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Thermal Evaluation of Railway Transformer Used in Autotransformer Feeding Systems

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Abstract—Globally, railway electrification is increasing at a fast rate and the overall power demand is expected to increase in the future to accommodate an increase in traffic volume and higher train speeds. However, due to dynamic train operation characteristics, the load profile at the feeder substation fluctuates and can reach beyond the transformer rating during peak timetable periods. This paper presents a multi-train simulation platform for an autotransformer feeding system using multi-conductor approach. Temperature is a key factor in the operational lifetime of a transformer, influencing the rate of insulation ageing processes. Sudden load variations seen in railway systems can pose a threat to the traction transformer's lifetime. Hence, a thermal model is integrated into the simulation, with parameters considered appropriate for the transformer, in order to assess the temperature of the hottest spot within the transformer. This model links temperature to load, and, given the propensity to push more trains through old assets as electrification of transport becomes *en vogue*, this relationship becomes ever more important. Consideration is given to the anticipated effect of the temperature profile on a traction transformer at a substation by making use of IEC standards for power transformers with appropriate modifications to traction transformers sourced from literature.

It is found through the use of this model that traction transformers are not at high risk of thermal failure, however, some mitigation may be required against dynamic behaviour that can lead to conditions conducive to dangerous flashover events.

Keywords—Autotransformer feeding, hottest spot temperature, multi-train simulator, transformer thermal modelling

I. INTRODUCTION

Electrical railway systems play critical roles in modern railway transport, and the development of railway electrification is undergoing fast growth [1]. Railway power-supply systems have been intensively studied for decades. Many previous studies focused on power quality issues such as system imbalance, reactive power and harmonics [2]. In addition, considering modern trains have a maximum power up to 20MW with operating speed more than 300km/h, autotransformer feeding (AT) is more generally adopted by newly-built high speed lines [1] as they can lower the current flow through the line. Some research began to look at how to manage the power flow more intelligently so as to achieve more efficient energy use. For example, authors in [3] proposed a novel co-phase feeding system to balance the load between adjacent feeding sections. The smart grid concept in

railways has also been brought to attention where renewable energy sources and energy storage devices are considered potentially usable in railways [4-6]. Nevertheless, even with a smart energy management, railway transformers still endure a rapid variation of load profile. Unfortunately, their thermal performance is sometimes ignored in traction system research.

In fact, transformers generate losses while in service. Such losses generate heat, and as a result the temperature of the transformer increases: the higher the transformer loading, the higher the losses, and hence the higher the transformer temperature. Note that the relationship between load and temperature is not linear. Generally, only the hottest spot temperature (HST) of the transformer is calculated, as this dominates the behaviour within the transformer.

The transformer HST is important as it can determine the end of life through several mechanisms; some long-term (LT) and some short-term (ST). The main mechanisms to be considered are:

- i. HST leads to ageing of transformer insulation through oxidation, hydrolysis and pyrolysis of insulation chemical bonds [7]. This is most prevalent at HST values above the rated transformer temperature, usually 98 °C. (LT)
- ii. HST exceeding a certain value can lead to failure through formation of bubbles [8]. This usually also requires a rapid rate of temperature increase. (ST)
- iii. Rapid decrease in temperature of oil may lead to gaseous bubble formation [9]. (ST)
- iv. HST may rise, then fall rapidly and so also the temperature of oil, leading to formation of free water in oil [10]. (ST)
- v. Temperature above flash point of oil leading to fire. (ST)

The rest of this paper is organised as follows. Section II introduces the methodologies used to build the simulation platform including the thermal model. A case study scenario is then created based on a 24-hour train timetable in Section III. Next, the results for railway thermal evaluation are shown in section IV. Discussion and conclusion are shown in Section V and VI, respectively.

II. METHODOLOGY

A. Symbols

a_{target}	Target acceleration rate [m/s ²]
A	Davis equation constant coefficient [N]

B	Davis equation linear term coefficient [N/(m/s)]
C	Davis equation quadratic term coefficient [N/(m/s ²)]
F_{T_target}	Target tractive force for each train [N]
g	Gravitational constant [m/s ²]
I_F	Current at auxiliary feeder conductor [A]
I_R	Current at rail conductor [A]
I_T	Current at catenary conductor [A]
I_{sub}	Injected current vector at feeder substation [A]
i_{train}	Current drawn by each train [A]
i	Node number
M	Total train mass [kg]
P_{elec}	Electrical power required by each train [W]
P_{mech}	Mechanical power on the wheel required by each train [W]
P_{sub}	Active power loading at feeder transformer [W]
P_{train}	Active power demand of each train [W]
Q_{sub}	Reactive power loading at feeder transformer [W]
Q_{train}	Reactive power demand of each train [W]
U_F	Voltage at auxiliary feeder conductor [V]
U_R	Voltage at rail conductor [V]
U_T	Voltage at catenary conductor [V]
U_i	Voltage vector at node 'i' [V]
u_R	Voltage of rail conductor at train position [V]
u_T	Train pantograph voltage [V]
v	Train operation velocity [m/s]
Y_{AT}	Autotransformer admittance matrix
$Y_{connect}$	Admittance matrix to connect different rail tracks
Y_{sub}	Feeder substation admittance matrix
α	Gradient angle [rad]
η	Efficiency between electrical and mechanical power conversion
σ	Rotary allowance

B. System Overview

Fig 1 shows an overview of the simulation platform implemented in MatLab environment. Data is first prepared and imported into three subsystems, namely Single Train Performance (STP), Electrical Network Simulation (ENS) and Transformer Thermal Simulation (TTS). For each time-step, which is set as five seconds in this study, STP calculates each train's tractive force and updates operating speed and distance. Then the simulation results are transferred to ENS where nodal analysis is applied to assess the current and voltage distribution. The substation power loading is calculated and fed into TTS. In this model, the thermal performance of traction transformer is then evaluated.

C. Dynamic Train Movement Simulation

A dynamic train movement simulator simulates the performance for each train. It repeatedly evaluates Newton's second law of motion based on individual train parameters to produce performance data such as speed, travelling distance and power demand. A look-up table is prepared beforehand

TABLE I. SIMULATION PARAMETERS FOR CLASS 390 PENDOLINO [11]

Variable	Value
Maximum Tractive force (kN)	203.7 ($0 \leq v < 24.47$ m/s) 5095.2/v (24.47 m/s $\leq v < 62.5$ m/s)
Maximum Braking force (kN)	331.3
Maximum Speed (m/s)	62.5
Resistive Force (kN)	A: 5.422 B: 0.069 C: 0.012
Maximum Power at Rail (kW)	5100
Maximum Regenerative Power at Rail (kW)	0 ($0 \leq v < 9.64$ m/s) 1579.02 ($9.64 \leq v < 11.06$ m/s) 202.74*v ($11.06 \leq v < 30.23$ m/s) 6153.66 ($30.23 \leq v < 62.5$ m/s)
Efficiency	85 %

including route information and reference speed profile. At each simulation time-step, the program uses each train's current speed against the reference and calculates the required tractive force to eliminate the speed error. This simulator can be divided into the three following blocks:

1) Mission Profile

In this paper, Class 390 Pendolino Electric Multiple Unit train is used with the simulation parameters listed in Table I [11]. The pre-defined speed profile for each train is firstly generated according to the timetable and stored in an MS Excel file. The program reads the reference speed according to train location and provides the data to the speed tracking block.

2) Speed Tracking Block

Given the speed profile as a reference, a speed error is calculated by comparing the reference with actual speed. Thus a target acceleration rate a_{target} is set by a proportional controller to track the reference speed, but it is limited by the maximum adhesion. The target tractive effort is calculated by using (1) and (2) [12].

$$F_{T_target} = M(1 + \sigma)a_{target} + F_R + Mgsin(\alpha) \quad (1)$$

$$F_R = A + Bv + Cv^2 \quad (2)$$

However, the tractive effort must conform to the maximum available tractive / braking torque of the train. The achieved tractive force F_{T_actual} , and achieved acceleration rate a_{actual} needs to be re-evaluated. Once this is obtained, train speed and distance are updated by using discrete time integration.

Fig 1. Overview of the simulation platform

Fig 2. Schematic diagram of AT Feeding system arrangement (a) and the corresponding chain representation (b).

3) Power Evaluation Block

When F_{T_actual} is determined, the mechanical power demand P_{mech} is calculated using (3). Hence, the electrical power demand is determined by multiplying / dividing by the efficiency, as shown in (4). The efficiency between mechanical and electrical power conversion is 85 % for Class 390. It is worth noting that the train auxiliary load is generally regarded as a relatively small and constant value compared to the train power demand, so it is not considered in this study.

$$P_{mech} = F_{T_actual} \cdot v \quad (3)$$

$$P_{elec} = \begin{cases} P_{mech}/\eta & \text{Motoring} \\ P_{mech} \cdot \eta & \text{Regenerating} \end{cases} \quad (4)$$

D. AT Feeding Network Modelling

Fig 2 (a) shows a typical arrangement of a 40 km double track AT system with parallel connections. This system consists of a feeder substation and four AT sites with 10 km spacing. It is shown in the figure that one train is operating on the up track and the other on the down track. The modelling parameters are listed in Table II. All the network components are represented as either series or shunt connected elements. Trains are modelled as dynamic current vectors varying with power demand and location. This approach is regarded as the chain circuit topology [13] and similar approaches have also been seen in [14, 15].

The feeder transformer and autotransformers are modelled as admittance matrices Y_{sub} and Y_{AT} respectively, the dimensions of which depend on the number of conductors being modelled. A track circuit contains several parallel conductors such as contact wire, messenger wire, positive feeder wire (F), and rails, with electromagnetic coupling effect extant across them. The formula for calculating self and mutual impedance is determined in [16]. However, it is necessary to reduce the dimension of Y_{line} otherwise it will be very bulky and affect the execution speed. This has been done by combining the contact wire and message wire as a catenary conductor (T). Two rails are combined to be a single conductor (R). Therefore, the overhead line system is reduced to six parallel conductors, as shown in Fig 3.

Fig 3. Schematic overview of multi-conductor feeding line model

TABLE II. PARAMETERS USED IN SIMULATION

Component	Parameter	Value
Feeder substation	Voltage ratio (kV)	400/2×25
	Rated power (MVA)	30
	Short-circuit impedance (%)	10
Autotransformer	Voltage ratio (kV)	50/2×25
	Rated power (MVA)	10
	Leakage impedance (Ω)	0.1+j0.45
Rail track geometry	Sourced from [13]	
OHL electrical parameters	Sourced from [17]	

To simulate the dynamic movement of train loads, the railway line is split into 400 small segments with individual length of 100 m. Hence, the total number of nodes is 401. At each time-step, the train is modelled as a constant current vector, $[I_{train}]$, using (5) and (6).

$$i_{train} = \left(\frac{P_{train} + jQ_{train}}{u_T - u_R} \right)^* \quad (5)$$

$$[I_{train}] = \begin{cases} [-i_{train}, i_{train}, 0, 0, 0, 0]^T & \text{(Up track)} \\ [0, 0, 0, -i_{train}, i_{train}, 0]^T & \text{(Down track)} \end{cases} \quad (6)$$

Fig 2 (b) illustrates the chain representation of Fig 2 (a). Y_{sub} and Y_{AT} are inserted into the system admittance matrix Y according to their position. In order to parallel connect both tracks, a shunt connected matrix $Y_{connect}$ is utilised. Besides, the power is fed from the substation node so an injected current vector $[I_{sub}]$ is calculated. $[I_{sub}]$ and $[I_{train}]$ are added into the current source vector and the nodal voltages can be calculated, as shown in (7).

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_{400} \\ U_{401} \end{bmatrix} = [Y]^{-1} \cdot \begin{bmatrix} I_{sub} \\ I_{train_1} \\ 0 \\ \vdots \\ I_{train_2} \\ 0 \end{bmatrix} \quad (7)$$

However, iteration is required as the train pantograph and rail voltages are unknown. The algorithm is shown in Fig 4. Firstly, the electrical parameters are imported and the system Y matrix is formulated. All train power demand and location information obtained from dynamic train performance simulation is then loaded. Based on the voltage at the corresponding node, the train current is computed and the current source vector is formed. After that, nodal voltage is updated using (7). The convergence is checked, which is set as 10^{-4} , against the error of critical node voltages compared with the previous iteration. The critical node is defined as the pantograph and rail voltage for all trains in the system. Under normal loading conditions (three trains running at maximum power on each track), no more than 6 iterations are required to reach convergence.

After obtaining node voltages, the current distribution is assessed by calculating current between each two adjacent nodes using (8).

$$\begin{bmatrix} I_{T1} \\ I_{R1} \\ I_{F1} \\ I_{T2} \\ I_{R2} \\ I_{F2} \end{bmatrix}_i = [Y_{line}]^{-1} \cdot \left(\begin{bmatrix} U_{T1} \\ U_{R1} \\ U_{F1} \\ U_{T2} \\ U_{R2} \\ U_{F2} \end{bmatrix}_{i+1} - \begin{bmatrix} U_{T1} \\ U_{R1} \\ U_{F1} \\ U_{T2} \\ U_{R2} \\ U_{F2} \end{bmatrix}_i \right) \quad (8)$$

($i = 1, 2, 3 \dots 400$)

The substation loading is evaluated by using the voltage and current values at the substation node, as in (9).

$$P_{sub} + jQ_{sub} = (U_{T1} - U_{R1}) \cdot I_{T1}^* + (U_{F1} - U_{R1}) \cdot I_{F1}^* + (U_{T2} - U_{R2}) \cdot I_{T2}^* + (U_{F2} - U_{R2}) \cdot I_{F2}^* \quad (9)$$

The benefit of using this approach is that the node number only depends on the length of line segment so the system Y matrix does not evolve with time. Only the current source vector has to be updated according the train location at each time-step. This simplifies the programming effort for multi-train simulation. Although trains can only be accommodated to fixed node positions, the accuracy is acceptable for transformer thermal performance study.

E. Thermal Modelling

Construction of the thermal model is done in accordance with IEC Standard 60076-7 [8]. This model is built by using the thermal diagram, where the hottest spot temperature within the transformer consists of three main parts: ambient temperature; top oil temperature (TOT) rise above ambient temperature; and HST rise above TOT.

Fig 4. Flowchart of the algorithm to calculate nodal voltage

Three transformer cases are considered: two cases ('base case' and 'future case') consider the same parameters (from [8] for power transformers with all-natural cooling), however the rating of the transformer is reduced in the 'future case' from 30 MVA to 20 MVA to imitate higher loads placed on the transformer due to electrification of transport. The final case (which uses the original 30 MVA rating) is an 'alternative case' which uses different parameters suggested in [18], thereby adopting a design more suited to a transformer that experiences frequent, large, short-term peak loads.

The model's inputs are: the above-mentioned thermal parameters; the ambient temperature (which is simulated for a September day); transformer load; and transformer rating. The latter two combine in ratio to give the load factor of the transformer.

The model then outputs a HST plot, which can then be interrogated to see if the transformer is likely to be at risk of failure through any of the five mechanisms outlined in Section I. TOT is also plotted as this informs considerations relating to failure mechanisms iii, iv and v outlined in the introduction.

III. CASE STUDY SCENARIO

A case study is used to assess the thermal performance of traction transformer in the feeder substation. The feeding system used in this study is the same as the one described in Section II. There are two train stations within the feeding section. For simplicity, station A is assumed at the substation and station B is set at the furthest AT site. Gradient variation is neglected. Trains departing from station A to station B are operating on the up track while those from station B to station A are running on the down track. Trains accelerate when they leave the station until maximum operating speed is

Fig 5. Line voltage distribution with 10 Class 390 trains drawing maximum power

TABLE III. DAILY TIMETABLE FOR 40 KM DOUBLE-TRACK RAILWAY LINE

Time	Up Track		Down Track	
	Number	Interval (s)	Number	Interval (s)
0:00-6:00	1	1020	1	1020
6:00-10:00	5	180	5	180
10:00-11:00	4	240	3	360
11:00-15:00	3	360	2	420
15:00-16:00	3	360	4	240
16:00-20:00	5	180	5	180
20:00-21:00	2	420	3	360
21:00-24:00	1	1020	1	1020

reached (200 km/h). All trains brake to stop at the destination station. The whole journey takes about 840 s to complete. It is assumed that both station A and B are intermediate stations, i.e. trains will continue beyond the destination station.

A daily timetable is given in Table III. Peak hours are set from 06:00 to 10:00 and 16:00 to 20:00. During peak hours, the interval between trains is 180 s so the maximum number of trains on each track is five. In order to assess whether the system can cope with ten trains without having under-voltage problems, a static analysis is conducted based on the worst case scenario. It is assumed that there are ten trains all drawing maximum power simultaneously. Trains are located at 8 km, 16 km, 24 km, 36 km and 40 km on up track and at 2 km, 10 km, 18 km, 26 km and 34 km on down track, with reference to the substation. The voltage distribution is shown in Fig 5. It shows that for parallel connected double-track system, the minimum voltage for up track and down track are both 20.3 kV, occurring at the furthest AT site. A comparison has been made by looking at voltage distribution without parallel connection. It can be seen the lowest voltage in the system is 19.9 kV at the furthest AT site. This confirms that parallel connected topology is able to withstand heavier loading conditions. The minimum voltage does not fall below the minimum allowable voltage (19.0 kV) [19], even in the worst case scenario. As a result, voltage dependent train operation is not seen in this study.

IV. RESULTS ANALYSIS

According to the timetable shown in Table II, a substation loading profile is obtained as shown in Fig 6. This load profile is assessed on a fineness of five seconds, and has the HST plot shown in Fig 7. These values are based on the ambient temperature of a September day, and so are not 'worst case', but are generally representative. Fig 7 also shows the TOT.

Fig 6. Substation daily loading profile

Fig 7. Traction transformer thermal performance evaluation

It is seen that there are two strong peaks in HST which coincide with the peak loading times (peak hours). The HST rises rapidly to a peak value during this period, before falling rapidly to a considerably lower value in the off-peak periods.

In the base case, maximum temperatures seen are far below the rated operating temperature of 98 °C. The rate of temperature increase is largest at just after 06:00 with a value of 3.1 K / min, and the rate of temperature decrease is greatest just after 10:00, at 2.1 K / min. This implies that the swift change from off-peak, to peak, and back again could be the main intervals where the transformer is placed at risk. For the alternative case, maximum temperatures and changes of temperature are seen to be much lower; for the future case, which considers additional loading, temperatures can be dangerously high (Fig. 7).

V. DISCUSSION

From the results of this modelling, there are several key areas of interest. All of these relate to the effect of temperature on the transformer lifetime, and specifically its propensity to fail under the loading conditions simulated. The output HST profile shown in Fig 7 suggests that the temperature is always significantly below the rated temperature of 98 °C. This is encouraging, as it suggests that transformer ageing will be below the designed for rate, and hence the transformer is unlikely to reach its useful end of life early.

Transformer ageing addresses long term concerns, yet short term failure of transformers can occur as identified in Section I. The absolute HST seen in the transformer here is not high enough to make bubble formation a concern, despite high rates of temperature increase.

Fig 8. Water saturation in mineral oil

In this study, it can be seen that, especially during the ‘future case’, there are two occasions of prolonged rapid temperature reductions in the transformer, from relatively high temperatures to much lower temperatures. Fig 7 shows how the TOT tracks the HST, and from this it can be concluded that formation of free water could be of concern – especially when viewed in collaboration with Fig 8 which indicates the amount of water that oil can hold at saturation at difference temperatures (shown for mineral oil). Using the temperatures of the high (77 °C) and low (45 °C) points in Fig 8, the difference in saturation is approximately 500 ppm. For transformers that have operated for a period of time and have hence become wetter, this may be a concern. Similarly, some consideration must be given here to gaseous generation through the same mechanism of rapid temperature reduction. Dissolved nitrogen (among other gases) will tend to be released from the oil as oil temperature decreases (as the transformer volume occupied by the oil is reduced due to a reduction of its density). In the same fashion as with water, when the rate of this temperature reduction is rapid, the gases may release as bubbles and this can endanger the transformer.

VI. CONCLUSION

Concerns about increasing railway traffic volume and power demands lead to the preference of AT feeding system. Although traction transformers are designed to operate at greater than normal loading conditions, they are under pressure from extreme loading conditions during peak hours. Therefore, a thermal assessment is necessary to address potential hazard to transformers. The results of this study show that traction transformers are at relatively low risk of thermal failures, but should be monitored to protect against potential free water formation which can lead to instantaneous failure due to flashover. Further, the potential for high instantaneous HST could be serious if additional loading of these transformers occurs.

This type of study has not been paid enough attention in traction systems. In this paper, an AT feeding simulation platform with integrated thermal modelling has been built in MatLab to meet this requirement. Future study will focus on improving the veracity of the model and include specific route analysis.

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