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DOI:
10.1016/j.jfranklin.2018.07.007

Document Version
Accepted author manuscript

Link to publication record in Manchester Research Explorer

Citation for published version (APA):

Published in:
Journal of the Franklin Institute

Citing this paper
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Modeling and switching control of air-breathing hypersonic vehicle with variable geometry inlet

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Abstract
In this paper, a multi-model switching control is developed for air-breathing hypersonic vehicle with variable geometry inlet (AHV-VGI). A variable geometry inlet with the translating cowl is adopted to capture the enough air mass flow for the scramjet engine, which can ensure a more powerful thrust. However, the using of VGI causes the unknown changes of the aerodynamics and thrust, making the model of AHV more complex. Therefore, we firstly analyze the thrust characteristic with the translating cowl and present the conception of optimal elongation distance of translating cowl (EDTC). Consequently, multiple different nonlinear aerodynamic models are constructed by curve fitting for each position of the translating cowl. Then, a switching mechanism dependent on EDTC is proposed and the adaptive RBF neural controllers are designed for velocity subsystem and altitude system of every model. Furthermore, the common Lyapunov functional is constructed to prove the stability of the multi-model switching process. Finally, numerical simulations are given to demonstrate the effectiveness of the proposed control approach for AHV-VGI.

Keywords: hypersonic vehicle, variable geometry inlet, multi model switching control, neural network

1. Introduction
Air-breathing hypersonic vehicle (AHV) may offer a reliable and more cost-efficient way to access space [1][2]. Due to the use of scramjet engine, quick response and global attack become possible[3]. In practice, hypersonic aircraft always fly within a wide flight envelope (range of flight conditions). However, for the AHV with fixed geometry inlet (AHV-FGI), once it is running at a low Mach, the shockwave would deviate away from the scramjet lip. This will lead to the scramjet engine could not get sufficient air stream, so that the thrust will be generated insufficiently. In order to solve the above issue, AHV with variable-geometry scramjet inlet (AHV-VGI) are popularly studied. For example, NASA investigates a rotary lip VGI for the X-43A hypersonic aircraft[4]. The Space and Astronautical Science institution of Japan developed a variable geometry axisymmetric inlet for the ATREX engine[5]. Besides, the Russian scholar Kuranov investigated the Magneto Hydrodynamic controlled inlet[6].

In recent years, investigators have carried on many researches on VGI characteristic of AHV. In Ref.[7], a new methodology using gas dynamic relations has been developed to obtain optimal geometry of scramjet inlet at different Mach numbers. Ref.[8] compares the properties of three kinds of VGI using the low-order control-oriented model, and designs a kind of inlet used in a wide range of Mach numbers. Ref.[9] designs a 2-D hypersonic VGI with movable lip along the flow direction, carries out three dimensional CFD numerical simulations. Comparison of the aerodynamic characteristics was made between VGI and FGI. Ref.[10] presents a design method of high-performance VGI, and obtains the adjusting rules and performance variation of VGI in various conditions.

Although there are many of researches on the configurature of AHV-VGI, the control system designed for AHV-VGI are investigated in the initial phase. The main reason is the complex structure of VGI system which is difficult to control well. A VGI scheme was proposed by [11][12]. By moving the translating cowl along the flow direction, the internal contraction ratio could be enlarged, and the propulsion efficiency could be improved. This VGI scheme is easy to operate, which only need to adjust the translating cowl to ensure that the shock wave can project right on the inlet lip. Thus, it provides a feasible idea for us to study the control design for AHV-VGI.

In the past decades, lots of efforts have also been put into flight control of AHV. The robust control of AHV is studied by introducing high gain observers to compensate the system uncertainties and additive disturbances, which improves the tracking control performance effectively[13]. To overcome the problem of system uncertainty, fuzzy logic system [14][15] and neural network [16] are employed due to their powerful ability of approximation for the smooth nonlinearities. Ref.[17] proposed a data-driven supplementary control approach with adaptive learning capability for AHV tracking control, which is suitable for hypersonic vehicle system with parameter uncertainties and disturbances. Recently, transient performance-based control design has become an important method for the research of uncertain nonlinear systems[18][19]. Ref.[20] proposed a novel estimation-free prescribed performance non-affine con-
control strategy for AHV to guarantee tracking error is limited to a predefined arbitrarily small residual set. In [21], a novel multivariable finite-time control algorithm is proposed and applied in attitude control of reusable launch vehicle for the first time. The developed method demonstrates better robustness, disturbance rejection and higher precision comparing with the PID control, and shows huge potentiality for application on complex systems subject to uncertainties and external disturbances [22][23][22][24]. Besides, Various techniques have been applied to hypersonic vehicles to deal with parametric uncertainties or bounded uncertainties and unmodeled system dynamics, such as the sliding model controller[25][26] and disturbance observer based control[27], the \( l_1 \) adaptive controller[28] and intelligent control algorithm[29], etc.

However, in the context of the aforementioned literature, AHV-FGI model was adopted for control design. Typically, the aerodynamic characteristics of AHV-FGI can be described by a mathematical model, while the AHV-VGA cannot be described by a single model due to highly nonlinear aerodynamic characteristics and uncertainties. For the control problem of AHV-VGI, Ref.[30] proposed a type-2 fuzzy sliding model control to estimate the uncertain influences induced by VGI with movable cowl. Ref.[31] presented a fuzzy disturbance observer-based control to reduce the effects on uncertain influences induced by VGI and disturbances. However, the above-mentioned methods need continue to estimate the uncertainties and nonlinear items by adaptive fuzzy method, which will cause computational complexity. For this reason, This paper will adopt the multi-model switching control (MMSC) method to design the controller for the AHV-VGI. The MMSC theory is an effective method to deal with the complex nonlinear systems. Therefore, the MMSC is also widely adopted to the morphing aircraft control. In Ref.[32], a switched polytopic model of variable swept wing aircraft was obtained and the stability analysis was accomplished by Lyapunov functional. However, the polytopic model is a kind of linear parameter varying model which can not fully reflect the nonlinear aerodynamic characteristics of AHV. Ref.[33] presented a novel adaptive mode switching scheme for hypersonic morphing aircraft. However, the nonlinear model of the hypersonic aircraft should be linearized by input/output at first. This means we need to know the AHV model accurately. The difficulty of designing a multi-nonlinear-model switching control system is the stability proof. Ref.[34] has proved that the existence of a common Lyapunov functional is the sufficient condition for the switching system to be asymptotic stability. The common Lyapunov functional approach is generally studied for switched linear systems [35] [36], however, there are few results on stability for switched nonlinear systems. In [37], an adaptive fuzzy attitude tracking control scheme based on switched nonlinear system is presented for a variable structure near space vehicle. According to the common Lyapunov functional theory, it is proved that the proposed controller can guarantee the uniform ultimate boundedness of all signals of the closed-loop system.

In this paper, we investigate the modeling and control for AHV-VGI with translating cowl. By adjusting the translating cowl, the AHV-VGI can capture more air mass flow, which will be favorable to extend the flight velocity range. However, the adjust of cowl will induce the unknown changes of the aerodynamic forces and moment, especially, the thrust will be changed greatly. These changes may be so large that cannot be regarded as small disturbances[38]. Therefore, the paper will establish multi models based on the translating cowl. And then, a multi-model switching controller is designed for the longitudinal dynamics of AHV-VGI to provide a stable tracking of velocity and altitude reference trajectories. This paper is organized as follows: Section 2 analyzes the effects of the translating cowl to the thrust, and constructs a longitudinal model of AHV-VGI. In Section 3, a switching mechanism of MMSC based on elongation distance of translating cowl, and control design is divided into two parts, and velocity controller and altitude controller are designed using a direct neural controller based on backstepping method. In Section 4, the stability of each subsystem and the switching process has been proved via Lyapunov theory. Some simulation results are presented in Section 5 to show its performance. The conclusion of the paper is drawn in Section 6.

2. Model of AHV-VGI With Translating Cowl

2.1. The model of AHV-VGI with translating cowl

As shown in Fig. 1, the AHV-VGI has a scramjet inlet with translating cowl which can move along the flow direction.

![Fig. 1. Model of AHV with translating cowl](image)

By applying the momentum theorem of fluid mechanics, the thrust generated by the scramjet engine is calculated as:

\[
T = \bar{m}_a(v_e - v_\infty) + (p_e - p_\infty)A_e - (p_1 - p_\infty)A_1
\]  

(1)

Where \( \bar{m}_a \) is the air mass flow captured by engine. \( A_1, p_1 \) are the area and pressure of the engine inlet entrance respectively. \( A_e, p_e \) and \( v_e \) are the area, pressure and velocity of the nozzle exit respectively. \( v_\infty \) and \( p_\infty \) are the velocity and pressure of
the freestream. As we can see in (1), the thrust generated by the scramjet engine has a relationship with the air mass flow $\dot{m}_a$, which is calculated by the engine. $\dot{m}_a$ is calculated as:

$$\dot{m}_a = p_\infty M_\infty \sqrt{\frac{\gamma}{RT_\infty}} D$$  \hspace{1cm} (2)

According to the flight condition, the capture area $D$ can be divided into two cases to solve.

2.1.1. Case one (fixed geometry inlet)

When the AHV works in a cruise condition, which is a small angle of attack (AOA) is small, the oblique shock wave will occur. As shown in Fig. 3, the blue dotted line is the oblique shock wave, $\theta_s$ is the shock wave angle. The red solid line is the free stream which hit against the oblique shock wave and then turn parallel to the lower forebody, $\delta_s$ is the flow turn angle. $\delta_s = \alpha + \tau_{1\ell}$, the shock wave angle $\theta_s$ is the middle root of the following shock angle polynomial:

$$\sin^6 \theta_s + b \sin^4 \theta_s + c \sin^2 \theta_s + d = 0 \hspace{1cm} (3)$$

where

$$b = \frac{M_{\infty}^2 - 2}{M_{\infty}^2} - 6 \sin^2(\delta_s)$$
$$c = \frac{2 M_{\infty}^2 + 1}{M_{\infty}^2} + \left[ \frac{(\lambda + 1)^2}{4} + \frac{\lambda - 1}{M_{\infty}^2} \right] \sin^2(\delta_s)$$
$$d = -\frac{\cos^2(\delta_s)}{M_{\infty}^2} \hspace{1cm} (4)$$

The scramjet can capture all the mass flow as long as the oblique shock wave can be sealed by the scramjet cowl. The capture area $D$ can be calculated as:

$$D = \frac{L_f \sin(\tau_{1\ell} + \alpha)}{\cos(\tau_{1\ell})} + h_i \cos(\alpha) \hspace{1cm} (5)$$

where the lower forebody angle $\tau_{1\ell}$ is 6.2deg. The engine inlet height $h_i$ is 3.5ft, the forebody $L_f = 47 ft$.

2.1.2. Case two (variable geometry inlet)

However, according to the shock expansion wave theory, when the AHV works in a low mach condition, the shockwave angle $\theta_s$ will be increased. Therefore, as shown in Fig. 4, if the cowl of the scramjet is fixed, the shock won’t be sealed by the cowl, which will cause the flow spillage (area $D_2$) and the engine will not get mass flow sufficiently ($D_1$) is the actual capture area. However, if the cowl can be adjusted to the position $\psi$ (shown in Fig. 4), the oblique shock wave can be sealed by the cowl again. Consequently, the inlet of scramjet will capture all mass flow of the area $D = D_1 + D_2$. The capture area can be calculated as

$$D_1 = \frac{h_i \sin(\theta_s) \cos(\tau_{1\ell})}{\sin(\theta_s - \alpha - \tau_{1\ell})} \hspace{1cm} D_2 = \frac{h_i + L_f \tan(\tau_{1\ell}) \sin(\theta_s)}{\sin(\theta_s - \alpha)} \hspace{1cm} (6)$$

The proper distance of the $\psi$ is a function of the AOA $\alpha$ and shockwave angle $\theta_s$:

$$\psi = L_f - (L_f \tan(\tau_{1\ell}) + h_i) \cot(\theta_s - \alpha) \hspace{1cm} (7)$$

Definition 1: The optimal elongation distance of translating cowl $\psi_e$ is the position where the inlet of scramjet can capture all mass flow of area $D$, which can be obtained from (7).

![Fig. 4. Engine capture the freestream partially](image)

Obviously, the captured mass flow in AHV-VGI with translating cowl will be increased. As shown in Fig. 5, for a given freestream velocity, air mass flow reach extreme at angle of attack 5deg in AHV-FGI, while the air mass flow increases continuously with angle of attack in AHV-VGI. The thrust of AHV is changed with the elongation distance of translation cowl (EDTC) $\psi_e$, which is shown in Fig. 6. By using the translating cowl, we can dramatically adjust the amount of air mass flow captured by inlet. It is helpful to reach a more powerful thrust. Besides, the aerodynamic forces and moments of the nacelle bottom will be influenced and ultimately the pitching moment of AHV will be changed. The above conclusions are detailed in [38].
2.2. Mathematical model of AHV with translating cowl

AHV with different translating distance cowl has different aerodynamic model. The dynamic model considered in this paper is developed by Bolender and Doman for the longitudinal dynamics of AHV [11]. The nonlinear motion equations are written as follow:

\[ \dot{V} = (T \cos \alpha - D)/m - g \sin \gamma \]  
(8)

\[ \dot{h} = V \sin \gamma \]  
(9)

\[ \dot{\gamma} = (T \sin \alpha + L)/(mV) - g \cos \gamma /V \]  
(10)

\[ \dot{\alpha} = Q - \gamma \]  
(11)

\[ \dot{Q} = M/I_{yy} \]  
(12)

where \( V, h, \gamma, \alpha \) and \( Q \) represent the velocity, the altitude, the flight-path angle (FPA), the angle of attack (AOA) and the pitch rate, respectively. \( I_{yy} \) and \( M \) denote the moment of inertia and the vehicle mass, respectively. Thrust \( T \), drag \( D \), lift \( L \) and pitching moment \( M \) are formulated as follow

\[ L \approx q_s C_l^k \]  
\[ D \approx q_s C_D^k \]  
\[ T \approx q (C_{T,\phi} \delta \phi + C_{T,\theta} \delta \theta) \]  
\[ M \approx q_s C_M^k + Z_f T \]  
(13)

where, \( C_l^k, C_D^k, C_M^k \) and \( C_{T,\phi} \) are the aerodynamic lift coefficients, drag coefficients, pitching moment coefficients and thrust coefficients. \( k = [0, 1, 2, \ldots, n] \) denotes the real position of EDTC \( \psi \); \( q = 1/2 \rho V^2 \) denotes the dynamic pressure. \( \rho, s, c, Z_f \) are the air density, reference area, the aerodynamic chord, and thrust moment arm respectively. The control inputs \( \delta \phi \) and \( \delta \theta \) are the elevator and the control surface deflection, respectively. The expressions of aerodynamic coefficients are given in (14). The detailed values of coefficients are obtained by curve fitted approximations, which are shown in [39].

\[ C_L^k = C_{L,a}^k \cdot \alpha + C_{L,Ma}^k \cdot Ma + C_{L,\delta \phi}^k \cdot \delta \phi + C_{L,\delta \theta}^k \]  
\[ C_D^k = C_{D,a}^k \cdot \alpha + C_{D,\delta \phi}^k \cdot \alpha^2 + C_{D,\delta \theta}^k \cdot \alpha \cdot \delta \phi + C_{D,\delta \phi}^k \cdot \delta \phi + C_{D,\delta \theta}^k \cdot \delta \theta \]  
\[ C_M^k = C_{M,a}^k \cdot \alpha + C_{M,\delta \phi}^k \cdot Ma + C_{M,\delta \phi}^k \cdot \delta \phi + C_{M,\delta \theta}^k \]  
\[ C_{T,\phi} = C_{T,\delta \phi}^k \cdot \alpha + C_{T,\delta \phi}^k \cdot Ma + C_{T,\delta \phi}^k \]  
\[ C_{T,\theta} = C_{T,\delta \theta}^k \cdot \alpha + C_{T,\delta \theta}^k \cdot Ma + C_{T,\delta \theta}^k \]  
(14)

3. Controller design

3.1. The regulation mode of translating cowl

In this paper, for improving the aerodynamic performance of AHV in low mach condition, the optimal translating distance of scramjet cowl varies with the flight speed continuously. However, continuous adjustment of the translating cowl may lead to the aerodynamic model of AHV become extremely complicated and difficult to control. Thus, we assume the cowl translates step by step, omit the transition time. The step length is 1 ft. As shown in Fig. 7, the critical mach and attack angle of each translating distance has been calculated. The area above the curve is the operating region for the AHV with translating cowl in different position.

![Fig. 7. Mach and attack angle of each each translating distance](image)

3.2. The switching mechanism of MMSC

The structure of multi-model switching control (MMSC) is shown in Fig. 8. The system includes three parts: model of AHV-VGI, model of AHV-FGI, and model of AHV-VGI. For the model of AHV-VGI, the real EDTC \( \psi \) can be measured. According to the every model with \( \psi \), NN controller is designed online. And multi controllers will be switched online by the switching rules. In this paper, we define \( n \) fixed position for the \( \psi_k = \psi(kt) \), \( k = 0, 1, 2, \ldots, n \). At every \( \psi_k \) position, the corresponding aerodynamic coefficients in (14) are obtained by curve fitted approximations.

Eq. (7) is used to theoretically calculate the optimal EDTC. However, actually we have to estimate the value of the optimal elongation distance by curve fitted approximations. From (3) and (7), we know that the \( \psi \) is a function of Mach number \( Ma \) and AOA \( \alpha \). The expression of fitting elongation distance \( \hat{\psi} \) is shown in (15).
where the detailed parameters value are shown in the Table. 1.

Assumption 1: The translating cowl can be infinitely fast adjusted to the desired elongation distance with the switching command (adjusting time is approximated to zero).

The switching mechanism is defined as follows: Assume the switching threshold \( \Delta \psi \) (in the paper, we define \( \Delta \psi = 0.5 \text{ft} \), and if \( \hat{\psi}_o \geq \psi_o + \Delta \psi \), generating the switching signal as adjust backward the cowl by 1 step, to \( \psi_{k-1} \); if \( \hat{\psi}_o \leq \psi_o - \Delta \psi \), generating the switching signal as adjust backward the cowl by 1 step, to \( \psi_{k-1} \). Then the MMSC system will switch to the corresponding aerodynamic model and controller in the meanwhile. For each subsystem, a direct neural controller will be designed.

Table 1: Fitting coefficients in elongation distance

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Values</th>
<th>Coefficients</th>
<th>Values</th>
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<tbody>
<tr>
<td>( C_f^o )</td>
<td>-0.2804</td>
<td>( C_f^q )</td>
<td>0.0281</td>
</tr>
<tr>
<td>( C_f^Mao )</td>
<td>-3.151</td>
<td>( C_f^q )</td>
<td>31.611</td>
</tr>
</tbody>
</table>

3.3. Radial basis function neural network (RBFNN)

In the paper, the RBFNN is adopted directly to design controller for AHV-VGI. The form of RBFNN is described as:

\[
y = W^T \xi(x)
\]

where \( x \in \mathbb{R}^n \) and \( y \in \mathbb{R} \) are the input vector and output vector respectively. \( W \in \mathbb{R}^m \) denotes the weight vector, \( n \) and \( m \) are the respective input number and node number. \( \xi(x) = [\xi_j(x)]^T \in \mathbb{R}^m \), and the \( \xi_j(x) \) is chosen as the commonly used Gaussian function, which has the form

\[
\xi_j(x) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{\|x - c_j\|^2}{2\sigma_j^2}\right), \quad j = 1, 2, \ldots, m
\]

where \( c_j \) and \( \rho_j \) are the center and the width of the basis function, respectively.

It has been demonstrated that the RBFNN can be employed to approximate any smooth nonlinear functions within arbitrary accuracy. Given a continuous function \( F(\cdot) \), there exists an ideal weight vector \( W^* \) such that \( W^{*T} \xi(x) \) can approximate the given function \( F(\cdot) \), and the approximate expression is

\[
F(x) = W^{*T} \xi(x) + \mu, \quad |\mu| \leq \mu_M
\]

where \( \mu \) and \( \mu_M \) denote the approximation error and its upper bound, respectively.

3.4. Model formulation

From (9)-(12), the velocity is mainly related to the FER, and the rate of altitude is controlled by the elevator deflection. So the dynamic can be decoupled into two subsystems: velocity subsystem and altitude subsystem. Given the tracking reference \( V_d \) and \( h_d \), we will design the velocity and altitude controller by neural network, separately. The whole control system is shown in Fig. 9. The velocity subsystem (9) can be rewritten as follows:

\[
\begin{cases}
V = f_V^k + g_V^k u_V \\
u_V = \phi
\end{cases}
\]

with

\[
\begin{align*}
f_V^k &= \bar{q}(C_{lD}^k \cos \alpha - S C_{lD}^k)/m - g \sin(\gamma) \\
g_V^k &= \bar{q} C_{lD}^k \cos \alpha/m
\end{align*}
\]

Eq. (10) shows that the altitude \( h \) and FPA \( \gamma \) have a one to one relationship, so we transform the altitude instruction \( h_d \) into FPA instruction \( \gamma_d \):

\[
\gamma_d = \arcsin\left[-k_h(h - h_d)/V\right]
\]

where \( e_h = h - h_d, k_h > 0 \) is the parameter to be designed, then the strict feedback equations of the altitude subsystem (10)-(12) are written as:

\[
\begin{cases}
\dot{y} = f_A^k + g_A^k \cdot \alpha \\
\dot{a} = f_A^k + g_A^k \cdot Q \\
Q = f_A^k + g_A^k \cdot u_A \\
u_A = \delta_e
\end{cases}
\]
where

\[ f'_y = \hat{q} \left[ (C_{Ld} \phi + C_{L} \delta_e) \sin \alpha \right] / (mV) \]
\[ + \hat{q} \left[ \left( C_{L,Ma} \alpha + C_{L} \delta_e \right) \sin \alpha + SC_{L\omega} \right] / (mV) + g \cos \gamma / V \]
\[ f'_{\alpha} = \hat{q} \left[ \left( C_{\alpha \delta} \delta_e + C_{\alpha} \right) \sin \alpha + \alpha C_{\alpha \delta} \right] / (mV) + g \cos \gamma / V \]
\[ f'_\phi = \hat{q} \left[ \left( C_{\phi \delta} \delta_e + C_{\phi} \right) \sin \alpha + \phi C_{\phi \delta} \right] / (mV) + g \cos \gamma / V \]
\[ g' = \hat{q} C_{L,Ma} / (mV) \]
\[ g'_{\alpha} = 1 \]
\[ g'_{\phi} = \hat{q} \left[ \alpha C_{\phi \delta} \sin \alpha + \phi C_{\phi \delta} \right] / (mV) \]
\[ - \hat{q} S \left[ \alpha C_{\phi \delta} \sin \alpha + \phi C_{\phi \delta} \right] / (mV) + g \cos \gamma / V \]

\[ \gamma \]

\[ \bar{\gamma} \]

\[ I_{yy} \]

\[ \bar{I}_{yy} \]

\[ S \]

\[ \sigma \]

\[ \alpha \]

\[ \mu \]

\[ \nu \]

\[ \xi \]

\[ \tau \]

\[ \gamma \]

\[ \bar{\gamma} \]

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\[ Q'_a = -\frac{1}{g_{\alpha}^a}(f^\alpha_k + u_a) - \left(\frac{1}{\epsilon_a g_{\alpha}^a} + \frac{1}{\epsilon_a (g_{\alpha}^a)^2} - \frac{g_{\alpha}^k}{2(g_{\alpha}^a)^2}\right)S_a \]  

(38)

Here, the RBF NN is employed to approximate \( Q^* \) directly

\[ Q'_a = W_{a}^T \xi_0(x_a) + \mu_a, |\mu_a| \leq \mu_{OM} \]  

(39)

where \( x_a = [V, \alpha, S_a, S_0/\xi_0], \mu_a \) and \( \mu_{OM} \) are the approximation error and its upper bound, respectively. Let \( \dot{W}_a \) be the estimate of \( W_a^* \). We obtain the following direct neural controller

\[ Q_d = \dot{W}_a^T \xi_0(x_a) \]  

(40)

with the following adaptive law

\[ \dot{W}_a = -\Gamma_1 [\xi_0(x_a) S_a + \sigma_a \dot{W}_a] \]  

(41)

where \( \Gamma_1 = \Gamma_1^T > 0 \) is an adaption gain matrix and \( \sigma_a \) is a positive constant.

3.6.3. Pitch-rate subsystem

Define the pitch-rate tracking error \( e_Q = Q - Q_d \) and error function \( S_Q \) as follow:

\[ S_Q = \left(\frac{d}{dt} + \lambda_Q\right) \int_0^t e_Q dt \]  

(42)

where \( \lambda_Q > 0 \) is a design parameter. The time derivative along (42) is formulated as

\[ S_Q = 0 + \lambda_Q e_Q - g_0^Q \phi + f_0^Q + v_Q \]  

(43)

where \( v_Q = \lambda_Q e_Q - \dot{Q}_d \). The desired ideal controller is designed as below

\[ \delta_e = -\frac{1}{g_0^Q}(f_0^Q + v_Q) - \left(\frac{1}{\epsilon_Q^b (g_0^Q)^2} - \frac{g_0^k}{2(g_0^Q)^2}\right)S_Q \]  

(44)

Here, the RBF NN is employed to approximate \( \delta_e \) directly

\[ \delta_e = W_{Q}^T \xi_0(x_Q) + \mu, |\mu| \leq \mu_{QOM} \]  

(45)

where \( x_Q = [V, \alpha, S_a, S_0/\xi_0], \mu \) and \( \mu_{QOM} \) are the approximation error and its upper bound, respectively. Let \( \dot{W}_Q \) be the estimate of \( W_Q^* \). We obtain the following direct neural controller

\[ \delta_e = \dot{W}_Q^T \xi_0(x_Q) \]  

(46)

with the following adaptive law

\[ \dot{W}_Q = -\Gamma_Q [\xi_0(x_Q) S_Q + \sigma_Q \dot{W}_Q] \]  

(47)

where \( \Gamma_Q = \Gamma_Q^T > 0 \) is an adoption gain matrix and \( \sigma_Q \) is a positive constant.

4. Stability analysis

The stability of switching process is proved by constructed common Lyapunov functional for velocity subsystem, flight-path angle subsystem, angle of attack subsystem and pitch-rate subsystem.

4.1. Velocity subsystem

Consider the following common Lyapunov candidate

\[ L_V = \frac{1}{2g_{\nu}} S_{\nu} + \frac{\tilde{W}_V^T \Gamma^{-1}_V \tilde{W}_V}{2} \]  

(48)

where \( \tilde{W}_V = \tilde{W}_V - \tilde{V}_V \). Substituting controller (28) into (25), we have

\[ S_V = g_0^b \tilde{W}_V^T \xi_0(x_V) + f_0^b + u_V + g_0^b \phi^* - g m \phi^* \]

\[ = -g_0^b \left( \frac{1}{g_0^b}(f_0^b + u_V) + \left(\frac{1}{\epsilon_0 V g_0^b} + \frac{1}{\epsilon_0 (g_0^b)^2} - \frac{g_0^k}{2(g_0^b)^2}\right)S_V \right) \]

\[ - g_0^b \tilde{W}_V^T \xi_0(x_V) + f_0^b + u_V - g_0^b \left[ \tilde{W}_V^T \xi_0(x_V) + \epsilon_0 \right] \]

\[ = g_0^b \tilde{W}_V^T \xi_0(x_V) - \left(\frac{1}{\epsilon_0 V g_0^b} + \frac{1}{\epsilon_0 (g_0^b)^2} - \frac{g_0^k}{2(g_0^b)^2}\right)g_0^b S_V - g_0^b \mu V \]  

(49)

Combining (29) and (49), the time derivate along (48) is described as

\[ L_V = \frac{1}{2g_{\nu}} S_{\nu} - \frac{g_0^b}{2(g_0^b)^2} S_{\nu} + \frac{\tilde{W}_V^T \Gamma^{-1}_V \tilde{W}_V}{2} \]

\[ = -\left(\frac{1}{\epsilon_0 V g_0^b} + \frac{1}{\epsilon_0 (g_0^b)^2}\right)S_{\nu} - \sigma_V \tilde{W}_V^T \tilde{W}_V - \mu V S_{\nu} \]  

(50)

Noting that

\[ 2 \tilde{W}_V^T \tilde{W}_V = \tilde{W}_V^T (W_V + W_V^T) + (W_V - W_V)^T \tilde{W}_V \]

\[ = \| \tilde{W}_V \|^2 + \| \tilde{W}_V \| - \| W_V \|^2 \geq \| \tilde{W}_V \|^2 - \| W_V \|^2 \]  

(51)

\[ | -\mu V S_{\nu} | \leq \frac{S_{\nu}^2}{2\epsilon_0 V g_0^b} + \frac{\epsilon_0 \mu^2 \tilde{W}_V}{2} \leq \frac{S_{\nu}^2}{2\epsilon_0 V g_0^b} + \frac{\epsilon_0 \mu^2 \tilde{W}_V}{2} \]  

(52)

and considering that \( |\mu V| \leq \mu_{QOM} \), we obtain

\[ L_V \leq -\frac{S_{\nu}^2}{2\epsilon_0 V g_0^b} + \frac{\epsilon_0 \mu^2 \tilde{W}_V}{2} + \frac{\epsilon_0 \sigma V^2}{2} \| \tilde{W}_V \|^2 \]  

(53)

Choosing the largest eigenvalue of \( \Gamma^{-1}_V \) as \( \gamma_\nu \), we have

\[ \tilde{W}_V \Gamma^{-1}_V \tilde{W}_V \leq \gamma_\nu \| \tilde{W}_V \|^2 \]. Then,

\[ L_V \leq -\frac{S_{\nu}^2}{2\epsilon_0 V g_0^b} + \frac{\epsilon_0 \mu^2 \tilde{W}_V}{2} + \frac{\epsilon_0 \sigma V^2}{2} \| \tilde{W}_V \|^2 \]  

(54)

where \( \kappa_V = \max(\epsilon_0, \gamma_\nu/\sigma_\nu) \). According to the Lemma B.5 in [42], the following inequality holds

\[ L_V \leq \left(\frac{\epsilon_0 \mu^2 \tilde{W}_V}{2} + \frac{\epsilon_0 \sigma V^2}{2} \| \tilde{W}_V \|^2 \right) \int_{t_0}^{t} e^{-\kappa_V t} dt + e^{-\kappa_V t} L_V(0) \]

\[ \leq \kappa_V \left(\frac{\epsilon_0 \mu^2 \tilde{W}_V}{2} + \frac{\epsilon_0 \sigma V^2}{2} \| \tilde{W}_V \|^2 \right) + e^{-\kappa_V t} L_V(0), \forall t \geq 0 \]  

(55)

Inequality (55) implies that \( S_V \) and \( \tilde{W}_V \) are bounded since \( L_V(0) \) is bounded. From (48), we have \( L_V \geq \frac{S_{\nu}^2}{2g_{\nu}^2} \). Furthermore, we have \( S_V \leq \sqrt{2g_{\nu}^2} L_V \). Therefore, combining \( \sqrt{a b} \leq \sqrt{a} + \sqrt{b} \) yields

\[ S_V \leq \sqrt{2g_{\nu}^2} \left(\frac{\epsilon_0 \mu^2 \tilde{W}_V}{2} + \frac{\epsilon_0 \sigma V^2}{2} \| \tilde{W}_V \|^2 \right) \frac{1}{\sqrt{2g_{\nu}^2} L_V(0), \forall t \geq 0} \]  

(56)

Thus, \( e_\nu \) can be uniformly ultimately bounded by the continuous controller.
4.2. Altitude subsystem

Substituting controller (34) (40) and (46) into (31) (37) and (43) respectively, yield

\[
\dot{S}_m = \frac{g}{S_m} \dot{\tilde{W}}_m^2 \bar{h}_m(S_m) - \left( \frac{1}{\epsilon m S_m} + \frac{1}{\epsilon(m (g_m)^2) - \frac{g}{2 (S_m)^2}} \right) S_m^2 S_m - \mu_m S_m \tag{57}
\]

where \( \bar{W}_m = \tilde{W}_m - \frac{2}{S_m} W_m \). Choose the following common Lyapunov function candidate

\[
L_m = \frac{1}{2} S_m^2 + \frac{\tilde{W}_m^2}{2} \gamma_m^2 \bar{W}_m - \mu_m S_m \tag{58}
\]

Combining (35) (41) (47), the time derivate along (58) are described as

\[
\dot{L}_m = \frac{1}{g_m} S_m \dot{S}_m - \frac{g}{S_m} \tilde{W}_m^2 \gamma_m^2 \bar{W}_m - \left( \frac{1}{\epsilon m S_m} + \frac{1}{\epsilon (m (g_m)^2) - \frac{g}{2 (S_m)^2}} \right) S_m^2 - \sigma_m \dot{W}_m \tilde{W}_m - \mu_m S_m \tag{59}
\]

Noting that

\[
2 \tilde{W}_m \tilde{W}_m \geq ||\tilde{W}_m||^2 - ||W_m||^2 \geq \frac{S_m^2 - \epsilon_m S_m^2}{2} - \frac{S_m^2}{2} - \mu_m S_m \tag{60}
\]

and considering that \( |\mu_m| \leq \mu_m \), we obtain

\[
L_m = - \frac{S_m^2}{2} - \sigma_m^2 ||W_m||^2 + \frac{\epsilon_m S_m^2 \tilde{W}_m^2}{2} + \frac{\mu_m^2 S_m^2}{2} \tag{61}
\]

Then, let \( \gamma, \gamma_Q \) and \( \bar{W}_Q \) be the largest eigenvalue of \( \Gamma \), \( \Gamma_{\theta} \), and \( \Gamma_Q \) respectively, we have \( \tilde{W}_m \Gamma \bar{W}_m \leq \gamma ||W_m||^2 \). Then,

\[
\dot{L}_m = \frac{1}{\epsilon_m} L_m + \frac{\epsilon_m S_m^2 \tilde{W}_m^2}{2} + \sigma_m^2 ||W_m||^2 \tag{62}
\]

where \( \epsilon_m = \max S_m \gamma_m / \sigma_m \). According to the Lemma B.5 in [42], the following inequality holds

\[
L_m \leq \left( \frac{\epsilon_m S_m^2 \tilde{W}_m^2}{2} + \frac{\epsilon_m S_m^2 \tilde{W}_m^2}{2} \right) \int_0^t e^{-(\epsilon_m + \epsilon_m) L_m(0)} + \epsilon_m^2 ||W_m||^2 \tag{63}
\]

Inequality (64) implies that \( S_m, S_m, S_Q \) and \( \bar{W}_Q(t), \tilde{W}_Q(t) \) are bounded since \( L_m(0), L_m(0), L_m(0) \) are bounded. From (58), we have \( L_m \geq S_m^2/(2 \epsilon_m) \). Furthermore, we get

\[
S_m \leq \sqrt{2 \epsilon_m L_m} \leq \sqrt{2 S_m/(\epsilon_m)} \sqrt{S_m} \tag{65}
\]

Thus, \( \epsilon_m (\gamma_{\theta}, \gamma_Q, \bar{W}_Q) \) can be uniformly ultimately bounded by the continuous controller.

Remark 1: Although the EDTC have a step change suddenly at the switching moment, its influences to aerodynamic and thrust are not big. Thus, the tracking performance will be affected slightly, while the stability at the switching moment will not be changed.

5. Simulation results

The control objective is to track the velocity and altitude commands and to adjust the real EDTC \( \psi_k \) in the meanwhile. All the parameters in simulation are on the basis of Ref.[11]. The system is initialized at V(0) = 6.8Mach = 6626 ft/s, h(0) = 75000 ft, \( \gamma(0) = 0 \deg, \alpha(0) = 0 \deg, Q(0) = 0 \degs \). The reference commands of velocity and altitude is generated by the filter

\[
F_V(s) = \frac{\omega_n \omega_n^2}{(s + \omega_n)(s^2 + 2 \omega_n \omega_n S + \omega_n^2)} \tag{66}
\]

\[
F_h(s) = \frac{\omega_n^3 \omega_n^3}{(s + \omega_n)(s^2 + 2 \omega_n \omega_n S + \omega_n^2)} \tag{67}
\]

where \( \omega_n = \omega_n = 0.1, \omega_n = \omega_n = 0.3, \epsilon_1 = 30, \epsilon_2 = 50 \).

To show the control performances and the advantages using AHV-VGI, we compare the control performances between AHV-VGI by using the proposed control method and AHV-FGI by the sliding control method [43]. The simulations are divided into three parts: The first simulation case is a slow acceleration process, which velocity is from 6.8 to 8 Mach and altitude is from 75000 ft to 76000 ft. The second case: the velocity increment will add up to (6.7-8.5Mach), and altitude is kept the same as case 1. The third case: the velocity increment will continue to add up to (6.8-9Mach). In the latter two cases, the tracking performance with VGI will be better than the performance with FGI.

With the changes of EDTC \( \psi_k \), the aerodynamic model of AHV-VGI will be switched correspondingly. For each submodel, we design corresponding controller with different parameters. For example, the design parameters at \( \psi_k = 5 \) are selected as follow: \( \epsilon_1 = 0.5, \epsilon_2 = 0.5, \epsilon_3 = 0.4, \epsilon_4 = 0.4, \lambda_1 = 0.1, \lambda_2 = 0.2, \lambda_3 = 0.4, \lambda_4 = 0.2, \Gamma_V = diag(10), \gamma_1 = diag(50), \Gamma_{\theta} = diag(100), \gamma_Q = diag(50), \sigma_{\psi} = 0.05, \sigma_\alpha = 0.1, \sigma_\gamma = 0.1, \sigma_\sigma = 0.2, k_h = 12 \). The initial values of \( \bar{W}_V, \bar{W}_Q, \bar{W}_Q \) are chosen as zero. For the other models, the controller’s parameters would have a little different, I shall not enumerate them here.

Case 1: The velocity is changed from 6.8 to 8Mach, and the altitude from 75000-76000 ft. The simulation results are shown in Fig. 10 ~ Fig. 13. Fig. 10 shows the changes of elongation distance of translating cowl during the simulation time. The blue line is the optimal EDTC obtained by (15), and red line the real EDTC obtained by the switching mechanism. The EDTC \( \psi_k \) is gradually decreased from 10 to 6. With the increase of velocity, the shock wave angle decreases, and the oblique shock

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Fig. 10. Elongation distance of translating cowl
wave generated by AHV forebody is assembled to the body, so the translating cowl correspondingly move back. Because the EDTC mainly affects thrust and velocity, the velocity causes some fluctuations within a narrow range at switching points shown in the Fig. 11. However the fluctuations don’t affect the tracking performance. As shown in Fig. 11, both of the AHV-VGI and AHV-FGI control system can fulfill the velocity and altitude tracking performance. And tracking errors remain remarkably small during the entire maneuver, and vanish asymptotically. Fig. 12 show flight-path angle, angle of attack and pitch rate range within their bounds. Fig. 13 show the control inputs of FER and the thrust $T$. Compared the FER curves of AHV-FGI and AHV-VGI, there are obvious difference shown in the Fig. 13. As one of control inputs of AHV system, the FER $\phi$ is the control variable of the thrust $T$. When the velocity command is given, the AHV-VGI can capture more free stream and offer greater thrust due to the existence of movable translating cowl. Therefore, the AHV-VGI can provide the required thrust with a smaller FER, which is obviously beneficial of the fuel saving. Similarly, the FER and thrust also cause some fluctuations within a narrow range at switching points shown in the Fig. 13.

Fig. 11. Velocity and altitude tracking

Fig. 12. Flight-path angle, AOA and pitch rate

Fig. 13. The control inputs of $\phi$ and the thrust

**Case 2:** The velocity is changed from 6.8 to 8.5Mach, which is a faster change process than the simulation case 1. Fig. 14 ~ Fig. 16 show the simulation results. From Fig. 14, it is shown that the AHV-VGI control system can accurately track a fast changing velocity command. In the fast acceleration process, the FER $\phi$ is large in order to provide the required thrust. However, considering $\phi \in [0, 1]$, the FER of the AHV-FGI will reach its saturated state during the time of 8-30s (shown in Fig. 15), which causes the velocity tracking error increased suddenly (shown in Fig. 14) and the response curve of the thrust is distorted (shown in Fig. 15). While the velocity tracking error of
AHV-VGI cause just a little wave at the switching time. Fig. 16 shows the switching process and the changes of EDTC during the simulation time.

**Case 3:** The velocity is changed from 6.5 to 9Mach (the velocity increment reaches to the 2.5Mach at the same time). In this simulation case, both of the systems can not ensure the velocity tracking performance. However, we can learn that the tracking error is smaller using AHV-VGI than the AHV-FGI (shown in Fig. 17). And the time duration of insufficient thrust for the AHV-FGI is longer than the AHV-VGI (shown in Fig. 18), because the control variable FER $\phi$ will keep more time at the saturated state. And during the saturated state of $\phi$, there are no fluctuation effect to the velocity and the thrust. Obviously, the switching process will be more frequent with being wide of the range of velocity.
6. Conclusion

This paper presented a multi-model switching control method for the AHV-VGI with actuating control. Different from the AHV-FGI, the AHV-VGI can avoid the flow spillage and extend the velocity range, which be shown in simulation results. By switching of multi-model, the uncertain changes of aerodynamic and thrust caused by translating cowl can be reduced. The work presented here did not consider the external disturbances. But in fact, the disturbances are unavoidable. Therefore, how to guarantee the smooth switching and choose the switching rules with uncertainties and disturbances can be the focus of future study.

Acknowledgments

This work was partially supported by the Natural Science Foundation of China under Grant 61673294 and Aeronautical Science Foundation of China (201707480003), joint Science Foundation of Ministry of Education of China (6141A02022328).

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