: a) Number of fireworks or particles: 100; b) Maximum number of iterations: 300; c) Number of function evaluations: 30,000.

The dimensional parameters of the composite EAS of subway vehicles in terms of each optimal *SEA* and the optimal *SEA* obtained using the algorithms are listed in Table 6.

Table 6 Optimisation results of the composite EAS using different algorithms.

	Iterations	t_1/mm	l/mm	t_2/mm	$SEA/(\text{kJ/kg})$
FWA	265	2.7177	1.0981	0.1569	19.0458

LDIW-PSO	187	2.7660	1.0000	0.1589	20.7081
HPSO	86	2.7720	1.0000	0.1600	20.8177

Figure 12 (a) shows the converging curve for the *SEA* of the composite EAS optimised by the algorithms. It can be concluded that the optimal *SEA* (20.8177 kJ/kg) was acquired by 86 iterations of the HPSO; applying LDIW-PSO, the optimal value of the structural *SEA* (20.7081kJ/kg) was obtained after 187 iterations and the optimal value of 19.0458 kJ/kg , as acquired by FWA reached optimal in 265 iterations. Figure 12(b) shows the boxplot (Tsai et al. 2010), which can graphically illustrate groups of data through their summaries including the smallest observation, upper quartile, median, lower quartile, the largest observation and indicate which observation might be an outlier. The closer the five-number summaries and outliers, the better the stability and convergence of the algorithm. It can intuitively be found that the stability and convergence of the FWA are inferiority to the HPSO and LDIW-PSO algorithms, and the optimised result of the LDIW-PSO is more uncertain than the HPSO due to the two extreme outliers. Obviously, the proposed HPSO performs the best and has the ability to find optimal solutions compared with other related methods (FWA and LDIW-PSO) for this optimisation problem.

Fig.12 (a)The convergence curves for the metrics of the *SEA* optimised by FWA, LDIW-PSO and HPSO algorithms (b) Boxplot for the metric of the structural *SEA*

A comparative analysis of the results optimised by using the three algorithms presents that the *SEA* by using the HPSO is optimal. Fig.13 shows the optimisation results in which the red

arrow refers to the trend in motion of the optimisation of particles while a black particle denotes the optimum solution.

Fig. 13 The optimisation result obtained based on the HPSO algorithm

To verify the optimisation result, Table 7 compares the optimisation result of SVR model and finite element analysis (FEA) result. It can be concluded that there is a relatively low discrepancy between the two results, which validates the optimisation result obtained by the HPSO algorithm.

Table 7 Verification of optimal solution of the composite EAS based on FEA.

	$SEA/(kJ/kg)$	F_{ip}/kN
Optimal value (SVR model)	20.8177	1499.5343
FEA	20.4271	1486.1653
Relative error %	1.9122	0.8996

Table 8 lists the results of optimal, and initial, designs: it can be found that the SEA and CFE of the optimised structure increase by 1.6514 and 1.1379 times, respectively, while the compression displacement decreases by 46.88%. Both the SEA and CFE of the composite EAS obtained by using optimal parameter combinations are both significantly improved, which indicates that the utilisation rate of the structure to materials increases. Although the aluminium honeycomb of the optimised EAS becomes dense, the quality of the overall structure shows no substantial increase.

Table 8 A comparison between results of initial, and optimal, designs.

Categories	$SEA/(kJ/kg)$	F_{ip}/kN	Displacement/mm	CFE
Initial design	8.0813	1390	576.09	0.29
Optimal design	21.4271	1486.1653	305.99	0.62
Optimized percentage (%)	165.14	6.92	-46.88	113.79

The SEA of the composite EAS, through optimal design, increases significantly compared with that under initial design conditions while the initial peak crushing force rises by a mere 6.92%. The main reason is that the inducement groove with 40 mm radius in the front end of the composite EAS does not change before or after the optimisation. The inducement groove reduces the initial peak crushing force of the structure and also effectively avoids secondary collision damage to passengers.

5 Conclusions

(1) A composite EAS installed on the front-end of the underframe of a subway vehicle was designed. The structural FEM was simulated in LS-DYNA, and was then efficiently validated by dynamic impact tests. Based on the verified FEM, response models, representing the relationship between design variables and crashworthiness indexes, were separately constructed using OED and approximate alternative methods (PRS-3, Kriging, RBF, and SVR). By analysing the comprehensive error evaluation criterion, the prediction accuracy of the SVR model for crashworthiness indexes of the composite EAS is higher than that of other surrogate models in this study.

(2) Considering the complexity of the mathematic formulae of the composite EAS and the shortcomings inherent to LDWI-PSO during the optimisation process, a HPSO algorithm was proposed. By comparing the optimised results of the mathematical model of the tension/compression spring obtained by employing the HPSO and the algorithms as published elsewhere (Yong et al. 2015; Arasomwan and Adewumi 2013; Yildiz AR 2009; Mahdavi et al. 2007; Huang et al. 2007; Coello and Montes 2002), it can be speculated that the HPSO algorithm only requires 30,000 function evaluations to find the best-known solution of 0.01266523 which is superior to the optimised results published elsewhere, and the HPSO algorithm offers rapid convergence and high-optimising accuracy.

(3) Through analysing the converging curves of the *SEA* obtained by using the algorithms including FWA, LDIW-PSO and HPSO, it is found that the HPSO algorithm can reach the optimal *SEA* (20.8177 kJ/kg) after a mere 86 iterations, which shows that the proposed HPSO performs better than all algorithms tested. In addition, it can be seen from the boxplot that the five-number summaries and outliers of the boxplot for the HPSO algorithm are the closest, which also shows that the HPSO has the best convergence and stability compared with FWA and LDIW-PSO algorithm for optimising the composite EAS.

(4) The mathematical model in Section 3.1 was optimised by employing the HPSO algorithm. The optimal configuration for the composite EAS is the thickness of quadrate-tapered thin-walled tube of 2.7720mm, the side length of the unit cell of the honeycomb of 1.0000mm and the wall thickness of the unit cell of 0.1600mm. The *SEA* of 20.4271 kJ/kg can be obtained by using the FEM based on the optimal parameter combination. Compared with the initial design, the values of *SEA*, F_{ip} , and *CFE* obtained through optimisation increase by 165.14%, 6.92%, and 113.79%, respectively; however, the optimised compression displacement of 305.99 mm decreases by 46.88% compared with that under initial design conditions. The optimisation results revealed that the utilisation rate of the structure for materials and impact loads was high, which fails to cause any reduction in energy absorption and instability of crash force. As shown in the increase amplitude of F_{ip} , the structure may not result in over-high deceleration. In this sense, the design method used for the structure is preponderant and the optimised composite EAS exerts a significant influence on the crashworthiness of subway vehicles.

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Appendix: The response model expressions

$$\begin{aligned}
SEA = & -949.2562e^{[-((t_1-1.5)^2+(l-1)^2+(t_2-0.04)^2)/8]} + 701.7790e^{[-((t_1-1.5)^2+(l-2)^2+(t_2-0.07)^2)/8]} + 598.7812e^{[-((t_1-1.5)^2+(l-3)^2+(t_2-0.1)^2)/8]} \\
& -1149.4894e^{[-((t_1-1.5)^2+(l-4)^2+(t_2-0.13)^2)/8]} + 1035.1016e^{[-((t_1-1.5)^2+(l-5)^2+(t_2-0.16)^2)/8]} + 2291.0571e^{[-((t_1-2)^2+(l-1)^2+(t_2-0.07)^2)/8]} \\
& -2297.4196e^{[-((t_1-2)^2+(l-2)^2+(t_2-0.1)^2)/8]} + 56.7215e^{[-((t_1-2)^2+(l-3)^2+(t_2-0.13)^2)/8]} + 1241.1849e^{[-((t_1-2)^2+(l-4)^2+(t_2-0.16)^2)/8]} \\
& -2598.5402e^{[-((t_1-2)^2+(l-5)^2+(t_2-0.04)^2)/8]} - 1452.4172e^{[-((t_1-2.5)^2+(l-1)^2+(t_2-0.1)^2)/8]} + 2625.0468e^{[-((t_1-2.5)^2+(l-2)^2+(t_2-0.13)^2)/8]} \\
& -3235.2708e^{[-((t_1-2.5)^2+(l-3)^2+(t_2-0.16)^2)/8]} + 2703.6099e^{[-((t_1-2.5)^2+(l-4)^2+(t_2-0.04)^2)/8]} + 1913.4500e^{[-((t_1-2.5)^2+(l-5)^2+(t_2-0.07)^2)/8]} \quad (20) \\
& -1667.6270e^{[-((t_1-3)^2+(l-1)^2+(t_2-0.13)^2)/8]} + 2171.7826e^{[-((t_1-3)^2+(l-2)^2+(t_2-0.16)^2)/8]} + 363.8051e^{[-((t_1-3)^2+(l-3)^2+(t_2-0.04)^2)/8]} \\
& -2427.5173e^{[-((t_1-3)^2+(l-4)^2+(t_2-0.07)^2)/8]} - 774.1893e^{[-((t_1-3)^2+(l-5)^2+(t_2-0.1)^2)/8]} + 2154.2208e^{[-((t_1-3.5)^2+(l-1)^2+(t_2-0.16)^2)/8]} \\
& -4115.0621e^{[-((t_1-3.5)^2+(l-2)^2+(t_2-0.04)^2)/8]} + 3274.2119e^{[-((t_1-3.5)^2+(l-3)^2+(t_2-0.07)^2)/8]} - 1048.0831e^{[-((t_1-3.5)^2+(l-4)^2+(t_2-0.1)^2)/8]} \\
& + 639.4714e^{[-((t_1-3.5)^2+(l-5)^2+(t_2-0.13)^2)/8]} - 0.00015694
\end{aligned}$$

$$\begin{aligned}
CFE = & -11.3532e^{[-((t_1-1.5)^2+(l-1)^2+(t_2-0.04)^2)/8]} + 14.8120e^{[-((t_1-1.5)^2+(l-2)^2+(t_2-0.07)^2)/8]} - 0.7069e^{[-((t_1-1.5)^2+(l-3)^2+(t_2-0.1)^2)/8]} \\
& -7.2826e^{[-((t_1-1.5)^2+(l-4)^2+(t_2-0.13)^2)/8]} + 5.9566e^{[-((t_1-1.5)^2+(l-5)^2+(t_2-0.16)^2)/8]} + 26.8433e^{[-((t_1-2)^2+(l-1)^2+(t_2-0.07)^2)/8]} \\
& -50.7002e^{[-((t_1-2)^2+(l-2)^2+(t_2-0.1)^2)/8]} + 21.9695e^{[-((t_1-2)^2+(l-3)^2+(t_2-0.13)^2)/8]} + 4.5661e^{[-((t_1-2)^2+(l-4)^2+(t_2-0.16)^2)/8]} \\
& -12.5050e^{[-((t_1-2)^2+(l-5)^2+(t_2-0.04)^2)/8]} - 8.7201e^{[-((t_1-2.5)^2+(l-1)^2+(t_2-0.1)^2)/8]} + 11.5366e^{[-((t_1-2.5)^2+(l-2)^2+(t_2-0.13)^2)/8]} \\
& + 6.2832e^{[-((t_1-2.5)^2+(l-3)^2+(t_2-0.16)^2)/8]} - 3.9383e^{[-((t_1-2.5)^2+(l-4)^2+(t_2-0.04)^2)/8]} + 8.9278e^{[-((t_1-2.5)^2+(l-5)^2+(t_2-0.07)^2)/8]} \quad (21) \\
& -21.1077e^{[-((t_1-3)^2+(l-1)^2+(t_2-0.13)^2)/8]} + 73.9078e^{[-((t_1-3)^2+(l-2)^2+(t_2-0.16)^2)/8]} - 73.7803e^{[-((t_1-3)^2+(l-3)^2+(t_2-0.04)^2)/8]} \\
& + 17.3019e^{[-((t_1-3)^2+(l-4)^2+(t_2-0.07)^2)/8]} - 0.7784e^{[-((t_1-3)^2+(l-5)^2+(t_2-0.1)^2)/8]} + 18.9962e^{[-((t_1-3.5)^2+(l-1)^2+(t_2-0.16)^2)/8]} \\
& - 60.2296e^{[-((t_1-3.5)^2+(l-2)^2+(t_2-0.04)^2)/8]} + 57.0276e^{[-((t_1-3.5)^2+(l-3)^2+(t_2-0.07)^2)/8]} - 16.0722e^{[-((t_1-3.5)^2+(l-4)^2+(t_2-0.1)^2)/8]} \\
& - 0.1643e^{[-((t_1-3.5)^2+(l-5)^2+(t_2-0.13)^2)/8]} + 0.000070585
\end{aligned}$$

$$\begin{aligned}
F_{ip} = & 6835.8680e^{[-((t_1-1.5)^2+(l-1)^2+(t_2-0.04)^2)/8]} - 50974.6406e^{[-((t_1-1.5)^2+(l-2)^2+(t_3-0.07)^2)/8]} + 43255.6059e^{[-((t_1-1.5)^2+(l-3)^2+(t_2-0.1)^2)/8]} \\
& - 29657.5978e^{[-((t_1-1.5)^2+(l-4)^2+(t_2-0.13)^2)/8]} + 4255.0217e^{[-((t_1-1.5)^2+(l-5)^2+(t_2-0.16)^2)/8]} + 3399.7037e^{[-((t_1-2)^2+(l-1)^2+(t_2-0.07)^2)/8]} \\
& + 116438.4361e^{[-((t_1-2)^2+(l-2)^2+(t_2-0.1)^2)/8]} - 70852.9481e^{[-((t_1-2)^2+(l-3)^2+(t_2-0.13)^2)/8]} + 38845.0350e^{[-((t_1-2)^2+(l-4)^2+(t_2-0.16)^2)/8]} \\
& + 13541.1670e^{[-((t_1-2)^2+(l-5)^2+(t_2-0.04)^2)/8]} - 27917.4190e^{[-((t_1-2.5)^2+(l-1)^2+(t_2-0.1)^2)/8]} - 88906.4254e^{[-((t_1-2.5)^2+(l-2)^2+(t_2-0.13)^2)/8]} \\
& + 6229.7421e^{[-((t_1-2.5)^2+(l-3)^2+(t_2-0.16)^2)/8]} + 16230.2918e^{[-((t_1-2.5)^2+(l-4)^2+(t_2-0.04)^2)/8]} - 39395.7577e^{[-((t_1-2.5)^2+(l-5)^2+(t_2-0.07)^2)/8]} \quad (22) \\
& + 12260.7972e^{[-((t_1-3)^2+(l-1)^2+(t_2-0.13)^2)/8]} + 48688.5739e^{[-((t_1-3)^2+(l-2)^2+(t_2-0.16)^2)/8]} + 8657.6748e^{[-((t_1-3)^2+(l-3)^2+(t_2-0.04)^2)/8]} \\
& - 20686.4121e^{[-((t_1-3)^2+(l-4)^2+(t_2-0.07)^2)/8]} + 20295.9443e^{[-((t_1-3)^2+(l-5)^2+(t_2-0.1)^2)/8]} + 15418.0245e^{[-((t_1-3.5)^2+(l-1)^2+(t_2-0.16)^2)/8]} \\
& - 47073.2019e^{[-((t_1-3.5)^2+(l-2)^2+(t_2-0.04)^2)/8]} + 40112.7689e^{[-((t_1-3.5)^2+(l-3)^2+(t_2-0.07)^2)/8]} - 24575.1546e^{[-((t_1-3.5)^2+(l-4)^2+(t_2-0.1)^2)/8]} \\
& + 9918.08293e^{[-((t_1-3.5)^2+(l-5)^2+(t_2-0.13)^2)/8]} - 0.0027
\end{aligned}$$

$$\begin{aligned}
F_{avg} = & -90049.9564e^{[-((t_1-1.5)^2+(l-1)^2+(t_2-0.04)^2)/8]} + 5072.6960e^{[-((t_1-1.5)^2+(l-2)^2+(t_2-0.07)^2)/8]} + 151010.6483e^{[-((t_1-1.5)^2+(l-3)^2+(t_2-0.1)^2)/8]} \\
& -186903.5460e^{[-((t_1-1.5)^2+(l-4)^2+(t_2-0.13)^2)/8]} + 105716.6446e^{[-((t_1-1.5)^2+(l-5)^2+(t_2-0.16)^2)/8]} + 267167.6072e^{[-((t_1-2)^2+(l-1)^2+(t_2-0.07)^2)/8]} \\
& -161084.3139e^{[-((t_1-2)^2+(l-2)^2+(t_2-0.1)^2)/8]} - 103470.2327e^{[-((t_1-2)^2+(l-3)^2+(t_2-0.13)^2)/8]} + 257555.9588e^{[-((t_1-2)^2+(l-4)^2+(t_2-0.16)^2)/8]} \\
& -257887.7089e^{[-((t_1-2)^2+(l-5)^2+(t_2-0.04)^2)/8]} - 197905.1519e^{[-((t_1-2.5)^2+(l-1)^2+(t_2-0.1)^2)/8]} + 305740.2096e^{[-((t_1-2.5)^2+(l-2)^2+(t_2-0.13)^2)/8]} \\
& -481509.8579e^{[-((t_1-2.5)^2+(l-3)^2+(t_2-0.16)^2)/8]} + 323578.3181e^{[-((t_1-2.5)^2+(l-4)^2+(t_2-0.04)^2)/8]} + 137802.4456e^{[-((t_1-2.5)^2+(l-5)^2+(t_2-0.07)^2)/8]} \\
& -246955.6966e^{[-((t_1-3)^2+(l-1)^2+(t_2-0.13)^2)/8]} + 301241.4774e^{[-((t_1-3)^2+(l-2)^2+(t_2-0.16)^2)/8]} + 187071.3895e^{[-((t_1-3)^2+(l-3)^2+(t_2-0.04)^2)/8]} \\
& -376267.3601e^{[-((t_1-3)^2+(l-4)^2+(t_2-0.07)^2)/8]} - 17173.4284e^{[-((t_1-3)^2+(l-5)^2+(t_2-0.1)^2)/8]} + 320346.5287e^{[-((t_1-3.5)^2+(l-1)^2+(t_2-0.16)^2)/8]} \\
& -573877.7356e^{[-((t_1-3.5)^2+(l-2)^2+(t_2-0.04)^2)/8]} + 383242.3155e^{[-((t_1-3.5)^2+(l-3)^2+(t_2-0.07)^2)/8]} - 102457.5937e^{[-((t_1-3.5)^2+(l-4)^2+(t_2-0.1)^2)/8]} \\
& + 57075.2454e^{[-((t_1-3.5)^2+(l-5)^2+(t_2-0.13)^2)/8]} - 0.0198
\end{aligned} \quad (23)$$

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