HIGH RESOLUTION TIME REVERSAL (TR) IMAGING BASED ON SPATIO-TEMPORAL WINDOWS

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Science and Engineering

2017

By
Victor Odedo
School of Electrical and Electric Engineering
# Contents

Abstract 22

Declaration 23

Copyright 24

Acknowledgements 25

1 Introduction 29

1.1 Radar imaging systems ........................................ 29
1.2 TWI .......................................................... 32
  1.2.1 TWI techniques ........................................ 32
  1.2.2 TWI Challenges ......................................... 33
1.3 Time Reversal (TR) ........................................... 37
  1.3.1 The TR method ......................................... 37
1.4 The DORT and MUSIC TR methods ......................... 38
  1.4.1 The TR methods for TWI ............................. 40
1.5 Aims and objectives .......................................... 42
  1.5.1 Aims .................................................. 42
  1.5.2 Objectives ............................................ 43
1.6 Outline ..................................................... 43
1.7 Summary .................................................... 45

2 The TR and FDTD methods 46

2.1 The Time Reversal (TR) method ........................... 46
2.2 The DORT and MUSIC methods ......................... 51
  2.2.1 The DORT methods ................................ 51
  2.2.2 The MUSIC methods ............................... 57
2.2.3 The similarities and differences between the DORT and MUSIC methods ........................................ 60
2.3 Finite Difference Time Domain (FDTD) method .................................................. 61
  2.3.1 Maxwell’s equations ............................................................................. 61
  2.3.2 FDTD method equations .................................................................. 63
  2.3.3 PML absorbing boundary condition .................................................... 65
2.4 Summary ....................................................................................................... 66
3 Imaging using the DORT and MUSIC methods .................................................. 67
  3.1 FDTD scenario with one PEC sphere scatterer ......................................... 67
    3.1.1 Analysing the singular value distribution and singular vectors ............. 72
    3.1.2 The CF-DORT and UWB-DORT images ........................................ 75
    3.1.3 The CF-MUSIC and UWB-MUSIC images ...................................... 77
    3.1.4 Comparisons between the DORT and MUSIC images ....................... 79
  3.2 FDTD scenario with two well-resolved PEC sphere scatterers .................. 80
    3.2.1 The CF-DORT and UWB-DORT images ........................................ 86
    3.2.2 The CF-MUSIC and UWB-MUSIC images ...................................... 87
    3.2.3 Comparisons between the DORT and MUSIC images ....................... 91
  3.3 FDTD scenario with one PEC sphere scatterer hidden behind a brick wall .... 95
    3.3.1 The DORT and MUSIC images ......................................................... 97
    3.3.2 The wall effect on the TR-based methods ......................................... 99
3.4 Summary ....................................................................................................... 101
4 The total sub-MDM algorithms ....................................................................... 102
  4.1 Time Reversal technique based on spatio-temporal windows ..................... 102
    4.1.1 Time-windowing ............................................................................ 103
    4.1.2 Space-windowing ........................................................................... 106
    4.1.3 Setting the window parameters ....................................................... 107
    4.1.4 The total sub-MUSIC imaging functional ....................................... 109
  4.2 The total sub-differential MDM algorithm ................................................. 112
    4.2.1 The total sub-differential MUSIC imaging functional.................... 113
  4.3 Summary ....................................................................................................... 113
5 TWI with the total sub-MDM algorithm ......................................................... 115
  5.1 FDTD scenario with one well-resolved PEC sphere .................................... 115
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1.1</td>
<td>Isolation of target signal using the time-windows</td>
</tr>
<tr>
<td>5.1.2</td>
<td>The sub-MUSIC images</td>
</tr>
<tr>
<td>5.1.3</td>
<td>The total sub-MUSIC images</td>
</tr>
<tr>
<td>5.2</td>
<td>FDTD scenario with clutter and target signal overlapped</td>
</tr>
<tr>
<td>5.2.1</td>
<td>The sub-MUSIC images</td>
</tr>
<tr>
<td>5.2.2</td>
<td>The total sub-MUSIC images</td>
</tr>
<tr>
<td>5.3</td>
<td>FDTD scenario with two targets</td>
</tr>
<tr>
<td>5.3.1</td>
<td>The sub-MUSIC images</td>
</tr>
<tr>
<td>5.3.2</td>
<td>The total sub-MUSIC images</td>
</tr>
<tr>
<td>5.4</td>
<td>Summary</td>
</tr>
<tr>
<td>6</td>
<td>Sub-differential MDM Imaging</td>
</tr>
<tr>
<td>6.1</td>
<td>FDTD scenario with two moving well-resolved scatterers</td>
</tr>
<tr>
<td>6.2</td>
<td>FDTD scenario with two buried expanding PEC oval targets</td>
</tr>
<tr>
<td>6.3</td>
<td>Imaging two moving targets in a known inhomogeneous FDTD scenario</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Imaging two moving targets</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Imaging two expanding targets</td>
</tr>
<tr>
<td>6.4</td>
<td>Summary</td>
</tr>
<tr>
<td>7</td>
<td>Conclusions and future work</td>
</tr>
<tr>
<td>7.1</td>
<td>Future works</td>
</tr>
<tr>
<td>A</td>
<td>Alternative plots for easier comparison</td>
</tr>
<tr>
<td>A.1</td>
<td>Figures from Section 5.1</td>
</tr>
<tr>
<td>A.2</td>
<td>Figures from Section 5.2</td>
</tr>
<tr>
<td>A.3</td>
<td>Figures from Section 5.3</td>
</tr>
</tbody>
</table>
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>The DORT and MUSIC methods compared</td>
<td>61</td>
</tr>
<tr>
<td>3.1</td>
<td>The accuracy and resolution of the DORT and MUSIC methods compared. The cross range resolution defined by the diffraction limit and the theoretical range resolution are 0.023 m and 0.12 m, respectively</td>
<td>81</td>
</tr>
<tr>
<td>3.2</td>
<td>The resolution of the DORT and MUSIC images compared. The cross range resolution defined by the diffraction limit is 0.012 m for target 1 and 0.023 m for target 2. The theoretical range resolution is 0.12 m for both targets</td>
<td>91</td>
</tr>
<tr>
<td>5.1</td>
<td>The highest accuracy and resolution obtained for the total sub-MUSIC methods. The cross range resolution defined by the diffraction limit and the theoretical range resolution are 0.2 m and 0.15 m, respectively</td>
<td>129</td>
</tr>
<tr>
<td>5.2</td>
<td>The highest accuracy and resolution obtained for the total sub-MUSIC methods. The cross range resolution defined by the diffraction limit and the theoretical range resolution are 0.1 m and 0.15 m, respectively</td>
<td>145</td>
</tr>
<tr>
<td>5.3</td>
<td>The highest resolutions obtained for the total sub-MUSIC methods. The cross range resolution defined by the diffraction limit is 0.2 m for target 1 and 0.225 m for target 2. The theoretical range resolution is 0.15 m for both targets</td>
<td>159</td>
</tr>
<tr>
<td>6.1</td>
<td>The highest resolutions obtained for the total sub-differential MUSIC methods. The cross range resolution defined by the diffraction limit is 0.2 m for target 1 and 0.225 m for target 2. The theoretical range resolution is 0.15 m for both targets</td>
<td>180</td>
</tr>
</tbody>
</table>
6.2 The major and minor axis of the oval targets in the scenario shown in Figure 6.18.
List of Figures

1.1 Block diagram illustrating the basic principle of radar imaging . . 30
1.2 The penetration and resolution compromise for TWI systems . . . 31
1.3 Typical system block diagram of a digital noise radar. The digital
correlator provides the complex cross-correlation between the time
delayed transmit waveform and the receive waveform . . . . . . . 31
1.4 An illustration of the received signal when scenarios containing
(a) a single target and (b) a target and a secondary scatterer, is
illuminated by an antenna in free-space, assuming the transmitted
signal strength is maximum in the direction of the target and zero
in other directions . . . . . . . . . . . . . . . . . . . . . . . . . . . 35
1.5 An illustration of the received signal when scenarios containing (a)
a single target and (b) a target and a side wall, is illuminated by an
antenna in free-space, assuming the transmitted signal strength is
maximum in the direction of the target and zero in other directions 36
2.1 The forward propagation of an excitation pulse in a scattering
medium . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 49
2.2 The backward propagation of the time reversed signal from a lim-
ited number of TRA antennas . . . . . . . . . . . . . . . . . . . 50
2.3 The increase of effective aperture from a media (a) with no mul-
tipaths \( a_e = a \) to a media (b) with multipaths created by two
PEC walls \( a_e > a \) and a media (c) with multipaths created by
discrete scatterers \( a_e > a \) . . . . . . . . . . . . . . . . . . . . . . . 50
2.4 Block diagram illustrating the implementation of the DORT and
MUSIC methods . . . . . . . . . . . . . . . . . . . . . . . . . . . 59
3.1 The geometry of the FDTD scenario, where \( \times \) represents the TRA
antennas . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 68
3.2 The source signal transmitted from an antenna location.

3.3 The normalised frequency spectrum of the excitation pulse as shown in Figure 3.2.

3.4 The signal at the first antenna when the source signal was emitted from the same antenna.

3.5 The singular values distribution obtained after SVD in (2.17) with signal to null subspace ratio taken as 10% of the first singular value.

3.6 The singular values at centre frequency $\omega_c$.

3.7 The ratio of consecutive singular values at centre frequency $\omega_c$.

3.8 The magnitude and phase of the first three singular vectors at centre frequency $\omega_c = 3.6$ GHz.

3.9 The CF-DORT image obtained using the first singular vector ($u_1(\omega_c)$) corresponding to the first singular value in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.10 The UWB-DORT image obtained using the first singular vector ($u_1(\omega_c)$) corresponding to the first singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.11 The CF-DORT and UWB-DORT images obtained using the second singular vectors ($u_2(\omega)$) corresponding to the second singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.12 The CF-DORT and UWB-DORT images obtained using the third singular vectors ($u_3(\omega)$) corresponding to the third singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.13 The CF-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.14 The UWB-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
3.15  The cross section at $y = 0.28$ m in Figure 3.11, Figure 3.12, Figure 3.13 and Figure 3.14 obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. The target is located at $x = 0.4$ m.

3.16  The range at $x = 0.4$ m in Figure 3.11, Figure 3.12, Figure 3.13 and Figure 3.14 obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. The target is located at $y = 0.28$ m.

3.17  The geometry of the FDTD scenario with two PEC spheres.

3.18  The singular values distribution obtained after SVD with signal to null subspace ratio taken as 10% of the first singular value.

3.19  The singular values at centre frequency $\omega_c$.

3.20  The ratio of consecutive singular values at centre frequency $\omega_c$.

3.21  The magnitude and phase of the first three singular vectors at centre frequency ($\omega_c = 3.6$ GHz).

3.22  The CF-DORT image obtained using the first singular vectors ($u_1(\omega)$) corresponding to the first singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the targets’ locations.

3.23  The UWB-DORT image obtained using the first singular vectors ($u_1(\omega)$) corresponding to the first singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the targets’ locations.

3.24  The CF-DORT image obtained using the second singular vectors ($u_2(\omega)$) corresponding to the second singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target(s) location(s).

3.25  The UWB-DORT image obtained using the second singular vectors ($u_2(\omega)$) corresponding to the second singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target(s) location(s).

3.26  The CF-DORT and UWB-DORT images obtained using the third singular vectors ($u_3(\omega)$) corresponding to the third singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target(s) location(s).
3.27 The CF-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

3.28 The UWB-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

3.29 The cross section of Figure 3.22, Figure 3.23, Figure 3.27 and Figure 3.28 of target 1 located at \( y = 0.2 \, \text{m} \) obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 1 is located at \( x = 0.544 \, \text{m} \).

3.30 The range section of Figure 3.22, Figure 3.23, Figure 3.27 and Figure 3.28 of target 1 located at \( x = 0.544 \, \text{m} \) obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 1 is located at \( y = 0.2 \, \text{m} \).

3.31 The cross section of Figure 3.24, Figure 3.25, Figure 3.27 and Figure 3.28 of target 2 located at \( y = 0.28 \, \text{m} \) obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 2 is located at \( x = 0.112 \, \text{m} \).

3.32 The range section of Figure 3.24, Figure 3.25, Figure 3.27 and Figure 3.28 of target 2 located at \( x = 0.112 \, \text{m} \) obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 2 is located at \( y = 0.28 \).

3.33 The geometry of the FDTD scenario with one target behind a brick wall.

3.34 The source signal transmitted from an antenna location.

3.35 The frequency spectrum of the excitation pulse Figure 3.34.

3.36 The reflected signal at the first antenna from the wall and the target.

3.37 The singular values distribution obtained after SVD with signal to null subspace ratio taken as 10% of the first singular value.

3.38 The singular values at centre frequency \( \omega_c \).

3.39 The magnitude and phase of the first three singular vectors at centre frequency (\( \omega_c = 2 \, \text{GHz} \)).
3.40 The CF-DORT and UWB-DORT images produced using the full-MDM with the first singular vector $u_1(\omega)$ where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.41 The CF-DORT and UWB-DORT images produced using the full-MDM with the second singular vector $u_2(\omega)$ where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.42 The CF-MUSIC and UWB-MUSIC images produced using the full-MDM, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

4.1 The Hanning window and the raised cosine window.

4.2 The normalised frequency spectrum of the Hanning window and the raised cosine shown Figure 4.1.

4.3 The summation of the first four time-windows has a magnitude of 1.

4.4 The spatial windowing of the TRA antennas, where $X$ represents the TRA antennas.

4.5 An illustration of the reduction in effective distance $\ell_e(l)$ between the centre of the TRA antennas and target illustrated due to spatial windowing, where $X$ represents the TRA antennas.

4.6 The propagation obtained (a) at the first antenna when a pulse was transmitted from the same antenna for the first and second sampling and (b) for the element, $k_{d11}(t)$ of the full-differential MDM $K_d(t)$.

5.1 The ratio of consecutive singular values at centre frequency $\omega_c$.

5.2 The signal observed at the first antenna in Figure 3.33 when a pulse was transmitted from the same antenna and line-of-sight signal is removed and the time-windows ($P = \bar{Y}$) which sum to give a magnitude of 1 when $k_{1,1}(t)$ is normalised.

5.3 The signal observed at the seventh antenna in Figure 3.33 when a pulse was transmitted from the same antenna and line-of-sight signal is removed and the time-windows ($P = \bar{Y}$) which sum to give a magnitude of 1 when $k_{7,7}(t)$ is normalised.

5.4 The singular value distribution of $K_{1,1}(\omega, \bar{Y}, N)$.

5.5 The singular values at centre frequency $\omega_c$ of $K_{1,1}(\omega, \bar{Y}, N)$.

5.6 The singular value distribution of $K_{12,1}(\omega, \bar{Y}, N)$.
5.7 The singular values at centre frequency $\omega_c$ of $K_{12,1}(\omega, \Upsilon, N)$ .... 120
5.8 The magnitude and phase of the first three singular vectors at centre frequency ($\omega_c = 2$ GHz) of $K_{11,1}(\omega, \Upsilon, N)$ .... 120
5.9 The magnitude and phase of the first three singular vectors at centre frequency ($\omega_c = 2$ GHz) of $K_{12,1}(\omega, \Upsilon, N)$ .... 121
5.10 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced for $K_{m,1}(\omega, \Upsilon, N)$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location .... 121
5.11 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{1,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location .... 123
5.12 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{12,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location .... 123
5.13 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{13,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location .... 124
5.14 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{14,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location .... 124
5.15 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{14,1}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location .... 125
5.16 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{14,7}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location .... 125
5.17 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = \Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location .... 126
5.18 The normalised cross section at $y = 1.15$ m in Figure 3.42 and Figure 5.17 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = \Upsilon$ and $N_s = 7$) and the reference. The target is located at $x = 0.75$ m .... 127

12
5.19 The normalised range section at \( x = 0.75 \) m in Figure 3.42 and Figure 5.17 obtained using the full-MDM MUSIC method, the total sub-MUSIC method and the reference. The target is located at \( y = 1.15 \) m.

5.20 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when \( P = 2\Upsilon \) and \( N_s = 7 \), where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

5.21 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when \( P = 4\Upsilon \) and \( N_s = 7 \), where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

5.22 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when \( P = 8\Upsilon \) and \( N_s = 7 \), where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

5.23 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when \( P = 16\Upsilon \) and \( N_s = 7 \), where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

5.24 The total sub-CF-MUSIC cross range resolution at \( y = 1.15 \) m, varying \( N_s \) and \( P \).

5.25 The total sub-UWB-MUSIC cross range resolution at \( y = 1.15 \) m, varying \( N_s \) and \( P \).

5.26 The total sub-CF-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \).

5.27 The total sub-UWB-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \).

5.28 The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \).

5.29 The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \).

5.30 The geometry of the FDTD scenario with one target behind a brick wall.

5.31 The time-windows \( (P = \Upsilon) \) which sum to give a magnitude of 1 and the normalised signals observed at (a) the first antenna when a pulse was transmitted from the same antenna \( (k_{1,1}(t)) \) and (b) the thirteenth antenna when a pulse was transmitted from the first antenna \( (k_{1,13}(t)) \) with the line-of-sight signal removed.
5.32 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{1,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

5.33 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{2,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

5.34 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{7,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

5.35 The CF-MUSIC and UWB-MUSIC images produced using the full-MDM, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

5.36 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = \Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

5.37 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 2\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

5.38 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 4\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

5.39 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 8\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

5.40 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 16\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

5.41 The normalised cross section at $y = 0.75$ m in Figure 5.35 and Figure 5.36 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = 8\Upsilon$ and $N_s = 7$) and the reference. The target is located at $x = 0.75$ m.

5.42 The normalised cross section at $x = 0.75$ m in Figure 5.35 and Figure 5.36 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = 8\Upsilon$ and $N_s = 7$) and the reference. The target is located at $y = 0.75$ m.
5.43 The total sub-CF-MUSIC cross range resolution at \( y = 0.75 \) m, varying \( N_s \) and \( P \) .................................................. 147

5.44 The total sub-UWB-MUSIC cross range resolution at \( y = 0.75 \) m, varying \( N_s \) and \( P \) .................................................. 148

5.45 The total sub-CF-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \) .................................................. 149

5.46 The total sub-UWB-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \) .................................................. 150

5.47 The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \) .................................................. 151

5.48 The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \) .................................................. 152

5.49 The geometry of the FDTD scenario with two PEC sphere behind a brick wall .................................................. 153

5.50 The signal observed at the seventh antenna in Figure 5.49 when a pulse was transmitted from the same antenna and line-of-sight signal is removed and the time-windows \( (P = \bar{\Upsilon}) \) which sum to give a magnitude of 1 when \( \hat{k}_{7,7}(t) \) is normalised .................................................. 154

5.51 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from \( K_{12,1}(\omega, \bar{\Upsilon}, 7) \), where \( \times \) represents the TRA antennas’ locations, \( \times \) represents the active antennas and \( \circ \) represents the target location 155

5.52 The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from \( K_{14,7}(\omega, \bar{\Upsilon}, 7) \), where \( \times \) represents the TRA antennas’ locations, \( \times \) represents the active antennas and \( \circ \) represents the target location 155

5.53 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when \( P = \bar{\Upsilon} \) and \( N_s = 7 \), where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the location of targets 156

5.54 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when \( P = 2\bar{\Upsilon} \) and \( N_s = 7 \), where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the location of targets 157

5.55 The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when \( P = 4\bar{\Upsilon} \) and \( N_s = 7 \), where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the location of targets 157
The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 8\upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas' locations and $\circ$ represents the location of targets...

The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 16\upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas' locations and $\circ$ represents the location of targets...

The total sub-CF-MUSIC cross range resolution at half maximum of $\mathbf{M}_\Gamma(\tau, \omega_c, P, N_s)$ at $y = 1.15$ m, varying $N_s$ and $P$...

The total sub-UWB-MUSIC cross range resolution at half maximum of $\mathbf{M}_{\Gamma_{UWB}}(\tau, P, N_s)$ at $y = 1.15$ m, varying $N_s$ and $P$...

The total sub-CF-MUSIC cross range resolution at half maximum of $\mathbf{M}_\Gamma(\tau, \omega_c, P, N_s)$ at $y = 1.25$ m, varying $N_s$ and $P$...

The total sub-UWB-MUSIC cross range resolution at half maximum of $\mathbf{M}_{\Gamma_{UWB}}(\tau, P, N_s)$ at $y = 1.25$ m, varying $N_s$ and $P$...

The total sub-CF-MUSIC range resolution at half maximum of $\mathbf{M}_\Gamma(\tau, \omega_c, P, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$...

The total sub-UWB-MUSIC range resolution at half maximum of $\mathbf{M}_{\Gamma_{UWB}}(\tau, P, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$...

The total sub-CF-MUSIC range resolution at half maximum of $\mathbf{M}_\Gamma(\tau, \omega_c, P, N_s)$ at $x = 1.0$ m, varying $N_s$ and $P$...

The total sub-UWB-MUSIC range resolution at half maximum of $\mathbf{M}_{\Gamma_{UWB}}(\tau, P, N_s)$ at $x = 1.0$ m, varying $N_s$ and $P$...

The geometry of the FDTD scenario with two moving PEC sphere behind a brick wall...

The singular value distribution of the full-differential MDM...

The singular value distribution of the full-differential MDM at centre frequency...

The ratio of consecutive singular values at centre frequency $\omega_c$ of the full-differential MDM...

The CF-MUSIC and UWB-MUSIC images produced using $\mathbf{K}(\omega)$ (without differencing or spatial windowing), where $\times$ represents the TRA antennas' locations and $\circ$ represents the target location...
6.6 The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{dl=1}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

6.7 The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{dl=4}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

6.8 The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{dl=7}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

6.9 The normalised cross section at $y = 1.15$ m (location of PEC Target 1) for images obtained using $K_{dl=1}(t, N_s = 3)$, $K_{dl=1}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 1 is located at $x = 0.6$ m.

6.10 The normalised cross section at $y = 1.25$ m (location of PEC Target 2) for images obtained $K_{dl=11}(\omega, N_s = 3)$, $K_{dl=7}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 2 is located at $x = 1.0$ m.

6.11 The normalised range section at $x = 0.6$ m (location of PEC Target 1) for images obtained using $K_{dl=1}(\omega, N_s = 3)$, $K_{dl=1}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 1 is located at $y = 1.15$ m.

6.12 The normalised range section at $x = 1.0$ m (location of PEC Target 2) for images obtained using $K_{dl=11}(\omega, N_s = 3)$, $K_{dl=7}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 2 is located at $y = 1.25$ m.

6.13 The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 3$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

6.14 The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

6.15 The CF-MUSIC and UWB-MUSIC images produced using the full-differential MDM, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
6.16 The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC cross range resolution at half maximum of $M_{d_{r}}(\tau, \omega, N_s)$ and $M_{d_{rUW}}(\tau, N_s)$ at $y = 1.15$ m for target 1 and at $y = 1.25$ m for target 2, varying $N_s$.

6.17 The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC range resolution at half maximum of $M_{d_{r}}(\tau, \omega, N_s)$ and $M_{d_{rUW}}(\tau, N_s)$ at $x = 0.6$ m for target 1 and at $x = 1.0$ m for target 2, varying $N_s$.

6.18 The geometry of the FDTD scenario with two expanding buried targets.

6.19 The CF-MUSIC and UWB-MUSIC images produced using $K(\omega)$ (without differencing or spatial windowing), where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

6.20 The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d_{l}=1}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

6.21 The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d_{l}=4}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

6.22 The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d_{l}=7}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

6.23 The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

6.24 The geometry of the FDTD scenario with two well-resolved targets in inhomogeneous scattering media.

6.25 The reflected signal from the target with multipath signal in the element $k_{d1,1}(t)$ of the full-differential MDM $K_{d1}(t, 13)$.
6.26 The total sub-differential CF-MUSIC and total sub-differential UWB-
MUSIC images produced when $N_s = 13$ (no spatial windowing),
where $\times$ represents the TRA antennas’ locations and $\circ$ represents the
target location ................................................. 188

6.27 The total sub-differential CF-MUSIC and total sub-differential UWB-
MUSIC images produced when $N_s = 7$, where $\times$ represents the
TRA antennas’ locations and $\circ$ represents the target location . . . 189

6.28 The total sub-differential CF-MUSIC and total sub-differential UWB-
MUSIC images produced when $N_s = 13$ (no spatial windowing)
when the background media is unknown and free-space steering
vectors are used, where $\times$ represents the TRA antennas’ locations
and $\circ$ represents the target location ................................................. 189

6.29 The normalised cross section in Figure 6.26 and Figure 6.27 of tar-
get 1 located at $y = 1.15$ m obtained using the total sub-differential
MUSIC method. Target 1 is located at $x = 0.6$ m . ............... 190

6.30 The normalised range section in Figure 6.26 and Figure 6.27 of tar-
get 1 located at $x = 0.6$ m obtained using the total sub-differential
MUSIC method. Target 1 is located at $y = 1.15$ m ............... 190

6.31 The normalised cross section in Figure 6.26 and Figure 6.27 of tar-
get 2 located at $y = 1.25$ m obtained using the total sub-differential
MUSIC method. Target 2 is located at $x = 1.0$ m ......... 191

6.32 The normalised range section in Figure 6.26 and Figure 6.27 of tar-
get 2 located at $x = 1.0$ m obtained using the total sub-differential
MUSIC method. Target 2 is located at $y = 1.25$ m ......... 191

6.33 The geometry of the FDTD scenario with two well-resolved oval
targets in inhomogeneous scattering media . ......................... 192

6.34 The total sub-differential CF-MUSIC and total sub-differential UWB-
MUSIC images produced when $N_s = 13$, where $\times$ represents the
TRA antennas’ locations and $\circ$ represents the target location . . . 193

6.35 The total sub-differential CF-MUSIC and total sub-differential UWB-
MUSIC images produced when $N_s = 7$, where $\times$ represents the
TRA antennas’ locations and $\circ$ represents the target location . . . 193

A.1 The total sub-CF-MUSIC cross range resolution at $y = 1.15$ m,
varying $N_s$ and $P$, previously plotted in Figure 5.24 . ............ 210
| A.2 | The total sub-UWB-MUSIC cross range resolution at \( y = 1.15 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.25 | 210 |
| A.3 | The total sub-CF-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.26 | 211 |
| A.4 | The total sub-UWB-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.27 | 211 |
| A.5 | The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \), previously plotted in Figure 5.28 | 212 |
| A.6 | The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \), previously plotted in Figure 5.29 | 212 |
| A.7 | The total sub-CF-MUSIC cross range resolution at \( y = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.43 | 213 |
| A.8 | The total sub-UWB-MUSIC cross range resolution at \( y = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.44 | 214 |
| A.9 | The total sub-CF-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.45 | 214 |
| A.10 | The total sub-UWB-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.46 | 215 |
| A.11 | The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \), previously plotted in Figure 5.47 | 215 |
| A.12 | The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \), previously plotted in Figure 5.47 | 216 |
| A.13 | The total sub-CF-MUSIC cross range resolution at half maximum of \( M_{\Gamma}(\tau, \omega_c, P, N_s) \) at \( y = 1.15 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.58 | 218 |
| A.14 | The total sub-UWB-MUSIC cross range resolution at half maximum of \( M_{\Gamma_{UWB}}(\tau, P, N_s) \) at \( y = 1.15 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.59 | 218 |
| A.15 | The total sub-CF-MUSIC cross range resolution at half maximum of \( M_{\Gamma}(\tau, \omega_c, P, N_s) \) at \( y = 1.25 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.60 | 219 |
A.16 The total sub-UWB-MUSIC cross range resolution at half maximum of $M_{\Gamma_{UWB}}(\tau, P, N_s)$ at $y = 1.25$ m, varying $N_s$ and $P$, previously plotted in Figure 5.61 ........................................ 219

A.17 The total sub-CF-MUSIC range resolution at half maximum of $M_{\Gamma}(\tau, \omega_c, P, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$, previously plotted in Figure 5.62 ........................................ 220

A.18 The total sub-UWB-MUSIC range resolution at half maximum of $M_{\Gamma_{UWB}}(\tau, P, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$, previously plotted in Figure 5.63 ........................................ 220

A.19 The total sub-CF-MUSIC range resolution at half maximum of $M_{\Gamma_{UWB}}(\tau, P, N_s)$ at $x = 1.0$ m, varying $N_s$ and $P$, previously plotted in Figure 5.64 ........................................ 221

A.20 The total sub-UWB-MUSIC range resolution at half maximum of $M_{\Gamma_{UWB}}(\tau, P, N_s)$ at $x = 1.0$ m, varying $N_s$ and $P$, previously plotted in Figure 5.65 ........................................ 221
Abstract

Through-the-wall Imaging (TWI) is crucial for various applications such as law enforcement, rescue missions and defense. TWI methods aim to provide detailed information of spaces that cannot be seen directly. Current state-of-the-art TWI systems utilise ultra-wideband (UWB) signals to simultaneously achieve wall penetration and high resolution. These TWI systems transmit signals and mathematically back-project the reflected signals received to image the scenario of interest. However, these systems are diffraction-limited and encounter problems due to multipath signals in the presence of multiple scatterers.

Time reversal (TR) methods have become popular for remote sensing because they can take advantage of multipath signals to achieve superresolution (resolution that beats the diffraction limit). The Decomposition Of the Time-Reversal Operator (DORT in its French acronym) and MUltiple SIgnal Classification (MUSIC) methods are both TR techniques which involve taking the Singular Value Decomposition (SVD) of the Multistatic Data Matrix (MDM) which contains the signals received from the target(s) to be located. The DORT and MUSIC imaging methods have generated a lot of interests due to their robustness and ability to locate multiple targets. However these TR-based methods encounter problems when the targets are behind an obstruction, particularly when the properties of the obstruction is unknown as is often the case in TWI applications.

This dissertation introduces a novel total sub-MDM algorithm that uses the highly acclaimed MUSIC method to image targets hidden behind an obstruction and achieve superresolution. The algorithm utilises spatio-temporal windows to divide the full-MDM into sub-MDMs. The summation of all images obtained from each sub-MDM give a clearer image of a scenario than we can obtain using the full-MDM. Furthermore, we propose a total sub-differential MDM algorithm that uses the MUSIC method to obtain images of moving targets that are hidden behind an obstructing material.
Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.
Copyright

i. The author of this thesis (including any appendices and/or schedules to this thesis) owns any copyright in it (the “Copyright”) and s/he has given The University of Manchester the right to use such Copyright for any administrative, promotional, educational and/or teaching purposes.

ii. Copies of this thesis, either in full or in extracts, may be made only in accordance with the regulations of the John Rylands University Library of Manchester. Details of these regulations may be obtained from the Librarian. This page must form part of any such copies made.

iii. The ownership of any patents, designs, trade marks and any and all other intellectual property rights except for the Copyright (the “Intellectual Property Rights”) and any reproductions of copyright works, for example graphs and tables (“Reproductions”), which may be described in this thesis, may not be owned by the author and may be owned by third parties. Such Intellectual Property Rights and Reproductions cannot and must not be made available for use without the prior written permission of the owner(s) of the relevant Intellectual Property Rights and/or Reproductions.

iv. Further information on the conditions under which disclosure, publication and exploitation of this thesis, the Copyright and any Intellectual Property Rights and/or Reproductions described in it may take place is available from the Head of School of Electrical and Electric Engineering (or the Vice-President).
Acknowledgements

I’d like to thank my family for their constant support throughout my research. This dissertation is dedicated to Ifeanyi (Dad), Nnedi (Mum), Uche (sister) and Arinze (brother). Their kind words and prayers kept me motivated throughout. I also owe my supervisor, Dr. Fumie Costen, a lot of gratitude for her guidance, support, availability and patience over the past few years. My fellow PhD also deserve a mention as we have been a team over the years despite our different research areas. I also thank my dear friends in Manchester who were always a welcome distraction when needed. Thank you Jesus.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full name</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>Absorbing boundary conditions</td>
</tr>
<tr>
<td>ADC</td>
<td>Analogue-to-digital converter</td>
</tr>
<tr>
<td>CF</td>
<td>Centre frequency</td>
</tr>
<tr>
<td>CFS</td>
<td>Complex frequency shifted</td>
</tr>
<tr>
<td>DORT (french acronym)</td>
<td>The decomposition of the time reversal operator</td>
</tr>
<tr>
<td>EVD</td>
<td>Eigenvalue decomposition</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>FDTD</td>
<td>Finite difference time domain</td>
</tr>
<tr>
<td>FD</td>
<td>Frequency dependency</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full width at half maximum</td>
</tr>
<tr>
<td>GPR</td>
<td>Ground penetrating radar</td>
</tr>
<tr>
<td>MTI</td>
<td>Moving target indication</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-input multi-output</td>
</tr>
<tr>
<td>MUSIC</td>
<td>MUltiple SIgnal Classification</td>
</tr>
<tr>
<td>MDM</td>
<td>Multistatic data matrix</td>
</tr>
<tr>
<td>PEC</td>
<td>Perfect electric conductor</td>
</tr>
<tr>
<td>PML</td>
<td>Perfectly matched layer</td>
</tr>
<tr>
<td>SAR</td>
<td>synthetic aperture radar</td>
</tr>
<tr>
<td>SLAR</td>
<td>Side-looking airborne radar</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular value decomposition</td>
</tr>
<tr>
<td>TWI</td>
<td>Through the Wall Imaging</td>
</tr>
<tr>
<td>TR</td>
<td>Time Reversal</td>
</tr>
<tr>
<td>TRA</td>
<td>Time reversal array</td>
</tr>
<tr>
<td>TRO</td>
<td>Time reversal operator</td>
</tr>
<tr>
<td>UWB</td>
<td>Ultra-wideband</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>Speed of light</td>
</tr>
<tr>
<td>( \ell )</td>
<td>Distance between target and TRA antennas</td>
</tr>
<tr>
<td>( a )</td>
<td>Antenna aperture</td>
</tr>
<tr>
<td>( a_e )</td>
<td>Effective aperture</td>
</tr>
<tr>
<td>( d_r )</td>
<td>Theoretical range resolution</td>
</tr>
<tr>
<td>( K_d )</td>
<td>Full-differential MDM</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Frequency</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Permittivity</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Permeability</td>
</tr>
<tr>
<td>( \tau_0 )</td>
<td>Scatterer’s location</td>
</tr>
<tr>
<td>( S )</td>
<td>Source signal</td>
</tr>
<tr>
<td>( n )</td>
<td>Identifies antenna</td>
</tr>
<tr>
<td>( *_t )</td>
<td>Convolution in time</td>
</tr>
<tr>
<td>( \tau_n )</td>
<td>( n )th antenna location</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Location in space</td>
</tr>
<tr>
<td>( d_c )</td>
<td>Diffraction limit cross range resolution</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Wavelength</td>
</tr>
<tr>
<td>( N )</td>
<td>Integer that gives the number of TRA antennas</td>
</tr>
<tr>
<td>( s )</td>
<td>Integer that identifies the antenna transmitting the signal</td>
</tr>
<tr>
<td>( r )</td>
<td>Integer that identifies the antenna observing the signal propagation</td>
</tr>
<tr>
<td>( K )</td>
<td>MDM</td>
</tr>
<tr>
<td>( k )</td>
<td>Elements of the MDM</td>
</tr>
<tr>
<td>( G )</td>
<td>Green’s function</td>
</tr>
<tr>
<td>( O )</td>
<td>Total number of scatterers</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Scattering coefficient</td>
</tr>
<tr>
<td>( g )</td>
<td>Steering vector</td>
</tr>
<tr>
<td>( o )</td>
<td>Identifies scatterer or target</td>
</tr>
<tr>
<td>( \dagger )</td>
<td>Complex conjugate transpose operation</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Singular value</td>
</tr>
<tr>
<td>( A )</td>
<td>Real diagonal matrix containing the singular values in descending order</td>
</tr>
<tr>
<td>( U )</td>
<td>Unitary matrix containing the left singular vectors</td>
</tr>
<tr>
<td>( V )</td>
<td>Unitary matrix containing the right singular vectors</td>
</tr>
<tr>
<td>( T )</td>
<td>Time reversal operator</td>
</tr>
<tr>
<td>( * )</td>
<td>Phase conjugation</td>
</tr>
<tr>
<td>( D )</td>
<td>DORT imaging functional</td>
</tr>
<tr>
<td>( \eta )</td>
<td>FDTD Timestep</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>Accuracy</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>Fourier transform</td>
</tr>
<tr>
<td>$t_{\text{foc}}$</td>
<td>Focusing time</td>
</tr>
<tr>
<td>$M$</td>
<td>MUSIC imaging functional</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Bandwidth of operation</td>
</tr>
<tr>
<td>$E$</td>
<td>Electric field</td>
</tr>
<tr>
<td>$H$</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>$\mathcal{D}$</td>
<td>Electric flux density</td>
</tr>
<tr>
<td>$B$</td>
<td>Magnetic flux density</td>
</tr>
<tr>
<td>$J$</td>
<td>Conduction current density</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Charge density</td>
</tr>
<tr>
<td>$P$</td>
<td>Polarization</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>Magnetization</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity of free space</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Permeability of free space</td>
</tr>
<tr>
<td>$j$</td>
<td>Square root of -1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Conductivity</td>
</tr>
<tr>
<td>$\varepsilon_S$</td>
<td>Static relative permittivity</td>
</tr>
<tr>
<td>$\varepsilon_\infty$</td>
<td>Optical relative permittivity</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>Relaxation time</td>
</tr>
<tr>
<td>$\mathcal{H}$</td>
<td>Normalised Hilbert transform</td>
</tr>
<tr>
<td>$W$</td>
<td>Window function</td>
</tr>
<tr>
<td>$P$</td>
<td>Window interval</td>
</tr>
<tr>
<td>$T$</td>
<td>Maximum value of time for the full-MDM</td>
</tr>
<tr>
<td>$m$</td>
<td>Integer that identifies a specific time-window</td>
</tr>
<tr>
<td>$M$</td>
<td>total number of time-windows</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>Shift in time</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of neighbouring antennas in a spatial-window</td>
</tr>
<tr>
<td>$l$</td>
<td>Integer that identifies a specific spatial-window</td>
</tr>
<tr>
<td>$L$</td>
<td>total number of spatial-windows</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Number of singular values in the signal subspace</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Entropy</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Through the Wall Imaging (TWI) methods are desirable for a range of applications including rescue missions, law enforcement and military applications [1, 2]. TWI is a promising research area which aims to provide detailed information of spaces that cannot be seen directly nor immediately accessed through traditional methods. Hence TWI methods attempt to provide vision into otherwise obscured areas. Various sensing techniques have been developed for different remote-sensing applications.

1.1 Radar imaging systems

Free-space imaging is typically employed in conventional radar [3] and synthetic aperture radar (SAR) techniques [4] since distortions in the atmosphere are often negligible. Figure 1.1 shows a block diagram illustrating the principle of radar imaging. The scene is illuminated with electromagnetic waves which reflect off any target in the medium and return to the receiving antenna giving information about the targets location. Conventional radar, sonar and optical image processes all utilise basic wave physics equations to provide focusing to individual points. For instance, radar applications sample data from an array of antennas which are mathematically integrated to provide equivalent focusing using free-space propagation assumptions. Hence, the received signals are compared to the predicted signals from points in the imaging space to focus on each point in that space, accurately imaging targets in free-space conditions.

Radar imaging systems achieve cross-range resolutions of targets through the aperture of an array of antennas used in the operation [5]. These systems also
attain the range resolutions of targets through the bandwidth of the signals \[6\].

Depending on the application requirements, an antenna array can contain a set of sparsely distributed antennas or a synthetic array can be used to construct a large virtual aperture from one or a small number of physical antennas. For instance, in side-looking airborne radar (SLAR) systems \[7\] an antenna is attached to an aircraft and the radar observes a scene it illuminates while in motion. A large synthetic aperture can achieve high cross-range resolution, even when operated from distance \[8\]. An antenna array aperture can be effectively constructed using bistatic and multi-input multi-output (MIMO) radar processing techniques \[9\].

Signals collected at different antennas are processed coherently or non-coherently. For coherent signal processing, the phase information of the signals are utilised which require calibrated positioning of the array antennas and high accuracy phase synchronisation. However, non-coherent signal processing does not require phase synchronisation although this yields inferior performance. Radar imaging techniques have also found applications for TWI.
CHAPTER 1. INTRODUCTION

TWR/TWI: Through the wall radar/imaging
UV: Ultraviolet
IR: Infrared radiation
FM: Frequency modulation
AM: Amplitude modulation

Figure 1.2: The penetration and resolution compromise for TWI systems [10]

Figure 1.3: Typical system block diagram of a digital noise radar. The digital correlator provides the complex cross-correlation between the time delayed transmit waveform and the receive waveform [11]
1.2 TWI

Remote sensing techniques in microwave wave range provide capacity for location and imaging of obscured targets. For instance, ground penetrating radar (GPR) techniques use electromagnetic radiation in the microwave band to detect land mines, underground pipes, tunnels and subsurface defects.

1.2.1 TWI techniques

Microwaves are also attractive for rescue missions and battlefield scenarios due to its ability to penetrate through forestation, fog, smoke, dust and walls to detect obscured vehicles and personnel as well as its ease of integration into multi-sensor (antenna array) systems. Figure 1.2 shows that although higher frequencies can attain finer spatial resolution, the signal penetration is more effective at lower signal frequency. Hence ultra-wideband (UWB) techniques are widely popular for TWI applications [12]. UWB refers to a sensing system with an effective signal bandwidth comprising of a wide spectrum of frequencies around the centre frequency (CF) [13], which is the frequency at which the frequency spectrum peaks. Hence UWB techniques utilise narrow pulses of sub-nanosecond duration, which generate a wide spectra, to attain precise location and high-resolution imaging of targets [14]. Therefore UWB signals simultaneously use the frequencies needed to penetrate walls and provide fine spatial resolution for imaging targets in TWI applications [15]. For military applications, there is a desire for the radar systems to be undetectable by hostile technology. To confound detection, UWB random noise radar technology [11] is usually employed for state-of-the-art TWI applications since random noise signals are difficult to detect or jam. Figure 1.3 shows the block diagram of a typical UWB random noise radar system which works by transmitting a random noise signal and cross-correlating the reflected signal received with a time-delayed and frequency-shifted replica of the source signal. The time delayed source signal is used to obtain range information while the frequency-shifted signal ensures phase coherence. An analogue-to-digital converter (ADC) may be used to digitise the source and received signal for digital correlation.

TWI systems are also capable of measuring motion using the popular Doppler effect. The Doppler effect was traditionally used for moving target indication (MTI) and pulse Doppler systems for bulk velocity measurements [16] such as
tracking the speed of an aircraft. Similar to conventional radar systems, a Doppler system transmits a signal to a target and receives the echo, using the time delay between the transmitted and received signals to determine the range of the target. However, when the target is moving the frequencies of the received signals are shifted from that of the transmitted signal due to the Doppler effect. The Doppler frequency shift is proportional to the transmitted signal frequency and the relative velocity between the target and the transmitter, and is commonly termed the “bulk” Doppler shift \cite{5} since it refers to the speed of the whole target. Since the Doppler frequency shift is measured from the phase change between the transmitted and received signal, a coherent system is required. Furthermore, if the target or any part of the target exhibits a micro-motion such as a vibration, an additional frequency modulation on the received signal is induced. The combination of the “bulk” Doppler frequency shift and the frequency modulation is known as the micro-Doppler signature \cite{16,17}. The micro-Doppler signatures may allow Doppler radar systems to locate human movement, such as breathing, through obstructing materials even when the human as a whole is stationary, hence expanding the application of TWI systems to earthquakes, avalanche and hostage scenarios among others.

1.2.2 TWI Challenges

Challenges remain for the full exploitation of electromagnetic signals for TWI techniques. Increased attenuation inside lossy materials and the often unknown and disordered nature in remote sensing scenarios, including secondary scatterers often lead to weak and distorted signals from targets which makes it difficult to locate and image them.

1.2.2.1 Attenuation and Dispersion

Signals impinging on a wall experience attenuation due to reflection loss, conductivity loss and multiple reflections within the wall. Loss that occurs due to reflections are a consequence of the contrast in the dielectric constant between the wall material and free-space as well as the angle of incidence. Conductivity losses can be significant especially at higher frequencies and when the wall is wet or moist \cite{18}. However this loss is not significant at microwave frequencies for typical wall materials and can be easily compensated by, for instance, increasing
the level of transmit power. Multiple reflections inside the wall structure are more significant when inhomogeneities (such as rebars) are present and when the wall thickness is much larger than the signal wavelength. UWB signals propagating through inhomogeneous walls and inhomogeneous media are significantly affected by the frequency-dependent properties of the materials comprising the propagation medium. Different materials exhibit diverse behaviours over a wide range of frequencies when interacting with electromagnetic waves. Hence electromagnetic waves propagating through an inhomogeneous media experience dispersion which is a phenomenon that causes different spectral components of a UWB signal to travel at different speeds. Hence walls can significantly alter the signal phase which cause image distortions. There have been research carried out to propose methods to mitigate the wall effect on signals propagating through walls for target detection [19] [20]. [21] investigated the effect of loss and dispersion on the location accuracy and implemented a compensation algorithm to correct the position of the target. These wall effects remains a research of interest for TWI techniques.

1.2.2.2 Multipath

The performance of traditional imaging techniques are known to degrade as multipath stemming from multiple scattering increases. Multipath may be caused by discrete scatterers or an extended scatterers. Discrete scatterers, also termed point-like scatterers, are small with respect to the wavelength and hence the resolution of the radar system. Extended scatterers, also commonly termed distributed scatterers refer to materials with extended surfaces such as a wall. Figure 1.4 illustrates the multipath problem due to secondary scatterer. In Figure 1.4(a) a signal is transmitted towards the target and the reflection is received by the antenna, assuming a perfectly directional antenna on transmission where the transmitted signal strength is maximum in the direction of the target and zero in other directions. In conventional imaging methods the received signal is utilised to locate the target using basic wave physics. For instance, the distance of the target from the antenna may be easily calculated since the speed of the transmitted signal is known. Figure 1.4(b) shows the target signal as well as the multipath signal received as a result of a secondary scatterer in the scene. Similarly, Figure 1.5 shows the multipath signal that occurs as a result of a side wall in the scene. Multipath signals can lead to the shadowing effect where non-existent
Figure 1.4: An illustration of the received signal when scenarios containing (a) a single target and (b) a target and a secondary scatterer, is illuminated by a antenna in free-space, assuming the transmitted signal strength is maximum in the direction of the target and zero in other directions.
Figure 1.5: An illustration of the received signal when scenarios containing (a) a single target and (b) a target and a side wall, is illuminated by an antenna in free-space, assuming the transmitted signal strength is maximum in the direction of the target and zero in other directions.
targets, commonly termed “ghost” targets, are located by imaging systems. In a realistic cluttered and highly scattering scenario containing multiple targets, the effect of multipath is magnified. Furthermore, realistic antennas are not perfectly directional and in some applications, omnidirectional antennas may be preferable which can amplify the multipath signals received. Signal processing techniques such as spatial filtering [22], compressed sensing [23] and image alignment procedures [24] have been used for TWI systems, primarily focusing on the removal of significant wall reflections. However current TWI devices still perform poorly in the presence of multiple targets and multipath issues remain a hindrance which results in missed detections or inaccurate estimates of the number of targets [25].

1.3 Time Reversal (TR)

Recent research have shown that we can utilise multipath components to obtain independent paths between the reflecting signal and the receiver in a remote sensing system [26] [27] hence converting previously hostile multipaths from secondary scatterers to performance-enhancing elements in cluttered environments.

1.3.1 The TR method

The time reversal (TR) method, introduced by Fink et al [28], is among such novel techniques which utilises the multipath components in the media of interest to achieve resolution that beats the classical diffraction limit which is also known as superresolution. Unlike in typical radar systems, where the signals are back-projected mathematically to image the scene, TR involves the physical or synthetic retransmission of signals acquired from say, a source or scatterer, in a time-reversed manner which is commonly termed back-propagation. The re-transmitted time-reversed (or phase conjugated in the frequency domain) signals re-propagate backwards through the same medium and undergo the same multiple reflections, scattering, refraction and dispersion that they experienced during the forward propagation. Thus the reversed signals coherently converge towards the original source or scatterer locations as if time were going backwards resulting in energy focusing at the initial source or scatterer locations at the “optimal focusing time”. TR relies on the invariance of the wave equation in stationary and lossless media, utilising UWB signals to exploit the advantages of simultaneously operating at both low frequency for more penetration into lossy materials...
and high frequencies for better resolution. TR was first successfully utilized in acoustics by Fink et al. and has since generated interest in the application of TR methods using EM waves.

1.4 The DORT and MUSIC TR methods

In media containing multiple discrete scatterers, the physical retransmission of time-reversed scattered waves results in focusing on all the scatterers simultaneously but more strongly on the dominant scatterers masking weaker scatterers [29]. The decomposition of the time reversal operator (DORT under its french acronym) [30, 31, 32] utilises the time reversal operator (TRO) to overcome this problem by isolating and classifying different scattering locations in the presence of multiple scatterers. The TRO is obtained from the multistatic data matrix (MDM) of a time reversal array (TRA) of antennas [33]. The MDM is obtained by transmitting a short duration pulse from each antenna of the TRA and receiving the reflected signals at all the antennas. Thereafter, the eigenvalue decomposition (EVD) of the TRO gives the eigenspace structure of the TRO, consisting of the eigenvalues and the corresponding eigenvectors, which contain information about the scattering scenario being considered. It has been shown that for a scene containing well-resolved discrete scatterers, each eigenvalue in the signal subspace which refers to non-zero eigenvalues and its corresponding eigenvector represent a particular scatterer in the medium of interest [30, 31]. Hence, the DORT method allows for selective focusing of discrete scatterers through the back-propagation of the eigenvectors corresponding to singular values in the signal subspace. By analysing the eigenvalues, the number of scatterers in the medium can be determined as long as they can be separated at the frequency of operation. Furthermore, since it is capable of isolating well-resolved scatterers, it can be used as a pre-processing step for radar imaging systems.

However, the performance of the DORT method degrades when the scatterers in the medium of interest are not well-resolved. When a pair of discrete scatterers are not well resolved, the eigenvectors in the signal subspace become linear combinations of the medium between the scatterers and the TRA antennas. Back-propagation of these signal subspace eigenvectors leads to broad focusing which hampers target localisation. However, even for closely spaced scatterers, the null subspace is orthogonal to the signal subspace and hence when the null subspace
eigenvectors are back-propagated, the signals would vanish only at the scatterers’ locations, which can be exploited for imaging applications. The utilisation of the null subspace eigenvectors rather than the signal subspace eigenvectors forms the basis of the MUltiple SIgnal Classification (MUSIC) method \cite{34, 35} which can achieve resolution that beats the classical diffraction limit even without multipath.

1.4.0.1 TR applications

Originally, TR was successfully utilized in acoustics by Fink et al. \cite{28} and they developed several devices to illustrate the feasibility of this concept \cite{36, 37, 38}. Thereafter, research and physical TR experiments were conducted using acoustics \cite{39, 40, 41} and ultrasonic waves \cite{32, 42, 31}. Theoretical analysis \cite{43, 44} and numerical simulations \cite{45, 46} have also been carried out to investigate the underlying principles of TR. TR has applications in different fields. The process of time reversal has been used in medicine to destroy kidney stones \cite{47}, hyperthermia \cite{48}, breast cancer detection \cite{49} and ultra focusing through the human skull \cite{50}, among others. TR has been used in construction to test for defective structures and materials \cite{51, 52, 32}. The concept has also been used under water to detect intruders and for echo enhancement \cite{53, 54}. Furthermore research on imaging with synthetic time-reversal, where the time reversed signals are not physically retransmitted but rather done numerically and computationally have also been studied \cite{55, 34, 16}. In synthetic TR, knowledge of the background Green’s function of the propagating medium is fundamental for imaging applications. In medicine for instance, \cite{56} showed how the known knowledge of the material properties of a human can help determine the time when the signal is maximum at a tumour location. The tumour response as well as some breast inhomogeneities were obtained from the total signal received at the antennas by subtracting from a different known case with skin and fat but without tumour effectively serving as a clutter suppression algorithm that successfully isolates tumour responses from the overall signal response \cite{57}. Some research has also been done on utilising EM waves for locating and imaging targets in discretely cluttered environments \cite{58, 59, 60}. TR may also be used for TWI.
1.4.1 The TR methods for TWI

The application of TR methods to TWI techniques is a relatively new research area. Theoretically, the TR method is very promising for TWI as it can utilise the multipath components, which is currently a hindrance in TWI systems, to achieve superresolution which is a resolution that transcends the classical diffraction limit. Furthermore, the TR-based DORT method is capable to selectively focusing on individual discrete targets. Particularly, the MUSIC method is capable of imaging targets that are not well resolved from each other. However, these TR methods have been previously considered to be unattractive for TWI as it requires the physical or synthetic back-propagation of the time-reversed signals. Physical back-propagation of the time-reversed signal is infeasible for TWI. Although the time reversed back-propagated signals focus on the scatterers location, it is impossible to obtain the scatterer location without sensors at every point in the medium of interest which is impractical for real life applications where the targets are inaccessible. Hence synthetic TR is usually more applicable. For synthetic TR, a rudimentary knowledge of the Green’s function, which is also termed response function, of the background media of interest is necessary. Although the free-space Green’s function is readily available, for applications involving inhomogeneous media the Green’s function is often not known in a deterministic manner. Furthermore, TR methods are adversely affected by lossy materials such as brick walls. Lossy materials reduce the power of the reflected signals received at the TRA. Thereafter, when the time-reversed signals are back-propagated, they experience further loss in power which may affect the precision of location when the signals are unable to reach the source of the reflection.

\[61\] is among other works \[62\] \[63\] done on implementing TR imaging method for TWI by back-propagating the signals when the entire scenario, including the wall, is perfectly modelled and simulated with a forward solver such as the finite difference time domain (FDTD) method \[64\] or the conjugate gradient fast Fourier transform method in the frequency domain \[65\]. By visually observing the back-propagation using a forward solver, the optimal focusing time may be obtained. \[60\] proposes target initial reflection method to obtain the optimal focusing time when the distance of the target from the antenna is known. Similarly, \[67\] employed TR for TWI but rather utilising the entropy criterion to obtain the
optimal focusing time for image production. The entropy criterion involves calculating the amount of randomness\(^1\) which is the entropy, for the images produced at each time. The time at which the image displays minimum entropy is selected as the optimal focusing time. \(^{29}\) proposes a TWI technique which obtains the Green’s function vectors with the saddle point method \(^{68}\) for a homogeneous wall, to image the scenario using the MUSIC method. In \(^{69}\) a similar method is investigated, utilising a single moving antenna similar to an SAR system. These methods demonstrate the possibilities for the TR for TWI techniques when the scenario and the material, size and location of the obstruction are known and the clutter signal as a result of the initial reflection from the obstruction is removed. Similar TR methods have been studied for GPR and successfully implemented for sensing buried targets \(^{70}\) \(^{71}\) \(^{72}\). For instance \(^{73}\) shows how sliding-windows space-frequency matrices can also be used in GPR to detect and image dielectric targets shallowly buried below the ground. The sliding windows are used after probing the scene with a single transceiver at different locations and receiving the reflections. The sliding windows are used to segregate the signals received. By taking the singular value decomposition (SVD) of the segregated signals received and comparing the singular vectors to those obtained from a similar surface and media without a target, the signals that represent the target can be extracted to reduce clutter. Hence, the clutter due to the initial surface scattering can be easily removed on account of the surface location being known. However, knowledge of the obstructing material is not always available, particularly in realistic TWI scenario.

The main problem with TR for TWI when knowledge of the obstruction is unknown is the performance degradation due to the presence of clutter signal from the obstructing surface between the antennas and the targets \(^{74}\) \(^{75}\). Hence the back-propagation of the time reversed signals focus on the obstruction rather than the targets behind. Prior studies have attempted to tackle the problem of an unknown obstruction in TWI for TR. The average trace subtraction method is among other works that utilises a single moving antenna parallel to an extended

\[ \xi = \min_t \left( \frac{\sum_{\tau} |E_{\tau}(t)|}{\sum_{\tau} |E_{\tau}(t)|^4} \right)^2 \]

where \( \xi \) is the minimum entropy \(^{67}\), \( |E_{\tau}(t)| \) is the modulus of the electric field, \( \tau \) is a point in space and \( t \) is time.
obstructing scatterer as both transmitter and receiver to ensure that clutter contributions vary minimally with antenna location \cite{76,77}. Therefore, the average of the signals received at different antenna locations are subtracted from the signal received at each antenna location to eliminate the clutter. However the configuration is only applicable to SAR systems as it does not obtain a full-MDM where multiple antennas are used and hence its application is limited. Furthermore, in a realistic scenario, the assumption that clutter contributions vary minimally with antenna location may not always hold true. The “time gating based on entropy criterion” method \cite{76} also uses the assumption that the clutter contribution will not really vary on each antenna. In this method, the signals from each receiving antenna are added up hence making the clutter signal maximum as they should be in phase. A threshold is then used to determine the time instances where there is clutter signal and force signals to zero at these time samples before time-reversal. The two-stage MUSIC algorithm proposed in \cite{78} is useful for identifying a weak scatterer which has been lost due to clutter from an obstacle. However the algorithm assumes having prior knowledge of the radii of the targets. After the initial MUSIC imaging is carried out, a new MDM will be formed which corresponds to just the strong scatterers that were located. This is then subtracted from the original MDM and the MDM obtained from this subtraction is then used to carry out the second stage of imaging. Compensation techniques \cite{79,80} have also been studied for the loss that occurs in TWI applications for TR methods.

1.5 Aims and objectives

1.5.1 Aims

The purpose of this research is to develop novel algorithms to image stationary and moving targets hidden behind an unknown obstruction. Current TWI systems have resolutions limited by the diffraction limit and suffer in the presence of multipath signals due to multiple targets and scatterers. The aim is to develop algorithms that produce images that can beat the diffraction limit and is robust in the presence of multipath signals.
CHAPTER 1. INTRODUCTION

1.5.2 Objectives

- Interpretation of the concepts and theory of the TR method on which the DORT and MUSIC methods are based
- Interpretation of the DORT and MUSIC methods
- Implementation of the DORT and MUSIC methods as well as analysis of their performance
- Development of the total sub-MDM algorithm which utilises the MUSIC methods to image stationary targets
- Development of the total sub-differential MDM algorithm which utilises the MUSIC methods to image moving targets
- Application of the total sub-MDM algorithm to TWI scenario containing targets hidden behind and obstructing brick wall
- Examination of the performance of the total sub-MDM algorithm
- Application of the total sub-differential MDM algorithm on TWI scenario containing moving targets hidden behind and obstructing brick wall or buried in the ground
- Examination of the performance of the total sub-differential MDM algorithm

1.6 Outline

The outline of the dissertation is as follows.

Chapter 2 details the theory of the TR method. We discuss how the TR method utilises multipath signal to increase spatial resolution. We discuss the theory of the TR-based DORT and MUSIC methods and their implementation. The specificity of the performance of the DORT and MUSIC methods were also reviewed as well as the differences between the methods. We also introduce the FDTD method which we utilise to simulate the scenarios considered in this dissertation.
Chapter 3 demonstrates and analyse the ability of the DORT and MUSIC methods to locate targets in free-space. We display the singular value distributions as well as the phase and magnitude distributions showing that the singular values in the signal subspace represent the number of well-resolved scatterers in a free-space medium. We also analyse the singular vectors which are used to produce images that locate the targets in the medium. The unique properties of the DORT and MUSIC methods are also discussed. The superiority of the MUSIC method over the DORT method are shown. Thereafter we investigate the difficulties the methods encounter due to obstruction from a brick wall that produce clutter signal in the MDM.

Chapter 4 introduces a novel TR-based imaging algorithm, utilising simultaneously both temporal and spatial windows, termed the total sub-MDM algorithm. The algorithm involves partitioning the elements of the full-MDM and hence the signals to create multiple sub-MDMs. The spatio-temporal windows isolate the responses from targets at different times and locations in the scenario and we obtain localised information in time and space. We select the MUSIC method, which uses the null subspace, ahead of the DORT due to the superior resolution and accuracy it provides. We apply the MUSIC method to the individual sub-MDMs to obtain the sub-MUSIC images. The summation of the sub-MUSIC images give the total sub-MUSIC image which provides a clearer image of the scenario than is possible using the full-MDM MUSIC method. Furthermore we expand our algorithm for moving targets applications by developing a total sub-differential MDM algorithm that utilises TR-based MUSIC method to produce images. The algorithm involves the successive probing of the medium to obtain two MDMs that only differ due to the displacement of a moving or live target. The difference of the successive MDMs gives the full-differential MDM. By using spatial windows, the full-differential MDM is segmented into sub-differential MDMs that contain localised information in space. The spatial windows partition the responses from the targets into sub-differential MDMs which combine to give a clearer image of the targets’ locations than is possible using the full-MDM without differencing or spatial windowing.

Chapter 5 considers a scenario containing a well resolved target located behind a brick wall and utilise the total sub-MDM algorithm to locate the target. We display the isolation of clutter and target signal that occurs in individual sub-MDMs by analysing their singular values and singular vectors as well as plotting
some of the sub-MUSIC images. A case with the target closer to the wall, where clutter and target signals overlap in some elements of the MDM, is considered. Finally we emphasize the advantage of the spatial windows by applying our algorithm to a scenario with two targets. The performance of the total sub-MDM algorithm in terms of (cross) range resolution and accuracy are displayed.

In Chapter 6 we apply the total sub-differential MDM algorithm on simple scenarios with moving or live (expanding) targets that are hidden behind a brick wall or buried in the ground. We show the total sub-differential MUSIC images which locate the targets and discuss the performance of the total sub-differential MDM algorithm in terms of (cross) range resolution. Furthermore, non-homogeneous highly scattering scenarios are considered.

Chapter 7 provides conclusions to the research and suggest future works.

### 1.7 Summary

In this chapter, we summarised the conventional radar imaging techniques and their applications. We discussed the popular TWI systems and their current limitations. The problems faced by TWI systems due to loss, dispersion and multipath were described. We briefly introduced the TR method, as well as the TR-based DORT and MUSIC methods which utilises multipath signal to achieve superresolution and is promising for TWI applications. We reviewed the applications of the TR-based methods. We discussed the current limitations for implementing the TR methods for TWI particularly due to clutter signal from an unknown obstruction. In the next chapter, we introduce the theory of TR and describe the implementation of the DORT and MUSIC methods in detail. Furthermore, we introduce the FDTD method which we utilise to simulate the scenarios considered in this dissertation.
Chapter 2

The TR and FDTD methods

In the Chapter 1, we discussed the current TWI systems and the challenges they face particularly in the presence of multiple scatterers that produce multipath signal. In this chapter, we detail the theory of TR and review the technical implementation of the DORT and MUSIC methods. Furthermore, we overview the FDTD method as we employ the method to simulate the various scenarios we consider in this dissertation.

2.1 The Time Reversal (TR) method

The concept of time reversal takes advantage of the invariance of the wave equation when time is reversed. For the wave equation

$$\nabla^2 E(\tau_0, t) - \mu(\tau_0)\epsilon(\tau_0) \frac{\partial^2}{\partial t^2} E(\tau_0, t) = 0$$

in a lossless media, $E(\tau_0, -t)$ along with $E(\tau_0, t)$ is a solution to the equation where $E(\tau_0, t)$ is the vector electric field, $\tau_0$ is the spatial location, $\epsilon$ is the permittivity of the medium and $\mu$ is the permeability of the medium. Hence the time reversal method is suitable for media that have the reciprocity property. (2.1) shows the reciprocal property which means that for every wave propagating away from a scatterer, there exist a reversed wave that can retrace the path of the original wave back to the scatterer [81]. A media possesses the reciprocal property when it is lossless. The losslessness of a media is governed by the conductivity of the media i.e. when conductivity is zero. It is possible to retrace the path of the original wave back to the scatterer it originates from in a reciprocal homogeneous
media and also in an inhomogeneous with the presence of other scatterers causing reflections, refraction and scattering of the wave. Ideally, wave propagating in all possible directions should be considered when retracing the path of the original wave back to the scatterer, so that the wave can be generated in its entirety. However, as it is impossible to completely surround the original scatterer and obtain all the wave fields propagated from it, a limited number of antennas, known as the TRA is used. The information lost by doing this can be recovered in some cases by using multipaths. Typically, the TRA antennas are used to record signals propagating from the original scatterer, reverse these signals in time and then re-transmit the time reversed signal into the medium. Assuming a point scatterer located at \( r_0 \) transmits a short UWB pulse \( S(t) \), the transmitted signal received by the \( n \)th antenna is

\[
 f_n(t) = S(t) *_t h_{r_0 r_n}(t) \quad (2.2)
\]

where \( *_t \) is convolution in time, \( h_{r_0 r_n}(t) \) is the impulse response between the scatterer located at \( r_0 \) and the TRA antenna located at \( r_n \). If the signals at each array element are recorded, reversed in time and transmitted back to the same medium, the time-reversed signal received at the original scatterer \( r_0 \) due transmission from the \( n \)th TRA antenna is

\[
 p_n(t) = S(-t) *_t h_{r_n r_0}(-t) *_t h_{r_n r_0}(t) \quad (2.3)
\]

because \( f_n(-t) = S(-t) *_t h_{r_0 r_n}(-t) \) where \( (h_{r_n r_0}(-t) *_t h_{r_n r_0}(t)) \) represent a correlation filter (time correlator) which has a maximum at \( t = 0 \) corresponding to \( \int |h_{r_n r_0}(t)|^2 dt \) which is the energy of \( h_{r_n r_0}(t) \). Therefore, having multiple antennas improves the performance of the TR system as each antenna will produce maximum energy at the original scatterer location which interfere constructively to improve the TR peak signal. For a TRA with \( N \) antennas, the received signal at the scatterer location becomes

\[
 p(r_0, t) = \sum_{n=1}^{N} S(-t) *_t h_{r_n r_0}(-t) *_t h_{r_n r_0}(t). \quad (2.4)
\]
CHAPTER 2. THE TR AND FDTD METHODS

TR acts as a time and space correlator because at any other point $\mathbf{r}$ in the medium, the signal becomes

$$p(\mathbf{r}, t) = \sum_{n=1}^{N} S(-t) \ast h_{\mathbf{r}_n, \mathbf{r}_0}(-t) \ast h_{\mathbf{r}_n, \mathbf{r}}(t). \quad (2.5)$$

Hence as the probe location $\mathbf{r}$ gets further away from the original source location, the uncorrelated terms tend to cancel out each other. Ideally, the scatterer transmitting the pulse should be completely surrounded with antennas to avoid loss of information. In reality, only a limited amount of antennas can be used to record the signal propagating from the scatterer. However, despite using a limited amount of antennas, it is still possible to back-propagate signals to focus on the original scatterer location. The focusing area size is dictated by the classical diffraction limit [82] which states that in a homogeneous medium, cross range resolution $d_c$ is

$$d_c = \frac{\lambda \ell}{a} \quad (2.6)$$

where $a$ is the TRA antenna aperture, $\lambda$ is the wavelength of operation and $\ell$ is the distance between the TRA and the scatterer. The cross range and range resolution together are referred to as the spatial resolution [46]. Theoretically, the range resolution [5] of an imaging system is

$$d_r = \frac{c_0 \Upsilon}{2} \quad (2.7)$$

where $c_0 = 3 \times 10^8$ m/s is the speed of light and $\Upsilon$ is the pulse duration of the source signal. However, TR can improve the spatial resolution in a highly scattering medium by utilising the multipath signals which increase the aperture. Figure 2.1 shows the forward propagation of an excitation pulse from a scatterer in a scattering medium with the TRA antennas. Figure 2.2 shows the backward propagation of the received signals from the TRA antenna. The backward propagated signals focus on the original scatterer location depending on the cross range resolution $d_c$ and the range resolution $d_r$. However the aperture of the antenna is effectively increased due to the multiple scattering as signals that would have otherwise been lost have been redirected to the antenna array. The increase in aperture is illustrated in Figure 2.3 which shows the transmission of a pulse signal
in (a) a homogeneous medium with no other scatterers, (b) a medium with two PEC side walls and (c) a medium with discrete scatterers. In Figure 2.1, the discrete scatterers are all the scatterers that are between the scatterer transmitting the pulse and the TRA antennas. If the PEC walls and discrete scatterers are located in a way that they redirect waves propagating away from the TRA toward the TRA antennas, thereby creating multipath, then the aperture $a$ of the TRA antenna is said to be increased. The increased TRA antenna aperture due to the multipaths enabling the collection of more information is known as the effective aperture ($a_e$) \[83\]. The increase in aperture by multipaths which interfere constructively yields the cross-range resolution

$$d_c = \frac{\lambda}{a_e}.$$ \hspace{1cm} (2.8)

Hence, in inhomogeneous media, it is possible to obtain better cross-range resolution than in homogeneous as long as the waves propagating away from the TRA are redirected to the TRA antennas thereby increasing the effective aperture ($a_e > a$). Media with rich multipath components help in achieving better correlation.
CHAPTER 2. THE TR AND FDTD METHODS

Figure 2.2: The backward propagation of the time reversed signal from a limited number of TRA antennas

Figure 2.3: The increase of effective aperture from a media (a) with no multipaths ($a_e = a$) to a media (b) with multipaths created by two PEC walls ($a_e > a$) and a media (c) with multipaths created by discrete scatterers ($a_e > a$) [81]
CHAPTER 2. THE TR AND FDTD METHODS

2.2 The DORT and MUSIC methods

When there are multiple scatterers, back-propagation of the TR wave fields results in the generation of focal points on all the scatterers but stronger on the dominant scatterer and the field distribution becomes much more localised on the dominant scatterer when the TR process is iterated. Focusing on weaker scatterers is only possible with standard TR when time-gating is used. Time-gating is performed by isolating a particular signal in time so that other signals are discarded but this can distinguish only well resolved scatterers. The well resolvedness criterion means that if there is more than one scatterer in a medium, the scatterers have to be a minimum distance apart from one another so that multiple scattering that occurs between the scatterers can be neglected. Hence the minimum distance between scatterers should be greater than or equal to the cross-range resolution \( (d_c) \). The DORT method is important as it isolates different scatterers without the need for iteration or time-gating. Although the DORT method still has to adhere to the well resolvedness criterion, it forms the basis for the much acclaimed MUSIC method. The DORT method uses the TRO which is obtained using the MDM of a time-reversal array which contains important information about the scattering scenario in its eigenspace structure. It was shown in the study that for well-resolved discrete isotropic scatterers, each eigenvalue and corresponding eigenvector of the TRO are associated with a distinct scatterer. Hence by carrying out the EVD of the TRO and transmitting the signals produced by signal subspace eigenvectors, selective focusing on point like scatterers becomes possible. The DORT method may be used as a pre-processing step which improves the efficiency of the inverse scattering algorithms. The process of selective focusing increases the Signal to Noise Ratio (SNR).

2.2.1 The DORT methods

In the DORT method, \( N \) TRA antennas are used to probe the medium. An excitation pulse is sent from an antenna and the signal is observed at the TRA antennas to obtain \( k_{s,r}(t) \) where \( s \) identifies the antenna transmitting the signal and \( r \) identifies the antenna observing the signal propagation. The elements
CHAPTER 2. THE TR AND FDTD METHODS

\( k_{s,r}(t) \) together give the MDM as

\[
K(t) = \begin{pmatrix}
k_{1,1}(t) & \ldots & k_{1,N}(t) \\
\vdots & \ddots & \vdots \\
\vdots & \ddots & \vdots \\
k_{N,1}(t) & \ldots & k_{N,N}(t)
\end{pmatrix}
\]

(2.9)

We convert \( k_{s,r}(t) \) to the frequency spectrum as

\[
k_{s,r}(\omega) = \mathcal{F}(k_{s,r}(t))
\]

(2.10)

where \( \mathcal{F} \) means the fast Fourier transform (FFT) with respect to time \( t \). Therefore FFT of \( K(t) \) gives the in the frequency spectrum as

\[
K(\omega) = \begin{pmatrix}
k_{1,1}(\omega) & \ldots & k_{1,N}(\omega) \\
\vdots & \ddots & \vdots \\
\vdots & \ddots & \vdots \\
k_{N,1}(\omega) & \ldots & k_{N,N}(\omega)
\end{pmatrix}
\]

(2.11)

For a well resolved and discrete scatterer, \( k_{s,r}(\omega) \) can be written as

\[
k_{s,r}(\omega) = \sum_{o=1}^{O} G(\tau_r, \tau_o, \omega) \tau_o(\omega) G(\tau_o, \tau_s, \omega) S(\omega)
\]

(2.12)

where \( G(\tau_r, \tau_o, \omega) \) is the Green’s function of the reciprocal medium between the TRA antenna location \( \tau_r \) and the \( o \)th scatterer location \( \tau_o \), \( O \) is the total number of scatterers, \( \tau_o(\omega) \) is the scattering coefficient of the \( o \)th scatterer and \( S(\omega) \) is the frequency spectrum representation of the excitation pulse from the transmitting TRA antenna \( s \). We note that the well resolvedness criterion means that the multiple scattering between the scatterers can be neglected. Hence, we include only the direct scattering from the scatterers in the medium in \( (2.12) \). Therefore, we can obtain the MDM \( K(\omega) \), which contains \( k_{s,r}(\omega) \) for \( s = 1, \ldots, N \) and \( r = 1, \ldots, N \), as

\[
K(\omega) = S(\omega) \sum_{o=1}^{O} \tau_o(\omega) g(\tau_o, \omega) g^T(\tau_o, \omega)
\]

(2.13)
where
\[
g(\mathbf{r}_o, \omega) = \left[ G(\mathbf{r}_1, \mathbf{r}_o, \omega), \ldots, G(\mathbf{r}_N, \mathbf{r}_o, \omega) \right]^T
\]  
(2.14)

where \( T \) represents the transpose of the matrix and \( g(\mathbf{r}_o, \omega) \) is the steering vector of the Green’s functions that connects the 0th scatterer to the TRA antennas. Thereafter, the TRO is defined as the self adjoining matrix
\[
\mathbf{T}(\omega) = \mathbf{K}^\dagger(\omega)\mathbf{K}(\omega) \tag{2.15}
\]

where \( \mathbf{K}^\dagger(\omega) \) is the complex conjugate transpose of \( \mathbf{K}(\omega) \) and \( \dagger \) represents the complex transpose operation. We utilise \( \mathbf{K}^\dagger(\omega) \) because a phase conjugation in the frequency corresponds to time reversal in the time domain. The eigenspace structure can be obtained from the singular values and singular vectors of the MDM. Hence by performing the singular value distribution (SVD) on \( \mathbf{K}(\omega) \) we obtain
\[
\mathbf{K}(\omega) = \mathbf{U}(\omega)\mathbf{A}(\omega)\mathbf{V}^\dagger(\omega) \tag{2.16}
\]

where \( \mathbf{A}(\omega) \) is the real diagonal matrix containing the singular values in descending order, \( \mathbf{U}(\omega) \) is a unitary matrix containing the left singular vectors and \( \mathbf{V}(\omega) \) is a unitary matrix containing the right singular vectors. The singular values matrix is written as
\[
\mathbf{A}(\omega) = \left( \begin{array}{ccc}
\alpha_1(\omega) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \alpha_N(\omega)
\end{array} \right) \tag{2.17}
\]

The singular values \( \alpha_n(\omega) \) are non-negative real numbers. The left singular vectors \( \mathbf{u}_n(\omega) \) are expressed as
\[
\mathbf{U}(\omega) = \left[ \mathbf{u}_1(\omega), \ldots, \mathbf{u}_N(\omega) \right] = \left( \begin{array}{ccc}
\mathbf{u}_{1,1}(\omega) & \cdots & \mathbf{u}_{1,N}(\omega) \\
\vdots & \ddots & \vdots \\
\mathbf{u}_{N,1}(\omega) & \cdots & \mathbf{u}_{N,N}(\omega)
\end{array} \right) \tag{2.18}
\]
where \( u_{s,r}(\omega) \) is a complex number. Similarly the right singular vectors \( v_n(\omega) \) are obtained as

\[
V(\omega) = \left[ v_1(\omega), \ldots, v_N(\omega) \right] = \begin{pmatrix}
    v_{1,1}(\omega) & \cdots & v_{1,N}(\omega) \\
    \vdots & \ddots & \vdots \\
    v_{N,1}(\omega) & \cdots & v_{N,N}(\omega)
\end{pmatrix}
\]

(2.19)

where \( v_{s,r}(\omega) \) is a complex number. Therefore by using the SVD of the MDM, the EVD of the TRO can be obtained as

\[
T(\omega) = V(\omega)A^\dagger(\omega)U^\dagger(\omega)U(\omega)A(\omega)V^\dagger(\omega).
\]

(2.20)

Since \( U^\dagger(\omega)U(\omega) \) gives the identity matrix, the TRO can be expressed as

\[
T(\omega) = V(\omega)A^\dagger(\omega)A(\omega)V^\dagger(\omega)
\]

(2.21)

where \( A^\dagger(\omega)A(\omega) \) is the real diagonal matrix of eigenvalues as written below

\[
A^\dagger(\omega)A(\omega) = \begin{pmatrix}
    \psi_1(\omega) & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & \psi_N(\omega)
\end{pmatrix}
\]

(2.22)

We note that the TRO eigenvalues is equal to the square of the singular values of the MDM

\[
\alpha_n^2(\omega) = \psi_n(\omega)
\]

(2.23)

Furthermore, the eigenvectors of the TRO are similar to the right singular vectors of the MDM. Hence simply obtaining the SVD of the MDM gives the eigenspace structure of the TRO. It is known that \( \alpha_n(\omega) \) is a singular value of \( K(\omega) \) if and only if

\[
K(\omega)v_n(\omega) = \alpha_n(\omega)u_n(\omega)
\]

(2.24)
and

\[ K^\dagger(\omega)u_n(\omega) = \alpha_n^*(\omega)v_n(\omega). \] (2.25)

Substituting (2.13) into (2.24) we obtain

\[ S(\omega) \sum_{o=1}^{O} \tau_o(\omega)g(\tau_o, \omega)g^T(\tau_o, \omega)v_n(\omega) = \alpha_n(\omega)u_n(\omega) \] (2.26)

where \( g^T(\tau_o, \omega)v_n(\omega) \) is a scalar identity. Therefore the left singular vector can be written as

\[ u_n(\omega) = \sum_{o=1}^{O} S(\omega)\tau_o(\omega)(g^T(\tau_o, \omega) \cdot v_n(\omega)) \frac{\alpha_n(\omega)}{\alpha_n^*(\omega)} g(\tau_o, \omega) \] (2.27)

where \( * \) represents the phase conjugation. We note that (2.27) and (2.28) satisfy \( u_n(\omega) = v_n^*(\omega) \) because \( K(\omega) \) is symmetrical. For well-resolved scatterers there exists a specific non-zero \( u_n(\omega) \) so that \( (g^T(\tau_o, \omega) \cdot u_n(\omega)) \) is zero when \( u_n(\omega) \neq g(\tau_o, \omega) \), that is,

\[ (g^T(\tau_o, \omega) \cdot u_n(\omega)) = \begin{cases} \|g(\tau_o, \omega)\|^2 & \text{when } u_n(\omega) = g(\tau_o, \omega) \\ 0 & \text{when } u_n(\omega) \neq g(\tau_o, \omega) \end{cases} \] (2.29)

where \( \|g(\tau_o, \omega)\| = \sqrt{(g^T(\tau_o, \omega) \cdot g(\tau_o, \omega))} \) is the norm of the vector \( g(\tau_o, \omega) \). Hence, we choose the left singular vectors which satisfies (2.24) as

\[ u_n(\omega) = \frac{g(\tau_o, \omega)}{\|g(\tau_o, \omega)\|} \text{ for } n = o = 1, ..., O \] (2.30)
Since $u_n(\omega) = v_n^*(\omega)$ the right singular vectors become

$$v_n(\omega) = \frac{g(\tau_\omega)}{||g(\tau_\omega)||} \text{ for } n = o, \ldots, O$$

where the corresponding singular values from (2.26) become

$$\alpha_n(\omega) = S(\omega)\tau_o(\omega)||g(\tau_\omega)||^2 \text{ for } n = o, \ldots, O$$

Hence, for a well-resolved discrete scatterer, each significant singular value and corresponding singular vector are associated with a single scatterer. Therefore the DORT method uses the singular vectors $(u_1(\omega), \ldots, u_{N_t}(\omega))$ corresponding to the significant singular values $(\alpha_1(\omega) > \ldots > \alpha_{N_t}(\omega) > 0)$ that are said to be in the signal subspace, where $N_t(\omega)$ is the number of singular values in the signal subspace and hence $N_t(\omega) = O$ when the scenario of interest contains only well-resolved discrete scatterers. The inner product (scalar product) of the steering vector and a singular vector in the signal subspace gives the DORT imaging functional as

$$D(\tau, \omega, n) = g^\dagger(\tau, \omega) \cdot u_n(\omega)$$

for $1 \leq n \leq N_t(\omega)$ where $g(\tau, \omega)$ is the steering vector of the Green’s functions $G(\tau_r, \tau, t)$ that connects the point in space $\tau$ to the TRA antennas for $1 \leq r \leq N$. $D(\tau, \omega, n)$ is the point spread function of the TRA that gives selective focusing on the scatterers as the singular vectors corresponding to singular values in the signal subspace represent scatterers and the steering vector represents the relative phase shifts to the centre of the TRA antennas a wave experiences through the medium in the scenario. For the CF-DORT image, the signal subspace singular vectors and the steering vectors at the centre frequency ($\omega_c$) are used. (2.33) becomes

$$D(\tau, \omega_c, n) = g^\dagger(\tau, \omega_c) \cdot u_n(\omega_c).$$

However for UWB-DORT, the signal subspace singular vectors and the steering vectors at all the frequency samples are used. To combine all frequencies, we project $D(\tau, \omega, n)$ in the time domain by taking its inverse Fourier transform

$$D(\tau, t, n) = \mathcal{F}^{-1}\{D(\tau, \omega, n)\}$$
CHAPTER 2. THE TR AND FDTD METHODS

where $\mathcal{F}^{-1}$ is the inverse Fourier transform. The focusing time $t_{foc}$ is defined as the time instant where the maximum value of $D(\vec{r}, t, n)$ over all points $\vec{r}$ in the imaging domain occurs. Finally the UWB-DORT imaging functional is given as

$$D_{UWB}(\vec{r}, n) = \frac{D(\vec{r}, t_{foc}, n)}{\max_{\vec{r}, t} (D(\vec{r}, t, n))}. \tag{2.36}$$

2.2.2 The MUSIC methods

The information of the scatterer location and strength are encoded by the singular values (and the corresponding singular vectors) of the signal subspace. These singular vectors are used to form the DORT images. However, the performance of the DORT method degrades when the well-resolvedness criterion is not met. When the well-resolvedness criterion is not met, the signal subspace singular vectors become linear combination of the steering vectors connecting the TRA antennas to the scatterers. Thus, the DORT imaging method, which uses the signal subspace singular vectors, creates an overlapped image field that gives broad focusing on the scatterers hampering resolution and target localisation. However, even for closely spaced scatterers, the singular vectors of the null subspace are orthogonal to the singular vectors of the signal subspace. That is to say, projection of any vector formed by the linear combination of signal subspace singular vectors onto the null subspace is zero \[81\]. As a result, inner products of the steering vectors with the null subspace singular vectors would vanish only at the locations of the scatterers. This essential property is the basis of the MUSIC method which provides better localisation and focusing properties than the DORT method regardless of the well-resolvedness criterion. The inverse of the inner products of the steering vectors with the null subspace singular vectors provides an imaging functional that can beat the diffraction limit \[84\]. The one drawback of the MUSIC method over the DORT method is the limitation of the MUSIC method in selective focusing. In the DORT method the singular values in the signal subspace are easy to analyse as these singular values represent different scatterers (providing the well-resolvedness criterion is met). The singular vectors can be used individually to create images depending on the targeted scatterer which is not possible with the singular vectors corresponding to singular values in the null subspace. Therefore, we use all the singular vectors corresponding to the singular values in the null subspace to create images that locate all scatterers
in the medium. At any scatterer location, due to the singular vectors of the null subspace being orthogonal to the singular vectors of the signal subspace, the inner product of the null subspace singular vectors and the steering vectors satisfies

$$
\sum_{n=N_t(\omega)+1}^{N} \mathbf{g}^\dagger(\mathbf{r}_o, \omega) \cdot \mathbf{u}_n(\omega) \approx 0
$$

(2.37)

Therefore, by taking the inverse of the inner products of the null subspace singular vectors with the steering vectors, focusing on the scatterer(s) is obtained. The MUSIC imaging function can be obtained by

$$
\mathbf{M}(\mathbf{r}, \omega) = \left[ \sum_{n=N_t(\omega)+1}^{N} \mathbf{g}^\dagger(\mathbf{r}, \omega) \cdot \mathbf{u}_n(\omega) \right]^{-1}
$$

(2.38)

where $\mathbf{M}(\mathbf{r}, \omega)$ is the MUSIC image functional. The distribution of $\mathbf{M}(\mathbf{r}, \omega)$ with respect to location ($\mathbf{r}$) gives an image with focus on the location of the scatterer as the singular vectors in the null subspace are orthogonal to the steering vector at the scatterer location. For the CF-MUSIC image, the signal subspace singular vectors and the steering vectors at the centre frequency ($\omega_c$) are used. (2.38) becomes

$$
\mathbf{M}(\mathbf{r}, \omega_c) = \left[ \sum_{n=N_t(\omega_c)+1}^{N} \mathbf{g}^\dagger(\mathbf{r}, \omega_c) \cdot \mathbf{u}_n(\omega_c) \right]^{-1}.
$$

(2.39)

However for UWB-MUSIC, the signal subspace singular vectors and the steering vectors at all the frequency samples of the bandwidth of operation are used. Hence we obtain

$$
\mathbf{M}_{UWB}(i, j) = \left[ \int_{\Omega} \sum_{n=N_t(\omega)+1}^{N} \mathbf{g}^\dagger(\mathbf{r}, \omega) \cdot \mathbf{u}_n(\omega) d\omega \right]^{-1}
$$

(2.40)

where $\Omega$ is the bandwidth of operation.
Figure 2.4: Block diagram illustrating the implementation of the DORT and MUSIC methods
2.2.3 The similarities and differences between the DORT and MUSIC methods

For both the DORT and MUSIC methods, the scenario of interest is illuminated from each TRA antenna and the reflections from any scatterers in the medium are received at all antennas, giving the MDM. Figure 2.4 shows a block diagram illustrating the implementation of the DORT and MUSIC methods. The SVD of the MDM give the singular values and their corresponding singular vectors. For a scenario containing only discrete well-resolved scatterers, the number of non-zero singular values gives the number of scatterers in the medium as each significant singular value represents a scatterer in the scene. Hence, the DORT images are obtained from the inner dot product of the singular vectors in the signal subspace and the steering vectors. For the MUSIC methods however, the null subspace singular vectors are used. As the inner dot product of the singular vectors in the null subspace and the steering vectors is zero at the scatterer location, the inverse of this inner product gives an image functional that focuses on the scatterers in the scenario. Table 2.1 shows the differences between the DORT and music methods. The MUSIC methods can beat the diffraction limit and give better contrast between the target location and free space than the DORT methods. We also note that because the steering vectors are not always ideal for a realistic scenario [85] [86] particularly for frequency dependent media, the UWB-DORT and UWB-MUSIC methods provides statistical stability that the CF-DORT and UWB-DORT methods does not. Unknown properties of the background medium means the steering vectors are not always ideal and hence susceptible to error. Therefore statistical stability is necessary to ensure that an error at a particular frequency sample does not have a huge effect on the final image. The UWB methods achieve this by combining the singular vectors and steering vectors at different frequencies which gives the UWB methods higher accuracy in locating targets than CF methods particularly for scenarios containing frequency dependent materials. In Chapter 3 we utilise the DORT and MUSIC methods to image targets whose locations are unknown. We utilise the FDTD method to simulate the scenarios we consider. Next, we introduce the theory behind the FDTD method and its application.
2.3 Finite Difference Time Domain (FDTD) method

The FDTD method [87], introduced by Yee in 1966 [64], uses a finite differencing scheme to find approximate solutions to Maxwell equations, in the differential form, in the time-domain [88]. Maxwell equations are solved either in the time domain or in the frequency domain. Time domain methods for solving Maxwell equations include the FDTD method and the finite element time domain method [89]. Frequency domain Maxwell equations solvers include the conjugate gradient fast transform method and the finite element method [90]. Frequency domain methods solve Maxwell equations at a single frequency in one simulation. Hence UWB applications require multiple simulations at various frequencies. However, the time domain methods can solve multiple frequencies in one simulation and the frequency response can be obtained by FFT. Since this dissertation deals with UWB signals for TWI the FDTD method is chosen. The FDTD method has become the preferable for finding numeric solutions over a large frequency range as it is a time-domain method. The method is fairly simple to implement, with straightforward calculations and simple mesh configuration. The radiation and temperature rise can also be observed in the time domain.

2.3.1 Maxwell’s equations

FDTD modeling is based around solving Maxwell’s equations. These equations are Faraday’s law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2.41)$$
Ampere’s law,
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \]  
(2.42)
Gauss’s law for the electric field,
\[ \nabla \cdot \mathbf{D} = \rho, \]  
(2.43)
and for the magnetic field,
\[ \nabla \cdot \mathbf{B} = 0. \]  
(2.44)
The Ohm’s law is
\[ \mathbf{J} = \sigma \mathbf{E}, \]  
(2.45)
where \( \mathbf{E} \) (Volt/Meter) is the Electric field, \( \mathbf{H} \) (Ampere/Meter) is the magnetic field, \( \mathbf{D} \) (Coulomb/[Meter]\(^2\)) is the electric flux density, \( \mathbf{B} \) (Tesla) is the magnetic flux density, \( \mathbf{J} \) (Ampere/[Meter]\(^2\)) is the conduction current density, and \( \rho \) (Coulomb/[Meter]\(^3\)) is the charge density. However, when located in a macroscopic media, the current density is the summation of the free current density, the magnetization current density, and the polarization current density. Hence,
\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \]  
\[ \mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}, \]
where \( \mathbf{P} \) is the polarization, \( \mathbf{M} \) is the magnetization, \( \varepsilon_0 \) (Farad/Meter) is the permittivity of free space and \( \mu_0 \) (Henry/Meter) is the permeability of free space (\( \mu_0 = 4\pi/10^7 \)). In a homogeneous medium, the equations must satisfy
\[ \mathbf{D} = \varepsilon \mathbf{E}, \]  
(2.46)
\[ \mathbf{B} = \mu \mathbf{H}, \]  
(2.47)
CHAPTER 2. THE TR AND FDTD METHODS

2.3.2 FDTD method equations

Firstly, (2.41) and (2.47) are re-written in a Cartesian vector manner as

\[ \nabla \times \mathbf{E} = \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{i}_y + \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \hat{i}_z = (2.48) \]

\[ \frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial (H_x + H_y + H_z)}{\partial t} \]

\[ \nabla \times \mathbf{H} = \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \hat{i}_y + \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{i}_z (2.49) \]

\[ = \epsilon \frac{\partial (E_x + E_y + E_z)}{\partial t} \]

where \( \hat{i}_x, \hat{i}_y \) and \( \hat{i}_z \) are the unit vectors in \( x, y \) and \( z \) directions. (2.48) is expressed in a scalar manner,

\[ \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \]

\[ (2.50) \]

\[ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t} \]

\[ (2.51) \]

\[ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial H_z}{\partial t} \]

\[ (2.52) \]

(2.49) is expressed in a scalar manner,

\[ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \epsilon \frac{\partial E_x}{\partial t} \]

\[ (2.53) \]

\[ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \epsilon \frac{\partial E_y}{\partial t} \]

\[ (2.54) \]

\[ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \epsilon \frac{\partial E_z}{\partial t} \]

\[ (2.55) \]
Discretizing (2.53), we have
\[
\frac{H_\eta^{\frac{1}{2}}(i-\frac{1}{2},j+\frac{1}{2},k+\frac{1}{2}) - H_\eta^{\frac{1}{2}}(i-\frac{1}{2},j-\frac{1}{2},k+\frac{1}{2})}{\Delta y} - \frac{H_\eta^{\frac{1}{2}}(i-\frac{1}{2},j,k+1) - H_\eta^{\frac{1}{2}}(i-\frac{1}{2},j,k)}{\Delta z} = (2.56)
\]
\[
\frac{\epsilon^\eta(i-\frac{1}{2},j,k+\frac{1}{2})E_x^\eta(i-\frac{1}{2},j+\frac{1}{2},k) - \epsilon^{\eta-1}(i-\frac{1}{2},j,k+\frac{1}{2})E_x^{\eta-1}(i-\frac{1}{2},j,k+\frac{1}{2})}{\Delta t}
\]
where \(\eta\) is timesteps. Similarly, (2.54) and (2.55) are also discretized. Discretizing (2.50) we have,
\[
\frac{E_\eta^\eta(i,j,k) - E_\eta^\eta(i,j-1,k)}{\Delta y} - \frac{E_y^\eta(i,j-\frac{1}{2},k+\frac{1}{2}) - E_y^\eta(i,j-\frac{1}{2},k-\frac{1}{2})}{\Delta z} = (2.57)
\]
\[
- \frac{\mu^\eta(\frac{1}{2},j-\frac{1}{2},k)H_x^{\eta+\frac{1}{2}}(i,j-\frac{1}{2},k) - \mu^{\eta-\frac{1}{2}}(i,j-\frac{1}{2},k)H_x^{\eta-\frac{1}{2}}(i,j-\frac{1}{2},k)}{\Delta t}
\]
Similarly, (2.51) and (2.52) are also discretized. From (2.57) yields, the equation for \(H\) is obtained as
\[
H_\eta^\eta(i,j,k) = \frac{\mu^{\eta-1}(i,j,k)H_x^{\eta-1}(i,j,k)}{\mu^\eta(i,j,k)} - \frac{\Delta t}{\mu^\eta(i,j,k)} \left[ \frac{E_\eta^\eta(i,j,k) - E_\eta^\eta(i,j-1,k)}{\Delta y} - \frac{E_y^\eta(i,j,k) - E_y^\eta(i,j,k-1)}{\Delta z} \right]
\]
\[
[i_{min} + 1 \leq i \leq i_{max} - 1, j_{min} + 1 \leq j \leq j_{max}, k_{min} + 1 \leq k \leq k_{max}]
\]
Likewise, (2.56) can be simplified to get the equation for the \(E\)
\[
E_x^{\eta+1}(i,j,k) = \frac{\epsilon^\eta(i,j,k)E_x^\eta(i,j,k)}{\epsilon^{\eta+1}(i,j,k)} + \frac{\Delta t}{\epsilon^{\eta+1}(i,j,k)} \left[ \frac{H_x^\eta(i,j+1,k) - H_x^\eta(i,j,k)}{\Delta y} - \frac{H_y^\eta(i,j,k+1) - H_y^\eta(i,j,k)}{\Delta z} \right]
\]
\[
[i_{min} + 1 \leq i \leq i_{max}, j_{min} \leq j \leq j_{max}, k_{min} \leq k \leq k_{max}]
\]

2.3.2.1 Frequency dependency (FD)-FDTD

In FDTD equations (2.58) and (2.59), the media parameters are frequency-independent. However in practical simulations, the media parameters depend on frequency. We use the Debye model to represent frequency dependent material as it is simple in implementation and very widely used. In Debye media, the
relative permittivity is obtained as

\[ \epsilon_r = \epsilon_\infty + \frac{\epsilon_S - \epsilon_\infty}{1 + j\omega \tau_D} - j\frac{\sigma}{\omega \epsilon_0} \]  

(2.60)

where \( j = \sqrt{-1} \) and \( \sigma, \epsilon_S, \epsilon_\infty \) and \( \tau_D \) are the conductivity, static relative permittivity, optical relative permittivity and relaxation time, respectively. We now re-write (2.42) as

\[
\begin{align*}
\frac{H_{z}^{\eta+1}(i,j+1,k) - H_{z}^{\eta}(i,j,k)}{\Delta y} - \frac{H_{z}^{\eta+1}(i,j,k+1) - H_{z}^{\eta}(i,j,k)}{\Delta z} = & \\
\frac{D_{x}^{\eta+1}(i,j,k) - D_{x}^{\eta}(i,j,k)}{\Delta t} + & \\
\Delta t \left[ \frac{H_{z}^{\eta+1}(i,j+1,k) - H_{z}^{\eta}(i,j,k)}{\Delta y} - \frac{H_{z}^{\eta+1}(i,j,k+1) - H_{z}^{\eta}(i,j,k)}{\Delta z} \right] + & \\
D_{x}^{\eta}(i,j,k).
\end{align*}
\]  

(2.61)

Similarly \( D_{y}^{\eta+1}(i,j,k) \) and \( D_{z}^{\eta+1}(i,j,k) \) are derived, After deriving \( \mathbf{D} \), \( \mathbf{E} \) can also be derived and is found to be

\[
E_{x}^{\eta+1}(i,j,k) = \frac{-\sigma (i,j,k)(\Delta t)^2 + 4\epsilon_0 \epsilon_\infty \tau_D + 2(\epsilon_0 \epsilon_S + \sigma (i,j,k) \tau_D)\Delta t}{2\epsilon_0 \epsilon_\infty \tau_D + 2(\epsilon_0 \epsilon_S + \sigma (i,j,k) \tau_D)\Delta t + \sigma (i,j,k)(\Delta t)^2} E_{x}^{\eta-1}(i,j,k) - \frac{2\epsilon_0 \epsilon_\infty \tau_D}{2(\Delta t + \tau_D)} E_{x}^{\eta}(i,j,k) + \frac{2\epsilon_0 \epsilon_\infty \tau_D + 2(\epsilon_0 \epsilon_S + \sigma (i,j,k) \tau_D)\Delta t + \sigma (i,j,k)(\Delta t)^2}{2\Delta t + 4\tau_D} D_{x}^{\eta+1}(i,j,k) - \frac{2\epsilon_0 \epsilon_\infty \tau_D + 2(\epsilon_0 \epsilon_S + \sigma (i,j,k) \tau_D)\Delta t + \sigma (i,j,k)(\Delta t)^2}{2\tau_D} D_{x}^{\eta}(i,j,k) + \frac{2\epsilon_0 \epsilon_\infty \tau_D + 2(\epsilon_0 \epsilon_S + \sigma (i,j,k) \tau_D)\Delta t + \sigma (i,j,k)(\Delta t)^2}{2\tau_D} D_{x}^{\eta+1}(i,j,k).
\]  

(2.62)

Similarly \( E_{y}^{\eta+1}(i,j,k) \) and \( E_{z}^{\eta+1}(i,j,k) \) are derived.

2.3.3 PML absorbing boundary condition

As it is impossible to simulate infinite space, absorbing boundary conditions (ABC) have to be used. The perfectly matched layer (PML) ABC, introduced by Jean-Pierre Bérenger in 1994 [91], bounds the FDTD space with an absorbing material medium. We use the complex frequency shifted (CFS) PML ABC [92].
to ensure that the reflections from the boundaries of the FDTD space are near zero so that the reflections from the point-like scatterers are not interfered with.

## 2.4 Summary

This chapter has reviewed the theory behind the TR method. We explained how TR acts as a time and space correlator. The way TR utilises multipath signals to improve resolution was also described. Furthermore, the DORT and MUSIC TR methods were also detailed. The implementation and advantages of the DORT and MUSIC methods were also presented. We also summarised the FDTD method which we use to simulate the scenarios considered in this dissertation. In the next chapter, we utilise the DORT and MUSIC methods to image targets in simple scenarios and display their current limitations.
Chapter 3

Imaging using the DORT and MUSIC methods

In Chapter 2, we detailed the TR-based methods and discussed their application for imaging discrete targets. In this chapter, we investigate the ability of the DORT and MUSIC TR Imaging methods to locate discrete isotropic targets in a medium. Here, we study the singular value distribution as well as the singular vectors when locating a single target in a medium. We also consider a scenario with two targets, displaying the strengths of each method. Finally, we investigate the problems that arise for the TR methods in locating targets when there is an obstructing extended scatterer such as a wall in the medium.

3.1 FDTD scenario with one PEC sphere scatterer

In Figure 3.1, we have a Perfect Electric Conductor (PEC) sphere (scatterer), of radius 4.8 mm (approximately λ/17), located in a homogeneous medium (free-space), where λ is the wavelength of the centre frequency (ωc) of the source signal, 3.6 GHz. An FDTD space of 500×300 cells was used with sampling time Δt = 3 ps and cell size 0.0016 m. A 10 cell PML layer was used for the FDTD space boundaries. The FDTD parameters for the PEC sphere are σ = 10^7 S/m, ε_s = 1, ε_∞ = 1 and τ_D = 0 s. The aim is to locate the scatterer using TR imaging methods assuming that the location of the scatterer and the actual number of scatterers in a two dimensional FDTD space are unknown. The N = 13 TRA antennas are 30
Figure 3.1: The geometry of the FDTD scenario, where × represents the TRA antennas.
cells (0.048 m, $\lambda/1.7$) apart from each other giving an aperture of 0.576 m, 6.9$\lambda$. The source signal, which is a first derivative of the Gaussian pulse, is transmitted from each antennas of the TRA, one at a time. Figure 3.2 shows the source signal transmitted ($\Upsilon = 0.8$ ns) and Figure 3.3 displays its normalised frequency spectrum where we observe that the centre frequency is $\omega_c = 3.6$ GHz. The pulse width is defined as $\Upsilon = t_2 - t_1$ where $t_2$ and $t_1$ satisfy $\mathcal{H}(S(t_1)) = \mathcal{H}(S(t_2)) = 0.1$ and $\mathcal{H}$ means the normalised Hilbert transform of the source signal $S(t)$ with respect to time $t$. Figure 3.3 also shows the frequency of interest which is defined as $\omega_1 \leq \omega \leq \omega_2$ where $\omega_1$ and $\omega_2$ satisfy $S(\omega_1) = S(\omega_2) = 0.1$ and $S(\omega)$ is the normalised spectrum of the excitation signal. Therefore Figure 3.3 gives $\omega_1 = 0.6$ and $\omega_2 = 8.2$. We simulate each transmission for 2000 timesteps (6 ns) and obtain the MDM using the TRA consisting of a set of 13 antennas. In the experiment, actual antennas are not modelled but instead soft point sources [93] and observation points were used to mimic an antenna. The soft source ensures that no extra scattering occurs at the location of the source during simulation, allowing only the source signal and reflected signal to be observed. For each transmission, the propagation of the signal in the FDTD space is observed at the neighbouring cells of all the antenna locations. Ideally, the observations should be done at the exact location of the transmitting antennas. However we observe at the neighbouring cells (2 cells away) to avoid inaccuracies that could occur due to the soft source implementation in FDTD space.

By observing the propagating signal very close to the antenna locations, we obtain $\hat{E}(i_r, j_r)_s(t)$ which gives the Green’s response function for the scenario containing the target that connects the transmitting antenna located at $\tau_s$ to the receiving antenna location $\tau_r$ as

$$\hat{G}(\tau_r, \tau_s, t) = \hat{E}(i_r, j_r)_s(t)$$

(3.1)

where $(i_r, j_r)$ is the observation location of the receiving antenna in the FDTD space and $\tau = xi + yj$. $i$ and $j$ are unit vectors in the $x$ and $y$ directions, respectively. Therefore $\hat{G}(\tau_r, \tau_s, t)$ contains the line-of-sight signal and the signal reflected from the scatterer.

Figure 3.4 shows a plot of $\hat{G}(\tau_1, \tau_1, t)$ for the scenario being considered containing the line-of-sight source signal as well as the reflected signal from the scatterer. However, since the elements of the MDM $k_{sr}(t)$ only require the reflected waves...
Figure 3.2: The source signal transmitted from an antenna location

Figure 3.3: The normalised frequency spectrum of the excitation pulse as shown in Figure 3.2
from the medium, the known line-of-sight signal needs to be removed. We accomplish this in our simulations by running a free-space scenario similar to the scenario shown in Figure 3.1 but without the scatterers and observe at the antenna locations to get $G(\vec{r}_r, \vec{r}_s, t)$ which contains the line-of-sight signal alone. By subtracting $G(\vec{r}_r, \vec{r}_s, t)$ from $\hat{G}(\vec{r}_r, \vec{r}_s, t)$, we obtain $k_{sr}(t)$ which retains the reflections alone in the MDM $K(t)$. It is also important to note that $G(\vec{r}_r, \vec{r}, t)$ obtained from the free-space scenario gives the background medium’s steering vectors $\mathbf{g}(\vec{r}, t)$ required for the DORT and MUSIC methods since we assume the material of the scatterer is unknown. Ideally, the steering vectors should be obtained by transmitting a signal from every single point in the space being considered and observing the signal propagation from each transmission at the TRA antennas. However, due to reciprocity property, the reverse can be done. That is, a signal was transmitted from each TRA antenna element and the propagation observed at every single point in the scenario space to obtain the steering vector as

$$\mathbf{g}(\vec{r}, t) = \left[ G(\vec{r}_1, \vec{r}, t), \ldots, G(\vec{r}_N, \vec{r}, t) \right]^T$$ (3.2)
3.1.1 Analysing the singular value distribution and singular vectors

We take the FFT of $K(t)$ to obtain $K(\omega)$ and the SVD of the $K(\omega)$ gives the singular values as well as the singular vectors of the MDM. We analyse the singular values and the singular vectors of the MDM at centre frequency ($\omega_c = 3.6$ GHz) to estimate the number of targets and the location of the targets relative to the antenna. The singular value distribution shows the singular values across the frequency bandwidth. By plotting the singular value distribution, we may estimate the number of targets in the medium simulated. The plot of the singular value distribution obtained for the scenario in Figure 3.1 is shown in Figure 3.5 which shows the singular values across frequency. Figure 3.6 shows the singular values at centre frequency $\omega_c = 3.6$ GHz. Figure 3.7 shows the ratio of consecutive singular values at $\omega_c$. In Figure 3.5, it can be seen that the second singular values are considerably lower (less than 10%) than the first singular values indicating that there is only one discrete scatterer in the medium. This is also true at the centre frequency of operation ($\omega_c = 3.6$ GHz). It is for this reason that the signal to null subspace threshold is typical said to be 10% of the first singular values.
CHAPTER 3. IMAGING USING THE DORT AND MUSIC METHODS

Figure 3.6: The singular values at centre frequency $\omega_c$

Figure 3.7: The ratio of consecutive singular values at centre frequency $\omega_c$
which conveniently puts the lower singular values in the null subspace. We also observe in Figure 3.7 that the ratio of consecutive singular values is highest when these values belong to different subspaces (signal and null subspace). The highest singular value (also called the first singular value) represents the strongest and only scatterer in the FDTD space. A signal to null subspace threshold of 10% is used as shown in Figure 3.5.

Figure 3.8 shows the magnitude and phase distributions against the TRA antennas for the first three singular vectors at \( \omega_c = 3.6 \text{ GHz} \). By observing the magnitude distribution of the \( u_1(\omega_c) \), the location of the PEC scatterer relative to the TRA antennas can be determined. When the magnitude distribution peaks smoothly, the relative location of the scatterer to the TRA antenna may be estimated \[81, 72, 71\]. For instance, the peak of the magnitude distribution of the first singular vector shows the relative location of the scatterer to the TRA antennas. The scatterer location is estimated to be directly in line with the centre (7th) antenna as is the case in the scenario in Figure 3.1. It is important to note that the 7th TRA antenna has the highest magnitude since it is closest to the target. The phase of the first singular vector is also at its highest at the 7th antenna of the TRA and peaks smoothly at this antenna. However, the other singular vectors have phase and magnitude distribution that are random and do not correlate to any particular TRA antenna as the corresponding singular values are in the null subspace and do not represent any scatterers.
CHAPTER 3. IMAGING USING THE DORT AND MUSIC METHODS

Figure 3.9: The CF-DORT image obtained using the first singular vector \((u_1(\omega_c))\) corresponding to the first singular value in the signal subspace, where \(\times\) represents the TRA antennas’ locations and \(\circ\) represents the target location.

### 3.1.2 The CF-DORT and UWB-DORT images

Section 3.1.1 shows that the PEC scatterer in the scenario in Figure 3.1 is represented by the first (highest) singular value and its corresponding singular vector. Hence, we use the first singular vector to create CF-DORT and UWB-DORT images. The distribution of \(D(\tau, \omega, n)\) with respect to locations \((\tau)\) gives an image that focuses on the location of the scatterer. In the case of Figure 3.5, \(N_t(\omega_c) = 1\).

Figure 3.9 and Figure 3.10 show the CF-DORT and UWB-DORT images, respectively, for the scenario in Figure 3.1. Both methods produce images which locate the PEC sphere in the FDTD space. It is also noticeable that the UWB-DORT image provides better (a more specific location of the target) spatial resolution than the CF-DORT image. It is also important to note the maximum magnitude of the CF-DORT image \(\max_{\tau} D(\tau, \omega_c, n)\) is lower than that of the UWB-DORT image \(\max_{\tau} D_{UWB}(\tau, n)\) as observed in Figure 3.9 and Figure 3.10. This is expected because the UWB-DORT method is obtained across multiple frequencies. However, the spatial resolution and accuracy of the images shown later in Section 3.1.4 are better metrics for comparing the TR methods as they compare the observable contrast between the magnitudes at the target location and free-space in a normalised manner. The images may also be compared visually since the colorbar effectively normalises images to its maximum magnitude.
CHAPTER 3. IMAGING USING THE DORT AND MUSIC METHODS

Figure 3.10: The UWB-DORT image obtained using the first singular vector \( \boldsymbol{u}_1(\omega) \) corresponding to the first singular values in the signal subspace, where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

Figure 3.11: The CF-DORT and UWB-DORT images obtained using the second singular vectors \( \boldsymbol{u}_2(\omega) \) corresponding to the second singular values in the signal subspace, where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.
Figure 3.12: The CF-DORT and UWB-DORT images obtained using the third singular vectors \( (u_3(\omega)) \) corresponding to the third singular values in the signal subspace, where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

Figure 3.11 shows the (a) CF-DORT and (b) UWB-DORT images obtained using the second singular vectors \( (u_2(\omega)) \) corresponding to the second singular values \( (\alpha_2(\omega)) \) in the signal subspace. Figure 3.12 shows the (a) CF-DORT and (b) UWB-DORT images obtained using the third singular vectors \( (u_3(\omega)) \) corresponding to the third singular values \( (\alpha_3(\omega)) \) in the signal subspace. By observing Figure 3.11 and Figure 3.12 it is observed that the DORT imaging method, whilst using the second and third singular vectors, fails to locate the PEC sphere in the scenario in Figure 3.1. Similar results were obtained using the singular vectors corresponding to \( \alpha_n(\omega) \) where \( n > 3 \). These singular vectors correspond to the singular values in the null subspace and do not represent any scatterers.

### 3.1.3 The CF-MUSIC and UWB-MUSIC images

In Section 3.1.2 we produced the DORT images for the scenario in Figure 3.1 with one PEC sphere in the FDTD space. In Section 2.2.2 we explained that the distribution of \( M(\tau, \omega) \) with respect to location \( (\tau) \) gives an image with focus on the location of the scatterer because the singular vectors in the null subspace are orthogonal to the steering vector at the scatterer location.

Figure 3.13 and Figure 3.14 show the CF-MUSIC and UWB-MUSIC images, respectively, for the scenario in Figure 3.1. The PEC sphere has been clearly located in the MUSIC images and with higher spatial resolution than the DORT
Figure 3.13: The CF-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where × represents the TRA antennas’ locations and ○ represents the target location.

Figure 3.14: The UWB-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where × represents the TRA antennas’ locations and ○ represents the target location.
images shown in Figure 3.9 and Figure 3.10. The MUSIC method also gives better contrast between the target location and free-space. We note that the maximum magnitude of the CF-MUSIC image \( \max_r M(r, \omega_c) \) is considerably higher than that of the UWB-MUSIC image \( \max_i M_{UWB}(i, j) \) as observed from Figure 3.13 and Figure 3.14. This is expected because the MUSIC methods uses the inverse of the inner products of the null subspace singular vectors and the steering vectors, and the UWB-MUSIC methods sums the inner products at multiple frequencies before obtaining the inverse. However we compare the performance of these methods in terms of spatial resolution and accuracy in a normalised manner in Section 3.1.4.

### 3.1.4 Comparisons between the DORT and MUSIC images

Figure 3.15 and Figure 3.16 show the cross range and range sections of the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC images. We easily observe that the MUSIC methods achieve images that give better contrast between the target location and other points in space, when compared to the DORT methods. Table 3.1 shows the cross range resolution, the range resolution and the accuracy of the each method. For analysis in this dissertation, we define the resolution as the full width at half maximum (FWHM) of the imaging functional across the section being considered. Additionally, the accuracy in metres is defined as the distance between the location of maximum of the imaging functional \( r_{\text{max}} \) and the actual location of the target \( r_o \) in space and is written as

\[
\varrho = |r_{\text{max}} - r_o|.
\]  

We note that the DORT and MUSIC images were plotted for every other point throughout this dissertation and thus the accuracy has an error of \( \Delta x \). (2.6) and (2.7) when \( \lambda = 0.083 \) m, \( a = 0.576 \) m, \( \ell = 0.16 \) m and \( \Upsilon = 0.8 \) ns gives the cross range resolution defined by the diffraction limit and the theoretical range resolution as 0.023 m and 0.12 m, respectively. Table 3.1 shows that the CF-MUSIC and UWB-MUSIC methods achieve perfect accuracy (0.0 m) in localisation of the target. The CF-DORT method displayed the worst accuracy (0.0544 m) in locating the target. The MUSIC methods also show clear advantage in the cross range
Figure 3.15: The cross section at $y = 0.28$ m in Figure 3.11, Figure 3.12, Figure 3.13 and Figure 3.14 obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. The target is located at $x = 0.4$ m and range resolution achieved, which beat both the expected cross range resolution defined by the diffraction limit and theoretical range resolution. However the achieved cross range of DORT methods were lower than the cross resolution defined by the diffraction limit. In particular, the DORT methods were unable to achieve range resolution within the space of the scenario as the imaging functions failed to fall below half of its maximum in the range section. We conclude that the MUSIC methods outperform the DORT methods in accuracy and spatial resolution for the scenario considered. In Section 3.2 we consider a scenario with 2 well-resolved targets.

### 3.2 FDTD scenario with two well-resolved PEC sphere scatterers

Figure 3.17 shows the geometry of the scenario with two PEC spheres (radius 4.8 mm) in a homogeneous medium (free-space). An FDTD space of 500×300 cells was used. We retain the FDTD parameters for the PEC sphere as $\sigma = 10^7$ S/m, $\varepsilon_s = 1$, $\varepsilon_\infty = 1$ and $\tau_D = 0$ s. As in Section 3.1 we transmit the source signal
Figure 3.16: The range at $x = 0.4$ m in Figure 3.11, Figure 3.12, Figure 3.13 and Figure 3.14 obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. The target is located at $y = 0.28$ m

<table>
<thead>
<tr>
<th>Method</th>
<th>Cross range resolution (m)</th>
<th>Range resolution (m)</th>
<th>Accuracy (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF-DORT</td>
<td>0.0961</td>
<td>N/A</td>
<td>0.0544</td>
</tr>
<tr>
<td>UWB-DORT</td>
<td>0.0500</td>
<td>N/A</td>
<td>0.0096</td>
</tr>
<tr>
<td>CF-MUSIC</td>
<td>0.0036</td>
<td>0.0061</td>
<td>0</td>
</tr>
<tr>
<td>UWB-MUSIC</td>
<td>0.0037</td>
<td>0.0057</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.1: The accuracy and resolution of the DORT and MUSIC methods compared. The cross range resolution defined by the diffraction limit and the theoretical range resolution are 0.023 m and 0.12 m, respectively.
Figure 3.17: The geometry of the FDTD scenario with two PEC spheres
Figure 3.18: The singular values distribution obtained after SVD with signal to null subspace ratio taken as 10% of the first singular value (shown in Figure 3.2), centred at 3.6 GHz (Figure 3.3), from 13 TRA antennas, one at a time to obtain $K(t)$.

The Fourier transform of all elements of $K(t)$ give $K(\omega)$ as shown in Section 3.1. The SVD of $K(\omega)$ gives the singular values $\alpha_n(\omega)$ and the corresponding singular vectors $u_n(\omega)$ which we use to plot the singular value distribution and the magnitude and phase of the singular vectors.

Figure 3.18 shows the singular value distribution obtained for the scenario with two targets as depicted in Figure 3.17. Figure 3.19 shows the singular values at centre frequency $\omega_c = 3.6$ GHz. By analysing the singular values of the MDM at centre frequency ($\omega_c = 3.6$ GHz), the number of targets is approximated. Figure 3.18 shows two singular values ($\alpha_1(\omega_c)$ and $\alpha_2(\omega_c)$) above the signal subspace to null subspace threshold at $\omega_c = 3.6$ GHz. The singular values below the threshold are at least 10% less than the highest $\alpha_1(\omega_c)$ and are said to be in the null subspace. Furthermore, we know that the targets are located in homogeneous media which means that there is no clutter signal represented by the singular values. Hence, the two singular values ($\alpha_1(\omega_c)$ and $\alpha_2(\omega_c)$) above the signal subspace to null subspace threshold represent two targets in the FDTD space.
Figure 3.19: The singular values at centre frequency $\omega_c$

Figure 3.20: The ratio of consecutive singular values at centre frequency $\omega_c$
Figure 3.21: The magnitude and phase of the first three singular vectors at centre frequency ($\omega_c = 3.6$ GHz)

Figure 3.20 shows the ratio of consecutive singular values at $\omega_c$. Figure 3.20 shows that the ratio of consecutive singular values is highest when these values belong to different subspaces (signal and null subspace). The phase and magnitude of the singular vectors are also plotted and observed to get approximate locations of the PEC spheres.

Figure 3.21 shows the magnitude and phase distributions against the TRA antennas for the first three singular vectors ($u_1(\omega_c)$, $u_2(\omega_c)$ and $u_3(\omega_c)$) at $\omega_c = 3.6$ GHz ($1 \leq n \leq 13$). By looking at the phase distribution of the $u_1(\omega_c)$, the location of the 1st PEC scatterer relative to the TRA antennas can be determined. The peak of the phase distribution of $u_1(\omega_c)$ shows the relative location of the scatterer to the TRA antenna. The first PEC scatterer location is estimated to be directly in line with the 10th antenna as is the case in the scenario in Figure 3.17. The magnitude of $u_1(\omega_c)$ is also at its highest at the 10th antenna of the TRA which confirms the presence of an target there. The phase distribution of the $u_2(\omega_c)$ peaks at the 1st TRA antenna. The magnitude of $u_2(\omega_c)$ is also at its highest at the 1st TRA antenna which confirms the location of the second PEC scatterer. However, $u_3(\omega_c)$ has phase and magnitude peaks that do not correlate to any particular TRA antenna as the corresponding singular value is in the null subspace and does not represent any scatterer. Similarly, all singular vectors corresponding to the singular values in the null subspace did not represent any scatterer.
Figure 3.22: The CF-DORT image obtained using the first singular vectors ($u_1(\omega)$) corresponding to the first singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the targets’ locations.

### 3.2.1 The CF-DORT and UWB-DORT images

By plotting $D(\tau, \omega_c, n)$ in (2.34), where $\omega_c = 3.6 \text{ GHz}$ and $N_t(\omega_c) = 2$, we obtain the CF-DORT image for the scenario in Figure 3.17. Similarly, we obtain the UWB-DORT image by plotting $D_{UWB}(\tau, n)$ in (2.36), where $\Omega$ is the bandwidth of operation which is from 0.6 GHz to 8.2 GHz, for the scenario in Figure 3.17.

Figure 3.22 and Figure 3.23 show the images obtained for the scenario in Figure 3.17 using $u_1(\omega)$ for the CF-DORT and UWB-DORT methods respectively. Similarly Figure 3.24 and Figure 3.25 show the images obtained for the scenario in Figure 3.17 using $u_2(\omega)$ for the CF-DORT and UWB-DORT methods respectively. Figure 3.26 shows the CF-DORT and UWB-DORT (from 0.6 GHz to 8.2 GHz) images obtained for the scenario in Figure 3.17 using $u_3(\omega)$ corresponding to the third singular values. Hence by using $u_1(\omega)$ and $u_2(\omega)$, the distribution of $D(\tau, \omega, n)$ with respect to locations $(i, j)$ gives an image with focus on the first and second scatterer respectively. We confirm that the first scatterer, which is located closer to the TRA antennas than the second scatterer, is represented by
the 1st singular value \( u_1(\omega) \). Furthermore, the images produced with the UWB-DORT method have a higher spatial resolution than the images produced using the CF-DORT method as was the case in Section 3.1. When the third singular vector \( u_3(\omega) \) was used to produce the DORT images, there was no focus on either of the PEC targets because \( u_3(\omega) \) is in the null subspace.

### 3.2.2 The CF-MUSIC and UWB-MUSIC images

By plotting \( M(\tau, \omega_c) \) in (2.39), where \( \omega_c = 3.6 \text{ GHz} \) and \( N_t(\omega_c) = 2 \), we obtain the CF-MUSIC image for the scenario in Figure 3.17. Similarly, we obtain the UWB-MUSIC image by plotting \( M_{UWB}(i, j) \) in (2.40), where \( \Omega \) is the bandwidth of operation which is from 0.6 GHz to 8.2 GHz, for the scenario in Figure 3.17.

Figure 3.27 and Figure 3.28 show the CF-MUSIC and UWB-MUSIC images, respectively, for the scenario in Figure 3.17. The PEC spheres have been clearly located by the MUSIC images and with narrower FWHM than the DORT images shown in Figure 3.22 and Figure 3.24. The MUSIC method also gives better contrast, between the target location and free space, than the DORT method.
Figure 3.24: The CF-DORT image obtained using the second singular vectors $(u_2(\omega))$ corresponding to the second singular values in the signal subspace, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target(s) location(s).
Figure 3.25: The UWB-DORT image obtained using the second singular vectors \((u_2(\omega))\) corresponding to the second singular values in the signal subspace, where \(\times\) represents the TRA antennas’ locations and \(\circ\) represents the target(s) location(s).

Figure 3.26: The CF-DORT and UWB-DORT images obtained using the third singular vectors \((u_3(\omega))\) corresponding to the third singular values in the signal subspace, where \(\times\) represents the TRA antennas’ locations and \(\circ\) represents the target(s) location(s).
Figure 3.27: The CF-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.

Figure 3.28: The UWB-MUSIC image obtained using the singular vectors corresponding to the singular values in the null subspace, where \( \times \) represents the TRA antennas’ locations and \( \circ \) represents the target location.
CHAPTER 3. IMAGING USING THE DORT AND MUSIC METHODS

<table>
<thead>
<tr>
<th>Method</th>
<th>Target 1</th>
<th>Target 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cross range resolution (m)</td>
<td>Range resolution (m)</td>
</tr>
<tr>
<td>CF-DORT</td>
<td>0.0864</td>
<td>0.2688</td>
</tr>
<tr>
<td>UWB-DORT</td>
<td>0.0512</td>
<td>0.2224</td>
</tr>
<tr>
<td>CF-MUSIC</td>
<td>0.0040</td>
<td>0.0048</td>
</tr>
<tr>
<td>UWB-MUSIC</td>
<td>0.0049</td>
<td>0.0064</td>
</tr>
</tbody>
</table>

Table 3.2: The resolution of the DORT and MUSIC images compared. The cross range resolution defined by the diffraction limit is 0.012 m for target 1 and 0.023 m for target 2. The theoretical range resolution is 0.12 m for both targets.

3.2.3 Comparisons between the DORT and MUSIC images

Figure 3.29 and Figure 3.30 show the cross range (at $y = 0.2$ m) and range sections (at $x = 0.544$ m) respectively, of the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC images. These figures display the contrast between target and free-space, the DORT and MUSIC methods achieve for target 1. Similarly, for target 2, Figure 3.31 and Figure 3.32 show the cross range (at $y = 0.28$ m) and range sections (at $x = 0.112$ m) respectively, of the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC images. Table 3.2 displays the cross range and range resolutions achieved for the two targets by each of the TR-based methods considered. Consistent with Section 3.1, we observe that the MUSIC methods outperform the DORT methods in terms of cross range and range resolution. Although the DORT methods produce individual images for the two targets in the scenario, it achieves wider FWHM than the MUSIC methods. The cross range and range resolution achieved by MUSIC methods both beat the expected cross range resolution defined by the diffraction limit and theoretical range resolution, respectively.
Figure 3.29: The cross section of Figure 3.22, Figure 3.23, Figure 3.27 and Figure 3.28 of target 1 located at \( y = 0.2 \) m obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 1 is located at \( x = 0.544 \) m.

Figure 3.30: The range section of Figure 3.22, Figure 3.23, Figure 3.27 and Figure 3.28 of target 1 located at \( x = 0.544 \) m obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 1 is located at \( y = 0.2 \) m.
Figure 3.31: The cross section of Figure 3.24, Figure 3.25, Figure 3.27 and Figure 3.28 of target 2 located at \( y = 0.28 \) m obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 2 is located at \( x = 0.112 \) m.

Figure 3.32: The range section of Figure 3.24, Figure 3.25, Figure 3.27 and Figure 3.28 of target 2 located at \( x = 0.112 \) m obtained from the CF-DORT, UWB-DORT, CF-MUSIC and UWB-MUSIC methods respectively. Target 2 is located at \( y = 0.28 \) m.
Sampling time ($\Delta t$) = 9.6 ps
Cell size ($\Delta x$) = 0.005 m

Central wavelength ($\lambda$) = 0.15 m

Figure 3.33: The geometry of the FDTD scenario with one target behind a brick wall
3.3 FDTD scenario with one PEC sphere scatterer hidden behind a brick wall

In Section 3.1 and Section 3.2 the ability of the TR-based DORT and MUSIC methods to image discrete targets in free-space was shown. However the performance of these methods are known to deteriorate dramatically when there is an unknown obstruction, typically an extended scatterer, in the medium. In this section, we investigate the problems that arise when the target targets are located behind a unknown brick wall as is common in TWI applications.

Figure 3.33 shows the geometry of the scenario with one PEC sphere of radius 15 mm (approximately $\lambda/10$) in a homogeneous medium (free-space) hidden behind a brick wall. An FDTD space of $300 \times 300$ cells was used. The FDTD parameters for the PEC sphere are $\sigma = 10^7$ S/m, $\epsilon_S = 1$, $\epsilon_\infty = 1$ and $\tau_D = 0$ s. The FDTD parameters for the brick wall are set to $\sigma = 0.02$ S/m, $\epsilon_S = 3.8$, $\epsilon_\infty = 3.8$ and $\tau_D = 0$ s as obtained from [95]. Figure 3.34 shows the source signal. In Figure 3.35 the normalised frequency spectrum of the signal in Figure 3.34 is shown. From Figure 3.35, we obtain $\omega_1 = 0.3$ GHz and $\omega_2 = 4.8$ GHz. We note that the FDTD cell size ($\Delta x = 0.005$ m) and sampling time ($\Delta t = 9.6$ ps) were increased from the previous scenarios, as observed in Figure 3.33, to reduce computational resources and runtime. Furthermore $\omega_2 = 4.8$ GHz gives the minimum wavelength of interest $\lambda_{min} = 0.0625$ m. Since $\lambda_{min}/10 = 0.00625$ m and $\lambda_{min}/10 \geq \Delta x$ these settings retain accuracy in the FDTD simulations. These settings were utilised for the rest of the scenarios considered in this dissertation.

The aim of this experiment is to locate the target, assuming its location in a two dimensional FDTD space is unknown. A first derivative Gaussian pulse, with a centre frequency of $\omega_c = 2$ GHz, is transmitted from 13 TRA antennas, one at a time. The TRA antennas were parallel to the wall and 10 cells (0.05 m, $\lambda/3$) apart from each other giving an aperture of 0.6 m, 4$\lambda$. In this experiment, actual antennas are not modelled but instead soft point sources and observation points were used to mimic an antenna as we did in Section 3.1. We simulate each transmission for 2000 timesteps (19.2 ns). The scenario was set up to avoid overlapping between clutter signal and target signal ensuring the wall and the target are well-resolved from one another.

We observe the reflections from the scenario at each antenna to obtain the elements of the MDM $k_{s,r}(t)$. Figure 3.36 shows the reflection from a wall and
Figure 3.34: The source signal transmitted from an antenna location

Figure 3.35: The frequency spectrum of the excitation pulse Figure 3.34
CHAPTER 3. IMAGING USING THE DORT AND MUSIC METHODS

-0.002
-0.001
0
0.001
0
1.92
3.84
5.76
7.68
9.6
Clutter signal from wall
Target signal from PEC target
Time [ns]
$E_z$

Figure 3.36: The reflected signal at the first antenna from the wall and the target PEC target observed at the first antenna when a Gaussian pulse ($\Upsilon = 1.008 \text{ ns}$) was transmitted from the same antenna ($k_{1,1}(t)$) where which contains the clutter signal from the wall as well as the target signal. We take the FFT of all elements of $K(t)$, to obtain $K(\omega)$ and apply the SVD on $K(\omega)$, which gives the singular values $\alpha_n(\omega)$ and the corresponding singular vectors $u_n(\omega)$.

3.3.1 The DORT and MUSIC images

Figure 3.37 shows the singular values distribution obtained for the scenario with one PEC target hidden behind a brick wall as depicted in Figure 3.33. It is known that for a free-space scenario containing well-resolved isotropic scatterers, the number of the singular values in the signal subspace gives an approximation of the number of the well-resolved scatterers in the medium as shown in Section 3.1.1. However, the clutter signal from the brick wall dominates the MDM and the singular values in the signal subspace. Furthermore we observe in Figure 3.37 that the singular values from $\alpha_1(\omega_c)$ to $\alpha_{13}(\omega_c)$ are less separated than for the scenarios without the brick wall obstruction previously shown in Figure 3.6 and Figure 3.19.

Figure 3.39 shows the magnitude and phase distributions against the TRA
Figure 3.37: The singular values distribution obtained after SVD with signal to null subspace ratio taken as 10% of the first singular value

Figure 3.38: The singular values at centre frequency $\omega_c$
antennas of the first three singular vectors. The singular vectors have phase and magnitudes peaks that do not correlate to any particular TRA antenna. Hence, it is also difficult to estimate the location of the scatterer relative to the TRA antennas as we did in Section 3.1.1. The difficulty arises due to the clutter signal contained in the MDM.

Figure 3.39 and Figure 3.41 show the CF-DORT and UWB-DORT (from 0.3 to 4.8 GHz) images obtained for the scenario in Figure 3.33 using the first singular vector \( u_1(\omega) \) and the second singular vector \( u_2(\omega) \) of the full-MDM, respectively. Figure 3.42 shows the CF-MUSIC and UWB-MUSIC (from 0.3 to 4.8 GHz) images obtained for the scenario in Figure 3.33 using the full-MDM. We observe that the DORT and MUSIC TR imaging methods fail to locate target hidden behind the brick wall.

3.3.2 The wall effect on the TR-based methods

We observe in Figure 3.40, Figure 3.41 and Figure 3.42 that both the DORT and MUSIC methods fail to locate the target hidden behind the brick wall. The wall dominates the MDM’s singular space because it is an extended scatterer and is closest to the TRA. When there is prior knowledge of the scenario, the clutter signal can easily be eliminated through time-gating. However, for an unknown scenario, time-gating could lead to loss of information and incomplete imaging.
Figure 3.40: The CF-DORT and UWB-DORT images produced using the full-MDM with the first singular vector $u_1(\omega)$ where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location

Figure 3.41: The CF-DORT and UWB-DORT images produced using the full-MDM with the second singular vector $u_2(\omega)$ where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location
Figure 3.42: The CF-MUSIC and UWB-MUSIC images produced using the full-MDM, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

3.4 Summary

In this chapter, we displayed the ability of the DORT and MUSIC methods to image well-resolved discrete targets in free-space. It was shown that, although the DORT methods can selectively focus on individual targets, the MUSIC methods outperform the DORT methods in terms of accuracy and spatial resolution. The performance of the MUSIC methods were shown to beat the diffraction limit for applications in free-space imaging. However, the performance of the TR-based methods was shown to deteriorate when there is an obstructing extended scatterer in the scene as is common in TWI applications. The clutter signal dominates the MDM and focusing on the target locations becomes impossible. In the next chapter, we introduce the novel spatio-temporal windows based algorithm for TWI using the MUSIC method which requires no knowledge of the scenario of interest.
Chapter 4

The total sub-MDM algorithms

Chapter 3 demonstrated that the MUSIC methods can image discrete targets in free-space. The superiority of the MUSIC methods over the DORT methods, in terms of accuracy and spatial resolution were also discussed. However, the methods were shown to be ineffective at imaging a well-resolved target located behind an obstructing brick wall. In this chapter, we introduce our novel spatio-temporal windows-based algorithm that uses the MUSIC method to image discrete targets even in the presence of an unknown obstructing extended scatterers.

4.1 Time Reversal technique based on spatio-temporal windows

The presence of clutter signal from an obstruction between the antennas and the target in the MDM $K(t)$ creates difficult for the TR methods. We showed in Chapter 3 that predicting the number of targets in the FDTD space is not possible as the signal subspace singular values also represent the obstruction. Furthermore, the location of the scatterer relative to the TRA antenna is not easily estimated by using the phase and magnitude distribution of the singular vectors. The clutter signal from an extended obstructing scatterer dominates the MDM since it is closer to the TRA. In the MUSIC methods particularly, the images obtained focus on the location of the wall albeit with less contrast than is achieved when there were only discrete targets in the medium. Since the target signals are retained in the MDM, it can be extracted but when the clutter signal is unknown extracting the target signal is difficult. The total sub-MDM algorithm
involves segregating the signals of the MDM in both time and space. By using temporal and spatial windows, we separate the signals contained in the $K(t)$, to multiple sub-MDMs. The SVD of the sub-MDMs gives singular values and singular vectors which we use with the CF-MUSIC and UWB-MUSIC methods to create images. Hence, rather than the global information contained in the MDM, which we term full-MDM for clarity, we obtain localised information from various parts of the scattering medium in the sub-MDMs, which reduces the amount of clutter signal processed in each SVD. The sub-MDMs containing clutter signals produce images that tend to focus on different locations of the obstruction, with poor contrast and low maximum magnitude comparative to discrete scatterers, as it is typical with extended scatterers. However the sub-MDMs containing target signals all produce images that focus on the target location although the (cross) range resolution is deteriorated due to the limited information contained within each sub-MDM. Since the scenario is unknown, we simply add all the images obtained from each sub-MDM to obtain the total sub-MDM image that locates the discrete target which has be shown to be impossible with the full-MDM. When all the sub-MDM images are added up, the entire information of the full-MDM is used but the effect of the clutter signal is limited to individual sub-MDMs allowing the total sub MDM image to give a clearer picture of the scenario than is possible with the full-MDM.

4.1.1 Time-windowing

To create a time-window, a window function $W(t)$ is used. A window function is a mathematical function with a zero value outside some specified time interval. Therefore, when the window function is multiplied by a function, the product is also zero-valued outside the interval.

Figure 4.1 shows the Hanning window [96] and the raised cosine window [97]. The Hanning window and the raised cosine window are considered for the total sub-MDM algorithm simply because they are easy to implement. Figure 4.2 shows the frequency spectrum of the Hanning window and the raised cosine window. It is observed that the frequency spectrum of the Hanning window and raised cosine window are similar. However the Hanning window has higher bandwidth than the raised cosine window. Hence, the Hanning window can resolve more independent information in individual sub-MDMs than is possible with the raised cosine window. Therefore, we proceed with the Hanning window for our
CHAPTER 4. THE TOTAL SUB-MDM ALGORITHMS

Figure 4.1: The Hanning window and the raised cosine window

Figure 4.2: The normalised frequency spectrum of the Hanning window and the raised cosine shown Figure 4.1
algorithm. In the future, other time windows can be considered. The Hanning window is obtained by

$$W(t) = \begin{cases} 
0.5(1 - \cos\left(\frac{2\pi t}{P}\right)) & \text{for } 0 \leq t \leq P \\
0 & \text{elsewhere}
\end{cases}$$

where $P$ is the window interval. By time shifting the time-windows, we obtain time-windows to cover the maximum value of time for $K(t)$. Hence, the number of time-windows needed is

$$M = \left\lfloor \frac{T}{P \cdot 0.5} - 1 \right\rfloor$$

where $T$ is the maximum value of $t$ for $K(t)$ and $\lfloor \cdot \rfloor$ represents a floor function that maps a real number to the largest previous integer. The time shifted time-window obtained is

$$W_m(t) = \begin{cases} 
0.5(1 - \cos\left(\frac{2\pi (t-\tau_m)}{P}\right)) & \text{for } \frac{P(m-1)}{2} \leq t \leq P + \frac{P(m-1)}{2} \\
0 & \text{elsewhere}
\end{cases}$$

while

$$\tau_m = \frac{P(m - 1)}{2}$$

where $m$ is an integer that identifies a specific time-window from the total number of time-windows $M$ used and $\tau_m$ is the shift in time. In order to avoid loss of data in the MDM when the elements of the MDM $k_{s,r}(t)$ are multiplied by the time-windows, the time-windows are shifted in time from one another in a manner that ensures that the addition of the time-windows gives a magnitude of 1. Figure 4.3 shows the first four time-windows as well as their summation which yields a magnitude of 1 ensuring that no vital information is lost from the MDM. By multiplying the time-windows $W_m(t)$ by the elements of the MDM $k_{s,r}(t)$, one at
a time, we obtain $M$ sub-MDMs as

$$\mathbf{K}_m(t, P) = \begin{pmatrix}
  k_{1,1}(t) \cdot W_{m}(t) & \ldots & k_{1,N}(t) \cdot W_{m}(t) \\
  \vdots & \ddots & \vdots \\
  k_{N,1}(t) \cdot W_{m}(t) & \ldots & k_{N,N}(t) \cdot W_{m}(t)
\end{pmatrix}$$  \hspace{1cm} (4.5)

### 4.1.2 Space-windowing

We further segment the $\mathbf{K}_m(t, P)$ in space by selecting elements of $\mathbf{K}_m(t, P)$ obtained from a set of $N_s$ neighbouring antennas where $1 \leq N_s \leq N$, as illustrated in Figure 4.4. Each set of neighbouring antenna contains information as observed from its antennas. Hence, for $\mathbf{K}_m(t, P)$, we obtain the $l$th sub-MDM as

$$\mathbf{K}_{m,l}(t, P, N_s) = \begin{pmatrix}
  k_{l,1}(t)W_{m}(t) & \ldots & k_{l,L}(t)W_{m}(t) \\
  \vdots & \ddots & \vdots \\
  k_{L,1}(t)W_{m}(t) & \ldots & k_{L,L}(t)W_{m}(t)
\end{pmatrix}$$  \hspace{1cm} (4.6)
where \( L = N_s - 1 + l \) and \( 1 \leq l \leq N + 1 - N_s \). Although segmenting the TRA reduces the aperture of the each sub-MDMs, the total sub-MDM images preserves the original aperture since all the sub-MDMs are used. Furthermore, each sub-MDM is able to achieve greater influence on the resolution on targets closer to the location of its segregated antenna as the distance from targets is effectively reduced as illustrated in Figure 4.5 where \( \ell_e(l) \) is the effective distance between the centre of the \( l \)th set of segregated \( N_s \) antennas and the target.

4.1.3 Setting the window parameters

The effectiveness of the spatio-temporal window algorithm depends on the size of the time window \( P \) and space window \( N_s \). A very small time window (\( P \ll \) the excitation pulse width) means that \( K_{m,l}(t,P,N_s) \) does not provide enough signal diversity to obtain reasonable images. However if \( P \) is large enough for a single \( K_{m,l}(t,P,N_s) \) to contain the entire information of the full-MDM, then the algorithm reduces to the standard TR imaging problems. Assuming the excitation pulse width is \( \Upsilon \), the minimum size of the time-window is chosen to be

\[
P \geq \Upsilon. \tag{4.7}
\]

\( (4.7) \) ensures that a single time window can cover the initial excitation signal transmitted from the TRA antennas.

Furthermore the minimum value of \( N_s \) for each scenario is dictated by the signal to null subspace threshold obtained from the full-MDM. Since the MUSIC
Figure 4.5: An illustration of the reduction in effective distance $\ell_e(l)$ between the centre of the TRA antennas and target illustrated due to spatial windowing, where X represents the TRA antennas.
methods utilise the singular vectors in the null subspace, \( N_s \) should be greater than the number of singular values in the signal subspace \( N_t \) of the full-MDM to ensure that we retain noise in the sub-MDMs. Therefore

\[
N_s > N_t. \tag{4.8}
\]

### 4.1.4 The total sub-MUSIC imaging functional

The sub-MDMs are employed in the CF-MUSIC and UWB-MUSIC methods. By performing the FFT on the elements of sub-MDM \( K_{m,l}(t, P, N_s) \) we obtain \( K_{m,l}(\omega, P, N_s) \). Furthermore by performing the SVD on \( K_{m,l}(\omega, P, N_s) \), we obtain

\[
K_{m,l}(\omega, P, N_s) = U(\omega, m, l)A(\omega, m, l)V^\dagger(\omega, m, l) \tag{4.9}
\]

where \( A(\omega, m, l) \) are \( N_s \times N_s \) real diagonal matrices containing the singular values in descending order and, \( U(\omega, m, l) \) and \( V(\omega, m, l) \) are \( N_s \times N_s \) unitary matrices containing the left and right singular vectors of \( u_n(\omega, m, l) \) and \( v_n(\omega, m, l) \) for \( 1 \leq n \leq N_s \) respectively. We know \( u_n(\omega, m, l) = v_n^*(\omega, m, l) \) when \( K_{m,l}(\omega, P, N_s) \) is symmetric \[81\].

The singular vectors obtained from \( K_{m,l}(\omega, P, N_s) \) are used to create images of the scenario by applying the MUSIC methods. The MUSIC methods involve obtaining the scalar product of the singular vectors in the null subspace and the steering vectors \( g^\dagger(\tau, \omega) \). The ratio of consecutive singular values corresponding to singular vectors in the same subspace (either signal or null subspace) tends to be lower than the ratio of consecutive singular values whose corresponding singular vectors are in different subspaces. Hence, for \( K(\omega) \), we can say the singular vectors, corresponding to the first \( N_t(\omega) \) singular values, are in the signal subspace if \( N_t(\omega) \) satisfies

\[
\frac{\alpha_{N_t(\omega)}(\omega)}{\alpha_{N_t(\omega)+1}(\omega)} = \max_{1 \leq n \leq N-1} \left( \frac{\alpha_n(\omega)}{\alpha_{n+1}(\omega)} \right). \tag{4.10}
\]

We propose (4.10) to replace the arbitrary 10% threshold typically used for the SVD based methods. We note that the (4.10) is similar to the 10% threshold in free-space but allows signal and null subspaces to be distinguished in scenarios containing extended scatterers. Therefore, by using the null subspace
singu lar vectors from the sub-MDMs for the MUSIC methods we define the sub-
CF-MUSIC imaging functional as
\[ M(\tau, \omega_c, m, l, P, N_s) = \left[ \sum_{n=N_t(\omega)+1}^{N_s} g^\dagger(\tau, \omega_c) \cdot u_n(\omega_c, m, l) \right]^{-1}. \] (4.11)

Similarly, the sub-UWB-MUSIC imaging functional, using the null subspace sin-
gular vectors from the sub-MDMs, is written as
\[ M_{UWB}(\tau, m, l, P, N_s) = \left[ \int_{\Omega} \sum_{n=N_t(\omega)+1}^{N_s} g^\dagger(\tau, \omega) \cdot u_n(\omega, m, l) d\omega \right]^{-1} \] (4.12)

The sub-MUSIC images give focusing on locations whose scattering information
is contained within the corresponding sub-MDM of the singular vectors utilised.
When the obstruction is a discrete scatterer, sub-MDMs which contain informa-
tion about its scattering give singular vectors and hence sub-MUSIC images that
focus at varying locations. However sub-MDMs which contain mostly target sig-
nal tend to produce sub-MUSIC images all focusing at the target’s location. The
sub-MDMs which contain well-resolved target signal also produce images with
higher maximum magnitude than the sub-MDMs dominated by clutter. Addi-
tionally, sub-MUSIC images produced from sub-MDMs containing target signals
whose antennas are closest to a target give higher magnitudes on the said target
location than sub-MDMs whose antennas are further away. Therefore it is possi-
ble to obtain images that focus on the target location rather than the obstruction
by taking the summation of the sub-MUSIC images. We define the total sub-
CF-MUSIC imaging functional as the summation of all sub-CF-MUSIC imaging
functionals to obtain
\[ M_{\Gamma}(\tau, \omega_c, P, N_s) = \sum_{l=1}^{L} \sum_{m=1}^{M} M(\tau, \omega_c, m, l, P, N_s). \] (4.13)

Correspondingly, the summation of the sub-UWB-MUSIC imaging functionals
gives the total sub-UWB-MUSIC imaging functional as
\[ M_{\Gamma UWB}(\tau, P, N_s) = \sum_{l=1}^{L} \sum_{m=1}^{M} M_{UWB}(\tau, m, l, P, N_s). \] (4.14)
Since the MUSIC imaging functional give magnitudes which is positive, the summation of the sub-MUSIC images can be carried out. Additionally, the summation of the sub-MUSIC images means that the total sub-MUSIC imaging functionals utilises the entire information in the full-MDM. We observe the total sub-MUSIC imaging functionals for $\Upsilon \leq P \leq T$. When $P = \Upsilon$ the total sub-MUSIC imaging functional give images that focus on the less dominant targets in the scenario. Furthermore, increasing $P$ may increase the (cross) range resolution obtained for the targets, since the each sub-MDM contains more information, but only as long as the time-windows remain small enough to resolve target signal from clutter signal. However, when the size of the time window is large enough for a single sub-MDM to contain the entire information of the full-MDM as $P \approx T$, the algorithm reduces to typical TR and gives focusing around the dominant obstruction’s locations. Therefore, the total sub-MUSIC imaging functionals give images that focus on the target as long as $P$ is small enough resolve clutter signal from target signal at different sub-MDMs. The total sub-MUSIC imaging functionals may attain increased (cross) range resolution when $N_t < N_s < N$ in scenarios with multiple targets or scenarios where the target is not centralised relative to the TRA antennas.
4.2 The total sub-differential MDM algorithm

The differential TR technique [98][99] has been shown to be capable of locating and tracking well-resolved moving targets in the presence of stationary targets using the DORT method. The differential TR technique is based on obtaining the differential-MDM, which reject signals from stationary scatterers, to locate the moving targets. The study also illustrates how a time-averaged MDM can be used to locate stationary targets in the presence of moving targets. However it was shown previously in Chapter 3 that the MUSIC methods have advantages over the DORT methods in terms of accuracy and (cross) range resolution. Here, we utilise the differential-MDMs technique for the MUSIC method to propose the novel total sub-differential MDMs algorithm that images targets in the presence of clutter signals by obstructing structures. The total sub-differential MDMs merges the differential TR technique with the total sub-MDM algorithm introduced in this chapter. The clutter signal from an obstruction between the antennas and the targets is easily eliminated when the targets are moving. We probe the medium at $t_1$ and then at a later time $t_2$, where the targets has either moved or changed in size, to obtain the MDMs $K|_{t=t_1}$ and $K|_{t=t_2}$ respectively. We subtract $K|_{t=t_1}$ from $K|_{t=t_2}$ to obtain the full-differential MDM as

$$K_d = K|_{t=t_2} - K|_{t=t_1}. \quad (4.15)$$

By spatial windowing, we catalogue the signals contained in the full-differential MDM, $K_d(t)$, to multiple sub-MDMs. We window the full-differential MDM in space by selecting its elements obtained from a set of $N_s$ antennas where $N_t < N_s \leq N$. Hence, we obtain the $l$th sub-differential MDM as

$$K_{dl}(t, N_s) = \begin{pmatrix}
  k_{dl,1}(t) & \cdots & k_{dl,L}(t) \\
  \vdots & \ddots & \vdots \\
  k_{dl,L}(t) & \cdots & k_{dl,L,L}(t)
\end{pmatrix}, \quad (4.16)$$

where $L = N_s - 1 + l$ and $1 \leq l \leq N + 1 - N_s$. Note that temporal-windowing has not been included here for the total sub-differential MDMs algorithm. Temporal-windowing may be included for the total sub-differential MDMs algorithm for scenarios where that contain numerous moving targets or a moving obstruction that gives clutter in the full-differential MDM. However such scenarios were not
considered within the scope of this dissertation.

### 4.2.1 The total sub-differential MUSIC imaging functional

The SVD of $K_d(t, N_s)$ give the singular values and singular vectors which we utilise with the MUSIC method to create images of the scenario. We obtain the sub-differential CF-MUSIC imaging functional using the null subspace singular vectors from the sub-differential MDMs as

$$M_d(\tau, \omega_c, l, N_s) = \left[ \sum_{n=N_l(\omega)+1}^{N_s} g^\dagger(\tau, \omega_c) \cdot u_n(\omega_c, l) \right]^{-1}. \quad (4.17)$$

The sub-differential UWB-MUSIC imaging functional, using the null subspace singular vectors from the sub-MDMs, is written as

$$M_{d\text{UWB}}(\tau, l, N_s) = \left[ \int_{\Omega} \sum_{n=N_l(\omega)+1}^{N_s} g^\dagger(\tau, \omega) \cdot u_n(\omega, l) d\omega \right]^{-1}. \quad (4.18)$$

The summation of the sub-differential CF-MUSIC images gives the total sub-differential CF-MUSIC imaging functional as

$$M_{d\Gamma}(\tau, \omega_c, N_s) = \sum_{l=1}^{L} M_d(\tau, \omega_c, l, N_s). \quad (4.19)$$

Similarly, the total sub-differential UWB-MUSIC imaging functional is defined as

$$M_{d\Gamma\text{UWB}}(\tau, N_s) = \sum_{l=1}^{L} M_{d\text{UWB}}(\tau, l, N_s). \quad (4.20)$$

### 4.3 Summary

In this chapter, we introduced the novel total sub-MDM algorithm that uses the MUSIC method to image discrete targets even in the presence of an unknown obstructing extended scatterers. The total sub-MDM algorithm allows the MUSIC method to image discrete targets hidden behind unknown obstructing extended scatterers which was not previously possible. The total sub-MDM algorithm
Chapter 4. The Total Sub-MDM Algorithms

The algorithm utilizes all information contained in the full-MDM, hence, no information is lost. The algorithm utilizes spatio-temporal windows to segregate the information contained in the full-MDM into sub-MDMs. The inner-product of the null-subspace singular vectors and the steering vectors give the sub-MUSIC images. The summation of all the sub-MUSIC images give images that focus on the targets despite the obstruction as long as the time windows are short enough to resolve the target signal from clutter signal. Furthermore, the total sub-MDM algorithm is extended for applications involving moving targets. The novel total sub-differential MDMs algorithm is proposed which is capable of locating moving targets in the presence of stationary obstructing scatterers. In Chapter 5, we investigate the performance of the total sub-MDM algorithm proposed on scenario previously considered in Figure 3.33 for different window settings. Thereafter, we consider a scenario where the target signal and clutter signal are not well-resolved for all elements of the full-MDM. Lastly, a scenario containing two stationary targets was considered.
Chapter 5

TWI with the total sub-MDM algorithm

In Chapter 3, we implemented the DORT and MUSIC methods and showed they were capable of locating discrete targets in free-space. The imaging advantages of the MUSIC methods in terms of resolution and accuracy, over the DORT methods were observed. We observed that for a scenario containing an obstructing brick wall, the performance of the TR methods deteriorates due to the clutter signals dominating the full-MDM. In this chapter, we implement the total sub-MDM algorithm, introduced in Chapter 4, which utilises the sub-MDMs collectively to obtain a MUSIC image that gives a clearer image of the scenario than is possible using the full-MDM.

5.1 FDTD scenario with one well-resolved PEC sphere

We consider the scenario in Figure 3.33 and the full-MDM obtained in Section 3.3. The ratio of consecutive singular values at centre frequency $\omega_c$ of the full-MDM is shown in Figure 5.1 that gives $N_t(\omega_c) = 2$ for (4.10). We recall that the maximum time for the full-MDM is $T = 19.2$ ns. The full-MDM $K(t)$ is multiplied by the time windows with initial size of $P = \Upsilon = 1.008$ ns ($T/19$) to obtain $K_m(t, \Upsilon)$. We note that $K_m(t, \Upsilon)$ is equivalent to $K_{m,1}(t, \Upsilon, N)$ since no spatial windowing has been applied i.e. $N_s = N$ and $L = 1$. Figure 5.2 and Figure 5.3 illustrate the segregation of signals by the time windows on $k_{1,1}(t)$ and $k_{7,7}(t)$ respectively.
We observe that the time-of-arrival of the clutter signal is the same for $k_{1,1}(t)$ and $k_{7,7}(t)$. However, the time-of-arrival of the target signal differs. The target signal is received earlier in $k_{7,7}(t)$ than in $k_{1,1}(t)$ since the target is closer to the 7th antenna. It is observed that the windows align with the targets signals differently for different elements $k_{s,r}(t)$. We take the SVD of $K_{m,1}(\omega, \Upsilon, N)$ to gain an insight into segregation of information that occurs due to the time windows.

### 5.1.1 Isolation of target signal using the time-windows

Figure 5.4 and Figure 5.6 show the singular value distributions of $K_{1,1}(\omega, \Upsilon, N)$ and $K_{12,1}(\omega, \Upsilon, N)$, respectively. Similarly, Figure 5.5 and Figure 5.7 show the singular values at centre frequency $\omega_c$ of $K_{1,1}(\omega, \Upsilon, N)$ and $K_{12,1}(\omega, \Upsilon, N)$, respectively. The difference between the highest and lowest singular values of $K_{12,1}(\omega, \Upsilon, N)$ is greater than that of $K_{1,1}(\omega, \Upsilon, N)$ because $K_{12,1}(\omega, \Upsilon, N)$ contains well resolved target signal while $K_{1,1}(\omega, \Upsilon, N)$ is dominated by clutter from the wall.

Figure 5.8 and Figure 5.9 show the magnitude and phase distributions against the TRA antennas of the first three singular vectors of $K_{1,1}(\omega, \Upsilon, N)$ and $K_{12,1}(\omega, \Upsilon, N)$, respectively. In Section 3.1.1 it was shown that the location of a well-resolved target could be estimated by observing the phase and magnitude distributions of the singular vectors. In Figure 5.8 the singular vectors have
Figure 5.2: The signal observed at the first antenna in Figure 3.33 when a pulse was transmitted from the same antenna and line-of-sight signal is removed and the time-windows ($P = \Upsilon$) which sum to give a magnitude of 1 when $k_{1,1}(t)$ is normalised.

phase and magnitude distribution that are random and do not correlate to any particular TRA antenna. However, we observe in Figure 5.9 that the phase and magnitude distribution of the first singular vector $u_1(\omega_c, 12, 1)$ peaked smoothly at the 7th TRA antenna due to $W_{12}(t)$ isolating information from the PEC target alone thereby allowing $K_{12,1}(\omega, \Upsilon, N)$ to contain only information from a well resolved discrete target. However, the total sub-MDM algorithm does not select individual sub-MDMs based on the singular values or singular vectors since an unknown scenario might contain multiple targets that are not well-resolved. Figure 5.10 shows the total sub-CF-MUSIC and total sub-UWB-MUSIC image produced by the summation of all sub-MUSIC images for $K_{m,l}(\omega, \Upsilon, N)$. We note that steering vectors consisting of the free-space Green’s response function is utilised for the total sub-MUSIC images since we assume the scenario is unknown. In Section 5.1.2 we observe some of the individual sub-MUSIC images to display the focusing locations produce by sub-MDMs that contain clutter signal, and sub-MDMs that contain target signal.
Figure 5.3: The signal observed at the seventh antenna in Figure 3.33 when a pulse was transmitted from the same antenna and line-of-sight signal is removed and the time-windows ($P = \Upsilon$) which sum to give a magnitude of 1 when $k_{7,7}(t)$ is normalised.

Figure 5.4: The singular value distribution of $K_{1,1}(\omega, \Upsilon, N)$.
Figure 5.5: The singular values at centre frequency $\omega_c$ of $K_{1,1}(\omega, \Upsilon, N)$

Figure 5.6: The singular value distribution of $K_{12,1}(\omega, \Upsilon, N)$
Figure 5.7: The singular values at centre frequency $\omega_c$ of $K_{12,1}(\omega, \Upsilon, N)$

Figure 5.8: The magnitude and phase of the first three singular vectors at centre frequency ($\omega_c = 2$ GHz) of $K_{1,1}(\omega, \Upsilon, N)$
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

Figure 5.9: The magnitude and phase of the first three singular vectors at centre frequency \((\omega_c = 2 \text{ GHz})\) of \(K_{12,1}(\omega, \Upsilon, N)\)

Figure 5.10: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced for \(K_{m,t}(\omega, \Upsilon, N)\), where \(\times\) represents the TRA antennas’ locations and \(\circ\) represents the target location
5.1.2 The sub-MUSIC images

We recall that the sub-MDMs $K_{m,l}(\omega, P, N_s)$, which we use to produce the sub-CF-MUSIC and the sub-UWB-MUSIC images, is obtained by applying the temporal-windows and the spatial-windows to the full-MDM. We initially set $P = \Upsilon$ and $N_s = 7$ to obtain $K_{m,l}(\omega, \Upsilon, 7)$ and plot some of the sub-MUSIC images to observe the focusing locations produced by the individual sub-MDMs that contain clutter signal, and sub-MDMs that contain target signal. Plotting $M(\tau, \omega_c, m, l, P, N_s)$ in (4.11), where $\omega_c = 2$ GHz, $N_t(\omega_c) = 2$ gives sub-CF-MUSIC image for the scenario in Figure 3.33. Similarly we obtain the sub-UWB-MUSIC image by plotting $M_{UWB}(\tau, m, l, P, N_s)$ in (4.12). Figure 5.11, Figure 5.12, Figure 5.13 and Figure 5.14 show the sub-CF-MUSIC and sub-UWB-MUSIC images obtained for the scenario in Figure 3.33 using the sub-MDMs $K_{1,4}(\omega, \Upsilon, 7)$, $K_{12,4}(\omega, \Upsilon, 7)$, $K_{13,4}(\omega, \Upsilon, 7)$ and $K_{14,4}(\omega, \Upsilon, 7)$ respectively. Figure 5.11 shows that the sub-MUSIC images fail to locate the PEC target when $K_{1,4}(\omega, \Upsilon, 7)$ is utilised. However Figure 5.12, Figure 5.13 and Figure 5.14 show that we begin to locate the PEC target when sub-MUSIC imaging functional were implemented using $K_{12,4}(\omega, \Upsilon, 7)$, $K_{13,4}(\omega, \Upsilon, 7)$ and $K_{14,4}(\omega, \Upsilon, 7)$. The maximum magnitude of the sub-MUSIC image produced using $K_{1,4}(\omega, \Upsilon, 7)$ is less than the sub-MUSIC image produced using $K_{12,4}(\omega, \Upsilon, 7)$, $K_{13,4}(\omega, \Upsilon, 7)$ and $K_{14,4}(\omega, \Upsilon, 7)$ which contain target signal.

Figure 5.15 and Figure 5.16 show the sub-CF-MUSIC and sub-UWB-MUSIC (from 0.3 to 4.8 GHz) images obtained for the scenario in Figure 3.33 obtained using $K_{14,1}(\omega, \Upsilon, 7)$ and $K_{14,7}(\omega, \Upsilon, 7)$ respectively. The images obtained using $K_{14,1}(\omega, \Upsilon, 7)$ and $K_{14,7}(\omega, \Upsilon, 7)$ both locate the PEC target but differ due to the location of the space-windowed TRA antennas, giving different perspective. We observe that the individual sub-MUSIC images give poor range resolution in general due to the limited information contained in each sub-MDM when the size of the time-windows is relatively small ($P = \Upsilon$). In Section 5.1.3 the total sub-MUSIC images are shown and the size of the time and spatial windows are varied to observe the impact on the (cross) range resolution and accuracy of location.

5.1.3 The total sub-MUSIC images

In Section 5.1.2 the capability of individual sub-CF-MUSIC and sub-UWB-MUSIC imaging functional to locate the PEC targets was demonstrated. However
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

Figure 5.11: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{1,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

Figure 5.12: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{12,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.
Figure 5.13: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{13,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

Figure 5.14: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{14,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.
Figure 5.15: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{14,1}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

Figure 5.16: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{14,7}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.
Figure 5.17: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = \Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from sub-MDMs dominated by clutter signal produced focusing around the wall’s location. In a realistic scenario, the location and amount of the target is unknown. Consequently the individual sub-MDMs do not give conclusive detail about the scenario. Plotting $M_{\Gamma}(\tau, \omega_c, P, N_s)$ and $M_{\Gamma, UWB}(\tau, P, N_s)$ give the total sub-CF-MUSIC and total sub-UWB-MUSIC images. Figure 5.17 shows the total sub-CF-MUSIC and total sub-UWB-MUSIC images obtained using $M_{\Gamma}(\tau, \omega_c, \Upsilon, 7)$ and $M_{\Gamma, UWB}(\tau, \Upsilon, 7)$ respectively. It is observed that the total sub-CF-MUSIC and total sub-UWB-MUSIC imaging functional which utilise all sub-MDMs obtained give an image that locates the target in the scenario which is not possible using the full-MDM as is done in standard TR methods.

Figure 5.18 shows the cross section of Figure 3.42 and Figure 5.17 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = \Upsilon$ and $N_s = 7$) and the cross section of the image of a reference. The reference is obtained by implementing the full-MDM MUSIC methods on a scenario similar to Figure 3.33 without a wall. Hence, the reference gives the resolution that the MUSIC methods achieves for the target without the obstructing wall. Figure 5.19 shows the range section of Figure 3.42 and Figure 5.17 obtained using the full-MDM MUSIC method, the total sub-MUSIC method and the cross section of
Figure 5.18: The normalised cross section at $y = 1.15$ m in Figure 3.42 and Figure 5.17 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = 7$ and $N_s = 7$) and the reference. The target is located at $x = 0.75$ m.

Figure 5.19: The normalised range section at $x = 0.75$ m in Figure 3.42 and Figure 5.17 obtained using the full-MDM MUSIC method, the total sub-MUSIC method and the reference. The target is located at $y = 1.15$ m.
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

The image of the reference. Although the total sub-MUSIC method locates the target, the range resolution is poor when compared to the reference. However, increasing the size of the time windows allows each sub-MDM to contain more information in time and can increase the range resolution in particular, as long as the time window remains small enough to resolve target signals from clutter signals. Figure 5.20, Figure 5.21, Figure 5.22, and Figure 5.23 show the total sub-CF-MUSIC and total sub-UWB-MUSIC (from 0.3 to 4.8 GHz) images when $N_s = 7$ as we vary the size of the time-window from $P = 2\Upsilon$, $P = 4\Upsilon$, $P = 8\Upsilon$ to $P = 16\Upsilon$, respectively. The increase in the size of the time windows yields images with higher range resolution until the time-window becomes large enough to contain the entire information of the full-MDM in a single sub-MDM. We note that the images obtained when $\Upsilon \leq P \leq 8\Upsilon$ all focus around the target’s location while the images obtained when $P \geq 16\Upsilon$ focus around the location of the obstructing wall. Figure 5.24 and Figure 5.25 show the cross range resolution of the total sub-CF-MUSIC and total sub-UWB-MUSIC images at $y = 1.15$ m, respectively, varying $N_s$ and $P$. Figure 5.26 and Figure 5.27 show the range resolution of the total sub-CF-MUSIC and total sub-UWB-MUSIC images at $x = 0.75$ m, respectively, varying $N_s$ and $P$. In general, the (cross) range resolution obtained by the total sub-MUSIC images is improved as $P$ increases, until $P \geq 16\Upsilon$ where the (cross) range resolution sharply reduces. When $P \geq 16\Upsilon$ the time window is large enough for a single sub-MDM to contain the entire information of the full-MDM, thus the algorithm reduces to the standard TR imaging and we are unable to locate the target due to clutter. Particularly, increasing $P$ from $\Upsilon$ to $8\Upsilon$ improves the range resolution for both the total sub-CF-MUSIC and total sub-UWB-MUSIC images. Figure 5.28 and Figure 5.29 show the accuracy of the total sub-CF-MUSIC and total sub-UWB-MUSIC images respectively, in locating the PEC target, varying $N_s$ and $P$. On average, the highest accuracy was obtained between $2\Upsilon \leq P \leq 8\Upsilon$. Table 5.1 shows the highest resolutions and accuracy achieved for the total sub-MUSIC methods (2.6) and (2.7) when $\lambda = 0.15$ m, $a = 0.6$ m, $\ell = 0.8$ m and $\Upsilon = 1.008$ ns gives the cross range resolution defined by the diffraction limit and the theoretical range resolution as 0.2 m and 0.15 m, respectively. The total sub-MUSIC methods obtain images that locate the target and beat both the diffraction limit and the theoretical range resolution. The sub-UWB-MUSIC method also achieves higher accuracy than the sub-CF-MUSIC method. We note that inaccuracies occur because the wall is unknown
Total sub-CF-MUSIC & Total sub-UWB-MUSIC \\
Highest cross range resolution (m) & 0.014 ($P = 2\Upsilon$, $N_s = 13$) & 0.017 ($P = 4\Upsilon$, $N_s = 11$) \\
Highest range resolution (m) & 0.043 ($P = 4\Upsilon$, $N_s = 3$) & 0.058 ($P = 8\Upsilon$, $N_s = 11$) \\
Highest accuracy (m) & 0.02 ($P = 2\Upsilon$, $N_s = 13$) & 0.01 ($P = 2\Upsilon$, $N_s = 10$) \\

Table 5.1: The highest accuracy and resolution obtained for the total sub-MUSIC methods. The cross range resolution defined by the diffraction limit and the theoretical range resolution are 0.2 m and 0.15 m, respectively and free-space steering vectors are used.
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

Figure 5.20: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 2\gamma$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

Figure 5.21: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 4\gamma$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
Figure 5.22: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 8\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

Figure 5.23: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 16\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
Figure 5.24: The total sub-CF-MUSIC cross range resolution at $y = 1.15$ m, varying $N_s$ and $P$. 
For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.25: The total sub-UWB-MUSIC cross range resolution at $y = 1.15$ m, varying $N_s$ and $P$
For $N_s = 3$  

Figure 5.26: The total sub-CF-MUSIC range resolution at $x = 0.75$ m, varying $N_s$ and $P$. 
Figure 5.27: The total sub-UWB-MUSIC range resolution at $x = 0.75$ m, varying $N_s$ and $P$
Figure 5.28: The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying $N_s$ and $P$
For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.29: The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying $N_s$ and $P$
5.2 FDTD scenario with clutter and target signal overlapped

In Section 5.1 the total sub-MDM algorithm was applied for a scenario with one well resolved target hidden behind a brick wall. Here, we simulate a scenario with some overlap between clutter signal and target signal in the full-MDM, by having the PEC target closer to the brick wall as shown in Figure 5.30. We retain the parameters of the wall, radius of the target and the source signal detailed in Section 3.3. Figure 5.31 shows $k_{1,1}(t)$ and $k_{1,13}(t)$ from the full-MDM obtained from the scenario with clutter and target signal overlapped. In $k_{1,1}(t)$ the target and clutter signal are well-resolved while in $k_{1,13}(t)$ the target and clutter signal overlap. We note that in Figure 5.31 $W_7(t)$ resolves the target signal for $k_{1,1}(t)$.
Figure 5.31: The time-windows \((P = \Upsilon)\) which sum to give a magnitude of 1 and the normalised signals observed at (a) the first antenna when a pulse was transmitted from the same antenna \((k_{1,1}(t))\) and (b) the thirteenth antenna when a pulse was transmitted from the first antenna \((k_{1,13}(t))\) with the line-of-sight signal removed but contains clutter signal for \(k_{1,13}(t)\).

5.2.1 The sub-MUSIC images

The spatio-temporal windows, with initial size of \(P = \Upsilon\) and \(N_s = 7\), are applied to the full-MDM to obtain sub-MDMs. We plot some of the sub-MUSIC images to observe the focusing locations produced by the individual sub-MDMs. Figures 5.32, Figure 5.33, and Figure 5.34 show the sub-CF-MUSIC and sub-UWB-MUSIC images obtained for the scenario in Figure 3.33 using the sub-MDMs \(K_{1,4}(\omega, \Upsilon, 7)\), \(K_{2,4}(\omega, \Upsilon, 7)\) and \(K_{7,4}(\omega, \Upsilon, 7)\) respectively. We observe that sub-MUSIC images obtained using \(K_{1,4}(\omega, \Upsilon, 7)\) and \(K_{2,4}(\omega, \Upsilon, 7)\) fail to locate the target and focus around the wall’s location because these sub-MDMs are dominated by clutter signal. However sub-MUSIC images obtained using \(K_{7,4}(\omega, \Upsilon, 7)\) locate the target despite Figure 5.31 showing that not all elements of \(K_{7,4}(\omega, \Upsilon, 7)\) contain well-resolved target signal.
Figure 5.32: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{1,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

Figure 5.33: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{2,4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.
Figure 5.34: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{7_4}(\omega, \Upsilon, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

Figure 5.35: The CF-MUSIC and UWB-MUSIC images produced using the full-MDM, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
Figure 5.36: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = \Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

Figure 5.37: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 2\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
Figure 5.38: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 4\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

Figure 5.39: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 8\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
Figure 5.40: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 16\gamma$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

### 5.2.2 The total sub-MUSIC images

The CF-MUSIC and UWB-MUSIC images obtained for the scenario using the full-MDM are shown in Figure 5.35, Figure 5.36, Figure 5.37, Figure 5.38, Figure 5.39 and Figure 5.40 show the total sub-CF-MUSIC and total sub-UWB-MUSIC images obtained for the scenario, with clutter and target signal overlap, when $N_s = 7$ as we vary the size of the time-window from $P = 2\gamma$, $P = 4\gamma$, $P = 8\gamma$ to $P = 16\gamma$, respectively. The total sub-CF-MUSIC and total sub-UWB-MUSIC images locate the target for $\gamma \leq P \leq 8\gamma$. Figure 5.41 shows the cross section of Figure 5.35 and Figure 5.36 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = 8\gamma$ and $N_s = 7$) and the cross section of the image of a reference. The total sub-MUSIC functionals give images that accurately locate the target. Figure 5.42 shows the range section of Figure 5.35 and Figure 5.36 obtained using the full-MDM MUSIC method ($P = 8\gamma$ and $N_s = 7$), the total sub-MUSIC method and the range section of the image of a reference. We observe that the total sub-UWB-MUSIC image attains higher accuracy in locating the target than the total sub-CF-MUSIC image. Figure 5.43 and Figure 5.44 show the cross range resolution of the total sub-CF-MUSIC and total sub-UWB-MUSIC images at $y = 0.75$ m, respectively, varying $N_s$ and $P$.

Similarly, Figure 5.45 and Figure 5.46 show the range resolution of the total
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

<table>
<thead>
<tr>
<th></th>
<th>Total sub-CF-MUSIC</th>
<th>Total sub-UWB-MUSIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest cross range resolution (m)</td>
<td>$0.011 \ (P = 8\Upsilon, N_s = 3)$</td>
<td>$0.029 \ (P = 2\Upsilon, N_s = 11)$</td>
</tr>
<tr>
<td>Highest range resolution (m)</td>
<td>$0.012 \ (P = \Upsilon, N_s = 3)$</td>
<td>$0.15 \ (P = 4\Upsilon, N_s = 3)$</td>
</tr>
<tr>
<td>Highest accuracy (m)</td>
<td>$0.03 \ (P = \Upsilon, N_s = 4)$</td>
<td>$0.01 \ (P = 2\Upsilon, N_s = 10)$</td>
</tr>
</tbody>
</table>

Table 5.2: The highest accuracy and resolution obtained for the total sub-MUSIC methods. The cross range resolution defined by the diffraction limit and the theoretical range resolution are 0.1 m and 0.15 m, respectively.

sub-CF-MUSIC and total sub-UWB-MUSIC images at $x = 0.75$ m, respectively, varying $N_s$ and $P$. Predominantly, the cross range resolution obtained by the total sub-MUSIC images is maintained as $P$ increases, until $P \geq 16\Upsilon$ where the cross range resolution sharply reduces. When $P \geq 16\Upsilon$ the time window is large enough for a single sub-MDM to contain the entire information of the full-MDM, hence the algorithm reduces to the standard TR imaging and we are unable to locate the target due to clutter. Additionally we observe that the range resolution obtained by the total sub-MUSIC images improves as $P$ increases, until $P \geq 16\Upsilon$ where the range resolution sharply reduces. Figure 5.47 and Figure 5.48 show the accuracy of the total sub-CF-MUSIC and total sub-UWB-MUSIC images respectively, in locating the PEC target, varying $N_s$ and $P$. Table 5.2 shows the highest resolutions and accuracy achieved for the total sub-MUSIC methods. (2.6) and (2.7) when $\lambda = 0.15$ m, $a = 0.6$ m, $\ell = 0.4$ m and $\Upsilon = 1.008$ ns gives the cross range resolution defined by the diffraction limit and the theoretical range resolution as 0.1 m and 0.15 m, respectively. The total sub-MDM algorithm produces images that beat the cross range resolution defined by the diffraction limit. The sub-UWB-MUSIC method also achieves higher accuracy than the sub-CF-MUSIC method.
Figure 5.41: The normalised cross section at $y = 0.75$ m in Figure 5.35 and Figure 5.36 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = 8\,\mu$ and $N_s = 7$) and the reference. The target is located at $x = 0.75$ m.

Figure 5.42: The normalised cross section at $x = 0.75$ m in Figure 5.35 and Figure 5.36 obtained using the full-MDM MUSIC method, the total sub-MUSIC method ($P = 8\,\mu$ and $N_s = 7$) and the reference. The target is located at $y = 0.75$ m.
Figure 5.43: The total sub-CF-MUSIC cross range resolution at $y = 0.75$ m, varying $N_s$ and $P$.
Figure 5.44: The total sub-UWB-MUSIC cross range resolution at $y = 0.75$ m, varying $N_s$ and $P$. 

For $N_s = 3$ 

For $N_s = 5$ 

For $N_s = 7$ 

For $N_s = 9$ 

For $N_s = 11$ 

For $N_s = 13$ 

Cross range resolution (m) 

Cross range resolution (m) 

Cross range resolution (m) 

Cross range resolution (m) 

Cross range resolution (m) 

$P \times \Upsilon^{-1}$ 

$P \times \Upsilon^{-1}$ 

$P \times \Upsilon^{-1}$ 

$P \times \Upsilon^{-1}$ 

$P \times \Upsilon^{-1}$ 

$0.01 \ 0.1 \ 1 \ 10 \ 2 \ 4 \ 6 \ 8 \ 10 \ 12 \ 14 \ 16$
For $N_s = 3$

Range resolution (m)

For $N_s = 5$

Range resolution (m)

For $N_s = 7$

Range resolution (m)

For $N_s = 9$

Range resolution (m)

For $N_s = 11$

Range resolution (m)

For $N_s = 13$

Range resolution (m)

Figure 5.45: The total sub-CF-MUSIC range resolution at $x = 0.75$ m, varying $N_s$ and $P$
Figure 5.46: The total sub-UWB-MUSIC range resolution at $x = 0.75$ m, varying $N_s$ and $P$. 
For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.47: The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying $N_s$ and $P$
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.48: The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying $N_s$ and $P$
5.3 FDTD scenario with two targets

In this section we consider a scenario containing more than one target. Figure 5.49 shows the geometry of a scenario with two well-resolved PEC spheres in a homogeneous medium (free-space) hidden behind a brick wall. We retain the parameters of the wall and radius of the target detailed in Section 3.3. We obtain the full-MDM of the scenario and apply the total sub-MDM algorithm to obtain the total sub-MUSIC images. Figure 5.50 shows the signals received in element $k_{7,7}(t)$ of the full-MDM where we observe clutter signal as well as signals reflected by the two targets.
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

We apply the spatio-temporal windows, with initial size of $P = \Upsilon$ and $N_s = 7$, to the full-MDM to obtain sub-MDMs. Here, some of the sub-MUSIC images are plotted to observe the focusing locations produced by the individual sub-MDMs. Figure 5.51 and Figure 5.52 show the sub-CF-MUSIC and sub-UWB-MUSIC images obtained for the scenario in Figure 5.49 using the sub-MDMs $K_{12,1}(\omega, \Upsilon, 7)$ and $K_{14,7}(\omega, \Upsilon, 7)$ respectively. The images obtained using $K_{12,1}(\omega, \Upsilon, 7)$ locate target 1 while the images obtained using $K_{14,7}(\omega, \Upsilon, 7)$ locate target 2. $K_{12,1}(\omega, \Upsilon, 7)$ and $K_{14,7}(\omega, \Upsilon, 7)$ locate different targets because their time-windows and the location of their space-windowed TRA antennas differ, hence giving different perspectives of the scenario. We observe that the individual sub-MUSIC images give poor range resolution due to the limited information contained in each sub-MDM when the size of the time-windows is relatively small ($P = \Upsilon$). In Section 5.3.2 the total sub-MUSIC images are shown and the size of the time and spatial windows are varied to observe the impact on the (cross) range resolution.
Figure 5.51: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{12,1}(\omega, \Upsilon, 7)$, where \(\times\) represents the TRA antennas’ locations, \(\times\) represents the active antennas and \(\circ\) represents the target location.

Figure 5.52: The sub-CF-MUSIC and sub-UWB-MUSIC images obtained from $K_{14,7}(\omega, \Upsilon, 7)$, where \(\times\) represents the TRA antennas’ locations, \(\times\) represents the active antennas and \(\circ\) represents the target location.
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

Figure 5.53: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = \Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the location of targets.

5.3.2 The total sub-MUSIC images

Figure 5.53, Figure 5.54, Figure 5.55, Figure 5.56, and Figure 5.57 show the total sub-CF-MUSIC and total sub-UWB-MUSIC images obtained for the scenario in Figure 5.49 when $P = \Upsilon$, $P = 2\Upsilon$, $P = 4\Upsilon$, $P = 8\Upsilon$ and $P = 16\Upsilon$ respectively for $N_s = 7$. The images locate the targets when $\Upsilon \leq P \leq 8\Upsilon$ which is not achieved by the Full-MDM. However when $P = 16\Upsilon$ the image is unable to locate the targets and focusing is observed around the wall’s location.

Figure 5.58 and Figure 5.59 show the cross range resolution of the total sub-CF-MUSIC and total sub-UWB-MUSIC images, respectively, at $y = 1.15$ m where PEC target 1 is located, varying $N_s$ and $P$. Similarly Figure 5.60 and Figure 5.61 show the cross range resolution of the total sub-CF-MUSIC and total sub-UWB-MUSIC images, respectively, at $y = 1.25$ m where PEC target 2 is located, varying $N_s$ and $P$. Figure 5.62 and Figure 5.63 show the range resolution of the total sub-CF-MUSIC and total sub-UWB-MUSIC images, respectively, at $x = 0.6$ m where PEC target 1 is located, varying $N_s$ and $P$. Similarly Figure 5.64 and Figure 5.65 show the range resolution of the total sub-CF-MUSIC and total sub-UWB-MUSIC images, respectively, at $x = 1.0$ m where PEC target 2 is located, varying $N_s$ and $P$. Generally, it is observed that for both targets, increasing $P$ from $\Upsilon$ to $4\Upsilon$ improves both the cross range and range resolution.
Figure 5.54: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 2\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the location of targets.

Figure 5.55: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 4\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the location of targets.
CHAPTER 5. TWI WITH THE TOTAL SUB-MDM ALGORITHM

Figure 5.56: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 8\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the location of targets.

Figure 5.57: The total sub-CF-MUSIC and total sub-UWB-MUSIC images produced when $P = 16\Upsilon$ and $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the location of targets.
Table 5.3: The highest resolutions obtained for the total sub-MUSIC methods. The cross range resolution defined by the diffraction limit is 0.2 m for target 1 and 0.225 m for target 2. The theoretical range resolution is 0.15 m for both targets.

Table 5.3 shows the highest resolutions achieved by the total sub-MUSIC methods. The results demonstrate that we can locate the targets with increased resolution, when the time windowed sub-MDMs are further segmented by spatial windows, i.e. \( N_s < 13 \), despite the reduction of the TRA antenna aperture for each sub-MDM as a negative effect. The spatial windows further segregate the signals from different targets into different sub-MDMs, leading to images which more accurately represent the scenario.
Cross range resolution (m)

For \( N_s = 3 \)

For \( N_s = 5 \)

For \( N_s = 7 \)

For \( N_s = 9 \)

For \( N_s = 11 \)

For \( N_s = 13 \)

Figure 5.58: The total sub-CF-MUSIC cross range resolution at half maximum of \( \mathbf{M}_T(\tau, \omega_c, P, N_s) \) at \( y = 1.15 \) m, varying \( N_s \) and \( P \)
Chapter 5. TWI with the Total Sub-MDM Algorithm

Cross range resolution (m)

For $N_s = 3$

Cross range resolution (m)

For $N_s = 5$

Cross range resolution (m)

For $N_s = 7$

Cross range resolution (m)

For $N_s = 9$

Cross range resolution (m)

For $N_s = 11$

Cross range resolution (m)

For $N_s = 13$

Figure 5.59: The total sub-UWB-MUSIC cross range resolution at half maximum of $\mathbf{M}_{\text{uwb}}(\tau, P, N_s)$ at $y = 1.15$ m, varying $N_s$ and $P$
For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.60: The total sub-CF-MUSIC cross range resolution at half maximum of $M_\Gamma(\tau, \omega_c, P, N_s)$ at $y = 1.25$ m, varying $N_s$ and $P$. 

Cross range resolution (m)
For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.61: The total sub-UWB-MUSIC cross range resolution at half maximum of $M_{\Gamma_{\text{UWB}}}(\tau, P, N_s)$ at $y = 1.25$ m, varying $N_s$ and $P$
For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.62: The total sub-CF-MUSIC range resolution at half maximum of $M_{T}(\tau, \omega, P, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$
For $N_s = 3$
$P \times \Upsilon^{-1}$

Range resolution (m)

2  4  6  8  10  12 14  16

Figure 5.63: The total sub-UWB-MUSIC range resolution at half maximum of $M_{\Gamma,\text{UWB}}(r, P, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$
For $N_s = 3$

For $N_s = 5$

For $N_s = 7$

For $N_s = 9$

For $N_s = 11$

For $N_s = 13$

Figure 5.64: The total sub-CF-MUSIC range resolution at half maximum of $M_{\Gamma_{w,u}}(\tau, P, N_s)$ at $x = 1.0$ m, varying $N_s$ and $P$
Figure 5.65: The total sub-UWB-MUSIC range resolution at half maximum of $M_{\text{UWB}}(r, P, N_s)$ at $x = 1.0$ m, varying $N_s$ and $P$.
5.4 Summary

In this chapter, the total sub-MDM algorithm was implemented successfully to image a single target located behind an obstructing brick wall which the full-MDM MUSIC method failed to achieve in Section 3.3. The time-windows limit the clutter signals to individual sub-MDMs allowing the total sub-MUSIC images to focus on the target’s location. We showed that the total sub-MUSIC images focus on the target location as long as the time-windows are small enough to isolate the target signals from clutter. Furthermore, increasing the time windows may increase the resolution and accuracy of the images obtained as long as the size of the time window is small enough to resolve target and clutter signal. A scenario where the target signal and clutter signal were not completely resolved at all elements of the full-MDM was also considered. It was observed that resolving target signal from clutter signal in only some elements of the full-MDM was enough for the total sub-MUSIC images to locate the targets. Finally, we applied the algorithm on a scenario containing two targets hidden behind an obstructing brick wall. The total sub-MUSIC images locate the target with highest cross range and range resolution when spatial windows were applied. In Chapter 6 we consider scenarios containing moving targets hidden behind obstructions and apply the total sub-differential MDM algorithm for TWI.
Chapter 6

Sub-differential MDM Imaging

In Chapter 5, the total sub-MDM algorithm was applied on scenarios with stationary targets hidden behind a brick wall. We observed that the time-windows limit the clutter signals to individual sub-MDMs allowing the total sub-MUSIC images to focus on the target’s location. When the targets are in motion, the clutter signal from stationary obstructions can be removed by probing the scenario of interest twice and obtaining the difference between the reflections received. In this chapter, we apply the total sub-differential MDM algorithm detailed in Section 4.2 on scenarios containing moving targets behind obstructions.

6.1 FDTD scenario with two moving well-resolved scatterers

Figure 6.1 shows the geometry of a scenario with two moving well-resolved PEC sphere (15 mm radius) in a free-space medium hidden behind a 5.5 cm thick brick wall. We retain the FDTD parameters for PEC and brick as detailed in Section 3.3. We probe the medium for $K|_{t=t_1}$ and $K|_{t=t_2}$, recording the propagation in $N = 13$ parallel TRA antennas. We assume that the acquisition time to record the MDM is short enough that the targets can be considered stationary throughout the recording period. Therefore, $K|_{t=t_1}$ is obtained when the target is at its original location shown in Figure 6.1 and $K|_{t=t_2}$ is obtained after the targets have moved 0.005 m towards the TRA. We subtract $K|_{t=t_1}$ from $K|_{t=t_2}$ to obtain $K_d(t)$ which contains information from any moving scatterers in the
medium. We note that $K_d(t)$ is equivalent to $K_{d1}(t, 13)$ since no spatial windowing has been applied i.e. $N_s = N = 13$ and $L = 1$. Figure 6.2 and Figure 6.3 show the singular value distributions and the singular values at centre frequency, respectively, of the full-differential MDM. The ratio of consecutive singular values at centre frequency $\omega_c$ of the full-differential MDM is shown in Figure 6.4 that gives $N_t(\omega_c) = 2$ for (4.10). By applying spatial-windows (set initially to $N_s = 7$) to the full-differential MDM $K_d(t)$ we obtain the sub-differential MDMs $K_{dl}(t, 7)$ where $1 \leq l \leq 7$.

Figure 6.5 shows the image obtained using the full-MDM $K|_{t=t_1}$ which struggles to locate the targets in the scenario. We note that using $K|_{t=t_2}$ gives similar images which fail to locate the targets. By using the sub-differential MDMs for the sub-differential MUSIC imaging functionals, we produce images for the scenario.
Figure 6.2: The singular value distribution of the full-differential MDM

Figure 6.3: The singular value distribution of the full-differential MDM at centre frequency
Figure 6.4: The ratio of consecutive singular values at centre frequency $\omega_c$ of the full-differential MDM

Figure 6.5: The CF-MUSIC and UWB-MUSIC images produced using $K(\omega)$ (without differencing or spatial windowing), where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
CHAPTER 6. SUB-DIFFERENTIAL MDM IMAGING

Figure 6.6: The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d=1}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

Figure 6.7: The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d=4}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.
Figure 6.8: The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{dl=7}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

in Figure 3.33, Figure 6.6, Figure 6.7 and Figure 6.8 show the sub-differential CF-MUSIC and UWB-MUSIC images obtained for the scenario in Figure 3.33 using $K_{dl=1}(\omega, 7)$, $K_{dl=4}(\omega, 7)$ and $K_{dl=7}(\omega, 7)$ respectively. Figure 6.6, Figure 6.7 and Figure 6.8 show that by using the sub-differential MDMs, we obtain images that locate the targets, which is not possible using the full-MDM. The individual sub-differential MUSIC imaging functionals give images from the perspective of its spatially windowed TRA antennas. Hence it is observed in Figure 6.6 that the sub-differential images obtained using $K_{dl=1}(\omega, 7)$ locate target 1 with greater magnitude than the magnitude achieved for target 2. Similarly Figure 6.8 shows that the sub-differential images obtained using $K_{dl=7}(\omega, 7)$ locate the target 2 with greater magnitude than the magnitude achieved for target 1.

Figure 6.9 shows the cross range comparison at $y = 1.15$ m between the images obtained using the sub-differential MDM of the first spatial windows $K_{dl=1}(\omega, N_s)$ varying $N_s$. Figure 6.10 shows the cross range comparison at $y = 1.25$ m between the images obtained using the sub-differential MDM of the last spatial windows $K_{dl=L}(\omega, N_s)$ varying $N_s$. Figure 6.9 and Figure 6.10 show that by using sub-differential MDMs, $K_{dl}(t, N_s)$, we obtain images of the scenario that locate the
Figure 6.9: The normalised cross section at $y = 1.15$ m (location of PEC Target 1) for images obtained using $K_{d_{l=1}}(t, N_s = 3)$, $K_{d_{l=7}}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 1 is located at $x = 0.6$ m

Figure 6.10: The normalised cross section at $y = 1.25$ m (location of PEC Target 2) for images obtained $K_{d_{l=11}}(\omega, N_s = 3)$, $K_{d_{l=7}}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 2 is located at $x = 1.0$ m
Figure 6.11: The normalised range section at $x = 0.6$ m (location of PEC Target 1) for images obtained using $K_{dl=1}(\omega, N_s = 3)$, $K_{dl=1}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 1 is located at $y = 1.15$ m.

Figure 6.12: The normalised range section at $x = 1.0$ m (location of PEC Target 2) for images obtained using $K_{dl=11}(\omega, N_s = 3)$, $K_{dl=7}(\omega, N_s = 7)$ and $K_d(\omega)$. Target 2 is located at $y = 1.25$ m.
Figure 6.13: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 3$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

target with greater cross range resolution than is possible when using the full-differential MDM, $K_d(t)$. Despite spatial windows reducing the effective aperture of the TRA antennas for the sub-differential MDMs, the departmentalising of the information contained in the full-differential MDM means we can locate the individual targets with greater resolution in different images. Figure 6.11 shows the range comparison at $x = 0.6$ m between the images obtained using the sub-differential MDM of the first spatial windows $K_{d1}(t, N_s)$ varying $N_s$. Figure 6.12 shows the range comparison at $x = 1.0$ m between the images obtained using the sub-differential MDM of the last spatial windows $K_{dL}(t, N_s)$ varying $N_s$. Figure 6.11 and Figure 6.12 show that the effect of spatial windowing is inconsistent on the range resolution obtained for the sub-differential MUSIC images.

The summation of the sub-differential MUSIC imaging functionals give the total sub-differential MUSIC images. Figure 6.13 and Figure 6.14 show the total sub-differential CF-MUSIC and UWB-MUSIC images obtained for the scenario in Figure 3.33 when $N_s = 3$ and $N_s = 7$ respectively. We observe that by using the total sub-differential imaging functionals, we obtain images that locate the targets, which is not possible using the full-MDM, $K(t)$. Similarly, Figure 6.15
Figure 6.14: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

Figure 6.15: The CF-MUSIC and UWB-MUSIC images produced using the full-differential MDM, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
shows the MUSIC images, obtained using the full-differential MDM, which locate the targets. Figure 6.16 shows the total sub-differential MUSIC cross range resolution of $M_d(\tau, \omega_c, l, N_s)$ and $M_{d_{uwb}}(\tau, l, N_s)$ at $y = 1.15$ m for target 1 and at $y = 1.25$ m for target 2, varying $N_s$. From Figure 6.16 we observe that spatial windowing improves the cross range resolution of the total sub-differential MUSIC images. Despite spatial windows reducing the effective aperture of the TRA antennas for the sub-differential MDMs, the departmentalising of the information means we can locate the individual targets with greater cross range resolution in the sub-differential MUSIC images. Consequently the summation of these images gives the total sub-differential MUSIC images that located both targets with greater resolution. However, Figure 6.16(b) shows that cross range resolution of the sub-differential UWB-MUSIC image drops for both targets when $N_s = 3$. Figure 6.17 shows the total sub-differential MUSIC range resolution at half maximum of $M_d(\tau, \omega_c, l, N_s)$ and $M_{d_{uwb}}(\tau, l, N_s)$ at $x = 0.6$ m for target 1 and at $y = 1.0$ m for target 2, varying $N_s$. Table 6.1 shows the highest resolutions achieved by the total sub-differential MUSIC images. The results demonstrates that we can locate the targets with increased (cross) range resolution by using spatial windows, i.e. $N_s < 13$. It is also observed that the total sub-differential MUSIC images achieve (cross) range resolution that beat both the cross range resolution defined by the diffraction limit and the theoretical range resolution.
Figure 6.17: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC range resolution at half maximum of \( M_{d\Gamma}(\tau, \omega_c, N_s) \) and \( M_{d\Gamma_{UWB}}(\tau, N_s) \) at \( x = 0.6 \) m for target 1 and at \( x = 1.0 \) m for target 2, varying \( N_s \).

Table 6.1: The highest resolutions obtained for the total sub-differential MUSIC methods. The cross range resolution defined by the diffraction limit is 0.2 m for target 1 and 0.225 m for target 2. The theoretical range resolution is 0.15 m for both targets.
CHAPTER 6. SUB-DIFFERENTIAL MDM IMAGING

<table>
<thead>
<tr>
<th>Target 1</th>
<th>major axis (mm)</th>
<th>minor axis (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>110 (\lambda/1.36)</td>
<td>50 (\lambda/3)</td>
</tr>
<tr>
<td>Target 2</td>
<td>90 (\lambda/1.67)</td>
<td>50 (\lambda/3)</td>
</tr>
</tbody>
</table>

Table 6.2: The major and minor axis of the oval targets in the scenario shown in Figure 6.18

6.2 FDTD scenario with two buried expanding PEC oval targets

We demonstrate the potential of using sub-differential MDMs to locate buried live humans for rescue missions by simulating a scenario, with two expanding PEC oval targets buried in brick as shown in Figure 6.18. To create the oval targets, the ellipse equation \[100\] is used. Table 6.2 shows the major and minor axis of the targets in the scenario shown in Figure 6.18. The FDTD parameters for the human skin was used for the oval targets and set as \(\sigma = 0.54\) S/m, \(\epsilon_S = 47.93\), \(\epsilon_\infty = 29.85\) and \(\tau_D = 44\) ps \[101\]. We first probe the medium to obtain \(K\mid_{t=t_1}\). Thereafter, we probe the medium at a later time, when the oval targets have expanded by 10 mm on the minor axis, to obtain \(K\mid_{t=t_2}\). We subtract \(K\mid_{t=t_1}\) from \(K\mid_{t=t_2}\) to obtain \(K_d(t)\) which contains information from any live targets in the medium. We set \(N_s = 7\) to obtain \(K_{dl}(t, N_s)\), which segments the TRA antennas in space. Steering vectors comprising of the Green’s response function of the brick medium is utilised for the MUSIC methods.

Figure 6.19 shows the image obtained from \(K(\omega)\), without using sub-differential MDMs algorithm, which fails to locate the targets buried in ground for the scenario in Figure 6.18. Figure 6.20, Figure 6.21 and Figure 6.22 show the sub-differential CF-MUSIC and UWB-MUSIC images obtained for the scenario in Figure 6.18 using \(K_{dl=1}(\omega, 7)\), \(K_{dl=4}(\omega, 7)\) and \(K_{dl=7}(\omega, 7)\) respectively. The sub-differential MUSIC images obtained using \(K_{dl=1}(\omega, 7)\), \(K_{dl=4}(\omega, 7)\) and \(K_{dl=7}(\omega, 7)\) differ due to the location of the space-windowed TRA antennas, giving different perspective of the scenario. Figure 6.23 shows total sub-differential CF-MUSIC and UWB-MUSIC images obtained for the scenario in Figure 6.18 when \(N_s = 7\). The images show that by using the total sub-differential MDM algorithm we locate the buried target in the scenarios.
FDTD Geometry

10 layer PML Region (0.05 m)  Sampling time (Δt) = 9.6 ps
Central wavelength (λ) = 0.15 m  Cell size (Δx) = 0.005 m

Figure 6.18: The geometry of the FDTD scenario with two expanding buried targets
Figure 6.19: The CF-MUSIC and UWB-MUSIC images produced using $K(\omega)$ (without differencing or spatial windowing), where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

Figure 6.20: The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d_{l=1}}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.
CHAPTER 6. SUB-DIFFERENTIAL MDM IMAGING

Figure 6.21: The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d_4}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.

Figure 6.22: The sub-differential CF-MUSIC and sub-differential UWB-MUSIC images produced using $K_{d_7}(\omega, 7)$, where $\times$ represents the TRA antennas’ locations, $\times$ represents the active antennas and $\circ$ represents the target location.
6.3 Imaging two moving targets in a known inhomogeneous FDTD scenario

In Chapter 1, the problems radar systems encounter due to multipath signals were discussed. However, TR methods are capable of taking advantage of multipath as long as the background medium is known. The requirement of the background knowledge is a limitation for many TWI applications since the scenario is often unknown. However, there remain law enforcement and military applications where the scenario is known but inaccessible and TWI may be applicable. The architecture of a building may be known but inaccessible due to hostile targets. In these applications, the scenario of interest may be modelled using a forward solver to obtain the Green’s response function for the steering vectors. There may also be applications for which the scenario is known but visual or camera monitoring is inappropriate.

6.3.1 Imaging two moving targets

We consider a inhomogeneous FDTD scenario consisting of randomly distributed brick scatterers behind a brick wall as shown in Figure 6.24 where the background
scenario including the brick scatterers is known but the targets locations are unknown and apply the total sub-differential MDM algorithm. The brick scatterers in Figure 6.24 were inserted in free-space randomly, with uniform distribution, so that 1% of the FDTD cells between \( y = 0.405 \) m and \( y = 1.05 \) m was set to brick material. The targets are two moving (0.005 m towards the TRA) well-resolved PEC spheres (15 mm radius). In this scenario, \( K|_{t=t_1} \) is obtained from the known background medium without the targets. We probe the scenario \((N = 13)\) that contains the targets to obtain \( K|_{t=t_2} \). We subtract \( K|_{t=t_1} \) from \( K|_{t=t_2} \) to obtain \( K_{d1}(t, 13) \) which contains information of the targets in the medium. Figure 6.25 shows the reflected target signal as well as the multipath signal in element \( k_{d1,1}(t) \) of the full-differential MDM. Figure 6.26 and Figure 6.27 show the total sub-differential MUSIC images obtained using \( K_{d1}(\omega, 13) \) and \( K_{d1}(\omega, 7) \) respectively. It is observed that the total sub-differential MDM algorithm produces images that locate the targets. For comparison, the total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images \((N_s = 13)\) produced when the background media is unknown and free-space steering vectors are used is shown in Figure 6.28. Figure 6.29 and Figure 6.30 show the cross range (at \( y = 1.15 \) m) and range sections (at \( x = 0.6 \) m) respectively, of the the total sub-differential MUSIC images for target 1. Similarly Figure 6.31 and Figure 6.32 show the cross range (at \( y = 1.25 \) m) and range sections (at \( x = 1.0 \) m) respectively, of the the total sub-differential MUSIC images for target 2. We observe that when the background scenario is known, higher (cross) range resolution is achieved for the total sub-differential MDM method. Furthermore, it is observed that spatial windowing adversely affects the (cross) range resolution when the background scenario is known.

### 6.3.2 Imaging two expanding targets

Here we apply the total sub-differential MDM algorithm to a scenario containing two expanding PEC oval targets hidden behind an brick wall and a inhomogeneous scattering medium. We consider a inhomogeneous FDTD scenario consisting of randomly distributed brick scatterers behind a brick wall as shown in Figure 6.33 where the background scenario is known. We retain the major and minor axis of the targets shown in Table 6.2 and their FDTD parameters detailed Section 6.2. We first probe the medium \((N = 13)\) to obtain \( K|_{t=t_1} \). Thereafter, we probe the medium at a later time, when the oval targets have expanded by
Sampling time ($\Delta t$) = 9.6 ps
Cell size ($\Delta x$) = 0.005 m
FDTD Geometry

Figure 6.24: The geometry of the FDTD scenario with two well-resolved targets in inhomogeneous scattering media
Figure 6.25: The reflected signal from the target with multipath signal in the element $k_{d1,1}(t)$ of the full-differential MDM $K_{d1}(t, 13)$

Figure 6.26: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 13$ (no spatial windowing), where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location
CHAPTER 6. SUB-DIFFERENTIAL MDM IMAGING

Figure 6.27: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 7$, where $\times$ represents the TRA antennas' locations and $\circ$ represents the target location.

Figure 6.28: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 13$ (no spatial windowing) when the background media is unknown and free-space steering vectors are used, where $\times$ represents the TRA antennas' locations and $\circ$ represents the target location.
Figure 6.29: The normalised cross section in Figure 6.26 and Figure 6.27 of target 1 located at $y = 1.15$ m obtained using the total sub-differential MUSIC method. Target 1 is located at $x = 0.6$ m.

Figure 6.30: The normalised range section in Figure 6.26 and Figure 6.27 of target 1 located at $x = 0.6$ m obtained using the total sub-differential MUSIC method. Target 1 is located at $y = 1.15$ m.
Figure 6.31: The normalised cross section in Figure 6.26 and Figure 6.27 of target 2 located at $y = 1.25$ m obtained using the total sub-differential MUSIC method. Target 2 is located at $x = 1.0$ m

Figure 6.32: The normalised range section in Figure 6.26 and Figure 6.27 of target 2 located at $x = 1.0$ m obtained using the total sub-differential MUSIC method. Target 2 is located at $y = 1.25$ m.
10 mm on the minor axis, to obtain $K|_{t=t_2}$. We subtract $K|_{t=t_1}$ from $K|_{t=t_2}$ to obtain $K_d(t)$ which contains information from any live targets in the medium as well as multipath signal. We utilise $K_d(t, N_s)$ for the total sub-differential MDM algorithm to obtain the total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images. Figure 6.34 and Figure 6.35 show the total sub-differential MUSIC images obtained using $K_d(\omega, 13)$ and $K_d(\omega, 7)$ respectively. It is observed that the total sub-differential MDM algorithm produces images that locate the targets in the scenario.
Figure 6.34: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 13$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.

Figure 6.35: The total sub-differential CF-MUSIC and total sub-differential UWB-MUSIC images produced when $N_s = 7$, where $\times$ represents the TRA antennas’ locations and $\circ$ represents the target location.
6.4 Summary

In this chapter, we applied the total sub-differential MDM algorithm to image two moving targets located behind an obstructing brick wall. We observed that the sub-differential MUSIC images allow the targets to be observed from the perspective of the spatially-windowed antennas. The total sub-differential MDM algorithm was applied to a scenario containing two buried expanding oval targets. We observed the total sub-differential MUSIC images locate the target illustrating the potential of the algorithm for tracking humans. We considered inhomogeneous scenarios containing targets hidden behind a scattering medium. We showed that the total sub-differential MDM algorithm is more effective at locating the targets when the background medium is known. Next, we conclude the dissertation by summarising the findings attained within this research and providing an outlook onto future works.
Chapter 7

Conclusions and future work

This dissertation proposed the novel total sub-MDM algorithm that utilises the highly acclaimed TR-based MUSIC method for TWI applications. Firstly, we summarised the theory behind radar imaging systems and its applications such as TWI. We described the current popular TWI systems, which are diffraction-limited. The problems typical TWI systems face due to multipath signals from multiple scatterers were discussed. Hence, we detailed the theory behind the TR method which can exploit multipath signal to achieve superresolution. The TR-based DORT and MUSIC methods which have recently become popular for remote sensing were also discussed. These methods involve taking the SVD of the full-MDM but differ in the subspaces used. The DORT method uses the signal subspace while the MUSIC method uses the null subspace.

We investigated and compared the performances of the DORT and MUSIC method in terms of accuracy and (cross) range resolution by considering a scenario containing a single PEC target located in free-space. By taking the SVD of the full-MDM, we showed the singular values distribution as well as the phase and magnitude distributions of the singular vectors which are important for estimating the number of well-resolved discrete scatterers under free-space conditions. We showed that by analysing the singular values, the number of well-resolved scatterers in the space of interest can be estimated. Moreover, by analysing the phase and magnitude distribution of the singular vectors the location of the scatterers relative to the TRA antennas may be estimated. Subsequently, a scenario containing two PEC targets located in free-space was also considered. We observed that although the DORT method is unique in its ability to selectively focus on well-resolved discrete targets, the MUSIC method outperforms the DORT method.
in terms of accuracy and (cross) range resolution even for multiple targets.

Next, we considered a scenario containing a single well-resolved PEC target located behind an obstructing brick wall and showed that both the DORT and MUSIC methods fail to image the target because the clutter signal from the brick wall dominates the full-MDM. We developed and applied a spatio-temporal window based algorithm to extract information on targets by segmenting the full-MDM into sub-MDMs. By applying the MUSIC method on the individual sub-MDMs, we obtained images that combine to localise the targets hidden behind a brick wall which was not achieved by the full-MDM MUSIC method. We showed that our algorithm produces images that locate the target as long as the size of the time-windows are small enough to resolve target signal from clutter signal. Furthermore we showed that because our algorithm utilises the MUSIC method, it achieves cross range resolution that beats the diffraction limit unlike typical TWI systems that are diffraction-limited. We considered a scenario where the target signal and clutter signal were not well-resolved in all elements of the MDM. We showed the algorithm produces images that locate the target as long as the time windows are small enough to resolve target signal and clutter signal in some elements of the MDM. Thereafter, we considered a scenario containing two targets located behind an obstructing brick wall and observed that our algorithm located both targets successfully, obtaining improved (cross) range resolution when the spatial windows were applied. The total sub-MDM algorithm is unique when compared to other TR-based TWI techniques because it does not require knowledge of the obstruction and it utilises the entire information contained in the full-MDM.

Additionally, we investigated the potential for applying the MUSIC imaging method for use in imaging moving targets such as humans for applications such as law enforcement, military operations as well as rescue missions in the event of earthquakes and avalanche. Therefore, this dissertation extended the total sub-MDM algorithm by proposing the total sub-differential MDM algorithm, which probes the scenario of interest at two time instances and obtains the difference in reflected signals received to eliminate clutter signal received from stationary obstructions in the differential-MDM. We apply spatial-windowing on the differential-MDM to obtain the sub-differential MDM. Firstly, we applied the total sub-differential MDM algorithm to a scenario containing two moving PEC targets hidden behind a brick wall and obtained images that successfully located
the targets. It was observed that utilising individual sub-MDMs whose spatial windowed antennas are closest to the target can give higher cross range resolution than is obtained without spatial windowing. Analysing the performance showed that the images produced by the total sub-differential MDM algorithm give (cross) range resolution that beats both the cross range resolution defined by the diffraction limit and the theoretical range resolution. We further applied the total sub-differential MDM algorithm to locate live oval-shaped targets buried in brick. The algorithm attained images that accurately located the targets in the scenario despite the presence of clutter. We noted that the steering vectors consisted of the Green’s response function of a homogeneous medium whose medium parameters are the same as those of a brick wall. Hence, when applying the total sub-differential MDM algorithm for practical GPR applications the background media should be approximated to improve accuracy. Utilising steering vectors consisting of free-space Green’s function for GPR applications is unsuitable since the reciprocal property is lost completely, leading to the images giving inaccurate locations of the targets.

Lastly, a scenario containing an obstructing brick wall and a inhomogeneous scattering medium was considered. We explained that to take advantage of multi-path signal for the total sub-differential MDM algorithm, the background medium must be known. Admittedly this is not always possible in many TWI applications. However there remain law enforcement and military applications where the architecture of a building is known. The continuing growth in computational resources mean complex buildings can be modelled with forward solvers such as the FDTD method to obtain the Green’s response function of the scenario of interest.

### 7.1 Future works

The immediate future works involve modelling more complex 3-dimensional scenarios with more realistic targets than the 2-dimensional scenarios and simple targets considered in this dissertation. Hence scenarios containing extended targets hidden behind an obstructing material can be considered. Furthermore, a scenario containing two adjacent discrete targets can be researched, changing the
distance between the targets to attain the specific performance of the total sub-MDM algorithm when the targets are not well-resolved from each other. Scenarios containing multiple moving targets which are not well-resolved in time and space can be considered, applying time-windowing to the total sub-differential MDM algorithm. Additionally, loss compensation techniques should be applied since the materials contained in typical TWI scenarios are lossy. A second stage can be added to the total sub-MDM algorithm once the algorithm has obtained the location of the targets. The second stage would involve using adaptive windows based on the information obtained from the images produced by the total sub-MDM algorithm. Adaptive windows involve time-shifting a particular time-window depending on the individual element of the full-MDM it is applied to, so that a sub-MDM can contain target information in all of its elements. The addition of adaptive windows could improve the range resolution, particularly when there are multiple targets.
Bibliography


Appendix A

Alternative plots for easier comparison

Here, we re-plot some figures shown in the main body of the dissertation for easier comparison albeit sacrificing clarity.

A.1 Figures from Section 5.1

Figure A.1 shows the cross range resolution of the total sub-CF-MUSIC images at \( y = 1.15 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.24. Figure A.2 shows the cross range resolution of the total sub-UWB-MUSIC images at \( y = 1.15 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.25. Figure A.3 shows the range resolution of the total sub-CF-MUSIC images at \( x = 0.75 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.26. Figure A.4 shows the range resolution of the total sub-UWB-MUSIC images at \( x = 0.75 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.27. Figure A.5 shows the accuracy of the total sub-CF-MUSIC images in locating the PEC target, varying \( N_s \) and \( P \), previously plotted in Figure 5.28. Figure A.6 shows the accuracy of the total sub-UWB-MUSIC images in locating the PEC target, varying \( N_s \) and \( P \), previously plotted in Figure 5.29.
APPENDIX A. ALTERNATIVE PLOTS FOR EASIER COMPARISON

Figure A.1: The total sub-CF-MUSIC cross range resolution at \( y = 1.15 \text{ m}, \) varying \( N_s \) and \( P \), previously plotted in Figure 5.24.

Figure A.2: The total sub-UWB-MUSIC cross range resolution at \( y = 1.15 \text{ m}, \) varying \( N_s \) and \( P \), previously plotted in Figure 5.25.
APPENDIX A. ALTERNATIVE PLOTS FOR EASIER COMPARISON

Figure A.3: The total sub-CF-MUSIC range resolution at $x = 0.75$ m, varying $N_s$ and $P$, previously plotted in Figure 5.26

Figure A.4: The total sub-UWB-MUSIC range resolution at $x = 0.75$ m, varying $N_s$ and $P$, previously plotted in Figure 5.27
APPENDIX A. ALTERNATIVE PLOTS FOR EASIER COMPARISON

Figure A.5: The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying $N_s$ and $P$, previously plotted in Figure 5.28.

Figure A.6: The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying $N_s$ and $P$, previously plotted in Figure 5.29.
Figure A.7: The total sub-CF-MUSIC cross range resolution at \( y = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.43

A.2 Figures from Section 5.2

Figure A.7 shows the cross range resolution of the total sub-CF-MUSIC images at \( y = 0.75 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.43. Figure A.8 shows the cross range resolution of the total sub-UWB-MUSIC images at \( y = 0.75 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.44. Figure A.9 shows the range resolution of the total sub-CF-MUSIC images at \( x = 0.75 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.45. Figure A.10 shows the range resolution of the total sub-UWB-MUSIC images at \( x = 0.75 \) m varying \( N_s \) and \( P \), previously plotted in Figure 5.46. Figure A.11 shows the accuracy of the total sub-CF-MUSIC images in locating the PEC target, varying \( N_s \) and \( P \), previously plotted in Figure 5.47. Figure A.12 shows the accuracy of the total sub-UWB-MUSIC images in locating the PEC target, varying \( N_s \) and \( P \), previously plotted in Figure 5.48.
APPENDIX A. ALTERNATIVE PLOTS FOR EASIER COMPARISON

Figure A.8: The total sub-UWB-MUSIC cross range resolution at \( y = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.44.

Figure A.9: The total sub-CF-MUSIC range resolution at \( x = 0.75 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.45.
Figure A.10: The total sub-UWB-MUSIC range resolution at $x = 0.75$ m, varying $N_s$ and $P$, previously plotted in Figure 5.46.

Figure A.11: The accuracy of the total sub-CF-MUSIC imaging functional in locating the PEC target varying $N_s$ and $P$, previously plotted in Figure 5.47.
Figure A.12: The accuracy of the total sub-UWB-MUSIC imaging functional in locating the PEC target varying \( N_s \) and \( P \), previously plotted in Figure 5.47.
A.3 Figures from Section 5.3

Figure A.13 shows the cross range resolution of the total sub-CF-MUSIC images at $y = 1.15$ m where PEC target 1 is located, varying $N_s$ and $P$, previously plotted in Figure 5.58. Figure A.14 shows the cross range resolution of the total sub-UWB-MUSIC images at $y = 1.15$ m where PEC target 1 is located, varying $N_s$ and $P$, previously plotted in Figure 5.59. Figure A.15 shows the cross range resolution of the total sub-CF-MUSIC images at $y = 1.25$ m where PEC target 2 is located, varying $N_s$ and $P$, previously plotted in Figure 5.60. Figure A.16 shows the cross range resolution of the total sub-UWB-MUSIC images at $y = 1.25$ m where PEC target 2 is located, varying $N_s$ and $P$, previously plotted in Figure 5.61. Figure A.17 shows the range resolution of the total sub-CF-MUSIC images at $x = 0.6$ m where PEC target 1 is located, varying $N_s$ and $P$, previously plotted in Figure 5.62. Figure A.18 shows the range resolution of the total sub-UWB-MUSIC images at $x = 0.6$ m where PEC target 1 is located, varying $N_s$ and $P$, previously plotted in Figure 5.63. Figure A.19 shows the range resolution of the total sub-CF-MUSIC images at $x = 1.0$ m where PEC target 2 is located, varying $N_s$ and $P$, previously plotted in Figure 5.64. Figure A.20 shows the range resolution of the total sub-UWB-MUSIC images at $x = 1.0$ m where PEC target 1 is located, varying $N_s$ and $P$, previously plotted in Figure 5.65.
Figure A.13: The total sub-CF-MUSIC cross range resolution at half maximum of $M_{\Gamma}(\tau, \omega_c, P, N_s)$ at $y = 1.15$ m, varying $N_s$ and $P$, previously plotted in Figure 5.58.

Figure A.14: The total sub-UWB-MUSIC cross range resolution at half maximum of $M_{\Gamma_{UWB}}(\tau, P, N_s)$ at $y = 1.15$ m, varying $N_s$ and $P$, previously plotted in Figure 5.59.
Figure A.15: The total sub-CF-MUSIC cross range resolution at half maximum of $M_{\Gamma}(r, \omega_c, P, N_s)$ at $y = 1.25$ m, varying $N_s$ and $P$, previously plotted in Figure 5.60.

Figure A.16: The total sub-UWB-MUSIC cross range resolution at half maximum of $M_{\Gamma_{UWB}}(r, P, N_s)$ at $y = 1.25$ m, varying $N_s$ and $P$, previously plotted in Figure 5.61.
**APPENDIX A. ALTERNATIVE PLOTS FOR EASIER COMPARISON**

<table>
<thead>
<tr>
<th>Range resolution (m)</th>
<th>$N_s = 3$</th>
<th>$N_s = 5$</th>
<th>$N_s = 7$</th>
<th>$N_s = 9$</th>
<th>$N_s = 11$</th>
<th>$N_s = 13$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_s = 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_s = 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_s = 7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_s = 9$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_s = 11$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_s = 13$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure A.17:** The total sub-CF-MUSIC range resolution at half maximum of $M_{\Gamma}(\tau, \omega_c, \Omega, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$, previously plotted in Figure 5.62.

**Figure A.18:** The total sub-UWB-MUSIC range resolution at half maximum of $M_{\Gamma_{UWB}}(\tau, P, N_s)$ at $x = 0.6$ m, varying $N_s$ and $P$, previously plotted in Figure 5.63.
Figure A.19: The total sub-CF-MUSIC range resolution at half maximum of \( M_{\Gamma_{\text{CF}}}(r, P, N_s) \) at \( x = 1.0 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.62.

Figure A.20: The total sub-UWB-MUSIC range resolution at half maximum of \( M_{\Gamma_{\text{UWB}}}(r, P, N_s) \) at \( x = 1.0 \) m, varying \( N_s \) and \( P \), previously plotted in Figure 5.65.