MICROWAVE IMAGING IN DISPERSIVE MEDIA USING TIME REVERSAL TECHNIQUES IN HIGH PERFORMANCE COMPUTING ENVIRONMENT

A THESIS SUBMITTED TO THE UNIVERSITY OF MANCHESTER FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE FACULTY OF SCIENCE AND ENGINEERING

2018

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Abstract

The Time Reversal (TR) techniques achieve spatio-temporal refocusing either by physical or synthetic retransmission of signals acquired by a set of transceivers in a time-reversed fashion which can be used in various applications, including microwave imaging of hidden targets. This is due to the invariance of wave equations in lossless space. However, the existence of dispersion and loss in the propagation medium breaks this invariance and the resultant TR focusing exhibits frequency and time-dependent degradation. Compensation methods can tackle this degradation to improve focusing resolution under such conditions.

In this thesis, we propose an algorithm that utilises inverse filters with threshold approach and different type of windows to compensate for this additional attenuation. The proposed threshold approach reduces the amplification of the unwanted noise in the received signals at the application of the inverse filters. Furthermore, optimum settings for window type and length in the Short Time Fourier Transform (STFT) method are obtained through a scanning operation in the propagation medium. While utilizing a large number of windows with short spatial lengths provides improved TR focusing performance, it also increases the overall cost and complexity of the imaging system. The threshold method introduced here achieves improved TR focusing performance without increasing the cost by utilising a lower number of inverse filters.

We also identify the limitation of the STFT compensation method in some human tissues. The limitation is overcame by our proposed compensation method with the Continuous Wavelet Transform (CWT) technique. The CWT method uses a size-adjustable window while the STFT method uses a fixed window. This thesis investigates the performance of conventional TR, TR MUltiple SIgnal Classification (TR-MUSIC) and total focusing techniques to detect a tumour in the lung after application of the CWT compensation method to the observed signals.
Declaration

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I would like to thank my supervisor Dr Fumie Costen for her continuous support, patience, motivation, and smile throughout my PhD study. Without her precious support, it would not be possible to conduct this research.

Many thanks also go to Dr Mehmet E. Yavuz for his valuable comments and help.

Thanks to RIKEN centre for providing the human phantom and the high performance computing resources.

Many thanks go to my friends and colleagues at the University of Manchester for all the support and the good times that we spend together.

I would like to thank my mother Suhair, my father Muwafaq and my sister Sarah for their continuous prayers, love, patience, encouragement and support throughout my entire life.

Finally, many thanks go to the love of my life and my precious wife Ula for her patience and support throughout my PhD study.
Dedication

To my parents, my wife and my kids
Yassir, Yazan and Misk
## Abbreviations

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<td>2D</td>
<td>Two Dimensional</td>
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<td>3D</td>
<td>Three Dimensional</td>
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<tr>
<td>CCF</td>
<td>Cross-Correlation Function</td>
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<td>CFS</td>
<td>Complex Frequency Shifted</td>
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<td>CPU</td>
<td>Central Processing Unit</td>
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<tr>
<td>CWT</td>
<td>Continuous Wavelet Transform</td>
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<tr>
<td>DHP</td>
<td>Digital Human Phantom</td>
</tr>
<tr>
<td>DORT</td>
<td>Time Reversal Operator Decomposition (french acronym)</td>
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<tr>
<td>DvT</td>
<td>Distance versus Time</td>
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<tr>
<td>EM</td>
<td>Electromagnetic</td>
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<tr>
<td>EVD</td>
<td>Eigenvalue Decomposition</td>
</tr>
<tr>
<td>FDTD</td>
<td>Finite Difference Time Domain</td>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
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<td>FLOPS</td>
<td>Floating point Operations Per Second</td>
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<td>FMC</td>
<td>Full Matrix Capture</td>
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<td>GPR</td>
<td>Ground Penetrating Radar</td>
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<td>GPU</td>
<td>Graphics Processing Unit</td>
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<td>HPC</td>
<td>High Performance Computing</td>
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<tr>
<td><strong>ICWT</strong></td>
<td>Inverse Continuous Wavelet Transform</td>
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<td><strong>IFFT</strong></td>
<td>Inverse Fast Fourier Transform</td>
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<tr>
<td><strong>MDM</strong></td>
<td>Multi-Dimensional Matrix</td>
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<tr>
<td><strong>MMW</strong></td>
<td>Millimeter Wave</td>
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<td><strong>MPI</strong></td>
<td>Message Passing System</td>
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<td><strong>MRI</strong></td>
<td>Magnetic Resonance Imaging</td>
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<td>MUltiple SIgnal Classification</td>
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<td><strong>NS</strong></td>
<td>Null Subspace</td>
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<td><strong>PEC</strong></td>
<td>Perfect Electric conductor</td>
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<td><strong>PML</strong></td>
<td>Perfectly Matched Layer</td>
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<td><strong>ROI</strong></td>
<td>Region Of Interest</td>
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<td><strong>SAR</strong></td>
<td>Synthetic Aperture Radar</td>
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<td><strong>SNR</strong></td>
<td>Signal to Noise Ratio</td>
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<td><strong>STFT</strong></td>
<td>Short Time Fourier Transform</td>
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<td><strong>SS</strong></td>
<td>Signal Subspace</td>
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<td><strong>SVD</strong></td>
<td>Singular Value Decomposition</td>
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<td><strong>TFM</strong></td>
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Chapter 1

Introduction

1.1 Microwave imaging techniques

1.1.1 Background and motivation

Microwave imaging is a technique used to localise hidden scatterers (targets) in a structure or media using electromagnetic (EM) waves at microwave frequencies (0.3 to 30) GHz and millimetre wave (MMW) frequencies (30 to 300) GHz [1]. Microwave imaging is achieved either by conventional radar [2] or by Synthetic Aperture Radar (SAR) techniques [3] by sending EM waves into a specific medium. These waves are reflected by the scatterers and the receiving antenna records the reflected waves which provide the required information to localise the targets. Unlike optical and infrared-based systems, both microwaves and MMWs can localise targets through dust, fog, smoke and forested environments [4]. Radar imaging techniques provide cross-range resolutions of the scatterers via both the aperture of antennas’ array utilised in the system and the bandwidth of the signals [5]. The aperture is either a real one constructed by a multiple antennas’ array or a virtual aperture which can be obtained by a synthetic array which consists of one or a few antennas. For example, a single antenna can be attached to a plane to image the earth surface while in motion for various science applications [6].

Localisation of targets behind structures or within obstructed media is of great interest in many microwave imaging applications such as detection of landmines [7], explosives [8] or tumours in the body [9] such as breast [10, 11, 12],...
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brain [13] and lung [14, 15]. X-rays, Magnetic Resonance Imaging (MRI) and ultrasound can also provide an effective tumours detection. However, X-rays have false negative results [16, 14] and have ionising effects [17] which are detrimental to patient health. MRI has some drawbacks such as strong magnetic fields [16]. Ground Penetrating Radar (GPR) and Through the Wall Imaging (TWI) methods are also examples of microwave imaging applications. Furthermore, human respiration detection method is used to localise multiple respiration motions [18] behind an obstacle by using UWB impulse radar [19]. GPR method can detect targets under the ground like mines [7] and pipes [20]. TWI techniques are desirable for a various applications like rescue missions [21] and military applications [5].

Both microwaves and MMWs suffer from attenuation inside lossy materials (media). They are affected by the frequency-dependent properties of the materials that are included in the propagation medium. Therefore, EM waves in random media experience dispersion under which each spectral component of the signal travels at different velocities. Furthermore, the propagation media are disturbed by clutter and multipath which causes phase shift (distortion) to the signals. The phase shift makes it difficult to localise the targets within the propagating medium. Multipath is caused by scatterers and degrades the performance of the conventional imaging techniques. Scatterers can be discrete (point-like scatterers) or extended (distributed scatterers). Discrete scatterers are considered to be small when compared with the wavelength of the propagated wave signal. Extended scatterers have extended surfaces such as a wall [5]. Multipath can cause the imaging techniques to localise non-existent targets. A highly cluttered medium that includes multiple scatterers and the usage of omnidirectional antennas enlarge the multipath’s effects. However, multipath can be utilised to obtain distinct routes between the target and the antennas in remote sensing system [4] to enhance the imaging process. A recent method in the microwave imaging domain that has been introduced to tackle the detection and localisation problem is the Time Reversal (TR) technique [22]. One important property of TR is that it achieves super-resolution by utilising the multipath propagation in the medium as discussed in [23, 24, 25]. Super-resolution is beating the classical diffraction limit. TR technique corresponds to phase conjugate in the frequency domain which has found applications in EMs where it is used to cancel distortions in the medium. Phase conjugate applies to continuous waves while TR technique utilises pulse
excitation [4]. On the other hand, as with other microwave imaging techniques, TR suffers from dispersion and losses in the propagating medium [26, 27] where TR invariance of the wave equations is broken. While the additional phase shift undergone due to dispersion can be compensated by the TR process itself [26], attenuation which affects the signals in both the forward and backward propagation stages are not compensated. This attenuation ultimately degrades the focusing resolution of the localisation.

TR techniques achieve spatio-temporal refocusing either by physical or synthetic retransmission of signals acquired by a set of transceivers in a time-reversed fashion. Conversely, the signals in conventional radar systems are back-projected mathematically to image the medium (e.g. Total Focusing Method (TFM) [28, 29, 30, 31] which is also known as the golden standard). TFM is a post-processing technique that utilises the Full Matrix Capture (FMC also known as Multi-Dimensional Matrix (MDM) elsewhere in the literature) to focus spatially at every point of the region of interest. FMC is used to acquire every possible transmit-receive combination for a set of array antennas.

In multiple targets environment, the physical retransmission of the time-reversed signals produces a focusing on all the scatterers. However, the focusing on each target occur at different time steps and the strongest focusing is around the dominant scatterer. By using the iterative time reversal method in [32] the spatial focusing becomes more and more around the dominant scatterer [4]. However, the iterative time reversal method does not permit spatial focusing around other scatterers. This problem is solved by using the decomposition of time reversal operator (DORT under its french acronym) [33, 34, 35, 27, 4, 36]. TR-DORT method is used for selective focusing at each target in the case of multiple targets. The TR-DORT method applies the TR Operator (TRO) to separate and allocate the targets locations in multiple scatterers media. The TRO is obtained from the FMC of a TR Array (TRA) of antennas. The eigenspace (eigenvalues and the corresponding eigenvectors) of the TRO is obtained by the EigenValue Decomposition (EVD) of the TRO. The eigenvalues and the corresponding eigenvectors of the TRO provide information about the scatterers in the examined medium. Each eigenvalue and its corresponding eigenvector in the Signal Subspace (SS) represent a specific scatterer in the medium. The number of targets in the medium can be estimated from the number of eigenvalue in the SS. One drawback of the TR-DORT technique is that the scatterers need to be well-resolved otherwise
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the performance of the imaging is degraded. However, the Null Subspace (NS) is orthogonal to the SS even for not well-resolved scatterers. The employment of NS eigenvectors rather than SS create the basis of the TR MUltiple SIgnal Classification (TR-MUSIC) method \[4, 37, 24, 31\]. TR-MUSIC can achieve the super-resolution even without the presence of multipath in the medium \[5\]. Both TR-DORT and TR-MUSIC are based on eigenspace analysis of the TRO to provide information about the scattering medium to localise the targets. The spatial focusing of the TR-MUSIC method is more accurate and has higher resolution than the TR-DORT and TFM.

1.1.2 Literature review

The first experiments that provided TR focusing were done by Fink \[38\] in 1989. Various physical TR experiments have been achieved by using acoustics \[39, 40, 41, 23\] and ultrasonic waves \[42, 43, 33, 34, 44\]. TR principles have been examined via theoretical analysis \[45\] and numerical simulations \[46\]. Many applications utilise the TR technique. For example in medicine, it has been used for the destruction of kidney stones \[47\], ultrasonic focusing through the skull \[48\] and microwave breast cancer detection \[10, 49\]. Similarly, some applications have utilised it for detecting defects in materials and structures \[50\]. Applications in geophysics and geoscience have employed TR for finding targets buried in the ground \[51\]. Underwater applications include sonar and acoustic communication \[52\]. Several experiments using the EM waves have been presented in target localisation \[53, 54, 55\], breast cancer detection \[56, 57\] and sensing for buried scatterers in \[58\]. TR-SAR imaging is applied in \[3\]. The application of TR with broadband microwave signals is explained in \[59\]. Telecommunications and wireless communications applications include the development of time-reversal based spatial-temporal matched filters to reduce channel dispersion and inter-symbol interference thereby increasing the capacity of the channel \[60, 25\]. The signals in the TR technique face attenuation in dispersive and lossy media. The attenuation degrades the imaging performance of the TR technique. The earliest works on the compensation of such attenuation involved a uniform absorption model over the entire frequency range of interest for acoustic propagation in skull \[48\] and multipath in ocean communication \[61\] with the so-called TR-DORT method. \[48\] and \[61\] applied the amplitude compensation method to compensate the losses of the TR signals and the eigenvectors of the TRO respectively. The amplitude
compensation technique estimates the loss by comparing the amplitude of the received waves at the TRA due to the medium attenuation with the corresponding waves obtained in the same conditions in homogeneous medium. However, these methods did not take the frequency dependent attenuation into account. A more recent compensation technique using the Short-Time Fourier Transform (STFT) method takes frequency dependency into account and tries to improve the resolution focusing of the TR technique in dispersive and lossy media [26]. Although this method proved to be a good candidate for compensation, the inverse filters utilised in the method also amplify unwanted oscillations and perturbations in the received signals, which affect the performance of TR technique. STFT-based method of [26] produced the inverse filters by comparing the solution of the wave equation in the applied dispersive medium against a corresponding non-dispersive test medium by transmitting a pulse and observing it at various propagation distances in both media. The process of creating the inverse filters is empiric and time consuming. The inverse filters in [26] need the non-dispersive version of the propagation media to create the inverse filter which in practical application we do not have the propagation media either. The STFT method uses a fixed time window length which provides a lower resolution in determining the attenuation when compared with variable time window methods [62, 63]. Later, in ultrasound imaging systems, [64] introduced a generalised TR-MUSIC that incorporates attenuation of the medium in the imaging function of the TR-MUSIC. However, the imaging resolution is degraded as noise is introduced to the recorded signals. In [65], Mason’s rule [66] is used to describe the wave propagation into the soil (texture is assumed to be known) in order to extract the phase change and create a compensation filter. Lately, two anti-dispersive filtering approaches are introduced in [67] to compensate the effects of dispersive homogeneous soil (no scatterers or targets) to the propagated Ultra-Wide Band (UWB) microwave signals. The first filter is represented by the inverse of the medium frequency response and the second filter is based on the inverse of attenuation and phase constants of the actual medium. Both filters outcomes are similar when the frequency response and the propagation constant are assumed to be known. When they are not well known, the filters may not cancel the frequency dependence due to the homogeneous dispersive medium. To overcome this issue, Cross-Correlation Function (CCF) between the excitation pulse and the compensated signal is used to aid the filters in achieving the compensation. However, this method fails when applied
to a random dispersive media because of the unknown distance travelled by the different reflected signals. Finally, amplitude compensation and CCF are used in \[68\] to compensate the channel attenuation. The amplitude compensation is used to solve the near-far problem and the CCF is used to reduce the perturbation interference caused by the system. The near-far problem occurs when the targets near the TRA antennas obtain more power than the targets that are far from the antennas; thus, far targets are not localised correctly.

1.1.3 Contributions

In addition to the studies mentioned in Section 1.1.2, we produced promising results for the compensation of signals in dispersive media for the application of TR. The summary of our contributions are

- We extended the method in \[26\] by adding a threshold approach to overcome the inverse filter amplification of the noise \[69\]. We applied different window types and lengths in \[69\] to obtain the optimum settings for the STFT technique. The reduction of unwanted noise amplification is achieved and we obtained improved results in terms of localisation and spatial distribution of the field around the target location. The STFT with threshold approach is 1.4 times more accurate than the simple TR technique. Additionally, through the optimal selection of window type and lengths used at the STFT stage, adaptive compensation is achieved depending on the scenario. We showed that the choice of Hanning window improved the localisation of the scatterers as compared to the Hamming window as it inherently avoids the discontinuity at the edges of the window.

- We did further investigations on the STFT threshold approach in \[70\]. We studied the robustness of our approach in the most dispersive tissue of the human body which is the muscle. We moved the target in the \(xy\)-plane in both directions to obtain the farthest distance that our imaging technique can achieve the focusing to localise the target.

- Our latest compensation method in \[71\] used adaptive window scheme based on the Continuous Wavelet Transform (CWT). The compensation method uses an inverse filter in the wavelet domain to overcome the attenuation and dispersion in the EM wave propagation due to the dispersive medium. The
CWT method is 5.1 times more accurate than the simple TR technique. The method uses adjustable-length windows by applying long time windows at low frequencies and short time windows at high frequencies [72]. Therefore, the optimisation of the time window length is not needed as was done in [69]. The inverse filters need only the relative permittivity at the centre frequency of one dominant medium to create the inverse filter. The inverse filters in the STFT method of [26, 69, 70] require more computation compared to the CWT compensation technique in [71], hence another advantage of CWT-filter over STFT-filter.

- Finally, we investigate the performance of TFM and TR-MUSIC techniques to detect a tumour in the lung after application of the CWT compensation method to the observed signals. Additionally, we introduce antenna location optimisation combined with compensation methods as a means to achieve further improved imaging results aimed for lung cancer detection. The implanted antennas of [71] surround the whole body which is financially inefficient. Therefore, we minimise the number of antenna elements and optimise their location, maintaining the imaging performance in the context of TR-MUSIC imaging functional and TFM technique.

1.1.3.1 List of publications


1.2 Aims and objectives

1.2.1 Aims

The purpose of this thesis is to contribute new compensation methods for microwave imaging techniques. In highly dispersive media, imaging techniques suffer from attenuation and distortion and previous compensation methods fail to recover the attenuated signals. The aim is to develop a robust compensation methods in order to achieve the optimum localisation and resolution of the targeted scatterers.

1.2.2 Objectives

- Build the conventional TR technique and apply it to several non-dispersive and dispersive media cases including homogeneous, random and practical simulations.
- Apply the current STFT compensation method to the TR signals when the propagation medium is lossy and dispersive.
- Develop the threshold approach to improve the performance of the STFT compensation method in terms of the accuracy and the resolution of the TR imaging.
- Apply the developed threshold approach to the TR imaging when the propagation medium is homogeneous and random.
- Develop a new compensation method based on the CWT method which uses adjustable window lengths rather than the fixed window length of the STFT method.
- Apply the CWT compensation method to the TR imaging when the propagation medium is dispersive in homogeneous, random and practical simulations.
- Build other imaging methods including the TR-MUSIC and the TFM techniques.
- Apply the CWT compensation method to the TR-MUSIC and the TFM techniques.
• Build OpenMP directives to speed up the running times of our simulations.

1.3 Outline of the thesis

The organisation of the thesis is as follow.

In Chapter 2 we study and discuss the conventional TR technique as well as the TR-DORT, TR-MUSIC and TFM. Chapter 3 introduces the details of our compensation methods which includes the threshold approach with the STFT method and the CWT compensation technique. We explain the details of the inverse filters that are used to compensate the losses caused by the dispersive media. In Chapter 4 we do the 2D and 3D simulations for TR, TR-STFT, TR-CWT, TR-MUSIC, TR-MUSIC-CWT, TFM and TFM-CWT. The simulations are either for canonical tests or practical tests using the Digital Human Phantom (DHP). The electric and magnetic fields for the simulations are calculated using the Finite Difference Time Domain (FDTD) method and the Complex Frequency Shifted Perfectly Matched Layer (CFS-PML) as the absorbing boundary condition. \( F \) represents the electric flux density which is the observed field throughout the thesis. Both the FDTD method and the CFS-PML absorbing boundary condition are explained in Appendix A. In Chapter 5 we apply the OpenMP directives to our serial code to parallelize it. Parallelization and High Performance Computing (HPC) can reduce the long running time of the serial code. Chapter 6 states the conclusion and the future works for our thesis.

1.4 Summary

In Chapter 1 we outlined different imaging techniques and gave some examples of their applications. The conventional TR, TR-DORT and TR-MUSIC methods are briefly introduced with their advantages and limitations. Furthermore, we summarised some of the TR techniques applications in different fields. Microwave imaging techniques face attenuation and distortion inside dispersive media. A compensation method is needed to be applied to the propagation signals to overcome attenuation and distortion. In Chapter 2 we explain in details the implementation of TR, TR-MUSIC and TFM techniques.
Chapter 2

Time Reversal (TR) Techniques

In this chapter, we review the details of different imaging techniques including simple TR, TR-DORT, TR-MUSIC and TFM.

2.1 Simple TR technique

For every wave propagating away from a source following a certain path, there is a time-reversed wave signal that can trace the same path back to the original point of the source. The path to the source can be traced back even if the propagation medium has scattering obstacles. Furthermore, the path to the source can be traced back even if the permittivity and permeability variations cause the wave to be reflected, scattered and refracted. If a scatterer located at \( r_p \) transmits a pulse \( b[n] \), the received signal by the \( o \)th antenna is

\[
F_o[n] = b[n] \ast_t h_{rp,ro}[n]
\]

where \( n \) is the discrete time, \( \ast_t \) is convolution in time, \( h_{rp,ro}[n] \) is the impulse response between the scatterer located at \( r_p \) and the TRA antenna located at \( r_o \).

If the recorded signal at the \( o \)th transceiver is time-reversed and retransmitted back to the medium, the observed signal at the target location is

\[
F_p[n] = b[-n] \ast_t h_{rp,ro}[-n] \ast_t h_{ro,rp}[n].
\]

Due to reciprocity \( h_{rp,ro}[n] = h_{ro,rp}[n] \). Therefore, \( h_{ro,rp}[-n] \ast_t h_{ro,rp}[n] \) represents a correlation filter which has a maximum at \( n = 0 \) corresponding to
\[
\sum_{n=0}^{N} |h_{r_p,r_p}[n]|^2 \quad \text{which is the energy of } h_{r_p,r_p}[n]. \quad \text{If the signals at } A \text{ TRA antennas are reversed in time and retransmitted back to the medium, the time-reversed waves constructively interfere with each other at } r_p \text{ to improve the TR signals focusing on the target as }
\]

\[ F_p[n] = \sum_{a=1}^{A} b[-n] * h_{r,a,r_p}[-n] * h_{r_a,r_p}[n]. \quad (2.3) \]

At any particular point \( r \) in the propagation medium, the signal becomes

\[ F_r[n] = \sum_{a=1}^{A} b[-n] * h_{r,a,r_p}[-n] * h_{r,r_p}[n]. \quad (2.4) \]

Therefore, TR can be considered as temporal and spatial correlator at the same time. As \( r \) gets further away from the scatterer location, the signals destructively interfere with each other.

TR technique consists of two parts, the forward propagation and the backward propagation. The forward propagation has two stages. The first stage of the forward propagation sends a signal from a source point. Figure 2.1 shows the first stage of the forward propagation. The source emits a signal into the scattering medium. Some of the scattered signals reach the target. The second stage of the forward propagation starts when the scattered signals are reflected from the target travelling back to the scattering medium. Figure 2.2 shows the second stage of the forward propagation. A set of TRA antennas receive the reflected and scattered signals. The received signals are recorded and time-reversed. In backward propagation, the time-reversed signals are sent back to the scattering medium without the target. Figure 2.3 shows the backward propagation part of the TR technique. In the backward propagation, each antenna of the TRA transmits the time-reversed signal back to the medium. The time-reversed signal faces the same scattering, reflection and refraction that the signal underwent through the forward propagation. Focusing region of the electric or magnetic field occurs around the target location at the time of the refocusing. The resolution of the focusing region around the target is

\[ R = \frac{\lambda L}{\alpha} \quad (2.5) \]
CHAPTER 2. TIME REVERSAL (TR) TECHNIQUES

Figure 2.1: The 1st stage of the forward propagation in the TR technique.

Figure 2.2: The 2nd stage of the forward propagation in the TR technique.
Figure 2.3: The backward propagation in the TR technique.

where $\lambda$ is the wavelength, $L$ is the distance between the target and the TRA antennas and $\alpha$ is the TRA aperture length.

Figure 2.4 compares three different scenarios to get better focusing resolution \[4\]. The source in Figure 2.4 represents the target in Figure 2.2. The signal emitted from the source represents the signal reflected from the target in Figure 2.2. Three TRA antennas are used to receive the emitted signal. The first scenario in Figure 2.4(a) has a homogeneous medium. The second scenario in Figure 2.4(b) has two Perfect Electric Conductor (PEC) walls. Furthermore, the third scenario in Figure 2.4(c) has discrete scatterers. In Figure 2.4(a) the waves do not suffer from multipath. The TRA antennas receive only the direct waves from the source. In Figure 2.4(b) and (c) the waves are guided to the TRA antennas by the PEC walls which act like a waveguide structure and by the discrete scatterers respectively. In both Figure 2.4(b) and (c) the waves propagating away from the TRA antennas are redirected back to the TRA antennas. The redirected waves increase the effective aperture $\alpha_e$ of the TRA. Increasing the TRA aperture leads to a higher resolution of the focusing region around the target. Both Figure 2.4(b) and (c) have effective aperture $\alpha_e$ larger than the TRA aperture $\alpha$ ($\alpha_e > \alpha$) \[4\].

$R$ for Figure 2.4(a) is in (2.5). $R$ for Figure 2.4(b) and (c) is

\[
\mathcal{R} = \frac{\lambda L}{\alpha_e}
\]  

(2.6)
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\[ \alpha_e = \text{TRA effective aperture} \]
\[ \alpha = \text{TRA aperture} \]

Figure 2.4: Effective aperture \( \alpha_e \) increases in media with multipath \cite{4}. The dashed-line arrows are the actual waves and the solid-line arrows represent the waves that formed the \( \alpha_e \).

\[ R \text{ in (2.6) is higher than } R \text{ in (2.5) because } \alpha_e > \alpha. \]

2.1.1 Simulation setup for the TR technique

2.1.1.1 Forward propagation without a scatterer

The medium used for the simulation setup of the TR technique is homogeneous, non-dispersive and lossless. The simulation setup is depicted in Figure 2.5. The medium parameters are \( \sigma = 0 \text{ S/m}, \epsilon_S = \epsilon_\infty = 28 \) (the value of \( \epsilon_\infty \) is based on muscle tissue) and \( \tau_D = 0 \) seconds. The simulations use the TM\(_z\) polarised 2D-FDTD method. The FDTD space is \( 150 \times 150 \), uniformly sampled with a spatial step of \( \Delta s \triangleq \Delta x = \Delta y = 1\text{mm} \). The time step \( \Delta t \) is set to 2.357ps. CFS-PML absorbing boundary conditions \cite{75} are used with a 9-cell layer. The TRA is composed of 15 transceivers antennas, equally spaced from each other and parallel to the \( x \)-axis. The TRA has the aperture of 70\( \Delta s \). The 1st TRA antenna is located at \( (40\Delta x, 25\Delta y) \) and the 15th is located at \( (110\Delta x, 25\Delta y) \). The 8th antenna \( (75\Delta x, 25\Delta y) \) of the TRA transmits a Gaussian pulse \( b[n] \) modulated at 3 GHz covering from 1.72 GHz to 4.2 GHz. The excitation pulse \( b[n] \) and its frequency domain representation \( b[k] \) are shown in Figure 2.6 where \( k \) represents the discrete frequency. The propagated pulse is observed and recorded by each TRA.
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Figure 2.5: Simulation setup diagram used for the TR technique without a PEC scatterer. $\Delta x = \Delta y = 1\text{mm}$.

Figure 2.6: The modulated Gaussian pulse with $f_c = 3\text{GHz}$. 
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Figure 2.7: The spatial distribution of $|F|$ in the $xy$-plane for the 1st stage of the forward propagation in the TR technique. The TRA antennas are represented by “x”. $\Delta x = \Delta y = 1$mm.

antenna. Figure 2.7 shows the spatial distribution of $|F[x, y]|$ after transmitting the pulse at different time steps.

2.1.1.2 Forward propagation with a scatterer

A PEC scatterer is located in the medium with a radius of 2 mm and centred at $(75\Delta x, 65\Delta y)$ as shown in Figure 2.8. Figure 2.8 is produced by adding the PEC scatterer to the scenario depicted in Figure 2.5. The remaining simulation settings are the same as in Section 2.1.1.1. The 8th TRA antenna transmits the same pulse as the one in Section 2.1.1.1 into the medium shown in Figure 2.8. The propagated pulse is observed and recorded by each TRA antenna. Figure 2.9
shows the spatial distribution of $|F(x, y)|$ at different time steps. Figure 2.9 (b), (c) and (d) show that the reflected signal from the scatterer is heading towards the TRA antennas.

### 2.1.1.3 Production of the time-reversed signal reflected by the scatterer

Figure 2.10 shows the observed signal at the 8th antenna of the TRA in the simulations performed in Section 2.1.1.1 and Section 2.1.1.2. We notice from Figure 2.10 that observed signal for Figure 2.8 has a small reflection. The reflection represents the reflected signal from the PEC scatterer. We produce the reflected signal $F[n]$ by subtracting the signal observed in Figure 2.8 from the signal observed in Figure 2.5 at each TRA antenna. Figure 2.11(a) shows $F[n]$ at the 8th TRA antenna. $F[n]$ at each TRA antenna is time-reversed. If the simulation time-step is $N = 1500$ then $F[n]$ at each TRA antenna is time-reversed as $F[1500 - n]$ as shown in Figure 2.11(b). Each time-reversed signal at each TRA antenna is sent back to the medium in Figure 2.5. A focusing of $F$ occurs around the scatterer location at $n = N - N_{\text{sca}}$ which is known as ($t = 0$). $N_{\text{sca}}$ is the time-step where the amplitude of the reflected signal at $(75\Delta x, 63\Delta y)$ (near the
Figure 2.9: The spatial distribution of $|F|$ in the $xy$-plane for the 2nd stage of the forward propagation in the TR technique. The TRA antennas are represented by “x”. $\Delta x = \Delta y = 1\text{mm}$.
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Figure 2.10: The observed signal at the 1st antenna of the TRA with and without scatterer as depicted in Figure 2.5 and Figure 2.8 respectively. Δt = 2.357ps.

Figure 2.11: The reflected signal from the PEC scatterer observed at the 1st antenna of the TRA. Δt = 2.357ps.
scatterer) has the highest peak as shown in Figure 2.12. We observe the signal at $(75\Delta x, 63\Delta y)$ for both simulations in Figure 2.8 and Figure 2.5. We get the reflected signal (Figure 2.12) by subtracting the signal observed in Figure 2.8 from the signal observed in Figure 2.5 at $(75\Delta x, 63\Delta y)$. The highest amplitude peak for the reflected signal is at $N_{sca} = 606$. The highest amplitude peak of the reflected signal needs $N_1 + N_{sca}$ to be observed by the 1st antenna where $N_1$ is the time-step needed for a signal to propagate from the scatterer to the 1st antenna. The highest amplitude peak of $F[N - n]$ is at $N - N_1 - N_{sca}$. The time-step needed for $F[N - n]$ to propagate from the 1st antenna to the scatterer is $N - N_1 - N_{sca} + N_1 = N - N_{sca} = 894$. Each $F[N - n]$ at each TRA antenna needs $894\Delta t$ to focus around the scatterer location. Figure 2.13 shows the spatial distribution of $|F[x, y]|$ for the time-reversed signals. All time-reversed signals propagate to focus around the scatterer location. Figure 2.13(d) shows the focusing around the scatterer location at $n = 894$. Finally, we need to mention that when we increase the number of antennas the value of the $F$ focusing around the scatterer is increased. Therefore when we use 30 antennas the value of $F$ focusing around the scatterer becomes double of that for the 15 antennas case.
Figure 2.13: The spatial distribution of $|F|$ in the $xy$-plane for the backpropagation in the TR technique. The TRA antennas are represented by “x”. $\Delta x = \Delta y = 1\text{mm}$. 

(a) $n = 550$
(b) $n = 650$
(c) $n = 750$
(d) $n = 894$
2.2 Multiple scattering effects

TR-DORT and TR-MUSIC exploit the TRO which is obtained using the FMC of the TRA. FMC is a process which records a set of observed signals by the TRA antennas. Each antenna of the TRA is sequentially used as a transmitter while all the TRA antennas are used as receivers. The eigenvalues and their corresponding eigenvectors of the TRO have useful information on the scatterers within the medium of interest. Each eigenvalue and its corresponding eigenvector of the TRO is associated with one of the scatterers in the medium for well-resolved point-like isotropic scatterers [4]. The TR-based imaging of DORT and MUSIC is achieved with the use of the SS and the NS respectively. The focusing resolution and accuracy around the scatterers are degraded in TR-DORT if the well-resolvedness criterion is not met [4]. However, in the TR-MUSIC method, the TRO NS is orthogonal to the TRO SS which means that the projection of one of the SS eigenvectors onto the NS is zero. Therefore the TR-based MUSIC method gives fine focusing resolution and accurate location around the scatterers even for not-well-resolved scatterers in the homogeneous medium [4]. Therefore we compare TR-based MUSIC with the TFM, the so-called gold standard in imaging [29] [31]. TFM uses the FMC data to create a spatial distribution of signals focusing on the scatterers locations in the FDTD space. TFM adds all the observed signals of the TRA antennas with proper delays at every point in the FDTD space by using delay and sum beamforming algorithm. Section 2.2.1 and Section 2.2.3 show the details of the TR-MUSIC and TFM methods respectively.

2.2.1 TR-DORT and TR-MUSIC

Due to reciprocity an $A \times A$ symmetric FMC is produced by the TRA of $A$ transceivers. The $A \times A$ FMC is denoted as

$$K[n] = \begin{pmatrix} k_{11}[n] & \cdots & k_{1A}[n] \\ \vdots & \ddots & \vdots \\ k_{A1}[n] & \cdots & k_{AA}[n] \end{pmatrix} \tag{2.7}$$

where $k_{ij}[n]$ corresponds to the signal received at the $i$th antenna when a pulse $b[n]$ is transmitted from the $j$th antenna as a sole transmitter. The Fourier transform
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of $K[n]$ yields

$$K[k] = \begin{pmatrix} k_{11}[k] & \cdots & k_{1A}[k] \\ \vdots & \ddots & \vdots \\ k_{A1}[k] & \cdots & k_{AA}[k] \end{pmatrix}. \quad (2.8)$$

For point-like and well-resolved scatterers $k_{ij}[k]$ can be written as

$$k_{ij}[k] = \sum_{p=1}^{P} G[r_i, r_p, k] \rho_p[k] G[r_p, r_j, k] b[k] \quad (2.9)$$

where $P$ is the number of scatterers in the medium, $G[r_i, r_p, k]$ is the medium Green’s function between the location of the $i$th antenna $r_i$ (for $i = 1, \ldots, A$) and the location of the $p$th scatterer $r_p$ (for $p = 1, \ldots, P$), $\rho_p$ is the scattering coefficient of the $p$th scatterer and $b[k]$ is the frequency domain of $b[n]$. $K[k]$ can be written as

$$K[k] = b[k] \sum_{p=1}^{P} \rho_p[k] g_s[r_p, k] g_s^T[r_p, k] \quad (2.10)$$

where $^T$ is the transpose and

$$g_s[r_p, k] = [G[r_1, r_p, k], \ldots, G[r_A, r_p, k]]^T \quad (2.11)$$

is the $A \times 1$ steering vector that associates the $p$th scatterer location $r_p$ to the TRA antennas. The TRO can be defined as the Hermitian matrix of

$$T[k] = K^\dagger[k] K[k] \quad (2.12)$$

where $K^\dagger$ is the Hermitian conjugate of $K$ and $\dagger$ stands for the complex conjugate transpose. TR corresponds to phase conjugation in frequency domain and hence the Fourier transform of $K[N - n]$ is $K^\dagger[k]$. The eigenstructure is obtained from the singular values and singular vectors of the FMC. The Singular Value Decomposition (SVD) of $K[k]$ is

$$K[k] = U[k] \Lambda[k] V^\dagger[k] \quad (2.13)$$
where

\[
\Lambda[k] = \begin{pmatrix}
\lambda_1[k] & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \lambda_A[k]
\end{pmatrix}
\] (2.14)

\[
U[k] = [u_1[k], \ldots, u_A[k]] = \begin{pmatrix}
u_{11}[k] & \cdots & u_{1A}[k] \\
\vdots & \ddots & \vdots \\
u_{A1}[k] & \cdots & u_{AA}[k]
\end{pmatrix}
\] (2.15)

\[
V[k] = [v_1[k], \ldots, v_A[k]] = \begin{pmatrix}
v_{11}[k] & \cdots & v_{1A}[k] \\
\vdots & \ddots & \vdots \\
v_{A1}[k] & \cdots & v_{AA}[k]
\end{pmatrix}
\] (2.16)

\(\Lambda[k]\) is the real diagonal matrix of the singular values in descending order, \(U[k]\) is a unitary matrix containing the left singular vectors \(u_A[k]\) and \(V[k]\) is a unitary matrix containing the right singular vectors \(v_A[k]\).

Using the SVD of the FMC the EVD of the TRO can be written as [4]

\[
T[k] = V[k]^{\dagger} \Lambda[k]^{\dagger} U[k]^{\dagger} U[k] V[k]^{\dagger}
\]

\[
= V[k]^{\dagger} \Lambda[k] \Lambda[k]^{\dagger} V[k]^{\dagger}
\]

\[
= V[k]^{\dagger} S[k] V[k]^{\dagger}
\] (2.17)

where \(I\) is the identity matrix and \(S[k]\) is the real diagonal matrix of eigenvalues \(\varrho_i[k]\) as

\[
S[k] = \Lambda[k]^{\dagger} \Lambda[k] = \begin{pmatrix}
\varrho_1[k] & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \varrho_A[k]
\end{pmatrix}
\] (2.18)

The eigenvalues of the TRO correspond to the square of the singular values of the FMC for \(i = 1, \ldots, A\) which is expressed as

\[
\varrho_i[k] = \lambda_i^2[k]
\] (2.19)

Furthermore, the eigenvectors of the TRO are equivalent to the right singular vectors of the FMC. Finding the singular value and the singular vectors of \(K[k]\) is needed to get the eigenvalues and eigenvectors of the TRO. First we recall the
singular system of
\[ K[k]v_i[k] = \lambda_i[k]u_i[k] \] (2.20)
and
\[ K^\dagger[k]u_i[k] = \lambda_i^*[k]v_i[k] \] (2.21)
where * denotes the phase conjugation. Substituting (2.10) into (2.20) yields
\[ b[k] \sum_{p=1}^{P} \rho_p[k]g_s[r_p, k]g^T_s[r_p, k]v_i[k] = \lambda_i[k]u_i[k] \] (2.22)
where \( g_s^T[r_p, k]v_i[k] \) is a scalar which allows us to get
\[ u_i[k] = \sum_{p=1}^{P} b[k]\rho_p[k](g_s^T[r_p, k] \cdot v_i[k]) \frac{g_s[r_p, k]}{\lambda_i[k]} \] (2.23)
where \( (g_s^T[r_p, k] \cdot v_i[k]) \) is the standard inner product and \( i = 1, \ldots, A \). (2.23) shows that \( u_i[k] \) is a linear combination of \( g_s[r_p, k] \) for \( p = 1, \ldots, P \) associating the scatterers to the TRA. Similarly
\[ v_i[k] = \sum_{p=1}^{P} b^*[k]\rho_p^*[k](g_s^*[r_p, k] \cdot u_i[k]) \frac{g_s^*[r_p, k]}{\lambda_i^*[k]} \] (2.24)
(2.23) and (2.24) satisfy \( u_i[k] = v_i^*[k] \) for \( i = 1, \ldots, A \) because \( K[k] \) is symmetric. For a particular \( u_i[k] \) that is not equal to zero such that \( (g_s^*[r_p, k] \cdot u_i[k]) \) is zero only when \( u_i[k] \neq g_s[r_p, k] \) as
\[ (g_s^*[r_p, k] \cdot u_i[k]) = \begin{cases} \|g_s[r_p, k]\|^2 & \text{when } u_i[k] = g_s[r_p, k] \\ 0 & \text{when } u_i[k] \neq g_s[r_p, k] \end{cases} \] (2.25)
where \( \|g_s[r_p, k]\| = \sqrt{(g_s^*[r_p, k] \cdot g_s[r_p, k])} \) is the norm of \( g_s[r_p, k] \). The choice of \( u_i[k] \) and \( v_i[k] \) that satisfy (2.20) and (2.21) is
\[ u_i[k] = \frac{g_s[r_p, k]}{\|g_s[r_p, k]\|} \quad \text{for } \ i = p = 1, \ldots, P \] (2.26)
and
\[ v_i[k] = \frac{g_s^*[r_p, k]}{\|g_s[r_p, k]\|} \quad \text{for } \ i = p = 1, \ldots, P. \] (2.27)
The corresponding singular value becomes

\[ \lambda_i[k] = b[k] \rho_p[k] \| g_s[r_p, k] \|^2 \quad \text{for} \quad i = p = 1, \ldots, P. \tag{2.28} \]

Each eigenvalue and its related eigenvector of the TRO are associated with a well-resolved point-like scatterer. We split the eigenvalues \( \rho_i \) and the corresponding eigenvectors into SS (\( \rho_1, \rho_2, \ldots, \rho_q \)) and NS (\( \rho_{q+1}, \rho_{q+2}, \ldots, \rho_A \)) where \( q \) is the number of eigenvalues that belongs to the SS which is selected based on a threshold. The threshold is set to 10\% \cite{4, 76, 31} of the largest eigenvalue; any eigenvalue below the threshold is in the NS. To produce the spatial focusing around the scatterer location we need to know the propagation media parameters at each point of the FDTD space. By transmitting \( b[n] \) from each TRA antenna and observing the signal at each point \( r \) of the FDTD space we get the steering vector

\[ g[r, n] = [F[r, n]_1 \cdots F[r, n]_A]. \tag{2.29} \]

The Fast Fourier Transform (FFT) of \( g[r, n] \) is

\[ g[r, k] = [F[r, k]_1 \cdots F[r, k]_A] \tag{2.30} \]

The TR-DORT imaging is \cite{4}

\[ I_{DORT}[r, f_c, i] = (g^*[r, f_c] \cdot v_i[f_c]) \tag{2.31} \]

and the TR-MUSIC imaging is \cite{4}

\[ I_{MUSIC}[r, f_c] = \left[ \sum_{i=q+1}^{A} (g^*[r, f_c] \cdot v_i[f_c]) \right]^{-1}. \tag{2.32} \]

Figure 2.14 shows the system diagram to summarise the equations of the TR-MUSIC algorithm.

2.2.2 Total Focusing Method (TFM)

TFM is a post-processing technique \cite{28, 29} that utilises the FMC that is captured by the TRA antennas to synthetically back-propagate at every point of the Region Of Interest (ROI) \cite{30}. In our simulations, the ROI represents all the points in the FDTD space excluding the PML region. Each point in the ROI has a scalar
data to represent the spatial distribution of the TFM focusing. The distance \( d_i \) from the transmitter antenna \( r_i \) to a particular point \( r \) in the ROI \((R_x \times R_y)\) is

\[
d_i = |r_i - r|
\]

for \( i = 1, \ldots, A \). The distance \( d_j \) from \( r \) to the receiver antenna \( r_j \) is

\[
d_j = |r - r_j|
\]

for \( j = 1, \ldots, A \). The TFM imaging is

\[
I_{TFM}[r, f_c] = \left\lceil \sum_{i=1}^{A} \sum_{j=1}^{A} k_{ij} \left( \frac{(d_i + d_j) \Delta s}{\mathcal{V}[f_c] \Delta t} \right) \right\rceil
\]

where \( \left\lceil \right\rceil \) represents the floor function and \( \mathcal{V} \) is the propagation speed within the medium as

\[
\mathcal{V}[f_c] = \frac{C}{\sqrt{\epsilon_r[f_c]}}
\]

and the time step \( \Delta t \) is calculated as

\[
\Delta t = \frac{\Delta s}{C \sqrt{2}}.
\]

Substituting (2.36) and (2.37) in (2.35)

\[
I_{TFM}[r, f_c] = \left\lceil \sum_{i=1}^{A} \sum_{j=1}^{A} k_{ij} \sqrt{2\epsilon_r[f_c]}(d_i + d_j) \right\rceil
\]
As shown in (2.38) the TFM Imaging $I_{TFM}$ of a point $\mathbf{r}$ is computed by the sum of $A$ samples of the observed signals at the TRA antennas. The summation is repeated for each $\mathbf{r}$ of the ROI for every transmitting and observing antennas pair.

### 2.2.3 Simulation setup for TR-MUSIC and TFM

The scenario is depicted in Figure 2.8. The simulation settings are the same as in Section 2.1.1.2. In the TR-MUSIC method, Figure 2.15 shows the singular values at the central frequency for the single PEC case. The number of PECs can be estimated from the values of the singular values. In Figure 2.15 the first singular value is larger than the threshold value which estimates that only one target exists in the medium. Figure 2.16 shows the first three singular values and the threshold that equals to 10% of the first singular value. The threshold split the singular values into SS domain (higher than the threshold) and into NS domain (lower than the threshold). Only the first singular value is in the SS domain and all the remaining singular values are in the NS domain. Figure 2.17(a) shows $I_{MUSIC}$ distribution for the TR-MUSIC method and Figure 2.17(b) shows $I_{TFM}$ distribution for TFM where the propagation medium is non-dispersive. Figure 2.17 shows how the scatterer is localised in the non-dispersive case with a fine spatial focusing resolution and accurate location.
Figure 2.16: The first three singular values and the threshold which split the singular values into SS and NS domains for Figure 2.8.

Figure 2.17: The spatial focusing for TR-MUSIC and TFM techniques for Figure 2.8.
2.3 Summary

Chapter 2 explained the details of the implementation of conventional TR, TR-MUSIC and TFM techniques. TR techniques can achieve spatial and temporal focusing on the target location due to the invariance of the wave equations. TR can utilise multipath waves to achieve super-resolution by beating up the diffraction limits. However, the invariance of the wave equations is broken inside dispersive and lossy media. Therefore, Chapter 3 introduces the STFT and CWT compensation methods to overcome and reduce the effects of the attenuation and the distortion that are caused by the dispersive media.
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A dispersive and lossy medium acts as a filter for signals propagating through it. Thus for compensation of such effects, inverse filters are needed. The real part of the dielectric permittivity $\epsilon$ of the medium causes a phase shift to the travelling waves during the forward propagation of the TR process. The phase shift is corrected in the backward propagation due to the fact that the signals in the backward propagation are phase-conjugated coherently along all the bandwidth [77]. On the other hand, the imaginary part of $\epsilon$ causes inevitable signal attenuation. The attenuation is a function of time (in other words, the duration that the signal has travelled in the dispersive medium) and frequency. Note that the more the signal propagates in a dispersive medium the more it is attenuated (duration dependency). Similarly, for certain wave packet that has travelled in the dispersive medium, different frequency components undergo different attenuation (frequency dependency).

3.1 STFT with threshold

We propose a method to compensate the attenuation as shown in Figure 3.1. Our method starts by time-windowing the signal $\chi[n]$ observed at each TRA antenna, setting

$$X_m[n] = \chi[n]W_m[n] \quad \text{for} \quad m = 1, \ldots, M$$

(3.1)

where $(0 \leq n \leq N)$, $N$ is the total time steps of $\chi[n]$, $W_m[n]$ is the $m$th window, $X_m[n]$ is the $m$th windowed time domain signal and $M$ is the total number of windows.
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Figure 3.1: The flowchart of the compensation method with variable windows utilised for improved imaging performance.

\[ M \text{ is selected based on } N \text{ as } M = \left\lceil \frac{2N}{l} \right\rceil \text{ where } \left\lceil \right\rceil \text{ represents the ceiling function. The window function is either a Hanning or Hamming window whose } W_m[n] \text{ is expressed as} \]

\[ W_m[n] = \frac{1}{2} \left( 1 - \cos \left( \frac{2\pi}{l}(n - \varsigma_m) \right) \right) \quad \varsigma_m \leq n \leq \varsigma_m + l \quad (3.2) \]

for the Hanning window and

\[ W_m[n] = 0.54 - 0.46 \cos \left( \frac{2\pi}{l}(n - \varsigma_m) \right) \quad \varsigma_m \leq n \leq \varsigma_m + l \quad (3.3) \]

for the Hamming window where \( \varsigma_m = \left\lfloor \frac{l(m - 1)}{2} \right\rfloor \) and \( l \) is the window’s length in time steps. \( W_m[n] = 0 \) beyond \( \varsigma_m \leq n \leq \varsigma_m + l \) limits.

Figure 3.2(a) shows examples of Hanning and Hamming windows with \( l = 512 \). Figure 3.2(b) shows three Hanning windows and their summation with \( l = 512 \) and an overlapping factor of 0.5. Overlapping guarantees that the original signal can be retrieved if all the windows are summed together. The Hanning window can avoid the abrupt change in the time-gated signals by approaching zero near the edges of the window. In [26] a Hamming window was used with a fixed \( l \) of 256, which is not optimised for the STFT method. The Hamming window causes an abrupt change in the time-gated signals near the edges of the window. Therefore,
we used Hanning and Hamming windows with $32 \leq l \leq 512$ to optimise $l$ for our compensation method. Another drawback of the compensation method in [26] is that the inverse filters amplify the noise in the observed signals. We propose a threshold approach to reduce the noise amplification that was inevitable in [26].

$X_m[n]$ is converted to the frequency domain $X_m[k]$. In our threshold approach, we set the threshold to $e^{-1}$ of the peak of the spectrum of the original signal $\chi[k]$. The threshold value is set to $e^{-1}$ to obtain the optimum signal to noise ratio (SNR) in different types of human tissues (propagation media) including fat (one of the least dispersive media) and muscle (one of most dispersive media). Therefore, we defined the highest frequency of interest as the frequency at which the spectrum is $e^{-1}$ relative to the spectrum maximum [73]. The inverse filters utilised in [26] amplify not only the signal of interest but also the noise leading to the reduction of the SNR and thus this lowers the accuracy of the localisation. When the threshold is set to $e^{-1}$ the noise amplification is reduced and the localisation is improved. The inverse filters amplify the noise (i.e. SNR is reduced) when the threshold value is set to less than $e^{-1}$. This reduces the amplitude of $X_m[k]$ relative to noise. This SNR reduction decreases the accuracy of the localisation. When the threshold value is set to higher than $e^{-1}$ the noise is not amplified. However, the inverse filters are not applied to the high frequency components of $X_m[k]$ within the frequency range of interest causing the SNR to drop again. We apply the compensation filter at those frequencies $k$ where the spectrum $X_m[k]$ is higher than the threshold. $X_m[n]$ has travelled different distances to reach the TRA antennas and each frequency component has gone
through different attenuation. Therefore, $X_m[k]$ has to be compensated with a different space and frequency dependent filter, $\Gamma_m[k]$. $\Gamma_m[k]$ is produced for the $m$th window by comparing observed signals in the dispersive medium with a corresponding non-dispersive medium. More details about space and frequency dependent filter are presented in Section 3.1.1. The filtered signal in the frequency domain is

$$Y_{\Gamma_m}[k] = X_m[k] \Gamma_m[k] \quad \text{for } m = 1, ..., M. \quad (3.4)$$

This frequency dependent filter acts like an inverse filter with respect to the attenuation of the dispersive medium. Due to the finite length of the filters, additional phase shifts may have to be introduced. These phase shifts do not affect the focusing on the scatterer location because all the observed signals go through the same filters [26]. We apply the inverse STFT to each filtered signal by converting $Y_{\Gamma_m}[k]$ to a time domain signal $Y_{\Gamma_m}[n]$ and sum all $Y_{\Gamma_m}[n]$ back together to obtain the compensated signal

$$X_{\Gamma}[n] = \sum_{m=1}^{M} Y_{\Gamma_m}[n]. \quad (3.5)$$

Varying $l$ from 32 to 512 the optimum $l$ is identified for the Hanning and Hamming windows by the following procedure:

Step 1) Find $(x_M, y_M)$ which satisfy

$$|F[x_M, y_M]| = \max_{x,y}|F[x, y]| \triangleq Q \quad (3.6)$$

where $Q$ represents the maximum value of the $|F|$ field in the entire FDTD space at the time of refocusing and $(x_M, y_M)$ is the location of $Q$ in the FDTD space.

Step 2) Select $l$ with which $(x_M, y_M)$ satisfy

$$(x_M - x_p)^2 + (y_M - y_p)^2 \leq D \quad (3.7)$$

which is regarded as a successful localisation where $D$ is the diameter of the scatterer and $(x_p, y_p)$ represents the centre of the scatterer location.
Step 3) Calculate \( S_l = \sum_{x,y} \frac{|F[x,y]|}{Q} \) where \((x, y)\) satisfy 
\[(x - x_p)^2 + (y - y_p)^2 \leq D.\]

Step 4) Choose \( l \) which gives the minimum \( S_l \).

### 3.1.1 The inverse filter production

The filter is presented and discussed in terms of a simulation example. To produce the filters, Distance versus Time (DvT) plots are used for both dispersive and non-dispersive medium. DvT plots are obtained by transmitting a short pulse from a source point and observing the pulse at increasing distances (see Figure 3.3). The simulation is performed using the 2D-FDTD method with TM\(_z\) polarisation. The simulation scenario is shown in Figure 3.3. CFS-PML absorbing boundary condition is used with 9 cells layer. The FDTD space is 150 \( \times \) 550 uniformly sampled with a spatial step \( \Delta x = \Delta y = 1 \, \text{mm} \) and a time step \( \Delta t = 2.357\, \text{ps} \). A source excitation is located at \((75\Delta x, 25\Delta y)\). The source transmits a Gaussian pulse modulated at \( f_c = 3\, \text{GHz} \) into the medium. The dispersive medium parameters are \( \sigma = 0.037 \, \text{S/m}, \epsilon_S = 5.53, \epsilon_\infty = 3.998 \) and \( \tau_D = 23.6 \, \text{ps} \). The non-dispersive medium parameters are \( \sigma = 0 \, \text{S/m}, \epsilon_S = \epsilon_\infty = 3.998 \) and \( \tau_D = 0 \, \text{seconds} \). Figure 3.4 shows five time-windows \( W_m[n] \) \((1 \leq m \leq 5)\) with overlapping factor of 0.5. Each \( W_m[n] \) corresponds to signals observed at a certain range of spatial
distances (spatial window). As $W_m[n]$ is shifted to cover later parts of signal $\chi[n]$ the spatial window is also shifted to a farther distance \cite{4}. Figure 3.5(a) shows the DvT plot for the non-dispersive medium and Figure 3.5(b) shows the DvT plot for the dispersive medium. The vertical dotted lines in Figure 3.4 and in Figure 3.5(b) represent the central point for each window at a specific time step. Central point for each window means half of the window’s length in time. Each vertical line in Figure 3.5(b) intersects with the highest amplitude of one of the recorded signals that observed at increasing distances. We draw a horizontal line at each intersection point in Figure 3.5(b). Each horizontal line represents the effective corresponding distance of propagation. For example, the vertical line at time = 241$\Delta t$ in Figure 3.5(b) intersects with the highest amplitude of the observed signal at distance = 27$\Delta y$. All the horizontal lines in Figure 3.5(b) are separated equally from each other because the windows have the same widths in time as shown in Figure 3.4. We draw the same horizontal lines of Figure 3.5(b) in Figure 3.5(a). Therefore each corresponding distance selected in the dispersive case is the same as those for the non-dispersive case. A corresponding signal is observed at each selected distance in both Figure 3.5(a) and (b). Each observed time domain signal $F_{m}^{\text{non}}[n]$ in Figure 3.5(a) is transformed to $F_{m}^{\text{non}}[k]$ in the frequency domain. Each observed time domain signal $F_{m}^{\text{disp}}[n]$ in Figure 3.5(b) is transformed to $F_{m}^{\text{disp}}[k]$ in the frequency domain. Each $F_{m}^{\text{non}}[k]$ is compared with $F_{m}^{\text{disp}}[k]$ to produce frequency dependent filters that correspond to the selected
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Figure 3.5: DvT of the propagating pulse in the dispersive and non-dispersive human’s fat. \( \Delta y = 1 \) mm and \( \Delta t = 2.357 \) ps.

\[
\Gamma_m[k] = \left( \frac{F_{m, \text{non}}[k]}{F_{m, \text{disp}}[k]} \right)^{1.5} \quad \text{for} \quad m = 1, \ldots, M \tag{3.8}
\]

where 1.5 is applied to compensate the propagated signals for both forward (from the source to the scatterer and from the scatterer back to the TRA antennas as shown in Figure 3.6 (a) and (b) respectively) and backward (from the TRA antennas to the scatterer as shown in Figure 3.6(c)) propagation of the TR technique.

Figure 3.7(a) shows examples of the observed signals for the dispersive and non-dispersive medium at 105\( \Delta y \) and 180\( \Delta y \). Figure 3.7(b) shows the frequency dependent filters obtained for the observed signals in Figure 3.7(a). A phase shift and attenuation are noticed in Figure 3.7(a) between the dispersive and non-dispersive signals for the same observation location. The phase shift and the attenuation increase as the observation location move farther away from the source point. We notice from Figure 3.7(b) the higher the frequency the higher the attenuation that signals face. As the frequency increases, \( \Gamma_m[k] \) value increases to compensate the dispersion.
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Figure 3.6: Forward and backward propagation for TR technique. $\Delta x = \Delta y = 1$ mm.
(a) Time-Domain signals observed at two specific distances for both DvTs.

(b) Frequency dependent filters obtained for two different distances.

Figure 3.7: Space and frequency dependent attenuation and phase shift in the dispersive case. $\Delta y = 1$ mm and $\Delta t = 2.357\text{ps}$. The dispersive medium parameters are $\sigma = 0.037$ S/m, $\epsilon_S = 5.53$, $\epsilon_\infty = 3.998$ and $\tau_D = 2.363 \times 10^{-11}$ seconds. The non-dispersive medium parameters are $\sigma = 0$ S/m, $\epsilon_S = 3.998$, $\epsilon_\infty = 3.998$ and $\tau_D = 0$ seconds.
3.2 Continuous Wavelet Transform (CWT)

The proposed method uses CWT technique which inherently incorporates both time and frequency variations. The wavelet function that we used for the CWT method is Morlet wavelet \[79\]. Morlet wavelet is considered in \[63\] as the proper choice for analysing the propagated wave in attenuating and dispersive media. The mother wavelet of Morlet in time domain \[80\] is

\[
\Psi_0[n] = \frac{1}{\sqrt{\pi}} e^{j f_\Psi n \Delta t} e^{-\frac{(n \Delta t)^2}{2}}
\] (3.9)

where \( f_\Psi \) is the central frequency of the mother wavelet. Note that without loss of generality, we have chosen to work with discrete signals instead of the continuous ones, hence the notation of \( \Psi_0[n] \) instead of \( \Psi_0(t) \). \( \psi_0[n] \) is expressed in scaled-frequency domain as

\[
\Psi_0(a_j \omega_k) = \frac{1}{\sqrt{\pi}} e^{-\frac{(a_j \omega_k - f_\Psi)^2}{2}}
\] (3.10)

where \( a_j \) is the dimensionless scaling factor of \( a_j = a_0 2^j \Delta_j \) for \( j = 0, 1, \ldots, J - 1 \), \( a_0 \) is the smallest scale, \( \Delta_j \) is the scale step, \( J \) is the largest scale \( J = \left\lceil \frac{1}{\Delta_j} \log_2 \left( \frac{N \Delta t}{a_0} \right) \right\rceil \), \( \lceil \rceil \) represents the ceiling function and the angular frequency \( \omega_k \) is \[80\]

\[
\omega_k = \begin{cases} 
0 & k = 1 \\
\frac{2\pi k}{N \Delta t} & 1 < k \leq \frac{N}{2} + 1 \\
-\frac{2\pi k}{N \Delta t} & \frac{N}{2} + 1 < k \leq N 
\end{cases}
\]

for \( k = 1, 2, \ldots, N \). \( a_0 \) is dimensionless and it is set to the value of \( \Delta t \) and \( \Delta_j \) is set to 0.025 based on the value chosen in \[80\]. Both \( \Psi_0[n] \) in (3.9) and \( \Psi_0(a_j \omega_k) \) in (3.10) are complex. In order for the analysing function \( \Psi_0[n] \) to be called wavelet it needs to be admissible \[81\]. Admissibility is one of the necessary conditions of the wavelet transform and it is achieved when \[81\]

\[
\lim_{N \to \infty} \sum_{n=-N}^{N} \Psi_0[n] = 0
\] (3.11)
and
\[
\Psi_0(a_j\omega_k) = \begin{cases} 
\frac{1}{\sqrt{\pi}}e^{-\frac{(a_j\omega_k - f))^2}{2}} & \omega_k > 0 \\
0 & \omega_k \leq 0.
\end{cases}
\]

Our compensation method starts with the conversion of the observed signal \(\chi[n]\) in the time domain to the wavelet domain. The CWT of \(\chi[n]\) is the convolution of the wave function \(\Psi^*\) and \(\chi[n]\) which is equivalent to the inverse Fourier transform of the product of \(\Psi^*(a_j\omega_k)\) and \(\chi[k]\) in the frequency domain where * represents the complex conjugate, \(\Psi^*(\frac{n}{a_j})\) is
\[
\Psi\left(\frac{n}{a_j}\right) = \sqrt{\frac{\Delta t}{a_j}}\Psi_0\left(\frac{n}{a_j}\right) \quad (3.12)
\]
and \(\Psi(a_j\omega_k)\) is
\[
\Psi(a_j\omega_k) = \sqrt{\frac{2\pi a_j}{\Delta t}}\Psi_0(a_j\omega_k). \quad (3.13)
\]
\(\Psi(a_j\omega_k)\) represents the normalised \(\Psi_0(a_j\omega_k)\) to guarantee that the CWTs at each \(a_j\) are comparable to each other and to the CWTs of other time series [80]. \(\Psi(a_j\omega_k)\) satisfies
\[
\sum_{k=1}^{N} |\Psi(a_j\omega_k)|^2 = N. \quad (3.14)
\]
The CWT for the observed signal \(\chi[n]\) at each TRA antenna is
\[
X[n,a_j] = \sum_{k=1}^{N} \left( \sum_{n=1}^{N} \chi[n]e^{-\frac{j2\pi nk}{N}}\Psi^*(a_j\omega_k) \right) e^{j\frac{2\pi nk}{N}}. \quad (3.15)
\]
The CWT method applies a long window (large \(a_j\)) at the lower end of the spectrum (low frequency) and a short window (small \(a_j\)) at the higher end of the spectrum (high frequency) to provide a perfect filter for time-dependent implementations in terms of time and frequency localisation. Furthermore, the CWT method provides us with an improved separation between the unwanted noise and our signal [63]. The inverse filters are applied to \(X[n,a_j]\) in the wavelet domain to compensate the attenuation caused by the dispersive medium. In order to derive the inverse filter, we need to calculate the attenuation. The solution of Maxwell
equations for a plane wave propagating in a dispersive medium [32] is

\[ E[n] = e^{j2\pi f \cdot n \Delta t} e^{-j2\pi f \sqrt{\epsilon_r d[n]}} \tag{3.16} \]

where \( f \) is the frequency of interest, \( d[n] \) is the distance between the excitation and the observation, \( \mu = \mu_0 \mu_r \) is the permeability of the medium, \( \mu_0 \) is the free space permeability and \( \mu_r \) is the relative permeability which is set to 1 as this thesis assumes a non-magnetically dispersive medium, \( \epsilon = \epsilon_0 \epsilon_r \) is the permittivity of the medium, \( \epsilon_0 \) is the free space permittivity, \( \epsilon_r = \epsilon_{\infty} + \frac{\epsilon_S - \epsilon_{\infty}}{1 + j2\pi f \tau_D} - j\frac{\sigma}{2\pi f \epsilon_0} \) is the complex relative permittivity of medium, \( \epsilon_{\infty} \) is the optical relative permittivity, \( \epsilon_S \) is the static relative permittivity, \( \tau_D \) is the relaxation time in seconds, \( \sigma \) is the static conductivity in S/m.

(3.16) can be rewritten as

\[
E[n] = e^{j2\pi f \cdot n \Delta t} e^{-j2\pi f \sqrt{\epsilon_r d[n]}} C \\
= e^{j2\pi f \cdot n \Delta t} e^{-j2\pi f \Re[\sqrt{\epsilon_r}] \frac{d[n]}{C}} e^{2\pi f \Im[\sqrt{\epsilon_r}] \frac{d[n]}{C}} \tag{3.17}
\]

\[ \equiv e^{j2\pi f \cdot n \Delta t} \cdot \Theta \cdot A \]

where \( C \) is the speed of light \( C = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \) in the vacuum, \( \Re[\sqrt{\epsilon_r}] \) and \( \Im[\sqrt{\epsilon_r}] \) are the real and the imaginary parts of \( \sqrt{\epsilon_r} \), \( A \) is the attenuation defined as

\[ A[n, f] = e^{2\pi f \Im[\sqrt{\epsilon_r}] \frac{d[n]}{C}} \tag{3.18} \]

and \( \Theta \) is the phase shift defined as

\[ \Theta[n, f] = e^{-j2\pi f \Re[\sqrt{\epsilon_r}] \frac{d[n]}{C}}. \tag{3.19} \]

Our filter is obtained from (3.18) and (3.19) as

\[ H[n, a_j] = \frac{1}{\Theta[n, a_j] \cdot A[n, a_j]} \]

\[ = e^{j2\pi \frac{f}{a_j} \Re[\sqrt{\epsilon_r}] \frac{d[n]}{C}} e^{-2\pi \frac{f}{a_j} \Im[\sqrt{\epsilon_r}] \frac{d[n]}{C}} \tag{3.20} \]

where \( a_j \) is a scaling factor which controls the actual frequency \( f(\equiv \frac{f_c}{a_j}) \) of the filter in order to have a variable window size [33]. \( H[n, a_j] \) uses \( \epsilon_r \) of the dominant
tissue in the propagating media at $f_c$. $H[n, a_j]$ is an exponential function which amplifies the noise in compensated signal and thus causes instability. Therefore $H$ should be stabilised [83] as

$$H_s[n, a_j] = \frac{A[n, a_j]}{A[n, a_j]^2 + B} \cdot \frac{1}{\Theta[n, a_j]} \quad (3.21)$$

where $H_s$ is the stabilised filter and $B$ is the stabilisation factor. The system is stable with a fixed value of $B = 10^{-3}$ for all our simulations. Alternatively we can adaptively set $B$ as [63]

$$B = \frac{\sigma_g^2}{\sigma_s^2} \quad (3.22)$$

where $\sigma_g$ and $\sigma_s$ are the variances for the noise and the signal respectively. By applying $H_s[n, a_j]$ to $X[n, a_j]$, we obtain the stabilised compensated wave $Y[n, a_j]$ in the wavelet domain as

$$Y[n, a_j] = X[n, a_j] H_s[n, a_j]. \quad (3.23)$$

To avoid the amplification of the high frequency noise, $H_s[n, a_j]$ is set to 1 for $0 \leq j \leq J_2$ (short window) which affects the higher part of the spectrum. Finally the compensated signal $X_c[n]$ is obtained by applying the inverse continuous wavelet transform (ICWT) to $Y[n, a_j]$ as [80]

$$X_c[n] = \frac{1}{C_\delta} \sum_{j=1}^{J} \frac{\mathcal{R}[Y[n, a_j]]}{\sqrt{a_j}} \quad (3.24)$$

where $C_\delta$ is a constant for each wavelet function. It is calculated as

$$C_\delta = \sum_{j=1}^{J} \frac{\mathcal{R}[X_\delta[a_j]]}{\sqrt{a_j}} \quad (3.25)$$

for the Morlet wavelet where $X_\delta$ is the CWT of a delta function ($\delta$) as

$$X_\delta[a_j] = \frac{1}{N} \sum_{k=1}^{N} \Psi^*(a_j \omega_k). \quad (3.26)$$

The parameters for the CWT and the ICWT were selected based on the theory in [80]. Our proposed method is depicted in Figure 3.8.
CHAPTER 3. COMPENSATIONS METHODS

Figure 3.8: Compensation method for the dispersive attenuation in the wavelet domain.

3.3 Summary

Chapter 3 explained the details of the STFT with threshold approach and the CWT compensation methods. The settings for the STFT technique are optimised by applying different window types and lengths. The STFT method uses fixed length windows while the CWT based compensation method uses adjustable-length window scheme by applying long time windows at low frequencies and short time windows at high frequencies. Therefore, the optimisation of the time window length is not needed as was done in the STFT method. The inverse filters of the compensation methods overcome the attenuation and dispersion in the EM wave propagation due to the dispersive medium. In Chapter 4 we applied different numerical simulations to show the results of the imaging techniques with and without compensation methods.
Chapter 4

Numerical Simulations

The purpose of these experiments is to compare the results of our compensation methods [69, 71] and the method in [26]. Our proposed threshold approach [69] reduces the amplification of the unwanted noise, resulting in more accurate spatial focusing. Furthermore, We expect that our proposed CWT compensation method [71] improves the localisation and the resolution of the conventional TR imaging because the inverse filters should compensate the losses caused by the dispersive media. We also investigate the performance of TR-MUSIC technique and TFM before and after applying the CWT filters. As a practical application of lung cancer detection, we introduce antenna location optimisation combined with CWT compensation methods.

4.1 TR canonical simulation with homogeneous medium

4.1.1 Radio environment setting

The simulation setup is depicted in Figure 4.1. The simulations use the TMz polarised 2D-FDTD method. The FDTD space is 150 × 150, uniformly sampled with a spatial step of Δs ≜ Δx = Δy = 1mm. The time step Δt is set to 2.357ps. CFS-PML absorbing boundary conditions [75] are used with a 9-cell layer. The TRA is composed of 15 transceivers antennas, equally spaced from each other and parallel to the x-axis. The TRA has the aperture of 70Δs. The 1st TRA antenna is located at (40Δx,25Δy) and the 15th is located at (110Δx,25Δy); they are all 20Δs or more from the boundary. A PEC scatterer is located in
the medium with a radius of 2 mm and centred at \((85 \Delta x, 85 \Delta y) \equiv (x_p, y_p)\). The human tissues are modelled using the one-pole Debye media \([84]\). The 5th antenna \((60 \Delta x, 25 \Delta y)\) of the TRA transmits a Gaussian pulse \(b[n]\) modulated at 3 GHz covering from 1.72 GHz to 4.2 GHz. The excitation pulse \(b[n]\) and its frequency domain representation \(b[k]\) are shown in Figure 4.2. As \(b[n]\) propagates through the dispersive medium, it is scattered by the PEC target and then observed by the TRA antennas. The observed signals on each TRA antenna are then

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**Figure 4.1:** Simulation setup.

**Figure 4.2:** The modulated Gaussian pulse with \(f_c = 3\text{GHz}\).
COMPENSATED, TIME-REVERSED AND TRANSMITTED BACK TO THE MEDIUM. THE TIME-
REVERSED SIGNALS AUTOMATICALLY FOCUS ON THE POSITION OF THE SCATTERER. WE APPLY
OUR COMPENSATION METHOD TO THE CASES WHERE THE FDTD SPACE IS HOMOGENEOUSLY
FILLED WITH MUSCLE \[85, 86\]. MUSCLE IS THE MOST DISPERSIVE TISSUE IN THE HUMAN
BODY. EACH OBSERVED SIGNAL \(\chi[n]\) AT EACH TRA ANTENNA IS CORRUPTED WITH ADDITIVE
WHITE GAUSSIAN NOISE. THE AVERAGE POWER OF THE ADDITIVE NOISE IS 20% OF THAT OF
\(\chi[n]\) WHICH IS CHOSEN TO MIMIC THE SYSTEM NOISE BASED ON \[87, 88\].

4.1.2 Numerical results

In order to select the optimum \(l\) and satisfy \((3.7)\) we measure the distance between
the location of the maximum value of \(|F[x_M, y_M]|\) at the time of refocusing and
the centre of the scatterer location of \((x_p, y_p)\) for our method in \[69\] and the
method in \[26\]. In the muscle medium, STFT with threshold approach satisfies
\((3.7)\) with \(l = 482\) based on our four-step procedure while the method in \[26\]
did not. The value of \(l = 482\) achieves the sharpest focusing in muscle using
Hanning window with the lowest possible number of filters. The inverse filters
in \[26\] amplify the noise (reducing the SNR) in highly dispersive tissues (such as
muscle). The amplification of the noise reduces the amplitude of \(X_c\) relative to
noise which reduces the accuracy of the localisation and \((x_M, y_M)\) becomes far
from \((x_p, y_p)\). In our threshold approach, the noise amplification is reduced as
the inverse filters are not applied to the high-frequency part of the spectrum.

Figure 4.3 shows the observed signals at the 8th antenna of the TRA for
the muscle medium using CWT, STFT with the threshold, STFT and without
any compensation method (simple TR). The oscillations in the simple TR signal
represent the 20% additive Gaussian noise. These oscillations are amplified by
the filters used in \[26\] and the noise level becomes large as shown in Figure 4.3
that the reconstructed signal is strongly distorted. However, the amplification
of the noise is reduced when our methods (STFT with threshold approach and
the CWT compensation method) are applied. Figure 4.4 (a), (b), (c) and (d)
show the normalised \(|F|\) in the \(xy\)-plane with conventional TR, with the CWT
method, with the method in \[26\] and with the threshold approach. To compare
the compensation methods (CWT \[71\], STFT with threshold \[69\] and STFT in
\[26\]), we calculate the accuracy \(\Xi\) as

\[\Xi = \frac{d_{tr}}{d_c}\] (4.1)
where \(d_{tr}\) and \(d_{c}\) are the distances between \((x_M, y_M)\) and \((x_p, y_p)\) for TR without any compensation and TR with a compensation method respectively. The CWT method and the STFT with threshold approach are \((\Xi =)5.1\) and \((\Xi =)1.4\) times more accurate than the simple TR technique respectively.

4.2 TR canonical simulation with random medium

We evaluate the accuracy and the resolution of our methods in a random medium \([89]\) to increase the complexity of the propagating medium. The random medium is of Gaussian distribution and the spatial random relative permittivity is defined as

\[
\epsilon_{rnd}[r, f] = \epsilon_r[r, f] + \mathcal{G}[r]
\]  

(4.2)

where \(\epsilon_r\) is calculated using the one-pole Debye \([84]\) parameters of the muscle tissue, \(\mathcal{G}\) is a function of space defining the random fluctuation on \(\epsilon_{rnd}[r]\) and is
Figure 4.4: The spatial distribution of $|F|$ in the $xy$-plane for the homogeneous canonical test with conventional TR (without applying any compensation method), with the method in [26] (STFT), with our threshold approach [69] and with our compensation method (CWT) [71]. The scatterer and the TRA antennas are represented by “◦” and “x” respectively. $\Delta x = \Delta y = 1$mm.
calculated as

\[ G[r] = \frac{1}{R_x R_y} \sum_{u=1}^{R_x} \sum_{v=1}^{R_y} \left( \sum_{x=1}^{R_x} \sum_{y=1}^{R_y} \mathcal{L}[r_1, r_2] e^{-j2\pi \left( \frac{ux}{R_x} + \frac{vy}{R_y} \right)} \times \mathcal{W} \cdot e^{-j2\pi \left( \frac{ux}{R_x} + \frac{vy}{R_y} \right)} \right) \cdot \left( \sum_{x=1}^{R_x} \sum_{y=1}^{R_y} \mathcal{L}[r_1, r_2] e^{-j2\pi \left( \frac{ux}{R_x} + \frac{vy}{R_y} \right)} \times \mathcal{W} \cdot e^{-j2\pi \left( \frac{ux}{R_x} + \frac{vy}{R_y} \right)} \right), \]  

(4.3)

where \( R_x \times R_y \) is the 2D FDTD size, \([x, y]\) is the Cartesian coordinates, \([u, v]\) is the Cartesian coordinates in the spatial frequency domain, \( \mathcal{W} \) is a Gaussian random number with zero mean and probability density function of \( \frac{1}{\sqrt{2\pi\eta}}e^{-\frac{\zeta^2}{2\eta}} \), \( \zeta \) is a random variable, \( \eta \) is the variance and \( \mathcal{L} \) is the correlation function between \( \epsilon_r \) at two different spatial points \( r_1 \) and \( r_2 \) given by a Gaussian function as

\[ \mathcal{L}[r_1, r_2] = \eta e^{-\left( \frac{|r_1 - r_2|}{l_s} \right)^2}, \]  

(4.4)

where \( l_s \) is the transverse correlation length. Gaussian function is chosen for its generalisation and mathematical properties \[89\]. The random media is characterised by \( l_s \) and \( \eta \). We chose muscle tissue because it is one of the most dispersive tissues in the human body.

### 4.2.1 Radio environment setting

The simulation setup depicted in Figure 4.5 is the same as the one in Figure 4.1 in Section 4.1.1 except propagation media which filled the FDTD space. Figure 4.6 shows the spatial distribution of \( \epsilon_r \), at 1.73 GHz (minimum frequency within the bandwidth), 3 GHz (central frequency) and 4.27 GHz (maximum frequency within the bandwidth) with \((l_s, \eta) = (\Delta s, 0.1\epsilon)\) for a single run. Figure 4.7 and Figure 4.8 show the spatial distribution of the average of \( \epsilon_r \) and the standard deviation of \( \epsilon_r \) respectively for 10000 random simulation run. The spatial distributions of both the average and the standard deviation of \( \epsilon_r \) are similar over the frequency range of interest. Therefore, \( \epsilon_r \) distribution at 3 GHz is the representative frequency of the frequency range of interest.
4.2.2 Numerical results

Figure 4.9 shows the example of the spatial distribution of $\epsilon_r$ for Figure 4.5 at 3 GHz when $l_s$ is set to $5\Delta s$ and $9\Delta s$. Figure 4.10 shows the observed signals at the 8th antenna of the TRA for the random medium using CWT, STFT with the threshold, STFT and without any compensation method (simple TR). The oscillations in the simple TR signal represent the 20% additive Gaussian noise. Part of these oscillations are amplified by the filters used in [26] and the noise level becomes large as shown in Figure 4.10 (i.e. the reconstructed signal is strongly distorted). The amplification of the noise is reduced when our method (STFT with threshold approach) is applied. However, the compensated signal has unwanted oscillations that were amplified by the filters even after applying the threshold approach. The filters of the CWT method did not amplify the oscillations of the noise in $X_c$. Figure 4.11 shows $|F|$ in the $xy$-plane with conventional TR, with our compensation method (CWT) [71], with the method in [26] and with our threshold approach [69]. Figure 4.11 is truncated by $10\Delta y$ away from the TRA antennas (starting at $35\Delta y$) because a high value near the antennas masks the field values around the target in Figure 4.11 (a), (c) and (d). The high value is caused by the random dispersive medium and the additive noise as shown in Figure 4.12 (The full version of Figure 4.11). Both STFT methods (Figure 4.11 (c) and (d)) could not remove the effects of the medium.
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(a) 1.73 GHz with minimum and maximum values of 11.44 and 43.59 respectively.

(b) 1.73 GHz with minimum and maximum values of 0.58 [S/m] and 0.89 [S/m] respectively.

(c) 3 GHz with minimum and maximum values of 12.03 and 43.03 respectively.

(d) 3 GHz with minimum and maximum values of 0.3 [S/m] and 1.16 [S/m] respectively.

(e) 4.27 GHz with minimum and maximum values of 12.8 and 42.31 respectively.

(f) 4.27 GHz with minimum and maximum values of -0.05 [S/m] and 1.5 [S/m] respectively.

Figure 4.6: The spatial distribution of $\varepsilon_r$ with $(l_s, \eta) = (5\Delta s, 0.15\varepsilon_\infty)$ for a single simulation.
CHAPTER 4. NUMERICAL SIMULATIONS

(a) 1.73 GHz with minimum and maximum values of 27.82 and 28.15 respectively.

(b) 1.73 GHz with minimum and maximum values of 0.745 [S/m] and 0.748 [S/m] respectively.

(c) 3 GHz with minimum and maximum values of 27.83 and 28.15 respectively.

(d) 3 GHz with minimum and maximum values of 0.74 [S/m] and 0.75 [S/m] respectively.

(e) 4.27 GHz with minimum and maximum values of 27.83 and 28.14 respectively.

(f) 4.27 GHz with minimum and maximum values of 0.73 [S/m] and 0.75 [S/m] respectively.

Figure 4.7: The spatial distribution of the average of $\varepsilon_r$ with $(l_s, \eta) = (5\Delta s, 0.15\varepsilon_\infty)$ for 10000 simulation run.
(a) 1.73 GHz with minimum and maximum values of 4.04 and 4.28 respectively.

(b) 1.73 GHz with minimum and maximum values of 0.03 [S/m] and 0.04 [S/m] respectively.

(c) 3 GHz with minimum and maximum values of 3.89 and 4.13 respectively.

(d) 3 GHz with minimum and maximum values of 0.1 [S/m] and 0.11 [S/m] respectively.

(e) 4.27 GHz with minimum and maximum values of 3.71 and 3.93 respectively.

(f) 4.27 GHz with minimum and maximum values of 0.19 [S/m] and 0.2 [S/m] respectively.

Figure 4.8: The spatial distribution of the standard deviation of $\epsilon_\tau$ with $(l_s, \eta) = (5\Delta s, 0.15\epsilon_\infty)$ for 10000 simulation run.
(a) \((l_s, \eta) = (9\Delta s, 0.1\epsilon_\infty)\) with minimum and maximum values of 17.93 and 40.59 respectively.

(b) \((l_s, \eta) = (9\Delta s, 0.1\epsilon_\infty)\) with minimum and maximum values of 0.46 and 1.09 respectively.

(c) \((l_s, \eta) = (5\Delta s, 0.16\epsilon_\infty)\) with minimum and maximum values of 12.03 and 43.03 respectively.

(d) \((l_s, \eta) = (5\Delta s, 0.16\epsilon_\infty)\) with minimum and maximum values of 0.3 and 1.16 respectively.

(e) \((l_s, \eta) = (9\Delta s, 0.16\epsilon_\infty)\) with minimum and maximum values of 15.29 and 40.07 respectively.

(f) \((l_s, \eta) = (9\Delta s, 0.16\epsilon_\infty)\) with minimum and maximum values of 0.39 and 1.08 respectively.

Figure 4.9: The spatial distribution of \(\epsilon_r\) for Figure 4.5 at 3 GHz.
Figure 4.10: The observed signal $\chi[n]$ at the 8th antenna of the TRA without compensation (TR), with our CWT compensation, with our threshold approach and with [26] compensation method (STFT).

and noise while the CWT method removed them as shown in Figure 4.11 (b). Figure 4.13 shows the spatial distribution of $|F|$ in the $xy$-plane for the canonical test with random propagation media using our compensation method (CWT) [71] when $(l_s, \eta) = (9\Delta s, 0.16\epsilon_\infty), (5\Delta s, 0.16\epsilon_\infty)$ and $(9\Delta s, 0.1\epsilon_\infty)$. Figure 4.14 shows the cross-section at $y = 85\Delta y$ of Figure 4.13. TR techniques utilise the multipath components generated by the random media to achieve more accurate focusing resolution than in homogeneous media. Both the high correlation length $l_s$ such as $9\Delta s$ and the low variance $\eta$ such as $0.1\epsilon_\infty$ reduce the randomness of the medium as both cases result in the reduction of the overall fluctuations of the permittivity and the conductivity of the propagation medium, hence its scatteredness. Reducing the randomness decreases the multipath components which lower the resolution of the TR focusing.

4.3 TR practical simulation

Lung cancer is identified as one of the deadliest cancers affecting people [90, 91]. Early-stage lung cancer is difficult to detect because the tumour is small and its electromagnetic properties are close to the parameters of the healthy
Figure 4.11: The spatial distribution of $|F|$ in the $xy$-plane for the random canonical test with conventional TR (without applying any compensation method), with our compensation method (CWT) \cite{71}, with the method in \cite{26} (STFT) and with our threshold approach \cite{69} when $(l_s, \eta) = (5\Delta s, 0.16\epsilon_\infty)$. The scatterer and the TRA antennas are represented by “◦” and “x” respectively. $\Delta x = \Delta y = 1$mm.
Figure 4.12: The spatial distribution of $|F|$ in the $xy$-plane for the random canonical test with conventional TR (without applying any compensation method), with our compensation method (CWT) [71], with the method in [26] (STFT) and with our threshold approach [69] when $(l_s, \eta) = (5\Delta s, 0.16\epsilon_\infty)$. The scatterer and the TRA antennas are represented by “◦” and “x” respectively. $\Delta x = \Delta y = 1\text{mm}$. 

(chapter 4. numerical simulations)

(a) TR.

(b) TR with CWT.

(c) TR with STFT.

(d) TR with STFT and threshold.
Figure 4.13: The spatial distribution of $|F|$ in the $xy$-plane for the random canonical test with our compensation method (CWT) \cite{71}. The scatterer and the TRA antennas are represented by “◦” and “x” respectively. $\Delta x = \Delta y = 1$mm.
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Figure 4.14: The cross-section of the normalised $|F|$ distribution at $y = 85\Delta y$ in the FDTD space after applying CWT for the cases $(l_s, \eta) = (9\Delta s, 0.16\epsilon_\infty), (5\Delta s, 0.16\epsilon_\infty)$ and $(9\Delta s, 0.1\epsilon_\infty)$.

lung. We apply the TR technique with our proposed compensation method to localise an early-stage tumour in a DHP\footnote{The DHP was provided by RIKEN (Saitama, Japan) under non-disclosure agreement between RIKEN and the University of Manchester. The usage was approved by RIKEN ethical committee.}. The DHP has the spatial resolution $\Delta s$ of 1 mm and contains 52 segmented tissue (see Appendix B). The one-pole Debye parameters of human tissues [92] have been fitted by our group using the measurement provided by [93, 94]. The Debye media parameters for human tissues are presented in [86]. Figure 4.15(a), (b) and (c) show the $265 \times 490 \times 1687$ DHP on $z = 380, y = 285$ and $x = 115$ planes respectively. The purpose of this numerical simulation is to evaluate the accuracy and the resolution of our method to detect a lung tumour in the human phantom medium.

4.3.1 Radio environment setting

Figure 4.16 shows the DHP with an early stage tumour represented by round scatterer with a 5 mm radius centred at $(115\Delta x, 285\Delta y, 95\Delta z)$. The FDTD space is $265 \times 490 \times 290$ including the 9 CFS-PML cells. The 102 TRA $z-$directed 14 mm dipole antennas [95] are placed in the fat tissue on $z = 95\Delta z$ assuming that we use the implantable antennas [96, 97, 98]. The human tissues are represented using the one-pole Debye model. We shifted the frequency response of the healthy lung tissue to match the complex permittivity of the lung cancer tissue presented
Figure 4.15: $265 \times 490 \times 1687$ DHP.
Figure 4.16: The human phantom with 5 mm radius round tumour. The TRA antennas are represented by “|”. $\Delta x = \Delta y = \Delta z = 1\text{mm}$. 
4.3.2 Numerical results

Figure 4.18 shows $|F|$ on $z = 95\Delta z$ plane within the 3D DHP with conventional TR without applying any compensation, with the CWT method [71], with the STFT method in [26] and with the STFT-threshold approach [69]. Figure 4.18 (c) and (d) show that the STFT methods in [26, 69] did not improve the conventional TR imaging or localise the tumour correctly. The inverse filters utilised in [26] not only amplify the signal of interest, but also the noise. This reduces the SNR ratio and in return lowers the accuracy of the localisation. The CWT filters [71] compensate the losses caused by the dispersive tissues. Therefore the application of the proposed CWT method to the TR technique gives more accurate spatial focusing than both the conventional TR and the STFT methods.
In order to reduce the complexity of the scenarios, we repeated the simulations using a limited number of antennas (15 TRA antennas) in front of the body as shown in Figure 4.19. Figure 4.20 shows $|F|$ on $z = 95\Delta z$ plane within the 3D DHP for the scenarios in Figure 4.19. In both cases (antennas all around the body or part of it) TR with CWT localised the tumour appropriately.

### 4.4 TR-MUSIC and TFM canonical simulations

#### 4.4.1 Radio environment setting

The purpose of this experiment is to study the TR-MUSIC and TFM in localisation of one and two scatterers. The simulation setup for the single PEC case is depicted in Figure 4.21. All simulations use the TM$_z$ polarised 2D-FDTD method. The FDTD space is $150 \times 150$ uniformly sampled with the spatial step of $\Delta s = \Delta x = \Delta y = 1\text{mm}$. The time step $\Delta t$ is set to 2.357ps. The CFS-PML absorbing boundary condition [75] is used with 9-cell layer. The FDTD space is filled with either fat (least dispersive tissue) or muscle (most dispersive tissue).

Figure 4.22 shows the real part of the relative permittivity and the conductivity of fat and muscle. A TRA composed of 15 transceivers antennas are equally spaced from each other and parallel to the $x$-axis. The TRA has aperture of $70\Delta s$. The 1st TRA antenna is located at $(40\Delta x, 25\Delta y)$ and the 15th is located at $(110\Delta x, 25\Delta y)$. A PEC scatterer(s) is located in the medium with a radius of 2 mm. A Gaussian pulse $b[n]$ modulated at 3 GHz is used as an excitation pulse. $b[n]$ and its frequency domain representation are shown in Figure 4.2.

#### 4.4.2 Numerical results for 1 PEC scatterer

The PEC is centred at $(85\Delta x, 85\Delta y)$. In the TR-MUSIC method, Figure 4.23 shows the singular values at the central frequency for the single PEC case. The number of PECs can be estimated from the values of the singular values. In Figure 4.23 the first singular value is larger than the threshold value which estimates that only one PEC exists in the medium. Figure 4.24 shows the first three singular values and the threshold that equals to 10% of the first singular value. The threshold split the singular values into SS domain (higher than the threshold) and into NS domain (lower than the threshold). Only the first singular
Figure 4.18: The spatial distribution of $|F|$ on $z = 95\Delta z$ plane for the human phantom with our compensation method, the method in [26] and without any compensation method applied to the scenario in Figure 4.16. The scatterer and the TRA antennas are represented by “◦” and “x” respectively. $\Delta x = \Delta y = \Delta z = 1$mm. In order to present $|F|$ inside the lung clearly, $|F|$ on the skin, muscle and fat are removed from the figures.
Figure 4.19: The simulation setup. The TRA antennas are represented by “|”. \( \Delta x = \Delta y = \Delta z = 1\text{mm} \).

The value is in the SS domain and all the remaining singular values are in the NS domain. Figure 4.25(a) and (b) show \( I_{\text{MUSIC}} \) distribution for TR-MUSIC method and Figure 4.25(c) and (d) show \( I_{\text{TFM}} \) distribution for TFM where the propagation medium is fat. Similarly, Figure 4.26 shows the distributions of \( I_{\text{MUSIC}} \) and \( I_{\text{TFM}} \) in muscle. Figure 4.25(a) and (b) show how the scatterer is localised in the fat case with a fine spatial focusing resolution and accurate location. In Figure 4.25(c) and (d) the focusing is degraded when compared to Figure 4.25(a) and (b) respectively. We notice that the highest peak of the focusing is not within the scatterer geometry for the case where TFM is used. However, the TFM with CWT compensation method in Figure 4.25(d) achieved a higher resolution focusing than Figure 4.25(c) around the scatterer. Figure 4.26(a) shows how the TR-MUSIC focusing is degraded around the scatterer in the muscle case. The TR-MUSIC imaging is improved after applying the CWT filter as shown in Figure 4.26(b). When the TFM is applied, the highest peak of the focusing is not within the scatterer geometry as in Figure 4.26(c) and (d). The TFM with CWT filters in Figure 4.26(d) has a higher resolution focusing than the case without any compensation in Figure 4.26(c) around the scatterer. Figure 4.25 and Figure 4.26 shows that TR-MUSIC technique has higher focusing resolution than the TFM approach.
Figure 4.20: The spatial distribution of $|F|$ on $z = 95\Delta z$ plane for the human phantom with our compensation method, the method in [26] and without any compensation method applied to the scenario in Figure 4.19. The scatterer and the TRA antennas are represented by “o” and “x” respectively. $\Delta x = \Delta y = \Delta z = 1$mm. In order to present $|F|$ inside the lung clearly, $|F|$ on the skin, muscle and fat are removed from the figures.
Figure 4.21: The simulation setup for the single PEC case.

Figure 4.22: $\Re[\varepsilon_r]$ and $\omega\varepsilon_0\Im[\varepsilon_r]$ for fat and muscle.
Figure 4.23: The singular values at $f_c$ for the single PEC case where the propagation medium is fat.

Figure 4.24: The first three singular values and the threshold which split the singular values into SS and NS domains for the single PEC case where the propagation medium is fat.
CHAPTER 4. NUMERICAL SIMULATIONS

Distance(\times \Delta y)

Distance(\times \Delta x)

0.77
1
0.55
0.33
0.11

Normalised |I_{MUSIC}[r, f_c]|

30
30
70
70
110
110

Normalised |I_{MUSIC}[r, f_c]|

0.6
1
0.4
0.2
0

Normalised |I_{TM}[r, f_c]|

30
30
70
70
110
110

Normalised |I_{TM}[r, f_c]|

0.2

(a) TR-MUSIC.

(b) TR-MUSIC with CWT.

(c) TFM.

(d) TFM with CWT.

Figure 4.25: The spatial focusing for TR-MUSIC and TFM with and without CWT compensation method in fat.
CHAPTER 4. NUMERICAL SIMULATIONS

Figure 4.26: The spatial focusing for TR-MUSIC and TFM with and without CWT compensation method in muscle.
4.4.3 Numerical results for 2 PEC scatterer

The simulations setup for the two PEC case are depicted in Figure 4.27. The four simulations in Figure 4.27 cover all the minimum cross-range distances that TR-MUSIC method and TFM need to distinguish between 2 PEC scatterers. The minimum cross-range distance between the two scatterers is 26 mm and 45 mm in fat for TR-MUSIC technique and TFM respectively. Furthermore, the minimum distance needed in the muscle tissue is 8 mm for TR-MUSIC method and 25 mm for TFM. These cross-range distances are utilised when the distance on the \( y \)-axis between the TRA and the scatterers is 60 mm and 40 mm for the TR-MUSIC technique and TFM respectively. The TFM fails to distinguish between the targets if the distance between the scatterers and the TRA transceivers is more than 40 mm. In TR-MUSIC method, Figure 4.28 shows the singular values...
Figure 4.28: The singular values at $f_c$ for the two PEC case where the propagation medium is fat.

Figure 4.29: The first three singular values and the threshold which split the singular values into SS and NS domains for the two PEC case where the propagation medium is fat.

at the central frequency for the two PECs case. The number of PECs can be estimated from the values of the singular values. We notice in Figure 4.28 that the first two singular values are larger than the 10% of the first singular value which estimates that two PECs exist in the medium. Figure 4.29 shows the first three singular values and the threshold that equals to 10% of the first singular value. The first two singular values are higher than the threshold around $f_c$. The first two singular values are in the SS domain and all the remaining singular values are in the NS domain. Figure 4.30, Figure 4.31, Figure 4.32 and Figure 4.33 show the spatial focusing for TR-MUSIC technique and TFM in fat and muscle with various cross-range distances between the scatterers.
Figure 4.30: The spatial focusing for TR-MUSIC and TFM in fat applied to the scenario in Figure 4.27 (c). The minimum cross-range distance between the scatterers is set for the TFM to localise both scatterers at the same time.
Figure 4.31: The spatial focusing for TR-MUSIC and TFM in fat applied to the scenario in Figure 4.27 (a). The minimum cross-range distance between the scatterers is set for the TR-MUSIC to localise both scatterers at the same time.
Figure 4.32: The spatial focusing for TR-MUSIC and TFM in muscle applied to the scenario in Figure 4.27 (d). The minimum cross-range distance between the scatterers is set for the TFM to localise both scatterers at the same time.
Figure 4.33: The spatial focusing for TR-MUSIC and TFM in muscle applied to the scenario in Figure 4.27 (b). The minimum cross-range distance between the scatterers is set for the TR-MUSIC to localise both scatterers at the same time.
In Figure 4.30 and Figure 4.31, the distance between the 2 PECs is the minimum cross-range distance for the TFM and TR-MUSIC respectively to distinguish between the scatterers in the fat medium. In Figure 4.30, the 2 PECs are localised using both the TR-MUSIC and the TFM. However, the CWT filters improved the localisation for TR-MUSIC method and TFM. Moreover, the TR-MUSIC technique has more accurate imaging localisation than the TFM. In Figure 4.31, the scatterers are located farther away from the TRA antennas and the cross-range distance between the PECs is smaller when compared with Figure 4.30. The focusing of TR-MUSIC imaging technique is degraded without the CWT filters. The CWT compensation method improved the localisation of the TR-MUSIC method. The TFM method failed to distinguish between the two PECs because the distance between the scatterers is less than the minimum cross-range distance for the TFM to localise the targets. In Figure 4.32 and Figure 4.33, the distance between the 2 PECs is the minimum cross-range distance for the TFM and TR-MUSIC respectively to distinguish between the scatterers in the muscle medium. The CWT method improved the accuracy of the localisation focusing of both methods as the focusing is surrounding the PECs and within the scatterers geometries. Finally, in Figure 4.33, the scatterers are farther away from the TRA and they are much closer to each other than the case in Figure 4.32. Only Figure 4.33 (b) of TR-MUSIC technique with the CWT compensation method was successful in localising the PECs when compared to Figure 4.33 (a), (c) and (d). The TFM failed to distinguish between the PECs as they are close to each other. Furthermore, the TR-MUSIC without any compensation filters also failed to localise the scatterers due to the high dispersive medium of muscle tissue. We noticed that in the medium filled with muscle, the microwave imaging methods needed smaller cross-range distances between the scatterers to achieve the localisation than those in the fat medium. The reason can be found in (2.6) where \( \lambda(=\frac{C}{k\sqrt{\varepsilon_r[k]}}) \) in muscle is shorter than \( \lambda \) in fat. Figure 4.34 shows the cross-section at \( y = 85\Delta y \) and \( y = 65\Delta y \) of the scenarios in Figure 4.27. Figure 4.34 shows that TR-MUSIC method has higher resolution than the TFM technique. The focusing of TR-MUSIC method is improved with the CWT compensation method. The focusing peaks at the targets locations are more accurate in terms of the intensity and localisation than without applying the CWT compensation method. In the fat medium (Figure 4.34 (a) and (c)), the resolution of the focusing without applying the CWT compensation method is
Figure 4.34: The cross-section of the normalised $I_{MUSIC}$ and $I_{TFM}$ distribution at $y = 85\Delta y$ and $y = 65\Delta y$ in the FDTD space before and after applying CWT compensation method.
Figure 4.35: DHP with a 5 mm radius round tumour whose centre is at $(115\Delta x, 280\Delta y)$. $\Delta x = \Delta y = 1\text{mm}$

higher than after applying the CWT filters. However, the corresponding focusing (Figure 4.30 and Figure 4.31) is faded and not accurate as the focusing after applying the CWT filters.

### 4.5 TR-MUSIC and TFM practical simulations

We tried a different set of antennas starting with a total of $A = 2$ TRA transceivers to check the minimum $A$ that we can use and have enough null space for the imaging methods. We also changed the distance between every two antennas to acquire the optimum distance for the localisation of the tumour. Moreover, we apply the transceivers in multiple locations to identify the tissues or regions in the human body that may cause reflection or loss to the signals which degrade the localisation focusing or fail to localise a tumour. Figure 4.35 (a) shows the 3D DHP excluding the head and the lower part for concise visualisation. Figure 4.35 (b) shows the cross-section of the DHP with a 5 mm radius round tumour whose centre is at $(115\Delta x, 280\Delta y)$ where $\Delta s = \Delta x = \Delta y = 1\text{mm}$. The medium parameters for the one-pole Debye relaxation model of each tissue in Figure 4.35 are presented in Table B.1.
4.5.1 Radio Environment Setting

The simulations use the TM\textsubscript{z} polarised 2D-FDTD method. The FDTD space is $290 \times 490$ including the 32 cells CFS-PML layer. The A TRA antennas are placed under the skin tissue assuming that we use the implantable antennas where $2 \leq A \leq 11$. We varied distance $d$ between antennas to localise the tumour accurately where $\frac{\lambda}{2} \leq d \leq 2\lambda$ and $\lambda = \frac{C}{f_c \sqrt{\epsilon_r [f_c]}} = 16.96$ mm in lung. Furthermore, for every combination of $A$ and $d$ we moved the location of antennas on the torso. Figure 4.17 shows the relative permittivity and the conductivity of the lung and the tumour. To calculate the speed of propagation for $I_{TFM}$ we used $\epsilon_r [f_c]$ of the lung. The remaining simulation settings such as $\Delta t$ and $\Delta s$ are the same as in Section 4.3.1.

4.5.2 Numerical Results

When the TRA antennas face the bone tissue, the imaging techniques did not localise the tumour accurately. The bone tissue reflects/scatters the signals and degraded the imaging process. Therefore, the focusing of the imaging techniques occurs on the bone tissue instead of the tumours. When the TRA antennas are implanted away from the bone tissue, the tumour is localised accurately. Figure 4.36 shows the DHP with a tumour represented by a round scatterer with a 5 mm radius centred at $(115\Delta x, 280\Delta y)$. Figure 4.36 (a) and (b) show two TRA antennas sets of 3 transceivers facing and avoiding the bone tissue respectively. In this section, we present the obtained results of multiple simulations for the optimum $A$, $d$ and TRA locations. The minimum number of the TRA antennas $A$ required to achieve accurate focusing is 3 antennas because we did not have enough NS for the case with 2 TRA antennas. Figure 4.37(a) and (b) show the first three singular values and the threshold for the cases of Figure 4.36 (a) and (b) respectively. In Figure 4.37(a), only the first singular value belong to the SS which is caused by the reflected/scattered signals by the bone tissue. We can notice in Figure 4.37(b) that the first and the second singular values belong to the SS and the third singular value is in the NS. The clutter of the human tissues reduced the number of the singular values that belong to the NS. Therefore, the minimum number of antennas required for this simulation is 3 to achieve an accurate focusing. The optimum distance between adjacent antennas is $d = 10$ mm for the cases of $3 \leq A \leq 11$. The ideal TRA antennas location is
Figure 4.36: The location of antennas in DHP relative to the tumour. The tumour and the TRA antennas are represented by “◦” and “x” respectively. $d = 10 \Delta s = 10$ mm and $A = 3$. 
Figure 4.37: The spatial focusing for TR-MUSIC method with CWT filters in the human body with 5 mm radius round tumour.
CHAPTER 4. NUMERICAL SIMULATIONS

achieved by avoiding the bone tissues as much as possible which can be noticed in Figure 4.36(a) where the 3 TRA antennas are at the edge of the chest bone. The reflections from the bone tissues reduce the accuracy of the imaging methods and make it harder to localise the early stage tumour. Figure 4.38(a) and (b) show $I_{MUSIC}$ distribution for the TR-MUSIC method with facing and avoiding the bone tissue respectively. Figure 4.39(a) and (b) show $I_{TFM}$ distribution for TFM with facing and avoiding the bone tissue respectively. Figure 4.38 shows that the scatterer is localised accurately and with a better spatial focusing resolution than Figure 4.39. TR-MUSIC is more accurate than the TFM in terms of localising the early stage lung tumour.

4.6 Summary

In Chapter 4, we ran multiple canonical and practical numerical simulations to show the results of the imaging techniques with and without compensation methods. Section 4.1.2, Section 4.2.2 and Section 4.3.2 show the conventional TR canonical simulations with homogeneous, random and practical DHP media respectively. The TR imaging results with the application of CWT compensation method are more accurate and have higher resolution than the results of STFT methods. Section 4.4.2 and Section 4.4.3 show the TR-MUSIC and TFM canonical simulations with one and two targets respectively. Moreover, Section 4.5.2 shows the practical DHP media simulations. TR-MUSIC is more accurate and has higher resolution than the TFM. The microwave imaging methods in muscle medium needed smaller cross-range distances between the scatterers to achieve the localisation than those in the fat medium. In the human phantom, the ideal TRA antennas location is achieved by avoiding the bone tissues as much as possible. The reflections from the bone tissues reduce the accuracy of the imaging methods and make it harder to localise the tumour. The 3D practical simulations take a long time to execute serially. Therefore, in Chapter 5 we apply the OpenMP directives to parallelize and speed up the simulation run time.
Figure 4.38: The spatial focusing for TR-MUSIC method with CWT filters in the human body with 5 mm radius round tumour.
Figure 4.39: The spatial focusing for TFM with CWT filters in the human body with 5 mm radius round tumour.

(a) TRA antennas facing the bone tissue.

(b) TRA antennas avoiding the bone tissue.
Chapter 5

High Performance Computing

5.1 Introduction

High Performance Computing (HPC) is the use of multiple Central Processing Units (CPUs) at the same time to solve dense calculations efficiently and quickly for the simulation of practical applications such as the propagation of electromagnetic waves in dispersive media. The intense computation is divided into independent parts to be handled simultaneously on different CPUs [100]. The CPUs might be of the same machine, multiple machines that are connected through the network, or both. Unlike single CPU, HPC can solve a large amount of computations within a realistic time. The utilised memory architectures in the parallel computations are either shared, distributed or shared-distributed memory (hybrid system) as shown in Figure 5.1 (a), (b) and (c) respectively. The data are written to the memory by a CPU in Figure 5.1 (a) are visible to all the other CPUs. The disadvantage of shared memory model is the memory bandwidth that limits the data transfer between the CPUs and the shared memory. The memory bandwidth problem is solved with the distributed memory model in Figure 5.1 (b) as each CPU has its own local dedicated memory. Therefore, the memory bandwidth is not shared by the CPUs anymore. However, any modification to the data in the memory that belongs to one of the CPUs is not available in the memory of other CPUs. For the data to be shared with other CPUs, a network link is needed to be established between the CPUs as shown in Figure 5.1 (b). The memory access time between one CPU to the memory on another CPU is longer than the access time between the CPU and its own memory. Modern machines apply both distributed memory and shared memory architectures [101] which is
Figure 5.1: Shared memory, distributed memory and hybrid system.
shown in Figure 5.1 (c). Each machine (node) in the hybrid system has a number of CPUs and a shared memory. All nodes are communicating via a network link in the hybrid system. A CPU has one or more cores to execute programs at a given time. The parallel part of a program can be divided into multiple threads. Each thread can be executed on one core of the CPU. For example, eight threads can be executed on an eight cores CPU simultaneously. Furthermore, Graphics Processing Units (GPUs) can be used side by side with CPUs to accelerate the calculations for the practical simulations. GPU was brought in by the American global technology company NVIDIA [102]. A GPU has a massively parallel architecture consisting of thousands of smaller, more efficient cores designed for handling multiple tasks simultaneously.

5.2 Parallel computing with OpenMP

OpenMP [103] and Message Passing Interface (MPI) [104] are the programming interfaces for parallel computation which are used with shared and distributed memory architectures respectively. MPI is more sophisticated than OpenMP and the data structure of the serial code need to be modified for the utilisation of MPI. However, due to our time limits (as mentioned in the future plan) we are only considering the OpenMP in this thesis. OpenMP consists of compiler directives and library routines to broaden the programming language (e.g. FORTRAN and C) to achieve parallelism. A serial program is converted to a parallel program by adding the OpenMP directives which control data handling and concurrency in parallel programs. Both the serial and the parallel programs execute the same computations which give the same numerical results. The OpenMP directives can be considered as a comment by the normal compiler (without using the OpenMP option). Therefore, programs that are created using OpenMP can be executed either in parallel or in serial. In FORTRAN the OpenMP directives start with “!$” which is considered as a comment unless an option (e.g. -openmp with the gfortran compiler) is added to the compiler. Thus, the programmer can easily switch between serial and OpenMP by adding or removing the compiler options. In the FDTD method, a large amount of the calculations of the field components are independent of each other. Therefore, OpenMP can parallelize the fields computations of the FDTD method. The management of the threads and the memory access times are approximately negligible to the program computational
time. Therefore, the speed of the serial code's computation is improved when converted into parallel OpenMP code. One more advantage of OpenMP programs over MPI is being portable from one platform to the other. OpenMP programs can be tested on one machine and can be imported and executed on a different one (e.g., high performance machine). However, the management of the threads increases when the number of the threads increases. Moreover, the memory bandwidth limits the computation speed when a large number of threads are used. The CPU provides a master thread to execute the serial parts of the OpenMP programs and a group of threads for the parallel region. Figure 5.2 shows the execution of an OpenMP program with $N$ threads. The serial part is executed by the master thread (thread 0) and the parallel part is executed by $N$ threads concurrently.

5.3 Implementation and optimisation of OpenMP

The main parts that we want to tackle in OpenMP programs are the loop iterations because they take up most of the calculation time. The allocation of
the loop iterations in OpenMP is controlled by a scheduling directive. The loop iterations are distributed on a number of threads to be executed simultaneously. Therefore, the computation time of the program is reduced. However, the overhead that is caused by the parallelization increases as well as the memory access time. Both OpenMP and serial programs calculation time are highly affected by the data access to the main memory. The speed of the data transfer between the memory and the CPU cores depends on how the requested data is stored in the memory. If the required data are stored in contiguous locations in the memory, the access time is improved. This improvement can be achieved by using the scheduling directive to adjust the allocation of loop iterations for each thread. The memory access time can be reduced by using the proper setting of the scheduling directive. Code 5.1 shows a parallelized loop using FORTRAN 90 without a scheduling directive.

![Without Scheduling:](image)

```fortran
! Without Scheduling:
$omp parallel default(shared), private(i)
   do i=1, 5000
      a(i)=b(i)+c(i)
   end do
$omp end do
$omp end parallel
```

Code 5.1: One dimensional loop in an OpenMP FORTRAN 90 program.

In Code 5.1, a, b and c are the names of arrays, i is a counter variable of the loop and !$omp parallel default(shared), private(i) and !$omp do are the OpenMP directives. To reduce the efforts and mistakes in the case of a large number of shared variables we can set default(shared) instead of including all of the variables one by one. Figure 5.3 shows 5000 iterations assigned successively to 10 threads where 500 iterations are allocated for each thread. In other words, iteration 1 to 500 is for thread 0, iteration 501 to 1000 is for thread 1, etc. Each thread needs to access the main memory to request a specific data such as b(1) to thread 0, b(501) to thread 1, b(1001) to thread 2, etc. The memory access time is ten times larger than the single memory transaction because each thread’s request is transferred one at a time (uncoalesced memory access). This issue is solved by adding the OpenMP scheduling directive which allocates the loop
iterations in the OpenMP programs in processing order \[101\] in order to achieve coalesced memory access. The requested data for the calculations are efficiently transferred using the OpenMP scheduling directive because they are in contiguous memory locations. Code 5.2 shows a parallelized loop using FORTRAN 90 with a scheduling directive.

\begin{verbatim}
! With Scheduling:

 !$omp parallel default(shared), private(j)
 !$omp do schedule(static, 1)
   do j=1, 5000
     d(j)=e(j)+f(j)
   end do
 !$omp end do
 !$omp end parallel

\end{verbatim}

Code 5.2: One dimensional scheduled loop in an OpenMP FORTRAN 90 program.

In Code 5.2, d, e and f are the names of arrays, j is a counter variable of the loop and \ !$omp do schedule(static, 1) is the OpenMP scheduling directives. static and 1 are parameters of schedule to indicate the type of scheduling methods and the segmentation size of the contiguous iterations respectively. The loop iterations are allocated for each thread using the segmentation size \[101\].

Figure 5.4 shows 5000 iterations assigned to 10 threads with segmentation size equals to 1 where the value of j for thread 0 is 1, 11, 21, \ldots, 4991, for thread 1 is 2, 12, 22, \ldots, 4992, etc. Each thread requests the value of the array element required for the calculations. The requested values by all threads transferred from
the main memory to the threads with coalesced memory access. Therefore, the memory access is efficient with the scheduled parallel OpenMP program. The program of the 3D-FDTD method has been parallelized with the OpenMP by our group which improved the computational speed of the FDTD method.

5.4 speedup and scaling

To calculate the overall theoretical speedup \( S \) that is achieved by the OpenMP parallelization, we use the Amdahl’s law as [105]

\[
S = \frac{1}{(1 - \mathcal{P}) + \frac{\mathcal{P}}{S_\mathcal{P}}}
\] (5.1)

where \( \mathcal{P} \) is the size of the parallelized part of the program and \( S_\mathcal{P} \) is the speedup obtained from the parallel region. The time needed to execute the serial code is \( T_s \). The execution time for the parallel form of the code has two parts. The serial part \((1 - \mathcal{P})\) of the code will execute in \((1 - \mathcal{P})T_s\), while the parallel part \( \mathcal{P} \) is executed in \( \frac{\mathcal{P}T_s}{S_\mathcal{P}} \). The total parallel execution time is \((1 - \mathcal{P})T_s + \frac{\mathcal{P}T_s}{S_\mathcal{P}}\). Amdahl’s law can be obtained by dividing the serial execution time by the parallel execution time [105]. The highest speedup performance is achieved when \( \mathcal{P} \) has the largest possible value (maximum theoretical value is 1). The performance of the application is controlled by the serial part of the program when the number of threads is increased. Scaling is achieved when the computational time decreases as the number of cores increases. In HPC there are two types of scaling, weak and strong. It is called strong scaling when we measure the change in the computational time for a fixed computational size in all simulations. Additionally, it is called weak scaling when the computational size varies in proportion to

![Figure 5.4: Scheduled iterations allocation for each thread with segmentation size equals to 1.](image-url)
The assigned computational size to a specific thread is decreased when the number of the threads is increased because the parallel part is distributed on the threads as shown in Figure 5.5. Therefore, the computational time of each thread is decreased because it depends on the computational size as shown in Figure 5.6.

However, the serial part of the program is fixed even if the number of threads is increased. Thus, the computational time of the serial part is fixed as well. The overall speedup of a parallel program is affected by the computational size of the serial part of the program. Figure 5.7 shows the speedup of a parallel program in strong scaling with different values of $0.7 \leq P \leq 1$ based on (5.1). The speedup of a parallel program increases with the increment of the threads number, for example, in (5.1) the speed up is linearly increased when $P = 1$. The case of $P = 1$ is the ideal situation when the whole program can be parallelized with no serial part. If the number of the threads reaches infinity, $\frac{P}{N}$ in (5.1) goes to zero.
Figure 5.6: Computational time in strong scaling with different number of threads.
and the maximum speedup is achieved at $\frac{1}{1-P}$. Furthermore, if $P = 0.9$ (90% of the computation size can be parallelized) then the speedup enhancement is restricted at 10 times faster than the speed of the serial program as $\frac{1}{(1 - 0.9)} = 10$. The overall performance of the strong scaling can be reduced by the speed of the memory access and the network communication which is restricted by the system specification and it is much slower than the computational speed of the system.

### 5.5 Applying OpenMP to the practical simulation

The simulations of the parallel OpenMP and serial programs are executed on the Application Computing Server with Large memory (ACSL) cluster in RIKEN. In June 2017, the 49th list edition of TOP500 [106] was released and RIKEN achieved rank 8 in the list. Table 5.1 shows the specification of the ACSL cluster. Both the parallel OpenMP and serial programs were compiled by Intel icf FORTRAN compiler with -lfftw3f option, -parallel and -qopenmp
<table>
<thead>
<tr>
<th></th>
<th>2 nodes of PRIMERGY RX4770 M1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nodes</strong></td>
<td></td>
</tr>
<tr>
<td><strong>CPU</strong></td>
<td>Intel Xeon E7-4880v2 (2.50 GHz) 2 units (8 CPUs, 120 cores)</td>
</tr>
<tr>
<td><strong>Theoretical peak</strong></td>
<td>2.4 TFLOPS (2.5 GHz × 8 floating-point operations × 15 cores × 8 CPUs)</td>
</tr>
<tr>
<td><strong>Theoretical peak</strong></td>
<td>1.2 TFLOPS</td>
</tr>
<tr>
<td><strong>performance</strong></td>
<td></td>
</tr>
<tr>
<td><strong>memory capacity</strong></td>
<td>2 TB (1TB × 2 units)</td>
</tr>
<tr>
<td><strong>Memory bandwidth</strong></td>
<td>85.3 GB/s/CPU</td>
</tr>
<tr>
<td><strong>memory bandwidth/FLOP</strong></td>
<td>0.28 Byte/FLOP</td>
</tr>
<tr>
<td><strong>Local disk capacity</strong></td>
<td>3.6 TB ((300 GB × 2 + 1.2 TB) × 2 units)</td>
</tr>
<tr>
<td><strong>Node interface</strong></td>
<td>FDR InfiniBand</td>
</tr>
<tr>
<td><strong>Node link bandwidth</strong></td>
<td>6.8 GB/s × 2 paths × 2 (bidirectional)</td>
</tr>
<tr>
<td><strong>Operating System</strong></td>
<td>Red Hat Enterprise Linux 6.5 (kernel version 2.6) 64-bit</td>
</tr>
</tbody>
</table>

Table 5.1: Specification of the ACSL cluster. FLOPS stands for Floating point Operations Per Second.
options are also used to compile OpenMP programs. The number of threads $2 \leq N \leq 16$ for the parallel OpenMP program is set by the environment variable `OMP_NUM_THREADS` in the command line. The simulation setup is shown in Figure 4.19 and the environment settings are in Section 4.3.1. Table 5.2 shows the specification of the RIKEN-GreatWave front end. The actual speedup [101] of a numerical simulation is

$$S_N = \frac{T_1}{T_N} \quad (5.2)$$

where $T_1$ and $T_N$ are the execution time of the serial and parallel (with $N$ threads) programs respectively. The actual parallel efficiency [101] of a numerical simulation is

$$E_N = \frac{S_N}{N} = \frac{T_1}{NT_N} \quad (5.3)$$

Figure 5.8 and Figure 5.9 shows $S_N$ and $E_N$ of our OpenMP parallel program and the ideal cases of strong scaling as references ($S$ for $P = 1$ and $E_N = 1$). Up to 120 cores in total are available for the computation and each core has 16GB. We utilised $1 \leq N \leq 16$ (2 threads per each core) and 5GB/core.

Code 5.3 shows an example of one of the loops’ iterations that are used to calculate the 3D-FDTD computations.

```c
!$omp parallel default(shared), private(i,j,k)
!$omp do schedule(static, 1)
do k=k1, k2
doi=j1, j2
doi=i1, i2
hx(i,j,k)=hx(i,j,k)+dt/(mu0*dz)*(ey2(i,j,k+1)&
```
Figure 5.8: Actual speedup of our OpenMP parallel program with the ideal case of strong scaling as reference.

Figure 5.9: Actual parallel efficiency of our OpenMP parallel program with the ideal case of strong scaling as reference.
&-ey2(i,j,k))-dt/(mu0*dy)*(ez2(i,j+1,k)-ez2(i,j,k))
end do
end do
end do
!$omp end do
!$omp end parallel

Code 5.3: One of the loops’ iterations that are used to calculate the 3D-FDTD computations.

where \( hx(i,j,k) = hx(i,j,k) + dt/(mu0*dz)*(ey2(i,j,k+1)- \cdots \) is the update equation of the magnetic field in the \( x \)–direction. Similar parallelization are applied to the update equations of the remaining electric and magnetic fields (Appendix A). Thread 0 is calculating \( hx(k1,k1,k1) \), thread 1 is calculating \( hx(k1+1,k1,k1) \), \ldots and thread 15 is calculating \( hx(k1+15,k1,k1) \) concurrently. Furthermore, thread 0 is calculating \( hx(k1+16,k1,k1) \), thread 1 is executing \( hx(k1+17,k1,k1) \), \ldots and thread 15 is calculating \( hx(k1+30,k1,k1) \) and so on till the \( hx \) field is updated for the whole 3D-FDTD domain for a specific time step. At the end of the time step, the matrix \( hx \) has the magnetic field values for all the 3D-FDTD grid. This process is repeated for each time step till the end of the simulation time.

The actual speedup and parallel efficiency of the OpenMP program is about the same as that of the ideal case when the number of threads is up to five. The parallelization accelerates the computational speed of the 3D CWT-TR method. MPI and/or GPU can be used instead of or in conjunction with OpenMP to improve the speedup and the parallel efficiency of 3D CWT-TR method.

### 5.6 Summary

In Chapter 5 we parallelize the fields components of the 3D-FDTD method because a large amount of the calculations of the field components are independent of each other. The management of the threads and the memory access times are approximately negligible to the program computational time. Therefore, the speed of the serial code’s computation is improved when converted into parallel OpenMP code. The actual speedup and parallel efficiency of the OpenMP program is about the same as that of the ideal case when the number of threads is up to five.
Chapter 6

Conclusion and Future Plan

6.1 Conclusion

The wave equation invariance of the time reversal technique is broken in human tissues due to their dispersive characteristics. We proposed a technique for improving a compensation scheme that uses the STFT method. The Hanning window improved the localisation of the scatterer because the Hanning can avoid the abrupt changes of the Hamming window by getting close to zero near the edges of the window. We obtained improved results in terms of localisation and spatial distribution of $F$ on the target location.

Since the dispersive attenuation is both time and frequency dependent, we have introduced CWT based compensation method that uses inverse filters in the wavelet domain to overcome this attenuation. The inherent time and frequency variations in a wavelet decompose the recorded signals into different time and frequency components to which different filters would be applied for compensation. CWT does not need optimisation of the time-window length as is the case with STFT. The proposed inverse filters need only the complex permittivity at the centre frequency of one dominant medium to create the inverse filter as opposed to the whole wide-band dispersion characteristic in the STFT method [26]. Therefore CWT technique can be applied in real-life scenarios while STFT method [26] [69] needs the non-dispersive version of the propagation media to create the inverse filter (in real-life, such an information might not be available readily). We applied our compensation methods to the TR signals in different scenarios to examine how our methods improved the TR imaging. In the canonical numerical simulation, our compensation methods improved the spatial focusing
of the TR techniques. The TR focusing around the target location is more accurate with our proposed methods than those with the method in [26] or without applying any method (i.e., with conventional TR imaging). Furthermore, the resolution is higher with the proposed methods than with the method in [26] or without any compensation method respectively. The tumour is localised accurately after applying our proposed compensation methods. Both 15 and 102 TR antennas localised the tumour inside the human DHP.

The TR-MUSIC technique can localise multiple targets with a shorter distance between the targets than the TFM. The $\lambda$ of the propagation wave in muscle is less than the $\lambda$ in fat. Therefore, the distance between two targets in fat tissue is larger than the distance in muscle tissue in order for the imaging techniques to localise the targets. In the practical numerical simulation, we obtained an optimised location of the TRA antennas, distance between adjacent antennas and the number of antennas required to locate a tumour. The minimum number of antennas required to accurately localise the tumour is 3 because the clutter of the human tissues reduces the NS. In order to get an accurate location of the tumour, we need to avoid implanting the TRA transceivers in front of the bone tissue. The chest bone scatter/reflect the signal and degrade the localisation process. The TR-MUSIC method is more accurate than the TFM method in localising the tumour.

In the FDTD method, a large amount of the calculations of the field components are independent of each other and can be parallelized with OpenMP. The OpenMP directives can be considered as a comment by the compiler (without using the OpenMP option). Therefore, programs that are created using OpenMP can be executed either in parallel or in serial. The memory access time can be reduced by using the proper setting of the OpenMP scheduling directive.

6.2 Future work

The future works involve the modification of the CWT compensation method to work with UWB TR-MUSIC technique. The current version of the CWT filters can only be applied with conventional TR and the central frequency TR-MUSIC. Furthermore, unlike the TR-MUSIC technique, the results of the TFM imaging technique did not distinguish between two adjacent targets even after the application of the inverse filters of the CWT compensation method. Therefore,
parameters of the inverse filters of the CWT compensation method need some tuning for further improvement of the TFM. In this thesis, our compensation methods are applied to the scenarios where the targets and transceivers are fixed. Therefore, as a future research, the CWT filters can be applied in various applications (e.g. TWI, SAR and GPR) where the targets and/or the transceivers are moving. These applications can be improved by using the robust CWT compensation filters. There is a high demand for the detection of multiple moving targets behind walls but the work in this field is scarce. Due to the lossy nature of the walls, our compensation methods could be extended to work in the detection of multiple moving targets behind the wall as it is one of the most challenging scenarios in TWI microwave imaging. Moreover, the dispersive and lossy behaviours of soils need compensation methods similar to the ones mentioned in this thesis. Therefore, we plan to apply the developed compensation techniques in microwave-based GPR application. The study of restricted cases can produce new algorithms. For example, update/modify the existing TR-MUSIC/TFM to use a single antenna to create the FMC and record the signals at different locations. By reducing the number of the TRA antennas, we can reduce the total cost of the microwave imaging technique. The application of TR-MUSIC and TFM in the 3D domain needs huge memory allocation and a very long simulation time. Therefore, MPI and GPU can be used to speed up the SVD and FFT calculations. The decomposition of SVD and FFT to parallel regions is a very complex process. However, speeding up the calculation of the SVD and FFT will reduce the simulation times of large 3D simulations. The next phase of our work can be applied to practical experiments and compare the results obtained with those of the practical simulations. Other methods such as the S-transform \[\text{[107]}\] can also be investigated in this framework as future works for the researchers in the field. S-transform is considered as the generalisation of the STFT method and the extension of the CWT technique.
Bibliography


Appendix A

Finite difference time domain method

A.1 FDTD method

The FDTD method is used to solve the Maxwell’s equations in time domain \[108\]. FDTD is the simplest method in terms of implementation \[109\]. The time domain Maxwell’s equations for non-dispersive and lossless media are

\[
\nabla \times \mathbf{E} = \frac{\partial}{\partial t} \mathbf{B}, \tag{A.1}
\]

\[
\nabla \times \mathbf{H} = \frac{\partial}{\partial t} \mathbf{D}, \tag{A.2}
\]

\[\mathbf{D} = \epsilon \mathbf{E}, \tag{A.3}\]

\[\mathbf{B} = \mu \mathbf{H}, \tag{A.4}\]

where \(\nabla\) is the Del operator (\(\nabla \equiv i_x \frac{\partial}{\partial x} + i_y \frac{\partial}{\partial y} + i_z \frac{\partial}{\partial z}\)) \[109\], \(i_x, i_y\) and \(i_z\) are the unit vectors in \(x, y\) and \(z\) directions, \(\mathbf{E}\) is the electric field in Volts/meter, \(\mathbf{B}\) is the magnetic flux density in Webers/meter\(^2\), \(\mathbf{H}\) is the magnetic field in Ampères/meter, \(\mathbf{D}\) is the electric flux density in Coulombs/meter\(^2\), \(\mu\) is the permeability in Henrys/meter and \(\epsilon\) is the permittivity in Farads/meter. \(\text{(A.1)}\) to \(\text{(A.4)}\) leads to six coupled partial differential equations as

\[
\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \left[ \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right], \tag{A.5}
\]
\[
\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \left[ \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right], \quad (A.6)
\]
\[
\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left[ \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right], \quad (A.7)
\]
\[
\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon} \left[ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right], \quad (A.8)
\]
\[
\frac{\partial E_y}{\partial t} = \frac{1}{\epsilon} \left[ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right], \quad (A.9)
\]
\[
\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon} \left[ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right]. \quad (A.10)
\]

(E.5) to (E.10) are discretized using the central finite difference to obtain the following FDTD equations

\[
H_x^{n+\frac{1}{2}} \left( i, j + \frac{1}{2}, k + \frac{1}{2} \right) = H_x^{n-\frac{1}{2}} \left( i, j + \frac{1}{2}, k + \frac{1}{2} \right)
+ \frac{\Delta t}{\mu(i, j + \frac{1}{2}, k + \frac{1}{2})} \left[ E_y^n \left( i, j + \frac{1}{2}, k + 1 \right) - E_y^n \left( i, j + \frac{1}{2}, k \right) \right] \nonumber
+ \frac{\Delta t}{\mu(i, j + \frac{1}{2}, k + \frac{1}{2})} \left[ E_z^n \left( i, j, k + \frac{1}{2} \right) - E_z^n \left( i, j + 1, k + \frac{1}{2} \right) \right], \quad (A.11)
\]

\[
H_y^{n+\frac{1}{2}} \left( i + \frac{1}{2}, j, k + \frac{1}{2} \right) = H_y^{n-\frac{1}{2}} \left( i + \frac{1}{2}, j, k + \frac{1}{2} \right)
+ \frac{\Delta t}{\mu(i + \frac{1}{2}, j, k + \frac{1}{2})} \left[ E_x^n \left( i + \frac{1}{2}, j, k + \frac{1}{2} \right) - E_x^n \left( i, j, k + \frac{1}{2} \right) \right] \nonumber
+ \frac{\Delta t}{\mu(i + \frac{1}{2}, j, k + \frac{1}{2})} \left[ E_z^n \left( i + \frac{1}{2}, j, k \right) - E_z^n \left( i + 1, j, k + 1 \right) \right], \quad (A.12)
\]

\[
H_z^{n+\frac{1}{2}} \left( i + \frac{1}{2}, j + \frac{1}{2}, k \right) = H_z^{n-\frac{1}{2}} \left( i + \frac{1}{2}, j + \frac{1}{2}, k \right)
+ \frac{\Delta t}{\mu(i + \frac{1}{2}, j + \frac{1}{2}, k)} \left[ E_x^n \left( i + \frac{1}{2}, j + 1, k \right) - E_x^n \left( i + \frac{1}{2}, j, k \right) \right] \nonumber
+ \frac{\Delta t}{\mu(i + \frac{1}{2}, j + \frac{1}{2}, k)} \left[ E_y^n \left( i, j + \frac{1}{2}, k \right) - E_y^n \left( i + 1, j + \frac{1}{2}, k \right) \right], \quad (A.13)
\]
\begin{align*}
E^{n+1}_x (i + \frac{1}{2}, j, k) &= E^n_x (i + \frac{1}{2}, j, k) \\
+ &\frac{\Delta t}{\epsilon (i + \frac{1}{2}, j, k)} \Delta y \left[ H^{n+\frac{1}{2}}_z (i + \frac{1}{2}, j + \frac{1}{2}, k) - H^{n+\frac{1}{2}}_z (i + \frac{1}{2}, j - \frac{1}{2}, k) \right] \\
+ &\frac{\Delta t}{\epsilon (i + \frac{1}{2}, j, k)} \Delta z \left[ H^{n+\frac{1}{2}}_y (i + \frac{1}{2}, j, k - \frac{1}{2}) - H^{n+\frac{1}{2}}_y (i + \frac{1}{2}, j, k + \frac{1}{2}) \right], \\
\text{(A.14)}
\end{align*}

\begin{align*}
E^{n+1}_y (i, j + \frac{1}{2}, k) &= E^n_y (i, j + \frac{1}{2}, k) \\
+ &\frac{\Delta t}{\epsilon (i, j + \frac{1}{2}, k)} \Delta z \left[ H^{n+\frac{1}{2}}_x (i, j + \frac{1}{2}, k + \frac{1}{2}) - H^{n+\frac{1}{2}}_x (i, j + \frac{1}{2}, k - \frac{1}{2}) \right] \\
+ &\frac{\Delta t}{\epsilon (i, j + \frac{1}{2}, k)} \Delta x \left[ H^{n+\frac{1}{2}}_z (i - \frac{1}{2}, j + \frac{1}{2}, k) - H^{n+\frac{1}{2}}_z (i + \frac{1}{2}, j + \frac{1}{2}, k) \right], \\
\text{(A.15)}
\end{align*}

\begin{align*}
E^{n+1}_z (i, j, k + \frac{1}{2}) &= E^n_z (i, j, k + \frac{1}{2}) \\
+ &\frac{\Delta t}{\epsilon (i, j, k + \frac{1}{2})} \Delta x \left[ H^{n+\frac{1}{2}}_y (i + \frac{1}{2}, j, k + \frac{1}{2}) - H^{n+\frac{1}{2}}_y (i - \frac{1}{2}, j, k + \frac{1}{2}) \right] \\
+ &\frac{\Delta t}{\epsilon (i, j, k + \frac{1}{2})} \Delta y \left[ H^{n+\frac{1}{2}}_x (i, j - \frac{1}{2}, k + \frac{1}{2}) - H^{n+\frac{1}{2}}_x (i, j + \frac{1}{2}, k + \frac{1}{2}) \right]. \\
\text{(A.16)}
\end{align*}

In order for (A.11) to (A.16) to be compatible with the computational systems, the equations are transformed into integers coordinates as

\begin{align*}
H^{n+1}_x (i, j, k) &= H^n_x (i, j, k) + \frac{\Delta t}{\mu (i, j, k)} \Delta z \left[ E^n_y (i, j, k + 1) - E^n_y (i, j, k) \right] \\
- &\frac{\Delta t}{\mu (i, j, k)} \Delta y \left[ E^n_z (i, j, k + 1) - E^n_z (i, j, k) \right], \\
\text{(A.17)}
\end{align*}

\begin{align*}
H^{n+1}_y (i, j, k) &= H^n_y (i, j, k) + \frac{\Delta t}{\mu (i, j, k)} \Delta x \left[ E^n_z (i + 1, j, k) - E^n_z (i, j, k) \right] \\
- &\frac{\Delta t}{\mu (i, j, k)} \Delta z \left[ E^n_x (i, j, k + 1) - E^n_x (i, j, k) \right], \\
\text{(A.18)}
\end{align*}
\begin{equation}
H_{z}^{n+1}(i, j, k) = H_{z}^{n}(i, j, k) + \Delta t \frac{1}{\mu(i,j,k)\Delta y} \left[ E_{x}^{n}(i, j + 1, k) - E_{x}^{n}(i, j, k) \right] - \Delta t \frac{1}{\mu(i,j,k)\Delta x} \left[ E_{y}^{n}(i + 1, j, k) - E_{y}^{n}(i, j, k) \right],
\end{equation}

\begin{equation}
E_{x}^{n+1}(i, j, k) = E_{x}^{n}(i, j, k) + \Delta t \frac{1}{\epsilon(i,j,k)\Delta y} \left[ H_{z}^{n}(i, j, k) - H_{z}^{n}(i, j - 1, k) \right] - \Delta t \frac{1}{\epsilon(i,j,k)\Delta z} \left[ H_{y}^{n}(i, j, k) - H_{y}^{n}(i, j, k - 1) \right],
\end{equation}

\begin{equation}
E_{y}^{n+1}(i, j, k) = E_{y}^{n}(i, j, k) + \Delta t \frac{1}{\epsilon(i,j,k)\Delta z} \left[ H_{x}^{n}(i, j, k) - H_{x}^{n}(i, j, k - 1) \right] - \Delta t \frac{1}{\epsilon(i,j,k)\Delta x} \left[ H_{z}^{n}(i, j, k) - H_{z}^{n}(i - 1, j, k) \right],
\end{equation}

\begin{equation}
E_{z}^{n+1}(i, j, k) = E_{z}^{n}(i, j, k) + \Delta t \frac{1}{\epsilon(i,j,k)\Delta x} \left[ H_{y}^{n}(i, j, k) - H_{y}^{n}(i - 1, j, k) \right] - \Delta t \frac{1}{\epsilon(i,j,k)\Delta y} \left[ H_{x}^{n}(i, j, k) - H_{x}^{n}(i, j - 1, k) \right].
\end{equation}

For a stable computation, $\Delta t$ must satisfy Courant-Friedrichs-Lewy (CFL) condition [110]

\begin{equation}
\Delta t \leq \frac{1}{\mathcal{V}} \left( \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}} \right)^{-1}
\end{equation}

where $\mathcal{V}$ is the speed of the wave propagation in a specific medium.

### A.2 FDTD method in dispersive and lossy media

The media parameters for the electromagnetic wave propagation in the human’s tissues depend on the frequency. Three main models represent the frequency dispersive materials: Debye model [84], Lorentz-Drude model [111] and Cole-Cole model [112]. Debye model is widely used for the FDTD method because of its simplicity in implementation [113]. In this thesis, the human tissues are...
assumed as one-pole Debye media where the relative permittivity $\epsilon_r$ is

$$\epsilon_r = \epsilon_\infty + \frac{\epsilon_S - \epsilon_\infty}{1 + j\omega \tau_D} + \frac{\sigma}{j\omega \epsilon_0}$$  \hspace{1cm} (A.24)$$

where $\epsilon_\infty$ is the optical relative permittivity, $\epsilon_S$ is the static relative permittivity, $\tau_D$ is the relaxation time, $\sigma$ is the static conductivity and $\omega$ is the angular frequency. Therefore, (A.3) can be written as

$$D = \epsilon_0 \left[ \epsilon_\infty + \frac{\epsilon_S - \epsilon_\infty}{1 + j\omega \tau_D} + \frac{\sigma}{j\omega \epsilon_0} \right] E.$$  \hspace{1cm} (A.25)$$

(A.25) can be written as

$$(j\omega)^2 \tau_D D + (j\omega) D = (j\omega)^2 \epsilon_0 \epsilon_\infty \tau_D E + j\omega (\epsilon_0 \epsilon_S + \sigma \tau_D) E + \sigma E.$$  \hspace{1cm} (A.26)$$

The time domain of (A.26) is

$$\tau_D \frac{\partial^2 D}{\partial t^2} + \frac{\partial D}{\partial t} = \epsilon_0 \epsilon_\infty \tau_D \frac{\partial^2 E}{\partial t^2} + (\epsilon_0 \epsilon_S + \sigma \tau_D) \frac{\partial E}{\partial t} + \sigma E.$$  \hspace{1cm} (A.27)$$

Discretizing (A.27) yields to

$$\tau_D \frac{D_{u+1}^{n+1} - 2D_u^n + D_{u-1}^{n-1}}{(\Delta t)^2} + \frac{D_{u+1}^{n+1} - D_u^n}{\Delta t} = \epsilon_0 \epsilon_\infty \tau_D \frac{E_{u+1}^{n+1} - 2E_u^n + E_{u-1}^{n-1}}{(\Delta t)^2} + (\epsilon_0 \epsilon_S + \sigma \tau_D) \frac{E_{u+1}^n - E_u^n}{\Delta t} + \sigma \frac{E_{u+1}^n + E_u^n}{2}, \hspace{0.5cm} (u = x, y, z).$$  \hspace{1cm} (A.28)$$

We obtain $E_{u+1}^{n+1}$ from (A.28) as

$$E_{u+1}^{n+1} = \nu_1 (\nu_2 E_u^n - \nu_3 E_u^{n-1} + \nu_4 D_u^{n+1} - \nu_5 D_u^n + \nu_6 D_u^{n-1})$$  \hspace{1cm} (A.29a)$$
where

\[ \nu_1 = (2\epsilon_0\epsilon_\infty \tau_D + 2(\epsilon_0\epsilon_S + \sigma\tau_D)\Delta t + \sigma(\Delta t)^2)^{-1}, \quad (A.29b) \]
\[ \nu_2 = 4\epsilon_0\epsilon_\infty \tau_D + 2(\epsilon_0\epsilon_S + \sigma\tau_D)\Delta t - \sigma(\Delta t)^2, \quad (A.29c) \]
\[ \nu_3 = 2\epsilon_0\epsilon_\infty \tau_D, \quad (A.29d) \]
\[ \nu_4 = 2(\Delta t + \tau_D), \quad (A.29e) \]
\[ \nu_5 = 2(\Delta t + 2\tau_D), \quad (A.29f) \]
\[ \nu_6 = 2\tau_D. \quad (A.29g) \]

In Debye model, \( (A.29) \) is the auxiliary equation for the FDTD technique. For non-magnetic medium, \( (A.1) \) to \( (A.4) \) leads to six partial differential equations as

\[ \frac{\partial H_x}{\partial t} = \frac{1}{\mu_0} \left( \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right), \quad (A.30) \]
\[ \frac{\partial H_y}{\partial t} = \frac{1}{\mu_0} \left( \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right), \quad (A.31) \]
\[ \frac{\partial H_z}{\partial t} = \frac{1}{\mu_0} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right), \quad (A.32) \]
\[ \frac{\partial D_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \quad (A.33) \]
\[ \frac{\partial D_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \quad (A.34) \]
\[ \frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \quad (A.35) \]

\( (A.30) \) to \( (A.35) \) are discretized using the central finite difference and transformed the coordinates into integers to obtain the following FDTD equations

\[ H_x^{n+1}(i, j, k) = H_x^n(i, j, k) + \frac{\Delta t}{\mu_0\Delta z} \left[ E_y^n(i, j, k + 1) - E_y^n(i, j, k) \right] - \frac{\Delta t}{\mu_0\Delta y} \left[ E_z^n(i, j + 1, k) - E_z^n(i, j, k) \right], \quad (A.36) \]

\[ H_y^{n+1}(i, j, k) = H_y^n(i, j, k) + \frac{\Delta t}{\mu_0\Delta x} \left[ E_z^n(i + 1, j, k) - E_z^n(i, j, k) \right] - \frac{\Delta t}{\mu_0\Delta z} \left[ E_x^n(i, j, k + 1) - E_x^n(i, j, k) \right], \quad (A.37) \]
\[ H_{n+1}^{z}(i,j,k) = H_n^{z}(i,j,k) + \frac{\Delta t}{\mu_0 \Delta y} [E_n^x(i,j+1,k) - E_n^x(i,j,k)] - \frac{\Delta t}{\mu_0 \Delta x} [E_n^y(i+1,j,k) - E_n^y(i,j,k)] , \quad (A.38) \]

\[ D_{n+1}^{x}(i,j,k) = D_n^{x}(i,j,k) + \frac{\Delta t}{\Delta y} [H_n^{n}(i,j,k) - H_n^{n}(i,j-1,k)] - \frac{\Delta t}{\Delta z} [H_n^{n}(i,j,k) - H_n^{n}(i,j,k-1)] , \quad (A.39) \]

\[ D_{n+1}^{y}(i,j,k) = D_n^{y}(i,j,k) + \frac{\Delta t}{\Delta z} [H_n^{n}(i,j,k) - H_n^{n}(i,j,k-1)] - \frac{\Delta t}{\Delta x} [H_n^{n}(i,j,k) - H_n^{n}(i-1,j,k)] , \quad (A.40) \]

\[ D_{n+1}^{z}(i,j,k) = D_n^{z}(i,j,k) + \frac{\Delta t}{\Delta x} [H_n^{n}(i,j,k) - H_n^{n}(i-1,j,k)] - \frac{\Delta t}{\Delta y} [H_n^{n}(i,j,k) - H_n^{n}(i,j-1,k)] . \quad (A.41) \]

The FDTD method has gained popularity due to several advantages it offers such as simplicity, accuracy and robustness. The FDTD method runs efficiently on parallel computing which is useful for practical applications [113]. The time needed for the FDTD calculations can be decreased by using parallel computing. An absorbing boundary condition is used to reduce the memory requirement for the FDTD computations.

### A.3 Absorbing boundary condition

The FDTD space is usually open. However, no computer can store infinite data. Therefore the FDTD method needs an Absorbing Boundary Condition (ABC). In order to give the FDTD space an extension to infinity, the ABC should absorb the reflection of the outgoing waves [113]. Mur’s ABC [114] is one of the oldest and popular ABCs. Mur’s ABC implementation is a straightforward but the accuracy of the Mur’s ABC need to be improved [113]. [115] introduced the perfectly matched layer (PML) ABC. The PML ABC has a thickness of few cells of an absorbing material medium terminating the FDTD space. The
APPENDIX A. FINITE DIFFERENCE TIME DOMAIN METHOD

PML is highly effective ABC because all the waves are matched at the boundary [113]. The success of the PML led to propose different versions such as Complex Frequency-Shifted Perfectly Matched Layer (CFS-PML) [75]. CFS-PML ABC is used through the simulations of this report. The 3D FDTD and PML FORTRAN code were provided by our research group.
Appendix B

Medium parameters for Debye relaxation model

Table B.1 includes the full details of the DHP (Figure B.1) 52 tissues represented in the one-pole Debye relaxation model.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$\sigma$ [S/m]</th>
<th>$\epsilon_S$</th>
<th>$\epsilon_\infty$</th>
<th>$\tau_D$ [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cerebral Cortex</td>
<td>0.595</td>
<td>56.444</td>
<td>33.057</td>
<td>35.197</td>
</tr>
<tr>
<td>White Matter</td>
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### Table B.1: Medium parameters for debye relaxation model

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<th>$\epsilon_S$</th>
<th>$\epsilon_\infty$</th>
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<td>Left Kidney</td>
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<td>Pharynx</td>
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<td>0.809</td>
<td>60.543</td>
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<td>17.706</td>
</tr>
</tbody>
</table>
Figure B.1: 3D DHP created by combining 1654 cross-sectional MRI scans of the healthy Japanese male subject.
Appendix C

Tumour and the TRA antennas locations

Code C.1 and Code C.2 show the FORTRAN 90 program and the csh shell script to insert the tumour and set the TRA antennas in the .pgm file respectively.

```fortran
PROGRAM M_TR
IMPLICIT NONE
character(len=2) :: magic ! magic number
character(len=64) :: comment
integer :: width, height, maxv ! maximum value
integer :: i,j,err,x,y,z
real s
integer, DIMENSION(:,,:), ALLOCATABLE :: mat_set
!#########################################################
open(unit=92,file=trim("DHP-2.pgm"),status="old",iostat=err)
read(92, '(A2)') magic
read(92, '(A64)') comment
read(92, *) width, height!the DHP dimentions in the .pgm file
read(92, *) maxv !the maximum value in the .pgm file
allocate(mat_set(1:width, 1:height), stat=err)
mat_set(1:width, 1:height) = 0
write(100, '(A2)') magic
write(100, '(A64)') comment
write(100, *) width, height!the DHP dimentions in the .pgm file
write(100, *) maxv+1
!#########################################################
```

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APPENDIX C. TUMOUR AND THE TRA ANTENNAS LOCATIONS

x=115 ! x value for the center of the tumour location
y=280 ! y value for the center of the tumour location
z=5 ! the radius of the object

#----------------------------------------------------------
do j = 1, height
do i = 1, width
read(92, *) mat_set(i,j)
if(mat_set(i,j)==48 .and. j>150 .and. j<380) write(99,*) i,j
! 48 represents the fat tissue and the implanted antennas is
! located between j=151 and j=379
s=sqrt(REAL((i-x)**2+(j-y)**2))
if(s.le.z) then ! to check for the tumour location
write(100,*) 59 ! 59 represents the tumour tissue
else
write(100,*) mat_set(i,j) ! normal tissues
endif
end do
end do

close(unit=92)
deallocate(mat_set, stat=err)
call system ("sort -g -k1 fort.99 > fort.999")
! Sorting is optional and it can be ascending or descending
! and based on the width (x-axis) or the height (y-axis) of
! the DHP based on the scenario

call system ("./xy-distance.sh")
! calculate the distance between each two antennas

END PROGRAM M_TR

---

#!/bin/csh

set o = 10 # the initial distance
while($o < 31) # the maximum distance
set x = 'cat fort.999 | awk 'NR == 1){print $1}''
# the 'x' value for the 1st location of the TRA antennas
set y = 'cat fort.999 | awk 'NR == 1){print $2}''
# the 'y' value for the 1st location of the TRA antennas
set n = 'cat fort.999 | awk '{print NR}' | tail -1'
#the total number of candidates locations of the TRA antennas
set j = 1 #the antenna's ID no.

echo Ant_$j $x $y > ant-loc-$o

set i = 2
while($i < $n)
  set xi = 'cat fort.999 | awk -v i=$i '{NR== i}{print $1}'
  set yi = 'cat fort.999 | awk -v i=$i '{NR== i}{print $2}''
  @ xt = ($x - $xi) * ($x - $xi)
  @ yt = ($y - $yi) * ($y - $yi)
  @ zt = $xt + $yt
  set d = 'echo sqrt "($zt)" | bc'
  #’d’ represents the distance between each two TRA antennas
  if($d > ($o - 1) * $j && $d < ($o + 1) * $j) then
    #’d’ must have the exact value of the initial distance ’o’
    @ j ++
    echo Ant_$j $xi $yi >> ant-loc-$o
  endif
  @ i ++
end

@ o ++
end
exit 0

Code C.2: The csh shell script to set the antenna location and the distance between each two antennas.