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DOI:

[10.1016/j.jfranklin.2018.08.015](https://doi.org/10.1016/j.jfranklin.2018.08.015)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Fang, X., Liu, F., & Ding, Z. (2018). Robust control of unmanned helicopters with high-order mismatched disturbances via disturbance-compensation-gain construction approach. *Journal of the Franklin Institute*, 355(15), 7158-7177. <https://doi.org/10.1016/j.jfranklin.2018.08.015>

Published in:

Journal of the Franklin Institute

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Robust control of unmanned helicopters with high-order mismatched disturbances via disturbance-compensation-gain construction approach

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Abstract

In this paper, a novel robust control strategy based on disturbance-compensation-gain (DCG) construction approach is proposed for small-scale unmanned helicopters in the presence of high-order mismatched disturbances. The overall control structure consists of two hierarchical layers. The inner-loop controller is to guarantee the stability of the unmanned helicopters subject to high-order mismatched disturbances. With the estimation of the disturbances and their successive derivatives via finite-time disturbance observer (FTDO), by properly designing some disturbance compensation gains, a novel robust controller is developed to remove the high-order mismatched disturbances from the output channels. The outer-loop controller is to produce flight commands for inner-loop system, as well as to track the reference trajectory, which is carried out with the dynamic inversion technique. The simulation results demonstrate that the unmanned helicopters are capable to perform flight missions autonomously with the proposed control strategy.

Keywords: Small-scale unmanned helicopters, mismatched disturbances, disturbance-compensation-gain (DCG) construction, finite-time disturbance observer (FTDO)

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1. Introduction

Recently, small-scale unmanned helicopters have aroused great attentions in industrial and academic areas. Small-scale unmanned helicopters are capable of vertical taking-off and landing, hovering, and rapid maneuvering. They are able to carry out work in dangerous environment, and also can be used as flight platforms for pure academic research [1, 2, 3, 4, 5].

However, small-scale unmanned helicopters are known as complex dynamic systems with under-actuation, strong coupling and multiple disturbances. More specifically, some disturbances cannot meet the “matching condition” that means the disturbances act on the system via the same channels as control inputs. That is to say, the unmanned helicopter system is subject to mismatched disturbances [6, 7]. Many research teams worldwide have attempted to design high-performance flight control systems for unmanned helicopters [8, 9].

During the past decades, a number of elegant control methods have been employed for unmanned helicopters, including H_∞ [10], μ -synthesis [11], sliding mode control (SMC) [12], model predictive control (MPC) [13], and so on. Note that all of these control methods deal with disturbances via a manner of feedback regulation, but do not counteract the influence of disturbances via feed-forward control directly. Therefore, these feedback control methods attenuate the disturbances in a relatively slow and gentle way, and may cannot handle the severe disturbances promptly and powerfully.

Disturbance-observer-based control (DOBC) method is regarded as a kind of disturbance-direct-rejection technique, providing an ideal way to deal with the disturbances [14]. DOBC method is referred to as two-degrees-of-freedom control structure: the disturbance observer is designed to estimate the disturbances and then compensate for them by feed-forward control technique, which is responsible to reject disturbances; the baseline feedback controller is designed to meet the requirements on stability and tracking performance specifications. The DOBC methods have been employed in a wide range of practical engineering systems, such as quadrotor system [15], hard disk drive system [16],

missile system [17] and robotic system [18]. Unfortunately, the majority of the DOBC methods only can be applied to the systems whose disturbances satisfy the so-called “matching condition”.

In recent years, some attempts have been made to extend DOBC methods
35 to attenuate mismatched disturbances. A set of SMC methods combining with disturbance observer have been developed to restrain mismatched disturbances [19, 20, 21]. Some output regulation methods are designed to suppress mismatched disturbances [22, 23]. The combination of dynamic surface control (DSC) technique and disturbance observer can deal with the mismatched dis-
40 turbances effectively [24]. Although the DOBC methods aforementioned can attenuate mismatched disturbances, these methods are binding to specific feedback control approaches. Therefore, they are poor in expansibility. Another important series of DOBC methods is to properly design the disturbance-compensation gain (DCG) to remove the effects of mismatched disturbances
45 from the output variables, which can be called as DCG methods [25, 26, 27]. The most key feature of the DCG methods is that they can be integrated with any kind of feedback control approaches, which implies that they have excellent capability of extension. However, the existing DCG methods are developed based on the assumption that the first derivatives of mismatched disturbances
50 go to zero in steady state. On the other hand, the mismatched disturbances acting on the unmanned helicopters do not meet this assumption, which are high-order functions of time.

In this paper, a robust control strategy based on novel DCG approach is proposed for small-scale unmanned helicopters to counteract the influence of the
55 high-order mismatched disturbances from output variables. The overall flight control scheme of unmanned helicopters consists of two hierarchical layers. The inner-loop controller is developed with the novel DCG approach to stabilize the unmanned helicopters in the presence of high-order mismatched disturbances. The outer-loop is designed via dynamic inversion technique to track the reference
60 trajectory. The simulation results demonstrate the superior control performance of the proposed control scheme.

The proposed novel DCG method mainly exhibits the following eminent features. First, the proposed DCG method is able to completely eliminate the effects of high-order mismatched disturbances from output variables in finite
65 time. Second, the proposed DCG method does not depend on the specific feedback control algorithm. Therefore, it is convenient to be extended to general cases. Third, the nominal performance can be maintained with the proposed DCG scheme. That is to say, its control performance is consistent with the baseline feedback control algorithm in the absence of disturbances.

70 This paper is organized as follows. The small-scale unmanned helicopter system is analyzed in Section 2. In Section 3, a robust flight control scheme is proposed for the unmanned helicopter system to reject mismatched disturbances. Section 4 presents the stability analysis of the closed-loop helicopter system. Numerical results are given in Section 5. Finally, some conclusions are
75 drawn in Section 6.

2. Problem statement

2.1. Nonlinear unmanned helicopter dynamics

The complete nonlinear unmanned helicopter dynamics will be established in this subsection. The unmanned helicopter model mainly contains three key
80 components: (1) translational dynamics, (2) rotational dynamics, and (3) main rotor flapping dynamics. Moreover, the external disturbances and model uncertainties are treated as the lumped disturbances. The schematic diagram of unmanned helicopter is shown in Figure 1.

First of all, two coordinate systems are defined for the unmanned helicopter.
85 $F_N = \{O_N, i_N, j_N, k_N\}$ and $F_B = \{O_B, i_B, j_B, k_B\}$ denote the inertial coordinate and body-fixed coordinate, respectively.

2.1.1. Translational dynamics

The translational dynamic model of the unmanned helicopter can be described by

$$\dot{P}_n = R(\Theta)V_b, \quad (1)$$

Figure 1: Schematic diagram of unmanned helicopter

$$m\dot{V}_b = -m\Omega_b \times V_b + mgR(\Theta)^T e_3 + f_b + f_d, \quad (2)$$

where $P_n = [x \ y \ z]^T$ and $\Theta = [\phi \ \theta \ \psi]^T$ denote the position and Euler angle vectors in the inertial coordinate, respectively. $V_b = [u \ v \ w]^T$ and $\Omega_b = [p \ q \ r]^T$ denote the velocity and angular rate vectors in the body-fixed coordinate, respectively. The notations of m and g denote the mass of the helicopter and the gravitational acceleration, respectively. $e_3 = [0 \ 0 \ 1]^T$.

The rotation matrix $R(\Theta)$ is defined by

$$R(\Theta) = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}, \quad (3)$$

where the compact notation c denotes for $\cos(\cdot)$, and s for $\sin(\cdot)$.

The force vector f_b acting on the helicopter can be given by

$$f_b = \begin{bmatrix} -T_m a \\ T_m b \\ -T_m \end{bmatrix} + \begin{bmatrix} 0 \\ -T_t \\ 0 \end{bmatrix} = \begin{bmatrix} -T_m a \\ T_m b - T_t \\ -T_m \end{bmatrix}, \quad (4)$$

where $T_m = K_m u_{col} + B_m$ and $T_t = K_t u_{ped} + B_t$ present the thrusts generated by main rotor and tail rotor respectively. The notations of u_{col} and u_{ped} denote the collective pitch servo input and rudder servo input respectively. The vector f_d denote the lumped disturbances applied to the helicopter system.

2.1.2. Rotational dynamics

The rotational dynamic model of the unmanned helicopter can be described by

$$\dot{\Theta} = S(\Theta)\Omega_b, \quad (5)$$

105

$$J\dot{\Omega}_b = -\Omega_b \times J\Omega_b + \tau_b, \quad (6)$$

where $J = \text{diag}(J_{xx}, J_{yy}, J_{zz})$ is the diagonal inertia matrix of the helicopter. The attitude kinematic transformation matrix $S(\Theta)$ is given by

$$S(\Theta) = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi/c_\theta & s_\phi/c_\theta \end{bmatrix}, \quad (7)$$

where t denotes for $\tan(\cdot)$.

The moment vector τ_b acting on the helicopter can be given by

$$\tau_b = \Sigma(T_m)v + \Delta(T_m), \quad (8)$$

110 where $v = [a \ b \ T_t]^T$. The notations of a and b represent the tip-path-plane (TPP) flapping angles of main rotor disc for longitudinal and lateral directions, respectively. The notations of $\Sigma(T_m) \in R^{3 \times 3}$ and $\Delta(T_m) \in R^{3 \times 1}$ denote the invertible matrix and vector for T_m , respectively.

2.1.3. Main rotor flapping dynamics

115 The main rotor flapping dynamics can be described by

$$\dot{a} = -q - \frac{1}{\tau_m}a + A_b b + A_{lon}u_{lon}, \quad (9)$$

$$\dot{b} = -p + B_a a - \frac{1}{\tau_m}b + B_{lat}u_{lat}, \quad (10)$$

with the parameters of τ_m , A_b , B_a , A_{lon} and B_{lat} . The notations of u_{lon} and u_{lat} denote the longitudinal cyclic servo input and lateral cyclic servo input, respectively.

120 *2.2. Linearized dynamic model of unmanned helicopter*

In order to develop the flight controller, the nonlinear dynamic model of the unmanned helicopter will be linearized at the operating conditions of interest. The linear dynamic model adopted in this paper has been verified that the model is accurate enough for a relatively wide range of the flight envelope around the
125 operating condition [1, 28].

The linear dynamic model is described by [1]

$$\dot{P}_n = R(\Theta)V_b, \quad (11)$$

and

$$\left\{ \begin{array}{l} \dot{u} = X_u u - g\theta + d_1, \\ \dot{v} = Y_v v + g\phi + d_2, \\ \dot{w} = Z_w w + Z_r r + Z_a a + Z_b b + Z_{col} u_{col} + d_3, \\ \dot{\phi} = p, \\ \dot{\theta} = q, \\ \dot{\psi} = r, \\ \dot{p} = L_u u + L_v v + L_b b, \\ \dot{q} = M_u u + M_v v + M_a a, \\ \dot{r} = N_v v + N_w w + N_p p + N_r r + N_{ped} u_{ped}, \\ \dot{a} = -q - \frac{1}{\tau_m} a + A_b b + A_{lon} u_{lon}, \\ \dot{b} = -p + B_a a - \frac{1}{\tau_m} b + B_{lat} u_{lat}, \end{array} \right. \quad (12)$$

with the parameters of $X_u, Y_v, Z_w, Z_r, Z_a, Z_b, Z_{col}, L_u, L_v, L_b, M_u, M_v, M_a, N_v, N_w, N_p, N_r, N_{ped}, \tau_m, A_b, A_{lon}, B_a$ and B_{lat} . The outputs of the unmanned
130 helicopter system are position $P_n = [x \ y \ z]^T$ and yaw angle ψ . The control inputs are $u_{col}, u_{ped}, u_{lon}$ and u_{lat} . The notations d_1, d_2 and d_3 are the lumped disturbances, which contain external disturbances, model uncertainties, couplings, and nonlinear dynamics neglected in linearization procedure, and so on. It is worth noting that the disturbances of d_1 and d_2 enter the unmanned
135 helicopter system via different channels from control inputs, so they are mismatched disturbances. Furthermore, these mismatched disturbances always are

Figure 2: The flight controller scheme of unmanned helicopter

high-order time-varying disturbances, which do not meet the condition that the mismatched disturbances go to constants in steady state [25, 26]. Therefore, the existing disturbance-compensation-gain (DCG) control methods cannot effectively reject these mismatched disturbances. In this paper, a novel DCG control method will be proposed for the unmanned helicopter system to remove the high-order mismatched disturbances from output variables.

3. Controller design

According to the time scale of the state variables of the unmanned helicopter system, a two-layer structure is proposed for the flight control system: (1) the inner loop is developed based on the novel DCG control method to stabilize the unmanned helicopter despite the presence of high-order mismatched disturbances; (2) the outer loop is designed with the dynamic-inversion technique to track the predefined reference trajectory $P_{nr} = [x_r(t) \ y_r(t) \ z_r(t)]^T$. The flight controller structure is shown in Figure 2.

3.1. Finite-time disturbance observer

First, some finite-time disturbance observers (FTDOs) will be designed to estimate the disturbances and their successive derivatives.

To develop the subsequent FTDOs, the following mild assumption is made for the helicopter system.

Assumption 1: The disturbances in helicopter system (12) satisfy that the mismatched disturbances $d_1(t)$ and $d_2(t)$ are at least 4-th order differentiable, and their fourth derivatives $d_1^{(4)}(t)$ and $d_2^{(4)}(t)$ have the Lipschitz constants L_1 and L_2 respectively. The matched disturbance $d_3(t)$ is at least 1-st order differentiable, and its first derivative $\dot{d}_3(t)$ has a Lipschitz constant L_3 .

A fourth-order FTDO is developed to estimate the mismatched disturbance d_1 and its successive derivatives of $d_1^{(1)}$, $d_1^{(2)}$ and $d_1^{(3)}$ [29]:

$$\begin{aligned}
\dot{\eta}_1^0 &= \epsilon_1^0 + (X_u u - g\theta), \\
\epsilon_1^0 &= -\lambda_1^0 L_1^{\frac{1}{5}} |\eta_1^0 - u|^{\frac{4}{5}} \text{sign}(\eta_1^0 - u) + \eta_1^1, \\
\dot{\eta}_1^1 &= \epsilon_1^1, \\
\epsilon_1^1 &= -\lambda_1^1 L_1^{\frac{1}{4}} |\eta_1^1 - \epsilon_1^0|^{\frac{3}{4}} \text{sign}(\eta_1^1 - \epsilon_1^0) + \eta_1^2, \\
\dot{\eta}_1^2 &= \epsilon_1^2, \\
\epsilon_1^2 &= -\lambda_1^2 L_1^{\frac{1}{3}} |\eta_1^2 - \epsilon_1^1|^{\frac{2}{3}} \text{sign}(\eta_1^2 - \epsilon_1^1) + \eta_1^3, \\
\dot{\eta}_1^3 &= \epsilon_1^3, \\
\epsilon_1^3 &= -\lambda_1^3 L_1^{\frac{1}{2}} |\eta_1^3 - \epsilon_1^2|^{\frac{1}{2}} \text{sign}(\eta_1^3 - \epsilon_1^2) + \eta_1^4, \\
\dot{\eta}_1^4 &= \epsilon_1^4, \\
\epsilon_1^4 &= -\lambda_1^4 L_1 \text{sign}(\eta_1^4 - \epsilon_1^3),
\end{aligned} \tag{13}$$

where η_1^0 is the estimate of state u . η_1^1 , η_1^2 , η_1^3 and η_1^4 are the estimates of the disturbance d_1 and its successive derivatives $d_1^{(1)}$, $d_1^{(2)}$ and $d_1^{(3)}$, respectively. $\lambda_1^i > 0 (i = 0, 1, 2, 3, 4)$ are the coefficients of the FTDO.

The mismatched disturbance d_2 and its successive derivatives of $d_2^{(1)}$, $d_2^{(2)}$ and $d_2^{(3)}$ are estimated by a similar fourth-order FTDO to (13).

Furthermore, a first-order FTDO is developed to estimate the matched disturbance d_3 :

$$\begin{aligned}
\dot{\eta}_3^0 &= \epsilon_3^0 + (Z_w w + Z_r r + Z_a a + Z_b b + Z_{col} u_{col}), \\
\epsilon_3^0 &= -\lambda_3^0 L_3^{\frac{1}{2}} |\eta_3^0 - w|^{\frac{1}{2}} \text{sign}(\eta_3^0 - w) + \eta_3^1, \\
\dot{\eta}_3^1 &= \epsilon_3^1, \\
\epsilon_3^1 &= -\lambda_3^1 L_3 \text{sign}(\eta_3^1 - \epsilon_3^0),
\end{aligned} \tag{14}$$

where η_3^0 is the estimate of state w . η_3^1 is the estimate of the disturbance d_3 . $\lambda_3^i > 0 (i = 0, 1)$ are the coefficients of the FTDO.

Considering the unmanned helicopter system (12), the error dynamics of the FTDO (13) understood in the Filippov sense is obtained by

$$\begin{aligned}
\dot{e}_1^0 &= -\lambda_1^0 L_1^{\frac{1}{5}} |e_1^0|^{\frac{4}{5}} \text{sign}(e_1^0) + e_1^1, \\
\dot{e}_1^1 &= -\lambda_1^1 L_1^{\frac{1}{4}} |e_1^1 - \dot{e}_1^0|^{\frac{3}{4}} \text{sign}(e_1^1 - \dot{e}_1^0) + e_1^2, \\
\dot{e}_1^2 &= -\lambda_1^2 L_1^{\frac{1}{3}} |e_1^2 - \dot{e}_1^1|^{\frac{2}{3}} \text{sign}(e_1^2 - \dot{e}_1^1) + e_1^3, \\
\dot{e}_1^3 &= -\lambda_1^3 L_1^{\frac{1}{2}} |e_1^3 - \dot{e}_1^2|^{\frac{1}{2}} \text{sign}(e_1^3 - \dot{e}_1^2) + e_1^4, \\
\dot{e}_1^4 &\in -\lambda_1^4 L_1 \text{sign}(e_1^4 - \dot{e}_1^3) + [-L_1, L_1],
\end{aligned} \tag{15}$$

where the FTDO estimation errors are defined by $e_1^0 = \eta_1^0 - u$, $e_1^1 = \eta_1^1 - d_1$,
175 $e_1^2 = \eta_1^2 - d_1^{(1)}$, $e_1^3 = \eta_1^3 - d_1^{(2)}$ and $e_1^4 = \eta_1^4 - d_1^{(3)}$. It follows from [29] that
the estimation errors $e_1^i(t)$ ($i = 0, 1, 2, 3, 4$) will converge to zero in finite time,
which means that there exists a time constant t_f such that $e_1^i(t) = 0$ for $t > t_f$.
Furthermore, it is true that $\eta_1^i(t) = \epsilon_1^{i-1}(t)$ for $t > t_f$.

The stability analysis of the FTDO (14) is similar to the FTDO (13) above.
180 Therefore, it is omitted here for brevity.

3.2. Inner-loop controller

The inner-loop controller is to stabilize the unmanned helicopter despite the
presence of high-order mismatched disturbances, which will be developed based
on the novel DCG control method.

185 Inspired by [1], in this subsection, the inner-loop dynamic model (12) will
be divided into two subsystems: horizontal subsystem Σ_1 and vertical-heading
subsystem Σ_2 :

$$\Sigma_1 : \begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u_1 + B_{1d} \omega_1, \\ y_1 = C_1 x_1, \end{cases} \tag{16}$$

$$\Sigma_2 : \begin{cases} \dot{x}_2 = A_2 x_2 + B_2 u_2 + B_{2d} \omega_2 + \varrho, \\ y_2 = C_2 x_2, \end{cases} \tag{17}$$

where $x_1 = (u \ v \ \theta \ \phi \ q \ p \ a \ b)^T$, $x_2 = (w \ \psi \ r)^T$, $u_1 = (u_{lon} \ u_{lat})^T$,
190 $u_2 = (u_{col} \ u_{ped})^T$, $\omega_1 = (d_1 \ d_2)^T$, $\omega_2 = d_3$ and $\varrho = (Z_a a + Z_b b \ 0 \ N_v v + N_p p)^T$.

$$\begin{aligned}
A_1 &= \begin{bmatrix} X_u & 0 & -g & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ M_u & M_v & 0 & 0 & 0 & 0 & M_a & 0 \\ L_u & L_v & 0 & 0 & 0 & 0 & 0 & L_b \\ 0 & 0 & 0 & 0 & -1 & 0 & -\frac{1}{\tau_m} & A_b \\ 0 & 0 & 0 & 0 & 0 & -1 & B_a & -\frac{1}{\tau_m} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0_{6 \times 2} & \\ A_{lon} & 0 \\ 0 & B_{lat} \end{bmatrix}, \\
B_{1d} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0_{6 \times 2} \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0_{6 \times 2} \end{bmatrix}^T, \quad A_2 = \begin{bmatrix} Z_w & 0 & Z_r \\ 0 & 0 & 1 \\ N_w & 0 & N_r \end{bmatrix}, \quad B_2 = \begin{bmatrix} Z_{col} & 0 \\ 0 & 0 \\ 0 & N_{ped} \end{bmatrix}, \\
B_{2d} &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.
\end{aligned}$$

3.2.1. Horizontal subsystem

195 The horizontal subsystem Σ_1 is subject to the high-order mismatched disturbances $\omega_1 = (d_1 \ d_2)^T$. However, the existing DCG control methods can only deal with the mismatched disturbances that tend to constants in steady state [25, 26]. They cannot fulfill the control task of unmanned helicopter system. Therefore, a novel DCG controller based on the FTDOs will be designed
200 for the horizontal subsystem to suppress these mismatched disturbances.

The novel DCG control law is designed as follows

$$u_1 = K_{x1}x_1 + K_{\omega_1}\hat{\omega}_1 + K_{\omega_1^{(1)}}\hat{\omega}_1^{(1)} + K_{\omega_1^{(2)}}\hat{\omega}_1^{(2)} + K_{\omega_1^{(3)}}\hat{\omega}_1^{(3)}, \quad (18)$$

where $\hat{\omega}_1 = (\eta_1^1 \ \eta_2^1)^T$, $\hat{\omega}_1^{(1)} = (\eta_1^2 \ \eta_2^2)^T$, $\hat{\omega}_1^{(2)} = (\eta_1^3 \ \eta_2^3)^T$ and $\hat{\omega}_1^{(3)} = (\eta_1^4 \ \eta_2^4)^T$ represent the estimates of the mismatched disturbances $\omega_1 =$

$(d_1 \ d_2)^T$ and their successive derivatives. $K_{x1} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{17} & k_{18} \\ k_{21} & k_{22} & \cdots & k_{27} & k_{28} \end{bmatrix}$

205 is the feedback control gain that should be properly designed to achieve satisfactory closed-loop control performance. K_{ω_1} , $K_{\omega_1^{(1)}}$, $K_{\omega_1^{(2)}}$ and $K_{\omega_1^{(3)}}$ are the

mismatched-disturbance-compensation gains, which are designed as

$$K_{\omega_1} = \begin{bmatrix} -\frac{k_{13}}{g} & \frac{k_{14}}{g} \\ -\frac{k_{23}}{g} & \frac{k_{24}}{g} \end{bmatrix}, \quad (19)$$

$$K_{\omega_1^{(1)}} = \begin{bmatrix} -\frac{k_{15}}{g} + \frac{1}{gA_{lon}} & \frac{k_{16}}{g} \\ -\frac{k_{25}}{g} & \frac{k_{26}}{g} - \frac{1}{gB_{lat}} \end{bmatrix}, \quad (20)$$

$$K_{\omega_1^{(2)}} = \begin{bmatrix} -\frac{k_{17}}{gM_a} + \frac{1}{\tau_m g M_a A_{lon}} & \frac{k_{18}}{gL_b} + \frac{A_b}{gL_b A_{lon}} \\ -\frac{k_{27}}{gM_a} - \frac{B_a}{gM_a B_{lat}} & \frac{k_{28}}{gL_b} - \frac{1}{\tau_m g L_b B_{lat}} \end{bmatrix}, \quad (21)$$

$$K_{\omega_1^{(3)}} = \begin{bmatrix} \frac{1}{gM_a A_{lon}} & 0 \\ 0 & -\frac{1}{gL_b B_{lat}} \end{bmatrix}. \quad (22)$$

3.2.2. Vertical-heading subsystem

The vertical-heading subsystem Σ_2 is only subject to matched disturbance $\omega_2 = d_3$. Therefore, a conventional DCG control method will be proposed for the vertical-heading subsystem to suppress this matched disturbance [25].

The conventional DCG control law is designed as follows

$$u_2 = K_{x_2}x_2 + K_{\omega_2}\hat{\omega}_2 - B_2^\dagger \varrho, \quad (23)$$

where $\hat{\omega}_2 = \eta_3^1$ represents the estimate of the matched disturbance $\omega_2 = d_3$. K_{x_2} is the feedback control gain that should be properly designed to achieve satisfactory closed-loop control performance. $K_{\omega_2} = \begin{bmatrix} -\frac{1}{Z_{col}} & 0 \end{bmatrix}^T$ is the matched-disturbance-compensation gain. $B_2^\dagger = (B_2^T B_2)^{-1} B_2^T$ is the pseudo-inverse of the matrix B_2 .

3.3. Outer-loop controller

The outer-loop controller is to drive the unmanned helicopter to track the predefined reference trajectory $P_{nr} = [x_r(t) \ y_r(t) \ z_r(t)]^T$, and to generate the reference signal for the inner-loop controller. The outer-loop controller will be developed based on the dynamic inversion control method.

According to the outer-loop dynamics (11), the dynamic inversion control law is designed as follows:

$$V_{br} = K_P R(\Theta)^T (P_n - P_{nr}), \quad (24)$$

where $V_{br} = (u_r \ v_r \ w_r)^T$ is the control input, as well as the reference signal of inner-loop. $K_P = \text{diag}(k_x, k_y, k_z)$ is the feedback control gain.

230 4. Stability analysis

The stability of the closed-loop unmanned helicopter system will be analyzed in this section.

Theorem: For horizontal subsystem Σ_1 (16) with the novel DCG control law (18), if the feedback control gain K_{x1} is selected such that $A_1 + B_1 K_{x1}$ is Hurwitz matrix, the system output y_1 will converge to the desired set-point asymptotically and the states will remain bounded. For vertical-heading subsystem Σ_2 (17) with the conventional DCG control law (23), if the feedback control gain K_{x2} is selected such that $A_2 + B_2 K_{x2}$ is Hurwitz matrix, the system output y_2 will converge to the desired set-point asymptotically and the states will remain bounded.

Proof: In order to give the stability analysis of the horizontal subsystem, some new state variables are defined as $\bar{\theta} = \theta - \frac{1}{g}\eta_1^1$, $\bar{\phi} = \phi + \frac{1}{g}\eta_2^1$, $\bar{q} = q - \frac{1}{g}\eta_1^2$, $\bar{p} = p + \frac{1}{g}\eta_2^2$, $\bar{a} = a - \frac{1}{gM_a}\eta_1^3$ and $\bar{b} = b + \frac{1}{gL_b}\eta_2^3$.

The equivalent system of (16) can be derived by

$$\begin{aligned} \dot{u} &= X_u u - g\theta + d_1 \\ &= X_u u - g(\theta - \frac{1}{g}\eta_1^1) + (d_1 - \eta_1^1) \\ &= X_u u - g\bar{\theta} - e_1^1, \end{aligned} \quad (25)$$

$$\begin{aligned} \dot{v} &= Y_v v + g\phi + d_2 \\ &= Y_v v + g(\phi + \frac{1}{g}\eta_2^1) + (d_2 - \eta_2^1) \\ &= Y_v v + g\bar{\phi} - e_2^1, \end{aligned} \quad (26)$$

$$\begin{aligned}
\dot{\bar{\theta}} &= \dot{\theta} - \frac{1}{g}\dot{\eta}_1^1 \\
&= q - \frac{1}{g}\dot{\eta}_1^1 \\
&= (q - \frac{1}{g}\eta_1^2) - \frac{1}{g}(\dot{\eta}_1^1 - \eta_1^2) \\
&= \bar{q} - \frac{1}{g}(\dot{e}_1^1 - e_1^2),
\end{aligned} \tag{27}$$

$$\begin{aligned}
\dot{\bar{\phi}} &= \dot{\phi} + \frac{1}{g}\dot{\eta}_2^1 \\
&= p + \frac{1}{g}\dot{\eta}_2^1 \\
&= (p + \frac{1}{g}\eta_2^2) + \frac{1}{g}(\dot{\eta}_2^1 - \eta_2^2) \\
&= \bar{p} + \frac{1}{g}(\dot{e}_2^1 - e_2^2),
\end{aligned} \tag{28}$$

$$\begin{aligned}
\dot{\bar{q}} &= \dot{q} - \frac{1}{g}\dot{\eta}_1^2 \\
&= M_u u + M_v v + M_a a - \frac{1}{g}\dot{\eta}_1^2 \\
&= M_u u + M_v v + M_a (a - \frac{1}{gM_a}\eta_1^3) - \frac{1}{g}(\dot{\eta}_1^2 - \eta_1^3) \\
&= M_u u + M_v v + M_a \bar{a} - \frac{1}{g}(\dot{e}_1^2 - e_1^3),
\end{aligned} \tag{29}$$

$$\begin{aligned}
\dot{\bar{p}} &= \dot{p} + \frac{1}{g}\dot{\eta}_2^2 \\
&= L_u u + L_v v + L_b b + \frac{1}{g}\dot{\eta}_2^2 \\
&= L_u u + L_v v + L_b (b + \frac{1}{gL_b}\eta_2^3) + \frac{1}{g}(\dot{\eta}_2^2 - \eta_2^3) \\
&= L_u u + L_v v + L_b \bar{b} + \frac{1}{g}(\dot{e}_2^2 - e_2^3),
\end{aligned} \tag{30}$$

$$\begin{aligned}
\dot{\bar{a}} &= \dot{a} - \frac{1}{gM_a}\dot{\eta}_1^3 \\
&= -q - \frac{1}{\tau_m}a + A_b b + A_{lon} u_{lon} - \frac{1}{gM_a}\dot{\eta}_1^3 \\
&= -\bar{q} - \frac{1}{\tau_m}\bar{a} + A_b \bar{b} + A_{lon} u_{lon} - \frac{1}{g}\eta_1^2 - \frac{1}{\tau_m g M_a}\eta_1^3 \\
&\quad - \frac{A_b}{gL_b}\eta_2^3 - \frac{1}{gM_a}\eta_1^4 - \frac{1}{gM_a}(\dot{e}_1^3 - e_1^4),
\end{aligned} \tag{31}$$

$$\begin{aligned}
\dot{\bar{b}} &= \dot{b} + \frac{1}{gL_b} \dot{\eta}_2^3 \\
&= -p + B_a a - \frac{1}{\tau_m} b + B_{lat} u_{lat} + \frac{1}{gL_b} \dot{\eta}_2^3 \\
&= -\bar{p} + B_a \bar{a} - \frac{1}{\tau_m} \bar{b} + B_{lat} u_{lat} + \frac{1}{g} \eta_2^2 + \frac{1}{\tau_m g L_b} \eta_2^3 \\
&\quad + \frac{B_a}{g M_a} \eta_1^3 + \frac{1}{gL_b} \eta_2^4 + \frac{1}{gL_b} (\dot{e}_2^3 - e_2^4). \tag{32}
\end{aligned}$$

The control inputs can be designed as

$$u_1 = K_{x1} \bar{x}_1 + \begin{bmatrix} A_{lon} & 0 \\ 0 & B_{lat} \end{bmatrix}^T \begin{bmatrix} \frac{1}{g} \eta_1^2 + \frac{1}{\tau_m g M_a} \eta_1^3 + \frac{A_b}{gL_b} \eta_2^3 + \frac{1}{g M_a} \eta_1^4 \\ -\frac{1}{g} \eta_2^2 - \frac{1}{\tau_m g L_b} \eta_2^3 - \frac{B_a}{g M_a} \eta_1^3 - \frac{1}{gL_b} \eta_2^4 \end{bmatrix}, \tag{33}$$

where $\bar{x}_1 = (u \ v \ \bar{\theta} \ \bar{\phi} \ \bar{q} \ \bar{p} \ \bar{a} \ \bar{b})^T$, and K_{x1} is the feedback control gain. Considering the definition of new state variables, the first item of (33) can be rewritten by

$$K_{x1} \bar{x}_1 = K_{x1} x_1 + \begin{bmatrix} -\frac{k_{13}}{g} \eta_1^1 - \frac{k_{15}}{g} \eta_1^2 - \frac{k_{17}}{g M_a} \eta_1^3 + \frac{k_{14}}{g} \eta_2^1 + \frac{k_{16}}{g} \eta_2^2 + \frac{k_{18}}{gL_b} \eta_2^3 \\ -\frac{k_{23}}{g} \eta_1^1 - \frac{k_{25}}{g} \eta_1^2 - \frac{k_{27}}{g M_a} \eta_1^3 + \frac{k_{24}}{g} \eta_2^1 + \frac{k_{26}}{g} \eta_2^2 + \frac{k_{28}}{gL_b} \eta_2^3 \end{bmatrix}. \tag{34}$$

Therefore, combining (33) and (34), the disturbance-compensation gains K_{ω_1} , $K_{\omega_1^{(1)}}$, $K_{\omega_1^{(2)}}$ and $K_{\omega_1^{(3)}}$ of (19)-(22) can be obtained.

By considering the control inputs (33), the closed-loop dynamics of the equivalent system (25)-(32) is governed by

$$\dot{\bar{x}}_1 = (A_1 + B_1 K_{x1}) \bar{x}_1 + \bar{\omega}_1, \tag{35}$$

where $\bar{\omega}_1 = [-e_1^1 \ -e_2^1 \ -\frac{1}{g}(\dot{e}_1^1 - e_1^2) \ \frac{1}{g}(\dot{e}_2^1 - e_2^2) \ -\frac{1}{g}(\dot{e}_1^2 - e_1^3) \ \frac{1}{g}(\dot{e}_2^2 - e_2^3) - \frac{1}{g M_a}(\dot{e}_1^3 - e_1^4) \ -\frac{1}{gL_b}(\dot{e}_2^3 - e_2^4)]^T$. e_i^j is the estimation error of the FTDOs, which will converge to zero in finite time. Therefore, the disturbance $\bar{\omega}_1$ will converge to zero in finite time too.

It will be shown that the bounded disturbance $\bar{\omega}_1$ will not drive the state variables $\bar{x}_1$ to infinity in finite time.

Define a finite-time bounded (FTB) function [30] as

$$V(\bar{x}_1) = \frac{1}{2} \bar{x}_1^T \bar{x}_1. \tag{36}$$

Taking the derivative of $V(\bar{x}_1)$ along the dynamics of (35), we can derive

$$\begin{aligned}
\dot{V}(\bar{x}_1) &= \bar{x}_1^T \dot{\bar{x}}_1 \\
&= \bar{x}_1^T ((A_1 + B_1 K_{x1}) \bar{x}_1 + \bar{\omega}_1) \\
&= \bar{x}_1^T (A_1 + B_1 K_{x1}) \bar{x}_1 + \bar{x}_1^T \bar{\omega}_1 \\
&\leq \lambda_{max}(A_1 + B_1 K_{x1}) \bar{x}_1^T \bar{x}_1 + (\frac{1}{2} \bar{x}_1^T \bar{x}_1 + \frac{1}{2} \bar{\omega}_1^T \bar{\omega}_1) \\
&\leq [\lambda_{max}(A_1 + B_1 K_{x1}) + \frac{1}{2}] \bar{x}_1^T \bar{x}_1 + \frac{1}{2} \bar{\omega}_1^T \bar{\omega}_1 \\
&\leq K_v V(\bar{x}_1) + L_v,
\end{aligned} \tag{37}$$

where $K_v = 2\lambda_{max}(A_1 + B_1 K_{x1}) + 1$, $L_v = \frac{1}{2} \bar{\omega}_1^T \bar{\omega}_1$, and $\lambda_{max}(A_1 + B_1 K_{x1})$ denotes the maximum eigenvalue of matrix $A_1 + B_1 K_{x1}$.

Therefore, we can know that the FTB function $V(\bar{x}_1)$, and so the state variables $\bar{x}_1$ will not tend to infinity in any finite time.

Next, we will prove that the state variables $\bar{x}_1$ will tend to zero asymptotically.

Since the disturbance $\bar{\omega}_1$ will tend to zero in a finite time $t > t_f$, the dynamics of (35) will reduce to

$$\dot{\bar{x}}_1 = (A_1 + B_1 K_{x1}) \bar{x}_1. \tag{38}$$

The matrix $A_1 + B_1 K_{x1}$ is a Hurwitz matrix, and therefore the state variables $\bar{x}_1$ will converge to zero asymptotically, which means that the system output y_1 will converge to the desired set-point asymptotically and the state variables x_1 will remain bounded.

The stability analysis of vertical-heading subsystem Σ_2 is similar to that of horizontal subsystem Σ_1 , so it is omitted here for space.

5. Simulation results

To validate the effectiveness of the proposed novel DCG control method for the unmanned helicopter system, some simulation results are presented in this section.

What needs to be particularly noted is that any feedback control technique that can stabilize the helicopter system (16) and (17) in the absence of disturbances can be employed for the system. In this paper, the linear-quadratic-regulator (LQR) method is adopted for the unmanned helicopter. Furthermore,
 280 in order to demonstrate the superiority of the proposed novel DCG method, both the conventional DCG method [25] and classical LQR method are employed as comparative methods.

The conventional DCG control law of the horizontal subsystem is designed
 285 as

$$u_1 = K_{x1}x_1 + K_{\omega_1}\hat{\omega}_1, \quad (39)$$

and the conventional DCG control law of the vertical-heading subsystem is the same as (23).

The classical LQR control laws of the horizontal subsystem and vertical-heading subsystem are designed as

$$u_1 = K_{x1}x_1, \quad (40)$$

290 and

$$u_2 = K_{x2}x_2, \quad (41)$$

respectively.

The controller parameters are selected as follows. The coefficients of the FTDOs are $\lambda_1^0 = \lambda_2^0 = 5$, $\lambda_1^1 = \lambda_2^1 = 3$, $\lambda_1^2 = \lambda_2^2 = 3$, $\lambda_1^3 = \lambda_2^3 = 3$, $\lambda_1^4 = \lambda_2^4 = 5$, $L_1 = L_2 = 10$, $\lambda_3^0 = \lambda_3^1 = 1$ and $L_3 = 3$. The feedback control
 295 gain and mismatched-disturbance-compensation gains of the horizontal subsystem are

$$K_{x1} = \begin{bmatrix} 3.0445 & 0.0571 & -7.4166 & 0.2018 & -0.3625 & 0.0241 \\ 0.0632 & -2.9777 & -0.0589 & -5.5381 & 0.0112 & -0.1337 \\ -7.4726 & -0.3689 \\ -0.3689 & -6.5958 \end{bmatrix}, K_{\omega_1} = \begin{bmatrix} 0.7583 & 0.0206 \\ 0.0060 & -0.5662 \end{bmatrix}, K_{\omega_1^{(1)}} = \begin{bmatrix} 0.0768 & 0.0025 \\ -0.0011 & -0.0534 \end{bmatrix},$$

$$K_{\omega_1^{(2)}} = \begin{bmatrix} 0.0034 & 0.0001 \\ -0.0002 & -0.0014 \end{bmatrix} \text{ and } K_{\omega_1^{(3)}} = \begin{bmatrix} 0.0001497 & 0 \\ 0 & -0.0000686 \end{bmatrix}. \text{ The}$$

feedback control gain and matched-disturbance-compensation gain of the vertical-
 heading subsystem are $K_{x2} = \begin{bmatrix} 3.1068 & 0.0134 & -0.0058 \\ 0.0175 & -3.1622 & -1.0044 \end{bmatrix}$ and $K_{\omega_2} = \begin{bmatrix} 0.0996 \\ 0 \end{bmatrix}$.
 The control gain of the outer-loop is $K_P = \text{diag}(-1.5, -1.2, -1.5)$.

In the simulation process, the ‘ $1 - \cos()$ ’ style disturbances are injected to x , y and z directions of the body-fixed frame with the amplitude of $8m/s$, $8m/s$ and $4m/s$, respectively [10, 31].

In order to validate the control performance of the proposed novel DCG control method comprehensively, two kinds of flight missions are simulated here.

5.1. Hovering flight simulation

To verify the hovering performance of the unmanned helicopter, a hovering flight simulation is conducted here.

The initial position of unmanned helicopter is set as $P_{n0} = [0 \ 0 \ -5]^T m$, and all the other state variables are set to zero. The control objective of the hovering flight simulation is to stabilize the unmanned helicopter to the position of $P_{nr} = [1 \ 1 \ -6]^T m$ despite the presence of high-order mismatched disturbances.

The simulation results of the hovering flight are shown in Figures 3-5. Figure 3 depicts the response curves of the position. We can observe that all the position of x , y and z can be stabilized at P_{nr} accurately with the proposed novel DCG controller. The position of x and y can almost be stabilized at the set-point under the conventional DCG method with some small amplitude fluctuations. The position of z can keep at the set-point accurately for that the vertical dynamics is only subject to matched disturbance, which can be compensated by conventional DCG method. However, the position of x , y and z with the classical LQR method suffer from fluctuations severely, which implies that the unmanned helicopter cannot hover in the sky steadily. Figure 4 shows the response curves of Euler angles. It can be seen that all the Euler angles under the three control methods are changed in the safety ranges. Furthermore, the response curves of the control inputs are illustrated in Figure 5. All the control input signals are within reasonable ranges.

Figure 3: Response curves of position in hovering simulation

Figure 4: Response curves of Euler angle in hovering simulation

Figure 5: Response curves of control inputs in hovering simulation

5.2. Maneuver flight simulation

330 In order to verify the tracking performance of the unmanned helicopter in multiple operating conditions that cover a wide range of flight envelope, a maneuver flight simulation is conducted here.

A desired reference trajectory is specially designed by:

$$\begin{aligned}
 x_r &= \begin{cases} 0 & t \leq 4s, \\ 3t - 12 & 4 < t \leq 8s, \\ 12 & 8 < t \leq 12s, \\ -3t + 48 & 12 < t \leq 16s, \\ 0 & t > 16s, \end{cases} \\
 y_r &= \begin{cases} 3t & t \leq 4s, \\ 12 & 4 < t \leq 8s, \\ -3t + 36 & 8 < t \leq 12s, \\ 0 & t > 12s, \end{cases} \\
 z_r &= -5 & t > 0s, \\
 \psi_r &= 0 & t > 0s.
 \end{aligned} \tag{42}$$

In this maneuver flight simulation, the helicopter is required to track a square
 335 trajectory in the horizontal direction, and to keep in a certain altitude all the
 time. Furthermore, the heading of the helicopter will remain constant through-
 out the maneuver flight simulation.

The simulation results of the maneuver flight are shown in Figures 6-10. The
 response curves of the position are illustrated in Figure 6, which demonstrates
 340 that the tracking performance of the proposed novel DCG method is superior
 to the conventional DCG method and classical LQR method. Specifically, the
 overshoots and settling times of the proposed novel DCG method are smaller
 than that of the other two methods. To show the tracking performance more
 explicitly, the response curves of the tracking errors are given in Figure 7. It
 345 can be seen that the tracking errors of the proposed novel DCG method are
 smaller than the other two methods of comparison. Furthermore, the root-
 mean-square (RMS) of the tracking errors are given in Table 1. The RMSs
 with novel DCG method are smaller than another two comparative methods,
 which means the novel DCG method is able to achieve more accurate tracking
 350 performance. Furthermore, the $x - y$ plane of the maneuver flight simulation
 is depicted in Figure 8, which verifies that the proposed novel DCG method

Figure 6: Response curves of position in maneuver simulation

outperforms the other two methods in the horizontal direction intuitively. The response curves of the Euler angle are shown in Figure 9. We can obtain that the Euler angle of the proposed novel DCG method is a little smoother than the other two methods. Figure 10 presents the response curves of the control inputs, which shows that all the control input signals are in the reasonable ranges.

Table 1: The root-mean-square (RMS) of tracking errors

Methods	$(e_x)_{RMS}$	$(e_y)_{RMS}$	$(e_z)_{RMS}$	$(e_\psi)_{RMS}$
Novel DCG method	0.2644m	0.3170m	0.0065m	0.0147rad
Conventional DCG method	0.7852m	0.9145m	0.0169m	0.0174rad
Classical LQR method	1.1953m	1.1863m	0.0342m	0.0175rad

Figure 7: Response curves of tracking error in maneuver simulation

Figure 8: x-y plane in maneuver simulation

Figure 9: Response curves of Euler angle in maneuver simulation

Figure 10: Response curves of control inputs in maneuver simulation

5.3. Discussion

This section presents the simulation results and comparative analyses among the proposed novel DCG, conventional DCG [25] and classical LQR methods for the small-scale unmanned helicopter system in the presence of high-order mismatched disturbances. From the simulation results, we can obtain that the proposed novel DCG method is able to remove the influence of the high-order mismatched disturbances from the outputs of helicopter system. Furthermore, through comparative analyses, we can acquire that the control performance of the proposed DCG method is superior to conventional DCG and classical LQR methods for its better disturbance rejection capability, more accurate tracking performance and milder transient process.

6. Conclusion

In this paper, a robust control strategy based on novel DCG approach for the small-scale unmanned helicopter system with high-order mismatched disturbances is investigated. The proposed flight control scheme consists of two hierarchical layers. The function of the inner-loop controller is to guarantee the stability of the unmanned helicopter system in the presence of high-order mismatched disturbances. The role of the outer-loop controller is to generate flight commands for inner-loop and track the reference trajectory. Finally, some simulation results demonstrate that the proposed flight control scheme exhibits excellent control performance. Future works include the experiment tests for the proposed DCG method on experimental helicopter platform and extending the DCG method to general systems.

Acknowledgements

This work is supported by the Natural Science Foundation of China under grant 61703118, and the Fundamental Research Funds for the Central Universities under grant JUS-RP11744.

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