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DOI: 10.1016/j.advwatres.2018.04.011

Document Version
Accepted author manuscript

Link to publication record in Manchester Research Explorer

Citation for published version (APA):

Published in:
Advances in Water Resources

Citing this paper
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Multi-phase SPH model for simulation of erosion and scouring by means of the Shields and Drucker-Prager criteria.

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ABSTRACT
A two-phase numerical model using Smoothed Particle Hydrodynamics (SPH) is developed to model the scouring of two-phase liquid-sediments flows with large deformation. The rheology of sediment scouring due to flows with slow kinematics and high shear forces presents a challenge in terms of spurious numerical fluctuations. This paper bridges the gap between the non-Newtonian and Newtonian flows by proposing a model that combines the yielding, shear and suspension layer mechanics which are needed to predict accurately the local erosion phenomena. A critical bed-mobility condition based on the Shields criterion is imposed to the particles located at the sediment surface. Thus, the onset of the erosion process is independent on the pressure field and eliminates the numerical problem of pressure dependant erosion at the interface. This is combined with the Drucker-Prager yield criterion to predict the onset of yielding of the sediment surface and a concentration suspension model. The multi-phase model has been implemented in the open-source DualSPHysics code accelerated with a graphics processing unit (GPU). The multi-phase model has been compared with 2-D reference numerical models and new experimental data for scour with convergent results. Numerical results for a dry-bed dam break over an erodible bed shows improved agreement with experimental scour and water surface profiles compared to well-known SPH multi-phase models.

1. Introduction

Sediment scouring and fluvial suspension due to liquid flows are common phenomena in many river flows. However, the construction of reservoirs interrupts the natural equilibrium of basins and leads to sedimentation. Siltation of artificial lakes affects water supply systems by reducing the storage capacity, and eventually affects other reservoir functions such as irrigation supply, energy production, navigation and flood control (Schleiss et al., 2016). More than 80% of the world’s total storage capacity of artificial reservoirs belongs to hydropower dams (ICOLD, 2012), in which sedimentation may also cause damages to turbines by abrasion or occlusion, and loss of power production.
The prediction of erosion rates is a key issue for siltation management and long-term operation plans of reservoirs, dams and other hydraulic structures. Flows involving sedimentation are characterised by a shear layer of soil particles under large relative motions at the interface between the sediment and the fluid. Numerical modelling enables prediction of the changes in the sediment balance upstream and downstream of a hydraulic structure, permitting different scenarios to be investigated quickly with relatively low cost, as well as supporting field solutions.

However, the presence of two phases, the combination of interfacial and free-surface flows, in addition to particle entrainment of the sediment by the fluid phase are the main challenges of traditional mesh-based methods to simulate erosion and sedimentation processes (Liu, 2003). In recent years, the alternative option of meshless methods has emerged (Gingold and Monaghan, 1977; Koshizuka and Oka, 1996; Oñate and Idelsohn, 1998). Of all the meshless methods now available, smoothed particle hydrodynamics (SPH) has unique advantages when modelling problems involving large deformation of interfacial and free surface flows or large sediment motions and mixing that occur during soil erosion.

SPH is a Lagrangian method that was initially developed to study astrophysical problems (Gingold and Monaghan, 1977; Lucy, 1977) and has been extended to a wide range of engineering applications. Geotechnical and geoenvironmental applications include flood and river dynamics (Prakash et al., 2014; Vacondio et al., 2012; Williams et al., 2016), soil mechanics (Bui et al., 2011, 2008; Chen and Qiu, 2014; Reyes et al., 2013), simulation of landslides (Manenti et al., 2016; Pastor et al., 2009; Ran et al., 2015; Tan and Chen, 2017; Xenakis et al., 2017) seeage problems (Bui and Fukagawa, 2013; Maeda et al., 2006) and sediment transport (Falappi et al., 2007; Fourtakas and Rogers, 2016; Manenti et al., 2012; Shi et al., 2017; Ulrich et al., 2013).

The simulation of sediment scour using SPH has developed steadily over the last decade. Falappi et al. (2007) simulated sediment scour in a reservoir flushing experiment using a pseudo-Newtonian approach based on Mohr-Coulomb parameters. Guandalini et al. (2012) used the Shields’ erosion criterion to predict scouring and in subsequent work Manenti et al. (2012) compared the Mohr-Coulomb pseudo-Newtonian approach with the Shields’ erosion criterion with experimental data. The results of Manenti et al. (2012) showed that the eroded volume and the profile evolution were better reproduced by the Shields’ criterion. However, the methodology treats the non-eroded sediment particles at subsurface layers as a fixed boundary and prevents the Shields’ approach from being able to simulate the rheology of the material at subsurface layers.

Ulrich et al. (2013) also used a yield criterion based on Mohr-Coulomb parameters to simulate ship-induced scouring near ports. The model combines a linear-elastic approach with a pseudo-Newtonian model for the un-yielded and yielded material, respectively. The model was supplemented by a suspension layer based on the Chezy-relation to account for the viscosity transition between the sediment and the water. Nevertheless, in order to avoid singularities on the viscosity value the pseudo-Newtonian treatment requires the definition of a maximum value for viscosity which may be case dependent.

Fourtakas and Rogers (2016), used the Drucker-Prager yield criterion which is also based on Mohr-Coulomb parameters, to simulate the rheology of sediment induced by rapid flows. The model uses a
pseudo-Newtonian approach based on the Bingham-type Herschel-Bulkley-Papanastasiou (HBP) model that allows simulating the rheology of the un-yielded and the yielded material without needing to define a maximum value for viscosity. The Vand equation (Vand, 1948) is used to account for sediment suspension at the surface layer. The model showed good results for problems involving impact of rapid flows in the bed of sediment.

However, numerical tests have shown spurious pressure fields in the sediment at areas of large deformation as, for example, near the wave fronts. At these large shear regions, deformation occurs as the kinematics of the liquid phase is sufficient to yield the sediment phase which exhibits slow kinematics and high shear forces. This discontinuity at the interface is known to cause the erroneous pressure due to the shear forces discontinuity which requires a second gradient summation for calculating the shear rates (Fatehi and Manzari, 2011). These slow kinematics exhibit by the sediment phase under high shear rate have been addressed in SPH more generally using diffusion-based techniques including the XSPH approach of Monaghan (1992) with an additional smoothing (Manenti et al., 2012; Ulrich et al., 2013) or by using the particle shifting algorithm of Lind et al. (2012), as employed in the work of Fourtakas and Rogers (2016). These techniques improve the particle distribution and avoid particle clustering which allows for a better sampled kernel. Nevertheless the issue is not entirely eliminated.

The aforementioned models use the so-called weakly compressible SPH (WCSPH) schemes where pressure is related to density through an equation of state. Alternatively, truly incompressible SPH (ISPH) formulations that couple the governing equations have also been applied to sediment transport problems (Ran et al., 2015). Comparisons of WCSPH and ISPH for free-surface flow have shown that improved velocity and pressure fields are obtained with the ISPH method (Lee et al., 2010, 2008; Xu et al., 2009).

Incompressible SPH formulations can be an alternative to improve the pressure fields which are essential to better reproduce the dynamics and erosion patterns of the sediment phase. In the work of Ran et al. (2015) an ISPH scheme and an interface erosion criterion which is based on the fluid and grain properties (similar to Shields’ criterion), is used to simulate erosion and entrainment of the sediment grain by the water. Non-eroded particles are treated as fixed boundaries, like in the model presented by Manenti et al. (2012). Satisfactory results with respect to experimental profiles were obtained by Ran et al. (2015). However, WCSPH formulations present some advantages over ISPH schemes. In terms of computational efficiency, ISPH generally produces reduced pressure fluctuations in the pressure field and larger time steps are possible. WCSPH schemes demand smaller time steps to keep density variations within acceptable levels, but on the other hand the formulation avoids solving the Poisson equation which is computationally expensive and numerical solutions are obtained by explicit algorithms.

In this work, a WCSPH scheme is used to simulate erosion and scouring of a sediment bed. A critical bedmobility condition based on the Shields criterion is imposed to the particles located at the sediment surface. Thus, the onset of the erosion process is independent on the pressure field and eliminates the problem of pressure dependant erosion at the interface.

The Mohr-Coulomb parameters refer to the mechanical properties of the soil. Thus, the yield criteria based on Mohr-Coulomb parameters are suitable to simulate the rheology of the subsurface domain of sediment. However, erosion at the fluid-sediment interface is dominated by the flux characteristics and physical
properties of the sediment (e.g. particle diameter and specific weight). The onset of the motion at the surface layer of a bed of sediment may be accurately modelled by the Shield’s parameter, as shown by Manenti et al. (2012) and Ghaitanellis et al. (2017). This paper is intended to bridge the gap between these mechanical and physical approaches by proposing a model that aims to predict erosion rates at the bed surface and scour profiles due to the effect of the water on the sediment. The two-phase WCSPH model of Fourtakas and Rogers (2016), applicable to rapidly varying flows, is complemented by the Shields’ theory to deal with the suspension of sediment under low velocity flow. These two criteria have been combined in order to maximise the applicability of the model to both the entrainment of soil by water, as occurs in fluvial environments, and the impact of rapidly varying flows in a bed of sediment.

The computational demands make the problem ideal for SPH development on graphics processing units (GPUs) to enable simulations with a large number of particles required for real engineering problems. The open source DualSPHysics solver (Crespo et al., 2015) that is accelerated by GPUs and extended by Fourtakas and Rogers (2016) for sediments has been used to implement the computational algorithm.

The paper is structured as follows: in Section 2, the computational model is presented. A detailed description of the equations for the water and sediment phases, including the erosion criteria and the suspension treatment is given. Numerical results are presented in Section 3. First, the implementation of the Shields’ criterion is compared to numerical and experimental results. Then, a flume experiment over an erodible bed of non-cohesive sediment is presented and the total amount of bed material transported by the wave is compared with the experimental results. Finally, the combined Drucker-Prager and Shields’ criteria approach is tested against experimental results of a dam break over erodible beds. Conclusions are presented in Section 4.

2. COMPUTATIONAL MODEL

The physical domain of the simulation may be represented by five regions, as shown in Figure 1. Region (1) represents the un-yielded sediment, which remains almost static due to its large apparent viscosities; region (2) consists of sediment with an apparent viscosity small enough to allow particle movement (according to a non-Newtonian flow model see Section 2.4.4.); region (3) represents the sediment that reached the critical Shields’ parameter and simulates bed load transport; region (4) is composed of particles suspended in the fluid with a variable viscosity; and region (5) represents only water particles. The description of how these regions are treated using SPH is presented in the following sections.
2.1. Governing equations and SPH formalism

In this Section we briefly describe the general governing equations and the SPH modelling technique for the fluid and sediment. Vectors and tensors are defined by reference to Cartesian coordinates using Greek superscripts, $\alpha$, $\beta$ and $\gamma$ to denote Cartesian tensor coordinate directions and Latin subscripts, $i$ and $j$, to identify particle locations. Einstein’s summation is employed over repeated Greek superscripts.

The governing equations of fluids are described by the Navier-Stokes equations.Written in Lagrangian form the conservation of mass (continuity) and momentum equations read

$$\frac{d \rho}{dt} + \rho \frac{\partial u^\alpha}{\partial x^\alpha} = 0,$$  \hspace{1cm} (1)

and

$$\frac{du^\alpha}{dt} = \frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta} + g^\alpha,$$ \hspace{1cm} (2)

respectively, where $\rho$ is the density, $u^\alpha$ is the velocity, $\sigma^{\alpha\beta}$ is the total stress tensor and $g^\alpha$ is the gravitational acceleration.

The total stress tensor $\sigma^{\alpha\beta}$ comprises of two parts, the isotropic pressure ($p$) and the viscous stresses $\tau^{\alpha\beta}$

$$\sigma^{\alpha\beta} = p \delta^{\alpha\beta} + \tau^{\alpha\beta}.$$ \hspace{1cm} (3)

where $\delta^{\alpha\beta}$ is Kronecker’s delta function, which equals 1 for $\alpha = \beta$ and 0 otherwise.

In order to relate pressure with density the fluid is considered to be weakly compressible and Tait’s equation of state is used to close the system of equations (Batchelor, 1967), i.e.
\[ p_i = \frac{c_i^2 \rho_i}{\gamma} \left[ \left( \frac{\rho_i}{\rho_0} \right)^\gamma - 1 \right], \]  

(4)

where \( p_i \) and \( \rho_i \) are the pressure and density of the particle, \( \gamma \) is the polytrophic index, set to 7 in this work, as suggested by Monaghan (2000), \( \rho_0 \) is the reference density and \( c_i \) is the numerical speed of sound that should be set to a value that guarantees small oscillations on pressure. Following Monaghan (1994), \( c_i \) is related to the maximum velocity magnitude in the domain, \( u_{\text{max}} \), as

\[ c_i \geq 10 u_{\text{max}} \]  

(5)

In this work, equations (1) to (5) are used for all the phases. However, the viscous part of the stress tensor in Equation (3) is modelled, depending on the phase (water or sediment) via a constitutive model and will be discussed in Sections 2.4 and 2.5.

2.2. General SPH approximation

SPH is a meshless particle method where particles represent the computational media and are assigned material properties. The method is based on the integral representation of a function \( f(x) \) over a domain \( \Omega \) at a point \( x \), by the follow identity:

\[ f(x) = \int_\Omega f(x') \delta(x - x') \, dx' \]  

(6)

where \( \delta(x - x') \) represents the Dirac delta function.

The continuous approximation of equation (6) is obtained by replacing the Dirac delta function by a normalized weighting or kernel function \( W_{ij} \). Then, the particle approximation is obtained by means of a summation over points inside the influence domain of the kernel function:

\[ < f(x_i) >= \sum_{j=1}^{N} f(x_i) W_{ij} V_j \]  

(7)

where the subscript \( i \) refers to the interpolating particle and \( j \) refers to neighbouring particles, \( V_j = m_j \rho_j \) is the discrete volume of the particle \( j \) and \( N \) is the number of particles within the influence domain of the kernel function. A more detailed description of the SPH formulation can be found in Violeau (2012).

The kernel function \( W_{ij} = W(x_i - x_j, h) \) is a smooth, even and isotropic function that has a finite radius (compact support) around \( x_i \). The smoothing length \( (h) \) defines the size of the support domain of the function. Following Fourtakas and Rogers (2016) the fifth-order Wendland kernel with compact support of \( 2h \) is used in the present study (Wendland, 1995)

\[ W(R, h) = a_5 \left( 1 - \frac{R}{2} \right)^4 \left( 2R + 1 \right), \]  

(8)
with \( R = |x_i - x_j| / h \) and the normalization constant \( a_d \) equals to \( 7 / 4 h \pi^2 \) and \( 21 / 16 h \pi^3 \) in 2-D and 3-D space, respectively.

### 2.3. SPH Discretisation of the Governing Equations

The transient evolution of the particle density and momentum within an SPH approximation is given by

\[
\frac{d \rho_i}{dt} = \sum_{j=1}^{N} \frac{m_j}{\rho_j} \frac{\partial W_{ij}}{\partial \rho}, \quad (9)
\]

and

\[
\frac{du_i^\alpha}{dt} = \sum_{j=1}^{N} \frac{m_j}{\rho_j} \left[ \sigma_{ij}^{\alpha\beta} + \sigma_{ij}^{\alpha\gamma} \right] \frac{\partial W_{ij}}{\partial x_i^\beta} + g_i^\alpha. \quad (10)
\]

respectively, where the subscript \( i \) refers to the interpolating particle and \( j \) refers to neighbouring particles, \( u_i^\alpha \) is the velocity vector (with relative velocity given by \( u_i^\alpha = u_i^\alpha - u_j^\alpha \)), \( x_i^\alpha \) is the position vector, \( \rho_i \) is the density, \( \sigma_{ij}^{\alpha\beta} \) is the total stress tensor, \( m_j \) is the mass, \( g_i^\alpha \) is the gravitational acceleration and \( W_{ij} \) is the smoothing or kernel function. Further information regarding the weakly compressible SPH approach (WCSPH) may be found in (Monaghan, 1994) and more recently in (Violeau and Rogers, 2016). The Equations (9) and (10) are for a general fluid description. In order to distinguish between the water and sediment phases, each one requires different treatment as now described below.

### 2.4. Fluid phase: Constitutive equation for the water

The fluid phase is treated as a simple Newtonian fluid thus the Newtonian constitutive equations are used. The viscous stresses of Equation (3) are computed as

\[
\tau_{ij}^{\alpha\beta} = 2 \mu \dot{\varepsilon}_{ij}^{\alpha\beta}, \quad (11)
\]

where \( \mu \) is the dynamic viscosity of water and \( \dot{\varepsilon}_{ij}^{\alpha\beta} \) is the deviatoric strain rate, defined as

\[
\dot{\varepsilon}_{ij}^{\alpha\beta} = \frac{1}{2} \left( \frac{\partial u_i^{\alpha}}{\partial x_j^{\beta}} + \frac{\partial u_i^{\beta}}{\partial x_j^{\alpha}} - \frac{1}{3} \frac{\partial u_i^{\gamma}}{\partial x_j^{\gamma}} \delta^{\alpha\beta} \right). \quad (12)
\]

where \( i \) and \( j \) refers to neighbouring particles, Greek superscripts \( \alpha \) and \( \beta \) are free indexes, \( \gamma \) is a dummy index and \( \delta^{\alpha\beta} \) is Kronecker’s delta function.

The SPH approximation of (12) may be written as

\[
\dot{\varepsilon}_{ij}^{\alpha\beta} = \frac{1}{2} \left( \sum_{k=1}^{N} \frac{m_k}{\rho_k} \frac{u_k^{\alpha}}{x_k^{\beta}} \frac{\partial W_{ik}}{\partial x_i^{\beta}} + \sum_{k=1}^{N} \frac{m_k}{\rho_k} \frac{u_k^{\beta}}{x_k^{\alpha}} \frac{\partial W_{ik}}{\partial x_i^{\alpha}} \right) - \frac{1}{3} \left( \sum_{k=1}^{N} \frac{m_k}{\rho_k} \frac{u_k^{\gamma}}{x_k^{\gamma}} \frac{\partial W_{ik}}{\partial x_i^{\gamma}} \right) \delta^{\alpha\beta} \quad (13)
\]

This model has been validated for a range of free-surface flows as described in Fourtakas and Rogers (2016)

### 2.5. Sediment phase: Constitutive equation for the sediment
The sediment phase is considered to be fully saturated and is modelled as a slightly compressible pseudo-Newtonian fluid in line with the works of (Fourtakas et al., 2013; Fourtakas and Rogers, 2016; Manenti et al., 2012; Ulrich et al., 2013). The viscous term of Equation (3) is obtained as a function of the apparent viscosity ($\mu_{app}$) as

$$\tau_{i,i}^{app} = 2 \mu_{app} \dot{\epsilon}_{i,i}^{app}.$$  \hfill (14)

Following Fourtakas and Rogers (2016) the apparent viscosity of the sediment is calculated using the Herschel-Bulkley-Papanastasiou (HBP) model. Thus, in this work the symbol $\mu_{HBP}$ is used to denote the apparent viscosity, i.e. $\mu_{app} = \mu_{HBP}$. The HBP model reads

$$\mu_{HBP} = \frac{\tau_y}{\ln \left( \frac{1}{\epsilon} \right)} + 2 \mu H_{II} \frac{\epsilon}{\tau_y},$$  \hfill (15)

where $m$ controls the exponential growth of stress and $n$ is a power-law index that enables simulation of shear thinning or shear thickening behaviour. Note that when $m = 0$ and $n = 1$ the model reduces to a Newtonian model, whereas when $m \to \infty$ and $n = 1$ a simple Bingham model is recovered. The parameter $\tau_y$ is the yield stress that should be defined by a yield criterion.

The advantage of the HBP model is that it provides information on the pre-yield and post-yield region and thereby avoids the need of setting a maximum threshold for the viscosity as required in other pseudo-Newtonian approaches (Manenti et al., 2012; Ulrich et al., 2013).

### 2.5.1. Shields’ erosion criterion

The Shields’ criterion is based on the balance of the drag force imposed by the fluid flow and the particle weight. The grain’s motion is triggered when a certain critical stress is exceeded. The dimensionless critical shear stress $\theta_\theta$ over a horizontal bed of uniform sediment is then defined as

$$\theta_\theta = \frac{\tau_{b,c optics}}{(\rho_s - \rho)gd},$$  \hfill (16)

where $\tau_{b,c optics}$ is the critical bed shear stress for incipient motion over horizontal bottom, $\rho_s$ is the saturated density of the sediment, $\rho$ is the fluid density, $g$ is the gravity and $d$ is the characteristic particle diameter, usually taken as the median diameter ($d_{50}$) for non-uniform materials.

Similar to Manenti et al. (2012), the present study adopts the analytical relation proposed by van Rijn (1993) to approximate the critical Shields parameter as a function of the sediment Reynolds number ($Re_c$)

$$\theta_\theta = 0.010595 \ln(Re_c) + \frac{0.110476}{Re_c} + 0.0027197 \text{ for } Re_c \leq 500,$$

$$\theta_\theta = 0.068 \text{ for } Re_c > 500,$$

with $Re_c = u_d \rho / \nu$, being $\nu$ the kinematic viscosity of water and $u_d$ the friction velocity related to the bed shear stress $\tau_y$ as (van Rijn, 1993)
Also, as in the work of Manenti et al. (2012) it is assumed that a turbulent boundary layer is developed along the sediment-fluid interface. A logarithmic velocity law is assumed without resolving the velocity field inside the turbulent boundary layer. **If smooth-flow conditions occur a viscous sub-layer is considered along the sediment-fluid interface. The definition of smooth-flow conditions is formulated in later time in the paper.**

The thickness, $\delta$, of this layer depends on the friction velocity and on the kinematic viscosity, i.e.

$$\delta = 11.6 \frac{v}{u_*}.$$  \hspace{1cm} (19)

The velocity profile is then defined as the combination of a linear function for the laminar sub-layer and the logarithmic velocity function for the turbulent layer

$$u_{(s)} = \frac{u_*^2}{v} \frac{z}{\delta} \quad \text{for} \quad z \leq \delta$$

$$\frac{u_{(t)}}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z}{z_o} \right) \quad \text{for} \quad z > \delta$$

where $z$ is the position of the water particle measured from the sediment surface (using Equation (26) given later) to the free surface, $u_{(s)}$ is the velocity of the water particle closest to the sediment particle for which the dynamic condition (eroded or at rest) is being evaluated, $\nu$ the kinematic viscosity of water, $\kappa = 0.41$ is the von Kármán constant and $z_o$ is the bottom roughness length-scale parameter which depends on the flow regime as

$$z_o = \begin{cases} 0.11 \frac{v}{u_*} & \frac{k u_*}{v} < 5 \quad \text{(smooth flow)} \\ 0.033k_s & \frac{k u_*}{v} > 70 \quad \text{(rough flow)} \\ 0.11 \frac{v}{u_*} + 0.033k_s & 5 \leq \frac{k u_*}{v} < 70 \end{cases}$$

where $k$ is the equivalent grain roughness, also known as Nikuradse’s roughness. It is an empirical parameter and its value depends on the sediment characteristics and the flow condition. For flat beds, it is considered to be on the order of the median grain diameter (van Rijn, 1993; Wilson, 1989).

To calculate $u_*$ from (20), it is necessary to know *a priori* the region (laminar or turbulent) where the water particle is located. Also, the dependency of $z_o$ on $u_*$ by Equation (21), implies that (20) must be solved iteratively for $u_*$. An iterative procedure suggested by Manenti et al. (2012) was implemented in DualSPHysics as, starting from the hypothesis that the smooth-flow condition occurs (i.e. $k u_* / v < 5$) as given in Equation (21) and that the water particle is located inside the viscous sub-layer (i.e. $z \leq \delta$) until convergence of $u_*$. Once $u_*$ is obtained, it is possible to calculate $\tau_s$ and $\tau_{bcr,0}$ from (18) and (16), respectively.
The gravitational influence for non-horizontal beds is considered by means of the correction factors \( k_\beta \) and \( k_\gamma \) deduced by van Rijn (1993) for small slopes.

\[
k_\beta = \frac{\sin(\phi \pm \beta)}{\sin \phi}, \quad (22)
\]

\[
k_\gamma = \cos \gamma \left( 1 - \frac{\tan^2 \gamma}{\tan^2 \phi} \right)^{0.5}, \quad (23)
\]

where \( \phi \) is the internal friction angle of the sediment, \( \beta \) and \( \gamma \) are the longitudinal and transverse slope, respectively. The minus sign in (22) holds for down-sloping and the plus for up-sloping flows.

For a combination of longitudinal and transverse slope, the critical bed stress is calculated as follows

\[
\tau_{bcv} = k_\beta k_\gamma \tau_{crv,0}. \quad (24)
\]

Finally, a sediment particle is considered to be yielded if

\[
\tau_p \geq \tau_{bcv}. \quad (25)
\]

In the DualSPHysics implementation, the closest water particle to a sediment particle is identified during the neighbour search at each time step and its distance to a sediment particle is stored. However, the calculation of the critical Shields parameter and the iterative process for calculating \( u_i \) is performed only for particles identified as being near the sediment surface using two criteria:

(i) at least one water particle must be within the support of the smoothing kernel of the sediment particle and

(ii) the summation of the mass of sediment particles within the support domain should be less than a fraction of the reference mass of sediment particles such as

\[
m_i = \sum_{j=1}^{N} m_j W_i V_j < m_f \quad (26)
\]

where \( i \) and \( j \) denote the interpolated sediment particle and the neighbour particle, respectively, and \( m_f \) represents a fraction of the reference mass of sediment particles. This approach has been previously used by Crespo et al. (2011) and Gómez-Gesteira et al. (2005) to identify the free surface. In order for a sediment particle be tagged to be near the surface, conditions (i) and (ii) above should be satisfied simultaneously. This procedure provides the sediment surface position that is used to define the value of \( z \) in equation (20).

### 2.5.2. Drucker-Prager yield criterion

The Drucker–Prager yield criterion is a pressure-dependent model for determining whether a material has failed or undergone plastic yielding. The yielding surface of the Drucker-Prager criterion may be considered depending on the state of stress and on mechanical properties of the sediment (Desai, 1984)

\[
\sqrt{J_{20}} - \alpha J_i - \kappa_s = 0, \quad (27)
\]
where $J_1$ is the first invariant (trace) of the stress tensor that equals to pressure for fluids in repose - in this particular case the sediment skeleton pressure - and $J_{20}$ is the second invariant of the deviatoric shear stress tensor defined as

$$J_{20} = \frac{1}{2} \tau^{ij} \tau^{ij}.$$  \hspace{1cm} (28)

The parameters $\alpha$ and $\kappa$ can be related to the Mohr-Coulomb parameters according to

$$\alpha = \frac{2 \sin \phi}{\sqrt{3(3 - \sin \phi)}},$$

$$\kappa = \frac{6 \cos \phi}{\sqrt{3(3 - \sin \phi)}}$$  \hspace{1cm} (29)

where $\phi$ and $c$ are the angle of internal friction and cohesion, respectively.

Yielding requires the square root of the second invariant $J_{20}$ of the deviatoric shear stress tensor to exceed a threshold value $(\tau_c)$, i.e.

$$\sqrt{J_{120}} \geq |\tau_c|.$$  \hspace{1cm} (30)

Using equations (27) and (30) a threshold criterion may be defined as

$$|\tau_c| = \alpha J_1 + \kappa,$$  \hspace{1cm} (31)

where $\tau_c$ is the critical shear stress that should replace $\tau_c$ in Equation (15) if the Drucker-Prager criterion is used to model the yielding mechanism of the sediment. The reader is directed to Fourtakas and Rogers (2016) for details of the methodology used herein.

Equation (31) means that the critical shear stress increases with pressure. Therefore, the apparent viscosity, calculated by Equation (15) will increase with depth, reducing the strain rate at points located at lower layers. In contrast to the Shields’ erosion criterion, the Drucker–Prager yield criterion enables computation of the stress tensor for all points at the sediment domain and update of the position and the velocity. Thus, while the critical Shield’s parameter provides information about the onset of erosion at the bed surface, the critical value of the shear stress defines the yielded or un-yielded state of particles over the entire sediment domain. This makes the approach ideal to reproduce the effect of the impact of rapid flows over subsurface layers of a bed of sediments, as occurs for example at dam spillways.

### 2.5.3. Suspension treatment

In line with Fourtakas and Rogers (2016), the viscosity of suspended particles is considered to be a function of a volumetric fraction. This non-dimensional concentration parameter $(\epsilon_v)$ is computed as the ratio of the volume of sediment particles to the total volume in the support domain of the kernel, i.e.
\[ c_{ij} = \frac{\sum_{k \in \text{mesh}} m_k}{\sum_{j \in \text{mesh}} \rho_j} \]  

The threshold for the onset of suspended transport is set to \( c_{ij} = 0.3 \) and the viscosity of the suspended material \( (\mu_{\text{susp}}) \) is computed using the Vand experimental colloidal equation (Vand, 1948) that reads

\[
\mu_{\text{susp}} = \mu e^{-\frac{c_i - c_i'}{0.3\mu}} \quad \text{if} \quad c_i \leq 0.3
\]

where \( \mu \) is the viscosity of water. Thus, when the volumetric concentration of a yielded sediment particle within the support domain of the SPH kernel is below 0.3, the sediment particle is treated as a pseudo-Newtonian flow with a variable viscosity that approximates to the viscosity of the water as \( c_{ij} \to 0 \). Consequently, as the density of the sediment is larger than that of the water, sediment particles tend to settle when their viscosity diminishes. This approach is also similar to the transition layer of suspended material proposed by Ulrich et al. (2013).

### 2.5.4. Modelling of the sediment subsurface and sediment-water interface of a two-phase sediment transport problem

In order to simulate the dynamics of the subsurface sediment region and to avoid the noisy pressure fields at the sediment surface the Drucker-Prager criterion describing the yield characteristics of the sediment phase was combined with the Shields’ criterion to model the fluvial suspension of the sediment at the interface.

Regions (1), (2) and (3) in Figure 1 are modelled as non-Newtonian fluids, with an apparent viscosity computed by the HBP model. The apparent viscosity of regions (1) and (2) is obtained by substituting the yield stress \( (\tau_y) \) in Equation (15) by the critical shear stress \( (\tau_c) \) computed from the Drucker-Prager criterion. The apparent viscosity of region (3) is obtained by replacing the yield stress \( (\tau_y) \) in Equation (15) by the critical bed stress \( (\tau_{bc}) \) computed from the Shields’ criterion.

Thus, the 3 regions are well defined. Region (1) is un-yielded with a large yield stress, region (2) which is yielded with an apparent viscosity derived from the HBP model and region (3) which is governed by the Shields’ criterion which is pressure independent.

The transition between regions (1) and (2) is not explicitly defined, but emerges naturally because of the increase in stress with depth using a pressure dependant yield criterion (see Equation(31)). Particles with higher critical shear stress \( (\tau_{bc}) \) exhibit larger values of apparent viscosity and remain static. The transition between regions (2) and (3) is defined by using the procedure to identify sediment particles located near the bed surface (see Section 2.5.1). In addition, in order for a particle to be located at region (3) the condition of Equation (25) should be assessed. If Equation (25) is valid, then the particle is considered to be at region (3) and the critical bed stress \( (\tau_{bc}) \) computed by the Shields’ criterion replace the yield stress \( (\tau_y) \) in the HBP model to calculate the apparent viscosity \( (\mu_{app}) \) through Equation (15), otherwise the particle is consider to
remain at region (1) or (2) and the critical shear stress ($\tau_c$) of the Drucker-Prager model is used to calculate $\mu_{sbr}$ by making ($\tau_c = \tau_r$) in Equation (15). The flowchart of Figure 2 shows the procedure for calculating the apparent viscosity of particles at different regions. Finally, The suspended sediment is identified by the concentration parameter ($c_r$) defined by Equation (32) and the viscosity is computed from Equation (33).

With this approach, the mechanical properties of the bed material are considered by means of the cohesion and friction angle, while the effect of the physical properties are introduced by the particle diameter and the density within the Shields’ erosion criterion. Thus, the model is able to reproduce the entrainment of the sediment by the fluid phase which is dominated by the physical properties of the sediment and the hydraulic characteristics of the flow, as well as the behaviour of the subsurface layers of sediment determined by the mechanical properties of the material.

![Figure 2 – Flow chart representation procedure for calculating the apparent viscosity of particles at different regions of the domain.](image)

2.6. DualSPHysics Implementation: Time integration and boundary conditions

The computational model herein presented has been implemented in the DualSPHysics code (Crespo et al., 2015), an SPH C++/CUDA open-source solver. The previous implementation of Fourtakas and Rogers (2016) was used as the starting point to implement the Shields criterion and combine it with the Druker-Prager yield criterion.

The predictor-corrector algorithm described by (Monaghan, 1989), bound by the Courant–Friedrichs–Lewy (CFL) condition of force, viscosity and speed of sound is employed for the time step integration. Wall boundaries are modelled using the dynamic boundary condition (DBC) of Crespo et al. (2007) implemented in DualSPHysics.

3. NUMERICAL RESULTS
This section presents numerical results. First, the implementation of the Shields’ criterion is validated using a 2-D flushing experiment and comparing the final profile with the experimental result and with numerical results of Manenti et al. (2012). Then, a flume experiment carried out by the authors with non-cohesive sediment bed under fluvial conditions is presented. This experiment was intended to evaluate the influence of an empirical parameter of the Shields’ formulation (the equivalent grain roughness, \( k_e \)) on the rate of erosion predicted by the Shields’ criterion. Thus, the flume experiment is simulated using only the Shields’ criterion at the surface, treating the non-eroded particles as a fixed boundary. Finally, the combined approach presented in Section 2.5.4 is used to simulate a 2-D dam-break problem over erodible bed. The results are qualitatively compared to results obtained by the Shield’ and Drucker-Prager criterion separately.

### 3.1. 2-D flushing experiment

The flushing experiment of Falappi et al. (2007) represents the scour of a sediment bed due to constant water discharge from a tank, in a similar manner to what happens when a water reservoir gets flushed. This is an ideal test case to assess the effectiveness of the Shields criterion in applications where erosion occurs at the bed surface only in the absence of sediment yielding. Therefore, the Drucker-Prager criterion is not necessary which deems this test case suitable for validation of the Shields criterion. This experiment has been widely used as a benchmark case for erosion models in SPH (Falappi et al., 2007; Manenti et al., 2012; Ulrich and Rung, 2010). The case is presented as numerical validation of the implementation of the Shields criterion in DualSPHysics. The geometry and dimensions of the experiment are shown in Figure 3.

![Figure 3 - Sketch of the experimental 2-D flushing test (Falappi et al., 2007)](image)

The bulk density of the sediment is \( \rho_s = 1750 \text{ kg/m}^3 \), the mean diameter \( d_{50} \) and the kinematic viscosity of the sediment are taken to be 0.1 mm and 0.75 m²/s, respectively. A constant water discharge of 0.0079 m³/s was imposed by setting the horizontal particle velocity to 0.658 m/s at the outlet. The parameters of the HBP model were set to \( n=1 \) and \( m=0 \) in order to reproduce the Newtonian behaviour of the eroded sediment employed by Manenti et al. (2012). In order to avoid excessive creeping of the sediment bed, non-eroded particles are treated as a fixed boundary. This is the main disadvantage of the use of the Shields’ criterion at the surface and justifies the need to use other complementary yield criteria to model the rheology of the subsurface sediment domain.

In order to provide a fair comparison the parameters for the numerical model and the material properties were set in accordance with Manenti et al. (2012). To assess the effect of resolution, three different particle spacing were tested: \( dp=0.01 \text{ m} \), \( dp=0.005 \text{ m} \), and \( dp=0.003 \text{ m} \) giving simulations with
20,249, 79,480 and 314,871 particles respectively. Figure 4 shows the bed profiles obtained with DualSPHysics at \( t = 48 \) s for each resolution. The profiles are plotted in Figure 5 together with the experimental profiles and the one found numerically by Manenti et al.(2012). It is observed that the profile computed with DualSPHysics is in close agreement with the experimental one at the upper part, although a small divergence can be seen at the base of the slope for the coarser resolutions. Although a direct comparison with Manenti et al. (2012) might indicate that the proposed method is less accurate, no convergence study was presented by Manenti et al. so the effect of particle size is unknown in that work. However, in this paper, with particle refinement it is demonstrated that the DualSPHysics result is in increasingly closer agreement with the experimental result indicating convergence.

![Figure 4. Bed profile of the 2-D flushing test case computed with DualSPHysics. a) \( dp=0.01 \) m; b) \( dp=0.005 \) m and c) \( dp=0.003 \) m.](image)

![Figure 5. Slope profile at 48 s](image)

### 3.2. Flume experiment

To simulate steady flow over a mobile bed, numerical simulations were compared with a flume experiment carried out at the facilities of the Lutheran University of Brazil (ULBRA), campus Palmas, TO. An experimental flume mark HM 160, produced by GUNT Hamburg was adapted in order to place a 5.5 cm bed of non-cohesive sediment. The flume has a closed water circuit and a trap of sediment was added downstream. The cross section of the channel is 86x300 mm and the side walls are made of tempered glass, which allows clear observation of the experiments. The schematic arrangement of the experiment is shown in Figure 6.
The water entered the flume by the inlet reservoir. The gate was completely open at the beginning of the experiment. The time elapsed from the instant that the water entered the channel and the instant the wave front reached the end of the channel was 8s. The experiment run for 10s after the wave front reached the end of the flume.

Figure 6 - Flume experiment.

The equipment provided a steady flow with a fixed discharge over a mobile bed. The bed was built by sand pluviation technique (Chaney and Mulilis, 1978). The bed material was a sand of 0.4 mm of mean diameter with a density of 2710 kg/m³. The grain size distribution curve is shown in Figure 7.
During the experiment the water discharge was kept to a constant value of 2 m$^3$/h and water level measurements were taken at the positions identified as P1, P2 and P3 in Figure 6. The time elapsed from the instant that the water entered the channel and the instant the wave front reached the end of the channel was 8s. The experiment was run for 10s after the wave front reached the end of the flume. Thus, the experiment was run for a total time of $t = 18$s. The test was performed three times in order to check for repeatability.

In the numerical simulation, an upstream reservoir filled with water particles was created. The reservoir had an orifice of 2 cm height at the entrance of the flume ($x = 0$). A constant water discharge was imposed by setting the horizontal particle velocity to 0.22 m/s at the water inlet ($x = 0$), which corresponds to the exit of the orifice of the tank, as shown in Figure 8. The imposed velocity was calibrated, considering the width of the flume, in order to obtain the mass flow rate of 2 m$^3$/h.

The HBP parameters were set to $n = 1$ and $m = 0$ in order to mimic a Newtonian behaviour of the eroded material.

A spatial convergence study in terms of the position of the wave front was conducted using the following initial particle spacings of $dp = 0.016$ m, 0.008m, 0.004 m and 0.002 m with the experimental results as the reference solution. Figure 9a shows the position of the wave front at $t = 8$ s for all particle resolutions.
Evidently the finer particle resolutions of $dp = 0.004$ m and $0.002$ m reproduced the experimental result (dashed line) as shown in Figure 9b, which shows a spatial convergence of the model concerning the viscous formulation and its ability to capture the physics of the flow. Hence, a minimum particle spacing of $dp = 0.004$ m is needed.

Thus, the numerical case with a particle spacing of $dp = 0.004$ m, leading to 24,909 water particles and 7,188 sediment particles has been used for the same physical time as with the experimental setup of 18s (10s after the wave front reach the end of the flume).

Figure 9 – Convergence study. a) Snapshots of the simulations at $t = 8$s. b) Position of the wave front at $t = 8$ s for different particle resolution and experimental result.
A collector was added for retaining the particles downstream the flume, as shown in Figure 8. Thus, the mass of the eroded sediment per meter wide at the end of the simulation may be calculated as

\[ m_e = d_p \cdot d_p \cdot \rho_s \cdot N_p \]  \hspace{1cm} (34)

where \( d_p \) is the particle spacing, \( \rho_s \) is the sediment density and \( N_p \) is the number of particles sediment particles collected at the end of the simulation. In order to verify the influence of the equivalent grain roughness \( k_s \) on the results, six different values were tested i.e. \( k_s/d_{s0} = 1, 2, 3, 5, 7, 10 \). The Shields’ criterion was applied at the surface layer of sediment. Sediment particles outside the surface layer were fixed to the domain by explicitly setting the velocity and acceleration to zero. The mass of eroded material per meter wide is plotted in Figure 10. The first three bars show the experimental results, while the last six bars present the results obtained by the SPH simulations, at \( t = 18 \) s which signifies the end of the experiment.

The numerical results show that the mass of eroded material increases with \( k_s \) until the ratio \( k_s/d_{s0} = 7 \). This is in agreement with the results reported by Manenti et al. (2012) and suggests a rough flow for \( k_s/d_{s0} \geq 7 \). Therefore, the grain is located inside the turbulent layer. For \( k_s/d_{s0} < 7 \) the hydraulic regime varies from transitional to smooth leading to a reduction of the eroded mass for smaller values of \( k_s \). The ratio that better reproduced the experimental results was \( k_s/d_{s0} = 3 \), as shown by the horizontal dotted line in Figure 8. However, no experimental evidence has been found to support this behaviour, since the experimental results show scattered data (Kamphuis, 1974; Millar, 1999; Sumer et al., 1996; Williams, 1967). A future study may consider the influence of the Shields’ parameter on \( k_s \). Experimental results suggest that for flat beds the values of \( k_s \) increase with the critical Shields’ parameter (Camenen et al., 2009; Sumer et al., 1996).
Note that in this approach, particles that do not reach the critical Shields’ parameter remain fixed to the domain. For that reason, this approach is appropriate for problems in which only the surface layer of sediment is affected by the flux, as in fluvial channels or when flushing sediment from artificial reservoirs.

### 3.3. 2-D dam-break over erodible bed

In order to show the ability of the combined approach presented in 2.5.4 to simulate the scouring and erosion caused by the impact of the water on a bed of sediment, the Louvain erosional dam-break of (Fraccarollo and Capart, 2002) is simulated. The experiment has been used as a benchmark for various researchers (Fourtakas and Rogers, 2016; Ulrich et al., 2013) and will be used herein to compare the behaviour of the computational model using the erosion criteria separately and to compare the combined approach to other SPH simulations.

The experiment consists of the collapse of a water column over a loose sediment bed in a prismatic channel. The bed was built with cylindrical PVC pellets with an equivalent spherical diameter of 3.5 mm and saturated density of 1270 kg/m³. In order to calculate the value of the saturated density, a volume fraction of 0.5 was assumed, following the suggestions of Fraccarollo and Capart (2002). Figure 11 shows the initial setup of the experiment.

Figure 11. Sketch of the erosional dambreak experiment of Fraccarollo and Capart (2002) (dimensions are in meters)

The 2D computational case was configured using a particle spacing of 0.002 m in order to compare with the same case reported in Fourtakas and Rogers (2016), leading to 25,000 water particles and 49,450 sediment particles. Three cases were run using different models for the sediment phase, i.e.:

A. Drucker-Prager yield criterion and HBP rheological model for the eroded sediment
B. Shields’ erosion criterion and Newtonian model for the eroded sediment.
C. Drucker-Prager yield criterion combined with Shields erosion criterion and HBP rheological model

The mechanical properties of the soil for the Drucker-Prager criterion (case A) were set to \( \varphi = 31° \) and \( c = 0 \) Pa in order to reproduce a cohesionless material as expected from PVC pellets, since the values are not given by Fraccarollo and Capart (2002).

Figure 12 shows the three cases at the same instant in time where the particles belonging to each phase are coloured blue for water and red for sediment in order to show the effect on the deformation of the cases A, B and C. The equivalent grain roughness \( (k_e) \) for the Shields criterion (case B) was set to 5 times the value of the mean diameter, which is expected to represent a transitional flow condition. For this case, sediment
particles are fixed to the domain unless they reach the erosion criterion (see Figure 12a), otherwise the impact of the water column over the soil produces excessive deformation of the sediment mass even if high viscosities are used (see Figure 12b). For the example shown in Figure 12b an upper limit of $\mu_{\text{max}} = 2500 \text{ Pa.s}$ following the suggestions of Ulrich et al. (2013) and Ulrich and Rung (2012). However, Manenti et al. (2012) set the upper limit to viscosity to several orders of magnitude higher than this, which suggests that the maximum value of the apparent viscosity is case dependent. The combination of the criteria (case C) eliminates the need of fixing the particles to the domain position, because the shear stress is computed at each sediment particle as a function of the mechanical soil properties, avoiding the excessive creeping resulting from a fluid model (see Figure 12c).

![Figure 12](image)

**Figure 12.** Behaviour of the sediment phase at $t = 0.25 \text{ s}$, blue and red colours represent the water and the sediment, respectively. a) Shields’ criterion: non-eroded particles are fixed to the domain position; b) Shields’ criterion: non-eroded particles behave as a Bingham type fluid with a prescribed maximum viscosity c) Drucker-Prager yield criterion: no need of fixing the particles to the domain.

The parameters of the HBP model used in cases A and B were set to $n = 1.2$ and $m = 100$ in order to represent a non-Newtonian shear thinning material and to avoid the numerical instabilities for zero shear strain rate.

Figure 13 to 16 show the snapshots of the erosional 2-D dam break experiments at times 0.25, 0.50, 0.75 and 1.00 s. The experimental snapshots were presented by Fraccarollo and Capart (2002) showing the estimated interfaces between the sediment bed, the eroded material and the water. Numerical experiments show three sub-domains. Blue particles represent the water phase, red particles are the un-yielded sediment and green particles denote the eroded material.
Figure 13. Interfaces of the Louvain experiment at $t = 0.25$ s; blue, red and green colours represent the water, the non-eroded sediment and the transport layer, respectively. a) Experimental result; b) Drucker-Prager (DP) yield criterion; c) Shields’ erosion criterion and d) Combined DP+Shields’ criterion.
Figure 14. Interfaces of the Louvain experiment at $t=0.50$ s; blue, red and green colours represent the water, the non-eroded sediment and the transport layer, respectively. a) Experimental result; b) Drucker-Prager (DP) yield criterion; c) Shields’ erosion criterion and d) Combined DP+Shields’ criterion.

Figure 15. Interfaces of the Louvain experiment at $t=0.75$ s; blue, red and green colours represent the water, the non-eroded sediment and the transport layer, respectively. a) Experimental result; b) Drucker-Prager (DP) yield criterion; c) Shields’ erosion criterion and d) Combined DP+Shields’ criterion.
Figure 16. Interfaces of the Louvain experiment at $t = 1.00$ s; blue, red and green colours represent the water, the non-eroded sediment and the transport layer, respectively. a) Experimental result; b) Drucker-Prager (DP) yield criterion; c) Shields’ erosion criterion and d) Combined DP+Shields’ criterion.

Two features may be analysed from the erosion patterns of Figure 13 to 16: the bed profile formed by the motionless particles and the transport layer formed by the moving sediment particles, represented by red and green particles in the numerical snapshots respectively. From the (a) snapshots in Figure 13 to 15 it may be observed that the bed profiles generated by the Drucker-Prager criterion are similar to the experimental profiles, reproducing the curve formed by the impact of the water in the bed of sediment at position $x \approx 0.05$ m. However, the model is not able to correctly simulate the transport layer. This effect is better reproduced by the Shields’ criterion, represented by the (b) snapshots where green particles show a well-defined layer of eroded sediment. The combined approach presented in the (c) snapshots keeps the positive features of each criterion, i.e. it captures the effect of the impact as well as the entrainment of the sediment particles by the fluid phase, showing satisfactory qualitative agreement between the numerical model and experimental observations. The plots of Figure 17 and Figure 18 confirm this behaviour for $t = 0.25$ s and $t = 1.00$ s. The Root Mean Square (RMS) error was used to quantify the deviation of the SPH interface location from the experimental interfaces. Accordingly, the acronyms RMSE(Zw), RMSE(Zs) and RMSE(Zb) in Figure 17 and Figure 18 represent the RMS error of the free surface (Zw), the interface between the water and the moving grains (Zs), and the interface between moving and immobile grains at the bed (Zb). The transport layer is defined by the region between the Zs and Zb interfaces.
Figure 17. Experimental and numerical interfaces at \( t = 0.25 \) s. a) Drucker-Prager (DP) yield criterion; b) DualSPHysics Shields’ criterion; c) DualSPHysics DP+Shields’ criterion.
Figure 18. Experimental and numerical interfaces at $t=1.00\ s$. a) Drucker-Prager (DP) yield criterion; b) DualSPHysics Shields’ criterion; c) DualSPHysics DP+Shields’ criterion.

The current model show closer agreement to the experimental data both for the bed surface ($Z_b$ interface) and for the transport layer (region between $Z_b$ and $Z_s$ interfaces). Comparing the profiles obtained by the Shields criterion with the profiles obtained by the combined approach here presented at $t=0.25\ s$, the current model demonstrate greater capability of capturing the yielding effect caused by the impact of the wave at the initial states (see the shape of the wave front in Figure 17b and 17c). The yielding near the dam toe is also well captured by the Drucker-Prager model, as observed in Figure 17a.

For $t=1.00\ s$, the Drucker-Prager yield criterion shows a tiny transport layer near the wave front, while the Shields criterion and the combined approach show reasonable agreement with the experimental data. The current model shows the smallest RMS error for the interface between the water and the moving grains ($Z_s$).
and for the interface between moving and immobile grains (Zb). This means that the scouring profile is satisfactorily predicted by the current model. The current model that combines the Drucker-Prager and the Shields’ criteria shows the smallest RMS error for the interface between the water and the moving grains (Zs) and for the interface between moving and immobile grains (Zb). This allows to conclude that the scouring profile is satisfactorily predicted by the current model.

4. Conclusions

In this paper, a novel approach that combines two erosion criteria with a non-Newtonian rheological model to simulate surface erosion is presented. The Drucker-Prager yield criterion was complemented by the Shields’ erosion criterion in order to model the relevant features of the erosive process by considering the mechanical and hydraulic properties of the sediment bed.

The implementation of the Shields’ criterion into the DualSPHysics code has been validated using experimental and numerical results. The analysis of the equivalent grain roughness parameter showed that it has a strong influence on the prediction of the amount of eroded material.

Significantly different behavior was observed when the erosion criteria were used separately to simulate a 2-D dam break experiment. The Drucker-Prager yield criterion was able to reproduce the effect of the impact of the water in the bed at the dam toe, while the Shields’ criterion showed better agreement on simulating the transport layer formed by the entrainment of the sediment particles by water at final stages. Satisfactory results are observed when both criteria are combined keeping the positive features of each criterion, i.e., the shape of the scouring profile and transport layer.

The combined approach here proposed is able to simulate the yielding of the sediment layer by considering the mechanical properties of the material. Thus, the rheology of the domain of sediment under the impact of rapidly varying flows is satisfactorily predicted. Also, the entrainment of soil by water is accurately simulated by including the physical properties of the sediment and the hydraulic characteristics of the flow through the Shields criterion.

5. Acknowledgements

The authors would like to thank the National Council of Technological and Scientific Development of Brazil (CNPq) and the Research Council of the United Kingdom for the financial support.

6. References


