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DOI:

[10.1016/j.ijimpeng.2017.02.016](https://doi.org/10.1016/j.ijimpeng.2017.02.016)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Kong, X. Z., Fang, Q., Li, Q. M., Wu, H., & Crawford, J. (2017). Modified K&C model for cratering and scabbing of concrete slab under projectile impact. *International Journal of Impact Engineering*, 108, 217-228. <https://doi.org/10.1016/j.ijimpeng.2017.02.016>

Published in:

International Journal of Impact Engineering

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Modified K&C model for cratering and scabbing of concrete slabs under projectile impact

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Abstract

The K&C material model, which was originally developed for the structural response subjected to blast **loadings**, is modified to improve its numerical prediction capability for cratering and scabbing phenomena in concrete slabs subjected to projectile impact. Four modifications are made for the parameters of the strength surfaces, the dynamic increase factor for tension, the relationship between yield scale factor and damage function, and the tensile damage accumulation. Single element tests of **the** unconfined uniaxial compression, triaxial compression, dynamic uniaxial tension as well as **the** biaxial and triaxial tensions are used to demonstrate the improved performances of the modified K&C model. The modified concrete strength model is implemented into the finite element code LS-DYNA through user-defined material model. The improved predictions for cratering and scabbing phenomena under projectile impact **using** the modified K&C model are demonstrated by comparisons with **the** perforation tests and **the** corresponding

simulation results from the original K&C model. Furthermore, sensitivity analyses of three parameters that control the dynamic tensile behavior of the modified K&C model are carried out to understand the influence of each individual parameter on the cratering and scabbing phenomena.

Keywords: Concrete; Cratering; Scabbing; Modified K&C model; Penetration

1 Introduction

When a concrete slab is subjected to projectile impact, cratering and scabbing phenomena are frequently observed on its impact and distal surfaces, respectively [1-2]. Cratering involves in the compressive stress wave propagation in the target, the reflected tensile stress wave on the impact surface of the slab and the associated material failure [3]. Scabbing is caused by the reflected tensile stress wave on the distal free surface. Numerical analyses of cratering and scabbing in concrete slabs subjected to projectile impact [3-8] have showed that the tensile strength, strain softening, fracture energy, and strain-rate effect on the tensile strength in a concrete model can greatly influence the accurate modeling of cratering and scabbing phenomena.

Tu and Lu [9] reviewed and evaluated several popular concrete models in commercial hydrocodes, in which the Holmquist-Johnson-Cook concrete model (HJC model) has been widely accepted for penetration studies because HJC model is valid mainly under the stress states dominated by compression [10] which happens in the penetration process. For the stress states dominated by tension, The HJC model uses the elastic, perfectly plastic constitutive equation, which is independent of fracture energy. Further modifications of the HJC model focused mainly on the compressive behavior [11] and the strain-rate effect [11-12]. Therefore, the HJC model and

above-mentioned modified HJC models are not suitable for the tension-dominated phenomena, e.g. cratering and scabbing. Huang et al. [4] employed a modified Taylor-Chen-Kuszmaul (TCK) continuum damage model together with an erosion technique to simulate cratering and scabbing of concrete slabs. The compressive behavior of the original TCK model [13] was replaced by the damaged Mohr-Coulomb yield surface [4] because the original one is governed by von-Mises yield surface, which is too simple to describe the concrete compressive behavior under high pressure. The modified TCK model [4] showed reasonable predictions of cratering and scabbing phenomena although the residual velocity was not well predicted, which was attributed to the inaccuracy of the damaged Mohr-Coulomb yield surface in compression. In order to resolve this issue, Liu et al. [5] employed the HJC and the TCK models to describe the dynamic compression and tension, respectively. The calculated residual velocity was improved in comparison with the predictions by Huang et al. [4]. However, the failure surface of the concrete material model by Liu et al. [5] is discontinuous due to the separate treatments of the compressive behavior and tensile behavior. Wang et al. [3] also presented a modified TCK model for numerical studies on cratering and scabbing, in which the compressive damage was introduced. The common feature of these studies is to apply the TCK model to describe the dynamic tensile behavior of concrete, while the von-Mises compressive yield surface in the original TCK model was replaced by the more suitable compressive yield surfaces (e.g. the Mohr-Coulomb or the HJC yield surfaces). Because the TCK tensile model [13] considers the effects of strain-softening, strain-rate and fracture energy under tensile loading, it is capable of predicting the brittle failure of concrete. However, the TCK tensile model has nine differential equations to be integrated in each time step in the implementation algorithm, which reduces the computational efficiency and may cause

68 numerical instability. Besides, some parameters in the TCK tensile model, such as the volumetric
69 strain-rate experienced by concrete at fracture, are practically difficult to determine. Recently,
70 Kong et al. [8] presented several improvements to the HJC model, including the modified yield
71 surface, the introduction of tensile damage, the introduction of Lode-angle dependency and the
72 modified strain-rate effect, for the numerical simulations of cratering and scabbing phenomena.
73 Although the predictions have reasonably good agreements with corresponding experimental
74 results, there are still some shortcomings of the model, i.e., (i) strain hardening was not considered;
75 (ii) strain softening is only dependent on the cutoff of fracture strain (named EFMIN in [10]) that is
76 usually difficult to determine.

77 In order to improve the numerical predictions of cratering and scabbing phenomena in
78 concrete, Leppänen [6] modified the RHT model [14] with a bi-linear tensile softening model
79 based on fracture energy and strain-rate effect in tension. Tu and Lu [7] further made several
80 enhancements to the modified RHT model by Leppänen [6], including the inclusion of the third
81 stress invariant in residual strength surface, and the improved tensile-to-compressive meridian
82 ratio. Both Leppänen [6] and Tu and Lu [7] assumed that the fracture energy is a constant and the
83 fracture strain decreases with the increase of strain-rate in tension. This assumption is
84 inconsistent with recent experiments on tensile strength and fracture energy from instrumented
85 spalling tests [15-16], which suggested that the fracture strain at high strain-rate is a constant and
86 the fracture energy increases with the increase of strain-rate in tension.

87 The Karagozian & Case concrete model [17] (K&C model, material #72REL3 in LS-DYNA)
88 was originally developed to analyze structural response subjected to blast loadings, which has
89 become one of the popular concrete models for the response of concrete targets under intensive

dynamic loads. However, this model still has some disadvantages, i.e., (i) In the version of Release III, the automatic generations of parameters are provided and widely adopted by users. These parameters, such as the parameters for the strength surfaces, are useful for relatively low pressure, but may be inappropriate for very high pressure; (ii) For dynamic tensile behavior, both the fracture strain and the fracture energy increase with the increase of the strain-rate in tension. The increase of fracture strain with the increase of strain-rate is inconsistent with the experimental observations [15-16], as discussed above; (iii) Tensile failure criterion is not available, which plays an important role for the simulations of tensile-dominated failure phenomena.

The present paper proposes a modified K&C model to improve its capability to predict tensile failure in concrete (e.g. cratering and scabbing). After a brief description of the original K&C concrete material model, modifications for the strength surfaces parameters, the dynamic increase factor for tension, the relationship between yield scale factor and damage function, and the tensile damage accumulation will be presented in Section 2. The improved performances of the modified K&C model are demonstrated by single element numerical tests. Simulation results and parametric analyses of concrete slabs subjected to projectile impact are presented in Sections 3 and 4, respectively, which is followed by conclusive remarks in Section 5.

2 Modified K&C model

The original K&C model is introduced briefly before the modified K&C model is presented where the improved performances due to the modifications are demonstrated by the single finite element numerical tests. All parameters are obtained using the automatic generation method offered by the original K&C model [18] unless stated otherwise.

112 2.1 The K&C model in LS-DYNA

113 Three independent strength surfaces of compressive meridians, namely, the initial yield
114 strength surface $\Delta\sigma_y$, the maximum strength surface $\Delta\sigma_m$ and the residual strength surface
115 $\Delta\sigma_r$ were used in the K&C model [17], i.e.,

$$116 \quad \Delta\sigma_y = \begin{cases} a_{0y} + p / (a_{1y} + a_{2y}p), & p \geq f_{yc} / 3 \\ 1.35T + 3p(1 - 1.35T / f_{yc}), & 0 \leq p \leq f_{yc} / 3 \\ 1.35(p + T), & p \leq 0 \end{cases} \quad (1)$$

$$117 \quad \Delta\sigma_m = \begin{cases} a_0 + p / (a_1 + a_2p), & p \geq f_c / 3 \\ (1.5 / \psi)(p + T), & \{0 \leq p \leq f_c / 3\} \text{ or } \{\lambda \leq \lambda_m \text{ and } -T \leq p \leq f_c / 3\} \\ 3(p / \eta + T), & p \leq 0 \text{ and } \lambda > \lambda_m \end{cases} \quad (2)$$

$$118 \quad \Delta\sigma_r = a_{0f} + p / (a_{1f} + a_{2f}p) \quad (3)$$

119 where a_i , a_{iy} and a_{if} ($i=0, 1, 2$) are constants determined from a suitable set of triaxial
120 compression test data. $f_{yc}=0.45f_c$, f_c and T are the **initial yield strength**, unconfined uniaxial
121 compressive strength and tensile strength, respectively. p is the **hydrostatic pressure**, and ψ
122 denotes the tensile-to-compressive meridian ratio, **which is determined by a linear interpolation of**
123 **the discrete points described by [17],**

$$124 \quad \psi(p) = \begin{cases} \frac{1}{2} & p \leq 0 \\ \frac{1}{2} + \frac{3T}{2f_c} & p = \frac{f_c}{3} \\ \frac{\alpha f_c}{a_0 + \frac{2\alpha f_c}{3a_1 + 2\alpha a_2 f_c}} & p = \frac{2\alpha f_c}{3}, \quad \alpha \approx 1.15 \\ 0.753 & p = 3f_c \\ 1 & p \geq 8.45f_c \end{cases} \quad (4)$$

125 **As observed from Eqs. (1-2), the initial yield strength surface and the maximum strength**
126 **surface are piecewise-defined functions. For the maximum strength surface, the first segment**

($p \geq f_c / 3$) is defined by available data of the unconfined uniaxial compression (UUC) test and the triaxial compression (TXC) test. If there is no available test data for the second segment ($\{0 \leq p \leq f_c / 3\}$ or $\{\lambda \leq \lambda_m \text{ and } -T \leq p \leq f_c / 3\}$) because the pressure is lower than the pressure domain of the UUC test, the second segment is obtained by dividing the tensile meridian with the tensile-to-compressive meridian ratio $\psi(p)$. For the third segment ($p \leq 0$ and $\lambda > \lambda_m$), strain-softening happens. Therefore, the pressure cutoff ($-T\eta$) is applied, and the maximum strength surface is a linear interpolation between the zero-pressure point at the second segment ($p=0, \Delta\sigma_m=3T$) and the pressure cutoff point ($p=-T\eta, \Delta\sigma_m=0$).

For the initial yield strength surface, the first segment ($p \geq f_{yc} / 3$) is defined by available TXC data, which suggest that $\Delta\sigma_y = 0.45\Delta\sigma_m$ [17]. If there is no available test data for the second segment ($0 \leq p \leq f_{yc} / 3$), the second segment is obtained as a linear interpolation between the yield point ($p=f_{yc}/3, \Delta\sigma_m=f_{yc}$) and the point proportional (45%) to the maximum strength surface at zero pressure ($p=0, \Delta\sigma_m=1.35T$). For the third segment ($p \leq 0$), the peak pressure can be $-T$ since the strain-hardening undergoes before yielding, and the initial yield strength surface is a linear interpolation between the triaxial tension point ($p=-T, \Delta\sigma_m=0$) and the 45% of the maximum surface at zero pressure ($p=0, \Delta\sigma_m=1.35T$). The definition of the residual strength surface is rather simple as shown in Eq. (3), which can be obtained by fitting TXC data since residual strength can be identified from stress-strain curves of TXC data. It is noted that a_{0f} should be zero since the residual strength should vanish for the UUC test.

Advantages of these strength surfaces, especially at $p < f_c / 3$, have been illustrated in [9], which will not be repeated here. A typical view of the strength surfaces for $f_c=50\text{MPa}$ is shown in Fig. 1, where the parameters a_i , a_{iy} and a_{if} are determined by the automatic generation

149 method [18].

150
151 Fig. 1 Three independent strength surfaces ($f_c=50\text{MPa}$)

152 The current failure surface was determined as follows,

$$153 \quad \Delta\sigma = \sqrt{3J_2} = \begin{cases} r' \left[\eta (\Delta\sigma_m - \Delta\sigma_y) + \Delta\sigma_y \right], & \text{strain hardening} \\ r' \left[\eta (\Delta\sigma_m - \Delta\sigma_r) + \Delta\sigma_r \right], & \text{strain softening} \end{cases} \quad (5)$$

154 where J_2 is the second deviatoric stress invariant. r' is the ratio of the current meridian to the
155 compressive meridian. η is the yield scale factor related to the damage function λ , which is
156 determined by,

$$157 \quad \lambda = \sum \begin{cases} \Delta\bar{\varepsilon}_p / (1 + p/T)^{b_1}, & p > 0 \\ \Delta\bar{\varepsilon}_p / (1 + p/T)^{b_2}, & p \leq 0 \end{cases} \quad (6)$$

158 where $\Delta\bar{\varepsilon}_p = \sqrt{(2/3)\Delta\varepsilon_{ij}^p \Delta\varepsilon_{ij}^p}$ is the effective plastic strain increment with $\Delta\varepsilon_{ij}^p$ being the tensor
159 of the plastic strain increment in a time step. The damage constants b_1 and b_2 usually have different
160 values for compression and tension with considering different damage evolutions of concrete. The
161 yield scale factor η follows a general trend, i.e., it varies from zero to unity when the current failure
162 surface changes from the initial yield strength surface to the maximum strength surface
163 corresponding to the strain hardening stage, and it varies from unity to zero when the current failure

surface changes from the maximum strength surface to the residual strength surface corresponding to the strain softening stage. It should be noted that no accumulative plastic strain occurs when the stress path is close to the triaxial tensile path where stress deviators approach zero. Consequently, both the damage function λ and the yield scale factor η remain zero, which cannot represent the actual situation. To overcome this problem, a volumetric damage scalar related to pressure was introduced [17], which will not be repeated here.

To account for the strain-rate effect, the current failure surface and the damage function λ were modified as follows,

$$\Delta\sigma = r_f \Delta\sigma(p/r_f), \quad \lambda = \lambda(p/r_f)/r_f \quad (7)$$

where r_f is the dynamic increase factor (DIF), which was defined separately for compression and tension, since experimental data showed that DIF for tension is much higher than that for compression. The determination of DIF will be discussed further in Section 2.3.

For the complete description of concrete behavior, a proper equation of state (EOS) is needed to relate pressure to volumetric strain. K&C model uses the tabulated compaction EOS (EOS #8) in LS-DYNA, which describes the current pressure p as a function of the volumetric strain μ

$$p = C(\mu) + \gamma_0 \theta(\mu) E_0 \quad (8)$$

where E_0 is the internal energy per initial volume, and γ_0 is the ratio of specific heat. $C(\mu)$ and $\theta(\mu)$ are the tabulated pressure evaluated along a 0-degree-Kelvin isotherm and tabulated temperature-related parameter as functions of the volumetric strain, respectively. In the loading stage, pressure is determined by extrapolating the tabulated function given in Eq. (8). As shown in Fig. 2, unloading occurs along the unloading bulk modulus to the pressure cutoff. Reloading

186 follows the unloading path to the point where unloading begins, and continues on the loading path.

Fig. 2 Equation of state

2.2 Modified strength surfaces parameters

In the original K&C model, the strength surfaces parameters were obtained from the triaxial compression data of 45.4MPa concrete and expressed as functions of concrete compressive strength [18], i.e.,

$$a_0 = 0.2956 f_c ; a_1 = 0.4463 ; a_2 = 0.0808 / f_c \quad (9a)$$

$$a_{0y} = 0.2232 f_c ; a_{1y} = 0.625 ; a_{2y} = 0.2575 / f_c \quad (9b)$$

$$a_{0f} = 0 ; a_{1f} = 0.4417 ; a_{2f} = 0.1183 / f_c \quad (9c)$$

Eq. (9) is suitable for relatively low pressure. However, for the projectile impact problem concerned here, the pressure around the projectile-target interface can reach a magnitude in the order of GPa. Consequently, the strength surfaces parameters should be re-determined. A large amount of triaxial compression data are shown in Fig. 3. It can be clearly observed that Eq. (9a) has good agreement with the experimental data when $p/f_c < 3$, beyond which the deviation increases with the increase of pressure. A new set of the maximum strength surface parameters is proposed from curve-fitting the data shown in Fig. 3 as follows,

$$a_0 = 0.4426f_c; \quad a_1 = 0.5698; \quad a_2 = 0.02516/f_c \quad (10a)$$

Fig. 3 Triaxial compression data as well as predictions from Eq. (9a) and Eq. (10a)

It should be pointed out that Eq. (10a) deviates from experimental data for high pressure using the least square method due to the lack of data. It is not clear whether Eq. (10a) is accurate enough for $p/f_c > 12$. However, the generality of above curve-fitting approach is not affected, and the maximum strength surface parameters can be re-determined when the shear strength-pressure experimental data for high pressure become available.

By assuming $\Delta\sigma_y = 0.45\Delta\sigma_m$, the initial yield surface parameters can be obtained following the same way proposed in [17],

$$a_{0y} = 0.2797f_c; \quad a_{1y} = 0.8989; \quad a_{2y} = 0.0685/f_c \quad (10b)$$

For the determination of the residual strength surface parameters, the residual strength from triaxial compression experiments is needed, which, however, is difficult to find in the relevant literatures. Noting that the residual strength surface should be parallel with the maximum strength surface for high pressure, the residual strength surface parameters are proposed as follows,

$$a_{0f} = 0; \quad a_{1f} = a_1; \quad a_{2f} = a_2 \quad (10c)$$

Eqs. (10a-c) are used for the modified K&C model in the following discussions. Another

concern is the accuracy of the automatically generated EOS parameters discussed below. For the EOS of concrete material, the available experimental data is limited. The isotropic compression data from Hanchak et al. [1] for 48MPa concrete as well as the full-scale explosive detonation and flyer-plate-impact data from Gebbeken et al. [25] for 51.2MPa concrete are plotted on Fig. 4. Using the automatically generated parameters of the original K&C model gives a satisfactory agreement with the experimental data. Therefore, the automatically generated EOS parameters are adopted in the present study.

Fig. 4 Pressure-volumetric strain test data and predicted EOS using automatically generated parameters

2.3 Modified DIFt

A previous study [8] showed that the dynamic increase factor for tension (DIFt) has a strong influence on the cratering and scabbing damage of concrete slabs subjected to projectile impact. Therefore, an accurate relationship between DIFt and strain-rate is essential. Numerical analysis [26] concluded that DIFt observed in the dynamic tensile tests is a genuine material effect, consequently, DIFt obtained from curve-fitting experimental data can be applied to material model directly.

So far, there are many suggested relationships between DIFt and strain-rate in the literatures [27-30], which are plotted on Fig. 5 along with the experimentally-obtained DIFt. It is found that, the CEB-FIB recommended DIFt, which was widely used in previous studies, greatly underestimates the test data. The empirical formula suggested by Xu and Wen [30] shows good agreement with the test data, and therefore, is used in the present study, i.e.,

$$\text{DIFt} = \left\{ \left[\tanh \left(\left(\log \left(\dot{\varepsilon} / \dot{\varepsilon}_0 \right) - W_x \right) S \right) \right] \left(F_m / W_y - 1 \right) + 1 \right\} W_y \quad (11)$$

where $\dot{\varepsilon}_0 = 1 \text{ s}^{-1}$ is the reference strain rate, $F_m=10$, $W_x=1.6$, $S=0.8$ and $W_y=5.5$ are the fitting constants.

This paper adopts the recommended dynamic increase factor for compression (DIFc) in K&C model (i.e., CEB model in compression [27]) for consistency. However, it has been demonstrate that DIFc obtained from the dynamic compressive tests includes the “structural effect” induced by the lateral confinement effect especially when the strain-rate is greater than a critical transition strain-rate between 10^1 and 10^2 s^{-1} for concrete [31-33]. The lateral confinement effect may significantly overestimate DIFc in a SHPB test for high strain-rates that happen frequently in impact and blast loaded concrete structures. The latest version of K&C model introduced a cutoff value of 2.94 for DIFc, which can avoid the overestimations of DIFc at high strain-rates, and it will be used in the present study.

Fig. 5 Experimental DIFt and suggested relations (test data reprinted from [15])

2.4 Modified relationship between η and λ

In the implementation of the original K&C model, the relationship between η and λ is taken as a piece-wise linear curve. If the automatically generated method is not adopted, 13 pairs of (λ_i, η_i) must be carefully defined, which is inconvenient for practical application.

A new relationship between η and λ is proposed as follows,

$$\eta(\lambda) = \begin{cases} \alpha\lambda / \lambda_m + (3-2\alpha)(\lambda / \lambda_m)^2 + (\alpha-2)(\lambda / \lambda_m)^3, & \lambda \leq \lambda_m, \text{strain hardening} \\ \frac{\lambda / \lambda_m}{\alpha_c (\lambda / \lambda_m - 1)^{\alpha_d} + \lambda / \lambda_m}, & \lambda > \lambda_m, \text{strain softening} \end{cases} \quad (12a, b)$$

where α , α_c and α_d are the constants govern the strain hardening and softening stages, respectively. λ_m is the corresponding value of λ when $\eta=1$. α , α_c , α_d and λ_m can be obtained by trial-and-error method until the predicted uniaxial stress-strain curve agrees well with the recommended uniaxial stress-strain relations, such as those in [27] and [34]. $\alpha=3$, $\alpha_c=0.29$, $\alpha_d=1.86$ and $\lambda_m=8.7e-5$ are suggested based on a large amount of trial calculations.

Attard and Setunge [34] developed an empirical analytical model for the full stress-strain relationships of confined and uniaxially-loaded concrete, which was shown to be applicable for a

269 broad range of concrete strengths between 20 MPa and 130 MPa. The stress-strain curves for
 270 50MPa concrete under different confinement pressure (P_{con}) obtained from the modified K&C
 271 model described in this section and the original K&C model are compared with the empirical
 272 formula suggested by Attard and Setunge [34], as shown in Fig. 6. It can be observed that the
 273 modified model shows good agreement with the empirical formula, while the original K&C model
 274 is stiffer in the loading stage and has larger softening gradient when compared with the empirical
 275 formula. In addition, the original K&C model overestimates the peak stress, while the modified
 276 model has good agreement with the empirical peak stress due to the use of the new strength
 277 surfaces parameters given in Eq. (10).

278
 279 Fig. 6 Comparisons of compressive stress-strain curves of 50MPa concrete developed by Attard
 280 and Setunge [34] with those by modified and original K&C models obtained from single element
 281 test

282 2.5 Modified tensile damage accumulation

283 A typical load-displacement curve for uniaxial tension is shown in Fig. 7(a) where δ is the
 284 deformation of the fracture zone and F is the tensile force. The subscripts m and $frac$ denote the
 285 corresponding quantities at the maximum strength and fracture, respectively. It should be pointed

286 out that δ cannot be represented by the total displacement in a uniaxial tension test as the
287 fracture occurs in a localized zone. A previous study [16] showed that δ is in the order of 0.1
288 mm.

289 In order to determine the relation between stress and strain that is necessary for material
290 model, a relation between the deformation δ and the tensile strain ε must be assumed.
291 Following the assumption made in [16], in which ε is defined as δ / l_{frac} , the corresponding
292 stress-strain curve can be obtained, as shown in Fig. 7(b), where l_{frac} denotes the length of the
293 fracture zone (crack width) [16] over which the microcracks are assumed to be uniformly spread.
294 Extensive static and dynamic tensile tests have been conducted previously, but there is little
295 quantitative information about the size of the fracture zone [16]. Based on the recommendation of
296 Bazant for static loading, the size of the fracture zone is typically 1-6 times of the maximum
297 aggregate size [17].

298 Since the static fracture energy G_f required to fail the concrete is dissipated within l_{frac} , the
299 specific strain energy in the entire cracking process should meet the following energy
300 conservation condition [16],

301
$$\int_{\varepsilon_m}^{\varepsilon_{frac}} \sigma d\varepsilon = G_f / l_{frac} \quad (13)$$

(a)

(b)

Fig. 7 Typical load-displacement and stress-strain curves for uniaxial tension: (a) load-displacement curve; (b) stress-strain curve

In **FE-simulations**, l_{frac} is always related to the characteristic length of the element h_c , which can be approximated by the cube root of the volume of an element in a 3-D analysis [7]. Previously, $l_{frac}=h_c$ was always used [7, 8] based on the assumption that the tensile fracture occurs in one element, as shown in Fig. 8(a). However, for the accurate simulations of cratering and scabbing phenomena, h_c is usually needed to be smaller than l_{frac} , which means that the fracture occurs in several elements, as shown in Fig. 8(b). Consequently, the element of size h_c will

314 dissipate fracture energy of $(h_c/l_{frac})G_f$, since (l_{frac}/h_c) of them are needed to fill the fracture zone
 315 and develop the fracture energy of G_f .

318 Fig. 8 Schematic diagram of the relationship between l_{frac} and h_c , tensile fracture occurs in (a) one
 319 element; (b) several elements

320 In the original K&C model, if not provided, l_{frac} can be taken as the element size h_c , but no
 321 less than three times of the maximum aggregate size of concrete. When $l_{frac}=10\text{cm}$ and G_f is
 322 around 100N/m , Eq. (13) is satisfied. For other lengths of the fracture zone, in order to satisfy Eq.
 323 (13) and maintain the static fracture energy around 100N/m at the same time, a relative element
 324 size parameter $rsizel_{frac}/(10\text{cm})$ was introduced to obtain the new damage function values for the
 325 strain softening stage as follows,

$$326 \quad \lambda'_i = \lambda_m + (\lambda_i - \lambda_m) / rsizel_{frac}/(10\text{cm}) \quad (14)$$

327 where the subscript i ranges from 6 to 13, corresponding to the strain softening stage. Then the
 328 relationship between η and λ for strain softening is taken as a piece-wise linear curve using new
 329 pairs of (λ'_i, η_i) shown in Eq. (14).

330 There are several weaknesses of the K&C tensile damage model, as discussed below,

331 (1) For the static fracture energy far from 100N/m , b_2 must be re-determined by adjusting its
 332 value until Eq. (13) is satisfied, which is time-consuming.

(2) Since the relationship between η and λ controls both the compressive and tensile behaviors of concrete material, as discussed in Section 2.1, the above treatment for ensuring the static fracture energy affects not only the tensile strain softening stage, but also the compressive strain softening stage. When the length of fracture zone is relatively small, the compressive strain softening will be not obvious, making concrete material relatively “hard”, which may be unrealistic for the FE-simulation of penetration where the compressive strain softening plays an important role in the penetration resistance.

(3) As will be shown in Fig. 9, for the original K&C model, both the predicted fracture strain and the fracture energy increase with the increase of the strain-rate. The increase of fracture strain with the increase of strain-rate is inconsistent with recent experimental observation [15-16] that the fracture strain at high strain-rate is almost a constant.

In order to address the three issues above, the tensile damage accumulation is modified as follows,

$$\lambda = \sum \begin{cases} \frac{\Delta \bar{\varepsilon}_p}{r_f (1 + p / r_f T)^{b_2}}, p < 0 & \text{and } \lambda < \lambda_m \\ \frac{\Delta \bar{\varepsilon}_p}{els}, & p < 0 \quad \text{and } \lambda \geq \lambda_m \end{cases} \quad (15)$$

where els is a constant related to the length of fracture zone and the static fracture energy.

According to Eq. (15), the tensile damage accumulation for the strain hardening stage is the same as that in the original K&C model while a modification is made for the strain softening stage.

Considering the elastic strain limit is relatively small, $\int_{\varepsilon_m}^{\varepsilon_{frac}} \sigma d\varepsilon \approx \int_{\varepsilon_m}^{\varepsilon_{frac}} \sigma d\varepsilon_p$ can be obtained. By

assuming that the tensile failure happens at $\lambda = n\lambda_m$, where the stress approaches zero, as shown in

Fig. 7(b), the left side of Eq. (13) can be replaced by,

353

$$\int_{\varepsilon_m}^{\varepsilon_{frac}} \sigma d\varepsilon \approx \int_{\varepsilon_m}^{\varepsilon_{frac}} \sigma d\varepsilon_p = \int_{\varepsilon_m}^{\varepsilon_{frac}} T \eta d\varepsilon_p \quad (16)$$

354 where η is given by Eq. (12b).355 Using Eq. (15), for the strain softening stage, $d\varepsilon_p = (els) d\lambda$. Consequently, Eq. (16) can be

356 further transformed into,

$$\int_{\varepsilon_m}^{\varepsilon_{frac}} T \eta d\varepsilon_p = \int_{\lambda_m}^{n\lambda_m} T \eta(els) d\lambda = T \times (els) \times \lambda_m \int_1^n \frac{x}{\alpha_c (x-1)^{\alpha_d} + x} dx \quad (17)$$

358 where Eq. (12b) is applied. Based on Eqs. (13, 17), els can be expressed by

$$els = \frac{G_f}{l_{frac} \times T \times \lambda_m \int_1^n \frac{x}{\alpha_c (x-1)^{\alpha_d} + x} dx} \quad (18)$$

360 Consequently, the fracture strain can be determined according to Eq. (15) as

$$\varepsilon_{frac} = n \lambda_m els \quad (19)$$

362 The length of fracture zone, static fracture energy and fracture strain are related by Eqs. (15,

363 18-19) directly in the proposed tensile damage model, which avoids the time-consuming

364 parameter calibration in the original K&C model. It is observed that els is proportional to G_f and365 inversely proportional to l_{frac} , T , λ_m and $\int_1^n x / [\alpha_c (x-1)^{\alpha_d} + x] dx$. Furthermore, els 366 decreases with the increase of n as a hyperbolic way since $\int_1^n x / [\alpha_c (x-1)^{\alpha_d} + x] dx$ approaches367 to a constant when n approaches to an infinite value. In addition, as shown in Eq. (15), the

368 proposed tensile damage model does not affect the compressive strain softening stage, which may

369 be more suitable than the original K&C model. In order to further gain insights into the modified

370 tensile damage accumulation, dynamic tensile stress-strain curves for 50MPa concrete under

371 different strain-rates are calculated. The static fracture energy is assumed to be 85 N/m which is

the same as the automatic generated one in the original K&C model. The length of fracture zone is assumed to be equal to the side length of the cubic element (10mm). Using Eqs. (18-19), ϵ_{ls} and ϵ_{frac} are obtained as 1.15 and 0.01, respectively. As will be discussed in Section 4.1, the parameter n has limited influence on the scabbing simulation results when it is larger than 100 where the tensile stress approaches zero. Consequently, it is set as 100. The predicted stress-strain curves at different strain-rates are shown in Fig. 9, where the results from the original K&C model are also presented for comparison purpose. It can be observed that, for the present model, the fracture strain is a constant and the fracture energy increases with the increase of the strain-rate, which is consistent with the experimental observations [15-16]. While for the original K&C model, both the fracture strain and the fracture energy increase with the increase of the strain-rate, which may overestimate the fracture energy at high strain-rate.

Fig.9 Comparisons of dynamic tensile stress-strain curves between modified and original K&C models obtained from single element test.

The predicted stress-strain curves for biaxial and triaxial tensions are also of interest and shown in Fig. 10. For the biaxial tension, it is found that, the strain softening using the original K&C model is almost linear, which does not give a good prediction. For the triaxial tension,

although the volumetric damage was considered in the original K&C model [17], the predicted stress-strain relation is **elastic, perfectly plastic**, indicating that there may be an error during the implementation of **the** volumetric damage into LS-DYNA. The modified model describes the strain softening well for both biaxial and triaxial tensions.

Fig.10 Stress-strain curves for biaxial and triaxial tensions of modified and original K&C models obtained from single element tests: (a) biaxial tension; (b) triaxial tension

The original K&C model has total 48 parameters, which can be generated automatically using the unconfined compressive strength if this function is opted. This is an attractive feature for engineering applications and case studies. It is worth pointing out that this attractive feature is also available in the modified K&C model, which has total 23 parameters that can all be calculated for a given f_c according to the above-mentioned approaches.

3 Numerical simulations of cratering and scabbing in concrete slabs subjected to projectile impact

The proposed modified concrete material model is implemented into **the** finite element code LS-DYNA [35] through user-defined material model. In order to ensure that the modified model can predict cratering and scabbing phenomena as well as projectile residual velocity at the same time, corresponding results from several experiments with varying projectile impact velocity and

concrete slab thickness should be realistically reproduced. Projectile perforation experiments into concrete slabs conducted by Wu et al. [2] are numerically simulated to validate the present modified model, in which a newly proposed erosion criterion is used. Furthermore, the corresponding simulation results of the original K&C model are also presented to show improved performances of the modified K&C model.

3.1 New erosion criterion

For the numerical simulations using the finite element approach, an erosion algorithm is essential and should be carefully introduced to capture the physical crush (compressive failure) and the fracture (tensile failure) of concrete material. When the specified variable of an element used to represent the compressive or tensile failure reaches its critical value, the element is deleted immediately. It is generally accepted that the compressive failure can be represented by the effective plastic strain [4-5] or the maximum principal strain [6-7, 11-12]. For the tensile failure, the criterion is still inconclusive. The tensile damage in the modified TCK model was considered in previous studies [3-5] as the criterion for the tensile failure. However, the value of tensile damage criterion was determined empirically as 0.5. Recently, Xu and Lu [36] as well as Li and Hao [37] proposed a tensile fracture strain criterion with an empirical critical value of 0.01 for the tensile failure of concrete material, which demonstrated good agreements with experimental results. However, it was observed that the simulation results for scabbing are sensitive to the critical tensile fracture strain [8], which affects its applications.

To resolve the empirical and sensitive issues in above tensile failure criteria, a new tensile failure criterion is proposed as follows,

$$\Delta\lambda_i = \begin{cases} 0, p > 0 \\ \Delta\bar{\varepsilon}_p / \left[r_f \left(1 + p / r_f T \right)^{b_2} \right], p < 0 \quad \text{and} \quad \lambda < \lambda_m; \quad \lambda_i = \sum \Delta\lambda_i \\ \Delta\bar{\varepsilon}_p / els, p < 0 \quad \text{and} \quad \lambda \geq \lambda_m \end{cases} \quad (20)$$

where the prefix Δ refers to the increment of a variable during a time step. When λ_i reaches $n\lambda_m$, the tensile failure occurs, and the element should be deleted immediately. Although an empirical constant n is introduced in the present tensile failure criterion, it will be shown in Section 4.1 that the influence of n on the scabbing simulation results is small when it is greater than 100 because the tensile stress approaches to zero when the strain is large, as shown in Fig. 9. The compressive failure is still represented by the effective plastic strain $\bar{\varepsilon}_p$.

3.2 Simulation results

Recently, we conducted twenty five shots of reduce-scaled projectile perforation tests on five configurations of monolithic and segmented RC panels with a layer of rear steel plate [2]. The cratering and scabbing size, residual velocity and the projectile acceleration were carefully recorded, which provide experimental data for the validation of the modified K&C model.

In our experiments, the projectiles were shot into cuboid 41MPa reinforced concrete (RC) targets with the impact surface size of 675mm×675mm. The steel projectiles have an ogive nose of caliber radius-head (CRH) 3.0, length of 152 mm, diameter of 25.3 mm, and mass of 0.428kg. In order to save the computational cost, only a quarter of the RC target and projectile are modeled, as shown in Fig. 11. The 3D solid element (Type-164 in LS-DYNA) is employed for the projectile and concrete target, and the beam element for the reinforcement bar, respectively. In order to save computational time and ensure computational precision at the same time, the local mesh refinement is employed for concrete target. Based on the mesh sensitivity analyses in [8] and further

449 verifications in this study, the minimum element size for the target is selected to be 2 mm near the
 450 impact location, which is enlarged to 4 mm after a distance beyond ten times of the projectile
 451 diameter. The eroding surface-to-surface contact is employed to define the contact behavior
 452 between the projectile and RC target without friction which has minor effect on the projectile
 453 penetration process [38]. As discussed in Section 3.1, the element erosion technique with the
 454 criteria based on $\bar{\varepsilon}_p$ and λ_t is adopted for the modified K&C model, i.e., the element will be
 455 deleted immediately when $\bar{\varepsilon}_p$ or λ_t is greater than critical values, which is achieved by
 456 user-defined material model. Since there is no available element erosion criteria in the original
 457 K&C model, the maximum principal strain criterion is adopted using *MAT_ADD_EROSION in
 458 LS-DYNA.

459
 460 Fig. 11 FE model of the RC target and projectile

461 The steel projectile is modelled as rigid material as little erosion and deformation of projectile
 462 have been observed. The reinforcement bar is modeled as the elastic, perfectly plastic material
 463 (Mat_003 in LS-DYNA) with the elastic modulus of 210GPa and yield stress of 400MPa. For the
 464 concrete material, the strength surfaces are defined by the parameters given in Eq. (10). The
 465 damage related parameters, i.e., α , α_c , α_d and λ_m , adopt the suggested values shown in Fig. 6.

Constant n for the tensile damage is set as 100. Using the recommended relation between the static fracture energy and the unconfined compressive strength of concrete given in [39] (i.e., $G_f = 73f_c^{0.18}$, f_c in MPa unit), G_f is found to be 140N/m for $f_c=41$ MPa. Since the length of the fracture zone is absent in this experiment, it is assumed to be the maximum diameter of aggregate [17], i.e., 10mm. Consequently, els can be obtained using Eq. (18). Other constants are obtained from the automatically generated method in the original K&C model. Table 1 presents all the parameters of the modified K&C model.

Table 1 Parameters of the modified K&C model for 41MPa concrete

Strength surface		Damage		Others	
a_0	1.73e7Pa	b_1	1.6	ρ	2300kg/ m ³
a_1	0.5698	b_3	1.15	f_c	41MPa
a_2	6.14e-10Pa ⁻¹	l_{frac}	10mm	G	12.8GPa
a_{0y}	1.15e7Pa	els	2	T	3.6MPa
a_{1y}	0.8989	G_f	140N/m	$\bar{\epsilon}_p$	0.5
a_{2y}	1.67e-9Pa ⁻¹	λ_m	8.7e-5	λ_t	8.7e-3
a_{0f}	0	n	100		
a_{1f}	0.5698	α	3		
a_{2f}	6.14e-10 Pa ⁻¹	$\alpha_c; \alpha_d$	0.29; 1.86		

In order to avoid repeated simulations, shot numbers 1-1, 2-3, 4-3 and 5-3, in which the thickness of slab and the impact velocity of projectile vary, are used to validate the modified model. The left of Fig. 12 presents the simulation results of the original and modified K&C models, including the cratering diameter, cratering depth, scabbing diameter, scabbing depth and the projectile residual velocity. The corresponding experimental results are shown on the right of Fig. 12. It can be clearly observed that, the predicted cratering size, scabbing size and residual velocity using the modified K&C model have good agreements with the corresponding experimental data. However, using the original K&C model, both the cratering and scabbing sizes are largely underestimated, which result from the overestimation of the fracture energy at high

strain-rate discussed in Section 2.4, and the absence of a tensile failure criterion. The predicted residual velocity agrees with the test data well, since it is mainly dominated by the compressive behavior of concrete material.

(a)

(b)

(c)

(d)

Fig. 12 Comparisons of simulation results with experimental data [2] (a) shot No. 1-1: slab thickness 300mm, projectile initial velocity 641m/s; (b) shot No. 2-3: slab thickness 200mm,

projectile initial velocity 544m/s; (c) shot No. 4-3: slab thickness 150mm, projectile initial velocity 643m/s; (d) shot No. 5-3: slab thickness 100mm, projectile initial velocity 486m/s.

The predicted acceleration, velocity and displacement of the projectile and the corresponding experimental data for shot No. 1-1 are shown in Fig. 13. The acceleration during the perforation was recorded using the newly developed small-caliber accelerometer, which was proved to be accurate and robust [2]. By integrating the acceleration-time curve, the velocity-time and displacement-time of projectile can be obtained. It is found that both the original and modified K&C models have good agreements with the test data, as they are mainly governed by the compressive behavior of concrete material.

Fig. 13 Comparisons of simulation results with experimental data (a) acceleration; (b) velocity and displacement of projectile

4 Parametric studies

The dynamic tensile behavior of concrete material, which is crucial for the correct modeling of cratering and scabbing phenomena, is governed by six parameters in the modified model, i.e., G_f , l_{frac} , T , n , $DIFt$ and b_2 . According to Fig.9, the difference of the predicted tensile behaviours

from original and modified K&C models is mainly in the strain softening region, the influence of parameter b_2 , which governs the strain hardening, on the predicted cratering and scabbing sizes should be small, and therefore, will not be discussed further. Sensitivity analyses of other five parameters, i.e., G_f , l_{frac} , T , n and $DIFt$, are carried out by varying one parameter by ± 25 and ± 50 percentages while other parameters are fixed. It should be noted that the purpose of parametric studies is not to compare the simulation results with the corresponding experimental results, but to investigate the influence of each individual parameter on cratering and scabbing phenomena. In order to save computational time, the simulation from Section 3.2 with projectile initial velocity of 544 m/s (shot No. 2-3) is chosen as a baseline.

4.1 Influence of parameter G_f

The influence of G_f on cratering and scabbing sizes is shown in Fig.14, in which their changing rates are defined as the relative changes of the quantities to their corresponding baseline values. It is observed that the predicted cratering diameter, cratering depth, scabbing diameter and scabbing depth decrease with the increase of G_f . Besides, the crack length decreases with the increase of G_f as a result of the increase of energy absorption capability of concrete material with the increase of G_f .

531

(a)

532

533

(b)

534 Fig.14 Effect of G_f on cratering and scabbing, (a) simulation pictures; (b) predicted results

535 4.2 Influence of parameter l_{frac}

536 Figure 15 shows the influence of the parameter l_{frac} on the cratering and scabbing sizes. In all
 537 simulations, it is found that the failure modes are always similar for the values of l_{frac} examined,
 538 and thus, the simulation pictures are not presented. It can be found that the cratering and scabbing
 539 sizes increase with the increase of l_{frac} . This is due to the fact that the increase of l_{frac} leads to the
 540 decrease of energy absorption capability in an element, i.e. each element of size hc dissipates a
 541 fracture energy of $(hc/l_{frac})G_f$ since the fracture energy is assumed to be dissipated uniformly over
 542 the length of the fracture zone (l_{frac}).

543

Fig.15 Effect of l_{frac} on cratering and scabbing sizes

4.3 Influence of parameter T

The influence of T on cratering and scabbing sizes is found to be evident, as shown in Fig.16. Even though the fracture energy is hold constant, the predicted cratering and scabbing sizes decrease with the increase of T since the yield surface enlarges with the increase of T .

Fig.16 Effect of T on cratering and scabbing sizes

4.4 Influence of parameter n

Figure 17 presents the influence of the parameter n on the cratering and scabbing sizes. It is observed that the cratering and scabbing sizes do not increase monotonously with the increase of n . On one hand, the increase of n results in the decrease of els according to Eq.(19). Meanwhile the decrease of els could be linked to either the decrease of G_f or the increase of l_{frac} or T according to Eq.(18). The decrease of G_f or the increase of l_{frac} (and thus the decrease of els) leads to the increase of the cratering and scabbing sizes. On the other hand, the change of T will lead to the change of both els and the yield surface, i.e. an increase of T will cause a decrease of els , making the cratering and scabbing sizes larger, and meanwhile an increase of T will cause an enlargement of the yield surface, making the cratering and scabbing sizes smaller. The overall changes of the predicted cratering and scabbing sizes reflect the competing of these two

mechanisms.

However, the cratering and scabbing sizes, especially the scabbing size, are not very sensitive to parameter n when it is larger than 100. This is due to the fact that the dynamic tensile stress approaches to zero when n reaches a relative large value, beyond which the contribution of dynamic stress-strain curve to the fracture energy is minor, as observed in Fig. 9. Therefore, without invoking the complex calibration of tensile failure parameters, the proposed erosion criterion with $\lambda_t > 100\lambda_m$ can be further used to describe other tensile-dominated failure phenomena, such as the spallation of concrete slab subjected to blast loadings.

Fig. 17 Effect of n on the cratering and scabbing sizes

4.5 Influence of DIFt

The influence of DIFt on cratering and scabbing phenomena is also found interesting. Fig.18 presents the different failure modes of simulation results using various DIFt, including the suggested ones by Malvar and Crawford [29], by Xu and Wen [30] (Eq. (10)), CEB [27] and CEB with a cutoff value of 10. It is found that the predicted cratering and scabbing phenomena are very similar using the suggested DIFt by Malvar and Crawford [29] and Xu and Wen [30], which are actually close to each other, as shown in Fig.5. However, using the CEB recommended DIFt

578 cannot reproduce the scabbing phenomenon well, since the CEB recommended DIF_t greatly
579 underestimates experimental data, as shown in Fig.5. The same conclusion was presented in our
580 previous study based on a modified HJC model [8], which indicates that this conclusion is
581 independent of concrete material model.

582
583 **Fig.18** Effect of DIF_t on cratering and scabbing

586 **5 Conclusions**

587 **The** original K&C concrete model was modified and applied to simulate cratering and
588 scabbing phenomena in concrete slabs subjected to projectile impact. The main contributions and
589 findings are,

590 (1) Based on a large amount of triaxial compression data, **the** new strength surfaces
591 parameters for K&C model were proposed.

592 (2) A new relationship between **the** yield scale factor η and **the** damage function λ was
593 presented, which is more convenient and accurate than the original K&C model.

594 (3) Based on the experimental observation that **the** fracture strain is a constant and **the**
595 fracture energy increases with the increase of strain-rate for high strain-rate, a new tensile damage
596 accumulation formula relevant directly to the static fracture energy and fracture zone length was
597 established.

598 (4) A new tensile failure erosion criterion was proposed and validated by comparing with
599 projectile perforation test data and parametric study.

600 (5) Based on several sets of projectile perforation experiments into concrete slabs with
601 varying projectile impact velocity and concrete slab thickness, advantages of the modified K&C
602 model for the simulations of cratering and scabbing phenomena were demonstrated.

603 (6) It was found that the scabbing size is sensitive to els and DIFt.

604 **Acknowledgements**

605 This study was supported by the National Natural Science Foundations of China (51210012,
606 51522813, 51238007, 11272060). Helpful advices on K&C model from Karagogian and Case Inc
607 through the cooperative project (51210012), and constructive comments from Dr S.R. Lan on the

608 paper were gratefully acknowledged.

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