EXPLOITATION OF THE
GYROELECTRIC EFFECT IN
DESIGNING MILLIMETRE-WAVE NONRECIPROCAL DEVICES

A thesis submitted to the University of Manchester
for the degree of Doctor of Philosophy
in the Faculty of Engineering and Physical Sciences

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By
Ghassan Nihad Jawad
School of Electrical and Electronic Engineering
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Abstract

EXPLOITATION OF THE GYROELECTRIC EFFECT IN DESIGNING
MILLIMETRE-WAVE NONRECIPROCAL DEVICES
Ghassan Nihad Jawad
A thesis submitted to the University of Manchester
for the degree of Doctor of Philosophy, 2016

Millimetre-wave nonreciprocal devices are vital elements in many modern radar and communication systems. Gyromagnetic behaviour in magnetised ferrite materials has been utilised for decades in the design of nonreciprocal devices. However, the effects of ferrite’s limited saturation magnetisation and high loss as the frequency of operation exceeds 40 GHz render such devices inadequate for millimetre-wave applications. On the other hand, solid plasma (such as semiconductors) are known to exhibit gyrotropic behaviour when they are biased with a steady magnetic field. This behaviour (which is referred to as gyroelectric) can extend up to the THz frequency ranges. Hence, magnetised semiconductors can be regarded as suitable candidates for realising millimetre-wave, sub-millimetre-wave and even THz nonreciprocal devices.

This thesis focuses on analysing different structures containing gyroelectric materials, and proposing millimetre-wave nonreciprocal devices based on the theoretical findings. Measurements and full wave electromagnetic simulation are used to validate and optimise the proposed designs where possible.

Before starting the electromagnetic analysis, the physical properties of a semiconductor plasma are studied, then a permittivity tensor is introduced to include the microscopic features of the magnetised semiconductors into a macroscopic model. Different semiconductor candidates for gyroelectric designs are also discussed and analysed.

Firstly, Semiconductor Junction Circulators (SJC’s) are analysed using a Green’s function approach. The same approach is then used to proposed new designs for broadband millimetre-wave SJC’s that require low magnetic bias using Indium
Antimonide (InSb) cooled down to 77 K. The possibility of realising planar nonreciprocal devices using a Molecular Beam Epitaxy (MBE) grown Two Dimensional Electron Gas (2-DEG) is also studied. Theoretical and simulation results prove the possibility of using this material to realise millimetre-wave resonators and circulators.

Then a novel type of circulator is realised by placing an InSb disk at 77 K in the middle of a three port waveguide junction. The structure is analysed by treating the junction as a resonator with a suspended axially magnetised gyroelectric rod placed in the middle. Electromagnetic analysis, simulations and measurements reveal the existence of counter rotating modes that degenerate or split at certain frequencies under specific magnetic bias conditions. Measuring this circulator reveals an isolation of 18 dB at 38.5 GHz when the InSb disk is biased with a D.C. magnetic flux of 0.55 T. This is the first time such a circulator has been demonstrated theoretically and experimentally.

In addition to the three port circulator, a model is developed for a rectangular waveguide loaded with layered dielectric and gyroelectric media. Mathematical analysis reveals the dispersion relations and field distributions for such a structure. High nonreciprocity in both phase and attenuation constants is observed from analysing a rectangular waveguide loaded with a transversely biased InSb slab at 77 K. The expected nonreciprocity is then verified, for the first time, by simulation and measurement of similar structures under the same conditions. More than 35 dB of isolation at $f = 35.6 \text{GHz}$ was obtained when loading a WR-28 rectangular waveguide with an InSb slab at 77 K, transversely biased with a magnetic flux of 0.8 T. Different effects on the isolation behaviour are also discussed theoretically and experimentally, including the effects of the slab’s thickness and length, the magnetic bias and the existence of a dielectric layer above the gyroelectric slab.

Theoretical and experimental outcomes of this thesis prove the possibility of using gyroelectric materials to develop a new class of component that meets the demands for millimetre-wave nonreciprocal devices. This will provide a significant improvement to the modern high frequency millimetre-wave systems.
Declaration

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At last, but by no means least, I like to express my deepest appreciation to my family, without their unconditional love, support and encouragement, I wouldn’t have been able to achieve anything in my life, let alone undertake this research.
Preface

Ghassan Nihad Jawad was born in Baghdad/Iraq in 1984. He received his B.Sc. and M.Sc. Degrees in Electrical Engineering from University of Baghdad in 2005 and 2009, respectively. From 2006 to 2009 he worked as a network switching engineer in the local mobile telecommunications company (Zain-Iraq). From 2009 to 2013 he worked as an assistant lecturer in the department of Electronics and Communication - University of Baghdad. Since January 2013, he has been pursuing his PhD degree in Electrical and Electronic Engineering from The University of Manchester with the research focus on gyroelectric behaviour in magnetised semiconductors and millimetre-wave nonreciprocal devices under the supervision of Prof. Robin Sloan.
List of Publications


- G. N. Jawad and R. Sloan, Millimetre wave semiconductor based isolators and circulators, in IET Colloquium on Millimetre-Wave and Terahertz Engineering Technology 2015, March 2015, pp. 18


# List of Variables

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<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Attenuation constant.</td>
</tr>
<tr>
<td>$a$</td>
<td>Radius of the semiconductor rod.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Phase constant.</td>
</tr>
<tr>
<td>$\beta_g$</td>
<td>Phase constant of the gyroelectric mode in a semiconductor rod.</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Flux density of the magnetic bias.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Complex propagation constant.</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Reflection coefficient.</td>
</tr>
<tr>
<td>$\Delta(\phi)$</td>
<td>Phase shift in a rectangular waveguide.</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between the electromagnet’s two poles.</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Permittivity tensor’s diagonal element.</td>
</tr>
<tr>
<td>$[\varepsilon]$</td>
<td>Permittivity tensor.</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity of free space.</td>
</tr>
<tr>
<td>$\varepsilon_d$</td>
<td>Dielectric constant of the external transmission line.</td>
</tr>
<tr>
<td>$\varepsilon_{\text{eff}}$</td>
<td>Effective permittivity.</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative permittivity.</td>
</tr>
<tr>
<td>$e$</td>
<td>Electron charge.</td>
</tr>
<tr>
<td>$f_A$</td>
<td>Frequency of $\varepsilon_{\text{eff}}$ first zero crossing.</td>
</tr>
<tr>
<td>$f_B$</td>
<td>Frequency of $\varepsilon_{\text{eff}}$ second zero crossing.</td>
</tr>
<tr>
<td>$f_C (\omega_c)$</td>
<td>Cyclotron (angular) frequency.</td>
</tr>
<tr>
<td>$f_r (\omega_r)$</td>
<td>Extraordinary wave resonance (angular) frequency.</td>
</tr>
<tr>
<td>$f_p (\omega_p)$</td>
<td>Plasma (angular) frequency.</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Permittivity tensor element in the direction of the magnetic bias.</td>
</tr>
<tr>
<td>$Z_d$</td>
<td>Impedance of the external transmission line.</td>
</tr>
<tr>
<td>$Z_{\text{eff}}$ ($\tilde{Z}_{\text{eff}}$)</td>
<td>Effective impedance.</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of the semiconductor rod.</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Wavenumber in the free space.</td>
</tr>
</tbody>
</table>
\( k_{\text{eff}} \) (\( \tilde{k}_{\text{eff}} \))

Effective wavenumber.

\( \theta_L \)

Phase difference between two calibration standards.

\( \kappa \)

Permittivity tensor’s off-diagonal element.

\( \lambda_0 \)

Wavelength in the free space.

\( \lambda_D \)

Debye Length.

\( \lambda_g \)

Wavelength of the gyroelectric mode in a semiconductor rod.

\( l_L \)

Length of the LINE calibration standard.

\( l_s \)

Length of the semiconductor layer inside a rectangular waveguide.

\( l_T \)

Length of the THRU calibration standard.

\( \mu_0 \)

Permeability of free space.

\( \mu_e \)

Electron mobility.

\( \mu_h \)

Hall mobility.

\( \mu_r \)

Relative permeability.

\( m \)

Electron mass.

\( m_e^* \)

Electron effective mass.

\( m_h^* \)

Hole effective mass.

\( n_i \)

Intrinsic carrier concentration.

\( N_D \)

Number of charged particles inside Debye sphere.

\( N_e \)

Electron concentration.

\( N_h \)

Hall concentration.

\( \nu_c \)

Collision frequency.

\( p_j \)

The \( j^{th} \) Eigenvalue.

\( [Q] \)

A matrix of Eigenvectors.

\( R \)

Radius of the resonator (or the circulator).

\( \sigma \)

Conductivity.

\( \tau \)

Relaxation time.

\( t_{i,j} \)

An element of the matrix \([T]\) in the \( i^{th} \) row and \( j^{th} \) column.

\( [T] \)

The overall transfer matrix for a waveguide filled with layered media.

\( T_d \)

Thickness of a dielectric layer inside a rectangular waveguide.

\( T_s \)

Thickness of a semiconductor layer inside a rectangular waveguide.

\( Y_d \)

Admittance of the external transmission line.

\( \hat{\phi} \)

A vector of field components inside a loaded waveguide.

\( v_d \)

Electron’s drift velocity.

\( w_i \)

Thickness of the \( i^{th} \) layer in the loaded rectangular waveguide.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi )</td>
<td>Transverse wavenumber inside a gyroelectric rod.</td>
</tr>
<tr>
<td>( X ) (( \tilde{X} ))</td>
<td>Normalised wavenumber.</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Coupling half angle of a resonator (or a circulator).</td>
</tr>
<tr>
<td>( \Psi )</td>
<td>A scalar potential.</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>2-DEG</td>
<td>Two Dimensional Electron Gas.</td>
</tr>
<tr>
<td>CST MWS</td>
<td>Computer Simulation Technology (CST®) Microwave Studio.</td>
</tr>
<tr>
<td>DUT</td>
<td>Device Under Test.</td>
</tr>
<tr>
<td>GaAs</td>
<td>Gallium Arsenide.</td>
</tr>
<tr>
<td>GaP</td>
<td>Gallium Phosphide.</td>
</tr>
<tr>
<td>GaSb</td>
<td>Gallium Antimonide.</td>
</tr>
<tr>
<td>Ge</td>
<td>Germanium.</td>
</tr>
<tr>
<td>InAlAs</td>
<td>Indium Aluminium Arsenide.</td>
</tr>
<tr>
<td>InAs</td>
<td>Indium Arsenide.</td>
</tr>
<tr>
<td>InGaAs</td>
<td>Indium Gallium Arsenide.</td>
</tr>
<tr>
<td>InP</td>
<td>Indium Phosphide.</td>
</tr>
<tr>
<td>InSb</td>
<td>Indium Antimonide.</td>
</tr>
<tr>
<td>LRL</td>
<td>Line-Reflect-Line.</td>
</tr>
<tr>
<td>MBE</td>
<td>Molecular Beam Epitaxy.</td>
</tr>
<tr>
<td>MOS</td>
<td>Metal Oxide Semiconductor.</td>
</tr>
<tr>
<td>MMIC</td>
<td>Microwave Monolithic Integrated Circuits.</td>
</tr>
<tr>
<td>Si</td>
<td>Silicon.</td>
</tr>
<tr>
<td>SJC</td>
<td>Semiconductor Junction Circulator.</td>
</tr>
<tr>
<td>TRL</td>
<td>Thru-Reflect-Line.</td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyser.</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Overview of Nonreciprocal Microwave Devices

Nonreciprocity in microwave devices can be described as the dependency of the device’s behaviour on the direction of the signal flow. Unlike reciprocal devices, nonreciprocal devices can be used in various applications where direction properties are needed. Circulators and isolators are the most widely known devices that rely on nonreciprocity to operate. However, phase shifters and filters can also exhibit nonreciprocal properties.

An isolator is a two port device with unidirectional transmission features. That is, it allows the signal to flow in one direction only [1]. Figure (1.1) shows a diagram of a basic isolator. Isolators are usually connected between a high power source and a load to prevent reflections from damaging the source. It can also be used for matching purposes although, unlike matching networks, the reflected power is totally absorbed by the isolator itself rather than reflected back to the load.

Circulators are multi-port devices with the ability to direct the signal flow in a certain direction. Three port circulators are the most widely known, but including more than three ports is also possible in a circulator design. Figure (1.2) shows the basic diagram of a three port circulator.

One of the major applications of a circulator is as a duplexer. In such arrangement, two of the circulator ports are connected to a transmitter and a receiver,
Figure 1.1: A diagram of a two port isolator. The arrow indicates the direction of signal flow.

Figure 1.2: A diagram of a three port circulator. The arrow indicates a clockwise direction of operation.
Figure 1.3: Operation of a clockwise three port circulator as a duplexer. Arrows indicate the direction of signal flow.

while the third port is connected to an antenna that is used for both transmission and reception, as shown in Figure (1.3). The clockwise circulator directs the transmitted signal to the antenna and no signal will be leaked in the counter clockwise direction to the receiver. When the antenna receives a signal, it will be directed to the receiver without reaching the transmitter. Hence, both transmitter and receiver can use the same antenna without causing any interference between signals.

A circulator can also be used as an isolator when two of its ports are connected to a high power sources and a load, and the third port is connected to a matched load, as shown in Figure (1.4). When a signal is reflected from the load, it will be directed to the matched load, which (ideally) absorbs all the incident energy. Hence, the source will be protected from the reflected power.

Some types of phase shifter can also exhibit nonreciprocity, where the signal undergoes different phase shift depending on the direction of its propagation through the device. Nonreciprocal phase shifters are used in phase array propagation and magneto-optical systems [2]. Filters can also be nonreciprocal, where isolation is added to the normal filtering features (e.g., band pass) so the signal will be filtered only when passed through the filter in a certain direction.
Figure 1.4: Operation of a clockwise three port circulator as an isolator. Solid arrows indicate the normal direction of signal flow, while the dashed arrows represent the direction of flow of reflected signals.

### 1.2 Ferrite Nonreciprocal Microwave Devices

Most microwave nonreciprocal devices are made of ferrite materials. These materials acquire the properties of a good insulator with high permeability, which differentiate them from other magnetic materials through which electromagnetic waves cannot propagate [2–4]. With the absence of a magnetic bias, ferrite materials are isotropic. However, when magnetically biased, they become axially symmetric with respect to that bias, and their permeability can be described as a tensor [5]. Such medium is called gyromagnetic.

The use of gyromagnetic materials to manipulate radio frequency electromagnetic signals dates back to the early 1950’s [6, 7]. During that period, it was found that a linearly polarised wave passing through a ferrite medium magnetically biased in the same direction of propagation rotates with a certain angle, this phenomenon is known as Faraday rotation. Hogan [8, 9] has used Faraday rotation to design two port nonreciprocal device with 180° phase difference between its ports; such device is called a Gyrator [10]. Gyrators were used as building blocks to design different types of devices, such as circulators, switches, variable attenuators and modulators [11].

Faraday rotation was also used to design nonreciprocal phase shifters, like the one shown in Figure (1.5). For this device, a dielectric quarter-wave plate is
used to convert the linearly polarised field incident at the input of the structure into a circularly polarised one. After that, the field interacts with the longitudinally magnetised cylindrical piece of ferrite, which gives rise to a certain phase shift. Another quarter wave plate is placed at the end of the structure to convert the field back from circular to linear polarisation. The same effect takes place when the transmission is reversed, though the phase shift will be different [1, 12]. Loading a rectangular waveguide with ferrite slabs leads to different behaviours: including electromagnetic field displacement, nonreciprocal phase shift and nonreciprocal attenuation [13–15]. A magnetised vertical ferrite slab causes the electromagnetic field in a rectangular waveguide to shift toward one side when the transmission is in a certain direction, and toward the opposite side when the transmission is reversed. An isolator can be designed by attaching a thin resistive absorbing sheet to the slab, as illustrated in Figure (1.6) [1, 16]. In this isolator, the $y$–component of the electric field$^1$ is not affected for forward propagation, but it is displaced towards the magnetised ferrite slab and its attached absorbing sheet when the propagation is reversed. Isolators can also be realised by exploiting the nonreciprocal attenuation in ferrite loaded waveguides. Figure (1.7) shows two examples of ferrite resonance isolators, where the gyromagnetic resonant behaviour of ferrite causes the electromagnetic fields to be highly attenuated for only one direction of propagation [13, 17]. Many types nonreciprocal phase shifters can be designed by utilising the nonreciprocity in

$^1$Assuming the rectangular waveguide is operating in the dominant TE$_{10}$ mode.
CHAPTER 1. INTRODUCTION

Figure 1.6: The geometry of a ferrite field displacement isolator [1]

Figure 1.7: Rectangular waveguides loaded with ferrite slabs [1]
CHAPTER 1. INTRODUCTION

Figure 1.8: A latching nonreciprocal ferrite phase shifter [1]

The phase shift exhibited by a rectangular waveguide loaded with magnetised ferrite slabs. Figure (1.8) shows a latching nonreciprocal phase shifter with a toroidal-shaped ferrite core placed in the middle of the waveguide. Magnetic bias is provided by a current passing through a wire extended in the centre of the core. The latching property of this phase shifter comes from the ability to use the hysteresis feature of magnetised ferrite materials to latch the toroid ferrite between magnetic states without the need of continuous current to be fed in the bias line [1].

Ferrite circulators were realised at first using Faraday rotation devices [18–20]. Figure (1.9) shows a four port circulator constructed using four rectangular waveguides and a single circular waveguide section [18]. In this circulator, axes of ports 1 and 3 are rotated by 45° from those of ports 2 and 4, respectively. The ferrite rod in the middle section provides a nonreciprocal rotation of 45° such that the signal flow will be as shown in the inset of Figure (1.9). Circulators can also be realised by connecting two hybrid junctions with a Gyrator [9], or by utilising field displacement phenomena to obtain circulation behaviour [18].

Inserting a piece of ferrite in the middle of a three port junction results into another type of circulators [21–23]. Waveguide junction circulators proved to have better performance and power handling and greater compactness. Early junction circulators were designed using symmetrical rectangular waveguide junctions [21,22]. Striplines were later used to realise compact circulators with higher isolation [24].

Figure (1.10) shows the configuration of a stripline ferrite junction circulator [25], it consists of three striplines connected to a circular metallic plate in the centre
Figure 1.9: A four port circulator based on Faraday rotation [18]

of the junction. Two axially-magnetised circular ferrite disks, each with radius \( R \), are placed on top and bottom of the junction. Metallic ground plates are placed in touch with the Ferrite disks [1,26–28]. This type of circulator has been the most widely used in practice [2]. Many researchers have participated in the effort of analysing this circulator and predicting its performance under different conditions [27–29]. However, Bosma’s seminal Green’s function analysis in [26] represents the most detailed approach to anticipate the performance of a stripline circulator.

The bandwidth of stripline circulators was improved empirically [30] and theoretically [31] to give wider bandwidth with higher isolation and minimum insertion loss. More improvements were applied later on the basic designs to increase the bandwidth to the ranges with negative effective permeability after modifying Bosma’s Green function formulation [32].

The subsequent improvements of ferrite nonreciprocal devices were made to mitigate the emerging demands in the microwave and communication systems industry. The growing interest in the millimetre-wave frequency ranges encouraged many researchers to design nonreciprocal devices working at higher frequencies [33,34]. In addition, the necessity of integrating nonreciprocal components with Microwave Monolithic Integrated Circuits (MMIC’s) urged other researchers to propose different techniques to make that possible [35–39]. Nevertheless, realising compact ferrite nonreciprocal devices working in the millimetre-wave frequency range still represents a significant challenge.
The two major difficulties in designing ferrite devices in the millimetre-wave frequency range are the high loss and limited saturation magnetisation above 40 GHz [40–42]. Figure (1.11) shows the transmission spectrum of a sintered hexaferrite\(^2\) sample [43]. High loss can be observed in the region 45 – 60 GHz due to the magnetic resonance. In addition, the dielectric loss causes the transmission to decrease at higher frequencies. These constraints affect the performance and limit the bandwidth of any millimetre-wave ferrite nonreciprocal device.

Moreover, compatibility of ferrite devices with MMIC’s is still a designing challenge due to the higher temperature requirements of ferrite film deposition, which is not tolerable by semiconductor substrates [2].

1.3 Gyroelectric Nonreciprocal Devices

Gyroelectric behaviour is a phenomenon resulted from internal interactions between charged particles inside magnetically biased plasma and an external electromagnetic field. It can be utilised to design nonreciprocal microwave devices in a similar way as gyromagnetic behaviour is used\(^3\).

\(^2\)Hexagonal ferrite (or hexaferrite) is a type of ferrite material with hexagonal crystal structure. It features high internal anisotropy without the need of external magnetic field bias [43,44].

\(^3\)Chapter (2) illustrates this behaviour in more details.
Semiconductors are considered as solid plasma, hence they exhibit gyroelectric behaviour when magnetically biased. In 1953, Suhl and Pearson [45] observed Faraday rotation in magnetised germanium at low temperature ($77 \, K$). Later, Rau and Gaspari [46] reported the same behaviour with magnetised germanium at room temperature in the X-band frequency range ($8.2 - 12.4 \, GHz$).

During the 1960’s, researchers started to investigate theoretically [47–49] and experimentally [50–52] the gyroelectric behaviour of magnetised semiconductors. In 1963, Allis, Buchsbaum and Bers analysed electromagnetic wave behaviour in waveguides loaded with magnetically biased plasma in their seminal book [53]. In 1965, Toda [52] proposed a $24 \, GHz$ field displacement isolator by inserting a vertical Indium Antimonide (InSb) slab at $77 \, K$ inside a rectangular waveguide. The isolation mechanism was based on displacing the electromagnetic field toward the bottom of the InSb slab, where lossy carbon powder is placed. Field displacement along the InSb sample was measured later by the same author [54] using the waveguide configuration shown in Figure (1.12). In 1967, Brodwin and Kahn [55] reported a $140 \, GHz$ waveguide junction circulator using an InSb rod aligned in the middle of the junction at $77 \, K$.

In 1968, May and McLeod [56,57] proposed a waveguide isolator based on Faraday rotation. Figure (1.13) shows a cross sectional view of this isolator, where a longitudinally magnetised InSb rod cooled to $75 \, K$ was used to rotate the electromagnetic field in a circular waveguide section. The reported isolation of this structure was more than $25 \, dB$ with less than $5 \, dB$ insertion loss at $35 \, GHz$.

Most of the aforementioned devices were designed using InSb cooled down to
Figure 1.12: A waveguide configuration for measuring the field displacement caused by a thin InSb slab [54]

Figure 1.13: The experimental configuration of an InSb isolator based on Faraday rotation [57]
low temperatures to reduce the loss. However, in 1969, Suzuki [58] proposed various nonreciprocal devices working at room temperature using very thin slabs of InSb. These devices were proved to operate in the K-band (18 − 26 GHz). Later, Suzuki and Hirota [59] presented similar nonreciprocal devices working in the W-band (80 − 140 GHz) at room temperature.

Electromagnetic analysis of gyroelectric filled waveguides was the focus of many researches during the 1970’s. Gardiol [60], Arnold and Rosenbaum [61], and Sorrentino [62] presented detailed treatments for rectangular waveguides partially or fully filled with transversely magnetised semiconductors. Theoretical results proved nonreciprocity in both attenuation and phase constants at higher frequency ranges.

The constant need of nonreciprocal devices compatible with microwave planar circuits encouraged many researchers during the 1980’s to investigate the possibility of using magnetised semiconductors for that purpose. Tedjini and Pic [63,64] reported nonreciprocity in the Ka-band (26.5−40 GHz) in a finline structure loaded with a thin slab of InSb between the fins. The structure was analysed using modified spectral domain analysis. Krowne et al. [65–67] presented a sophisticated mathematical analysis based on Fourier transform matrix that can be applied to finlines, slotlines and microstrip lines with mixed dielectric and gyroelectric substrates.

In 1981, Bolle and Talisa [68–70] investigated the possibility of designing gyroelectric isolators and phase shifters working in the sub-millimetre-wave frequency range (> 350 GHz) using a layered media of semiconductors and dielectrics, as shown in Figure (1.14). This analytical and numerical study was based on the Drude-Zener model for magnetised semiconductors [71]. In this model, the permittivity of the semiconductor layer is described as a tensor which elements depend on the electrons behaviour in the existence of steady magnetic bias.

The idea of designing junction circulators using magnetised semiconductors instead of ferrite materials was first proposed by Davis and Sloan [40,72] in 1993. The design followed Bosma’s [26] Green function approach for electromagnetic analysis. Using InSb at 77 K, they predicted the performance of a semiconductor junction circulator working in the millimetre-wave frequency range. Theoretical proof of broadband operation for the same device was reported later [41].

In 2001, Yong et al. [73] presented the first measured results of a Ka-band
CHAPTER 1. INTRODUCTION

Figure 1.14: A layered structure of a magnetised semiconductor and a dielectric [68]

(26.5 – 40 GHz) semiconductor junction circulator which validated the previous theoretical analyses. To perform the measurements, they used the finline structure shown in Figure (1.15) to provide the necessary electromagnetic excitation of an InSb disc in the middle of the junction. In 2004, the same theoretical approach was used by Zee M. Ng et al. [74] to design and measure a semiconductor junction circulator working in the V-band (50 – 75 GHz) also using InSb at 77 K.

A relatively recent theoretical treatment for a circular waveguide filled with magnetised plasma was presented in 2009 by Alshannaq and Rojas [75]. This work was based on two and three dimensional Finite Element Method (FEM) simulation to investigate Faraday rotation in various semiconductors at 77 K. The same authors also proposed a gyroelectric isolator [76] and circulator [77] that theoretical analysis predicted their work at room temperature.

In 2012, Sounas et al. [78] reported experimental results that showed a small Faraday rotation exhibited by a Graphene layer biased with high magnetic field (up to 5 T). Electromagnetic analysis were carried out using a two dimensional tensor conductivity for the magnetised Graphene layer.

Modelling and electromagnetic simulation of gyromagnetic and gyroelectric materials were made possible in the 2012 (and subsequent) versions of the commercial simulation package CST® Microwave Studio (CST MWS). This simulation upgrading gave a significant advantage to this field of research by allowing the user to model and simulate materials with a tensor permittivity or permeability. This
Figure 1.15: Schematic diagram of finline components used to measure a semiconductor junction circulator [73]
introduced a vital tool in verifying and optimising gyromagnetic and gyroelectric microwave devices. More information about electromagnetic simulation of gyroelectric materials is included in Section (2.5).

1.4 Thesis Review

1.4.1 Research Objectives

Ferrite nonreciprocal devices have been used for decades in different radar and communication systems. They provide high isolation, good power handling and low insertion loss. However, as the frequency of operation gets higher, the performance of these devices starts to deteriorate [40,42]. During the last decade, many applications have emerged that require compact nonreciprocal devices working in the millimetre-wave and THz frequency ranges. Examples of such applications include the sub-millimetre-wave radar for concealed person-borne weapons [79,80], automobile anti-collision radar [81], E-band backhaul ultra-fast communication links [82,83] and stabilisation of THz Quantum Cascade Laser (QCL) [84,85].

This project aims to explore the possibility of utilising the gyroelectric behaviour in magnetised semiconductors to design and fabricate nonreciprocal devices working in the millimetre-wave and THz frequency ranges.

As a start, it is important to improve and optimise the previous designs based on InSb at 77 K by expanding the mathematical understanding and using the recent advances in the three dimensional simulation packages. Based on that, the theoretical analysis should be extended to investigate the possibility of using other types of materials, such as the high mobility Two Dimensional Electron Gas (2-DEG) to realise millimetre-wave nonreciprocal devices.

Another objective of this research is to explore the conditions at which a gyroelectric equivalent to the well-known ferrite turnstile circulator [86] can be realised theoretically and experimentally. This will allow designing a new type of gyroelectric circulators with a simple, easy to fabricate structure.

Other types of nonreciprocal devices are also included within the research objectives. By considering rectangular waveguides loaded with semiconductor slabs, the research intends to investigate, mathematically and experimentally, the nonreciprocal behaviour when magnetically biasing the semiconductors with a D.C.
magnetic field. The outcomes of this part of the research can be of special importance for realising a new class of gyroelectric isolators and phase shifters.

1.4.2 Research Contributions

There are several important areas where this research makes an original contribution to the knowledge. Firstly, the already-existing semiconductor junction circulator design technology is deeply analysed and optimised, which results in a better understanding of the electromagnetic principles of both gyroelectric and gyromagnetic devices. In addition, optimising the design methods allows the future researchers to get the full benefit from the designed gyroelectric circulators. This research also offers a new insight into the possibility of using a two-dimensional material, namely the 2-DEG, to realise fully-integrated gyroelectric resonators and circulators. Unlike the majority of the previously reported gyroelectric designs which focused on bulky semiconductors such as the InSb, using the 2-DEG is considered an important step towards realising fully integrated millimetre-wave nonreciprocal devices.

Another contribution of this research is investigating the possibility of realising a gyroelectric circulator by suspending a semiconductor rod inside a three port waveguide junction. In an endeavour to undertake this investigation, a full electromagnetic modal analyses is performed for an axially magnetised gyroelectric cylinder with open boundaries and placed inside a rectangular waveguide structure. This is the first time such approach is attempted mathematically and experimentally.

In addition to the above, this study makes a significant contribution to research on rectangular waveguides loaded with magnetised semiconductors. Even though this topic has been reported in the literature before, it is the first time the modal analysis of such a structure is verified using electromagnetic simulations and high frequency measurements. These findings will have an important impact on the future designs of millimetre-wave waveguide isolators and phase shifters.

1.4.3 Outline of Chapters

A general overview of the concept of nonreciprocal components and their applications has been given in this chapter. A survey of the previously published works related to the gyromagnetic and gyroelectric nonreciprocal components for the
last six decades has also been covered. This chapter also featured the main aims of this research and an overview of the rest of the thesis.

In Chapter 2, the physical principles of plasma and its criteria are covered. The chapter also includes a general overview of the electronic properties of semiconductors, and it examines the applicability of the plasma criteria on a group of important semiconductors. The chapter then illustrates the derivation of a mathematical model that contains the macroscopic features of magnetised plasma using the single particle approximation. Finally, simulation of gyroelectric materials in the CST MWS simulation package is discussed in terms of the used mathematical model and numerical solver.

Chapter 3 covers the electromagnetic analysis of a resonator that consists of a thin semiconductor disk biased in its axial direction with an electric wall on its side and magnetic walls on its top and bottom. After deriving the necessary expressions for the resonant conditions, two ports are introduced on the side wall of the same resonators, and the scattering parameters for this arrangement are derived. Next, the same resonator is used as a circulator (referred to as the Semiconductor Junction Circulator, or the SJC) by introducing three equally spaced ports on its side wall. After deriving expressions for the SJC’s scattering parameters, the perfect circulation conditions are derived and plotted to facilitate the SJC designs in the next chapter.

In Chapter 4, the theory of SJC is used to verify a previous design that works in the Ka-band by calculating and measuring the scattering parameters. Then another SJC is designed to work at the sub-millimetre-wave frequency range using a low value of magnetic bias. Next, an algorithm is proposed to optimise the design of broadband SJC’s that work at high frequencies using low values of magnetic bias. The chapter ends with theoretical investigation of the resonant behaviour of magnetised 2-DEG based resonators, then a circulator design using the same material is proposed and analysed.

Chapter 5 begins with a mathematical analysis of an infinitely long semiconductor rod magnetised in its axial direction with open boundaries. After exploring the propagating modes along this rod, inserting a piece of such rod (as a disk) inside a three port waveguide junction is investigated. The possibility of designing circulators using this arrangement is then discussed, followed by the simulation and measurements of the same structure using an InSb disk at 77 K.

Chapter 6 introduces the theoretical analysis of rectangular waveguides loaded
with dielectric, gyroelectric or mixed layers. After finding the characteristic equation that reveals the propagation constants, different types of loadings are considered and the resulting modes are discussed. InSb at 77 $K$ is used to demonstrate the nonreciprocal behaviour when using it to load a rectangular waveguide with the existence of a magnetic bias in the transverse direction.

In Chapter 7, the anticipated results from theoretical analysis in Chapter 6 are validated by simulating rectangular waveguides with different loadings. The nonreciprocal behaviour of InSb loaded waveguides is also validated by measuring the scattering parameters of a waveguide loaded with different InSb samples, biased in the transverse direction and cooled down to 77 $K$.

Chapter 8 is the concluding chapter, which highlights the main findings of the preceding chapters and evaluates the fulfilment of the research objectives. Finally, some suggestions for further work are presented.
Chapter 2

Gyroelectric Behaviour of Magnetised Semiconductor Plasma

2.1 Introduction

It is well known that there are two types of media that can be referred to as gyro-rotropic: the first is the ferrite, where its spinning electrons align with an external D.C. magnetic field and precess about their axes. These electrons interact with an A.C. magnetic field resulting into what is called the gyromagnetic behaviour [1]. The other type of gyrotropic media is the plasma, where its charged particles interact with a D.C. magnetic field and start to move in a circular motion. The interaction of magnetised plasma with an A.C. electric field is described as the gyroelectric behaviour.

The aim of this chapter is to investigate the prospects of modelling semiconductors as solid plasma. It also aims to introduce a mathematical model for the magnetised semiconductor to facilitate its electromagnetic analysis and components design.

The chapter will begin with introducing the basic physical features of a plasma, highlighting some important quantities such as the plasma frequency and Debye length. The criteria for any medium to be considered as plasma will also be illustrated and discussed.

Next, a generic physical description of semiconductors is given, emphasizing some important properties such as the effective mass, permittivity, relaxation time and
mobility. Then the plasma criteria will be tested for some selected intrinsic semiconductors at room temperature.

To realise a mathematical model for magnetised semiconductors, the Drude-Zener model will be adapted alongside with the single particle approximation. In this approach, the motion of a single electron in the presence of electric and magnetic fields is analysed. The velocity components of this electron are derived in terms of the various forces exerted on it. Based on that, the current density components will be found. To include the effects of both conduction and displacement currents, a permittivity tensor is introduced. The elements of this tensor are then analysed and their behaviour with frequency is discussed.

Finally, the possibility of modelling gyroelectric materials in the CST MWS simulation package will be analysed. This is done by comparing the permittivity tensor elements resulted from the simulation software with those calculated using the aforementioned mathematical model.

2.2 Physical Properties of Plasma

A plasma is an aggregation of positively and negatively charged particles of approximately the same density [87]. It was first identified by Heaviside as an ionised layer in the atmosphere [53], and later it was found that most of the universe consists of plasma, mostly in gaseous state. However, plasma can also exist in solid and liquid forms [88]. The most familiar forms of plasma are fire and the ionised gas in neon tubes.

Plasma is sometimes referred to as the fourth state of matter, after solids, liquids and gases. It can be formed by one of the basic three states of matter after gaining a significant amount of energy. In that case, negative electrons will be separated from their atoms, leaving positive ions behind in a process known as ionisation [89].

The basic feature that identifies plasma from any system of ionised particles is the interaction between the particles according to Coulomb-force law [88]. According to this law, plasma interacts with neighbouring media, giving the plasma a cohesive behaviour similar to that of jelly\(^1\). This feature becomes more important.

\(^1\)When Langmuir and Tonks were studying the electrical discharges in gases in 1929, they observed that electron clouds were shining and wiggling, similar to the jelly-like blood plasma. Therefore they used the term "plasma" to describe these electron clouds [90].
when the interaction is between the free electrons and the relatively heavier positive ions. If the electrons were displaced from their initial position by applying an instantaneous electric field to the plasma, they will be attracted back to their ions due to the ion-electron force. However, due to their inertia, they will start to overshoot and oscillate around their equilibrium position until they are damped by collisions. The frequency of this harmonic motion is usually described as the plasma frequency, and it is expressed by [91]:

$$\omega_p = \sqrt{\frac{N_e e^2}{m \epsilon_0}} \quad (2.1)$$

Where $N_e$ is the concentration of electrons (in $m^{-3}$), $e$ is the electron charge (in Coulomb), $m$ is the electron mass (in kg) and $\epsilon_0$ is the permittivity of free space. Plasma frequency is critical for the electromagnetic propagation through the plasma. Hence, its value should exceed that of the collision frequency, such that these oscillations will not be strongly damped by collisions [91].

Plasma is also characterised by the ability of the charged particles to rearrange their positions to shield any arbitrary charge inside the plasma or an electrostatic field at its surface [88]. Such shield will cancel any electrostatic field within a distance called Debye Length ($\lambda_D$), and this overall effect is called Debye shielding. Debye length is expressed in terms of the temperature and carrier concentration, as follows:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{2e^2 N_e}} \quad (2.2)$$

Where $k_B$ is Boltzmann constant and $T$ is the temperature (in Kelvin).

Not every mixture of charged particles can be considered as plasma. Instead, the matter should fulfil some important criteria: first, the concentration of electrons and ions should be approximately the same. As such, the plasma should exhibit a tendency to become electrically neutral if it is left to itself. This is usually referred to as the quasi-neutrality condition [91].

In addition, the physical dimensions of the system should be larger than its Debye length ($\lambda_D$). Otherwise there will be no enough room for the electrons for the shielding to occur [88]. Finally, there must be enough electrons within the a distance of $\lambda_D$ from the position of any disturbance to produce the shielding. Hence, the distance between the electrons should be small as compared to the Debye length. In other words, the number of electrons ($N_D$) within a sphere
of radius ($\lambda_D$) should be much greater than 1. By rearranging (2.2), this is equivalent to [88]:

$$N_D = \frac{4}{3} N_e \pi \lambda_D^3 >> 1$$

(2.3)

Given that a plasma can be defined as a large collection of positive and negative charged particles that comply with the above criteria, many liquids and solids can be treated as plasmas [89]. Hence, metals (such as iron and tungsten), semimetals (like bismuth and antimony) and semiconductors can be defined as solid plasmas because they consist of electrons and holes [88]. In the next section, the properties of semiconductors that are relevant to the plasma will be considered and discussed.

2.3 Semiconductors as Solid Plasma

As stated in the previous section, semiconductors that comply with the plasma criteria can be regarded as a type of solid plasma. In this section, the general physical features of semiconductors will be illustrated first. Then, the similarities between plasma and a selected group of semiconductors will be discussed.

2.3.1 General Properties of Semiconductors

Solid state materials can have three basic forms: conductors, insulators and semiconductors. Conductors, such as silver and copper, are characterised by their ability to conduct the electric current efficiently, their conductivity ($\sigma$) is typically as high as $10^8 \, Sm^{-1}$. Insulators, on the other hand, are known to have very low conductivity (of the order of $10^{-16} - 10^{-6} \, Sm^{-1}$), such as glass and fused quartz [92].

Semiconductors have conductivity between those of insulators and conductors, and it highly depends on many factors, including the temperature, illumination, magnetic field and impurity doping level. Hence, semiconductors are very important for many electronic applications.

Semiconductors can be divided into two broad categories: element semiconductors, which comprise of the same kind of atoms, such as silicon and germanium, and compound semiconductors, which are compositions of two or more elements. Compound semiconductors can be a combination of two different groups in the periodic table. For example, the III-V semiconductors (such as Gallium Arsenide
and Indium Antimonide) are realised by combining elements from the group IIIA (all metals) and those in group VA. On the other hand, II-VI compounds (like cadmium sulphide, and zinc sulphid) are combinations of those from the group IIB and VIA [93].

Like conductors and insulators, electric conductivity of semiconductors is usually explained in terms of the energy bands\(^2\). In metals (also called conductors), the conduction band is either partially filled (as in the upper portion of part (a) in Figure (2.1)) or it overlaps the valence band (as shown in the lower part of the same figure). Consequently, the electrons in conductors can move with only small value of applied electric field. Insulators, as shown in part (c) of Figure (2.1), are characterised by a large energy gap \( E_g \) due to the strong bonds between neighbouring atoms. This will make only very few electrons able to move to the conduction band, which results into high resistivity [92].

As for the semiconductors, the energy gap is much smaller than that of the insulators, as shown in part (b) of Figure (2.1). At absolute zero temperature, the valence band is fully occupied and the conduction band is empty, hence the semiconductor can be considered as an insulator. However, as the temperature increases, electrons will start to be thermally excited and moved to the conduction band, leaving vacancies in the valence band, which are called (holes). The increased number of electrons will necessarily increase the conductivity of the

\(^2\)A detailed illustration of the theory of energy bands will not be given here. Details can be found in many textbooks, including Chapter (2) in [92], and Chapter (1) in [93].
semiconductor [92].

Semiconductors that have no added atoms are called *intrinsic* semiconductors, these types have equal numbers of electrons and holes ($N_e = N_h = n_i$). In many cases, however, a certain impurity atoms are added to the semiconductor in a process called the (doping). Doped semiconductors are called *extrinsic*. There are two types of doping: the first is when the impurity atoms have excess of electrons to be added to the semiconductor, these atoms are called the *donors*, and the resulted semiconductor will have more electrons than holes and it is called (n-type). The other type of doping is when the impurities act as (acceptors) and they accept electrons from the semiconductor, rendering more holes than electrons. The semiconductor in this case is called the (p-type).

Semiconductors have numerous properties, many of which are outside the scope of this thesis. However, since the designs and analyses in the following chapters depend mainly on the plasma properties of semiconductors, the following aspects have special importance and they deserve to be illustrated with some details:

**Effective Mass**

Atoms in semiconductor materials are usually arranged in a three-dimensional periodic fashion, in an arrangement called a *lattice* [92]. Therefore the movement of electrons in a semiconductor is different from its movement in free space. This is because it will be affected by the interaction with the periodically arranged atoms in the semiconductor’s lattice.

Consequently, the electrons’ motion cannot be analysed in a classical manner. Instead, they are analysed by applying the principles of quantum physics to the system, which includes solving Schrödinger equation given the periodic lattice [94,95]. The solution to Schrödinger equation will result into the wavenumber of the electron ($k$) in terms of its energy ($E$), as plotted in part (a) of Figure (2.2)\(^3\).

By considering the electrons as (wave packets) that move in the periodic lattice structure, their group velocity can be derived and expressed by [94]:

$$\nu = \frac{2 \pi dE}{\hbar \frac{d}{dk}} \quad (2.4)$$

\(^3\)For more details about the electron’s particle-wave duality and the solution of Schrödinger equation for periodic lattice, please refer to [95].
Figure 2.2: (a) Energy, (b) velocity and (c) effective mass as functions of the electron wave vector $k$ for an electron moving in a semiconductor crystal [94].
where $h$ is the Planck constant.

As can be seen from part (b) of Figure (2.2), the group velocity depends on the slope of the $E - k$ curve, and it reaches zero at the top and bottom of the band. By assuming a certain force ($F$) exerted on the electrons by an external field, and by virtue of (2.4), it was found that the acceleration of the electrons ($a$) will be expressed as [94]:

$$a = \frac{4\pi^2 F d^2 E}{h^2}$$  \hfill (2.5)

It is clear that the expression in (2.5) is very different from the classical Newtonian law of motion ($F = ma$). However, it is still possible to consider the electrons as free and use the classical law of motion after taking into consideration the effects of the internal forces inside the semiconductor in the effective mass ($m^*_e$). By comparing (2.5) with above Newton’s law, the effective mass can be expressed as [94]:

$$m^*_e = \frac{h^2}{4\pi^2 d^2 E}$$  \hfill (2.6)

Part (c) of Figure (2.2) shows that $m^*_e$ has constant values at the top and bottom of the band. Knowing that the electrons in the conduction band stay near the bottom of the band [96], it is possible to consider these electrons free with an effective mass of $m^*_e$.

**Relaxation Time and Mobility**

In a perfect lattice, an electron moves freely and accelerates from $k = 0$ under the influence of an external field ($\mathcal{E}$) until reaching the end of the zone, where it suffers a reflection, which returns it to the other end and it goes to the $k = 0$ point and so on. Hence, the electron motion will be governed by the following relation [93]:

$$-e\mathcal{E} = m^*_e \frac{dv}{dt}$$  \hfill (2.7)

Where $v$ is the electron velocity (in $ms^{-1}$), and the effective mass ($m^*_e$) is used to include the lattice effects.

However, in real crystals, there are many imperfections that change the periodic potential. When the electron passes near one of these imperfections, its direction

---

4This behaviour is coming from the wave-particle duality; when treating an electron as a wave, it will be reflected and form a standing wave when its wave vector has a certain relationship to the spacing of the lattice. Hence, it would comply with the **Bragg condition** for electromagnetic waves [94].
will be changed and hence, be scattered. At a certain temperature, the electrons move randomly due to the thermal energy, while suffer from scattering from the lattice imperfections. When an electric field is applied, the electrons, still moving randomly, will drift in the direction opposite to that of the electric field with what is called the drift velocity. As the velocity of the electrons increases, they will suffer more collisions per unit time. Hence, they cannot be accelerated continuously and, instead, face more resistive force. This will give the medium a viscous nature. Therefore a new term is introduced into (2.7) to describe the average drift velocity by the following equation:

\[-e\mathcal{E} = m_e \frac{dv_d}{dt} + m_e \frac{v_d}{\tau}\]  \hspace{1cm} (2.8)

Where \(\tau\) is a constant. Assuming that the electric field is removed at \(t = 0\), (2.8) will become a differential equation with the solution at any time \((t)\) expressed as:

\[v_d(t) = v_d(0) e^{-\frac{t}{\tau}}\]  \hspace{1cm} (2.9)

Where \((v_d(0))\) is the average drift velocity when the electric field is removed. Equation (2.9) indicates that \(\tau\) has the significance of a relaxation time. In the steady state, the average drift velocity becomes constant. Hence, the term \(\left(\frac{dv_d}{dt}\right)\) will vanish and, by virtue of (2.8), \(v_d\) will become:

\[v_d = -e\mathcal{E} \frac{\tau}{m_e}\]  \hspace{1cm} (2.10)

From (2.10), it can be seen that the average drift velocity is directly proportional to the applied electric field, and it can be written as:

\[v_d = -\mu_e \mathcal{E}\]  \hspace{1cm} (2.11)

Where \(\mu_e\) is the mobility of the electron, and it is a measure of the electron’s average drift velocity per unit electric field. By comparing (2.11) with (2.10), electron mobility is expressed by:

\[\mu_e = \frac{e\tau}{m_e}\]  \hspace{1cm} (2.12)
Similarly, the mobility of holes can be expressed by:

\[ \mu_h = \frac{e\tau}{m^*_h} \] (2.13)

Where \( m^*_h \) is the effective mass of the hole. The constant \( \tau \) is sometimes referred to as the collision time, and it can be defined as the mean time between two collisions [97]. Hence, a new parameter can be introduced as the collision frequency \( (\nu_c) \), and it is described as:

\[ \nu_c = \frac{1}{\tau} = \frac{e}{m^*_e\mu_e} \] (2.14)

It is worthwhile to notice that mobility of both electrons and holes are related to the conductivity of the semiconductor \( (\sigma) \) via the following relation [94]:

\[ \sigma = e \left( N_e\mu_e + N_h\mu_h \right) \] (2.15)

Despite their part in the current conduction, the low mobility of the holes makes their contribution less significant than those of the electrons in the conduction band.

**Permittivity**

Electric field has an effect on the insulators in a way that it polarises the electrons around the nuclei. This effect depends on the material itself, and is called the permittivity. Permittivity of a certain material depends on its polarizability (its ability to be polarised), which is a function of frequency [98]. Even though the polarizability of a material depends on the many factors, its values for semiconductors depend entirely on the electrons.\(^5\)

The electronic polarizability is not effective below the optical frequency ranges (around \( 10^{14} \) Hz). Hence, it is safe to assume that the dielectric constant of a semiconductor at zero frequency is the same in the millimetre and sub-millimetre-wave frequency ranges.

\(^5\)There are, in general, three types of polarizability, namely the electronic, ionic and dipolar. Since semiconductors are in general nonionic and nondipolar, only electronic polarizability is effective [99].
2.3.2 Plasma Features of Semiconductors

It was shown in Section (2.2) that any solid material with positive and negative charged particles, such as semiconductors, can be treated as plasma if they comply with some specific criteria.

Table (2.1) shows the properties of some selected intrinsic semiconductors at 300 $K$, which were cited from [100]. These semiconductors are tested for the plasma criteria according to their electronic characteristics as follows:

- **Debye length ($\lambda_D$):** It was shown before that Debye length should be smaller than the overall size of the system of the plasma. From Table (2.1) it is shown that the calculated values of $\lambda_D$ for the selected intrinsic semiconductors vary, although small values appear for Indium Phosphide (InP), Indium Arsenide (InAs) and Indium Antimonide (InSb). Therefore it can be concluded that smaller plasma devices can be designed using these semiconductors.

- **Number of charged particles ($N_D$):** The number of charged particles in a Debye sphere should be much larger than 1. This is to ensure that there are enough electrons in the material to shield any potential inside the plasma or on its surface. This was calculated for the semiconductors in Table (2.1) using equation (2.3)$^6$. It is shown that for few semiconductors (e.g., GaP, InAs and InSb), $N_D$ is around 1. However, as stated in [88], this is the case for most of the solids for usual lattice temperatures (around 300 $K$). This is because the temperature of the electrons is as high as three or four times that of the plasma due to the quantum-mechanical effects$^7$. Hence, the actual value of $N_D$ should be high enough for the shielding effect to take place.

- **Quasi-neutrality:** It is known that in any plasma, the number of positively and negatively charged particles should be almost the same [91]. This condition is fulfilled for intrinsic semiconductors since the number of electrons and holes is the same. As for extrinsic semiconductors, for each free electron there is a positively charged donor atom, so the condition is fulfilled as well.

$^6$The values of $N_D$ were rounded down to the nearest integer.
$^7$To see the full proof using the uncertainty principle, pleas refer to pages 4-5 of [88].
Table 2.1: Basic parameters for some selected intrinsic semiconductors at 300 K [100].

<table>
<thead>
<tr>
<th>Semiconductor</th>
<th>Group</th>
<th>(\frac{m^*}{m_0})</th>
<th>(\frac{m^*_h}{m_0})</th>
<th>(\mu_e)</th>
<th>(\mu_h)</th>
<th>(\epsilon_r)</th>
<th>(\nu_c) ((\times 10^{12} \text{ s}^{-1}))</th>
<th>(n_i) ((m^{-3}))</th>
<th>(\lambda_D) ((\mu m))</th>
<th>(N_D)</th>
<th>(N_p) ((rad m^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>IV</td>
<td>0.26</td>
<td>0.49</td>
<td>0.135</td>
<td>0.048</td>
<td>11.9</td>
<td>5.01</td>
<td>1.5 (\times 10^{16})</td>
<td>23.81</td>
<td>848</td>
<td>1.84 (\times 10^9)</td>
</tr>
<tr>
<td>Ge</td>
<td>IV</td>
<td>0.12</td>
<td>0.28</td>
<td>0.390</td>
<td>0.190</td>
<td>16.0</td>
<td>3.76</td>
<td>2.4 (\times 10^{19})</td>
<td>0.69</td>
<td>33</td>
<td>9.32 (\times 10^{10})</td>
</tr>
<tr>
<td>GaP</td>
<td>III-V</td>
<td>0.13</td>
<td>0.6</td>
<td>0.030</td>
<td>0.015</td>
<td>10.2</td>
<td>45.1</td>
<td>3 (\times 10^{22})</td>
<td>0.02</td>
<td>1</td>
<td>8.49 (\times 10^{12})</td>
</tr>
<tr>
<td>GaAs</td>
<td>III-V</td>
<td>0.067</td>
<td>0.45</td>
<td>0.85</td>
<td>0.04</td>
<td>13</td>
<td>3.09</td>
<td>9 (\times 10^{13})</td>
<td>322.45</td>
<td>12639</td>
<td>0.57 (\times 10^9)</td>
</tr>
<tr>
<td>GaSb</td>
<td>III-V</td>
<td>0.042</td>
<td>0.4</td>
<td>0.5</td>
<td>0.085</td>
<td>15.7</td>
<td>8.38</td>
<td>5 (\times 10^{17})</td>
<td>4.74</td>
<td>223</td>
<td>4.91 (\times 10^{10})</td>
</tr>
<tr>
<td>InP</td>
<td>III-V</td>
<td>0.07</td>
<td>0.4</td>
<td>0.4</td>
<td>0.06</td>
<td>12.1</td>
<td>6.28</td>
<td>1.0 (\times 10^{20})</td>
<td>0.29</td>
<td>10</td>
<td>6.31 (\times 10^{11})</td>
</tr>
<tr>
<td>InAs</td>
<td>III-V</td>
<td>0.028</td>
<td>0.33</td>
<td>2.26</td>
<td>0.02</td>
<td>12.5</td>
<td>2.78</td>
<td>1.0 (\times 10^{21})</td>
<td>0.09</td>
<td>3</td>
<td>3.02 (\times 10^{12})</td>
</tr>
<tr>
<td>InSb</td>
<td>III-V</td>
<td>0.013</td>
<td>0.18</td>
<td>8.0</td>
<td>0.075</td>
<td>17.7</td>
<td>1.69</td>
<td>1.1 (\times 10^{22})</td>
<td>0.03</td>
<td>1</td>
<td>1.23 (\times 10^{13})</td>
</tr>
</tbody>
</table>
• Plasma frequency \( (\omega_p) \) higher than collision frequency \( (\nu_c) \): This condition should be fulfilled because if the collision frequency is high, the movement of the electrons will be heavily damped and the material will lose the plasma features. For the semiconductors shown in Table (2.1), only InAs and InSb fulfil this condition at 300 K. However, more semiconductors should fulfil it at lower temperatures because of the increased mobility.

It can be seen from Table (2.1) that InSb is the most likely to be treated as a solid plasma since it complies with the plasma criteria more than the other semiconductors in the list. In addition, it features high electron mobility, low hole mobility and high hole effective mass. Therefore the effects of holes can be safely neglected.

Based on that, InSb can be considered as the most suitable candidate for designing devices based on the plasma properties. These results make it understandable for the InSb to be extensively used in the literature for designing semiconductor based nonreciprocal components [40, 47, 73, 74, 77, 101, 102].

In [100], an undoped InSb sample was characterised by measuring its electrons concentration and mobility at different temperatures, as shown in Figure (2.3). It can be seen that as the temperature is reduced to 77 K, which is associated to the liquid nitrogen’s boiling point, the mobility of the InSb sample is increased more than ten times to reach about 58 m²V⁻¹s⁻¹. Electron concentration, on the other hand, is reduced to around 1.1 \times 10^{20} m^{-3} at \( T = 154 K \) (−119°C), and stays constant till \( T = 77 K \).

2.4 Mathematical Model for Magnetised Semiconductors

There are many approaches used to analyse plasmas, such as the single particle approximation, the hydrodynamic approximation and the kinetic equation approximation [88]. The first approach is regarded as the same as the Drude-Zener model adopted by some previous researchers to analyse gyroelectric materials [46, 100, 103]. This model will be used here to translate the microscopic
Figure 2.3: Measured mobility ($\mu_e$) and electron concentration ($N_e$) versus the temperature ($T$) for the undoped InSb sample [73].
2.4.1 Motion of Electrons in Semiconductor Plasma

Motion of charged particles in semiconductors in presence of external electric and magnetic fields affects the properties of the material. Consider the electron shown in Figure (2.4), with charge of $-e$ and effective mass of $m_e^*$ moving with velocity of $\nu$ in a certain direction. In the presence of an A.C. electric field ($E$) and a magnetic field with a flux density of $B_0$, three forces will affect the motion of the electron: electric force ($F_E$), magnetic force ($F_B$) and retarding force ($F_D$). These forces can be expressed as follows [87, 93, 104]:

$$F_E = eE$$  \hfill (2.16)
$$F_B = e\nu B_0$$  \hfill (2.17)
$$F_D = \frac{m_e^*\nu}{\tau}$$  \hfill (2.18)

where $\tau$ is the relaxation time of the electron, which can be found from the electron’s mobility using (2.14) [105].

Figure (2.4) shows that the electric force direction is opposite to the retarding
force and the electric field (due to the negative charge of the electron), and in the same direction of motion. On the other hand, magnetic force is perpendicular to the direction of motion. Here, the effects of the A.C. magnetic field associated with the electric field are neglected [100].

Motion of the electron in Figure (2.4) can be expressed in terms of the following equation [53]:

$$m_e \frac{d\vec{v}}{dt} = -e(\vec{E} + \vec{v} \times \vec{B}_0) - m_e^* \nu_c \vec{v}$$  \hspace{1cm} (2.19)

Assuming that the velocity has a time dependence of $e^{j\omega t}$, and the steady magnetic flux with a density of $(B_0)$ is in the $z-$direction, the relation in (2.19) can be solved for the three spatial components of $\vec{v}$, yielding [100]:

$$\nu_x = \frac{je}{m_e^*} \left( \omega - j\nu_c \right) E_x - \frac{e^2}{m_e^*} \omega_c E_y$$  \hspace{1cm} (2.20a)

$$\nu_y = \frac{e}{m_e^*} \omega_c E_x + \frac{je}{m_e^*} \left( \omega - j\nu_c \right) E_y$$  \hspace{1cm} (2.20b)

$$\nu_z = \frac{j e^2}{m_e^*} E_z$$  \hspace{1cm} (2.20c)

Where $(\omega_c)$ is the angular *cyclotron frequency* of the electron, and it is expressed by:

$$\omega_c = \frac{e}{m_e^*} B_0$$  \hspace{1cm} (2.21)

The cyclotron frequency can be defined as the ratio of the electron’s circular motion due to the presence of the steady magnetic field. An important condition for the material to show a gyroelectric behaviour is for the cyclotron frequency to exceed the collision frequency $(\omega_c > \nu_c)$.

As a result of the motion of electrons, a current density $(\vec{J})$ occurs, and it is related to the electron’s velocity vector as follows [104]:

$$\vec{J} = -eN_e \vec{v}$$  \hspace{1cm} (2.22)

Note that the direction of the current density is opposite to that of the electrons’ movement.

Now, by virtue of (2.20), the current density components can be found as follows:
\[ J_x = \epsilon_0 \frac{-j\omega_p^2(\omega - j\nu_c)E_x + \omega_p^2\omega_cE_y}{(\omega - j\nu_c)^2 - \omega_c^2}, \]  
\[ (2.23a) \]

\[ j_y = \epsilon_0 \frac{-\omega_p^2\omega_cE_x - j\omega_p^2(\omega - j\nu_c)E_y}{(\omega - j\nu_c)^2 - \omega_c^2}, \]  
\[ (2.23b) \]

\[ J_z = \epsilon_0 \frac{-j\omega_p^2E_z}{(\omega - j\nu_c)} \]  
\[ (2.23c) \]

Here, \((\omega_p)\) is the angular plasma frequency for the semiconductor, and it can readily be found from the expression in (2.1) after replacing \(m\) with \(m^*_c\).

### 2.4.2 The Permittivity Tensor

Consider Maxwell’s equation:

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \]  
\[ (2.24) \]

There are two types of current in the above equation: conduction current \((\vec{J})\) and displacement current \(\left(\frac{\partial \vec{D}}{\partial t}\right)\). Displacement current plays a role in the microwave frequency range and it cannot be neglected. Hence, for an electric field \((\vec{E})\) varying with time as \(e^{j\omega t}\), substituting \(\epsilon_0\epsilon_r\vec{E}\) instead of \(\vec{D}\) will result into the following expression:

\[ \frac{\partial \vec{D}}{\partial t} = j\omega\epsilon_0\epsilon_r\vec{E} \]  
\[ (2.25) \]

The two currents in (2.24) can be expressed in one term by introducing a permittivity tensor \([\epsilon]\) into the equation. Hence, it can be rewritten as follows:

\[ \nabla \times \vec{H} = j\omega\epsilon_0[\epsilon]\vec{E} \]  
\[ (2.26) \]

where:

\[ [\epsilon] = \begin{bmatrix} \epsilon & -j\kappa & 0 \\ j\kappa & \epsilon & 0 \\ 0 & 0 & \zeta \end{bmatrix} \]  
\[ (2.27) \]
The elements of the tensor in (2.27) are derived by virtue of (2.25) and (2.23), and they are expressed by:

\[
\varepsilon = \varepsilon_r - \frac{\omega_p^2(\omega - j\nu_c)}{\omega[(\omega - j\nu_c)^2 - \omega_c^2]} \\
\kappa = \frac{\omega_p^2\omega_c}{\omega[(\omega - j\nu_c)^2 - \omega_c^2]} \\
\zeta = \varepsilon_r - \frac{\omega_p^2}{\omega(\omega - j\nu_c)}
\]

(2.28)  
(2.29)  
(2.30)

Permittivity tensor elements in (2.28-2.30) take into account the loss due to the collision frequency \((\nu_c)\) in their imaginary parts. Effect of loss can be reduced by decreasing the temperature of the semiconductor, and hence, decreasing the collision frequency. Lossless case can be assumed by discarding the \(\nu_c\) terms in (2.28-2.30).

Figure (2.5) shows the variation of each of the tensor elements with frequency (assuming a lossless semiconductor).

The variation in \(\varepsilon\) with frequency shown in Figure (2.5a) indicates that it starts from a certain value \((\varepsilon_1)\) at \(\omega = 0\), and then it increases until reaching \(\infty\) at \(\omega_c\). Above that frequency, \(\varepsilon\) increases from \(-\infty\) and tends to reach \(\varepsilon_r\) as the frequency increases, while crossing zero at \(\omega = \omega'_1\), where \([100]\):

\[
\omega'_1 = \sqrt{\omega_c^2 + \frac{\omega_p^2}{\varepsilon_r}}
\]

(2.31)

Figure (2.5b) shows that \(\kappa\) starts from \(-\infty\) at zero frequency, and increases until reaching a turning point at \(\omega'_2\), where \([100]\):

\[
\omega'_2 = \frac{\omega_c}{\sqrt{3}}
\]

(2.32)

After that, the value of \(\kappa\) decreases with increased frequency till \(\omega = \omega_c\), where it undergoes a discontinuity. Above the cyclotron frequency, \(\kappa\) decreases from \(\infty\), approaching zero at higher frequencies.

As for \(\zeta\), its variation shown in Figure (2.5c) indicates that its behaviour does not depend on the cyclotron frequency as its value start from \(-\infty\) and increases with frequency, approaching \(\varepsilon_r\) as the frequency approaches \(\infty\), crossing zero at \(\omega = \frac{\omega_p}{\sqrt{\varepsilon_r}}\).
Figure 2.5: Variation of the permittivity tensor elements with frequency for a lossless semiconductor.
2.5 Modelling Gyroelectric Materials in CST MWS

The CST® Microwave Studio electromagnetic simulation package (CST MWS) has introduced the possibility to simulate materials with gyrotropic dielectric dispersion model (which are equivalent to the gyroelectric materials) in its 2012 (and subsequent) versions. Using this simulation package, different components can be modelled as magnetised plasma within a general microwave model. This section will illustrate the details of the gyrotropic model used in this package and the available numerical solver used for the simulation.

When creating a new component in a CST MWS project, the material of this component can be chosen from a library, where many standard materials have been characterised and stored. When it is required to simulate a non-standard material, the software allows the user to create one with any specific parameters.

To create a new gyroelectric material, a gyrotropic dielectric dispersion model is chosen from the ”Dispersion” tab in the material parameters window, as shown in Figure (2.6). The relative dielectric constant is entered in the ”Epsilon infinity” box. Plasma, collision, and cyclotron frequencies are entered in their respective boxes shown in Figure (2.6), while the magnetic bias direction can be set in Cartesian coordinates from the bottom boxes.

To verify the use of this model to design gyroelectric microwave components, elements of the tensor permittivity resulted from this software were compared to those calculated using the results of the mathematical model illustrated in the previous section.

The gyroelectric material considered for this comparison was the previously measured InSb sample at 77 K, biased with a magnetic flux \((B_0)\) of 0.5 \(T\) in the \(x\)--direction. As can be seen from Figure (2.3), the electron concentration and mobility for this sample at 77 K are \(1.1 \times 10^{20} \, m^{-3}\) and \(58 \, m^2 V^{-1} s^{-1}\), respectively. These values result into a plasma frequency of \(6.44 \times 10^{12} \, rad \, s^{-1}\) and a collision frequency of \(2.51 \times 10^{11} \, s^{-1}\). Cyclotron frequency was found using (2.21) to be \(6.76 \times 10^{12} \, rad \, s^{-1}\).

From the relations in (2.28) to (2.30), the values of \(\varepsilon\), \(\kappa\) and \(\zeta\) were calculated and plotted in Figure (2.7). It can be seen from Figures (2.7a) and (2.7b) that there is a significant increase in the imaginary part of both \(\varepsilon\) and \(\kappa\) at the cyclotron
frequency \( f_C = 1060 \, GHz \). On the other hand, \( \zeta \) is expected to cross zero at \( \frac{\omega p}{\sqrt{\epsilon_r}} \) for the lossless case. However, the real and imaginary parts will behave differently when the loss is included, as shown in Figure (2.7c).

The same material parameters used to calculate the tensor permittivity elements were used to create a material in the CST MWS. After setting the frequency range from 0 to 4000 \( GHz \), the software calculated the tensor permittivity elements for the same frequency range. It is obvious from the comparison in Figure (2.7) that there is a perfect match between the calculated and simulated results. From above, it can be concluded that gyroelectric materials modelled in the CST MWS simulation package based on the same mathematical model resulted from analysing the motion of electron in a magnetised plasma illustrated in Section (2.4).

CST MWS offers a wide range of numerical solvers, such as time domain solver, frequency domain solver, Eigenmode solver, integral equation solver, .. etc. Each of these solvers has certain advantages for specific electromagnetic problems. However, when a gyroelectric material is used in a model, the frequency domain solver is the only one that can be used.
Figure 2.7: Calculated and simulated variations of the permittivity tensor elements with frequency for InSb at 77 K, biased with $B_0 = 0.5 \, T$. 
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(a) A cylinder model in CST with 275 tetrahedrons

(b) A cylinder model in CST with 5000 tetrahedrons.

Figure 2.8: Mesh view of a cylinder model in CST MWS with different number of tetrahedrons

The frequency domain solver solves Maxwell’s equations after transferring the fields into the frequency domain. Scattering parameters of a model that contains gyroelectric material are found between two pre-defined ports that support a specific type of electromagnetic modes. However, these ports cannot be in touch with the gyroelectric materials. The solver can also calculate the electromagnetic field distributions at selected frequency samples.

When the frequency domain solver is started, it divides the calculation domain into smaller cells on which Maxwell’s equations should be solved. This is done either using orthogonal Cartesian grids or tetrahedral grids. Only the latter is allowed to be used with gyroelectric materials. The generation of a tetrahedral grid is done first by meshing the edges and faces of the whole model. After that, the volume mesh is created to be consistent with the surface mesh. Once the initial mesh is created, an optimising process is applied to the model for smoothing the mesh and improving the quality of the tetrahedrons.

Accuracy of the frequency domain solver results highly depends on the quality and density of the tetrahedral mesh. For example, Figure (2.8a) shows a cylinder model in CST with primitive mesh of 275 tetrahedrons. To have more accurate results, the local mesh settings for this cylinder can be changed to have a better mesh as shown in Figure (2.8b) where the number of tetrahedrons was increased to 5000.

The mesh properties of each object in a CST MWS model can be changed independently. In addition, number of tetrahedrons depends on the dimensions of

\[8\] This limitation has special significance in some cases, as will be shown in Chapter (4).
the objects and the frequency range of simulation. High number of tetrahedrons prolongs the simulation time, hence a trade off between the desired accuracy and the simulation period can be made when setting the mesh properties of a simulation model.

2.6 Conclusions

This chapter has introduced the basic concepts of plasma, and featured some of the general characteristics of this matter including quasi-neutrality, Debye shielding and plasma frequency. Based on these principles, the criteria for any material to be treated as plasma were illustrated and discussed.

Since some of the semiconductors can be considered as solid plasma, their relevant features including the effective mass, mobility and permittivity were analysed.

Next, the criteria of plasma were applied to a number of selected semiconductors. Among the considered semiconductors, Indium Antimonide (InSb) was proved to be the most suitable candidate for realising plasma based devices since it features low Debye length \(\lambda_D\) and high mobility at liquid nitrogen temperatures (77 K) according to previously measured data.

The mathematical model necessary for designing and analysing gyroelectric devices was introduced after adopting the single particle approximation model for the semiconductor. The mathematical model originates from the basic equation of motion of an electron in the presence of electric and magnetic fields. The effect of different forces on the electron’s velocity in the spatial domain were derived.

The current density was then found from the velocity components.

After that, a permittivity tensor was introduced to include the effects of the displacement current alongside the conduction current. Expressions for the elements of the permittivity tensor were derived and their variations with frequency for a lossless semiconductor were analysed.

Finally, the use of the CST MWS simulation package to simulate gyroelectric materials was illustrated. This was done by calculating the complex values of the permittivity tensor elements over a certain range of frequencies for InSb at 77 K, then comparing the results with those of the CST for the same material. It was shown that this simulation package uses the same approach illustrated before for modelling the gyroelectric materials. Despite some limitations in the solver,
CST MWS represents a significant advantage to the design and verification of gyroelectric devices, as will be shown in the next chapters.
Chapter 3

Theory of Gyroelectric Resonators and Circulators

3.1 Introduction

This chapter illustrates the electromagnetic analysis of gyroelectric disks used to design resonators and circulators. The assumed disk comprises of an axially magnetised semiconductor with electric wall covering the sides and magnetic walls on its top and bottom. The thickness of the disk is assumed to be small enough such that any field change in its axial direction is neglected. Following previously reported modal analysis of similar structures [26,40,100,106], resonant conditions are derived and analysed.

The gyroelectric disk is used next to design a two port resonator by considering two ports connected to the disk on opposite sides. Using a Green’s function approach, expressions for the scattering parameters are derived and used to verify the existence of the previously expected gyroelectric modes.

Finally, a Semiconductor Junction Circulator (SJC) is realised by coupling the gyroelectric disk with three ports 120° apart. After deriving expressions for its scattering parameters, the conditions at which the circulation occurs are found and plotted to facilitate the SJC design procedure in the next chapter.
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3.2 Modal Analysis of an Axially Magnetised Gyroelectric Disk

Consider the structure shown in Figure (3.1), it consists of a semiconductor disk, biased with a magnetic field ($B_0$) in the $z$-direction. The curved side of the disk is coated with conducting material forcing the tangential electric field to be zero, resulting into an electric wall. Top and bottom of the disk are assumed to be open to air, which can be approximated to be magnetic walls.

Electromagnetic analysis will be applied to this structure to find the resonant modes arise from the gyroelectric behaviour of the disk. These modes depend on the magnetic bias ($B_0$) and the radius of the disk ($R$).

3.2.1 Electromagnetic Analysis

As illustrated in the previous chapter, when applying a steady magnetic field ($B_0$) in the $z$-direction to a semiconductor, the permittivity of the semiconductor will be described by the following tensor:

$$
\epsilon = \begin{bmatrix}
\varepsilon & -j\kappa & 0 \\
 j\kappa & \varepsilon & 0 \\
 0 & 0 & \zeta
\end{bmatrix}
$$  \hspace{1cm} (3.1)

Analysing a gyroelectric material requires the consideration of Maxwell’s equations after replacing the normal scalar permittivity ($\epsilon$) with the tensor in (3.1),
as follows:

\[ \nabla \times \vec{E} = -j\omega\mu_0\vec{H} \]  
(3.2)

\[ \nabla \times \vec{H} = j\omega\epsilon_0[\epsilon]\vec{E} \]  
(3.3)

Here, \( e^{-j\beta z} \) dependence is assumed for both \( \vec{E} \) and \( \vec{H} \). It is also assumed that the material has a unity relative permeability (\( \mu_r = 1 \)).

Projecting the three electric and magnetic fields to cylindrical coordinates will result into:

\[
\frac{1}{r} \frac{\partial E_z}{\partial \phi} + j\beta E_{\phi} = -j\omega\mu_0 H_r, \quad \text{(3.4a)}
\]

\[
-j\beta E_r - \frac{\partial E_z}{\partial r} = -j\omega\mu_0 H_{\phi}, \quad \text{(3.4b)}
\]

\[
\frac{1}{r} \left[ \frac{\partial (rE_\phi)}{\partial r} - E_r \right] = -j\omega\mu_0 H_z, \quad \text{(3.4c)}
\]

\[
\frac{1}{r} \frac{\partial H_z}{\partial \phi} + j\beta H_{\phi} = j\omega\epsilon_0 \left[ \epsilon E_r - j\kappa E_\phi \right], \quad \text{(3.4d)}
\]

\[
-j\beta H_r - \frac{\partial H_z}{\partial r} = j\omega\epsilon_0 \left[ j\kappa E_r + \epsilon E_\phi \right], \quad \text{(3.4e)}
\]

\[
\frac{1}{r} \left[ \frac{\partial (rH_\phi)}{\partial r} - \frac{\partial H_r}{\partial \phi} \right] = j\omega\epsilon_0\zeta E_z \quad \text{(3.4f)}
\]

After some manipulations, another set of equations is yielded in terms of \( E_z \) and \( H_z \):

\[
\nabla^2 H_z + \left[ k_e^2 \left( 1 - \left( \frac{\kappa}{\epsilon} \right)^2 \right) \right] H_z = j\omega\epsilon_0 \zeta \kappa \beta E_z, \quad \text{(3.5a)}
\]

\[
\nabla^2 E_z + \left[ \frac{\zeta}{\epsilon} \left( k_e^2 - \beta^2 \right) + \beta^2 \right] E_z = -j\omega\mu_0 \frac{\kappa}{\epsilon} \beta H_z, \quad \text{(3.5b)}
\]

\[
E_r = \frac{\beta}{\omega\mu_0} \left( j\theta_r \frac{\partial E_z}{\partial r} - \frac{1}{r} \frac{\kappa}{\epsilon} \frac{\partial E_\phi}{\partial \phi} \right) + \left( \frac{\kappa}{\epsilon} \frac{\partial H_z}{\partial r} + j\theta_r \frac{\partial H_\phi}{\partial \phi} \right), \quad \text{(3.5c)}
\]

\[
E_\phi = \frac{\beta}{\omega\mu_0} \left( \frac{\kappa}{\epsilon} \frac{\partial E_z}{\partial r} + j\theta_r \frac{\partial E_\phi}{\partial \phi} \right) - \left( j\theta_r \frac{\partial H_z}{\partial r} - \frac{1}{r} \frac{\kappa}{\epsilon} \frac{\partial H_\phi}{\partial \phi} \right), \quad \text{(3.5d)}
\]

\[
H_r = \frac{j}{\omega\mu_0} \left( \frac{1}{r} \frac{\partial E_z}{\partial \phi} + j\beta E_\phi \right), \quad \text{(3.5e)}
\]

\[
H_\phi = -\frac{j}{\omega\mu_0} \left( \frac{\partial E_z}{\partial r} + j\beta E_r \right) \quad \text{(3.5f)}
\]
Where:

\[ k_e^2 = \omega^2 \mu_0 \epsilon_0 \varepsilon \]  

(3.6)

and

\[ \theta_e = 1 - \frac{\beta^2}{k_e^2} \]  

(3.7)

Due to the inter-coupling between \( E_z \) and \( H_z \) in equations 3.5a and 3.5b, there is no pure Transverse Magnetic (TM), nor Transverse Electric (TE) modes supported by the material in the above conditions. However, the following assumptions have been made to simplify the case:

1. The gyroelectric disk is thin enough such that there is no field variation along the \( z \) axis (\( \frac{\partial}{\partial z} = 0 \)). This is equivalent to making \( \beta = 0 \).

2. Only TE mode is considered through electromagnetic analysis. This is simply because electromagnetic field arrangement in TM modes does not give rise to gyroelectric behaviour (since \( \vec{E} \) is in the same direction as \( B_0 \), and there is no \( \kappa \) term in the modal expression).

The above two assumptions reduces equation (3.5a) to:

\[
\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \phi^2} + k_{eff}^2 H_z = 0 \]  

(3.8)

Where \( k_{eff} \) is the effective wavenumber, and it is defined as:

\[ k_{eff}^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_{eff} \]  

(3.9)

where:

\[ \epsilon_{eff} = \frac{\varepsilon^2 - \kappa^2}{\varepsilon} \]  

(3.10)

is the effective permittivity of the gyroelectric material.

Applying the above assumptions to (3.5c) and (3.5d) yields:

\[
E_r = -\frac{1}{j \omega \varepsilon_0 \epsilon_{eff}} \left( \frac{1}{r} \frac{\partial H_z}{\partial \phi} - \frac{\kappa}{\varepsilon} \frac{\partial H_z}{\partial r} \right), \]  

(3.11a)

\[
E_\phi = -\frac{1}{j \omega \varepsilon_0 \epsilon_{eff}} \left( \frac{\kappa}{\varepsilon} \frac{1}{r} \frac{\partial H_z}{\partial \phi} + \frac{\partial H_z}{\partial r} \right) \]  

(3.11b)
By looking at equation (3.8), it can be concluded that its solution requires the separation of variables. Hence, the general solution for $H_z$ for positive permittivity is:

$$H_{z,n}(r,\phi) = a_n J_n(k_{eff}r)e^{in\phi} \quad (3.12)$$

Where $n$ is an integer, $a_n$ is a constant, and $J_n$ is a first kind of Bessel function of the $n^{th}$ order. For negative values of $\epsilon_{eff}$, effective propagation constant ($k_{eff}$) is purely imaginary, namely $\tilde{k}_{eff} = jk_{eff}$, where:

$$\tilde{k}_{eff} = \omega \sqrt{\mu_0 \epsilon_0 |\epsilon_{eff}|} \quad (3.13)$$

Hence, Helmholtz equation in (3.8) expressed as:

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \phi^2} - \tilde{k}_{eff}^2 H_z = 0 \quad (3.14)$$

And the solution in this case will be:

$$H_{z,n}(r,\phi) = a_n I_n(\tilde{k}_{eff}r)e^{in\phi} \quad (3.15)$$

Where $I_n(x)$ is the modified Bessel function of the first kind.

For positive $\epsilon_{eff}$, substituting (3.12) in (3.11a) and (3.11b) will yield:

$$E_{r,n}(r,\phi) = a_n e^{in\phi} Z_{eff} \left[ \frac{n}{x} J_n(x) - \frac{\kappa}{\epsilon} J'_n(x) \right], \quad (3.16a)$$

$$E_{\phi,n}(r,\phi) = ja_n e^{in\phi} Z_{eff} \left[ J'_n(x) - \frac{\kappa}{\epsilon} \frac{n}{x} J_n(x) \right] \quad (3.16b)$$

Similarly, for negative $\epsilon_{eff}$, substituting (3.15) in (3.11a) and (3.11b) will yield the following expressions:

$$E_{r,n}(r,\phi) = -a_n e^{in\phi} \tilde{Z}_{eff} \left[ \frac{n}{\tilde{x}} I_n(\tilde{x}) - \frac{\kappa}{\epsilon} I'_n(\tilde{x}) \right], \quad (3.17a)$$

$$E_{\phi,n}(r,\phi) = -ja_n e^{in\phi} \tilde{Z}_{eff} \left[ I'_n(\tilde{x}) - \frac{\kappa}{\epsilon} \frac{n}{\tilde{x}} I_n(\tilde{x}) \right] \quad (3.17b)$$

Where:

$$Z_{eff} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_{eff}}} \quad (3.18)$$

$$x = k_{eff}r \quad (3.19)$$

$$\tilde{Z}_{eff} = \sqrt{\frac{\mu_0}{\epsilon_0 |\epsilon_{eff}|}} \quad (3.20)$$
3.2.2 Regions of Operation

From the previous electromagnetic analysis, it is now concluded that the gyroelectric terms $\kappa$ and $\epsilon_{eff}$ determine the behaviour of the gyroelectric material at any given frequency. Hence, the frequency range can be divided into a number of regions of operation depending on the behaviour of each gyroelectric term with the frequency. Introducing these regions will make the analysis of the gyroelectric structures more convenient.

For a lossless semiconductor ($\nu_c = 0$), the behaviour of $\epsilon_{eff}$ with frequency is shown in Figure (3.2). The curve is seen to be rising from $-\infty$ to $+\infty$, crossing zero at $f = f_A$. There is a discontinuity in the graph at $f = f_r$, where:

$$f_r = \sqrt{f_C^2 + \frac{f_p^2}{\sqrt{\epsilon_r}}}$$  \hspace{1cm} (3.22)
Figure 3.3: Regions of operation for $\frac{\kappa}{\epsilon}$

Where $f_C$ and $f_p$ are the cyclotron and plasma frequencies, respectively. $f_r$ is referred to as the extraordinary wave resonance frequency. Above the discontinuity point, $\epsilon_{eff}$ rises from $-\infty$ and approaches $\epsilon_r$ as the frequency approaches infinity, crossing $\epsilon_{eff} = 0$ point at $f = f_B$. The frequencies at which $\epsilon_{eff} = 0$ are expressed as:

$$f_A = \frac{f_C}{2} \left[ \sqrt{1 + \frac{4f_p^2}{\epsilon_r f_C^2}} - 1 \right], \quad (3.23a)$$

$$f_B = \frac{f_C}{2} \left[ \sqrt{1 + \frac{4f_p^2}{\epsilon_r f_C^2}} + 1 \right] \quad (3.23b)$$

Figure (3.3) shows a plot of $\frac{\kappa}{\epsilon}$ versus frequency. This quantity also shows an asymptote at $f = f_r$. It rises from $-\infty$ to reach $-1$ at $f = f_A$, then it increases further until reaching a turning point at $f = f_t$, then it goes down to $-\infty$ crossing $\frac{\kappa}{\epsilon} = -1$ line at $f = f_C$. Above $f_r$, its value decreases from $+\infty$ to approach zero as the frequency increases, crossing $\frac{\kappa}{\epsilon} = +1$ line at $f = f_B$.

The turning frequency of $\frac{\kappa}{\epsilon}$ ($f_t$) can be found by making $\frac{\partial}{\partial f} \frac{\kappa}{\epsilon} = 0$, and solving
for frequency [100].

\[ f_t = \frac{f_r}{\sqrt{3}} \quad (3.24) \]

From (3.23), it is shown that \( f_C = f_B - f_A \). That is, for a fixed plasma frequency, when increasing the cyclotron frequency (by increasing steady magnetic bias, \( B_0 \)), \( f_A \) will decrease, and \( f_B \) will increase. In addition, by examining equations (3.22) and (3.23), it can be concluded that when the cyclotron frequency approaches zero (i.e., when decreasing \( B_0 \)), \( f_A, f_B \) and \( f_r \) will approach \( \frac{f_p}{\sqrt{\epsilon}} \).

It is shown in Figure (3.3) that the frequencies at which \( \frac{\kappa}{\epsilon} \) crosses \(-1\) are not fixed. The reason for that is when \( f_p < \sqrt{2\epsilon_r} f_C \), the \( \frac{\kappa}{\epsilon} \) curve reaches \(-1\) from \(-\infty\) at \( f = f_A \). However, when \( f_p > \sqrt{2\epsilon_r} f_C \), the curve reaches \(-1\) for the first time at \( f = f_C \) instead, as indicated between the two brackets in Figure (3.3). It should be noted that the zero crossing frequencies of \( \epsilon_{eff} \) do not change for the above cases.

Based on the above, regions of operation should be assigned differently depending on the relationship between \( f_C \) and \( f_A \), as follows:

1. **\( f_C > f_A \): Black arrows** in Figure (3.3) indicate these regions, they can be defined as follows:

   **Region I** defined from \( f = 0 \) to \( f = f_A \). Where \( \frac{\kappa}{\epsilon} < -1 \), \( \epsilon_{eff} < 0 \) and the effective wavenumber \( (k_{eff}) \) is imaginary.

   **Region II** defined from \( f = f_A \) to \( f = f_r \). Where \( \frac{\kappa}{\epsilon} < 0 \), \( \epsilon_{eff} > 0 \) and the effective wavenumber \( (k_{eff}) \) is real.

   **Region III** defined from \( f = f_r \) to \( f = f_B \). Where \( \frac{\kappa}{\epsilon} > +1 \), \( \epsilon_{eff} < 0 \) and the effective wavenumber \( (k_{eff}) \) is imaginary.

   **Region IV** defined for the frequencies above \( f_B \). Where \( 0 < \frac{\kappa}{\epsilon} < +1 \), \( \epsilon_{eff} > 0 \) and the effective wavenumber \( (k_{eff}) \) is real.

Division of the frequency range in the above way has been considered and previously reported in the literature [41,100,107].

2. **\( f_C < f_A \):** In this case, an additional region will appear where \( \epsilon_{eff} \) is negative, and \( \frac{\kappa}{\epsilon} \) is between -1 and 0; this region is denoted as (I-B).

   All the regions in this case are marked by **red arrows** in Figure (3.3), they are defined as:
Region I-A Defined from $f = 0$ to $f = f_C$. Where $\frac{\kappa}{\varepsilon} < -1$, $\epsilon_{eff} < 0$ and the effective wavenumber ($k_{eff}$) is imaginary.

Region I-B Defined from $f = f_C$ to $f = f_A$. In this region, $0 > \frac{\kappa}{\varepsilon} > -1$, $\epsilon_{eff} < 0$ and the effective wavenumber ($k_{eff}$) is imaginary.

Region II Defined from $f = f_A$ to $f = f_r$. In this region, $\frac{\kappa}{\varepsilon} < -1$, $\epsilon_{eff} > 0$ and the effective wavenumber ($k_{eff}$) is real.

Region III and IV They are the same as those defined for $f_C > f_r$.

### 3.2.3 Resonant Modes

The presence of electric wall on the sides of the gyroelectric disk in Figure (3.1) forces the tangential electric field to be zero at the disk periphery (at $r = R$). By considering the expressions for $E_\phi$ in (3.16b) and (3.17b), the following equations should be satisfied to make $E_\phi = 0$:

\begin{align}
J'_n(X) - \frac{\kappa}{\varepsilon} \frac{n}{X} J_n(X) &= 0 \quad \text{for } \epsilon_{eff} > 0, & (3.25a) \\
I'_n(\tilde{X}) - \frac{\kappa}{\varepsilon} \frac{n}{\tilde{X}} I_n(\tilde{X}) &= 0 \quad \text{for } \epsilon_{eff} < 0 & (3.25b)
\end{align}

Where $X$ and $\tilde{X}$ are the \textit{normalised wavenumbers}, and they are expressed as:

\begin{align}
X &= k_{eff} R, & (3.26a) \\
\tilde{X} &= \tilde{k}_{eff} R & (3.26b)
\end{align}

Resonance in each of the different regions of operation defined before has been analysed using an iterative algorithm implemented in Matlab to find the values of $X$ (or $\tilde{X}$) that satisfy the resonant conditions in (3.25) for a specific value of $n$ and a range of $\frac{\kappa}{\varepsilon}$.

The modes presented here have been identified as $n[l, m]$, where $l$ is the value of $n$, and $m$ is the index of the set. For example, $n[3, 2]$ stands for a mode in the second set where $n = 3$. Resonant conditions for each region are summarized as follows:

**Region I, and I-A** Since $k_{eff}$ is imaginary in this case, the program is set to search for the values of $\tilde{X}$ that satisfy equation (3.25b). It is found that for each $n$ (where $n < 0$), there is only one solution of the equation. Results
Figure 3.4: Resonant conditions for Regions I and I-A.

for \( n = -1, -2 \) and \(-3\) are shown in Figure (3.4). No solution exists for \( n \geq 0 \).

**Region I-B** Equation (3.25b) should be used in this region since \( k_{\text{eff}} \) is imaginary. However, no solution is found for this case for all \( n \), hence, no resonance occurs in this region.

**Region II** \( k_{\text{eff}} \) is real in this region. Hence, equation (3.25a) is solved. Infinite number of solutions is found for each value of \( n \), first two sets of solutions for \( n = 0, \pm 1, \pm 2 \) and \( \pm 3 \) are shown in Figure (3.5).

**Region III** This region has almost the same properties of Region I, except that \( \frac{\varepsilon}{k} > 1 \). As with Region I, only one set of \( \tilde{X} \) solutions is found, but this time is for positive values of \( n \) only as shown in Figure (3.6).

**Region IV** This region also similar to Region II, the only difference here is that \( 0 < \frac{\varepsilon}{k} < 1 \), that will make the graphs for \( X \) solution sets in Figure (3.7) look like a mirror image of those in Figure (3.5)
Figure 3.5: Resonant conditions for region II.

Figure 3.6: Resonant conditions for region III.
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(a) First set

(b) Second set

Figure 3.7: Resonant conditions for Region IV.

3.3 Two Port Gyroelectric Resonators

This section is about analysing the electromagnetic fields and resonant conditions for a gyroelectric resonator that is coupled to other transmission lines via two ports. This approach has been used first by Bosma [26] to analyse ferrite stripline circulators, and it was also used to analyse gyroelectric resonators [100, 106] and circulators [40]. Here, the same method is used to derive detailed expressions for the scattering parameters of a two port gyroelectric resonator.

Consider the resonator shown in Figure (3.8), where the central locations of the two ports are defined at $\phi = -\pi/2$ and $\phi = \pi/2$. Axial magnetic field is expressed in equations (3.12) and (3.15) for $\epsilon_{\text{eff}} > 0$ and $< 0$, respectively. Expressions for $E_\phi$ and $E_r$ has been shown in equations (3.16) and (3.17) for similar conditions.

In this case, however, the electric wall at $r = R$ is not continuous for all $\phi$. Instead, $E_\phi$ has certain value(s) at the positions of the two ports. These new boundary conditions are expressed as:

$$E_\phi = \begin{cases} 
A & -\frac{\pi}{2} - \psi < \phi < -\frac{\pi}{2} + \psi \\
B & \frac{\pi}{2} - \psi < \phi < \frac{\pi}{2} + \psi \\
0 & \text{Elsewhere}
\end{cases} \quad (3.27)$$

Where $A$ and $B$ are constants represent the value of tangential electric field across ports 1 and 2, respectively. These values are regarded as complex numbers since fields at the two ports may not be in phase.
3.3.1 Finding the Scattering Parameters

Performance of the two port gyroelectric resonator can be determined through its scattering parameters. These parameters are found by knowing the relationships between electromagnetic field components at both ports when exciting one of them with incident electric field and terminating the other with a matched load. Boundary conditions in equation (3.27) are used here to determine $H_z$ at any point on the gyroelectric disk. A Green’s function $(G(r, \phi; r', \phi'))$ is introduced as a weighting function to quantify the contribution of tangential electric field at any port $(E_{\phi}(R, \phi'))$ to the axial magnetic field anywhere on the disk $(H_z(r, \phi))$, as follows:

$$H_z(r, \phi) = \int_{-\pi}^{\pi} G(r, \phi; R, \phi')E_{\phi}(R, \phi')d\phi'$$  \hspace{1cm} (3.28)

Since the values of axial magnetic field are only required on the boundaries (at $r = R$), the function $G(r, \phi; R, \phi')$ and the field $(H_z(r, \phi))$ can be reduced to $G(\phi; \phi')$ and $H_z(\phi)$, respectively. This will make equation (3.28):

$$H_z(\phi) = \int_{-\pi}^{\pi} G(\phi; \phi')E_{\phi}(R, \phi')d\phi'$$  \hspace{1cm} (3.29)
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Substituting $E_\phi$ from the boundary conditions in (3.27) into (3.29) will yield:

$$ H_z(\phi) = \int_{-\frac{\pi}{2}-\psi}^{\frac{\pi}{2}+\psi} G(\phi; \phi') A d\phi' + \int_{\frac{\pi}{2}-\psi}^{\frac{\pi}{2}+\psi} G(\phi; \phi') B d\phi' $$ (3.30)

The average of axial magnetic field across each port ($H_{z1,2}$) is obtained by integrating $H_z(\phi)$ over each port:

$$ H_{z1} = \frac{1}{2\psi} \int_{-\frac{\pi}{2}-\psi}^{-\frac{\pi}{2}+\psi} H_z(\phi) d\phi, \quad (3.31a) $$

$$ H_{z2} = \frac{1}{2\psi} \int_{\frac{\pi}{2}-\psi}^{\frac{\pi}{2}+\psi} H_z(\phi) d\phi \quad (3.31b) $$

Substituting $H_z(\phi)$ from (3.30) into (3.31) will result into a double integration of the Green’s function. Amplitudes of tangential electric fields ($A$ and $B$) are assumed constants across ports 1 and 2, respectively. Hence, they are not included in the integration. In result, integrated Green’s function can be defined as:

$$ \bar{G}(\phi; \phi') = \frac{1}{2\psi} \int_{-\frac{\pi}{2}-\psi}^{\frac{\pi}{2}+\psi} \int_{\frac{\pi}{2}-\psi}^{\frac{\pi}{2}+\psi} \bar{G}(\phi; \phi') d\phi' d\phi $$ (3.32)

Now, let’s find an expression for the Green’s function from the modal analysis in Section (3.2) and the boundary conditions. By considering equation (3.27) as periodic function with a period of $2\pi$, $E_\phi$ at disk periphery can be expressed as an infinite sum of terms defined by Fourier series.

$$ E_\phi(R, \phi) = \sum_{n=-\infty}^{\infty} D_n e^{jn\phi} $$ (3.33)

Where:

$$ D_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} E_\phi(R, \phi') e^{-jn\phi'} d\phi' $$ (3.34)

On the other hand, $E_\phi$ is also expressed as an infinite series of components expressed in (3.16b) and (3.17b) for positive and negative $\epsilon_{eff}$, respectively. The concept of boundary conditions can be applied to find $a_n$ in equation (3.16b) by
equating it to (3.33)\(^1\). Hence, the constant \(a_n\) will become\(^2\):

\[
a_n = \frac{D_n}{jZ_{\text{eff}} [J'_n(X) - \frac{\kappa}{\varepsilon} \frac{n}{X} J_n(X)]} = \frac{\int_{-\pi}^{\pi} E_\phi(R, \phi') e^{-jn\phi'} d\phi'}{j2\pi Z_{\text{eff}} [J'_n(X) - \frac{\kappa}{\varepsilon} \frac{n}{X} J_n(X)]} \tag{3.35}
\]

Now, \(a_n\) can be used to find an expression for the \(n\)th component of \(H_z\) in (3.12) for \(r = R\):

\[
H_{z,n}(R, \phi) = e^{jn\phi} J_n(X) \int_{-\pi}^{\pi} E_\phi(R, \phi') e^{-jn\phi'} d\phi' \tag{3.36}
\]

Total axial magnetic field on disk periphery \((H_z(R, \phi))\) is the sum of infinite terms of (3.36), and it is expressed as:

\[
H_z(R, \phi) = \int_{-\pi}^{\pi} \left[ -\frac{j}{2\pi Z_{\text{eff}}} \sum_{n=-\infty}^{\infty} J_n(X) e^{jn(\phi - \phi')} \right] E_\phi(R, \phi') d\phi' \tag{3.37}
\]

Comparing equation (3.37) with the definition of the Green’s function in (3.29), the following expression for the Green’s function will result:

\[
G(\phi; \phi') = \frac{-j}{2\pi Z_{\text{eff}}} \sum_{n=-\infty}^{\infty} J_n(X) e^{jn(\phi - \phi')} \tag{3.38}
\]

Knowing that \(J_{-n}(X) = (-1)^n J_n(X)\), (3.38) can be rewritten as:

\[
G(\phi; \phi') = \frac{-j J_0(X)}{2\pi Z_{\text{eff}} J'_0(X)} + \frac{1}{\pi Z_{\text{eff}}} \sum_{n=1}^{\infty} \frac{\frac{\kappa}{\varepsilon} \frac{n}{X} J_n^2(X) \sin[n(\phi - \phi')] - j J'_n(X) J_n(X) \cos[n(\phi - \phi')]}{[J'_n(X)]^2 - \left[\frac{\kappa}{\varepsilon} \frac{n}{X} J_n(X)\right]^2} \tag{3.39}
\]

As shown before, the average axial magnetic field over each port can be found by integrating \(H_z\) over the port’s span angle. Hence, Green’s function has to be integrated as in (3.32) to describe the averaged \(H_z\) at each port. The integrated

\(^1\)Positive \(\varepsilon_{\text{eff}}\) is considered first.
\(^2\)Note that \(E_\phi\) in (3.16b) is in terms of \(r\) and \(x = k_{\text{eff}}r\), while it will be substituted here with \(r = R\) and \(X = k_{\text{eff}}R\).
Green’s function \( \tilde{G}(\phi; \phi') \) would be:

\[
\tilde{G}(\phi; \phi') = \frac{-jJ_0(X)\psi}{\pi Z_{eff}J'_0(X)} + \frac{2}{\pi Z_{eff}} \sum_{n=1}^{\infty} \frac{\sin^2(n\psi)}{n^2\psi} \frac{\frac{2}{\pi} n J_n^2(X) \sin[n(\phi - \phi')] - j J'_n(X) J_n(X) \cos[n(\phi - \phi')]}{[J'_n(X)]^2 - [\frac{2}{\pi} n J_n(X)]^2}
\]

\( (3.40) \)

The above derivation can be repeated for \( \epsilon_{eff} < 0 \) starting from finding \( a_n \) by equating (3.27) to (3.17b) instead of (3.16b). Following the same procedure will yield the following expression for integrated Green’s function in terms of modified Bessel function:

\[
\bar{G}(\phi; \phi') = \frac{jI_0(\tilde{X})\psi}{\pi Z_{eff}I'_0(\tilde{X})} - \frac{2}{\pi Z_{eff}} \sum_{n=1}^{\infty} \frac{\sin^2(n\psi)}{n^2\psi} \frac{\frac{2}{\pi} I_n(\tilde{X}) \sin[n(\phi - \phi')] - j I'_n(\tilde{X}) I_n(\tilde{X}) \cos[n(\phi - \phi')]}{[I'_n(\tilde{X})]^2 - [\frac{2}{\pi} I_n(\tilde{X})]^2}
\]

\( (3.41) \)

Deriving scattering parameters of the two port resonator starts with expressing averaged axial magnetic field across each port in terms of Green’s functions and the tangential electric fields at the two ports.

\[
H_{z1} = \tilde{G}_{1,1} A + \tilde{G}_{1,2} B, \quad \text{H}(3.42a)
\]

\[
H_{z2} = \tilde{G}_{2,1} A + \tilde{G}_{2,2} B \quad \text{H}(3.42b)
\]

Where \( \tilde{G}_{1,1} \) is the integrated Green’s function that express the contribution of \( E_{\phi} \) at port 1 to \( H_{z1} \), in the same way \( \tilde{G}_{1,2} \) expresses the contribution of \( E_{\phi} \) at port 2 to \( H_{z1} \). The same applies to \( \tilde{G}_{2,1} \) and \( \tilde{G}_{2,2} \).

As mentioned before, the centres of the two ports are assumed to be at \( \phi = -\pi/2 \) and \( \pi/2 \). Due to the cyclic symmetry of the resonator, only two integrated Green’s functions may be defined as follows:

\[
\tilde{G}_1 = \tilde{G}(-\frac{\pi}{2}; \frac{\pi}{2}) = \tilde{G}(\frac{\pi}{2}; \frac{\pi}{2}), \quad (3.43a)
\]

\[
\tilde{G}_2 = \tilde{G}(-\frac{\pi}{2}; \frac{\pi}{2}) = \tilde{G}(\frac{\pi}{2}; \frac{\pi}{2}) \quad (3.43b)
\]
Hence, the average axial magnetic fields at each port in (3.42) can be redefined as:

\[ H_{z1} = \tilde{G}_1 A + \tilde{G}_2 B, \quad (3.44a) \]
\[ H_{z2} = \tilde{G}_2 A + \tilde{G}_1 B \quad (3.44b) \]

The scattering matrix of a symmetrical two port resonator can be expressed as:

\[
[S] = \begin{bmatrix}
S_{11} & S_{12} \\
S_{12} & S_{11}
\end{bmatrix}
\]

(3.45)

Assume a signal characterized by \((E_i \hat{\phi} \text{ and } H_i \hat{z})\) is fed to port 1 of the two port resonator in Figure (3.8). Port 2 is assumed to be terminated with a matched load. Tangential electric field and axial magnetic field can be expressed in terms of scattering parameters as:

\[ A = (1 + S_{11})E_i \quad (3.46) \]
\[ H_{z1} = (1 - S_{11})H_i \quad (3.47) \]

Assuming that the incident wave is coming from a surrounding media with dielectric constant of \(\epsilon_d\), and admittance of \(Y_d\), the ratio between incident electric and magnetic wave should be:

\[ \frac{H_i}{E_i} = -Y_d \quad (3.48) \]

The minus signal in (3.48) indicates a wave incident to the gyroelectric disk according to the chosen coordinate system.

At port 2, since there is no reflected wave (as assumed), the ratio between axial magnetic and tangential electric fields is given by:

\[ \frac{H_{z2}}{B} = Y_d \quad (3.49) \]

By virtue of (3.44b) and (3.49), electrical field at port 2 \((B)\) can be expressed in terms of the field at port 1 \((A)\) as:

\[ \frac{B}{A} = \frac{\tilde{G}_2}{(Y_d - G_1)} \quad (3.50) \]
Substituting (3.50) in (3.44a) will yield:

\[ H_{z_1} = \bar{G}_1 A + \frac{\bar{G}_2^2 A}{(Y_d - \bar{G}_1)} \]  

And using (3.47) and (3.48) for \( H_{z_1} \), the following equation will result:

\[ -Y_d (1 - S_{11}) E_i = A \left[ \bar{G}_1 + \frac{\bar{G}_2^2}{(Y_d - \bar{G}_1)} \right] \]  

Finally, using (3.46) will yield:

\[ \frac{S_{11} - 1}{S_{11} + 1} = \frac{\bar{G}_1 (Y_d - \bar{G}_1) + \bar{G}_2^2}{Y_d (Y_d - \bar{G}_1)} \]  

Algebraic manipulation of (3.53) will result into the following expression for \( S_{11} \):

\[ S_{11} = \frac{Y_d^2 - \bar{G}_1^2 + \bar{G}_2^2}{Y_d^2 - 2Y_d \bar{G}_1 + \bar{G}_1^2 - \bar{G}_2^2} \]  

\( S_{12} \) in (3.45) can be defined as the ratio between the tangential electric field at port 2 to the incident electric field at port 1, or:

\[ S_{12} = S_{21} = \frac{B}{E_i} \]  

Now, (3.50) is substituted in (3.55) to result:

\[ S_{12} = A \left[ \frac{\bar{G}_2}{Y_d - \bar{G}_1} \right] \]  

And by virtue of (3.46), an expression for \( S_{12} \) will result:

\[ S_{12} = (1 + S_{11}) \left[ \frac{\bar{G}_2}{Y_d - \bar{G}_1} \right] = \frac{2Y_d \bar{G}_2}{Y_d^2 - 2Y_d \bar{G}_1 + \bar{G}_1^2 - \bar{G}_2^2} \]  

It is more convenient to express scattering parameters in dB, thus, \( S_{11} \) and \( S_{12} \) can be defined as:

\[ S_{11} = 20 \log \left| \frac{Y_d^2 - \bar{G}_1^2 + \bar{G}_2^2}{Y_d^2 - 2Y_d \bar{G}_1 + \bar{G}_1^2 - \bar{G}_2^2} \right| \]  

(3.58)
3.3.2 Performance of an InSb Resonator

The scattering parameters were calculated here for a lossless two port InSb resonator using the Green’s function approach. The measured electronic parameters of InSb illustrated in Chapter (2) were used. Here, the considered two port resonator has a radius ($R$) of (50 µm), coupled\(^3\) to a surrounding medium of $\epsilon_d = 20$ via two ports with coupling half angle ($\psi$) of 0.05 rad. Biasing the lossless InSb with $B_0 = 0.5$ T in $z$-direction will result $\xi$ and $\epsilon_{eff}$ to behave with frequency as shown in Figure (3.9). For the given value of $B_0$, $f_A$ is found to be = 52.69 GH$z$ and $f_C = 1.07$ TH$z$. These two frequency points represent the limits of a part of Region II, where $\epsilon_{eff}$ is positive and $-1 \leq \frac{\xi}{\epsilon} \leq 0$.

The resonant mode of the resonator can be predicted by calculating the value of $k_{eff}R$ for the desired frequency range, and then projecting the $\frac{\xi}{\epsilon} - k_{eff}R$ line on the mode chart of Region II previously shown in Figure (3.5). Here, the $\frac{\xi}{\epsilon} - k_{eff}R$

\(^3\)The values of $\epsilon_d$ and $\psi$ were chosen to minimise the coupling effects.
Figure 3.10: Projection of $k_{eff}R$ for the InSb resonator on the first set of resonant conditions for Region II. Points of intersection are indicated by arrows.

line is plotted against the first set of the resonant regions. It can be seen that $\frac{\kappa}{\varepsilon} - k_{eff}R$ line crosses four of the resonant lines, as indicated by the numbers in Figure (3.10).

Using the Green’s function approach, the scattering parameters for this resonator were calculated and plotted in Figure (3.11)\(^4\). The results illustrate the resonances at the four frequencies as points of maximum transmission ($S_{12,21}$) and minimum reflection ($S_{11,22}$), as indicated by the numbers.

### 3.4 Semiconductor Junction Circulators

Figure (3.12) shows a schematic diagram of a Semiconductor Junction Circulator (SJC). It consists of a circular shaped semiconductor of a radius ($R$) surrounded by a metallic wall which forms an electric boundary. The metallic boundary cease to exist at three 120° apart coupling ports around the circular disk.

As with the gyroelectric resonator, a magnetic wall is assumed to be on the top and bottom of the SJC disk. In addition, the disk is assumed to be thin enough

\(^4\)Thirteen Fourier series terms were used to calculate these parameters, i.e., $n = 0, \pm 1, \pm 2, \pm 3, \ldots , \pm 6$. 
Figure 3.11: Calculated scattering parameters for the lossless InSb two port resonator, relating the resonance frequencies with the points of intersection in Figure (3.10).

Figure 3.12: A diagram of the Semiconductor Junction Circulator (SJC).
so that the axial change in the electromagnetic fields is neglected. Another assumption was made by not considering the $TM$ modes into calculation since they do not give rise to any gyroelectric behaviour in the magnetised semiconductor.

### 3.4.1 Finding the Scattering Parameters

As shown in Figure (3.12), it is evident that the only difference between the gyroelectric circulator discussed here and the resonator illustrated in Figure (3.8) is the boundary conditions. Here, ports are assumed to be at $\phi = \pi, \frac{\pi}{3},$ and $-\frac{\pi}{3}$, and each port spans for an angle of $2\psi$. Hence, boundary conditions are assumed to be as follows:

$$E_{\phi}(R, \phi) = \begin{cases} 
A & \pi - \psi < \phi < \pi + \psi \\
B & \frac{\pi}{3} - \psi < \phi < \frac{\pi}{3} + \psi \\
C & \frac{\pi}{3} - \psi < \phi < -\frac{\pi}{3} + \psi \\
0 & \text{Elsewhere}
\end{cases}$$

(3.60)

Where $A$, $B$, and $C$ can be expressed as complex numbers since electric field intensities at the three ports are (generally) different in phase. Electromagnetic field analysis in Section (3.3) can be applied here without any change. Hence, the axial magnetic field and tangential electric fields on the circulator can be expressed the same way as in equations (3.12-3.17b) in terms of normal and modified Bessel functions. The Green’s function that expresses the axial magnetic field on the disk in terms of electric field on the port does not change from that depicted in (3.40) and (3.41) either. However, the different boundary conditions in the circulator case changes the derivation of the scattering parameters.

To find the expressions for the scattering parameters, the same procedure in (3.3.1) is followed. First, the average axial magnetic field at each port is expressed as follows:

$$H_{z1} = \tilde{G}_{1,1}A + \tilde{G}_{1,2}B + \tilde{G}_{1,3}C,$$  \hspace{1cm} (3.61a)

$$H_{z2} = \tilde{G}_{2,1}A + \tilde{G}_{2,2}B + \tilde{G}_{2,3}C,$$  \hspace{1cm} (3.61b)

$$H_{z3} = \tilde{G}_{3,1}A + \tilde{G}_{3,2}B + \tilde{G}_{3,3}C$$  \hspace{1cm} (3.61c)
Where $\tilde{G}_{i,j}$ represents the effect of tangential electric field at port $(j)$ on the axial magnetic field at port $(i)$. Due to the cyclic symmetry of the circulator, it is possible to define only three integrated Green’s functions:

$$\tilde{G}_1 = \tilde{G}(\pi; \pi) = \tilde{G}(\frac{\pi}{3}; \frac{\pi}{3}) = \tilde{G}(-\frac{\pi}{3}; -\frac{\pi}{3}), \quad (3.62a)$$
$$\tilde{G}_2 = \tilde{G}(\pi; \frac{\pi}{3}) = \tilde{G}(\frac{\pi}{3}; -\frac{\pi}{3}) = \tilde{G}(-\frac{\pi}{3}; \pi), \quad (3.62b)$$
$$\tilde{G}_3 = \tilde{G}(\pi; -\frac{\pi}{3}) = \tilde{G}(-\frac{\pi}{3}; \frac{\pi}{3}) = \tilde{G}(\frac{\pi}{3}; \pi) \quad (3.62c)$$

Thus, equation (3.61) can be replaced by:

$$H_{z1} = \tilde{G}_1 A + \tilde{G}_2 B + \tilde{G}_3 C, \quad (3.63a)$$
$$H_{z2} = \tilde{G}_2 A + \tilde{G}_1 B + \tilde{G}_3 C, \quad (3.63b)$$
$$H_{z3} = \tilde{G}_3 A + \tilde{G}_2 B + \tilde{G}_1 C \quad (3.63c)$$

The scattering matrix for a three port junction is described as:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad (3.64)$$

by assuming a symmetric three port circulator that is circulating counter clockwise, the following approximations can be made:

$$S_{11} = S_{22} = S_{33}, \quad (3.65a)$$
$$S_{12} = S_{23} = S_{31}, \quad (3.65b)$$
$$S_{13} = S_{21} = S_{32} \quad (3.65c)$$

Hence, scattering matrix in (3.64) will be:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{13} & S_{11} & S_{12} \\ S_{12} & S_{13} & S_{11} \end{bmatrix} \quad (3.66)$$

To find the scattering parameters in (3.66), the same previously reported procedure [40, 100, 108] is repeated here. First, assuming a wave represented by tangential electric field ($E_i$) and axial magnetic field ($H_i$) is applied to port 1.
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Assuming all other ports are terminated with matched loads (i.e., no incoming waves coming from ports 2 and 3), the following relationships for electric and magnetic fields at the matched ports will apply:

\[ B = \frac{H_{z2}}{Y_d} \quad (3.67a) \]
\[ C = \frac{H_{z3}}{Y_d} \quad (3.67b) \]

The same ratio applies to the incident electric and magnetic fields as:

\[ -Y_d = \frac{H_i}{E_i} \quad (3.68) \]

As a result, the total electric and magnetic fields at port 1 will be:

\[ A = (1 + S_{11})E_i, \quad (3.69a) \]
\[ H_{z1} = (1 - S_{11})H_i \quad (3.69b) \]

By virtue of (3.68), the ratio between magnetic and electric field at the input port will be:

\[ \frac{H_{z1}}{A} = \frac{(1 - S_{11})}{(1 + S_{11})} Y_d \quad (3.70) \]

Now, from (3.63b) and (3.63c) and by virtue of (3.67), \( B \) and \( C \) can be expressed in terms of \( A \) as follows:

\[ B = \left[ \frac{(Y_d - \tilde{G}_1)\tilde{G}_3 + \tilde{G}_2^2}{(Y_d - \tilde{G}_1)^2 - \tilde{G}_2\tilde{G}_3} \right] A \quad (3.71) \]
\[ C = \left[ \frac{(Y_d - \tilde{G}_1)\tilde{G}_2 + \tilde{G}_3^2}{(Y_d - \tilde{G}_1)^2 - \tilde{G}_2\tilde{G}_3} \right] A \quad (3.72) \]

Substituting (3.71) and (3.72) in (3.63a) will yield:

\[ \frac{H_{z1}}{A} = \frac{\tilde{G}_3^3 + \tilde{G}_2^3 + \tilde{G}_1^3 + \tilde{G}_1Y_d^2 + 2\tilde{G}_2\tilde{G}_3Y_d - 2\tilde{G}_1^2Y_d - 3\tilde{G}_1\tilde{G}_2\tilde{G}_3}{(Y_d - \tilde{G}_1)^2 - \tilde{G}_2\tilde{G}_3} \quad (3.73) \]

Equating (3.73) and (3.70) can result into an expression for \( S_{11} \) as:

\[ S_{11} = -1 - \frac{j\pi Z_{eff}Y_d [Q_1^2 - Q_2Q_3]}{[Q_1^2 + Q_2^3 + Q_3^3 - 3Q_1Q_2Q_3]} = S_{22} = S_{33} \quad (3.74) \]
Where:

\[ Q_1 = \frac{1}{2} (\tilde{G}_1 - Y_d)(j\pi Z_{eff}), \]  
\[ Q_2 = \frac{j\pi \tilde{G}_2 Z_{eff}}{2}, \]  
\[ Q_3 = \frac{j\pi \tilde{G}_3 Z_{eff}}{2} \]

(3.75a)  
(3.75b)  
(3.75c)

\( S_{12} \) and \( S_{13} \) can be defined as the ratios between electric field in port 3 and 2, respectively, and the incident electric field \( E_i \), such that:

\[ S_{12} = \frac{B}{E_i} \]  
\[ S_{13} = \frac{C}{E_i} \]

(3.76)  
(3.77)

By virtue of (3.71) and (3.69), an expression for \( S_{12} \) will yield:

\[ S_{12} = -j\pi Z_{eff} Y_d \left[ \frac{Q_2^2 - Q_1 Q_3}{Q_1^2 + Q_2^2 + Q_3^2 - 3Q_1 Q_2 Q_3} \right] = S_{23} = S_{31} \]

(3.78)

Similarly, (3.72) and (3.69) can be used to produce an expression for \( S_{13} \):

\[ S_{13} = -j\pi Z_{eff} Y_d \left[ \frac{Q_3^2 - Q_1 Q_2}{Q_1^2 + Q_2^2 + Q_3^2 - 3Q_1 Q_2 Q_3} \right] = S_{21} = S_{32} \]

(3.79)

As with the two port resonator, it is useful to have expressions for the scattering parameters in decibels (dB’s) to compare theoretical performance results with practical ones.

\[ S_{11\,dB} = 20 \log |S_{11}|, \]  
\[ S_{12\,dB} = 20 \log |S_{12}|, \]  
\[ S_{13\,dB} = 20 \log |S_{13}| \]

(3.80a)  
(3.80b)  
(3.80c)

All the above analysis has been done assuming a counter clockwise circulator. For a counterclockwise circulator, the exact same procedure can be repeated by swapping \( S_{12} \) with \( S_{13} \).
3.4.2 Perfect Circulation Conditions

The first step in designing a circulator at a specific frequency is to find parameter values at which the junction can work as an ideal circulator. Hence, the conditions for such circulator should be derived.

An ideal circulator has zero reflection coefficient ($S_{11}$) and isolation ($S_{12}$), and a unity transmission coefficient ($S_{13}$). Hence, the scattering matrix should be as follows:

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$  \hspace{1cm} (3.81)

Perfect circulation condition can be found by equating $S_{12}$ in equation (3.78) to zero, which reveals the following condition for $S_{12}$ to vanish:

$$Q_2^2 = Q_1 Q_3$$  \hspace{1cm} (3.82)

Simplifying the expression in (3.82) is done by separating $Q_{1,2,3}$ into real and imaginary parts. Which results into [100]:

$$Q_1 = R + jS, \hspace{1cm} (3.83a)$$
$$Q_2 = V + jW, \hspace{1cm} (3.83b)$$
$$Q_3 = V - jW \hspace{1cm} (3.83c)$$

Where:

$$R = \frac{\psi J_0(X)}{2J'_0(X)} + \sum_{n=1}^{\infty} \frac{\sin^2(n\psi)}{n^2\psi} \frac{J_n(X)J'_n(X)}{[J'_n(X)]^2 - [\frac{n}{X} J_n(X)]^2}, \hspace{1cm} (3.84a)$$

$$S = -\frac{\pi Z_{eff}}{2Z_d}, \hspace{1cm} (3.84b)$$

$$V = \frac{\psi J_0(X)}{2J'_0(X)} + \sum_{n=1}^{\infty} \frac{\sin^2(n\psi)}{n^2\psi} \frac{J_n(X)J'_n(X)\cos(\frac{2\pi n}{3})}{[J'_n(X)]^2 - [\frac{n}{X} J_n(X)]^2}, \hspace{1cm} (3.84c)$$

$$W = -\sum_{n=1}^{\infty} \frac{\sin^2(n\psi)}{n^2\psi} \frac{\frac{n}{X} J_2^2(X)\sin(\frac{2\pi n}{3})}{[J'_n(X)]^2 - [\frac{n}{X} J_n(X)]^2} \hspace{1cm} (3.84d)$$

Assuming a clockwise circulator, for a counter clockwise circulator, $S_{13}$ should be zero.
Substituting equations (3.83a - 3.83c) into (3.82) will yield the following two equations:

\[ R = V \frac{V^2 - 3W^2}{V^2 + W^2} \]  \hspace{1cm} (3.85)

\[ S = W \frac{3V^2 - W^2}{V^2 + W^2} \]  \hspace{1cm} (3.86)

Equations (3.85) and (3.86) are referred to as the **first** and **second** perfect circulation conditions, respectively.

An iterative Matlab program was written to find the value(s) of \( X \) that satisfies equation (3.85) over a range of \( \frac{\epsilon}{\varepsilon} \) for different coupling half angles (\( \psi \)).

Solutions for first circulation condition plotted in Figure (3.13a) were obtained by considering the circulator operation to be in the parts of Regions II and IV where \( \epsilon_{eff} \) is positive, which results into using normal Bessel function in equations (3.84a - 3.84d). Since the solution for \( X \) is the same for positive and negative \( \frac{\epsilon}{\varepsilon} \), values of \( \frac{\epsilon}{\varepsilon} \) were considered as magnitudes between 0 and 1.

For the above values of \( \frac{\epsilon}{\varepsilon} \), there are many solutions for \( X \) that satisfy (3.85). However, only the smallest values \( X \) were considered here. These set of values represent the **first mode of circulation**. Higher modes of circulation are also possible for \( 0 < |\frac{\epsilon}{\varepsilon}| < 1 \), and have been reported previously for gyromagnetic [109] and gyroelectric [110] circulators.

Values of \( X \) resulted from the solution of the first perfect circulation condition were applied to the second circulation condition in (3.86), where it was solved for the ratio between slotline impedance (\( Z_d \)) and effective impedance of the gyroelectric material (\( Z_{eff} \)). The resulted solutions form the second circulation conditions, and they are plotted in Figure (3.13b) for \( 0 < |\frac{\epsilon}{\varepsilon}| < 1 \).

When considering the case when \( \epsilon_{eff} \) is negative, modified Bessel function has to be used to find the solution of (3.85). In this case, only one set of (\( \tilde{X} \)) is found, since there is only one value of (\( \tilde{X} \)) satisfies equation (3.85) for each \( \frac{\epsilon}{\varepsilon} \) and \( \psi \).

First and second perfect circulation condition graphs for this case are shown in Figures (3.14).

### 3.5 Conclusions

In this chapter, the theory of using a thin gyroelectric disk to realise resonators and circulators was illustrated. Assuming a semiconductor disk with electric wall
CHAPTER 3. THEORY OF GYROELECT. RES.S AND CIR.S

(a) First circulation conditions.

(b) Second circulation conditions.

Figure 3.13: Perfect circulation conditions for positive $\epsilon_{eff}$, where normal Bessel function is used.
CHAPTER 3. THEORY OF GYROELECT. RES.S AND CIR.S

(a) First circulation conditions. 

\[
\psi = 0.1 \text{ rad} \quad \psi = 0.3 \text{ rad} \quad \psi = 0.5 \text{ rad} \quad \psi = 0.7 \text{ rad} \quad \psi = 1.0 \text{ rad}
\]

\[\text{Normalized Wavenumber, } \tilde{\chi} \]

\[\frac{|K|}{\epsilon} \]

(b) Second circulation conditions.

\[
\psi = 0.1 \text{ rad} \quad \psi = 0.3 \text{ rad} \quad \psi = 0.5 \text{ rad} \quad \psi = 0.7 \text{ rad} \quad \psi = 1.0 \text{ rad}
\]

\[\left| \frac{Z_{\Delta \psi}}{Z_{\psi}} \right| \]

\[\frac{|K|}{\epsilon} \]

Figure 3.14: Perfect circulation conditions for negative $\epsilon_{eff}$, where modified Bessel function is used.
on its side and magnetic walls on its top and bottom, Maxwell’s equations were solved, given a magnetic bias ($B_0$) in the disk’s axial direction. The disk was also assumed to be thin enough so that no field change occurs along its axis.

Modal analysis revealed that this disk resonates under certain conditions, where two quantities (namely, $\epsilon_{\text{eff}}$ and $\kappa_\varepsilon$) play a major role to characterise them. To utilise these resonances, a two port resonator was considered by introducing two ports on either side of the disk. Expressions for the scattering parameters for this resonator were derived by using a Green’s function approach. Gyroelectric modes were verified when calculating the scattering parameters of a lossless InSb disk of 50 $\mu m$ radius, axially biased with $B_0 = 0.5 \, T$.

The semiconductor disk structure was also used to design Semiconductor Junction Circulators (SJC’s) by assuming equally spaced three ports on the sides of the disk. After deriving expressions for the scattering parameters using the Green’s function approach, the design parameters with which circulation occurs were derived and plotted. These conditions are important for designing SJC’s, as will be shown in the next chapter.
Chapter 4

Semiconductor Junction Circulators

4.1 Introduction

After covering theoretical basics of the gyroelectric resonators and circulators in Chapter (3), this chapter will apply these principles to design different types of Semiconductor Junction Circulators (SJC’s).

The chapter starts by illustrating a general overview of the procedure used to find the design parameters of a SJC, namely the magnetic bias ($B_0$), coupling half angle ($\psi$) and radius ($R$). Based on this approach, two SJC designs using InSb at 77 K will be illustrated. The difference between these two circulators is the frequency at which they operate. For the first one, it was previously designed [73] to work below the extraordinary wave resonant frequency ($f_r$). The SJC is realised and measured at 77 K with high value of magnetic bias ($B_0$). The second design, on the other hand, is aimed to work above $f_r$ to minimise the required magnetic bias for operation in the sub-millimetre-wave frequency range.

CST MWS simulation package is used to simulate the latter SJC. Next, an algorithm is suggested to realise SJC’s with broader bandwidth. This is done by tracking the perfect circulation conditions in a systematic manner after choosing an optimum value of magnetic bias ($B_0$).

Finally, the possibility of using magnetised Two Dimensional Electron Gas (2-DEG) layers to design resonators and circulators is discussed. Because of the unique electronic properties of this material, the perfect circulation conditions have to be extended to allow the 2-DEG SJC to be realised.
CHAPTER 4. SEMICONDUCTOR JUNCTION CIRCULATORS

4.2 Design and Implementation of Basic SJC’s

A gyroelectric circulator can be designed to work at a certain frequency by finding the normalised wavenumber (\( X = k_{\text{eff}}R \)) and coupling half angle (\( \psi \)) necessary to achieve circulation. This is done by using the values of \( \kappa \), \( \epsilon \), and \( Z_{\text{eff}} \) and following these steps:

1. After specifying the frequency of operation, the value of \( \frac{\kappa}{\epsilon} \) and \( Z_{\text{eff}} \) are found at that frequency for a certain value of magnetic bias (\( B_0 \)). Impedance of the surrounding slotline (\( Z_d \)) can also be found from its dielectric constant (\( \epsilon_d \)).

2. Knowing \( \frac{\kappa}{\epsilon} \) and \( \frac{Z_d}{Z_{\text{eff}}} \), the second circulation condition graph is used to find the point at which the values of those two parameters intersect. Coupling half angle (\( \psi \)) is found to be that associated with the closest (\( \frac{Z_d}{Z_{\text{eff}}} \)) line passing by the crossing point, as shown in Figure (4.1a).

3. By using the first circulation conditions graph, the value of \( X \) can be found as the value of the line associated with \( \psi \) (from the last step) at the known \( \frac{\kappa}{\epsilon} \) point, as shown in Figure (4.1b).

4. After finding \( X = k_{\text{eff}}R \), the disk radius (\( R \)) is found after calculating \( k_{\text{eff}} \) from the material properties at the desired frequency of circulation.
The above points can be applied to the cases where \( \epsilon_{eff} \) is negative by using the parameters \( \tilde{X} \) and \( \frac{Z_d}{Z_{eff}} \) instead of \( X \) and \( \frac{Z_d}{Z_{eff}} \), respectively, in Figure (3.14).

A narrowband circulator is resulted from designing a circulator to work at a single frequency. This type of circulators is characterised by intersecting the perfect circulation conditions in Figure (3.14) (or (3.13)) at a single point only. On the other hand, broad band circulators are feasible by careful choice of the coupling half angle (\( \psi \)) and the dielectric constant of the surround (\( \epsilon_d \)) such that the change in \( k_{eff}R \) and \( \frac{Z_d}{Z_{eff}} \) with \( \epsilon_{eff} \) will track the change in \( X \) and \( \frac{Z_d}{Z_{eff}} \) in the first and second circulation conditions, respectively [32, 41]. Tracking the perfect circulation conditions is not restricted to the frequencies where \( \epsilon_{eff} \) is positive. It can also extend to enter frequency ranges where \( \epsilon_{eff} \) is negative, i.e., Regions I and III.

### 4.2.1 SJC Design Below the Resonance Frequency (\( f_r \))

To demonstrate the operation of a SJC, a previously designed circulator with an InSb disk at 77 K with \( R = 1 \) mm and a thickness of 0.72 mm is used [73]. As discussed in Chapter (3), the design procedure depends on the behaviour of \( \frac{\psi}{\epsilon} \) and \( \epsilon_{eff} \) within the frequency range of interest. Here, the circulator was designed to work below the extraordinary wave resonance frequency (\( f_r \)) within the Ka frequency band\(^1\) (i.e., in Regions I and II).

Using the measured electronic characteristics of InSb at 77 K illustrated in Chapter (2), biasing the sample with \( B_0 = 0.8 \) T in the \( z \)-direction will result into a change in \( \frac{\psi}{\epsilon} \) and \( \epsilon_{eff} \) with frequency as shown in Figure (4.2).

For the given conditions, the extraordinary wave resonance frequency (\( f_r \)) is found to be (1.74 \( THz \)) and the real value of \( \epsilon_{eff} \) crosses zero for the first time at \( f_A = 33.87 \) GHz. Assuming the InSb disk to have the boundary conditions shown in Figure (3.1), it will exhibit the resonant characteristics of Region I for \( f = 26.5 - 33.87 \) GHz, and Region II for the rest of the band (till \( f = 40 \) GHz). The insets of Figure (4.2) show that, within the Ka-band, the real value of \( \frac{\psi}{\epsilon} \) changes between \(-1.3\) and \(-0.85\). Assuming the surrounding medium to have a dielectric constant (\( \epsilon_d \)) of 2.2, the quantities \( k_{eff}R \) and \( \frac{Z_d}{Z_{eff}} \) can be found for the above range of \( \frac{\psi}{\epsilon} \). Figure (4.3) shows the projection of \( k_{eff}R, \tilde{k}_{eff}R \) and \( \frac{Z_d}{Z_{eff}} \) on the first and second perfect circulation conditions (for both positive

\(^1\) \( f = 26.5 - 40 \) GHz
Figure 4.2: The change of $\kappa / \epsilon$ and $\epsilon_{\text{eff}}$ with frequency for InSb at 77 K biased with $B_0 = 0.8 \, T$ in the $z$–direction.
CHAPTER 4. SEMICONDUCTOR JUNCTION CIRCULATORS

(a) First circulation conditions.

(b) Second circulation conditions.

Figure 4.3: The intersection of $k_{eff} R$ and $\frac{Z_d}{Z_{eff}}$ with the first and second perfect circulation conditions for the Ka-band circulator.
and negative $\epsilon_{eff}$). The projection shows that both quantities track the perfect circulation conditions for a value of $\psi$ of less than 0.1 rad. Hence, it was assumed that circulation within the Ka frequency band is achievable using the given InSb sample when the coupling half angle is 0.07 rad [73]. Now, after obtaining all the circulator’s design parameters, a Matlab code is written to find the scattering parameters for the designed SJC using the Green’s function approach illustrated in Chapter (3). Figure (4.4) shows the calculated scattering parameters for the designed SJC at 77 K.

The calculated results show that the differential isolation ($|S_{12} - S_{13}|$) is relatively high for most of the band due to the tracking of the perfect circulation conditions. The frequency of maximum isolation at $f = 29 \text{ GHz}$ is associated with an intersection point with the two conditions.

To measure the designed circulator, the assumptions related to the exciting fields at the input ports and the boundary conditions in the SJC theory illustrated in Chapter (3) should be maintained. To do this, a three port waveguide junction containing a finline circuit with $\epsilon_d = 2.2$ was used, as shown in Figure (4.5). The dimensions of the waveguides were based on the WR-28 waveguide standard, which supports the $\text{TE}_{10}$ mode within the Ka frequency band, as illustrated in Table (4.1). The finline components used for the structure in Figure (4.5) were
Table 4.1: Specifications of the WR-28 rectangular waveguide standard.

<table>
<thead>
<tr>
<th>Frequency Limits (GHz)</th>
<th>Inside Dimensions (mm)</th>
<th>Frequency Band</th>
<th>TE_{10} mode cut off frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.5 – 40</td>
<td>a: 7.11, b: 3.56</td>
<td>Ka</td>
<td>21.1</td>
</tr>
</tbody>
</table>

Figure 4.5: A photograph of the waveguide junction and finline circuit used for measuring the Ka-band SJC.

depicted in Figure (1.15), and they are repeated in Figure (4.6) for clarity.

The three finlines are tapered to provide a smooth transition between a WR-28 rectangular waveguide and the input ports of the circulator [100]. Such arrangement will guarantee exciting the InSb disk with electric field in the $\phi$–direction and magnetic field in the $z$–direction. The metallisation of the taper were assumed to provide the necessary electric wall boundaries on the InSb disk’s sides.

to perform the measurements, two of the junction’s waveguide ports were connected to a Vector Network Analyser (VNA)\(^2\), while the third port was terminated with a matched load \(^3\). To fulfil theoretical assumptions, the circulator was immersed in liquid nitrogen during the measurements, and placed axially between

\(^2\)Keysight’s 8510 VNA was used.

\(^3\)The matched load is made to maintain high absorption over the frequency range of operation, as illustrated in Appendix (B).
Figure 4.6: Schematic diagram of the finline components used for the measurements [100].

the poles of an electromagnet, as shown in Figure (4.7).

Thru-Reflect-Line (TRL) calibration was used to bring the reference plane to the waveguide junction’s input ports\(^4\). The value of the provided magnetic bias \((B_0)\) was set by controlling the feeding DC current to the electromagnet, keeping the separation between the poles constant\(^5\).

Figure (4.8) shows the measured scattering parameters for the designed SJC when \(B_0 = 0.8\ T\), compared to the calculated ones under the same conditions. Measured data show a differential isolation of more than 15 \(dB\) with about 3 \(dB\) insertion loss. Comparing the measured scattering parameters with the calculated ones reveals high similarity between the general behaviour of both results. However, the measured insertion loss is 3 to 4 \(dB\)'s higher than that resulted from calculations. This results from the effect of the lossy finline taper. The existence of the waveguide to finline transition also affects the measured reflection coefficient, where it exceeds the calculated \(S_{11}\).

4.2.2 SJC Design Above the Resonance Frequency \((f_r)\)

It can be noted from the previous design that the required magnetic bias to achieve circulation in the Ka-band is relatively high \((B_0 = 0.8\ T)\). It was the same for the previously measured designs [73, 74], where the required magnetic

\(^{4}\)More information about the TRL calibration is given in Appendix (A).

\(^{5}\)More information about the electromagnet is given in Appendix (C).
Figure 4.7: A photograph of the experimental set up for the SJC measurements.

Figure 4.8: A comparison between the calculated and measured scattering parameters for the Ka-band SJC.
bias exceeded 0.5 $T$ for both cases. Such values of magnetic bias are usually obtained using an electromagnet or a large permanent magnet, which may represent a hindrance towards adopting a SJC design for practical use [111].

In the last decade, millimetre and sub-millimetre-wave frequency ranges found their way into many applications in telecommunication [112,113] and imaging [79] systems. However, the lack of compact nonreciprocal components that work in the same frequency ranges imposes a significant challenge to realise such systems. For example, the sub-millimetre-wave imaging system in [79] works at a frequency of 675 $GHz$ for high resolution concealed weapon detection. A massive beam splitter is used as a duplexer because of the unavailability of circulators working at the same frequency range.

Because of the above, designing circulators to work in the sub-millimetre-wave frequency range becomes a necessity. The size of such circulators is also important so they can comply with the practical circuit requirements. That’s why $B_0$ should be small enough to be provided by small permanent magnets.

Using InSb at 77 $K$, designing a low loss SJC in Regions I and II working above 500 $GHz$ requires a significantly high magnetic bias. However, much lower $B_0$ is needed to design the same circulator working in Region IV (for frequencies above $f_B$).

Figure (4.9) shows the change of $\frac{\kappa}{\varepsilon}$ and $\epsilon_{eff}$ with frequency for InSb at 77 $K$ when $B_0 = 0.2$ $T$ [111].
can be seen that the imaginary parts of both $\kappa$ and $\epsilon_{eff}$ are high for the frequencies close to $f_r$. This indicates high loss, and a careful choice should be made to ensure higher real part at the frequency of operation. For the given case, the circulator is designed to work at $f = 650 \text{GHz}$ where $Re(\kappa) = 0.3$ and $Re(\epsilon_{eff}) = 12$, and the imaginary parts of both of the quantities fall below 0.1. Assuming the surrounding dielectric constant ($\epsilon_d$) to be 2.2, the ratio $\frac{Z_d}{Z_{eff}}$ at the the desired frequency will be 2.1. Applying this to the second perfect circulation conditions in Figure (3.13b) will reveal an optimum coupling half angle ($\psi$) of 0.1 rad. After that, $k_{eff}R$ is found from the first circulation conditions graph in Figure (3.13a) to be 1.9. After calculating $k_{eff}$ at the frequency of operation, the radius of the semiconductor disk ($R$) is found to be 40 $\mu$m.

After finding the design parameters, the scattering parameters can be calculated using the Green’s function approach explained in Chapter (3). Figure (4.10) shows the calculated scattering parameters for the designed circulator at 77 K.

Because of the SJC’s small size and the high frequency of operation, it is a challenge to fabricate and measure this circulator. However, it can be simulated using the CST MWS simulation package. Figure (4.11) shows the CST model used for in the simulation. The gyroelectric material is characterised by setting the values of the plasma frequency ($\omega_p$), cyclotron frequency ($\omega_c$) and the effective
collision frequency \( (\nu_c) \), as explained in Chapter (2). The background material is set to be a dielectric with \( \epsilon_d = 2.2 \), as assumed in the design procedure.

Simulating the structure in Figure (4.11) resulted into the scattering parameters shown in Figure (4.12). The behaviour of these parameters can be seen to be very similar to the expected ones in Figure (4.10), where the differential isolation at \( f = 650 \text{ GHz} \) is 15 \( \text{dB} \) with insertion loss of 2.15 \( \text{dB} \). The reflection \( (S_{11}) \) is below \(-10 \text{ dB}\) at the circulation frequency.

Beside the scattering parameters, CST simulation also reveals the distribution of the different electromagnetic field components at the frequency of circulation. Figure (4.13a) shows the distribution of the axial magnetic field \( (H_z) \). In this figure, dark and light regions represent highly positive and negative field intensities, respectively. It can be seen that \( H_z \) is distributed such that port 3 is isolated, while ports 1 and 2 are coupled with 180\(^\circ\) phase difference. Plotting the power flow resulting from the CST simulation also confirms the clockwise circulation process since the power is directed from port 1 to port 2, while port 3 is isolated, as shown in Figure (4.13b).
Figure 4.12: Simulated scattering parameters for the 650 GHz circulator [111].

Figure 4.13: Distribution of axial magnetic field ($H_z$) and power for the 650 GHz circulator at the frequency of circulation.
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4.3 Bandwidth Optimisation of a SJC

The SJC design below \( f_r \), discussed in the previous section was based on tracking the perfect circulation conditions in both Regions I and II. Hence, the tracking takes place when \( \epsilon_{\text{eff}} \) is negative and positive. Despite the fact that measurements revealed consistent results with calculations, the performance was not as expected, especially in terms of insertion loss.

The tracking technique was previously done by carefully choosing the design parameters (such as \( R \), \( \psi \), and surrounding material) until getting the required tracking [73, 100]. In [73], Yong et al. have proposed a broadband circulator to have more than 20 dB differential isolation for the frequency range from 40 to 130 GHz, as shown in Figure (4.14), where the following design parameters were used: \( R = 0.24 \text{ mm}, \psi = 0.5 \text{ rad}, B_0 = 0.35 \text{ T} \) and \( \epsilon_d = 20 \).

Using the CST MWS simulation package, a circulator with the same design parameters reported in In [73] was simulated. Figure (4.15) shows scattering parameters resulted from the CST simulation.

It is evident from the CST results that the simulation outcomes do not match theoretical expectations shown in Figure (4.14). The difference between the two results can be attributed to the degree of approximation in the scattering parameters calculation code, which involves limited number of Fourier terms. CST
CHAPTER 4. SEMICONDUCTOR JUNCTION CIRCULATORS

Figure 4.15: Resulted scattering parameters from applying theoretical broadband design in [73] into CST MWS.

Simulation results also reveal the difference in magnitude in the frequency range where $\epsilon_{eff}$ is negative ($f < 71$ GHz).

The above remarks imply that a significant difference between the calculated and simulated performance when the operation extends to frequency ranges with negative $\epsilon_{eff}$.

The above limitations of the existing technique for extending the bandwidth of the designed circulator necessitate the development of a systematic algorithm for designing broadband circulators. Such algorithm should result into the design parameters of a circulator working across the broadest possible band and centred at the highest possible frequency for a given surrounding material (i.e., $\epsilon_d$). In addition, the required magnetic bias ($B_0$) for the circulator should be minimised [114].

4.3.1 The Proposed Algorithm

First, let’s inspect the general behaviour of $|\frac{\sigma}{\epsilon}|$ with frequency for a lossless semiconductor. It can be shown from Figure (4.16) that $|\frac{\sigma}{\epsilon}|$ starts from $\infty$ at $f = 0$ Hz and decreases until crossing 1 and 0.5 at $f_A$ and $f'_A$, respectively. At $f_t$, $|\frac{\sigma}{\epsilon}|$ starts to increase to cross 0.5 and 1 again at $f'_C$ and $f_C$ (the cyclotron
Figure 4.16: The change of $|\frac{\kappa}{\varepsilon}|$ with frequency for a magnetised semiconductor frequency), respectively. A discontinuity occurs at $f = f_r$, where $|\frac{\kappa}{\varepsilon}|$ goes to $\infty$ and then decreases to cross 1 and 0.5 at $f_B$ and $f'_B$, respectively.

As shown in Chapter (3), SJC’s regions of operation where $\epsilon_{eff}$ is positive are either assigned as Region II ($f_A < f < f_r$) or Region IV ($f > f_B$).

As illustrated before, the circulator design starts with the second perfect circulation condition. To track the perfect circulation lines in its graph, the value of $\frac{Z_d}{Z_{eff}}$ with $\frac{\kappa}{\varepsilon}$ should follow the tendency of the same parameter in the second perfect circulation conditions. The tendency of $\frac{Z_d}{Z_{eff}}$ with $\frac{\kappa}{\varepsilon}$ can be concluded from the its change and that of $|\frac{\kappa}{\varepsilon}|$ with the frequency. For positive $\epsilon_{eff}$, $\frac{Z_d}{Z_{eff}}$ tends to increase with frequency for $f < f_r$. On the other hand, Figure (4.16) shows that $|\frac{\kappa}{\varepsilon}|$ decreases with frequency for $f < f_t$ and $f > f_r$, and increases for $f_t < f < f_r$.

Hence, it can be concluded that the $\frac{Z_d}{Z_{eff}} - \frac{\kappa}{\varepsilon}$ curve decreases when $|\frac{\kappa}{\varepsilon}|$ decreases with frequency, and it increases when $|\frac{\kappa}{\varepsilon}|$ increases with the frequency.

From above, tracking the second circulation curves in Figure (3.13b) is possible within three possible regions (highlighted in Figure (4.16)), as follows:

**Region A** defined from $f = f_A$ to $f = f_A'$. Where $0.5 < |\frac{\kappa}{\varepsilon}| < 1$ and the $\frac{Z_d}{Z_{eff}} - \frac{\kappa}{\varepsilon}$ curve is decreasing.

**Region B** defined from $f = f_t$ to $f = f_C'$. Where $0 < |\frac{\kappa}{\varepsilon}| < 0.5$ and the $\frac{Z_d}{Z_{eff}} - \frac{\kappa}{\varepsilon}$ curve is increasing.

**Region C** defined from $f = f_B$ to $f = f_B'$. Where $0.5 < |\frac{\kappa}{\varepsilon}| < 1$ and the $\frac{Z_d}{Z_{eff}} - \frac{\kappa}{\varepsilon}$ curve is decreasing.
Operating the circulator in each of the defined regions above will allow tracking part of the second circulation conditions, as shown in Figure (4.17). One region should be chosen for the broadband circulator to be designed. However, the closeness of Regions B and C to \( f_r \) makes them less desirable for design because of the high loss. Moreover, a high magnetic bias (\( B_0 \)) is required to make Region B include higher frequencies. Therefore the widest band that can be achieved with minimum magnetic biasing (\( B_0 \)) is Region A, which will be used in this algorithm.

To make Region A extend to broader bandwidth, the value of \( \frac{\kappa}{\epsilon} \) at \( f_t \) should be equal to 0.5 (i.e., it merges with \( f'_A \)). The following steps are taken to determine the value of \( B_0 \) required to achieve this: First, from the definition of \( \kappa \) and \( \epsilon \) in (2.29) and (2.28), respectively, it can be shown that:

\[
\frac{\kappa}{\epsilon} = \frac{\omega_p^2 \omega_c}{\epsilon_r \omega^3 - \epsilon_r \omega_c^2 - \omega \omega_p^2}
\] (4.1)

As illustrated in (3.24), the turning frequency (\( f_r \)) can be found by differentiating (4.1) with respect to \( \omega \) and equating the result to zero, the radian frequency \( \omega_t = 2\pi f_t \) is expressed as:

\[
\omega_t = \sqrt{\frac{\omega_c^2 + \omega_r^2}{3}} = \frac{\omega_r}{\sqrt{3}}
\] (4.2)
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Where $\omega_r$ is the extraordinary wave resonance radian frequency. The value of $\frac{\kappa}{\varepsilon}$ at $\omega_t$ is found by virtue of (4.2) and (4.1) [114]:

$$\frac{\kappa}{\varepsilon}(\omega_t) = -\frac{3\sqrt{3}}{2} \frac{\omega_p^2}{\varepsilon_r \omega_c^2} \left( 1 + \frac{\omega_p^2}{\varepsilon_r \omega_c^2} \right)^{-\frac{3}{2}}$$  \hspace{1cm} (4.3)

Equating $\frac{\kappa}{\varepsilon}(\omega_t)$ to 0.5 will yield a cubic function in terms of $\frac{\omega_p^2}{\varepsilon_r \omega_c^2}$. This is then solved to result into one realistic root equal to 0.278 [114]. After finding $\omega_c, B_0$ can be calculated from the definition of $\omega_c$ in (2.21). This value of magnetic bias will give the widest frequency range from $f_A$ to $f'_A$. Next, by knowing the dielectric constant of the surround ($\epsilon_d$), the change in $\frac{Z_d}{Z_{eff}}$ versus $\frac{\kappa}{\varepsilon}$ for the design is found.

To find the coupling half angle, the second perfect circulation equation in (3.86) is solved for a range of $\psi$ from $\frac{\kappa}{\varepsilon} = 0.5$ to 1. For each $\psi$, the $\frac{Z}{Z_{eff}} - \frac{\kappa}{\varepsilon}$ line is compared to the same quantity calculated for the design by finding the distance between them for a certain range of $\frac{\kappa}{\varepsilon}$. If the distance was below a certain predefined threshold, the range is saved as a possible tracking range.

For each possible tracking range, the first perfect circulation equation in (3.85) is solved, then the resulted $X = k_{eff}R$ is compared to $k_{eff}$ of the design multiplied by a range of radii ($R$). If there is a value of $R$ at which the distance is less than a pre-defined threshold, then the considered range of $\frac{\kappa}{\varepsilon}$ (and hence, frequencies) is said to support broadband circulation for the found $\psi$ and $R$ since both perfect circulation conditions are fulfilled within the same range.

Many possible ranges of $\frac{\kappa}{\varepsilon}$ can result from the above procedure. The bandwidth for each range is calculated by the end of the process, then $R$ and $\psi$ associated with the one with widest bandwidth are chosen accordingly. A flow chart illustrating the bandwidth optimisation procedure is shown in Appendix (D).

It is worthwhile to remark that the proposed algorithm is used to find the optimum design parameters ($B_0, \psi$ and $R$) for a certain semiconductor surrounded by a material with a dielectric constant of $\epsilon_d$. It is possible to find the optimum $\epsilon_d$ as well using the algorithm. However, the surrounding material depends on the transmission lines to which the circulator will be connected. Therefore it is less likely in practice to be able to choose a transmission line with a specific value of dielectric constant.
4.3.2 Simulation Results and Discussion

By considering the InSb at 77 K with the measured electronic parameters illustrated in Chapter (2), the magnetic bias \( B_0 \) required to maximise the bandwidth of Region A is found to be 0.214 \( T \). The behaviour of \( \epsilon_{\text{eff}} \) and \( \frac{\kappa}{\epsilon} \) for the InSb under these conditions is shown in Figure (4.18).

It can be noted that the bandwidth of Region A extends from 100 \( GHz \) to 300 \( GHz \). However, tracking the perfect circulation conditions is not always possible for the whole range, as will be shown later.

To demonstrate the effect of the surrounding material on the perfect circulation tracking possibility, different values of dielectric constants \( \epsilon_d \) are considered as follows:

**Dielectric Constant of the Surround \( (\epsilon_d) = 2.2 \)**

Here, bandwidth optimisation is demonstrated when using the dielectric surround of \( \epsilon_d = 2.2 \), which was used in the SJC’s reported in Section (4.2). It can be noted from Figure (4.18) that the value of \( \epsilon_d \) is much lower than that of \( \epsilon_{\text{eff}} \) within the frequency range of interest. Hence, the surrounding impedance \( (Z_d) \) is higher than the circulator’s effective impedance \( Z_{\text{eff}} \), and the ratio \( \frac{Z_d}{Z_{\text{eff}}} \) is found to vary...
between 1 and 3 as shown in Figure (4.19a).

The algorithm starts by finding the optimum coupling half angle \( \psi \) that results into a close \( \frac{Z_d}{Z_{eff}} \) line to the one produced from the design. In this case, the angle was found to be 0.11 rad. Applying this parameter to the first circulation condition, however, does not reveal any line close to \( k_{eff} \) multiplied by any value of \( R \), as shown in Figure (4.19a). For this reason, tracking both circulation conditions is not attainable for such low value of \( \epsilon_d \). However, it is possible to obtain broadband circulation with low differential isolation \( |S_{12} - S_{13}| \) and high reflection\(^6\).

Figure (4.20a) shows the CST simulation results of a SJC with \( \psi = 0.11 \text{ rad} \) and \( R = 107 \mu m \), which are resulted from applying the bandwidth optimisation algorithm after setting the threshold of the comparison to a high value, allowing the detection of more tracking possibilities.

**Dielectric Constant of the Surround \( (\epsilon_d) = 10 \)**

Let’s consider the value of \( \epsilon_d \) to be 10 in this case. Applying the bandwidth optimisation algorithm reveals that a coupling half angle \( \psi \) of 0.42 rad results into the closest tracking to the second perfect circulation conditions. The first circulation conditions can also be tracked when the radius of the circulator \( R \) is set to 106.7 \( \mu m \). Simulating this design with CST results into the scattering parameters shown in Figure (4.20b). It is obvious that the bandwidth of the circulation is considerably improved over the previous case.

**Dielectric Constant of the Surround \( (\epsilon_d) = 20 \)**

From the previous two cases, it can be concluded that the closer tracking to the first perfect circulation conditions is achieved when the coupling half angle is relatively high. This is because the lines of this conditions have the same tendency of \( k_{eff}R \) of the designed circulator. To obtain higher value of \( \psi \), \( \frac{Z_d}{Z_{eff}} \) should be decreased in order to be closer to the higher \( \psi \) lines in the second circulation conditions graph. This implies that the surrounding impedance \( Z_d \) should be decreased or, effectively, the dielectric constant \( \epsilon_d \) should be increased.

\(^6\)This is expected since tracking the perfect circulation lines essentially means preserving the match between the circulator and the surrounding circuitry. Loosing the track means that the match is being compromised.
Figure 4.19: Variation of $\frac{Z_d}{Z_{eff}}$ and $k_{eff}R$ with $|\frac{\kappa}{\xi}|$ for optimum $\psi$ alongside with the first and second perfect circulation lines for different values of $\epsilon_d$ [114].
Figure 4.20: Simulated scattering parameters for the designed SJC’s with optimum bandwidths for different values of \( \epsilon_d \) [114].
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Table 4.2: Outcomes of the tracking algorithm for an InSb sample at 77 K [114].

<table>
<thead>
<tr>
<th>$\epsilon_d$</th>
<th>$R$ ($\mu m$)</th>
<th>$\psi$ (radians)</th>
<th>10dB Bandwidth (%)</th>
<th>20dB Bandwidth (%)</th>
<th>Max. Insertion Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>107</td>
<td>0.11</td>
<td>66</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>106.7</td>
<td>0.42</td>
<td>87</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>141</td>
<td>0.565</td>
<td>90</td>
<td>48</td>
<td>2</td>
</tr>
</tbody>
</table>

When choosing $\epsilon_d = 20$, close tracking is achieved for both perfect circulation conditions, as shown in Figure (4.19c). It should be noted that the tracking does not include the whole possible band of Region A assigned before mainly because of the difference in tendencies in some parts of the band. Using the design parameters resulted from the algorithm ($\psi = 0.565$ rad and $R = 141$ $\mu m$), the designed circulator was simulated using CST. Results shown in Figure (4.20c) show about 48% 20 dB bandwidth with 2 dB maximum insertion loss.

Table (4.2) summarises the design parameters and the bandwidth resulted from CST simulation for the previous three designs. It should be noted here that the circulation bandwidth is mainly determined by the gyroelectric properties of the semiconductor used in the design. The tracking frequency range can also shifted down by simply increasing the magnetic bias ($B_0$).

4.4 Two Dimensional Electron Gas (2-DEG) based Resonators and Circulators

Most of the previously fabricated and tested SJC's were parts of three dimensional structures. However, most modern microwave circuits are planar, with all the active and passive components lie on the same plane. Therefore designing circulators to work at higher frequencies, which can be integrated into microwave circuits, represents an increasing necessity. Here, the possibility of using a two dimensional material to design and realise sub-millimetre-wave gyroelectric resonators and circulators is investigated. This material is the Two Dimensional Electron Gas (2-DEG), which is a two dimensional layer that appears in some semiconductor structures.

Exploiting the gyroelectric properties of the 2-DEG differs from that of the InSb
analysed in the previous sections. The reason behind that is the high carrier concentration of this material, which makes the plasma frequency much higher than that of any other semiconductor previously used to design gyroelectric devices. The high plasma frequency makes the frequency point $f_A$ higher than $f_C$, which results into a different assignment of the regions of operation, as illustrated in Chapter (3) [115].

### 4.4.1 Electronic Properties of the 2-DEG

The 2-DEG layer occurs when the energy of electrons and holes in a semiconductor become quantised in one dimension and free in the other two dimensions [116]. This behaviour is observed in many structures, such as the Metal Oxide Semiconductor (MOS) diodes, heterojunctions [117], super lattices [116] and the electrons on the surface liquid of helium [118]. In MOS diodes, the 2-DEG layer results from accumulating mobile electrons in the semiconductor side of a semiconductor-insulator interface when a positive voltage ($V > V_{\text{Threshold}}$) is applied to the gate.

In a heterojunction, on the other hand, the 2-DEG layer is formed as a result of combining two semiconductors of two different band gaps, such as a GaAs and AlGaAs [117]. In this type of structures, electrons from the semiconductor with wider bandgap (such as the AlGaAs) transfer to the semiconductor with the narrower bandgap (such as the GaAs) when they are brought together. This will leave a depletion region in the wide bandgap semiconductor. On the other hand, the electrons will accumulate in the other side of the interface forming an accumulation region. Electrons in this area are confined in one direction and
form a two dimensional electron gas (2-DEG) region, as shown in Figure (4.21). The 2-DEG has many interesting physical properties that can be used in many applications. The most important of these features is the high mobility, which occurs because of the separation between the electrons and the donor impurities [120]. This high mobility depends on many factors, such as the temperature and the carrier concentration. Many researches were dedicated to investigate and report the 2-DEG mobility under different conditions [120–125]. However, to design a 2-DEG based component, experimental up-to-date data have to be collected for the structure used in the design.

Here, Molecular Beam Epitaxy (MBE) technique is used to fabricate GaAs/AlGaAs and InGaAs/InAlAs heterostructures. Table (4.3) shows the experimental data of the MBE grown 2-DEG for both heterostructures [126].

<table>
<thead>
<tr>
<th>Materials</th>
<th>$\epsilon_d$</th>
<th>Electron Mobility (cm$^2$V$^{-1}$s$^{-1}$)</th>
<th>Carrier Concentration (cm$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>300K</td>
<td>77K</td>
</tr>
<tr>
<td>GaAs/AlGaAs</td>
<td>13</td>
<td>8000</td>
<td>130000</td>
</tr>
<tr>
<td>InGaAs/InAlAs</td>
<td>13</td>
<td>8240</td>
<td>23876</td>
</tr>
</tbody>
</table>

It can be noticed from the 2-DEG data that the carrier concentration values are in cm$^{-2}$, to find the equivalent 3D values, the following formula is used:

$$(N_s)_{3D} = [(N_s)_{2D}]^\frac{3}{2}$$  \hspace{1cm} (4.4)

Using (4.4), the three-dimensional equivalent carrier concentration for the 2-DEG materials in Table (4.3) are found to be $1.25 \times 10^{17}$cm$^{-3}$ and $9.3 \times 10^{16}$cm$^{-3}$ for GaAs/AlGaAs and InGaAs/InAlAs, respectively. Consequently, plasma frequency for GaAs/AlGaAs will be 12.2 THz, and 10.37 THz for InGaAs/InAlAs. As mentioned before, high plasm frequencies change the relationship between $f_A$ and $f_C$. When $f_A$ exceeds $f_C$, regions of operation are designated differently. This occurs when the plasma frequency is higher than $\sqrt{2\epsilon_r}$ times the cyclotron frequency. For the MBE grown 2-DEG illustrated in Table (4.3), to make $f_A$
smaller than $f_C$, the latter has to be increased to be equal to or more than 8.6 THz for GaAs/AlGaAs and 7.3 THz for InGaAs/InAlAs. These two values require magnetic bias ($B_0$) of 20.5 T and 17.4 T, respectively. These values are clearly non-realistic, therefore the cyclotron frequency ($f_C$) is expected to be smaller than $f_A$ for nominal values of $B_0$.

To investigate the possibility of designing resonators and circulators using the MBE grown 2-DEG, the theory of gyroelectric resonators illustrated in Chapter (3) should be applied. From Table (4.3), it is shown that the carrier concentration of the 2-DEG in the InGaAs/InAlAs heterostructure is slightly lower than that in the GaAs/AlGaAs heterostructure. However, the electron mobility at 77 $K$ of the latter is much higher than that of the former. Hence, the MBE grown 2-DEG in the GaAs/AlGaAs at 77 $K$ will be used for the resonator and circulator designs. Assuming a magnetic bias ($B_0$) of 0.5 T, Figure (4.22) shows the calculated change of $\frac{\xi}{\varepsilon}$ and $\epsilon_{eff}$ with frequency of the MBE grown 2-DEG of the GaAs/AlGaAs heterostructure. Figure (4.22a) shows that $\frac{\xi}{\varepsilon}$ increases from $-\infty$ at low frequencies and reaches the value of $-1$ at the cyclotron frequency ($f_C = 208.8$ GHz). On the other hand, Figure (4.22b) shows that $\epsilon_{eff}$ stays highly negative till $f = f_A = 3296.5$ GHz. Therefore, the millimetre and sub-millimetre-wave frequency ranges fall in Regions I-A and I-B illustrated in (3.2.2).
4.4.2 A 2-DEG Based Resonator

To analyse a 2-DEG gyroelectric resonator, the uncoupled structure shown in Figure (3.1) will be considered first, with the 2-DEG material\(^\text{10}\) forming a disk of radius \((R)\), surrounded by an electric wall from the sides and magnetic walls on the top and the bottom. By reviewing the modal analysis in (3.2.3), it is noticed that equation (3.25b) (using modified Bessel’s function of the first kind) has to be applied to satisfy the boundary conditions at \(r = R\). From the same section, it was found that no resonant modes exist in Region I-B for any value of \(n\). Hence, the resonator analysis here will be applied to Region I-A, where \(\epsilon_{eff}\) is negative and \(\frac{\xi}{\epsilon} < -1\).

For \(n = -1, -2\) and \(-3\), the solutions of of the characteristic equation (3.25b) are shown in Figure (3.4). For any uncoupled gyroelectric resonator to resonate in Region I-A, the \(\tilde{k}_{eff} R - \frac{\xi}{\epsilon}\) line has to intersect one of the lines in Figure (3.4). Using the GaAs/AlGaAs 2-DEG data from Table (4.3), a magnetically biased 2-DEG resonator with radius of 5 \(\mu m\) is analysed in terms of intersections with the solutions of the characteristic equation (3.25b). It is shown that for different values of magnetic bias \((B_0)\), the \(\tilde{k}_{eff} R - \frac{\xi}{\epsilon}\) track for the resonator intersects with the \(n[-1, 1]\) line in Figure (3.4) once. The intersection point occurs at almost the same value of \(\frac{\xi}{\epsilon}\). However, \(\frac{\xi}{\epsilon}\) represents different frequency for each value of \(B_0\).

The intersection points for \(B_0 = 0.5, 0.6\) and \(0.7\) are shown in Figure (4.23). By assuming a loosely coupled two port resonator with the structure depicted in Figure (3.8), the Green’s function approach illustrated in Chapter (3) is used to find the scattering parameters, assuming a coupling half angle \((\psi)\) of 0.001 \(rad\) and a dielectric constant of the surround \(\epsilon_d\) to be 20 to minimise the coupling effects. The same structure was simulated using the CST MWS package, where the magnetically biased 2-DEG layer was modelled according to the data shown in Table (4.3). The thickness of the MBE grown 2-DEG layer was set to be 100 \(nm\) [126].

Figure (4.24) shows the calculated and simulated reflection coefficient \((S_{11})\) for the considered resonator for different values of magnetic bias \((B_0)\). It is evident from the dip in \(S_{11}\) that the resonant frequency shifts with \(B_0\) as expected from the uncoupled resonator intersections shown in Figure (4.23). It can also be noted that there is a reduction in the value of \(S_{11}\) with increased \(B_0\) due to the increase in \(\epsilon_{eff}\) (it approaches zero from the negative side).

\(^\text{10}\)The 2-DEG is assumed to be at 77 \(K\) from here onward.
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Figure 4.23: Intersections between the normalised wavenumbers $\tilde{k}_{\text{eff}} R$ and a solution of the characteristic equation in Region I-A [115].

Figure 4.24: Reflection coefficients for the 2-DEG two port resonator using both Green’s function calculation approach and the CST MWS simulation for different values of $B_0$ [115].

These results prove the existence of resonant modes in Region I-A, where the negative value of $\eta$ indicates a counter clockwise rotation of the fields. Such behaviour
can be used to design circulators after some modifications to the mathematical design approach, as will be illustrated next.

4.4.3 A 2-DEG Based Circulator

Consider the circulator diagram in Figure (3.12). Here, a 2-DEG disk, axially magnetised with $B_0$, is surrounded by an electric wall from all the sides except for three ports, each extends for $2\psi$ radians. The dielectric constant of the ports ($\epsilon_d$) is assumed to be 13, which corresponds to the GaAs. Because of the negative values of $\epsilon_{eff}$, the design procedure will make use of the perfect circulation conditions shown in Figure (3.14).

Because of the highly negative value of $\epsilon_{eff}$ in this case, the effective impedance of the 2-DEG area is very low. This unusual situation dictates studying the change of the ratio $\left|\frac{Z_d}{Z_{eff}}\right|$ with frequency first. Given $B_0 = 0.5 \ T$, Figure (4.25) depicts this change.

Considering the Region I-A ($f < f_C$), it can be seen that $\left|\frac{Z_d}{Z_{eff}}\right|$ in this range is higher than the upper limit of the same quantity in the second perfect circulation conditions shown in Figure (3.14b). Hence, these circulation conditions are not suitable for a circulator design with such high values of $\left|\frac{Z_d}{Z_{eff}}\right|$. Since it is not

![Figure 4.25: The Change of $\left|\frac{Z_d}{Z_{eff}}\right|$ with Frequency for a 2-DEG disk magnetised with $B_0 = 0.5T$ [115].](image-url)
possible to find other set of circulation conditions when $\epsilon_{eff}$ is negative\(^{11}\), the only solution is to extend the already existed perfect circulation conditions to include higher values of $\left| \frac{Z_d}{Z_{eff}} \right|$. By following this approach, the extended perfect first and second circulation conditions were found and plotted in Figure (4.26) for $2 < \left| \frac{\kappa}{\epsilon} \right| < 3$.

It can be seen that the values of $\left| \frac{Z_d}{Z_{eff}} \right|$ reaches as high as 8 in the extended perfect circulation conditions. However, as shown in Figure (4.25), the values of $\left| \frac{Z_d}{Z_{eff}} \right|$ in Region I-A ($f < f_C$, where $f_C = 208.8 \text{ GHz}$) are higher than that. To apply the extended perfect circulation conditions, the cyclotron frequency ($f_C$) has to be increased such that Region I-A will include frequencies associated with lower values of $\left| \frac{Z_d}{Z_{eff}} \right|$, which is achieved by increasing the magnetic bias ($B_0$).

For the given 2-DEG data, $B_0$ has to be increased to 2.5 T for $\left| \frac{Z_d}{Z_{eff}} \right|$ to become around 8 at $f = 200 \text{ GHz}$.

Now, the usual procedure is followed to find the circulator’s design parameters. Given the above value of $B_0$, the coupling half angle ($\psi$) is found to be 0.075 rad, and the radius of the circulator to be 14 $\mu$m. The scattering parameters for this

\(^{11}\)Unlike the perfect circulation conditions where $\epsilon_{eff}$ is positive, where many sets can be found from solving equations (3.85) and (3.86) using normal Bessel functions [100]
design were calculated using the Green’s function approach, and by simulating the circulator using the CST MWS simulation package. The resulted parameters from both methods are illustrated in Figure (4.27).

From the calculated scattering parameters, it can be seen that more than 12 dB differential isolation is expected between \( f = 150 \) and 600 GHz. The CST results, on the other hand, show almost a constant differential isolation of around 10 dB. However, both the reflection and insertion losses are higher than those expected by the calculation. This is attributed to two reasons: first, a limited number of terms for the Fourier series were used in calculating the Green’s function. Secondly, the CST MWS has limitations regarding the simulation of materials with gyrotropic dispersion (as discussed in Chapter (2)); the input ports to any gyroelectric structure cannot be in touch with the gyroelectric material. Hence, in this design, a gap had to be left between each port and the magnetised 2-DEG layer, as shown in Figure (4.28).

Because of the high difference between the dielectric constant of the surround \( (\epsilon_d = 13) \) and the effective permittivity of the 2-DEG \( (\epsilon_{eff}) \), the change of impedance here will be more pronounced than the previously designed circulators, and that renders the reflection and insertion losses higher than expected. However, by inspecting the behaviour of \( S_{11} \) and \( S_{12} \) in Figure (4.27), it can be
Figure 4.28: Port alignment for the 2-DEG circulator model in CST showing the distance between the waveguide port and the gyroelectric disk [115].

noted that the parameters resulted from CST have the same tendencies as the calculated ones and the difference, as expected, originates from the high reflection at each port.

The reported results here are to prove the possibility of using magnetically biased 2-DEG to realise resonators and circulators. Despite its limitations, the CST MWS simulation package had a significant role in validating theoretical expectations. Nevertheless, measuring the designed resonator and circulator can reveal more about the potential of this new class of nonreciprocal devices.

Using the MBE facility in the University of Manchester, both the 2-DEG resonator and circulator were fabricated, as shown in Figure (4.29). However, measuring these structures faces many challenges, such as coupling them to suitable test fixtures that cover the frequency range of interest, while maintaining the high magnetic bias and liquid nitrogen temperature requirements to meet theoretical expectations.

4.5 Conclusions

This chapter has discussed different types of Semiconductor Junction Circulator (SJC) and their performance in terms of scattering parameters and field distributions. After illustrating the design procedure, two types of basic SJC’s were analysed using InSb at 77 K. The first SJC was previously designed to work
below the extraordinary wave resonance frequency \( f_r \) by following the perfect circulation conditions in Regions I and II. Calculated scattering parameters in the Ka frequency band showed more than 15 \( dB \) differential isolation with 1 \( dB \) insertion loss when the magnetic bias \( (B_0) \) was 0.8 \( T \). Measured results for this SJC showed almost the same differential isolation and higher insertion loss (about 3 \( dB \)) due to the effects of the finline circuit used for coupling the InSb disk to the waveguide ports. The other basic SJC was designed by utilising the frequencies above \( f_r \) (in Region IV). This facilitates circulation in the sub-millimetre-wave frequency range \( (f = 650 \, GHz) \) using a low magnetic bias \( (B_0 = 0.2 \, T) \). Both calculated and simulated scattering parameters confirmed a differential isolation of 15 \( dB \) with insertion loss of 2.15 \( dB \) at the frequency of circulation.

Next, an algorithm to optimise the bandwidth of a SJC was proposed. The algorithm uses an iterative process to find the design parameters that guarantee tracking the perfect circulation conditions for the widest possible bandwidth. Analysis showed that the optimum bandwidth depends highly on the dielectric constant of the surround \( (\epsilon_d) \), and a maximum possible 10 \( dB \) bandwidth of 90% with 2 \( dB \) maximum insertion loss was achieved using InSb at 77 \( K \), biased with \( B_0 = 0.214 \, T \) when \( \epsilon_d = 20 \).

Finally, the MBE grown Two dimensional Electron Gas (2-DEG) at 77 \( K \) was
considered for designing resonators and circulators. Due to the high carrier concentration of this material, the cyclotron frequency \( f_C \) is found to be lower than \( f_A \) for normal values of \( B_0 \). Therefore the regions of operation were assigned differently. By setting \( B_0 = 0.5 \, T \), calculations and CST simulations proved the existence of a resonant mode at 90 GHz. This mode was tuned up and down in frequency when \( B_0 \) was changed.

On the other hand, designing a 2-DEG SJC required the perfect circulation conditions to be extended to include higher values of \( \left| \frac{Z_d}{Z_{eff}} \right| \). A high value of magnetic bias was also required to achieve circulation at \( f = 200 \, GHz \) using a 2-DEG disk at 77 K with \( R = 14 \, \mu m \) and \( \psi = 0.075 \, rad \).

Circulation was proved by calculating the scattering parameters, which showed 12 dB differential isolation within the frequency range of interest. CST simulation showed similar differential isolation, but with reflection and insertion losses of 10 and 7.5 dB, respectively.

Results and the analysis presented in this chapter prove the possibility of using a magnetised InSb at 77 K to achieve broadband circulation at high frequency ranges. In addition, results showed the possibility of achieving fully planar resonators and circulators working at high frequency ranges using magnetically biased MBE grown 2-DEG layers at 77 K.
Chapter 5

Waveguide Junction Gyroelectric Circulators

5.1 Introduction

This chapter examines the possibility of designing gyroelectric waveguide circulators. Contrary to the Semiconductor Junction Circulators (SJC’s) discussed in the previous chapters, the gyroelectric disk in this circulator is not covered with an electric wall. Instead, the disk is suspended inside the H-plane waveguide junction, as shown in Figure (5.1). Despite its simplicity, analysing this structure is significantly complicated. The reason behind that is the open boundaries on the sides of the semiconductor sample, which renders many of the previously made assumptions not valid.

Here, the gyroelectric behaviour of the structure shown in Figure (5.1) is analysed by treating the waveguide junction as a cavity loaded with a disk of an axially magnetised semiconductor. This treatment is similar to that of a ferrite turnstile circulator [127].

This chapter starts with a full electromagnetic analysis of a suspended semiconductor rod magnetised in its axial direction, which is the same direction of propagation. Based on this analysis, the conditions under which a split between two counter rotating modes are discussed.

Next, some of these modes are excited by placing an InSb disk inside a rectangular waveguide. The shift in the eigenmode resonance frequencies of the structure when changing the axial magnetic bias is simulated and measured.
Based on the modal analysis, a waveguide junction gyroelectric circulator is realised using a WR-28 waveguide junction. The performance of this circulator is explored via simulations and measurements. In addition, the possibility of obtaining circulation with reduced magnetic bias by including dielectric disks above and below the InSb disk is investigated.

5.2 Electromagnetic Analysis of Axially Magnetised Gyroelectric Rods

This section will deal with analysing the propagating modes in a suspended semiconductor rod magnetically biased in its axial direction. First, let’s assume a lossless unbounded semiconductor magnetically biased in the $z$–direction\(^1\). As discussed in Chapter (2), the permittivity of a lossless magnetically biased

\(^1\)It is more convenient to use Cartesian coordinates at this stage of the analysis.
semiconductor in $z$–direction is described using the following tensor:

$$[\epsilon] = \begin{bmatrix} \varepsilon & -j\kappa & 0 \\ j\kappa & \varepsilon & 0 \\ 0 & 0 & \zeta \end{bmatrix}$$  \hspace{1cm} (5.1)

Assuming $\mu_r = 1$, Maxwell’s equations for a source free gyroelectric media are described given the tensor in (5.1) as [128]:

\begin{align*}
\nabla \times \vec{E} &= -j\omega \mu_0 \vec{H}, \hspace{1cm} (5.2a) \\
\nabla . \mu_0 \vec{H} &= 0, \hspace{1cm} (5.2b) \\
\nabla \times \vec{H} &= j\omega \varepsilon_0 [\epsilon] \vec{E}, \hspace{1cm} (5.2c) \\
\n\nabla . \epsilon_0 [\epsilon] \vec{E} &= 0 \hspace{1cm} (5.2d)
\end{align*}

By letting the free space wavenumber $k_0$ be $\omega \sqrt{\mu_0 \varepsilon_0}$, expanding the equations in (5.2) yields:

\begin{align*}
\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y + jk_0 H_x &= 0, \hspace{1cm} (5.3a) \\
\frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z + jk_0 H_y &= 0, \hspace{1cm} (5.3b) \\
\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x + jk_0 H_z &= 0, \hspace{1cm} (5.3c) \\
\frac{\partial}{\partial x} H_x + \frac{\partial}{\partial y} H_y + \frac{\partial}{\partial z} H_z &= 0, \hspace{1cm} (5.3d) \\
\frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y - jk_0 \varepsilon E_x + k_0 \kappa E_y &= 0, \hspace{1cm} (5.3e) \\
\frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z - k_0 \kappa E_x - jk_0 \varepsilon E_y &= 0, \hspace{1cm} (5.3f) \\
\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x - jk_0 \zeta E_z &= 0, \hspace{1cm} (5.3g) \\
\varepsilon \left( \frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y \right) + j\kappa \left( \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \right) + \zeta \frac{\partial}{\partial z} E_z &= 0 \hspace{1cm} (5.3h)
\end{align*}

Electric and magnetic field components normal to the direction of biasing can be eliminated by differentiating (5.3a) with respect to $y$ and (5.3b) with respect to
\[
\begin{align*}
\frac{\partial^2}{\partial y^2} E_z - \frac{\partial^2}{\partial z \partial y} E_y + jk_0 \frac{\partial}{\partial y} H_x &= 0, \\
\frac{\partial^2}{\partial z \partial x} E_x - \frac{\partial^2}{\partial x^2} E_z + jk_0 \frac{\partial}{\partial x} H_y &= 0
\end{align*}
\] (5.4a)
\[
\begin{align*}
\frac{\partial^2}{\partial x^2} E_x - \frac{\partial^2}{\partial y^2} E_z - \frac{\zeta}{\varepsilon} \frac{\partial^2}{\partial z^2} E_z + k_0^2 \zeta E_z + k_0 \frac{\kappa}{\varepsilon} \frac{\partial}{\partial z} H_z &= 0,
\end{align*}
\] (5.5)
\[
\begin{align*}
\frac{\partial^2}{\partial x^2} H_z + \frac{\partial^2}{\partial y^2} H_z + \frac{\partial^2}{\partial z^2} H_z + k_0^2 \varepsilon_{\text{eff}} H_z + k_0 \zeta \frac{\kappa}{\varepsilon} \frac{\partial}{\partial z} E_z &= 0
\end{align*}
\] (5.6)

Where \( \varepsilon_{\text{eff}} = \varepsilon - \frac{\kappa^2}{\varepsilon} \).

Assuming a transverse differential operator \( (\nabla_t) \) to be:
\[
\nabla_t = \nabla - \frac{\partial}{\partial z} \hat{z}
\] (5.7)

We can now rewrite (5.5) and (5.6), respectively, as:
\[
\left( \nabla_t^2 + g_0 \frac{\partial^2}{\partial z^2} + k_0^2 \zeta \right) E_z + k_0 g_e \frac{\partial}{\partial z} H_z = 0,
\] (5.8a)
\[
\left( \nabla^2 + k_0^2 \varepsilon_{\text{eff}} \right) H_z - k_0 \zeta g_e \frac{\partial}{\partial z} E_z = 0
\] (5.8b)

Where \( g_0 = \xi \) and \( g_e = \frac{\xi}{\varepsilon} \).

As concluded in Chapter (3), it is clear from (5.8) that no pure TE or TM mode can be obtained in such medium. By multiplying (5.8a) by \( (\nabla^2 + k_0^2 \varepsilon_{\text{eff}}) \) and (5.8b) by \( k_0 g_e \frac{\partial}{\partial z} \) and subtracting the results, the field component \( H_z \) will be eliminated, and the following equation in terms of \( E_z \) only is obtained.
\[
\left[ \nabla_t^4 + g_0 \frac{\partial^4}{\partial z^4} + \nabla_t^2 \frac{\partial^2}{\partial z^2} (1 + g_0) + \nabla_t^2 \left( k_0^2 \zeta + k_0^2 \varepsilon_{\text{eff}} \right) + \frac{\partial^2}{\partial z^2} \left( 2k_0^2 \zeta \right) + k_0^4 \varepsilon_{\text{eff}} \right] E_z = 0
\] (5.9)
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Equation (5.9) can be expressed in terms of a differential operator $L$ as:

$$L (E_z) = 0 \quad (5.10)$$

Similarly, eliminating $E_z$ from the system in (5.8) will result into:

$$L (H_z) = 0 \quad (5.11)$$

By manipulating the solutions of Maxwell’s equations, an expression similar to (5.10) and (5.11) is obtained for the field components transverse to the direction of magnetic bias$^2$. Hence, it can be assumed that both electric and magnetic field vectors obey the same equation, or:

$$L (\bar{E}) = 0 \quad (5.12a)$$

$$L (\bar{H}) = 0 \quad (5.12b)$$

The expressions in (5.12) represent fourth order differential equations. To solve such equations, it is more convenient to use scalar potentials that can be easily converted back into the different field components by means of a simple derivative process [129]. For simpler media, two scalar potentials can be found: one for TE or H-modes and another for TM or E-modes. In this case, however, the gyrotropy of the media prevents the use of such functions. Hence, another type of scalar function which is neither TE or TM should be introduced [128].

Following the same method used for the scalar function for ferrite materials in [130], a scalar function can be introduced given the following relation between the field components $E_x, E_y$ and their derivatives in both the $x$ and $y$-directions, as follows:

$$\frac{\partial}{\partial x} (F_1 (E_x, E_y)) = \frac{\partial}{\partial y} (F_2 (E_x, E_y)) \quad (5.13)$$

A scalar function $(\Psi)$ is defined to be related to both field components as:

$$\frac{\partial}{\partial y} \Psi = F_1 (E_x, E_y), \quad (5.14a)$$

$$\frac{\partial}{\partial x} \Psi = F_2 (E_x, E_y) \quad (5.14b)$$

$^2$This can be done by projecting Maxwell’s two curl equations into two components: one in $z$-direction to obtain equations in terms of $E_z$ and $H_z$, and the other in the transverse direction, i.e., $E_t$ and $H_t$. Chapter 5 of [128] (page 120) contains a similar derivation.
Now, the relation in (5.13) should be found from the previously derived relations between the different field components. By differentiating (5.5) with respect to \( z \), and by virtue of (5.3h) and (5.3c), the following relation is obtained:

\[
\frac{\partial}{\partial x} \left[ E_x \left( \frac{1}{g_0} \nabla_i^2 - \frac{\partial^2}{\partial z^2} + k_0^2 \right) + E_y \left( j \frac{\kappa}{\zeta} \nabla_i^2 + j k_0^2 \kappa \right) \right] = \frac{\partial}{\partial y} \left[ E_x \left( j \frac{\kappa}{\zeta} \nabla_i^2 + j k_0^2 \kappa \right) - E_y \left( \frac{1}{g_0} \nabla_i^2 + \frac{\partial^2}{\partial z^2} + k_0^2 \right) \right]
\]

(5.15)

The above equation can be rewritten as:

\[
\frac{\partial}{\partial x} (T E_x + j S E_y) = \frac{\partial}{\partial y} (j S E_x - T E_y)
\]

(5.16)

Where \( T = \frac{1}{g_0} \nabla_i^2 + \frac{\partial^2}{\partial z^2} + k_0^2 \) and \( S = \frac{\kappa}{\zeta} \nabla_i^2 + k_0^2 \kappa \).

Based on the definition in (5.14), the scalar potential \( \Psi \) is related to the field components \( E_x \) and \( E_y \) as:

\[
(T^2 - S^2) \frac{\partial}{\partial y} \Psi = T E_x + j S E_y,
\]

(5.17a)

\[
(T^2 - S^2) \frac{\partial}{\partial x} \Psi = j S E_x - T E_y
\]

(5.17b)

It was more convenient to include the term \( (T^2 - S^2) \) in (5.17) so that the expressions for \( E_x \) and \( E_y \) in terms of \( \Psi \) are more abstract, as follows:

\[
E_x = T \frac{\partial}{\partial y} \Psi + j S \frac{\partial}{\partial x} \Psi,
\]

(5.18a)

\[
E_y = j S \frac{\partial}{\partial y} \Psi - T \frac{\partial}{\partial x} \Psi
\]

(5.18b)

Using (5.3h), the electric field component \( E_z \) is found in terms of \( \Psi \) to be:

\[
E_z = j \frac{\kappa}{\zeta} \frac{\partial}{\partial z} \nabla_i^2 \Psi
\]

(5.19)

Similarly, an expression for \( H_z \) is found by substituting (5.18) in (5.3c):

\[
H_z = -j \frac{1}{k_0} T \nabla_i^2 \Psi
\]

(5.20)
And from (5.3a) and (5.3b), similar expressions can be found for the remaining magnetic field components:

\[ H_x = N \frac{\partial^2}{\partial y \partial z} \Psi + jM \frac{\partial^2}{\partial x \partial z} \Psi, \]  
\[ H_y = jM \frac{\partial^2}{\partial y \partial z} \Psi - N \frac{\partial^2}{\partial x \partial z} \Psi \]  

(5.21a)  
(5.21b)

Where \( N = k_0 \kappa \) and \( M = \frac{1}{k_0} \left( \frac{1}{g_0} \nabla_t^2 + \frac{\partial^2}{\partial z^2} + k_0^2 \epsilon \right) \).

By using the relations (5.12a) and (5.12b) for any of the field components described in (5.18) to (5.21), it is found that the scalar function \( \Psi \) satisfies:

\[ L(\Psi) = 0 \]  

(5.22)

Now, let’s assume a harmonic dependence on the \( z \)-direction in the form \( e^{-j\beta z} \). This makes the operator \( \frac{\partial}{\partial z} = -j\beta \) and \( \frac{\partial^2}{\partial z^2} = -\beta^2 \). This allows separating the function \( \Psi \) into a transverse function \( \psi \) which depends on the two transverse coordinates\(^3\) and a harmonic function \( Z \) which depends on \( z \) only.

Given (5.22), the operator \( L \) can now be written as:

\[ L = \nabla_t^4 + \beta^4 g_0 - \beta^2 \nabla_t^2 (1 + g_0) + \nabla_t^2 \left( k_0^2 \zeta + k_0^4 \epsilon_{eff} \right) - \beta^2 \left( 2k_0^2 \zeta \right) + k_0^4 \epsilon_{eff} \]  

(5.23)

(5.23) can be rearranged to become:

\[ L = \nabla_t^4 + A \nabla_t^2 + B \]  

(5.24)

Where:

\[ A = k_0^2 \left( \zeta + \epsilon_{eff} \right) - \beta^2 \left( g_0 + 1 \right), \]  
\[ B = \zeta \left[ k_0^4 \epsilon_{eff} - 2k_0^2 \beta^2 + \frac{\beta^4}{\epsilon} \right] \]  

(5.25a)  
(5.25b)

It is possible to represent the fourth order differential equation in (5.23) as a product of two second order differential equations, as follows [128,130]:

\[ L = (\nabla_t^2 + \chi_1^2) (\nabla_t^2 + \chi_2^2) \]  

(5.26)

\(^3\)This can be \( x \) and \( y \) for Cartesian coordinates or \( r \) and \( \phi \) for cylindrical coordinates.
The two squared variables $\chi_{1,2}^2$ are related to differential equation arguments in (5.25) as:

$$\chi_1^2 + \chi_2^2 = A,$$  \hspace{1cm} (5.27a)  

$$\chi_1^2 \chi_2^2 = B$$ \hspace{1cm} (5.27b)  

The transverse scalar function $\psi$ is related to the operator in (5.26) by [128]:

$$\nabla_t^2 \psi + \chi_{1,2}^2 \psi = 0$$ \hspace{1cm} (5.28)  

By solving the biquadratic equation in (5.24), expressions for the roots $\chi_{1,2}^2$ are found [128,131]:

$$\chi_{1,2}^2 = \frac{1}{2} \left[ k_0^2 (\zeta + \epsilon_{eff}) - (1 + g_0) \beta^2 \right] \pm \sqrt{\frac{1}{4} \left[ k_0^2 (\zeta - \epsilon_{eff}) + (1 - g_0) \beta^2 \right]^2 + \beta^2 k_0^2 \zeta g_0^2}$$ \hspace{1cm} (5.29)  

Now, let’s consider a cylindrical coordinate system, the transverse scalar function can now be written as $\psi (r, \phi)$. Applying this to the differential equation in (5.28) will yield:

$$\frac{\partial^2}{\partial r^2} \psi + \frac{1}{r} \frac{\partial}{\partial r} \psi + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \psi + \chi_{1,2}^2 \psi = 0$$ \hspace{1cm} (5.30)  

The two variables $r$ and $\phi$ in (5.30) can be separated into two functions:

$$\psi(r, \phi) = R(r) \Phi(\phi)$$ \hspace{1cm} (5.31)  

The function $R(r)$ should have a finite value at $r = 0$, hence it satisfies Bessel’s function of the first kind ($J_n(\chi_{1,2}r)$). Due to the cyclic symmetry, the function $\Phi(\phi)$ can have a solution of the form $e^{in\phi}$, where $n$ in both equations is an integer ($n = 0, \pm 1, \pm 2, ...$). A positive value of $n$ correspond to a wave with the field polarisation that rotates in the clockwise direction and the negative values correspond to the counter clockwise rotated waves [128].

Now, let’s consider the infinitely long cylindrical rod in Figure (5.2). The
boundary conditions for this structure are:

\[
\begin{align*}
E_{g,z} &= E_{s,z} \\
E_{g,\phi} &= E_{s,\phi} \\
H_{g,z} &= H_{s,z} \\
H_{g,\phi} &= H_{s,\phi}
\end{align*}
\]

for \( r = a \) \hspace{1cm} (5.32)

Where the subscript \((g)\) indicates a field component inside the gyroelectric rod, and the subscript \((s)\) indicates the field in the free spaces on the sides of the rod. As mentioned before, the fields inside the rod cannot be a pure TE or TM mode. Therefore the boundary conditions in (5.32) cannot be satisfied unless considering both TE and TM waves outside the rod, and a summation of two partial waves with different wavenumber \((\chi)\) and the same order \((n)\) inside the rod. Based on that, the scalar potential function \(\Psi\) can be described as:

\[
\Psi = [G_{1,n}J_n(\chi_1 r) + G_{2,n}J_n(\chi_2 r)] e^{j(n\phi - \beta z)}
\]

where \(G_{1n,2n}\) are arbitrary amplitudes for the two transverse waves inside the gyroelectric rod.

Electric and magnetic field components inside the gyroelectric rod are related to
Ψ in (5.33) using equations (5.18) to (5.21). These can be written in an abstract form as:

\[
\begin{align*}
\vec{E}_g &= \vec{U} \nabla \Psi, \\
\vec{H}_g &= \frac{\partial}{\partial z} \vec{V} \nabla \Psi
\end{align*}
\] (5.34a, 5.34b)

Where:

\[
\begin{align*}
\vec{U} &= \begin{bmatrix} jS & T & 0 \\ -T & jS & 0 \\ 0 & 0 & jW \end{bmatrix}, \\
\vec{V} &= \begin{bmatrix} jM & N & 0 \\ -N & jM & 0 \\ 0 & 0 & jR \end{bmatrix}
\end{align*}
\] (5.35a, 5.35b)

The operators used in (5.35) are defined as:

\[
\begin{align*}
S_{1,2} &= \frac{\kappa}{\zeta} \chi_{1,2} + k_0^2 \kappa, \\
T_{1,2} &= \frac{1}{g_0} \chi_{1,2}^2 - \beta^2 + k_0^2 \varepsilon, \\
W_{1,2} &= \frac{\kappa}{\zeta} \chi_{1,2}^2, \\
M_{1,2} &= \frac{1}{k_0} \left( \frac{1}{g_0} \chi_{1,2}^2 + \beta^2 + k_0^2 \varepsilon \right), \\
N &= k_0 \kappa, \\
R &= - \frac{T}{k_0^2 \chi_{1,2}^2}
\end{align*}
\] (5.36a, 5.36b, 5.36c, 5.36d, 5.36e, 5.36f)

The fields on the sides of the gyroelectric rod are assumed to decay with distance away from its sides. These waves are described using the following scalar potentials:

\[
\Pi_{TE} = A_n K_n (k_t r) e^{i(\omega t + n\phi - \beta z)}, \\
\Pi_{TM} = B_n K_n (k_t r) e^{i(\omega t + n\phi - \beta z)}
\] (5.37a, 5.37b)

Where \(k_t = \sqrt{\beta^2 - k_0^2}\) and \(K_n(.)\) is the modified Bessel’s function of the second kind.
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Using (5.37), the electromagnetic field components outside the rod ($\vec{E}_s$ and $\vec{H}_s$) can be derived as [132]:

\[
\vec{E}_s = \begin{bmatrix}
-\frac{1}{r} \frac{\partial}{\partial \phi} \Pi_{TE} + \frac{\beta}{\omega_0} \frac{\partial}{\partial r} \Pi_{TM}

\frac{\partial}{\partial r} \Pi_{TE} + \frac{\beta}{\omega_0 r} \frac{\partial}{\partial \phi} \Pi_{TM}

-\frac{j \tau^2}{\omega_0} \Pi_{TM}
\end{bmatrix},
\]

(5.38a)

\[
\vec{H}_s = \begin{bmatrix}
\frac{\beta}{\omega_0} \frac{\partial}{\partial r} \Pi_{TE} + \frac{1}{r} \frac{\partial}{\partial \phi} \Pi_{TM}

\frac{\beta}{\omega_0 r} \frac{\partial}{\partial \phi} \Pi_{TE} - \frac{\partial}{\partial r} \Pi_{TM}

-\frac{j \tau^2}{\omega_0} \Pi_{TE}
\end{bmatrix},
\]

(5.38b)

Where $\tau = \sqrt{\beta^2 - k_0^2}$.

Matching the four field components $E_{z,\phi}$ and $H_{z,\phi}$ at the surface of the rod (i.e., $r = a$) results into four equations for the four variables $G_{1n,2n}$, $A_n$ and $B_n$ as follows:

\[
0 A_n + \frac{-j \tau}{\omega_0} K_n(\tau a) B_n + C_{13} G_{1,n} + C_{14} G_{2,n} = 0, \quad (5.39a)
\]

\[
\frac{-j \tau}{\omega_0} K_n(\tau a) A_n + 0 B_n + C_{23} G_{1,n} + C_{24} G_{2,n} = 0, \quad (5.39b)
\]

\[
\tau K_n'(\tau a) A_n + \frac{j \beta n}{\omega_0 a} K_n(\tau a) B_n - C_{33} G_{1,n} - C_{34} G_{2,n} = 0, \quad (5.39c)
\]

\[
\frac{j \beta n}{\omega_0 a} K_n(\tau a) A_n - \tau K_n'(\tau a) B_n + C_{43} G_{1,n} + C_{44} G_{2,n} = 0. \quad (5.39d)
\]
Where:

\[ C_{13} = \frac{\beta \kappa \chi_1^2}{\zeta} J_n(\chi_1 a), \quad (5.40a) \]

\[ C_{14} = \frac{\beta \kappa \chi_2^2}{\zeta} J_n(\chi_2 a), \quad (5.40b) \]

\[ C_{23} = \frac{j \beta^2 T_1 \chi_1^2}{k_0} J_n(\chi_1 a), \quad (5.40c) \]

\[ C_{24} = \frac{j \beta^2 T_2 \chi_2^2}{k_0} J_n(\chi_2 a), \quad (5.40d) \]

\[ C_{33} = T_1 \chi_1 J_n'(\chi_1 a) + \frac{S_1 n}{a} J_n(\chi_1 a), \quad (5.40e) \]

\[ C_{34} = T_2 \chi_2 J_n'(\chi_2 a) + \frac{S_2 n}{a} J_n(\chi_2 a), \quad (5.40f) \]

\[ C_{43} = j \beta N \chi_1 J_n'(\chi_1 a) + \frac{j \beta M_1 n}{a} J_n(\chi_1 a), \quad (5.40g) \]

\[ C_{44} = j \beta N \chi_2 J_n'(\chi_2 a) + \frac{j \beta M_2 n}{a} J_n(\chi_2 a) \quad (5.40h) \]

The primed Bessel functions \((J'_n(.)\) and \((K'_n(.))\) represent a derivation with respect to \(r\). After few manipulations, the system in (5.39) is identified as:

\[
\begin{vmatrix}
0 & 1 & 1 & 1 \\
1 & 0 & \frac{j \beta T_1 \zeta}{k_0 \kappa} & \frac{j \beta T_2 \zeta}{k_0 \kappa} \\
-j \omega \mu_0 \frac{k_0' \chi_1}{K_n(\chi_1 a)} & -\frac{\beta n}{a} & \frac{-T_1 \zeta J_n'(\chi_1 a)}{\beta \kappa \chi_1 J_n(\chi_1 a)} + \frac{S_1 n \zeta}{\beta \kappa \chi_1^2} - \frac{T_2 \zeta J_n'(\chi_2 a)}{\beta \kappa \chi_2 J_n(\chi_2 a)} - \frac{S_2 n \zeta}{\beta \kappa \chi_2^2} \\
- \frac{-\beta n}{a} & - j \omega \epsilon_0 \frac{K_n'(\chi_1 a)}{K_n(\chi_1 a)} & \frac{j N \zeta J_n'(\chi_1 a)}{\kappa \chi_1 J_n(\chi_1 a)} + \frac{j n M_1}{\kappa \chi_1^2} - \frac{j N \zeta J_n'(\chi_2 a)}{\kappa \chi_2 J_n(\chi_2 a)} - \frac{j n M_2}{\kappa \chi_2^2}
\end{vmatrix} = 0
\]

(5.41)

A propagating mode is identified by the propagation constant \((\beta)\) at which the determinant in (5.41) vanishes for a rod with a radius \((a)\) at a certain frequency.
5.3 Propagating Modes in an Axially Magnetised Lossless InSb Rod

In this section, we will apply the algorithm explained in the previous section to a lossless InSb rod. The material has the same electronic properties stated in Chapter (2).

The main thing to analyse here is the two transverse wavenumbers $\chi_{1,2}$ expressed in (5.29). Propagating modes through the InSb rod can be classified according to the nature of these two wavenumbers as being purely real, imaginary or complex. Let’s rearrange the expressions for $\chi_{1,2}$ in (5.29) as follows:

\[
\begin{align*}
\chi_1^2 &= X + \sqrt{Y} \\
\chi_2^2 &= X - \sqrt{Y},
\end{align*}
\]

Where

\[
\begin{align*}
X &= \frac{1}{2} \left[ k_0^2 \left( \zeta + \epsilon_{eff} \right) - (1 + g_0) \beta^2 \right] \\
Y &= \frac{1}{4} \left[ k_0^2 \left( \zeta - \epsilon_{eff} \right) + (1 - g_0) \beta^2 \right]^2 + \beta^2 k_0 \zeta g_e^2,
\end{align*}
\]

Depending on the signs of $X$ and $Y$ and the value of each of them, the nature of the mode specified by $\chi_1$ and $\chi_2$ can be determined. When a wavenumber is purely imaginary, the mode change in $r-$direction will be determined by a modified Bessel function of the first kind, and hence the fields will be increasing till reaching a maximum value at $r = a$. This mode can be called a surface mode (since most of the field is shifted towards the surface of the rod). Another type of modes is identified when the wavenumber is real, so the field behaviour will be determined according to a Bessel function of the first kind, hence most of the field will be concentrated inside the rod. Such mode is called a volume mode.

Here, a mode is assigned as volume-volume when both its wavenumbers are real. On the other hand, it is assigned as surface-surface when both wavenumbers are purely imaginary. Surface-volume or volume-surface denote the mode when one of the wavenumber is real and the other is imaginary. In Table (5.1), different regions of operation are defined based on the nature of the two wavenumbers$^4$.

$^4$It should be noted here that these regions are different from those defined in Chapter (3) to divide the frequency range according to the values of $\frac{\pi}{2}$ and $\epsilon_{eff}$.
Table 5.1: Regions of operation for the two wavenumbers $\chi_{1,2}$ based on the signs of the two variables in equation (5.43).

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
<th>Region</th>
<th>Condition</th>
<th>$\chi_1$</th>
<th>$\chi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ve</td>
<td>+ve</td>
<td>I – A</td>
<td>$X &gt; \sqrt{Y}$</td>
<td>Real</td>
<td>Real</td>
</tr>
<tr>
<td>+ve</td>
<td>−ve</td>
<td>II – A</td>
<td>$X &gt; \sqrt{Y}$</td>
<td>Complex</td>
<td>Complex</td>
</tr>
<tr>
<td>−ve</td>
<td>+ve</td>
<td>III – A</td>
<td>$X &gt; \sqrt{Y}$</td>
<td>Imaginary</td>
<td>Imaginary</td>
</tr>
<tr>
<td>−ve</td>
<td>−ve</td>
<td>IV – A</td>
<td>$X &gt; \sqrt{Y}$</td>
<td>Complex</td>
<td>Complex</td>
</tr>
</tbody>
</table>

Inspecting (5.42) and (5.43) reveals that the values of the two wavenumbers depend on $\beta$ and the gyroelectric properties of the material. Hence, at a certain frequency, the values of $\chi_{1,2}$ depend mainly on $\beta$. Analysing the relation in (5.43) yields the limits of $\beta$ at which the transitions between the regions depicted in Table (5.1) occur.

The limits between the regions I, II, III and IV are found by setting $X = 0$ or $Y = 0$ in (5.43a) and (5.43b), respectively. The results are quadratic equations in $\beta$ that can be solved numerically.

Setting $X = \sqrt{Y}$ allows finding the limits between the sub-regions (that is, regions A or B within any of the four main regions). The result of this is a biquadratic equation that can also be solved numerically.

Figure (5.3) shows the boundaries between the regions for a lossless semiconductor rod axially biased with a fixed magnetic field ($B_0$). The discontinuity seen in the $X = 0$ line occurs at $f_g = \frac{\omega_g}{2\pi}$, where $\omega_g$ is the the real root of the following polynomial:

$$\omega^4 - \left(\frac{\epsilon_r \omega_c^2 + \omega_p^2}{\epsilon_r}\right) \omega^2 + \frac{\omega_p^2 \omega_c^2}{2\epsilon_r} = 0 \quad (5.44)$$

It can be seen that for $f < f_g$, low values of $\beta$ are associated with negative $X$ and positive $Y$. Hence, the operation lies in region III. Types of the modes depend on the position of the operation point $(f, \beta)$ with respect to the $|X| = \sqrt{Y}$ line (i.e., being in region III – A or III – B ). Increasing $\beta$ causes a transition to region IV, where the $\chi_{1,2}$ are the complex conjugate of each other. After crossing

\footnote{The measured electronic properties depicted in Chapter (2) are used here while assuming $\nu_c = 0$.}
both $X = 0$ and $Y = 0$ lines, the operation enters region $I - A$, where both transverse wavenumbers are real. The operation in this region is limited till the value of $X$ exceeds $\sqrt{Y}$, then region $I - B$ (real $\chi_1$ and imaginary $\chi_2$) is entered. The operation stays in this region as $\beta$ increases.

For $f > f_g$, the parameter $X$ is positive for $k_0 \leq \beta \leq \beta|_{X=0}$. Hence, the operation starts in region $I - A$. It can also be noted that the variable $Y$ doesn’t cross zero above a certain frequency, therefore there is no operation in regions $II$ and $IV$ above that frequency.

Studying the values of the determinant in (5.41) for an InSb rod of a fixed radius ($a$) shows that they do not converge to zero for regions $II$ and $IV$ for real $\beta$. Region $III - A$ (where both wavenumbers are imaginary) does not reveal any convergence either.

When $\chi_1$ is real and $\chi_2$ is imaginary (in regions $I - B$ and $III - B$), the real part of the determinant does not converge for real valued $\beta$. However, it converges when $\beta$ includes an imaginary component.

Based on all the above, for a lossless InSb rod, a *volume-volume* mode is obtained.
in region I – A, and a surface-volume mode is obtained in either region I – B or III – B.

Now, let’s evaluate the determinant in (5.41) for a lossless InSb rod of 1 mm radius, biased with \( B_0 = 0.5 \, T \). Results show that a number of volume-volume modes can exist in region I – A. However, the determinant does not vanish in the regions that support surface-volume modes unless an imaginary component is included in \( \beta \). It is worthwhile to note that such modes with imaginary components in the phase constant are purely mathematical since the considered semiconductor is lossless.

Figure (5.4) shows the change of the \( n = 1 \) volume-volume modes for the above InSb rod in the frequency range 30 – 40 GHz, where the region I – A exists. The subscripts given to the modes are based on the number of times the Bessel function crosses zero for each of the two transverse wavenumbers. Another subscript was added to each mode related to the value of \( n \).

Figure (5.4) shows that for most cases, both clockwise (\( n = +1 \)) and counterclockwise (\( n = -1 \)) modes degenerate, except for one mode (\( v_6 - v_0 \)), where the
two counter-rotating modes split above a certain frequency. This occurs because the determinant in (5.41) vanishes at two different frequencies for the same $\beta$. Propagation of counter rotating modes with different $\beta$'s causes a linearly polarised wave to be \textit{rotated} with a certain angle, in a phenomena well known as \textit{Faraday Rotation}. This behaviour was observed before in circular waveguides loaded with axially magnetised semiconductor rods [57,102]. Mode splitting can be further analysed by observing the modes’ behaviour when changing the magnetic bias. Figure (5.5) depicts the change in the phase constant at the frequency of $40 \text{GHz}$ when increasing the magnetic bias from 0.5 to 1 T within the limits of region $I - A$. First, increasing the magnetic bias has the effect of decreasing the upper and lower limits of region $I - A$, hence it will cause a reduction in the phase constant of all the propagating volume-volume modes. Moreover, for $B_0 = 0.5 \text{T}$, all the modes shown in Figure (5.4) at $f = 40 \text{GHz}$ can be observed\(^6\). The negative branch of the split mode $v_6 - v_0$ reaches the region’s upper limit as the magnetic bias increase. By exiting this region, it will enter the region $I - B$, where one of the two wavenumbers is imaginary, and hence the mode will be converted to (surface-volume)\(^7\). Other modes, such as $v_5 - v_0$ and $v_5 - v_1$ start to split as the magnetic bias increase.

5.4 **Exciting Counter-Rotating Resonant Modes in an InSb Rod**

Exciting and measuring the modes travelling through a suspended, axially magnetised semiconductor rod is challenging. However, it is possible to observe the counter rotating modes as they affect the resonant behaviour of a rectangular waveguide loaded with a gyroelectric disk. Inspecting the changes in the measured transmission parameters for the loaded waveguide when changing the magnetic bias can give an indication for this modal behaviour. In addition, field plots from the CST MWS simulation package can be used to confirm the existence of certain modes when simulating the same structure.

\(^6\) The mode $v_3 - v_2$ coincides with the lower limit of the region I-A, and it was removed for clarity.

\(^7\) The propagation constant for such modes will contain an imaginary component. Similar modes were identified before in ferrite rods, as reported in [130]
Figure 5.5: Changes in the volume-volume modes for a lossless InSb rod of 1 mm radius for different values of axial magnetic bias ($B_0$).
Figure 5.6: Waveguide configuration used to excite counter rotating modes in a semiconductor disk with fields distribution of the TE\textsubscript{10} mode in the cross section of the same waveguide.

Here, modal behaviour of a magnetised InSb disk was tested by simulating and measuring the structure shown in Figure (5.6). By operating the rectangular waveguide in the TE\textsubscript{10} mode, $E_y$, $H_x$ and $H_z$ will propagate through it. These field components have the following expressions [1]:

\begin{align*}
    H_z &= A_{10} \cos \left( \frac{\pi x}{a} \right), \\
    H_x &= \frac{j \beta' \pi}{k_c^2 a} A_{10} \sin \left( \frac{\pi x}{a} \right), \\
    E_y &= \frac{-j \omega \mu_0 \pi}{k_c^2 a} A_{10} \sin \left( \frac{\pi x}{a} \right)
\end{align*}

Where $A_{10}$ is an arbitrary amplitude, $k_c$ and $\beta'$ are the cut-off wavenumber and phase constant of the TE\textsubscript{10} mode, respectively.

The InSb disk is treated here as a suspended resonator that interacts with the dominant mode fields inside the waveguide. It can be seen from (5.45) that the magnetic field component $H_z$ changes as a cosine function with the $x$–direction; it has zero value at $x = \frac{a}{2}$, and reaches two opposite values at the waveguide edges (that is, $x = 0$ and $x = a$). This means that the semiconductor sample in Figure (5.6) is excited with a tangential magnetic field that changes polarity along its axial direction. Gyroelectric modes with harmonic change along the semiconductor’s axis will be excited in this arrangement.

Assuming the phase constant of the gyroelectric mode propagating through the
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Figure 5.7: Simulated scattering parameters for a WR-28 waveguide loaded with an InSb disk of \( h = 0.72 \ mm \) and \( a = 1 \ mm \) at 77 \( K \), axially magnetised with \( B_0 = 0.5 \ T \). Insets: plots of the axial magnetic field component \( H_z \) on the InSb disk, arrows indicate the direction of rotation.

The structure resonates when the tangential fields on the top and bottom of the semiconductor disk match those in the free space waveguide regions above and below it. Hence, the resonant frequency will depend on the wavenumber of these regions, as well as the wavelength of the gyroelectric modes excited in the sample \( (\lambda_g) \).

For a certain waveguide, a sample with a fixed length will resonate when its electrical length \( h_e = \frac{h}{\lambda_g} \), where \( \lambda_g = \frac{2\pi}{\beta_g} \).

Resonant frequency is directly proportional to the electrical length of the sample. Therefore resonance can be tuned down in frequency by either decreasing the height of the sample \( (h) \) or increasing the gyroelectric mode’s wavelength \( (\lambda_g) \). The latter is achieved by decreasing the phase constant \( \beta_g \) which occurs when increasing the magnetic bias \( (B_0) \), as discussed in the previous section.

Figure 5.7 shows the scattering parameters resulted from simulating a WR-28 waveguide loaded with an InSb sample at 77 \( K \). The sample’s height is 0.72 \( mm \).
and radius is 1 mm, and was biased with a magnetic field of 0.5 T. CST MWS package allows viewing and animating the fields in the simulated structure. Using this feature, a clockwise rotating mode was noticed to be propagating at $f = 35 \text{ GHz}$, and a counter clockwise mode starts to propagate at $f = 39 \text{ GHz}$. The insets of Figure (5.7) show plots of the magnetic field component $H_z$ on the gyroelectric disk. Because of the included loss in the simulated InSb, both wavenumbers ($\chi_{1,2}$) are expected to be complex valued. Hence, the distribution of the field on the disk does not indicate a pure volume-volume mode.

The same structure was measured at 77 K by inserting the above InSb sample inside an 11 mm long waveguide, and applying different values of magnetic bias across it (in the axial direction to the InSb disk). Figure (5.8) shows the measured transmission parameters ($S_{12,21}$) at different values of $B_0$ alongside with the simulated parameters under the same conditions. It is clear that both sets of curves behave in a similar way, and the dip that indicates a clockwise rotating is shifting down in frequency as the magnetic bias is increased.

\footnote{It is not possible to model a gyroelectric material with $\nu_e = 0$ using the CST MWS simulation package.}

\footnote{More details about the waveguide used in these measurements is given in Appendix (B).}
Figure 5.9: Simulated transmission parameters for a WR-28 waveguide loaded with an InSb disk of $a = 1\ mm$ at $77\ K$, biased with $B_0 = 0.6\ T$ for different values of height ($h$).

The effect of increasing the height of the sample was tested by simulating the structure for different heights of the InSb sample. Figure (5.9) shows that the resonance frequency for the clockwise rotating mode is shifting up when the height of the sample is increased. In addition, field plots shown in Figure (5.10) indicate that the number of changes along the height of the sample is related to the gyroelectric modes discussed in the previous section.

5.5 Designing Waveguide Junction Circulators Using Axially Magnetised InSb Disks

It can be concluded from Chapter (3) that circulation is achieved in a certain junction as a result of a superposition of counter rotating modes at the circulation frequency. For a three port junction, this superposition isolates one of the ports and allows the transmission to be in one direction between the other two ports.
Figure 5.10: Tangential magnetic field distribution in a gyroelectric disk inside a waveguide at $f = 35 \text{GHz}$ and with a magnetic bias of $0.6 \text{T}$ for different disk heights ($h$).

The counter rotating modes observed in the previous section indicate the possibility of achieving circulation when these modes incorporate to form a standing wave when placing the semiconductor disk in a three port waveguide junction. Using a junction of three WR-28 waveguides, the circulator depicted in Figure (5.1) was realised by utilising the same InSb sample used in the modal analysis. It is expected to obtain circulation in the mid-frequency between the two points at which the counter rotating modes are excited.

Figure (5.11) shows a photograph of the measured fixture. It consists of the same three port WR-28 waveguide junction used to measure the SJC in Chapter (4). However, this time no finline circuit is used, and the InSb sample is fixed in the middle of the junction using a transparent tape that adds minimum possible loss. During the measurements, one of this fixture’s ports was terminated with a matched waveguide load $^{10}$. The two remaining ports were connected to the Vector Network Analyser (VNA) via waveguide to coaxial transitions. TRL calibration was used to set the reference plane at the two input ports of the fixture in Figure (5.11)$^{11}$.

As with the measured SJC in Chapter (4), the fixture was immersed into a liquid nitrogen container and placed axially between the poles of an electromagnet, as shown in Figure (4.7).

$^{10}$More information about this waveguide load is provided in Appendix (B).

$^{11}$More information about the TRL calibration is given in Appendix (A).
Figure 5.11: A photograph of the measured waveguide circulator structure. Inset: the InSb sample (dimensions are in mm)

Figure (5.12) shows the measured scattering parameters for the structure at room temperature when the magnetic bias is 1 T. These parameters indicate that no significant isolation can be obtained at room temperature even with relatively high magnetic bias. The reason behind this is the high collision frequency ($\nu_c$) associated with this semiconductor at higher temperatures, as discussed in Chapter (2).

Figure (5.13) shows the measured scattering parameters for the fixture at 77 K when the magnetic bias is 0.55 T. CST simulation results for the same structure are plotted in the same figure. Measured results show an isolation between the two ports of the circulator of 18.75 dB, with insertion loss of 2.6 dB at $f = 38.5 \text{ GHz}$. This confirms the prediction about the circulation occurrence between the clockwise and counter clockwise frequencies. Measured reflection loss in both Figure (5.12) and (5.13) seems to be different for the two ports, this is attributed to the asymmetry of the InSb sample’s position, in addition to the existence of the tape used to fix the sample in place.

As stated in the previous section, increasing the magnetic bias causes the excitation of the modes to occur at lower frequencies. Hence, circulation can also be
Figure 5.12: Measured scattering parameters for an InSb waveguide junction circulator at room temperature with $h = 0.72$ mm and $a = 1$ mm. The InSb sample is axially biased with 1 $T$.

Figure 5.13: Measured scattering parameters for a waveguide junction circulator with InSb sample at 77 $K$ with $h = 0.72$ mm and $a = 1$ mm. The sample is axially biased with 0.55 $T$. 
Figure 5.14: Measured differential isolation ($|S_{12} - S_{21}|$) for a waveguide junction circulator with InSb sample at 77 K with $h = 0.72$ $mm$ and $a = 1$ $mm$ for different values of magnetic bias.

achieved at lower frequency when the bias is increased.

Figure (5.14) shows the differential isolation (the absolute difference between the transmission parameters) for the tested structure for different values of $B_0$. It is clear that as the magnetic bias is increased, the circulation frequency shifts down. This can be observed by tracking the shift in the frequency at which the two transmission parameters cross (when the difference reaches zero).

All the above simulation and measurement results were taken when the InSb sample is suspended inside an empty waveguide. Increasing the dielectric constant of the spaces above and below the sample causes the wavenumber in these regions to increase. Hence, the tangential field components there will match those in the gyroelectric sample at lower frequencies.

Here, we aim to reduce the required magnetic bias for circulation at a certain frequency. To do this, the circulation obtained for a certain magnetic bias can be tuned down to a lower frequency range by including dielectric disks above and below the InSb sample, as shown in Figure (5.15). As indicated in Figure (5.8), decreasing the magnetic bias shifts up the frequencies at which the counter rotating modes are excited. Hence, it is expected that the circulation frequency
Figure 5.15: A photograph of the InSb sample sandwiched between the two dielectric disks.

increases for low values of $B_0$. Figure (5.16) shows the measured scattering parameters for the circulator using the InSb disk only when the magnetic bias is $0.25 \ T$. It is clear that the circulation frequency in this case is increased beyond the limits of frequency band of interest (i.e., the Ka-band).

To shift down the circulation frequency, two dielectric disks, each of 0.55 mm thickness and dielectric constant of 6.15 were included below and above the InSb sample\textsuperscript{12}. Biasing this structure with $B_0 = 0.25 \ T$ now results in 29.5 dB isolation at $f = 35.5 \ GHz$, as shown in Figure (5.17). The high insertion and reflection losses (5 dB and 7 dB, respectively) are caused by the increased size of the circulator’s filling. Better results can be achieved by changing the thickness and the dielectric constant of the disks.

5.6 Conclusions

In this chapter, the possibility of designing gyroelectric circulators in a waveguide junction was investigated for the first time. Despite the simplicity of the structure, calculating the exact circulation frequency is difficult because of the fields’ non negligible change in the $z-$direction. Hence, an approximate approach was followed by analysing a lossless suspended axially magnetised semiconductor rod

\textsuperscript{12}Both disks are cut from Rogers RO3006 dielectric substrate after removing the metallisation.
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Figure 5.16: Measured scattering parameters for the InSb waveguide circulator at 77 K without dielectric disks when $B_0 = 0.25 \, T$.

Figure 5.17: Measured scattering parameters for the InSb waveguide circulator at 77 K with dielectric disks when $B_0 = 0.25 \, T$. 
with a certain radius.

The outcome of the mathematical analysis showed that the suspended rod can support counter rotating modes splitting under certain frequency and magnetic bias conditions. These modes can be excited in an axially magnetised semiconductor disk placed inside a rectangular waveguide. Each mode will be resonant when the sample’s electrical length (with respect to that mode) reaches a certain value that guarantees matching with the fields in the spaces above and below the sample.

A superposition between the counter rotating modes was then utilised to induce a standing wave that isolates one port out of a three port waveguide junction, when the sample is placed in the middle of it.

Using a 0.72 mm thick InSb disk, cooled down to 77 K, it was possible to achieve circulation at \( f = 38.5 \, GHz \) with a magnetic bias of 0.55 T. Despite some differences due to the experimental errors, measured results matched simulation results to high degree.

To achieve circulation with lower values of magnetic bias, the InSb sample was sandwiched between two dielectric disks. This allowed the wavelength in the space above and below the InSb disk to be reduced and hence, match the fields in the counter rotating modes at a lower frequency. Measured results for this case showed more than 25 dB isolation at 35.5 GHz with \( B_0 = 0.25 \, T \), which cannot be achieved for a stand-alone sample with a magnetic bias less than 0.7 T. These results are therefore highly promising despite the relatively high insertion and reflection losses, which can be improved by reducing the size of the dielectric disks.
Chapter 6

Electromagnetic Analysis of Partially Loaded Rectangular Waveguides

6.1 Introduction

The aim of this chapter is to demonstrate the electromagnetic analysis of rectangular waveguides partially loaded with dielectric, gyroelectric or mixed media. The chapter starts by considering a general model of a waveguide with rectangular cross section loaded with layered media. Dispersion relations and field distributions of such structures are then found by solving Maxwell’s equations for each layer and matching the fields at their interfaces. This approach followed what Gardiol [60], Sorrentino [62] and Krowne [65] have used for waveguides loaded with layered gyrotropic media.

After that, an algorithm is developed to find the complex propagation constant for all the modes that can be excited for a particular waveguide loading. Using this algorithm, propagating modes through a WR-28 waveguide loaded with dielectric layers are found and analysed. Next, the same waveguide is analysed when it is symmetrically and asymmetrically loaded with transversely biased gyroelectric layers. The possibility of realising reciprocal and nonreciprocal devices based on each loading is then discussed.

Finally, the developed algorithm is used to find the propagating modes and their characteristics for a waveguide loaded with mixed dielectric and gyroelectric layers.


6.2 A General Model of Loaded Rectangular Waveguides

Consider the model shown in Figure (6.1), it consists of a metallic waveguide of $a \times b$ cross section$^1$, filled with $f$ layers, aligned in parallel to the $x-z$ plane. The material of each layer is characterised by a tensor permittivity ($[\epsilon]$). Free space or dielectric materials are considered by assuming the off-diagonal tensor elements to be zero and the diagonal elements to be unity for the former, and $\epsilon_d$ for the latter.

The material of all the layers are assumed to be uniform along the direction of propagation (the $z-$direction). It is also assumed that there is no surface current flowing between the layers (except for $y = 0$ and $b$). Hence, the fields components tangential to the $x-z$ plane are considered to be continuous along the boundaries between the layers.

If the $i^{th}$ layer of the loading is represented by a tensor permittivity $[\epsilon]_i$, Maxwell’s

$^1$It is assumed that the walls of the waveguide are perfect conductors.
two curl equations for this layer can be written as (assuming $e^{j\omega t}$ time dependence):

\[ \nabla \times \vec{E} - k\eta \vec{H} = 0, \]
\[ \nabla \times (\eta \vec{H}) - k[\epsilon]_1 \vec{E} = 0 \]  

(6.1a, 6.1b)

Where $k = \omega \sqrt{\epsilon_0 \mu_0}$ and $\eta = -j \sqrt{\mu_0 / \epsilon_0}$ are the free space wavenumber and wave impedance, respectively.

In this analysis, the field components of all the modes are assumed to be sinusoidal in the $x$-direction and described as:

\[ A_{j,n}(y) \sin\left(\frac{n\pi x}{a}\right) + B_{j,n}(y) \cos\left(\frac{n\pi x}{a}\right) e^{-\gamma z} \]  

(6.2)

Where the subscript $j$ refers to the field component ($x$, $y$ or $z$) and $n$ is a non zero integer. $\gamma$ is the propagation constant for the specific mode, given the propagation is in the direction of increasing $z$.

By applying the boundary conditions at $x = 0$ and $a$ for the model shown in Figure (6.1), electric field components parallel to the those boundaries can be expressed as:

\[ E_y = A_{2,n}(y) \sin(hx)e^{-\gamma z}, \]
\[ E_z = A_{3,n}(y) \sin(hx)e^{-\gamma z} \]  

(6.3a, 6.3b)

Where $h = \frac{n\pi}{a}$.

The field behaviour described in (6.3) limits the properties of the considered media in the $i^{th}$ layer to be symmetric in the $x$-direction. In other words, the semiconductor considered here should be magnetically biased in the $x$-direction, and described using the tensor:

\[ \begin{bmatrix} \zeta & 0 & 0 \\ 0 & \varepsilon & j\kappa \\ 0 & -j\kappa & \varepsilon \end{bmatrix} \]  

(6.4)

\[ ^2 \text{The permeability } \mu_r \text{ is assumed to be 1 in all cases.} \]
By solving the two curl equations in (6.1) after substituting the tensor permittivity in (6.4), the expressions for the remaining field components can be found\(^3\).

Now the fields are expressed as:

\[
\{E_y, E_z, \eta H_x\} = \{\hat{E}_y, \hat{E}_z, \eta \hat{H}_x\} \sin(hx)e^{-\gamma z}, \quad (6.5a)
\]

\[
\{E_x, \eta H_y, \eta H_z\} = \{\hat{E}_x, \eta \hat{H}_y, \eta \hat{H}_z\} \cos(hx)e^{-\gamma z} \quad (6.5b)
\]

Where \(\hat{E}_{x,y,z}\) and \(\hat{H}_{x,y,z}\) are functions of \(y\) only.

The solution of Maxwell’s equations given the field expressions in (6.5) yields six homogeneous equations in terms of all the field components. They can be rearranged in matrix form as follows:

\[
\begin{bmatrix}
0 & -\gamma & -\nabla_y & k & 0 & 0 \\
\gamma & 0 & h & 0 & k & 0 \\
\nabla_y & -h & 0 & 0 & 0 & k \\
k\zeta & 0 & 0 & 0 & -\gamma & -\nabla_y \\
0 & k\varepsilon & jk\kappa & \gamma & 0 & h \\
0 & -jk\kappa & k\varepsilon & \nabla_y & -h & 0
\end{bmatrix}
\begin{bmatrix}
\hat{E}_x \\
\hat{E}_y \\
\hat{E}_z \\
\eta \hat{H}_x \\
\eta \hat{H}_y \\
\eta \hat{H}_z
\end{bmatrix}
= 0 \quad (6.6)
\]

Where \(\nabla_y\) represents the derivative with respect to \(y\). By eliminating the terms not subjected to derivative, (6.6) yields the following expression (after some rearrangements):

\[
\begin{bmatrix}
\hat{E}_x \\
\hat{E}_y \\
\hat{E}_z \\
\eta \hat{H}_x \\
\eta \hat{H}_y \\
\eta \hat{H}_z
\end{bmatrix}
= \begin{bmatrix}
0 & -j\frac{bh}{k}\varepsilon & -\frac{\gamma h}{k}\varepsilon & \frac{\gamma h}{k}\varepsilon & k - \frac{h^2}{k}\varepsilon & 0 \\
0 & \frac{j\gamma k}{k}\varepsilon & \frac{\gamma h}{k}\varepsilon & -\frac{\gamma h}{k}\varepsilon & -\frac{\gamma k}{k} & \frac{jbh}{k} \\
k\zeta & -\frac{\gamma h}{k} & \frac{\gamma h}{k} & 0 & 0 & \frac{jbh}{k}
\end{bmatrix}
\begin{bmatrix}
\hat{E}_x \\
\hat{E}_y \\
\hat{E}_z \\
\eta \hat{H}_x \\
\eta \hat{H}_y \\
\eta \hat{H}_z
\end{bmatrix} \quad (6.7)
\]

It is possible to write the relation in (6.7) in matrix form as:

\[
\nabla_y [\hat{\phi}] = [A][\hat{\phi}] \quad (6.8)
\]

It can be seen that (6.8) is a differential equation, which has a solution of the form \([60, 133]\):

\(^3\)The change of each field component with respect to the \(x\)-direction can be related to either \(E_y\) or \(E_z\) after rearranging all the tangential components in terms of the longitudinal components. More details are given in page (189) of [53]
for $y_{i-1} \leq y \leq y_i$.

The solution in (6.9) indicates that the set of fields ($\hat{\phi}$) at any point $y$ is related to that at the interface directly before it (i.e., at $y_{i-1}$) by the exponential of the matrix $[A]$ multiplied by the distance between $y$ and $y_{i-1}$. Hence, the fields at two consecutive interfaces are related to each other by means of a transfer matrix ($T_i$), which is related to the material properties between the two interfaces as:

$$[T_i] = e^{[A]w_i} \quad (6.10)$$

Where $w_i$ is the width of the layer $i$ sandwiched between the two interfaces.

Since the field components are assumed to be continuous across the interfaces, their values at the last interface (at $y_f = b$) can be found in terms of those at the first interface (at $y_0 = 0$) by multiplying the transfer matrices of all the layers between them. For a waveguide filled with $f$ layers, the overall transfer matrix is expressed as:

$$[\hat{\phi}]_{y_f} = [T_f][T_{f-1}]...[T_{i+1}][T_i][T_{i-1}]...[T_2][T_1][\hat{\phi}]_{y_0} = [T][\hat{\phi}]_{y_0} \quad (6.11)$$

Applying boundary conditions at $y = 0$ and $b$ will make $\hat{E}_x$ and $\hat{E}_z$ vanish at these two positions. This will yield the following equations:

$$\hat{E}_x(b) = t_{13}\eta \hat{H}_x(0) + t_{14}\eta \hat{H}_z(0) = 0, \quad (6.12a)$$
$$\hat{E}_z(b) = t_{23}\eta \hat{H}_x(0) + t_{24}\eta \hat{H}_z(0) = 0 \quad (6.12b)$$

Where $t_{ij}$ is an element of the matrix $[T]$ that lies in the $i^{th}$ row and $j^{th}$ column. The two equations in (6.12) can be expressed in matrix form as follows:

$$\begin{bmatrix} t_{13} & t_{14} \\ t_{23} & t_{24} \end{bmatrix} \begin{bmatrix} \hat{H}_x(0) \\ \hat{H}_z(0) \end{bmatrix} = 0 \quad (6.13)$$

For a nontrivial solution to exist, the determinant in (6.13) has to vanish, or:

$$t_{13}t_{24} - t_{14}t_{23} = 0 \quad (6.14)$$
Dispersion relations for the structure shown in Figure (6.1) with any number of layers are found by solving (6.14) for \( n \geq 1 \). Setting \( n \) to 0 will include the modes of a parallel plate waveguide with the a space of \( b \) between the two plates filled with the multiple layers\(^4\). By considering the TE\(_{0,m}\) mode\(^5\), a simpler matrix \( \tilde{A} \) is resulted for the \( n = 0 \) case:

\[
\nabla_y \begin{bmatrix} \hat{E}_x \\ \eta \hat{H}_z \end{bmatrix} = \begin{bmatrix} \begin{array}{cc} 0 & -k \\ k \zeta + \frac{\omega^2}{c^2} & 0 \end{array} \end{bmatrix} \begin{bmatrix} \hat{E}_x \\ \eta \hat{H}_z \end{bmatrix} = [\tilde{A}] \begin{bmatrix} \hat{E}_x \\ \eta \hat{H}_z \end{bmatrix} \tag{6.15}
\]

The transfer matrix for this TE\(_{0,m}\) mode \( ([\tilde{T}] ) \) can then be realised, as before, by finding the exponential of the matrix \( \tilde{A} \). Dispersion relation is then found from the equation:

\[
\bar{t}_{12} = 0 \tag{6.16}
\]

Where \( \bar{t}_{12} \) is the element in first row and second column of the matrix \( \tilde{T} \).

### 6.3 Computing the Dispersion Relations

To find the propagation constant \((\gamma)\) for any mode, the exponential of the matrix \([A]\) in (6.10) should be found. Given that \([A]\) is not singular (which is the case most of the time), the exponential term in the transfer matrix of the layer \( i \) \( ([T_i]) \) can be found from the relation \([133]\):

\[
[T_i] = e^{[A]w_i} = [Q][\text{diag } e^{p_j w_i}][Q^{-1}] \tag{6.17}
\]

where \( (\text{diag}) \) represents a diagonal matrix, \( p_j \) are the eigenvalues of the matrix \([A]\) and \([Q]\) is a matrix whose columns are the eigenvectors of \([A]\).

Finding the propagation constant for a certain mode is done by considering each layer of the structure shown in Figure (6.1) by itself, starting from the first one at the bottom. After finding the transfer matrices for all the layers, they are multiplied to get the overall transfer matrix \( ([T]) \). Next, an algorithm uses \( ([T]) \) to find the zeros of the function \( f(\gamma) \), where:

\[
f(\gamma) = t_{13}t_{24} - t_{14}t_{23} \tag{6.18}
\]

\(^4\)This is because making \( n \) zero will force \( h = \frac{n\pi}{a} \) to be zero as well. This makes the case equivalent to setting \( a = \infty \) which will also make \( h = 0 \).

\(^5\)In this case, the ratio between the waveguide’s dimensions (\( \frac{a}{b} \)) is assumed to be larger than 1.
The search algorithm starts by assuming two initial values of $\gamma$. These values are usually a large imaginary number to allow finding the propagating modes through the complex $\gamma$ domain. Using the two initial values, the algorithm follows an equi-phase line to reach a zero of the function $f(\gamma)$ by using the following relation:

$$
\gamma_{m+1} = \gamma_m - \frac{f_m(\gamma_m - \gamma_{m-1})/((f_m - f_{m-1})l)}{|(\gamma_m - \gamma_{m-1})/((f_m - f_{m-1})|}
$$

(6.19)

Where $\gamma_m$ and $\gamma_{m-1}$ are the two previous points, $f_m = f(\gamma_m)$, $f_{m-1} = f(\gamma_{m-1})$ and $l$ is the step length for the search process.

The search process stops when the phase is highly changed (i.e., when the zero point is crossed), $\gamma$ at that point is considered as the solution for the equation. To find the next modes, the initial values of $\gamma$ are set to those associated with the maximum value of $f(\gamma)$.

After finding $\gamma$ for a certain mode, its field patterns along the $x$ and $y$—directions are found. This is started by setting the initial values of the fields vector $[\hat{\phi}]$ at $y = 0$. After that, the field components at each interface along the $y$—direction are found using the transfer matrix of the layer before it, until reaching $y = b$.

The amplitudes of the fields for each mode are then normalised to carry a certain power ($P_t$). This is done by multiplying the fields amplitudes vector by $\sqrt{\frac{P_z}{P_t}}$, where $P_z$ is the average power along the $z$—direction found from:

$$
P_z = 0.5 \left| \Re \int_0^b \int_0^a E_t^* H_t^* \, dx \, dy \right|
$$

(6.20)

Where $E_t$ and $H_t$ are the transverse electric and magnetic fields, respectively.

The above steps are repeated to find the dispersion characteristics for each frequency point in the desired frequency range. In addition, fields amplitude patterns for each mode can be found for any desired frequency point. A flow chart showing the main steps of this algorithm is shown in Appendix (D).

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6The search can start from, for example, $\gamma = j20000$ to find the propagating modes with the highest phase constant first.
6.4 Field Analysis of Rectangular Waveguides Loaded with Layered Media

In this section, the previously illustrated algorithm will be implemented to find the dispersion relations and fields patterns for rectangular waveguide structures filled with multiple layers of free space, dielectric and gyroelectric media.

To demonstrate the operation of the algorithm, a WR-28 waveguide with the specifications illustrated in Table (4.1) is used. The waveguide is assumed to contain $N$ layers with equal thickness, each of which has its own dielectric (or gyroelectric) properties. All the layers are aligned parallel to the $x-z$ plane, as shown in Figure (6.1).

Air filled waveguide is considered first. Here, the change of the propagation constant versus frequency for the dominant TE$_{10}$ mode is calculated theoretically from the following relation [1]:

$$\beta = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2} \quad (6.21)$$

The same mode can also be found by applying the developed algorithm after assuming all the layers to have a unity dielectric constant. Figure (6.2) compares between the propagation constants found using the algorithm and that found by applying the relation in (6.21). It is clear that there is a perfect match between the two results.

Three types of loading are considered here, namely dielectric, gyroelectric and a mix of both. The developed algorithm is applied to few types of each case to demonstrate the dispersion properties of such structures.

6.4.1 Dielectric Loading

The first case is when a lossless dielectric layer with $\varepsilon_r = 6$ fills 25% of the waveguide’s height. To implement this case to the algorithm, it is assumed that the number of layers $N = 64$. 16 of these layers have the dielectric constant of 6, the remaining 48 layers are with $\varepsilon_r = 1$. This number of layers is chosen to give accurate values of propagation constants and field distributions within a reasonable computation time.

By checking the propagating modes for each $n$ applied in the algorithm, four
CHAPTER 6. EM ANALYSIS OF PARTIALLY LOADED RWG

Figure 6.2: Propagation constant for the TE_{10} mode in a WR-28 air filled rectangular waveguide calculated using two approaches.

Table 6.1: Description of the propagating modes for a WR-28 waveguide loaded with one dielectric layer of $\epsilon_r = 6$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Cut-off Frequency (GHz)</th>
<th>n</th>
<th>m</th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$E_z$</th>
<th>$H_x$</th>
<th>$H_y$</th>
<th>$H_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE_{01}</td>
<td>32.27</td>
<td>0</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>EH_{11}</td>
<td>18.1</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>HE_{11}</td>
<td>35.3</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>EH_{21}</td>
<td>31.15</td>
<td>2</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
</tbody>
</table>

Hybrid\(^7\) modes were found to be transmitted through the structure. The hybrid modes are denoted as EH_{nm} or HE_{nm}, where $n$ is the variable used in (6.2) and it represents the number of changes in the $x$-direction. $m$ is the order of the mode for a specific $n$. The terms EH or HE are used when the mode has dominant electric field or magnetic field components, respectively.

Table (6.1) illustrates the characteristics of each mode, including its cut-off frequency and the components of electric and magnetic fields that appear in it. The behaviour of each mode with frequency in the Ka-band is shown in Figure

\(^7\)A mode is described as hybrid when both longitudinal components $E_z$ and $H_z$ exist.
Figure 6.3: Variation of the propagation constants of a WR-28 rectangular waveguide loaded with a $\epsilon_r = 6$ dielectric filling 25% of its height.

(Beside finding the propagation constant at each frequency, the developed algorithm allows finding the distribution of each field component in both $x$ and $y$—directions. To verify the results, a similar structure was modelled using the CST MWS simulation package, where a WR-28 waveguide loaded with the same dielectric was simulated and analysed. Simulation results confirmed those of the algorithm in terms of fields’ change along the $y$—direction at $f = 35\, GHz$, as shown in Figure (6.4) for the EH$_{11}$ mode.  

The same structure is analysed again after placing another layer with equal thickness and half the dielectric constant ($\epsilon_r = 3$) on top of the existing one. The new mixed loading is now covering half of the waveguide’s height. By analysing this structure, seven modes are found to propagate through it in the Ka-band. The characteristics of these modes are shown in Table (6.2) and Figure (6.5).

It can be concluded from comparing the propagating modes in the previous cases that the cut-off frequencies of all the modes for a single-layer loading are

---

8Here, the value of power was normalised to 0.5 $W$, which is the default power for the input ports for the frequency domain solver in the CST MWS simulation package.
Figure 6.4: Electromagnetic field components distribution along the $y$–direction for the $EH_{11}$ mode of a rectangular waveguide loaded with a dielectric of $\varepsilon_r = 6$ at $f = 35\;GHz$ (all the components were normalised for the mode to carry 0.5 W)
Table 6.2: Description of the propagating modes for a WR-28 waveguide loaded with two dielectric layers of $\epsilon_r = 6$ and 3.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Cut-off Frequency (GHz)</th>
<th>n</th>
<th>m</th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$E_z$</th>
<th>$H_x$</th>
<th>$H_y$</th>
<th>$H_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{HE}_{01}$</td>
<td>26.4</td>
<td>0</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>$\text{EH}_{11}$</td>
<td>15.6</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\text{HE}_{11}$</td>
<td>29.2</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\text{EH}_{12}$</td>
<td>30.4</td>
<td>1</td>
<td>2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\text{EH}_{21}$</td>
<td>26.2</td>
<td>2</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\text{HE}_{21}$</td>
<td>35.8</td>
<td>2</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$\text{EH}_{31}$</td>
<td>34.3</td>
<td>3</td>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

shifted down when adding another layer. This conclusion can be useful when analysing the gyroelectric filling, as will be show in the next section.

### 6.4.2 Gyroelectric Loading

Now, let’s consider the WR-28 waveguide to be loaded with an InSb slab at 77 K magnetically biased in the $x$–direction\(^9\). The slab thickness ($T_s$) is 0.9 mm, which corresponds to a 25% filling of the waveguide’s height (given the value of $b$ in Table (4.1)).

In this case of loading, the developed algorithm reveals complex-valued $\gamma$ as a solutions to the characteristic equation in (6.14). Here, a mode is considered to be propagating when $\text{Im}(\gamma) > \text{Re}(\gamma)$.

Figure (6.6) shows the propagating modes through the structure with magnetic bias of 1 $T$ within the Ka-band frequency range. Only $n = 1$ case is considered here\(^{10}\), although other modes can propagate within the band as will be shown later. At the beginning of the band, a mode is propagating with a propagation constant close to the TE\(_{10}\) of an unloaded waveguide. Electric field components dominate in this mode, hence it is called $\text{EH}_{11}$.

Field distributions of this mode show that most of the fields are confined in the free space above the slab for both directions of propagation, as shown in Figures (6.7b) and (6.7d). In addition to this mode, Figure (6.6) shows another mode

\(^9\)Measured electronic properties for InSb at 77 K illustrated in Chapter (2) were used throughout this chapter.

\(^{10}\)These modes have special importance since they are most likely to be excited by the TE\(_{10}\) mode of an empty waveguide.
Figure 6.5: Variations of the propagation constants of a WR-28 rectangular waveguide loaded with two layers of $\epsilon_r = 6$ and 3, both filling 50% of its height. ($EH_{12}$) with higher propagation constant that exists for only one direction\textsuperscript{11}. In this mode, electromagnetic field components are shifted towards the bottom of the gyroelectric layer, as shown in Figure (6.7a)\textsuperscript{12}.

At $f = 30$ GHz, the mode $EH_{11}$ in the forward direction (can be denoted as $EH_{11+}$) starts to decrease with frequency until $f = 34$ GHz where it starts to increase again. During this frequency range, the attenuation of this mode undergoes a significant increase and decrease. However, the same mode does not show this behaviour when observed on the opposite direction of propagation ($EH_{11-}$). Above this range of frequencies, a new $EH_{11-}$ mode starts to propagate, while the previously tracked $EH_{11-}$ mode changes to become similar to the $EH_{12}$ mode in the forward direction ($EH_{12+}$), where most of the field is confined in the gyroelectric slab. Although the fields in this mode are shifted towards the top of the slab, as shown in Figure (6.7f). A very similar phenomenon was observed by Bernardi [135] and Gardiol [60] with rectangular waveguides loaded with transversely magnetised ferrite slabs. In that case, two modes are defined

\textsuperscript{11}However, analysis shows that there is a similar, highly attenuated mode that exists on the other direction for the same frequency range.

\textsuperscript{12}Similar modes were expected by Seshadri [134] as TEM edge modes for gyroelectric filled parallel plate waveguides. They were shown to be reciprocal and propagate in both directions.
Figure 6.6: Complex propagation constants of the propagating modes through a WR-28 waveguide loaded with a 0.9 mm InSb slab at 77 K transversely biased with 1 T.
Figure 6.7: Calculated distribution of the electric field component \( E_y \) along the \( y \)-direction of the propagating modes in a WR-28 waveguide loaded with a 0.9 \( mm \) thick InSb slab at 77 \( K \) transversely magnetised with 1 \( T \). Shaded areas represent the location of the InSb slab.
to be coupled to each other, one of them undergoes an increased attenuation and
the other shows a decreased attenuation. When they are coupled together, the
overall attenuation looks like a (hump) at a certain frequency. In the same time,
the phase constant of one of the modes changes polarity. Here, however, both
modes are regarded as \((EH_{11})\) because the field behaviour is the same, as can be
concluded by inspecting the field plots in Figure (6.7).

From Figure (6.6), a decrease and increase in the phase constant is also noted in
the higher mode \(EH_{12^+}\), and it is associated with an increase in the attenuation
for the same range of frequencies. The distribution of the field components re-
main to be the same for the whole range of frequencies.

Now, let’s analyse the effect of magnetic bias \((B_0)\) on the modes shown in Fig-
ure (6.6). Figure (6.8) shows the change in the complex propagation constant
resulted by setting the magnetic bias to different values at \(f = 35\ GHz\).

It is shown in Figure (6.8a) that the mode \(EH_{11}\) is totally reciprocal when
\(B_0 = 0\ T\), this mode resembles that appears in the dielectric loading case. As the
magnetic bias increases to 0.5 \(T\), the higher mode \(EH_{12^+}\) starts to propagate (in
the forward direction only). The opposite directed mode \((EH_{12^-})\) is highly atten-
uated under the above conditions. The modes shown in Figure (6.8c) illustrate
the effect of setting bias to 1 \(T\), where the mode \(EH_{11^-}\) is changed to become
\(EH_{12^-}\), as discussed before. Both \(EH_{11}\) and \(EH_{12}\) tend to become reciprocal as
the magnetic bias is increased from 1.5 \(T\) to 2.5 \(T\), as shown in Figures (6.8d) to
(6.8f).

The nonreciprocity noticed before also depends on the thickness of the gyro-
electric slab loading the waveguide. Keeping the frequency and magnetic bias as
constants (35 \(GHz\) and 1 \(T\), respectively), increasing the thickness of the slab
causes different modes to propagate through the structure, some of them show
nonreciprocity.

Starting with a thin slab that covers 12.5% of the waveguide’s cross section, we
can notice from Figure (6.9a) that the mode \(EH_{11}\) is reciprocal at \(f = 35\ GHz\)
and with \(B_0 = 1\ T\). Increasing the thickness to cover 25% will introduce the
mode \(HE_{01}\) which has no change in the \(x-\)direction and is reciprocal, as shown
in Figure (6.9b). The higher mode \(EH_{12}\) also appears with high nonreciprocity.
This mode features high field concentration at the opposite sides of the slab,
similar to that shown in Figure (6.7a) and (6.7f). The mode \(EH_{21^+}\) also appears
at this thickness, although the one in the opposite direction \((EH_{21^-})\) is highly
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Figure 6.8: The change in the complex propagation constants of a waveguide loaded with a 0.9 mm thick InSb slab at 77 K with the transverse magnetic bias ($B_0$).
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(a) 12.5% filling  
(b) 25% filling  
(c) 37.5% filling  
(d) 50% filling  

Figure 6.9: The change in the complex propagation constants for a gyroelectric loaded waveguide with the thickness of the InSb slab when \( f = 35 \) GHz and \( B_0 = 1 \) T.

Increasing the thickness of the slab to cover 37.5% of the cross section almost causes the two modes \( EH_{11} \) and \( HE_{01} \) to degenerate. Phase constant of the other two modes in the opposite direction starts to increase, as shown in Figure (6.9c). This same situation remains when the slab fills half of the waveguide, as shown in Figure (6.9d). More modes will be included as the slab gets thicker, until filling the waveguide. Then the normal mode \( EH_{11} \) disappears (because there is no space above the slab), and the higher mode \( EH_{12} \) dominates. This mode will be totally reciprocal.

Loading the waveguide with two symmetrical transversely magnetised gyroelectric slabs, as shown in Figure (6.10), also result into reciprocal behaviour. Figure (6.11) shows the behaviour of the complex propagation constant of a WR-28 waveguide symmetrically loaded with two InSb slabs at 77 K, each of
0.9 mm thickness (total 50% loading). Due to the reciprocity of the mode $EH_{12}$, the modes’ annotation is different in this case, where the mode that appears at $f = 35 \, GHz$ is denoted as $EH_{13}$.

We notice here that there is a region of high attenuation for both directions of propagation. This region can be tuned by changing the magnetic bias and/or the thickness of the slabs. However, there is a certain (cut-off) thickness of the slabs, below which only one mode is propagated with minimum attenuation. Symmetrical gyroelectric loading of a waveguide does not reveal any nonreciprocal behaviour. However, it can be used to design other types of tunable microwave components, such as reciprocal phase shifters and filters, as will be illustrated in the next chapter.

### 6.4.3 Mixed Loading

As shown in (6.4.1), adding a dielectric layer results into a reduction in the cut off frequency of the structure’s propagating modes. Here, the effect of adding a dielectric layer above the magnetised semiconductor slab is analysed. Consider a WR-28 waveguide loaded with a $T_s = 0.9 \, mm$ InSb slab at 77 K (covers 25% of the cross section), transversely magnetised with 1 T. A dielectric layer of a certain thickness ($T_d$) and a dielectric constant of $\epsilon_d$ is placed above the InSb slab, as shown in Figure (6.12). The developed algorithm is used to find the complex propagation constants for the propagating modes in both directions.
(a) Imaginary part.

(b) Real part.

Figure 6.11: Complex propagation constants of the propagating modes through a WR-28 waveguide symmetrically loaded with two 0.9 mm InSb slabs at 77 K transversely biased with 1 T.
As described in (6.4.2), the propagating mode \((EH_{11})\) shows high degree of non-reciprocity within the considered frequency range. Hence, the effect of adding a dielectric layer is studied by inspecting the behaviour of this mode when including different dielectric layers.

Figure (6.13) shows a plot of the phase and attenuation constants of the \(EH_{11}\) mode for forward direction when the dielectric layer is 0.9 \(mm\) thick \((T_d = T_s)\) and have a dielectric constant of 3 and 6.

By comparing with the gyroelectric only loading \((\epsilon_d = 1)\), increasing the dielectric constant of the dielectric layer causes the \(EH_{11}\) mode to shift down in frequency, as can be seen from the change in the range at which the phase constant decreases. Similar behaviour can be noted by inspecting the attenuation constant in Figure (6.13b), where the frequency of highest attenuation shifts down in frequency with \(\epsilon_d\), until it shifts below the frequency range of operation when \(\epsilon_d = 6\).

Now, let’s inspect the effect of varying the thickness of the dielectric layer for a fixed value of dielectric constant. Figure (6.14) shows the effect of reducing the thickness of the dielectric layer \((T_d)\) while keeping \(\epsilon_d = 6\).

Results plotted in Figure (6.14a) show that the nonreciprocal region of the mode \(EH_{11}\) shifts up in frequency when reducing the thickness of the dielectric layer from 0.9 \(mm\) to 0.2 \(mm\). The same conclusion can be made by inspecting the change in the attenuation constant shown in Figure (6.14b). Such behaviour is
Figure 6.13: Forward-direction phase and attenuation constants for a WR-28 waveguide loaded with a 0.9 mm thick transversely biased InSb slab at 77 K ($B_0 = 1 \, T$) and dielectric layers of $T_d = 0.9 \, mm$ and different dielectric constants.
Figure 6.14: Forward-direction phase and attenuation constants for a WR-28 waveguide loaded with a 0.9 mm thick transversely biased InSb slab ($B_0 = 1 T$) and dielectric layers of $\epsilon_d = 6$ and different thicknesses.
expected since the effect of the dielectric layer is reduced as it gets thinner. From above, it can be deduced that for a certain value of magnetic bias ($B_0$), it is possible to achieve nonreciprocity at lower frequencies by adding a dielectric layer above the gyroelectric slab. However, there are many effects of doing this, such as inducing higher modes and increasing the reflection and insertion losses.

### 6.5 Conclusions

This chapter has illustrated the electromagnetic analysis of a rectangular waveguide partially filled with layered media. By solving Maxwell’s equations for each layer and matching the fields at the boundaries, a determinant was found which value vanishes for certain values of propagation constants. A special algorithm was used to search for the solutions of this determinant and find the propagation constants for the propagating modes through the structure.

Using the developed algorithm, the propagating modes through a partially loaded WR-28 waveguide were found. Three types of loadings were considered: dielectric, gyroelectric and mixed.

By analysing the rectangular waveguide partially filled with transversely biased InSb at 77 K, nonreciprocity was found in at least one propagating mode under certain biasing conditions within a specific frequency range. Adding a dielectric layer above the semiconductor causes this region to shift down in frequency for the same value of magnetic bias. The amount of shift depends on the dielectric constant and the thickness of the dielectric layer.

The developed algorithm was also used to investigate the behaviour of a waveguide symmetrically loaded with two identical gyroelectric slab. The outcomes of such analysis are useful to develop novel tunable gyroelectric reciprocal devices.
Chapter 7

Simulation and Measurement of Semiconductor Loaded Rectangular Waveguides

7.1 Introduction

This chapter presents the results of simulating and measuring rectangular waveguides with different dielectric and gyroelectric loadings. Simulation results are obtained from the CST MWS simulation package, while measurements are conducted using a Vector Network Analyser (VNA) with the measured fixtures cooled down to 77 K.

The chapter is divided into two main sections: the first discusses the CST simulation results of waveguides with different loadings, and compares the transmission behaviour and field distributions to those calculated using the algorithm developed in the previous chapter for similar structures. The possibility of scaling up the frequency of operation is demonstrated by simulating an isolator working above 100 GHz. This section also deals with symmetrical loading of waveguides and the feasibility of designing reciprocal tunable devices.

The second section presents measurement results of a WR-28 waveguide with different types of loading. Slabs with various lengths cut from an InSb wafer are used to load the waveguide, and the results of measurements at liquid nitrogen temperature are compared with simulation and theoretical expectations. The possibility of shifting down isolation frequency for a certain magnetic bias ($B_0$) is also investigated by measuring the InSb loaded waveguide at 77 K topped with
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dielectric layers of different thickness.

7.2 Simulation of Gyroelectric Loaded Waveguides

As shown in Chapter (6), reciprocity in gyroelectric loaded waveguides depends on the type of loading and the value of magnetic bias ($B_0$). Here, two types of structures are considered for simulation, namely nonreciprocal and reciprocal. The former is realised by asymmetrically loading a waveguide with transversely biased InSb at 77 K. As for the latter, two transversely biased InSb slabs at 77 K are used to produce reciprocal modes in the loaded waveguide.

7.2.1 Nonreciprocal Structures

It was shown in the previous chapter that magnetic bias ($B_0$) and slab thickness ($T_s$) can control the frequency range of nonreciprocity in a waveguide loaded with a transversely magnetised semiconductor. CST MWS electromagnetic simulation package allows verifying the existence of the nonreciprocal modes anticipated by theoretical analysis. This is done here by creating a 25 mm long WR-28 waveguide model loaded with InSb at 77 K biased in the $x-$direction\textsuperscript{1}, as shown in Figure (7.1). Due to the limitations in the simulation package, the waveguide’s input ports cannot be in touch with the gyroelectric material\textsuperscript{2}. As a result, a space should be left between the input ports and the beginning of the loaded waveguide section.

The first simulated structure consists of a loaded section of the length ($l_s = 24 mm$). The InSb slab has a thickness ($T_s$) of 0.9 mm (equivalent to 25% of the waveguide’s height) and is transversely biased with a magnetic field ($B_0$) of 1 T. Figure (7.2) shows the resulting scattering parameters of this structure alongside with the calculated nonreciprocal attenuation constant for the $EH_{11}^+$ mode. It can be seen that more than 50 dB isolation is obtained within the frequency range at which the $EH_{11}$ mode is nonreciprocal. Reflection is relatively high, but it is less than $-10$ dB for all the band. The effect of higher modes on both transmission and reflection is apparent at higher frequencies (above the isolation

\textsuperscript{1}Plasma, collision and cyclotron frequencies required to characterise the material were calculated using the measured electronic properties of InSb at 77 K illustrated in Chapter (2).

\textsuperscript{2}This is the same limitation mentioned before in Chapter (4).
Figure 7.1: A CST model of a WR-28 rectangular waveguide loaded with a transversely magnetised InSb slab at 77 K.

Propagating modes in the simulated structure shown in Figure (7.1) can be verified by inspecting the distribution of the electric field component ($E_y$) along the $z$-direction at the frequency of maximum isolation ($f = 31 \, GHz$), as shown in Figure (7.3). When the input port is 1, it is expected for two modes to propagate, namely $EH_{11+}$ and $EH_{12+}$, as shown in Figure (6.6). However, at $f = 31 \, GHz$, the mode $EH_{11+}$ is highly attenuated ($\alpha \approx 371 \, Np \, m^{-1} = 3222.5 \, dB \, m^{-1}$), so it is not expected to observe any propagation from this mode. The other mode ($EH_{12+}$) is propagating with a phase constant of 2464 rad m$^{-1}$ at 31 GHz, which is associated with a wavelength of 2.55 mm. By inspecting Figure (7.3a), it is obvious that there is one mode propagating through the slab with a profile very similar to that illustrated in Figure (6.7a) for the $EH_{12+}$ mode. Furthermore, the observed wavelength is almost equal to the one calculated from the mode’s phase constant at the same frequency.

When the input port is 2, modal analysis show that only one mode is propagating at $f = 31 \, GHz$, which is $HE_{11-}$. This mode has a phase constant of 568 rad m$^{-1}$, which results into a wavelength of 11 mm. Inspecting Figure (7.3b) confirms this prediction by showing one mode propagating with a wavelength very close to the above value. The distribution of $E_y$ also confirms the existence of the $EH_{11-}$ mode since it resembles its calculated field distribution plotted in Figure (6.7d). By inspecting $S_{21}$ in Figure (7.2), local minima are seen within the frequency range of isolation ($f = 28 - 35 \, GHz$). These fluctuations are attributed to the interferences between the two modes $EH_{11+}$ and $EH_{12+}$ that exist when the input is from port 1. Such interference is affected by the length of the gyroelectric slab ($l_s$).
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Figure 7.2: Simulated scattering parameters and calculated $EH_{11+}$ mode attenuation for a 25 mm long WR-28 waveguide loaded with an InSb slab at 77 K with $l_s = 24$ mm and $T_s = 0.9$ mm and biased in the $x-$direction with $B_0 = 1$ T.

Figure (7.4) illustrates the simulated differential isolation ($|S_{12} - S_{21}|$) of the loaded waveguide shown in Figure (7.1) for different values of $l_s$. Results show that the amount of isolation decreases as the length of the slab is reduced, which is expected since the attenuation will have less effect on shorter loaded sections. In addition, the interference between the two existing modes ($EH_{11+}$ and $EH_{12+}$) causes shifting in the frequencies of maximum isolation. Insertion loss, on the other hand, decreases when reducing the slab length, as shown in Figure (7.5).

As noted in the previous chapter, the region of high attenuation of the propagating modes can be controlled by changing the magnetic bias ($B_0$). Hence, it is expected for the frequency of isolation to increase when the magnetic bias is reduced. Figure (7.6) shows the effect of magnetic bias on the differential isolation for the structure shown in Figure (7.1) with $T_s = 0.9$ mm and $l_s = 24$ mm.

As expected, the isolation frequency range caused by the $EH_{11+}$ mode attenuation shifts down in frequency as the magnetic bias gets higher. Moreover, Figure (7.6) indicates that the frequency range of isolation becomes outside the Ka-band as $B_0$ is reduced. Hence, there is a limit of magnetic bias below which it is not possible to obtain nonreciprocity in this frequency band.
Figure 7.3: Simulated distribution of the electric field component $E_y$ along the $z$-direction for a WR-28 waveguide loaded with a 0.9 mm thick InSb slab at 77 K transversely biased with 1 T at 31 GHz for both directions of propagation.
Figure 7.4: Simulated differential isolation ($|S_{12} - S_{21}|$) for different values of $l_s$

Figure 7.5: Simulated insertion loss for different values of $l_s$. Inset: The values of insertion loss at the frequency of isolation.
Practical needs require the magnetic bias ($B_0$) to be as small as possible in order to maintain the small size of the designed component. As illustrated in the previous chapter, this can be done by shifting down the frequency of nonreciprocal attenuation when adding a dielectric layer above the gyroelectric slab. Hence, isolation can be achieved within the frequency range of interest using lower values of magnetic bias.

To inspect this effect, the CST model shown in Figure (7.7) was created. It contains the same transversely magnetised InSb at 77 K of $T_s = 0.9 \text{ mm}$ and $l_s = 24 \text{ mm}$ covered with a dielectric layer of thickness ($T_d$) and dielectric constant ($\epsilon_d$). Simulated scattering parameters for this model with $T_d = 0.2 \text{ mm}$, $B_0 = 1 \text{ T}$ and $\epsilon_d = 6$ are shown in Figure (7.8).

By comparing these results with those of the same structure without the dielectric layer (shown in Figure (7.2)), it can be seen that the maximum isolation is shifted down from 31 GHz to 28 GHz. However, the insertion loss at the frequency of isolation was increased from 1.3 dB (in the absence of the dielectric) to 1.5 dB. The calculated attenuation for the $EH_{11+}$ mode plotted in Figure (7.8) confirms theoretical expectations since the nonreciprocal attenuation occurs at the same frequency range of isolation.

It is also expected for the shift to be tuned up in frequency as the value of $\epsilon_d$ of
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Figure 7.7: A CST model of a WR-28 rectangular waveguide loaded with a transversely magnetised InSb slab at 77 K and a dielectric layer.

Figure 7.8: Simulated scattering parameters and calculated $EH_{11+}$ mode attenuation for a 25 mm long WR-28 waveguide loaded with an InSb slab at 77 K and a dielectric layer of $\epsilon_d = 6$. The length of the loaded section $l_s = 24$ mm, thickness of the InSb slab is $T_s = 0.9$ mm and the dielectric is $T_d = 0.2$ mm. Magnetic bias is in the $x$-direction and equal to 1 T.
Figure 7.9: Simulated differential isolation ($|S_{12} - S_{21}|$) for the structure shown in Figure (7.7) for different values of $\epsilon_d$.

Table 7.1: Specifications of the WR-08 rectangular waveguide standard.

<table>
<thead>
<tr>
<th>Frequency Limits (GHz)</th>
<th>Inside Dimensions (mm)</th>
<th>Frequency Band</th>
<th>TE$_{10}$ mode cut off frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
<td></td>
</tr>
<tr>
<td>90 − 140</td>
<td>2.32</td>
<td>1.02</td>
<td>F</td>
</tr>
</tbody>
</table>

the dielectric layer decreases. Figure (7.9) illustrates this effect by showing the differential isolation of the structure under the same conditions with different $\epsilon_d$.

So far, all the analysed and simulated models work within the operating frequency range of the WR-28 waveguide used in the designs (26.5 – 40 GHz). Nevertheless, the main advantage of the gyroelectric devices is the possibility of working at higher frequencies with low magnetic bias ($B_0$). To demonstrate this ability, an isolator is designed to work in the F-band frequency range using a WR-08 waveguide with the specifications shown in Table (7.1).

Using an InSb slab of $T_s = 0.2$ mm at 77 K, modal analysis shows that maximum attenuation for the $EH_{11+}$ mode can be achieved at $f = 105$ GHz when
biasing the slab with \( B_0 = 0.27 \, T \). Figure (7.10) shows the calculated \( EH_{11^+} \) mode phase and attenuation constants under these conditions.

By loading a 10 \( mm \) long WR-08 waveguide with a slab of the above thickness and the length \( l_s = 1.5 \, mm \), simulation results show differential isolation of 33.8 \( dB \) with 1.16 \( dB \) insertion loss at \( f = 105 \, GHz \). Reflection for the whole frequency range of interest does not exceed \(-10 \, dB\), as shown in Figure (7.11).

Despite the effects of higher modes, which are evident in this case for \( f > 115 \, GHz \), this design proves the possibility of realising high frequency waveguide isolators of small size with low magnetic bias requirements.

### 7.2.2 Reciprocal Structures

In addition to the nonreciprocity in phase shift and attenuation, gyroelectric loading of a waveguide can provide reciprocal behaviour when the waveguide is symmetrically loaded with the gyroelectric slabs, as shown in Figure (7.12). The resulting components will exhibit tuning capability by the magnetic bias (\( B_0 \)). This can be useful to realise high frequency tunable reciprocal phase shifters and filters.
Figure 7.11: Simulated scattering parameters for a 10 mm long WR-08 waveguide loaded with an InSb slab at 77 K with $l_s = 1.5$ mm and $T_s = 0.2$ mm and biased in the $x-$direction with $B_0 = 0.27$ T.

Simulating a waveguide loaded with two slabs of $T_s = 0.9$ mm (each) and $l_s = 24$ mm, and biased with $B_0 = 1$ T will result into the scattering parameters shown in Figure (7.13). It is clear that the structure is affected by high attenuation for both directions of propagation at the same calculated frequency range for the high attenuation in the $EH_{11\pm}$ modes. It can also be seen that the structure suffers from high insertion loss and high reflection because both slabs cover 50% of the waveguide cross section.

To realise a reciprocal phase shifter, the insertion and reflection losses should be minimised. Hence, the high attenuation in the $EH_{11}$ mode should not be excited within the desired frequency range of operation. This is achieved by carefully choosing the thickness of the slabs ($T_s$). In addition, the length of the slabs ($l_s$) should be sufficient to provide the necessary phase shift with minimum insertion loss.

The above requirements can be realised using two 10 mm long and 0.3 mm thick slabs (they both cover 20% of the waveguide’s cross section) magnetically biased with 1 T. Figure (7.14) shows the simulated scattering parameters for this structure. The insertion and reflection losses do not exceed 1 and 15 dB.
Figure 7.12: A CST model of a WR-28 rectangular waveguide symmetrically loaded with two transversely magnetised InSb slabs at 77 K.

Figure 7.13: Simulated scattering parameters for a symmetrically loaded waveguide with two 0.9 mm thick InSb slabs at 77 K, biased with $B_0 = 1$ T with the calculated attenuation constant for the $EH_{11}$ mode.
Figure 7.14: Simulated scattering parameters for a symmetrically loaded waveguide with two 0.3 mm thick and 10 mm long InSb slabs at 77 K, biased with $B_0 = 1 T$.

respectively, for the frequency range of interest.

Using the calculated phase constants for different values of $B_0$, it is possible to calculate the phase shift ($\Delta \phi$) between loaded and unloaded sections of the same length. The same parameter (i.e., $\Delta \phi$) was found by simulating the loaded waveguide for the same values of magnetic bias. Both results are shown in Figure (7.15). Simulation and calculation results show that the phase shift ($\Delta \phi$) can be tuned within wider range at higher frequencies. It is also possible to obtain higher phase shifts when increasing the length of the loaded section ($l_s$).

### 7.3 Measurements of Gyroelectric Loaded Waveguides

The waveguide loading arrangements analysed in Chapter (6) and simulated in Section (7.2) are now measured using a 25 mm long WR-28 waveguide loaded with different slabs cut from an InSb wafer of 0.85 mm thickness, as shown in
Figure 7.15: Calculated and simulated phase shift ($\Delta\phi$) resulted from symmetrically loading a 10 mm long WR-28 waveguide with two 0.2 mm thick InSb slabs at 77 K for different values of transverse biasing ($B_0$).

Figure (7.16)\(^3\).

It is known that InSb is a very brittle material, and an InSb slab can easily break when trying to cut and grind it. Hence, it is a challenge to obtain a sample with dimensions that exactly match the one in the simulated structures.

Here, most of the measured samples have a thickness ($T_s$) of 0.85 mm (the same as the InSb wafer’s thickness), and they are cut to match the dimensions of those in the related simulations as close as possible. After cutting a sample and smoothing its edges using a very fine sand paper\(^4\), it is fixed to the bottom of the waveguide shown in Figure (7.16), 12.5 mm from its edge, using a very small amount of glue (covering less than 1 mm\(^2\) of the sample’s base with a thickness of less than 0.1 mm).

Figure (7.17) shows a photograph the measurements set up. The magnetic bias ($B_0$) is provided by the electromagnet (GMW5403), where the value of the field depends on the input D.C. current and the separation between the poles\(^5\).

The liquid nitrogen Styrofoam container has the dimensions of ((40 \times 140 \times

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\(^3\)Electronic properties for InSb illustrated in Chapter (2) were measured using this wafer.

\(^4\)This sand paper has an average particle diameter of less than 100 $\mu$m.

\(^5\)More information about the electromagnet is given in Appendix (C).
Figure 7.16: A photograph of the WR-28 waveguide used to perform the measurements alongside with an InSb slab.

Figure 7.17: A photograph of the measurements setup for the loaded waveguide structures.
and the waveguide with the sample inside it is fit in the container and then liquid nitrogen is poured in it. Liquid nitrogen level inside the container has to be maintained to continuously vaporise and cool the waveguide during the measurements. The separation between the poles when measuring the waveguide structures was 40 mm. Hence, the maximum possible magnetic bias is limited to 0.8 \( T \). TRL calibration was used to bring the reference planes to the edges of the waveguide.

The first measured structure was the 25 mm long WR-28 waveguide loaded with an InSb slab of \( l_s = 8 \) mm and \( T_s = 0.85 \) mm. Figure (7.18) shows the measured reflection and transmission parameters when the loaded waveguide was biased with 0.8 \( T \) at room temperature. Results show an insertion loss between 0.5 and 3 \( dB \), and a reflection below −10 \( dB \) for the whole frequency range. No isolation appears because of the high temperature of the sample, and the structure can be considered as a simple dielectric loaded waveguide. The differences between \( S_{11} \) and \( S_{22} \) are attributed to the slight asymmetry of the sample’s cut and alignment inside the waveguide.

After pouring liquid nitrogen into the container and waiting for the structure to cool down, the scattering parameters shown in Figure (7.19) were obtained. An isolation of 39.6 \( dB \) can be seen at \( f = 35.6 \) GHz with insertion loss of 1.83 \( dB \). Reflection in this case is also less than −10 \( dB \) for the whole frequency band.

Figure (7.20) shows the scattering parameters resulted from CST simulation of the same structure alongside with the calculated attenuation constant for the \( EH_{11+} \) mode. Comparison reveals that isolation in both simulation and measurement occur within the region of the calculated nonreciprocal isolation (34 – 40 GHz). However, there is an extra local minimum (a dip at 38.5 GHz) in the simulated results that does not appear in the measurements. As noted in the previous section, such behaviour in \( S_{21} \) results from the interference between the existing modes (\( EH_{11+} \) and \( EH_{12+} \)), such interference causes a ripple in the transmission within the region at which it is highly attenuated. This was clear in Figure (7.4), where the differential isolation shows many local maxima that shift as the length of the slab (\( l_s \)) changes.

To investigate the effect of \( l_s \) on the measured isolation, another two structures were measured for the same magnetic bias (\( B_0 = 0.8 \) \( T \)). Figure (7.21) shows the transmission parameters (\( S_{12,21} \)) for the same waveguide when loaded with

\(^6\)More details about the calibration are found in Appendix (A)
(a) Reflection parameters.

(b) Transmission parameters.

Figure 7.18: Measured scattering parameters for a WR-28 waveguide loaded with an InSb slab of \( l_s = 8 \text{ mm} \) and \( T_s = 0.85 \text{ mm} \) and transversely biased with 0.8 \( T \) at room temperature.
Figure 7.19: Measured scattering parameters for a WR-28 waveguide loaded with an InSb slab of $l_s = 8 \text{ mm}$ and $T_s = 0.85 \text{ mm}$ and transversely biased with $0.8 \text{ T}$ at $77 \text{ K}$.
InSb slabs of \( l_s = 2.5 \, \text{mm} \), \( 4 \, \text{mm} \) and \( 8 \, \text{mm} \) with the calculated nonreciprocal attenuation constant plotted against \( S_{21} \).

It can be seen from Figure (7.21a) that the insertion loss increases with the length of the InSb slab \( (l_s) \). On the other hand, the interference between the modes causes the frequency of maximum isolation to shift down for longer slabs. As the length of the slabs gets smaller, the isolation frequency gets closer to the point of maximum attenuation of the \( EH_{11+} \) mode, as depicted in Figure (7.21b). Now, let’s analyse the measured insertion phase for both directions of propagation \( (\angle S_{12,21}) \). By considering the \( l_s = 4 \, \text{mm} \) case, Figure (7.22) shows the measured unwrapped insertion phase for the loaded section\(^7\) when \( B_0 = 0.8 \, T \).

Inspecting the results show that the change in \( \angle S_{21} \) from \( f = 37 \, \text{GHz} \) till \( f = 38 \, \text{GHz} \) correlates with the decrease in the phase constant of the \( EH_{11+} \) mode. Besides, as in the increase of the phase constant, this effect only appears for one direction of propagation.

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\(^7\)Here, the effect of the empty waveguide section on both sides of the slab is removed by multiplying the complex transmission parameter by \( e^{-2\beta_g l_g} \), where \( \beta_g \) is the phase constant of the empty waveguide and \( l_g \) is the length of the empty section on either side of the slab (in this case \( l_g = 10.5 \, \text{mm} \)). More details are given in page (180) of [1]
Figure 7.21: Measured transmission parameters for a WR-28 waveguide loaded with InSb slabs at 77 K of $T_s = 0.85 \text{ mm}$ and different lengths and biased with 0.8 $T$. Calculated attenuation constant for the $EH_{11+}$ mode is plotted against $S_{21}$. 
As illustrated earlier, changing the thickness of the measured InSb slabs is challenging because of the difficulty of grinding the surface of the slab without breaking it. However, it was possible to slightly reduce the thickness ($T_s$) of an InSb sample with $l_s = 2.5\ mm$ from 0.85 $mm$ to 0.7 $mm$ by carefully grinding it with a fine sand paper. Figure (7.23) compares between the measured transmission parameters resulted from loading the WR-28 waveguide with each sample.

From the calculated attenuation constants plotted in Figure (7.23b), the frequency of maximum isolation is expected to increase as the slab gets thinner, which is confirmed by the measured $S_{21}$ plotted in the same figure. All the previous results were recorded for a magnetic bias of 0.8 $T$, which results from maximum current with the given distance between the electromagnet’s poles. Figure (7.24) illustrates the effect of reducing the magnetic bias ($B_0$) on the measured transmission parameters using an InSb sample of $l_s = 4\ mm$ and $T_s = 0.85\ mm$.

The results show that the shift in the maximum isolation frequency matches that of maximum calculated attenuation constant for the same values of magnetic bias ($B_0$). Furthermore, the value of the insertion loss at the frequency of maximum

Figure 7.22: Unwrapped measured insertion phase of the 4 $mm$ long InSb loaded waveguide section at 77 $K$ for both directions of propagation with the calculated phase constant of the $EH_{11+}$ mode.
Figure 7.23: Measured transmission parameters for a WR-28 waveguide loaded with InSb slabs at 77 K of $l_s = 2.5$ mm and biased with 0.8 $T$ for different thicknesses ($T_s$). Calculated attenuation constant for the $EH_{11+}$ mode is plotted against $S_{21}$. 

(a) $S_{12}$

(b) $S_{21}$, with the calculated attenuation constant ($\alpha$) of the $EH_{11+}$ mode.
Figure 7.24: Measured transmission parameters for a WR-28 waveguide loaded with InSb slabs at 77 K of $l_s = 4$ mm and $T_s = 0.85$ mm for different values of magnetic bias ($B_0$). Calculated attenuation constant for the $EH_{11+}$ mode is plotted against $S_{21}$.
isololation is approximately the same ($\approx 1 \, dB$) for all the values of $B_0$, as shown in Figure (7.24a).

Measured results shown in Figure (7.24) indicate that it is not possible to achieve isolation within the Ka-band with magnetic bias less than 0.6 $T$ using the given InSb slabs. As explained in Section (7.2), isolation can be realised with lower values of $B_0$ when adding a dielectric layer above the InSb slab. Here, a dielectric material of thickness ($T_d$) of 0.5 $mm$ and $\epsilon_d = 3.55$ is used\textsuperscript{8} to investigate this effect.

The scattering parameters shown in Figure (7.25) were measured for the WR-28 waveguide loaded with an $l_s = 8 \, mm$ and $T_s = 0.85 \, mm$ InSb slab at 77 $K$, and

\textsuperscript{8}The dielectric layer was cut from Rogers RO4005c substrate after removing the metallisation layers from both sides.
biased with 0.65 $T$, compared to the parameters resulted from adding one and two layers of the $\epsilon_d = 3.55$ dielectric ($T_d = 0.5$ and 1 $mm$, respectively).

Figure (7.25d) shows the behaviour of $S_{21}$ alongside with the calculated attenuation constant of the $EH_{11+}$ mode for each mixed loading. Adding a dielectric layer of $T_d = 0.5$ $mm$ above the InSb slab causes the isolation frequency to shift down from $f = 38.5 \, GHz$ (for InSb only loading) to $f = 34.5 \, GHz$. Increasing $T_d$ to 1 $mm$ results into another shift in the isolation frequency to 29.5 $GHz$. The observed isolation frequency shift matches the expected shift in the calculated frequency of maximum nonreciprocal attenuation.

This isolation tuning ability adds more flexibility to the design of gyroelectric isolators, although it comes with a cost in terms of insertion loss and reflection. Figures (7.25a), (7.25b) and (7.25c) show that both the reflection and insertion losses increase with the thickness of the dielectric layer. The differences between the measured $S_{11}$ and $S_{22}$ are due to the asymmetry in both the InSb slab and the alignment of the dielectric layers.

### 7.4 Conclusions

This chapter demonstrated the characteristics of a rectangular waveguide loaded with transversely magnetised InSb slabs at 77 $K$. Electromagnetic simulation package CST MWS was used to illustrate the match between the frequency range of isolation and that of the calculated nonreciprocal attenuation. It was also shown that tunable reciprocal behaviour can be obtained in both attenuation and phase shift when the waveguide is symmetrically loaded with two InSb slabs. Operation at higher frequencies was also demonstrated by simulating an F-band isolator using a WR-08 waveguide loaded with InSb slab at 77 $K$ transversely biased with 0.27 $T$. Differential isolation of more than 30 $dB$ was obtained with 1.2 $dB$ insertion loss at 105 $GHz$.

In addition to CST simulations, this chapter reported the measurements of a WR-28 waveguide loaded with transversely biased InSb and dielectrics at liquid nitrogen temperatures. Using an 8 $mm$ long InSb slab, differential isolation of 39.5 $dB$ with insertion loss of 1.8 $dB$ was obtained at $f = 35.6 \, GHz$. High values of isolation were also achieved within the Ka frequency band using different lengths of InSb. Moreover, the effects of magnetic bias ($B_0$) and slab thickness ($T_s$) were investigated, and it was proved that they follow theoretical expectations.
The effect of adding one or more dielectric layers with a certain dielectric constant \( \epsilon_d \) was also investigated through measurements. It was shown that adding such layers shifts down the isolation frequency for any given magnetic bias \( B_0 \).

Simulation and measurement results reported in this chapter prove, for the first time, the possibility of realising millimetre-wave reciprocal and nonreciprocal gyroelectric devices based on rectangular waveguides loaded with semiconductors parallel to their H-plane.
Chapter 8

Conclusions

Nonreciprocal devices are vital parts of many communications and radar systems. The use of ferrite materials to design this type of device faces many challenges as the frequency of operation reaches the millimetre and sub-millimetre-wave frequency ranges. The main challenges are the limited saturation magnetisation and the high loss.

This thesis has investigated the possibility of utilising the gyroelectric behaviour in magnetised semiconductors to realise nonreciprocal devices working in the frequency ranges at which the performance of the ferrite based devices deteriorate. The work presented here is a continuation of the work previously undertaken by many other researchers, and it gives a new perspective to the various possibilities of this type of device.

Gyroelectric behaviour is observed in magnetically biased plasma, where the electrons show circular motion due to the existence of a D.C. magnetic bias normal to the direction of their motion. By definition, plasma is a collection of positively and negatively charged particles. Hence, semiconductors can be considered as solid state plasma, since they basically consist of electrons and holes.

In an endeavour to determine the general features that identify a plasma, some of the basic concepts have been clarified, such as the plasma frequency and Debye shielding. Based on these features, the criteria with which any material is identified as plasma have been discussed.

After illustrating the basic physical properties of semiconductors, the plasma criteria have been tested for a list of semiconductors. The different semiconductors have fulfilled the criteria to different extents. Indium Antimonide (InSb) has been found to be the most suitable candidate for realising plasma based devices because
of its high degree of compliance with the plasma criteria. This choice has given further impetus by the fact that the majority of the previously reported research has chosen the same semiconductor to demonstrate the gyroelectric properties of solid state plasma. In one of these research publications, the measured electronic properties of InSb at different temperatures have been cited.

Using the single particle approximation of the plasma, which is basically the same as the Drude-Zener model under the assumed conditions, the velocity of the electron in the presence of an A.C. electric field and a D.C. magnetic field has been analysed. Solving the equation of motion for the electron has resulted in finding expressions for the velocity and current density components. To complete the macroscopic model, the effect of displacement current have been taken into account by introducing a permittivity tensor to be used with the electromagnetic analysis.

In addition to the mathematical approach, recent advances allow the use of three dimensional electromagnetic simulators to validate and optimise the designs of gyroelectric devices. The CST MWS simulation package has introduced the possibility of simulating gyroelectric materials in its 2012 (and subsequent) version. However, before using this package, its model should be validated by comparing it to the one derived from the microscopic model. The validation has been done by matching the permittivity tensor elements resulted from the simulation software with those calculated from the previously derived mathematical model.

After illustrating the macroscopic model, electromagnetic analysis has been undertaken for different structures. The first structure was a thin semiconductor disk with electric wall on the sides and magnetic walls on the top and bottom. No field change has been assumed in the axial direction, which is the same direction of the magnetic bias. Under these conditions, Maxwell’s equations have been solved for cylindrical coordinates given the tensor permittivity of the semiconductor.

Electromagnetic analysis has revealed that the structure supports different types of resonant modes that feature clockwise and counter clockwise rotation of the polarised fields. The existence of these modes depends on the values of two important quantities, namely $\epsilon_{eff}$ and $\kappa_{\epsilon}$. To facilitate the analysis, the frequency range has been divided into different regions depending on the values of the two aforementioned quantities.
CHAPTER 8. CONCLUSIONS

After identifying the possible modes that can be supported by the analysed structures, the effects of introducing two ports opposite to each other on the resonator perimeter have been studied and analysed. Using a Green’s function approach, expressions for the scattering parameters for this resonator have been derived in terms of the material properties of the outer transmission lines and the coupling angles of the ports.

After that, a Semiconductor Junction Circulator (SJC) has been introduced by assuming the existence of three equally spaced ports on the sides of the gyroelectric resonator. By using the same Green’s function approach, expressions for the SJC’s scattering parameters have been derived. By setting the expression for the transmission parameter to one, and that for the isolation to zero, the conditions at which the SJC exhibits perfect circulation have been found.

Finding the perfect circulation conditions are considered the main step towards the design procedure of SJC’s. Consequently, a previously designed Ka-band SJC has been reviewed in terms of tracking the perfect circulation conditions in the frequencies below the extraordinary wave resonance frequency \( f_r \) using InSb cooled down to 77 K. Calculated scattering parameters revealed differential isolation of more than 15 dB with around 1 dB insertion loss. The same structure have been measured by placing the InSb sample in a tapered finline circuit fitted inside a WR-28 waveguide junction. Measured scattering parameters has shown the same behaviour as the calculated ones, although with higher insertion and reflection losses.

Another design has been considered to work above \( f_r \), where higher frequency of operation can be realised with smaller value of magnetic bias \( (B_0) \). By using the perfect circulation conditions approach, an InSb SJC (at 77 K) working at 650 GHz with \( B_0 = 0.2 \) T has been designed and simulated. Calculated and simulated scattering parameters have shown a differential isolation of 15 dB with 2.15 dB insertion loss. This design proves the possibility of achieving circulation using InSb at 77 K in the sub-millimetre-wave frequency range with low magnetic bias.

To improve the SJC design procedure, an algorithm has been proposed to find the design parameters for a broadband SJC working at the highest frequency while being biased with the minimum possible magnetic field. The algorithm is based on a systematic approach to track the perfect circulation conditions between specific values of \( \frac{\kappa}{\varepsilon} \) that have been derived accordingly. The algorithm results in
the design parameters, namely the radius of the gyroelectric disk \((R)\) and the coupling half angle \((\psi)\), with which the SJC will reveal the optimum results for a given semiconductor, while connected to a transmission line with a certain dielectric constant \((\epsilon_d)\). The operation of the algorithm has been demonstrated using InSb at 77 K with different possible values of \(\epsilon_d\). After finding the optimum magnetic bias \((= 0.214 \, T)\), the algorithm has shown that a SJC can be designed to work at 200 GHz with 10 dB bandwidth of 90\% when \(\epsilon_d = 20\).

Beside the InSb at 77 K, the thin gyroelectric resonator have been assumed to be made of magnetised MBE-grown 2-DEG layer at 77 K. This type of material features high electron concentration that requires a different assignment for the previously described regions of operation. Using the same analysis procedure, it has been shown that only modes rotating in the counter clockwise direction can be excited in a 2-DEG resonator. This has been demonstrated by calculating and simulating a loosely coupled 2-DEG resonator, where the resonance frequency has shifted with changing the magnetic bias, as expected from the theory. Designing 2-DEG based circulators, on the other hand, has required extending the previously calculated perfect circulation conditions because of the low impedance of the 2-DEG layer. To realise circulation at 200 GHz, the circulator would have to be biased with a magnetic flux as high as 2.5 T. Calculated scattering parameters revealed 12 dB differential isolation within the frequency range of interest. CST simulation revealed similar behaviour in the scattering parameters, but with higher reflection and insertion losses. These results prove the possibility of using the 2-DEG, which can be integrated into many microwave circuits, to realise fully planar nonreciprocal devices under certain conditions.

Due to the assumed boundary conditions, the aforementioned SJC can only be realised using finline structures. Such arrangements deteriorate the performance of the device by increasing the insertion and reflection losses. On the other hand, suspended semiconductor disks can be placed inside a three port waveguide junction with no added circuit, resulting into a waveguide junction circulator. Despite its simplicity, analysing this structure is challenging due to the open boundaries of the semiconductor disk. In addition, the change along the axis of the disk cannot be assumed zero in this case.

Here, a scalar potential analytical approach has been followed to solve the differential equation resulted from solving Maxwell’s equations in an infinitely long
gyroelectric rod with open boundaries. After finding expressions for the different field components inside the rod, they have been matched to those outside it (in the free space). This has resulted into a system of equations that can be expressed as a matrix, the determinant of which should vanish for specific values of the propagation constant for the modes propagating along the rod. This electromagnetic analysis has been applied to the case of a lossless axially biased InSb rod suspended in free space. Different modes have been found, some of them have been noticed to split into clockwise and counter clockwise rotating modes under certain conditions.

By placing a semiconductor disk in the middle of a waveguide junction, the split modes can be incorporated into the overall resonance of the cavity where the three waveguides meet. Circulation can be achieved by exciting the semiconductor disk at the point where two counter rotating modes split. To meet this condition, the space above and below the disk should have a certain wavenumber that matches that of the disk.

The waveguide junction circulator has been realised for the first time by placing an InSb disk at 77 K in the middle of a WR-28 waveguide junction. Simulation results have shown differential isolation of 18 dB with 2.6 dB insertion loss at 38.5 GHz when the magnetic bias was 0.55 T. Circulation frequency has been proved to shift down in frequency as the magnetic bias increases, as expected from theory.

By understanding the role of the wavenumber in the regions above and below the InSb disk, it has been concluded that reducing it will make the resonance occur at lower frequency for a fixed magnetic bias. To achieve this, two dielectric disks have been placed on top and bottom of the InSb disk. With the dielectric constant of the disks set to 6.15, circulation at 35.5 GHz has been achieved with a magnetic bias as low as 0.25 T. Despite the high insertion and reflection losses (5 dB and 7 dB, respectively), these novel results are very promising since they prove the possibility of realising simple gyroelectric waveguide circulators with low values of magnetic bias.

Beside the resonant circular structures, the focus of this thesis has expanded to explore the gyroelectric effects on the propagation in rectangular waveguide loaded with transversely magnetised semiconductors. Such arrangements are considered very useful to design other novel type of nonreciprocal devices, such as isolators and phase shifters.
The considered model consists of a waveguide with rectangular cross section, loaded with layered media. Each layer has been characterised as dielectric (with a scalar permittivity) or gyroelectric (magnetised semiconductor with a permittivity tensor).

By assuming that the semiconductor layers are transversely magnetised (in the direction normal to both the directions of electric field and propagation), Maxwell’s equations have been solved for each layer. The solution has resulted into a transfer function that relates the tangential fields at the bottom of each layer to those at its top. By multiplying the transfer matrices of all the layers, a final transfer matrix results will connect the fields at the bottom of the first layer to the top of the last one. Given that the tangential electric field components should vanish at the top and bottom metallic walls, the determinant of the final transfer matrix should vanish for specific values of the propagation constant.

An algorithm has been written to search the solutions of the derived determinant. Using this algorithm, the propagating modes for rectangular waveguides loaded with different dielectric, gyroelectric and mixed layers have been found.

When analysing the propagating modes in a WR-28 waveguide loaded with a single InSb slab at 77 K, magnetically biased in the transverse direction, a strong nonreciprocity in both phase and attenuation constants has been found. It has also been found that this region of nonreciprocity shifts up and down the frequency with the magnetic bias, thickness of the slab and by adding dielectric layers above it.

The same loaded waveguide structures have been simulated using the CST MWS simulation package. Tunable and nonreciprocal attenuation have been validated by showing high isolation behaviour at the frequencies anticipated by theoretical analysis. The effects of changing the slab length, thickness and magnetic bias have also been tested and they have followed the mathematically predicted behaviour.

In addition, symmetrical loading of a rectangular waveguide with two identical transversely biased InSb slabs at 77 K has resulted into tunable attenuation and/or phase shift, depending on the thickness of the slabs and the value of the magnetic bias. Moreover, isolation behaviour has been demonstrated at higher frequencies by simulating an InSb isolator working at 105 GHz with more than 30 dB isolation and less than 2 dB insertion loss, given a magnetic bias of 0.27 T and operation temperature of 77 K.
Further validation for the waveguide loaded structures has come from measuring a WR-28 waveguide loaded with different InSb slabs at 77 K for the first time. Using a 0.8 mm long InSb slab of 0.85 mm thickness, more than 39 dB differential isolation has been achieved at $f = 35.6 \text{ GHz}$ with insertion loss of 1.83 dB when the slab was transversely biased with $B_0 = 0.8 \text{ T}$. In addition, the nonreciprocal phase constant behaviour has been validated by noticing the phase behaving in an increasing manner at the same expected frequency range.

In addition to the gyroelectric loading, adding a dielectric layer above the semiconductor slab has resulted into a reduction in the frequency of isolation for a fixed magnetic bias. Hence, adding dielectric layers above the InSb slab has revealed a significant shift down in the measured frequency of maximum isolation. These results prove the possibility of reducing the magnetic bias required for designing waveguide isolators at a certain frequency by adding dielectric layers above the gyroelectric slabs.

In conclusion, the initial aims of the projects have been met, as summarised in the following points:

1. The mathematical analysis has been investigated to explore the possibility of realising Semiconductor Junction Circulators (SJC’s) working in the millimetre and sub-millimetre-wave frequency ranges. This has been demonstrated by proposing a SJC design working above the extraordinary wave resonance frequency ($f_r$) using InSb at 77 K.

2. The SJC design approach has been extended to realise broadband circulation with minimum magnetic bias. The proposed algorithm is based on the basic perfect circulation conditions tracking after studying the requirements for optimum operation.

3. It has been shown that it is possible to design resonators and circulators using MBE grown 2-DEG layers. Despite the limitations of the designs, this possibility is encouraging for further investigations into the design of fully planar nonreciprocal devices.

4. Circulation has been proved in three port waveguide junctions loaded with suspended InSb disks at 77 K analytically and experimentally for the first time. This novel circulator has the advantage of ease of fabrication over the classical SJC. A big step towards full theoretical analysis has also been taken, although further investigations are still required.
5. In addition to resonant structures, gyroelectric loaded waveguides have also been investigated. Strong nonreciprocity in both propagation and attenuation has been anticipated from mathematical analysis. These expectations have been validated by the simulations and measurements of waveguides loaded InSb at 77 K. Such behaviour can be exploited to design novel isolators and phase shifters working at higher frequency ranges.

Some of the results reported in this thesis can be considered as starting points for further investigations in terms of mathematical analysis and measurements. In addition, the potential of gyroelectric behaviour in magnetised solid plasma can be utilised in a wider range of structures that deserve further consideration. The following section will illustrate a few suggestions for future research.

\section*{8.1 Future Work}

1. After recognising the electronic properties of a semiconductor that are effective to the gyroelectric behaviour, material science can be incorporated to realise semiconductor materials with customized features to enhance the performance of the designed gyroelectric devices. Operation at room temperature is considered as one of the major goals to be fulfilled by improving the semiconductor’s electronic properties.

2. The high loss associated with the finline taper in the measured results of the Ka-band SJC in Chapter (4) can be mitigated by improving the finline taper design using materials with lower loss. Results can also be improved by using thinner InSb samples with a metallic yoke that guarantees the fulfilment of the boundary conditions.

3. Promising results of significant nonreciprocity have been obtained when simulating finline structures loaded with a transversely biased InSb slab at 77 K placed under one side of the finline’s slot. This behaviour is attributed to the high loss applied to only one direction of propagation. It is suggested here to analyse this behaviour theoretically and exploit it to design nonreciprocal finline devices. A good starting point for theoretical investigation is the work of Krowne et al. [65, 66, 136].

4. After proving the possibility of realising 2-DEG based nonreciprocal devices, the next step should be testing the fabricated structures by incorporating
it with the modern waveguide microfabrication technology, such as 3-D printed waveguides.

5. Due to the nature of the 2-DEG layer, its electron concentration can be tuned by changing an electrical bias at the bottom of the heterostructure. This will allow an alternative way to tune the gyroelectric resonance in the 2-DEG structures. However, the electrical bias shouldn’t highly reduce the 2-DEG layer’s electron concentration, otherwise the Debye shielding criteria of the plasma illustrated in Chapter (2) will not be fulfilled.

6. Mathematical derivation of the axially magnetised suspended semiconductor rod illustrated in Chapter (4) can be taken into another level by adopting the Green’s function approach and applying the boundary conditions of the waveguide junction. This way, perfect circulation conditions for the waveguide junction circulators can be derived, which guarantees better designs in the future.

7. Expressions for the scattering parameters for the waveguide loaded structures illustrated in Chapters (6) and (7) can be derived by applying the mode matching technique between the loaded and unloaded waveguide sections, taking into consideration all the propagating modes on both sides.

8. By applying the same waveguide loading model in Chapter (6) to a parallel plate waveguide filled with a transversely biased semiconductor, it has been found that the electric and magnetic fields along the waveguide are strongly displaced towards the top and bottom plates for forward and reverse propagation, respectively. Theoretical investigation showed that the boundary conditions in this case cannot be fulfilled without considering two TEM modes propagating opposite to each other [134]. This phenomenon can be used to design field displacement isolators by loading a rectangular waveguide with a thin, transversely biased semiconductor slab parallel to the E-plane, with an absorber attached to one of its sides. Initial theoretical investigation for gyroelectric filled parallel plate waveguide has been reported before by Seshadri [134], and basic experimental results about waveguides loaded with InSb slabs parallel to the E-plane have been reported by Hirota and Suzuki [137].

9. Gyroelectric features of Graphene have been investigated before [138,139].
It was shown that magnetically biased chemically doped Graphene can exhibit Faraday rotation in circular waveguides [78]. Despite the small angle of rotation and high magnetic bias requirements, these results are promising and deserve further investigation in conjunction with the future advances in the Graphene technology.
Appendix A

Thru-Reflect-Line (TRL)
Rectangular Waveguide Calibration

A Vector Network Analyser (VNA) measures the scattering parameters as ratios between complex voltage amplitudes. The reference plane for these measurements is usually somewhere inside the VNA. Hence, any measurements will include the loss and phase delay effects of the components used to connect the VNA to the Device Under Test (DUT) [1]. A calibration process is usually used to correct the scattering parameters from the errors caused by the imperfections inside the analyser itself, in addition to the added effects of the connectors, cables and transitions used to connect the VNA to the DUT. Such process would effectively shift the reference plane to the input ports of the DUT [1, 140].

A common way to calibrate a VNA is to connect a set of known standards (such as a short, open, load and through). The VNA then calculates the necessary parameters to characterise an error model that is used later to remove the undesired effects from the measurements [141]. However, for non-coaxial measurements, it is difficult to obtain a set of standards with well characterised impedances, and they usually end up adding errors into the measurements. For this reason, the Thru-Reflect-Line (TRL) calibration is often used for non-coaxial measurements since it does not rely on known standards. It relies, instead, on three types of transmission lines that allow characterising the errors [142].

To bring the reference planes to the waveguide ports of the DUT, a TRL calibration process, as implied by its name, follows three basic steps [140]:

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1. **THRU**: It is about connecting the two ports directly, or via a short transmission line (of length $l_T$).

2. **REFLECT**: To connect identical high reflection coefficient devices to each port.

3. **LINE**: About inserting a certain length of a transmission line ($l_L$) between the two ports.

There are some requirements for the TRL calibration standards: first, the amplitude of the reflection coefficient for the SHORT should be close to 1 (a perfect short). Secondly, the phase difference between the THRU and LINE connections ($\theta_L$) should satisfy the condition $20^\circ < \theta_L < 160^\circ$ [140]. This difference (in degrees) is calculated from the following relation:

$$\theta_L = \frac{360}{\lambda_g} (l_L - l_T)$$  \hspace{1cm} (A.1)

Where $\lambda_g$ is the guided wavelength in m.

Most measurements throughout this thesis require the use of a massive electromagnet, which can affect any electronic equipment in its vicinity. Therefore it was necessary to keep the experimental setup at a safe distance from the VNA by using two coaxial cables, each of 120 cm long.

Since all the measured components work within the Ka frequency band and are made of WR-28 waveguides, the cables were connected to a couple of WR-28 waveguide transitions via 2.92 mm connectors. To keep the transitions outside the liquid nitrogen container, two 10 cm waveguides were used to connect the transitions and the DUT, as shown in Figure (A.1).

The TRL calibration standards used here consist of a 4.8 mm thick piece of brass, used as a short and a waveguide of $l_T = 20.25$ mm as a nonzero THRU\(^1\). As for the LINE standard, a 23.25 mm long waveguide was used.

To check for the validity of these standards, the phase difference ($\theta_L$) between the THRU and LINE connections was calculated, as shown in Table (A.1). It is obvious that the phase differences at the edges of the Ka-band are within the aforementioned limits.

To calibrate the Keysight (HP8510) VNA, a calibration kit has to be defined and installed to the analyser. Defining the kit requires providing information about

\(^1\)This type of calibration is sometimes referred to as Line-Reflect-Line (LRL) [108].
Figure A.1: A photograph of the cables, transitions and calibration waveguide standards used for the measurements.

Table A.1: Phase differences between the THRU and LINE standards within the Ka-band.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Free Space Wavelength, $\lambda_0$ (mm)</th>
<th>Guided Wavelength, $\lambda_g$ (mm)</th>
<th>Phase Difference $\theta_L$ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.5</td>
<td>11.32</td>
<td>18.7</td>
<td>57.7</td>
</tr>
<tr>
<td>40</td>
<td>7.49</td>
<td>8.81</td>
<td>139.3</td>
</tr>
</tbody>
</table>

each standards in terms of its type, offset delay, impedance and the frequency range of operation.

The delay for the THRU and LINE are defined by considering a speed of light propagation through the waveguide sections. The calibration kit for the WR-28 waveguide is illustrated in Table (A.2).

The TRL calibration process was performed after defining a calibration kit with the parameters in Table (A.2) and connecting the standards shown in Figure
To verify the calibration, the scattering parameters for a 25 mm long waveguide were measured after two types of calibration: the first was the WR-28 waveguide TRL calibration illustrated above. The other type was done by removing the effects of only the 120 cm coaxial cables by performing a full two ports calibration at the 2.92 mm connector ends (using a standard calibration kit using short, open, load and through connections).

Figure (A.2) compares between the measured insertion loss for the 25 mm long waveguide after each calibration. It is shown that the TRL calibration results an insertion loss of no more than 0.06 dB for the whole frequency band. On the other hand, more than 0.6 dB of insertion loss results from measuring the same waveguide after the two ports coaxial calibration.

Measured reflection loss shown in Figure (A.3) reveals that the TRL calibration shows more than 30 dB of reflection loss for most of the frequency range of operation. However, less than 20 dB of reflection loss is resulted from the measurements after the two ports coaxial calibration.

(A.1) sequentially while the VNA calculated the calibration parameters. For detailed information about performing the TRL calibration for Keysight’s HP8510 VNA, please refer to p. 21 of [140].
Figure A.2: Measured insertion loss for a 25 mm long WR-28 waveguide with two different calibration processes.

Figure A.3: Measured reflection for a 25 mm long WR-28 waveguide with two different calibration processes.
Appendix B

Measurements of the Supporting Components

This appendix illustrates the measured performance of the locally made WR-28 waveguide components\(^1\) used in the measurements throughout the thesis. All the components were measured after setting the reference planes at their input ports by performing the TRL calibration process illustrated in Appendix (A).

B.1 Empty Waveguide

This waveguide was used to measure the resonant behaviour of an axially magnetised InSb disk at 77 K. The waveguide is of 11 cm length, and it consists of two halves that are fitted together by ten screws and two alignment pins, as shown in Figure (B.1).

It can be seen from the measured scattering parameters in Figure (B.2) that the insertion loss for this structure is around 0.35 dB, while the reflection is below −20 dB over the Ka-band. Despite the fluctuations in the measured parameters, these results were considered acceptable given the type of the undertaken measurements.

\(^1\)These components were fabricated by the mechanical workshop in the School of Electrical and Electronics Engineering, The University of Manchester.
APPENDIX B. MEAS. OF THE SUPPORTING COMP.S

Figure B.1: A photograph of the 11 cm long WR-28 waveguide used in the measurements ($a = 7.11$ mm and $b = 3.56$ mm).

Figure B.2: Measured insertion and reflection losses for the 11 cm long WR-28 waveguide.

(a) Insertion loss.  
(b) Reflection loss.
B.2 Matched Load

This load was used to match one of the circulators’ three ports during the measurements. It consists of a 6 cm long piece of waveguide with an open end, as shown in Figure (B.3). A tapered absorber sheet with the same length is fitted inside the waveguide. The absorber is made of a carbon loaded foam sheet with peel-and-stick backing\(^2\).

It can be seen from Figure (B.4) that the measured reflection coefficient at the input of the load did not exceed $-20\ dB$. This was considered acceptable for the loading purpose since less than 1% of the incident power is reflected back to the input port.

\(^2\)0.125" thick C-RAM MT-26/PSA by CUMING MICROWAVE LTD.
B.3 Three Port Waveguide Junction

This structure consists of a simple junction of three waveguides, each of 4 cm long. The structure consists of two halves to be fitted together using six screws and three alignment pins, as shown in Figure (B.5).

Assuming the input waveguide to have a characteristic impedance of $Z_w$, and loaded with two identical waveguides in parallel. The parallel loading gives a total effective impedance of $\frac{Z_w}{2}$. The reflection coefficient at the input waveguide can be calculated as [100]:

$$\Gamma = \frac{Z_w}{Z_w} \frac{Z_w}{2} = \frac{-1}{3} \quad (B.1)$$

$S_{11}$ can then be found as:

$$S_{11} = 10 \log |\Gamma|^2 = 10 \log \left( \frac{1}{3} \right)^2 = -9.5 \, dB \quad (B.2)$$
Figure B.5: A photograph of the WR-28 three port waveguide junction ($a = 7.11 \text{ mm}$ and $b = 3.56 \text{ mm}$).

Assuming that the junction is symmetric and lossless, the rest of the scattering parameters are found to be [100]:

$$S_{12} = S_{13} = 10 \log \left[ \frac{1 - |\Gamma|^2}{2} \right] = -3.5 \text{ dB} \quad (B.3)$$

The structure shown in Figure (B.5) was measured by terminating one of the ports with the waveguide matched load illustrated in Section (B.2), and measuring the scattering parameters between the remaining two ports.

Figure (B.6) shows that the insertion loss is $3.7 \text{ dB}$ and reflection loss is $9.4 \text{ dB}$ on average. These results are very close to the calculated expectations. The differences between $S_{11}$ and $S_{22}$ are caused by slight asymmetry in the measured structure.
APPENDIX B. MEAS. OF THE SUPPORTING COMP.S

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(a) Insertion loss

(b) Reflection loss.

Figure B.6: Measured insertion and reflection losses for the WR-28 three port waveguide junction.

B.4 Three Port Waveguide Junction with a Fin-line Circuit

This is the same waveguide junction illustrated in the previous section, but with a finline circuit fitted between the junction’s two halves, as shown in Figure (B.7).

The scattering parameters for this structure were measured the same way as with the previous waveguide junction. The results in Figure (B.8) show higher insertion loss (around 9 dB) and lower reflection loss (about 4.25 dB). This is expected because of the added mismatch by the circular cut (the sample’s position) in the middle of the finline circuit.
Figure B.7: A photograph of the WR-28 three port waveguide junction with a finline circuit ($a = 7.11 \text{ mm}$ and $b = 3.56 \text{ mm}$).

Figure B.8: Measured insertion and reflection losses for the WR-28 three port waveguide junction with a finline circuit.

(a) Insertion loss.  
(b) Reflection loss.
Appendix C

Characterisation of the GMW5403 Electromagnet

This appendix illustrates the excitation graphs of the GMW5403 electromagnet with 35 mm pole face diameter used to provide magnetic biasing for the measured gyroelectric devices in this thesis. The system consists of an electromagnet and a D.C. power supply to provide D.C. excitation current, as shown in Figure (C.1). The coils of this electromagnet model require constant reduction of the temperature, which is done by circulating water through specifically designed copper tubes.

The value of the flux density provided by this electromagnet depends on the input D.C. current from the power supply and the gap between the two poles ($d$). The user’s manual for this model [143] provides excitation plots for a number of gap values, as shown in Figure (C.2).

Another set of excitation plots were measured using an a Nano-Tesla magnetic field magnetometer [144] for the gap widths used in the measurements. The results are plotted in Figure (C.3).
Figure C.1: A photograph of the GMW5403 electromagnet with its power supply.

Figure C.2: Excitation plots for the GMW5403 electromagnet with 35 mm pole face diameter as provided by the manufacturer [143]
Figure C.3: Measured excitation plots for the GMW5403 electromagnet with 35 mm pole face diameter.
Appendix D

Flow Charts for the Used Algorithms

Flow charts that describe the algorithms introduced in the thesis are shown here. Figure (D.1) shows a flow chart for the proposed Semiconductor Junction Circulator (SJC) bandwidth optimisation algorithm, while the flow chart in Figure (D.2) describes the algorithm used to find the propagation constants and field plots for a rectangular waveguide filled with layered media.
APPENDIX D. FLOW CHARTS FOR THE USED ALGORITHMS

Figure D.1: A flow chart for the proposed bandwidth optimisation algorithm
Figure D.2: A flow chart for the algorithm used to find the propagation constants of a rectangular waveguide loaded with layered media.
Bibliography


