Extracting foreground-obscured $\mu$-distortion anisotropies to constrain primordial non-Gaussianity

M. Remazeilles* and J. Chluba†
Jodrell Bank Centre for Astrophysics, Alan Turing Building, School of Physics and Astronomy, The University of Manchester, Oxford Road, Manchester, M13 9PL, U.K.

Accepted 2018 –. Received 2018 February 26

ABSTRACT
Correlations between cosmic microwave background (CMB) temperature, polarization and spectral distortion anisotropies can be used as a probe of primordial non-Gaussianity. Here, we perform a reconstruction of $\mu$-distortion anisotropies in the presence of Galactic and extra-galactic foregrounds, applying the so-called Constrained ILC component separation method to simulations of proposed CMB space missions (PIXIE, LiteBIRD, CORE, PICO). Our sky simulations include Galactic dust, Galactic synchrotron, Galactic free-free, thermal Sunyaev-Zeldovich effect, as well as primary CMB temperature and $\mu$-distortion anisotropies, the latter being added as correlated field. The Constrained ILC method allows us to null the CMB temperature anisotropies in the reconstructed $\mu$-map (and vice versa), in addition to mitigating the contaminations from astrophysical foregrounds and instrumental noise. We compute the cross-power spectrum between the reconstructed (CMB-free) $\mu$-distortion map and the ($\mu$-free) CMB temperature map, after foreground removal and component separation. Since the cross-power spectrum is proportional to the primordial non-Gaussianity parameter, $f_{NL}$, on scales $k \approx 740$ Mpc$^{-1}$, this allows us to derive $f_{NL}$-detection limits for the aforementioned future CMB experiments. Our analysis shows that foregrounds degrade the theoretical detection limits (based mostly on instrumental noise) by more than one order of magnitude, with PICO standing the best chance at placing upper limits on scale-dependent non-Gaussianity. We also discuss the dependence of the constraints on the channel sensitivities and chosen bands. Like for $B$-mode polarization measurements, extended coverage at frequencies $\nu \lesssim 40$ GHz and $\nu \gtrsim 400$ GHz provides more leverage than increased channel sensitivity.

Key words: cosmic microwave background – inflation – early universe – methods: analytical

1 INTRODUCTION
Interactions between cosmic microwave background (CMB) photons and matter generate temperature and polarization anisotropies (Sunyaev & Zeldovich 1970b; Peebles & Yu 1970) as well as spectral distortions of the CMB radiation (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1970c). Precisely characterizing the CMB thus enables us to probe different stages in the evolution of the Universe, thereby learning about the main cosmological parameters and the nature of dark matter and dark energy (e.g., Spergel et al. 2003; Planck Collaboration et al. 2016).

In the late Universe ($z < 10-100$), large-scale structures modify the primordial CMB light by gravitational effects and Thomson scattering, while the hot gas inside galaxy clusters distorts the CMB radiation through the Sunyaev-Zeldovich (SZ) effect (Zeldovich & Sunyaev 1969; Carlstrom et al. 2002), generating $y$-distortion anisotropies over a wide range of scales. A similar type of spectral distortion can also be created in the pre-recombination era ($z > 10^3$) when no structures were present yet, but most excitingly, in the early Universe ($z > 10^3$), other types of spectral distortions can be formed. One prominent example is the $\mu$-distortion resulting from energy injection to the CMB (e.g., Sunyaev & Zeldovich 1970c; Burigana et al. 1991; Hu & Silk 1993a; Chluba & Sunyaev 2012), e.g., by annihilating or decaying particles (Sarkar & Cooper 1984; McDonald et al. 2001; Hu & Silk 1993b; Chluba 2013a), primordial black holes (Carr et al. 2010; Ali-Haïmoud & Kamionkowski 2017; Poulin et al. 2017) or the dissipation of small-scale acoustic modes (Sunyaev & Zeldovich 1970a; Daly 1991; Hu et al. 1994). Other types of distortions, with more rich spectral structure, can be formed through photon injection (Chluba 2015) or in the partially Comptonized regime (Chluba & Sunyaev 2012; Khatri & Sunyaev 2012; Chluba 2013b). Although so far undiscovered, $\mu$-distortions will open a new window to the early Universe, potentially shedding light on the nature of dark matter, particle and inflation physics, all from behind the last-scattering surface well into the

* E-mail: mathieu.remazeilles@manchester.ac.uk
† E-mail: jens.chluba@manchester.ac.uk

© 0000 RAS
pre-recombination-era at $z > 10^3$ (e.g., Chluba & Sunyaev 2012; Sunyaev & Khatri 2013; De Zotti et al. 2016; Chluba 2016).

Aside from average CMB distortions, primordial distortion fluctuations (sometimes referred to as spectral-angular anisotropies) can be created through anisotropic heating mechanisms, as discussed by Chluba et al. (2012b). Aside from late time effects (e.g., related to SZ clusters), in $\Lambda$CDM these anisotropies are usually expected to be small ($\Delta T/\mu \lesssim 10^{-5} - 10^{-4}$). Exotic mechanisms could in principle be connected to anisotropic dark matter annihilation during structure formation (density square enhancement), dissipation of large-scale primordial magnetic fields, energy injection by decaying particles in cosmologies with iso-curvature perturbations at small scales or heating by primordial black hole clusters, to name a few possibilities that come to mind.

One mechanism capable of creating sizable primordial distortion anisotropies is through the damping of primordial small-scale acoustic modes in the presence of (enhanced) primordial non-Gaussianity in the ultra-squeezed limit, as first discussed by Pajer & Zaldarriaga (2012). There it was demonstrated that by measuring intrinsic spatial correlations between $\mu$-distortion and CMB temperature anisotropies we can place new constraints on the primordial non-Gaussianity at small scales with wavenumbers corresponding to $k > 10^{-5} - 10^{4}$ Mpc$^{-1}$.

At face value, the expected theoretical limits are fairly weak, corresponding to $|f_{\text{NL}}| \lesssim 10(10^3)$ on the (local-type) non-Gaussianity parameter for PIXIE-like experiments (Pajer & Zaldarriaga 2012; Ganc & Komatsu 2012). However, as stressed again later (Pajer & Zaldarriaga 2013; Biagetti et al. 2013; Emami et al. 2015), these apply to currently unconstrained scales, thus allowing to place novel bounds on the scale-dependence of non-Gaussianity, for which theoretical motivation can be given (e.g., Dimastrogiovanni & Emami 2016). One of the most important aspects here is the huge lever arm between typical CMB scales, $k \approx 10^{-2}$ Mpc$^{-1}$, and those relevant to the creation of the $\mu$-distortion anisotropies, $k \approx 740$ Mpc$^{-1}$, which in principle can lead to greatly enhanced distortion anisotropies while being consistent with CMB temperature anisotropy limits $|f_{\text{NL}}| \lesssim 5$ (Planck Collaboration et al. 2014). In addition, correlations between temperature and $y$-distortion anisotropies can be created from modes dissipating at scales $k \approx 10$ Mpc$^{-1}$ (Emami et al. 2015). Similarly, correlations of distortion signals with polarization anisotropies can be used to further disentangle different contributions (Ota 2016; Ravenni et al. 2017). Higher order statistics can also be affected (Bartolo et al. 2016; Shiraishi et al. 2016). Finally, the overall level of the anisotropic distortion signal also depends on the average value of the dissipation-induced distortion (Chluba et al. 2017a), which in principle can be strongly enhanced in non-standard early-Universe models (e.g., Chluba et al. 2012a).

Here, we wish to address the question about the detectability of these distortion signals in the presence of real world limitations caused by foregrounds. For the average CMB distortion signals it was recently shown that foregrounds indeed pose a serious challenge, with one of the biggest limitations caused by lack of low-frequency coverage (Abitbol et al. 2017). Also, like for the detection of primordial $B$-mode polarization signals, one always is faced with questions about the robustness of the analysis with respect to biases (Remazeilles et al. 2016, 2017).

To obtain unbiased constraints on the non-Gaussianity parameter, $f_{\text{NL}}$, from the $T\mu$ cross-angular power spectrum between CMB temperature anisotropies and $\mu$-distortion anisotropies, it is essential to cancel out residual CMB temperature anisotropies in the reconstructed map of $\mu$-distortion anisotropies. This can be robustly achieved by using the Constrained ILC component separation method (Remazeilles et al. 2011a). Utilizing the known spectral signature of $\mu$-distortion and CMB temperature anisotropies, we apply the Constrained ILC component separation method to map the $\mu$-distortion anisotropies while nulling the CMB temperature contamination, and vice versa. With this approach, we assess the performance of the proposed CMB space missions PIXIE (Kogut et al. 2016), LiteBIRD (Matsumura et al. 2016), CORE (De Labrouille et al. 2017), and PICO (Shaul Hanany, priv. comm.) in this context. We also discuss the optimization of future CMB experiments in terms of sensitivity and frequency bands to allow a detection of anisotropic $\mu$-distortions for average $(\mu) \approx 2 \times 10^{-8}$ and $|f_{\text{NL}}(k \approx 740 \text{ Mpc}^{-1})| \lesssim 4500$. For $f_{\text{NL}}(k_0) = 5$ at pivot scale, $k_0 = 0.05$ Mpc$^{-1}$, this would impose a limit of $f_{\text{NL}} \lesssim 1.6$ on the spectral index of $f_{\text{NL}}(k) \approx f_{\text{NL}}(k_0)(k/k_0)^{n_{\text{NL}} - 1}$ for scale-dependent non-Gaussianity, providing a new way to constrain non-standard early-universe model (e.g., multi-field inflation).

We do not consider higher order statistics of the $\mu$-distortion field here, but the method can be easily extended. This might become relevant if indeed large values of $f_{\text{NL}}(k > 10^3)$ are indicated by future data, but a more careful assessment is beyond the scope of this paper. We also do not include any effects from the residual ($r$-type) distortion signal created by heating at $10^4 \lesssim z \lesssim 2 \times 10^5$ (e.g., Chluba & Jeong 2014). Just like for primordial $\mu$ and $y$, this would lead to $r-T$ correlations dominated by acoustic damping of perturbations with $k = 50$ Mpc$^{-1}$, but at a level that is about one order of magnitude smaller than the primordial $\mu$ or $y$ distortion signals. In refined analysis, this signal could cause another contamination that should to be considered.

The paper is organized as follows. In Sect. 2 we describe our simulations of $\mu$-distortions and foregrounds for different CMB experiments. In Sect. 3, we first discuss the details of the component separation method employed for the analysis (Sect. 3.1). We then present our results on the reconstruction of the $\mu-T$ correlation signal and detection limits on $f_{\text{NL}}$ after foreground removal for the different CMB satellite concepts (Sect. 3.2). In Sect. 4, we discuss some important issues to be addressed in order to optimize for the detection of $\mu$-distortions. We conclude in Sect. 5.

2 SIMULATIONS

In this section we outline the various ingredients of our simulations, starting with the distortion anisotropies (Sect. 2.1), then discussing foregrounds (Sect. 2.2) and closing with the various mission specifications (Sect. 2.3).

2.1 Correlated $\mu$ distortion and CMB temperature anisotropies

To simulate the CMB $\mu$-distortion anisotropies, we need to provide a description of its auto- and cross-power spectra. We mainly use the compact expressions given by Emami et al. (2015) and Chluba et al. (2017a), which link the various power spectra to the $f_{\text{NL}}$ parameter and average $\mu$-distortion, $(\mu)$, in a simple manner. To capture the $\ell$-dependence of the temperature and $\mu$-distortion cross-

---

1 A small additional enhancement of the average distortion caused by scale-dependent non-Gaussianity (Chluba et al. 2017a) was taken into account for this estimate.
power spectrum, $C^{\mu T}_{\ell}$, corrections beyond the Sachs-Wolfe limit originally used by Pajer & Zaldarriaga (2012) have to be included. These were first considered by Ganc & Komatsu (2012) and at large angular scales ($2 \leq \ell \leq 200$) can be represented using analytical expressions (Chluba et al. 2017a). More accurate computations of the $\mu$-T cross-power spectrum (extending to small scales) was recently carried out by Ravenni et al. (2017). These results were published while we were completing this work, so that most of the results derived here are based on the modelling of Chluba et al. (2017a). However, both models have very similar amplitude and scale-dependence at large angular scales ($\ell \leq 200$) probed in our analysis, so that our results on foreground removal and signal reconstruction are consistent for both models (see Sect. 3.3).

Depending on the model for the $\mu$-T cross-power spectrum, we simulate full-sky maps of $\mu$-distortion anisotropies and CMB temperature anisotropies as correlated fields using the following prescription: the auto-power spectrum of CMB temperature anisotropies in the Sachs-Wolfe (SW) limit is given by

$$C^{TT}_{\ell,SW} = \frac{2\pi T_{\text{CMB}}^2 A^2}{25} \ell (\ell + 1)$$

(1)

in thermodynamic temperature units, where the CMB blackbody temperature is $T_{\text{CMB}} = 2.7255$ K and the amplitude of the curvature perturbation power spectrum is $A_s = 2.4 \times 10^{-9}$ under the assumption of scale-invariance ($n_s = 1$).

In the description of Chluba et al. (2017a), the cross-power spectrum between $\mu$-distortion anisotropies and CMB temperature anisotropies on angular scales $2 \leq \ell \leq 200$ is then given by:

$$C^{\mu T}_{\ell} = 12 C^{TT}_{\ell,SW} \rho(\ell) f_{\text{NL}}(\mu),$$

(2)

where the monopole of the $\mu$-distortion is set to $\langle \mu \rangle = 2 \times 10^{-8}$ (Chluba 2016) and $f_{\text{NL}}$ is the primordial non-Gaussianity parameter. The scale-dependence of the correlation is approximated by:

$$\rho(\ell) = 1.08(1 - 0.0227 \ell - 1.72 \times 10^{-4} \ell^2 + 2.00 \times 10^{-6} \ell^3 - 4.56 \times 10^{-9} \ell^4).$$

(3)

We adopt the opposite sign convention to that of WMAP (Komatsu & Spergel 2001): $f_{\text{NL}} = -f_{\text{NL}}^{\text{WMAP}}$. Thus, for positive $f_{\text{NL}}$, $\mu$ and $T$ are correlated at the largest angular scales. In Sect. 3.3.1, we show that this is not a limitation, as the results directly apply to negative values of $f_{\text{NL}}$ too, making this choice unimportant.

Equation (2) clearly illustrates the dependence of the cross-correlation on $f_{\text{NL}}$ and $\langle \mu \rangle$, showing that the obtained limits from anisotropy measurements alone only constrain the product $f_{\text{NL}}(\mu)$ (Chluba et al. 2017a). To break the degeneracy, an absolute measurement using a PIXIE-type experiment is required. Thus, a combination of imager (providing angular resolution) and spectrometer (providing spectral coverage) might be one viable avenue forward towards clear detections and constraints.

In the more complete computation of Ravenni et al. (2017), the $\mu$-T cross-power spectrum at all angular scales is given by:

$$C^{\mu T}_{\ell} = 4\pi \left( \frac{12}{5} \right) \int \frac{k^2 dk}{2\pi^2} T_{\ell}^j(k) P(k) \times \int \frac{d\ell_1 dq_1}{2\pi^2} P^\ell(q_1, k) P(q_1),$$

(4)

where $T_{\ell}^j(k)$ denotes the radiation transfer function for CMB temperature, $P^\ell(q_1, q_1, k)$ is the transfer function for the $\mu$-distortion, $j_1$ is the spherical Bessel function of the first kind, $r_0$ is the comoving distance to the last-scattering surface, and $P(k)$ is the primordial power spectrum. The resultant cross power spectrum is illustrated in Fig. 1, exhibiting several oscillations towards small angular scales. However, the added signal-to-noise is limited at $\ell \geq 200$ even for PICO (see Sect. 3.3), so that the approximation Eq. (2) is sufficient for our main estimates.

The $\mu$-distortion auto-power spectrum in the Gaussian limit is negligible in comparison to the noise-level of future CMB imagers (Pajer & Zaldarriaga 2012; Ganc & Komatsu 2012). For the non-Gaussian contribution we use (Emami et al. 2015)

$$C^{\mu\mu}_{\ell} = 144 C^{TT,SW}_{\ell} f_{\text{NL}}^2(\mu)^2,$$

(5)

which is in good agreement with the original estimates (Pajer & Zaldarriaga 2012). For illustration, we adopt different values of primordial non-Gaussianity in our simulations:

$$f_{\text{NL}} = 4.5 \times 10^3; 10^4; 10^5.$$  

(6)

The first value is close to the estimated detection limit of a PIXIE-like experiment without foregrounds in more detail (Chluba et al. 2017a). The last value, putting us into the large signal-to-noise regime, is mainly chosen to validate the method as it reaches the limits of perturbation theory.

Figure 1. Angular power spectra of anisotropies from theory (red, blue, and green lines) and simulations (black lines), for average $\langle \mu \rangle = 2 \times 10^{-8}$ and $f_{\text{NL}} = 4500$: CMB TT and $\mu$-T distortion auto-power spectra (upper panel), and $\mu \times T$ cross-power spectrum (lower panel). The upper panel illustrates the large dynamic range between CMB temperature anisotropies (red line) and $\mu$-distortion anisotropies (blue line).
The covariance matrix of CMB temperature anisotropies and \( \mu \)-distortions anisotropies is then given by:

\[
C = \begin{pmatrix}
C_{TT} & C_{T\mu} & C_{T\nu}
\end{pmatrix}.
\]

(7)

By computing the singular-value decomposition (SVD) of the covariance matrix, \( C \), we can first generate two independent Gaussian random fields, then reproject them in the appropriate basis in order to obtain a simulated CMB map and a simulated \( \mu \)-distortion map that are correlated according to Eq. (7).

The spectral energy distribution (SED) of CMB temperature anisotropies is the derivative of the blackbody spectrum with respect to temperature, thus in thermodynamic units corresponding to a constant spectrum across frequency, \( \nu \):

\[
a_T(\nu) = T_{CMB}.
\]

(8)

This expression neglects higher order terms \( \propto (\Delta T/T)^2 \), which introduce a \( \gamma \)-type spectral dependence (e.g., Chluba & Sunyaev 2004) that are negligible in our discussion. Conversely, the SED of \( \mu \)-distortion anisotropies, \( a_\mu \), scales across frequencies as:

\[
a_\mu(\nu) = T_{CMB} \left( \frac{x}{2.79} - 1 \right), \quad x \equiv \frac{h\nu}{kT_{CMB}},
\]

(9)

in thermodynamic temperature units. The distinct spectral signatures of CMB temperature and \( \mu \)-distortion anisotropies should allow us to separate the two signals by using multi-frequency observations from CMB satellite experiments. We use Eqs. (8) and (9) in our simulations to integrate the CMB and \( \mu \)-distortion template maps over the frequency bands of different CMB experiments.

Figure 1 shows both the auto- and cross-angular power spectra of the simulated CMB and \( \mu \) maps (black lines), plotted against the theory power spectra (coloured lines) for the model of Ravenni et al. (2017). CMB temperature anisotropies are a significant foreground to \( \mu \)-distortion anisotropies, dominating the signal at all angular scales by more than six orders of magnitude for \( f_{NL} \lesssim 10^3 \). Conversely, the \( \mu-T \) cross-correlation signal is only about three orders of magnitude lower than the CMB \( TT \) power spectrum, therefore providing a potentially more accessible target for future CMB experiments (Pajer & Zaldarriaga 2012).

The resulting maps of correlated CMB temperature and \( \mu \)-distortion anisotropies are shown in Fig. 2 for \( (\mu) = 2 \times 10^{-8} \), which is close to the value expected within the standard \( \Lambda \)CDM (Chluba 2016), and \( f_{NL} = 4.5 \times 10^4 \). The top panel shows typical degree-scale fluctuations of CMB temperature anisotropies over the sky, while the bottom panel shows that bulk of the \( \mu \)-distortion fluctuations are present at large angular scales, giving the impression of a low-resolution version of the temperature map.

2.2 Foregrounds

We use the PSM (Planck Sky Model; Delabrouille et al. 2013) software to simulate foregrounds and instrumental noise. We include both Galactic and extragalactic foregrounds in our sky simulations: thermal dust emission, synchrotron radiation, Galactic free-free emission, and thermal Sunyaev-Zeldovich (SZ) effect (\( \gamma \)-distortion) from galaxy clusters. We neglect potential effects of line-of-sight and beam averaging on the SEDs of the different components (Chluba et al. 2017b). We also do not include any intrinsic \( y-T \) correlations and focus only on the \( \mu-T \) signal described in Sect. 2.1. As shown in Ravenni et al. (2017), \( y-T \) correlations and also correlations of distortions with CMB polarization signals can help us to separate different contributions, but we leave a more detailed analysis to future work.

One risk of ignoring \( y-T \) correlations in the analysis of real data would be that residual SZ emission in the reconstructed \( \mu \)-distortion map might bias the measurement of the \( \mu-T \) correlation signal. Here, masking and the explicit scale-dependence of the SZ-\( T \) and SZ-\( E \) correlations can be utilized to separate contributions (Creque-Sarbinowski et al. 2016; Ravenni et al. 2017). The primordial \( y-T \) correlation signal itself is about one order of magnitude smaller than the \( \mu-T \) signal (e.g., Ravenni et al. 2017), so that residual \( y-T \) contamination will also be much lower than the measured \( \mu-T \) signal, while in contrast spurious residual \( TT \) correlations might be larger than the signal itself. Thus, the main enemy here is rather residual CMB temperature anisotropies in the reconstructed \( \mu \)-distortion map, which, if not canceled, will add spurious \( T-T \) correlations at a larger level than residual primordial \( y-T \) correlations to the measured \( \mu-T \) signal (see Fig. 9).

Galactic thermal dust emission (top left panel in Fig. 3) arise from small dust grains of various sizes in the interstellar medium (silicates and carbonaceous grains, molecules of polycyclic aromatic hydrocarbon) that are heated by the emission from stars and re-emit photons at infra-red wavelengths. This is the dominant astrophysical foreground at high frequencies (> 100 GHz) in CMB observations. We use the publicly released Planck GNILC dust all-sky map at 353 GHz (Planck Collaboration Int. XLVIII 2016) as a template for the simulation of the Galactic thermal dust emission. The GNILC dust map does not suffer from contamination by...
Figure 3. Simulation of Galactic and extragalactic foregrounds observed in the 280 GHz frequency band of LiteBIRD: thermal dust (top left); synchrotron (top right); free-free (middle left); SZ (middle right); LiteBIRD instrumental noise (bottom left); total observation map at 280 GHz (bottom right), which includes foregrounds, CMB anisotropies, µ-distortion anisotropies, and noise.

High-energy cosmic ray electrons spiraling Galactic magnetic fields are responsible for Galactic synchrotron radiation. Since those magnetic fields extend outside the Galaxy, synchrotron emission is present even at high Galactic latitudes in the sky (top right panel in Fig. 3), and is the main astrophysical foreground at radio frequencies (< 100 GHz) in CMB observations. As a template of Galactic synchrotron emission, we use the reprocessed Haslam et al. (1982) 408 MHz all-sky map of Remazeilles et al. (2015), in which extragalactic radio sources and other systematic effects have been subtracted. The 408 MHz map is scaled across the frequency bands of CMB experiments through a power-law emission law in Rayleigh-Jeans brightness temperature units

\[ I_\nu^\text{synch} = I_{408\text{MHz}} \left( \frac{\nu}{408} \right)^{\beta_s} \]

with a variable spectral index, \( \beta_s \), over the sky. The synchrotron spectral index map is taken from Miville-Deschênes et al. (2008),

### Cosmic Infrared Background Anisotropies

Cosmic infrared background anisotropies thanks to filtering by the GNILC algorithm (Remazeilles et al. 2011b). The dust template is then integrated over the frequency bands of the considered CMB experiment assuming a modified blackbody emission law

\[ I_{\nu}\text{dust} = I_{353\text{GHz}} \left( \frac{\nu}{353} \right)^{\beta_d} \frac{B_\nu(T_d)}{B_{353}(T_d)} \]

with variable emissivity, \( \beta_d \), and temperature, \( T_d \), over the sky. \( B_\nu(T_d) \) being the Planck’s law for blackbody radiation. The released Planck GNILC maps of dust temperature and emissivity, with average values over the sky of \( \langle \beta_d \rangle = 1.6 \) and \( \langle T_d \rangle = 19.4 \text{K} \), are used for the spectral scaling of the dust template map.
which has an average value of \( \beta_\ell = -3 \) over the sky. Similar variations of the spectral index are expected along the line of sight, leading to higher order curvature terms (Chluba et al. 2017b), which are not included here.

Free electrons lose energy through Coulomb interactions with heavy ions, which results in a Bremsstrahlung emission of photons in \( \text{H}_2 \) regions of the Galactic plane. This emission is also termed as Galactic free-free emission, which, while fainter than synchrotron and thermal dust emissions, is still a significant foreground in star-forming regions of the Galaxy at low frequencies \( \lesssim 100 \) GHz for CMB observations (middle left panel in Fig. 3). Galactic free-free emission is simulated from the \( \text{H}_2 \) emission map corrected for dust extinction (Dickinson et al. 2003), and scaled across frequency bands through a power-law emission law in brightness temperature units, with a uniform spectral index of \( \beta_g = -2.1 \) over the sky:

\[
I_\ell^\text{ff} = 90 \, \text{mK} \left( \frac{T_e}{K} \right)^{-0.35} \left( \frac{v}{\text{GHz}} \right)^{\beta_g} \left( \frac{H_\alpha}{\text{cm}^{-6} \text{pc}} \right),
\]

where \( T_e \) is the electronic temperature in K, \( v \) is the frequency in GHz, and the \( H_\alpha \) emission measure is in \( \text{cm}^{-6} \text{pc} \) units (electron density squared along the line of sight).

The hot gas of electrons residing in galaxy clusters scatter CMB photons, generating \( y \)-distortions of the CMB blackbody emission at the location of galaxy clusters (middle right panel in Fig. 3). This is well-known as the thermal Sunyaev-Zeldovich (SZ) effect (Sunyaev & Zeldovich 1972). While dominant at small angular scales, clustering generates thermal SZ emission on all angular scales in the sky in the low-redshift Universe, and is recognised as an important foreground to early \( \mu \)-distortion anisotropies (Khatri & Sunyaev 2015). Thermal SZ emission from galaxy clusters is generated in our simulations by mapping the MCXC catalogue of galaxy clusters (Piffaretti et al. 2011) from ROSAT (Böhringer et al. 2004) and the SDSS catalogue of galaxy clusters (Koester et al. 2007). The model of \( y \)-Compton parameter flux is derived from the universal cluster pressure profile of (Arnaud et al. 2010), while the scaling of the thermal SZ \( y \)-map across frequency bands in the non-relativistic limit is given by (Sunyaev & Zeldovich 1972):

\[
I_\ell^\text{SZ} = y T_{\text{CMB}} \left( x \coth \left( \frac{x}{2} \right) - 4 \right), \quad x \equiv \frac{h v}{kT_{\text{CMB}}},
\]

in thermodynamic temperature units. We do not include extragalactic compact radio and infra-red sources in our simulations because we are interested in quite large angular scales (\( \ell \lesssim 200 \)) to extract the \( \mu-T \) correlation signal.

Figure 3 provides an overview of all the simulated maps of Galactic and extragalactic foregrounds in the 280 GHz frequency band of the LiteBIRD experiment, as well as the instrumental noise at this frequency, and the total observation sky map at 280 GHz, which consists of foreground emissions, instrumental noise, and CMB temperature and \( \mu \)-distortion anisotropies.

### 2.3 Instrumental specifications

Motivated by the success of the Planck space mission (Planck Collaboration I 2016) in mapping CMB temperature anisotropies with high precision over the full sky, a certain number of future high-sensitive CMB satellite experiments are now being considered around the world, mainly to detect the primordial CMB \( B \)-mode polarization at very large angular scales (Remazeilles et al. 2016, 2017). Future CMB satellites will also allow to probe temperature anisotropies with unprecedented sensitivity at those very large angular scales, where bulk of the \( \mu-T \) correlation signal lies (CORE Collaboration et al. 2016).

**Table 1.** Instrumental specifications for PIXIE (A. Kogut, priv. comm.). The aggregated sensitivity in temperature is \( 4.7 \, \mu \text{K.arcmin} \).

<table>
<thead>
<tr>
<th>Frequency [GHz]</th>
<th>Beam FWHM [arcmin]</th>
<th>( I ) noise r.m.s [( \mu \text{K.arcmin} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>96</td>
<td>219.8</td>
</tr>
<tr>
<td>60</td>
<td>96</td>
<td>59.0</td>
</tr>
<tr>
<td>90</td>
<td>96</td>
<td>29.3</td>
</tr>
<tr>
<td>120</td>
<td>96</td>
<td>19.3</td>
</tr>
<tr>
<td>150</td>
<td>96</td>
<td>14.9</td>
</tr>
<tr>
<td>180</td>
<td>96</td>
<td>13.0</td>
</tr>
<tr>
<td>210</td>
<td>96</td>
<td>12.4</td>
</tr>
<tr>
<td>240</td>
<td>96</td>
<td>12.6</td>
</tr>
<tr>
<td>270</td>
<td>96</td>
<td>13.5</td>
</tr>
<tr>
<td>300</td>
<td>96</td>
<td>15.2</td>
</tr>
<tr>
<td>330</td>
<td>96</td>
<td>17.7</td>
</tr>
<tr>
<td>360</td>
<td>96</td>
<td>21.3</td>
</tr>
<tr>
<td>390</td>
<td>96</td>
<td>26.4</td>
</tr>
<tr>
<td>420</td>
<td>96</td>
<td>33.5</td>
</tr>
<tr>
<td>450</td>
<td>96</td>
<td>43.3</td>
</tr>
<tr>
<td>480</td>
<td>96</td>
<td>57.3</td>
</tr>
<tr>
<td>510</td>
<td>96</td>
<td>76.4</td>
</tr>
<tr>
<td>540</td>
<td>96</td>
<td>103.5</td>
</tr>
<tr>
<td>570</td>
<td>96</td>
<td>142.1</td>
</tr>
<tr>
<td>600</td>
<td>96</td>
<td>197.7</td>
</tr>
<tr>
<td>630</td>
<td>96</td>
<td>277.9</td>
</tr>
<tr>
<td>660</td>
<td>96</td>
<td>393.7</td>
</tr>
<tr>
<td>690</td>
<td>96</td>
<td>564.3</td>
</tr>
<tr>
<td>720</td>
<td>96</td>
<td>810.3</td>
</tr>
<tr>
<td>750</td>
<td>96</td>
<td>1175.2</td>
</tr>
<tr>
<td>780</td>
<td>96</td>
<td>1718.3</td>
</tr>
<tr>
<td>810</td>
<td>96</td>
<td>2528.6</td>
</tr>
<tr>
<td>840</td>
<td>96</td>
<td>3742.0</td>
</tr>
<tr>
<td>870</td>
<td>96</td>
<td>5557.9</td>
</tr>
<tr>
<td>900</td>
<td>96</td>
<td>8315.6</td>
</tr>
<tr>
<td>930</td>
<td>96</td>
<td>12473.4</td>
</tr>
<tr>
<td>960</td>
<td>96</td>
<td>18837.3</td>
</tr>
<tr>
<td>990</td>
<td>96</td>
<td>28510.5</td>
</tr>
<tr>
<td>1020</td>
<td>96</td>
<td>43274.9</td>
</tr>
<tr>
<td>1050</td>
<td>96</td>
<td>66185.2</td>
</tr>
<tr>
<td>1080</td>
<td>96</td>
<td>101399.0</td>
</tr>
<tr>
<td>1110</td>
<td>96</td>
<td>155705.0</td>
</tr>
<tr>
<td>1140</td>
<td>96</td>
<td>240113.3</td>
</tr>
<tr>
<td>1170</td>
<td>96</td>
<td>371231.0</td>
</tr>
<tr>
<td>1200</td>
<td>96</td>
<td>576999.0</td>
</tr>
</tbody>
</table>
Table 2. Instrumental specifications for LiteBIRD. The aggregated sensitivity in temperature is 1.7 µK.arcmin.

<table>
<thead>
<tr>
<th>Frequency [GHz]</th>
<th>Beam FWHM [arcmin]</th>
<th>1 noise r.m.s. [µK.arcmin]</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>69</td>
<td>26.0</td>
</tr>
<tr>
<td>50</td>
<td>56</td>
<td>16.7</td>
</tr>
<tr>
<td>60</td>
<td>48</td>
<td>13.8</td>
</tr>
<tr>
<td>68</td>
<td>43</td>
<td>11.2</td>
</tr>
<tr>
<td>78</td>
<td>39</td>
<td>9.4</td>
</tr>
<tr>
<td>89</td>
<td>35</td>
<td>8.1</td>
</tr>
<tr>
<td>100</td>
<td>29</td>
<td>6.4</td>
</tr>
<tr>
<td>119</td>
<td>25</td>
<td>5.3</td>
</tr>
<tr>
<td>140</td>
<td>23</td>
<td>4.1</td>
</tr>
<tr>
<td>166</td>
<td>21</td>
<td>4.5</td>
</tr>
<tr>
<td>195</td>
<td>20</td>
<td>4.0</td>
</tr>
<tr>
<td>235</td>
<td>19</td>
<td>5.3</td>
</tr>
<tr>
<td>280</td>
<td>24</td>
<td>9.2</td>
</tr>
<tr>
<td>337</td>
<td>20</td>
<td>13.5</td>
</tr>
<tr>
<td>402</td>
<td>17</td>
<td>26.1</td>
</tr>
</tbody>
</table>

The aggregated overall sensitivity of ≃ 7 µK.arcmin in polarization. In polarization mode, two beams observe the sky, while in intensity mode for spectral distortion monopoles, only one beam observes the sky resulting in lower sensitivity. However, we are interested in anisotropies of spectral distortions in our analysis, so that the two beams for polarization can be used in differential mode and averaged, giving effectively an aggregated sensitivity of ≃ 4.7 µK.arcmin in intensity for temperature anisotropies. Table 1 summarizes the instrumental specifications of PIXIE in this chosen configuration (A. Kogut, private communication).

LiteBIRD (Matsumura et al. 2016) is a proposed Japanese CMB satellite experiment, selected for Phase-A study by JAXA (Japan Aerospace Exploration Agency). In its current proposed design (Suzuki et al. 2018), LiteBIRD must observe the full sky through 15 frequency channels ranging from 40 to 402 GHz in order to remove foreground contamination, with a small cross-Dragone telescope of 40 cm diameter, thus providing an overall beam resolution of δθ ≈ 20 arcmin in CMB channels. The LiteBIRD satellite concept is an imager with 2622 detectors composed of Transition Edge Sensor (TES) bolometers, providing a combined sensitivity of about 1.7 µK.arcmin in polarization (1.7 µK.arcmin in intensity).2 The instrumental specifications of LiteBIRD used for our simulations are summarized in Table 2.

CORE (Delabrouille et al. 2017) is a European CMB space mission concept that was proposed to ESA (European Space Agency) in 2016 as a medium-class (M) mission. CORE is an imager allowing to observe the full sky within 19 frequency bands ranging from 60 to 600 GHz for foreground subtraction, and with a cross-Dragone telescope diameter of 120 cm diameter, providing a beam resolution of δθ ≈ 10 arcmin in CMB channels. The focal plane of the CORE satellite would be composed of 2100 sensitive Kinetic Inductance Detectors (KIDs), providing an aggregated sensitivity from all channels of 1.7 µK.arcmin in polarization (1.2 µK.arcmin in intensity). While not selected by ESA, a set of ten papers of the CORE mission concept currently being discussed for proposal to NASA, which would observe the sky in 21 frequency bands ranging from 21 to 800 GHz with 12060 TES detectors, thus providing a combined sensitivity of 0.8 µK.arcmin in polarization (0.8 µK.arcmin in intensity). CORE is thus the most sensitive CMB satellite experiment among the ones investigated in this work, and has the largest sensitivity.

Table 3. Instrumental specifications for CORE. The aggregated sensitivity in temperature is 1.2 µK.arcmin.

<table>
<thead>
<tr>
<th>Frequency [GHz]</th>
<th>Beam FWHM [arcmin]</th>
<th>1 noise r.m.s. [µK.arcmin]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>17.87</td>
<td>7.5</td>
</tr>
<tr>
<td>70</td>
<td>15.39</td>
<td>7.1</td>
</tr>
<tr>
<td>80</td>
<td>13.52</td>
<td>6.8</td>
</tr>
<tr>
<td>90</td>
<td>12.08</td>
<td>5.1</td>
</tr>
<tr>
<td>100</td>
<td>10.92</td>
<td>5.0</td>
</tr>
<tr>
<td>115</td>
<td>9.56</td>
<td>5.0</td>
</tr>
<tr>
<td>130</td>
<td>8.51</td>
<td>3.9</td>
</tr>
<tr>
<td>145</td>
<td>7.68</td>
<td>3.6</td>
</tr>
<tr>
<td>160</td>
<td>7.01</td>
<td>3.7</td>
</tr>
<tr>
<td>175</td>
<td>6.45</td>
<td>3.6</td>
</tr>
<tr>
<td>195</td>
<td>5.84</td>
<td>3.5</td>
</tr>
<tr>
<td>220</td>
<td>5.23</td>
<td>3.8</td>
</tr>
<tr>
<td>255</td>
<td>4.57</td>
<td>5.6</td>
</tr>
<tr>
<td>295</td>
<td>3.99</td>
<td>7.4</td>
</tr>
<tr>
<td>340</td>
<td>3.49</td>
<td>11.1</td>
</tr>
<tr>
<td>390</td>
<td>3.06</td>
<td>22.0</td>
</tr>
<tr>
<td>450</td>
<td>2.65</td>
<td>45.9</td>
</tr>
<tr>
<td>520</td>
<td>2.29</td>
<td>116.6</td>
</tr>
<tr>
<td>600</td>
<td>1.98</td>
<td>358.3</td>
</tr>
</tbody>
</table>

Table 4. Instrumental specifications for PICO (S. Hanany, priv. comm.). The aggregated sensitivity in temperature is 0.8 µK.arcmin.

<table>
<thead>
<tr>
<th>Frequency [GHz]</th>
<th>Beam FWHM [arcmin]</th>
<th>1 noise r.m.s. [µK.arcmin]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>40.9</td>
<td>35.4</td>
</tr>
<tr>
<td>25</td>
<td>34.1</td>
<td>23.3</td>
</tr>
<tr>
<td>30</td>
<td>28.4</td>
<td>15.8</td>
</tr>
<tr>
<td>36</td>
<td>23.7</td>
<td>10.6</td>
</tr>
<tr>
<td>43</td>
<td>19.7</td>
<td>6.4</td>
</tr>
<tr>
<td>52</td>
<td>16.4</td>
<td>4.9</td>
</tr>
<tr>
<td>62</td>
<td>13.7</td>
<td>3.5</td>
</tr>
<tr>
<td>75</td>
<td>11.4</td>
<td>2.8</td>
</tr>
<tr>
<td>90</td>
<td>9.5</td>
<td>2.3</td>
</tr>
<tr>
<td>110</td>
<td>7.9</td>
<td>2.1</td>
</tr>
<tr>
<td>130</td>
<td>6.6</td>
<td>1.9</td>
</tr>
<tr>
<td>155</td>
<td>5.5</td>
<td>1.8</td>
</tr>
<tr>
<td>185</td>
<td>4.6</td>
<td>2.5</td>
</tr>
<tr>
<td>225</td>
<td>3.8</td>
<td>3.7</td>
</tr>
<tr>
<td>270</td>
<td>3.2</td>
<td>6.4</td>
</tr>
<tr>
<td>320</td>
<td>2.7</td>
<td>11.3</td>
</tr>
<tr>
<td>385</td>
<td>2.2</td>
<td>22.6</td>
</tr>
<tr>
<td>460</td>
<td>1.8</td>
<td>53.0</td>
</tr>
<tr>
<td>555</td>
<td>1.5</td>
<td>155.6</td>
</tr>
<tr>
<td>665</td>
<td>1.3</td>
<td>777.8</td>
</tr>
<tr>
<td>800</td>
<td>1.1</td>
<td>7071.1</td>
</tr>
</tbody>
</table>

2 The sensitivities and resolutions of the proposed CMB satellites used in our simulations are just indicative and subject to discussion during the design study of the experiment.

© 0000 RAS, MNRAS 000, 000–000
number of frequency bands and broadest frequency range for foreground subtraction among the CMB imagers considered. The PICO satellite concept will be composed of an open-Drakegon telescope with a diameter of 140 cm, providing a beam resolution of $\delta \theta \approx 7$° in CMB channels. The possibility of adding a small spectrometer that will complement the imager for absolute calibration is still being discussed. The proposed configuration of PICO for our simulations is summarized in Table 4.

For the aforementioned instrumental configurations, the simulated sky components are integrated over the dedicated frequency bands, co-added, and convolved of by a Gaussian beam of full-width half-maximum (FWHM) depending on the frequency channel. Instrumental white noise maps are co-added to the sky frequency maps, depending of the sensitivity of each CMB experiment in each frequency band. These simulations built the starting point of our analysis and can be shared upon request.

### 3 RECONSTRUCTION OF ANISOTROPIC $\mu$-DISTORTIONS

The detection of $\mu$-distortion anisotropies will rely on component separation methods to subtract foreground contamination and reconstruct the signal. Most component separation techniques, either parametric fitting approaches (e.g. Eriksen et al. 2008) or blind variance minimization methods (e.g. Delabrouille et al. 2009; Fernández-Cobos et al. 2012), aim at minimizing the global foreground contamination. However, while the residual contamination is minimized, it is not completely nulled in the reconstructed signal. Also specific residuals might be more of an issue for measuring the $\mu$-$T$ correlation signal than others. For instance, any non-zero residual of CMB temperature anisotropies in the reconstructed $\mu$-distortion map adds spurious $T$-$T$ correlations in the $\mu$-$T$ cross-correlation measurement, therefore largely biasing the signal (see further discussion in Sect. 4.1). Indeed, the spectral template of temperature anisotropies correlates with $\mu$-distortions at a similar level as for example $y$-signals (Chluba & Jeong 2014). Thus, instead of just minimizing the global foreground contamination, it is beneficial to null the unwanted CMB temperature anisotropy component in the reconstructed $\mu$-distortion map, at the expense of slightly more contamination by other foregrounds and noise.

#### 3.1 Component separation methodology

To achieve this, we apply the Constrained ILC component separation method (Remazeilles et al. 2011a) on the set of simulations. Because this technique allows one to reconstruct the $\mu$-distortion anisotropies, while simultaneously nulling CMB temperature anisotropies (and vice versa), it will allow us to minimize potential biases created by temperature fluctuations. It also automatically removes temperature fluctuations related to the integrated-SW of clusters (Creque-Sarbinowski et al. 2016; Ravenni et al. 2017). From the foreground-cleaned, CMB-free, $\mu$-distortion map, $\vec{\mu}$, and the $\mu$-free CMB map, $\vec{T}$, we will then compute the cross-power spectrum, $C^{\mu T}$, between the two maps. The reconstructed cross-power spectrum will allow us to derive forecasts for the detection of the local-type non-Gaussianity parameter, $f_{NL}$, in the presence of foregrounds for the different CMB satellite experiments like PIXIE, LiteBIRD, CORE, and PICO.

For a given frequency band $i$, the sky observation map $x_i$ can be modelled as the combination of different emission components:

$$x_i(\hat{\theta}) = a_{i\mu} s_{\mu}(\hat{\theta}) + a_{iT} s_{CMB}(\hat{\theta}) + n_i(\hat{\theta}),$$

where $s_{\mu}(\hat{\theta})$ is the $\mu$-distortion anisotropy at pixel $\hat{\theta}$, $s_{CMB}(\hat{\theta})$ is the CMB temperature anisotropy in the same direction, and $n_i(p)$ is a “nuisance” term including instrumental noise and Galactic foregrounds in the frequency channel $i$. The $\mu$-distortion and CMB temperature anisotropies scale with frequency through distinct emission laws that are parameterized by the vectors $a_{i\mu}$ and $a_{iT}$ (Eq. 9), with $n_i$ dimensions accounting for the number of frequency bands of the CMB experiment.

Like the standard NILC method (Delabrouille et al. 2009; Remazeilles et al. 2013), the Constrained ILC method (Remazeilles et al. 2011a) constrains a minimum-variance weighted linear combination of the sky maps:

$$\delta_{\mu}(\hat{\theta}) = w^T x(\hat{\theta}) = \sum_{i=1}^{N_i} w_i x_i(\hat{\theta}) (w^T = \text{the transpose of } w),$$

under the condition that the scalar product of the weight vector, $w$, and the $\mu$-distortion SED vector, $a_{\mu}$, is equal to unity, i.e. $\sum_{i=1}^{N_i} w_i a_{\mu i} = 1$. This guarantees the full conservation of the $\mu$-distortion anisotropies in the reconstruction.

However, the Constrained ILC (Remazeilles et al. 2011a) generalizes the standard NILC method by offering an additional constraint for the ILC weights to be orthogonal to the CMB emission law, $a_{iT}$, while guaranteeing the conservation of the $\mu$-distortion component. The Constrained ILC estimate of the CMB-free map of $\mu$-distortion anisotropies, $\hat{s}_{\mu}(\hat{\theta}) = w^T x(\hat{\theta})$, is thus a solution of the minimization problem:

$$\min_w E [s_{\mu}^2], \quad \min_w w^T a_{\mu} = 1, \quad w^T a_{iT} = 0. \tag{15c}$$

Benefiting from the knowledge of the spectral shapes of $\mu$-distortion and CMB temperature anisotropies, the weights of the Constrained ILC are thus adjusted to yield simultaneously unit response to the $\mu$-distortion emission law $a_{\mu}$ (Eq. 15b) and zero response to the CMB emission law $a_{iT}$ (Eq. 15c).

The method is blind in the sense that no parametrization or assumption is made on the foregrounds, but just for the $\mu$ and temperature signals. The two-dimensional constraint (Eqs. 15b, 15c) allows us to null CMB temperature anisotropies, while preserving $\mu$-distortion anisotropies. The residual contamination from Galactic foregrounds and instrumental noise is also controlled through the minimum-variance condition (Eq. 15a). The exact expression for the Constrained ILC weights was derived in Remazeilles et al. (2011a) by solving the minimization problem Eqs. (15a)–(15c), which for a CMB-free reconstruction of $\mu$-distortion anisotropies is given by:

$$\delta_{\mu}^{\text{CMB-free}}(\hat{\theta}) = \left( a_{\mu}^T C^{-1} a_{\mu} \right)^{-1} a_{\mu}^T C^{-1} \left( a_{\mu}^T C^{-1} a_{\mu} \right)^{-1} \left( a_{\mu}^T C^{-1} a_{\mu} \right)^{-1} x(\hat{\theta}). \tag{16}$$

Here $C^{\mu}$ is the covariance matrix of the sky channel maps. Similarly, a $\mu$-free estimate of CMB anisotropies can be derived by exchanging $a_{\mu}$ and $a_{iT}$ in the formula Eq. (16).

One can also easily verify that applying the Constrained ILC...
weights (Eq. 16) to the frequency maps (Eq. 14) yields:

\[
\hat{x}_\mu^{\text{free}} = \left( \frac{\hat{a}_\mu^1}{C_{\mu\mu}^{-1/2}} \right) \left( \frac{\hat{a}_\mu^1}{a_\mu^1} \right) a_\mu^{-1} \hat{a}_\mu - \left( \frac{\hat{a}_\mu^1}{a_\mu^1} \right) \left( \frac{\hat{a}_\mu^1}{a_\mu^1} \right) a_\mu^{-1} \hat{a}_\mu.
\]

\[
+ \frac{\left( \frac{\hat{a}_\mu^1}{C_{\mu\mu}} \right) \left( \frac{\hat{a}_\mu^1}{a_\mu^1} \right) a_\mu^{-1} \hat{a}_\mu - \left( \frac{\hat{a}_\mu^1}{a_\mu^1} \right) \left( \frac{\hat{a}_\mu^1}{a_\mu^1} \right) a_\mu^{-1} \hat{a}_\mu}{a_\mu^1 a_\mu^{-1} a_\mu^1 a_\mu^{-1} a_\mu^1 a_\mu^{-1} a_\mu^1 a_\mu^{-1}} \right) \times \text{SCMB}.
\]

\[
= 1 s_\mu + 0 a_\mu + w' n.
\]

In other words, the \(\mu\)-distortion signal, \(s_\mu\), is reconstructed without any bias by this projection (first term of Eq. 17), while the CMB temperature signal is not just mitigated but cancelled out owing to the orthogonal weighting (second term of Eq. 17). Residual foregrounds and instrumental noise are minimized by the minimum-variance weighting (third term of Eq. 17).

It should be noted that additional constraints can in principle be implemented in the Constrained ILC pipeline, for example to also cancel the contaminations by \(y\)-distortion anisotropies. However, any added constraint usually comes with the cost of further increasing the noise in the reconstructed \(\mu\)-distortion map, since less information is available for reducing the noise variance. To address the limitations caused by this aspect, we plan more comprehensive simulations also including information from CMB polarization.

Lastly, the Constrained ILC method also operates on a needlet (spherical wavelet) frame (Narcovich et al. 2006) because the localization properties of the needlets allows the weights of components to re-adjust depending on the local conditions of foreground contamination both over the sky and over the angular scales.

3.2 Main results

The result of foreground removal and reconstruction of a “\(\mu\)-free” CMB temperature anisotropy map by the Constrained ILC method is shown in Fig. 4 for LiteBIRD. We can clearly see that the method nicely recovers the input temperature sky map, as expected from the huge signal-to-noise in this case.

Similarly, Fig. 5 shows the reconstruction of the “CMB-free” map of \(\mu\)-distortion anisotropies for the specifications of LiteBIRD (as an example), and for different values of the non-Gaussianity parameter: \(f_{\text{NL}} = 10^4\) (top panels), \(f_{\text{NL}} = 10^6\) (middle panels), and \(f_{\text{NL}} = 10^8\) (bottom panels), the latter being chosen mainly for illustration. While guaranteed by the Constrained ILC method, the total absence of residual CMB temperature \((\leftrightarrow \mu\)-distortion) anisotropies in the reconstructed \(\mu\)-map \((\leftrightarrow \text{CMB map})\) is not obvious by looking at the maps, but will be clearly established when considering the angular power spectra (see e.g. Sect. 4.1). We see from Fig. 5 how foreground removal for faint \(\mu\)-distortion anisotropies becomes challenging for decreasing values of \(f_{\text{NL}}\), despite the 15 frequency bands of LiteBIRD. For instance, the \(\mu\)-map for \(f_{\text{NL}} = 10^4\) is clearly still contaminated by strong residual foregrounds, while we obtain accurate reconstruction of the \(\mu\)-distortion anisotropies for \(f_{\text{NL}} \gtrsim 10^5\). However, even for \(f_{\text{NL}} = 10^6\) the \(\mu\)-distortion anisotropies could still be detected by cross-correlation with CMB temperature anisotropies, as we will show below.

As mentioned several times, one advantage of the Constrained ILC reconstruction is that the resulting \(\mu\)-distortion map is free from any residual CMB temperature anisotropies, and vice versa, so that we can measure the cross-power spectrum, \(C_{\ell}^{\mu\mu}\), between \(\mu\)-distortion and CMB temperature anisotropies without suffering from spurious T-T correlations arising from CMB residuals in the \(\mu\) map, and thus obtain unbiased estimates of \(f_{\text{NL}}\). Figures 6 and 7 present our results from the calculation of the cross-power spectrum between the reconstructed CMB and \(\mu\)-distortion maps (red points) after component separation with the Constrained ILC, for the different CMB experiments and different values of \(f_{\text{NL}}\). The multipole bin width is \(\Delta \ell = 30\), and the uncertainty on \(C_{\ell}^{\mu\mu}\) has been computed in each bin using (Tristram et al. 2005):

\[
\sigma(C_{\ell}^{\mu\mu}) = \sqrt{\frac{C_{\ell}^{\mu\mu} C_{\ell}^{TT} + C_{\ell}^{\mu\mu}}{2(2\ell + 1)f_{\text{sky}}}},
\]

therefore including residual foregrounds leaking in the power spectra of the reconstructed CMB temperature and \(\mu\)-distortion maps. The fraction of the sky used for component separation is \(f_{\text{sky}} = 0.66\). The reconstructed \(C_{\ell}^{\mu\mu}\) can be compared to the cross-power spectrum of the input CMB and \(\mu\) realizations of the simulation (black lines) and and the theory \(C_{\ell}^{\mu\mu}\) (green lines).

From the reconstructed \(\mu\)-T cross-power spectrum after component separation, we can derive Fisher forecasts on the primordial non-Gaussianity parameter, \(f_{\text{NL}}\), in the presence of foregrounds. The \(1\sigma\) uncertainty on \(f_{\text{NL}}\) is computed using the Fisher information as:

\[
\sigma(f_{\text{NL}}) = [\sum_{\ell} \left( \frac{1}{\sigma_{\ell}} \frac{1}{\sigma_{\ell}(C_{\ell}^{\mu\mu})} \sigma_{\ell}(C_{\ell}^{\mu\mu}) \right)^{1/2} ]^{1/2},
\]

where \(\sigma_{\ell}(C_{\ell}^{\mu\mu})\) are the \(1\sigma\) error bars on the reconstructed cross-power spectrum (assuming a diagonal covariance matrix across
Figure 5. CMB-free reconstruction of \( \mu \)-distortion anisotropies \( (\langle \mu \rangle = 2 \times 10^{-8}) \) with the Constrained ILC component separation method at 69\(^\circ\) resolution corresponding to LiteBIRD, and for increasing values of the non-Gaussianity parameter: \( f_{NL} = 10^4 \) (top), \( f_{NL} = 10^5 \) (middle), \( f_{NL} = 10^6 \) (bottom). Left panels: input \( \mu \)-map realizations. Right panels: Constrained ILC \( \mu \)-map reconstructions. The case with \( f_{NL} = 10^6 \) is mainly shown as a demonstration that the method can recover the anisotropic \( \mu \)-distortion signal.

In addition to the uncertainty limit, \( \sigma(f_{NL}) \), we quantify the bias on \( f_{NL} \) due to foreground residuals by computing the maximum-likelihood estimate:

\[
\hat{f}_{NL} = \frac{\sum \tilde{C}_\mu^\mu \times T(f_{NL} = 1) / \sigma^2}{\sum \left[ C_\mu^\mu \times T(f_{NL} = 1) \right] / \sigma^2},
\]

where \( C_\mu^\mu \times T(f_{NL} = 1) \equiv \partial C_\mu^\mu \times T / \partial f_{NL} \) is the theoretical cross-power spectrum from Eq. (2) for the fiducial value of \( f_{NL} = 1 \). Our forecasts on \( f_{NL}(k = 740 \text{ Mpc}^{-1}) \) are summarized in Tables 5 and 6 for the four CMB satellite configurations.

3.2.1 Results without foregrounds
In Chluba et al. (2017a), the detection limit on \( f_{NL} \) from the measurement of anisotropic \( \mu \)-distortions by future CMB satellites have been estimated to be of order \( f_{NL} \lesssim 4500 \) for \( \langle \mu \rangle = 2 \times 10^{-8} \) in the absence of foregrounds. In order to verify this assertion, let us first consider PIXIE, LiteBIRD, CORE, and PICO simulations without foregrounds, therefore including only instrumental noise and cor-

© 0000 RAS, MNRAS 000, 000–000
Figure 6. Measurement of the cross-power spectrum, $\hat{C}^{\mu \times T}_\ell$, for $f_{NL} = 4500$ in the absence of foregrounds (left panels) and with foregrounds (right panels) for PIXIE (first row), LiteBIRD (second row), CORE (third row), and PICO (last row): theory (green), input realization (black), Constrained ILC reconstruction (red). The beam resolution adopted for the reconstruction is 96′ for PIXIE, 69′ for LiteBIRD, and 60′ for CORE and PICO.
Figure 7. Similar to Fig. 6 with foregrounds included and for $f_{NL} = 10^4$ (left panels) and $f_{NL} = 10^5$ (right panels).
related CMB temperature and \( \mu \)-distortion anisotropies in each frequency band, with fiducial values \( f_{\text{NL}} = 4500 \) and \( (\mu) = 2 \times 10^{-8} \). The left panels of Fig. 6 show the reconstructed \( \mu \)-T cross-power spectrum after component separation in this case. Indeed, using the Constrained ILC method, without foregrounds we do recover the scale-dependence and amplitude of the cross-power spectrum \( C_{\ell}^{\mu \times T} \) for all CMB satellite experiments. Due to limited angular resolution, \textit{PIXIE} quickly loses leverage for modes at \( \ell > 10^2 \), while these are still accessible for the other configurations.

The derived limits on \( f_{\text{NL}} \) in the absence of foregrounds are listed in the last column of Table 5 when integrating over multipoles \( 2 \leq \ell \leq 200 \). The most sensitive experiment, \textit{PICO}, could measure \( f_{\text{NL}} = 4500 \) with 12\( \sigma \) significance in the absence of foregrounds, while the least sensitive experiment, \textit{PIXIE}, would detect \( f_{\text{NL}} = 4500 \) at 1.2\( \sigma \), consistent with earlier estimations (Chluba et al. 2017a). In the absence of foregrounds, \textit{LiteBIRD} and \textit{CORE} could detect \( f_{\text{NL}} = 4500 \) at 5\( \sigma \) and 7\( \sigma \) significance, respectively.

3.2.2 Results with foregrounds

In the presence of foregrounds, the bias and uncertainty on the reconstructed \( \mu \)-distortion signal increase dramatically. The right panels of Fig. 6 show the reconstructed \( \mu \)-T cross-power spectrum after foreground cleaning for all CMB experiments assuming \( f_{\text{NL}} = 4500 \). In this case, the measurement of the \( \mu \)-T correlation signal is of poorer quality at all angular scales, with a reconstructed \( C_{\ell}^{\mu \times T} \) compatible with zero for \textit{PIXIE}, \textit{LiteBIRD} and \textit{CORE}. Conversely, due to a higher sensitivity and large number of frequencies, \textit{PICO} is the only CMB experiment among those considered here which is able to detect a signal for \( f_{\text{NL}} = 4500 \) in the presence of foregrounds (bottom right panel of Fig. 6), with a measurement of \( f_{\text{NL}} = 4500 \) at \( \sim 2 \sigma \) significance (third column of Tables 5 and 6).

For increasing \( f_{\text{NL}} \geq 10^4 \) (Fig. 7), \textit{LiteBIRD}, \textit{CORE}, and \textit{PICO} clearly detect the \( \mu \)-T correlation signal even after foreground cleaning with Constrained ILC. Because of increased sensitivity and number of frequency bands, \textit{PICO} performs better than \textit{CORE} and \textit{LiteBIRD} in the presence of foregrounds. Typically, \textit{PICO} could allow to constrain \( f_{\text{NL}} = 10^4 \) at \( \approx 3 \sigma \) (fourth column of Table 5) after foreground cleaning (third column of Table 5), while \textit{LiteBIRD} and \textit{CORE} could detect \( f_{\text{NL}} = 10^4 \) at \( \approx 1.5 \sigma \). Conversely, mainly due to lower sensitivity, \textit{PIXIE} cannot detect \( f_{\text{NL}} \leq 10^4 \) after foreground cleaning with the Constrained ILC.

Lastly, if primordial non-Gaussianity is as large as \( f_{\text{NL}} = 10^4 \) on scales \( k \approx 740 \text{Mpc}^{-1} \), then the four CMB experiments would be able to measure the \( \mu \)-T correlation signal (Fig. 7), with \textit{PIXIE} detecting \( f_{\text{NL}} = 10^4 \) with 2.5\( \sigma \) significance, \textit{LiteBIRD} and \textit{CORE} with \( \approx 12 \sigma \) significance, and \textit{PICO} with \( \approx 18 \sigma \) significance (second column of Table 5). Again due to its lower angular resolution, \textit{PIXIE} looses modes at \( \ell > 10^2 \) (see upper row of Fig. 7).

It should be noted that the relative sensitivity between different CMB experiments in the presence of foregrounds depends on the foreground complexity assumed in the simulations. For much more complex foregrounds than in our simulations, \textit{LiteBIRD}, \textit{CORE} and \textit{PICO} may face additional difficulties in capturing the full signal complexity for detecting \( \mu \)-distortions. In this case, spectrometer concepts like \textit{PIXIE} could make a difference, being able to provide additional information thanks to the large number of available frequency bands, though at lower overall channel sensitivity. Conversely, in the case of basic foregrounds, where the foreground subtraction can be controlled decently with 15 to 20 frequencies, it is the overall sensitivity that makes a difference.

3.3 Results on the updated model of \( \mu \)-T correlations

As mentioned above, while this work was being completed, a more accurate modelling of the \( \mu \)-T cross-power spectrum on a wider range of angular scales has been obtained by Ravenni et al. (2017). In this new model, the crossing point at which \( \mu \)-distortion anisotropies anti-correlate with CMB temperature anisotropies is at a slightly smaller angular scale (\( \ell \approx 50 \) versus \( \ell \approx 40 \)) compared to the model used in this work. At large scales, the overall amplitude of the signal is the same in the two models, however, the improved model allows one to include more modes beyond \( \ell \approx 200 \), which we do find to improve the constraints at the level of \( \approx 20 - 30 \% \).

To complete our analysis, we performed additional sky simulations in which the \( \mu \)-distortion map and the CMB map are simulated as correlated fields according to this improved model. In this simulation, the foregrounds and the noise remain unchanged with respect to the other simulations used in this work. We only considered the most challenging case investigated in this work, i.e. \( f_{\text{NL}} = 4500 \), and we adopt the instrumental configuration of \textit{PICO}, for which we have obtained the best results throughout this work.

Figure 8 (upper panel) shows the reconstruction of \( C_{\ell}^{\mu \times T} \) by the Constrained ILC for the \textit{PICO} simulation based on the new

<table>
<thead>
<tr>
<th>( f_{\text{NL}} ) (fiducial)</th>
<th>10^5</th>
<th>10^4</th>
<th>4500</th>
<th>4500 w/o foregrounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{PIXIE}</td>
<td>((1.11 \pm 0.40) \times 10^5)</td>
<td>((2.17 \pm 3.90) \times 10^4)</td>
<td>((1.5 \pm 3.9) \times 10^4)</td>
<td>((4778 \pm 3868))</td>
</tr>
<tr>
<td>\textit{LiteBIRD}</td>
<td>((2.5 \sigma))</td>
<td>((1.5 \sigma))</td>
<td>((1.2 \sigma))</td>
<td>((4.6 \sigma))</td>
</tr>
<tr>
<td>\textit{CORE}</td>
<td>((0.98 \pm 0.08) \times 10^5)</td>
<td>((0.91 \pm 0.68) \times 10^4)</td>
<td>((4272 \pm 6788))</td>
<td>((4753 \pm 930))</td>
</tr>
<tr>
<td>\textit{PICO}</td>
<td>((0.97 \pm 0.08) \times 10^5)</td>
<td>((1.35 \pm 0.74) \times 10^4)</td>
<td>((5692 \pm 6397))</td>
<td>((4336 \pm 653))</td>
</tr>
<tr>
<td>\textit{PICO}</td>
<td>((1.2 \sigma))</td>
<td>((1.4 \sigma))</td>
<td>((6.9 \sigma))</td>
<td>((12.1 \sigma))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( f_{\text{NL}} ) (fiducial)</th>
<th>(-4500)</th>
<th>0</th>
<th>4500</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{PICO}</td>
<td>(-2996 \pm 2112)</td>
<td>1325 \pm 2114</td>
<td>5698 \pm 2121</td>
</tr>
</tbody>
</table>
The adopted beam resolution for the reconstruction is $41^\prime$ (et al. (2017) after foreground cleaning with the Constrained ILC: theory

Figure 8. Simulated PICO measurement of the cross-power spectrum, $C_{\ell}^{\mu\mu}$, for $f_{NL} = \{4500, 0, -4500\}$ based on the full model of Ravenni et al. (2017) after foreground cleaning with the Constrained ILC: theory (green), input realization (black), Constrained ILC reconstruction (red). The adopted beam resolution for the reconstruction is $41^\prime$.

input model. Clearly, the reconstruction is of similar quality as presented in Fig. 6, but extending to slightly smaller angular scales. The constraints for different fiducial values of $f_{NL}$ are summarized in Table 6. The resulting 1σ uncertainty of PICO for using the model of Ravenni et al. (2017) is $\sigma(f_{NL} = 4500) = 3059$, when including modes at $2 \leq \ell \leq 200$. Comparing this to the previous constraint, i.e., $\sigma(f_{NL} = 4500) = 2929$ from Table 5, shows that the conclusion is not affected significantly. Also adding modes with $200 \leq \ell \leq 500$, we find $\sigma(f_{NL} = 4500) = 2121$. It highlights that the additional gain from modes at $\ell > 200$ is about $\pm 30\%$, improving the expected detection significance for $f_{NL} = 4500$ from $\approx 1.5\sigma$ to $\approx 2\sigma$ after foreground cleaning.

It has been shown by Ravenni et al. (2017) that correlations between $\mu$-distortions and CMB $E$-mode polarization anisotropies could add constraining power to $f_{NL}$ measurements because of more available modes. However, this assertion has to be confronted to the presence of strongly polarized foregrounds, for which foreground cleaning for polarization might be more challenging than for intensity. This will be investigated in a future work.

3.3.1 Constraints on $f_{NL} \leq 0$

As mentioned above, we also checked the detection significance for $f_{NL} < 0$. For $f_{NL} = -4500$, The reconstructed cross-power spectrum is shown in the lower panel of Fig. 8, and Table 6 provides the expected PICO constraint. Comparing the result with that for $f_{NL} = 4500$ shows that the reconstruction is fairly insensitive to the sign of $f_{NL}$: the estimated errors are comparable and the biases are consistent with statistical fluctuations. Note that $f_{NL} = -4500$ corresponds to $f_{NL}^{\text{WMAP}} = 4500$ in the standard WMAP definition, so that our sign convention is not expected to affect the main conclusions significantly.

We also ran a simulation without including any $\mu$-signal in the simulations. The reconstruction is shown in the middle panel of Fig. 8, clearly being consistent with $f_{NL} = 0$ at all $\ell$. The obtained constraint is $|f_{NL}| < 2114$ (see Table 6), which is consistent with the 1σ error in the simulations with non-zero signal, $|f_{NL}| = 4500$.

4 DISCUSSION

4.1 Standard ILC versus Constrained ILC

As mentioned earlier, most of the component separation techniques aim at minimizing the variance of the global foreground contamination in the reconstructed signal, e.g., as for standard ILC methods like NILC (Delabrouille et al. 2009). However, in the context of reconstructing the $\mu$-T correlation signal, CMB temperature anisotropies are a main foreground to the anisotropic $\mu$-distortion signal, and this CMB foreground could in addition be spatially correlated to the $\mu$-distortion signal due to primordial non-Gaussianity, so that residuals of CMB temperature anisotropies in the reconstructed $\mu$-distortion map after component separation will add residual $T$-$T$ correlations in the $\mu$-$T$ correlation measurement. Even if CMB temperature anisotropies are minimized in the $\mu$-distortion map by standard component separation techniques, the spurious $T$-$T$ correlation can be large enough to bias the measurement of the $\mu$-$T$ correlation signal. It is therefore essential to eliminate CMB temperature anisotropies in the component separation process rather than minimizing the global foreground contamination. This strong constraint is made possible by the Constrained ILC method (Remazeilles et al. 2011a) through the knowledge of the SED of both CMB temperature and $\mu$-distortion anisotropies.

To illustrate our point, in Fig. 9 we compare the reconstruction of the cross-power spectrum $C_{\ell}^{\mu\mu}$ between the standard ILC method (blue) and the Constrained ILC method (red), in the case of LiteBIRD for $f_{NL} = 10^5$. Without the extra orthogonality condition on the CMB SED, the standard ILC reconstruction of $C_{\ell}^{\mu\mu}$ shows a significant bias at all angular scales, due to spurious $T$-$T$ correlations between the CMB temperature anisotropies and the

© 0000 RAS, MNRAS 000, 000–000
will differ from the physical SED of the foregrounds on the sky because of finite-size pixelization and beam convolution, which both lead to an average of the spectral parameters from different lines-of-sight. Due to averaging, say different power-law SEDs (with varying spectral indices for different lines-of-sight), the effective SED within a pixel is no longer described by a power-law. Similarly, the effect of averaging along one line-of-sight is unavoidable.

Therefore, the effective foreground SED in each pixel differs from the physical SED in each line-of-sight. As shown by Remazeilles et al. (2017), the spurious curvature created by averaging effects, if ignored in the parametric modelling, is significant enough to bias the reconstruction of the faint primordial CMB B-mode signal at a level of \( r \approx 10^{-5} \), which is the sensitivity target of current CMB satellite concepts. Thus, due to the very large dynamic range between the foregrounds and the signals (\( \mu \)-distortions and CMB B-modes), the impact of foreground mismodelling can no longer be ignored. Beyond simple pixel/beam averaging effects, the spherical harmonic transforms that are performed on the maps by most component separation algorithms effectively also is a weighted average of different SEDs (Chluba et al. 2017b), thus adding even further complications on foreground cleaning.

With these subtle averaging effects in mind, it is useful to implement blind component separation approaches, such as the Constrained ILC, in which there is no parametrisation of the foreground SEDs in the process, thus making it less susceptible to mismodelling. However, for a detection of faint cosmological signals such as \( \mu \)-type distortion and primordial B-modes one will always want to compare and/or combine independent component separation methods, both parametric and blind, to ensure the robustness of the obtained result.

### 4.3 Optimization: more detectors or more frequencies?

In this section, we open a discussion on the optimization of the configuration of a CMB experiment to improve the quality of the \( C_\ell^{\mu B} \) measurement in the presence of foregrounds. The main question we address is: do we need more frequencies or more detectors (\( \leftrightarrow \) sensitivity)? A similar optimization is also highly relevant to ongoing and planned B-mode searches. In this regard, we considered a “super-LiteBIRD” experiment having the same frequency range and distribution (40–402 GHz) as LiteBIRD but 100 times more detectors per frequency, therefore an overall sensitivity increased by a factor 10 (i.e. \( \sim 0.2 \mu K \text{arcmin} \)). While applied here to spectral distortions, the conclusions are instructive for B-mode searches.

In Fig. 10, we show the reconstructed cross-power spectrum \( C_\ell^{\mu B} \) for \( f_{\text{NL}} \approx 4500 \), with a “super-configuration” of LiteBIRD. When increasing the number of detectors by a factor of 100, the uncertainties on \( C_\ell^{\mu B} \) are slightly reduced by about 8% when compared to the baseline configuration presented in Fig. 6. Nevertheless the reconstructed signal is still consistent with zero, even for this large boost in sensitivity. This suggests that astrophysical foregrounds, not the instrumental noise, set the ultimate limit to which the \( \mu \)-T correlation signal can be measured. With more frequencies (21–800 GHz) but lower overall sensitivity (0.8 \( \mu K \text{arcmin} \)), PICO (bottom right panel of Fig. 6) actually obtains a higher level of detection of the \( \mu \)-distortion signal than a super-LiteBIRD concept. This indeed demonstrates that the uncertainty after foregrounds cleaning by the Constrained ILC will decrease more significantly if we increase the number of frequency bands and the frequency range rather than the number of detectors.

We note that PIXIE has the largest number of frequency bands among all the proposed CMB experiments, nevertheless PIXIE

![Figure 9. Standard ILC (blue) versus Constrained ILC (red), on the example of LiteBIRD and \( f_{\text{NL}} = 10^5 \). The constraint of orthogonality of the Constrained ILC weights to the SED of CMB anisotropies guarantees the absence of any CMB contamination in the reconstructed \( \mu \)-distortion map. The Constrained ILC approach is essential to avoid spurious/unphysical \( \mu \)-T correlations that are inherent to standard component separation methods because of residual CMB contamination in the reconstructed signal.](image-url)
results are of poorer quality compared to CORE, LiteBIRD, and PICO. One reason is that the overall sensitivity of PIXIE is significantly lower (e.g., more than 4 times lower than CORE). This results in a degradation of the ILC foreground cleaning due to the trade-off between minimizing the variance of foregrounds and the variance of noise. However, it is also the lack of angular resolution that diminishes the constraining power of PIXIE even at large angular scales. Considering a simulation where PIXIE has 40’ resolution (instead of 96’) in each frequency band, while keeping the same sensitivity (4.7 μK.arcmin) and frequency coverage, we find that the constraint on $f_{NL} = 10^6$ from multipoles 2 $\leq \ell \leq 200$ is improved by more than 50%, leading to $\sigma(f_{NL} = 10^6) = 1.78 \times 10^4$ (compare with Table 5). PIXIE would thus benefit from higher resolution in each spectral band to allow the use of the spatially-correlated information during component separation and access valuable additional modes at $\ell \simeq 100 – 200$.

Last, we are interested in knowing which part of the frequency range (low- or high-frequencies) is the most important for the reconstruction of the $\mu$-distortion signal by the Constrained ILC. We thus investigate two descoped configurations of PICO, one without low-frequency channels (i.e. 17 frequencies from 43 to 800 GHz) and the other one without high-frequency channels (i.e. 18 frequencies from 21 to 460 GHz). We find that, in the absence of frequencies above 460 GHz the detection of $f_{NL} = 4500$ by PICO would be degraded by about 7%, while in the absence of frequencies below 43 GHz, the detection would be degraded by about 30%, lowering the detection of $f_{NL} = 4500$ to 1σ significance (2 $\leq \ell \leq 200$). Therefore, for the given sensitivities of PICO we conclude that frequencies below 43 GHz are more essential than frequencies above 460 GHz for a detection of $\mu$-T correlation signals.

4.4 Calibration uncertainties and the imperfect knowledge of the CMB monopole temperature

Although future CMB imagers like PICO are in a good position to detect $\mu$-distortion anisotropies for $f_{NL} \gtrsim 4500$, the interpretation has to rely on knowledge of the average sky spectrum and its distortion. Indeed, the $\mu$-T correlation signal is proportional to the product $f_{NL}(\mu)$, causing a direct degeneracy between $f_{NL}$ and the monopole distortion $\langle \mu \rangle$ that can only be broken through absolute measurements. In this respect, Fourier transform spectrometers like PIXIE might be essential for disentangling different effects (see also Chluba et al. 2017).

In addition, relative inter-channel calibration errors may impact the component separation. It has been shown by Dick et al. (2010) that 1% calibration errors on the CMB spectrum in Planck channels can dramatically degrade the reconstruction of CMB temperature anisotropies by an ILC method in the high signal-to-noise regime, because of the mismatch between the actual CMB SED in the miscalibrated experiment and the assumed CMB SED in the ILC weights. For current CMB temperature measurements (0.1% calibration errors for Planck), this seems to be sufficiently under control (Dick et al. 2010), but for the large dynamic range encountered with $\mu$-distortion signals additional studies are required.

We therefore ran several simulations with different levels of calibration uncertainties (covering $\simeq 0.01\% - 1\%$) and different levels of non-Gaussianity ($f_{NL} = 10^5$ and $f_{NL} = 4500$). For calibration errors $\simeq 0.1\%$ in PICO channels, we find a significant bias/loss on the reconstruction of the $\mu$-T correlated signal, even for a large $f_{NL} = 10^6$ value, because of the partial erasing by the ILC of the variance of the CMB temperature map in the high signal-to-noise regime (Dick et al. 2010). The sensitivity of future CMB satellite experiments like PICO is about an order of magnitude larger than the Planck sensitivity, therefore increasing the high signal-to-noise regime for CMB temperature anisotropies, which makes the CMB $T$ signal reconstruction with PICO more sensitive to miscalibration than with Planck. We thus find that the allowed calibration uncertainty for PICO has to be $\leq 0.01\%$, which is the typical level of precision expected for CORE (e.g. Burigana et al. 2017). The conclusion holds also for lower $f_{NL} = 4500$ values, for which we find that 0.01% calibration errors guarantee the full recovery of the CMB $T$ signal, while only slightly impacting the reconstruction of the $\mu$ signal, with less than 0.6σ bias on $f_{NL} = 4500$, i.e. an error of $\Delta f_{NL} = 10^3$. This is also found by extrapolation from earlier estimate by Ganc & Komatsu (2012).

The absolute error in the CMB monopole temperature, $T_{CMB}$, which currently is known to $\Delta T_{CMB} \simeq 1$ mK (Fixsen et al. 1996; Fixsen 2009), will furthermore result in an error of the differential SED of CMB temperature anisotropies and therefore could also impact the component separation processes. For the $\mu$-distortion SED, $\Delta T_{CMB}$ implies a tiny relative error ($\simeq 0.1\%$) on a small signal, which can thus be neglected. However, the much larger CMB temperature anisotropies, will lead to an extra term $\Delta T_{CMB}/CMB$ in Eq. (14) that could cause spurious (correlated) residuals in the $\mu$-distortion map with the Constrained ILC method. This is because the assumed CMB temperature SED now has a new (γ-type) frequency dependence, $\Delta T_{CMB}/CMB$ in Eq. (15c). For enhanced $\mu$-anisotropies studied here, we estimate this to matter at the level of $\Delta f_{NL} \simeq 10^5$.

While this level of precision for $f_{NL}$ from $\mu-T$ correlations is still futuristic, the current uncertainty in the CMB monopole could become a serious challenge for recently discussed methods that are meant to use differential measurements in frequency (without requiring an absolute measure of the CMB monopole flux) to extract average CMB distortion signals (Mukherjee et al. 2018). The CMB monopole temperature could be measured to $\Delta T_{CMB} \simeq f \times 10^5$ precision with a spectrometer like PIXIE (Abitbol et al. 2017), which would certainly eliminate this potential problem. For similar reasons could a combination of absolutely calibrated maps (using a PIXIE-like spectrometer) with future CMB imagers allow us to greatly reduce calibration uncertainties, potentially opening a way.
to access small spectral-spatial signals, e.g., caused by resonance scattering terms (Basu et al. 2004).

5 CONCLUSIONS

Using sky simulations with foregrounds, we demonstrated that future CMB satellite experiments could be able to achieve a detection of $\mu$-distortion anisotropies caused by primordial non-Gaussianity by cross-correlating with CMB temperature anisotropies. In particular, the CMB satellite concept PICO, with its broad frequency range and high sensitivity for accurate foreground removal, among the considered satellite concepts is in the best position to detect primordial $\mu$-T correlations for $|f_{NL}(k \approx 740 \text{ Mpc}^{-1})| \gtrsim 4500$. While this is far off the limits obtained with CMB measurements at scales $k \approx 10^{-2} \text{ Mpc}^{-1}$, we emphasize again that this still provides interesting new constraints on the scale-dependence of non-Gaussianity (see also Biagetti et al. 2013; Emami et al. 2015; Ravenni et al. 2017). Our forecasts on the primordial non-Gaussianity parameter $f_{NL}$ for the four different CMB satellite concepts: PIXIE, LiteBIRD, CORE and PICO are given in Tables 5 and 6. This work thus provides the first realistic predictions for the detection of $\mu$-distortion anisotropies in the presence of foregrounds, showing that PICO could place an upper limit $|f_{NL}(k \approx 740 \text{ Mpc}^{-1})| < 4200$ (95% c.l.) on local-type non-Gaussianity in the ultra-squeezed limit.

We demonstrated the necessity of nulling CMB temperature anisotropies in the reconstruction of the $\mu$-distortion anisotropies during the component separation process in order to avoid biasing the measurement of the $\mu$-T correlation signal by residual $T$-$T$ correlations (Sect. 4.1). In this regard, we proposed a tailored component separation method, the Constrained ILC, to reconstruct (CMB-free) $\mu$-distortion maps. Our simulations did not include additional information from (primordial) $\gamma$-$T$ and distortion-polarization correlations, which have been shown to improve (foreground-free) detection limits significantly (e.g., Ravenni et al. 2017). Thus, the limits derived here could still improve noticeably. However, when adding information from polarization new foreground challenges do appear, such that it is hard to anticipate the outcome. We plan to generalize our method to take these effect into account and then answer this question in future work.

We argued that for future CMB imagers a broad frequency coverage and large number of bands (with the decent sensitivity) is preferred over hugely improved sensitivity in a limited frequency range when attempting to extract $\mu$-distortion anisotropies by the Constrained ILC method (see Sect. 4.3). In particular, coverage at low frequencies seems important, while for the simulations carried out here high-frequency channels can be sacrificed. This conclusion depends strongly on the type of the signal and the assumed complexity of foregrounds. For example, when comparing to $B$-mode searches, the (unpolarized) $\mu$-distortion signal has a spectral dependence with focus towards longer wavelength, enhancing the importance of low- over high-frequency channels. However, to obtain similar constraints when allowing for increased complexity of the dust model (e.g., caused by averaging processes or presence of CIB) might still require extended coverage at high frequencies. We note that, although not directly transferable to $B$-mode searches, our analysis is still quite instructive, exploring the CMB signal extraction in the (extremely) low signal-to-noise regime.

Finally, we also highlighted the need for absolute measurement of the monopole distortion even for the measurement of anisotropies of $\mu$-distortions to break the parameter degeneracy $c_{f}^{\mu,T} \propto f_{NL}(\mu)$ of the amplitude of the $\mu$-T cross-power spectrum. Fourier-transform spectrometer concepts similar to PIXIE would be very useful in this respect. Similarly, the channel inter-calibration and possible errors from imperfect knowledge of the CMB monopole temperature have to be carefully considered to safely extract possible spectral-spatial distortion signals from the very early phases of our Universe (see Sect. 4.4).

ACKNOWLEDGEMENTS

We cordially thank Andrea Ravenni for useful discussions on the modelling of anisotropic $\mu$- and $\gamma$-type spectral distortions and for providing us with his $\mu - T$ cross-power spectra. We also thank Eiichiro Komatsu and Enrico Pajer for their useful comments on the manuscript, and the referee for their suggestions that helped improve the paper. Some of the results in this paper have been derived using the HEALPix package (Górski et al. 2005). We also acknowledge the use of the PSM package (Delabrouille et al. 2013), developed by the Planck working group on component separation, for making the simulations used in this work. MR acknowledges funding from the European Research Council Consolidator Grant (CMBSPEC, No. 725456). IC is supported by the Royal Society as a Royal Society University Research Fellow at the University of Manchester, UK.

REFERENCES

Bartolo N., Liguori M., Shiraishi M., 2016, JCAP, 3, 029
Burgiana C. et al., 2017, arXiv:1704.05764, accepted by JCAP
CORE Collaboration et al., 2016, ArXiv:1612.08270
De Zotti G., Negrello M., Castex G., Lapi A., Bonato M., 2016, JCAP, 3, 047
Delabrouille J. et al., 2017, arXiv:1706.04516, accepted by JCAP
Dimastrogiovanni E., Emami R., 2016, JCAP, 12, 015
Hu W., Silk J., 1993b, Physical Review Letters, 70, 2661
Khatri R., Sunyaev R., 2015, JCAP, 9, 026
Khatri R., Sunyaev R. A., 2012, JCAP, 9, 16
Kogut A. et al., 2011, JCAP, 7, 25
Matsumura T. et al., 2016, Journal of Low Temperature Physics, 184, 824
Pajer E., Zaldarriaga M., 2013, JCAP, 2, 036
Poulin V., Lesgourgues J., Serpico P. D., 2017, JCAP, 3, 043
Ravenni A., Liguori M., Bartolo N., Shiraishi M., 2017, JCAP, 9, 042
Shiraishi M., Bartolo N., Liguori M., 2016, JCAP, 10, 015
Sunyaev R. A., Khatri R., 2013, JMPD, 22, 30014
Sunyaev R. A., Zeldovich Y. B., 1972, comas, 4, 173
Suzuki A. et al., 2018, ArXiv:1801.06987