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Optical side-band generation in THz Fabry-Perot laser cavities

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Optical nonlinearities in semiconductor laser cavities can be exploited to characterize the properties of laser radiation or perform high speed frequency conversion operations. For example, nonlinear up-
10 conversion inside the cavity of quantum cascade lasers allows the use of near infrared optical components to measure high-speed terahertz or mid-infrared optical effects. This letter investigates two aspects of cavity up-conversion which control both the bandwidth and up-converted power; waveguide dispersion and cavity feedback. Specifically, we up-convert multi-mode Fabry Perot terahertz laser emission and detect each THz mode as a sideband signal on an optical carrier in the near infrared. Analysis of these
15 results shows that a single frequency near infrared laser can up-convert terahertz modes spanning a bandwidth of approximately 220 GHz; limited by the group index mismatch between the near infrared and terahertz waves. Secondly, transfer matrix techniques are used to study strong cavity feedback on all three waves, which produces etalon-like resonances in the sideband power. This can significantly enhance the efficiency of the conversion process, in agreement with experiments. It is thus possible to achieve
20 high up-conversion efficiency in quantum cascade lasers for both characterizing broadband laser sources and performing frequency conversion in the near infrared.

25 The high optical power inside semiconductor laser cavities makes them an appealing medium for nonlinear optics. Possible applications include nonlinear frequency conversion [1,2], generation of terahertz (THz) light [3,4], and mixing of THz radiation with near infrared (NIR) waves for detection using telecoms components [5–9]. However, nonlinear optics in semiconductor lasers comes with two design challenges: waveguide dispersion and facet feedback. Firstly, phase matching (PM) in nonlinear processes is highly sensitive to the refractive indices experienced by the waves involved [10,11]. Therefore, the combined material and waveguide dispersion of the laser cavity controls both the frequency and bandwidth of the nonlinear process. If these are not well understood then serious limitations are placed on the effectiveness of the conversion technique. Secondly, the use of a resonant laser cavity also induces feedback on pump, idler, and signal waves, thereby altering the efficiency of nonlinear processes [12,13]. Indeed, when the frequencies of the mixed waves are also resonant with the laser cavity it may be possible to significantly enhance the conversion efficiency. This letter examines the effect of both dispersion and cavity feedback in the up-conversion of THz waves to the NIR within the cavity of a quantum cascade laser (QCL). Crucially, we show that waveguide dispersion allows simultaneous up-conversion of THz frequencies spanning 220 GHz, and that the nonlinear process is resonantly enhanced by cavity feedback.

Using cavity up-conversion to detect THz signals in the NIR is appealing as it avoids the challenges of direct detection in the far infrared [2,5,6,14,15]. In THz QCLs we can exploit the large $\chi^{(2)}$ nonlinear susceptibility of the GaAs active region for intra-cavity up-conversion. The lack of birefringence makes achieving PM (equation 1) between pump (f_p), idler (f_i), and signal waves ($f_s = f_p \pm f_i$) difficult [10];

$$45 \quad n_s f_s = n_p f_p \pm n_i f_i \quad (1)$$

However, by exploiting a combination of the anomalous dispersion introduced by the Reststrahlen band and modal phase matching [16], one can achieve efficient PM between THz and NIR waves ($f_p = f_{\text{NIR}}$, $f_i = f_{\text{THz}}$) [11]. In practice, this is accomplished by integrating a NIR waveguide into the active region of the

QCL, leading to a strong optical field overlap and hence high conversion efficiency [2,5,6]. The result is
50 conversion of the THz wave into a NIR sideband signal. The sideband encodes information about any
temporal variations in the frequency, amplitude and phase of the THz wave, allowing for coherent
characterisation of QCLs with high-responsivity telecoms components [7–9]. Here, multi-frequency
emission from a Fabry Perot (FP) THz QCL is up-converted to the NIR, which allows measurement of
the up-conversion properties of each THz mode simultaneously.

55 Our results show that each THz mode of the FP laser has a distinctly measurable NIR frequency where
the three waves are phase matched and all the process produces the most power (the so-called phase
matching point). This is surprising given the relatively small THz mode spacing (~ 8 GHz). By measuring
the changes in f_{THz} and associated phase matched f_{NIR} , we can calculate that the up-conversion process
spans a frequency range of ~ 220 GHz. This bandwidth is ultimately limited by the large difference in
60 group indices between THz and sideband signals, which could in principle be managed using waveguide
dispersion engineering. Additionally, a transfer matrix model (TMM) of the nonlinear process is used to
study etalon fringes present in the up-converted sideband power. These fringes are a consequence of FP
feedback on the NIR waves (both pump and signal), and significantly enhances the conversion efficiency.
This combination of experiment and simulation provides a complete picture of up-conversion in FP laser
65 cavities, and suggests that up-conversion in QCLs can be broadband with moderate efficiency. Our work
opens a route to characterisation of broadband mode locked THz QCLs, as well as applications in
frequency conversion for both multicasting and de-multiplexing of communication signals [17].

Two QCL ridges, with a $12.8 \mu\text{m}$ -high GaAs/ $\text{Al}_{0.15}\text{Ga}_{0.85}\text{As}$ bound-to-continuum active region, containing
an integrated NIR guiding layer, were fabricated on a single chip [5,18]. These two $200 \mu\text{m}$ -wide semi-
70 insulating surface plasmon (SI-SP) waveguides were cleaved to form ~ 4.6 mm long laser cavities. The
choice of SI-SP waveguides (as opposed to double-metal waveguides) significantly simplified device
fabrication. Devices were operated in pulsed mode (repetition rate f_{rep} , peak current I_{QCL}), well above
threshold and at a heatsink temperature of 10K. Current pulses were provided by an Agilent 8144A pulse

generator, along with an Avtech AVX-MRB2 pulse amplifier. Both QCLs showed similar THz emission

properties and were characterized with a Bruker Vertex 80 Fourier transform infrared (FTIR) spectrometer and a calibrated thermopile (placed directly in front of a laser facet) while mounted in a Janis ST100 cryostat. The fiber optic setup used to generate and detect QCL radiation in the NIR is illustrated in Figure 1(a) [7,18]. NIR light was generated by a tunable external cavity laser (Yenista OSICS T100 – power 3 mW) and injected into the NIR waveguide embedded within the QCL [5] via a single mode optical fiber (SMF) butt-coupled to a QCL facet inside a Janis ST500 micromanipulation cryostat. Fiber to QCL input/output coupling losses are approximately 21 dB. THz light was generated inside the electrically driven QCL, which subsequently mixed with the injected NIR wave, generating sum and difference frequency sidebands. To optimize the sideband intensity the injected NIR light was TE polarized, with a NIR laser wavelength chosen to lie close to the PM point of the up-conversion process ($\sim 1.3 \mu\text{m}$). The NIR sidebands were collected alongside the main NIR pump mode using a second butt-coupled SMF (at the other QCL facet) and detected using an optical spectrum analyzer (OSA) (Yokogawa AQ6370C).

Our simulations of the nonlinear properties of this system were based on a nonlinear TMM method [19,20]. This extends our previous linear transfer matrix techniques (which use the Fresnel coefficients for each of the three waves) [21] by introducing a second-order polarization $P^{\text{NL}} = 2\chi^{(2)}E_pE_i$ and solving the coupled wave equations for the signal (E_s) generated by the pump (E_p) and idler (E_i) fields. First, linear propagation matrices are defined for the pump and idler waves using a conventional transfer matrix in terms of input and output waves:

$$\begin{pmatrix} E_{ut}^+ \\ E_{ut}^- \end{pmatrix} = \mathbf{T}_u \begin{pmatrix} E_{u0}^+ \\ E_{u0}^- \end{pmatrix} \quad (2)$$

where $\mathbf{T}_u$ ($u=p,i$ representing pump and idler waves respectively) is a 2×2 matrix for the electric field E_u within the FP cavity. The components ($T_{u(ij)}$) of this matrix define reflection ($\rho_u = -T_{u(21)}/T_{u(22)}$). For E_s the

transfer matrix includes two terms; one describing propagation through the layer analogous to $\mathbf{T}_u$, the second describing the generated field as per the nonlinear wave equation:

$$\begin{pmatrix} E_{st}^+ \\ E_{st}^- \end{pmatrix} = \mathbf{T}_s \begin{pmatrix} E_{s0}^+ \\ E_{s0}^- \end{pmatrix} + \begin{pmatrix} r^+ \\ r^- \end{pmatrix} \quad (3)$$

100 In equation 3, E_{st} and E_{s0} are the plane wave coefficients at the output and input, the sign + (-) denotes the forward (backward) propagating wave. $\mathbf{T}_s$ (2×2) defines the linear transfer matrix of E_s , whereas r^+ and r^- define the fields generated in the layer due to the mixing of E_p and E_i . Detailed expressions for r^+ and r^- are described in references [19,20]. From equation (3) it is possible to derive expressions for the output efficiency ($\eta_i = (E_{st}^+)^2 / (E_{p0}^+)^2$) in terms of the incident pump and idler waves [19,20]:

$$105 \quad \eta_i = \left[-\frac{T_{s(12)}}{T_{s(22)}} (M_{12}^+ + M_{12}^-) + (M_{11}^+ + M_{11}^-) \right] \frac{I_i}{A} \quad (4)$$

Here P_i is the power of the idler wave, A is the effective cross section area, and $\mathbf{M}^+$ ($\mathbf{M}^-$) is a 2×1 nonlinear vector with elements M_{11}^+ and M_{12}^+ (M_{11}^- and M_{12}^-), defined by:

$$\mathbf{M}^\pm = \left(-\frac{2k_{s0}^2 \chi^{(2)}}{k_{sn}^2 - (k_{pn} \pm k_{in})^2} \right) (G_0^{-1} G_{p\pm i} Q_{p\pm i} - G_0^{-1} G_s Q_s G_s^{-1} G_{p\pm i}) \begin{pmatrix} (T_{p11} + \rho_p T_{p12})(T_{i11} + \rho_i T_{i12}) \\ (T_{p21} + \rho_p T_{p22})(T_{i21} + \rho_i T_{i22}) \end{pmatrix} \quad (5)$$

110 In equation (5) k_{s0} is the free-space signal wave wavenumber, whereas k_{pn} , k_{in} and k_{sn} are the pump, idler and signal wavenumbers within the cavity. Finally G_0 , G_s , G_{p+i} and G_{p-i} are scattering matrices and Q_s , Q_{p+i} and Q_{p-i} are phase matrices defined as:

$$G_0 = \begin{pmatrix} 1 & 1 \\ n_0 & -n_0 \end{pmatrix}, \quad G_s = \begin{pmatrix} 1 & 1 \\ k_{sn}/k_{s0} & -k_{sn}/k_{s0} \end{pmatrix}, \quad G_{p\pm i} = \begin{pmatrix} 1 & 1 \\ (k_{pn} \pm k_{in})/k_{s0} & -(k_{pn} \pm k_{in})/k_{s0} \end{pmatrix}$$

$$Q_s = \begin{pmatrix} \exp(jk_{sn}L) & 0 \\ 0 & -\exp(jk_{sn}L) \end{pmatrix}, \quad Q_{p\pm i} = \begin{pmatrix} \exp(j(k_{pn} \pm k_{in})L) & 0 \\ 0 & -\exp(j(k_{pn} \pm k_{in})L) \end{pmatrix}$$

We assume that the length of the cavity is $L = 4.6$ mm, the injected THz power $P=40$ mW, the effective mode overlap $1/A = 4 \times 10^8 \text{ m}^{-2}$ [6], and the second order nonlinear susceptibility $\chi^{(2)}$ of GaAs is 100 pm/V [10]. Unlike in previous work, the transfer matrix approach accounts for cavity enhancement of the electromagnetic field, so the measured THz power (Fig. 1(b)) could be used as the input for simulations. Simulations use the refractive index of bulk GaAs from reference [22], with a small offset (-0.00065) to shift the PM wavelength from 1.1 μm to 1.3 μm , as observed experimentally. To account for the effect of material gain on the terahertz wave dispersion [23], we estimated a Lorentzian gain profile with full width half maximum of 200 GHz, and a peak gain product of $gL=1$ at 2.85 THz. This difference between simulation and experiment arises from the presence of free carriers, waveguide dispersion and cryogenic operation in the QCL, all of which change the refractive index of the active region.

Figure 1. Up-conversion of THz waves to the NIR. a) the experimental setup and a schematic of the laser ridge, b) electrical and THz output power behaviour of the QCL, c) a FTIR spectrum of the QCL at $I_{QCL} = 1870$ mA and d) the experimentally measured optical spectrum in the OSA. Highlighted are the three modes analyzed in Figure 2.

Figures 1(b) and 1(c) respectively show the current-voltage curve and multimode emission spectrum of Device A ($I_{QCL} = 1850$ mA, duty cycle = 10%), the latter measured directly by FTIR. Figure 1d shows the optical spectrum collected from the OSA when the THz signal is up converted to the NIR ($I_{QCL} =$

1870mA, duty cycle = 15%). This spectrum shows the NIR carrier wave (pump) and the up-converted THz sideband (signal). The sideband shows the up-converted multi-mode emission of the QCL, with a maximum conversion efficiency of 5×10^{-6} (measured power approximately 63pW), comparable to previously reported results [5]. The converted power is linearly related to the pump and idler powers in our experiments, as predicted by theory for sum and difference frequency generation [10]. It is worth pointing out that in our case, unlike the earlier experiments, a large number of NIR signals are obtained in the conversion process, corresponding to each THz idler mode from the QCL (see highlighted modes i-iii) [5]. The low power of the up-converted modes could be improved by using double metal QCL waveguides [6], few mode QCLs with higher spectral power density, or by using optical amplification techniques. Furthermore, the use of coherent detection techniques can help overcome the low power of these signals for both THz characterization and telecoms applications [9].

We now study the properties of this process by measuring both sideband power (P_s) and the transmitted pump power (P_p) to calculate the conversion efficiency (P_s/P_p). This parameter can be directly compared between simulations and experiments, as is presented in Figure 2. Specifically, we can isolate each individual mode up-converted from the THz to NIR to study the influence of phase matched pump frequency as a function of idler frequency, shown in Figure 2(a). Each NIR signal frequency has a distinct PM frequency, which makes it possible to study the dispersion of the nonlinear process. The inset of Figure 2a shows the fitted PM pump frequency as a function of THz (idler) mode frequency. An approximately linear relationship is observed between the THz mode frequency and the PM NIR pump, with a df_p/df_i of ~ 27 . This plot, and its slope, can be directly related to the group indices of each of the three waves at the phase matched frequency. Taking the derivative of equation (1) for sum frequency generation with respect to f_i , results in the equation;

$$\frac{df_p}{df_i} = \frac{n_i^g - n_s^g}{n_s^g - n_p^g} \quad (6)$$

Where n_s^g , n_p^g , n_i^g are the group indices of the signal, pump and idler waves respectively. In general equation (6) should lead to nonlinear behavior in Figure 2a. However, the relatively large difference between n_i^g (~ 3.87 – measured from mode spacing) and n_s^g (~ 3.6), means that equation (6) is relatively insensitive to small perturbations in n_i^g or n_s^g (such as those seen in simulations). Furthermore, the dispersion of GaAs in the near infrared is almost linear, meaning that n_s^g and n_p^g are similar and show comparable frequency dependence, so equation 6 is approximately constant. This is in general agreement with simulations, where a small nonlinearity is observed due to the gain induced change in refractive index. This calculation can provide realistic limits to the frequency range (Δf_i) of terahertz modes which can be phase matched simultaneously using a fixed frequency pump laser. As the behavior of Figure 2a is approximately linear we can write $\Delta f_i = \Delta f_p \times (1 / df_p / df_i)$ and calculate that $\Delta f_i = 220$ GHz. Equation 6 shows that this bandwidth is limited by the difference in group indices between the three waves [24] – notably between the THz and infrared waves.

Figure 2. Experimental and simulated PM data. a) Measured efficiency data for three NIR sideband frequencies from Device A, with a frequency spacing $\Delta f = 90$ GHz, each fitted using a sinc^2 function (solid lines). The three sinc^2 fits shown reveal a full width half maximum PM bandwidth of $\Delta f_p = 5.985$ THz. Inset: phase matched pump frequency as a function of idler frequency for devices A, B and simulations. b) Comparison between PM efficiency curves from TMM calculations and experiments. Note the efficiency has been normalised to peak power, accounting for the duty cycle. Inset shows high-resolution experimental data and TMM calculation of etalon fringes, measured for $\Delta f \sim 0.65$ GHz.

We note the scattering on the experimental results presented in Figure 2(a). This is a consequence of the cavity effect on the nonlinear process, and can be explained by comparing with the TMM results, shown in Figure 2(b). Due to the high refractive index of GaAs and the presence of cleaved facets, the laser cavity produces etalon interference (reflectivity on each facet ~ 0.29 , etalon reflectivity ~ 0.7) on the NIR pump and signal waves. Numerical results reveal etalon-like fringes [25] in the efficiency curve and a significant enhancement in signal (i.e. the sideband) power compared to the results for the bulk material without a cavity. These fringes are clearly resolved in experiments by measuring sideband conversion efficiency over a narrow pump frequency range (Figure 2(a) inset). The influence of the cavity can be seen more clearly by comparing raw signal power (not efficiency) between simulations and experiments. Figure 3(a) shows a simulation of the output power relative to the input pump wave, revealing a significant enhancement in signal power compared with the bulk. Moreover, the feedback introduces an additional modulation to the sinc^2 -like PM curve (shown for the sum frequency signal). This modulation is comparable to the product of the cavity transmission at the pump and signal frequencies ($T(f_p) \times T(f_s)$); suggesting that the nonlinear process is enhanced by feedback on pump and signal waves. The properties of this interference effect are clearly seen in Figure 3(b) – where the power shows both single and double etalon fringes depending on the choice of pump frequency range. These effects can be observed experimentally by instead measuring different THz modes as f_{NIR} is finely tuned (Figure 3(c)). The results show similar double interference fringes, validating the TMM calculations. Crucially – our results show that up-conversion inside a THz laser is strongly enhanced by cavity feedback, as long as the pump frequency is accurately chosen.

Figure 3. Sideband power as a function of pump frequency. a) Numerically calculated sideband power and the numerically calculated product between pump transmission ($T(f_p)$) and signal transmission ($T(f_s)$), b) etalon patterns generated within three different pump frequency ranges, c) measured experimental data from the three studied THz modes of device A.

In this letter, we have demonstrated multi-mode up-conversion of THz waves inside a QCL surface-plasmon waveguide. Conversion efficiencies for this nonlinear mixing process show that each up-converted NIR signal has a distinct PM curve. A THz idler bandwidth of 220 GHz can be measured using a single pump frequency; a span which could be further expanded with dispersion management techniques. The generated signal power is subject to the resonant properties of the etalon; by suitably choosing an appropriate resonant idler frequency it is possible to significantly enhance the measured conversion efficiency. This suggests that intra-cavity up-conversion offers an appealing route to both characterising THz sources, and applications in frequency conversion for both multicasting and de-multiplexing.

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