LOAD SIDE ACTIVE STABILISATION TECHNIQUES FOR DC SYSTEMS

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## Superscript

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<tr>
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## General

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<tr>
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<td>A</td>
<td>State matrix</td>
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<td>Input matrix</td>
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<tr>
<td>$\frac{d}{dt}$</td>
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<td>$g(x)$</td>
<td>PCH input matrix</td>
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<tr>
<td>$H(x)$</td>
<td>Hamiltonian function</td>
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<tr>
<td>$H_d(x)$</td>
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<tr>
<td>$I$</td>
<td>Identity matrix</td>
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<tr>
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<td>Imaginary part of the complex number $\chi$</td>
</tr>
<tr>
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<td>Imaginary unit</td>
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<td>$J_d(x)$</td>
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<td>Discrete time step</td>
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<td>$L_i(s)$</td>
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<td>$Z_{sOUT}$</td>
<td>Source subsystem output impedance</td>
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<td>Frequency</td>
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<td>$</td>
<td>\chi</td>
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<td>$\angle\chi$</td>
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<td>RLC input filter capacitor</td>
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<tr>
<td>$C_1$</td>
<td>PBSC small-signal stabilisation gain</td>
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<tr>
<td>$C_{IB}(s)$</td>
<td>IM closed-loop current control with IBSC</td>
</tr>
<tr>
<td>$C_{PB}(s)$</td>
<td>IM closed-loop current control with PBSC</td>
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<tr>
<td>$C_s(s)$</td>
<td>IBSC band-pass filter transfer function</td>
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<tr>
<td>$D(s)$</td>
<td>DSC computational delay transfer function</td>
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<td>$D_{tot}(s)$</td>
<td>DSC computational delay combined with zero-order hold transfer function</td>
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<td>IM stator voltage frequency</td>
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<tr>
<td>$f_s$</td>
<td>Inverter switching frequency</td>
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<td>$F_G(s)$</td>
<td>RLC input filter grid current transfer function</td>
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<td>$F_G(s)$</td>
<td>RLC input filter grid current transfer function with PBSC stabilisation gain</td>
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<td>IM electromechanical gain</td>
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<td>Symbol</td>
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<td>RLC input filter PCH input matrix</td>
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<td>RLC input filter desired PCH Hamiltonian function</td>
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<tr>
<td>$H_f(x)$</td>
<td>RLC input filter PCH Hamiltonian function</td>
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<td>$i_{as}$</td>
<td>IM phase $a$ stator current</td>
</tr>
<tr>
<td>$i_{bs}$</td>
<td>IM phase $b$ stator current</td>
</tr>
<tr>
<td>$i_{cs}$</td>
<td>IM phase $c$ stator current</td>
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<tr>
<td>$i_{dr}$</td>
<td>IM $d$ axis rotor current</td>
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<tr>
<td>$i_{ds}$</td>
<td>IM $d$ axis stator current</td>
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<tr>
<td>$i_{ds}^{\ast}$</td>
<td>FOC $d$ axis current IP filter output</td>
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<td>$i_g$</td>
<td>DC grid current</td>
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<td>$i_l$</td>
<td>DC load current</td>
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<td>$I_P(s)$</td>
<td>FOC current IP filter transfer function</td>
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<td>$i_{qr}$</td>
<td>IM $q$ axis rotor current</td>
</tr>
<tr>
<td>$i_{qs}$</td>
<td>IM $q$ axis stator current</td>
</tr>
<tr>
<td>$i_{qs}^{\ast}$</td>
<td>FOC $q$ axis current IP filter output</td>
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<tr>
<td>$i_{qs}^{\text{sat}}$</td>
<td>FOC speed controller output saturation</td>
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<td>$f_{fd}(x)$</td>
<td>RLC input filter desired PCH interconnection matrix</td>
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<tr>
<td>$f_f(x)$</td>
<td>RLC input filter PCH interconnection matrix</td>
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<td>$J_m$</td>
<td>IM machine inertia</td>
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<td>$K_D$</td>
<td>Computational delay and zero-order hold compensation gain</td>
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<td>$k_{ic}$</td>
<td>FOC current PI controller integral gain</td>
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<td>$k_{i\omega}$</td>
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<td>$K_{pc}$</td>
<td>PBSC phase compensator gain</td>
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<td>$k_{p\omega}$</td>
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<td>$L$</td>
<td>RLC input filter inductor</td>
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<td>$L_{cpl}(s)$</td>
<td>Transfer function of the small-signal minor loop gain with CPL</td>
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<tr>
<td>$L_r$</td>
<td>IM rotor self-inductance</td>
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<tr>
<td>$L_{lr}$</td>
<td>IM rotor leakage inductance</td>
</tr>
<tr>
<td>$L_{ls}$</td>
<td>IM stator leakage inductance</td>
</tr>
<tr>
<td>$L_m$</td>
<td>IM magnetising inductance</td>
</tr>
<tr>
<td>$L_s$</td>
<td>IM stator self-inductance</td>
</tr>
<tr>
<td>$M_e(s)$</td>
<td>IM electrical transfer function</td>
</tr>
<tr>
<td>$M_m(s)$</td>
<td>IM mechanical transfer function</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>--------</td>
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</tr>
<tr>
<td>( p )</td>
<td>IM number of poles</td>
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<td>( P_c(s) )</td>
<td>PBSC phase compensator transfer function</td>
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<tr>
<td>( P_{Ic}(s) )</td>
<td>FOC current PI controller transfer function</td>
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<td>( P_{IM} )</td>
<td>IM nominal power</td>
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<td>( P_l )</td>
<td>Load power</td>
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<tr>
<td>( P_u )</td>
<td>PBSC stabilising power</td>
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<td>( K_{1,2,3,A} )</td>
<td>Small-signal FOC IM drive DC current gains</td>
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<td>( K_c )</td>
<td>IBSC band-pass filter gain</td>
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<td>( R )</td>
<td>RLC input filter resistor</td>
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<tr>
<td>( R_{1,2} )</td>
<td>PBSC damping injection constants</td>
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<tr>
<td>( R_{fd}(x) )</td>
<td>RLC input filter desired PCH damping matrix</td>
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<td>( R_f(x) )</td>
<td>RLC input filter PCH damping matrix</td>
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<tr>
<td>( R_g )</td>
<td>PBSC virtual RLC input filter resistor</td>
</tr>
<tr>
<td>( R_r )</td>
<td>IM rotor resistance</td>
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<td>( R_s )</td>
<td>IM stator resistance</td>
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<tr>
<td>( S_c(s) )</td>
<td>IM small-signal compensating speed sensitivity transfer function</td>
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<tr>
<td>( S_c^P(s) )</td>
<td>IM small-signal compensating speed sensitivity transfer function with PBSC</td>
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<tr>
<td>( S_{cpl}(s) )</td>
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<td>( T_e )</td>
<td>IM electromagnetic torque</td>
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<td>( T_l )</td>
<td>Load torque</td>
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<tr>
<td>( T_s )</td>
<td>DSC sampling period</td>
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<tr>
<td>( u_{PB} )</td>
<td>PBSC control law</td>
</tr>
<tr>
<td>( V_{as} )</td>
<td>FOC phase ( a ) stator voltage</td>
</tr>
<tr>
<td>( V_{bs} )</td>
<td>FOC phase ( b ) stator voltage</td>
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<tr>
<td>( V_c )</td>
<td>ASC ( q ) axis stabilisation voltage</td>
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<tr>
<td>( V_{cs} )</td>
<td>FOC phase ( c ) stator voltage</td>
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<td>( V_{dqs}^{sat} )</td>
<td>FOC current controller output saturation</td>
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<td>( V_{dr} )</td>
<td>IM ( d ) axis rotor voltage</td>
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<tr>
<td>( V_{ds} )</td>
<td>IM ( d ) axis stator voltage</td>
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<tr>
<td>( V_{dqs} )</td>
<td>IM ( d ) axis stator decoupled voltage</td>
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<td>( V_{dsc} )</td>
<td>IM ( d ) axis stator decoupling voltage</td>
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<td>( V_g )</td>
<td>DC grid voltage source</td>
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<td>( V_l )</td>
<td>DC-link voltage</td>
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<td>$V_{l0}$</td>
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<tr>
<td>$V_{qs}$</td>
<td>IM $q$ axis stator voltage</td>
</tr>
<tr>
<td>$V'_{qs}$</td>
<td>IM $q$ axis stator decoupled voltage</td>
</tr>
<tr>
<td>$V_{qsc}$</td>
<td>IM $q$ axis stator decoupling voltage</td>
</tr>
<tr>
<td>$V_{as}$</td>
<td>FOC $a$ axis stator voltage</td>
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<tr>
<td>$V_{bs}$</td>
<td>FOC $b$ axis stator voltage</td>
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<td>$x_1$</td>
<td>RLC input filter inductor flux linkage</td>
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<td>$x_2$</td>
<td>RLC input filter capacitor electric charge</td>
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<td>$Y_C(s)$</td>
<td>IM small-signal compensating admittance</td>
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<td>$Y^{PB}_C(s)$</td>
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<td>$Y_{cpp}(s)$</td>
<td>CPL small-signal input admittance model transfer function</td>
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<td>$\xi_\omega$</td>
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<td>IBSC band-pass filter phase shift at $\omega_{nf}$</td>
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<td>$\theta_{PB}$</td>
<td>PBSC phase compensator phase shift at $\omega_{nf}$</td>
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<td>$\varphi_{r0}$</td>
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<td>IBSC band-pass filter high corner frequency</td>
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<td>$\omega_{cz}$</td>
<td>Zero corner frequency of the IM small-signal compensating admittance transfer function</td>
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<td>$\omega_e$</td>
<td>IM stator magnetic field frequency</td>
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<td>$\omega_{sl}$</td>
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### Units

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ABSTRACT

The increased penetration of distributed generators in power systems with the transport electrification trend combined with significant advances in power electronics and control has led to DC systems being proposed for applications such as microgrids, electric vehicles and more-electric aircrafts and ships.

Stability, where for example, oscillations in the DC-bus voltage exceed protection levels causing the network to be disconnected, was identified as one of the main issues related with DC systems due to the increase in dynamic interactions in multi-converter networks. System stability degradation due to constant power loads was especially highlighted. Constant power loads often consist of tightly regulated power converters and motor drives. In this thesis, active stabilisation control techniques for a field oriented controlled induction motor drive connected to a feeder system, are proposed.

An impedance-based stabilisation control is first considered and a tuning method based on an impedance stability criterion, enabling a control design in term of stability margins, is proposed. The control is thoroughly analysed using frequency-domain and time-domain simulations.

A passivity-based stabilisation control is then proposed. The control scheme is derived from an energy principle, which enables a more intuitive tuning procedure, whilst still providing a design in term of stability margins by the means of a phase compensator. Extensive frequency-domain and time-domain simulations of the stabilisation control are presented.

Both stabilisation strategies are implemented on a low cost digital signal controller and are then demonstrated, evaluated and compared using a control hardware-in-the-loop emulation platform. The active stabilisation controls exhibit a significantly improved system relative stability whilst minimising the degradation of the system performance. Control sensitivity to parameter and design errors is carefully investigated. Finally, the benefits and limitations of each stabilisation scheme are discussed.
DECLARATION

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DEDICATION

To Emma for her unconditional support during these years. The last six months would have been a far more difficult challenge without your endless encouragement and reassurance. Despite the ups and downs of my research, your absolute dedication gave me the strength and inspiration to reach my goals. There are no words to describe how much I owe you.

À ma sœur, Laurie et mes parents, Marie-Pierre et Alain pour l’indéfectible soutien qu’ils m’ont apporté jusqu’à présent. Toutes ces années durant, malgré les échecs, vous n’avez jamais douté de la confiance que vous m’accordiez, pour cela je vous serais éternellement reconnaissant. Votre exemple fut chaque jour une source d’inspiration qui m’a aidé à gravir chaque échelon pour aujourd’hui, je l’espère, vous rendre un peu de ce que je vous dois tant.
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Chapter One

INTRODUCTION

This Chapter first provides some background and in doing so aims to provide a context within which the thesis can be placed. The research objectives and the thesis structure are then presented.

1.1 BACKGROUND AND MOTIVATION

In the late 19th century two rival concepts for electric power transmission systems were introduced: the Direct Current (DC) system, proposed by Thomas Edison and the Alternating Current (AC) system invented by Nikola Tesla. The DC system was first implemented in 1882, whilst the AC system was introduced in 1888 with the help of the American entrepreneur George Westinghouse. From then on, a conflict also known as the “War of Currents”, between the Edison Electric Light Company and the Westinghouse Electric Company, went on to impose their patented system as a standard, to mainly power the lighting in homes and streets [1]. The main challenge faced by Edison’s DC concept was that the DC electrical power had to be generated and distributed at the same voltage level as that used by the customers, which resulted in limited transmission distances due to high losses; whereas, the advent of transformers enabled an increase in AC power transmission length using high voltage to
significantly reduce the losses. However, some controversy over the safety of AC high voltage lines existed. Eventually, Tesla’s vision was popularised and the AC system established itself as the electric power network standard for more than a century, owing to the fundamental AC characteristic of rotating electric generators and motors, the lower loss long distance power transmission capability and the ease of AC voltage magnitude transformation [2].

With the 20th century came the era of the AC grid, which consists of interconnected AC systems with centralised power generation, supporting the forever growing need for electrical energy to power lighting, electric motors, railways, and later, loads such as air conditioning, televisions and computers. This trend was sustained by the significant increase in generator sizes and transmission distances due to high voltage AC (HVAC) lines [3]. The AC grid offered increased reliability, and more efficient, cheaper power generation with large centralised plants [4].

Today, the conventional types of electric power generation are fossil fuel based (coal, gas and oil) and nuclear fission based turbine generators, and renewable energy sources (RESs), including hydroelectric, wind, solar, geothermal, biomass and waste [5, 6]. The global net electric generation reached 21.5 trillion kilowatt-hours (kWh) in 2012 and is predicted in the International Energy Outlook 2016 [6] to increase by 69% in 2040 to reach 36.5 trillion kWh. According to [6], the electric power generation by energy source in 2012 was shared as shown in Figure 1.1. The coal, oil and gas powered generation resulted in carbon dioxide emissions from 67% of the total electricity produced in 2012.
In recent years, growing concerns have been raised about the human impact on the environment, especially greenhouse gases (GHG) emissions responsible for global warming. In 2010, the equivalent of 49.5 GtC\textsubscript{2} of GHG was released into the atmosphere, 25\% of which were attributed to electric and heat production [7], as shown in Figure 1.2. The general consensus is that to limit global warming to 2 degrees Celsius above pre-industrial levels, GHG emissions must be at least halved by 2050 with respect to the 2008 level [8]. This consensus was adopted by the global agreement COP21, which was ratified by 174 countries in 2015. The United Kingdom is aiming to even further reduce its GHG emissions with an 80\% cut by 2050 on the 2008 level [9]. To meet these targets, low carbon technologies undeniably have a key role to play.
Although nuclear and hydroelectric power generation have a low carbon footprint there are concerns over the nuclear waste treatment issue, which is yet to be solved, and the impact on the environment caused by large hydroelectric reservoirs [5], and so many researchers suggest [4, 10, 11] distributed generators (DGs) such as wind turbines, fuel cells and photovoltaic (PV) electric production, present a greater potential. The increased penetration of DGs in the power system has significantly modified the way electric energy has been previously distributed, resulting in the emergence of the microgrid (MG) concept [12-15].

The concept of microgrid can be defined as a low voltage (LV) distribution system consisting of interconnected DGs, energy storage devices, such as flywheels, energy capacitors and batteries, and flexible loads [15]. One of the main features of a MG is the capability of operating either connected or disconnected, also known as “islanded”, from the main AC grid [10]. Research in the area of microgrids, reviewed in [16-18] has underlined the benefits of MGs over centralised power systems such as the ease of DG integration, increased local reliability and efficiency, local voltage support, voltage sag correction and uninterruptible power supply functions [12, 19, 20].
The electrification of transport forms another challenge to achieve the sustainability targets set by the COP21, since 14% of the global GHG emissions in 2010 were attributed to this sector as shown in Figure 1.2. Hybrid electric vehicles (HEVs), fuel cell vehicles (FCVs) and electric vehicles (EVs) [21-26] have been thoroughly investigated in the last decade as replacements for conventional fossil fuel powered vehicles. The more-electric aircraft (MEA) has also been a research topic broadly discussed in the literature, where conventional non-propulsive aircraft systems, which depend on hydraulic, pneumatic or mechanical power, are tending to be replaced by electrically powered systems so as to reduce their size and weight, and improve fuel efficiency [27-30]. A similar trend is observed in marine, with the concept of the all-electric ship, detailed in [31-33], which optimises the overall efficiency, and therefore reduces the GHG emissions, by generating electricity on-board with a combination of fossil fuel based generators, fuel cells, energy storage systems, and RES, to power a full electric propulsion system. The more-electric transport trend has resulted in a significant increase in the size and complexity of on-board electric distributed power systems (DPSs), which also can be seen as on-board islanded MGs.

The research focused on MG and DPS, due to the need for increased DGs integration to power systems and greener transport, combined with significant advancements made in power electronics and control in the last decade, has restarted the debate over AC or DC systems [34-36]. In fact, the penetration of DGs is projected to have an average growth rate of 3.1% per year, to reach 11 trillion kWh by 2030 [6]. This combined with the expected increase in energy storage systems [37] and DC loads, especially DC chargers for plug-in HEVs and EVs [39], has renewed the interest in DC MGs since it reduces the amount of conversion stages required to connect the abovementioned systems and therefore improves the overall MG efficiency. Furthermore DC MG systems offer improved power quality [35], enables power transmission and distribution with a reduction of power losses and voltage drop, and increases the line capacity and transmitted power [40]. On-board DC DPS for a MEA also provides significant

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1 Solar PVs have the largest expected average year growth (8.9%) among RESs, followed by geothermal (6.5%) and wind (6.1%) [6].

2 Plug-in HEVs and EVs are predicted to increase from 1.26 million in 2016 to 140 million by 2030 in the Global EV Outlook 2016 [38].
benefits such as reduced losses, component volumes and weights (fewer cables and power converters are needed compared to an equivalent AC system) [27, 41]. Similarly the DC DPS concept enables the simplification of the power system cabling, and increases the transmitted power on ships [42]. Furthermore it allows a reduction in the number of transformers on-board, conventionally required for ship AC DPS, therefore decreasing the overall weight and cost [43].

A lack of guidelines and standards has been one of the key challenges associated with DC systems [36]. DC protection is another challenge which is the focus of ongoing research, the main challenge being the non-zero crossing characteristic of the current in DC systems [44, 45]. Some of the advancements made in MG, MEA and ship DC protection have been reported in [46-50]. The stability of DC MGs and DPSs has also been an area of concern due to the increased interactions between systems with a multi-converter system [51]. Interactions with constant power loads (CPLs) have been highlighted as one of the main stability issues with DC systems [52-54]. CPLs often consist of tightly regulated converters and motor drives, which exhibit a destabilising characteristic known as the incremental negative resistance [52, 54]. Such phenomena have been thoroughly detailed in the literature, affecting various applications such as, telecommunication networks [55], high voltage DC (HVDC) systems [56], sea and undersea vehicles [54, 57], HEVs, EVs and FCVs [52, 58, 59], MEAs [60-63] and DC MGs [64-66].

In 2011, 46% of the global electric consumption was attributed to electric motors, which are mainly used in applications such as compressors, mechanical movement, pumps and fans [67]. A significant amount of the motors used in industry are induction motors (IM), due to their robustness, low-cost, maintenance-free operation, maturity of the technology and the high-performance enabled by field oriented control (FOC) techniques. Induction motors are also used for electric propulsion of HEVs, EVs and FCVs [68, 69] and ship propulsion [69]. Although the use of a conventional IM drive in the MEA is limited due to its low fault tolerance owing to the natural phase coupling existing in IM [30], recent advances in multi-phase IM [70, 71] have widen its range of applications. When tightly regulated, IM drives behave as constant power loads and therefore present a considerable stability issue due to their widespread applications in DC MGs and DPSs.
The aim of this thesis is to devise, demonstrate and evaluate active stabilisation techniques for DC systems which can be implemented using existing load side hardware systems: in this thesis the load side system is a FOC IM drive.

1.2 Research Objectives

The objectives of this research are:

- Investigate ideal and non-ideal CPL stability.

- Devise and evaluate load side stabilisation control for DC systems.

- Compare the performance of the proposed stabilisation control using a standard industrial digital signal controller and a control hardware-in-the-loop emulator.

1.3 Thesis Structure

The remainder of the thesis is organised as follows:

- Chapter 2 contains a literature review of stabilisation techniques for CPLs. The concept of CPL and the definitions of stability are presented. The impedance-based stabilisation criteria are summarised, and the concept of passivity-based control is described. Existing source, interface and load side active stabilisation techniques are then reviewed. Finally, a brief review of IP-based FOC for induction machine drives is presented.

- Chapter 3 first presents the stability analysis mathematical tools which are used throughout the thesis. The stability analysis of an ideal CPL interconnected with a feeder system is then discussed. Finally, the stability of a non-ideal CPL, consisting of an IP-based FOC IM, connected with a feeder system is described.

- Chapter 4 proposes an impedance-based stabilisation control (IBSC) using band-pass filtering for an IP-based FOC IM drive connected to a feeder system. The system frequency-domain model is first presented,
then the stabilisation control tuning is discussed and evaluated using frequency-domain analysis. Finally, simulation results comparing the stabilised and non-stabilised system are discussed.

- **Chapter 5** proposes a passivity-based stabilisation control (PBSC). The method is first derived for an ideal CPL connected to a feeder system. The PBSC is then adapted for an IP-based FOC IM drive using phase compensation and frequency-domain analysis. Simulation results of the stabilised and non-stabilised system are compared.

- **Chapter 6** provides a comparison of the IBSC and PBSC schemes using a control hardware-in-the-loop (HIL) emulation platform. The HIL system used is described. The discretisation of both controls is then addressed. The PBSC and IBSC emulation results are presented. Finally, the benefits and limitations of the proposed stabilisation schemes are discussed.

- **Chapter 7** provides concluding remarks, highlights the contribution to knowledge of this thesis and discusses future research opportunities.

- **Appendix A** provides the IP-based FOC IM drive model, including the IM model, the IP-based FOC and the PWM inverter model. A small-signal model of the drive is also derived.

- **Appendix B** details the calculations of the IBSC band-pass filter corner frequencies.

- **Appendix C** presents the simulation results of the PBSC technique for an ideal CPL connected to a feeder system.

- **Appendix D** details the discretisation of the IP filters used for the IP-based FOC.

- **Appendix E** presents the ASCs C-code, which are implemented in the TI DSC.
Chapter Two

LITERATURE REVIEW

This chapter provides an overview of the stabilisation techniques published for CPLs. The concept of CPL is first briefly introduced, and stability definitions and analysis tools such as small-signal state-space and impedance criteria are summarised. The concept of passivity-based control is then introduced, and the main impedance-based and passivity-based, source, interface and load side active stabilisation techniques are reviewed. Finally, the IP-based FOC technique for an induction machine drive is briefly presented.

2.1 CONSTANT POWER LOAD

The concept of a constant power load (CPL) is discussed in this section and the negative incremental resistance destabilising characteristic is explained with simple examples based on the ideal CPL principle.

2.1.1 PRINCIPLE

Multiple loads such as electric motors, actuators and power converters have to be controlled in order to maintain constant output power [52]. Figure 2.1 shows an example of a CPL which consists of an electric motor controlled via an inverter using a tight speed regulation. The motor is loaded with a rotating load
which exhibits a one-to-one torque-speed characteristic, implying that the torque remains constant for a constant speed since there is only one torque for every speed at which the load can operate. Therefore when the speed is regulated to operate at a constant reference speed, the motor drive output power is constant as it is the product of the speed and torque. Assuming a constant efficiency of the drive, the electric motor drive is seen as a CPL from the DC bus since its input power is constant.

\[
P_{\text{out}} = \omega_m T = \text{cst}
\]

Controller to maintain constant speed

\[
P_{m} = V_i i_j = \text{cst}
\]

Figure 2.1: CPL, Tightly regulated speed electric motor drive

Figure 2.2 shows a DC/DC converter feeding a resistor where the voltage across the resistor is tightly regulated by the DC/DC converter. Resistors have a one-to-one voltage-current characteristic; therefore by maintaining the voltage to a constant value causes the current to be constant. This again results in a constant output power. Again by assuming constant converter efficiency, this system is seen as a CPL from the DC bus.

\[
P_{\text{out}} = V_r i_r = \text{cst}
\]

Controller to maintain constant voltage

\[
P_{\text{in}} = V_i i_j = \text{cst}
\]

Figure 2.2: CPL, Tightly regulated voltage DC/DC converter with resistive load
2.1 CONSTANT POWER LOAD

The challenge with a CPL is when there is a change in DC bus voltage, for example if the bus voltage reduces, then the CPL will draw more current, to maintain its power, which may further decrease the DC bus voltage and potentially may reach minimum voltage protection levels. A similar case occurs for an increase in DC bus voltage, with the CPL reducing its current which may further increase the voltage, and potentially to the upper protection level.

This inherent destabilising characteristic of the CPL is known as the negative incremental resistance. Although the CPL direct impedance is positive \((V/i > 0)\), its incremental resistance is negative \((\Delta V/\Delta i < 0)\) as shown for an ideal CPL case in Figure 2.3. The impact of such characteristic on the system stability is discussed in Section 2.1.2.

![Figure 2.3: CPL negative incremental resistance](image)

**2.1.2 NEGATIVE INCREMENTAL IMPEDANCE**

Figure 2.4 shows a simplified system, where an ideal CPL is connected to a DC-link represented by \(E\), which is assumed ideal. The transmission line between the CPL and the DC-link is modelled by an ideal inductor.

![Figure 2.4: Simplified system feeding an ideal CPL](image)
The system is at equilibrium when the DC-link voltage $E$ is equal to the CPL voltage $V$. Here, the system is said to be stable when the original operating point is recovered after a source or load disturbance. Assuming that the system is operating at steady state, its stability can be considered around the operating point A shown in Figure 2.5. Supposing a disturbance occurs which produces a slight current increase, $+\Delta i$, this will cause a lower CPL voltage $V$ to maintain a constant power. This will result in a positive voltage across the inductor, forcing the current to further increase. The operating point will therefore drift away from point A in Figure 2.5. For a small decrease in current ($-\Delta i$) due to a disturbance, the resulting voltage across the inductor will be negative, forcing the current to further decrease, and so the operating point will again move away from point A. According to this demonstration the operating point A is unstable.

![Figure 2.5: CPL destabilising characteristic](image)

This simple example highlights the intrinsic destabilising characteristic of ideal CPLs due to their negative incremental resistance. Furthermore, CPLs result in reduced system damping and stability margins [53]. However it was shown in [72] that the ideal CPL assumption is not necessary the worst-case condition. Therefore system stability analysis tools enabling a more detailed examination of stability issues due to non-ideal CPL are introduced in Section 2.2.

## 2.2 System Stability

Different stability concepts are reviewed in Section 2.2.1 to clarify the meaning used throughout this thesis. Then analytical tools for system stability analysis, such as small-signal state-space and impedance criteria, are introduced in Section 2.2.2.
2.2 System Stability

2.2.1 Stability Definition

The concept of stability, which first appeared in mechanics, has been extensively investigated during the past century, though this has resulted in numerous definitions of system stability that need to be clarified [73-77]. In control theory a system is considered stable “if starting the system somewhere near its desired operating point implies that it will stay around the point ever after.”, as stated in [77]. Conversely, a system described as unstable will diverge from the operating point. Another definition can be found in [73], “A stable system is a dynamic system with a bounded response to a bounded input.”, in other words the system response is limited in magnitude when exposed to a limited input or disturbance. For instance, a DC-link voltage oscillating between 100 V and 300 V, around its desired operating point of 200 V, will be described as stable since it satisfies the definitions from [73] and [77], however, from a practical perspective this behaviour is not desirable and stability as characterised here does not fulfill the system requirements. The above definitions of stability are actually definitions of absolute stability [73]. A system exhibiting absolute stability is also referred to as marginally stable. In the remainder of the discussion the marker ‘absolute’ or ‘marginal’ will be omitted, and a system displaying absolute or marginal stability will be denoted as stable, for conciseness.

Asymptotic stability is defined in [74] as: “An equilibrium point is stable if all solutions starting at nearby points stay nearby; otherwise, it is unstable. It is asymptotically stable if all solutions starting at nearby points not only stay nearby, but also tend to the equilibrium point as time approaches infinity.”. In the case of the DC-link voltage example, if the oscillations decay from ±100 V to ±50 V around the equilibrium point (desired operating point), within one hour, the system is considered asymptotically stable. Nevertheless, asymptotic stability still does not ensure satisfactory system performance as the DC-link voltage may reach voltage protection levels.

A specific case of asymptotic stability is defined as exponential stability in [77], which can quantify “how rapidly” the system will exponentially converge to the equilibrium point. However, fast convergence may result in large overshoots, which may not be acceptable in some applications.
In order to evaluate the degree of stability for a system described as stable, according to the previous definitions, a fourth concept is introduced in [73], and is referred to as **relative stability**. The relative stability of a system is defined using different stability criteria, which will be reviewed in Section 2.2.2.2, and can provide an assessment of the degree of stability of the system. With the notion of relative stability, systems are no longer classified as stable or unstable but exhibit a certain degree of stability.

Despite the concepts of absolute, asymptotic and exponential stability being well-defined, engineering applications require a deeper evaluation of the notion of system stability, which can be provided by determining the relative stability using specific tools, to deliver a qualitative analysis of the system stability. Some of the main analytical tools dedicated to address system stability are reviewed in Section 2.2.2.

### 2.2.2 Stability Analysis

Small-signal state-space analysis techniques are discussed in Section 2.2.2.1, with different impedance-based criteria being presented in Section 2.2.2.2.

#### 2.2.2.1 Small-Signal State-Space Analysis Techniques

The \(d\bar{q}\) approach for stability analysis has been widely used, based on the fact that switching converters can be represented by time-varying transformers [78]. This technique has been used to analyse controlled PWM rectifiers [79, 80], or simplified electric power systems with controlled and uncontrolled rectifiers feeding CPLs [81, 82]. In [83], the dynamics of controlled electromechanical actuators are taken into account in order to analyse the effect of a non-ideal CPL on electric power systems stability.

Other state-space modelling techniques for small-signal stability analyses of power-electronic based power systems, combining AC and DC power systems can be found in the literature. The main one is the generalized state-space averaging (GSSA) method, which has been introduced in [84] to extend the state-space averaging method to more general cases such as switched circuits or resonant converters. This approach has been used for different DC/DC converters studies [52, 54, 85, 86] or uncontrolled single phase rectifier [87]. Furthermore, the stability problems encountered in automotive more
electric/hybrid systems are discussed in [51], where a multi-converter power electronics system study using the same modelling technique has been addressed. The GSSA method produces complex mathematical models which limits the order of the system studied. However, it was demonstrated in [88] that the GSSA model can be reduced to the same set of state equations as the \(d^*q\) model, for the same assumptions which are introduced by the \(d^*q\) technique, i.e., a balanced three-phase systems. The stability prediction of both models was experimentally validated in [88], using an inverter-controlled Permanent Magnet Synchronous Motor (PMSM) as the non-ideal CPL.

Large-signal stability analysis techniques have also been investigated in [60, 89]. However, large signal methods may be inapplicable to complex systems due to the dimension of the systems.

Although small-signal state-space analysis techniques provide a good insight to system stability, as with large-signal modelling the complexity of the mathematical model significantly increases with additional components.

### 2.2.2.2 Impedance Criteria

The increased size and complexity of MG and DPS has led to these systems being designed as a set of smaller subsystems. Therefore, one of the main concerns for subsystems integration resides in the stability of the whole DPS or MG: in fact, subsystems with well-defined stability margins could lead to instabilities when integrated as a whole system [90], due to interactions between subsystems. The stability of interconnected subsystems can be addressed using a linear feedback model as shown in Figure 2.6.

![Figure 2.6: Linear feedback model of interconnected subsystems](image)

In Figure 2.6, \(Z_{LIN}\) represents the equivalent input impedance of the load subsystem, and \(Z_{SIN}\) and \(Z_{SOUT}\) are the input and output impedance of the source subsystem, respectively. For instance, in Figure 2.1 and Figure 2.2 the load subsystem is represented by the CPL, whilst in these simple systems the source...
subsystem is represented by the DC bus; in a more realistic scenario the CPL would be interfaced to the DC bus, via an input filter, cable impedance and/or DC-link capacitance which would also need to be considered when characterising the source subsystem.

The stability of the interconnected subsystems, Figure 2.6, can be analysed using the minor-loop gain \( L_i(s) \), given by:

\[
L_i(s) = \frac{Z_{OUT}}{Z_{LIN}}
\]  

(2.1)

In the literature, several stability criteria have been derived from the frequency-domain analysis of the minor-loop gain \( L_i(s) \) by defining forbidden regions for the Nyquist contour in the complex plane [53, 91].

The first criterion proposed was the so-called Middlebrook criterion, [92]. It was originally introduced to study the stability of a feedback controlled power electronic converter with an input filter. This criterion requires the magnitude of the minor-loop gain \( L_i(s) \) to lie within the unit circle, so the complex coordinate \((-1+j0)\) is not encircled, and then system stability is ensured. To fulfil a relative stability requirement, this criterion can be extended to provide a specific gain margin, as shown in (2.2).

\[
\left| \frac{Z_{OUT}(s)}{Z_{IN}(s)} \right| \leq \frac{1}{GM}
\]  

(2.2)

Figure 2.7 shows the forbidden region, defined by the Middlebrook criterion, for a specific relative stability. The limit of stability is defined by a circle centred at the origin with a radius equal to \( 1/GM \).
The Middlebrook criterion is only based on the magnitude of the input and output impedances of the subsystems, and so the design is only limited by the gain margin. This results in a very conservative system design, since an infinite phase margin is imposed by the criterion. Furthermore, stable configurations achieving satisfactory gain and phase margins can be found within the forbidden region defined by the Middlebrook criterion.

To relax the conservative system design from the Middlebrook criterion, further criteria were introduced, using both the gain and phase margins. The gain and phase margin (GMPM) criterion was detailed in [93]. A more general criterion which was developed in [94], known as the opposing argument criterion (OAC) shown in Figure 2.8.

A less conservative relative stability criteria than the Middlebrook or OAC, is the energy source analysis consortium (ESAC) [95, 96] criterion. The ESAC criterion accounts for both phase and gain margins, and defines a smaller forbidden region than the aforementioned methods. Furthermore the ESAC
criterion allows the load impedance to be defined, using a 3-D load admittance constraint representation in term of frequency, magnitude and phase, which provides a tool to assess regional stability. The limit of stability is obtained by defining three points: the first point lies on the Real axis and defines the system gain margin, the two other points are complex conjugate and are positioned on the unit circle, therefore defining the system phase margin.

![Figure 2.9: ESAC criterion](image)

The Middlebrook, GMPM, OAC and ESAC criteria have all been used to design input filters for a CPL to satisfy a desired relative stability [97, 98]. However, it is shown in Chapter Three that passive stabilisation methods are often undesirable due to their impact on system reliability, efficiency and weight. The ESAC criterion, by defining the load input admittance specification for stability, provides a tool to allow the rigorous design of active stabilisation controllers (ASCs). A stabilisation design based on the ESAC criterion is described in Chapter Four and Chapter Five.

Existing impedance-based ASCs for both source and load subsystems, also referred to as active damping controls or negative resistance compensators, which are derived from the impedance criteria are discussed in Section 2.4.

In Section 2.3 a different approach of system stability control based on the principle of passivity, and known as passivity-based control, is introduced.

### 2.3 Passivity-Based Control

The passivity-based control (PBC) technique, which is derived from an energy-based principle, was first introduced in 1989 [99]. The concept of
passivity is defined in [100] as: “In passive systems the rate at which the energy flows into the system is not less than the increase in storage. In other words, a passive system cannot store more energy than is supplied to it from the outside, with the difference being the dissipated energy.”. According to this definition, in a passive system, a finite amount of input energy will result in a bounded amount of output energy which is not exceeding the amount of input energy [100, 101]. This therefore insures the stability of passive systems. In the remainder of this section the physical interpretation of the definitions of the concept of passivity and stability are presented. Rigorous mathematical definitions and proofs of these concepts can be found in [100-103], though they are omitted here for brevity.

The fundamental principle of the PBC technique is that a closed-loop system can be rendered passive with respect to a desired energy storage function by the means of a control law [101, 104], this is also referred to as energy shaping. Furthermore, virtual damping can be injected using the PBC, which therefore achieves asymptotic stabilisation [101, 104]. This combination of energy shaping and damping injection not only enables the system stabilisation but also allows the achievement of performance objectives [101].

In the literature, PBC techniques are often applied to port-control Hamiltonian systems (PCH) [105-107]. The model of PCH systems is expressed as given in (2-3).

\[
\frac{dx}{dt} = [J(x) - R(x)] \frac{\partial H(x)}{\partial x} + g(x)u
\]

where \(J(x)\) is the \(n\) by \(n\) interconnection matrix and \(R(x)\) is the \(n\) by \(n\) damping matrix, representing the energy exchange and the dissipation in the physical system, respectively. \(\frac{\partial H(x)}{\partial x}\) is the partial derivative of the Hamiltonian function of the system, which represents the total energy stored in the system. The input vector, \(u\), affects the system dynamics via the input matrix \(g(x)\).

PCH models offer a representation of the system in terms of physical characteristics such as energy stored, energy exchange and dissipation. This formalisation enables an easier implementation of the abovementioned PBC method, since the damping terms and the dependence on the energy function are made explicit.
The PBC technique for PCH systems is often referred in the literature as interconnection and damping assignment (IDA) [104, 105, 108, 109]. In fact, the IDA-PBC enables the modification of both the energy exchange and the dissipation of the system. The system closed-loop can be shaped, assuming a control law $u = \beta(x)$ satisfying (2.4), where $\beta(x)$ is a function of the state variables of the system.

$$\frac{dx}{dt} = [J_d(x) - R_d(x)] \frac{\partial H_d(x)}{\partial x} \quad (2.4)$$

where $J_d(x)$ is the desired interconnection matrix, $R_d(x)$ is the desired damping matrix and $\partial H_d(x)/\partial x$ is the partial derivative of the desired total energy stored function. The energy function, $H_d(x)$, must exhibit a minimum at the desired operating point so as to achieve the stabilisation of the system.

The IDA-PBC was originally introduced to control mechanical systems [110, 111]. Later, the approach was also applied to control processes [112, 113], electromechanical systems [114-117], three-phase rectifiers [118-120] and DC/DC converters [121]. Literature concerning the mitigation of instabilities due to CPLs using the IDA-PBC method is reviewed in Section 2.4. A stabilisation design method using the PBC technique is also proposed in Chapter Five.

### 2.4 Active Stabilisation Control

Distributed DC power systems are often described as multi-converter systems due to the volume of individual converters in each of the subsystems. Such a topology allows stabilisation at the source, interface or load converter level. Existing active stabilisation techniques are reviewed in Section 2.4.1 for the source side and interface converters, and in Section 2.4.2 for the load side converters.

#### 2.4.1 Source Side and Interface Converter

The stabilisation techniques at the source and interface level have been mainly proposed for DC/DC converters with output voltage control. Stabilisation of interconnected source side and interface DC/DC converters with a CPL are often addressed assuming an ideal CPL in the literature, as shown in Figure 2.10 and
2.4 Active Stabilisation Control

Figure 2.11 respectively. In Figure 2.10 the ideal voltage source \( E \) is often used as a simple model of a solar PV array, energy storage battery system or supercapacitor. The ideal CPL is then fed via a DC bus.

![Diagram](image1)

**Figure 2.10**: Source side converter feeding an ideal CPL

In Figure 2.11 the interface converter enables DC voltage adjustment between the main DC bus and the secondary DC bus, which feeds the ideal CPL.

![Diagram](image2)

**Figure 2.11**: Interface converter feeding an ideal CPL

### 2.4.1.1 Impedance-Based Techniques

Several methods were proposed for a source side converter feeding an ideal CPL, [55, 122, 123], in order to fulfil the source output impedance constraint imposed by the previously introduced impedance criteria. Moreover, stabilisation control based on the emulation of virtual passive elements was proposed for a source side DC/DC converter [64, 124, 125] and interface DC/DC converter [66], feeding
an ideal CPL. This technique allows output voltage stabilisation by shaping the source output impedance. DC bus voltage stabilisation was experimentally demonstrated with these techniques.

2.4.1.2 Passivity-Based Techniques
An ASC for a source side DC/DC boost converter with an ideal constant power load, using IDA-PBC, was proposed in [126]. The method is tuned using a virtual circuit derived from the desired structure of the PCH model of the converter. The control is designed at a certain CPL power to achieve a desired damping ratio of the new PCH structure. A complementary PI controller is introduced to cancel the output voltage steady state error due to the CPL power variation. DC voltage control is experimentally demonstrated, however, no comparison against non-stabilised responses is provided. Furthermore, the load dynamics are not taken into account since the experiment is run with an ideal CPL emulated with a programmable DC electronic load.

2.4.1.3 Others Techniques
A source side stabilisation control using a nonlinear feedback linearisation method for DC/DC converters feeding ideal CPLs was introduced in [127-129]. The feedback linearisation is used to cancel the intrinsic nonlinear characteristic of the CPL, and thus mitigate the negative incremental resistance behaviour of the CPL. The stabilisation control is tuned by choosing a feedback gain to ensure the desired relative stability. Stabilisation is experimentally demonstrated with a buck and boost DC/DC converters feeding ideal CPLs.

A simple control technique modification for source side DC/DC converters was proposed in [130-132], to improve the relative stability when feeding CPLs. The conventional PWM was replaced by a pulse adjustment, which regulates the output voltage of the controller generating high and low power pulses, to control the switches. It was demonstrated that the relative stability of a DC/DC converter feeding a CPL could be improved, by increasing the number of high-power pulses as well as increasing the duty cycle of the high-power pulses. However, the range of CPL power which can be stabilised with this technique is restricted by the choice of high and low power pulse duty cycles. The duty cycles are chosen by making a trade-off between the output voltage control and stabilisation, which results in a significantly degraded voltage control for
satisfactory stabilisation. The proposed control was experimentally validated with a buck-boost converter feeding an ideal CPL.

In [133] both impedance-based and passivity-based principles are merged with the definition of a new passivity margin criterion to enable the shaping of the converter admittance with respect to a desired phase margin. The new criterion is demonstrated using a source side DC/DC boost converter used to connect solar PV to a DC microgrid. The proposed method uses a simple output voltage feed-forward summed to the output of the inner current loop controller to achieve passivation. Satisfying stabilisation of the microgrid DC bus is experimentally demonstrated. However the technique requires an increase in the output capacitor value to improve the system phase margin. It was discussed in Section 2.2.2.2 that increased passive components are often undesirable for system reliability, efficiency, weight and volume reasons.

In [62] a multi-load DC system stabilisation was proposed. The system consists of interconnected source and interface converters feeding ideal CPLs. The stabilisation is achieved with a centralised controller generating stabilising power references for each load which must be drawn by the CPL to stabilise the system. The stabilising power references are obtained from a Lyapunov equation derived from the mathematical model of the whole DC system. DC bus voltage stabilisation is experimentally demonstrated. This method requires the extensive use of sensors and a complex communication system, however these issues were solved with the implementation of an observer in [134] for the same DC system. Again stabilisation is achieved with a centralised controller generating stabilising power references for each load from a Lyapunov equation of the whole system. This method was also experimentally validated. The main drawback of multi-loads system stabilisation is that a detailed mathematical model of the entire system is required, which renders the stabilisation scheme sensitive to every single source, converter and load. Furthermore, the complexity of the control strategy exponentially increases with additional sources and loads. Finally one of the main features of future DC MGs and DPSs could be high flexibility, with reconfigurable system structure and plug-and-play sources and loads, therefore limiting the use of centralised stabilisation controllers due to possible significant changes in the system configuration.
The stabilisation techniques presented in this section for source side and interface converters are often derived and demonstrated with ideal CPLs. However it was shown in [72] that the ideal CPL assumption is not necessary the worst-case condition in term of dynamics. In fact, it is demonstrated that there is an interrelationship between the source and load bandwidth which can lead to instability in the presence of a CPL. With the assumption of an ideal CPL, the load inherits the time constant of the DC-link passive components which can results in misleading analysis. According to this, stabilisation control implementations at the source and interface side converter are not considered further in this thesis.

The aforementioned remarks emphasise the need for techniques enabling a stabilisation control at the load side converter. Such stabilisation control can be designed accounting for the load dynamics and their interactions with the feeder system. In this configuration each load autonomously ensures its stable operation when connected to the system. Therefore load side active stabilisation provides significant advantages in term of system flexibility. Existing load side ASCs are reviewed in Section 2.4.2.

2.4.2 Load Side Converter

In this section ASC techniques for the load side converter of a non-ideal CPL are discussed. Most of the techniques presented in the literature consist of a DC/AC converter interfaced electrical machine connected to a source impedance, which can represent an input filter, cable impedance and/or DC link capacitance.

2.4.2.1 Impedance-Based Techniques

The first stabilisation scheme for a DC/AC converter interfaced electrical machine was introduced in [135, 136], using a nonlinear system stabilising control (NSSC) technique. In [135], the load input impedance of a field oriented controlled (FOC) induction machine is modified using a nonlinear control law, which consists of the ratio between the actual DC-link voltage and its filtered component. The filter is designed using a root locus in order to provide a satisfactory relative stability. The NSSC scales the torque producing component reference, assuming a perfect torque control, as shown in Figure 2.12. The NSSC is experimentally validated and demonstrates stabilisation of the DC-link voltage. However, this technique is limited at low torque due to the small
current reference, and is ineffective at zero torque. Furthermore, since the dynamic behaviour of the induction machine is not considered, the NSSC method significantly degrades the induction motor performance. The main drawback of CPL active damping techniques is to degrade the system performances; since the stabilisation techniques increase the CPL sensitivity to DC-link voltage changes.

![Diagram](image)

**Figure 2.12**: NSSC stabilisation control for motor drive

A simple way to satisfy the stability criteria mentioned in Section 2.2.2.2 is to increase the size of the input filter impedance. Since passive stabilisation is undesirable, stabilisation schemes using virtual passive elements have been proposed in the literature, [137, 138]. In [137], a large-signal DC-link voltage stabilisation using a virtual capacitor injection (VCI) technique is proposed. The method modifies the CPL power to emulate a virtual capacitor, and so the system behaves in the same way as if an additional capacitor was physically implemented. The stabilisation control is demonstrated in a permanent-magnet synchronous motor (PMSM) drive system. The CPL power is modified by adding an emulated virtual capacitor power reference, obtained from a control law function of the DC-link voltage, to the torque producing current reference. Therefore, the DC-link voltage behaves as if the capacitor of the input filter was physically increased, which significantly enhances the relative stability of the system. Although the concept of this technique is intuitive, consisting of designing an input filter with two parallel capacitors, the proposed method to analyse the impact of the additional control, a nonlinear analysis using a candidate Lyapunov function to estimate the domain of attraction, is complex.
DC-link voltage stabilisation is experimentally demonstrated, however, the degradation of the motor torque due to the modification of the torque reference is not addressed.

In a similar manner a virtual resistor injection (VRI) stabilisation control is introduced in [138]. Similar to the VCI control, the stabilisation is realised by virtually modifying the topology of the input filter. In this case a virtual resistor is emulated in parallel with the original resistor and inductor. To do so, the current which would be drawn by the virtual resistor if it was physically implemented is emulated by modifying the current drawn by the CPL. This method is implemented in a permanent-magnet synchronous motor (PMSM) drive. The extra current due to the virtual resistor is generated by modifying the \( q \) axis voltage reference of the PMSM. To calculate the additional resistor current reference, both DC-link voltage and source voltage are required. To cancel the need for an extra sensor, a closed-loop estimator is proposed, however, this is sensitive to input filter or line impedance parameter changes. Furthermore, the damping injection requires a change in reference frame in order to optimise the DC-link voltage stabilisation while maintaining satisfactory motor control performance. Although significant DC-link voltage stability improvement is experimentally demonstrated, the complexity of the motor control is considerably increased.

A linear technique to compensate for the negative incremental resistance characteristic (NIRC) of the CPL was described in [56, 139, 140]. The NIRC is implemented in [139] for a three-phase brushless DC drive, and in a PMSM drive system in [56, 140]. The NIRC principle is shown in Figure 2.13 for a generic electrical drive system, and consists of modifying the conventional current reference of the speed controller by adding a compensating current reference. The compensating current reference is generated applying a high-pass filter to the DC-link voltage. Under steady state conditions the current reference is not modified, however when the DC-link voltage oscillates around its operating point, a portion of the voltage change is fed into the motor drive control loop via the high-pass filter. This technique improves the damping of the input filter by modifying the speed control of the motor drive. In fact, to stabilise the DC-link voltage, the NIRC is shaping the motor drive impedance to behave as a positive impedance. The corner frequency and the gain of the
2.4 Active Stabilisation Control

High-pass filter are derived using the frequency-domain small-signal input admittance of the motor drive, to enable the compensation to be most effective around the resonant frequency of the input filter; later in Chapter 3 it is shown that instabilities are more likely around the resonant frequency due to the large input filter gain. The NIRC is validated experimentally, and significant improvement on system stability is demonstrated. However, due to the high-pass filter stabilisation strategy, the NIRC is sensitive to measurement noise on the DC-link voltage, and this may even degrade the motor drive control loops, potentially leading to instabilities.

In [141], a combination of the NSSC and NIRC is proposed for a FOC induction motor drive. The load input impedance is shaped by modifying the $q$ axis torque producing component with a multiplicative (NSSC) and an additive (NIRC) compensating signal. Stabilisation of the DC-link voltage was experimentally demonstrated. Although the speed sensitivity to DC-link voltage changes was improved compare to the single NSSC method, the sensitivity to measurement noise remains due to the high-pass filter of the NIRC.

To overcome these issues related to high-frequency measurement noise, the high pass-filter of the NIRC was replaced by a band-pass filter (BPNIRC), in [142]. The compensation block was implemented using a PMSM drive system, and the controller was designed using frequency-domain small-signal input admittance analyses. Significant DC-link voltage stabilisation was experimentally demonstrated. In [143], a BPNIRC for a FOC induction machine drive with a LC input filter stability was investigated. Stabilisation was achieved by modifying

![Figure 2.13: NIRC stabilisation control for motor drive](image)
the $q$ axis current reference with a compensation component which used the filtered DC-link voltage. The controller was tuned in the same manner as in [142]. However, the control scheme was not experimentally demonstrated.

It was demonstrated in [144], that the performance of the BPNIRC [142, 143] method was affected by the current control bandwidth. In fact, it is assumed that the frequency region within which the motor drive input admittance must be compensated, is contained in the bandwidth of the current control for the compensation to be effective, however, in practice the current control bandwidth must often be reduced [141] to limit the control sensitivity to measurement noise, which then degrades the stabilisation action. Furthermore, accurate current tracking implies a high-gain current control, and so the compensating block increases the speed and torque control sensitivity to DC-link voltage variations. Compensators were proposed in [144] to inject a stabilisation component to the $q$ axis voltage reference, which is generated by the $q$ axis current controller. This reduced the sensitivity of the speed and torque controls to DC-link voltage changes, whilst ensuring negative incremental resistance compensation within the desired frequency region. The first method proposed uses a portion of the DC-link capacitor current change to form a reference-voltage-based compensator (CRVBC), whilst the second is based on the DC-link voltage (VRVBC). The CRVBC technique emulates a voltage drop proportional to the DC-link capacitor current which is fed to a low-pass filter to generate the compensating component. The VRVBC stabilisation signal is produced as described in [142] using a band-pass filter. Figure 2.14 shows the control implementation of the CRVBC and VRVBC techniques. Both stabilisation schemes were validated experimentally, demonstrating significant improvement in term of DC-link voltage stabilisation and reduced sensitivity. However the CRVBC control requires an extra current sensor, for DC-link current.
An experimental comparison of the NIRC, BPNIRC and VRVBC, was described in [145]. The stabilisation controls were implemented with a high-speed magnetically suspended brushless DC (BLDC) motor. Similar stabilised DC-link voltage responses to a motor torque step were achieved for the different controllers. However, the benefits of the VRVBC over NIRC and BPNIRC technique were demonstrated, with significantly reduced speed sensitivity to DC-link voltage change. Furthermore, the speed response to a torque step was also considerably enhanced with the VRVBC compensator.

Small-signal admittance based stabilisation schemes were also investigated for an indirect self-control induction motor with input LC filter, in [146-148]. The stabilisation was formulated in terms of a $H_\infty$ optimisation problem, in order to ensure desired stability margins while minimising the motor-drive performance degradation. However, the controller tuning is very complex and less intuitive.

### 2.4.2.2 Passivity-Based Techniques

Load side active stabilisation using PBC was also investigated. It was shown in Section 2.1.1 that a DC/DC converter feeding a resistive load with tight voltage regulation behaved as a CPL. In [65, 149, 150] the stabilisation of buck, boost and buck-boost DC/DC converters feeding a resistive load was proposed. The PBC technique with damping injection resulted in a linear PD controller for the buck converter, and a non-linear PD controller for the boost and buck-boost converters. Significant DC-link voltage stability improvement was experimentally demonstrated for each converter. However PD controllers
increase the control sensitivity to noise due to the differentiator. Furthermore the PD controller could result in output voltage steady state error due to the inductor’s parasitic resistance. To overcome this issue a complementary PID controller to the PD-PBC controller was detailed in [151]. The proposed control scheme was experimentally validated for a boost converter feeding a resistive load. The aforementioned techniques have not been developed using the PCH formalisation, rendering the design of such controllers more complex and less intuitive.

Load side active stabilisation was also developed using the PBC method for a BLDC motor drive system connected to an input LC filter in [152, 153], where an input filter state feed-forward stabilising controller (SFSC) is derived from the PBC method. The SFSC control law is obtained from the ideal CPL case, using PBC damping injection. The control law generates a stabilising power reference which must be drawn by the load in order to stabilise the system. The SFSC is a function of both grid current and DC-link voltage. The SFSC is then applied to the motor drive and achieves stabilisation by modifying the current reference given by the speed controller by adding a compensating current reference as shown in Figure 2.15. The controller is designed based on the input admittance frequency-domain model. DC-link voltage stabilisation is experimentally demonstrated. Again the PCH formalisation is not applied in this case which results in a less intuitive tuning. Furthermore, unlike the impedance-based stabilisation techniques reviewed in Section 2.4.2.1 using filters, the PBC controller does not provide a phase compensation mechanism of the load input admittance. This could limit the effectiveness of the stabilisation control as discussed in Chapter Five. Another drawback of this control is the implementation as a current reference injection based stabilisation control, which could limit the effectiveness of the stabilisation scheme due to the current control bandwidth, as mentioned in Section 2.4.2.1.
Although satisfactory stabilisation of the DC-link voltage is provided by the techniques discussed in this section, the review has underlined the lack of a systematic design approach for load side ASC techniques. The sensitivity of the stabilisation controller to changes in operating point or parameter errors has not been addressed. Furthermore, the main drawback of load side ASCs is the unavoidable load performance degradation, which could reach unacceptable levels due to an approximate tuning of the stabilisation controller, changes in operating point and system parameter errors. A rigorous approach based on the ESAC criterion introduced in Section 2.2.2.2 is proposed in this thesis, with impedance-based and passivity-based stabilisation controllers for an IP-based FOC IM drive introduced in Chapter Four and Chapter Five, respectively. The proposed approach allows a rigorous tuning in terms of gain and phase margin, whilst ensuring a minimal performance degradation.

The induction machine IP-based FOC used in this thesis is discussed in Section 2.5.

### 2.5 Induction Machine IP-Based FOC Control

In has been shown in the literature that the zero introduced by the PI controller in the closed-loop transfer function can affect the dynamic performance of the system [154]. In fact, in the physical system, the time response and the overshoot will be degraded as the zero moves towards the imaginary axis,
resulting in the desired natural frequency and damping ratio of the control closed-loop previously set not being achieved. IP control was introduced so as to cancel the influence of this zero, and recover the desired performance. PI and IP control were first compared for DC drive speed control [155-157], underlining the improved performance resulting from the IP control. The speed overshoot was reduced, whilst maintaining the tight disturbance rejection of the PI control. The benefits of IP over PI control were also demonstrated for position control of synchronous [158-162] and induction machines [163], enabling no overshoot and zero steady state error with a pre-defined rise time for position tracking. Speed control for induction machine was also investigated using the IP control [164, 165].

More recently, an IP based field oriented control for induction motor drive was introduced in [166], demonstrating improved performance in terms of speed tracking, whilst retaining robust disturbance rejection; the $d$-$q$ stator voltages and currents are enhanced, resulting in a much smoother DC-link power. IP control is usually implemented applying the integral factor on the error between the reference and the measured variable, whilst the proportional factor is only applied to the measured variable. In [166], the proposed implementation consists of IP filters added to the conventional field oriented PI control structure.

A validation of the IP-based FOC can be found in [166], using a control hardware-in-the-loop (HIL) emulation system controlled with a digital signal controller (DSC); this system is the same as that discussed in Chapter Six. A comparison of the IP and PI based control is carried out for current and speed control, highlighting the benefits of the former. It is shown that IP control improves the speed tracking and produces smoother stator voltages and currents with lower peak values. Enhanced stator voltages and currents result in an improved DC-link power quality.

### 2.6 Summary

This chapter has first presented the concept of a CPL and its destabilising negative incremental resistance characteristic. Stability definitions were outlined and stability analysis tools were reviewed. The impedance-based
stability criterion ESAC was introduced to form the basis of a systematic approach for the design of active stabilisation schemes presented in Chapter Four and Chapter Five. The passivity-based control technique, which is used in Chapter Five to develop an ASC, was presented. Stabilisation controllers for source side and interface converters feeding CPLs were reviewed. However, the limitations of these stabilisation schemes due to the unmodeled load dynamics (ideal CPL) were underlined. It was shown that load side active stabilisation was the most suitable candidate for the mitigation of CPL instabilities. Both existing impedance-based and passivity-based load side active stabilisation were reviewed and the lack of a systematic approach in the design of the controllers was highlighted. Finally, the IP-based FOC for the induction motor drive used in the remainder of this thesis was presented.
Chapter Three

STABILITY ANALYSIS

This chapter briefly introduces some of the stability analysis mathematical tools which are used throughout the thesis. The interactions between a feeder system and an ideal CPL, and the resulting destabilising characteristic, are investigated using poles and frequency-domain analysis. An IP-based FOC IM which is used throughout the thesis to develop and compare load side ASC techniques is then introduced and its stability is investigated.

3.1 BACKGROUND

This section discusses several concepts of stability and then briefly reviews some of the well-known stability criteria for linear and time invariant systems, which will be used for qualitative analysis and control design throughout this thesis.

3.1.1 MATHEMATICAL MODELLING

The first step in describing the behaviour of any dynamic system is the derivation of a mathematical model, which is generally characterised by a set of differential equations. Once such a model is formulated, stability, performance, frequency response or time response can be analysed using numerous methods.
In the remainder of Section 3.1 a linear\(^3\) second-order system is considered; which is defined in (3·1) as a transfer function. A transfer function is described by the Laplace transform of the output divided by the Laplace transform of the input and provides an input-output relationship describing the dynamic behaviour of the system \([76]\).

\[
C(s) = \frac{Out(s)}{In(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{3·1}
\]

where \(\omega_n\) characterises the undamped natural frequency; and \(\xi\), the damping ratio of the system.

The system described by (3·1), can also be represented in the time-domain using the state-space method \([73, 76]\), as shown in (3·2). The state-space representation is a first order vector-matrix differential equation, which combines the first-order differential equations which describe the system behaviour. The system dynamic behaviour is described by the state variables of the system. The state-space method simplifies the modelling and analysis of complex systems with multi-inputs and multi-outputs, since the complexity of the equation does not increase with the number of state variables, inputs or outputs.

\[
\frac{dx(t)}{dt} = Ax(t) + Bu(t) \tag{3·2}
\]

where \(x(t)\), represents the state variable vector, \(\frac{d}{dt}\) is the time derivative of the state variable vector, \(u(t)\) characterises the input\(^4\) and \(A\) and \(B\) are the state and input matrix, respectively.

Both, the transfer function and state-space methods will be used throughout the thesis, and the main stability criteria dedicated to those types of mathematical models will be introduced in Section 3.1.2.

\(^3\) Nonlinear systems can also be represented by linear mathematical models using linearisation approximations. The model is then only valid around the operating point considered.

\(^4\)A single input is considered in this example but the state-space representation allows modelling multi-input systems.
3.1.2 Stability Criteria

This section aims to briefly summarise some of the principle stability criteria, which will be used in this thesis to assess relative stability. The review is not exhaustive and focusses on the main methods applied to linear and time-invariant systems. Although it is well known that most physical systems display nonlinear characteristics, they can often be linearised around an operating point \[77\], so that local stability can be investigated using linear techniques.

3.1.2.1 Time-Response Analysis

The dynamic behaviour of a second order system, represented by the transfer function given in (3.1), is characterised by the undamped natural frequency, \( \omega_n \), and the damping ratio, \( \xi \). The transient response to a unity step exhibits oscillations as shown in Figure 3.1. The damping ratio quantifies how much the oscillations due to a disturbance are attenuated (damped), and the system is underdamped for \( 0 < \xi < 1 \). If \( \xi = 1 \) or \( \xi > 1 \) the system is defined as critically damped and overdamped, respectively; in both cases the transient response is not oscillatory. If \( \xi = 0 \), the response does not converge and the system is marginally stable. The transient response is unstable for negative damping ratios since the oscillations will diverge from the operating point.

![Figure 3.1: Transient response to unity step of a 2nd order system](image)

The damping ratio of a second order system provides information on the relative stability of the dynamic response. However, in practical applications additional poles or zeros often degrade the dynamic response, and their influence on system stability needs to be considered.
**3.1.2.2 Root Locus Analysis**

The stability of a system is closely correlated to the location of its poles. The poles of a system are defined by the roots of the characteristic equation, which is represented by the denominator of the transfer function which models the system. In the case of an underdamped second-order system in (3·1), the poles are complex conjugate and expressed as:

\[ p_1 = -\xi \omega_n + j\omega_n\sqrt{1-\xi^2} \]
\[ p_2 = -\xi \omega_n - j\omega_n\sqrt{1-\xi^2} \]  

(3·3)

For the system to be stable the real part of the poles in (3·3) must remain negative. In this case the real part of the poles becomes positive for a negative damping ratio, which as suggested in Section 3.1.2.1, results in diverging oscillations, leading the system to be unstable. The poles of the characteristic equation can be plotted on the complex plane by varying system parameters. This method is called the Root Locus method, and it shows the impact on stability of the system parameters. The relative stability of the system will be degraded with poles in the left-hand side of the complex plane moving towards the imaginary axis; conversely, relative stability will be improved if the poles move away from the imaginary axis. The Root Locus method is a powerful tool to evaluate relative stability. It is also used to analyse and design feedback control systems, as it allows pole placement by adjusting controllers gains to satisfy stability requirement, and can also be used to investigate the influence of zeros on system stability.

However, for transfer functions greater than second order, it can be complex to derive the roots of the characteristic equation. In this case the state-space method, introduced in Section 3.1.1, is more convenient, as the poles of the system are represented by the eigenvalues of the matrix \( A \) in (3·2). The eigenvalues are obtained by satisfying the following equation:

\[ |\lambda I - A| = 0 \]  

(3·4)

where the vertical bars indicate the matrix determinant, \( \lambda \) is the eigenvalue matrix and \( I \) is the \( n \) by \( n \) identity matrix. Stability is achieved for eigenvalues with a negative real part and the Root Locus method can be used to plot these values.
3.1.2.3 Frequency-Response Analysis

The frequency response of a system shows its steady state response to a sinusoidal input. The response of a linear system to a sinusoidal input is sinusoidal, and only varies from the input signal by changes in the amplitude and phase angle. Therefore, in the frequency domain transfer functions are represented as:

\[ G(j\omega) = |G(j\omega)|e^{j\phi(\omega)} \]  

where \(|G(j\omega)|\) indicates the gain and \(\phi(\omega)\) represents the phase shift introduced by the transfer function \(G(s)\). By varying the input frequency, the behaviour of the system can be studied throughout the frequency range.

A well-known technique to show the evolution of the gain and phase shift introduced by the transfer function throughout the frequency range is the Bode plot. A Bode plot shows the gain and phase shift from low to high frequencies using a logarithmic scale of frequency. The phase angle (phase shift) is calculated using:

\[ \text{Phase shift} = \tan^{-1}\left(\frac{\text{Im}[G(j\omega)]}{\text{Re}[G(j\omega)]}\right) \]

The gain is stated in decibels (dB), which represents the logarithm of the magnitude defined as:

\[ \text{Logarithmic gain} = 20\log|G(j\omega)| \]

The Bode plot of the transfer function given in (3.1), for damping ratios from 0.1 to 1.6 with steps of 0.3, is shown in Figure 3.2.
To investigate relative stability the gain and phase margin can be deduced from the Bode plot. As introduced in the previous section, the system becomes unstable if the real part of one of its poles becomes positive. A system with an unstable pole will display a Bode plot with a gain greater than 0 dB and a phase below -180° or above +180° at a specific frequency. For a 0 dB gain and ±180° phase shift, the system is marginally stable. Therefore, the gain margin quantifies how much the system gain can increase, when the phase is ±180°, before it reaches 0 dB. Similarly the phase margin quantifies how much the phase shift introduced by the system can increase, when the gain is 0 dB, before it reaches ±180° [73]. To assess the relative stability of a system, both margins need to be considered, as an acceptable phase margin will not be sufficient to fulfil relative stability requirements if a satisfactory gain margin is not achieved, and vice-versa.

The magnitude of a transfer function response to a sinusoidal input can be plotted versus its phase angle on the complex plane, throughout the frequency range, using polar plots. The benefit of a polar plot is that the gain and phase margins can be viewed on the same plot, which enables the absolute and relative stability of the system to be evaluated. The polar plot which will be used throughout this thesis is the Nyquist plot. In the Nyquist plot, the real part of the transfer function \( G(j\omega) \) from (3·5) is projected on the \( x\)-axis (real axis), while the imaginary part of (3·5) is projected on the \( y\)-axis (imaginary axis), defining a
contour by varying the frequency, \( \omega \), from 0 to infinity. Therefore, for a given frequency, \( \omega \), the magnitude of the transfer function is equal to the norm of the vector going from the origin to the polar coordinate \( \text{Re}[G(j\omega)], \text{Im}[G(j\omega)] \), and the phase shift introduced by the transfer function is represented by the angle between the vector and the positive real axis. The 0 dB gain and phase shift of \( \pm 180^\circ \) in the Bode plot is then represented by the complex coordinate \((-1, 0)\) in the Nyquist plot. Thus, the Nyquist contour must not encircle the complex coordinate \((-1, 0)\), to ensure absolute stability. Relative stability can be assessed from the Nyquist plot, as it improves the further the Nyquist contour is from the complex coordinate \((-1, 0)\).

Frequency-response analyses are used to investigate the relative stability of a linear feedback system by evaluating the open-loop frequency response. Control systems can also be designed using frequency-response analyses by adjusting controller gains in order to ensure sufficient relative stability; this is known as loop-shaping.

### 3.2 Ideal Constant Power Load

This section investigates the interactions between an ideal CPL and a feeder system, to examine the stability of such systems. The feeder system is characterised by a RLC impedance which can represent an input filter, cable impedance and/or DC link capacitance. In Figure 3.3 an ideal CPL is connected to an ideal DC grid voltage source \( (V_g) \) through an input filter consisting of a series connected resistor \( (R) \), inductor \( (L) \) and a DC-link capacitor \( (C) \). The DC grid current is \( i_g \), and \( i_l \) and \( V_l \) are the load current and voltage respectively.

![Figure 3.3: Ideal Constant Power Load with input filter](image)

The relative stability of the system shown in Figure 3.3 will be examined from its mathematical model using root locus and frequency-domain analyses.
### 3.2.1 Mathematical Model

The state equations of the system are derived by applying Kirchhoff’s current and voltage laws to the circuit in Figure 3.3.

\[
\begin{align*}
L \frac{di_g}{dt} &= V_g - R_i g - V_l \\
C \frac{dV_l}{dt} &= i_g - i_l
\end{align*}
\]  

(3-8)

The ideal CPL can be modelled as an ideal current source using (3-9).

\[
i_l = \frac{P_l}{V_l}
\]

(3-9)

Due to the nonlinear behaviour of the ideal current source, (3-9) must be linearised around an equilibrium point, in order to apply linear analysis to the system in Figure 3.3. Assuming a constant power operation and using the first order term of the Taylor expansion, the linearisation of (3-9) is expressed as:

\[
\delta i_l = -\frac{P_l}{V_{l0}^2} \delta V_l
\]

(3-10)

where the subscript “0” designates the steady state values and the prefix “\( \delta \)” denotes the small-signal variable.

Equation (3-10) shows that the ideal CPL behaves like a negative incremental impedance at positive power values. In fact, to maintain a constant power the load current must increase if the DC-link voltage drops, conversely, \( i_l \) must decrease if \( V_l \) increases; this behaviour degrades the stability of the system as shown in Section 2.1.

### 3.2.2 Stability Analysis

Using the linearised equation of the current drawn by the CPL, (3-10), and the differential equations (3-8), the characteristic equation of the system (3-11) can be derived.

\[
s^2 + \left( R - \frac{P_l}{CV_{l0}^2} \right) s + \frac{V_{l0}^2 - R P_l}{L C V_{l0}^2} = 0
\]

(3-11)

For the system to be stable, the poles of the characteristic equation given in (3-11) must be located in the left hand-side of the complex plane, in other words
the real part of the characteristic equation poles must be negative, as introduced in Section 3.1.2.2. Thus, a stability boundary for the system can be derived from (3.11) as shown in (3.12). This criterion can be used to predict the stability of the system shown in Figure 3.3, [141].

\[ P_l \leq \frac{RC}{L} V_{l0}^2 \quad (3.12) \]

It can be deduced from this expression that the system is naturally stable if \( P_l \) is negative even for a reduced resistor \( R \) and DC-link capacitor \( C \) or large inductor \( L \). The CPL generating mode for the system in Figure 3.3 is therefore inherently stable. Hence, this case will not be further discussed in the remainder of this thesis.

Equation (3.12) can be rewritten as (3.13) so that the maximum power for a stable CPL can be calculated. If \( P_l = P_{lmax} \), the system is marginally stable and the oscillations will not die out, beyond this limit the system becomes unstable.

\[ P_{lmax} = V_g^2 \left( \frac{RC}{L + R^2C} - \frac{R^3C^2}{(L + R^2C)^2} \right) \quad (3.13) \]

Using the input filter parameters given in Table 3.1 in (3.13), gives a maximum power for this configuration of 8.3267 kW. In Table 3.1 the grid voltage \( V_g \) is set to 540 V, as it was suggested in [27] that on-board DC DPS systems could potentially evolved to 540 V DC in order to reduce the current levels and therefore minimise the weight of the wiring for MEAs. The resistor value given in Table 3.1 was selected according to the measured line resistance of the Intelligent Electric Power Network Evaluation Facility (IEPNEF) [167] at The University of Manchester. The IEPNEF is a flexible 100 kW 540 V DC power network, which is representative of a potential new on-board architecture. According to this measurement a line impedance of 10 mΩ as given in Table 3.1 represents a cable length of 10 meters. The capacitor and inductor were calculated according to [168] to achieve a 0.2% steady state DC-link voltage ripple in the case of a 20 kHz inverter controlled motor drive.

Figure 3.4 shows the DC-link voltage \( V_l \), obtained from a nonlinear Matlab/Simulink simulation of (3.8) and (3.9) using the parameters in Table

---

5 In the sense of absolute stability as defined in Section 2.2.1.
3.1. At 10 s the power of the CPL is stepped from 8 kW to 8.327 kW, which is just above the stability limit defined in (3.13). As predicted by the stability boundary, after the power step occurred, the DC-link voltage contains an oscillation diverging from the operating point, which results in the system being unstable.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>10 mΩ</td>
</tr>
<tr>
<td>$L$</td>
<td>140 μH</td>
</tr>
<tr>
<td>$C$</td>
<td>400 μF</td>
</tr>
<tr>
<td>$V_g$</td>
<td>540 V</td>
</tr>
</tbody>
</table>

Table 3-1: Input filter parameters

![Figure 3.4: DC-link voltage unstable example](image)

To investigate the influence of the RLC filter parameters on the system's relative stability, different root loci of the system described in Figure 3.3 are shown in Figure 3.5. The root loci are obtained with the eigenvalues of the state matrix $A_f$ (3.14), which is derived from (3.8).

$$A_f = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ -\frac{1}{C} & \frac{P_l}{CV_{i_o}^2} \end{bmatrix} \quad (3.14)$$

The standard filter parameters for the system are given in Table 3-1, and the power of the CPL is set to 8 kW. For each passive element, $C$, $L$ and $R$, a root locus is formed as shown in Figure 3.5 a), b), c), respectively. The root loci are plotted by varying the value of the concerned passive element while the other parameters are kept as given in Table 3-1. In Figure 3.5 a), the capacitor increases from 100 μF to 600 μF in steps of 100 μF. As deduced from the
3.2 Ideal Constant Power Load

stability criterion (3.12), the relative stability improves as the capacitance increases, as the poles are drifting towards the left-hand side of the complex plane. However, it can be seen that the system is unstable if the capacitance is below 400 μF, since the poles exhibit positive real parts. Figure 3.5 b) shows the impact of the RLC filter inductance on relative stability; its value varies from 80 μH to 180 μH with steps of 20 μH. The stability is degraded as the inductance rises and the system becomes unstable for inductors greater than 140 μH; again this is consistent with (3.12). The consequence of resistor variation is shown in Figure 3.5 c), with R increments from 4 mΩ to 14 mΩ by steps of 2 mΩ; the system is unstable at 4 mΩ and becomes stable for resistors equal or greater than 10 mΩ.

The filter resistor R directly affects the systems relative stability since the damping of the system improves by increasing R, thus more power can be
drawn by the CPL and the relative stability is improved. However a larger resistor will cause more power losses which will degrade the system efficiency. The value of the capacitor also influences the system stability, as the relative stability improves for higher values of stored energy in \( C \) which is given by \( E = CV_i^2/2 \). The rate of change of the energy stored in the capacitor is equal to the input power \( (P_i = V_g I_g) \) minus the power consumed by the CPL \( (P_l) \), therefore a greater value of capacitance will increase the maximum power which can be fed to the CPL. The capacitor acts as a power buffer between the source and the CPL. Nevertheless in applications such as traction, space and aviation, capacitors tend to be kept small due to volume, weight and reliability constraints [27]. In practical systems, CPLs are likely to be power electronic converters or electric motor drives as tightly regulated power converters often exhibit a negative incremental impedance [52, 54]. Consequently, the inductance \( L \) must be relatively large in order to cancel the harmonic content introduced by the CPL high-frequency switching. This is in contradiction with the stability criterion, (3-12), and the observations made from Figure 3.5, which require a small \( L \) to ensure system stability, and therefore further justify the investigation of active stabilisation techniques in order to meet requirements in terms of both system performance and stability.

The observations made from the stability criterion and the root loci plots highlight the unavoidable trade-off between relative stability and performance: according to (3-12) and Figure 3.5 system stability requires a large resistor \( R \) and capacitor \( C \), conversely bulky resistive and capacitive elements are undesirable to optimise the system performance since the former increases the power losses while the latter tend to be minimised for reliability purposes and volume and weight requirements. Furthermore, to meet regulations on harmonics suppression, the inductance \( L \) has to be relatively large, which degrades the system stability. This compromise between stability and performance limits the system design and the overall performances. Although a large variety of input filter topologies, [97], with different advantages can be implemented, the design is challenging due to limitations on the passive elements. To further evaluate the behaviour of CPLs, and to provide an understanding which will form the basis of the next chapters, a frequency domain analysis of the system shown in Figure 3.3 is discussed in Section 3.2.3.
3.2.3 Impedance Analysis

This section focuses on characterising the behaviour of the CPL in the frequency domain as this forms the basis for the impedance-based stabilisation control (IBSC) and passivity-based stabilisation control (PBSC) presented in Chapter Four and Chapter Five, respectively.

From the differential equations given in (3.8), and the load current linearisation (3.10), the small-signal model of the system shown in Figure 3.3 can be represented by the following linear expression:

$$\delta V_l = Z_1(s)\delta V_g - Z_2(s)\delta i_l$$

(3.15)

with

$$Z_1(s) = \frac{\omega_{nf}^2}{s^2 + 2\xi_f \omega_{nf}s + \omega_{nf}^2}$$

(3.16)

$$Z_2(s) = \frac{(Ls + R)\omega_{nf}^2}{s^2 + 2\xi_f \omega_{nf}s + \omega_{nf}^2}$$

where the undamped natural frequency $\omega_{nf}$ and the damping ratio $\xi_f$ are defined as:

$$\omega_{nf} = \frac{1}{\sqrt{LC}}, \quad \xi_f = \frac{R}{2\sqrt{C}}$$

(3.17)

This expression defines the interaction between the RLC filter and the CPL, as the DC-link voltage response to variations in voltage source and load current are described by the transfer functions $Z_1(s)$ and $Z_2(s)$, respectively. Again, using the input filter parameters given in Table 3.1, Figure 3.6 shows the Bode plot of the transfer functions $Z_1(s)$ and $Z_2(s)$. The gains of both transfer functions are relatively low for frequencies above 7 krad/s which will ensure good suppression of high-frequency harmonics potentially introduced by the CPL high-frequency switching, or from supply voltage high-frequency disturbances. As discussed in Section 3.2.2, the value of resistance and capacitance are kept small for efficiency and reliability purposes, which results in a poorly damped filter as shown by the resonant peak in Figure 3.6. The DC-link voltage will be sensitive to supply voltage and load current variations.
around the resonant frequency (undamped natural frequency) of the RLC filter, due to the large gains displayed by both transfer functions. Thus, the system is likely to be unstable around this operating point when the RLC filter is coupled to a practical CPL, since inherent oscillations due to the destabilising characteristic of the CPL will be amplified by the input filter. In the low-frequency region (<1 krad/s), the low gain presented by the transfer functions $Z_1(s)$ and $Z_2(s)$, should be sufficient to ensure DC-link voltage stability for supply voltage and load current variations, respectively. The phase of the transfer function $Z_1(s)$, shows a typical second order system phase shift of $0^\circ$ in the low frequency range and $-180^\circ$ for high frequencies. The effect of the zero in the transfer function $Z_2(s)$ can be seen as it introduces a phase shift of $+90^\circ$.

![Frequency response of $Z_1(s)$ and $Z_2(s)$](image)

**Figure 3.6:** Frequency response of $Z_1(s)$ and $Z_2(s)$

According to equation (3-15), the small-signal linear feedback model of the system shown in Figure 3.3 can be derived as shown in Figure 3.7. This linear feedback representation enables the linear methods introduced in Section 3.1.2.3, to be applied so the relative stability of the system can be evaluated.
where $Y_{cpl}(s)$ is the linear small-signal input admittance model of the CPL, derived from (3·10) as:

$$Y_{cpl}(s) = -\frac{P_l}{V_{f0}^2} \quad (3·18)$$

Assuming that $Y_{cpl}(s)$ is stable, then the stability of the system is fulfilled when the Nyquist plot of the minor-loop gain $L_{cpl}(s) = Y_{cpl}(s)Z_2(s)$ does not encircle the (-1+j0) point, [146], as discussed in Section 3.1.2.3. Using the RLC filter parameters in Table 3·1, Nyquist plots of the minor loop gain $L_{cpl}(s)$ are shown in Figure 3.8 for variable CPL power. The CPL power is increased from 2 kW to 12 kW, in 2 kW steps, and the system is originally stable as the contour does not encircle the point (-1+j0), and is predicted to become unstable between 8 kW and 10 kW. This correlates with the calculation in Section 3.2.2, where the limit of stability was 8.3267 kW for the parameters in Table 3·1.

Figure 3.8: Nyquist plot for an increase in CPL power

Figure 3.9 shows Bode plots of the minor loop gain $L_{cpl}(s)$ for a CPL power of 5 kW, and 8 kW for the parameters in Table 3·1. As indicated by the Nyquist
plot, the gain margin is reduced at higher power, going from approximately 12.4 dB at 2 kW to approximately 4.4 dB at 5 kW. The phase margin is infinite for both CPL values and so the gain margin becomes the main criteria to assess the relative stability of the system.

![Frequency response of \( L_{\text{cpl}}(s) \) highlighting the gain margin](image)

The phase displayed by the transfer function \( Z_2(s) \) in Figure 3.6, is between ±90°, and provides a 90° phase margin, and an infinite gain margin as the phase shift does not reach ±180°. However, the introduction of the linear small-signal input admittance model of the CPL, \( Y_{\text{cpl}}(s) \), characterised by a negative gain, has degraded the relative stability of the feedback system given in Figure 3.7, since \( Y_{\text{cpl}}(s) \) has introduced a +180° phase\(^6\), which is added to the original phase of \( Z_2(s) \) and makes the phase of the minor-loop gain \( L_{\text{cpl}}(s) \) greater than +180° in the lower part of the frequency range as seen in Figure 3.9.

The gain, \(-R_l/V_{fo}^2\), has reduced the maximum gain of the minor loop gain \( L_{\text{cpl}}(s) \), resulting in an infinite phase margin, since the magnitude plot does not cross the 0 dB gain line for a CPL power below the limit of stability. However the gain margin which was infinite for \( Z_2(s) \) in Figure 3.6, can now be calculated at the frequency, \( \omega_{gm} \), at which the phase shift of the minor loop gain \( L_{\text{cpl}}(s) \) reaches +180° as given in (3.19).

\(^6\) Intuitively, it can be understood that a negative gain will introduce a ±180° phase shift, since the phase of a sinusoidal input would be simply inverted by a negative gain.
\[ \angle L_{cpl}(j\omega_{gm}) = \angle Y_{cpl}(j\omega_{gm}) + \angle Z_2(j\omega_{gm}) = 180^\circ + \tan^{-1}\left(\frac{\text{Im}[L(j\omega_{gm})]}{\text{Re}[L(j\omega_{gm})]}\right) = 180^\circ \] (3-19)

It can be deduced from (3-19) that the phase angle of \( Z_2(j\omega_{gm}) \) must be 0° for the phase shift of the minor loop gain \( L_{cpl}(j\omega_{gm}) \) to be +180°, which implies that the imaginary part of \( L_{cpl}(j\omega_{gm}) \) must be zero, so:

\[ \text{Im}[L_{cpl}(j\omega_{gm})] = L_{cpl}\omega_{gm}\left(1 - \frac{\omega_{gm}}{\omega^2_{nf}}\right) - 2R\xi_f \frac{\omega_{gm}}{\omega_{nf}} = 0. \] (3-20)

Equation (3-20) is satisfied if:

\[ \omega_{gm} = \omega_{nf} \sqrt{1 - \frac{2R\xi_f}{L\omega_{nf}}} \] (3-21)

Using the frequency, \( \omega_{gm} \), the gain margin can be calculated using (3-22), as the inverse of the minor loop gain.

\[ GM \equiv \frac{1}{|L_{cpl}(j\omega_{gm})|} = \frac{V^2_{i0}}{P_l} \sqrt{\frac{\left(1 - \frac{\omega^2_{gm}}{\omega^2_{nf}}\right)^2 + \frac{4\xi^2_f}{\omega_{nf}} \frac{\omega^2_{gm}}{\omega^2_{nf}}}{R^2 + L^2 \omega^2_{gm}}} \] (3-22)

From (3-22), the gain margins shown in the Bode plots in Figure 3.9, can be calculated and for the 5 kW CPL the gain margin is 4.432 dB, and 0.3478 dB for the 8 kW CPL.

Figure 3.10 shows the gain margin of the minor loop gain \( L_{cpl}(s) \) from (3-22) against the CPL power. As expected from the previous Nyquist and Bode plots, the gain margin decreases as the CPL power increases. The gain margin is equal to 0 dB at 8.327 kW, which corresponds to the maximum power at which the system is marginally stable, as calculated in Section 3.2.2. As the power increases, the magnitude of the minor loop gain \( L_{cpl}(s) \), at \( \omega_{gm} \), is increasing with the factor \( P_l/V^2_{i0} \) as shown in (3-22), which reduces the gain margin and so degrades the relative stability.
The linear feedback model in Figure 3.7 has been used to show that the system is stable if the minor loop gain $L_{cpl}(s)$ complies with the Nyquist criterion. However, the introduction of a negative gain in the minor loop gain $L_{cpl}(s)$, due to the linear input admittance model of the CPL, $Y_{cpl}(s)$, degrades the relative stability of the system. This is underlined by the derivation of the gain margin illustrated in Figure 3.10. The phase of the transfer function $Z_2(s)$, originally constrained between $+90^\circ$ and $-90^\circ$, is deteriorated by $Y_{cpl}(s)$ which introduces a phase shift of $+180^\circ$, resulting in a minor-loop gain phase shift bounded between $+250^\circ$ and $+90^\circ$.

Systems exhibiting a phase shift exceeding $\pm 90^\circ$ are often defined as non-passive [133]. As introduced in Section 2.3, a passive system can be described as a system where the energy stored cannot be greater than the energy supplied into the system and the difference between the supplied and stored energy is dissipated [169]. The intrinsic dissipative characteristic of passive systems ensures their absolute stability. This observation forms the basis of the ASCs which are introduced in Chapter Four and Chapter Five. In fact, stabilisation controls perform a passivation of the CPL. Therefore, stable operation can be ensured and the desired relative stability can be achieved by adjusting the controller gains. In the next section a non-ideal CPL, an induction motor drive which will be used to demonstrate the proposed ASC techniques, is introduced and its relative stability is investigated.
3.3 **Non-Ideal Constant Power Load**

This section investigates the interactions between a feeder system and a non-ideal CPL. Again the feeder system is characterised by a RLC impedance which can represent an input filter, cable impedance and/or DC link capacitance. It has been discussed in Chapter Two that motor drive systems, when tightly regulated, exhibit a negative incremental resistance, which characterises a CPL. As explained in Chapter Two, an induction motor drive has been selected as the non-ideal CPL in this thesis.

A typical induction motor drive system is shown in Figure 3.11. The drive consists of a sinusoidal\(^7\) PWM controlled inverter feeding a cage rotor (squirrel cage) induction machine. Field-oriented vector control is used to regulate the speed of the induction motor. The DC-link of the inverter is connected to a RLC filter.

![Figure 3.11: Induction motor drive with input filter](image)

The system in Figure 3.11, will be used as a case study throughout the thesis in order to compare active stabilisation techniques. Appendix A.1 describes the non-linear model, used in this research, of the adjustable speed induction motor drive which is connected to an RLC filter. The induction machine is controlled using a field oriented control technique, using both a classic proportional integral (PI) control structure and an IP control structure. The design of both control methods is fully discussed in Appendix A.2, and formed the basis of a

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\(^7\) Sinusoidal PWM is chosen for simplicity, however, as the proposed stabilisation techniques have no dependency on the PWM generation they may also be combined with space vector PWM techniques.
paper presented at the 2016 IEEE IECON conference [166]. The simplified PWM inverter model is discussed in Appendix A.3. A linearised small-signal model is derived from the non-linear model and is shown in Appendix A.4 and this model is used to investigate the relative stability of the FOC induction motor drive.

This section discusses the stability analysis of the adjustable speed induction motor (IM) drive connected to an input filter shown in Figure 3.11, using the \( d-q \) small-signal model discussed in Appendix A.4. The stability of the speed and \( q \) current control loops is investigated, and their impact on the overall stability of the system is studied.

Experimentally determined parameters of a 350 V 4.3 kW induction motor, given in Table 3-2, are used in this section. The parameters of the input filter remain as stated in Table 3-1, and the DC grid voltage \( V_g \) is set to 540 V.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance</td>
<td>( R_s )</td>
<td>895 mΩ</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>( L_{ls} )</td>
<td>4.4 mH</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>( R_r )</td>
<td>723 mΩ</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>( L_{lr} )</td>
<td>4.4 mH</td>
</tr>
<tr>
<td>Magnetising inductance</td>
<td>( L_m )</td>
<td>85 mH</td>
</tr>
<tr>
<td>Machine inertia</td>
<td>( J_m )</td>
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</tr>
<tr>
<td>Friction coefficient</td>
<td>( B_m )</td>
<td>0.0011 kgm(^2)</td>
</tr>
<tr>
<td>Number of poles</td>
<td>( p )</td>
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</tr>
<tr>
<td>Nominal power</td>
<td>( P_{IM} )</td>
<td>4.3 kW</td>
</tr>
<tr>
<td>Rated speed</td>
<td>( \omega_{mIM} )</td>
<td>183 rad/s</td>
</tr>
<tr>
<td>Rated torque</td>
<td>( T_{IM} )</td>
<td>23.5 Nm</td>
</tr>
</tbody>
</table>

### 3.3.1 Limit of Stability

In this section the rotor flux (\( \phi_r \)) is set to 0.8998 Wb so the 4.3 kW induction machine can achieve rated flux at synchronous speed. The gains of the speed and \( d-q \) current PI controllers are calculated using the expressions in Table A-1 in Appendix A.2.1, to achieve a damping ratio \( \xi_\omega \) and \( \xi_c \) of 0.707 for the
3.3 Non-Ideal Constant Power Load

closed-loop control with a bandwidth $\omega_{n_c}$ and $\omega_{n_\omega}$ of 10 Hz and 500 Hz, respectively. The proportional and integral gains for the current controllers are $k_{pc} = 36.58$ and $k_{ic} = 8.47e4$, and for the speed control $k_{p\omega} = 0.98$ and $k_{i\omega} = 43.43$. The time constant for the speed and $d$-$q$ current IP filters are 44.9 ms and 431.8 $\mu$s, respectively; the time constants are calculated using the gains in (A-21) and (A-22) in Appendix A.2.2. For both the IP and PI time-domain simulations, the current control output saturation is set to 421 V using (A-24) in Appendix A.2.2, whilst the speed control output saturation is set to 70.39 A using (A-23) in Appendix A.2.2. These parameters are listed in Table 3-3.

Figure 3.12 shows the maximum torque-speed characteristic of the IM drive and the torque-speed limit beyond which the system becomes unstable. The limit of stability is obtained using the eigenvalues of the linearised small-signal model of the system shown in Figure 3.11, given in (A-26), Appendix A.4. The system is considered to be unstable when the DC-link voltage $V_l$ is diverging from its steady-state value for a certain IM torque-speed operating point. However, it can be seen that the torque-speed characteristic of the induction machine remains within the stable region, ensuring stable operation across the

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor flux</td>
<td>$q_r$</td>
<td>0.8998 Wb</td>
</tr>
<tr>
<td>Closed-loop current control damping ratio</td>
<td>$\xi_c$</td>
<td>0.707</td>
</tr>
<tr>
<td>Closed-loop current control bandwidth</td>
<td>$\omega_{n_c}$</td>
<td>500 Hz</td>
</tr>
<tr>
<td>Current PI controller proportional gain</td>
<td>$k_{pc}$</td>
<td>36.58</td>
</tr>
<tr>
<td>Current PI controller integral gain</td>
<td>$k_{ic}$</td>
<td>8.47e4</td>
</tr>
<tr>
<td>Current IP filter time constant</td>
<td>$k_{pc}/k_{ic}$</td>
<td>431.8 $\mu$s</td>
</tr>
<tr>
<td>Closed-loop speed control damping ratio</td>
<td>$\xi_{\omega}$</td>
<td>0.707</td>
</tr>
<tr>
<td>Closed-loop speed control bandwidth</td>
<td>$\omega_{n_\omega}$</td>
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<td>Speed PI controller proportional gain</td>
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<tr>
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<td>$V_{dqs}^{\text{sat}}$</td>
<td>421 V</td>
</tr>
<tr>
<td>Speed controller output saturation</td>
<td>$i_{q\text{sat}}$</td>
<td>70.39 A</td>
</tr>
</tbody>
</table>
torque-speed range. In fact, the analysis of the eigenvalues provides the same maximum power of 8.3267 kW as calculated in Section 3.2, which is significantly larger than the IM rated power of 4.3 kW.

![Figure 3.12: Torque-speed induction machine characteristic](image)

Although the system is defined as stable according to Figure 3.12, disturbances might affect the system performance if the relative stability is not sufficient. Figure 3.13, shows a time domain simulation result of the DC-link voltage $V_l$, responding to a disturbance in grid voltage $V_g$ from a Matlab/Simulink based model of the non-linear decoupled model of the IP-based adjustable speed IM drive with input filter: the IM and filter parameters are given in Table 3.1 and Table 3.2. The IM is operating at a speed of 150 rad/s and a load torque of 20 Nm. At time=2 s, the grid voltage steps from 540 V to 530 V. This results in oscillations on the DC-link voltage, which degrades the DC-power quality. Section 3.2 demonstrated that these oscillations are due to the low damping of the RLC input filter, which results in a reduced system relative stability.

The remainder of this chapter investigates the relative stability of the IM drive with RLC input filter in Figure 3.3 and evaluates the impact of PI and IP control implementations and different controller bandwidths.
3.3.2 Field Oriented Control Stability Analysis

A comparison between IP-based and PI-based FOC IM drive with RLC input filter is first undertaken. The eigenvalues corresponding to the small-signal IM q-axis current reference $\delta i_{qs}^*$ and decoupled voltage reference $\delta V_{qs}^*$ for both the IP-based and PI-Based FOC are displayed in Figure 3.14 a). The eigenvalues of the IP-based and PI-based FOC IM system are obtained from the linearised small-signal state-space system derived in (A-27), Appendix A.4. For both cases, the rotor flux and control gains are kept as stated in Section 3.3.1. The operating IM speed is 150 rad/s and the load torque is 20 Nm. Figure 3.14 a) shows that the IP FOC marginally degrades the stability of the cascaded speed and q current control loops, since the complex-conjugate eigenvalues are slightly closer to the imaginary axis, this is due to the short delays in the control loops from the IP filters. Figure 3.14 b) shows that the eigenvalues corresponding to the small-signal grid current and DC-link voltage, are identical for both the IP-based and PI-based FOC, which means that the relative stability of $\delta i_g$ and $\delta V_l$ is similar for both control techniques.
Although the control loop stability is slightly degraded using IP filters, these controllers provide a significant speed tracking improvement, whilst retaining the robust PI disturbance rejection as shown in Figure 3.15 a). This figure displays the IM speed response simulation using the non-linear model derived in Appendix A and shown in Figure A.3, Figure A.4, Figure A.5 and Figure A.8. The speed is stepped from 150 rad/s to 170 rad/s at 1 s and from 170 rad/s to 130 rad/s at 1.5 s. The torque is initially 0 Nm and then at 1.25 s the torque is stepped to 15 Nm, and to -30 Nm at 1.75 s. It can be seen that the IP speed response presents significantly smoother transients to change in speed reference, whilst disturbance rejections are kept the same, resulting in significantly reduced oscillations in the DC-link voltage as shown in Figure 3.15 b). This is due to improved stator $d$-$q$ currents and voltages with lower peak values using IP filters [166]. However, despite the substantial improvement in DC-link voltage transient, IP-based FOC does not provide an increased relative stability of $V_l$ as shown in Figure 3.14 b). The reduced oscillations on the DC-link voltage are only the result of a smoother load current, and do not imply improved overall system stability. More details about the benefits of IP control over PI control for a field oriented control induction machine can be found in [166], where the controllers are implemented on a low-cost digital signal controller (DSC) system and compared using a control hardware-in-the-loop (HIL) emulation system. In the remainder of this discussion the IP control will be used for the FOC IM drive.
3.3 Non-Ideal Constant Power Load

Figure 3.15: Comparison between IP-based and PI-based FOC system response to change in IM speed and torque

Figure 3.16 shows the root loci of the eigenvalues corresponding to the small-signal IM $q$-axis current reference $\delta i_{qs}$ and decoupled voltage reference $\delta V_{qs}^{*}$ for a change in the natural frequency (bandwidth) of the current and speed control loop in a) and b), respectively. The damping of both closed-loop control is set to 0.707 and the rotor flux set to 0.8998 Wb. The control proportional and integral gains, and the IP filter time constants are recalculated for each natural frequency value. The induction machine speed is fixed to 150 rad/s with a load torque of 20 Nm. In Figure 3.16 a), the bandwidth of the current control, $\omega_{nc}$, increases from 500 Hz to 3 kHz with steps of 500 Hz, and as the natural frequency rises the relative stability of the control loops is improved since the eigenvalues are drifting away from the imaginary axis. An increased current control bandwidth offers a better relative stability due to a faster tracking of the reference value; however in practice, the current control bandwidth must often be reduced to limit the control sensitivity to measurement high-frequency noise. Figure 3.16 b), displays the root locus for a change in speed control bandwidth, $\omega_{n\omega}$ going from 5 Hz to 20 Hz with steps of 3 Hz with the $q$ current control at a
fixed 500 Hz bandwidth. The relative stability of the cascaded control loops decreases with an increased $\omega_n$. In fact, the low time constant of the IM mechanical dynamic behaviour, limits the speed control bandwidth. Figure 3.16 c) shows that the eigenvalues of the small-signal variables $\delta i_q$ and $\delta V_i$ for current bandwidths of 500 Hz to 3 kHz in 500 Hz steps, and speed bandwidths of 5 Hz to 20 Hz in 3 Hz steps, are identical for all values of current and speed bandwidth tested, which therefore means that their relative stability is not affected by the changes in FOC parameters.

![Root loci of eigenvalues for change in controller bandwidth](image)

Figure 3.16: Root loci of eigenvalues for change in controller bandwidth

Figure 3.17 shows the influence of the speed and $q$ current closed-loop control damping ratios on the relative stability of the cascaded control loops, using the eigenvalues corresponding to the small-signal variables $\delta i_{qs}$ and $\delta V_{qs}$. The bandwidths of the speed and $q$ current control are fixed at 10 Hz and 500 Hz,
respectively. The controller gains and IP filters time constant are recalculated for each damping ratio value. The induction machine operates at a speed of 150 rad/s with a load torque of 20 Nm. Figure 3.17 a), shows the root locus for a rise in current control damping ratio from 0.5 to 1.5 in steps of 0.2 with $\xi_\omega = 0.707$. For underdamped cases ($\xi_c < 1$) the relative stability is improved with an increased damping ratio. However, for overdamped damping ratios ($\xi_c > 1$) the eigenvalues are moving towards the imaginary axis. In fact, for damping ratios above 1 the time response of the current control loop is slowed down. Therefore, the response delay introduced degrades the relative stability of the current control loop. The response is optimal for a damping ratio of 1, providing a fast current time response with no overshoot.

![Root locus for change in controller damping ratio](image1)

**Figure 3.17:** Root loci of eigenvalues for change in controller damping ratio

Figure 3.17 b), shows the root locus for an increase in speed control damping ratio from 0.5 to 1.5 with steps of 0.2 with the $q$ current $\xi_c = 0.707$. The relative

![Root locus for speed control damping ratio](image2)
stability deteriorates as the damping ratio increases. However, as a speed control with a small damping ratio will exhibit a significant overshoot the $\xi_\omega$ is often set between 0.7 and 1 to ensure no overshoot. Again the relative stability of the DC-link voltage shown in Figure 3.17 c) is not affected by the change in control parameters, as the eigenvalues remain at the same value.

The benefits of IP-based FOC on DC-link voltage transient have been demonstrated. Furthermore, investigations show that the IP-based FOC relative stability is sensitive to changes in control parameters, highlighting the unavoidable trade-off between IM control loop stability and performance. However improving the stability of IM FOC drive does not provide an overall increased relative stability. Implementation of ASCs, which are introduced in Chapter Four and Chapter Five, is therefore required to improve the overall system stability.

**3.4 SUMMARY**

This chapter has briefly introduced stability analysis mathematical tools which are used throughout the thesis. The stability of an ideal CPL connected to a feeder system, which can be characterised by an input filter, cable impedance and/or DC link capacitance, was investigated using poles and frequency-domain analysis. An absolute stability limit was defined for this system and the influence of input filter passive elements on the relative stability was addressed.

Passive stabilisation was discussed, but as this introduce efficiency and reliability issues, this further strengthens the motivation to investigate active control solutions. An IP-based field oriented control induction motor drive system which is used to develop and compare the ASCs proposed in Chapter Four and Chapter Five was introduced. Finally, the stability of the IP-based FOC was investigated.
Chapter Four

**IMPEDANCE-BASED CONTROL**

This chapter introduces a design methodology for an impedance-based active stabilisation control (IBSC) which can be applied to an IP-based field oriented controlled induction motor drive, as discussed in Chapter 3. A frequency-domain model of the induction machine drive is first derived. Then the impedance-based design method is described. Finally the effectiveness of the proposed stabilisation scheme is demonstrated using a Matlab/Simulink time-domain simulation.

4.1 **IM Frequency-Domain Model**

The impedance-based ASC techniques, described in Section 2.4 require a detailed knowledge of the CPL input impedance/admittance. This section derives the input admittance for an IP-controlled FOC IM drive with a RLC input filter as shown in Figure 3.11, Section 3.3.

4.1.1 **Small-Signal Model**

In this section the small-signal model of the IP-based field-oriented control induction motor (IM) drive presented in Chapter Three is derived, using the decoupled $d$-$q$ induction motor model from Appendix A.2. To simplify the
analysis, the inverter behaviour is approximated as a unity gain, and so it neglects the PWM high-frequency harmonic content, the switching and conduction losses and the delay due to the switching mechanism. Therefore, the averaged IM drive DC current over a switching period given in (A-25), Appendix A.3, can be rewritten as:

$$i_t = \frac{3}{2V_l} [i_{ds} (V_{ds}' + V_{dsc}) + i_{qs} (V_{qs}' + V_{qsc})] \quad (4-1)$$

where $V_{dqs}$, $V_{dsc}$ are the reference $d$-$q$ decoupled stator voltages and decoupling terms defined Appendix A.2, $i_{dqs}$ are the $d$-$q$ stator currents and $V_l$ is the DC-link voltage. The decoupling terms must be included in the load current expression in order to accurately describe the behaviour of the grid current and DC-link voltage.

Using (4-1) and assuming constant rotor flux operation, the small-signal DC-current can be derived using the first term of the Taylor expansion, so:

$$\delta i_t = K_1 \delta \omega_m + K_2 \delta i_{qs} + K_3 \delta V_{qs}' + K_4 \delta V_l \quad (4-2)$$

where $K_{1,2,3,4}$ have been defined in (A-28), Appendix A.4.

Equation (4-2) and the decoupled $d$-$q$ induction motor and IP-based FOC models presented in Appendices A.2 and A.2.2, respectively, can be used to form the small-signal model of the FOC IM drive connected to an input filter shown in Figure 4.1. The IM rotor flux is assumed constant and the inverter behaviour is approximated by a unity gain, as previously stated. Furthermore, the IM input admittance derivation is focused around the resonant frequency of the RLC input filter, since instabilities are likely to occur around this frequency region. Therefore the speed control loop is omitted from Figure 4.1, since the RLC input filter resonant frequency is normally considerably higher than the speed control bandwidth.
The stabilisation control block $C_s(s)$, applies a gain to the small-signal DC-link voltage $\delta V_l$, and is then summed with the $q$ axis voltage reference in Figure 4.1, as from Section 2.4.2, this implementation offers better performance (reduced IM speed deviations during DC-link voltage variations) than if $C_s(s)$ is combined with the $q$ axis current reference, and enables the stabilisation control to operate well at low or zero speed. The design of $C_s(s)$ will be detailed in Section 4.2.

The other blocks shown in Figure 4.1 are expressed given in (4.3).

$$M_m(s) = \frac{1}{J_m s + B_m}, \quad M_e(s) = \frac{1}{\lambda_1 s + \lambda_2}$$

$$IP_c(s) = \frac{1}{k_{pc} k_{ic} s + 1}, \quad PI_c(s) = k_{pc} + \frac{k_{ic}}{s} \quad \text{(4.3)}$$

$$G = \frac{3p L_m}{22 L_r}$$

The closed-loop transfer function (4.4) between the small-signal DC-link voltage $\delta V_l$ and the small-signal $q$-axis current $\delta i_{qs}$ can be derived from Figure 4.1. The transfer function is obtained by considering the current reference $\delta i_{qs}^*$ as a disturbance which means the current IP filter does not affect the $q$-axis current response to change in DC-link voltage.

$$C_{cl}^{IR}(s) = \frac{\delta i_{qs}}{\delta V_l} = \frac{M_e(s)}{1 + PI_c(s) M_e(s)} C_s(s) \quad \text{(4.4)}$$
The small-signal model in Figure 4.1 can then be simplified to Figure 4.2 by substituting in $C_{CL}^{IB}(s)$.

![Simplified induction machine small-signal model](image)

**Figure 4.2: Simplified induction machine small-signal model**

### 4.1.2 Input Admittance

The IM input admittance can be derived from a small-signal representation of the system. The small-signal input admittance $Y_{cpl}(s)$ (4.5) of the IP-based FOC IM is derived from Figure 4.2, and consists of the sum of two terms, $Y_{IM}(s)$ and $Y_C(s)C_s(s)$.

$$Y_{cpl}(s) = rac{\delta i}{\delta V} = Y_{IM}(s) + Y_C(s)C_s(s)$$  \hspace{1cm} (4.5)

The term $Y_{IM}(s)$, given by $K_4$ in Appendix A.4, (A.28), represents the inherent admittance of the system since it does not depend on the stabilisation block, $C_s(s)$. The admittance $Y_C(s)$ (4.6), corresponds to the compensating admittance via which the total input admittance is shaped to a desired value by tuning the active stabilisation block, $C_s(s)$.

$$Y_C(s) = \frac{M_e(s)}{1 + P L_c(s)M_e(s)} \left[ \varphi r_0 G K_1 M_m(s) + K_2 + K_3 M_e(s)^{-1} \right]$$  \hspace{1cm} (4.6)

The inherent admittance $Y_{IM}(s)$ is negative at positive power, which characterises the negative incremental resistance behaviour of the IM drive.

Substituting the transfer functions in (4.3) into (4.6) enables the admittance $Y_C(s)$ to be written as:

$$Y_C(s) = \frac{s}{s(\lambda_1 s + \lambda_2) + k_{pe}s + k_{ic}} \left[ \varphi r_0 G K_1 \frac{f_m s + B_m}{m s + B_m} + K_2 + K_3 (\lambda_1 s + \lambda_2) \right]$$  \hspace{1cm} (4.7)
The mechanical transfer function $M_m(s)$ of the IM, typically exhibits a low cut-off frequency equal to $B_m/J_m$ and so its behaviour can be neglected since it does not affect the admittance $Y_c(s)$ in the mid-frequency region, around the input filter resonant frequency. The denominator of the first term in (4·7) is equal to the characteristic equation of the closed-loop current control and so it can be expressed as a function of the damping ratio and natural frequency, which were introduced section A.2.1. The simplified transfer function of $Y_c(s)$ is expressed in (4·8) and it exhibits two complex conjugate poles, a zero and a zero at the origin.

$$Y_c(s) = K_3 \frac{s(s + \omega_{cz})}{s^2 + 2\xi_c \omega_n s + \omega_n^2}$$  \hspace{1cm} (4·8)

The natural frequency of the complex conjugate poles is $\omega_n$, and it is set by the design of the stator current control loop. The corner frequency of the zero, $\omega_{cz}$, is from (4·7):

$$\omega_{cz} = \frac{K_3 \lambda_2 + K_2}{K_3 \lambda_1}$$  \hspace{1cm} (4·9)

$\omega_{cz}$, is a function of the linearised DC load current coefficients, and so changes with the operating point, which will modify the frequency response of $Y_c(s)$. The frequency $\omega_{cz}$ is low at high-torque, whilst it significantly increases for low-torque values.

For $\omega_n \gg \omega_{cz}$, the compensating admittance $Y_c(s)$ exhibits a high-pass frequency characteristic. An asymptotic approximation of gain and phase frequency responses for $Y_c(s)$ is presented in Figure 4.3 a). The gain increases with a slope of +20 dB in the low-frequency region due to the zero at the origin, and has a gain slope of +40 dB between the corner frequency of the second zero and the natural frequency of the complex conjugate poles, the gain becomes constant, $|Y_c(s)| = K_3$, beyond the natural frequency of the complex conjugate poles. The zero at the origin makes the asymptotic phase approximation start at +90°, and then jump to +180° due to the second zero. The phase asymptotically approaches zero in the high-frequency region as the second order low-pass filter, characterised by the complex conjugate poles, compensate the positive phase introduced by the zeros. The blue-dashed line represents an approximation of the actual phase shift introduced by the compensating admittance.
For $\omega_{nc} \ll \omega_{cz}$, Figure 4.3 b) shows that the zero at the origin introduces a gain slope of +20 dB in the low-frequency region. The gain decreases at a rate of ~20 dB beyond the natural frequency of the complex conjugate poles, and then becomes constant ($|Y_c(s)| = K_3$) when the frequency reaches $\omega_{cz}$. The phase shift profile is also modified in this configuration, since it is now asymptotically approximated to ~90° between $\omega_{nc}$ and $\omega_{cz}$, and is 0° in the high-frequency region. This, significantly changes the approximation of the actual phase angle, as highlighted by the blue-dashed line in Figure 4.3 b).

As mentioned in Section 2.4.2, the bandwidth of the IM current control is often reduced to limit the control sensitivity to noise and so the natural frequency of the RLC input filter $\omega_{nf}$ is larger than $\omega_{nc}$. For the low torque case in Figure 4.3 a), $\omega_{nf}$ is then higher than $\omega_{cz}$ and $\omega_{nc}$, whereas in the low torque case in Figure 4.3 b), $\omega_{nf}$ is higher than $\omega_{nc}$ but lower than $\omega_{cz}$. Considering Figure 4.3, around the resonant frequency of the input filter, the gain and phase provided by the compensating admittance will vary depending on $\omega_{cz}$. In addition, the gains of the inherent and compensating admittance are also changing with the operating point. The changes in $Y_{IM}(s)$ and $Y_c(s)$ mean the stabilisation control will be sensitive to changes in operating point, in the frequency region where instabilities are likely to occur.
To highlight the features shown in Figure 4.3, some simulation results are presented in the following section.

### 4.1.2.1 Simulation Results

Using the experimentally determined parameters of a 350 V 4.3 kW induction motor listed in Section 3.3, Table 3-2, the gain and phase of the compensating admittance, at the natural frequency (4.23 krad/s) of the input filter ($\omega_{nf}$), across the range of operating points are shown in Figure 4.4. The induction machine rotor flux ($\phi_r$) and the gains of the $q$ current PI controller calculated to achieve a damping ratio of 0.707 for the closed-loop control with a bandwidth of 500 Hz, remain as listed in Section 3.3.1, Table 3-3. The input filter parameters used to calculate $\omega_{nf}$ are given in Section 3.2.2, Table 3-1. For both gain and phase plots the speed varies from 0 to 180 rad/s and the torque range from 0 to 70 Nm. Both maps remain within the maximum IM torque-speed characteristic shown in Section 3.3.1, Figure 3.12. Figure 4.4 is obtained using the gain and phase expressions of $Y_c(s)$ at $\omega_{nf}$ derived from (4-8) as:

\[
|Y_c(j\omega_{nf})| = K_3 \sqrt{\frac{\omega_{nf}^4 + \omega_{cz}^2\omega_{nf}^2}{\omega_{nc}^2 - \omega_{nf}^2 + 4\xi_c^2\omega_{nc}^2\omega_{nf}^2}}
\]

\[
\angle Y_c(j\omega_{nf}) = \tan^{-1}\left(-\frac{2\xi_c\omega_{nc}\omega_{nf}^2 + \omega_{cz}(\omega_{nc}^2 - \omega_{nf}^2)}{\omega_{nf}(\omega_{nc}^2 - \omega_{nf}^2 - 2\xi_c\omega_{nc}\omega_{cz})}\right)
\]

(4-10)

As suggested by the asymptotically approximated frequency response given in Figure 4.3, the gain and phase provided by the compensating admittance significantly vary with the operating point. Figure 4.4 a) shows that the gain decreases from -23.52 dB, at a speed of 10 rad/s and a 70 Nm torque, to -33.7 dB at a speed of 180 rad/s and 0 Nm. Similarly, the phase provided by the compensating admittance goes from 61.09°, at a speed of 10 rad/s and a torque of 70 Nm, to -22.53°, at 180 rad/s and 0 Nm.
The most critical operating points in terms of stability arise at high-power as shown in Section 3.2.3, Figure 3.8, and hence lie along the 4.3 kW speed-torque characteristic defined in Section 3.3.1, Figure 3.12. Although the induction machine is operating at approximately 4.3 kW, the power seen from the DC side will be larger for higher torque values, since the induction machine rotor and stator losses increase with the torque producing current component. Figure 4.5 shows the Bode plot of the compensating admittance for three operating points, located close to the 4.3 kW limit. The frequency responses are plotted for a speed of 60 rad/s, 120 rad/s and 180 rad/s with a load torque of 70.4 Nm, 35.8 Nm and 23.9 Nm, respectively. The current control bandwidth ($\omega_{nc}$) is equal to 3141.6 rad/s (500 Hz) and the corner frequency of the zero ($\omega_{cz}$) in the numerator of $Y_c(s)$, is equal to 796.6 rad/s, 2069.7 rad/s and 4182.9 rad/s at a speed of 60 rad/s, 120 rad/s and 180 rad/s, respectively. Therefore, at a speed of 60 rad/s and a torque equal to 70.4 Nm, the frequency response of the compensating admittance is similar to the asymptotic approximation depicted in Figure 4.3 a). For a speed-torque operating point of 120 rad/s and 35.8 Nm, and 180 rad/s and 23.9 Nm, $\omega_{cz}$ and $\omega_{nc}$ are located in the same frequency region. Therefore the phase frequency response exhibited by $Y_c(s)$ is similar to a first order high-pass transfer function, since the zero and one of the complex conjugate poles cancel each other in the mid-frequency region.
4.1 IM Frequency-Domain Model

Figure 4.5: Bode plot of the compensating admittance at three high power operating conditions

At the resonant frequency of the input filter, $\omega_{nf}$ in Figure 4.5, the gain of the compensating admittance decreases from -23.36 dB at 60 rad/s, to -28.43 dB at 120 rad/s to finally reach -29.86 dB at 180 rad/s. Likewise, the phase shift introduced by $Y_c(s)$ reduces as speed increases, going from 55.97° to 40.89° and 22.08° at 60 rad/s, 120 rad/s and 180 rad/s, respectively.

4.1.3 Sensitivity Function

The introduction of the active stabilisation block in Figure 4.1, increases the sensitivity of the IM speed to DC-link voltage changes, since the speed or torque producing voltage $q$ axis reference is generated by the sum of the $q$ axis current controller output and a portion of the DC-link voltage change. To investigate this feature, the small-signal sensitivity transfer function $S_{cpl}(s)$ (4-11) can be derived from Figure 4.2, with the small-signal DC-link voltage and speed as input and output, respectively.

$$S_{cpl}(s) = \frac{\delta \omega_m}{\delta V_l} = S_c(s)C_s(s) \quad (4-11)$$

The transfer function results in the product of the compensation block $C_s(s)$, and the sensitivity function $S_c(s)$ introduced by the IBSC given in (4-12).

$$S_c(s) = \frac{M_e(s)}{1 + PL_c(s)M_e(s)\varphi r0GM_m(s)} \quad (4-12)$$
By substituting for the transfer functions in (4.12), the sensitivity function introduced by the compensating scheme is:

\[ S_c(s) = \frac{s}{s(\lambda_1 s + \lambda_2) + k_{pc}s + k_{ic}f_ms + B_m} \frac{\varphi_{r0}G}{1} \]  

(4.13)

Due to the low cut-off frequency of the pole introduced by the IM mechanical transfer function its effect in the mid-frequency and high-frequency regions can be neglected since it is cancelled by the zero at the origin. As for the input admittance, the numerator of the first term in (4.13) can be replaced by the characteristic equation of the closed-loop current control. Thus, the sensitivity transfer function introduced by the IBSC can be simplified as:

\[ S_c(s) = \frac{\varphi_{r0}G}{J_m\lambda_1} \frac{1}{s^2 + 2\xi_c\omega_{nc}s + \omega_{nc}^2} \]  

(4.14)

The sensitivity function \( S_c(s) \), displays a characteristic low-pass transfer function. Figure 4.6, presents the asymptotic approximation of the \( S_c(s) \) gain frequency response. The compensating sensitivity gain is given by \( |S_c(s)| = \varphi_{r0}G/(J_m\lambda_1) \) in the low frequency region and decays at a -40 dB rate above \( \omega_{nc} \). Both the gain and the cut-off frequency of the compensating sensitivity function \( S_c(s) \) in (4.14) are independent of the operating point. Hence, the speed control degradation (total sensitivity function, \( S_{cp}(s) \)) will only be sensitive to changes in stabilisation block parameters.

![Figure 4.6: Asymptotic approximation of the compensating sensitivity function frequency response](image)

Using the same induction machine parameters, rotor flux, and current controller gains as in Section 4.1.2.1 the gain, in the low-frequency region, of the compensating admittance is equal to -51.2 dB.
4.2 **IBSC Tuning**

The IBSC concept provides stabilisation by shaping the load input admittance via a compensating term, which consists of the product of the stabilisation block $C_s(s)$ and the compensating admittance $Y_c(s)$, to satisfy one of the criteria presented in Section 2.2.2.2. The stabilisation scheme focuses on the mid-frequency region, around the resonant frequency of the input filter, where instabilities are likely to arise. As the gain and phase of the compensating admittance were shown in Section 4.1.2 to vary with the operating point, at $\omega_{nf}$, the stabilisation action provided by the IBSC will be sensitive to changes in operating point, and this must be taken into consideration in the design of IBSCs to guarantee the effectiveness of the stabilisation control. The detailed design of the IBSC and its sensitivity to changes in operating point have not been addressed in the literature. Furthermore, the designs reviewed in Section 2.4 do not account for stability margins, and are only derived in terms of DC-link voltage oscillation damping and motor speed sensitivity to changes in DC-link voltage.

Here, a stabilisation design based on the ESAC criterion described in Section 2.2.2.2 is proposed. This design method accounts for both the desired phase and gain margin, and enables a fine tuning of the stabilisation block depending on the operating point.

4.2.1 **Principle**

The ESAC criterion enables a forbidden region to be defined, in terms of the desired phase and gain margins, for the Nyquist contour of the minor-loop gain $L_{cpl}(s) = Y_{cpl}(s)Z_2(s)$ of the feedback system was shown in Section 3.1.2.3, Figure 3.7, in the case of an ideal CPL. Here, the minor-loop gain $L_{cpl}(s)$ consists of the product of the load input admittance $Y_{cpl}(s)$ (IM input admittance derived in Section 4.1.2) and the source output transfer function $Z_2(s)$ (output transfer function of the input filter given in Section 3.2.3, (3-16)). To satisfy the ESAC criterion, the Nyquist plot of the minor-loop gain $L_{cpl}(s)$ must remain outside the boundary defined in terms of gain and phase margin, as shown in Figure 4.7.
From Section 3.2.3, the source output transfer function (or RLC filter output transfer function), $Z_2(s)$, exhibits a large gain and a phase contained between $\pm 90^\circ$ around the resonant frequency of the input filter. Figure 4.7 a), shows an approximated Nyquist plot of the minor-loop gain $L_{cpl}(s)$ for a non-stabilised CPL. In the mid-frequency region (around the filter resonant frequency) the Nyquist contour enters the forbidden area due to the large gain and excessive phase shift of $L_{cpl}(s)$. Therefore, the desired relative stability of the system cannot be achieved.

A sufficient condition to satisfy the ESAC criterion would be to reduce the phase shift introduced by the minor-loop gain around the filter resonant frequency. In doing so, the Nyquist plot of $L_{cpl}(s)$ would be kept outside the forbidden area as shown in Figure 4.7 b). Hence, despite the $L_{cpl}(s)$ gain being larger than unity at some frequencies, the desired relative stability is preserved by minimising the IM input admittance ($Y_{cpl}(s)$) phase in the mid-frequency region.

In order to minimise the IM input admittance $Y_{cpl}(s)$ phase at $\omega_{nf}$, the phase introduced by the compensating term $Y_c(s)C(s)$ must be kept to a minimum or set to a desired value, by the IBSC, at this frequency. Then by increasing the gain of the stabilisation block ($|C_s(s)|$), the compensating term in (4-5) becomes dominant over the intrinsic admittance, $Y_{IM}(s)$, leading the IM input admittance ($Y_{cpl}(s)$) phase to approach the phase shift provided by the compensating term. Additionally, the stabilisation contribution of the IBSC rises with the stabilisation gain, which improves the damping of the DC-link voltage. However, although increasing the stabilisation block gain enhances the damping and improves the phase margin, an excessive gain could lead to a
4.2 IBSC Tuning

violation of the ESAC criterion, since the Nyquist contour will be increased. This shows that greater damping of the DC-link voltage does not necessarily translate to improved stability margins. Furthermore, a large stabilisation gain will affect the sensitivity of speed to changes in DC-link voltage, resulting in performance degradation. Therefore, the IBSC design is a trade-off between stability margins, damping effectiveness, and speed sensitivity.

In [135, 139, 140], the stabilisation gain selection is achieved using a root loci of the system. This method enables an optimal stabilisation gain to be identified; however, it can be unintuitive to tune the IBSC gain using root loci, and very detailed mathematical modelling of the system, including the derivation of a high-order linearised system is required.

Both high-pass and band-pass filter implementations for IBSC have been investigated in the literature, see Section 2.4.2. The high-pass filter method is undesirable due to an increased sensitivity to measurement noise. The proposed design method is derived using a band-pass filter, owing to its reduced sensitivity to measurement noise when compared to high-pass filter, in the following section. The proposed IBSC tuning is realised in two phases. First, the phase of the compensating term $Y_c(s)C(s)$ must be minimised or set to a desired value by appropriately adjusting the filter corner frequencies, around the selected operating point to satisfy the ESAC criterion. Then, the stabilisation gain achieving the desired compromise must be calculated.

4.2.2 Band-pass Filter Based IBSC

Band-pass filters for impedance-based IBSC schemes have been detailed in the literature as reviewed in Section 2.4.2. The band-pass compensation block allows stabilisation by modifying the voltage control references using the mid-frequency component of the DC-link voltage.

The band-pass filter used in this section is defined by (4.15).

$$C_s(s) = \frac{K_c s}{s^2 + (\omega_{cl} + \omega_{ch})s + \omega_{cl}\omega_{ch}}$$  (4.15)

where $\omega_{cl}$ is the low corner frequency, $\omega_{ch}$ is the high corner frequency and $K_c$ is the stabilisation gain.
Figure 4.8 shows the asymptotic approximation of the gain and phase frequency response of the band-pass filter. In the low frequency region, the gain of $C_s(s)$ increases at a rate of +20 dB and the phase shift is close to +90°, due to the zero at the origin. Above the low corner frequency, $\omega_{cl}$, the low-frequency pole and the zero at the origin compensate each other and so the gain is fixed at $K_{BP}$, and the asymptotic approximation of the phase is set 0°. Above $\omega_{cH}$ the influence of the high-frequency pole, causes the gain of $C_s(s)$ decreases at a rate of -20 dB and the phase approaches -90° in the high-frequency region. The red dashed-line represents an approximation of the actual phase shift introduced by the filter. The actual gain $K_{BP}$ in the mid-frequency region is given by (4·16).

$$K_{BP}(\omega) = \frac{K_c \omega}{\sqrt{\left(\omega_{cl} \omega_{cH} - \omega^2\right)^2 + \left(\omega_{cl} + \omega_{cH}\right)^2 \omega^2}}$$  \hspace{1cm} (4·16)

The negative phase shift exhibited by the band-pass filter, from the mid-frequency to high-frequency region, means the phase of the compensating admittance, $Y_c(s)$, can be further minimised, by selecting appropriate values for the corner frequencies $\omega_{cl}$ and $\omega_{cH}$.

Figure 4.9 shows the asymptotic gain approximation and actual phase approximation frequency responses for the compensating admittance, $Y_c(s)$, stabilisation block $C_s(s)$ and product $Y_c(s)C_s(s)$. Both high-torque ($\omega_{cz} \ll \omega_{nc}$) and low-torque ($\omega_{cz} \gg \omega_{nc}$) configurations are presented in Figure 4.9 a) and Figure 4.9 b), respectively. Figure 4.9 a) indicates that, at high torque, the phase of the compensating term $Y_c(s)C_s(s)$ can be decreased by the band-pass
filter, around the resonant frequency of the input filter, \( \omega_{nf} \), to improve the phase margin of the system in order to satisfy the ESAC criterion. However, the low-torque configuration, Figure 4.9 b) shows that, for identical band-pass filter low and high corner frequencies, the phase of the compensating term is significantly degraded in the mid-frequency region and this high phase may affect the system phase margin leading to a violation of the ESAC criterion. Therefore the tuning of the band pass corner frequencies for the IBSC are dependent on the IM operating point.

\[ \omega_{n_c} \gg \omega_{cz}; \text{ high-torque} \]
\[ \omega_{n_c} \ll \omega_{cz}; \text{ low-torque} \]

Figure 4.9: Asymptotic approximation of the frequency response of \( Y_c(s)C_s(s) \) for band-pass filter based IBSC

Considering the discussion in Section 4.2.1, the phase introduced by the term \( Y_c(s)C_s(s) \) in (4.6) must be kept to a minimum at the resonant frequency of the input filter. To achieve this the value of the high corner frequency, \( \omega_{ch} \), can be calculated to achieve a desired compensating term phase shift at \( \omega_{nf} \), for a given low corner frequency, \( \omega_{cl} \), using (4.17), for high torque operations.

\[ \omega_{ch} = \omega_{nf} \frac{\omega_{cl} \tan(\theta_{IB}) + \omega_{nf}}{\omega_{cl} - \omega_{nf} \tan(\theta_{IB})} \]  \hspace{1cm} (4.17)

where,

\[ \theta_{IB} = -\angle Y_c(j\omega_{nf}) + \angle Y_c C_s(j\omega_{nf}) \]  \hspace{1cm} (4.18)
\( \angle Y_c(j\omega_{nf}) \) is the phase of the compensating admittance given by (4-10), and \( \angle Y_c C_s(j\omega_{nf}) \) is the desired compensating term phase shift at the input filter resonant frequency.

At low torque the phase of the compensating admittance at \( \omega_{nf} \) is negative as shown in Figure 4.9 b), and so to achieve the desired phase shift for the compensating term the low corner frequency must be modified. By setting \( \omega_{ch} \) relatively high, \( \omega_{cl} \) can be obtained using (4-19).

\[
\omega_{cl} = \omega_{nf} \frac{\omega_{ch} \tan(\theta_{IB}) + \omega_{nf}}{\omega_{ch} - \omega_{nf} \tan(\theta_{IB})}
\]  

(4-19)

The derivations of (4-17), (4-18) and (4-19) are detailed in Appendix B.

The IBSC design is a trade-off between stability margins, damping effectiveness and speed sensitivity. By setting the desired compensating term phase shift at the resonant frequency of the input filter \( \angle Y_c C_s(j\omega_{nf}) \), to 0°, will provide a significant phase margin, allowing a considerable increase in the stabilisation gain whilst maintaining the Nyquist contour outside the forbidden region defined by the ESAC criterion. This will result in a substantial improvement in the damping of the DC-link voltage oscillations; however, will cause the IM speed to be significantly more sensitive to changes in DC-link voltage.

To manage this sensitivity, (4-20) can be used to tune the stabilisation gain to achieve a desired induction machine speed sensitivity at \( \omega_{nf} \) to changes in DC-link voltage. Equation (4-20) is derived from (4-11).

\[
K_c = \left| \frac{S_{cpl}(j\omega_{nf})}{S_c(j\omega_{nf})} \right|^* \sqrt{(\omega_{cl} \omega_{ch} - \omega_{nf}^2)^2 + (\omega_{cl} + \omega_{ch})^2 \omega_{nf}^2} \omega_{nf}
\]  

(4-20)

where \( |S_{cpl}(j\omega_{nf})|^* \) characterises the desired gain of the IM speed sensitivity function, and \( |S_c(j\omega_{nf})| \) represents the compensating sensitivity function given as:

\[
|S_c(j\omega_{nf})| = \frac{\varphi_{ra} G}{f_m \lambda} \frac{1}{\sqrt{(\omega_{n_c}^2 - \omega_{nf}^2)^2 + 4\xi_c^2 \omega_{n_c}^2 \omega_{nf}^2}}
\]  

(4-21)
4.2 IBSC TUNING

By decreasing $|S_{cpl}(j\omega_{nf})|^*$, the stabilisation gain is reduced, which reduces the speed sensitivity to changes in DC-link voltage, degrades the DC-link voltage damping, and increases the gain margin. Conversely, an increased $|S_{cpl}(j\omega_{nf})|^*$ will result in higher speed sensitivity and improved damping with a reduced gain margin.

The resulting gain approximation frequency response of the sensitivity function, $S_c(s)$, is given in Figure 4.10. The sensitivity function exhibits a band-pass transfer function and so the IM speed sensitivity to changes in DC-link voltage increases with the stabilisation gain $K_c$ in the mid-frequency region. The frequency range across which the sensitivity is maximal, increases or decreases with the corner frequencies. The band-pass characteristic of the sensitivity function shows that the IBSC scheme cannot excite the mechanical resonant mode of the induction machine, providing the low corner frequency of the band-pass filter is sufficiently higher than the natural frequency of the IM mechanical system.

![Figure 4.10: Frequency response approximation of the sensitivity function gain](image)

Frequency-domain analysis for the IP-control FOC IM drive with input RLC filter is shown in the next section, with and without the IBSC.

### 4.2.3 FREQUENCY-DOMAIN ANALYSIS

The experimentally determined parameters of a 350 V 4.3 kW induction motor, listed in Section 3.3, Table 3-2, are used in this section. The induction machine rotor flux ($\varphi_r$) and the gains of the $q$ current PI controller calculated to achieve a damping ratio of 0.707 for the closed-loop control with a bandwidth of 500 Hz, remain as listed in Section 3.3.1, Table 3-3. The input filter parameters used are given in Section 3.2.2, Table 3-1.
The gain and phase margin for practical systems are often set to 6 dB and 60° [91], respectively, as these margins must account for model approximations and uncertainties, such as linearisations and neglected dynamic behaviours, nonlinearities or delays, while ensuring stable operation. These margin values are used in this section to define the ESAC criterion forbidden region.

**4.2.3.1 Non-Stabilised System**

Figure 4.11 shows the Nyquist plot of the minor-loop gain $L_{cpl}(s)$, for a change in FOC induction machine operating point, with the IBSC inactive. Three operating points are selected along the 4.3 kW speed-torque characteristic of the induction machine: 60 rad/s and 70.4 Nm, 120 rad/s and 35.8 Nm and 180 rad/s and 23.9 Nm. The IM input power is approximately 4.3 kW and in all cases the Nyquist contour enters the ESAC forbidden region and so the desired relative stability cannot be achieved. At reduced IM power, the system inherently exhibits acceptable stability margins.

![Nyquist plot](image)

Figure 4.11: Nyquist plot of the minor-loop gain $L_{cpl}(s)$ for non-stabilised 4.3 kW operating points

For identical IM mechanical powers, the DC input power is larger for the high-torque condition due to the increased IM rotor and stator losses. This explains the larger contour for the low-speed condition, which results in a worse relative stability. The gain margins exhibited are 2.9 dB, 4.6 dB and 5.1 dB at an IM speed of 60 rad/s to 120 rad/s and 180 rad/s, respectively. The phase margin is infinite, since the gain of the loop $L_{cpl}(s)$ never reaches unity.
4.2.3.2 STABILISED SYSTEM

The first step in the design of the IBSC is to reduce the phase shift introduced by the compensating term $Y_c(s)C_s(s)$, around the resonant frequency of the input filter, $\omega_{nf}$. To do this, (4.17) and (4.18) can be used to calculate the appropriate IBSC band-pass filter corner frequencies. The low corner frequency $\omega_{cl}$ is set to 300 rad/s, whilst the high corner frequencies $\omega_{ch}$ calculated to achieve a desired phase shift of $Y_c(s)C_s(s)$ at $\omega_{nf}$ of $0^\circ$ are listed in Table 4·1 for the same three operating points along the 4.3 kW IM torque-speed characteristic as in Section 4.2.3.1. The phase Bode plot for the three operating points listed in Table 4·1 are plotted in Figure 4.12. Since $\theta_{IB}$ decreases with torque, the phase compensation required to achieve $0^\circ$ reduces as the speed increases along the 4.3 kW IM torque-speed characteristic. This causes the high corner frequency $\omega_{ch}$ to increase at high speed as shown in Table 4·1, which therefore increases the frequency range across which the speed is sensitivity to changes in DC-link voltage.

![Phase Bode Plot](image)

**Figure 4.12**: Phase bode plot of the compensating term for $\angle Y_cC_s(\omega_{nf})=0^\circ$

At 60 rad/s IM speed the high corner frequency $\omega_{ch}$ of the band pass filter is below the resonant frequency $\omega_{nf}$ (4.23 krad/s) of the RLC input filter. Nevertheless, it does not significantly affect the IBSC since both the input filter resonant and band-pass high-corner frequencies are still within the same region.

---

8 According to Figure 4.9 a) the band-pass low corner frequency $\omega_{cl}$ must satisfy $\omega_{cl} \ll \omega_{nf}$ to enable an appropriate compensation of the phase of $Y_c(s)C_s(s)$ at $\omega_{nf}$. 

Table 4·1: Band-pass filter high corner frequency for 4.3 kW operating points

<table>
<thead>
<tr>
<th>( \omega_m ) (rad/s)</th>
<th>( T_l ) (Nm)</th>
<th>( \omega_{ch} ) (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 ( \cdot ) 70.4</td>
<td>2407</td>
<td></td>
</tr>
<tr>
<td>120 ( \cdot ) 35.8</td>
<td>4236</td>
<td></td>
</tr>
<tr>
<td>180 ( \cdot ) 23.9</td>
<td>8553</td>
<td></td>
</tr>
</tbody>
</table>

The second step to design the IBSC is to define a stabilisation gain to achieve the desired trade-off of stability margins against IM speed control degradation. The stabilisation gains are calculated using (4·20) and (4·21), and the \( \omega_{ch} \) values in Table 4·1 (\( \omega_{cl}=300 \) rad/s) and are listed in Table 4·2 for the same operating conditions as in Table 4·1. The stabilisation gains are calculated for three different values of the desired gain of the IM speed sensitivity function, \( |S_{cpl}(j\omega_{nf})|^\ast \), at the resonant frequency of the input filter \( \omega_{nf} \). The gains, \( K_{s1} \), \( K_{s2} \) and \( K_{s3} \) correspond to sensitivity function gains of \( \cdot 45 \) dB, \( \cdot 40 \) dB and \( \cdot 35 \) dB, respectively.

Table 4·2: Stability gains for 4.3 kW operating points

<table>
<thead>
<tr>
<th>( \omega_m ) (rad/s)</th>
<th>( K_{s1} )</th>
<th>( K_{s2} )</th>
<th>( K_{s3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 ( \cdot ) 70.4</td>
<td>2.06e4</td>
<td>3.66e4</td>
<td>6.51e4</td>
</tr>
<tr>
<td>120 ( \cdot ) 35.8</td>
<td>2.53e4</td>
<td>4.50e4</td>
<td>8.01e4</td>
</tr>
<tr>
<td>180 ( \cdot ) 23.9</td>
<td>4.03e4</td>
<td>7.17e4</td>
<td>1.28e5</td>
</tr>
</tbody>
</table>

Figure 4.13 shows the Nyquist plot of the minor-loop gain \( (L_{cpl}(s) = Y_{cpl}(s)Z_2(s)) \), for the same three 4.3 kW IM operating points listed in Table 4·1. For each operating point the IBSC is designed using the corner frequencies given in Table 4·1 and the three gains derived in Table 4·2. Significant improved stability margins are provided by the IBSC across the range of proposed operating point. As previously stated the operating points along the 100% nominal power characteristic of the induction machine represent the most critical operation in term of stability.
As the IM speed increases in Figure 4.13 the Nyquist contours recede further from the forbidden region as the reduced gain of the compensating admittance $Y_c(s)$ at lower torque (shown in Figure 4.4 a)) enables a higher system gain margin. This suggests the stabilisation gain may be further increased to increase the damping effectiveness, however this could destabilise the IM speed control due to an increased sensitivity to changes in DC-link voltage. This highlights the benefit of designing the stabilisation gain using (4.20), as it constrains the IM speed performance degradation to the specified level.

**4.2.3.3 IBSC Sensitivity to Change in Operating Point**

In Section 4.2.2 the design of the IBSC scheme was shown to be dependent on the specific IM operating point. Figure 4.14 shows the Nyquist plot of the
minor-loop gain $L_{cp}(s)$ for the same three IM speeds. The design of the IBSC is performed as listed in Section 4.2.3.2 for the IM operating at 100% of its nominal power and a sensitivity gain of -35 dB. However, in this case the load torque is actually set to 0 Nm to demonstrate the impact of a poorly tuned IBSC. The three Nyquist contours enter the forbidden region defined by the ESAC criterion, for all speeds tested, resulting in reduced stability margins.

Figure 4.14: Nyquist plot of the minor-loop gain $L_{cp}(s)$ for stabilised 0 kW operating points with a poorly tuned IBSC

Figure 4.15 shows the Nyquist plot of the minor-loop gain for the same operating points as in Figure 4.14 but with the IBSC designed for 0 kW operation and the desired gain of the IM speed sensitivity function is set to -35 dB. The gain and phase margin are acceptable for all speeds with the appropriate IBSC tuning. Since the phase of the compensating admittance is the same across the speed range at 0 Nm (as shown in Figure 4.4 b) and the compensating sensitivity function $S_c(s)$ does not depend on the operating point, the corner frequencies of the band-pass filter calculated using (4-18) and (4-19) and the stabilisation gain calculated using (4-20) are identical at, $\omega_{cl} = 2.9$ krad/s, $\omega_{ch} = 20$ krad/s$^9$ and $K_c = 3.3e5$ for all IM speeds at 0 Nm. The wide frequency range of the band pass filter means the range of operating conditions when the IM speed is at maximum sensitivity to DC-link voltage changes, is significantly increased at zero torque.

$^9$ According to Figure 4.9 b) the band-pass high corner frequency $\omega_{ch}$ must satisfy $\omega_{ch} \gg \omega_{nf}$ to enable an appropriate compensation of the phase of $Y_c(s)C_s(s)$ at $\omega_{nf}$.
The final section of this chapter discusses Matlab/Simulink time domain simulation results to evaluate the performance of the IBSC using the more detailed IM drive model from Appendix A.

4.3 **TIME-DOMAIN SIMULATION**

In this section time-domain simulations of the band-pass filter based IBSC are presented, using the averaged non-linear model of the IP-based FOC induction motor with RLC input filter described in Appendix A. The experimentally determined parameters of a 350 V 4.3 kW induction motor remain unchanged, as given in Section 3.3, Table 3-2. The parameters of the input filter are those listed in Section 3.2.2, Table 3-1, and the DC grid voltage $V_g$ is set to 540 V. The induction machine rotor flux ($\varphi_r$), the speed and $d$-$q$ current IP filter time constants and control output saturations, and the gains of the speed and $d$-$q$ current PI controllers calculated to achieve a damping ratio of 0.707 for the closed-loop control with a bandwidth of 10 Hz and 500 Hz, respectively, are listed in Section 3.3.1, Table 3-3.

4.3.1 **EVALUATION OF THE IBSC AT RATED INDUCTION MACHINE POWER**

Figure 4.16 shows the DC-link voltage $V_l$ and IM speed $\omega_m$ response for a 70.4 Nm torque step at 1 s with the IBSC active and disabled. The IM speed reference is set to 60 rad/s, and the torque is initially zero. The IM speed of
60 rad/s is chosen as Figure 4.13 indicated this speed has the smallest gain margin of the three speed investigated. The IBSC low and high corner frequencies of the band-pass filter are set to 300 rad/s and 2407 rad/s respectively according to Table 4·1. To disable the IBSC the stabilisation gain is set to 0, whilst for the IBSC active case, \( K_s \) is set to the value \( K_{s3} \) given in Table 4·2, corresponding to a desired sensitivity gain at the resonant frequency of the input filter of \(-35 \) dB. Figure 4.16 a) shows that the DC-link voltage oscillations are significantly reduced when the IBSC is active. The frequency range of the injected stabilisation signal is significantly higher than the speed control bandwidth, and so the speed response is not degraded by the IBSC as shown in Figure 4.16 b).

![Figure 4.16: Responses to a 0 to 70.4 Nm torque step at 1 s: 60 rad/s IM speed](image)

Figure 4.17 shows a magnified view of the IBSC DC-link voltage response displayed in Figure 4.16 a) together with the responses for the stabilisation gains \( K_{s1} \) and \( K_{s2} \), from Table 4·2: \( K_{s1} \) and \( K_{s2} \) correspond to a desired sensitivity gain at the resonant frequency of the input filter equal to \(-45 \) dB and \(-40 \) dB, respectively. The stabilisation gain increases with the sensitivity gain, resulting
in an improved DC-link voltage response for greater values of $K_s$, since the stabilisation control contribution is increased. This improvement in terms of damping effectiveness causes a reduced gain margin as shown in Figure 4.13 a), due to the increased stabilisation gain. However as the Nyquist contours remain outside the limit defined by the ESAC criterion, the system has acceptable relative stability.

Figure 4.17: DC-link voltage responses to a 0 to 70.4 Nm torque step at 1 s: 60 rad/s IM speed

Figure 4.18 shows DC-link voltage and IM speed responses to step in speed reference from 60 rad/s to 50 rad/s at 1.5 s with the IBSC active and disabled. The induction machine torque is fixed at 70.4 Nm. Again the low and high corner frequencies of the IBSC band-pass filter remain as given in Table 4.1 for this operating point. The stabilisation gain is set to 0 and $K_{s3}$ for the IBSC disabled and active IM drive, respectively. The damping of the DC-link voltage is significantly improved with the IBSC active as shown in Figure 4.18 a). The speed tracking response is again similar for both IBSC active and disabled as shown in Figure 4.18 b), as the bandwidth of the speed control is lower than the frequency of the injected component.
A magnified view of the IBSC active DC-link voltage response shown in Figure 4.18 a) is shown in Figure 4.19 together with the responses for the stabilisation gains of $K_1$ and $K_2$ from Table 4·2. As for Figure 4.17, the oscillation damping is improved as the stabilisation gain increases.

Figure 4.19: DC-link voltage response to a 60 rad/s to 50 rad/s speed step at 1.5 s; load torque is 70.4 Nm

Figure 4.20 shows DC-link voltage and IM speed responses to change in grid voltage from 540 V to 530 V at 1.5 s for the IBSC active and disabled. The speed
4.3 Time-Domain Simulation

reference is set to 60 rad/s and the load is 70.4 Nm. The low and high corner frequencies of the IBSC band-pass filter are set to 300 rad/s and 2407 rad/s respectively as given in Table 4-1. The stabilisation gain is set to 0 and $K_{s3}$ for the IBSC disabled and active IM drive, respectively. The DC-link voltage with the IBSC active is considerably enhanced compared to the IBSC disabled as shown in Figure 4.20 a). The voltage peak deviation is reduced from -9.98V with the IBSC disabled IM drive to -6.38V for the IBSC active IM drive. The settling time at ±5% of the steady state value of the DC-link voltage is drastically improved with the IBSC active, reducing from 206ms (IBSC disabled) to 6ms (IBSC active). The penalty however of the IBSC is a degradation of the IM speed response as shown in Figure 4.20 b).

A magnified view of the DC-link voltage and IM speed response is provided in Figure 4.21 a) and b), respectively, for the stabilisation gains $K_{s1}$, $K_{s2}$ and $K_{s3}$, from Table 4-2. Although the settling time is similar for each gain, at around 6ms, the voltage peak deviation from the steady state value of the DC-link voltage is reduced from -8.48V to -7.58V and -6.38V as the stabilisation gain
value increases from $K_{s1}$ to $K_{s3}$. Conversely the speed response is degraded for greater values of $K_s$ with the deviation going from 0.26 rad/s to 0.44 rad/s and 0.78 rad/s for stabilisation gain equal to $K_{s1}$, $K_{s2}$ and $K_{s3}$, respectively. This highlights the design trade-off between DC-link voltage oscillation damping and IM performance.

Figure 4.21: Magnified responses to a 540 V to 530 V grid voltage step at 1.5 s: IM speed is 60 rad/s and load torque is 70.4 Nm

The most critical operating points in terms of IM drive stability arise at high-power, and especially at high-torque, since the phase and the gain of the CPL input admittance, are a maximum in these cases, as shown in Figure 4.4, which results in reduced stability margins. Therefore it is demonstrated with the operating point previously selected (IM speed of 60 rad/s and load torque equal to 70.4 Nm) that the band-pass filter based IBSC provides a significantly improved system overall relative stability. However, the IBSC design is highly dependent on the operating point as shown in Section 4.1.2.1.
4.3.2 IBSC Sensitivity to Change in Operating Point

The DC-link voltage response to a change in grid voltage from 540 V to 530 V at 1.5 s for the IBSC active and IBSC disabled IM drive, is shown in Figure 4.22. The speed reference is set to 60 rad/s, and the load torque is zero. The IBSC is designed around the operating point \( \omega_m = 60 \text{ rad/s} \) and \( T_l = 70.4 \text{ Nm} \). The low and high corner frequencies of the IBSC band-pass filter \( \omega_{cl} \) and \( \omega_{ch} \) are set as given in Table 4-1 and the stabilisation gain is set to the value \( K_{s3} \), given in Table 4-2. This IBSC design causes more DC-link voltage oscillations as shown in Figure 4.22. Furthermore the settling time at ±5% of the steady state value of the DC-link voltage is increased from 87ms (IBSC disabled) to 149ms with the IBSC active. The settling time for the IBSC inactive IM drive is much smaller at zero torque, since the intrinsic relative stability of the system is naturally superior at reduced IM power.

![Figure 4.22: DC-link voltage response to grid voltage step for a poorly tuned IBSC at 0 kW](image)

The same operating condition in Figure 4.22 is shown in Figure 4.23 for the IBSC properly designed around the operation point \( \omega_m = 60 \text{ rad/s} \) zero load torque. Using (4·18) and (4·19) the low and high corner frequencies of the band pass filter are set to 2.9 kHz and 20 kHz, respectively, and the stabilisation gain is fixed to 3.3e5 using (4·20). Figure 4.23 a) shows that the DC-link voltage response with the IBSC is improved, with a reduction of the settling time, at ±5% of the steady state value of the DC-link voltage, from 87ms for the IBSC disabled IM drive to 12ms for the IBSC active system. The penalty of the improved DC-link voltage response is a slight degradation in the IM speed response as shown in Figure 4.23 b), with oscillations between 60.27 rad/s and 59.67 rad/s.
The time-domain simulations presented in this section together with the frequency-domain analysis in Section 4.2.3, illustrate the significant improvement in terms of stability and DC-link voltage oscillation damping, provided by the IBSC. Performance degradations were also discussed, and the IBSC sensitivity to changes in operating point was also addressed. It was demonstrated that although the IBSC scheme substantially improves the system stability, its design is highly dependent on the selected operating point. This can cause reduced system stability when operating outside the area where the stabilisation control has been designed. Furthermore it was shown throughout this chapter that the IBSC design is closely dependent on the system parameters. Therefore in a practical system, changes in operating point and parameter variation could reduce the effectiveness of the IBSC, or even cause reduced relative stability. This suggests that the implementation of the IBSC requires the use of adaptive control techniques.
4.4 **SUMMARY**

The design of an impedance-based active stabilisation control for an IP-based field-oriented controlled induction motor drive was presented in this chapter. The proposed IBSC design was based on the ESAC criterion to account for the stability margins in the tuning of the controller. The resulting expressions allow fine tuning of the stabilisation scheme. Frequency-domain analysis and time-domain simulation have both demonstrated the effectiveness of the IBSC method. However, limitations in terms of performance degradation and IBSC sensitivity to changes in operating point were highlighted. Furthermore, it was shown in this chapter that the design of an impedance-based IBSC is dependent on system parameters, which could lead to implementation challenges in a practical system.
Chapter Five

PASSIVITY-BASED CONTROL

This chapter introduces the concept of passivity-based active stabilisation control (PBSC) for an IP-based field oriented controlled induction motor drive, which was discussed in Chapter 3. The control law is first derived and a tuning method is proposed based on an ideal CPL case. The PBSC is then adapted for the FOC IM with the implementation of a phase compensator and a low torque tuning method. Relative stability is discussed using frequency-domain analysis. Finally the effectiveness of the proposed stabilisation scheme is demonstrated using a Matlab/Simulink time-domain simulation.

5.1 STABILISING METHOD

In this section a passivity-based stabilisation control law is derived based on the system shown in Figure 5.1, consisting of an ideal CPL connected to an ideal DC grid voltage source ($v_g$) through an impedance which can represent an input filter, cable impedance and/or DC link capacitance. Here, $R$, $L$ and $C$ are the RLC input filter equivalent series connected resistor, inductor and capacitor, respectively. The load current is $i_l$, the load voltage is $V_l$ and the DC grid current is $i_g$. The power drawn by the CPL consists of the sum of the demanded load power, $P_l$, and the power resulting from the action of the stabilisation control, $P_u$. 

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As mentioned in Section 2.3, in order to derive the PBSC law, the system must be expressed as a Port-Controlled Hamiltonian (PCH) system. The PCH model of the system in Figure 5.1 is derived in Section 5.1.1. The design of the controller is derived from the PCH model in Section 5.1.2, and a tuning enabling to achieve desired performance is presented in 5.1.3.

### 5.1.1 Port-Controlled Hamiltonian Model

The system in Figure 5.1 is described by the set of differential equations given in (5-1). Assuming an infinite PBSC bandwidth, the power drawn by the CPL due to the stabilisation control, is equal to the power reference given by the stabilisation controller. Therefore in (5-1), $u_{PB}$ represents the stabilisation control law.

\[
\begin{align*}
\frac{di_g}{dt} &= -\frac{R}{L}i_g - \frac{1}{L}V_l + \frac{1}{L}V_g \\
\frac{dV_l}{dt} &= \frac{1}{C}i_g - \frac{P_l}{CV_l} - \frac{u_{PB}}{CV_l}
\end{align*}
\]  

(5-1)

To derive the PCH model of the system, a change in variables is required, as:

\[
\begin{align*}
x_1 &= Li_g \\
x_2 &= CV_l
\end{align*}
\]  

(5-2)

where $x_1$ and $x_2$ are the inductor flux linkage and the electric charge in the capacitor, respectively.

Substituting (5-2) in (5-1), the set of differential equations describing the system in Figure 5.1 can be rewritten as given in (5-3).
\( \frac{dx_1}{dt} = -\frac{R}{L} x_1 - \frac{1}{C} x_2 + V_g \)
\( \frac{dx_2}{dt} = \frac{1}{L} x_1 - \frac{p_l C}{x_2} - \frac{u_{PB} C}{x_2} \)  \hspace{1cm} (5\cdot3) 

The Hamiltonian function representing the total energy stored in the system described by (5\cdot3) is expressed as:

\[ H_f(x) = \frac{1}{2L} x_1^2 + \frac{1}{2C} x_2^2 \]  \hspace{1cm} (5\cdot4) 

Using (5\cdot3) and (5\cdot4) the PCH model of the system in Figure 5.1 is given in (5\cdot5).

\( \frac{dx}{dt} = \left[ J_f(x) - R_f(x) \right] \frac{\partial H_f(x)}{\partial x} + g_f(x) u_{PB} + \zeta_f \)  \hspace{1cm} (5\cdot5) 

where \( J_f(x) \) is the \( n \) by \( n \) interconnection matrix and \( R_f(x) \) is the \( n \) by \( n \) damping matrix, which represent the energy exchange and the dissipation in the physical system, respectively. \( \frac{\partial H_f(x)}{\partial x} \) is the partial derivative of the Hamiltonian function of the system. The stabilisation control reference, \( u_{PB} \), affects the system dynamics via the vector \( g_f(x) \). The vector \( \zeta_f \) represents the system inputs. The interconnection matrix satisfies \( J_f(x) = -J_f^T(x) \), and the damping matrix satisfies \( R_f(x) = R_f^T(x) \). \( J_f(x), R_f(x), \frac{\partial H_f(x)}{\partial x}, g_f(x) \) and \( \zeta_f \) are detailed in (5\cdot6), (5\cdot7), (5\cdot8), (5\cdot9) and (5\cdot10), respectively.

\[ J_f(x) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \]  \hspace{1cm} (5\cdot6) 

\[ R_f(x) = \begin{pmatrix} R & 0 \\ 0 & \frac{P_l C^2}{x_2^2} \end{pmatrix} \]  \hspace{1cm} (5\cdot7) 

\[ \frac{\partial H_f(x)}{\partial x} = \begin{pmatrix} \frac{x_1}{L} \\ \frac{x_2}{C} \end{pmatrix} \]  \hspace{1cm} (5\cdot8) 

\[ g_f(x) = \begin{pmatrix} 0 \\ -\frac{C}{x_2} \end{pmatrix} \]  \hspace{1cm} (5\cdot9) 

\[ \zeta_f = \begin{pmatrix} V_g \\ 0 \end{pmatrix} \]  \hspace{1cm} (5\cdot10)
Using the PCH model derived in (5.5), an interconnection and damping assignment (IDA) PBC design is presented in Section 5.1.2.

### 5.1.2 IDA-PBC DESIGN

As introduced in Section 2.3, the principle of the IDA-PBC technique is to assign a desired PCH structure to the system closed-loop. First, a desired energy function $H_{fd}(x)$ with a minimum at the desired values is selected, based on the Hamiltonian function $H_f(x)$; this is also referred to as energy shaping. Then, the desired interconnection and damping matrices can be chosen as $J_{fd}(x)$ and $R_{fd}(x)$, respectively, in order to achieve the desired stability margins or performance. This enables the modification of the energy exchange and damping of the system. Finally, the system closed-loop can be shaped, assuming a control law $u_{PB} = \beta(x)$ satisfying (5.11), where $\beta(x)$ is a function of the state variables of the system.

\[
\frac{dx}{dt} = \left[ J_{fd}(x) - R_{fd}(x) \right] \frac{\partial H_{fd}(x)}{\partial x} \tag{5.11}
\]

The desired energy function (5.12) to stabilise the system in Figure 5.1 is derived from (5.4). \[ H_{fd}(x) = \frac{1}{2L} (x_1 - x_1^*)^2 + \frac{1}{2C} (x_2 - x_2^*)^2 \] (5.12)

where $x_1^*$ is the desired inductor flux linkage and $x_2^*$ is the desired electric charge in the capacitor, and these variables can be related to the desired grid current and load voltage, by:

\[
x_1^* = L i_g^* \tag{5.13}
\]

\[
x_2^* = C V_l^* \tag{5.13}
\]

The partial derivative of the desired energy function in (5.12) is:

\[
\frac{\partial H_{fd}(x)}{\partial x} = \begin{pmatrix}
\frac{x_1 - x_1^*}{L} \\
\frac{x_2 - x_2^*}{C}
\end{pmatrix} \tag{5.14}
\]

Note that $H_{fd}(x)$ in (5.12) has a minimum at $V_l = V_l^*$ and $i_g = i_g^*$. Therefore the system stability is ensured by assigning this Hamiltonian function to the system.
closed-loop. Appropriate selection of \( J_{fd}(x) \) and \( R_{fd}(x) \) allows the desired system response to be achieved.

The desired damping matrix \( R_{fd}(x) \) is given in (5.15). \( R_1 \) and \( R_2 \) are the damping injection constants (with zero power dissipation) that enable the tuning of the PBSC by virtually modifying the damping of the system. The selection of appropriate values for \( R_1 \) and \( R_2 \) is addressed in Section 5.1.3.

\[
R_{fd}(x) = \begin{pmatrix} R + R_1 & 0 \\ 0 & 1/R_2 \end{pmatrix} \quad (5.15)
\]

The structure of the energy exchange within the system is not modified, and so the desired interconnection matrix is given as:

\[
J_{fd}(x) = J_f(x) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (5.16)
\]

A control law \( u_{PB} = \beta(x) \) can now be derived by satisfying the equation given in (5.17), which is formed by substituting (5.5) into (5.11).

\[
\left[ J_f(x) - R_f(x) \right] \frac{\partial H_f(x)}{\partial x} + g_f(x)u_{PB} + \zeta_f = \left[ J_{fd}(x) - R_{fd}(x) \right] \frac{\partial H_{fd}(x)}{\partial x} \quad (5.17)
\]

By substituting (5.6) to (5.10) and (5.14) to (5.16) into (5.17), the expression can be rewritten as:

\[
\left[ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} R & 0 \\ 0 & P_t C^2 \end{pmatrix} \right] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -C \end{pmatrix} u_{PB} + \begin{pmatrix} V_G \\ 0 \end{pmatrix} = \left[ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} R + R_1 & 0 \\ 0 & 1/R_2 \end{pmatrix} \right] \begin{pmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{pmatrix} \quad (5.18)
\]

Equations (5.19) and (5.20) are obtained from (5.18).

\[
\frac{R}{L} x_1 - \frac{1}{C} x_2 + V_G = -\frac{R + R_1}{L} (x_1 - x_1^*) - \frac{1}{C} (x_2 - x_2^*) \quad (5.19)
\]

\[
\frac{1}{L} x_1 - \frac{P_t C}{x_2} - u_{PB} = \frac{1}{L} (x_1 - x_1^*) - \frac{1}{R_2 C} (x_2 - x_2^*) \quad (5.20)
\]

The expression of the desired electric charge in the capacitor \( x_2^* \) can be derived from (5.19), as:
\[ x_2^* = \frac{(R + R_1)C}{L} (x_1 - x_1^*) - \frac{RC}{L} x_1 + CV_g \]  \hspace{1cm} (5.21)

The control law \( u_{PB} \) is obtained rearranging (5.20) as expressed in (5.22).

\[ u_{PB} = \frac{x_2}{C} \left( \frac{x_1^*}{L} + \frac{(x_2 - x_2^*)}{R_2C} \right) - P_l \]  \hspace{1cm} (5.22)

Equations (5.21) and (5.22) can be rewritten in term of grid current, \( i_g \), and DC-link voltage, \( V_l \), by substituting in (5.2) and (5.13), so:

\[ V_l' = (R + R_1)(i_g - i_g^*) - R i_g + V_g \]  \hspace{1cm} (5.23)

\[ u_{PB} = V_l \left( i_g^* + \frac{V_l - V_l'}{R_2} \right) - P_l \]  \hspace{1cm} (5.24)

Substituting the desired DC-link voltage expression (5.23) into (5.24) enables the control law to be written as:

\[ u_{PB} = V_l \left( i_g^* - \frac{R + R_1}{R_2} (i_g - i_g^*) + \frac{1}{R_2} (V_l + R i_g - V_g) \right) - P_l \]  \hspace{1cm} (5.25)

The desired grid current \( i_g^* \), is set to the grid current steady state value, \( i_{g0} \), which can be expressed as given in (5.26).

\[ i_g^* = i_{g0} = \frac{P_l}{V_{l0}} \]  \hspace{1cm} (5.26)

Using (5.1), (5.25) and (5.26) the final control law is given in (5.27).

\[ u_{PB} = -\frac{V_l}{R_2} \left( R_g (i_g - i_{g0}) + \frac{Ldi_g}{dt} \right) + \left( \frac{V_l}{V_{l0}} - 1 \right) P_l \]  \hspace{1cm} (5.27)

where

\[ R_g = R + R_1 \]  \hspace{1cm} (5.28)

The tuning of the control law in (5.27) is proposed in Section 5.1.3.

### 5.1.3 PBSC Tuning

The control law (5.27) can be applied to the system shown in Figure 5.1 to enable a new set of differential equations describing the system when controlled with the proposed method to be obtained as:
5.1 Stabilising Method

\[
\frac{di_g}{dt} = -\frac{R}{L}i_g - \frac{1}{L}V_i + \frac{1}{L}V_g \tag{5-29}
\]

\[
\frac{dV_l}{dt} = \frac{R + R_1}{R_2C}i_g - \frac{R + R_1 + R_2}{R_2C}i_{g_0} - \frac{1}{R_2C}V_i + \frac{1}{R_2C}V_g \tag{5-30}
\]

In (5-30) the nonlinearity introduced by the CPL is cancelled using (5-25). Therefore the intrinsic negative incremental resistance characteristic of the CPL is fully compensated and the DC-link voltage can be stabilised.

The desired DC-link voltage performance in terms of overshoot and time-response can be achieved by selecting appropriate values for the damping injection constants \(R_1\) and \(R_2\).

The system shown in Figure 5.1, described by (5-29) and (5-30) when controlled with (5-25), can be represented by the linear expression given in (5-31).

\[
V_l = Z_1^*(s)V_g - Z_2^*(s)i_{g_0} \tag{5-31}
\]

where, \(Z_1^*(s)\) and \(Z_2^*(s)\) are the desired input and output transfer functions of the RLC filter, which can be shaped with \(R_1\) and \(R_2\), as given in (5-32).

\[
Z_1^*(s) = \frac{1}{R_2C}s + \omega_{nf}^2
\]

\[
Z_2^*(s) = \frac{(Ls + R)\omega_{nf}^2}{s^2 + 2\xi^*\omega_{nf}s + \omega_{nf}^2} \tag{5-32}
\]

where, \(\omega_{nf}^*\) is the desired undamped natural frequency and \(\xi^*\) is the desired damping ratio of the input RLC filter, defined as:

\[
\omega_{nf}^* = \sqrt{\frac{R + R_1 + R_2}{R_2LC}} \tag{5-33}
\]

\[
\xi^* = \frac{1}{2\sqrt{R_2LC(R + R_1 + R_2)}} \tag{5-34}
\]

The expressions for the damping injection constants \(R_1\) and \(R_2\) to achieve a desired undamped natural frequency and damping ratio for the system described by the linear expression in (5-31) can be derived from (5-33) and (5-34) as given in (5-35) and (5-36).
\[ R_1 = \omega_{nf}^2 R_2 L C - R - \frac{L}{2\xi_f \omega_{nf}^2 L C - RC} \] \hspace{1cm} (5.35)

\[ R_2 = \frac{L}{2\xi_f \omega_{nf}^2 L C - RC} \] \hspace{1cm} (5.36)

The tuning proposed in this section consists of shaping the impedance of the input RLC filter in order to achieve desired performance. This method is intuitive and does not require any information on the load, since the CPL behaviour is fully cancelled in (5.30) when using the control law proposed in (5.27). Simulation results of the PBSC for an ideal CPL with input RLC filter are discussed in Appendix C.1.

5.2 IM PBSC

In this section, the control law derived in Section 5.1.2 and given in (5.27), for an ideal CPL with input RLC filter, is implemented in an IP-based FOC IM drive. The control law implementation is first described in Section 5.2.1, and then a small-signal model of the system with the PBSC is derived in Section 5.2.2. A tuning method for low torque operations and a phase compensator are proposed in Sections 5.2.4 and 5.2.3, respectively. Finally the impact on the stability and performance of the overall system of the proposed PBSC for IP-based FOC IM is thoroughly analysed in the frequency-domain in Section 5.2.4.

5.2.1 Control Law Implementation

5.2.1.1 Control Law Simplification

The control law, (5.27), requires knowledge of the load power, \( P_l \), which in a practical system will necessitate extra sensors or the implementation of an observer. The former solution is costly and not always achievable, whilst the latter increases the control complexity and computational cost. The control law, (5.27), however can be simplified assuming that the DC-link voltage response to a disturbance exhibits a negligible deviation from its steady state value. This results in \( V_l/V_{l0} \approx 1 \), which enables (5.27) to be rewritten as:

\[ u_{PB} = -\frac{V_l}{R_2} \left( R_g (i_g - i_{g0}) + L \frac{di_g}{dt} \right) \] \hspace{1cm} (5.37)
The PBSC design proposed in Section 5.1.2 remains valid for (5·37) under the assumption \( V_l/V_{l0} \approx 1 \) as demonstrated in Appendix C.2. Furthermore it is shown in Appendix C.1 that the PBSC is most effective for \( \omega_{n_f}^* = \omega_{n_f} \) since the additional power which has to be drawn by the CPL in order to stabilise the system is reduced when the desired undamped natural frequency of the RLC input filter is set to its actual undamped natural frequency value. According to (5·33), \( \omega_{n_f}^* = \omega_{n_f} \) implies that \( R_1 = -R \), and then using this result in (5·28) \( R_g = 0 \). (5·37) can then be simplified to (5·38).

\[
  u_{PB} = -\frac{LV_i di_g}{R_2} dt
\]

### 5.2.1.2 PBSC FOR IM DRIVE

It was shown in Section 2.4.2 that the stabilisation is improved and the system performance is less degraded when the stabilisation control signal is summed with the \( q \) axis voltage reference instead of the \( q \) axis current reference. The former implementation was used in Chapter Four, since it reduces the IM speed sensitivity to changes in DC-link voltage. Furthermore it was shown that the current control bandwidth limited the effectiveness of the stabilisation scheme in the case of \( q \) current reference injection based stabilisation. Therefore in this chapter a \( q \) voltage reference injection PBSC is proposed.

The control law in (5·38) was derived assuming an infinite control bandwidth. Therefore the power drawn by the IM due to the stabilisation control, is assumed to be equal to the power reference given by \( u_{PB} \). Then using the power conservation law, the load power \( P_l \) plus the PBSC power \( u_{PB} \) is equal to the IM power.

\[
  P_l + u_{PB} = \frac{3}{2} \left( V_{ds}i_{ds} + i_{qs}(V_{qs} + V_c) \right)
\]

where \( V_c \) is the \( q \) axis stabilisation voltage.

Assuming an ideal inverter, so neglecting the losses, switching and delays, the \( q \) axis stabilisation voltage reference \( V_{c*} \) can be expressed as given in (5·40) using (5·39) and (5·38).
\[ V_c^* = -\frac{2L}{3R_2} \frac{dV_l}{i_{qs}^*} dt \] (5.40)

5.2.2 IM Frequency-Domain Model

In order to address the impact of the PBSC on the system stability and the performance degradation, a frequency-domain model of the IP-based FOC IM drive with RLC input filter is derived in this section.

5.2.2.1 Small-Signal Model

The small-signal model of the IP-based FOC IM drive with PBSC is derived in this section using the decoupled \( d \)-\( q \) induction motor model given in Figure A.3, Appendix A.2.

The small-signal DC-current expression remains as given in (4.2), Section 4.1.1, assuming constant rotor flux operation and approximating the inverter behaviour by a unity gain; so neglecting the PWM high-frequency harmonic content, the switching and conduction losses and the delay due to the switching mechanism.

Using the first term of the Taylor expansion the \( q \) axis stabilisation voltage reference \( V_c^* \) in (5.40) can be linearised as given in (5.41).

\[
\delta V_c^* = -\frac{2L}{3R_2} \left( \frac{1}{i_{qs}^*} \frac{dV_l}{dt} + \frac{V_{l0}}{i_{qs}^*} \frac{d\delta i_g}{dt} - \frac{V_{l0}}{i_{qs0}^*} \frac{d\delta i_g}{dt} \right) \] (5.41)

where the subscript “0” denotes the steady state value and the symbol “\( \delta \)” denotes deviations from the steady state values.

From \( \frac{d\delta i_q}{dt} = 0 \), (5.41) can be simplified to:

\[
\delta V_c^* = -\frac{2L}{3R_2} \frac{d\delta i_g}{i_{qs0}^*} dt \] (5.42)

The small-signal model of the FOC IM drive with PBSC connected to a RLC input filter is shown in Figure 5.2, and it is formed from (4.2), from Section 4.1.1, (5.42), the decoupled \( d \)-\( q \) induction motor given in Figure A.3, Appendix A.2, and the IP-based FOC model presented in Figure A.5, Appendix A.2.2. Figure 5.2 is identical to Figure 4.1 apart from the PBSC.
replaces $C_s(s)$ in Figure 4.1, and its input is different, and $q$ axis current IP filter is also omitted since it was shown in Section 4.1.1 that it does not affect the IM admittance or speed sensitivity function for $q$ voltage reference injection stabilisation schemes.

As previously stated the IM rotor flux is assumed constant and the inverter behaviour is approximated by a unity gain. The analysis of the CPL input admittance is focused around the resonant frequency of the input filter, since instabilities are likely to arise around this frequency. Hence, the speed control loop is omitted from Figure 5.2, since the input filter resonant frequency is considerably higher than the speed control bandwidth.

In Figure 5.2, $K_{1,2,3,4}$ are the linearised DC-link current gains defined in (A-28), Appendix A.4. The small-signal grid current, $\delta i_g$, is obtained from the small-signal DC-link and grid voltages via the transfer function $F_g(s)$ which for Figure 5.1 is (5-43). The derivative of $\delta i_g$ is obtained using the complex variable $s$. For the PBSC block in Figure 5.2 the block $P_c(s)$, is a phase compensator which will be introduced in Section 5.2.3.

$$F_g(s) = \frac{1}{Ls + R} \quad (5-43)$$

The gain $G$, the IM electrical and mechanical transfer function $M_e(s)$ and $M_m(s)$, respectively, and the transfer function of the PI current controller $P_Ic(s)$ are given in (4-3), Section 4.1.1. The gain $C_1$ is given in (5-44).
Considering the $q$ axis current reference and the grid voltage as disturbances in Figure 5.2, the closed-loop transfer function between the small-signal DC-link voltage and the small-signal $q$ axis current can be obtained, as:

$$
C_{CL}^{PB}(s) = \frac{\delta i_{qs}}{\delta V_l} = \frac{M_e(s)}{1 + P_I(s)M_e(s)} F_G(s) P_c(s)
$$

(5.45)

where,

$$
F_G(s) = C_1 s F_g(s) = \frac{C_1 s}{L s + R}
$$

(5.46)

Using (5.45) the small-signal model in Figure 5.2 can be simplified to that shown in Figure 5.3.

![Diagram](image)

**Figure 5.3: Simplified induction machine small-signal model with PBSC**

### 5.2.2.2 Input Admittance

The small-signal input admittance $Y_{cpl}(s)$ of the IP-based FOC IM with PBSC is derived from Figure 5.3, as (5.47). The admittance consists of the sum of the inherent admittance of the system $Y_{IM}(s)$, and the compensating term $Y_c^{PB}(s) P_c(s)$.

$$
Y_{cpl}(s) = \frac{\delta i_l}{\delta V_l} = Y_{IM}(s) + Y_c^{PB}(s) P_c(s)
$$

(5.47)

where,

$$
Y_c^{PB}(s) = F_G(s) Y_c(s)
$$

(5.48)
The inherent and compensating admittances, $Y_{IM}(s)$ and $Y_c(s)$ are the same as derived in Section 4.1.2, for the input admittance of the IP-based FOC IM with IBSC. $Y_{IM}(s)$ is given by $K_4$, defined in (A-28), Appendix A.4. The compensating admittance $Y_c(s)$ is first given in (4-7), Section 4.1.2. However due to the low mechanical cut-off frequency equal to $B_m/J_m$, the behaviour of the IM mechanical transfer function $M_m(s)$ can be neglected, since it does not affect the input admittance in the mid-frequency region, around which instabilities are likely to occur. Therefore, the compensating admittance $Y_c(s)$ can be simplified to (4-8), Section 4.1.2.

Figure 5.4 shows the asymptotic gain approximation and actual phase approximation frequency responses for the compensating admittance, $Y_c(s)$, the transfer function $F_G(s)$ and the product of both, $Y_c^{PB}(s)$, which defines the PBSC compensating admittance. The PBSC compensating admittance is the function via which the phase compensator can modify the IM input admittance. High and low torque cases are shown in Figure 5.4 a) and b), respectively.

![Figure 5.4: Asymptotic approximation of the PBSC compensating admittance frequency response](image)

$Y_c(s)$ is identical to that shown in Section 4.1.2, Figure 4.3, and at high torque the undamped natural frequency of the current control loop $\omega_{n_c}$ is significantly larger than the corner frequency of the zero $\omega_{cz}$ of the compensating admittance; conversely at low torque $\omega_{cz} \gg \omega_{n_c}$. The transfer function $F_G(s)$ exhibits a characteristic high-pass filter frequency response with a corner frequency at $R_s/L_s$ and a gain equal to $C_1$ beyond this corner frequency. The gain of $F_G(s)$
changes with the operating point, however its corner frequency is fixed assuming no variation in IM parameters.

Figure 5.4 a) shows the resulting gain and phase frequency response approximation of the PBSC compensating admittance $Y_{cPB}(s)$ at high-torque. The asymptotic gain approximation exhibits a high-pass characteristic with a gain above $\omega_{nc}$ equal to $C_1 K_3$. The phase of $Y_{cPB}(s)$ approaches $+180^\circ$ and $0^\circ$ in the low and high frequency region, respectively, and is positive at the resonant frequency, $\omega_{nf}$, of the input RLC filter. At low-torque the asymptotic gain approximation of $Y_{cPB}(s)$ increases at a rate of $+40$ dB/decade until $R_s/L_s$, then at $+20$ dB/decade for frequencies greater than $R_s/L_s$ but below $\omega_{nc}$. The gain decreases at a rate of $-20$ dB/decade to reach a plateau of magnitude $C_1 K_3$ for frequencies greater than $\omega_c$. The phase of $Y_{cPB}(s)$ also approaches $+180^\circ$ and $0^\circ$ in the low and high frequency region, however, it is negative at $\omega_{nf}$.

Figure 5.4 shows that the phase provided by the PBSC compensating admittance, $Y_{cPB}(s)$, at the input filter resonant frequency varies with the operating point. This is also true in Figure 4.4, Section 4.1.2.1, showing that the gain and phase provided by the compensating admittance $Y_c(s)$ at the input filter resonant frequency significantly changes with the operating point. However no phase compensation mechanism is provided by the control law derived in (5.40). This could lead the proposed PBSC method for IM drive to be ineffective, since the excess phase exhibited by $Y_{cPB}(s)$, at $\omega_{nf}$ could lead to the violation of the ESAC criterion despite a well-tuned control law. To ensure the PBSC is effective, a phase compensator, which will be introduced in Section 5.2.3, is proposed.

The gain $C_1$ given in (5.44) is inversely proportional to the torque producing current component, which results in a large gain at low torque. According to Figure 5.4 b), a significantly large gain of $Y_c(s)F_G(s)$ could lead the gain frequency response of the minor-loop gain$^{10}$, $L_{cpl}(s) = Y_{cpl}(s)Z_2(s)$, crossing the $0$ dB line at a low frequency, which would significantly reduce the system phase margin, since the phase of $Y_{cpl}(s)$ approaches $+180^\circ$ in the low frequency region. This would significantly degrade the systems relative stability at low torque despite a well-tuned control law and accurate phase compensation, leading to a

---

$^{10}$ $Y_{cpl}(s)$ and $Z_2(s)$ are given in (5.47) and (3.16), Appendix 3.2.3, respectively.
violation of the ESAC criterion. Therefore the contribution of the PBSC must be minimised at low torque, in order to maintain a satisfactory stability margins, though this will also lessen the effectiveness of the DC-link voltage oscillation damping.

5.2.2.3 Sensitivity Function

The implementation of the PBSC with phase compensator, shown in Figure 5.2, modifies the speed or torque producing voltage reference by generating an additional stabilising $q$-axis voltage reference which is function of the grid current, $i_g$, and DC-link voltage, $V_l$. The grid current relates to the DC-link voltage via the transfer function $F_g(s)$ given by (5-43). Therefore the IM speed sensitivity to changes in DC-link voltage will be increased with the PBSC. In order to analyse the impact of the stabilisation scheme on the system performance, a small-signal sensitivity transfer function $S_{cpl}(s)$ can be obtained from Figure 5.3 and (5-45) as given in (5-49), with small-signal DC-link voltage and speed as input and output, respectively.

$$S_{cpl}(s) = \frac{\delta \omega_m}{\delta V_l} = S_c^{PB}(s)P_c(s) \quad (5-49)$$

where,

$$S_c^{PB}(s) = S_c(s)F_G(s) \quad (5-50)$$

The sensitivity transfer function consists of the product of the PBSC compensating sensitivity function $S_c^{PB}(s)$ and the phase compensator $P_c(s)$. $S_c(s)$ is given by (4-12) in Section 4.1.3, as the behaviour of the IM mechanical transfer function in the mid-frequency region can be neglected, since its effect is cancelled by a zero at the origin.

Figure 5.5 shows the asymptotic gain approximation frequency response for the compensating sensitivity function $S_c(s)$, the transfer function $F_G(s)$ and the product of both, $S_c^{PB}(s)$. The gain frequency response of the transfer function $F_G(s)$ remains as shown in Figure 5.4. The compensating sensitivity function $S_c(s)$, is not dependent on the operating point so Figure 5.5 is valid across the full operating range. The PBSC compensating sensitivity function term $S_c^{PB}(s)$ represents the function via which the phase compensator affects the IM speed sensitivity to changes in DC-link voltage. The resulting gain
approximation frequency response of the term $S_c^{PB}(s)$ exhibits a band pass characteristic with a +20 dB/decade slope and -40 dB/decade slope in the low-frequency and high-frequency region, respectively. The maximum gain magnitude of $S_c^{PB}(s)$ varies with the operating point since it is function of $C_1$, thus the IM speed sensitivity to change in DC-link voltage will be increased for lower values of torque, which confirms the comment made in Section 5.2.2.2.

Figure 5.5: Asymptotic approximation of the PBSC compensating sensitivity function frequency-response

### 5.2.2.4 Simulation Results

The gain and phase expressions of the PBSC compensating admittance $Y_c^{PB}(s)$ at $\omega_{nf}$ are derived as:

$$|Y_c^{PB}(j\omega_{nf})| = |F_G(j\omega_{nf})||Y_c(j\omega_{nf})|$$

$$\angle Y_c^{PB}(j\omega_{nf}) = \angle F_G(j\omega_{nf}) + \angle Y_c(j\omega_{nf})$$  \hspace{1cm} (5-51)

where $|Y_c(j\omega_{nf})|$ and $\angle Y_c(j\omega_{nf})$ are given in (4-10), Section 4.1.2.1, and the gain and phase of $F_G$ are expressed as:

$$|F_G(j\omega_{nf})| = \frac{C_1\omega_{nf}}{\sqrt{L^2\omega_{nf}^2 + R^2}}$$

$$\angle F_G(j\omega_{nf}) = \tan^{-1}\left(\frac{R}{\omega_{nf}L}\right)$$  \hspace{1cm} (5-52)

The gain and phase of the PBSC compensating admittance, $Y_c^{PB}(s)$, are represented in Figure 5.7, across the range of operating points for the experimentally determined parameters of a 350 V 4.3 kW induction motor, given in Table 3-2, Section 3.3, and the steady state value of the $q$-axis stator
current given in (A-29), Appendix A.4. The induction machine rotor flux ($\psi_r$) and the gains of the $q$ current PI controller calculated to achieve a damping ratio of 0.707 for the closed-loop control with a bandwidth of 500 Hz, are given in Table 3-3, Section 3.3.1. The damping injection constant $R_2$ is set to achieve a desired damping ratio, $\xi_f$, for the input RLC filter of 0.3, using (5.36). The parameters of the input filter are listed in Table 3-1, Section 3.2.2. For both gain and phase plots the speed goes from 0 to 180 rad/s and the torque range from 0 to 70 Nm. For clarity both speed and torque axis have been reversed in Figure 5.6 a) compared to Figure 5.6 b).

As mentioned in Section 5.2.2.2, the gain $C_1$ significantly increases at low torque which results in a high gain of $Y_c^{PB}(s)$ at low torque as shown in Figure 5.6 a). For IM speeds of 60 rad/s, 120 rad/s and 180 rad/s, at 100% of the IM nominal power\(^{11}\), the gain of $Y_c^{PB}(s)$ rises from -1.13 dB, -0.32 dB and 1.84 dB respectively, to 39.56 dB at 0 Nm load torque for all three speeds. This gain increase will limit the effectiveness of the PBSC at low torque since the contribution of the stabilisation scheme will have to be reduced in order to maintain a satisfactory stability margins as explained in Section 5.2.2.2. The phase shift exhibited by the PBSC compensating admittance $Y_c^{PB}(s)$ at $\omega_{nf}$ remains similar to the phase of $Y_c(s)$ presented in Figure 4.4 a), Section 4.1.2.1, since $\angle F_G(j\omega_{nf}) \approx 0$.

\(^{11}\) 100% of the IM nominal power corresponds to a load torque of 70.4 Nm, 35.8 Nm and 23.3 Nm for a speed of 60 rad/s, 120 rad/s and 180 rad/s, respectively.
The PBSC compensating sensitivity function gain at the resonant frequency of the input RLC filter is shown in Figure 5.7 across the range of operating points, for the same FOC IM, input filter and PBSC parameters as used in Figure 5.6. Both speed and torque axes have been reversed. Figure 5.7 is obtained using the gain for $S_{cPB}(s)$ at $\omega_{nf}$, which is given in (5-53), which has been derived from equation (5-50).

$$|S_{cPB}(j\omega_{nf})| = |F_G(j\omega_{nf})||S_c(j\omega_{nf})|$$

(5-53)

where $|F_G(j\omega_{nf})|$ and $|S_c(j\omega_{nf})|$ are given by (5-52) and (4-21), Section 4.2.2, respectively.

Figure 5.7 shows that the sensitivity gain is considerably increased at low torque due to the gain $C_1$ as introduced in Section 5.2.2.3. This will result in significantly degraded speed performance at low torque due to the increased speed sensitivity to changes in DC-link voltage. Therefore the contribution of the PBSC must be reduced at low torque in order to maintain satisfactory IM speed control performance.

![Figure 5.7: Gain map for the PBSC compensating sensitivity function at $\omega_{nf}$](image)

Figure 5.6 and Figure 5.7 show that the tuning method introduced in Section 5.1.3 is limited for low torque operation in terms of stability margins and performance. Therefore a tuning method for low torque operation is proposed in Section 5.2.4.
5.2.3 Phase Compensator

At the end of Section 5.2.2.2 the potential ineffectiveness of the PBSC due to the phase shift introduced by the compensating admittance $Y_c(s)$ not being compensated by the control law in (5·40) was explained. This could result in the desired relative stability not being achieved, with a violation of the ESAC criterion due to an excessive phase of $Y_{cpl}(s)$. In fact, it was shown in Section 4.2.1 that a sufficient condition to satisfy the ESAC criterion was to minimise the phase of the CPL input admittance at the resonant frequency of the input filter, $\omega_{nf}$. This highlights the need to add a phase compensation mechanism to the control law in (5·40). The design of the proposed phase compensator in (5·54) is derived in this section.

$$P_c(s) = K_{pc} \frac{s + \omega_{p1}}{s + \omega_{p2}}$$  \hspace{1cm} (5·54)

where $\omega_{p1}$ and $\omega_{p2}$ are the corner frequencies of the zero and pole respectively, whilst $K_{pc}$ is the gain of the compensator.

Figure 5.8 shows the asymptotic approximation of the gain and phase frequency response of the phase compensator. For $\omega_{p1} > \omega_{p2}$, in the low frequencies the gain of $P_c(s)$ is equal to $K_{pc}\omega_{p1}/\omega_{p2}$ and the phase shift is close to 0° as shown in Figure 5.8 a). Due to the pole, the gain of $P_c(s)$ decays at a rate of -20 dB/decade above $\omega_{p2}$ (but bellows $\omega_{p1}$) and the phase asymptotic approximation is equal to -90°. In the high frequency region ($\omega > \omega_{p1}$) both pole and zero cancel each other; the gain reaches a plateau at $K_{pc}$ and the phase asymptotic approximation is again equal to 0°. For $\omega_{p2} > \omega_{p1}$, the low frequency region gain is still $K_{pc}\omega_{p1}/\omega_{p2}$, but as $\omega_{p1} < \omega_{p2}$ this gain is now less than $K_{pc}$, and the phase of $P_c(s)$ remain identical to the $\omega_{p1} > \omega_{p2}$ case. Under the influence of the zero the gain rises at a rate of +20 dB/decade for $\omega_{p1} < \omega < \omega_{p2}$ and the phase asymptotic approximation is equal to 90°. Again both pole and zero cancel each other in the high frequency region, resulting in a gain equal to $K_{pc}$ and a phase asymptotic approximation of 0°. The red dashed line represents an approximation of the actual phase shift introduced by the phase compensator.
Figure 5.8: Phase compensator frequency response approximation

Figure 5.8 shows that the phase compensator can exhibit both positive and negative phase shifts by appropriately selecting the corner frequencies $\omega_{p1}$ and $\omega_{p2}$. It therefore enables the cancellation of the phase of the PBSC compensating admittance $Y_{cPB}(s)$ for both high-torque and low-torque configurations, since, as it was shown in Figure 5.4, the phase of $Y_{cPB}(s)$ is positive for high-torque and negative for low-torque operations, at the resonant frequency of the RLC input filter $\omega_{nf}$.

The phase compensation technique is highlighted in Figure 5.9, where the asymptotic gain approximation and actual phase approximation frequency responses of the transfer functions $Y_{cPB}(s)$, $P_c(s)$ and $Y_{cPB}(s)P_c(s)$ are shown. Both cases, high and low torque are presented in Figure 5.9 a) and b), respectively. Figure 5.9 a) shows that the negative phase exhibited by the phase compensator at high-torque enables the compensation of the phase of $Y_{cPB}(s)F_G(s)$. Similarly at low-torque the phase compensator positive phase shift can accurately compensate the phase of $Y_{cPB}(s)$ as shown in Figure 5.9 b). This will result in an improved phase margin of the system in order to satisfy the ESAC criterion.
5.2 IM PBSC

A band-pass filter as implemented for the IBSC in Chapter Four would not have been suitable for the phase compensation of $Y_c^{PB}(s)$ since it would have further increased the phase shift in the low frequency region, so it approaches $+270^\circ$ due to the $+90^\circ$ phase of the band-pass filter for low-frequencies. This would have significantly reduced the phase margin.

Figure 5.9 shows that an appropriate selection of $\omega_{p1}$ and $\omega_{p2}$ for the phase compensator enables the minimisation of the phase of the term $Y_c(s)F_G(s)$. The value of the phase compensator zero corner frequency, $\omega_{p1}$, can be calculated to achieve a desired compensating term phase shift at $\omega_{nf}$, for a given phase compensator pole corner frequency, $\omega_{p2}$, using equation (5.55).

$$\omega_{p1} = \frac{\omega_{nf}\omega_{p2} - \omega_{nf}^2\tan(\theta_{PB})}{\omega_{nf} + \omega_{p2}\tan(\theta_{PB})} \quad (5.55)$$

where,

$$\theta_{PB} = -\angle F_G(j\omega_{nf}) - \angle Y_c(j\omega_{nf}) + \angle Y_c^{PB} P_c^*(j\omega_{nf}) \quad (5.56)$$

where $\angle F_G(j\omega_{nf})$ is the phase of the transfer function $F_G$, $\angle Y_c(j\omega_{nf})$ is the phase of the compensating admittance given by (4.10) in Section 4.1.2.1, and $\angle Y_c^{PB} P_c^*(j\omega_{nf})$ is the desired compensating term phase shift, at the RLC input filter resonant frequency.
The phase of $P_c(s)$ can perfectly compensate the phase of the term $Y_c^{PB}(s)$ at $\omega_{nf}$, when its phase is either increasing or decreasing as shown in Figure 5.9. Depending on the chosen configuration, the phase of $Y_c^{PB}$ can not only be compensated at $\omega_{nf}$ but also at higher or lower frequencies. This characteristic can be defined by an appropriate selection of $\omega_{p2}$. At high torque, setting $\omega_{p2} \ll \omega_{p1}$ also minimises the phase of $Y_c^{PB}P_c(s)$ in the low frequency region due to the negative phase of $P_c(s)$ in this frequency region. However at low torque setting $\omega_{p1} \ll \omega_{p2}$ insures no further increase in the phase of the compensating term for low frequencies, which would reduce the system phase margin, due to the increase in gain $C_1$ as introduced in Section 5.2.2.2.

For the stabilisation scheme to be most effective the impact of the phase compensator on the magnitude of the $q$ axis stabilisation voltage reference must be minimised around the resonant frequency of the input filter. This implies that the gain of the phase compensator must be equal to 1 at $\omega_{nf}$, resulting in (5.57).

$$K_{P_c} = \frac{\sqrt{\omega_{nf}^2 + \omega_{p2}^2}}{\sqrt{\omega_{nf}^2 + \omega_{p1}^2}}$$ (5.57)

The resulting gain approximation frequency response of the sensitivity function, $S_{cpl}(s)$ given in (5.49) is shown in Figure 5.10. The asymptotic gain approximation frequency responses of the transfer functions $S_c^{PB}(s)$, $P_c(s)$ and $S_{cpl}(s)$ are presented. At high torque, the gain of $S_{cpl}(s)$ decays at a rate of $-20$ dB/decade, for frequencies greater than $\omega_{p2}$, $-60$ dB/decade, for frequencies greater than $\omega_{nc}$, and $-40$ dB/decade, for frequencies greater than $\omega_{p1}$, as shown in Figure 5.10 a). Furthermore it was shown in Section 5.2.2.4 that the gain $C_1$ is small at high torque, which reduces the maximum gain magnitude of the compensating sensitivity function. Hence a low sensitivity gain will be ensured$^{12}$ at $\omega_{nf}$, which will significantly limit the speed performance degradation due to the fact that the PBSC is designed for a desired undamped natural frequency equal to $\omega_{nf}$, and therefore injects a stabilisation signal at this frequency. However the gain approximation frequency response

$^{12} \omega_{nf} > \omega_{nc}$, for the case studied in this thesis.
of $S_{\text{cpl}}(s)$ in Figure 5.10 b) and the increase of $C_1$ at low torque shown in Figure 5.7, indicates that the sensitivity gain magnitude will be significantly increased around $\omega_{nf}$, at low torque. This will result in an increased IM speed sensitivity to changes in DC-link voltage at low torque. The phase compensator does not affect the sensitivity gain at $\omega_{nf}$, since it is equal to 1.

$$R_2 = \frac{\left|S_c(j\omega_{nf})\right|}{\left|S_{\text{cpl}}(j\omega_{nf})\right|} \frac{\omega_{nf}}{\sqrt{L^2\omega_{nf}^2 + R^2}} \frac{2LV_{10}}{3i_{q0}}$$

(5.58)

**5.2.4 PBSC TUNING AT LOW TORQUE**

The tuning of the PBSC for IM low torque operations proposed in this section is similar to the technique developed in Section 4.2.2 for the IBSC, as it aims to achieve a desired IM speed sensitivity function gain at the resonant frequency of the RLC input filter, $\omega_{nf}$. The control law remains as given by (5.40), thus, the desired undamped natural frequency of the input filter remains as $\omega_{nf} = \omega_{nf}$ since $R_1 = -R$ in (5.40). The tuning consists of selecting a value of the damping injection constant $R_2$ to achieve the desired $S_{\text{cpl}}(s)$ gain at $\omega_{nf}$. Alternatively, this method involves reducing the desired damping ratio $\xi_f$ of the input RLC filter to limit the gain $C_1$ so as to enable satisfactory stability margins and performance at low torque. However in either case it will lessen the effectiveness of the PBSC to dampen DC-link voltage oscillations for the IM low torque operation. The expression given in (5.58) allows $R_2$ to be tuned to achieve a desired gain of $S_{\text{cpl}}(s)$ at $\omega_{nf}$. As introduced in Section 5.2.3 the phase compensator magnitude is equal to 1 at $\omega_{nf}$. 

![Figure 5.10: Frequency response approximation of the sensitivity function gain](image-url)
where, $|S_{cpl}(j\omega_{nf})|^*$ is the desired PBSC compensating sensitivity function gain at the resonant frequency of the input RLC filter, $\omega_{nf}$.

Figure 5.11 shows the IM speed sensitivity function gain at the resonant frequency of the input RLC filter across the full operating range, for the same FOC IM, input filter and PBSC parameters as used in Section 5.2.2.4. The desired IM speed sensitivity function gain $|S_{cpl}(j\omega_{nf})|$ is set to -21 dB. The PBSC is first tuned using the method proposed in Section 5.1.3, which enables the tuning in terms of desired DC-link voltage response. However for operating points resulting in $|S_{cpl}(j\omega_{nf})|>|S_{cpl}(j\omega_{nf})|^*$ with this method, the PBSC is redesigned with the low torque tuning proposed in this section, which reduces the contribution of the stabilisation scheme to maintain satisfactory speed control performance. This is highlighted in Figure 5.11 as $|S_{cpl}(j\omega_{nf})|$ reaches a plateau at -21 dB for low torque operating points (torque< 22 Nm).

![Figure 5.11: Gain map for the IM sensitivity function at $\omega_{nf}$ with low torque tuning](image)

The low torque tuning limits the gain $C_1$ at low torque, which will enable a satisfactory stability margins as introduced in Section 5.2.2.2. This is highlighted in Figure 5.12, which shows the gain of the PBSC compensating admittance, $Y_c^{PB}(s)$, across the range of operating points, and confirms the high gain at low torque shown in Figure 5.6 a) is now cancelled.
5.2 IM PBSC

5.2.5 FREQUENCY-DOMAIN ANALYSIS

A frequency-domain analysis of the IP-based FOC IM drive with PBSC connected to a RLC input filter is carried out in this section using the small-signal input admittance model derived in Section 5.2.2.2. The experimentally determined parameters of the 350 V 4.3 kW induction motor used are listed in Section 3.3, Table 3-2. The input filter parameters used are given in Section 3.2.2, Table 3-1. The induction machine rotor flux ($\varphi_r$) and the gains of the $q$ current PI controller calculated to achieve a damping ratio of 0.707 for the closed-loop control with a bandwidth of 500 Hz, remain as listed in Section 3.3.1, Table 3-3. The stability margins defining the ESAC criterion are the same as used in Chapter Four, a phase margin of 6 dB and a gain margin of 60°.

The frequency-domain analysis is first presented without the phase compensator in Section 5.2.5.1, whilst the effect of the phase compensator is included in Section 5.2.5.2.

5.2.5.1 PBSC WITHOUT PHASE COMPENSATOR

Figure 5.13 shows the Nyquist plot of the minor-loop gain $L_{cpl}(s) = Y_{cpl}(s)Z_2(s)$ using the expression of $Y_{cpl}(s)$ and $Z_2(s)$ given in (5.47) (omitting the phase compensator) and (3.16), Section 3.2.3, respectively. Three operating points along the 4.3 kW IM torque-speed characteristic are selected: 60 rad/s and 70.4 Nm, 120 rad/s and 35.8 Nm, 180 rad/s and 23.9 Nm as shown in Figure 5.13 a), b) and c), respectively. In fact, it was shown in Section 4.1.2.1 that the

Figure 5.12: Gain map for the PBSC compensating admittance at $\omega_{nf}$ with low torque tuning
operating points at 100% of the nominal power of the induction machine represent the most critical operating points in term of stability. For each operating point the damping injection constant $R_z$ is calculated using (5.36) to achieve a desired damping ratio $\xi_f^*$ of the input RLC filter of 0.1, 0.3 and 0.5. The values of $R_z$ for the different $\xi_f^*$ are listed in Table 5.1. As shown in Section 5.1.3, the tuning of the PBSC is independent of the IM operating point since the nonlinearity introduced by the CPL is cancelled in (5.30).

Figure 5.13: Nyquist plot of the minor-loop gain $L_{cpl}(s)$ for stabilised 4.3 kW operating points without phase compensator

Figure 5.13 a), b) and c) show that in most cases the desired relative stability cannot be achieved by the control law in (5.40) without the phase compensator. This is due to the fact that (5.40) does not provide any phase compensation
mechanism to minimise the excessive phase exhibited by the term $Y_c(s)F_G(s)$ as explained in Section 5.2.2.2.

<table>
<thead>
<tr>
<th>$\xi_0^*$</th>
<th>$R_z$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.2311</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0146</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6018</td>
</tr>
</tbody>
</table>

This section justifies the implementation of a phase compensator with the PBSC as introduced in Section 5.2.3.

### 5.2.5.2 PBSC With Phase Compensator

In order to accurately compensate the phase of the PBSC compensating admittance $Y_c^{PB}$ around the resonant frequency of the RLC input filter, $\omega_{nf}$, the phase compensator corner frequencies are calculated using (5.55). The gain $K_{Pc}$ can then be calculated from the obtained corner frequencies using (5.57). Table 5.2 gives the gain $K_{Pc}$ and the zero corner frequency $\omega_p$ for a pre-selected pole corner frequency $\omega_{p2}$ of 200 rad/s\(^{13}\), for the same three operating points along the 4.3 kW IM torque-speed characteristic used in Section 5.2.5.1. The desired compensating term phase $\angle Y_c^{PB}P_c^* (j\omega_{nf})$ is set to $0^\circ$ in order to satisfy the ESAC criterion as mentioned in Section 4.2.1.

<table>
<thead>
<tr>
<th>$\omega_p$ (rad/s)</th>
<th>$K_{Pc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 rad/s</td>
<td>7305</td>
</tr>
<tr>
<td>120 rad/s</td>
<td>4160</td>
</tr>
<tr>
<td>180 rad/s</td>
<td>2053</td>
</tr>
</tbody>
</table>

Figure 5.14 shows the phase Bode plot of the compensating term, demonstrating that the phase of $Y_c^{PB}(s)$ can be accurately compensated at the resonant frequency of the input RLC filter with the phase compensator for IM speeds of 60 rad/s, 120 rad/s and 180 rad/s at 4.3 kW. As introduced in Section 5.2.3, the phase is also compensated in the low frequency region due to a low pole corner frequency of the phase compensator. The phase of the compensating term for a

\(^{13}\) According to Figure 5.9 a) the pole corner frequency $\omega_{p2}$ must satisfy $\omega_{p2} \ll \omega_{nf}$ to enable an appropriate compensation of the phase of $Y_c(s)C(s)$ at $\omega_{nf}$.\)
speed of 60 rad/s and 70.4 Nm load torque presents a larger phase in low and high frequency regions; this is due to the phase shift introduced by $Y_c(s)$ as shown in Figure 4.5, Section 4.1.2.1.

Figure 5.14: Phase bode plot of the compensating term for $\angle Y^p_p P_c^* (j\omega_n) = 0^\circ$

Figure 5.15 shows the Nyquist plot of the minor-loop gain $L_{cpl}(s) = Y_{cpl}(s)Z_2(s)$ for the PBSC with phase compensator for the same system parameters and operating conditions used in Figure 5.13. The phase compensator gain and zero corner frequency used are listed in Table 5.2 for each operating point, whilst the pole corner frequency is set to 200 rad/s. In contrast to Figure 5.13, Figure 5.15 shows significantly improved stability margins, with all operating conditions comfortably meeting the ESAC criterion due to the combined effect of the PBSC with phase compensator.
a) $\omega_m = 60 \text{ rad/s}$  

b) $\omega_m = 120 \text{ rad/s}$  

c) $\omega_m = 180 \text{ rad/s}$

Figure 5.15: Nyquist plot of the minor-loop gain $L_{cp}(s)$ for stabilised 4.3 kW operating points with phase compensator

Table 5·3 list the sensitivity function $S_{cp}(s)$ gain in dB at the frequency of the input filter, $\omega_{nf}$, for the same operating points at 4.3 kW. As expected the sensitivity gain rises as the desired damping ratio of the input RLC filter increases, since the contribution of the PBSC is increased. The $q$ axis stator current decreases for lower values of torque resulting in an increased gain $C_1$, which therefore justifies the larger sensitivity function gain at higher speeds.

<table>
<thead>
<tr>
<th>$\omega_m$ (rad/s)</th>
<th>$\xi_f = 0.1$</th>
<th>$\xi_f = 0.3$</th>
<th>$\xi_f = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>-45.34 dB</td>
<td>-35.28 dB</td>
<td>-30.74 dB</td>
</tr>
<tr>
<td>120</td>
<td>-39.5 dB</td>
<td>-29.44 dB</td>
<td>-24.9 dB</td>
</tr>
<tr>
<td>180</td>
<td>-36.02 dB</td>
<td>-25.96 dB</td>
<td>-21.42 dB</td>
</tr>
</tbody>
</table>
5.2.5.3 PBSC WITH PHASE COMPENSATOR AT ZERO TORQUE

A low torque tuning method was introduced in Section 5.2.3 in order to reduce the gain $C_1$, which ensures satisfactory stability margins and limits the IM performance degradation at low torque. Figure 5.16 shows the Nyquist plot of the open $L_{cpl}(s)$ for three IM speeds of 60 rad/s, 120 rad/s and 180 rad/s all with a load torque set to 0 Nm. The gain $C_1$ is the largest at 0 Nm load torque with the tuning proposed in Section 5.1.3, which therefore represents the worst case for stabilisation at low torque. The damping injection constant $R_2$ is calculated using the low torque tuning method in (5.58) to achieve a desired IM speed sensitivity function gain, $|S_{cpl}(j\omega_{nf})|$, of -21 dB. The values of $R_2$ together with the resulting RLC input filter damping ratios $\xi_f$ are given in Table 5.4. Since the phase of the compensating admittance is the same across the range of speed at 0 Nm torque, as shown in Figure 4.4 b), Section 4.1.2.1, the design of the phase compensator is identical for each IM speed. Using (5.55) the zero corner frequency $\omega_{p1}$ of the phase compensator is calculated as 6.386 krad/s for a pre-selected pole corner frequency $\omega_{p2}$ of 20 krad/s$^{14}$ for the three IM speeds. The gain $K_P$ is calculated using (5.57) and is 2.6694 for the three IM speeds. Figure 5.16 shows that the low torque tuning method enables satisfactory stability margins at speeds with zero torque since all Nyquist contours remain outside the forbidden region defined by the ESAC criterion.

Figure 5.16: Nyquist plot of the minor-loop gain $L_{cpl}(s)$ for stabilised 0 kW operating points with PBSC low torque tuning

$^{14}$ According to Figure 5.9 b) the pole corner frequency $\omega_{p2}$ must satisfy $\omega_{p2} \gg \omega_{nf}$ to enable an appropriate compensation of the phase of $Y_c(s)C_z(s)$ at $\omega_{nf}$. 
Table 5.4 shows that in order to maintain satisfactory stability margins and limit the IM speed control performance degradation, the damping ratio of the input RLC filter is significantly reduced. This will result in reduced effectiveness of the damping of DC-link voltage oscillations with the PBSC at low torque.

<table>
<thead>
<tr>
<th>$\omega_m$ (rad/s)</th>
<th>$R_2$</th>
<th>$\xi_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>209.3</td>
<td>0.0099</td>
</tr>
<tr>
<td>120</td>
<td>104.65</td>
<td>0.0113</td>
</tr>
<tr>
<td>180</td>
<td>69.77</td>
<td>0.0127</td>
</tr>
</tbody>
</table>

### 5.2.5.4 PBSC WITH PHASE COMPENSATOR SENSITIVITY TO CHANGE IN OPERATING POINT

It was shown in Section 5.2.3 that the design of the phase compensator is dependent on the operating point. Figure 5.17 shows the Nyquist plot of the minor-loop gain $L_{cp1}(s)$ for the same zero torque operating points used in Section 5.2.5.3, Figure 5.16. However, here the phase compensator is tuned for operation at 100% of the IM nominal power and so is as designed in Section 5.2.5.2. The gain $K_{Pc}$ and the zero corner frequency $\omega_p1$ for a given pole corner frequency $\omega_p2$ of 200 rad/s, are given in Table 5.2. The three Nyquist contours enter the forbidden region defined by the ESAC criterion, resulting in a reduced stability margin compared to Figure 5.16.

![Nyquist plot](image-url)
These observations underline the phase compensator design sensitivity to changes in operating point. However it shows that an appropriate design of the phase compensator and the control law in (5.40), enable satisfactory stability margins across the range of IM torque-speed characteristics. Time-domain simulations, presenting the enhanced performance and limitations of the PBSC are discussed in the following section.

5.3 **TIME-DOMAIN SIMULATION**

In this section Matlab/Simulink time-domain simulations of the IP-based FOC induction motor drive with input filter and PBSC, are presented. The non-linear model of the IP-based FOC induction motor drive is described in Appendices A.1, A.2 and A.3. The nonlinear model of the RLC input filter is given in (5.1). The experimentally determined parameters of a 350 V 4.3 kW induction motor, remain as listed in Section 3.3 Table 3-2. The input filter parameters are those listed in Section 1.2.2, Table 3-1, and the DC grid voltage $V_g$ is set to 540 V. The induction machine rotor flux ($\varphi_r$), the speed and $d$-$q$ current IP filter time constants and control output saturations, and the gains of the speed and $d$-$q$ current PI controllers calculated to achieve a damping ratio of 0.707 for the closed-loop control with a bandwidth of 10 Hz and 500 Hz, respectively, are listed in Section 3.3.1, Table 3-3.

5.3.1 **EVALUATION OF THE PBSC AT RATED INDUCTION MACHINE POWER**

The DC-link voltage and IM speed response for a 70.4 Nm torque step at 1 s with the PBSC active and disabled are shown in Figure 5.18. In the active case, the phase compensator is also enabled. The IM speed reference is set to 60 rad/s, since as shown in Figure 5.15, this speed has the smallest stability margin of the three speeds investigated, at 100% of the IM nominal power. The torque is initially zero. For the PBSC active case, the phase compensator zero and pole corner frequencies, $\omega_{p1}$ and $\omega_{p2}$ are set to 7305 rad/s and 200 rad/s using (5.55) as given in Table 5-2, in order to fix the phase of the term $Y_c(s)F_G(s)P_c(s)$ at the resonant frequency of the RLC input filter, $\omega_{nf}$, to 0°. As mentioned in Section 5.2.3, for the control law to be the most effective, the phase compensator gain must be equal to 1 at $\omega_{nf}$. To do so $K_{PC}$ is set to 0.5013 using (5.57).
Furthermore in the case of active PBSC the damping injection constant $R_2$ is calculated using (5.36), to achieve a desired damping ratio $\xi_f^*$ of the input RLC filter of 0.5, and is given in Table 5.1. Figure 5.18 a) shows that the DC-link voltage settling time is significantly improved when the PBSC is active. Furthermore, the maximum deviation from the DC-link voltage steady state value is slightly improved, reducing from ~0.54V to ~0.37V with the PBSC disabled and enabled respectively. The frequency range of the injected stabilisation signal is significantly higher than the speed control bandwidth, and so the speed response is not degraded by the PBSC as shown in Figure 5.18 b).

![DC-link voltage response](image1)

![Speed response](image2)

Figure 5.18: Responses to torque step at 4.3 kW

Figure 5.19 shows a magnified view of the DC-link voltage response shown in Figure 5.18 a). The response with disabled PBSC is omitted. DC-link voltage responses for three desired damping ratio $\xi_f^*$ of the input RLC filter of 0.1, 0.3 and 0.5 are presented. The corresponding damping injection constants $R_2$ are given in Table 5.1. As the desired damping ratio increases, the maximum deviation and the DC-link voltage oscillations are reduced, which correlates
with the results obtained from the ideal CPL simulation in Appendix C.1, Figure C.1.

![DC-link voltage response to torque step at 4.3 kW](image)  
**Figure 5.19:** DC-link voltage response to torque step at 4.3 kW

Figure 5.20 shows the DC-link voltage and IM speed response for a step in speed reference from 60 rad/s to 50 rad/s at 1.5 s with the PBSC active and disabled. The load torque is set to 70.4 Nm. The design of the PBSC and phase compensator is identical to that used for Figure 5.18. The DC-link voltage oscillations are significantly reduced when the PBSC and phase compensator are active as shown in Figure 5.20 a). The speed tracking performance is identical for both active and disabled PBSC as presented in Figure 5.20 b), again this is due to the frequency of the stabilisation injection being above the speed control bandwidth.
A magnified view of the DC-link voltage response given in Figure 5.20 a) is presented in Figure 5.21. The response for the disabled PBSC is omitted. The damping injection constants $R_2$ are calculated to achieve a desired damping ratio $\xi_f^*$ of the input RLC filter of 0.1, 0.3 and 0.5, as given in Table 5·1. The DC-link voltage oscillations are significantly improved as $\xi_f^*$ increases.

Figure 5.21: DC-link voltage response to speed step at 4.3 kW

Figure 5.22 shows the DC-link voltage and IM speed response for a step in grid voltage from 540 V to 530 V at 1.5 s with the PBSC active and disabled. The IM
speed and load torque are set to 60 rad/s and 70.4 Nm, respectively. Again the PBSC and phase compensator designs are unchanged. The desired damping ratio $\xi_f^*$ of the RLC input filter is set to 0.5 for the active PBSC case. The corresponding damping injection constant $R_2$ is given in Table 5.1. The DC-link voltage response is significantly improved with the PBSC and phase compensator active as shown in Figure 5.22 a). The maximum DC-link voltage deviation from its steady state value is reduced from 9.98V with the PBSC disabled to 3.88V with PBSC active. Furthermore the settling time at $\pm 5\%$ of the steady state value of the DC-link voltage is drastically improved with the PBSC reducing from 206ms to 0.9ms. The cost however of the significantly improved stabilisation is a degradation in the IM speed response as shown in Figure 5.22 b).

Magnified views of the DC-link voltage and IM speed response presented in Figure 5.22 are shown in Figure 5.23 a) and b), respectively. The response for the disabled PBSC is omitted. Again responses are presented for desired
damping ratio $\xi_f^*$ of the RLC input filter of 0.1, 0.3 and 0.5. The damping injection constants $R_2$ are given in Table 5-1.

The settling time is reduced from 6ms to 1.6ms and 0.9ms for a $\xi_f^*$ of 0.1, 0.3 and 0.5, respectively. The maximum DC-link voltage deviation from its steady state value is reduced from -7.98V to -5.32 and -3.88V as the desired damping ratio increases from 0.1 to 0.5. Whereas the speed response is more degraded for greater values of $\xi_f^*$ with the deviation increasing from 0.06 rad/s to 0.15 rad/s and 0.21 rad/s for desired damping ratio of 0.1, 0.3 and 0.5, respectively. Even with the smallest $\xi_f^*$ tested, the DC voltage deviation is less than that with the PBSC and phase compensator inactive, and the settling time is an order of magnitude smaller, and the resulting speed transient is within 0.1% of the steady state speed.

Figure 5.23: Magnified responses to grid voltage step at 4.3 kW
5.3.2 **EVALUATION OF PBSC AT LOW TORQUE**

Figure 5.24 shows the DC-link voltage and IM speed response for a step in grid voltage from 540 V to 530 V at 1.5 s with the PBSC active and disabled. The speed reference is set to 60 rad/s and the load torque is zero, this is the worst case operating point. The phase compensator zero corner frequency, $\omega_{p1}$, is set to 6.386 krad/s using (5.55) for a pre-selected pole corner frequency, $\omega_{p2}$, of 20 krad/s. The phase compensator gain, $K_{Pc}$, is set to 2.6694 using (5.57). The PBSC is designed using the low torque tuning method proposed in Section 5.2.3. The damping injection constant $R_2$ is therefore derived from (5.58) for a given desired IM speed sensitivity function gain at the resonant frequency of the RLC input filter, $|S_{cpl}(j\omega_{nf})|^*$, of -21 dB. The value of $R_2$ and the corresponding input RLC filter damping ratio for an IM speed of 60 rad/s at 0 Nm load torque is given in Table 5.4.

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According to Figure 5.9 b) the pole corner frequency $\omega_{p2}$ must satisfy $\omega_{p2} \gg \omega_{nf}$ to enable an appropriate compensation of the phase of $Y_c(s)C_s(s)$ at $\omega_{nf}$.
Figure 5.24 a) shows that the oscillations in the DC-link voltage are slightly reduced with the active PBSC. However the system performance is worse with the low torque tuning in comparison with the tuning proposed in Section 5.1.3, due to the reduction of the stabilisation scheme contribution in order to maintain satisfactory stability margin and IM speed control performance, as shown by Figure 5.11. The IM speed degradation is increased with the low torque tuning as shown in Figure 5.24 b), since the PBSC compensating sensitivity function gain is at its maximum.

Both the DC-link voltage and speed responses for the PBSC active in Figure 5.24 are distorted. This can be explained by the saturation of the sum of the q axis voltage reference, \( V_{qs}^* \), and the q axis stabilisation voltage reference \( V_c^* \) as shown in Figure 5.25. In fact, at zero torque, \( i_{qs}^* \) is close to 0 A which gives a significantly high \( V_c^* \) as suggested by \((5\cdot40)\), resulting in \( V_{qs}^* + V_c^* \) reaching the voltage reference saturation. This shows the limitation of the PBSC at low torque, since despite a considerable reduction in the stabilisation scheme contribution resulting in a reduced improvement in the DC-link voltage oscillations, the stabilisation signal \( V_c^* \) remains significantly large. However this 0 Nm load torque case represents the worst case, and the PBSC provides satisfying results for most of the low torque operating points.

![Graph showing voltage response](image)

Figure 5.25: q axis voltage reference response for a speed step at 0 kW

### 5.3.3 Phases Compensator Sensitivity to Change in Operating Point

Figure 5.26 shows the DC-link voltage response for a step in grid voltage from 540 V to 530 V at 1.5 s with the PBSC active and disabled, for the same operating point as in Section 5.3.2 (0 Nm torque). The PBSC design remains the
same; however, the phase compensator is designed for operation at 100% of the IM nominal power as used in Section 5.3.1. The phase compensator corner frequencies and gain are given in Table 5.2. This design results in a degraded DC-link voltage response as shown in Figure 5.26, and therefore highlights the phase compensator design sensitivity to changes in operating point.

Both time-domain simulations presented in this section and the frequency-domain analysis discussed in Section 5.2.4, demonstrate the significant improvement in terms of stability and DC-link voltage oscillation damping, provided by the PBSC with phase compensator. The performance degradation introduced by the PBSC and the phase compensator design sensitivity to change in operating point have also been addressed. The PBSC tuning method proposed in Section 5.1.3 allows an intuitive tuning in terms of desired damping ratio and undamped natural frequency of the RLC input filter resulting in substantial DC-link voltage response improvement and minimised IM speed control degradation at high torque. However the limitations of the PBSC at low torque were also presented. Furthermore throughout this chapter it was shown that the PBSC design at low torque and the phase compensator tuning are closely dependent on the system parameters. Therefore in practical cases, changes in operating point and parameters variation could reduce the effectiveness of the stabilisation control, or even lead to degraded relative stability. This suggests that the implementation of the PBSC with phase compensator requires only tuning in practical cases.
5.4 SUMMARY

The design of a passivity-based active stabilisation control for an IP-based field-oriented controlled induction motor drive was described in this chapter. An intuitive tuning method, simplifying the design of the stabilisation scheme was proposed based on an ideal CPL case. The PBSC was then adapted for a FOC IM drive with the implementation of a phase compensator. Limitations of the original tuning method proposed were shown at low torque and an alternative tuning technique was then proposed for low torque operating points. The system relative stability was thoroughly analysed in the frequency domain with a small-signal model of the system with PBSC. It was demonstrated that the PBSC with phase compensator satisfied the EASC criterion, and therefore provided desired system relative stability. Simulation results proved the effectiveness of the PBSC with phase compensator to dampen DC-link voltage oscillations with minimal IM speed control degradation. However limitations at low torque were highlighted. Finally it was shown that in practical cases, online tuning might be required in order to extend the range of operating points which can be stabilised by the PBSC since the design of the phase compensator and the PBSC tuning at low torque are closely dependent on the system parameters.
Chapter Six

**HARDWARE-IN-THE-LOOP VERIFICATION**

This chapter provides a comparison of the IBSC and PBSC schemes for IP-based FOC induction motor drive using a control Hardware-In-the-Loop (HIL) emulation system. The system consisting of an emulation platform and a digital signal controller (DSC) card hosting the control is first presented. The discretisation of the FOC with IBSC and PBSC is then derived. Finally, results of both stabilisation strategies are compared and discussed.

### 6.1 **SYSTEM SETUP**

The system used to demonstrate the implementation of the control from Chapter Four and Chapter Five, and to enable their performance to be evaluated is a Typhoon HIL 600 real-time emulation platform which is purpose designed to enable safe testing of power electronic converter control system. The real-time HIL system simulates in real-time the power hardware, and the converter under test is controlled using physical hardware, in this case a TI DSC card (TM320F28335), which is interfaced with the Typhoon system through a docking station as shown in Figure 6.1.
6.1.1 HIL Platform and DSC

The Typhoon HIL 600 real-time emulation platform is a 6-core processor with a 20ns simulation step. The HIL system is interfaced with the DSC via 16 analogue outputs, 8 analogue inputs, 32 digital inputs and 32 digital outputs. These signals are scaled via the docking station to interface the Typhoon system with the DSC. A schematic editor enables the model of the power electronic system to run in real-time on the HIL hardware platform to be built from Typhoon library component which includes sources, passive elements, converters and machines to be designed. Figure 6.2 a) shows the induction motor drive system with input filter from the schematic editor. A control panel, Figure 6.2 b), is also available to monitor the system performance via oscilloscope and capture functionalities, and enables the simulation parameters to be defined. Furthermore, automated testing procedure of the system can be carried out using Python scripts.
The DSC interfaced with the Typhoon HIL 600 is a TI microcontroller, TMS320F28335, with a Central Processing Unit (CPU) clock rate of 150MHz, 24 Analogue-to-Digital Converter (ADC) channels, 12 PWM channels and 2 Quadrature Encoder Pulse (QEP) channels. The QEP provides the motor rotor position from the shaft position encoder pulses. More information about the DSC ADC, PWM and QEP can be found in [170-174]. The TMS320F28335 was selected as the control hardware as this is a low-cost, widely available DSC, which is marketed for use in power electronic converter based industrial applications.

6.1.2 SYSTEM IMPLEMENTATION

The induction motor drive system with input filter introduced in Section 3.3 is emulated by the Typhoon HIL 600, whilst the discretised IP-based FOC with IBSC and PBSC developed in Chapter Four and Chapter Five, respectively, are implemented on the TI DSC.
The drive consists of a sinusoidal PWM controlled inverter feeding a cage rotor (squirrel cage) three-phase induction machine. The DC-link of the inverter is connected to an ideal DC voltage source, $V_g$, through a RLC filter. The stator 3-phase voltages are generated by the PWM voltage source inverter (VSI). The VSI consists of three legs, one per phase, with two switches per leg, synthesising the DC-link voltage, $V_l$, shown in Figure 6.3, into sinusoidal phase voltages. The switches each have an antiparallel diode and only one switch per leg is turned on at any time. The switches are controlled by sinusoidal PWM phase shifted by 120° for each phase, generated by the TI DSC. Sinusoidal PWM is chosen for simplicity, however, as the proposed stabilisation techniques have no dependency on the PWM generation they may also be combined with space vector PWM techniques.

The voltage and current sensors are emulated by the Typhoon HIL 600, and each are mapped to separate ±5 V analogue output signals. The signal is then scaled via the docking station to the standard DSC ADC input level of 0 V to 3.3 V. Both the voltage and current sensors are assumed to have an infinite
bandwidth. The emulated Typhoon IM speed position absolute encoder delivers output digital signals (2048 pulses per revolution) which are directly connected to the DSC QEP input channel.

The selected DSC PWM configuration consists of symmetrical up and down counters, per phase, which are compared to the phase stator voltage reference to produce the PWM command signals for the inverter switches. The obtained command signal for each phase is used to control the top switch of the leg, whilst the inverted command signal is used for the bottom switch of the leg. Rising and falling edge dead-bands are generated on the command signal in order to prevent two switches in the same leg to be closed simultaneously.

To enable the new switch command signals to be calculated a DSC program interruption occurs every time the PWM up counter of the phase \(a\) reaches its maximum value. Once the program interruption is enabled, the stator phase currents, \(i_{a,b,c,s}\), the grid current, \(i_g\), and the DC-link voltage, \(V_l\), values are first obtained via the ADC. The IM speed, \(\omega_m\), is calculated by a speed calculation function using the motor rotor position provided by the QEP. The speed calculation function is detailed in [175]. The \(q\) axis stator current reference, \(i_{qs}^*\), is then generated by the discrete IP speed controller, which is detailed in Appendix D. The \(d\) axis stator current reference, \(i_{ds}^*\), is directly set in order to achieve the desired constant rotor flux. The \(d'q\) axis decoupled stator voltage references, \(V_{ds}'^*\) and \(V_{qs}'^*\), are generated by the discrete IP current controllers given in Appendix D. For the \(q\) axis, the stabilising voltage reference \(V_c^*\), generated by the ASC, is summed to the output of the controller. The discrete IBSC and PBSC are detailed in Sections 6.2.2 and 6.2.3, respectively. The stator voltage references, \(V_{ds}^*\) and \(V_{qs}^*\), are calculated with a feedforward decoupling function using (A-14) and (A-15), given in Appendix A.2. The \(d'q\) axis measured stator currents are obtained from the \(\alpha\beta\) axis measured stator currents via the direct Park transform. The \(\alpha\beta\) axis measured stator currents are generated by the direct Clarke transform from the three-phase currents, \(i_{a,b,c,s}\). Conversely the phase voltage references \(V_{a,b,c,s}^*\) are obtained from \(V_{d,q,s}^*\) via the successive inverse Park and inverse Clarke transforms. The new switch command signals obtained from the PWM using \(V_{a,b,c,s}^*\) are activated at the start of the next interruption. The direct and inverse Park and Clarke transforms functions are
given in [175]. The direct and inverse Park transforms require the knowledge of the rotor flux position. This is obtained from a rotor flux position calculation function given in [175].

6.2 ASC DISCRETISATION

To enable the IP-based FOC with IBSC and PBSC to be implemented on the TI DSC TMS320F28335 these controllers must be discretised.

From Section 6.1.2, the outputs of the digital control, the PWMs, enable the control of the inverter switches, and this PWM mechanism is actually a digital-to-analogue conversion [170]. The behaviour of such conversion can be represented by a zero-order hold (ZOH), which describes the effect of converting a discrete-time signal to a continuous time signal by holding each sample value for one sample interval [176]. The ZOH introduces a phase shift and affects the magnitude of the command signals. Furthermore it was introduced in Section 6.1.2 that the new switch command signals obtained from the PWM are only activated at the start of the next interruption, resulting in a time delay of a sampling period, which also introduces a phase shift on the command signals. Therefore a computational time delay and ZOH compensation is presented in Section 6.2.1, which is then combined with the ASC schemes in Sections 6.2.2 and 6.2.3 to enable them to be more effective.

6.2.1 COMPUTATIONAL TIME DELAY AND ZERO-ORDER HOLD COMPENSATION

In Chapter Four and Chapter Five the design of the proposed IBSC and PBSC both required knowledge of the phase of the equivalent load compensating admittance at the resonant frequency of the RLC input filter, \(\omega_{nf}\). It was also highlighted that both strategies are sensitive to changes in operating point. Therefore an imprecise design could reduce the effectiveness of the stabilisation schemes. This implies that the behaviour of the computational time delay and ZOH must be carefully compensated in order to achieve the desired system relative stability.
The expression of the computational time delay equal to a sampling period and the equivalent ZOH expression in the Laplace domain can be written as (6·1) and (6·2), respectively according to [176].

\[ D(s) = e^{-sT_s} \quad (6·1) \]

\[ ZoH(s) = \frac{1 - e^{-sT_s}}{sT_s} \quad (6·2) \]

where \( T_s \) is the sampling period equal to \( 1/f_s \), with \( f_s \) being the switching frequency.

The total expression in the Laplace domain \( D_{tot}(s) \) (6·3), which needs to be compensated, is obtained from (6·1) and (6·2)

\[ D_{tot}(s) = \frac{1 - e^{-sT_s}}{sT_s} e^{-sT_s} \quad (6·3) \]

Figure 6.4 shows the gain and phase frequency response of the computational delay combined with ZOH (\( D_{tot}(s) \)) using expressions (6·5) and (6·6) which are derived from (6·3), and the equivalence given in (6·4), for a switching frequency of 20 kHz. The time delay \( D(s) \) does not affect the magnitude of the signal since its gain is equal to 1 for all frequencies. The gain frequency response shows that the gain magnitude is significantly attenuated by the ZOH, whilst the phase shift oscillates between ± 120°, in the high frequency region. For a resonant frequency of the input RLC filter, \( \omega_{nf} \), of 4.23 krad/s calculated with the parameters listed in Table 3·1, Section 3.2.2, the gain and phase of \( D_{tot}(s) \) at \( \omega_{nf} \) are -0.0162 dB and -18.16°, respectively.

\[ e^{-j\omega T_s} = \cos(\omega T_s) - j \sin(\omega T_s) \quad (6·4) \]

\[ \angle D_{tot}(j\omega) = \tan^{-1} \left( \frac{1 - \cos(\omega T_s)}{\sin(\omega T_s)} \right) + \tan^{-1} \left( \frac{-\sin(\omega T_s)}{\cos(\omega T_s)} \right) \quad (6·5) \]

\[ |D_{tot}(j\omega)| = \sqrt{\frac{\sin^2(\omega T_s) + (1 - \cos(\omega T_s))^2}{\omega T_s}} \quad (6·6) \]
In order to compensate for the slight gain attenuation due to the ZOH at the resonant frequency of the input RLC filter, the stabilisation voltage reference, $V_c^*$, generated by the IBSC and PBSC, must be scaled up by the factor $K_D$ given in (6.7).

$$K_D = \frac{1}{\left| D_{tot}(j\omega_{nf}) \right|} = \frac{\omega_{nf}T_s}{\sqrt{\sin^2(\omega_{nf}T_s) + (1 - \cos(\omega_{nf}T_s))^2}}$$  \hspace{1cm} (6.7)

Phase compensation mechanisms are already employed in both the IBSC and PBSC techniques, at the resonant frequency of the RLC input filter, as discussed in Chapter Four and Chapter Five. Therefore the phase shift introduced by the computational delay and ZOH can be directly compensated, at $\omega_{nf}$, by accounting for it in the design process of the band-pass filter for the IBSC and phase compensator for the PBSC. Thus expressions (4.18), Section 4.2.2, and (5.56), Section 5.2.3, can be rewritten as given in (6.8) and (6.9), respectively.

$$\theta_{IB} = -\angle D_{tot}(j\omega) - \angle Y_c(j\omega_{nf}) + \angle Y_cC^*(j\omega_{nf})$$  \hspace{1cm} (6.8)

$$\theta_{PB} = -\angle D_{tot}(j\omega) - \angle F_G(j\omega_{nf}) - \angle Y_c(j\omega_{nf}) + \angle Y_c^{PB}P_c^*(j\omega_{nf})$$  \hspace{1cm} (6.9)

where $\theta_{IB}$ and $\theta_{PB}$ are first defined in Section 4.2.2 and Section 5.2.3, respectively, for the calculation of the IBSC band-pass filter and PBSC phase
compensator corner frequencies in order to achieve desired compensating admittance phase shift at $\omega_{nf}$, $\angle Y_c C^*(j\omega_{nf})$ and $\angle Y_{PB} P^*(j\omega_{nf})$.

### 6.2.2 IBSC

To enable the IBSC strategy introduced in Chapter Four to be coded directly on the DSC, the band-pass filter introduced in Section 4.2.2 must be discretised. The discretisation is performed by applying the trapezoidal approximation $z$ transform (6.10), also known as the Tustin transform.

$$s = \frac{2z - 1}{T_s z + 1} \quad (6.10)$$

Applying (6.10) to (4.15), in Section 4.2.2, the IBSC band-pass filter in the $z$ domain is given by:

$$C_a(z) = \frac{V_c^*(z)}{V_l(z)} = \frac{C_{b_0}z^2 + C_{b_2}}{C_{a_0}z^2 + C_{a_1}z + C_{a_2}} \quad (6.11)$$

where,

$$C_{a_0} = \frac{4}{T_s^2} + (\omega_{cl} + \omega_{ch}) \frac{2}{T_s} + \omega_{cl}\omega_{ch}$$

$$C_{a_1} = 2\omega_{cl}\omega_{ch} - \frac{8}{T_s^2}$$

$$C_{a_2} = \frac{4}{T_s^2} - (\omega_{cl} + \omega_{ch}) \frac{2}{T_s} + \omega_{cl}\omega_{ch} \quad (6.12)$$

$$C_{b_0} = K_c \frac{2}{T_s}$$

$$C_{b_2} = -K_c \frac{2}{T_s}$$

The low and high corner frequencies of the band-pass filter, $\omega_{cl}$ and $\omega_{ch}$, are calculated accounting for the computational delay and ZOH phase shift using (6.8) from Section 6.2.1.

For the IBSC scheme to be implemented in the TI DSC, the IBSC stabilising voltage reference recursive formula is derived from (6.11) as (6.13) when the computational delay and ZOH gain compensation is included.
\[ V_c^*(k) = \left( \frac{C_{b_0}}{C_{a_0}} V_i(k) + \frac{C_{b_2}}{C_{a_0}} V_i(k-2) - \frac{C_{a_1}}{C_{a_0}} V_c^*(k-1) + \frac{C_{a_2}}{C_{a_0}} V_c^*(k-2) \right) K_D \tag{6.13} \]

where, \( k \) represents the discrete time step.

### 6.2.3 PBSC

Similarly the PBSC scheme derived in Chapter Five must be discretised in order to be coded in the DSC. First the control law in (5.38), Section 5.2.1.2, is discretised as (6.15) by using the difference as an equivalent of the time derivative in the \( z \) domain (6.14).

\[ Z \left( \frac{d}{dt} \right) = \frac{1}{T_s} \frac{z-1}{z} \tag{6.14} \]

\[ u_{PB}(z) = -\frac{L}{R_2} V_i \frac{1}{T_s} \frac{z-1}{z} i_g \tag{6.15} \]

The phase compensator introduced in Section 5.2.3 is then discretised as (6.16) using the Tustin approximation in (6.10).

\[ P_c(z) = \frac{P_{b_0} z + P_{b_1}}{P_{a_0} z + P_{a_1}} \tag{6.16} \]

where,

\[ P_{a_0} = K_{p_c} \left( \omega_{p1} + \frac{2}{T_s} \right) \]

\[ P_{a_1} = K_{p_c} \left( \omega_{p1} - \frac{2}{T_s} \right) \tag{6.17} \]

\[ P_{b_0} = \omega_{p2} + \frac{2}{T_s} \]

\[ P_{b_1} = \omega_{p2} - \frac{2}{T_s} \]

Again the zero and pole corner frequencies of the phase compensator, \( \omega_{p1} \) and \( \omega_{p2} \), are calculated by accounting for the computational delay and ZOH phase shift using (6.9) as discussed in Section 6.2.1.
The PBSC discrete stabilising voltage reference can be expressed as (6.18) according to (5.40), in Section 5.2.1.2, when the computational delay and ZOH gain compensation is included.

\[
V_c^*(z) = \frac{2K_D}{3i_{qs}} u_{PB}(z) P_c(z)
\]  
(6.18)

As in Section 6.2.2, the PBSC stabilising voltage reference recursive formula must be derived to enable it to be implemented on the TI DSC. First the control law recursive formula, \( u_{PB}(k) \), is derived from (6.15) as:

\[
u_{PB}(k) = -\frac{L}{R} V_i(k) \frac{1}{T_s} (i_g(k) - i_g(k - 1))
\]  
(6.19)

Then the PBSC stabilising voltage reference recursive formula (6.20) is obtained using (6.15),(6.16) and (6.19) when the computational delay and ZOH gain compensation is included.

\[
V_c^*(k) = \frac{2K_D}{3i_{qs}} \left( \frac{P_{ba}}{P_{ao}} u_{PB}(k) + \frac{P_{ba}}{P_{ao}} u_{PB}(k - 1) - \frac{P_{a1}}{P_{ao}} V_c^*(k - 1) \right)
\]  
(6.20)

### 6.3 HARDWARE-IN-THE-LOOP RESULTS

In this section, results obtain from the control HIL emulation of the IP-based FOC induction motor drive with input filter and stabilisation control, are presented and compared to a Matlab/Simulink time-domain averaged-value simulation results. The HIL platform controlled by the TI DSC setup remains as described in Section 6.1. The non-linear model of the IP-based FOC induction motor is described in Appendices A.1, A.2 and A.3. The nonlinear model of the input RLC filter is given in (5.1). For both the control HIL emulation and the Matlab/Simulink simulation, the experimentally determined parameters of a 350 V 4.3 kW induction motor, remain as listed in Table 3·2, Section 3.3. The input filter parameters are those listed in Section 1.2.2, Table 3·1, and the DC grid voltage \( V_g \) is set to 540 V. The induction machine rotor flux \( \varphi_r \), the speed and \( d-q \) current IP filter time constants and control output saturations, and the gains of the speed and \( d-q \) current PI controllers calculated to achieve a
damping ratio of 0.707 for the closed-loop control with a bandwidth of 10 Hz and 500 Hz, respectively, are listed in Section 3.3.1, Table 3-3. The switching frequency of the inverter $f_s$ is set to 20 kHz and the PWM dead-time is fixed to 130 ns in the Typhoon HIL 600. The non-stabilised system results are first presented, and then the IBSC and PBSC responses are compared. Finally the ASCs design sensitivity to changes in operating points are discussed.

### 6.3.1 Non-Stabilised Responses

Figure 6.5 shows a comparison of the DC-link voltage and IM speed response between the simulation and HIL emulation to change in grid voltage. The speed reference is stepped from 0 rad/s to 60 rad/s at 0.5 s and the induction machine load torque is 0 Nm. At 1.5 seconds the grid voltage decreases from 540 V to 530 V.

![Figure 6.5](image)

Figure 6.5: Responses to a 540 V to 530 V grid voltage step at 1.5 s; IM speed is 60 rad/s and load torque is 0 Nm

Figure 6.5 a) shows that although the DC values for both simulation and HIL emulation DC-link voltage responses agree, the HIL response exhibits high-frequency oscillations. This is due to the inverter high frequency switching,
which is omitted from the non-linear simulation model. The speed response is almost identical for both the simulation and HIL emulation, as shown in Figure 6.5 b). This is due to the speed response being considerably less affected by the high frequency switching since the speed control bandwidth (10 rad/s) is much lower than the switching frequency; it therefore results in a significantly better match between the simulation and HIL emulation responses.

The magnified view of the DC-link voltage transient at 1.5 s shown in Figure 6.5 is shown in Figure 6.6, demonstrates the oscillatory response due to the step in grid voltage. The HIL response agrees with the response obtained from the non-linear simulation.

![Figure 6.6: Magnified DC-link voltage response to a 540 V to 530 V grid voltage step at 1.5 s: IM speed is 60 rad/s and load torque is 0 Nm](image)

DC-link voltage and IM speed responses to a change in grid voltage for both simulation and HIL emulation are shown in Figure 6.7. The speed reference and the induction machine load torque are stepped from 0 rad/s to 60 rad/s at 0.5 s and from 0 Nm to 70.4 Nm at 1 s, respectively. The grid voltage reduces from 540 V to 530 V at 1.5 seconds. As observed in Figure 6.5, the emulated DC-link voltage exhibits high-frequency oscillations due to the inverter high-frequency switching as shown in Figure 6.7 a). These are significantly increased after the load torque step at 1 s. This is suggested by Figure 3.8, Section 3.2.3, since the relative stability of the system is significantly degraded as the load power increases, resulting in a considerably worse DC-link voltage response. Again the speed response from both HIL emulation and simulation show an excellent correlation as shown in Figure 6.7 b).
a) DC-link voltage response

Figure 6.7: Responses to a 540 V to 530 V grid voltage step at 1.5 s: IM speed is 60 rad/s and load torque is 70.4 Nm

Figure 6.8 shows a magnified view of the DC-link voltage response in Figure 6.7 a) to the change in grid voltage at 1.5 s. The responses are similar for both the HIL emulation and simulation. As previously mentioned the relative stability is degraded at high power, which results in increased oscillations compared to Figure 6.6.

b) Speed response

Figure 6.8: Magnified DC-link voltage response to a 540 V to 530 V grid voltage step at 1.5 s: IM speed is 60 rad/s and load torque is 70.4 Nm
Despite the oscillations due to the inverter high switching frequency, exhibited by the HIL DC-link voltage responses in Figure 6.5 a) and Figure 6.7 a), this section validates the IP-based FOC induction motor non-linear model described in Appendices A.1, A.2 and A.3, as shown by Figure 6.5 b), Figure 6.6, Figure 6.7 b) and Figure 6.8. Moreover, the high-frequency oscillations observed on the HIL emulation responses further justify the need for a stabilisation control in order to improve the system relative stability, while also enhancing the system power quality.

### 6.3.2 Comparison Between IBSC and PBSC Responses

In this section a comparison between the IBSC and PBSC response is presented. The stabilisation controllers are implemented in the TI DSC using (6·13) for the IBSC and (6·19) and (6·20) for the PBSC. In order to fairly compare the two stabilisation schemes, they are both tuned to achieve the same speed sensitivity gain at the resonant frequency of the input filter, $\omega_{nf}$. The IBSC tuning method is detailed in Section 4.2. For the PBSC, the tuning in terms of speed sensitivity was introduced in Section 5.2.3 for low torque operating points since the original tuning method described in Section 5.1.3 was ineffective at low torque. Although the low torque PBSC tuning method is less intuitive, it is extended to all operating points in this section for comparison purposes. Both the IBSC band-pass filter and PBSC phase compensator are tuned accounting for the computational delay and ZOH phase shift discussed in Section 6.2.1, whilst $K_D$ is set to 1.0019 using (6·7).

Comparison of the stabilisations schemes is carried out for IM speed of 60 rad/s and 120 rad/s in Sections 6.3.2.1 and 6.3.2.2, respectively, at 100%, 50% and 0% of the IM nominal power, corresponding to a load torque of 70.4 Nm, 35.8 Nm and 0 Nm at 60 rad/s, and 35.8 Nm, 17.9 Nm and 0 Nm at 120 rad/s. Table 6·1 lists the IBSC band-pass corner frequencies, $\omega_{cl}$ and $\omega_{ch}$, and the gain, $K_c$, calculated using (4·17), (4·19) and (4·20), from Section 4.2.2, and (6·8), for a desired gain of the induction machine speed sensitivity function at $\omega_{nf}$, $|S_{cpl}(j\omega_{nf})|^*$, of -35 dB, for each operating point.
Table 6.1: IBSC parameters accounting for the computational delay and ZOH compensation

<table>
<thead>
<tr>
<th>$\omega_m$ = 60 rad/s</th>
<th>$T_i$ (Nm)</th>
<th>$\omega_{cl}$ (rad/s)</th>
<th>$\omega_{ch}$ (rad/s)</th>
<th>$K_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.4</td>
<td>300</td>
<td>4664</td>
<td>84200</td>
<td></td>
</tr>
<tr>
<td>35.8</td>
<td>300</td>
<td>5651</td>
<td>94404</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>5531</td>
<td>20000</td>
<td>449330</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\omega_m$ = 120 rad/s</th>
<th>$T_i$ (Nm)</th>
<th>$\omega_{cl}$ (rad/s)</th>
<th>$\omega_{ch}$ (rad/s)</th>
<th>$K_c$</th>
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<tr>
<td>35.8</td>
<td>300</td>
<td>8381</td>
<td>125570</td>
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</tr>
<tr>
<td>17.9</td>
<td>300</td>
<td>21452</td>
<td>292510</td>
<td></td>
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<tr>
<td>0</td>
<td>5531</td>
<td>20000</td>
<td>449330</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2 gives the PBSC phase compensator corner frequencies, $\omega_p1$ and $\omega_p2$, gain $K_{Pc}$, and the damping injection constant $R_2$ and its corresponding desired RLC input filter undamped natural frequency, $\xi_f^*$, for the same value of $|S_{cpi}(j \omega_{nf})|^*$, for each operating point, using (5.58), from Section 5.2.3, (5.55) and (5.57), from Section 5.2.3, and (6.9).

Table 6.2: PBSC parameters accounting for the computational delay and ZOH compensation

<table>
<thead>
<tr>
<th>$\omega_m$ = 60 rad/s</th>
<th>$T_i$ (Nm)</th>
<th>$R_2$ ((\Omega))</th>
<th>$\xi_f^*$</th>
<th>$\omega_p1$ (rad/s)</th>
<th>$\omega_p2$ (rad/s)</th>
<th>$K_{Pc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.4</td>
<td>0.9823</td>
<td>0.3096</td>
<td>3778</td>
<td>200</td>
<td>0.7464</td>
<td></td>
</tr>
<tr>
<td>35.8</td>
<td>1.9301</td>
<td>0.1617</td>
<td>3116</td>
<td>200</td>
<td>0.8057</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1049</td>
<td>0.0087</td>
<td>3343</td>
<td>20000</td>
<td>3.7937</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\omega_m$ = 120 rad/s</th>
<th>$T_i$ (Nm)</th>
<th>$R_2$ ((\Omega))</th>
<th>$\xi_f^*$</th>
<th>$\omega_p1$ (rad/s)</th>
<th>$\omega_p2$ (rad/s)</th>
<th>$K_{Pc}$</th>
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<tbody>
<tr>
<td>35.8</td>
<td>1.9264</td>
<td>0.1620</td>
<td>2095</td>
<td>200</td>
<td>0.8969</td>
<td></td>
</tr>
<tr>
<td>17.9</td>
<td>3.8391</td>
<td>0.0855</td>
<td>803</td>
<td>200</td>
<td>0.9835</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>524.4856</td>
<td>0.009</td>
<td>3343</td>
<td>20000</td>
<td>3.7937</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.9 shows the DC-link voltage response to a change in grid voltage for the non-stabilised and stabilised with both IBSC and PBSC, IP-based FOC motor drive. At 1.5 seconds the grid voltage is stepped down from 540 V to 530 V. The induction motor operates at 100% of its nominal power, that is, the IM speed and load torque are set to 60 rad/s and 70.4 Nm, respectively, in Figure 6.9 a), and 120 rad/s and 35.8 Nm, respectively, in Figure 6.9 b). The corresponding ASCs parameters are listed in Table 6.1 and Table 6.2, for a desired gain of the induction machine speed sensitivity function at $\omega_{nf}$, $|S_{cpi}(j \omega_{nf})|^*$, of -35 dB, for each operating point. Figure 6.9 a) and b) show that the oscillations around the DC-link voltage DC value are significantly reduced with the IBSC and PBSC, resulting in a considerably improved DC-link voltage...
response for the two operating points. For IM speeds of 60 rad/s and 120 rad/s, the oscillations around the DC-link voltage steady state value, are approximately ±8 V and ±2.5 V respectively with no ASC, and are within ±1.5 V and ±1 V with the IBSC and PBSC respectively, as shown in Figure 6.9 a) and b), respectively. Furthermore the response settling time to a change in grid voltage is significantly reduced. This will be discussed in more detail in Sections 6.3.2.1 and 6.3.2.2. Without the stabilisation, the oscillations are larger for the IM operating at 60 rad/s and 70.4 Nm, due to the fact that the losses are increased for higher torque values resulting in an increased power drawn from the DC side, which therefore further degrades the system relative stability as explained in Section 3.2.3.

Figure 6.9: DC-link voltage responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s

More detailed analysis is presented in Sections 6.3.2.1 and 6.3.2.2, for an induction machine operating at a speed of 60 rad/s and 120 rad/s, respectively. In these sections the IBSC and PBSC schemes are only compared for a DC-link voltage and speed responses to a change in grid voltage. The ASCs cannot be properly compared for a change in IM torque or speed since the degradation
imposed on the DC-link voltage is smaller than the oscillations introduced by the inverter high frequency switching

### 6.3.2.1 Speed of 60 rad/s

Figure 6.10 shows the HIL emulation DC-link voltage and IM speed response for a step in grid voltage from 540 V to 530 V at 1.5 s with no ASC, IBSC and PBSC. The IM is loaded at 70.4 Nm. The corresponding ASCs parameters are listed in Table 6·1 and Table 6·2, for \(|S_{cpl}(j\omega_n)|^*\)=-35 dB.

![DC-link voltage response](image1)

![Speed response](image2)

Figure 6.10: Responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s: IM speed is 60 rad/s and load torque is 70.4 Nm

The DC-link voltage response is significantly improved with both the IBSC and PBSC as shown in Figure 6.10 a). The maximum DC-link voltage deviation from its steady state value is reduced from -9.1V with no ASC to -8.4V with the PBSC and -6.2V with the IBSC. Although the PBSC response exhibits a greater initial deviation, it has a reduced settling time and enables a smoother steady state DC-link voltage with reduced spikes magnitude compared to the IBSC. The response settling time is approximately 2ms and 1ms with the IBSC and PBSC, respectively. It is however difficult to compare the DC-link voltage results with
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The no ASC response due to the amount of oscillations shown in Figure 6.9 a). Significantly less oscillation is evident on the speed response shown in Figure 6.10 b). Figure 6.10 b) shows that the impact of the stabilisation control on the IM speed is greater with the IBSC, displaying a maximum speed deviation of -0.65 rad/s as opposed to 0.19 rad/s with the PBSC. However, it is important to notice that the speed error introduced by the IBSC is only around 1%.

A magnified view of the DC-link voltage and IM speed response presented in Figure 6.10 are shown in Figure 6.11 a) and b), respectively at 1.5 s. The no ASC response is omitted for clarity, whilst both IBSC and PBSC responses obtained from the non-linear Matlab/Simulink simulation are included.

![Figure 6.11: Magnified responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s: IM speed is 60 rad/s and load torque is 70.4 Nm](image)

Figure 6.11 b) shows a good correlation between the HIL emulation and simulation results for the speed response with both IBSC and PBSC. However there is slightly more difference between the DC-link voltage responses presented in Figure 6.11 a). This is due to the inverter high-frequency switching mechanism, which is omitted from the simulation model, and the fact the
inverter high-frequency switching has a greater impact on the DC-link voltage than on the IM speed due to the low natural frequency of the IM mechanical system. This therefore results in a better match between the HIL emulation and simulation for the speed response.

HIL emulation DC-link voltage and IM speed response for non-stabilised and stabilised with IBSC and PBSC motor drive are presented in Figure 6.12, for a step in grid voltage from 540 V to 530 V at 1.5 s. The IM speed is kept at 60 rad/s, whilst the load torque is now set to 35.8 Nm. The corresponding ASCs parameters can be found in Table 6-1 and Table 6-2 for $|S_{cpl}(j\omega_{nf})|^* = -35$ dB.

Both IBSC and PBSC again result in a considerably enhanced DC-link voltage response as shown in Figure 6.12 a), with a significant reduction in the magnitude of the oscillations. The maximum DC-link voltage deviation from its steady state value is reduced from $\cdot 10.1$ V with no ASC to $\cdot 8.2$ V with the IBSC and $\cdot 7.7$ V with the PBSC, whilst the settling time is significantly reduced from approximately 40 ms with no ASC to 2 ms and 1 ms with the IBSC and PBSC,
respectively. Here the response settling time with no ASC can be obtained since the response contains reduced oscillations with the IM operating at 50% of its nominal power. The cost of the stabilisation schemes is highlighted in Figure 6.12 b), where the slight degradation in the speed response due to the action of the ASCs is visible, and the maximum deviation is -0.65 rad/s and 0.18 rad/s for the IBSC and PBSC, respectively.

Figure 6.13 shows a magnified view of the DC-link voltage and IM speed response presented in Figure 6.12 at 1.5 s; the IBSC and PBSC simulation responses are added and the no ASC response is omitted. Both DC-link voltage and speed responses in Figure 6.13 a) and b), respectively, show an excellent correlation between the HIL emulation and the simulation results.

The simulation results will be omitted in the remainder of this chapter since good correlation between the HIL emulation and Matlab/Simulink simulation results have been demonstrated in this section.

![DC-link voltage response](image1)

![Speed response](image2)

**Figure 6.13:** Magnified responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s; IM speed is 60 rad/s and load torque is 35.8 Nm
Figure 6.14 shows the HIL emulation DC-link voltage and IM speed response for a step in grid voltage from 540 V to 550 V at 1.5 s with no ASC, IBSC and PBSC for the same IM operating condition as in Figure 6.10. Again the ASCs parameters are calculated for $|S_{cp}(j\omega_n)| = 35$ dB as listed in Table 6.1 and Table 6.2.

Both the IBSC and PBSC result in significantly improved DC-link voltage response as shown Figure 6.14 a) with a reduction of the maximum DC-link voltage deviation from its steady state value from $-9$ V with no ASC to $-7.3$ V with the PBSC and $-6.7$ V with the IBSC. Again the PBSC response enables a smoother steady state DC-link voltage with reduced spikes magnitude compared to the IBSC, despite a greater initial deviation. The response settling time is significantly reduced from approximately 42 ms with no ASC to 2.5 ms and 1.5 ms with the IBSC and PBSC, respectively. The impact of the stabilisation control on the IM speed is presented in Figure 6.14 b), showing a maximum
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speed deviation of 0.55 rad/s and -0.24 rad/s with the IBSC and PBSC, respectively. Figure 6.14 shows that the proposed ASCs result in significantly improved DC-link voltage response with a speed error below 1%, for a step up in grid voltage, however, in the remainder of this chapter grid voltage step down cases are investigated since according to (3.12) in Section 3.2.2, grid voltage step down is a worse case in terms of stability as it reduces the system relative stability.

6.3.2.2 SPEED OF 120 RAD/S

Figure 6.15 shows the HIL emulation DC-link voltage and IM speed response for the non-stabilised and stabilised IM motor drive with IBSC and PBSC for a reduction in grid voltage from 540 V to 530 V at 1.5 s. The IM speed and load torque are set to 120 rad/s and 35.8 Nm, respectively. Table 6.1 and Table 6.2 list the ASCs parameters for this operating point, to achieve a desired gain of the induction machine speed sensitivity function at \( \omega_{nf} \), \( |S_{cp}(j\omega_{nf})| \), of -35 dB. Figure 6.15 a) shows that the DC-link voltage response is markedly enhanced by both the IBSC and PBSC, which exhibit a significant reduction in the time to reach steady state. The response settling time is reduced from around 40ms with no stabilisation control to approximately 2.5 ms with both IBSC and PBSC schemes. The DC-link voltage deviation from its steady state value is reduced from -9.4 V with no ASC to -8.2 V with the PBSC and -6.6 V with the IBSC. The impact of the stabilisation control on the IM speed is presented in Figure 6.15 b), showing a maximum speed deviation of -0.70 rad/s and -0.30 rad/s with the IBSC and PBSC respectively; the speed error introduced by both the stabilisation schemes is below 1%.
The HIL emulation DC-link voltage and IM speed response for the non-stabilised and stabilised IM drive with IBSC and PBSC are presented in Figure 6.16, for a step in grid voltage from 540 V to 530 V at 1.5 s. The IM speed is kept at 120 rad/s, however the load torque is now constant at 17.9 Nm. The corresponding ASCs parameters are listed in Table 6·1 and Table 6·2 for $|S_{cpi}(j\omega_{nf})|^2=35$ dB. Again both IBSC and PBSC provide a noticeably improved DC-link voltage response as shown in Figure 6.12 a). The settling time is significantly reduced from approximately 35 ms with no ASC, to 2.5 ms for both IBSC and PBSC, respectively; the maximum DC-link voltage deviation from its steady state value is reduced from ~9.4 V with no ASC to ~8.9 V with the IBSC and ~8.5 V with the PBSC. The DC-link voltage stabilisation results in a small speed deviation of ~0.7 rad/s with the IBSC and ~0.5 rad/s with the PBSC.
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Figure 6.16: Responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s; IM speed is 120 rad/s and load torque is 17.9 Nm

6.3.2.3 ZERO TORQUE

Figure 6.17 shows the HIL emulation DC-link voltage and IM speed response for a step in grid voltage from 540 V to 530 V at 1.5 s with the induction machine operating at 60 rad/s and 0 Nm load torque. The IBSC and PBSC parameters for this operating point are listed in Table 6-1 and Table 6-2, for $|S_{cpl}(j\omega_n)|^{-35}$ dB. The DC-link voltage response is enhanced with the IBSC, with a settling time reduction from approximately 50 ms to 12 ms, however, the PBSC does not provide any improvement as shown in Figure 6.17 a). Figure 6.17 b) shows that the IM speed control is slightly affected by the IBSC with a maximum deviation of 0.3 rad/s, whilst the PBSC does not introduce any speed error.
This can be explained by considering Section 5.2.2.4 which showed the PBSC stabilisation gain is inversely proportional to the torque producing current component, which results in a large gain at low torque, and a reduction in the system phase margin which degrades the system relative stability. Furthermore, it was shown in Figure 5.7, Section 5.2.2.4, that the increase in stabilisation gain resulted in a significantly degraded speed response at low torque due to the increased speed sensitivity to change in DC-link voltage. To overcome these issues a PBSC low torque tuning was introduced in Section 5.2.4, in order to maintain satisfying stability margins and performance at low torque. However, despite the low torque tuning the limitations of the PBSC at zero torque were underlined in Section 5.3.2. Although simulation results shown that for a desired gain of the induction machine speed sensitivity function at $\omega_{nf}$, $|S_{cpl}(j\omega_{nf})|^*$, of -21 dB, the oscillations were slightly reduced as presented in Figure 5.24 a), Section 5.3.2; this resulted in increased speed degradation in Figure 5.24 b), Section 5.3.2, due to the large stabilisation gain leading to a saturation of the $q$ axis voltage reference as highlighted in Figure...
5.25, Section 5.3.2. Here, an increase in $|S_{cpl}(j\omega_{nf})|^*$ results in an even more degraded speed control and DC-link voltage response due to the influence of the high frequency PWM and dead-time, which are neglected from the simulation. The PBSC $|S_{cpl}(j\omega_{nf})|^*$ must therefore be kept within reasonable limits in order to maintain good speed control performance. This shows that the PBSC scheme is ineffective at zero torque, however, 0 Nm load torque represents the worst case, and the PBSC provides improved results for operating points above 5% of the IM nominal power.

### 6.3.3 ASC Sensitivity to Changes in Operating Point

In this section the ASCs design sensitivity to changes in IM operating point is first considered in Section 6.3.3.2, then the impact on the stabilisation schemes of design error due and changes in input RLC filter parameters are discussed in Sections 6.3.3.3 and 6.3.3.3.

#### 6.3.3.1 Speed Transient

Figure 6.18 shows the HIL emulation DC-link voltage and IM speed response for a step in grid voltage from 540 V to 530 V at 1.5 s with no ASC, IBSC and PBSC. The IM load torque is set to 35.8 Nm, whilst the speed originally set to 60 rad/s is stepped to 120 rad/s at 1.45 s, so that the grid voltage step occurs during the speed transient. The ASCs are designed for the IM operating condition 60 rad/s and 35.8 Nm as listed in Table 6.1 and Table 6.2, for $|S_{cpl}(j\omega_{nf})|^*$=−35 dB. Figure 6.18 a) shows that the DC-link voltage response is significantly improved with both the IBSC and PBSC, with a reduction of the maximum DC-link voltage deviation from its steady state value from ≈12.5 V with no ASC to ≈9.9 V with the PBSC and ≈7.2 V with the IBSC. Again the response settling time is significantly reduced from approximately 46 ms with no ASC to 2 ms with both the IBSC and PBSC. Figure 6.18 b) shows that due to the speed transient the speed error resulting from the stabilisation control is not noticeable. Figure 6.18 demonstrates the robust performance of the proposed ASCs, enabling a considerable enhancement of the DC-link voltage response during a speed transient without a noticeable speed error.
6.3.3.2 Errors in IM Operating Point

Figure 6.19 shows the HIL emulation DC-link voltage and speed response for a step in grid voltage from 540 V to 530 V at 1.5 s. The induction machine is operating at a speed of 60 rad/s and at 50% of its nominal power ($T_1 = 35.8 \text{ Nm}$). However the ASCs are designed for the operating point 60 rad/s and 70.4 Nm as given in Table 6-1 and Table 6-2, for $|S_{cp}(j\omega_{nf})| = -35 \text{ dB}$. Figure 6.19 a) shows that despite a 50% error on the operating point around of which the ASCs are designed, the DC-link voltage response is still significantly improved with both IBSC and PBSC schemes. The DC-link voltage settling time is reduced from approximately 42 ms with no stabilisation control to approximately 2 ms with the IBSC and PBSC. The maximum deviation on the IM speed caused by the IBSC and PBSC are -0.63 rad/s and 0.33 rad/s, respectively.
6.3 Hardware-in-the-Loop Results

In Figure 6.20 the design of the ASCs remain unchanged; however the induction machine is now operating at a speed of 60 rad/s and at 25% of its nominal power ($T_l = 17.9$ Nm), which represents a 75% ASC design error. Figure 6.20 a) shows that the DC-link voltage response to a step in grid voltage from 540 V to 530 V at 1.5 s, remains considerably enhanced with the IBSC, exhibiting a settling time reduction from approximately 40 ms with no stabilisation control to 4 ms. Although the PBSC DC-link voltage response is worse than the IBSC, it still provides an improvement in term of settling time, shortening it to 11 ms, when compared to no stabilisation control. However, Figure 6.20 b) shows that the speed degradation is increased with the PBSC, with a maximum deviation of $-1$ rad/s. The maximum speed error introduced by the IBSC is $-0.69$ rad/s.
The degradation of the responses with the PBSC is due to the saturation of the $q$ axis voltage reference shown in Figure 6.21. Section 5.2.2.4 it was shown that the PBSC stabilisation signal is inversely proportional to the torque producing current component, resulting in a large gain at low torque. Furthermore it is shown in Table 6.2 that the PBSC designs at higher torque values, for $|S_{cp}(j\omega_{nf})|^2=35\,\text{dB}$, result in an increased contribution of the stabilisation scheme\textsuperscript{16}. Therefore the low torque operating point, combined with a PBSC design for high-torque operations leads to a significant increase of the PBSC contribution which results in saturation of the $q$ axis voltage reference. The $q$ axis voltage reference generated with the IBSC remains within the saturation limit.

\textsuperscript{16} The contribution of the stabilisation control increases with the desired undamped natural frequency of the input RLC filter, $\xi_f$. 

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\textbf{Figure 6.20:} Responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s; IM speed is 60 rad/s and load torque is 35.8 Nm; 75\% error on the operating point used for the design.
6.3 HARDWARE-IN-THE-LOOP RESULTS

6.3.3.3 CHANGE IN FILTER PARAMETERS

The DC-link voltage and speed response obtained from the HIL emulation, for a step in grid voltage from 540 V to 530 V at 1.5 s, are presented in Figure 6.22. The induction machine speed is set to 60 rad/s and the load torque is 35.8 Nm. Table 6-1 and Table 6-2 list the ASCs parameters for this operating point, for $|S_{cpl}(j\omega_n)|^*=-35$ dB. The resistor and inductance of the input RLC filter are reduced by 25% to 7.5 mΩ and 105 μH, respectively, whilst the capacitor is kept as listed in Table 3-1, Section 1.2.2; this scenario represents uncertainty in filter parameters or cable length between the motor drive and the DC bus. Both the IBSC and PBSC strategies are designed with the original input RLC filter parameters listed in Table 3-1, Section 1.2.2. Figure 6.22 a) shows that despite the change in input filter parameters, the IBSC and PBSC provide a noticeable reduction in the oscillations with a minimisation of the settling time from 42 ms with no stabilisation control to 2 ms for both stabilisation schemes. The maximum DC-link voltage deviation from its steady state value is reduced from $-11.2$ V with no ASC to $-9.2$ V with the PBSC and $-8.5$ V with the IBSC. The maximum speed deviation introduced by the IBSC and PBSC are $-0.58$ rad/s and $0.24$ rad/s, respectively, as shown in Figure 6.22 b). In this case, the resonant frequency of the input RLC filter with the modified parameters is equal to 4.88 krad/s, whilst the resonant frequency with the original parameters is 4.23 krad/s. In Chapter Four and Chapter Five the ASCs are designed to be the most effective around the resonant frequency of the input filter, and so both frequencies are within the same region, the ASCs performances are not significantly affected by the change in parameters.
Figure 6.22: Responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s; IM speed is 60 rad/s and load torque is 35.8 Nm; 25% reduction of the resistance and inductance of the RLC filter

Figure 6.23 shows the HIL emulation DC-link voltage and speed response for the same IM operating point, and with the same ASCs design as in Figure 6.22. However in this case, the resistance and inductance of the input RLC filter are reduced by 50% to 5 mΩ and 70 μH, respectively, whilst the capacitor is kept as listed in Table 3.1, Section 1.2.2. Figure 6.23 a) shows that the performance of the IBSC and PBSC are both degraded, although they still result in a slightly improved DC-link voltage response. The settling time is 18 ms and 30 ms for the IBSC and PBSC, respectively, and 35 ms for the non-stabilised system. The impact on induction machine speed resulting from the stabilisation controls is shown in Figure 6.23 b), where the maximum speed deviation is -0.6 rad/s for the IBSC, and -0.43 rad/s for the PBSC.
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In the case of the IBSC, the performance degradation is due to the fact that, as detailed in Section 4.2.2, the band-pass filter is designed around the resonant frequency of the input RLC filter which is originally 4.23 krad/s, however with the reduction by 50% of the resistance and inductance the resonant frequency is increased to 5.98 krad/s. Therefore the stabilising signal, which must be injected at the resonant frequency of the input RLC filter for the control to be optimal, is attenuated by the band-pass filter resulting in less effective stabilisation performance. The PBSC does not use band-pass filtering, which should therefore maintain satisfactory stabilisation even though the stabilisation scheme has been designed around a different frequency. However, it was shown in Figure C.3 and Figure C.4, of Appendix C.1, that the stabilisation signal reference significantly increases with the resonant frequency of the input filter. This leads the $q$ axis voltage reference reaching the saturation, with the PBSC, as shown in Figure 6.24, resulting in degraded stabilisation.

Figure 6.23: Responses with ASCs to a 540 V to 530 V grid voltage step at 1.5 s: IM speed is 60 rad/s and load torque is 35.8 Nm; 40% reduction of the resistance and inductance of the RLC filter.
The DC-link voltage and speed response obtained from the HIL emulation to a step in grid voltage from 540 V to 530 V at 1.5 s, for the same IM operating point, and with the same ASCs design as in Figure 6.22 and Figure 6.23, are presented in Figure 6.25 for the scenario where the capacitor is kept as listed in Table 3-1, Section 1.2.2, whilst the resistance and inductance of the input RLC filter are increased by 400% to 40 mΩ and 560 μH, respectively. The settling time of the DC-link voltage response is improved from 62 ms without stabilisation control to 20 ms with the PBSC as shown Figure 6.22 a). However the IBSC strategy results in a worse DC-link voltage response. The impact of the ASCs on the IM speed is presented in Figure 6.22 b), with a maximum speed deviation of ~0.16 rad/s with the PBSC whilst the IBSC results in speed oscillations around the desired value of 0.5 rad/s peak to peak magnitude.
In this case the PBSC maintain a good stabilisation of the DC-link voltage despite the changes in filter parameters. As previously mentioned, the PBSC does not use band-pass filtering, which therefore lessens its sensitivity to change in resonant frequency. Furthermore, the increase in filter parameters results in a decreased resonant frequency equal to 2.11 krad/s, which reduces the stabilisation contribution of the PBSC as shown in Figure C.3 and Figure C.4, Appendix C.1. This decrease in resonant frequency significantly degrades the response of the system with the IBSC. In fact, the IBSC band-pass filter introduces a +90° phase shift in the low frequency region, therefore, for fixed band pass corner frequencies, as the resonant frequency of the input filter reduces, the phase of the system at the resonant frequency increases leading to a reduced system phase margin as shown in Figure 4.9, Section 4.2.2.
### 6.3.4 Discussion

The results for a grid voltage step down presented in Section 6.3.2 have been summarised in Table 6.3 in terms of settling time reduction and speed deviation ratio. Both methods significantly improve the DC-link voltage response for an IM operating at 100% and 50% of its nominal power, with a settling time reduction to a step in grid voltage ranging from 91% to around 98%. The IBSC introduces a small speed deviation going from 0.6% to 1.1% for these operating points, whilst the PBSC causes a reduced speed degradation with a deviation maintained below 0.5%. Significant DC-link voltage response enhancement was also demonstrated for a grid voltage step up, with a settling time reduction of 94% and 96%, for a speed deviation of 0.9% and 0.4% with the IBSC and PBSC, respectively, for an IM operating at 60 rad/s and 100% of its nominal power. It was shown in Section 6.3.2.3 that the IBSC scheme maintains satisfactory performance at zero torque, with a 76% and 75% reduction of the settling time for a speed deviation of 0.5% and 0.3% at 60 rad/s and 120 rad/s, respectively. However the PBSC is ineffective at zero torque as shown in Figure 6.17.

<table>
<thead>
<tr>
<th>Settling time reduction</th>
<th>Speed deviation</th>
<th>Settling time reduction</th>
<th>Speed deviation</th>
<th>Settling time reduction</th>
<th>Speed deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBSC</td>
<td>&gt;=98%</td>
<td>1.1%</td>
<td>91%</td>
<td>0.6%</td>
<td>93%</td>
</tr>
<tr>
<td></td>
<td>&gt;=98%</td>
<td>0.3%</td>
<td>92%</td>
<td>0.4%</td>
<td>94%</td>
</tr>
<tr>
<td>PBSC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

The ASC selection could therefore depend on the IM operating point at which the system must be stabilised and the tolerable speed degradation. Furthermore, although the PBSC requires an extra sensor or the use of an observer for the grid current, the stabilisation control can be tuned to achieve the desired DC-link voltage performance using the method introduced in Section 5.1.3, whilst satisfactory stability margins can be obtained with the phase...
compensator introduced in Section 5.2.3; whereas, the IBSC only allows tuning in term of stability margins.

For applications with varying IM torque or speed the ASC sensitivity to change in IM operating point must also be taken into account. The robust performance of the proposed ASCs were demonstrated in Section 6.3.3.1 demonstrated, as both the IBSC and PBSC enabled a 96% reduction of the DC-link voltage response settling time during a speed transient without a detectable speed error. It was shown in Figure 6.19 that both ASCs provide good results for an error of 50% in the IM operating point, with a 95% reduction of the DC-link voltage response settling time. The stabilisation caused a 0.6% and 1.1% speed deviation with the PBSC and IBSC, respectively. With a 75% error in the IM operating point, the IBSC maintain a satisfactory stabilisation with a 90% reduction in the DC-link voltage response settling time for a 1.2% speed deviation. However the PBSC performance is degraded with a 72% reduction of the DC-link voltage response settling time for a 1.7% speed deviation. Therefore depending on the range of operating point at which the system operates, the IBSC might be more suitable, although it is important to notice that the PBSC results in a smaller speed deviation up to at least a 50% error in IM operating point. Moreover the ASC sensitivity to error in IM operating point could be improved using online tuning techniques, which have not been investigated in this thesis.

The sensitivity of the ASCs to changes in the input filter resistance and inductance values was considered in Section 6.3.3.3. Although an input filter has been considered in this thesis, the resistance and inductance can also represent cable line impedance. Therefore a change in resistance and inductance values could characterise a network reconfiguration leading to different line impedance, which could affect the stabilisation control. Satisfactory DC-link voltage stabilisation was achieved for a 25% reduction of the resistance and inductance as shown in Figure 6.22, with the IBSC and PBSC, resulting in a 95% reduction of the DC-link voltage response settling time for a 0.97% and 0.4% speed deviation, respectively. As the resistance and inductance are further decreased to 50% of their original values both schemes performance is significantly degraded. For a 400% resistor and inductor value increase the PBSC exhibits acceptable performance as the speed deviation is
maintained below 0.3% to achieve a 67% reduction of the DC-link voltage response settling time, as shown in Figure 6.25. However it was shown that due to the band-pass filter the IBSC introduced significant oscillations on both speed and DC-link voltage, by degrading the phase margin. The PBSC has a reduced sensitivity to changes in input filter or line impedance parameters, compared to the IBSC, especially for an increase in resistor and inductor value. It was shown in Chapter Four and Chapter Five, that the IBSC and PBSC techniques are designed to be the most effective around the resonant frequency of the input filter, thus, for applications where the resonant frequency is likely to vary due to change in line impedance, online parameter estimation might be required to maintain a satisfactory system relative stability.

6.4 Summary

The impedance-based and passivity-based stabilisation control techniques introduced in Chapter Four and Chapter Five, respectively, were validated and compared in this chapter. The HIL emulation platform and the DSC card used for the validation of the stabilisation schemes were first presented. The discretisation of the ASCs for the implementation into the digital controller was then described. Computational time delay and ZOH compensation was added to both ASCs, to enable accurate cancellation of its phase shift. The IBSC and PBSC were validated and thoroughly evaluated and compared using HIL emulation results. Moreover, the sensitivity of the stabilising controllers to changes in system parameters was investigated. Finally the benefits and limitations of each method were discussed and possible enhancements of the stabilisation controls were proposed.
Chapter Seven

CONCLUSION

This chapter summarises the PhD findings and provides the concluding remarks in Section 7.1. The contribution to knowledge of this thesis is highlighted in Section 7.2. Finally, future work opportunities are discussed in Section 7.2.

7.1 GENERAL CONCLUSIONS

The increased penetration of DGs with the transport electrification trend, combined with significant advances in power electronic converters and their control has led to DC systems being proposed for applications such as MGs, HEVs, EVs, FCVs, MEAs and ships. The benefits of DC over AC for those applications essentially being the reduced amount of conversion stages and losses, improved efficiency and power quality, increased transmitted power and minimised weight and size of on-board DPSs, as highlighted in Chapter One. Stability, where for example, oscillations in the DC-bus voltage exceed protection levels causing the network to be disconnected, was identified as one of the main challenges related with DC systems due to the increased dynamic interactions in multi-converter systems: system stability degradation due to CPL was especially highlighted.
Almost 50% of the global electrical energy consumption was attributed to electric motors in 2011, a significant portion of those motors being IM, due to their robustness, low-cost, maintenance-free operation, maturity of the technology and the high-performance enabled by FOC techniques. It was also shown that IMs are used for electric propulsion in application such as HEVs, EVs, FCVs and ship. This has therefore motivated the research presented in this thesis, which aimed to devise, demonstrate and evaluate load side active stabilisation techniques for DC systems; in this thesis the candidate load side system was a FOC IM drive as this form a common CPL.

The concept of a CPL and its destabilising negative incremental resistance characteristic were presented in Chapter Two. It was shown that due to the multi-converter configuration of DC MGs and DPSs, stabilisation could be achieved at the source, interface or load converter level. Source and interface side stabilisation techniques proposed in the literature were reviewed. Multi-loads configurations were also addressed using centralised stabilisation controllers; however this technique requires a detailed mathematical model of the entire system and an extensive communication network. It was concluded that the unmodelled load dynamics, due to the assumption made of an ideal CPL for the design of the abovementioned controllers, could significantly limit the effectiveness of the stabilisation schemes in practical cases. Load side active stabilisation techniques proposed in the literature were then outlined. The benefits of load side active stabilisation control were emphasised, however the performance degradation introduced by load side ASCs, and the lack of a systematic and rigorous approach in the design of the controllers was highlighted. The impedance-based criteria reviewed in Chapter Two showed that the ESAC criterion could form the basis of a systematic and rigorous design approach of load side ASCs, accounting for system gain and phase margins. The ESAC criterion was used in Chapter Four for the design of an impedance-based active stabilisation controller (IBSC) and was combined with the passivity-based control technique, reviewed in Chapter Two, to propose a passivity-based active stabilisation controller (PBSC) in Chapter Five.

The stability of an ideal CPL connected to a feeder system, which can be characterised by an input filter, cable impedance and/or DC link capacitance, was addressed in Chapter Three. System poles analysis showed that the feeder
system impedance directly affects the system relative stability. In the case of a RLC input filter, an increased capacitor or resistor results in improved relative stability, however, it was shown that large resistive and capacitive elements are undesirable to optimise the system performance since the former increases the power losses, while the latter tends to be minimised for reliability purposes and volume and weight requirements. Furthermore, the poles analysis showed that small values of inductance enhance the relative stability of the system, whilst this in contradiction with the regulations on harmonics suppression which require a large filtering inductor. These remarks highlighted the unavoidable trade-off between relative stability and performance, and further justified the investigation of ASC techniques. Frequency-domain analysis of an ideal CPL connected to a RLC input filter was also presented, outlining the fundamental destabilising characteristic of CPLs when interacting with feeder systems due to the degraded stability margins. Finally, the stability of a non-ideal CPL, consisting of an IP-based FOC IM, connected with a feeder system was presented.

The IBSC technique using band-pass filtering for an IP-based FOC IM drive connected to a feeder system, consisting of an ideal DC voltage source and a RLC input filter, was proposed in Chapter Four. A rigorous tuning method was presented based on the ESAC criterion, which enables the definition of a forbidden region in terms of the desired phase and gain margins for the Nyquist contour of the minor-loop gain function (product of the source output transfer function and IM input admittance) of the linear feedback model of the CPL with a RLC input filter. It was shown that one of the main conditions to satisfy the ESAC criterion was to reduce the phase shift introduced by the IM input admittance around the RLC input filter resonant frequency. Equations enabling the calculation of the IBSC band-pass filter corner frequencies to achieve a desired phase shift of the IM compensating admittance (admittance which allows to modify the total IM input admittance) at the RLC input filter resonant frequency, were proposed. The gain of the IBSC band-pass filter was calculated to achieve a desired magnitude of the IM speed sensitivity function to changes in DC-link voltage at the resonant frequency of the input RLC filter. The proposed IBSC design performed a trade-off between stability margins, damping effectiveness and speed sensitivity. Minimising the minor-loop gain function phase shift at the resonant frequency of the input filter, provided a significant
phase margin, allowing a considerable increase in the stabilisation gain of the IBSC band-pass filter whilst maintaining the Nyquist contour outside the forbidden region defined by the ESAC criterion. Furthermore, the speed control degradations were kept to predefined limits. The proposed IBSC technique was evaluated using extensive frequency-domain and time-domain simulations.

The PBSC technique for an IP-based FOC IM drive connected to the same feeder system as in Chapter Four was proposed in Chapter Five. The control scheme was first derived for an ideal CPL and an intuitive tuning method was proposed for the PBSC damping injection constants, in terms of desired undamped natural frequency and damping ratio of the input RLC filter. The control law was then adapted for the IP-based FOC IM drive and simplified for implementation purposes. The control law did not provide any phase compensation mechanism to minimise the minor-loop gain function phase shift at the resonant frequency of the RLC input filter, therefore leading to a violation of the ESAC criterion. To overcome this issue a phase compensator was introduced. The phase compensator was designed using the same rigorous method as in Chapter Four, based on the ESAC criterion, to achieve a desired phase shift of the IM compensating admittance at the RLC input filter resonant frequency. In order to minimise the effect of the phase compensator on the stabilisation control, its gain at the RLC input filter resonant frequency was set to unity. It was shown that the original tuning method proposed was ineffective at low torque since it resulted in significantly degraded stability margins and speed sensitivity to changes in DC-link voltage. Therefore a low torque tuning technique was proposed: enabling the desired magnitude of the IM speed sensitivity function to changes in DC-link voltage at resonant frequency of the input RLC filter, to be achieved by calculating the appropriate PBSC damping injection constant. This technique is similar to the design method proposed in Chapter Four. The proposed PBSC technique and tuning methods were thoroughly evaluated using frequency-domain and time-domain simulations.

The IBSC and PBSC techniques were demonstrated and compared using a control hardware-in-the-loop (HIL) emulation platform. Both controllers were discretised and a computational time delay and zero-order hold block compensation was proposed to optimise the effectiveness of the ASCs. Significant improvement in terms of relative stability was demonstrated for
both stabilisation techniques with a settling time reduction of the DC-link voltage response to changes in grid voltage, of at least 90% for the IM operating at 50%, and above, of its nominal power. The IBSC technique introduced a small IM speed control degradation with speed deviation ranging from 0.6% to 1.1% for different IM operating points at 50%, and above, of its nominal power. For the same operating conditions, the PBSC enabled further reductions in the speed deviation to around 0.3%. However, it was shown that the PBSC was ineffective at zero torque whilst the IBSC resulted in DC-link voltage response settling time of approximately 75% for a small speed deviation comprise between 0.3% and 0.5% for the IM operating points investigated. ASCs sensitivity to design and parameter errors was thoroughly examined, highlighting the robustness of the ASC schemes. However, it was shown that in practical cases, online tuning might be required in order to extend the range of operating points which can be stabilised by both the IBSC and PBSC.

The aim of this thesis which was to devise, demonstrate and evaluate load side active stabilisation techniques for DC systems using a FOC IM drive, has been achieved. The main contributions to knowledge of this thesis are listed in Section 7.2.

### 7.2 Main Contributions to Knowledge

The main contributions to knowledge of this thesis are:

- Detailed review of existing active stabilisation control techniques for source, interface and load converter level in DC MG and DPS.

- Devised an IBSC technique, with rigorous tuning based on the ESAC criterion and speed sensitivity function magnitude, for FOC IM drives.

- Developed a PBSC with phase compensation technique for FOC IM drives, with tuning of the control law in terms of desired undamped natural frequency and damping ratio of the input RLC filter, phase compensator design based on the ESAC criterion and a low torque tuning using speed sensitivity function magnitude.
- Practical implementation of the IBSC and PBSC on an industry standard low-cost digital signal controller to provide results from a control HIL system to complement the extensive time and frequency domain study of the proposed controllers.

- More generally this thesis demonstrated that the ESAC criterion based tuning method provided a systematic and rigorous approach for the design of ASCs.

### 7.3 Future Work

One of the future work opportunities is the demonstration of the IBSC and PBSC for FOC IM drive with the Intelligent Electric Power Network Evaluation Facility (IEPNEF) [167]. The IEPNEF is a 100 kW 540 V DPS which consists of real hardware and HIL developed at The University of Manchester to enable the safe emulation of an aircraft power system. The control HIL platform will be used to command existing emulation hardware to enable system level performance of the ASC to be evaluated. A key capability of the demonstrator facility is the flexible DC power network, which is representative of a potential new on-board architecture.

Both the IBSC and PBSC scheme can also be extended to the stabilisation of others CPLs such as PMSM and BLDC motor drives and DC/DC converters feeding resistive loads.

It was shown in Chapter Six that in order to extend the range of operating points which can be stabilised by both the IBSC and PBSC the use of online tuning might be required. The tuning of these ASCs is dependent on the impedance values of the feeder system for the stabilisation to be the most effective; however it was discussed in Chapter Two that one of the main features of future DC MGs and DPSs could be their high flexibility, with a reconfigurable system structure that will result in varying line impedances. Therefore adaptive techniques for ASCs enabling online tuning using estimated value of feeder system parameters is an area which must be investigated in order to widen the range of applications for ASCs.
Finally, with the potential significant increase in size and complexity of DC MG and DPS, for example, in the hybrid aircraft concept, stability will remain one of the main challenges. This thesis proposed stabilisation techniques which are based on the modification of existing control systems, however future control schemes that enable both the performance and stability specification to be achieved at the design level must be investigated.
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Appendix A

INDUCTION MOTOR DRIVE MODEL

A.1 INDUCTION MOTOR MODEL

A.1.1 REFERENCE FRAME

Assuming a balanced three-phase induction motor, the phase quantities can be transformed into a stationary two-phase \(d^s-q^s\) reference frame using the Clarke transform given in (A-1) [177]. The two axis are 90° angle apart.

\[
\begin{bmatrix}
X_q^s \\
X_d^s \\
X_0^s
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos \theta & \cos \left( \theta - \frac{2\pi}{3} \right) & \cos \left( \theta + \frac{2\pi}{3} \right) \\
\sin \theta & \sin \left( \theta - \frac{2\pi}{3} \right) & \sin \left( \theta + \frac{2\pi}{3} \right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
X_a \\
X_b \\
X_c
\end{bmatrix}
\]  

(A-1)

where \(\theta\) is the angle between the three-phase and the \(d^s-q^s\) reference frames, and \(X_0^s\) is the zero sequence component.

The stationary \(d^s-q^s\) reference frame can be transformed into a rotating \(d^q\) reference frame using the Park transform given in (A-2) [177]. The benefits of such a reference frame will be later introduced.

\[
\begin{bmatrix}
X_q \\
X_d
\end{bmatrix} = \begin{bmatrix}
\cos \theta_e & -\sin \theta_e \\
\sin \theta_e & \cos \theta_e
\end{bmatrix} \begin{bmatrix}
X_q^s \\
X_d^s
\end{bmatrix}
\]  

(A-2)

A.1.2 INDUCTION MACHINE EQUATIONS

The induction machine consists of two parts, a stationary one, the stator, and a rotating one, the rotor. The stator is connected to a 3-phase AC source, and presents windings evenly distributed to form several poles, and the rotor is usually short-circuited. Both parts are separated by an airgap. The sinusoidal 3-phase voltage applied to the stator will result in a rotating flux in the induction machine airgap. The stator flux rotates at synchronous frequency. This magnetic field will induce voltages across the rotor conductors, which will produce AC currents if the rotor is short-circuited. The interaction between the
stator flux and the rotor flux generated by these currents produces torque, which aims to align both fields. However, rotation of the rotor at synchronous frequency would result in no induced rotor current, since no change in magnetic field would appear across the rotor conductors. Hence, when motoring the rotor magnetic field frequency is slightly reduced compare to the synchronous frequency, and so the rotor speed is slightly below the synchronous speed. This difference between rotor and stator flux frequencies is called slip frequency, $\omega_{sl}$, which is given by:

$$\omega_{sl} = \omega_e - \omega_r,$$

where $\omega_e$ is the frequency of the stator magnetic field and $\omega_r$ represents the frequency of the rotor magnetic field.

The rotor can either be wound or made of bars short-circuited by end rings (squirrel cage). A wound rotor must have the same number of poles as the stator windings, while the squirrel cage automatically matches it [178]. The squirrel cage rotor enables high efficiency and is cheaper to manufacture than the wound rotor due to its simple construction. However, a wound rotor is more suitable for high-torque applications, since the rotor-slip frequency can be controlled to increase the torque [178, 179].

The number of poles of the stator windings influence the speed of the rotor shaft [178], since the stator magnetic field rotates through 2 poles for a complete revolution of current. Therefore, the speed decreases as the pole number increases, as for instance the stator magnetic field of a 6 poles induction machine will need 3 cycles of current to complete a single rotation. The stator flux frequency or synchronous frequency can thus be defined as follow,

$$\omega_e = \frac{4\pi f}{p}$$

where $f$ is the stator voltage frequency, and $p$ the number of poles of the stator windings.

The squirrel cage induction machine can be modelled by representing the rotor cage as an equivalent short-circuited sinusoidal 3-phase winding [179]. In doing so, the induction machine can be considered as a transformer with a rotating short-circuited secondary, where the transformer ratio constantly changes with
the rotor position, [177]. However, this method introduces a time-varying mutual inductance, which increases the complexity of the model. Using the transformation presented in Section A.1.1, the model of the 3-phase induction machine can be reduced to a corresponding 2-phase machine with constant parameters, since variables in \(d'q\) frame are seen as DC components due to the synchronous rotation of the \(d'q\) reference frame. From this, the well-known dynamic T-model of the induction machine for the direct and quadrature axis, introduced in [177], can be derived as shown in Figure A.1. Core loss and saturation are both neglected.

In Figure A.1, \(R_s\) and \(R_r\) represent the stator and rotor resistors, respectively; \(L_{ls}\) and \(L_{lr}\) are the linkage inductances, \(L_m\) the mutual inductance, and the symbol \(\varphi\) denotes the flux. The symbols \(s, r, d\) and \(q\) denote variables referred to the stator, rotor, \(d\) axis and \(q\) axis, respectively.

It can be seen that an emf is introduced at the stator due to the rotation of the \(d'q\) axis at the synchronous frequency \(\omega_e\). Similarly, an emf is included at the rotor, at the slip frequency \(\omega_{sl}\), which is the relative frequency of the rotor flux.
compared to the reference frame. From Figure A.1, the following set of differential equations can be written:

\[ V_{ds} = R_s i_{ds} + \frac{d\varphi_{ds}}{dt} - \omega_e \varphi_{qs} \]
\[ V_{qs} = R_s i_{qs} + \frac{d\varphi_{qs}}{dt} + \omega_e \varphi_{ds} \]
\[ V_{dr} = R_r i_{dr} + \frac{d\varphi_{dr}}{dt} - \omega_s \varphi_{qr} = 0 \]
\[ V_{qr} = R_r i_{qr} + \frac{d\varphi_{qr}}{dt} + \omega_s \varphi_{dr} = 0 \]

where,

\[ \varphi_{ds} = L_s i_{ds} + L_m i_{dr} \]
\[ \varphi_{qs} = L_s i_{qs} + L_m i_{qr} \]
\[ \varphi_{dr} = L_r i_{dr} + L_m i_{ds} \]
\[ \varphi_{qr} = L_r i_{qr} + L_m i_{qs} \]

with, \( L_s = L_{ls} + L_m \) and \( L_r = L_{lr} + L_m \)

The mechanical dynamic of the induction motor can be described as

\[ \frac{d\omega_m}{dt} + B_m \omega_m = \frac{T_e - T_l}{J_m} \]

where \( \omega_m \) is the mechanical speed of the rotor shaft, \( T_l \) is the load torque, \( J_m \) is the machine inertia, \( B_m \) is the coefficient of friction and the induction machine torque is defined as:

\[ T_e = \frac{3p}{2} \frac{L_m}{L_r} (\varphi_{dr} i_{qs} - \varphi_{qr} i_{ds}) \]

The induction machine mechanical transfer function can be determined from (A-7) as:

\[ \frac{\omega_m(s)}{T(s)} = \frac{1}{J_m s + B_m} \]

where \( T = T_e - T_l \).
A.2 FIELD-ORIENTED CONTROL

A strong coupling is apparent between the $d$ and $q$ axis equations (A-5) to (A-8), which makes the control complex. The idea behind field-oriented control (FOC), first introduced in the 1970s, is to control the induction machine like a separately excited dc motor [177], so that the torque and the flux producing components can be controlled independently. This control is realisable when the stator direct current, $i_{ds}$, is oriented in the direction of the rotor flux $\varphi_r$, and $i_{qs}$ is oriented orthogonally to it, with their reference frame rotating at synchronous frequency, as shown in Figure A.2.

![Figure A.2: Rotating $d$-$q$ frame oriented in the direction of rotor flux](image)

It can be seen from Figure A.2 that in such a configuration the $q$ axis rotor flux component is zero. By cancelling the term, $\varphi_{qr}$, in (A-5) the rotor voltage expressions can be rewritten considering $\varphi_r = \varphi_{dr}$, so:

$$\frac{d\varphi_r}{dt} + R_r i_{dr} = 0$$  \hspace{1cm} (A-10)

$$\omega_{sl} \varphi_r + R_r i_{qr} = 0$$

Expressions for the rotor currents can be derived by rearranging (A-6), so:

$$i_{dr} = \frac{1}{L_r} \varphi_r - \frac{L_m}{L_r} i_{ds}$$ \hspace{1cm} (A-11)

$$i_{qr} = -\frac{L_m}{L_r} i_{qs}$$

Since the rotor currents are not measurable they can be substituted in (A-10) by (A-11), and the resulting equations rearrange to give rotor flux and the slip frequency expressions (A-12).
\[
\frac{d\varphi_r}{dt} + \frac{R_r}{L_r} \varphi_r = \frac{R_r L_m}{L_r} i_{ds}
\]

\[
\omega_{st} = \frac{R_r L_m}{\varphi_r L_r} i_{qs}.
\]

To obtain the stator magnetic field position, the slip frequency given by (A-12) is summed with the rotor frequency and then integrated. This is defined as indirect FOC as oppose to direct FOC where the stator magnetic field position is calculated using electromagnetic variables.

From (A-12), the transfer function of the rotor current can then be derived as:

\[
\frac{\varphi_r(s)}{i_{ds}(s)} = \frac{L_m}{\tau_r s + 1}
\]

where \(\tau_r = \frac{L_r}{R_r}\).

From equations (A-12), it can be noted that the rotor flux \(\varphi_r\) only depends on \(i_{ds}\). Likewise, the slip frequency generating the torque is only dependent on \(i_{qs}\), for constant flux operation, which is prerequisite for accurate speed control [177]. Hence, decoupling the flux producing component, \(i_{ds}\), and torque producing component, \(i_{qs}\), has been achieved by orienting the synchronous rotating frame to the direction of the rotor flux. Therefore, a change in \(i_{qs}\) will change the torque while the flux will remain constant.

The stator voltage equations also need to be decoupled so the stator current components can be controlled individually by controlling the terminal voltages of the induction machine [180]. Using (A-5) and (A-6), and assuming a loss-less inverter, the stator voltages can be split in to two terms, a decoupled voltage denoted by an apostrophe, and a decoupling term denoted by subscript ‘c’, so:

\[
V_{ds} = V'_{ds} + V_{dsc}
\]

\[
V_{qs} = V'_{qs} + V_{qsc}
\]

Where \(V_{dsc}\) and \(V_{qsc}\) are the decoupling terms, as follow,

\[
V_{dsc} = -\omega_e \left( L_s - \frac{L_m^2}{L_r} \right) i_{qs} - R_r \frac{L_m}{L_r^2} \varphi_r
\]

\[
V_{qsc} = \omega_e \left( L_s - \frac{L_m^2}{L_r} \right) i_{ds} - R_r \frac{L_m^2}{L_r^2} i_{qs} + \omega_e \frac{L_m}{L_r} \varphi_r
\]
where \( \omega_e = \frac{p}{2} \omega_m + \frac{L_m}{\tau_r \varphi_r} i_{qs} \).

The decoupled voltages on the \( d \) and \( q \) axis are given by:

\[
V'_{ds} = \left( R_s + R_r \frac{L_m^2}{L_r^2} \right) i_{ds} + \left( L_s - \frac{L_m^2}{L_r} \right) \frac{di_{ds}}{dt}
\]

\[
V'_{qs} = \left( R_s + R_r \frac{L_m^2}{L_r^2} \right) i_{qs} + \left( L_s - \frac{L_m^2}{L_r} \right) \frac{di_{qs}}{dt}
\]

(A-16)

The transfer function of the decoupled \( d-q \) stator voltages can be derived from (A-16) as:

\[
\frac{i_{ds}(s)}{V'_{ds}(s)} = \frac{i_{qs}(s)}{V'_{qs}(s)} = \frac{1}{\lambda_1 s + \lambda_2}
\]

(A-17)

where \( \lambda_1 = L_s - \frac{L_m^2}{L_r} \), and \( \lambda_2 = R_s + R_r \frac{L_m^2}{L_r^2} \).

Equations (A-8), (A-9), (A-13) and (A-17), can be used to form the decoupled model of the FOC induction machine shown in Figure A.3.

![Figure A.3: Decoupled d-q induction motor model](image)

**A.2.1 PI Control**

The field oriented control (FOC) is usually implemented using inner current control loops for the \( d \) and \( q \) axis stator currents, cascaded with outer speed and flux control loops. Conventionally, FOC is implemented with proportional-integral (PI) controllers, with their general expression given by:

\[
PI(s) = k_{px} + \frac{k_{ix}}{s}
\]

(A-18)
where the subscript ‘\(x\)’ is ‘\(c\)’ for the current control, and ‘\(\omega\)’ for the speed loop, and \(k_{px}\) and \(k_{ix}\) are the proportional and integral gains respectively.

Figure A.4 shows the inner current and outer speed control structure for a FOC induction motor. The feed-forward decoupling is neglected in Figure A.4, since the induction machine model in Figure A.3 is decoupled. The rotor flux controller has been omitted since we assume that the induction machine operates at constant flux to ensure precise speed control, as previously mentioned.

In Figure A.4, \(i^*_{ds}\), \(i^*_{qs}\) and \(\omega^*_m\) are the reference values for the \(d\)-\(q\) currents and speed controls respectively, \(V^*_{d,qs}\), are the \(d\)-\(q\) stator voltage references used to generate the PWM signals.

The current controllers are designed by deriving the closed-loop current control transfer functions (A·19) by combining the PI equation (A·18) and the decoupled \(d\)-\(q\) stator voltages transfer function (A·17), and assuming an ideal inverter.

\[
\frac{i^*_{ds}(s)}{i_{ds}(s)} = \frac{i^*_{qs}(s)}{i_{qs}(s)} = \frac{k_{pc}s + k_{ic}}{\lambda_1s^2 + (\lambda_2 + k_{pc})s + k_{ic}} \tag{A·19}
\]

Assuming \(k_{pc} \ll k_{ic}\), the zero in (A·19) can be neglected. Therefore, the closed-loop current control transfer function can be associated to a second order system to define the controller gains to achieve the desired current control loop natural frequency, \(\omega_{nc}\), and damping ratio, \(\xi_c\).

Similarly, the speed PI gains can be derived from the closed-loop speed control transfer function given in (A·20), which is formed by combining (A·9) and (A·18). It is assumed that the current control bandwidth is considerably higher than
the speed control bandwidth. Thus, assuming an ideal drive and inverter, the current control can be approximated to a unity gain.

\[
\frac{\omega_m^*(s)}{\omega_m(s)} = \frac{k_{p\omega}s + k_{i\omega}}{J_m s^2 + (B_m + k_{p\omega})s + k_{i\omega}} \quad (A-20)
\]

Again, assuming \( k_{p\omega} \ll k_{i\omega} \), the zero in (A-20) can be neglected and the speed PI gains are derived from the standard second order transfer function, as function of the speed control loop desired natural frequency, \( \omega_{n_\omega} \), and damping ratio, \( \xi_\omega \).

Table A-1 shows the PI gain equations for both speed and current controllers.

<table>
<thead>
<tr>
<th>Current loop</th>
<th>Proportional gain</th>
<th>Integral gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed loop</td>
<td>( k_{pc} = 2\xi_c\lambda_1\omega_{n_c} - \lambda_2 )</td>
<td>( k_{ic} = \lambda_1\omega_{n_c}^2 )</td>
</tr>
<tr>
<td></td>
<td>( k_{p\omega} = 2\xi_\omega J_m\omega_{n_\omega} - B_m )</td>
<td>( k_{i\omega} = J_m\omega_{n_\omega}^2 )</td>
</tr>
</tbody>
</table>

### A.2.2 IP CONTROL

The IP-based field oriented control structure used in this thesis is shown in Figure A.5. \( i_{ds}^* \), \( i_{qs}^* \) and \( \omega_m^* \) are the output of the \( d^*q^* \) current and speed IP filters, respectively. The motivation of this design was to allow the zero in the control closed-loop transfer function to be cancelled in simple manner, using existing PI based FOC.

![Figure A.5: IP-based field oriented control structure (neglecting decoupling)](image)

The IP control is designed by setting the denominator of the filter to perfectly cancel the zero introduced by the PI control in both the current (A-19) and the speed (A-20) closed-loop transfer functions. The IP filter is shown in (A-21) and (A-22) for both \( d^*q^* \) stator currents and speed control. The IP filters gain is then set according to the proportional and integral gains.
The structure of the IP-based speed control shown in Figure A.6, and the \(d-q\) current control if decoupling is neglected is shown in Figure A.7. Saturation blocks are added to the speed and \(d-q\) current control to ensure safe operation. The equations in Table A-1 can be used to determine the proportional and integral gains as in the PI case.

\[
\begin{align*}
\frac{i_{d^*}}{i_{ds}}(s) &= \frac{i_{q^*}}{i_{qs}}(s) = \frac{1}{k_{pc}\frac{k_{ic}}{s} + 1} \\
\frac{\omega_{m^*}}{\omega_m}(s) &= \frac{1}{k_{p\omega}\frac{k_{i\omega}}{s} + 1}
\end{align*}
\] (A-21) (A-22)

The IP filter does not modify the disturbance rejection performance of the PI control since the closed-loop transfer function between the perturbation and the system output remains unchanged. Therefore, the IP control improves the dynamic performance of the system whilst retaining the typical robust disturbance rejection of the conventional PI control.
The expressions enabling the calculation of the saturations, for the speed and current controller outputs are given in (A-23) and (A-24), respectively.

\[ i_{qs}^{sat} = 3 \frac{P_{IM}}{\omega_{m,IM}} \]  
\[ V_{dqs}^{sat} = \frac{2}{\pi} \frac{\sqrt{3}}{2} V_l \]

where \( P_{IM} \) and \( \omega_{m,IM} \) are the induction machine, nominal power and rated speed, respectively, and \( V_l \) is the DC-link voltage.

## A.3 PWM INVERTER

The stator 3-phase sinusoidal voltages are generated by a pulse width modulation (PWM) voltage source inverter (VSI). The VSI consists of three legs, one per phase, with two switches per leg, converting the DC-link voltage, \( V_l \), shown in Figure 3.11, into sinusoidal phase voltage. The switches each have an antiparallel diode and only one switch per leg is turned on at any time. The switches are controlled by sinusoidal PWM phase shifted by 120° for each phase. The inverter allows the stator sinusoidal phase voltage magnitude and frequency to be controlled. The PWM of each phase is generated by comparing the per unit stator \( d'q \) phase voltage reference, given by the FOC, to a triangular waveform, to produce a square signal between 0 and 1, with a sinusoidal duty cycle. For instance, when the PWM of phase \( a \) is equal to 1, the top switch of leg \( a \) is turned on, whilst the bottom switch of leg \( a \) is turned off. The resulting stator phase voltage will contain high frequency harmonics introduced by the switching. The switching frequency is determined by the frequency of the triangular waveform, and must be significantly higher than the stator frequency in order to obtain smooth stator currents.

To model the inverter it is assumed that the switches are ideal, and so the switching and conduction losses can be neglected. Furthermore, only the PWM fundamental is considered by neglecting the high frequency harmonic content. PWM over modulation, duty cycle limits and dead time are also neglected and so the inverter can be represented by the per unit stator \( d'q \) phase voltage references \( V_{d'qs}^{sat} \) multiplied by the DC-link voltage \( V_l \). The delay due to the
switching mechanism is also neglected, however in practice, for the implementation of ASCs the phase shift introduced by delays must be compensated as shown in Section 6.2.1. The simplified inverter model is shown in Figure A.8, and the modified stator voltages $V'_{dqs}$ can be directly connected to Figure A.3.

The DC current drawn by the induction machine drive can be determined from the PWM inverter model, using the power conservation law. The decoupling terms in (A-15) need to be reintroduced in order to accurately describe the behaviour of the grid current and DC-link voltage using the model for the RLC filter derived in Section 3.2.1. Thus, the averaged induction machine drive DC current over a switching period can be expressed as:

$$i_l = \frac{3}{2V_l} \left[ i_{ds}(V'_{ds} + V_{dsc}) + i_{qs}(V'_{qs} + V_{qsc}) \right]$$  \hspace{1cm} (A-25)

**A.4 LINEARISED SMALL-SIGNAL MODEL**

The induction machine model in Appendix A.1 is both complex and exhibits non-linear behaviour and so to enable the relative stability to be examined a state-space model is formed. This model is then used to form a linearised small-signal model of the induction machine drive shown in Figure 3.3.

Equations (3-8), (A-7), (A-8), (A-12), (A-16), (A-18), (A-21) and (A-22) from Appendix A.1 and Chapter Three have been rearranged where necessary to form (A-26) which describes the non-linear behaviour of the induction motor drive system.

$$\frac{di_g}{dt} = -\frac{R}{L} i_g - \frac{1}{L} V_l + \frac{1}{L} V_g$$  \hspace{1cm} (A-26)
A.4 Linearised Small-Signal Model

\[ \frac{dV_i}{dt} = \frac{1}{C_l}i_g - \frac{1}{C_l}i_l \]

\[ \frac{d\omega_m}{dt} = -\frac{B_m}{J_m} \omega_m + 3p \frac{L_m}{2} \frac{1}{L_r} \frac{1}{J_m} \varphi_r i_{qs} - \frac{1}{J_m} T_l \]

\[ \frac{d\varphi_r}{dt} = -\frac{1}{T_r} \varphi_r + \frac{L_m}{T_r} i_{ds} \]

\[ \frac{di_{ds}}{dt} = \frac{1}{\lambda_1} V_{ds} V_l - \frac{\lambda_2}{\lambda_1} i_{ds} \]

\[ \frac{di_{qs}}{dt} = \frac{1}{\lambda_1} V_{qs} V_l - \frac{\lambda_2}{\lambda_1} i_{qs} \]

As the induction machine control in Appendix A.2 achieves constant rotor flux operation, the \( d \) axis decoupled voltage and current, and the rotor flux can be approximated to their steady state values. Furthermore, neglecting the delay introduced by the PWM inverter and approximating the converter behaviour by a unity gain, a 6th order linearised small-signal model of the IM drive connected to the RLC filter can be derived and is shown in state-space form in (A-27).

\[ \frac{d\delta i_g}{dt} = -\frac{R}{L} \delta i_g - \frac{1}{L} \delta V_l + \frac{1}{L} V_g \]  
(A-27)

\[ \frac{d\delta V_l}{dt} = \frac{1}{C} \delta i_g - \frac{K_1}{C} \delta \omega_m - \frac{K_2}{C} \delta i_{qs} - \frac{K_3}{C} \delta V_{qs} - \frac{K_4}{C} \delta V_l \]

17 Using the first term of the Taylor expansion.
where the subscript “0” designates the steady state values; $K_1$, $K_2$, $K_3$ and $K_4$ are gains from the linearisation of the IM drive DC current and are expressed as follow,

\[
K_1 = \frac{3}{2V_{l0}} \frac{pL_m}{2L_r} \varphi_{r0} i_{q0}
\]

\[
K_2 = \frac{3}{2V_{l0}} \left[ V_{q0}^{**} + \frac{pL_m}{2L_r} \varphi_{r0} \omega_{m0} \right]
\]

\[
K_3 = \frac{3}{2V_{l0}} i_{q0}
\]

\[
K_4 = -\frac{3}{2V_{l0}^2} \left[ i_{d0} V_{d0}^{**} - \frac{L_m}{L_r^2} i_{d0} \varphi_{r0} + i_{q0} V_{q0}^{**} + \frac{pL_m}{2L_r} \varphi_{r0} \omega_{m0} i_{q0} \right]
\]

where the steady state values are obtained by setting the derivatives in (A-27) as given in (A-29).

\[
i_{d0} = \frac{\varphi_{r0}}{L_m}
\]

\[
i_{q0} = \frac{4L_r(T_l + B_m \omega_{m0})}{3pL_m \varphi_{r0}}
\]

\[
V_{d0}^{**} = \lambda_2 i_{d0}
\]

\[
V_{q0}^{**} = \lambda_2 i_{q0}
\]
The stability of the adjustable speed induction motor drive can be now investigated by considering the eigenvalues of the 6 by 6 matrix \( A \) of the linearised state-space representation of the system represented as:

\[
\frac{d\delta x}{dt} = A\delta x + Bu
\]  \hspace{1cm} (A-30)

where,

\[
\delta x = [\delta i_g \ \delta V_i \ \delta \omega_m \ \delta i_{qs} \ \delta i_{qs}^* \ \delta V_{qs}^{**}]
\]  \hspace{1cm} (A-31)

\[
u = [V_g \ T_l \ \omega_m^*]
\]
Appendix B

IBSC Band-Pass Filter Corner Frequencies Calculation

Equation (B·1) is obtained using (4·15), from Section 4.2.2.

\[ C_s(j\omega_{nf}) = \frac{j\omega_{nf}K_C}{\omega_{cl}\omega_{ch} - \omega_{nf}^2 + j\omega_{nf}(\omega_{cl} + \omega_{ch})} \]  
(B·1)

Equation (B·1) can be written as:

\[ C_s(j\omega_{nf}) = \frac{1}{(\omega_{cl}\omega_{ch} - \omega_{nf}^2)^2 + \omega_{nf}^2(\omega_{cl} + \omega_{ch})^2} \left( -\omega_{nf}^2K_C(\omega_{cl} + \omega_{ch}) + j\omega_{nf}K_C(\omega_{cl}\omega_{ch} - \omega_{nf}^2) \right) \]  
(B·2)

The phase shift, \( \theta_{IB} \), (B·3) of the band-pass filter (4·15), from Section 4.2.2, at the resonant frequency, \( \omega_{nf} \), of the input RLC filter, is derived from (B·2).

\[ \theta_{IB} = \tan^{-1}\left( -\frac{\omega_{cl}\omega_{ch} - \omega_{nf}^2}{\omega_{nf}(\omega_{cl} + \omega_{ch})} \right) \]  
(B·3)

The high corner frequency, \( \omega_{ch} \), (B·4) and the low corner frequency, \( \omega_{cl} \), (B·5) resulting in a phase shift, \( \theta_{IB} \), of the band-pass filter at the resonant frequency, \( \omega_{nf} \), of the input RLC filter, are obtained from (B·3) for a given low corner frequency, \( \omega_{cl} \), and high corner frequency, \( \omega_{ch} \), respectively.

\[ \omega_{ch} = \omega_{nf} \frac{\omega_{cl}\tan(\theta_{IB}) + \omega_{nf}}{\omega_{cl} - \omega_{nf}\tan(\theta_{IB})} \]  
(B·4)

\[ \omega_{cl} = \omega_{nf} \frac{\omega_{ch}\tan(\theta_{IB}) + \omega_{nf}}{\omega_{ch} - \omega_{nf}\tan(\theta_{IB})} \]  
(B·5)
Appendix C
PBSC FOR IDEAL CPL RESULTS

C.1 SIMULATION RESULTS

In this section time-domain simulations of the system shown in Figure 5.1 in Section 5.1, consisting of an ideal CPL connected via a RLC input filter to a fixed DC grid voltage, are discussed. Results are obtained from a Matlab/Simulink model of the non-linear model given in (5-1), Section 5.1, using the control law derived in (5-25), Section 5.1.2. The damping injection constants are calculated using (5-35) and (5-36), Section 5.1.3. The parameters of the input filter are those listed in Section 3.2.2, Table 3-1.

Figure C.1 shows the DC-link voltage and control law response for a step in CPL power from 0 W to 8 kW occurring at 0.5 s. The grid voltage is set to 540 V. Responses are given for a desired RLC input filter damping ratio $\xi_f^*$ of 0.1, 0.3 and 0.5, whilst the desired undamped natural frequency $\omega_{nf}^*$ is fixed to be equal to the undamped natural frequency of the RLC filter, $\omega_{nf}^* = \omega_{nf} = 1/\sqrt{LC}$. Damping injection constants achieving $\xi_f^*$ and $\omega_{nf}^*$ are listed in Table C-1. Note that for $\omega_{nf}^* = \omega_{nf}$, (5-33) implies that $R_1 = -R$.

<table>
<thead>
<tr>
<th>$\xi_f^*$</th>
<th>$\xi_f^*=0.1$</th>
<th>$\xi_f^*=0.3$</th>
<th>$\xi_f^*=0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.01Ω</td>
<td>0.01Ω</td>
<td>0.01Ω</td>
</tr>
<tr>
<td>$R_2$</td>
<td>3.231Ω</td>
<td>1.015Ω</td>
<td>0.602Ω</td>
</tr>
</tbody>
</table>

Table C-1: Damping injection constants for $\xi_f^*$ equal to 0.1, 0.3, 0.5 when $\omega_{nf}^* = \omega_{nf}$

Figure C.1 a) shows that the DC-link voltage response is improved with an increased damping ratio. The maximum deviation from the steady state value is reduced from -7.7V for $\xi_f^*=0.1$ to -5.9V for $\xi_f^*=0.3$ and -4.9V for $\xi_f^*=0.5$. Furthermore the time response is significantly reduced for a larger value of the desired damping ratio. However, the control law response increases as the desired damping ratio $\xi_f^*$ increases, as shown in Figure C.1 b). In fact the peak power which must be drawn by the load in order to stabilise the system increases from -1.4 kW for $\xi_f^*=0.1$ to -4.3 kW for $\xi_f^*=0.5$. 

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Figure C.1: DC-link voltage and control law response for a step in CPL from 0 kW to 8 kW at 0.5 s for fixed $\omega_{nf}^*$ and variable $\xi_f^*$.

Figure C.2 shows the DC-link voltage and control law response to a step in grid voltage from 540 V to 530 V at 1 s. The power drawn by the CPL is set to 8 kW, and the damping injection constants achieving a desired RLC input filter damping ratio of 0.1, 0.3 and 0.5 with $\omega_{nf}^* = \omega_{nf}$, remain as listed in Table C-1. The DC-link voltage time response at ±5% of the steady state value is reduced from 7.5 ms for $\xi_f^*=0.1$ to 2.4 ms for $\xi_f^*=0.3$ and 1.6 ms for $\xi_f^*=0.5$. Furthermore, the maximum deviation from the steady state value goes from -7.6 V to -2.9 V for $\xi_f^*=0.1$ and $\xi_f^*=0.5$, respectively. However, Figure C.2 b) shows that the contribution of the stabilisation control scheme is increased with the damping ratio, resulting in a peak stabilising power of 9.1 kW, 5.5 kW and 1.7 kW, which must be absorbed by the CPL for desired damping ratios of , 0.1, 0.3 and 0.5, respectively.
C.1 Simulation Results

Figure C.2: DC-link voltage and control law response to a step in grid voltage from 540 V to 530 V at 1 s for fixed $\omega_{nf}^*$ and variable $\xi_f^*$

Figure C.1 and Figure C.2 demonstrate that the proposed control law in (5.25) enables a significant improvement of the DC-link voltage response to changes in ideal CPL load and grid voltage. As expected, the relative stability of the system is enhanced with an increased desired damping ratio, by reducing the DC-link voltage oscillations. The cost of a large desired damping ratio is to increase the power which must be absorbed by the CPL in order to achieve the stabilisation. This could lead to significant load performance degradation in the case of a non-ideal CPL. Therefore the desired damping ratio selection is a trade-off between system stability and performance.

DC-link voltage and control law responses for a step in CPL power from 0 W to 8 kW at 0.5 s are shown in Figure C.3. The grid voltage is set to 540 V. Responses are given for a desired RLC input filter undamped natural frequency equal to $0.5\omega_{nf}$, $\omega_{nf}$ and $2\omega_{nf}$, whilst the desired damping ratio is fixed to 0.5. The damping injection constants to achieve $\xi_f^*$ and $\omega_{nf}$ are listed in Table C.2.
Table C-2: Damping injection constants for $\omega_{nf}^*$ equal to $0.5\omega_{nf}$, $\omega_{nf}$ and $2\omega_{nf}$ when $\xi_f^* = 0.5$

<table>
<thead>
<tr>
<th>$\omega_{nf}^*$</th>
<th>$\omega_{nf}^*$=0.5$\omega_{nf}$</th>
<th>$\omega_{nf}^*$=0$\omega_{nf}$</th>
<th>$\omega_{nf}^*$=2$\omega_{nf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>-0.93Ω</td>
<td>-0.01Ω</td>
<td>0.3Ω</td>
</tr>
<tr>
<td>$R_2$</td>
<td>1.22Ω</td>
<td>0.602Ω</td>
<td>0.89Ω</td>
</tr>
</tbody>
</table>

Figure C.3 a) shows that the maximum DC-link voltage response overshoot increases with the desired undamped natural frequency, going from -2.5V for $\omega_{nf}^*=0.5\omega_{nf}$ to -4.9V for $\omega_{nf}^*=\omega_{nf}$ and -9.6V for $\omega_{nf}^*=2\omega_{nf}$. In addition to the $\omega_{nf}^*=2\omega_{nf}$ exhibiting the largest overshoot, it also causes the largest power to be absorbed by the CPL as shown in Figure C.3 b), with a power peak at 23.98 kW.

The control law response causes the smallest deviation from the steady state value for $\omega_{nf}^*=\omega_{nf}$ than for $\omega_{nf}^*=0.5\omega_{nf}$.

![DC-link voltage response](image1)

![Control law response](image2)

**Figure C.3**: DC-link voltage and control law responses for a step in CPL power from 0 W to 8 kW at 0.5 s for fixed $\xi_f^*$ and variable $\omega_{nf}^*$

A response to a step in grid voltage from 540 V to 530 V at 1 s, of the DC-link voltage and the control law is shown in Figure C.4. The CPL power is set to 8 kW, and the damping injection constants to achieve a desired undamped natural frequency equal to $0.5\omega_{nf}$, $\omega_{nf}$ and $2\omega_{nf}$, with $\xi_f^*=0.5$ remain as
C.1 Simulation Results

specified in Table C-2. Figure C.4 a) shows that the DC-link voltage overshoot is similar for the three undamped natural frequencies. The response time increases with lower values of \( \omega_{nf} \) as expected. As in Figure C.3 b), Figure C.4 b) shows that the lowest additional stabilising power is required for \( \omega_{nf}^* = \omega_{nf} \), whilst the power peak is significantly increased for \( \omega_{nf}^* = 2\omega_{nf} \).

Figure C.4: DC-link voltage and control law response to a step in grid voltage from 540 V to 530 V at 1 s for fixed \( \xi_f \) and variable \( \omega_{nf}^* \)

Figure C.3 and Figure C.4 show that the additional power drawn by the ideal CPL in order to stabilise the system, is lowest when the desired undamped natural frequency of the input filter set to its actual undamped natural frequency value. This is due to the fact that to impose \( \omega_{nf}^* \neq \omega_{nf} \), the contribution of the stabilisation scheme is significantly increased as the control is trying to modify the physical characteristic of the system. In the case of a non-ideal CPL, setting damping injection constants to satisfy \( \omega_{nf}^* \neq \omega_{nf} \) could lead to markedly degraded system performance due to the increase in additional power which must be absorbed.
C.2 PBSC Simplification Validation

Equations (C-1) and (C-2) are obtained by substituting the simplified control law (5-37), from Section 5.2.1.1, into (5-1), from Section 5.1.1, (C-1) and (C-2) describe the dynamic behaviour of the system shown in Figure 5.1, Section 5.1 when controlled with (5-37), Section 5.2.1.1.

\[
\frac{di_g}{dt} = - \frac{R}{L} i_g - \frac{1}{L} V_l + \frac{1}{L} V_g \tag{C-1}
\]

\[
\frac{dV_l}{dt} = \frac{R_1 + R_2}{R_2 C} i_g - \frac{R + R_1}{R_2 C} i_{g0} - \frac{1}{R_2 C} V_l - \frac{P_l}{C V_l} + \frac{1}{R_2 C} V_g \tag{C-2}
\]

Unlike in (5-30), in Section 5.1.3, the nonlinearity introduced by the CPL is not directly cancelled in (C-2).

The system described by (C-1) and (C-2) can be expressed by the linear expression given in (C-3).

\[
V_l = Z_1^*(s) V_g - Z_2^*(s) i_{g0} + \frac{Z_2^*(s)}{L C \omega_{nf}^*} i_{g0} - \frac{Z_2^*(s)}{L C \omega_{nf}^* V_l} P_l \tag{C-3}
\]

where \(Z_1^*(s)\) and \(Z_2^*(s)\), and \(\omega_{nf}^*\) are given by (5-32) and (5-33), Section 5.1.3, respectively.

Using (5-26), in Section 5.1.2, (C-3) can be rewritten as:

\[
V_l = Z_1^*(s) V_g - Z_2^*(s) i_{g0} + \frac{Z_2^*(s)}{L C \omega_{nf}^*} \left( \frac{1}{V_{l0}} - \frac{1}{V_l} \right) P_l \tag{C-4}
\]

Assuming that the DC-link voltage response to a disturbance exhibits a negligible deviation from its steady state value, that is \(V_l \approx V_{l0}\), (C-4) is equivalent to the expression given in (5-31) in Section 5.1.3, which was derived with the non simplified control law given in (5-27), Section 5.1.2. This shows that for \(V_l \approx V_{l0}\) the PBSC tuning proposed in Section 5.1.3 remains valid for the simplified control law given in (5-37), Section 5.2.1.1.

Figure C.5 shows the DC-link voltage response to a step in grid voltage from 540 V to 530 V at 1 s. The results are obtained from a Matlab/Simulink model of the non-linear model given in (5-1), Section 5.1.1, using both, the standard control law derived in (5-27) in Section 5.1.2, and the simplified control law.
given by (5.37), in Section 5.2.1.1. The parameters of the input filter are those listed in Section 3.2.2, Table 3.1. The power drawn by the CPL is set to 8 kW. In Figure C.5 a) the damping injection constants, \( R_1 \) and \( R_2 \) are calculated using (5.35) and (5.36), to achieve a desired RLC input filter damping ratio, \( \xi_f^* \), of 0.1 and desired undamped natural frequency \( \omega_{nf}^* \) equal to \( \omega_{nf} \). In Figure C.5 b) the desired damping ratio and undamped natural frequency of the input RLC filter are set to 0.3 and 0.5\( \omega_{nf} \), respectively. The corresponding values of \( R_1 \) and \( R_2 \) are given in Table C.1 and Table C.2. The responses for both standard and simplified PBSC are similar. The maximum deviation of the simplified PBSC from the standard PBSC response is 176.6 mV and 175.6 mV in Figure C.5 a) and b), respectively. This shows that as suggested in (C.4), the PBSC tuning proposed in 5.2.1.1 remains valid for \( V_I = V_{I0} \).

Figure C.5: DC-link voltage response to a step in grid voltage from 540 V to 530 V at 1 s comparing simplified PBSC and non-simplified PBSC
Appendix D

IP FILTER DISCRETISATION

Applying the Tustin approximation given in (6.10) in Section 6.2.2, to (A.21) and (A.22) in Appendix A.2.2, the \(dq\) current and speed IP filters in the \(z\) domain are given in (D.1) and (D.2), respectively.

\[
\frac{i_{sd}^{**}(z)}{i_{sd}^{*}(z)} = \frac{i_{sq}^{**}(z)}{i_{sq}^{*}(z)} = \frac{k_{ic}T_s (z - 1)}{(2k_{pc} + k_{ic}T_s)z - 2k_{pc} + k_{ic}T_s} \tag{D.1}
\]

\[
\frac{\omega_{r}^{**}(z)}{\omega_{r}^{*}(z)} = \frac{k_{i\omega}T_s (z - 1)}{(2k_{p\omega} + k_{i\omega}T_s)z - 2k_{p\omega} + k_{i\omega}T_s} \tag{D.2}
\]

For the IP filters to be implemented in the TI DSC, the \(dq\) current and speed IP filter recursive formulas are derived from (D.1) and (D.2) as (D.3) and (D.4), respectively.

\[
i_{sd,q}^{**}(k) = \frac{k_{ic}T_s}{2k_{pc} + k_{ic}T_s} \left( i_{sd,q}^{*}(k) + i_{sd,q}^{*}(k - 1) \right) - \frac{k_{ic}T_s - 2k_{pc}}{2k_{pc} + k_{ic}T_s} i_{sd,q}^{*}(k - 1) \tag{D.3}
\]

\[
\omega_{r}^{**}(k) = \frac{k_{i\omega}T_s}{2k_{p\omega} + k_{i\omega}T_s} \left( \omega_{r}^{*}(k) + \omega_{r}^{*}(k - 1) \right) - \frac{k_{i\omega}T_s - 2k_{p\omega}}{2k_{p\omega} + k_{i\omega}T_s} \omega_{r}^{*}(k - 1) \tag{D.4}
\]
Appendix E
ASCs C-code

E.1 IBSC C-CODE

/*============================================================================
File name:       IBSC_CONTROL.H
============================================================================*/

#ifndef __IBSC_H__
#define __IBSC_H__

typedef struct {
   _iq Out;       // IBSC output
   _iq u_p1;      // IBSC output k+1
   _iq u_p2;      // IBSC output k+2
   _iq Vdc;       // DC-link voltage
   _iq Vdc_p1;    // DC-link voltage k+1
   _iq Vdc_p2;    // DC-link voltage k+2
   _iq K1;        // 1st numerator term
   _iq K2;        // 2nd numerator term
   _iq K3;        // 1st denominator term
   _iq K4;        // 2nd denominator term
   _iq K5;        // 2nd denominator term
} IBSC_CONTROLLER;

Default initializer for the IBSC object.

#define IBSC_CONTROLLER_DEFAULTS {
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
   _IQ(0.0),
}

IBSC Macro Definition

#define IBSC_MACRO(v)
/* Tustin */
    v.Vdc_p2 = v.Vdc_p1;
    v.Vdc_p1 = v.Vdc;

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APPENDIX E ASCS C-CODE

E.2 PBSC C-CODE

/*============================================================================
* File name:       PBSC_CONTROL.H
*============================================================================*/

#ifndef __PBSC_H__
#define __PBSC_H__

typedef struct {
            _iq Out;     // PBSC output
            _iq u;       // PBSC control law
            _iq u_p1;   // PBSC control law k+1
            _iq Vdc;    // DC-link voltage
            _iq ig;     // Grid current
            _iq ig_p1;  // Grid current k+1
            _iq dig;    // Grid current derivative
            _iq dig_p1; // Grid current derivative k+1
            _iq vdig;   // Current derivative * DC-link voltage
            _iq vdig_p1; // Current derivative * DC-link voltage k+1
            _iq isqr;   // q axis current reference
            _iq C;      // L/R2
            _iq T;      // Sampling period
            _iq K1;     // 1st numerator term
            _iq K2;     // 2nd numerator term
            _iq K3;     // 1st denominator term
            _iq K4;     // 2nd denominator term
        } PBSC_CONTROLLER;

#define PBSC_CONTROLLER_DEFAULTS {
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
            _IQ(0.0),
        }

*/

#endif // __IBSC_H__
PBSC Macro Definition

#define PBSC_MACRO(v)  
/* Current derivative calculation */  
v.dig = _IQdiv(v.ig-v.ig_p1,v.T);
/* Current derivative * Voltage calculation */  
v.vdig = _IQmpy(v.Vdc,v.dig);
/* Control law Tustin */  
/* CPBSC output */  
v.Out = _IQdiv(_IQmpy(_IQdiv(2,3),v.u),v.isqr);

v.vdig_p1 = v.vdig;
v.ig_p1 = v.ig;
v.dig_p1 = v.dig;
v.u_p1 = v.u;
#endif  // __PBSC_H__