DESIGN AND ANALYSIS OF GREEN MOBILE COMMUNICATION NETWORKS

A dissertation submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

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By
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Abstract

Increasing energy consumption is a result of the rapid growth in cellular communication technologies and a massive increase in the number of mobile terminals (MTs) and communication sites. In cellular communication networks, energy efficiency (EE) and spectral efficiency (SE) are two of the most important criteria employed to evaluate the performance of networks. A compromise between these two conflicting criteria is therefore required, in order to achieve the best cellular network performance. Fractional frequency reuse (FFR), classed as either strict FFR or soft frequency reuse (SFR), is an inter-cell interference coordination (ICIC) technique applied to manage interference when more spectrum is used, and to enhance the EE. A conventional cellular model’s downlink is designed as a reference in the presence of inter-cell interference (ICI) and a general fading environment. Energy-efficient cellular models, such as cell zooming, cooperative BSs and relaying models are designed, analysed and compared with the reference model, in order to reduce network energy consumption without degrading the SE. New mathematical models are derived herein to design a distributed antenna system (DAS), in order to enhance the system’s EE and SE. DAS is designed in the presence of ICI and composite fading and shadowing with FFR. A coordinate multi-point (CoMP) technique is applied, using maximum ratio transmission (MRT) to serve the mobile terminal (MT), with all distributed antenna elements (DAEs), transmit antenna selection (TAS) being applied to select the best DAE and general selection combining (GSC) being applied to select more than one DAE. Furthermore, a Cloud radio access network (C-RAN) is designed and analysed with two different schemes, using the high-power node (HPN) and a remote radio head (RRH), in order to improve the EE and SE of the system. Finally, a trade-off between the two conflicting criteria, EE and SE, is handled carefully in this thesis, in order to ensure a green cellular communication network.
Declaration

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and in The University’s policy on presentation of Theses
With deepest appreciation and affection, I dedicate this thesis to

my parents,

I gratefully thank you that you agree to let me travel away from you to have this degree. I ask ALLAH to bless you and guide me to serve you the rest of my life and I ask ALLAH forgive my father and elevate his station among those who are guided.

my wife,

I appreciate your support, patient and inspiration. I know that this study makes me busy. I will be insha’a ALLAH very close to you and all members of my beloved family.
Acknowledgements

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List of Abbreviations

2G  Second Generation
3G  Third Generation
4G  Fourth Generation
5G  Fifth Generation
AA  Antenna Adaptation
ASE  Area Spectral Efficiency
AGE  Area Green Efficiency
AF  Amplify-and-Forward
A/D  Analogue/Digital
AP  Access Point
AWGN  Additive White Gaussian Noise
BER  Bit Error Rate
BS  Base Station
BBU  Baseband Unit
COTS  Commercial Off The Shelf
CCI  Co-Channel Interference
CDMA  Code Division Multiple Access
CoMP  Coordinated Multi-Point
CPRI  Common Public Radio Interface
C  Capacity
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>CO</td>
<td>Central Office</td>
</tr>
<tr>
<td>CAS</td>
<td>Collocated Antenna System</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Density Function</td>
</tr>
<tr>
<td>CAPEX</td>
<td>Capital Expenditure</td>
</tr>
<tr>
<td>CAS</td>
<td>Collocated Antenna System</td>
</tr>
<tr>
<td>CAGR</td>
<td>Compound Annual Growth Rate</td>
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<td>C-RAN</td>
<td>Cloud-Radio Access Network</td>
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<td>CAN</td>
<td>Cellular Access Networks</td>
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<tr>
<td>CSI</td>
<td>Channel State Information</td>
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<tr>
<td>C4ISR</td>
<td>Command, Control, Computer, Communication, Intelligence, Surveillance and Reconnaissance</td>
</tr>
<tr>
<td>D2D</td>
<td>Device to Device</td>
</tr>
<tr>
<td>DAS</td>
<td>Distributed Antenna System</td>
</tr>
<tr>
<td>DAE</td>
<td>Distributed Antenna Elements</td>
</tr>
<tr>
<td>DMF</td>
<td>Demodulate and Forward</td>
</tr>
<tr>
<td>DF</td>
<td>Decode-and-Forward</td>
</tr>
<tr>
<td>DU</td>
<td>Digital Unit</td>
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<tr>
<td>DTX</td>
<td>Discontinuous Transmission</td>
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<tr>
<td>EE</td>
<td>Energy Efficiency</td>
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<td>ESE</td>
<td>Energy-Spectral Efficiency</td>
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<tr>
<td>EE</td>
<td>Energy Efficiency</td>
</tr>
<tr>
<td>FD</td>
<td>Full Duplex</td>
</tr>
<tr>
<td>FCC TAC</td>
<td>Federal Communications Commission Technological Advisory Council</td>
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<tr>
<td>FR</td>
<td>Frequency Reuse</td>
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<td>FFR</td>
<td>Fractional Frequency Reuse</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>FRF</td>
<td>Frequency Reuse Factor</td>
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<td>FC</td>
<td>Femto Cell</td>
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<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
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<td>G</td>
<td>Gain</td>
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<td>GSC</td>
<td>General Selection Combining</td>
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<td>HetNet</td>
<td>Heterogeneous Network</td>
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<td>HPN</td>
<td>High Power Node</td>
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<td>HMT</td>
<td>HPN Mobile Terminal</td>
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<td>H-CRAN</td>
<td>Heterogeneous C-RAN</td>
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<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
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<tr>
<td>ITU</td>
<td>International Telecommunications Union</td>
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<tr>
<td>IID</td>
<td>Independent Identically Distribution</td>
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<tr>
<td>IND</td>
<td>Independent Non-identically Distribution</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
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<tr>
<td>ICI</td>
<td>Inter Cell Interference</td>
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<tr>
<td>ICIC</td>
<td>Inter-Cell Interference Coordination</td>
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<tr>
<td>IFU</td>
<td>Indoor FC User</td>
</tr>
<tr>
<td>ICT</td>
<td>Information and Communications Technology</td>
</tr>
<tr>
<td>iDAS</td>
<td>indoor DAS</td>
</tr>
<tr>
<td>IMT-Advanced</td>
<td>International Mobile Telecommunications-Advanced</td>
</tr>
<tr>
<td>JSLNR</td>
<td>Joint Signal-to-Leakage-Noise-Ratio</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-of-Sight</td>
</tr>
<tr>
<td>LC</td>
<td>line Card</td>
</tr>
<tr>
<td>LNA</td>
<td>Low Noise Amplifier</td>
</tr>
<tr>
<td>LSPE</td>
<td>Large Scale Propagation Effects</td>
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<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
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<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>MIMO</td>
<td>Multi-Input Multi-Output</td>
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<tr>
<td>MU-MIMO</td>
<td>Multi User MIMO</td>
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<tr>
<td>MIL-STD</td>
<td>Military Standards</td>
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<tr>
<td>MT</td>
<td>Mobile Terminal</td>
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<td>M2M</td>
<td>Machine to Machine</td>
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<tr>
<td>MRT</td>
<td>Maximum Ratio Transmission</td>
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<td>MCS</td>
<td>Mote Carlo Simulation</td>
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<tr>
<td>MC</td>
<td>Macro Cell</td>
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<tr>
<td>N</td>
<td>Noise</td>
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<tr>
<td>NP</td>
<td>Nondeterministic Polynomial time</td>
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<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<tr>
<td>oDAS</td>
<td>outdoor DAS</td>
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<tr>
<td>OMU</td>
<td>Outdoor MC User</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
</tr>
<tr>
<td>OPEX</td>
<td>Operational Expenditure</td>
</tr>
<tr>
<td>ONU</td>
<td>Optical Network Unit</td>
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<tr>
<td>PAR</td>
<td>Peak-to-Average Ratio</td>
</tr>
<tr>
<td>P</td>
<td>Transmit Power</td>
</tr>
<tr>
<td>PC</td>
<td>Power Control</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PCI</td>
<td>Physical Cell ID</td>
</tr>
<tr>
<td>PCM</td>
<td>Power Consumption Model</td>
</tr>
<tr>
<td>PON</td>
<td>Passive Optical Network</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>RAT</td>
<td>Radio Access Technologies</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RRH</td>
<td>Remote Radio Head</td>
</tr>
<tr>
<td>RMT</td>
<td>RRH Mobile Terminal</td>
</tr>
<tr>
<td>RS</td>
<td>Relay Station</td>
</tr>
<tr>
<td>RV</td>
<td>Random Variable</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>SFR</td>
<td>Soft Frequency Reuse</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SSPE</td>
<td>Small Scale Propagation Effects</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to Interference plus Noise Ratio</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal to Interference Ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SE</td>
<td>Spectral Efficiency</td>
</tr>
<tr>
<td>SUE</td>
<td>Spectrum Utilization Efficiency</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>TWDM</td>
<td>Time-Wavelength-Division Multiplexing</td>
</tr>
<tr>
<td>TAS</td>
<td>Transmit Antenna Selection</td>
</tr>
<tr>
<td>UFR</td>
<td>Unity Frequency Reuse</td>
</tr>
<tr>
<td>UE</td>
<td>User Equipment</td>
</tr>
<tr>
<td>UDN</td>
<td>Ultra Dense Networks</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>WiMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless Sensor Networks</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength-Division Multiplexing</td>
</tr>
</tbody>
</table>
List of Mathematical Notations

\[ \Pi \] The product symbol
\[ \Sigma \] The summation symbol
\[ \eta_{EE} \] The energy efficiency
\[ CO_2 \] The carbon dioxide
\[ P_t \] The transmit power
\[ P_{edg} \] The cell edge transmit power
\[ P_{cent} \] The cell centre transmit power
\[ P_r \] The receive power
\[ R_t \] The threshold distance
\[ d_0 \] The polar distance between MT and its home BS
\[ d_i \] The polar distance between MT and the interfering BS
\[ d_{s,r} \] The polar distance between source and relay
\[ d_{r,d} \] The polar distance between relay and destination
\[ d_{i,r} \] The polar distance between interferers and relay
\[ d_{i,d} \] The polar distance between interferers and destination
\[ u \] The instantenous distance between MT and it home BS
\[ \alpha \] The power amplification factor
\[ \beta \] The path loss exponent
\[ N \] The number of interfering cells
\[ R \] The cell radius
$D$  The reuse distance
$G$  The gain of the relay station
$T$  The Time
$R_C$  The DAS cell radius
$C(.)$  The spectral efficiency
$C_b$  The achievable rate
$n_{s,r}$  The additive white gaussian noise between source and relay
$n_{r,d}$  The additive white gaussian noise between relay and destination
$N_0$  The Noise Spectral Density of the AWGN
$N_{s,r}$  The Noise Spectral Density between source and relay
$N_{r,d}$  The noise spectral density between relay and destination
$\eta_{EE}$  The energy efficiency
$\exp(x)$  The exponential function $e^x$
$\phi_i$  The angles between the MT and the co-channel interfering cells
$I_i$  The set of interfering BSs
$\theta$  The MT location angle referring to its Home BSs
$F$  The cluster total sub-bands
$F_{cent}$  The cell-centre sub-bands
$F_{edg}$  The cell-edge sub-bands
$\nabla_{edg}$  The cell-edge FRF
$\nabla_{cent}$  The cell-centre FRF
$C$  The number of co-channel Cells
$\xi_0$  The shadow fading of the useful Signal
$\xi_i$  The shadow fading of the interfering signals
$\xi_{c,n}$  The shadow fading of the interfering DAE$_n$ at cell $c$
$\rho$  The signal-to-noise ratio
\( \rho_1 \) The SNR between source and relay
\( \rho_2 \) The SNR between relay and destination
\( \delta \) The binary number represents selected antenna
\( g \) The larg scale signal level
\( x \) The trasnitted symbol
\( r_n \) The DAE\(_n\) position referring to its home cell centre
\( w \) The channel complex weight
\( m \) The nakagami-m fading parameter
\( \sigma \) The shadowing standard deviation
\( h_0 \) The small scale channel gain of the useful signal
\( h_i \) The small scale channel gain of the interfering signal
\( h_{c,n} \) The small scale channel gain of the interfering DAE\(_n\) at cell c
\( h_{s,r} \) The small scale channel gain between source and relay
\( h_{r,d} \) The small scale channel gain between relay and destination
\( h_{i,r} \) The small scale channel gain between interferers and relay
\( h_{i,d} \) The small scale channel gain between interferers and destination
\( a_{s,r} \) The signal channel complex between source and relay
\( a_{r,d} \) The signal channel complex between relay and destination
\( a_{i,r} \) The signal channel complex between interferers and relay
\( a_{i,d} \) The signal channel complex between interferers and destination
\( d_v \) The distance between MT and cooperative RRH\(_v\)
\( \mathcal{R} \) The set of interfering RRH
\( \mathcal{H} \) The set of interfering HPN
\( \mathcal{F} \) The set of cooperative RRH
\( R_L \) The remainder of Gauss-Hermite Quadrature
\( w_l \) The weight of \( L^{th} \) order Hermite polynomial
$x_l$  The Abscissas of $L^{th}$ order Hermite polynomial
$f(.)$  The probability density function
!  The factorial operator
$Pr(x)$  The probability of $x$
$log_x(.)$  The logarithmic function to base $x$
$ln(.)$  The natural logarithm
$\max$  The argument of the maximum
$E_b$  The energy-per-bit
$P_c$  The circuit components power consumption
$P_{PA}$  The PA power consumption
$P_o$  The total overhead power consumed
$P_{\Sigma}$  The total power consumed
$P_s$  The static power consumption
$\Delta_p$  The power amplifier efficiency
$\zeta$  The dynamic power consumption per throughput
$\varsigma$  The peak-to-average ratio
Chapter 1

Introduction

The energy consumption of cellular equipment is a huge concern for both manufacturers and operators of mobile networks. Increasing energy consumption is a result of the rapid growth in cellular communication technologies and a massive increase in the number of mobile terminal (MT) communication sites. Several projects, such as GreenTouch and energy-aware radio and network technology (EARTH), are currently focused on researching new technologies that may minimise the energy consumption of cellular networks, in order to address associated climate and environmental concerns. The University of Manchester, through the school of Electrical and Electronic Engineering, is a member of the GreenTouch project and provides full support and investment in this regard.

The target of GreenTouch project is to reduce the energy consumption of mobile networks by up to 98% by 2020 [3] according to a GreenTouch White Paper released in June 2015. The target of the EARTH project is to reduce the energy consumed by 4G mobile networks by a factor of about 50% compared to existing networks, by providing applicable solutions and cost-efficient broadband wireless services. EARTH has gone beyond the target by providing integrated solutions and thus providing a power saving in the range of 70% [4].

Different methods have been proposed to improve the energy efficiency (EE) of mobile cellular networks. Ericsson’s mobility report, released in November 2015, stated that 85% of mobile subscriptions would be using mobile broadband by 2021 [5]. The same report stated that mobile broadband subscriptions are growing by about 25% year-on-year globally, and increased
by approximately 160 million in the third quarter of 2015 alone. The total number of mobile broadband subscriptions is forecast to reach 7.7 billion by 2021.

New innovations and applications such as the Internet of Things (IoT), which allows communications between all home appliances, further increases demand on the frequency spectrum. Pressure on the frequency spectrum is further increased by other the daily aspects of our lives, such as emergency services, television and sound broadcasting, air travel, defence equipment, utilities management and environmental monitoring \[6\]. The Cellular Telephone Industries Association released a report in April 2016 about the American wireless industry, in which it stated that it had granted almost $195 billion of domestic economic value in 2014 and more than $282 billion to the US gross domestic product (GDP) \[7\]. Services and products using mobile data communications machine-to-machine (M2M) along with IoT are making huge steps in efficiency and productivity. The McKinsey Global Institute bottom-up analysis estimated the potential economic impact of IoT applications on the world economy at $3.9 trillion to $11.1 trillion a year by 2025 \[8\].

The EE of a cellular network can be improved by switching off some base stations during off-peak periods, when traffic demand is low \[9\]. Customer behaviour and traffic intensity are the main factors that influence where and when to apply this technique. For example, during off-peak periods, BSs with lower data traffic levels can be put into sleep mode \[10\], and MTs whose parent BSs are put to sleep can be served by either the remaining active BSs, which zoom out, or by relay stations which consume considerably lower power than BSs. Furthermore, power control and cooperative communications can be applied to extend network coverage and serve areas where BSs are switched off \[11\]. Distributed antenna systems (DASs) and Cloud radio access networks (C-RAN) are energy-efficient techniques that can be implemented to reduce the power consumed by conventional BSs and improve networks’ EE.
1.1 Motivation

The rapid growth in the number of communication sites and mobile terminals has caused a massive increase in the energy consumption of mobile communication networks. Many projects are dedicated to reducing the energy consumption of the ICT sector, and specifically mobile cellular networks, for example GreenTouch and EARTH. These projects aim at reducing energy costs and subsequently the CO₂ emissions which are responsible for global warming. However, given that traffic demand is currently increasing exponentially, any reduction in energy consumption should not come at the expense of much-needed network capacity. Data traffic in mobile networks increased by 81% in 2013 alone, and this data demand is expected to reach about 24.3 Exabytes per month by 2019 (one Exabyte equals one billion Gigabytes), a ten-fold increase over five years [12]. Furthermore, the frequency spectrum is a precious but scarce resource which should be used effectively to enhance its utilisation by different technologies and applications [13]. Furthermore, it is finite, and the usable spectrum is relatively congested and cannot physically increase in size [6]. Many researchers have investigated ways to optimally utilise the frequency spectrum, for example through the employment of opportunistic reuse using cognitive radio.

The number of communication sites is expected to increase to more than 11 million by 2020 [14], and equivalent CO₂ emissions are predicted to increase by a factor of three by 2020. Therefore, reducing the number of BSs is one of the best ways to reduce power consumption and to enhance the EE of mobile broadband systems. The influence of customer behaviour can be used to minimise energy consumption by applying different energy-efficient techniques across different day-night time periods, such as cell zooming, cooperative BSs, relaying distributed antenna system (DAS) and C-RAN, in order to enhance EE of systems; for example, cells which have lower data traffic or fewer numbers of MTs (a threshold might be applied) are put into sleep mode [10]. The remaining active BSs can increase their transmit power to cover the areas that were originally covered by these BSs in sleep mode. Reducing the number of active BSs and increasing network energy efficiency would eliminate climate change concerns by reducing associated CO₂ emissions. The trade-off between EE and SE is extensively investigated in this thesis, by using different energy-efficient cellular communication models.
1.2 Aims and Objectives

This research aims at investigating the trade-off between EE and SE for different and recently proposed energy-efficient cellular communication models, in order to reduce energy consumption and enhance the throughput of mobile cellular communication networks. In order to achieve this aim, the following are addressed herein:

- Design and analysis of a downlink conventional cellular communication model that will be used as a reference to investigate other proposed energy-efficient cellular models. Mathematical models are developed and analysed for three different energy-efficient cellular communication networks, namely cell zooming, cooperative BSs and relaying models. The trade-off between EE and SE is investigated for these models and compared with the conventional cellular model. FFR and SFR are considered with Nakagami-$m$ fading, which is applied in the conventional, cell zooming, cooperative BS models, while Rayleigh fading is applied in the relaying model.

- Develop novel mathematical models for DAS, in order to create a cellular network with low power consumption and better spectral efficiency. The trade-off between EE and SE is calculated by using maximum ratio transmission (MRT), transmit antenna selection (TAS) and general selection combining (GSC) schemes. For each scheme, FFR and SFR are applied to calculate the trade-off between the network’s EE and SE. A composite Nakagami-$m$ and log-normal shadowing environment is considered, in order to produce a realistic cellular network environment. DAS is designed to solve cell-edge performance degradation and to cover dead spots in the network. Moreover, this research will reduce the distance between the BS and the MT, which in turn will result in radiated power and improve the EE of the system. Distributed antenna elements (DAEs) which are deployed around the cell will reduce the number of traditional BSs, extend coverage and improve network performance.

- Design and analysis of two different C-RAN schemes, to reduce the power consumed by the cellular network and improve the network’s SE. Novel mathematical models are developed for C-RAN, to calculate the
trade-off between EE and SE, by using MRT, TAS and GSC schemes. For each scheme, FFR is applied to calculate the trade-off between EE and SE with composite Nakagami-$m$ and log-normal shadowing environments. The C-RAN model was selected for this task because it is a 5G network and has recently been proposed to improve the EE and SE of cellular networks. A visualised and soft BS is implemented with BBU and integrated onto the same server in C-RAN.

1.3 Thesis Organisation

This thesis has three main chapters, namely Chapters 3, 4 and 5. Each of these main chapters is divided into three major parts: i) designing green cellular models and analysing the SE of each model, ii) developing a realistic power consumption model (PCM) to calculate the energy consumption of the network and iii) investigating the trade-off between EE and SE for each model.

Chapter 2 Background

Existing works on spectral efficiency (SE) and energy efficiency (EE) are reviewed in this chapter. Some energy-efficient models of cellular networks are summarised. Definitions and metrics in relation to SE and EE are described in addition to the FFR and SFR models of cellular networks.

Chapter 3 Energy Efficient Cellular Models in FFR

In this chapter, mathematical approaches of the SE are derived, calculated and compared for different cellular communication models, such as conventional, cell zooming, cooperative BSs and relaying. FFR and SFR are considered to calculate SE and EE, where Nakagami-$m$ fading is considered in the conventional, cell zooming and cooperative BS models, while Rayleigh fading is applied in the relaying model.

Chapter 4 Distributed Antenna Systems in FFR

Mathematical analysis of the SE for DAS is derived in this chapter. EE is calculated using MRT, TAS and GSC schemes. For each scheme, FFR and SFR are applied to calculate spectral and energy efficiencies, and a composite Nakagami-$m$ and log-normal shadowing environment is considered.
CHAPTER 1. INTRODUCTION

Chapter 5 Cloud Radio Access Networks in FFR

In this chapter, SE mathematical approaches of C-RAN is derived. EE is calculated using MRT, TAS and GSC schemes. For each scheme, FFR is applied to calculate spectral and energy efficiencies, and a composite Nakagami-$m$ and log-normal shadowing environment is considered.

Chapter 6 Conclusions and Future Directions

The thesis is concluded in this chapter, and ideas for further work are discussed. In this chapter, energy harvesting is proposed to extend my work to develop new energy-efficient cellular models by exploiting ambient interference. A D2D cellular network and wireless fronthaul for C-RAN are also proposed in this chapter, in order to extend my work.

1.4 Achievements and Major Contributions

The main contributions of this thesis are as follows:

- A novel mathematical approach is developed to investigate the downlink performance of DAS, using FFR and MRT techniques. A novel mathematical approach to investigate the downlink performance of DAS, using FFR and MRT techniques, is also presented. New expressions for an ergodic SE in the cell-centre and cell-edge regions are derived in a composite Nakagami-$m$ and log-normal shadowing environment for an arbitrary number of DAEs in each cell and for an arbitrary number of cells and interferers. The analysis is further simplified by using the MGF-based approach. The trade-off between EE and SE for the DAS approach using an MRT scheme is also investigated.

- New mathematical approaches are derived for DAS, using FFR and TAS techniques in the presence of interference. New expressions for the ergodic SE in the cell-centre and cell-edge regions are derived in a composite Nakagami-$m$ and log-normal shadowing environment for an arbitrary number of DAEs in each cell and for an arbitrary number of cells and interfering BSs. The analysis is further simplified by using the MGF-based approach. The trade-off between EE and SE for the DAS approach, using the TAS scheme, is investigated.
• For DAS and C-RAN, GSC is applied to investigate the trade-off between EE and SE for M number of transmitting antennas, the results of which are then compared with TAS and MRT. GSC is a solution sitting between MRT, which selects all transmitting antennas, and TAS, which selects the best transmitting antenna.

• Design and analysis of different C-RAN models to calculate the trade-off between EE and SE, by using MRT, TAS and GSC with FFR, for an arbitrary number of DAEs in each cell and for any number of cells and interferers. The models are designed and analysed in the presence of ICI and by using a composite Nakagami-\(m\) and log-normal shadowing environment.

• Design and analysis of a general approach for an amplify-and-forward relaying system in the presence or absence of interference with FFR for an arbitrary user and an arbitrary number of cells and interfering BSs. The trade-off between EE and SE for the relaying approach is investigated within a Rayleigh fading environment.

• Design and analysis of a general approach for a conventional cellular system, using FFR for an arbitrary user and an arbitrary number of cells and interfering BSs. The trade-off between EE and SE for the relaying approach is investigated in a Nakagami-\(m\) fading environment.

• Design and analysis of cooperative BSs and cell zooming systems, using FFR for an arbitrary user and an arbitrary number of cells and interfering BSs. The trade-off between EE and SE for the relaying approach is investigated in a Nakagami-\(m\) fading environment.

1.5 List of Publications

1.5.1 Journals


1.5.2 Conference Papers


Chapter 2

Background Theory and Literature Review

GLOBAL energy consumption is expected to grow by about 56% between 2010 and 2040 [15]. The ICT sector contributes significantly to this growth, due to the increase in the number of mobile communication subscribers and sites. Improving the energy efficiency of mobile communication networks, however, should not be at the cost of network throughput and quality of service (QoS). Increasing cellular network throughput to fulfil subscriber demand requires more frequency spectrum.

Nowadays, the demand on the frequency spectrum has increased massively, due to an increase in the applications using it: “Spectrum underpins our modern lives. We can’t see or feel it, but without it we would have no mobile phones, no TV and radio, no radar, no safety of life services... the list goes on. We know we cannot make more of it, though we can use it more efficiently” [6]. In this thesis, the trade-off between EE and SE is investigated. A performance analysis of cellular networks’ downlink is investigated with the use of fractional frequency reuse (FFR), and the trade-off between the SE and EE of cellular communication systems is extensively discussed, to determine the optimal SE at the highest EE for different energy-efficient cellular models or deployments such as cell zooming, cooperative base stations (BSs), relaying models, DASs and C-RAN.

In this chapter, the literature on the performance analysis of SE and EE across the physical layer is presented. *Strict* FFR and soft frequency reuse (SFR) are applied to improve the trade-off between EE and SE for the whole network. During off-peak traffic periods, as illustrated in Fig. 2.1, many
energy-efficient techniques might be applied, in order to enhance communication network EE.

\subsection*{2.1 Energy Efficiency}

Increasing energy cost and demand has resulted into the need for more energy efficient cellular systems which is a central focus of research in the mobile communication industry. [16, 17]. This creates a need to design future networks with higher energy efficiency, which means that energy requirements will not grow at the same rate as network data demand [18]. The joint Guadalajara ICT declaration [ICT-Cop16], “Guadalajara ICT Declaration for Transformative Low-Carbon Solutions,” was presented and signed by 41 ICT companies at the UN climate change conference Cop16, committing to support the solutions to reducing the CO\textsubscript{2} footprint target by about 5\% under the Kyoto Protocol and to improve EE [2, 19].

\subsubsection*{2.1.1 EE Definition}

EE in general means producing the same services while using less energy [20, 21]. In other words, it involves using the same energy input to deliver more
services [22]. In cellular communication systems, EE involves the efficient utilisation of the energy within the system, i.e. it is total capacity to total power consumed by cellular network deployment models [2].

2.1.2 EE Metrics

SE metrics provides no indication about how to utilise energy efficiently. Accordingly, the metrics of EE, such as bits-per-Joule (bit/J) capacity, are applied to provide this insight [2]. According to [23], bits-per-Joule capacity is defined as the maximum number of bits that the energy-limited wireless network can transmit per joule of consumed energy. Therefore, EE can be expressed as the ratio of the system’s capacity to total power consumed by system $P_{Σ}$, such that

$$\eta_{EE} = \frac{C_{b}}{P_{Σ}}, \quad (2.1)$$

where $C_{b}$ (bit/s) is the system’s achievable rate. In addition, EE can be measured in terms of capacity-per-unit cost [24] or rate-per-energy [25–27]. Capacity-per-unit cost can be viewed as a special case of bits-per-Joule capacity when average power is represented by cost.

2.2 Spectral Efficiency

The spectrum is an invisible medium used to connect different wireless communication devices; it is finite, and physically we cannot increase its usable range [6]. The SE is one of the main performance indicators for optimising and designing wireless networks, and it has been extensively studied as an evaluation measurement for the effectiveness of cellular communication networks [28–30].

This demand on the frequency spectrum makes it one of the most important and expensive parameters in cellular communication networks, due to the limitations of the frequency bandwidth (BW) and the massive increase in the number of users and their data rate needs. Therefore, the frequency spectrum should be used efficiently by all applications. In this thesis, SE is extensively investigated for different cellular communication models, in order to provide a more efficient cellular network.
Figure 2.2: The AWGN channel EE-SE trade-off for different values of overhead power consumption [2].

2.2.1 SE Definition

SE or spectrum utilization efficiency (SUE) is defined by International Telecommunications Union (ITU) as the ratio of the amount of transferred information over a distance to the spectrum utilisation factor [31]. Moreover, SE can be expressed differently as the efficient way of using the frequency spectrum or BW for transmitting data over a wireless channel while achieving an acceptable quality of service (QoS). Furthermore, SE can be defined as “the ratio of information rate that can be transmitted over a channel to the channel bandwidth” [2], which indicates how efficiently the limited frequency spectrum is utilised.

2.2.2 SE Metrics

The usage of frequency spectrum determines the way of measurement for each system. Therefore, U.S. government agencies have applied different efficiency metrics for different developed system categories [32]. For example, The U.S. Federal Communications Commission Technological Advisory Council (FCC TAC) has applied six categories for efficiency metrics: four
for terrestrial systems, including terrestrial point-to-point systems, terrestrial hybrid systems, terrestrial broadcast system, satellite broadcast systems and terrestrial personal communication systems, while the other two are for satellite systems, namely satellite broadcast systems and point-to-point satellite systems.

There are many ways to calculate and measure SE, such as bit per second per hertz \([b/s/Hz]\) [13]. Furthermore, area SE is measured as bits per second per hertz per unit area \([b/s/Hz/area]\). Accordingly, data transferred in available amounts of spectrum can be easily calculated.

The SE can be expressed in an average of a logarithmic function of SINR, which has many random variables as in [33, 34]

\[
C = \mathbb{E}\left[\log_2 (1 + \text{SINR})\right]. \quad \text{b/s/Hz (2.2)}
\]

The SE average can be solved by multi-integrations, depending on the number of random variables. The following lemma is applied to solve the complicated integrations for 2.2.

**Lemma 1**: From [35] for any \(t > 0\)

\[
\ln (1 + t) = \int_0^\infty \frac{1}{s} \left(1 - e^{-st}\right) e^{-s} ds, \quad t \geq 0. \quad (2.3)
\]

The derivation of lemma in 2.3 is provided in Appendix A as a reference.

### 2.3 The Trade-off between Energy and Spectral Efficiency

The trade-off between EE and SE is investigated in [36] by a using simple and effective method employing a single scenario extended to a multi-cell environment. It shows that a single cell with a low path-loss exponent has higher EE. The trade-off between EE and SE can be described simply as the expression of EE as a function of SE for an available bandwidth. The authors in [37] investigated the energy-spectral efficiency (ESE) trade-off in virtual MIMO cellular systems, but they did not consider the signal-to-interference ratio (SIR) or compare different models. In the meantime, the authors in [38] applied cooperative base stations to improve the SE and EE of a downlink cellular model, while [39] did not consider co-channel interference but instead
investigated EE performance and the influence of SE on EE in multiple-input multiple-output (MIMO) systems over Rayleigh fading channels.

There are two different ways to express EE. If we assume that $C_b$ (bit/s) is the achievable rate and $P_\Sigma$ (Watts) is the total power consumption of the transmitted data, at this rate EE can be expressed in terms of energy-per-bit $E_b$ as $E_b = P_\Sigma/C_b$, or in terms of bits-per-Joule $\eta_{EE}$ as $\eta_{EE} = C_b/P_\Sigma$ [40]. Note that in some related theoretical works [29] [41] about EE-SE trade-off it is assumed that $P_\Sigma = P$, where $P$ (Watts) is the transmit power of the system. Therefore, in practice, the power consumed by the cellular system does not only accounted for the transmit power. According to Shannon’s capacity theorem [33], the channel capacity per-unit-bandwidth $C$ or equivalently the maximum achievable SE, $C$ (bits/s/Hz), is a function in SNR, $\gamma$ such that

$$C = f(\rho),$$ \hspace{1cm} (2.4)

where $\rho = P/N$ is the SNR, $P$ is the transmit power and $N$ is the noise power, and $N = N_0 F$, where $N_0$ (Joule) is the noise spectral efficiency and $F$ (Hz) is the bandwidth. In general case, capacity $f(\rho)$ can be described as an increasing function of $\rho$, where $\rho \in [0, \infty)$ and $C \in [0, \infty)$. In addition, as long as $f(\gamma)$ is a bijective function [2], the invertible function of $f(\rho)$ is

$$f^{-1}(C) = \rho,$$ \hspace{1cm} (2.5)

where $f^{-1}: C \in [0, \infty) \to \rho \in [0, \infty)$ is the inverse function of $f$. Shannon’s capacity theorem $f(\gamma)$ is expressed as

$$f(\rho) = \log_2 (1 + \rho),$$ \hspace{1cm} (2.6)

and $f^{-1}(\rho)$ is expressed as

$$f^{-1}(C) = 2^C - 1,$$ \hspace{1cm} (2.7)

for AWGN as given in [33]

In [42], transmit power $P$ can be expressed as $E_bC_b$ and SNR, $\rho$ can be
re-expressed as a function of EE, $\eta_{EE}$ and achievable SE, $C$ such that

$$\rho = \frac{P}{N_0 F} = \frac{E_b C_b}{N_0 F} = \frac{C}{N_0 \eta_{EE}}. \quad (2.8)$$

By using (2.7) and (2.8), and where $\rho = f^{-1}(C)$ and $N_0 = \frac{N}{F}$, the trade-off between EE and SE can be expressed simply as

$$\eta_{EE} = \frac{F}{N} \frac{C}{f^{-1}(C)}. \quad (2.9)$$

Total consumed power is assumed to be $P_\Sigma = P$ as an ideal power consumption model (PCM) [2]. For more generic PCM, $P_\Sigma = P + P_o$, where $P_o$ is the total overhead power consumed. Therefore, the trade-off between EE and SE in a simple formula [2,43] can be reformulated as

$$\eta_{EE} = \frac{C}{f^{-1}(C) + P_o}, \quad (2.10)$$

where $C$ is system throughput.

The EE-SE trade-off for the additive white Gaussian noise (AWGN) channel, using different values of overhead power consumption, is illustrated in Fig. 2.2, where $\eta_{EE}$ converges to $1/(\ln 2)$ when $C$ approaches zero. On the other side, $\eta_{EE}$ approaches zero when $C$ goes to infinity.

The trade-off between EE and SE can be calculated by using a realistic PCM as described in [2,14,48].

$$\eta_{EE} = \frac{F}{N} \frac{C}{(\Delta_p f^{-1}(C) + P_o/N) N}, \quad (2.11)$$

where $F$ is the total available bandwidth, $N$ is the noise power, $N$ is the number of active BSs and $\Delta_p$ is power amplifier (PA) efficiency. The above (2.11) is applied to the trade-off between EE and SE for all energy-efficient models developed in chapter 3 for a fair comparison between all of the models.
2.4 Energy-Efficient Cellular Techniques

Manufacturers spend a lot of time and money on improving the performance of their equipment, in order to reduce power consumption, and communication operators and planning engineers try to come up with creative techniques in this respect. Cell distribution structures help operating engineers apply the best sleep mode criteria according to the shape of the distributed cells on the served area during network data off-peak traffic periods.

From the outset, planning engineers should consider the best distribution for cells according to many geographical and demographic factors and human habits and activities. Cell zooming, which is one of the techniques that might be applied to networks in order to increase power consumption efficiency, can be applied through different ways or techniques, as we shall see in the following subsection.

2.4.1 Cell Zooming

Cell zooming is a technique applied to reduce the power consumed by cellular communication networks, by reducing the number of active base stations during off-peak traffic periods [44]. It is applied also for adjusting cell size according to many parameters, such as channel conditions, user requirements and traffic load; furthermore, cell zooming has the potential to reduce power consumption and provide traffic balance [45]. Cell zooming can also be applied when cells with low amounts of traffic are turned to sleep mode, in which case its users are serviced by the adjacent cell which will be zoom out to cover a larger area [46]. Many challenges stand in the way of this technique, such as coverage holes caused by the sleep BSs and tracing the exact traffic load balance of the active cells covering new areas. In addition, frequency management and radio spectrum planning are also problematic in this respect.

2.4.1.1 Transmit Power Adaptation

Transmit power adaptation can be used in multi-user orthogonal frequency division multiplexing (OFDM) to increase the system data rate by applying
spectral diversity and multi-user diversity [47]. In cell zooming, the transmitted power of active cells can be increased to serve the mobile terminal (MT) located in the adjacent sleeping cell. Moreover, optimal transmit power helps to maximise the battery lifetime of wireless sensor networks (WSNs) [48].

2.4.1.2 Changing Physical Parameters

This can be achieved by increasing the antenna height of the active cell, to extend the covered area. Moreover, adjusting the antenna’s tilt might reduce or increase the area covered by the BSs to have load balancing in the network [49]. Using an electrical antenna, tilted to select the right antenna tilt angle, provides the best network performance, while choosing an antenna tilt angle smaller than the optimal causes interference with neighbouring cells [50]. Reducing this interference by choosing the optimal antenna tilt angle increases system capacity [51].

2.4.2 Cooperative BSs

Traffic stream densities in cellular communication networks vary significantly during the day, depending on many factors, including human activities and user habits. However, during network off-peak periods, a cooperative BSs scheme serving the users of sleep cells offers a suitable method for increasing EE. A cooperative BS model is applied to improve the EE of cellular networks [38, 52]. Applying cooperative BSs in multi-cell processing has tremendous potential to increase the data rate of a cellular network [53]. During the sleep cell model, two or more active base stations might cooperate to serve MTs in the sleeping cell area, which helps to extend the covered area and reduce the power consumption of the whole network during off-peak periods [54]. The optimal number of cooperative BSs provides better energy efficiency compared to a fixed relay scheme [55].
2.4.3 Relay Stations

Fixed relay stations have the capability to enhance a system’s SE and EE [56], and deploying relay or low-power BSs in macro BSs is expected to minimise the total energy consumed by cellular networks [57]. This technique can be applied by deploying relay stations in the turned off macro BSs during off-peak periods, to reduce the power consumption of the system. Relay stations can also balance traffic by transferring traffic from congested cells to their non-congested counterparts.

2.4.4 Heterogeneous Networks

The heterogeneous network (HetNet) has garnered significant attention as a technique for improving SE and EE [58]. HetNet is one of the most efficient ways of cutting power consumption and increasing the maximum achievable capacity of cellular communication networks [59], since macro cells can be replaced by a certain number of micro cells which consume less power. In dense urban areas, it is possible to replace micro cells with a number of small cells, to overcome coverage holes or blind spots [60], depending on many factors such as data rate, number of MTs, geographical area and interference.

2.4.5 Distributed Antenna System

Providing a uniformly distributed high capacity in cellular networks is a challenge due to many factors, such as path loss, fading and interference, but this problem can be solved partially by deploying many transmit points throughout the cell and connecting to the BS via coaxial or fibre cables instead of one transmitting point [61]. The DAS has been used in wireless communication networks to improve coverage, increase capacity and reduce power consumption. In this technique, the base station is surrounded by an appropriate number of distributed antennae (e.g. six) or according to the need to cover a larger cell area. The DAS provides considerable gain in capacity and coverage at lower cost compared to decreasing cell size [62].
2.4.6 Small Cell Deployment

Many factors encourage wireless communication operators to migrate from macro to small cells which will improve capacity and reduce cost [59, 63]. Using small cells to cover tens or a few hundreds of metres is useful for higher frequencies, which in turn are suitable for higher data rates [64]. We still need to use macro cells in low-density and suburban areas, though. Small cells serve fewer numbers of users compared to macro cells; however, using higher frequencies – and by default gaining a higher data rate – provides users with more bandwidth and definitely fulfils their requirements.

Small cell base stations consume much less power compared to macro cells, but there is a significant increase in the number of small cell base stations that are required to cover the same area covered by macro cells. This means that there might be no significant difference in power consumption from migrating to a small cell approach. Consequently, we have to apply techniques to improve the ability of small cell base stations to activate and deactivate the sleep mode, depending on the traffic in each cell [10]. In small cells, there are many important actions that we can perform in order to reduce power consumption, such as reducing transmission power compared to the macro cells or replacing traditional macro cells in hotspot zones that have higher traffic and therefore need a higher data rate. Moreover, small cells serve fewer numbers of users, which makes it more amenable to applying the sleep mode to reduce power consumption [59]. Furthermore, small cells serve users experiencing coverage problems due to the weakness of the received signals on the very edge of the macro cells [65]. If there is need to use macro cells, such as in rural areas, the region can therefore be surrounded by small cells which help to cover the dead and hotspot areas and meet any increase in traffic demand [66]. Reducing cell size or using smaller cells in cellular communication networks will cause an increase in spectral efficiency [67].

2.4.7 Cloud-Radio Access Networks

The Cloud-Radio Access Network (C-RAN) is a novel cellular and mobile network architecture which can overcome many challenges faced by network operators trying to extend and upgrade their networks to fulfil end user requirements [68]. It consists of three main parts: a baseband unit (BBU) pool,
a remote radio head (RRH) and a fronthaul network which might be a fibre
network or a microwave link. This architecture provides more flexibility to
operators to extend and upgrade their networks. Moving a processing func-
tion to a BBU pool reduces the amount of equipment in an RRH site and
reduces air-conditioning requirements and overall site power consumption.

2.5 Frequency Reuse (FR)

The increase in the number of users demanding more data traffic means the
deployment of more dense base stations as one of the features of sophisti-
cated multi-cellular systems. Orthogonal frequency division multiple access
(OFDMA) is applied to increase the throughput of a network where inter-cell
interference is the primary source of interference (which affects users closer
to the cell boundary), while intra-cell users are assumed to be orthogonal to
each other [69]. To mitigate the interference and improve the performance of
the network, inter-cell interference coordination (ICIC) is applied to minimise
interference and maximise the spatial reuse of the frequency by dedicating
specific resources to each cell. FFR techniques have been proposed as an
ICIC approach in OFDMA to overcome signal-to-interference-plus-noise ratio
(SINR) potential coverage problems [70]. Recent relevant research, e.g. [71],
has investigated strict FFR and SFR, in order to enhance the area spectral
efficiency (ASE) of full and partial loaded systems.

In cellular systems, and due to the limitation of frequency spectrum avail-
ability, the frequency is reused at a certain distance, in order to avoid in-
terference. The geographical region is divided into different clusters where
the frequency cannot be reused. The number of cells per cluster $K$ satisfies
$K = i^2 + ij + j^2$ i.e. 3, 4, 7, 9 which can be determined depends on the reuse
distance between the co-channel cells, as illustrated in Fig. 2.3, where $i = 2,$
$j = 1$ and $K = 7$.

There are two different types of FFR deployment model, namely . These
common types are the Strict FFR and the SFR, depending on the method of
partitioning and reusing the frequency spectrum.
2.5.1 *Strict Fractional Frequency Reuse*

*Strict* FFR is an enhancement of traditional frequency reuse and is applied extensively in cellular networks [70]. In this model the total available bandwidth is partitioned into two disjointed sets of non-overlapping frequency bands between the cell-edge $F_{edg}$ and the cell-centre $F_{cent}$. Total bandwidth $F$ and the frequency reuse factor (FRF) for our model equate to $\nabla^{FFR}_{edg} = 3$ for cell edge users and $\nabla^{FFR}_{cent} = 1$, which means that $F_{edg}$ is divided into three non-overlapping sub-bands where $F_{edg} = F_A + F_B + F_C$, which are dedicated to cell-edges users, while the other $F_{cent}$ sub-bands are dedicated to cell-centre users, as illustrated in Fig. 2.4a. For a fair FFR spectrum allocation it is common to assume that $F_{cent} = \left\lceil F \left( \frac{R_t}{R} \right)^2 \right\rceil$, where $R_t$ represents the threshold distance that identifies the cell centre region and $R$ is the cell radius [69].

2.5.2 *Soft Frequency Reuse*

SFR is more efficient in the usage of the bandwidth compared to *Strict* FFR, which results in more interference for both cell-centre and cell-edge users [72]. SFR applies the same bandwidth partitioning strategy to the cell-edge regions...
Figure 2.4: Spectrum resource allocation for strict FFR in a hexagonal system with an FRF of three. (a) One sector of three cells strict FFR resource allocation. (b) Cell-edge and cell-centre sub-bands over 19 cells.
Figure 2.5: Spectrum resource allocation for SFR in a hexagonal system with FRF of three. (a) One sector of three cells SFR resource allocation. (b) Cell-edge and cell-centre sub-bands over 19 cells.
as illustrated in FFR, while cell-centre users are allowed to share sub-bands with the edge users of other cells. In SFR, the bandwidth is partitioned into three non-overlapping sub-bands \( F = F_A + F_B + F_C \). The cell-centre uses two sub-bands and the cell-edge uses one sub-band, as illustrated in Fig. 2.5a. Cell-centre users in SFR transmit at lower power to avoid interfering with edge users sharing the same sub-bands in adjacent cells [73]. The FRF for the cell-edge of the SFR is \( \nabla_{edg}^{SFR} = 3 \), and the FRF for the cell centre is \( \nabla_{cent}^{SFR} = 3/2 \) for the most common SFR schemes [71].
Chapter 3

Energy Efficient Cellular Models in FFR

In this chapter, different energy-efficient cellular models are designed and analysed, in order to reduce the energy consumption faced by the ITC sector. Conventional, cell zooming, cooperative BSs and amplify-and-forward relaying models are designed and analysed and the trade-off between EE and SE for these models are investigated. A massive increase in data traffic has led to the deployment of more dense base stations (BSs) as one of the features of sophisticated multi-cellular systems. The compound annual growth rate (CAGR) of BS deployment is expected to increase by 41% between 2012 and 2017 [74]. Since BS sites consume 80% of the power used by cellular network operators [14], reducing the number of active BSs in one area will in turn help to reduce the power consumed by cellular communication networks and improve the energy efficiency (EE) of the ICT sector. This increase in the number of BSs and mobile subscribers requires an enlarged frequency spectrum, in order to increase network throughput. Orthogonal frequency division multiple access (OFDMA) is applied to increase the throughput of a network, where the inter-cell interference is the primary source of interference (which affects users closer to the cell boundary), while intra-cell users are assumed to be orthogonal to each other [69]. To mitigate interference and to improve the performance of the network, inter-cell interference coordination (ICIC) is applied to minimise interference and maximise the spatial reuse of the frequency by dedicating specific resources to each cell. Fractional frequency reuse (FFR) techniques have been proposed as an ICIC approach in OFDMA to overcome potential signal-to-interference-plus-noise ratio (SINR) coverage problems [70].
CHAPTER 3. ENERGY EFFICIENT CELLULAR MODELS IN FFR

Recent relevant research (e.g. [71]) has investigated FFR and SFR to enhance the area spectral efficiency (ASE) of full and partial loaded systems. The authors in [37] investigated the energy-spectral efficiency (ESE) trade-off in virtual MIMO cellular systems but did not consider signal-to-interference (SIR) or compare different models. The authors in ( [39]) did not consider co-channel interference but instead investigated EE performance and the influence of spectral efficiency (SE) on EE in multiple-input multiple-output (MIMO) systems over Rayleigh fading channels.

The SE of femtocells overlapping with conventional macrocells is investigated in [75], in which the SE for two different tier networks is derived for single users. It is shown that an increase in the number of femtocells provides higher cell SE. More details for the derivation of system’s SE, however, are needed, as the authors did not apply any other techniques to improve it other than increasing the number of femtocells.

One simple and effective method was applied by using a single cell scenario [36] to investigate the trade-off between EE and SE, which extended to a multi-cell environment where interference was considered. Furthermore, the authors applied many techniques to mitigate interference effects, showing that a single cell with a low path-loss exponent has higher EE. The authors did not apply any frequency reuse techniques or any energy-efficient models such as cell zooming or cooperative BSs during off-peak traffic periods to improve the EE of the proposed system.

The authors in [76] calculated the area spectral efficiency (ASE) of cellular mobile radio systems. They derived an expression for efficiency as a function of reuse distance for worst and best case interference configurations, and they also developed a Monte Carlo simulation to estimate the value of this efficiency and investigated a fully and a partially loaded cellular system. Users’ random location effects and the impact of log-normal shadowing and Nakagami multipath fading were also applied. The authors used a two-slope path-loss model to obtain the average received power as a function of distance, and they calculated the efficiency for frequency division multiple access (FDMA) and time division multiple access (TDMA) systems as a function of reuse distance. Finally, they produced different results due to many factors, such as carrier frequency, the path-loss exponent and worst and best case interference configurations.
The authors in [77] investigated the amount of energy consumed by cells by providing criteria to reduce the number of active cells in each area according to certain parameters and factors. One of the factors was day-night customer behaviour, and the other was customer movement in the early morning and at the end of the day. This means that customers move in the morning to their workplaces and go back to their residential areas after work. When we reduce the number of active cells we expect an increase in output power by the base stations of the remaining active cells to cover areas that were covered by the sleep cells and to guarantee quality of service in these areas. Any increase in the output power of the active cells might be negligible compared to the power consumption of the turned-off BSs. The authors applied some typical network configurations, such as hexagonal cells with omni-directional antennae, hexagonal cells with tri-sectorial antennae, omni-directional antennae, crossroad cells with omni-directional antennae and the Manhattan layout. They did not, however, apply any other model such as cooperative or cell zooming.

The area green efficiency (AGE) of two-tier heterogeneous cellular networks is presented in [78]. The authors used uniformly distributed femtocells on the edge of macro cells to save energy, reduce co-channel interference and enhance the ASE. They presented a mathematical analysis and derived analytical expressions to evaluate the ASE under best and worst interference scenarios, and they also proposed a threshold radius to determine the macro cell-served area and the best location for the mobile to hand over to the femtocells. In addition, they found that there is good enhancement on the ASE when two-tier heterogeneous cellular networks are compared only to macrocells. Finally, they deduced that there is an obvious power saving by applying femtocells located on the edge of the macro cell.

In [79], the authors investigated different techniques for energy-efficient wireless communication systems, such as MIMO and relay techniques. They mentioned that MIMO techniques have been shown to make effective improvements in the capacity and SE of wireless communication systems. In addition, there are two different types of relays, namely a pure relay system and a cooperative relay system. The authors highlighted the relationship between the signal-to-noise ratio (SNR) and energy efficiency (EE) and between the EE and the data rate of multi-hop relays, but they did not indicate the trade-off
between energy efficiency and spectral efficiency for different techniques.

In [39], the authors adopted a realistic power model to design a practical MIMO by considering independent Rayleigh fading and semi-correlated fading channels. They also analysed the effect of increasing the number of antennae on EE and the relationship between EE and SE, deriving in the process the optimum value of SE at the maximum value of EE when the number of antennae is fixed. However, they did not consider co-channel interference or compare their results with different models.

Making improvements in the area of spectral efficiency for heterogeneous networks, by integrating femto-macro cells, is investigated in [65], who derived an area of spectral efficiency for their proposed two-tier heterogeneous network and applied low-cost, low-power consumption and user-deployed cells on the edge of the macro cells, in order to increase the capacity of the system. Furthermore, they proposed two schemes that would be dependent on the number of channels dedicated to each cell. The authors found that any gain in area spectral efficiency \([\text{bps/Hz/km}^2]\) increases by increasing the number of femtocells until they reach a certain number (10 cells), following which the gain will start to decrease by increasing the number of femtocells.

In [10], the authors applied small cell BSs over existing macrocells, in order to maximise the data rate of the network, and they then moved on to propose an algorithm to manage the criteria involved in switching off small cells, in order to reduce the power consumed as a result of the tremendous increase in the number of small cells. They presented three strategies for sleep mode regulations in small cells according to the small cells, the core network and user equipment. Their algorithm showed an approximate power saving of between 13% to 56% for the mix of voice and data models; however, they did not provide any details on the trade-off between energy and area spectral efficiencies for different models.

BS supply power consumption is reduced in [80] by using different energy-saving strategies such as minimising the time that a BS can transmit at maximum output. The proposed algorithm has the capability to reduce BS supply power consumption by about 25 to 40%, depending on the load of the system. Their combination of discontinuous transmission (DTX) and power control (PC) provides better energy efficiency compared to the exclusive operation of
either PC or DTX. The energy-saving strategies applied include antenna adaptation (AA), by reducing the number of radio frequency (RF) chains, PC, which minimises the transmit power of each resource unit, and DTX, where the BS does not radiate it is maximum transmit power.

Coordinated multi-point (CoMP) transmission was applied in [38] to enhance SE and exploit traffic fluctuations, with a view to increasing the idle time of the BSs and the EE of the downlink cellular network. SE and EE trade-off was investigated in [40] by using realistic and non-realistic PCM for the Rayleigh fading channel via MIMO and SISO techniques. They concluded that MIMO has better EE compared to the SISO technique, and SE increases with the theoretical EE and the number of antennae in a high SE regime.

In [81–83], the authors investigated the outage probability of dual-hop systems at different SNR values. In [81], amplify and forward (AF) and decode and forward (DF) techniques were applied, while [83] used the average SNR-per-hop value to calculate outage probability, albeit they did not investigate the effect of using dual-hop systems on the energy and spectral efficiency of cellular systems. The authors in [84], [85] and [86] investigated the ergodic capacity of dual-hop amplify and forward, and [87], [88] and [89] investigated outage probability but did not investigate energy efficiency. The authors in [90] investigated a full duplex (FD) massive MIMO AF relay system, using three different power scaling schemes, and concluded that one of the schemes had the ability to restrict loop interference.

3.1 General System Model

In this chapter, a general mathematical model for a downlink cellular communications network with a general number of cells is derived. In order to determine our numerical results, a 19 cell is considered for the system model, as illustrated in Fig. 2.4b and Fig. 2.5b, for which the energy and spectral efficiencies of four models are calculated: conventional, cooperative BSs, cell zooming and relaying.

FFR is applied to mitigate interference at the edge of the cell, where it is divided into two regions, namely the cell-centre and the cell-edge. The users of each region are served by a portion of the cell bandwidth. Therefore, users at the cell edge do not interfere with the other users at the edge of the adjacent
cell. Furthermore, interference created and received by the cell-centre users is reduced, and extra bandwidth frequency is used more than conventional reused frequency. FFR calls to improve the trade-off between total cellular network throughput and spectral efficiency and the rate and coverage of the cell-edge users.

3.2 Conventional Cellular Model

In this section, a downlink conventional cellular system is considered with three cells per cluster, as illustrated in Fig. 2.4a. The performance of this conventional system is then used as a norm to study the possible improvements offered by using base station cooperation and cell zooming during an under-utilisation situation in the cellular network. Without any loss of generality, we consider a reference MT to be an arbitrary MT in an arbitrary cell (which will be called the ‘home cell’). In the conventional model it is assumed that the home cell is active, and therefore the useful signal is received from the home base station.
3.2.1 Fractional Frequency Reuse

For this conventional cellular model, both strict FFR and SFR are applied to manage the ICI, enhance SE and EE which will improve the trade-off between EE and SE of the proposed system.

3.2.1.1 Strict Fractional Frequency Reuse

In Strict FFR, the dedicated bandwidth for each sector is denoted by $F$ and divided into four non-overlapping sub-bands where $F_{edg} = F_A + F_B + F_C$, which are dedicated to cell-edge users, while the other $F_{cent}$ sub-bands are dedicated to cell-centre users, as illustrated in Fig. 2.4a. The BS transmits two different powers for the cell-edge users and cell-centre users. Cell-edge transmit power is given by $P_{edg} = \alpha P$ while cell-centre transmit power is given by $P_{cent} = (2 - \alpha) P$, where $P$ is the BS transmit power and $\alpha$ is the FFR cell-edge power amplification factor and $1 < \alpha < 2$. During the non-power control scheme, $\alpha = 1$ [69, 71, 91]. Fig. 2.4b shows the interfering BSs for the cell-edge and cell-centre users. For the cell centre users the set of interfering BSs are $I_1 \in \{2, 3, 4, 5, 6, 7\}$, representing first-tier interfering BSs, and $I_2 \in \{8, 9, 10, \ldots, 19\}$, representing second-tier interfering BSs, which use the same cell-centre sub-band. For the cell-edge users, MT will receive interference from the set of BSs $I_3 \in \{9, 11, 13, 15, 17, 19\}$, which use the same cell-edge sub-band, as illustrated in Fig 2.4b. Therefore, the received signal can be expressed as

$$y^{FFR}_{cent} = \underbrace{(2 - \alpha) \sqrt{P_0} x_0 \sqrt{|u|^{-\beta} h_0}}_{Useful-signals} + (2 - \alpha) \sum_{i \in I_1} \sqrt{P_i} x_i \sqrt{|d_e - u|^{-\beta} h_i} + (2 - \alpha) \sum_{i \in I_2} \sqrt{P_i} x_i \sqrt{|d_e - u|^{-\beta} h_i} + n,$$

where $x_i$ and $x_0$ represent the interfering and useful transmitted symbols, respectively. For the sake of SE analysis, it is assumed that $x_0$ and $x_i$ are complex Gaussian with zero mean and unity variance. It is also assumed that the home BS and interfering BSs transmit the same power, whereby $P_0 = P_i$. 
Moreover, $d_{i,d}$ is the distance between the MT and interfering BS, $u$ is the distance between the MT and its home BS and $d_c$ is the reuse distance. $|h_0|^2$ is the channel gain of the useful signal, $|h_i|^2$ are the channel gains of the interfering signals and $\beta$ is the path-loss exponent, while $n$ is the additive white Gaussian noise (AWGN) at the reference MT and satisfies $\mathbb{E}[|n|^2] = N_0$.

The distance between MT and interfering BS $i$ can be calculated as

$$d_{i,d} = |d_c \left(\cos(\theta + \phi_i) + j \sin(\theta + \phi_i)\right) - u|,$$

where $\theta$ is the angle between the instantaneous location of MT and the interfering BS and $\phi$ is the angle between two adjacent BSs in the same tier where $\phi_i = 0$ for the first interfering BS and $\phi_i \in \{0, \phi, 2\phi, 3\phi, \ldots, (N-1)\phi\}$ and $N$ is the number of interfering BSs in the same tier.

Therefore, the SINR for the MT at the cell-centre when FFR is applied can be expressed as

$$\text{SINR}_{\text{FFR cent}}^{\text{FFR}} = \frac{(2 - \alpha) P_0 |h_0|^2 |u|^{-\beta}}{(2 - \alpha) \sum_{i \in \mathcal{I}_1} P_i |h_i|^2 |d_c - u|^{-\beta} + (2 - \alpha) \sum_{i \in \mathcal{I}_2} P_i |h_i|^2 |d_c - u|^{-\beta} + N_0}.$$  

(3.3)

Moreover, the MT received signal when it located in the cell-edge can be expressed as

$$y_{\text{FFR edge}}^{\text{FFR}} = \alpha \frac{\sqrt{P_0 x_0 \sqrt{|u|^{-\beta}} h_i}}{\text{Useful--signals}} + \alpha \sum_{i \in \mathcal{I}_3} \sqrt{P_i x_i \sqrt{|d_c - u|^{-\beta}} h_i} \quad \text{Interfering--signals} + n,$$

(3.4)

and the cell-edge SINR for the MT using FFR can be written as

$$\text{SINR}_{\text{edge}}^{\text{FFR}} = \frac{\alpha P_0 |h_0|^2 |u|^{-\beta}}{\alpha \sum_{i \in \mathcal{I}_3} P_i |h_i|^2 |d_c - u|^{-\beta} + N_0}.$$  

(3.5)

### 3.2.2 Soft Frequency Reuse

SFR makes it more complicated to define interfering BSs compared to FFR. Therefore, SFR will be applied in the following derivations to obtain the downlink SE for the proposed conventional cellular communication model.
When the MT is located at the cell-centre of its home cell, interference will be received from sets $I_4$ and $I_5$ when SFR scheme is applied and Fig. 2.5b the set of interfering BSs. Therefore, the received signal can be expressed as

$$y_{\text{SFR cent}} = \frac{(\nabla_{\text{edg}}^{\text{SFR}} - \alpha)}{2} \sqrt{P_0 x_0} \sqrt{|u|^{-\beta} h_0}$$

$$+ \frac{(\nabla_{\text{edg}}^{\text{SFR}} - \alpha)}{2} \sum_{i \in I_4} \sqrt{P_i x_i} \sqrt{|d_c - u|^{-\beta} h_i} + \alpha \sum_{i \in I_5} \sqrt{P_i x_i} \sqrt{|d_c - u|^{-\beta} h_i} + n,$$

where $I_4$ and $I_5$ are the sets of interfering BSs when the SFR technique is applied and when the MT is located at its home cell-centre. Fig. 2.5b, illustrates the set of interfering BSs using the MT sub-band at their cell-centres $I_4 \in \{3, 5, 7, 8, 9, 11, 12, 13, 15, 16, 17, 19\}$ and the set of interfering BSs using the same sub-band at their cell-edges $I_5 \in \{2, 4, 6, 10, 14, 18\}$.

Therefore, the SINR experienced by the reference MT at the cell-centre during SFR mode takes the form

$$\text{SINR}_{\text{SFR cent}} = \frac{\frac{(\nabla_{\text{edg}}^{\text{SFR}} - \alpha)}{2} P_0 |h_0|^2 |u|^{-\beta}}{\frac{(\nabla_{\text{edg}}^{\text{SFR}} - \alpha)}{2} \sum_{i \in I_4} P_i |h_i|^2 |d_c - u|^{-\beta} + \alpha \sum_{i \in I_5} P_i |h_i|^2 |d_c - u|^{-\beta} + N_0}.$$  

(3.6)

When MT is located at the cell-edge of its home cell, the received signal can be expressed as

$$y_{\text{SFR edge}} = \alpha \sqrt{P_0 x_0} \sqrt{|u|^{-\beta} h_0}$$

$$+ \frac{(\nabla_{\text{edg}}^{\text{SFR}} - \alpha)}{2} \sum_{i \in I_6} \sqrt{P_i x_i} \sqrt{|d_c - u|^{-\beta} h_i} + \alpha \sum_{i \in I_7} \sqrt{P_i x_i} \sqrt{|d_c - u|^{-\beta} h_i} + n.$$

(3.7)
The SINR for the cell-edge user using the SFR scheme can be expressed as

\[
\text{SINR}_{\text{SFR edge}} = \frac{\alpha P_0 |h_0|^2 |\mathbf{u}|^{-\beta}}{\left(\frac{\nabla_{\text{SFR edge}} - \alpha}{2}\right) \sum_{i \in I_6} P_i |h_i|^2 |d_c - \mathbf{u}|^{-\beta} + \alpha \sum_{i \in I_7} P_i |h_i|^2 |d_c - \mathbf{u}|^{-\beta} + N_0}.
\]

For the cell-edge MT, the set of interfering BSs using the same sub-band are \( I_6 \) and \( I_7 \), where \( I_6 \) is the set of BSs using the same sub-band at their cell-centre and \( I_6 \in \{2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 16, 18\} \) and \( I_7 \) is the set of interfering BSs using the same sub-band at their cell-edges where \( I_7 \in \{9, 11, 13, 15, 17, 19\} \).

\[
C_{\text{SFR cent}}(\mathbf{u}) = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\left(\frac{\nabla_{\text{SFR edge}} - \alpha}{2}\right) |h_0|^2 |\mathbf{u}|^{-\beta}}{\sum_{i \in I_4} I_i + \alpha \sum_{i \in I_5} I_i + \frac{1}{\rho}} \right) \right], \quad [\text{b/s/Hz}] \quad (3.10)
\]

where \( I = P|h|^2 |d - \mathbf{u}|^{-\beta} \) for identical BSs, \( \rho = \frac{P}{N_0} \) is the SNR and for the proposed model \( \nabla_{\text{SFR edge}} = 3 \).

The expectation in (3.10) is taken with respect to the \(|I_4| + |I_5| + 1\) non-negative random variables \( \{|h_0|^2, |h_1|^2, \ldots, |h_{N-1}|^2, \mathbf{u}\} \). Classical methods for evaluating such averages require at least \(|I_4| + |I_5| + 1\)-fold numerical integrations, which makes the process very complicated. In order to reduce the computational complexity of (3.10) a non-direct method is invoked which greatly simplifies the required computation.

**Lemma 1**: From [35] for any \( t > 0 \)

\[
\ln(1 + t) = \int_0^\infty \frac{1}{s} \left(1 - e^{-st}\right) e^{-s} ds, \quad t \geq 0. \quad (3.11)
\]

Since the channel gains \(|h|^2\) are assumed to be independent, the following
is an expression for the conditional average

\[ C_{\text{cent}}^{SFR}(u) = \mathbb{E} \left[ \log_2 \left( 1 + \frac{(\nabla \text{SFR}_\text{edg} - \alpha)^2 |h_0|^2 |u|^{-\beta}}{2 \sum_{i \in I_4} I_i + \alpha \sum_{i \in I_5} I_i + \frac{1}{\rho}} \right) \right] \]

\[ = \log_2 (e) \int_0^\infty \frac{1}{z} e^{-\frac{z}{m}} \left( 1 - \mathbb{E} \left[ e^{-\frac{z (\nabla \text{SFR}_\text{edg} - \alpha)^2 |u|^{-\beta}}}{2} \right] \right) \]

\[ \times \prod_{i \in I_4} \mathbb{E} \left[ e^{-\frac{z (\nabla \text{SFR}_\text{edg} - \alpha)^2 |d_c - u|^{-\beta}}}{2} \right] \]

\[ \times \prod_{i \in I_5} \mathbb{E} \left[ e^{-z} \right] dz. \]  

(3.12)

For Nakagami-\(m\) fading, PDF follows the central chi-square distribution [92] which is given by

\[ f_{|h|^2}(x) = \frac{2m^m}{\Gamma(m)(\sigma^2)^m} x^{2m-1} e^{-\left( \frac{m}{\sigma^2} x^2 \right)}, \]  

(3.13)

where the square \(|h|^2\) of a Nakagami random variable has Gamma distribution with the following moment generating function (MGF) which can be expressed as

\[ \mathbb{E} \left[ e^{-z|h_0|^2|u|^{-\beta}} \right] = \left( 1 + \frac{z}{m} |u|^{-\beta} \right)^{-m}, \]  

(3.14)

and \(\Gamma(\bullet)\) is the Euler Gamma function, while \(m\) is the Nakagami-\(m\) fading parameter with the value range \((\frac{1}{2} \leq m \leq \infty)\) and represents channel severity and \(\sigma\) is the shadowing severity parameter which has a typical value of between 2 dB and 9 dB, depending on the physical environment.

Therefore, the SE of the conventional model [93] for the cell-centre user by applying SFR is expressed as

\[ C_{\text{cent}}^{SFR}(u) = \log_2 (e) \int_0^\infty \frac{1}{z} \left( 1 - \left( 1 + \frac{(\nabla \text{SFR}_\text{edg} - \alpha)^2 |d_c - u|^{-\beta}}{m} \right)^{-m} \right) \]

\[ \times \prod_{i \in I_4} \left( 1 + \frac{\frac{(\nabla \text{SFR}_\text{edg} - \alpha)^2 |d_c - u|^{-\beta}}{m}}{2} \right)^{-m} \]

\[ \times \prod_{i \in I_5} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} e^{-\frac{z}{\rho}} dz. \]  

(3.15)
The SE for a typical MT when it is located at the cell-edge using SINR (3.9) can be expressed as

\[
C_{edge}^{SFR}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \left( 1 + \frac{\alpha z}{m} |u|^{-\beta} \right)^{-m} \right) \times \prod_{i \in I_6} \left( 1 + \frac{(SFR_{edge} - \alpha)}{2} z d_c - u \right)^{-m} \times \prod_{i \in I_7} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} e^{-\frac{z}{\rho}} dz. \tag{3.16}
\]

The SE for a typical MT at the cell-centre and cell-edge regions using the SFR mode can be obtained by averaging out (3.15) and (3.16) with respect to \( u \), where the probability density functions (PDF) of the MT when it is located at the cell-edge of the home cell are given by [76]

\[
f_{|u|}(u) = \frac{2(u - R_t)}{(R - R_t)^2}, \quad R_t \leq u \leq R, \tag{3.17}
\]

and

\[
f_{\theta|u} = \frac{1}{2\pi} \quad 0 \leq \theta \leq 2\pi, \tag{3.18}
\]

where \( R_t \) is the threshold distance, which determines the closest distance between the cell-edge MT and its home BS, \( R \) is the home BS radius, and \( u \) is the instantaneous random distance between the MT and its home BS.

### 3.3 Cooperative BSs

To have a more energy-efficient cellular communication system, some BSs during off-peak periods, as illustrated in Fig. 2.1 might be turned to the sleep mode, in order to reduce the power consumed by the cellular system [54]. Therefore, a cooperative BS model is applied during the off-peak period to improve the EE of the cellular networks [38, 52]. During off-peak times, when some BSs are in an under-utilisation situation which might be turned to sleep mode, two or more active BSs cooperate to serve MTs located in the sleep BS areas.
3.3.1 Soft Frequency Reuse

In this scheme, we assume that the home BS in cell number 1, as illustrated in Fig. 3.2.b, is turned to the sleep mode and the reference MT is served by set $\mathcal{I}_8$ of the neighbouring BSs, where $\mathcal{I}_8 \in \{5, 7\}$, by using their cell-edge sub-bands. Therefore, the sets of interfering BSs are $\mathcal{I}_9 \in \{2, 4, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19\}$, which use the same sub-bands at their cell-centre, and $\mathcal{I}_{10} \in \{3, 8, 12, 16\}$, which use the same sub-bands at their cell-edge.

In this case, it can be shown that the SE of the cooperative model [9] [94] can be expressed as

$$C_{SFR}^{Coop}(u) = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\alpha \sum_{l \in \mathcal{I}_8} |h_l|^2 w_l |d_c - u|^{-\beta}}{\left( \nabla_{SFR} - \alpha \right) \sum_{i \in \mathcal{I}_9} I_i + \alpha \sum_{i \in \mathcal{I}_{10}} I_i + \frac{1}{\rho}} \right) \right],$$

b/s/Hz (3.19)

where $w_l = \frac{h_l^*}{\sqrt{\sum_{l \in \mathcal{I}_8} |h_l|^2}}$ is the weight of the channel coefficient received from set $\mathcal{I}_8$ of the cooperative BSs and $1 < \alpha < 2$.

In order to evaluate the expectation in (3.19) the same process as in

Figure 3.2: Cooperative BS system model in SFR, where the MT is served by BSs 5 and 7 using their cell-edge sub-band (A).
Lemma 1 is invoked.

Therefore, the SE of the cooperative BSs is then given by

\[
C_{\text{Coop}}^{SFR}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \prod_{i \in I_8} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} \right) \\ \times \prod_{i \in I_9} \left( 1 + \frac{\nabla_{\text{edg}}^{SFR} - \alpha}{m} z |d_c - u|^{-\beta} \right)^{-m} \\ \times \prod_{i \in I_{10}} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} e^{-\frac{z}{\rho}} dz. \quad (3.20)
\]

### 3.3.2 Strict Fractional Frequency Reuse

In our cooperative model using FFR schemes, cell number 1 in Fig. 3.3 is assumed to be turned to the sleep mode and its MT is served by set \( I_8 \in \{5, 7\} \) of the cooperative BSs, as illustrated in Fig. 3.3, using their cell-edge frequency sub-band. Therefore, the sets of interfering BSs are \( I_{10} \in \{3, 8, 12, 16\} \) and the SE of this scheme can be expressed as
\[ C_{C_{\text{Coop}}}^{\text{FFR}}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \prod_{i \in I_8} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} \right) \times \prod_{i \in I_{10}} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} e^{-z} \, dz. \] (3.21)

3.4 Cell Zooming

In cellular networks the cell size is fixed in general based on the estimated traffic load. However, the load can fluctuate significantly based on many parameters [45]. The EE of the cellular network can be improved by switching off some base stations during off-peak periods [9]. Customer behaviour and traffic intensity are the main factors that help to deduce where and when to apply the sleep mode and cell zooming techniques. For example, cells which have lower data traffic levels are turned to sleep mode [10], and remaining active cells zoom out to cover the areas of sleeping BSs. Therefore, the SE of the proposed cellular model during cell zooming can be expressed as below, for SFR and strict FFR.

3.4.1 Soft Frequency Reuse

In the cell zooming model it is assumed that cell number 1 in Fig. 2.5 is turned to the sleep mode. Therefore, cell number 1’s users will be served by one of the adjacent cells using their cell-edge sub-bands. We assume that the MT at cell number 1 is served by cell number 2 and is interfered with by the BSs using the same sub-bands. By using the SFR mode the interfering BSs are the set of BSs using the same sub-band at their cell-centres \( I_{11} \in \{3, 5, 7, 8, 9, 11, 12, 13, 15, 16, 17, 19\} \) and the set of BSs using the same sub-band at their cell-edges \( I_{12} \in \{4, 6, 10, 14, 18\} \). The SE of the MT in the cell zooming
Table 3.1: Conventional System Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>19</td>
</tr>
<tr>
<td>Number of cells per cluster</td>
<td>3</td>
</tr>
<tr>
<td>R</td>
<td>1000 m</td>
</tr>
<tr>
<td>$\Delta_p$</td>
<td>1.5, 2.6, 3.5</td>
</tr>
<tr>
<td>$R_t$</td>
<td>0.4R</td>
</tr>
<tr>
<td>$\nabla_{edg}$</td>
<td>3</td>
</tr>
<tr>
<td>Path-loss exponent $\beta$</td>
<td>3</td>
</tr>
</tbody>
</table>

model, using the SFR mode, is expressed as

$$C^{SFR}_{\text{zoom}}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \left( 1 + \frac{\alpha z}{m} |u|^{-\beta} \right)^{-m} \right)$$

$$\times \prod_{i \in \mathcal{I}_{11}} \left( 1 + \frac{\left( \frac{\nabla_{edg}^{SFR}}{2} \right)^z}{m} |d_c - u|^{-\beta} \right)^{-m}$$

$$\times \prod_{i \in \mathcal{I}_{12}} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} e^{-z} d z. \quad (3.22)$$

### 3.4.2 Strict Fractional Frequency Reuse

In this scheme, it is assumed that cell number 1 has been turned to the sleep mode, in which case cell number 2 will take responsibility for serving cell number 1’s users. In this case, the interfering signals will be received from BSs at cells $\mathcal{I}_{12} \in \{4, 6, 10, 14, 18\}$; therefore, the SE of the cell zooming model, using the FFR scheme, is written as

$$C^{FFR}_{\text{zoom}}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \left( 1 + \frac{\alpha z}{m} |u|^{-\beta} \right)^{-m} \right)$$

$$\times \prod_{i \in \mathcal{I}_{12}} \left( 1 + \frac{\alpha z}{m} |d_c - u|^{-\beta} \right)^{-m} e^{-z} d z. \quad (3.23)$$
3.5 Spectral Efficiency Analysis of Fractional Frequency Reuse

In strict FFR, a unity frequency reuse factor (FRF) is used in cell-centres, whereas a non-unity FRF is assigned to cell-edges. Let the total available bandwidth be partitioned into two disjointed sets of non-overlapping frequency bands denoted by $F_{cent}$ and $F_{edge}$. In strict FFR, the BS transmits two different power levels for the cell-edge users and cell-centre users. Cell-edge transmit power is given by $P_{edge} = \alpha P$, while the cell-centre transmit power is given by $P_{cent} = (2 - \alpha)P$ [91] where $1 < \alpha < 2$ is the cell-edge power amplification factor, and $\alpha = 1$ during the non-power control scheme [69] and $P$ is BS transmit power. In SFR, cell-edge transmit power is given by $P_{edge} = \alpha P$, while cell-centre transmit power is given by $P_{cent} = \frac{(\nabla_{SFR} - \alpha)}{2} P$.

From section 3.2, the SE of the MT in the cell-centre region for the conventional model, using the SFR mode, is expressed as

$$C^{SFR}_{cent}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \left( 1 + \frac{\nabla_{SFR} - \alpha}{2} \frac{z}{m} |u|^{-\beta} \right)^{-m} \right)$$

$$\times \prod_{i \in I_4} \left( 1 + \frac{\nabla_{SFR} - \alpha}{2} \frac{z}{m} |d_i - u|^{-\beta} \right)^{-m}$$

$$\times \prod_{i \in I_5} \left( 1 + \frac{\alpha z}{m} |d_5 - u|^{-\beta} \right)^{-m} e^{-\frac{z}{\rho} dz}. \quad (3.24)$$

On the other hand, the SE of the MT in the cell-edge region is expressed as

$$C^{SFR}_{edge}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \left( 1 + \frac{\alpha z}{m} |u|^{-\beta} \right)^{-m} \right)$$

$$\times \prod_{i \in I_6} \left( 1 + \frac{\nabla_{SFR} - \alpha}{2} \frac{z}{m} |d_i - u|^{-\beta} \right)^{-m}$$

$$\times \prod_{i \in I_7} \left( 1 + \frac{\alpha z}{m} |d_7 - u|^{-\beta} \right)^{-m} e^{-\frac{z}{\rho} dz}. \quad (3.25)$$

Therefore, the SE for an arbitrary user at position $u$ for a conventional
cellular system using the SFR mode can be written as

$$C_{\text{conv}}^{\text{SFR}}(u) = \begin{cases} C_{\text{cent}}^{\text{SFR}}(u), & |u| \leq R_t \\ C_{\text{edg}}^{\text{SFR}}(u), & R_t < |u| \leq R. \end{cases} \quad (3.26)$$

Assuming circular cells of radius $R$, it can be shown that when MTs are uniformly distributed in the cell the joint PDF of the position vector $u$ is given by

$$f_{|u|,\arg u}(x,\theta) = \frac{x}{\pi R^2}, \quad 0 < x \leq R, \quad 0 < \theta \leq 2\pi. \quad (3.27)$$

We arrive at the following expression for the SE of the conventional model using the SFR mode

$$C_{\text{conv}}^{\text{SFR}}(u) = \frac{1}{\pi R^2} \int_0^{2\pi} \left( F_{\text{cent}} \int_0^{R_t} C_{\text{cent}}^{\text{SFR}}(u)(xe^{j\theta})x \, dx \right. \\
+ \left. F_{\text{edg}} \int_{R_t}^{R} C_{\text{edg}}^{\text{SFR}}(u)(xe^{j\theta})x \, dx \right) d\theta. \quad (3.28)$$

### 3.6 Relaying System Model

In this chapter, a cellular communications network is considered, for which the energy and spectral efficiencies of dual-hop relay stations is calculated. During the off-peak period, some cells might turned to sleep mode to reduce the power consumed by the cellular communication systems which helps to improve the EE. The proposed model as shown in the Fig. 2.5 is a dual-hop amplify and forward relay station to serve the MT when it’s home BS turned to sleep mode. The relay and the MT affected by interference from the other BSs depends on the sub-band used by the MT. Both FFR and SFR techniques are applied to improve the SE and enhance the EE of the proposed model. In relaying model, there are four probabilities according to the location of the relay and the MT and the sub-band they will use which will determine the interfering BSs.
CHAPTER 3. ENERGY EFFICIENT CELLULAR MODELS IN FFR

3.6.1 Relaying Model Fractional Frequency Reuse

Two different types of FR deployment models, namely *Strict* FFR and SFR, are applied to the relaying system model, in order to calculate the SE and EE of the scheme.

3.6.1.1 *Strict* Fractional Frequency Reuse

Fig. 2.4 shows interfering BSs for the cell-edge and cell-centre users. We assume that cell number 1 is turned to sleep mode and the relay station receives the signal from the BS of cell number 2 using its cell-edge sub-band. The relay station and the MT will use the same sub-band used at the edge of cell number 2. Therefore, set $\mathcal{I}_{13}$ is the interfering BS on the relay and MT, where $\mathcal{I}_{13} \in \{4, 6, 10, 14, 18\}$.

3.6.2 Soft Frequency Reuse

In this scheme, it is assumed that the relay station and MT use the frequency sub-band (B), which is used at the edge of cell number 2. Therefore, sets $\mathcal{I}_{14}$
and \( I_{15} \) of the interfering BSs, where \( I_{14} \in \{3, 5, 7, 8, 9, 11, 12, 13, 15, 16, 17, 19\} \), use the same frequency sub-band at their cell-centre, and \( I_{15} \in \{4, 6, 10, 14, 18\} \) use the same sub-band at their cell-edges.

### 3.6.3 Dual-Hop AF Relays in the Presence of Interference at the Relay and the Destination and Noise at the Relay

The dual-hop relay, which extends the coverage of a network, is not particularly complex in its implementation [86]. According to the relay system model shown in Fig. 3.4, the source station transmits a signal to a destination via a relay station. It is assumed that there is no direct communication between S and the destination [95, 96]. There are two phases of communication, the first of which is between the source and the relay station, and the second is between the relay station and the destination. The relay node has no effect on the signal, apart from amplifying and re-transmitting it to the destination. It is assumed that interference is received at both the relay station and MT. The distance between the source and the relay station is fixed, while the MT is uniformly distributed around the relay station within its coverage area. The signal transmitted by the source is expressed as

\[
P_r = \sqrt{P_s a_{s,r}} + n_{s,r}, \tag{3.29}
\]

where \( a_{s,r} \) is the small-scale channel gain for the link between the transmitter and the relay, \( P_s \) is the source transmitted power and \( n_{s,r} \sim \mathcal{CN}(0, N_0) \) denotes the additive white Gaussian noise between the source and the relay. The signal transmitted by the relay station is

\[
y = G(\sqrt{P_s a_{s,r}} + n_{s,r} + \sum_{i=1}^{N} \sqrt{P_{i,r} a_{i,r}}), \tag{3.30}
\]

where \( G \) is the gain of the relay station and \( P_{i,r} \) is the average power for each interfering signal at the relay, while \( a_{i,r} \) is the interfering signal channel complex at the relay [82]. Therefore,

\[
G = \sqrt{\frac{y^2}{P_s|a_{s,r}|^2 + \sum_{i=1}^{N} P_{i,r}|a_{i,r}|^2 + N_{s,r}}}, \tag{3.31}
\]
where $n_{s,r}$ is the AWGN at the relay station and satisfies $\mathbb{E}[|n_{s,r}|^2] = N_{s,r}$ and $|a_{s,r}|^2 = |h_{s,r}|^2 |d_{s,r}|^{-\beta}$, where $d_{s,r}$ is the distance between the home BS and the relay, $\beta$ is the path-loss exponent and $N_{s,r}$ is the noise spectral density of the additive white Gaussian noise for the signal between the source and the relay.

Substituting (3.29) in (3.31) gives

$$v = \sum_{i=1}^{M} a_{i,d} \sqrt{P_{i,d}} + n_{r,d} G a_{r,d} (a_{s,r} \sqrt{P_s} + n_{s,r} + \sum_{i=1}^{N} \sqrt{P_{i,r}} a_{i,r}), \quad (3.32)$$

where $a_{r,d}$ is the channel complex for the link between the relay and the destination, and $|a_{r,d}|^2 = |h_{r,d}|^2 |u|^{-\beta}$. Also, $P_{i,d}$ represents the average power for each interfering signal at the destination, while $|a_{i,d}|$ is the interfering signal channel complex at the destination, and $|a_{i,d}|^2 = |h_{i,d}|^2 |d_{i,d}|^{-\beta}$, where $\{|h_0|^2, |h_1|^2, \ldots, |h_N|^2\}$ are the small-scale channel gains. In this case, Rayleigh fading is considered, where $\{|h_0|^2, |h_1|^2, \ldots, |h_N|^2\}$ are independent and exponentially-distributed random variables and $d_{i,d}$ is the distance between the MT and the interfering BS. It is assumed that the AWGN between the relay and the destination $n_{r,d}$ is neglected.

The SINR is expressed as follows

$$\text{SINR} = \frac{P_s G^2 |a_{s,r}|^2 |a_{r,d}|^2}{G^2 N_{s,r} |a_{r,d}|^2 + G^2 |a_{r,d}|^2 \sum_{i=1}^{N} P_{i,r} |a_{i,r}|^2 + \sum_{i=1}^{M} P_{i,d} |a_{i,d}|^2}$$

$$= \frac{G^2 |a_{r,d}|^2}{G^2 |a_{r,d}|^2 \beta + G^2 |a_{r,d}|^2 \sum_{i=1}^{N} |a_{i,r}|^2 + \sum_{i=1}^{M} |a_{i,d}|^2}, \quad (3.33)$$

where $\rho_1 = \frac{P}{N_{s,r}}$ is the signal-to-noise ratio and $P_s = P_{i,r} = P_{i,d} = P$.

We denote $\gamma_1 = |h_{s,r}|^2 |d_{s,r}|^{-\beta}, \gamma_2 = |h_{r,d}|^2 |u|^{-\beta}, \delta_{i,r} = \sum_{i=1}^{N} |h_{i,r}|^2 |d_{i,r}|^{-\beta}$ and $\delta_{i,d} = \sum_{i=1}^{M} |h_{i,d}|^2 |d_{i,r} - u|^{-\beta}$. Then,

$$\text{SINR} = \frac{G^2 \gamma_1 \gamma_2}{G^2 \gamma_1 + G^2 \gamma_2 \delta_{i,r} + \delta_{i,d}} = \frac{\gamma_1}{\delta_{i,r} + \frac{\delta_{i,d}}{\gamma_2 \rho_1} + \frac{1}{\rho_1}}, \quad (3.34)$$

Therefore, by applying Lemma 1 to (3.34), the SE for a dual-hop relay achieved by an arbitrary user can be estimated by using the average of the
Shannon formula [35] as follows

\[ C(u) = \frac{1}{2} \mathbb{E} \left[ 1 + \frac{\gamma_1}{\delta_{t,r}} + \frac{\delta_{i,d}}{7G^2} + \frac{1}{\rho_i} \right] \]

\[ = \frac{1}{2} \mathbb{E} \left[ \int_0^\infty \frac{1}{z} (1 - e^{-z\gamma_1}) e^{-z\delta_{i,r}} e^{-z\delta_{i,d}} e^{-z\rho_1} \right] dz \]

\[ = \frac{1}{2} \int_0^\infty \frac{1}{z} (1 - \mathcal{M}_1(z)) \mathcal{M}_2(z) \mathcal{M}_3(z) e^{-z} dz. \quad (3.35) \]

It is possible to represent the averages \( \mathbb{E} [e^{-z\gamma_1}], \mathbb{E} [e^{-z\delta_{i,r}}] \) and \( \mathbb{E} [e^{-z\delta_{i,d}}] \) by the MGF. Therefore the MGFs of \( \mathcal{M}_1(z) \) and \( \mathcal{M}_2(z) \) are expressed below

\[ \mathcal{M}_1(z) = \mathbb{E} [e^{-z\gamma_1}] = \mathbb{E} \left[ e^{-z|a_{s,r}|^2|d_{s,r}|^{-\beta}} \right] = \frac{1}{1 + z|d_{s,r}|^{-\beta}}, \quad (3.36) \]

and

\[ \mathcal{M}_2(z) = \mathbb{E} [e^{-z\delta_{i,r}}] = \prod_{i=1}^N \left( \frac{1}{1 + z|d_{i,r}|^{-\beta}} \right). \quad (3.37) \]

To calculate the MGF of \( \mathcal{M}_3(z) \) where

\[ \mathcal{M}_3(z) = \mathbb{E} \left[ e^{-z\delta_{i,d}} \right] = \prod_{i=1}^M \mathbb{E} \left[ e^{-z|h_{i,d}|^2|d_{i,d}|^{-\beta}} \right], \quad (3.38) \]

we denote that \( x = |h_{i,d}|^2, w = |h_{r,d}|^2, \kappa = |d_{i,d} - u|^{-\beta}, \varphi = |u|^{-\beta} \) and \( y = 1/w \). Therefore,

\[ \mathcal{M}_3(z) = \prod_{i=1}^M \mathbb{E} \left[ e^{-\frac{z}{w\varphi^\beta}} \right] = \int_0^\infty \int_0^\infty e^{-\frac{z}{w\varphi^\beta}} p_X(x)p_Y(y)dx dy. \quad (3.39) \]

The PDF of \( y \) is \( \frac{1}{y^2} e^{-\frac{1}{y}} \) [83] and

\[ \mathcal{M}_3(z) = \prod_{i=1}^M \int_0^\infty \int_0^\infty \frac{1}{y^2} e^{-\frac{z}{w\varphi^\beta}} e^{-\frac{1}{y}} dy dx. \quad (3.40) \]

Let us assume that \( x \) is a constant for instant and integrate with respect to
y and let \( s = -\frac{1}{y} \) and \( ds = \frac{1}{y^2} dy \). Therefore

\[
\int_0^\infty \frac{1}{y^2} e^{-\left(\frac{\sqrt{z \kappa \wp}}{\sqrt{G}} + \frac{1}{y}\right)} dy = \int_{-\infty}^0 s^2 e^{\left(\frac{\sqrt{z \kappa \wp}}{\sqrt{G}} + s\right)} \frac{1}{s^2} ds = \int_{-\infty}^0 e^{\left(\frac{\sqrt{z \kappa \wp}}{\sqrt{G}} + s\right)} ds = \int_0^\infty e^{-\left(\frac{\sqrt{z \kappa \wp}}{\sqrt{G}} + s\right)} ds. \tag{3.41}
\]

From [97, 3.324], the above function can be rewritten as below

\[
\int_0^\infty e^{-\left(\frac{\sqrt{z \kappa \wp}}{\sqrt{G}} + s\right)} ds = \frac{2 \sqrt{\frac{z \kappa \wp}{G}} K_1 \left(\frac{2 \sqrt{z \kappa \wp}}{G}\right)}{G}, \tag{3.42}
\]

where \( K_1(x) \) is the modified Bessel function of the first kind of order 1.

From [97, 6.643.6, 9224] and [98, 5.1.1], the integration of (3.42) can be expressed as

\[
\int_0^\infty 2 \frac{\sqrt{\frac{z \kappa \wp}{G}} K_1 \left(\frac{2 \sqrt{z \kappa \wp}}{G}\right)}{G} dx = \left(1 + \frac{e^{\frac{z \kappa \wp}{\sqrt{G}}}}{\sqrt{G}^2} z \kappa \wp Ei \left[-\frac{z \kappa \wp}{\sqrt{G}}\right]\right). \tag{3.43}
\]

Therefore, from (3.41), (3.42) and (3.43), the MGF in (3.40) can be rewritten as

\[
\mathcal{M}_3(z) = \prod_{i=1}^M \left(1 + \frac{e^{\frac{z |u|^{-\beta}}{d_{i,r} - u^{-\beta} G^2}} z |u|^{-\beta} Ei \left[-\frac{z |u|^{-\beta}}{d_{i,r} - u^{-\beta} G^2}\right]}{d_{i,r} - u^{-\beta} G^2}\right), \tag{3.44}
\]

where \( Ei \) is an exponential integral function.

Therefore, from (3.35) - (3.37) and (3.44), the SE of the dual-hop relay in the presence of interference at the relay and the destination as shown in Fig.
3.4 is written as

\[
C(u) = \frac{1}{2} \log_2 \left( e \int_0^\infty \frac{1}{z} \left( 1 - \frac{1}{1 + z|d_{s,r}|^{-\beta}} \right) \prod_{i=1}^N \left( 1 + z|d_{i,r}|^{-\beta} \right) \right.
\]

\[
\left. \times \sum_{i=1}^N \left( e^{\frac{|u|^{-\beta}}{G^2} z|u|^{-\beta} Ei \left[ -\frac{z|u|^{-\beta}}{|d_{i,r} - u|^{-\beta} G^2} \right]} \right) e^{-\frac{z}{\rho_1}} dz. \right) \quad (3.45)
\]

3.6.4 Dual-Hop AF Relays in the Presence of Noise at the Relay and the Destination

The average amount power received by the relay is expressed as

\[
P_r = \sqrt{P_s a_{s,r} + n_{s,r}}, \quad (3.46)
\]

where \(a_{s,r}\) is the channel coefficient for the link between the transmitter and the relay while \(P_s\) is the power transmitted by the source where \(n_{s,r}\) denotes additive white Gaussian noise. The signal transmitted by the relay station is

\[
y = G(\sqrt{P_s a_{s,r} + n_{s,r}}), \quad (3.47)
\]

where \(G\) is a constant gain of the relay station [82].

\[
G^2 = \frac{y}{P_s |a_{s,r}|^2 + N_{s,r}}, \quad (3.48)
\]

where \(n_{s,r}\) is the AWGN at the relay station and satisfies \(\mathbb{E}[|n_{s,r}|^2] = N_{s,r}\) and \(a_{s,r}\) is the channel coefficient for the link between the relay and the destination \(|a_{s,r}|^2 = |h_{s,r}|^2 d_{sr}^{-\beta}\), where \(d_{s,r}\) is the distance between the home BS (source) and the relay, whilst \(\beta\) is the path-loss exponent.

\[
v = G h_{r,d}(a_{r,d} \sqrt{P_s + n_{s,r}}) + n_{rd}, \quad (3.49)
\]

where \(a_{r,d}\) is the channel coefficient for the link between the relay and the destination where \(|a_{r,d}|^2 = |h_{r,d}|^2 |u|^{-\beta}\) and \(d_{r,d}\) is the distance between the relay and the destination MT.
Figure 3.5: Dual-hop AF relay system model in the presence of noise only relay and the destination (MT).

SNR is expressed as follows:

\[
\text{SNR} = \frac{P_s G^2 |a_{s,r}|^2 |a_{r,d}|^2}{G^2 \sigma_{s,r}^2 |a_{r,d}|^2 + N_{r,d}}.
\] (3.50)

We assume that gain G is constant and we consider dominant noise at the relay and the destination.

\[
\text{SNR} = \frac{G^2 |a_{s,r}|^2 |a_{r,d}|^2}{\frac{\rho_1}{\rho^2}} + \frac{1}{\rho_2},
\] (3.51)

where \(\rho_1\) , \(\rho_2\) represent the SNR for the link between the source and the relay and SNR for the link between the relay and the destination, respectively.

\[
\text{SNR} = \frac{G^2 |h_{s,r}|^2 |d_{s,r}|^\beta |h_{r,d}|^2 |u|^{-\beta}}{\frac{\rho_1}{\rho^2} + \frac{1}{\rho^2}}.
\] (3.52)

If we assume \(\gamma_1 = |h_{s,r}|^2 |d_{s,r}|^{-\beta}\) and \(\gamma_2 = |h_{r,d}|^2 |u|^{-\beta}\)
\[ \text{SNR} = \frac{G^2 \gamma_1 \gamma_2}{\rho_1} + \frac{1}{\rho_2} = \frac{\gamma_1}{\gamma_2 G^2 \rho_2} + \frac{1}{\rho_1}, \quad (3.53) \]

where,

\[
\ln \left( 1 + \frac{x}{y + 1} \right) = \int_0^\infty \frac{1}{z} \left( 1 - e^{-zx} \right) e^{-zy} e^{-z} dz.
\]

Therefore, the SE of the relaying system in the absence of interference can be expressed as

\[
C(u) = \frac{1}{2} \mathbb{E} \left[ 1 + \frac{\gamma_1}{\gamma_2 G^2 \rho_2} + \frac{1}{\rho_1} \right] = \frac{1}{2} \mathbb{E} \left[ \int_0^\infty \frac{1}{z} \left( 1 - e^{-z\gamma_1} \right) e^{-z\gamma_2 G^2 \rho_2} + \frac{1}{\rho_1} dz \right] = \frac{1}{2} \int_0^\infty \frac{1}{z} \left( 1 - \mathcal{M}_1(z) \right) \mathcal{M}_2(z) e^{-z} dz. \quad (3.54)
\]

We can represent the averages \( \mathbb{E} \left[ e^{-z\gamma_1} \right] \) and \( \mathbb{E} \left[ e^{-z\gamma_2 G^2 \rho_2} \right] \) by the moment generation functions \( \mathcal{M}_1(z) \) and \( \mathcal{M}_2(z) \). Therefore,

\[
\mathcal{M}_1(z) = \mathbb{E} \left[ e^{-z\gamma_1} \right] = \mathbb{E} \left[ e^{-z|h_{s,r}|^2|d_{s,r}|^{-\beta}} \right] = \frac{1}{1 + z|d_{s,r}|^{-\beta}}, \quad (3.55)
\]

and

\[
\mathcal{M}_2(z) = \mathbb{E} \left[ e^{-\frac{z}{\gamma_2 G^2 \rho_2}} \right] = \mathbb{E} \left[ e^{-\frac{|x|}{\gamma_2 G^2 \rho_2}} \right] = 2 \frac{\sqrt{\frac{|x|}{\rho_2}} K_1 \left( 2 \sqrt{\frac{|x|}{\rho_2}} \right)}{G}. \quad (3.56)
\]

Therefore, the SE of the dual-hop relay in the absence of interference as
Table 3.2: Relaying System Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>19</td>
</tr>
<tr>
<td>Number of cells per cluster</td>
<td>3</td>
</tr>
<tr>
<td>BS radius</td>
<td>1000 m</td>
</tr>
<tr>
<td>Relay station radius</td>
<td>300 m</td>
</tr>
<tr>
<td>Relay station $\Delta_p$</td>
<td>1.5, 2.6, 3.5</td>
</tr>
<tr>
<td>$R_t$</td>
<td>$0.4R$</td>
</tr>
<tr>
<td>$\nabla_{edg}$</td>
<td>3</td>
</tr>
<tr>
<td>Path-loss exponent $\beta$</td>
<td>3, 3.5, 3.75</td>
</tr>
</tbody>
</table>

illustrated in Fig. 3.5 is written as

\[
C(u, \theta) = \frac{1}{2} \int_0^\infty \frac{1}{z} \left(1 - \frac{1}{1 + z|d_{s,r}|^{-\beta}}\right) \times \left(2 \sqrt{\frac{z|u|^\beta}{\rho_2}} K_1 \left(\frac{2z|u|^{\beta} K_{1,1}}{G}\right) e^{-\frac{z}{\rho_1}} dz.
\]

The values of $C_{edg}(u)$ and $C_{cen}(u)$ are considered, depending on the applied relay model and whether it is in the SFR or FFR mode. Equations 3.24-3.28) in the previous section are applied to calculate the cell-centre and cell-edge SE for the relaying system model, using both the SFR and the FFR mode.

3.7 Trade-off between EE and SE

In this chapter, the trade-off between EE and SE is calculated using (2.11) in chapter 2 for all of the developed energy-efficient models, in order to make a fair comparison. Different parameters, such as BSs PA drain efficiency $\Delta_p$ and the path-loss exponent $\beta$, are changed to determine how they affect the EE for each model.
3.8 Simulations and Numerical Results

In this section, numerical and simulation results are presented. The simulation parameters are shown in Tables 3.1 and 3.2. The numerical results obtained by using the theoretical approach and verified by Monte Carlo simulations. In the conventional cellular model, the FFR and SFR schemes are applied to improve the energy efficiency of the system. Fig. 3.6 illustrates EE against normalized distance for the FFR and SFR scheme at different values of PA drain efficiency and at SNR = 10 [dB] and shows that SFR has better EE compared to the FFR scheme at a distance near to the cell-centre, while it degrades at the cell-edge due to the increase in the interference when SFR is applied.

The trade-off between EE and SE for conventional system is illustrated in Fig. 3.7 using different values of PA drain efficiency and depicted that the conventional system using SFR scheme has better EE at different values of $\Delta_p$ compared to FFR scheme. The cell zooming model is applied to improve the EE of the system, and it appears in Fig. 3.8 which shows that cell zooming with the SFR mode enhances EE compared to the conventional cellular...
Figure 3.7: EE against SE for the conventional cellular model for $\beta = 3$, $R_t = 0.4R$ at different $\Delta_p$ using SFR and FFR.

Figure 3.8: EE against SE for conventional and zooming models for $\beta = 3$, $R_t = 0.4R$ at different $\Delta_p$ using SFR.
Figure 3.9: EE against SE for conventional and zooming models for $\beta = 3$, $R_t = 0.4R$ at different $\Delta_p$ using SFR.

Figure 3.10: EE against SE for conventional and zooming models for $\beta = 3$, $R_t = 0.4R$ at different $\Delta_p$ using SFR.
Figure 3.11: EE against SE for conventional and cooperative BSs for $\beta = 3$, $R_t = 0.4R$ at different $\Delta p$.

system. The same figure shows that the cell zooming model provides the best EE at 33 kbpJ at PA drain efficiency $\Delta p = 1.5$ compared to 22 kbpJ for the conventional system at same PA efficiency value. Fig. 3.9 shows that the conventional system provides SE = 1.645 bps/Hz at the optimal point of EE while Fig. 3.10 depicts that the zooming model provides SE =1.25 bps/Hz at the optimal point of the EE. Therefore, due to the trade-off between the EE and SE, the enhancement in EE might degrades the SE of the system.

The cooperative BS model provide better solution where it improves EE with an acceptable optimal SE at 1.48 bps/Hz, as illustrated in Fig.3.11, compared to 1.25 bps/Hz for the cell zooming model shown in Fig. 3.8 and enhanced the EE to about 30 kbpJ compared to 22 kbpJ for the conventional system at the same value of PA drain efficiency.

For the relaying model, the first scenario, highlighted in Fig. 3.4, is applied in the presence of interference at both the relay and MT, while the second scenario in Fig. 3.5 is applied by considering noise only at both the relay and the MT. Fig. 3.12 demonstrates EE against SNR for both scenarios at different values of $\beta$ and illustrates that EE increases linearly with SNR for the second scenario due to the absence of interference. Also, EE increases at higher $\beta$ due
Figure 3.12: SE against SNR for a relay with/without interference for different values of $\beta$.

The increase in SE, which is expected because the relay radius is less than 0.5 [35]. The SE against EE is shown in Fig. 3.13 for FFR and SFR at different values of PA efficiency $\Delta_p$ and illustrates that SFR improves the system’s SE optimal point to 1.95 bps/Hz compared to 1.7 bps/Hz for the FFR mode. Also, the SFR mode enhances the EE of the system from about 14 kbpJ, when FFR is applied compared to 19 kbpJ when FFR scheme is applied.

In FFR, SE improves at higher values of $\beta$, which is expected because the distance between the relay is less than 0.5, which increases the numerator of the SINR, while the distance to the interfering BSs is greater than the one which degrades the denominator of the SINR [35].

### 3.9 Conclusions and Chapter Summary

In this chapter, the performance evaluations of the SE and EE for different cellular communication models are greatly simplified by using the proposed analytical approaches. Cooperative BSs and cell zooming systems are shown
Figure 3.13: SE against EE for relay station in the presence of interference using FFR at $\beta = 3$ and different $\Delta_p$.

Figure 3.14: SE against EE for relay station in the presence of interference using FFR for different values of $\beta$. 
to improve the EE of cellular communication networks. The conventional system using the SFR mode provides better SE, while cell zooming provides better EE. Cooperative BSs provide a solution in between, with an enhanced EE compared to the conventional system and an acceptable SE compared to the cell zooming model. Dual-hop AF relays are applied to improve the EE and enhance the SE of the cellular system during off-peak periods. The trade-off between SE and EE is greatly improved by using the SFR mode compared to its FFR counterpart. Adjusting power amplification factor $\alpha$ in the SFR mode for the relaying model improves the SE of the system. An increase in the path-loss exponent might improve the SE of small cells with a radius less than 0.5 km, as described above in the numerical results. Also, an increase in the overhead power consumption or power amplifier efficiency of the system’s equipment degrades the EE of cellular networks. Using dual-hop AF relays instead of macro base stations can be spatial-temporally optimised to improve their EE and reduce CO$_2$ emissions while maintaining SE performance.

In this chapter, BS cooperation, cell zooming and a relaying model were introduced as techniques that might be applied by cellular communication network operators and operating engineers to increase the energy efficiency of a cellular network. The trade-off between SE and EE was investigated, and mathematical models were developed to calculate the average capacity of four different models, including a conventional cellular network model, and then validating our results through a simulation. The three models were applied to serve the MT at the adjacent sleep cells, while the conventional model was used as a reference, following which we calculated the SINR and SE for each model. Small-scale Nakagami-$m$ fading for an urban area in the presence of inter-cell interference (ICI) was considered for conventional, cooperative BS and cell zooming, while Rayleigh fading was considered for a relaying model. Both the FFR and the SFR techniques were applied for the more efficient use of the available frequency spectrum and to enhance the EE of all four proposed models.
Chapter 4

Distributed Antenna Systems in FFR

DEPLOYING small BSs or antennas with low power consumption is a potential solution to reducing the energy consumed by cellular communication networks. In conventional cellular networks, and due to inter-cell interference (ICI), SE and EE are very low, especially at the cell-edge [99]. Recently, in the development of a new generation of cellular communication systems, the cell-edge has been focused on, in order to solve the serious performance degradations at the cell-edge [100]. The distributed antenna system (DAS) is traditionally applied to overcome dead spot problems in indoor wireless communication systems. DAS has been recently considered as a technique to reduce the radiated power of BSs which mitigate ICI and to help solve the cell-edge coverage problems seen in outdoor cellular communication systems while enhancing SE [100].

The rapid revolution in cell phone technologies and their applications is driving the need for enhanced mobile communication network infrastructure, to fulfil huge data traffic demands. Network SE is one of the most important performance measures of communications networks. Many researchers have proposed different methods and techniques to increase the SE of cellular communication networks. DAS is applied in this regard whereby remote antennae are spatially distributed within a structure or across a geographical area to reduce the distance between the antennae (RA) and a mobile terminal (MT), which by default enhances, modifies and extends the coverage of the cell. DAS is one of the solutions proposed to increase SE and improve coverage in dead spots [101,102]. Ergodic capacity is evaluated in Rayleigh fading and log-normal shadowing, by considering the signal-to-interference
ratio (SIR) for both cooperative and non-cooperative RAs [101]. The downlink capacity of both the blanket transmission and selective diversity strategies is investigated in [102], and it is shown that the use of DAS leads to a reduction in inter-cell interference in a multi-cell environment. Improvement in SE can mitigate cell-edge coverage problems in both indoor DAS (iDAS) [103] and outdoor DAS (oDAS) environments [100]. In [103], DAS is proposed as a method to enhance the coverage of indoor wireless communications. Furthermore, in [100], DAS is applied to improve capacity performance in the cell-edge when compared to a collocated antenna system (CAS).

In [104], selective transmission (ST) is used in DAS to optimise the DAE locations, with or without the central antenna, and to maximise the expected signal-to-noise ratio (SNR) lower bound in each region. Cooperative DAS is described in [105] for three different types of coverage, depending on the MT location. The authors concluded that, at certain distances away from the cell-centre, cooperative DAS techniques improve system capacity, but they also concluded that a superior signal-to-interference-plus-noise ratio (SINR) is achieved, compared to that of non-CoMP DAS transmissions. In [106], downlink joint antenna selection and transmit beamforming in a single-cell DAS is investigated as a method to calculate the satisfied user ratio. The authors in [107] use fractional frequency reuse (FFR) and unity frequency reuse (UFR), and they apply coordinated multipoint (CoMP) cooperation to maximise cell throughput. It is shown in [108] that DAS helps macrocells (MC) and femtocells (FC) to coexist within the same area, which in turn helps to achieve high SE for outdoor MC users (OMU) and indoor FC users (IFU), both of which use unity frequency reuse (UFR) and FFR. The authors in [109] investigated the joint signal-to-leakage-noise-ratio (JSLNR) of CoMP-aided DAS with FFR and assumed imperfect CSI and synchronisation errors. The authors in [110] investigated combining both transmit antenna selection and receiver maximal-ratio combining (the TAS/MRC scheme) in flat Rayleigh fading channels, by using a multiple-input-multiple-output (MIMO) scheme to compare between TAS/MRC and a space-time block code (STBC) scheme, using diversity order and the same number of receive antennae. The SE of a downlink DAS is investigated in [111], using blanket transmission and selective transmission for a single user in each DAS with a fixed cell boundary. A Poisson point process (PPP) was applied for the distribution of antenna
ports in the cell, by comparing regular DAS SE with user-centric DAS, where
the cell cell boundary was not fixed. The authors found that user-centric
DAS with selective transmission provides higher network SE. Applying FFR
or SFR techniques might increase the SE of the proposed system. The authors
in [112] investigated the trade-off between EE and SE for DAS in FFR, using
MRT. In this chapter, the trade-off between EE and SE for DAS in FFR, using
TAS, MRT and general selection combining (GSC), is investigated, following
which the results for each aspect are compared to one another.

4.1 TAS System Model

Consider a mobile cellular system where each cell is equipped with \( N \) dis-
tributed antenna elements (DAEs). We assume that one of the DAEs in each
cell is collocated with the BS at the centre of the cell, whereas the remaining
\((N-1)\) DAEs are arbitrarily distributed at the cell-edge. Both cell-edge and
cell-centre DAEs are connected to the BS via fronthaul fibre optic cables.

Consider an arbitrary MT as a reference MT, and let it be located at an
arbitrary location \( u \) in an arbitrary cell called cell\#0. Without any loss of
generality, we assume that the origin is at the BS of cell\#0. In this case, the
signal received by the reference MT in the presence of co-channel interference
(CCI) from \( C \) neighbouring cells can be expressed as

\[
y = \sqrt{P} x_0 \mathcal{H}(u) + \sum_{c=1}^{C} \sum_{n=1}^{N} \delta_{c,n} \sqrt{P} x_{c,n} \sqrt{g_{c,n}} h_{c,n} + n,
\]

(4.1)

where \( \mathcal{H}(u) \) is the complex channel of the selected DAE received by MT from
all DAEs in the same region and using the same sub-band, with a property
that \( \mathcal{H}(u) = \max_n \left\{ \sqrt{x_{0,n}} \sqrt{g_{0,n}} |h_{0,n}| \right\} \). \( x_{0,n}, x_{c,n} \) which represent the inter-
fering and useful transmitted symbols, respectively. For the sake of SE ana-
lysis, it is assumed that \( x_0 \) and \( x_{c,n} \) are complex Gaussian with zero mean
and unity variance. \( |h_{c,n}|^2 \) are the channel gains from all interfering DAEs
whereas \( |h_{0,n}|^2 \) are the channel gains from the home DAEs. \( a \) is the additive
white Gaussian noise (AWGN) at the reference MT and satisfies \( \mathbb{E}[|a|^2] = \sigma^2 \),
and \( P \) is the transmit power from the selected DAE. \( n = \{0,1,2,...,N\} \), where
\( N \) is the number of DAEs in each cell.
In (4.1), \( g_{c,n} = |d_c + r_n - u|^{-\beta} \xi_{c,n} \) are large-scale signal levels and \( \xi_{c,n} \) is the log-normal shadowing of the interfering signals. The path loss of the interfering signal is expressed as \( |d_c + r_n - u|^{-\beta} \). Here, \( \xi \sim \log N(0, \sigma^2) \) represents the random channel shadow fading and \( \beta \) is the path-loss exponent. \( r_n \) represents the position of the DAE\(_n\) referring to the centre of its home cell and, \( d_c \) represents the position of the centre of the co-channel cell \( c \).

Fig. 4.1 depicts a scenario where the \( N \) DAEs are located at \( r_n = \{ r e^{j0}, r e^{j\varnothing}, r e^{j2\varnothing}, ..., r e^{j(q-1)\varnothing} \} \) and \( r \) as the system parameter represents the distance between MT and the centre of it is home cell. \( \varnothing \) is the angle between the two adjacent DAEs at the same tier relative to the centre of their home cell, where \( \varnothing = \frac{2\pi}{q} \) and \( q \) is the number of DAEs in the cell.

The large scale signal level of the interfering DAEs can be expressed as

\[
g_{c,n} = |d_c (\cos(\theta + \Theta_c) + j \sin(\theta + \Theta_c)) + r_n (\cos(\varnothing_n) + j \sin(\varnothing_n)) - u|^{-\beta} \xi_{c,n}, \tag{4.2}
\]

where \( d_c \) is the reuse distance and \( \theta \) is the angle between the instantaneous location of MT and the first interfering BS and \( \Theta \) is the angle between two adjacent interfering BSs in the same tier where \( \Theta = 0 \) for the first interfering BS and \( \Theta_c \in \{ 0, \Theta, 2\Theta, 3\Theta, ..., (C-1)\Theta \} \) and \( C \) is the total number of interfering BSs in the same tier. Moreover, \( \varnothing \) is the angle between two adjacent DAEs in the same cell, where \( \varnothing = 0 \) for the first interfering BS and \( \varnothing_n \in \{ 0, \varnothing, 2\varnothing, 3\varnothing, ..., (N-1)\varnothing \} \) and \( N \) is the number of interfering DAEs in the same cell.

The SINR can be written as

\[
\text{SINR} = \frac{|\mathcal{H}(u)|^2}{\sum_{c=1}^{C} \sum_{n=1}^{N} \delta_{c,n} g_c h_{c1}^2 + 1/\rho}, \tag{4.3}
\]

where \( \rho = \frac{P}{N_0} \) is the SNR.

The variable \( \delta_{c,n} \) in (4.3) is a binary random variable represents the selected antenna \( n \) in the interfering cell \( c \) as

\[
\delta_{c,n} = \begin{cases} 
1 & \text{if DAE}\#n \text{ in cell } c \text{ is transmitting,} \\
0 & \text{otherwise.}
\end{cases} \tag{4.4}
\]
In TAS, only one DAE in each cell is transmitting at any one time. Therefore, \( \sum_{n=1}^{N-1} \delta_{c,n} = 1 \), for all \( c \in \{1, 2, \ldots, C\} \).

The SE achieved by an arbitrary user, at position \( u \), can be estimated by the average of Shannon’s capacity formula \([35]\)

\[
C(u) = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\|H(u)\|^2}{\sum_{c=1}^{C} \sum_{n=0}^{N} \delta_{c,n} |r_n + d_c - u|^{-\beta} \xi_{c,n} |h_{c,n}|^2 + 1/\rho} \right) |u| \right],
\]

(4.5)

where the expectation is to be taken with respect to the set of \( 2C \) non-negative random variables \( \{|h_{c,n}|^2, \xi_{c,n}, c = 0, 1, \ldots, C\} \).

**Lemma:** By using \([35]\) for any \( u, v > 0 \)

\[
\ln \left( 1 + \frac{u}{v} \right) = \int_0^{\infty} \frac{1}{s} \left( 1 - e^{-su} \right) e^{-v} ds.
\]

(4.6)

Using (4.6), we obtain from (4.7) the following explicit expression for SE

\[
C(u) = \log_2 (e) \int_0^{\infty} \frac{1}{z} (1 - W(z, u)) \mathcal{U}_c(z, u) e^{-\frac{z}{\rho}} dz,
\]

(4.7)
Figure 4.2: Resource allocation for FFR in a hexagonal system with FRF of three. (a) Three-cell cluster and (b) CCIs for the cell-centre and cell-edge MTs.
where
\[ W(z, u) = \mathbb{E} \left[ e^{-z|h(u)|^2} \right], \quad (4.8) \]

and
\[ U_c(z, u) = \sum_{n=1}^{N} \mathbb{E} \left[ e^{-z \sum_{m=1}^{N} \delta_{c,m} |r_n + d_c - u|^2 \xi_{c,n} |h_{c,n}|^2} \right] = \sum_{n=1}^{N} p_n \mathcal{V}(z | r_n + d_c - u|^{-\beta}), \quad (4.9) \]

where \( \mathcal{V}(z, u) = \mathbb{E} \left[ e^{-z \xi_{c,n} |h_{c,n}|^2} \right] \) is the MGF of the composite fading random variable \( \xi_{c,n} |h_{c,n}|^2 \) and \( p_n = P_r(\delta_{c,n} = 1) \) with \( \sum_{n=1}^{N} p_n = 1 \).

As far as the statistics of the random variables \( \{\delta_{c,1}, \delta_{c,2}, \ldots, \delta_{c,N-1}\} \) are concerned, note that these depend on the random position of the active user in cell c and the instantaneous channel conditions. An exact expression for \( p_n \) can be derived straightforwardly which require at least two-fold integrations (in order to average out the random user’s location and channel gains). However, when the \( N - 1 \) DAEs are uniformly distributed over a ring in the outer region then, owing to the symmetry of the problem, the reference MT in cell #0 will see each DAE in cell #c as being equally likely to be selected at any time. Therefore, assuming circular cells of radius \( R \), let \( r \) be the radius of the inner region, in which case it is plausible to approximate \( p_n \) as follows
\[ p_n = \begin{cases} \frac{R^2}{R^2}, & n = 0 \\ \frac{1 - \frac{R^2}{R^2}}{N-1} & n = 1, 2, \ldots, N-1. \end{cases} \quad (4.10) \]

Composite fading is modelled by a composite Nakagami-Lognormal distribution, in which case the PDF of the product \( \xi_{c,n} |h_{c,n}|^2 \) is given by [113, eq. 2.31]
\[ p_{\xi_{c,n} |h_{c,n}|^2}(x) = \int_{-\infty}^{\infty} \frac{m^m x^{m-1}}{\psi^m \Gamma(m)} \exp \left( -\frac{m x}{\psi} \right) \frac{10 (\ln 10)^{-1}}{\psi \sqrt{2\pi \sigma^2}} 
\times \exp \left( -\frac{(10 \log_{10} \psi - \mu)^2}{2\sigma^2} \right) d\psi, \quad (4.11) \]
where \( m \) is the Nakagami-\( m \) fading parameter with a value range \( (\frac{1}{2} \leq m \leq \)
∞) representing channel severity, μ is the mean power in (dB) and σ is the shadowing standard deviation of x. σ has a typical value between 2 dB and 9 dB, depending on the physical environment. In this case the MGF V(z, u) becomes

\[ V(z, u) = \int_{-\infty}^{\infty} \left( 1 + \frac{\psi}{m} \right)^{-m} \frac{10 \log_{10} 10}{\psi \sqrt{2 \pi \sigma}} \exp \left( -\frac{(10 \log_{10} \psi - \mu)^2}{2 \sigma^2} \right) d\psi. \] (4.12)

Assume \( u = \frac{(10 \log_{10} \psi - \mu)}{\sqrt{2} \sigma} \), \( \psi = \frac{\sqrt{2} \sigma u + \mu}{10} \), and \( d\psi = \frac{y \sigma \ln 10}{5 \sqrt{2}} \). Therefore

\[ V(z, u) = \int_{-\infty}^{\infty} \left( 1 + \frac{\psi}{m} \right)^{-m} \exp \left( u^2 \right) du. \] (4.13)

The equation in (4.13) can be efficiently calculated by using the Gauss-Hermite quadrature as applied in [98, (25.4.46)], which represents

\[ \int_{-\infty}^{\infty} \exp (-\varphi^2) f(\varphi) d\varphi = \sum_{t=1}^{T} w_t f(\varphi_t), \]

where \( w_t \) and \( f(\varphi_t) \) are the weight and abscissas factors of \( T \) values according to [98, Table 25.10], which provides the following:

\[ V(z, u) = \sum_{l=1}^{L} \frac{w_l}{\sqrt{\pi}} \left( 1 + z \frac{e^{\sqrt{2} \sigma x_l + \mu}}{m} \right)^{-m} + R_L, \] (4.14)

where \( w_l \) and \( x_l \) are the weight and abscissas of the \( L \)th order Hermite polynomial as tabulated in [98, table 25.10], and \( R_L \) is the remainder.

Equation (4.7) gives the SE for a specific user at location \( u \). The SE for an arbitrary user can be obtained by averaging out \( C(u) \) in (4.7), with respect to the distribution of \( u \) as can be seen in the next section.

Therefore, by using (4.14), (4.9) reduces to

\[ \mathcal{U}_c(z, u) = \sum_{n=1}^{N} \mathcal{P}_n \sum_{l=1}^{L} \frac{w_l}{\sqrt{\pi}} \left( 1 + z \frac{e^{\sqrt{2} \sigma x_l + \mu}}{m} \right)^{-m} + R_L. \] (4.15)

As far as \( W(z, u) \) in (4.8) is concerned, note that in DAS, using the TAS technique, the reference MT chooses the maximum signal received from the DAEs in its home cell. To find the maximum signal received at the location of
the MT, the MGF of the maximum received signal is
\[
E[e^{-zx}] = \int_0^\infty e^{-zx} f_{\text{max}}(x) \, dx. \tag{4.16}
\]

Therefore, (4.16) can be written as
\[
E[e^{-zx}] = \int_0^\infty ze^{-zx} P_r \{x_{\text{max}} > x\} \, dx, \tag{4.17}
\]
and
\[
W(z, u) = E[e^{-z|H(u)|^2}] = \int_0^\infty ze^{-zx} P_r \{|H(u)|^2 > x\} \, dx, \tag{4.18}
\]
where
\[
P_r \{|H(u)|^2 > x\} = P_r \left\{\max_n \{|r_n - u|^{-\beta} \xi_{0,n} h_{0,n}|^2\}^N_{n=1} > x\right\}
= 1 - \prod_{n=1}^N P_r \{|r_n - u|^{-\beta} \xi_{0,n} h_{0,n}|^2 < x\}
= 1 - \prod_{n=1}^N Y(\{|r_n - u|\beta x\}). \tag{4.19}
\]

Therefore, from (4.18) and (4.19), we have
\[
W(z, u) = 1 - z \int_0^\infty e^{-zx} \left(1 - \prod_{n=1}^N Y(\{|r_n - u|\beta x\})\right) \, dx
= \int_0^\infty ze^{-zx} \prod_{n=1}^N Y(\{|r_n - u|\beta x\}) \, dx, \tag{4.20}
\]
where
\[
Y(x) = P_r \{\xi_{0,n} h_{0,n}|^2 < x\} \tag{4.21}
\]
and
\[
P_r \{\xi_{0,n} h_{0,n}|^2 < x|\xi_{0,n}\} = P_r \left\{|h_{0,n}|^2 < \frac{x}{\xi_{0,n}}\right\}. \tag{4.22}
\]
In this research, \(|h_{0,n}|^2\) has a gamma distribution, and the probability to receive the maximum signal \(P_r\) is given by

\[
P_r \left\{ |h_{0,n}|^2 < \frac{x}{\xi_{0,n}} \right\} = 1 - \frac{\Gamma(m, mx e^{-y})}{\Gamma(m)},
\]

where \(\xi_{0,n} = e^y\) and

\[
\mathcal{Y}(x) = 1 - \int_{-\infty}^{\infty} \frac{\Gamma(m, mx e^{-y})}{\Gamma(m)} \frac{10 (\ln 10)^{-1}}{y \sqrt{2\pi\sigma}} \exp \left( -\frac{(10 \log_{10} y - \mu)^2}{2\sigma^2} \right) dy.
\]

Assume \(u = \frac{(10 \log_{10} y - \mu)}{\sqrt{2\sigma}}\), \(y = \sqrt{2\sigma u + \mu} 10\), and \(dy = \frac{\sigma}{\sqrt{2\pi}} du\). Therefore

\[
\mathcal{Y}(x) = 1 - \int_{-\infty}^{\infty} \frac{\Gamma(m, mx e^{-\sqrt{2\sigma u + \mu} 10})}{\Gamma(m)} \frac{1}{\sqrt{\pi}} e^{-u^2} du.
\]

The equation in (4.25) can be efficiently calculated by using the Gauss-Hermite quadrature as applied in [98, (25.4.46)], which represents

\[
\int_{-\infty}^{\infty} \exp \left( -\phi^2 \right) f(\phi) d\phi = \sum_{t=1}^{T} w_t f(\phi_t),
\]

where \(w_t\) and \(f(\phi_t)\) are the weight and abscissas factors of \(T\) values according to [98, Table 25.10], which provides the following:

\[
\mathcal{Y}(x) = 1 - \frac{1}{\sqrt{\pi} \Gamma(m)} \sum_{t=1}^{T} w_t \Gamma \left( m, mx e^{-\sqrt{2\sigma u + \mu} 10} \right).
\]

The incomplete Gamma function can be replaced by the following by using [115, (10.60)] as below

\[
\Gamma(m, \Theta) = (m - 1)! e^{-\Theta} \sum_{k=0}^{m-1} \frac{k^{\Theta-k}}{k!}, \quad m \in \mathbb{Z}^*,
\]

where \(\Theta = mx e^{-a\sqrt{2\sigma}}\) and \(\mathbb{Z}^*\) represents the set of all integer numbers.

Therefore,

\[
\mathcal{Y}(x) = 1 - \frac{(m - 1)!}{\sqrt{\pi} \Gamma(m)} \sum_{t=1}^{T} w_t e^{-\Theta} \sum_{k=0}^{m-1} \frac{k^{\Theta-k}}{k!}.
\]

Equation (4.7) with (4.15), (4.20) and (4.28) gives the SE for a specific MT.
Table 4.1: DAS Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>6</td>
</tr>
<tr>
<td>N</td>
<td>7</td>
</tr>
<tr>
<td>Number of cells per cluster</td>
<td>3</td>
</tr>
<tr>
<td>( R, R_t )</td>
<td>750m, ( R/3 )</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>0.35</td>
</tr>
<tr>
<td>( \varsigma )</td>
<td>0.1 W/bps</td>
</tr>
<tr>
<td>Power amplifier efficiency</td>
<td>38%</td>
</tr>
<tr>
<td>Path-loss exponent ( \beta )</td>
<td>2.5, 3.5, 3.75</td>
</tr>
<tr>
<td>Shadow fading factor ( \sigma )</td>
<td>8</td>
</tr>
<tr>
<td>Static power</td>
<td>5, 10, 15 W</td>
</tr>
</tbody>
</table>

at location \( u \). The SE for an arbitrary user can be obtained by averaging out \( C(u) \) in (4.7) with respect to the distribution of \( u \).

### 4.2 MRT System Model

In MRT, the signal received by the reference MT in the presence of CCI from \( C \) neighbouring cells, can be expressed as [112]

\[
y = \sqrt{P}x_0 \left( \sum_{n=0}^{N-1} \sqrt{g_{0,n} h_{0,n} w_{0,n}} \right) + \sum_{c=1}^{C} \sqrt{P}x_c \left( \sum_{n=0}^{N-1} \sqrt{g_{c,n} h_{c,n} w_{c,n}} \right) + n, \tag{4.29}
\]

where

\[
w_{0,n} = \frac{\sqrt{g_{0,n} h_{0,n}^*}}{\sqrt{\sum_{n=0}^{N-1} g_{0,n} |h_{0,n}|^2}}, \tag{4.30}
\]
\[ w_{c,n} = \frac{\sqrt{g_{c,n} h_{c,n}^*}}{\sqrt{\sum_{n=0}^{N-1} g_{c,n} |h_{c,n}|^2}} \quad (4.31) \]

and \( \sum_{n=0}^{N-1} |w_{c,n}|^2 = 1 \) and \( \sum_{n=0}^{N-1} |w_{0,n}|^2 = 1 \).

All parameters in (4.29) are as described in Section 4.1.

From (4.29), the SINR can be written as
\[
\text{SINR} = \frac{\sum_{n=0}^{N-1} g_{0,n} |h_{0,n}|^2}{\sum_{c=1}^{C} \left[ \sum_{n=0}^{N-1} \sqrt{g_{c,n} h_{c,n} w_{c,n}} \right]^2 + \frac{1}{\rho}},
\]

where \( \rho = \frac{P}{N_0} \) is the SNR.

The SE achieved by an arbitrary user at position \( u \) can be estimated from Shannon capacity’s formula [35]
\[
C(u) = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\sum_{n=0}^{N-1} g_{0,n} |h_{0,n}|^2}{\sum_{c=1}^{C} \left[ \sum_{n=0}^{N-1} \sqrt{g_{c,n} h_{c,n} w_{c,n}} \right]^2 + \frac{1}{\rho}} \right) |u| \right]. \quad (4.33)
\]

In order to evaluate the average in (4.33), the Cauchy-Schwarz inequality [116] is invoked
\[
\left| \sum_{n=0}^{N-1} \sqrt{g_{c,n} h_{c,n} w_{c,n}} \right|^2 \leq \left( \sum_{n=0}^{N-1} g_{c,n} |h_{c,n}|^2 \right) \left( \sum_{n=0}^{N-1} |w_{c,n}|^2 \right) = \sum_{n=0}^{N-1} g_{c,n} |h_{c,n}|^2. \quad (4.34)
\]

Therefore, a lower bound on the SE can be obtained as follows
\[
C(u) \geq \mathbb{E} \left[ \log_2 \left( 1 + \frac{\sum_{n=0}^{N-1} (r_{n} - u)^{-\beta} |\xi_{0,n}| |h_{0,n}|^2}{\sum_{c=1}^{C} \sum_{n=0}^{N-1} |d_c + r_{n} - u|^{-\beta} |\xi_{c,n}| |h_{c,n}|^2 + \frac{1}{\rho}} \right) |u| \right], \quad (4.35)
\]

where the expectation is to be taken with respect to the set of \( 2N(C+1) \) non-negative random variables \( \{ h_{0,n}^2, \xi_{c,n}, c = 0, 1, \ldots, C \) and \( n = 0, 1, \ldots, N-1 \} \).

Classical methods employed to evaluate such an average require at least \( N(C+1) \)-fold numerical integrations. In order to reduce the computational complexity of (4.35), a non-direct method can be used, based on the following lemma, which greatly simplifies computational complexity.
Using (4.6), we obtain from (4.35) the following explicit expression for SE

\[
C(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \mathbb{E} \left[ e^{-z \sum_{n=0}^{N-1} |r_n - u|^{-\beta} \xi_{0,n} |h_{0,n}|^2} \right] \right) \\
\times \mathbb{E} \left[ e^{-z \sum_{c=1}^C \sum_{n=0}^{N-1} |d_c + r_n - u|^{-\beta} \xi_{c,n} |h_{c,n}|^2} \right] e^{-z \rho d} dz. \quad (4.36)
\]

For MRT \cite{112}, the calculation of the interfering signals transmitted from the other DAEs in the interfering cells with MGF \( \mathbb{E} \left[ e^{-z \sum_{c=1}^C \sum_{n=0}^{N-1} |d_c + r_n - u|^{-\beta} \xi_{c,n} |h_{c,n}|^2} \right] \) and the useful signal with MGF \( \mathbb{E} \left[ e^{-z \sum_{n=0}^{N-1} |r_n - u|^{-\beta} \xi_{0,n} |h_{0,n}|^2} \right] \).

As above, in the special case of independent random variables, (4.36) reduces down to the following simple expression

\[
C(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \prod_{n=0}^{N-1} U(z | r_n - u|^{-\beta}) \right) \\
\times \prod_{c=1}^C \prod_{n=0}^{N-1} U(z | d_c + r_n - u|^{-\beta}) e^{-z \rho d} dz, \quad (4.37)
\]

where

\[
U(z) = \mathbb{E} \left[ e^{-z \xi_{c,n} |h_{c,n}|^2} \right]. \quad (4.38)
\]

By using the mathematical manipulations demonstrated in \cite{112}, \( U(z, u) \) can be written as

\[
U(z, u) = \sum_{l=1}^L \frac{w_l}{\sqrt{\pi}} \left( 1 + z \frac{e^{2 \sigma^2 x_l + \mu}}{m} \right)^{-m} + R_L. \quad (4.39)
\]

The SE in the case of MRT, for a specific MT at location \( u \) is obtained by using (4.36) and (4.39).

### 4.3 Generalized selection combining

In Section 4.1, a mathematical equation is derived to calculate the maximum received signal from many transmitting DAEs, using a TAS model. In the section, generalized selection combining (GSC) is applied to select a general
number of the best antennae \([117, 118]\). By using the GSC method, the best
two antennae will be selected and compared with the two extreme methods
discussed herein, whereby the best antenna is selected using TAS, and where
all transmit antennae are selected using MRT. As mentioned in the TAS sys-
tem model \(4.1\) above, the total number of DAEs per cell is \(N\) and the number
of DAEs at the cell-edge is \((N - 1)\).

The MT at the cell-edge will be served by a number of DAEs when GSC is
applied to select the best DAEs. Let

\[
\gamma_n = G_{0,n} |h_{0,n}|^2, \quad n = 1, 2, 3, ..., N - 1, \quad (4.40)
\]

be the signal level received from DAE\#\(n\) at cell \(0\) and let the order statistics
of \(\{\gamma_1, \gamma_2, \gamma_3, ..., \gamma_{N-1}\}\) be ordered such that \(\gamma^{(1)} \geq \gamma^{(2)} \geq \gamma^{(3)}, ..., \geq \gamma^{(N-1)}\).

Therefore, the SINR for the system using GSC, can be expressed as

\[
\text{SINR} = \frac{\sum_{m=1}^{M} \gamma^{(m)}}{\sum_{c=1}^{C} \sum_{n=1}^{N-1} \delta_{c,n} |r_n + d_c - u|^{-\beta} \xi_{c,n} |h_{c,n}|^2 + 1/\rho}, \quad (4.41)
\]

where \(\sum_{n=1}^{N} \delta_{c,n} = M\).

The SE can be evaluated by using the approach presented in the previous
section

\[
C(u) = \log_2 (e) \int_{0}^{\infty} \frac{1}{z} (1 - \Phi_M (z)) \prod_{c=1}^{C} A_c (z) e^{-\frac{z}{\rho}} dz, \quad (4.42)
\]

where \(\Phi_M (z) = \mathbb{E} \left[ e^{-z \sum_{m=1}^{M} \gamma^{(m)}} \right] \) and

\[
A_c (z) = \mathbb{E} \left[ e^{-z \sum_{c=1}^{C} \sum_{n=1}^{N-1} \delta_{c,n} |r_n + d_c - u|^{-\beta} \xi_{c,n} |h_{c,n}|^2} \right]. \quad (4.43)
\]

The joint PDF of the \(\{\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}, ..., \gamma^{(M)}\}\) is given by

\[
f_{\gamma^{(1)},\cdots,\gamma^{(M)}} (v_1, v_1, ..., v_M) = \sum_{n_1, n_2, ..., n_M} f_{n_1} (v_1) \cdots f_{n_M} (v_M) \prod_{l' = M + 1}^{N-1} F_{n_{l'}} (v_{M}), \quad (4.44)
\]
where \( \nu_1, \nu_1, \ldots, \nu_M \) satisfies \( \nu_1 \geq \nu_1, \ldots, \nu_M \geq 0 \), and 
\( n_1, n_2, \ldots, n_M (1 \leq n_1 \leq n_2, \ldots, n_M \leq (N - 1)) \).

It has been shown in [117] that in the case of non-identical independent order statistics

\[
\Phi_M(z) = \sum_{\substack{n_1, n_2, \ldots, n_M \geq 0 \atop n_1 \leq n_2 \leq \cdots \leq n_M}} \sum_{\substack{n_1, n_2, \ldots, n_M \geq 0 \atop n_1 \leq n_2 \leq \cdots \leq n_M}} \int_0^{\infty} e^{-zx} f_{nM}(x) \prod_{l=1}^{M-1} \Phi_{n_l}(z, x) \prod_{l'=M+1}^{N-1} F_{n'l'}(x) \, dx,
\]

where \( \Phi_{n_l}(z, x) = \int_{-\infty}^{\infty} f_{n_l}(\nu) e^{-z\nu} d\nu \) and \( f_{n_l}(\nu) \) is the PDF of the instantaneous SINR \( \gamma_{n_l} \).

It can be shown that \( \Phi_M(z) \) reduces down to

\[
\Phi_{\gamma_m}(z) = \sum_{\substack{n_1, n_2, \ldots, n_M \geq 0 \atop n_1 \leq n_2 \leq \cdots \leq n_M}} \sum_{\substack{n_1, n_2, \ldots, n_M \geq 0 \atop n_1 \leq n_2 \leq \cdots \leq n_M}} \left[ \sum_{l=1}^{L} \frac{w_l}{\sqrt{\pi}} \left( 1 + z \frac{e^{\sqrt{2}x_l + \mu}}{m} \right)^{-m} + R_L \right] \prod_{l=M+1}^{N-1} \left[ \sum_{l=1}^{L} \frac{w_l}{\sqrt{\pi}} \left( 1 + z \frac{e^{\sqrt{2}x_l + \mu}}{m} \right)^{-m} + R_L \right].
\]

As far as the evaluation of \( A_c(z) \) is concerned, we assume that the DAEs in the outer region are uniformly distributed over a ring. Therefore, due to the symmetry of the problem, it is plausible to assume that the active \( M \) DAEs in any interference cell are adjacent to one another.

In this case

\[
A_c(z) = \mathbb{E} \left[ e^{-z \sum_{c=1}^{C} \sum_{m=1}^{N-1} \delta_{c,n} |r_n + d_c - u|^{-\beta} h_{c,n}^2} \right] = \frac{1}{N-1} \sum_{n=1}^{N-1} \prod_{m=n}^{(n+M) \mod(N-1)} U \left( z |d_c + r_n - u|^{-\beta} \right),
\]

where \( U(z) \) is given in (4.39).
4.4 Spectral Efficiency Analysis of Fractional Frequency Reuse

In FFR, a unity FRF is used in the cell-centres, whereas a non-unity FRF is assigned to the cell-edges. Let the total available bandwidth be partitioned into two disjoint sets of non-overlapping frequency bands, denoted by $F_{\text{cent}}$ and $F_{\text{edg}}$. All cell-centre DAEs use the same frequency sub-band $F_{\text{cent}}$, whereas the sub-band $F_{\text{edg}}$ is divided between the cell-edges in the cluster, such that adjacent cell-edges would have distinct frequencies where $F_{\text{edg}} = F_A + F_B + F_C$. For fair FFR spectrum allocations, it is common to assume that $F_C = \lceil F \left( \frac{R}{R_t} \right)^2 \rceil$ [69], where $R_t$ represents the threshold distance that identifies the cell-centre region, $F$ is the total system bandwidth, mean $F = F_{\text{cent}} + F_{\text{edg}}$ and $R$ is the cell radius. The cell-centre and cell-edge power profiles are given by $P_{\text{cent}} = (2 - \alpha) P$ and $P_{\text{edg}} = \alpha P$, respectively, where $\alpha > 1$ is the FFR cell-edge amplification factor and $\alpha = 1$ during a non-power control scheme [69, 71]. The set of interfering cells will be changed depending on the location of the MT. In (4.1), $C = \{2, 3, 4, 5, ..., 19\}$ when the MT is located at the cell-centre and $C = \{9, 11, 13, 15, 17, 19\}$ when the MT is located at the cell-edge, as shown in Fig. 4.2, whilst CCIs outside the 19 cells are neglected.

We assume that all of the MTs at the cell-edge receive the maximum signal from one of the $(N - 1)$ DAEs at the cell-edge by using the TAS strategy, as analysed in section 4.1. Therefore, using the method presented in the previous section, it can be shown that, when the MT is in the cell-edge region

$$C_{\text{TAS-FFR-E}}(u) = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \int_0^\infty z e^{-zx} \prod_{n=1}^N Y(|r_n - u|^\beta x) \, dx \right)$$

$$\times \prod_{c \in C_E} \sum_{n=1}^N P_n \mathcal{V}(z|r_n + d_c - u |^{-\beta}) e^{-\frac{z}{\beta}} \, dz, \quad (4.48)$$

where $C_E$ is the set of relevant CCI cells.

On the other hand, an MT in the central region is served by a single DAE at the centre of the cell, and it would be subjected to CCI from all neighbouring
cells. It can be verified that in this case

\[
C_{\text{TAS}}^{\text{FFR-C}}(u) = \log_2 (e) \int_0^\infty \frac{1}{z} \left( 1 - \int_0^\infty z e^{-z x} \mathcal{V} \left( \{ |u|^\beta x \} \right) dx \right) \times \prod_{c \in C_k} \mathcal{V}(z | r_n + d_c - u |^{-\beta}) e^{-z^2} dz. \quad (4.49)
\]

For the MRT model, the cell-edge’s SE will written as

\[
C_{\text{MRT}}^{\text{FFR-E}}(u) = \log_2 (e) \int_0^\infty \frac{1}{z} \left( 1 - \prod_{n=1}^{N-1} U \left( z | r_n - u |^{-\beta} \right) \right) \times \prod_{c \in C_k} \prod_{n=1}^{N-1} U \left( z | d_c + r_n - u |^{-\beta} \right) e^{-z^2} dz, \quad (4.50)
\]

while the cell-centre’s SE can be represented by

\[
C_{\text{MRT}}^{\text{FFR-C}}(u) = \log_2 (e) \int_0^\infty \frac{1}{z} \left( 1 - U \left( z | u |^{-\beta} \right) \right) \prod_{c=1}^C U \left( z | d_c - u |^{-\beta} \right) e^{-z^2} dz. \quad (4.51)
\]

Spectral efficiency in (4.7) and (4.37) is in a general form, and so it will be calculated by using \( \mathcal{W}(z, u), \mathcal{U}_c(z, u) \) and \( U(z, u) \) for the useful and interfering signals from the previously derived equations, according to either the MRT or the TAS technique.

Therefore, the SE for an arbitrary user at position \( u \) can be written as

\[
C_{\text{FFR}}(u) = \begin{cases} 
C_{\text{FFR-C}}(u), & |u| \leq R_C \\
C_{\text{FFR-E}}(u), & R_C < |u| \leq R. 
\end{cases} \quad (4.52)
\]

Assuming circular cells of radius \( R \), it can be shown that, when MTs are uniformly distributed in the cell, the joint PDF of the position vector \( u \) can be given by

\[
f_{|u|,\arg u}(x, \theta) = \frac{x}{\pi R^2}, \quad 0 < x \leq R, \quad 0 < \theta \leq 2\pi. \quad (4.53)
\]
We therefore arrive at the following expression for SE

\[
C_{\text{FFR}}(u) = \frac{1}{\pi R^2} \int_0^{2\pi} \left( F_C \int_0^{R_C} C_{\text{FFR-C}}(xe^{j\theta}) x \, dx + \frac{F_{\text{edge}}}{\nabla} \int_{R_C}^R C_{\text{FFR-E}}(xe^{j\theta}) x \, dx \right) \, d\theta,
\]

where \( \nabla > 1 \) is the FRF and \( C_{\text{FFR-E}}(u) \), and \( C_{\text{FFR-C}}(u) \) will be substituted depending on the either the TAS or the MRT model.

### 4.5 Energy and spectral efficiencies trade-off

In DAS, it is assumed that each cell has one BS located at the cell-centre and connected to the DAEs via fronthaul fibre optic cables. Power consumed by the BS is considered in addition to the DAEs’ power consumption. Power consumed by a DAS system can be classified in two major ways: power consumed by the PA, denoted by \( P_{PA} \), and power consumed by the remaining circuit components, denoted by \( P_c \) [43].

PA power consumption can be expressed as \( P_{PA} = (1 + \tau) P \), where \( \tau = \frac{\varsigma}{\wp} - 1 \) with \( \varsigma \) being the peak-to-average ratio (PAR), which is defined as the ratio between average and maximal power where \( \wp \) is the drain efficiency of the radio frequency (RF) power amplifier.

Circuit power \( P_c \) can be expressed as a linear function of system throughput [119], [43]

\[
P_c = P_s + \zeta C(u),
\]

where \( P_s \) is DAS static power consumption, \( \zeta \) is a constant representing the dynamic power consumption per throughput, and \( C(u) \) is the total throughput of the system.

Therefore, the trade-off between EE and SE can be evaluated by using the following general and realistic expression [40], [120] and [43].

\[
\eta_{\text{EE}} = \frac{FC(u)}{P_{\text{BS}} + N(N(1 + \tau)f^{-1}(C) + P_s + \zeta C(u))},
\]

(4.56)
where $P_{BS}$ is base station power consumption, $F$ is the total available bandwidth and $N$ is the number of active DAEs which will be changed according to whether the MRT or the TAS technique is used.

4.6 Simulations and Numerical Results

In this section, we present numerical and simulation results; the simulation parameters are shown in Table 4.1. Numerical results obtained by using the theoretical approach are verified by Monte Carlo simulations. Nakagami-$m$ fading and log-normal shadowing are considered in this chapter, in order to show the exact results for the simulations. Different path-loss exponents are applied in the research, with different static power consumptions, to find the effect of both parameters on SE and EE for both the TAS and the MRT technique.

Fig. 4.3 depicts the relationship between EE and SNR for MRT at different shadow fading parameters values $\sigma$ and shows that the system has an optimal point for EE at a certain value of SNR and how EE might degrade after this
Figure 4.4: EE against SE for MRT at the path-loss exponent $\beta = 3$ and DAS radius = 333 m for a different static power $P_s$.

Figure 4.5: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3$ and a DAS radius = 250 m for $P_s = 5$ Watts.
Figure 4.6: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3$ for $P_s = 10$ Watts and a DAS radius = 250 m.

Figure 4.7: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3$ for $P_s = 15$ Watts and a DAS radius = 250 m.
Figure 4.8: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3.5$ for $P_s = 5$ Watts and a DAS radius = 250 m.

Figure 4.9: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3.5$ for $P_s = 10$ Watts and a DAS radius = 250 m.
Figure 4.10: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3.5$ for $P_s = 15$ Watts and a DAS radius = 250 m.

Figure 4.11: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3.75$ and $P_s = 5$ Watts for a DAS radius = 250 m.
Figure 4.12: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3.75$ and $P_s = 10$ Watts for a DAS radius = 250 m.

Figure 4.13: SE against EE for MRT and TAS at the path-loss exponent $\beta = 3.75$ and $P_s = 15$ Watts for a DAS radius = 250 m.
Figure 4.14: EE against SNR for MRT and TAS at the path-loss exponent $\beta = 3$ and $P_s = 5$ Watts for a DAS radius = 250 m.

Figure 4.15: EE against SNR for MRT and TAS at the path-loss exponent $\beta = 3.75$ and $P_s = 15$ Watts for a DAS radius = 250 m.
Figure 4.16: EE against SNR for MRT and TAS at the path-loss exponent $\beta = 3.5$ and $P_s = 10$ Watts for a DAS radius = 250 m.

point. The trade-off between EE and SE for MRT is illustrated in Fig. 4.4 at different values of static power $P_s$ and depicts that any increase in $P_s$ will result in the system’s EE degrading.

The trade-off between SE and EE for MRT, TAS and GSC for path-loss exponent $\beta = 3$ and different static power $P_s$ are illustrated in Fig. 4.5. This figure shows that the TAS has better EE compared to MRT and GSC, due to the reduction in the number of active DAEs in each cell. MRT has better SE compared to the GSC and TAS. Moreover, GSC has better EE compared to MRT and better SE compared to the TAS model. Furthermore, TAS has an optimal EE for the system at higher SE compared to MRT and GSC. For example, at $\beta = 3$ and $P_s = 5$ W, the optimal point for the MT of the TAS model provided at SE equates to 6.4 bps/Hz, which gives the best EE at about $2.6 \times 10^5$ b/Joule, while MRT has an optimal point for the MT provided at SE equating to 4.2 bps/Hz, which gives the best EE at about 1.75 b/Joule. GSC provides a solution between the two extreme models for SE, EE and the optimal point, which is given at 5.5 bps/Hz. The same results are evidenced in Figs. 4.6 and 4.7, where TAS has better EE for the system. We see in Figs. 4.5-4.7 that an increase in static power $P_s$ degrades system EE for MRT, TAS
Figure 4.17: EE against SNR for TAS at the different path-loss exponent $\beta$ and different $P_s$ for a DAS radius = 250 m.

and GSC.

Figs. 4.8-4.10 illustrate the trade-off between EE and SE for TAS and MRT at the path-loss exponent $\beta = 3.5$ and with different static power, and they also show that the increase in $P_s$ reduces EE for both the TAS and the MRT model. Furthermore, Figs. 4.11-4.13 illustrate the trade-off between EE and SE for TAS, MRT at the path-loss exponent $\beta = 3.75$ and with different static power, highlighting that the increase in $P_s$ reduces EE for both the TAS and the MRT model.

Figs. 4.14-4.15 illustrate the EE of the system against SNR, using TAS, MRT and GSC, and they show that the increase in SNR degrades EE for the system, especially when MRT is applied, while there is slight degradation when TAS is applied. Also, the increase in the path-loss exponent results in higher degradation in TAS and GSC. EE against SNR for TAS and MRT at the path-loss exponent $\beta = 3.5$ and $P_s = 10$ W is shown in Fig. 4.16. The trade-off between EE and SE for the TAS model is illustrated in Fig. 4.17 for different $\beta$ and different $P_s$, highlighting that the increase in $P_s$ reduces the EE of the system even if the path-loss exponent $\beta$ increases. For MRT, the trade-off between EE and SE at different values of $P_s$ and $\beta$ is illustrated in
Figure 4.18: EE against SNR for MRT at the different path-loss exponent $\beta$ and different $P_s$ for a DAS radius = 250 m.

Figure 4.19: SE against EE for TAS at $\beta = 3$ and different $P_s$ for a DAS radius = 250 m.
Figure 4.20: SE against EE for MRT at $\beta = 3$ and different $P_s$ for a DAS radius = 250 m.

Fig. 4.18 and shows that EE degrades more at higher SNR compared to TAS in Fig. 4.17.

For the TAS technique, Fig. 4.19 shows the trade-off between SE and EE at $\beta = 3$ and different $P_s$, and it illustrates that the lower the value of $P_s$, the better EE. In this figure, the system shows improved EE at about $2.59 \times 10^5$ b/Joule and SE = 6.4 bps/Hz, when the system has lower $P_s = 5$ W. On the other hand, the trade-off between EE and SE for the MRT technique at $\beta=3$ and different $P_s$ is illustrated in Fig. 4.20, which shows also that the higher $P_s$ we have, the lower EE. Furthermore, it illustrates that the TAS technique provides EE at $2.59 \times 10^5$ b/Joule and SE at 6.4 bps/Hz SE, compared to $1.7 \times 10^5$ b/Joule for the MRT EE at 4.2 bps/Hz SE when $P_s = 5$ W and same path-loss exponent $\beta$.

In Figs. 4.21a and 4.21b it is evidenced that using DAS improves SE at the edge of the cell, especially at a distance where the DAEs are deployed around the cell-edge. Moreover, these figures illustrate the effects when $P_s$ is increased and how it degrades SE over the normalised distance toward the cell-edge for both the TAS and the MRT model.

Furthermore, Fig. 4.22 illustrates EE against the normalised distance for
MRT, TAS and GSC and how SE is enhanced at the cell-edge as a benefit of applying the DAS system.

4.7 Conclusions and Chapter Summary

The performance evaluation of DAS in FFR mode, using TAS and MRT techniques, was greatly simplified through the use of a new and unified analytical approach. The trade-offs between EE and SE for both the TAS and the MRT technique were discussed, and it was concluded that TAS has better EE, while the MRT had better SE. Therefore, if we need higher SE without any concern for energy consumption, the best choice is to apply the MRT technique in the cellular network. On the other hand, if we are indeed concerned about energy consumption, and we need to improve EE, the TAS will be a better choice to be applied with DAS and FFR techniques, to gain better EE and acceptable SE. Furthermore, the potential of DAS to improve SE at the edge of the cells and to enhance the EE of a cellular network was investigated. From the numerical results, it can be concluded that the increase in the $P_s$ degrades the EE of the network; furthermore, a specific SNR might be selected for both TAS and MRT, to give SE which provides the best EE.

In this chapter, new mathematical approaches were derived for DAS, using different methods – TAS, MRT and GSC – where trade-offs between EE and SE were achieved. When MT is located at the cell-edge, we have many choices to serve it either by all DAEs, using the same sub-band by applying the MRT model, or by selecting the best transmit DAE by applying the TAS model. Instead of using the two extreme cases MRT and TAS, however, GSC can be applied as a solution to serve the MT by the best DAEs, which will be more than one and fewer than the total number of available DAEs.

According to our objective, either improving the SE or enhancing the EE of the network will help in deciding which model will be applied, in order to provide the best network performance. For each model, trade-off between EE and SE was investigated and the optimal points was calculated, in order to find a balance between the two conflicting metrics.
Figure 4.21: SE against normalised distance for TAS and MRT at a different path-loss exponent $\beta$ and SNR = 15 [dB] for a DAS radius = 250 m.
Figure 4.22: SE against normalised distance for MRT and TAS at path loss exponent $\beta = 3.75$ and SNR = 15 [dB] for DAS radius = 250 m.
Chapter 5

Cloud-Radio Access Networks in FFR

Operators looking to support smartphones and data-eating wireless devices in their cellular networks face a real dilemma when it comes to network coverage extension and capacity expansion. The data traffic of mobile networks increased by 81% in 2013 alone, and data demand is expected to reach about 24.3 Exabyte per month by 2019 (one Exabyte equals one billion gigabytes), which means that data traffic will have experienced almost a tenfold increase over just half a decade [12]. On the other hand, the availability of the frequency spectrum is limited and it is value massively increases depends on a range of parameters such as geographical location, time of purchase, physical band characteristics and the effect of neighbouring band interference.

C-RAN is an alternative solution to reducing consumed power and increasing network throughput. C-RAN architecture is different from the traditional cellular communication network, in that it is simpler and cheaper compared to the traditional RAN. In addition to enhancing network performance, the EE of the network is also improved. Moreover the CAPEX/OPEX of the cell site will be reduced. In C-RAN, the BBU (baseband unit) is responsible for the processing task of many RRHs (radio remote heads) deployed over a wide area, while in the traditional BS each RRH has a dedicated BBU. Fibre optic networks are deployed to connect RRH with BBU over high-speed fibre links, using Gbps or even Tbps switches [121]. Fibre optic networks are applied as fronthaul solutions which provide very huge data rates compared to wireless
networks. Fronthaul might use dedicated fibre (dark fibre), wavelength division multiplexing (WDM) passive optical network (PON), time-wavelength-division multiplexing (TWDM) PON \[122\] to connect an optical network unit (ONU) at an RRH site with a digital unit (DU) at the BBU pool site. The ONU sends digital baseband signal packets to a DU cloud, using common public radio interface (CPRI), and same wavelength in TWDM-PON can be shared by more than one ONU \[123\].

5.1 Background

In C-RAN, a virtualised and soft BS is implemented and integrated with a multiple BBU on the same server while supporting multiple radio access technologies (RATs) \[124\]. The objectives of 5G, to improve the EE and SE of cellular networks, can be achieved by a soft end-to-end solution from the core network to the RAN \[124\]. Different on/off small cell schemes were investigated in \[125\] and evaluated energy saving and throughput increase for ultra-dense networks (UDN) with centralised C-RAN architecture, using adaptive base band unit sharing. The authors investigated SE advantages of the C-RAN by using cooperative transmission and its power consumption privileges, which may improve the EE of the system by using a simple and efficient pre-coding scheme to reduce the complexity of cooperative transmission. SE advantages of the C-RAN cooperative transmission and its associated power consumption, as well as its effect on EE, are investigated in \[126\] using an efficient and simple per-code scheme. Outage probability at different transmit powers is investigated in \[127\] for C-RAN with different numbers of RRH, using MRT and TAS. The authors in \[128\] investigated the advantages of both heterogeneous networks and C-RAN for energy and spectral efficiency and applied RRH to a high-power node (HPN) to mitigate inter-tier interference in C-RAN.

5.2 C-RAN Architecture

C-RAN is call known as a centralised processing, Cloud, cooperative radio and clean (green) infrastructure. It has a BBU, RRH and fronthaul network, such as a fibre optic, for high-speed communication between RRH and BBU.
C-RAN is designed to be capable of performing most scenarios found in a typical RAN, from femtocell to macro cells [129]. The C-RAN architecture is also more convenient in maintenance compared to traditional RAN [68], and it is defined in two ways: fully centralised and partially centralised, according to the functions of RRH and BBU. In full centralisation C-RAN, layer 1, 2 and 3 functions are located at the BBU, while layer 2 and 3 functions are only performed by BBU in a partial centralisation C-RAN where layer one functions are located in RRH. RRH consists of a power amplifier, an antenna, an A/D converter and a low-noise amplifier (LNA). Since it is not responsible for all baseband processing functions, this makes cell sites consume lower amounts of power compared to traditional BSs site. This fact provides the opportunity to increase the number of RRHs in urban areas where traffic is very high. Centralised processing will reduce power consumed by C-RAN cell sites due to the reduction in the amount of equipment in RRH, while air-conditioning requirements will also be reduced. The required transmit power will be decreased due to the cooperative radio technology in the C-RAN and the reduction in the distance between the MT and RRHs. Furthermore, the separation in some functions between RRH and BBU will help avoid delays in the high-speed fronthaul and also avoid system service degradation [130].

5.2.1 Base Band Unit Pool

The BBU pool is placed at a remote and centralised site and has real-time precessing capabilities. The distance between the BBU pool and RRH can be extended to about 40 km, using either a fibre optic network or a microwave link, but the limitation comes from propagation and processing delays [68]. In a software-defined BBU, the signal processing resources can by dynamically allocated and processing capabilities reconfigured according to MT traffic and radio channel time variations [131]. Furthermore, the joint decoding and decompression functions are executed by the BBU pool [132].

5.2.2 Remote Radio Head

Remote radio head elements are located at remote sites and distributed over serviced areas. An RRH consists of an RF amplifier and an electro-optical conversion unit [126]. Baseband processing capabilities have moved to the
BBU pool unit, while in traditional RANs this feature is integrated at BS [68]. RRH acts as a soft relay in C-RAN by forwarding the baseband signal from the MT to the centralised BBU, thereby utilising the high-speed fronthaul network [133].

### 5.2.3 Fronthaul

C-RAN fronthaul can be equipped with high-capacity and low-cost E-band transport microwave and as an advanced architecture application [134]. E-band is a line-of-sight (LOS) and point-to-point microwave radio. E-band radio operates at 71 – 86 GHz and has many advantages, such as installation time, price and the ease of deployment, compared to fronthaul fibre optic networks. Moreover, fibre optic networks are very expensive, and they take longer time to install, find difficulty in covering some geographical areas and are not flexible as far as mobility is concerned, which means they do not offer the optimal solution. However, they are still one of the best solutions as an infrastructure for large-scale networks [134].

Fibre optic networks are considered as fronthaul in this thesis, where CPRI is used to connect RRHs with the BS over fronthaul fibre optic cables.
C-RAN SYSTEM MODEL

In C-RAN, all types of BS (RRH, small, micro and macro) in the system are aggregated in a large BBU pool, which helps to share channel state information (CSI) signalling and data for system users. Furthermore, it makes it much easier to implement algorithms applied to mitigate ICI and enhance SE [134]. For example, CoMP processing technology can be applied easily in a C-RAN infrastructure.

In this chapter, we propose a general system with an arbitrary number of high point node (HPN) cells per cluster, and each HPN has an arbitrary number of spatially distributed RRHs. In this proposed work, HPN cells are deployed to enhance coverage and serve MTs located in dead spot areas [128] or which cannot be served by distributed RRHs. HPN covers a circular area with two different frequency bands for cell-centres and cell-edges. Total HPN cell radius is $R$ and cell-centre radius is $R_t$, and we propose that each RRH covers a circular area with radius $R_t$, as illustrated in Fig. 5.2. All HPNs and RRHs are connected to the BBU pool via a fronthaul fibre optic network. The reference MT is arbitrarily located at distance $u$ and angle $\theta$, where $u$ is the distance between the MT and the centre of its home cell and $\theta$ is a random angle where $\theta \in [0, 360^\circ]$, as illustrated in Fig. 5.2.

The bandwidth in this chapter is divided between the HPN and RRH cell regions, where the sub-band dedicated to each RRH equals the sub-band of the HPN cell-centre. In fractional frequency reuse (FFR), HPN’s BW is divided between the cell-edge and cell-centre according to $F_{cent} = \lceil F_{HPN} \left( \frac{R_t}{R} \right)^2 \rceil$, where $F_{cent}$ is the HPN cell-centre, $F_{HPN}$ is the total BW dedicated to the HPN.
cell and $R_t$ is the threshold distance. The distance between the reference MT and the interfering RRH or HPN will be calculated as discussed in chapter 4 equation (4.2).

### 5.4 C-RAN Spectral Efficiency Analysis

In this chapter, it is assumed that we have two different schemes to represent the C-RAN network as illustrated in the Fig. 5.4.

#### 5.4.1 Scheme A

##### 5.4.1.1 SE for the MT of HPN

In this scheme, FFR is applied to the HPN as shown in Fig. 5.3 and the dedicated frequency spectrum for the HPN cells are divided between the cell-edge and the cell-centre, where FRF for the cell edge is $\nabla_{FFR}^{edg} = 3$ and the FRF for the cell-centre is $\nabla_{FFR}^{cent} = 1$. Two out of three RRHs in each HPN cell reuse the same frequency sub-band of the adjacent HPN cell, while these RRHs are relocated as closely as possible to mitigate the ICI of the system. It is assumed that $\mathcal{R}$ denotes the sets of groups representing interfering RRHs and $\mathcal{H}$ denotes the sets of groups representing interfering HPNs.

It is proposed that all RRHs transmit an equal power $(2 - \alpha) P$. For HPN, cell-edge transmit power is given by $P_{edg} = \alpha P$, while cell-centre transmit...
power is given by \( P_{\text{cent}} = (2 - \alpha) P \), where \( 1 < \alpha < 2 \) is the FFR cell-edge power amplification factor [69,71].

For the HPN cell-centre, the signal received by the reference MT is expressed as

\[
S_{\text{HPN,cent}}^{\text{cent}} = (2 - \alpha) \sqrt{P} x_0 \sqrt{g_0 h_0} + (2 - \alpha) \sqrt{P} \sum_{i \in \mathcal{H}} x_i \sqrt{g_i h_i} + n + n_q, \quad (5.1)
\]

where \( x_0 \) and \( x_i \) represent transmitted symbols of useful and interfering interfering HPN.

The SINR for MT in the HPN cell-centre can be represented as

\[
\text{SINR}_{\text{HPN,cent}} = \frac{(2 - \alpha) g_0 |h_0|^2}{(2 - \alpha) \sum_{i \in \mathcal{H}} g_i |h_i|^2 + \frac{1}{\rho} + \frac{1}{\rho_q}}. \quad (5.2)
\]

For notation simplicity, all useful signal parameters are subscripted by \( (0) \) for all RRHs and HPNs for both the cell-edge and cell-centre MT.

In (5.2), \( g_0 = |u|^{-\beta} \xi_0 \) are the large-scale signal levels and \( \xi_0 \) is the log-normal shadowing of the useful signals. Here, \( \xi \sim \log N(0, \sigma^2) \) represents random channel shadow fading and \( \beta \) is the path-loss exponent. \( |h_0|^2 \) is the channel gain of the useful signal, whereas \( |h_i|^2 \) is the channel gains of the interfering HPNs. Moreover, \( \rho = \frac{P}{N} \) is the SNR and \( \rho_q = \frac{P}{N_q} \) where \( N \) is the power density of the AWGN \( n \) at MT and satisfies \( N = \mathbb{E}[|n|^2] \) and where \( N_q \) is the power spectral density of quantisation noise over the fibre optic fronthaul, due to the signal compression and quantization \( n_q \) [135], and satisfies \( N_q = \mathbb{E}[|n_q|^2] \). Quantisation noise in this chapter is assumed to be stationary with zero mean and power spectral density (PSD) \( N_q \) [136].

In (5.2), \( g_i = |d_i - u|^{-\beta} \xi_i \) are the large-scale signal levels and \( \xi_i \) is the log-normal shadowing of the signals received from the interfering HPN. The path-loss of the interfering signal is expressed as \( |d_i - u|^{-\beta} \). Here, \( \xi \sim \log N(0, \sigma^2) \) represents the random channel shadow fading and \( \beta \) is the path-loss exponent.
For the HPN cell-edge, the signal received by the reference MT is expressed as

\[ y_{\text{HPN,edge}} = \alpha \sqrt{P} x_0 \sqrt{g_0} h_0 + \alpha \sqrt{P} \sum_{i \in H} x_i \sqrt{g_i} h_i + (2 - \alpha) \sqrt{P} \sum_{j \in R} x_j \sqrt{g_j} h_j + n + n_q. \]  

(5.3)

The SINR for the MT when its located at the cell-edge of HPN can be represented as

\[ \text{SINR}_{\text{HPN,edge}} = \frac{\alpha g_0 |h_0|^2}{\alpha \sum_{i \in H} g_i |h_i|^2 + (2 - \alpha) \sum_{j \in R} g_j |h_j|^2 + \frac{1}{\rho} + \frac{1}{\rho_q}}, \]  

(5.4)

where \( g_j |h_j|^2 \) represents the large- and small-scale fading of the interfering RRHs and HPNs in the adjacent HPNs which use the same sub-band as the home HPN cell-edge.

5.4.1.2 SE for the MT of RRH

In this scenario, it is assumed that the MT uses frequency sub-band F3, as illustrated in Fig. 5.3 which means that interference will be received from all HPNs and RRHs using the same sub-band and will be represented by \( H \) and \( R \), respectively.

Therefore, the signal received by the reference MT of RRH is written as

\[ y_{\text{RRH}} = (2 - \alpha) \sqrt{P} x_0 \sqrt{g_0} h_0 + \alpha \sqrt{P} \sum_{i \in H} x_i \sqrt{g_i} h_i + (2 - \alpha) \sqrt{P} \sum_{j \in R} x_j \sqrt{g_j} h_j + n + n_q, \]  

(5.5)

and the SINR is presented as

\[ \text{SINR}_{\text{RRH}} = \frac{(2 - \alpha) g_0 |h_0|^2}{\alpha \sum_{i \in H} g_i |h_i|^2 + (2 - \alpha) \sum_{j \in R} g_j |h_j|^2 + \frac{1}{\rho} + \frac{1}{\rho_q}}, \]  

(5.6)
Figure 5.4: Three-cell sector for scheme B with colour codes representing the frequency of sub-bands applied to each covered area using FFR with three HPNs and nine RRHs.

where $H$ and $R$ are the set of groups of HPN and RRH using the same sub-band of the home RRH, respectively.

### 5.4.2 Scheme B

In scheme B, all RRHs in the same HPN cell use the same frequency sub-band. FFR is applied in the HP, where the dedicated frequency spectrum for each HPN cells are divided between the cell-edge and the cell-centre as illustrated in Fig. 5.4 and discussed in chapter 2.

For the HPN cell-centre, the signal received by the reference MT will be the same as equation (5.1), while the signal received by the reference MT located at the HPN cell-edge will expressed as

$$y_{\text{edge,HPN}}^\text{HPN,A} = \alpha \sqrt{P} x_0 \sqrt{g_0 h_0} + \alpha \sqrt{P} \sum_{i \in H} x_i \sqrt{g_i h_i} + n + n_q,$$

and the SINR of the HPN cell-edge MT is expressed as

$$\text{SINR}_{\text{edge,HPN,A}} = \frac{\alpha g_0 |h_0|^2}{\alpha \sum_{i \in H} g_i |h_i|^2 + \frac{1}{\rho} + \frac{1}{\rho_p}}.$$

5.4.2.1 MRT Channel Model and Received Signal

The signal received by the reference MT is expressed as

\[ y_{\text{RRH}}^0 = (2 - \alpha) \sum_{v \in \mathcal{F}} \sqrt{P_x} g_v h_v w_v + \alpha \sqrt{P} \sum_{j \in \mathcal{R}} x_j \sqrt{g_j} h_j w_j + n + n_q, \]  

(5.9)

where \( \mathcal{F} \) denotes the set of groups representing CoMP RRHs in one HPN, \( x_j \) is the useful transmitted symbol and \( |h_j|^2 \) is the channel gains of useful signals between the reference MT and RRHs.

In (5.3), \( g_v = |d_v|^{-\beta} \xi_v = |r_v - u|^{-\beta} \xi_v \) is the large-scale signal level of the useful signal received from RRH \( v \), \( \xi_v \sim \log N(0, \sigma^2) \) represents the random shadowing and \( \beta \) is the path-loss exponent. \( u \) represents the random position of the MT relative to the centre of its home cell, as shown in Fig. 5.2. Variable \( r_v \) represents the positions of the CoMP RRH relative to the centre of the home HPN cell, which can be determined using Euler’s formula, where \( r_v = \{re^{j0}, re^{j\varnothing}, re^{j2\varnothing}, \ldots re^{j(q-1)\varnothing}\} \) and \( r \) is the distance between the centre of the HPN cell and RRH and \( d_v \) is the distance between the MT and the serving RRH. Therefore, the large-scale signal level can be expressed as

\[ g_v = |r_v (\cos(\theta + \varnothing_i) + j \sin(\theta + \varnothing_i)) - u|^{-\beta} \xi_v, \]  

(5.10)

where \( \theta \) is the angle between the instantaneous location of the MT and RRH in the same cell where \( \theta \in [0, 360^\circ] \), and \( \varnothing \) is the angle between two adjacent RRHs in the same cell, where \( \varnothing_i = 0 \) for the first RRH and \( \varnothing_i \in \{0, \varnothing, 2\varnothing, 3\varnothing, \ldots, (q-1)\varnothing\} \) and \( q \) is the total number of RRHs in the same cell.

Furthermore, \( g_j = |d_j - u|^{-\beta} \xi_j \) is the large-scale signal level of the signal received from the interfering RRH \( j \) and \( d_j \) is the distance between the MT and the interfering RRH \( j \).

In (5.9), \( w_v \) and \( w_j \) are the signal power weights [62] of the useful and interfering RRH when MRT is applied, and they are given by

\[ w_v = \frac{\sqrt{g_v h_v^*}}{\sqrt{\sum_{v \in \mathcal{F}} g_v |h_v|^2}}. \]  

(5.11)
and

\[ w_j = \frac{\sqrt{g_j^* h_j^*}}{\sqrt{\sum_{j \in R} g_j |h_j|^2}}. \]  \hspace{1cm} (5.12)

For the sake of SE analysis, it is assumed that \( x_j \) and \( x_j \) are zero mean complex Gaussian RVs. The SINR can therefore be written as

\[ \text{SINR}^{B, \text{MRT}}_{\text{RRH}} = \frac{\left(2 - \alpha\right) \sum_{v \in F} g_v |h_v|^2}{\left(2 - \alpha\right) \left| \sum_{j \in R} \sqrt{g_j h_j w_j} \right|^2 + \frac{1}{\rho} + \frac{1}{\rho_q}}, \]  \hspace{1cm} (5.13)

where \( \rho = \frac{P}{N} \) and \( \rho_q = \frac{P}{N_q} \).

### 5.4.2.2 MRT Spectral Efficiency Analysis

The SE achieved by an arbitrary user, at position \( u \), can be estimated by calculating the average of Shannon’s capacity formula [35]

\[ C(u)_{\text{RRH}}^{B, \text{MRT}} = \mathbb{E} \left[ \log_2 \left( 1 + \frac{\left(2 - \alpha\right) \sum_{v \in F} g_v |h_v|^2}{\left(2 - \alpha\right) \left| \sum_{j \in R} \sqrt{g_j h_j w_j} \right|^2 + \frac{1}{\rho} + \frac{1}{\rho_q}} \right) \right] \]  \hspace{1cm} (5.14)

In order to evaluate the average in (5.14), Cauchy-Schwarz inequality is invoked as in 4.34. Therefore, the SE of the system is expressed as

\[ C(u)_{\text{RRH}}^{B, \text{MRT}} = \log_2 (e) \int_0^\infty \frac{1}{z} \left( 1 - \sum_{v \in F} U_v \left( z (2 - \alpha) |r_v - u|^{-\beta} \right) \right) \times \sum_{j \in R} U_j \left( z (2 - \alpha) |r_j - u|^{-\beta} \right) e^{-\frac{z}{\rho}} e^{-\frac{z}{\rho_q}} dz. \]  \hspace{1cm} (5.15)

Gauss-Hermite quadrature as in [98, 25.4.46] is applied to calculate \( U(z) \) (5.15), which we present as

\[ U(z) = \sum_{l=1}^L \frac{w_l}{\sqrt{\pi}} \left( 1 + \frac{e^{\sqrt{2 \sigma x_l} + \mu}}{m} \right)^{-m} + R_L, \]  \hspace{1cm} (5.16)

where \( w_l \) and \( x_l \) are the weight and abscissas of the \( L^{th} \) order Hermite polynomial, which are tabulated in [98, table 25.10], and \( R_L \) is the remainder.

Equation (5.15) gives the SE for a specific user at location \( u \). SE for an
arbitrary user can be obtained by averaging out $C(u)$ in (5.15) with respect to the distribution of $u$.

**5.4.2.3 C-RAN System Model using TAS**

In the TAS model, SE is calculated depending on the signal received from the best antenna available in the HPN cell. The best SINR depends on the combination of channel state information and the distance between MT and the transmitting RRH. In the TAS model, it is also proposed that all RRHs transmit equal power \((2 - \alpha) P\), the same as the HPN cell-centre, where \(1 < \alpha < 2\) is the FFR cell-edge power amplification factor [69,71], as discussed in detail in Chapter 3.

**5.4.2.4 TAS Channel Model and Received Signal**

The signal received by the MT from the best RRH, using the TAS model in scheme B, is represented by

\[
y_{\text{RHH}}^{B,\text{TAS}} = \sqrt{(2 - \alpha) P x H(u)} + \sqrt{(2 - \alpha) P \sum_{j \in R} x_j \sqrt{g_j |h_j| w_j} + n + n_q}, \tag{5.17}
\]

where $H(u) = \max_x \{ \sqrt{x} \sqrt{\xi_v |h_v|} \}_{v=1}^q$ and $q$ is the total number of RRHs in the HPN cell.

In (3.9), $g_v$, $g_i$, and $g_j$ are described in Section 5.4.1 and 5.4.2.1.

Therefore, for the sake of SE analysis, it is assumed that $x_v$, $x_i$ and $x_j$ are Gaussian, and so the SINR the MT of RRH using the TAS model can be written as

\[
\text{SINR}_{\text{RHH}}^{B,\text{TAS}} = \frac{(2 - \alpha) |H(u)|^2}{(2 - \alpha) \sum_{j \in R} g_j |h_j|^2 + 1/ \rho + 1/ \rho_q}. \tag{5.18}
\]

**5.4.2.5 TAS Spectral Efficiency Analysis**

The SE achieved by an arbitrary user, at position $u$, can be estimated by the average of Shannon’s capacity formula [35]

\[
C(u)^{B,\text{TAS}}_{\text{RHH}} = \mathbb{E} \left[ \log_2 \left( 1 + \frac{(2 - \alpha) |H(u)|^2}{(2 - \alpha) \sum_{j \in R} g_j |h_j|^2 + 1/ \rho + 1/ \rho_q} \right) |u| \right]. \tag{5.19}
\]
The expectation in (5.19) is applied with respect to the \(2 + 2q\) non-negative random variables.

By using (3.11), we obtain, from (5.19), the following explicit expression for the SE

\[
C(u)_{RRH}^{B,TAS} = \log_2(e) \int_0^\infty \frac{1}{z} \left( 1 - \sum_{v \in \mathcal{F}} U_v \left( z (2 - \alpha) |r_v - u|^{-\beta} \right) \right) \times \sum_{j \in \mathcal{R}} U_j \left( z (2 - \alpha) |r_j - u|^{-\beta} \right) e^{-\frac{z}{\rho} e^{-\frac{x}{\rho q}}} dz. \tag{5.20}
\]

As applied in 4.1 to derive (4.27), the above function is manipulated to find the expectation for the random variables. Therefore,

\[
U(x) = 1 - \frac{1}{\sqrt{\pi} \Gamma(m)} \sum_{t=1}^T w_t \Gamma \left( m, mxe^{-a\sqrt{2}\sigma x_t} + \mu \right). \tag{5.21}
\]

The incomplete Gamma function can be replaced by the following using [115, (10.60)] as below

\[
\Gamma(m, \Theta) = (m - 1)!e^{-\Theta} \sum_{k=0}^{m-1} \frac{\kappa \Theta^{-k}}{k!}, \quad m \in \mathbb{Z}^*, \tag{5.22}
\]

where \(\Theta = mxe^{-a\sqrt{2}\sigma x_t} + \mu\).

By using (3.19), \(U(x)\) will be written as

\[
U(x) = 1 - \left( \sum_{t=1}^T w_t \frac{(m - 1)!e^{-(mxe^{-a\sqrt{2}\sigma x_t} + \mu})}{\sqrt{\pi} \Gamma(m)} - \sum_{k=0}^{m-1} \frac{(mxe^{-a\sqrt{2}\sigma x_t} + \mu)^{-k}}{k!} \right). \tag{5.23}
\]

### 5.4.2.6 General Selection Combining

In Scheme B is assumed that there are \(q\) RRHs using the same sub-band with the capability to cooperate in serving the MT in the same HPN cell by using the MRT model, as discussed in section 5.4.2.1. Moreover, MT can receive the best RRH when the TAS model is applied, as in section 5.4.2.3. General selection combining (GSC) is another solution between the extreme cases of MRT and TAS. In GSC, MT will receive the best \(M\) number of antennae, where \(1 < M < q\) and \(q\) is the total number of RRH available in HPN and using the same sub-band. The joint PDF of the useful signals received from the best
**Table 5.1: C-RAN Simulation Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RRH</td>
<td>63</td>
</tr>
<tr>
<td>HPN</td>
<td>19</td>
</tr>
<tr>
<td>HPN per cluster</td>
<td>3</td>
</tr>
<tr>
<td>RRH per HPN cell</td>
<td>3</td>
</tr>
<tr>
<td>RRH cell radius</td>
<td>200 m</td>
</tr>
<tr>
<td>HPN cell radius</td>
<td>450 m</td>
</tr>
<tr>
<td>Path-loss exponent</td>
<td>$\beta = 3.5$</td>
</tr>
<tr>
<td>Shadow fading factor</td>
<td>$\sigma = 8$</td>
</tr>
<tr>
<td>HPN static power</td>
<td>5, 10, 15 W</td>
</tr>
<tr>
<td>RRH static power</td>
<td>2, 4, 6 W</td>
</tr>
</tbody>
</table>

RRHs can be calculated as in section 4.47.

### 5.5 Spectral Efficiency Analysis

In this chapter, FFR is applied for HPN cells, and unity FRF is applied for the cell-centres whereas non-unity FRF is assigned to the cell-edges. The BW allocated for the HPN cell is $F_{HPN}$ and the cell-centre sub-band is calculated according to $F_{cent} = \lceil F_{HPN} \left( \frac{R_t}{R_R} \right)^2 \rceil$, where $F_{cent}$ is the HPN cell-centre and $R_t$ is the threshold distance. Therefore, the cell-edge sub-band can be calculated as $F_{edge} = F_{HPN} - F_{cent}$. The sub-band allocated for each RRH is equal to $F_{cent}$.

The SE of the C-RAN for the HPN cell-edge and cell-centre, either for scheme A or for scheme B, will be calculated using (4.54) and the equations in 5.4.1 and 5.4.2.

### 5.6 C-RAN Energy Efficiency

In C-RAN, the number of BSs will be reduced due to the use of a centralised processing feature, in which case any equipment consuming more power,
such as air-conditioning, will actually be reduced [134]. Therefore, small cells such as RRH with low power consumption are deployed, while network capacity and coverage are enhanced.

In the proposed approach, the sector has three cells, each of which has one HPN and three RRHs. We tried to calculate the power consumed by the C-RAN system, using realistic PCM and including all parameters in the system. The energy consumed by C-RAN is consolidated in different terms, such as BBU housing facilities, cell site network devices and central office (CO) network devices [123]. Housing facilities need an amount of energy to run certain equipment (e.g. air conditioning) which is important to cool network devices in the facility. The power consumed by cell site, either by HPN or BBU, is described respectively below [123] [43].

\[ P_{HPN} = (P_s^H + N(1 + \tau)f^{-1}(C) + P_{ONU} + \zeta C_H(u)) |\mathcal{H}|, \] (5.24)

and

\[ P_{RRU} = (P_s^R + N(1 + \tau)f^{-1}(C) + P_{ONU} + \zeta C_R(u)) |\mathcal{R}|, \] (5.25)

where \( P_s^H, C_H(u), P_s^R \) and \( C_R(u) \) are static power and throughput for the HPN and RRU units respectively. \(|\mathcal{H}|\) and \(|\mathcal{R}|\) are the total numbers of HPNs and BBUs in the system. Power consumed by the optical network unit is denoted by \( P_{ONU} \). Therefore, the trade-off between EE and SE of the C-RAN can be expressed as

\[ \lambda_{EE} = \frac{F \mathcal{C}(u)}{P_E} \text{ bits/Joule}, \] (5.26)

where \( P_E \) is substituted by either \( P_{HPN} \) or \( P_{RRU} \) and \( F \) represents the total available bandwidth.

In this chapter, in order to produce numerical results and make a comparison between different proposed models, we applied at least two different schemes as follows.
CHAPTER 5. CLOUD-RADIO ACCESS NETWORKS IN FFR

Figure 5.5: EE against the SE of HPN for scheme A with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and for several values of $P_s$.

Figure 5.6: EE against the SE of RRH for scheme A with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and for several values of $P_s$. 
Figure 5.7: EE against the SE of HPN for scheme B with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and for several values of $P_s$.

Figure 5.8: EE against the SE of RRH for scheme B using MRT with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and for several values of $P_s$. 
Figure 5.9: EE against the SE of RRH for scheme B using CT-2 with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and for several values of $P_s$. 

Figure 5.10: EE against the SE of RRH for scheme B using TAS with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and for several values of $P_s$. 
In this section, we present the numerical and simulation results. The simulation parameters are shown in Table 5.1. The numerical results, obtained by using the theoretical approach, are verified by Monte Carlo simulations. Fig. 5.5 depicts the trade-off between EE and SE for the RRH of scheme A for several values of static power $P_s$ and a shadow fading parameter $\sigma = 8$ with a path-loss exponent $\beta = 3.5$, demonstrating that a reduction in static power improves the EE of the system. In this figure, the optimal point for HPN, achieved at about 2.25 bps/Hz. Fig. 5.6, illustrates the trade-off between EE and SE for the RRH of scheme A for several values of static power $P_s$ and a shadow fading parameter $\sigma = 8$ with a path-loss exponent $\beta = 3.5$, depicting that RRH has an optimal point at about 2.35 bps/Hz of SE which provides better value compared to HPN for the same system. Fig. 5.7 depicts the trade-off
Figure 5.12: EE against the SE of RRH of scheme A and B with different antenna diversity at a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and $P_S = 6$ W.

between EE and SE for scheme B at several values of $P_s$ and shows reducing static power improves EE. Moreover, the HPN of scheme B has a wider range of SE compared to the HPN of scheme A, which in turn has lower EE due to the misuse of the available frequency spectrum in scheme B. Also, the HPN of scheme A has a better optimal point at about $2.38$ bps/Hz compared to $2.2$ bps/Hz for the HPN of scheme B. Trade-off between EE and SE for the RRH of scheme B using MRT is illustrated in Fig. 5.8 for different $P_s$ values and shows the effect of reducing the static power of the RRH. Furthermore, Fig. 5.9 and 5.10 show the trade-off between EE and SE for the RRH of scheme B, using CT-2 and TAS, and also illustrate that a reduction in static power improves the EE of the system.

In Figs. 5.11-5.13, MRT has the best SE and lowest EE due to the large number of active RRHs for scheme B. Scheme A has better EE compared to scheme A due to the efficient use of the available frequency spectrum. Finally, Fig. 5.14 depicts the EE against normalised distance for the RRH of scheme B, using TAS and scheme A, and shows that the RRH of scheme A has an EE optimal point at 0.27 of the normalised distance, while scheme B using
Figure 5.13: EE against SE of RRH of scheme A and B with different antenna diversity at shadow fading parameter $\sigma = 8$ at path loss exponent $\beta = 3.5$ and $P_S = 6$ W.

Figure 5.14: EE against the normalised distance of RRH of scheme A and B using TAS with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and $P_S = 2$ W and SNR $= 10$ [dB].
Figure 5.15: SE against the normalised distance of RRH of scheme A with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and $P_S = 2$ W and SNR = 10 [dB].

TAS has an EE optimal point at about 0.12 of the normalised distance. These results are supported by Figs. 5.15-5.16.

Fig. 5.11 illustrate a comprehensive comparison for an EE-SE trade-off for RRH between scheme A and the three models in scheme B at static power $P_s = 2$ W, whereby scheme A has better EE at about 0.41 MbpJ compared to the TAS model of scheme B, which achieves EE at about 0.393 MbpJ. The MRT model provides better SE at about 2.52 bps/Hz compared to 2.49 bps/Hz for scheme A and 2.37, 2.45 bps/Hz for TAS and CT-2, respectively. Moreover, Fig. 5.12 illustrates a comparison for an EE-SE trade-off of RRH between scheme A and the three models in scheme B at static power $P_s = 4$ W, highlighting that scheme A has better EE at about 0.396 MbpJ compared to the TAS model in scheme B, which achieves EE at about 0.387 MbpJ. Also, Fig. 5.13 illustrates a comparison for the EE-SE trade-off of RRH between scheme A and the three models in scheme B at static power $P_s = 6$ W, showing that scheme A has better EE at about 0.393 MbpJ compared to the TAS model in scheme B, which achieves EE at about 0.384 MbpJ.
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Figure 5.16: EE against the normalised distance of RRH of B using TAS with a shadow fading parameter $\sigma = 8$ at a path-loss exponent $\beta = 3.5$ and $P_S = 2$ W and SNR = 10 [dB].

5.8 Conclusions and Chapter Summary

The potential of C-RAN to improve the SE and EE of a cellular network was investigated in this chapter. The performance evaluation of C-RAN in FFR mode, using TAS, MRT and general selection combining, was greatly simplified by using an analytical approach. From the numerical results, MRT improves the system’s SE by using either RRH or HPN compared to TAS or GSC with the best two antennae as applied in scheme B, while the TAS model improves the EE of the system. Efficient use of the available frequency spectrum helps to improve system EE as applied in scheme A. FFR is applied on HPN for both schemes A and B, which helps to improve the EE of the system. Furthermore, an increase in static power degrades the EE of the network. Finally, a specific SNR might be selected to give a certain throughput, which provides the best EE for the system.

In this chapter, two different C-RAN schemes were investigated, in order to enhance the trade-off between EE and SE. In both schemes, the most efficient ways to use the available frequency spectrum were applied and compared. Two different sizes of antenna, namely RRH and HPN, with different specifications were applied, in order to gain better coverage and quality of
service for the MT. FFR was applied to HPN cells for the more efficient use of the frequency spectrum and to improve the EE of the system. Three models of antenna diversity, MRT, TAS and GSC, were applied and compared to find the best model.
Chapter 6

Conclusions and Future Directions

This chapter presents conclusion remarks in relation to the main results in this thesis. In addition, possible areas for further research are proposed and discussed.

6.1 Conclusions

At the beginning of Chapter 2, the metrics and definitions of EE and SE are presented. Section 2.2 defines the SE of the cellular network as an efficient way of using the frequency spectrum or bandwidth to transmit data over a wireless channel with an acceptable QoS. The SE metric is derived and expressed as kbps/Hz in many existing works. Section 2.3 presents an overview of network EE in bits/Joule, which a number of other studies express as Joules/bit. In Section 2.4, the trade-off between EE and SE is discussed, using realistic and non-realistic PCM. Section 2.5 defines some energy-efficient cellular communication models, some of which are studied in this thesis. In Section 2.6, two types of FFR, namely strict FFR and SFR, are presented. In strict FFR, the available system BW is partitioned into two disjointed sets of non-overlapping frequency bands between the cell-edge and cell-centre regions, and a cell-edge sub-band is divided into three sub-bands. In SFR, the available BW is divided into three non-overlapping sub-bands, where cell-centre users are allowed to share sub-bands with the edge users of other cells.

Chapter 3 concerns four different energy-efficient cellular communication models. Section 3.4 presents the derivation of the mathematical equation for the SE of the conventional cellular model in the presence of ICI and
Nakagami-\( m \) fading, in which strict FFR and SFR are applied. The trade-off between EE and SE is calculated at different system static power and path-loss exponent values. Subsections 3.5 and 3.6 present the SE of cooperative BSs and cell zooming models. Section 3.8 concerns a relaying model in the presence and absence of ICI. Rayleigh fading and FFR are applied in this model at different path-loss exponent values.

Chapter 4 defines DAS with two different schemes to improve and increase SE and EE. In this chapter, composite Nakagami-\( m \) fading and log-normal shadowing are considered as well as strict FFR in the presence of interference. Section 4.2 presents a TAS system model and derives a mathematical equation for SE. An EE and SE trade-off is calculated for several values of path-loss exponent, and EE is calculated at different values of DAS static power Ps. Section 4.3 presents the mathematical derivation of the DAS system, using MRT, in order to calculate SE. The trade-off between EE and SE is calculated and compared with the results in Section 4.2 at different values of path-loss exponents. SE for both schemes’ analysis with strict FFR is presented in Section 4.4. A realistic PCM is applied to calculate EE for the two schemes of the DAS model. The effect of the DAS model on SE at the edge of the cell is presented in some detail.

In Chapter 5, C-RAN is presented as a proposed 5G cellular model, in order to enhance the SE and EE of the communication network. Section 5.3 defines a C-RAN architecture, which consists of the RRH, BBU pool and network fronthaul. A C-RAN general system model is described in Section 5.4. Subsection 5.4.1 presents a C-RAN model, using an MRT scheme, and a TAS scheme is presented in Subsection 5.4.2 in the presence of interference and composite Nakagami-\( m \) fading, in which log-normal shadowing is also considered. Section 5.5 defines BW allocation for the RRH and HPN when FFR is applied. The trade-off between SE and EE for C-RAN is presented in Section 5.6, and the power consumption for both RRH and HPN is calculated for several path-loss exponents and RRH and HPN static power values. Two different schemes are proposed depending on frequency resource allocation for the RRH and HPN. FFR is applied in both schemes, in order to enhance the trade-off between EE and SE.
6.2 Future Directions

There are many interesting topics to be studied in the area of energy efficiency and spectral efficiency for cellular networks. Here, we suggest a few possible research extensions to the work carried out in this thesis.

6.2.1 Energy Harvesting

Energy harvesting is one of the hot topics currently drawing the significant attention of researchers [137]. This new technology allows MT to collect energy from ambient environments that act as a part of renewable energy sources such as wind and solar systems. Therefore, the signal radiated by the surrounded transmitters can be treated as a proposed source for energy harvesting [138, 139]. Furthermore, interference can be exploited as a source of energy harvesting, which would help enhance system throughput [140]. Therefore, energy harvesting is a proposed subject to extend our previous cellular networks presented in Chapters 3, 4 and 5 into wireless power, cooperative cellular communication networks by adding one hybrid access point (AP) to each cluster. The effects on system performance, such as EE and SE, will be analysed, and the trade-off between energy and spectral efficiency will be extensively investigated.

6.2.2 EE of C-RAN Multiple Antenna RRH and HPN

In Chapter 5, the C-RAN model is presented by using spatial antenna diversity via HPN and RRH, where each option is equipped with one antenna. In order to enhance system performance further, and improve SE and EE, HPN and RRH might be equipped with multiple antennae. The effect of this proposal will be investigated specially on a TAS model using one antenna. The proposal has great potential to improve the SE of TAS, which will be compared with the MRT scheme with spatial distributed antennae. The effect on EE and the trade-off between EE and SE will also be investigated.

6.2.3 D2D Cellular Networks

In order to continue researches in an attempt to improve the EE of communication networks, D2D could potentially enhance the trade-off between EE
and SE. D2D communication systems can be energy-harvesting based on exploiting ambient interference in cellular networks [141]. With the support of 5G communication networks, D2D allows communication directly between devices with low transmit power and high data rates, which is an appealing solution for energy-efficient cellular networks [142]. In this proposal and by using smart phones, the MT will be less dependent on the BSs or RRH when C-RAN is applied. With this in mind, the D2D system will be analysed, and the SE and EE trade-off will be investigated and compared with previously developed and proposed models.

### 6.2.4 Wireless Fronthaul

C-RAN fronthaul can be equipped with a high-capacity and low-cost E-band transport microwave and used as an advanced architecture application [134]. E-band is a line-of-sight (LOS) and point-to-point microwave radio. E-band radio operating at 71-86 GHz has many advantages, such as installation time, price and the ease of deployment, compared to fronthaul fibre optic networks. Moreover, fibre optic networks are very expensive and they takes longer to install, find difficulty in covering some geographical areas and are net-fixable in mobility, which does not make them an optimal solution. Nonetheless, they are still one of the best solutions as infrastructure for large-scale networks [134]. Wireless front-haul in a C-RAN model might be considered instead of a fibre optic network, in order to provide more flexibility for cellular networks. In this case, HPN and RRH in the C-RAN will work as relaying stations. Therefore, the air segment between BBU and RRH or HPN will be extensively analysed, to investigate the effect of large-scale and small-scale fading on network performance.
Bibliography


[84] I. Trigui, S. Affes, and A. Stephenne, “On the ergodic capacity of amplify-and-forward relay channels with interference in nakagami-m


[93] M. Aldosari and K. Hamdi, “Trade-off between energy and area spectral efficiencies of cell zooming and bss cooperation,” in Intelligent and
Advanced Systems (ICIAS), 2014 5th International Conference on, June 2014, pp. 1–6.


Appendix A

Lemma 1 Derivation

The formal proof of the following lemma is reproduced as in [143]

\[ \ln (1 + x) = \int_{0}^{\infty} \frac{1}{z} \left(1 - e^{-zx}\right) e^{-z} dz \]  
(A.1)

The following series of \((1 + x)\) is valid for \(x \geq 0\) [98, Eq. 4.1.25]

\[ \ln (1 + x) = \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x}{1+x}\right)^n, \quad x \geq 0. \]  
(A.2)

By using the identity as in [97, Eq. 3.381.4]

\[ (x)^n = \int_{0}^{\infty} s^{n-1} \frac{e^{-\left(\frac{x}{s}\right) \Gamma(n)}}{\Gamma(n)} ds, \quad s, x > 0. \]  
(A.3)

Therefore, by applying A.3, then A.2 can be derived as

\[ \ln (1 + x) = \sum_{n=1}^{\infty} \frac{1}{n} \int_{0}^{\infty} s^{n-1} \frac{e^{-\left(\frac{1+x}{s}\right) \Gamma(n)}}{\Gamma(n)} ds \]

\[ = \int_{0}^{\infty} \left\{ \sum_{n=1}^{\infty} \frac{1}{n} \frac{s^{n-1}}{\Gamma(n)} \right\} e^{-s \left(\frac{1+x}{s}\right) \Gamma(n)} ds \]  
(A.4)

where \(\Gamma(n) = (n-1)!\) and \(e^s = \sum_{n=1}^{\infty} \frac{s^n}{n!}\), we have

\[ \ln (1 + x) = \int_{0}^{\infty} \left\{ \frac{1}{s} \left( e^s - 1 \right) \right\} e^{-s \left(\frac{1+x}{s}\right)} ds \]  
(A.5)
APPENDIX A. LEMMA 1 DERIVATION

by substitute \( s = zx \), therefore, we have

\[
\ln(1 + x) = \int_0^\infty \left\{ \frac{1}{zx} (e^{zx} - 1) \right\} e^{-zx} \frac{1}{x} x dz
\]

(A.6)

Finally, we obtained A.1.

\[
\ln(1 + x) = \int_0^\infty \frac{1}{z} (1 - e^{-zx}) e^{-z} dz
\]

(A.7)

If we apply the above lemma for \( x = \frac{a}{b+c} \) for any \( a, b, c > 0 \). We have

\[
\ln \left( 1 + \frac{a}{b+c} \right) = \int_0^\infty \frac{1}{t} \left( 1 - e^{-t(\frac{a}{b+c})} \right) e^{-t} dt
\]

(A.8)

Assume \( t = z \frac{b}{b+c} \). Therefore,

\[
\ln \left( 1 + \frac{a}{b+c} \right) = \int_0^\infty \frac{1}{z} (1 - e^{-za}) e^{-zb} e^{-zc} dz
\]

(A.9)

and

\[
\ln \left( 1 + \frac{a}{b+c} \right) = \int_0^\infty \frac{1}{z} (1 - e^{-za}) e^{-zb} e^{-zc} dz
\]

(A.10)

By using logarithmic basics where \( \ln(a) = \log_e(a) = \frac{\log_2(a)}{\log_2(e)} \). From (A.10), we have

\[
\log_2 \left( 1 + \frac{a}{b+c} \right) = \log_2(e) \int_0^\infty \frac{1}{z} (1 - e^{-za}) e^{-zb} e^{-zc} dz
\]

(A.11)

Now, we find the average of (A.11) where \( a \) and \( b \) are independent random variables and \( c \) is a constant number.

\[
\mathbb{E} \left[ \log_2 \left( 1 + \frac{a}{b+c} \right) \right] = \mathbb{E} \left[ \log_2(e) \int_0^\infty \frac{1}{z} (1 - e^{-za}) e^{-zb} e^{-zc} dz \right]
\]

(A.12)

where \( \mathbb{E}[e^{-za}] \) and \( \mathbb{E}[e^{-zb}] \) are the MGF of the random variables \( a \) and \( b \).
Appendix B

Moment Generating Function

The moment generating function (MGF) of a random variable $X$ can be expressed as the expected value $E[e^{Xt}]$ which is finite and exist for all real numbers determined by the close interval $[-a, a] \subseteq \mathbb{R}$ and $a > 0$. Therefore, we can say that $X$ has MGF as

$$M_X(t) = E[e^{Xt}] \quad (B.1)$$

To calculate the MGF of exponential random variable $[144, 145]$, we have

$$E[e^{Xt}] = \int_{-\infty}^{\infty} e^{Xt} f_X(x) \, dx \quad (B.2)$$

where $f_X(x)$ is the probability density function of $X$ and can be expressed as

$$f_X(x) = \begin{cases} \lambda e^{-(\lambda x)}, & \text{if } x \in R_X \\ 0, & \text{if } x \notin R_X \end{cases} \quad (B.3)$$
APPENDIX B. MOMENT GENERATING FUNCTION

where \( \lambda \) is a positive number and \( R_X = [0, \infty) \). Therefore,

\[
\mathbb{E}[e^{Xt}] = \int_{-\infty}^{\infty} e^{xt} f_X(x) \, dx
\]

\[
= \int_0^\infty e^{xt} \lambda e^{-(\lambda x)} \, dx
\]

\[
= \lambda \int_0^\infty e^{(x(t-\lambda))} \, dx \text{ (only finite if } t < \lambda) \]

\[
= \lambda \left[ \frac{1}{t-\lambda} e^{(x(t-\lambda))} \right]_0^\infty
\]

\[
= \lambda \left[ 0 - \frac{1}{t-\lambda} \right]
\]

\[
= \frac{\lambda}{\lambda - t} \quad \text{(B.4)}
\]

The MGF of Gamma distributed random variable where the pdf is expressed as [144]

\[
f_X(x) = \begin{cases} 
\frac{\lambda e^{-(\lambda x)} (\lambda x)^{n-1}}{(n-1)!}, & \text{if } x \geq 0 \\
0, & \text{if } x < 0
\end{cases}
\]

\[
\text{(B.5)}
\]

therefore

\[
\mathbb{E}[e^{Xt}] = \int_{-\infty}^{\infty} e^{xt} f_X(x) \, dx
\]

\[
= \int_0^\infty e^{xt} \frac{\lambda e^{-(\lambda x)} (\lambda x)^{n-1}}{(n-1)!} \, dx
\]

\[
= \frac{\lambda^n}{\Gamma(n)} \int_0^\infty e^{-(x(t-\lambda))} (x)^{n-1} \, dx \quad \text{(B.6)}
\]

Gamma function can be defined by Euler Integral [97, 8.310]

\[
\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} \, dx, \quad n > 0
\]

\[
\text{(B.7)}
\]

Assume \( u = x(\lambda - t) \), \( dx = \frac{du}{(\lambda - t)} \) and \( x = \frac{u}{(\lambda - t)} \), therefore from B.6 and B.7,
\( \mathbb{E}[e^{Xt}] \) can be written as

\[
\mathbb{E}[e^{Xt}] = \frac{\lambda^n}{\Gamma(n)} \int_0^\infty e^{-u} \left( \frac{u}{\lambda - t} \right)^{n-1} \frac{1}{(\lambda - t)} du
\]

\[
= \frac{\lambda^n}{\Gamma(n) (\lambda - t)^n} \int_0^\infty e^{-u} (\lambda - t)^{n-1} du
\]

\[
= \frac{\lambda^n}{\Gamma(n) (\lambda - t)^n} \Gamma(n)
\]

\[
= \left( \frac{\lambda}{\lambda - t} \right)^n
\]

(B.8)