Modeling Phase Interactions in the Dual-Interleaved Buck Converter Using Sampler Decomposition

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Abstract—The averaged small-signal model of the dual-interleaved buck converter is extended to include the phase interaction effects that arise from the interleaved sampling of the phase currents. Sampler decomposition techniques are used to extend the averaged model, revealing a slow-scale instability that can place significant restrictions on the choice of controller parameters. The model is confirmed by simulations and measurements using a 60kW dual-interleaved prototype with Inter-Phase Transformer (IPT), however the analysis is equally applicable to interleaved converters without magnetic coupling.

Index Terms—Average current, control, DC-DC converters, interleaved converters, modelling, small-signal.

I. INTRODUCTION

INTERLEAVING in DC-DC converters is a well-established technique to increase input and output ripple frequencies, reduce passive component size and spread the thermal load. This technique is used in many applications ranging from low voltage power supplies to the high power converters within an electric vehicle power train [1]–[4]. Individual average phase current feedback is often used to balance the phase-currents in interleaved systems [5]–[8], but can result in phase interaction and instability, which is not predicted by standard average-value modelling techniques. The instabilities can be at quite low frequencies, around 1 kHz, and can place significant limitation on the parameters for the phase-current controllers. One approach to analyse these effects is through the extension of the standard average-value model using sampler decomposition techniques [9]. This approach, which was generalized for converters with two or more phases and demonstrated for a dual-interleaved boost converter, has the advantage of being more straightforward than the highly detailed sampled-data methods. This papers builds on the work in [9] to show that the modelling technique proposed for the dual-interleaved boost converter can also be applied successfully to the interleaved buck converter in continuous conduction mode, revealing a slow-scale instability that restricts the choice of control parameters. The analysis is presented for a converter with interphase transformer, but is equally applicable to uncoupled converters where similar slow-scale instabilities can arise.

II. SMALL-SIGNAL MODELLING OF INTERLEAVED CONVERTERS WITH AVERAGE-CURRENT MODE CONTROL

A. Small-signal averaged model of the dual-interleaved buck converter

Fig. 1 shows the dual-interleaved buck converter with Inter-phase transformer (IPT). Assuming continuous conduction operation, then by substitution of the converter switch networks with the averaged PWM switch model [10], the averaged DC and small-signal model shown in Fig. 2 is obtained, where the IPT has been modelled using the windings’ self-inductances \(L_1\) and \(L_2\) and mutual inductance \(L_m\) [11]. The upper case variables in Fig. 2 denote steady-state components whilst the lower case variables are the small-signal components. \(D_1\) and \(D_2\) are the duty-ratios of \(Q_1\) and \(Q_2\). Neglecting the DC components of Fig. 2 and under the assumption that the components comprising the converter phases are identical, \(L_1 = L_2 = L_c\), the small-signal variations...
of the converter phase-currents $\tilde{i}_1(s)$ and $\tilde{i}_2(s)$ can be expressed in terms of the control inputs $\tilde{d}_1(s)$ and $\tilde{d}_2(s)$ as:

$$\tilde{i}_1(s) = G_{d_1}(s)\tilde{d}_1(s) + G_{d_2}(s)\tilde{d}_2(s)$$  (1)

$$\tilde{i}_2(s) = G_{d_1}(s)\tilde{d}_1(s) + G_{d_2}(s)\tilde{d}_2(s)$$  (2)

where $G_{d_1}(s)$ and $G_{d_2}(s)$ are the converter duty ratio-to-phase current and the converter duty ratio-to-opposite phase current transfer functions which are defined in the Appendix.

**B. Small-signal model of the converter with digital average current feedback control**

The closed-loop, small-signal model of the dual-interleaved buck converter with digital average current mode control is shown in Fig. 3. The sampling of the average phase current is represented by the samplers, $S$. Given the symmetry of the converter waveforms in continuous conduction mode, the average phase current can be obtained by sampling once in the middle of the corresponding transistor conduction interval at any point of operation [9, 12, 13]. The phase-shifted operation of the phase-2 sampler with respect to the phase-1 sampler, $S$, is modelled by means of the time delay and advance units, $e^{-\tau \phi}$ and $e^{-\tau \theta}$ according to the sampler decomposition method [9]. The controllers are represented by $C(z)$, and the computational delay of the control algorithms is modelled by $e^{-\tau \phi}$ where $\tau = t_{comp}/T$ and $t_{comp}$ and $T$ are the computational delay and the converter switching period respectively. The digital PWM operation is modelled using the zero-order-hold extrapolator transfer function $G_{OH}(s) = (1-e^{-sT})/s$. The closed-loop reference-to-phase current transfer functions of this system can be found to be:

$$G_{i_1}(z) = \frac{i_1(z)}{\tilde{i}_1(z)} = \frac{C(z)G_{d_1}(z) + C(z)G_{d_2}(z) + C^2(z)G_{d_1}(z) - G_{d_2}(z)G_{d_2}(z)}{1 + 2C(z)G_{d_1}(z) + C^2(z)G_{d_1}(z) - G_{d_2}(z)G_{d_2}(z)}$$  (3)

$$G_{i_2}(z) = \frac{i_2(z)}{\tilde{i}_2(z)} = \frac{C(z)G_{d_1}(z) + C(z)G_{d_2}(z) + C^2(z)G_{d_1}(z) - G_{d_2}(z)G_{d_2}(z)}{1 + 2C(z)G_{d_1}(z) + C^2(z)G_{d_1}(z) - G_{d_2}(z)G_{d_2}(z)}$$  (4)

where the $z$-domain transfer functions $G_{d_1}(z)$, $G_{d_2}(z)$ and $G_{d_2}(z)$ are obtained using the modified $z$-transform to account for the fractional time-delay:

$$G_{d_1}(z) = Z_m\{G_{d_1}(s)G_{d_1}(s)\}_{s^{-1}/(T\phi)}$$  (5)

$$G_{d_2}(z) = Z_m\{G_{d_2}(s)G_{d_2}(s)\}_{s^{-1}/(T\phi)}$$  (6)
The closed-loop reference-to-phase current transfer functions may also be derived assuming non-delayed operation of the samplers in the control-loops, which is the conventional modelling approach [5, 7, 8], resulting in identical transfer functions for each phase since the phases are assumed to have the same component values:

\[
    \begin{align*}
    G_{\text{ref}} (z) &= z Z_n \left\{ G_{\text{in}} (s) G_{\text{m}} (s) \right\} \bigg|_{s \rightarrow (z \tau / T)} \\
    \end{align*}
\]  

(7)

The closed-loop reference-to-phase current transfer functions may also be derived assuming non-delayed operation of the samplers in the control-loops, which is the conventional modelling approach [5, 7, 8], resulting in identical transfer functions for each phase since the phases are assumed to have the same component values:

\[
    G_{\text{ref}} (z) = G_{\text{ref}_{2\text{est}}} (z) = \frac{C(z) \left[ G_{\text{in}} (z) - G_{\text{m}} (z) \right]}{1 + C(z) \left[ G_{\text{in}} (z) - G_{\text{m}} (z) \right]} 
\]  

(8)

where \( G_{\text{m}} (z) = Z_n \left\{ G_{\text{in}} (s) G_{\text{m}} (s) \right\} \bigg|_{s \rightarrow (z \tau / T)} \).

III. COMPARISON BETWEEN THE INTERLEAVED AND THE CONVENTIONAL MODELS

A. Performance of the phase current to step changes

The set of unit step responses of the phase current, Fig. 4, is used to illustrate the difference between the predictions of the interleaved model and the conventional model. The first column of Fig. 4 shows SABER simulation results using a switched model that includes the interleaved sampling of the phase currents in the digital controller. The sample times and sample period (13.33 µs) were identical in the model and simulations. The second and third columns show the transfer-function predictions from the interleaved, (3), and the conventional small-signal model (third column).

Fig. 4. Time-domain response of the phase-1 current to small step-increments in the reference input obtained using: the SABER switched model (first column), the interleaved small-signal model (second column) and the conventional small-signal model (third column).

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>CONVERTER COMPONENTS AND PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
<td>Symbol</td>
</tr>
<tr>
<td>Output inductor</td>
<td>( L )</td>
</tr>
<tr>
<td>Output inductor stray resistance</td>
<td>( R_L )</td>
</tr>
<tr>
<td>IPT self-inductance</td>
<td>( L_1, L_2 )</td>
</tr>
<tr>
<td>IPT mutual inductance</td>
<td>( L_m )</td>
</tr>
<tr>
<td>IPT coupling coefficient</td>
<td>( k )</td>
</tr>
<tr>
<td>Output capacitance</td>
<td>( C_o )</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>( f )</td>
</tr>
<tr>
<td>Switching/sampling period</td>
<td>( T )</td>
</tr>
<tr>
<td>Computational delay</td>
<td>( \tau )</td>
</tr>
</tbody>
</table>
**IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS**

**TABLE II**  
COMPARISON OF POLES AND ZEROS FROM THE CLOSED-LOOP, REFERENCE-TO-PHASE CURRENT TRANSFER FUNCTIONS*

<table>
<thead>
<tr>
<th></th>
<th>Conventional</th>
<th>Interleaved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poles</td>
<td>$G_{i1}(z)$ &amp; $G_{i2}(z)$</td>
<td>$G_{i1}(z)$ &amp; $G_{i2}(z)$</td>
</tr>
<tr>
<td></td>
<td>$0.189\pm0.864j$ (16.2 kHz)</td>
<td>$0.180\pm0.872j$ (16.4 kHz)</td>
</tr>
<tr>
<td></td>
<td>$0.945$</td>
<td>$0.991\pm0.095j$</td>
</tr>
<tr>
<td></td>
<td>$0.943$</td>
<td>$0.7369$</td>
</tr>
<tr>
<td></td>
<td>$0.138$</td>
<td>$0.5$</td>
</tr>
<tr>
<td></td>
<td>$0.0510$</td>
<td>$-0.028$</td>
</tr>
</tbody>
</table>

*Point of operation: $V_{in} = 400$ V, $R_{load} = 1.8$ Ω with PI controller gains $K_p = 50(T)$ and $K_i = 50$.

The results from the transfer functions show a close correspondence with the simulation results with virtually-identical rise time, natural frequency (16.66 kHz) and damping ratio. However a lower slightly damped natural frequency (ranging from 1 kHz to 1.6 kHz) is evident in many of the responses from the interleaved model, but is completely absent in the conventional model results. The same natural frequency is also observable in the SABER results. Additionally, the high-frequency oscillations present in the simulation results were attributed to PWM quantization and current-sampling effects.

**B. Pole-zero locations of the system and stability**

To illustrate the difference between the conventional and interleaved model transfer functions, the values of the poles and zeros predicted by both models are compared in Table II. The converter parameters used to obtain these results are listed in Table I.

![Comparison of the stability-range predicted by the conventional/non-interleaved model and the interleaved when a digital PI compensator is used to regulate the current-feedback control-loops at (a) $V_{in} = 400$ V, $R_{load} = 1.8$ Ω; (b) $V_{in} = 700$ V, $R_{load} = 2.7$ Ω.](image)

It can be observed that the conventional and the interleaved models have four poles situated in similar locations: two real poles at $\approx0.94$ and $\approx0.1$, and a pair of high-frequency complex poles at $\approx0.18\pm0.87j$ ($\approx16$ kHz). The high-frequency oscillations observed in the step responses are attributed to the latter. Furthermore, the interleaved model contains an additional pair of complex poles, $0.993\pm0.097j$, which are almost cancelled by a pair of complex zeroes present in both the reference-to-phase current transfer functions. These poles are responsible for the low-frequency oscillations observed in the transient responses ($\approx1.1$ kHz) and become unstable when the controller gain is chosen to be at least two times larger than the proportional gain.

Finally, the $K_p / K_i$ controller design spaces shown in Fig. 5 are used to illustrate the difference in the stability range predicted by the interleaved model and the conventional model. The dark shaded areas indicate the stable combinations of $K_p$ and $K_i$ predicted by the interleaved model whilst the lighter shaded areas are the additional regions where

---

conventional, (8), models respectively. SABER and small-signal models exclude all losses except $R_L$, the series resistance of the output inductor. The converter parameters are listed in Table I. PI compensators were used to regulate the phase currents. In the first row in Fig. 4, the PI integral gain, $K_i$, is varied from 10 to 100, and in the second row the input voltage is varied from 100 V to 700 V.

The results from the transfer functions show a close correspondence with the simulation results with virtually-identical rise time, natural frequency (16.66 kHz) and damping ratio. However a lower slightly damped natural frequency (ranging from 1 kHz to 1.6 kHz) is evident in many of the responses from the interleaved model, but is completely absent in the conventional model results. The same natural frequency is also observable in the SABER results. The additional high-frequency oscillations that occur in the simulation results were attributed to PWM quantization and current-sampling effects.

![Comparison of the stability-range predicted by the conventional/non-interleaved model and the interleaved when a digital PI compensator is used to regulate the current-feedback control-loops at (a) $V_{in} = 400$ V, $R_{load} = 1.8$ Ω; (b) $V_{in} = 700$ V, $R_{load} = 2.7$ Ω.](image)
the conventional model suggests that the system operation will be stable. These regions were generated numerically by calculation of the system poles over a systematic sweep of the controller parameters. The patterns are similar to those found in [9] for the dual-interleaved boost converter, and are consistent with the fact that the conventional model over predicts the system stability limits.

IV. EXPERIMENTAL VALIDATION

The experiments were undertaken using a 60 kW, 75 kHz SiC MOSFET-based dual-interleaved converter with input and output voltages up to 700 V and 350 V respectively, Fig. 6. The semiconductor modules used for the prototype power stage are CAS300M12BM2 from Wolfspeed (1200V@300A). The prototype passive component values are listed in Table I.

Two single-sample, average current-mode control-loops were implemented on a Texas Instruments TMS320F28377 digital signal controller to regulate the converter phase-currents, [14]. The sampling instants of each control-loop are strategically positioned in the middle of the transistor on-state intervals to acquire the phase current average value. The controller gains were selected as a compromise between rise-time, overshoot percentage and settling-time, the values used were 0.5 ms, <5% and 2 ms respectively.

To verify the accuracy of the model, the measured response of the converter phase-1 current to a 15 A step-change in the reference was compared to that obtained from switched simulations and the interleaved model, Fig. 7. The waveforms in the top plot, correspond to the measured phase-current and its instantaneous moving average value, whilst the waveforms in the bottom plot are from the SABER simulation and the interleaved model. Fig. 7(a) shows the phase-current response when \( V_{in} = 400 \text{ V}, \ R_{load} = 1.8 \text{ } \Omega, \ K_p = 50(T), K_i = 50 \) and Fig. 7(b) corresponds to \( V_{in} = 700 \text{ V}, \ R_{load} = 2.7 \text{ } \Omega, \ K_p = 50(T), K_i = 30 \). These results show that the model is able to predict the phase current behavior correctly.
were used, later at 
and 
showed that a low-frequency natural mode is present in the 
verified experimentally and by simulation. The analysis 
emphasized current control. The predictions of the enhanced model were 
has been shown to be applicable to the dual-interleaved buck 
oscillations appear in both phase currents with the same 
instability observed in the phase current. The unstable 
oscillations appear in both phase currents with the same 
magnitude, but are out of phase, therefore they are not 
observable in the converter output current. The converter 
output voltage was also stable. This suggests phase-current 
estimation algorithms using a single current sensor, [15]–[17], 
might not be suitable for interleaved converters with this form 
of phase current control as they may not detect these phase 
current instabilities.

V. CONCLUSION
Enhanced averaged modelling using sampler decomposition has been shown to be applicable to the dual-interleaved buck 
converter with inter-phase transformer using digital average-
current control. The predictions of the enhanced model were 
verified experimentally and by simulation. The analysis 
showed that a low-frequency natural mode is present in the 
system that is not predicted by standard average-value models. 
The natural mode is attributed to the interaction between the 
phases and can result in low frequency oscillations in the 
phase currents that are unobservable in the converter input and 
output currents. \( K_p \) / \( K_i \) controller design-space plots were 
generated to aid in the visualization of the stability of the 
system and with the PI parameter selection. These plots are 
similar to those presented for the interleaved boost converter 
in that the conventional model over predicts the stability range of the system [9].

Finally, this modelling technique can be further applied to 
converters with more than two phases by appropriately 
modelling the phase-delayed sampling of the individual 
current control loops using the time delay and time advance 
units, \( e^{\text{time}} \) and \( e^{\text{time}} \), where \( N \) is the total number of phases and \( n = 1 \ldots N \) the phase index.

APPENDIX
The duty ratio-to-phase current transfer function is defined as:
\[
G_{d1i}(s) = G_{d2i}(s) = G_{di}(s) = \frac{(L + L_i)V_{in}}{L_{tot}} \left( s^2 + a_d s + a_{d0} \right),
\]
where:
\[
L_{tot} = 2L(L_i + L_m) + (L_i^2 - L_m^2),
\]
\[
a_{d1} = \frac{(L + L_i)L_{tot} + C_i(2L + L_m - L_n)(L_i + L_m)R_mR_{load}}{(L + L_i)C_iL_{tot}R_{load}},
\]
\[
a_{d0} = \frac{(2L + L_m - L_n)(L_i + L_m)(R_m + R_{load})}{(L + L_i)C_iL_{tot}R_{load}},
\]
\[
b_1 = \frac{2L(R_m + 2L_mR_m)}{L_{tot}C_iR_{load}},
\]
\[
b_2 = \frac{2(L_i + L_m)(R_m + R_{load})}{L_{tot}C_iR_{load}}.
\]

The duty ratio-to-opposite phase current transfer function is defined as:
\[
G_{d1s}(s) = G_{d2s}(s) = G_{ds}(s) = \frac{(L_m - L)V_{in}}{L_{tot}} \left( s^2 - a_{ds} s + a_{ds0} \right),
\]
where:
\[
a_{ds0} = \frac{(2L + L_m - L_n)(L_i + L_m)(R_m + R_{load})}{(L_m - L)C_iL_{tot}R_{load}},
\]
\[
a_{ds1} = \frac{(L - L_m)L_{tot} + C_i(2L + L_m - L_n)(L_i + L_m)R_mR_{load}}{(L_m - L)C_iL_{tot}R_{load}}.
\]
REFERENCES


