Risk

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Risk: An R Package for Financial Risk Measures

by

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Abstract: A new R contributed package written by the authors is introduced. The package is believed to be the most comprehensive one to date for financial risk measures. It computes twenty six financial risk measures for any continuous distribution. The use of the package is illustrated.

Keywords: Beyond value at risk, Expected shortfall, Value at risk

1 Introduction

A risk measure say ρ (from a class of random variables to the positive real line) is said to be a coherent risk measure [Artzner et al., 1999] if it satisfies the following axioms:

1. for all X and all real numbers α , we have $\rho(X + \alpha) = \rho(X) - \alpha$ (Translation),
2. for all X_1 and X_2 , $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$ (Sub-additivity),
3. for all $\lambda \geq 0$ and all X , $\rho(\lambda X) = \lambda \rho(X)$ (Positive homogeneity),
4. for all X, Y with $X \leq Y$, $\rho(Y) \leq \rho(X)$ (Monotonicity).

A risk measure is deemed to be good if it is a coherent risk measure.

Value at Risk (VaR) and Expected Shortfall (ES) are the two most popular measures of financial risk. The VaR is due to Till Guldumann in the late 1980s. He was then the head of global research at J. P. Morgan. The ES is due to Artzner et al. [1999].

Both the VaR and the ES have some disadvantages. The VaR not only ignores any loss beyond the VaR level and also can not satisfy the axiom of sub-additivity. Some other disadvantages of the VaR are mentioned in Yamai and Yoshida [2002]. A disadvantage of the ES is that backtesting (a method for checking forecastability of the measure) is difficult. Another disadvantage of the ES is that its estimate may not be as accurate as that of the VaR, as shown by Yamai and Yoshida [2002].

There are however backtesting methods for the ES. Two methods based on excesses are proposed in Embrechts et al. [2005]. Other backtesting methods can be found in the excellent book McNeil et al. [2015].

In the last ten years or so, many other measures of financial risk have been proposed. But there have been hardly any applications of these measures. The aim of this paper is to provide a software for these relatively new measures. The software also computes some risk measures predating the VaR that are not well known. This software could encourage applications and also development of further risk measures.

We are not aware of any R [R Development Core Team, 2017] package for risk measures other than the VaR and the ES. There are not comprehensive R packages even for the VaR and the ES. The ones we are aware of are actuar [Goulet et al., 2017], ghyp [Luethi and Breymann, 2016], PerformanceAnalytics [Peterson and Others, 2014], crp.CSFP [Jakob et al., 2016], fAssets [Rmetrics Core Team et al., 2014a], fPortfolio [Rmetrics Core Team et al., 2014b], CreditMetrics [Wittmann, 2009], fExtremes [Wuertz, 2013], rugarch [Ghalanos, 2015], evir [Pfaff et al., 2012], QRM [Pfaff and Others, 2016] and VaRES [Nadarajah et al., 2013]. Except for the last one, most of these packages compute the VaR and the ES for restricted distributions like the normal distribution. VaRES [Nadarajah et al., 2013] computes the VaR and the ES for over 100 distributions.

The newly introduced R package computes twenty six risk measures for any user specified continuous distribution. The risk measures computed are: VaR, ES, tail conditional median, expectiles, beyond value at risk, expected proportional shortfall, elementary risk measures, Omega, Sortino ratio, Kappa, Wang’s risk measure, Stone’s risk measures, Luce’s risk measures, Sarin’s risk measures and Bronshtein and Kurelenkova’s risk measures.

The details of the package are given in Section 2. Its practical use is illustrated in Section 3. Some concluding remarks are given in Section 4.

2 Financial risk measures and R functions

Throughout, X denotes the financial variable of interest (for example, X = investment return). We shall assume X is a continuous random variable. We let F_X denote the cumulative distribution function of X , f_X denote the probability density function of X and $[u]^+ = \max(0, u)$.

We give mathematical definitions of the twenty six risk measures and calling sequences to compute them. The spec (a character string) specifies the distribution corresponding to the probability density function, $f_X(\cdot)$, and the cumulative distribution function, $F_X(\cdot)$. The distribution should be one that is recognized by R. It could be one of the distributions implemented in the R base package or one of the distributions implemented in an R contributed package or one freshly written by a user. In any case, there should be functions dspec, pspec and qspec, computing the probability density function, cumulative distribution function and quantile function of the F_X distribution.

Some examples of spec are: spec = "norm", meaning that $f_X(x) = (1/\sigma)\psi((x - \mu)/\sigma)$ and $F_X(x) = \Phi((x - \mu)/\sigma)$, where $\psi(\cdot)$ and $\Phi(\cdot)$ denote, respectively, the probability density function and the cumulative distribution function of a standard normal random variable; spec = "lnorm", meaning that $f_X(x) = \{1/(\sigma x)\}\psi((\log x - \mu)/\sigma)$ and $F_X(x) = \Phi((\log x - \mu)/\sigma)$; spec = "exp", meaning that $f_X(x) = \lambda \exp(-\lambda x)$ and $F_X(x) = 1 - \exp(-\lambda x)$.

Value at risk:

$$\text{VaR}_\alpha(X) = \inf \{x : F_X(x) > \alpha\},$$

$$\text{varg}(\text{spec}, \alpha, \dots),$$

where $0 < \alpha < 1$. That is, the VaR is the smallest number such that the probability that the loss X exceeds it is at most $1 - \alpha$. If X is an absolutely continuous random variable then $\text{VaR}_\alpha(X) = F_X^{-1}(\alpha)$, where $F_X^{-1}(\cdot)$ denotes the inverse function of $F_X(\cdot)$. A comprehensive review of the properties and applications of the VaR is given in Nadarajah and Chan [2016]. The VaR is not sub-additive and so not a coherent measure.

Expected shortfall due to Artzner et al. [1999]:

$$\begin{aligned} \text{ES}_\alpha(X) &= E[X|X \geq \text{VaR}_\alpha(X)], \\ \text{esg}(\text{spec}, \alpha, \dots), \end{aligned}$$

where $0 < \alpha < 1$. $\text{ES}_\alpha(X)$ can be interpreted as the average loss in the 100α percent worst cases of the returns. **If X is an absolutely continuous random variable then**

$$\text{ES}_\alpha(X) = \frac{1}{\alpha} \int_\alpha^1 F_X^{-1}(p) dp,$$

where $F_X^{-1}(\cdot)$ denotes the inverse function of $F_X(\cdot)$. A comprehensive review of the properties and applications of the ES is given in Nadarajah et al. [2014]. The ES is a coherent risk measure, see, for example, Acerbi et al. [2001] for a proof. The ES is also known as conditional VaR and average VaR. The ES is highly model dependent, and particularly sensitive to modeling assumptions on the extreme tails of loss distributions, because the computation of the ES relies on these extreme tails Kou et al. [2013].

Tail conditional median due to Kou et al. [2013]:

$$\begin{aligned} \text{TCM}_\alpha(X) &= \text{median}[X|X \geq \text{VaR}_\alpha(X)], \\ \text{tcm}(\text{spec}, \alpha, \dots), \end{aligned}$$

where $0 < \alpha < 1$. **Since X is continuous, $\text{TCM}_\alpha(X) = \text{VaR}_{\frac{1+\alpha}{2}}(X)$.** According to Kou et al. [2013], the TCM is more robust than the ES by at least four tools in robust statistics: (i) influence functions, (ii) asymptotic breakdown points, (iii) continuity of statistical functional, and (iv) finite sample breakdown points. Kou et al. [2013] also showed that sub-additivity may be satisfied by the TCM in situations where it is not satisfied by the VaR. Furthermore, the TCM is less sensitive to tail behavior than the ES and changes in the TCM are narrower than those in the ES [Kou et al., 2013].

Expectiles due to Newey and Powell [1987]:

$$\begin{aligned} \text{argmin}_\ell \left\{ \alpha E \left[((X - \ell)^+)^2 \right] + (1 - \alpha) E \left[((\ell - X)^+)^2 \right] \right\}, \\ \text{expp}(\text{spec}, \alpha, a, b, \dots), \end{aligned}$$

where $0 < \alpha < 1$, and a and b denote, respectively, the lower and upper end points of X . $E \left[((X - \ell)^+)^2 \right]$ is the average of the squared deviations of the losses exceeding the level ℓ . $E \left[((\ell - X)^+)^2 \right]$ is the average of the squared deviations of the losses not exceeding the level ℓ . The minimum is taken over all possible values of ℓ . α is the weight given to deviations of the losses exceeding the level ℓ as opposed to deviations of the losses not exceeding the level ℓ . If $\alpha > 0.5$ deviations of the losses exceeding the level ℓ are given more weight. If $\alpha < 0.5$ deviations of the losses not exceeding the level ℓ are given more weight. If $\alpha = 0.5$ they are given equal weights.

Expectiles summarize the distribution of X in the same way as quantiles summarize the distribution of X . Expectiles are determined by tail expectations while quantiles are determined by the distribution function. However, expectiles are more difficult to interpret than quantiles. Expectiles are location and scale equivariant. They are coherent risk measures, as shown by Bellini et al. [2014]. Expectiles are also referred to as “graviles”, “heftiles”, “loadiles” as well as “projectiles”.

Beyond value at risk due to Longin [2001]:

$$\begin{aligned} \text{BVaR}_\alpha(X) &= E[X \mid X < \text{VaR}_\alpha(X)], \\ \text{bvar}(\text{spec}, \alpha, a, \dots), \end{aligned}$$

where $0 < \alpha < 1$, and a denotes the lower end point of X . Beyond value at risk is the mean of losses not exceeding the VaR. It incorporates both the frequency of losses beyond the VaR and the size of the losses beyond the VaR [Longin, 2001]. Unlike the VaR, the BVaR is sub-additive. Basak and Shapiro [1999] showed that a BVaR sort of constraint for investors would lead to optimal strategies for which the magnitude of the extreme losses is under control. Such a control is important for positions invested in financial assets characterised by heavy tailed distributions or for positions including options [Longin, 2001]. The BVaR is of interest to financial regulators as well [Longin, 2001]. If X is a standard normal random variable, then asymptotically $\text{BVaR} \approx \text{VaR} + 2^{-1}\text{VaR}^{-1}$. If X is a Pareto random variable with characteristic exponent $\alpha > 1$, then asymptotically $\text{BVaR} \approx \text{VaR} + \frac{\text{VaR}}{\alpha-1}$.

Expected proportional shortfall due to Belzunce et al. [2012]:

$$\begin{aligned} \text{EPS}_\alpha(X) &= (1 - \alpha) \frac{\text{ES}_\alpha(X)}{\text{VaR}_\alpha(X)}, \\ \text{epsg}(\text{spec}, \alpha, \dots), \end{aligned}$$

where $0 < \alpha < 1$ and X is assumed to have finite mean. The EPS is scale invariant, that is, $\text{EPS}_\alpha(X) = \text{EPS}_\alpha(cX)$ for all α . The EPS does not depend on the base currency, so it can be used to compare risks of different nature [Belzunce et al., 2012]. Given that the stop-loss function is decreasing, the EPS is also decreasing [Belzunce et al., 2012]. The EPS measures the upper tail from the VaR onwards, but it is proportional to the VaR [Belzunce et al., 2012].

Expected loss:

$$\begin{aligned} E(X) &= \int_a^b x f_X(x) dx, \\ \text{expect}(\text{spec}, a, b, \dots), \end{aligned}$$

where a and b denote, respectively, the lower and upper end points of X . Expected loss as a measure for risk is related to the pure premium principle used in actuarial science. It is not scale invariant. But it is a coherent risk measure. Obviously, the larger the expected loss the more extreme the risk is.

An elementary risk measure due to Ahmadi-Javid [2012]:

$$\begin{aligned} E(X) + \alpha \sqrt{\text{Var}(X)}, \\ \text{expvar}(\text{spec}, \alpha, a, b, \dots), \end{aligned}$$

where $\alpha > 0$ and a and b denote, respectively, the lower and upper end points of X . This measure is known as the mean standard deviation. Larger values of α correspond to more extreme risks. The measure is related to the variance premium principle used in actuarial science. It is not scale invariant and not a coherent risk measure.

Omega due to Shadwick and Keating [2002]:

$$\frac{\int_a^b [1 - F_X(x)] dx}{\int_a^\alpha F_X(x) dx},$$

$$\text{omegag}(\text{spec}, \alpha, a, b, \dots),$$

where $-\infty < \alpha < \infty$ is the investor's minimum acceptable return and a and b denote, respectively, the lower and upper end points of X . As a function of α , the Omega maps from (a, b) to $(0, \infty)$. It is a monotonic decreasing function. The value the Omega at $\alpha = E(X)$ is always 1, independent of the distribution. The Omega is invariant to changes in location and scale. The Omega provides corrections to mean-variance measures by taking all higher moment information into account [Shadwick and Keating, 2002], it is a powerful tool for the capture of higher moment effects [Shadwick and Keating, 2002]. It also takes into account the level of return against which a given outcome will be viewed as a gain or loss [Shadwick and Keating, 2002]. The Omega is a function that may be evaluated at any value in the range of possible returns, so that it allows performance comparisons with respect to any risk threshold in this range [Shadwick and Keating, 2002]. The Omega may be used to rank manager performance, without the need to introduce utility functions [Shadwick and Keating, 2002]. Agarwal and Naik [1999] showed how the Omega can be extended for optimal asset allocation.

Sortino ratio due to Rollinger and Hoffman [2013]:

$$[E(X) - \alpha] \left[\int_a^\alpha (\alpha - x)^2 f_X(x) dx \right]^{-1/2},$$

$$\text{sortinog}(\text{spec}, \alpha, a, b, \dots),$$

where $\alpha > 0$ is the investor's minimum acceptable return and a and b denote, respectively, the lower and upper end points of X . The Sortino ratio is a modification of the Sharpe ratio, using downside deviation rather than standard deviation as the measure of risk – i.e., only those returns falling below a user specified target or required rate of return are considered risky [Rollinger and Hoffman, 2013]. It takes into account both the frequency of below target returns as well as the magnitude of them [Rollinger and Hoffman, 2013]. The Sortino ratio is especially good when measuring and comparing the performance of managers whose programs exhibit positive skew in their return distributions [Rollinger and Hoffman, 2013]. The Sortino ratio is not scale invariant.

Kappa due to Kaplan and Knowles [2004]:

$$[E(X) - \alpha] \left[\int_a^\alpha (\alpha - x)^n f_X(x) dx \right]^{-1/n},$$

$$\text{kappag}(\text{spec}, \alpha, n, a, b, \dots),$$

where $\alpha > 0$ is the investor's minimum acceptable return, n is a positive integer, and a and b denote, respectively, the lower and upper end points of X . The Omega and the Sortino ratio measures are particular cases of the Kappa. The Omega is the particular case for $n = 1$. The Sortino ratio is the particular case for $n = 2$. Kappa values should be treated as simple ordinals: for a given value of the n parameter, a higher Kappa is always better than a lower, but the size of the difference between two Kappa values is not subject to simple interpretation [Kaplan and Knowles, 2004]. The rate of change in the Kappa as a function of α is inversely proportional to n [Kaplan and Knowles, 2004]. The n parameter can be interpreted as a measure of skewness risk “appetite” for threshold returns below the mean [Kaplan and Knowles, 2004]. The n parameter can also be interpreted as a measure of kurtosis risk aversion for return thresholds below the mean [Kaplan and Knowles, 2004].

Values for the first four moments of a return distribution are sufficient in many cases to enable a robust estimation of the Kappa [Kaplan and Knowles, 2004]. It is not necessary to know the

individual data points in the distribution [Kaplan and Knowles, 2004]. The Kappa is not scale invariant.

Wang's risk measures [Wang, 1998]:

$$\frac{1}{2} \int_a^b \{[1 - F_X(x)]^\alpha - [1 - F_X(x)]\} + \frac{1}{2} \int_a^b \{F_X^\alpha(x) - F_X(x)\},$$

wangg1(spec, alpha, a, b, ...),

and

$$\int_a^b \{[1 - F_X(x)]^\alpha - [1 - F_X(x)]\},$$

wangg2(spec, alpha, a, b, ...),

where $0 < \alpha < 1$ and a and b denote, respectively, the lower and upper end points of X . The smaller the value of α the more extreme the risk will be. The second measure is known as the right tail deviation. Both measures outperform the standard deviation and Gini mean and have great potential for actuarial applications [Wang, 1998]. Both measures are coherent risk measures. The advantages of the right tail deviation over standard deviation include: it accounts for all moments without explicit calculation [Wang, 1998]; it is more discriminative than the standard deviation [Wang, 1998]; it is additive for excess-of-loss layers and thus yields the same premium no matter how the risk is divided into sublayers [Wang, 1998]; it preserves common ordering of risks such as first and second stochastic dominance, while the standard deviation does not [Wang, 1998]. The right tail deviation can be interpreted as follows [Wang, 1998]: Suppose that there are two hidden independent and identical variables Y_1 and Y_2 such that $X = \min(Y_1, Y_2)$; then the right tail deviation of X is half the expected absolute difference between Y_1 and Y_2 , or the expected difference between Y_1 and X .

Stone's risk measures [Stone, 1973]:

$$\int_a^b |x - x_0|^k f_X(x) dx,$$

stoneg1(spec, x0, k, a, b, ...)

and

$$\left[\int_a^b |x - x_0|^k f_X(x) dx \right]^{1/k},$$

stoneg2(spec, x0, k, a, b, ...),

where x_0 is a reference level of wealth from which deviations are measured, k is the power to which deviations in wealth from the reference level are raised, and a and b denote, respectively, the lower and upper end points of X . k accounts for relative importance of large deviations with respect to small deviations [Stone, 1973]:

- $k > 1$ corresponds to large deviations assuming relatively more importance than small deviations;
- $k = 1$ corresponds to all deviations being equally weighted;

- $0 < k < 1$ corresponds to small deviations given a relatively greater weight than large deviations.

a and b determine deviations to be counted in specifying the risk measures [Stone, 1973]:

- $b = \infty$ corresponds to all deviations being considered;
- $b = w_0$ corresponds to only deviations below a reference level being considered;
- $b < w_0$ corresponds to only deviations worse than some level being included, so small adverse deviations and all favorable deviations will not be counted;
- $b > w_0$ corresponds to all adverse and some (but not all) favorable deviations being counted.

The second of the two measures is scale invariant. The first is not scale invariant. So, the second measure can be preferred to the first one [Stone, 1973]. The variance, semi variance and mean absolute deviation are particular cases of the first of the two measures. Both measures are monotonic increasing functions of b . The first risk measure may reach a minimum with respect to k if $k > 1$ [Stone, 1973].

Luce's risk measures [Luce, 1980]:

$$d - c \int_a^b f_X(x) \log f_X(x) dx,$$

$$\text{luceg1}(\text{spec}, a, b, c, d, \dots),$$

$$c \int_a^b [f_X(x)]^{1-d} dx,$$

$$\text{luceg2}(\text{spec}, a, b, c, d, \dots),$$

$$d + c \int_a^b \log x f_X(x) dx,$$

$$\text{luceg3}(\text{spec}, a, b, c, d, \dots)$$

and

$$c \int_a^b x^d f_X(x) dx,$$

$$\text{luceg4}(\text{spec}, a, b, c, d, \dots),$$

where $c > 0$, $d \geq 0$ and a and b denote, respectively, the lower and upper end points of X . The first of these four measures is related to the concept of Shannon entropy [Shannon, 1948, 1951]. The second is related to the concept of Rényi entropy Renyi [1961]. The third is based on the logarithmic moment. The fourth is based on the d th moment. All of these measures are based on the assumption that the risk either increases additively or multiplicatively when its density is multiplied by a constant [Luce, 1980]:

- The additive assumption is that if one moves from a domain all in dollars to one in units of one hundred dollars, then the risk of all transformed gambles will change by a constant amount depending upon the change in units [Luce, 1980];

- The multiplicative assumption is that if one moves from a domain all in dollars to one in units of one hundred dollars, then the risk of all transformed gambles will be multiplied by a constant amount depending upon the change in units [Luce, 1980].

Different aggregation rules are used for transforming the density into a single number. None of the four measures are scale invariant.

Sarin's risk measures [Sarin, 1987]:

$$k \int_a^b \exp(gx) f_X(x) dx,$$

$$\text{saring1}(\text{spec}, a, b, k, g, \dots),$$

$$d \int_0^b f_X(x) dx + e \int_a^0 f_X(x) dx + cE[\log |X|] - \frac{c^2}{2} \text{Var}[\log |X|],$$

$$\text{saring2}(\text{spec}, a, b, c, d, e, \dots)$$

and

$$d \int_0^b |x|^c f_X(x) dx + e \int_a^0 |x|^c f_X(x) dx + \frac{c}{2(c-1)} E[|X|^{2c}] - \frac{1}{2} \text{Var}[|X|^c],$$

$$\text{saring3}(\text{spec}, a, b, c, d, e, \dots),$$

where $c > 0$, $d > 0$, $e > 0$, and either $k > 0$, $g < 0$ or $k < 0$, $g > 0$. Further, a and b denote, respectively, the lower and upper end points of X . The first of these three measures is related to the concept of moment generating function. The second is based on logarithm of the absolute values of the losses. The third is based on powers of the absolute values of the losses. These measures contain Luce's risk measures as particular cases. The measures are based on the assumption that the risk either increases, decreases or remains constant when its density is multiplied by a constant [Sarin, 1987]. As in Luce [1980], different aggregation rules are used for transforming the density into a single number. Also as in Luce [1980], none of the three measures are scale invariant.

Bronshtein and Kurelenkova's risk measures Bronshtein and Kurelenkova [2009]:

$$\text{VaR}_\alpha(X - E(X)),$$

$$\text{BKg1}(\text{spec}, \alpha, a, b, \dots),$$

$$\text{ES}_\alpha(X - E(X)),$$

$$\text{BKg2}(\text{spec}, \alpha, a, b, \dots),$$

$$\text{ES}_\alpha(X) - \beta E[|X - E(X)|],$$

$$\text{BKg3}(\text{spec}, \alpha, a, b, \beta, \dots)$$

and

$$\text{VaR}_\alpha(X) + \text{ES}_\alpha(X) - \beta E[|X - E(X)|],$$

$$\text{BKg4}(\text{spec}, \alpha, a, b, \beta, \dots),$$

where $0 < \alpha < 1$, $\beta \geq 0$ and a and b denote, respectively, the lower and upper end points of X . These measures are combinations of the VaR, the ES, expected loss, standard deviation, average of absolute deviation and their modifications. The first of the four measures is a deviation between the VaR and the expected loss. The second is a deviation between the ES and the expected loss. The third is a deviation between the ES and the average of absolute deviation. The fourth is a deviation between the sum of the VaR and the ES and the average of absolute deviation. The second and third measures are coherent risk measures. The first and fourth measures are not coherent risk measures. However, none of the four measures are scale invariant. Bronshtein and Kurelenkova [2009] found that large values of β have the effect of making the profits of optimal portfolios stable. Bronshtein and Kurelenkova [2009] also found that for short time periods the third risk measure is more efficient than the fourth one.

The new package Risk [Nadarajah and Chan, 2017] computes each of the twenty six risk measures. The package has been uploaded to CRAN, see

<https://cran.r-project.org/web/packages/Risk/index.html>.

3 Illustrations

Here, we provide three illustrations of the practical use of the package Risk [Nadarajah and Chan, 2017].

The first illustration plots the tail conditional median of the Student's t distribution. We have taken the degree of freedom parameter to be one of 1, 2, 5 or 10. The following codes will produce Figure 1, the plot of the tail conditional medians of the Student's t distribution.

```
R> alpha <- seq(0.01, 0.99, 0.01)
R> y1 <- tcm("t", alpha, df = 1)
R> y2 <- tcm("t", alpha, df = 2)
R> y3 <- tcm("t", alpha, df = 5)
R> y4 <- tcm("t", alpha, df = 10)
R> xrange <- range(alpha)
R> yrange <- range(0, 20)
R> plot(alpha, y1, type = "l", xlab = expression(alpha),
+ ylab = "TCM", col = "black", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y2, type = "l", xlab = "", ylab = "",
+ col = "red", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y3, type = "l", xlab = "", ylab = "",
+ col = "blue", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y4, type = "l", xlab = "", ylab = "",
+ col = "brown", xlim = xrange, ylim = yrange)
R> legend(0.5, 20,
+ legend = c("df = 1", "df = 2", "df = 5", "df = 10"),
+ col = c("black", "red", "blue", "brown"), lty = 1)
```

Figure 1: Tail conditional medians of the Student's t distribution with degrees of freedom 1, 2, 5 and 10.

The second illustration plots the expected proportional shortfall of the normal distribution for varying parameter values. We have taken the mean to be equal to 1 and standard deviation equal to be one of 1, 2, 3 or 4. The following codes will produce Figure 2, the plot of the expected proportional shortfalls of the normal distribution.

```
R> alpha <- seq(0.01, 0.99, 0.01)
R> y1 <- epsg("norm", alpha, mean = 1, sd = 1)
R> y2 <- epsg("norm", alpha, mean = 1, sd = 2)
R> y3 <- epsg("norm", alpha, mean = 1, sd = 3)
R> y4 <- epsg("norm", alpha, mean = 1, sd = 4)
R> xrange <- range(alpha)
R> yrange <- range(y1, y2, y3, y4)
R> plot(alpha, y1, type = "l", xlab = expression(alpha),
+ ylab = "EPS", col = "black", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y2, type = "l", xlab = "", ylab = "",
+ col = "red", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y3, type = "l", xlab = "", ylab = "",
+ col = "blue", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y4, type = "l", xlab = "", ylab = "",
```

```

+ col = "brown", xlim = xrange, ylim = yrange)
R> legend(0.6, -100,
+ legend = c("mean = 1, sd = 1", "mean = 1, sd = 2",
+ "mean = 1, sd = 3", "mean = 1, sd = 4"),
+ col = c("black", "red", "blue", "brown"), lty = 1)

```

Figure 2: Expected proportional shortfalls of the normal distribution.

The third illustration plots the beyond value at risk of the logistic distribution for varying parameter values. We have taken the mean to be equal to zero and scale equal to be one of 1, 2, 5 or 10. The following codes will produce Figure 3, the plot of the beyond values at risk of the logistic distribution.

```

R> alpha <- seq(0.01, 0.99, 0.01)
R> y1 <- bvar("logis", alpha, -Inf, scale = 1)
R> y2 <- bvar("logis", alpha, -Inf, scale = 2)
R> y3 <- bvar("logis", alpha, -Inf, scale = 5)
R> y4 <- bvar("logis", alpha, -Inf, scale = 10)
R> xrange <- range(alpha)
R> yrange <- range(y1, y2, y3, y4)
R> plot(alpha, y1, type = "l", xlab = expression(alpha),
+ ylab = "BVaR", col = "black", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y2, type = "l", xlab = "", ylab = "",

```

```

+ col = "red", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y3, type = "l", xlab = "", ylab = "",
+ col = "blue", xlim = xrange, ylim = yrange)
R> par(new = TRUE)
R> plot(alpha, y4, type = "l", xlab = "", ylab = "",
+ col = "brown", xlim = xrange, ylim = yrange)
R> legend(0.4, -30,
+ legend = c("scale = 1", "scale = 2", "scale = 5", "scale = 10"),
+ col = c("black", "red", "blue", "brown"), lty = 1)

```

Figure 3: Beyond values at risk of the logistic distribution.

4 Conclusions

We have developed a new R package for computing twenty six financial risk measures. This package is the most comprehensive one known to date for financial risk measures of continuous random variables. The package can be updated if new risk measures are developed. **A future work is to extend the package for financial risk measures for discrete random variables and multivariate random variables.**

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