BEAMFORMING TECHNIQUES FOR MILLIMETER WAVE RELAY NETWORKS

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Science and Engineering

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Abstract

The energy and data rate requirements for the next generation cellular networks urge the need for innovative solutions. Inspired by its massive bandwidth, millimeter wave (mmWave) band is thought-out to be one of the key elements to meet the aspirations. However, mmWave links are known to have short coverage distance due to the propagation losses introduced at high frequencies. The proposed solutions to overcome the transmission challenges include using large arrays with improved directivity, adopting smaller cells, and relying on cooperative networks to extend the mmWave link and avoid shadowing areas.

This work aims to improve the connectivity of the mmWave link in the outdoor environments. One of the cost effective methods is to increase the array gain by using Analogue Beamforming (ABF). The performance of the ABF system in the presence of phase quantization error has been analytically investigated. The study also includes comparing three different channel sounding techniques, namely: exhaustive search, side-to-side search, and n-tier search. The time overhead related to each method and their energy consumption are calculated. The numerical results assist in determining the optimum search period to obtain a reasonable spectral efficiency using minimal power consumption. The results also help identify the minimum number of quantization bits required to produce about ninety percent of the optimistic results.

In order to extend the coverage further, relay networks are considered an essential component in mmWave communications. The performance of a single hybrid beamforming full-duplex relay system and multi-relay networks were investigated. The design algorithms for the processors in the network are proposed based on the greedy pursuit approach. The performance of the proposed algorithms is analysed under various scenarios. The analysis highlights the influence of the array size, the number of RF chains, and the length of the channel sounding period. The performance of the proposed systems is compared from both the
spectral efficiency and power consumption prospects. The results also establish that the number of antennas at the source and the relay receiver arrays have a superior impact on the system performance than the sizes of the array at the destination and the relay transmitter.
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Dedication

To you, dad.
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
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<tbody>
<tr>
<td>3GPP</td>
<td>Third Generation Partnership Project</td>
</tr>
<tr>
<td>4G</td>
<td>Fourth Generation</td>
</tr>
<tr>
<td>5G</td>
<td>Fifth Generation</td>
</tr>
<tr>
<td>ABF</td>
<td>Analogue Beamforming</td>
</tr>
<tr>
<td>ADC</td>
<td>Analogue-to-Digital Converter</td>
</tr>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Arrival</td>
</tr>
<tr>
<td>AoD</td>
<td>Angle of Departure</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>AWV</td>
<td>Antenna Weight Vector</td>
</tr>
<tr>
<td>CAR</td>
<td>Collision Avoidance Radar</td>
</tr>
<tr>
<td>CCDF</td>
<td>Complementary Cumulative Distribution Function</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
<td>--------------</td>
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</tr>
<tr>
<td>CIR</td>
<td>Channel Impulse Response</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal-Oxide-Semiconductor</td>
</tr>
<tr>
<td>CS</td>
<td>Channel Sounding</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital-to-Analogue Converter</td>
</tr>
<tr>
<td>DBF</td>
<td>Digital Beamforming</td>
</tr>
<tr>
<td>DEV</td>
<td>Device</td>
</tr>
<tr>
<td>ETSI</td>
<td>European Telecommunications Standards Institute</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>FD</td>
<td>Full-Duplex</td>
</tr>
<tr>
<td>GSCM</td>
<td>Geometric Based Stochastic Channel Model</td>
</tr>
<tr>
<td>HBF</td>
<td>Hybrid Beamforming</td>
</tr>
<tr>
<td>HD</td>
<td>Half-Duplex</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>Independent and Identically Distributed</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>ITU-R</td>
<td>International Telecommunication Union - Radiocommunication Sector</td>
</tr>
<tr>
<td>LNA</td>
<td>Low-Noise Amplifier</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
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</tr>
<tr>
<td>LOS</td>
<td>Line-Of-Sight</td>
</tr>
<tr>
<td>MAC</td>
<td>Medium Access Control</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MMIC</td>
<td>Monolithic Microwave Integrated Circuit</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>mmWave</td>
<td>Millimeter Wave</td>
</tr>
<tr>
<td>MTC</td>
<td>Machine-Type-Communication</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non-Line-Of-Sight</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OMP</td>
<td>Orthogonal Matching Pursuit</td>
</tr>
<tr>
<td>P2P</td>
<td>Point-to-Point</td>
</tr>
<tr>
<td>PAN</td>
<td>Personal Area Network</td>
</tr>
<tr>
<td>PHY</td>
<td>Physical</td>
</tr>
<tr>
<td>PLE</td>
<td>Path Loss Exponent</td>
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<tr>
<td>PQE</td>
<td>Phase Quantization Error</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>Rx</td>
<td>Receiver</td>
</tr>
<tr>
<td>S-V</td>
<td>Saleh-Valenzuela</td>
</tr>
<tr>
<td>S2S</td>
<td>Side-to-Side</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input Single-Output</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>STA-AP</td>
<td>Station-to-Access Point</td>
</tr>
<tr>
<td>STA-STA</td>
<td>Station-to-Station</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>Tx</td>
<td>Transmitter</td>
</tr>
<tr>
<td>ULA</td>
<td>Uniform Linear Array</td>
</tr>
<tr>
<td>WiGig</td>
<td>Wireless Gigabit Alliance</td>
</tr>
<tr>
<td>WINNER</td>
<td>Wireless World Initiative New Radio Project</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
<tr>
<td>WPAN</td>
<td>Wireless Personal Area Network</td>
</tr>
</tbody>
</table>
List of Mathematical Notations

\((\cdot)^*\) Conjugate transpose

\((\cdot)^T\) Transpose

\(U[a,b]\) Uniform distribution with parameters \(a\) and \(b\)

\(\Gamma(\cdot)\) Gamma function

\(\Gamma(a,b)\) Complementary gamma function

\(I\) Identity matrix

\(\exp(x)\) Exponential Function, \(e^x\)

\(\ln\) Natural logarithm

\(\log_x\) Logarithmic function to base \(x\)

\(\mathcal{CN}(\mu,\sigma^2)\) Complex normal distribution with mean \(\mu\) and \(\sigma^2\)

\(\Pi\) Production symbol

\(\Sigma\) Summation symbol

\(\text{argmax}\) Argument of the maximum
List of Mathematical Notations

$||X||_p$ The $p$-norm of $X$

$||\cdot||_F$ Frobenius norm

$P(x)$ Probability of $x$

$||\cdot||$ Norm of a vector

argmin Argument of the minimum

bd[·] Block-diagonal matrix

diag(·) Diagonal matrix
### List of Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Codebook size</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of relays</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of paths in the cluster</td>
</tr>
<tr>
<td>$M_x$</td>
<td>Number of antennas at the relay, $x \in {t, r}$</td>
</tr>
<tr>
<td>$M_{RF}^x$</td>
<td>Number of RF chains at the relay, $x \in {t, r}$</td>
</tr>
<tr>
<td>$N_x$</td>
<td>Number of antennas, $x \in {t, r, s}$</td>
</tr>
<tr>
<td>$N_{RF}^x$</td>
<td>Number of RF chains, $x \in {t, r}$</td>
</tr>
<tr>
<td>$P$</td>
<td>Number of clusters in the channel</td>
</tr>
<tr>
<td>$T$</td>
<td>Slot duration</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Channel sounding duration</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Performance measure factor</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Beamforming gain</td>
</tr>
<tr>
<td>$\Omega_x$</td>
<td>Array gain $x \in {t, r}$</td>
</tr>
</tbody>
</table>
List of Variables

\( \alpha_{p\ell} \) The complex small-scale fading gain of the \( \ell \)-th sub-path in the \( p \)-th cluster

\( \beta \) Relay Tx-to-Rx antenna ratio

\( \beta_{RF} \) Relay Tx-to-Rx RF chain ratio

\( H \) Channel matrix

\( H_s \) Self-interference channel

\( a_x \) Array response vector, \( x \in \{t, r\} \)

\( f \) Transmitter beamforming vector

\( n \) Noise vector

\( w \) Receiver combining vector

\( A_x \) A matrix contain the array response vectors of the channel, \( x \in \{T, R\} \)

\( \chi \) The ray decay factor

\( \chi_c \) The cluster decay factor

\( \gamma_x \) Signal to noise ration \( x \in \{t, r\} \)

\( \kappa \) Path Loss Exponent

\( \lambda \) Wavelength

\( \nabla \) Array Factor
List of Variables

$\omega$ Hybridization ratio

$\phi$ The required rotation angle of the beam

$\psi^x_i$ Quantization error for the $(i+1)$–th antenna of the array. $x \in \{r, t\}$

$\rho$ Transmitted power

$\rho_r$ Received power

$\tau_0$ Minimum excess delay time

$\tau_{RMS}$ RMS delay spread

$\theta^x_{pl}$ Angle of the $l$th path of the $p$th cluster. $x \in \{r, t\}$

$\varphi^x_{rpl}$ Elevation angle of the $l$th path of the $p$th cluster. $x \in \{r, t\}$

$\vartheta$ Quantised phase of the array antenna

$\zeta$ Frequency-loss factor

$c$ Speed of light

$d$ The space between the antennas in an array

$d_x$ Distance between transmitter and receiver

$q$ Number of quantisation bits

$s, s$ Transmitted information signal

$\mathbf{F}_x$ Beamforming matrix at transmitter $x \in \{RF, BB, u\}$
List of Variables

\begin{itemize}
  \item $G_x$ Beamforming matrix at the relay $x \in \{R, BB, T, u\}$
  \item $W_x$ Beamforming matrix at receiver $x \in \{RF, BB, u\}$
  \item $X^{opt}$ Optimum matrix in a hybrid processor
  \item $X_u$ The unconstrained matrix (fully digital processor)
  \item $X_{BB}$ Digital matrix in a hybrid processor
  \item $X_{RF}$ Analogue matrix in a hybrid processor
  \item $\mathcal{F}_{RF}$ Set of feasible RF precoders. Similarly defined is: $\mathcal{W}_{RF}, \mathcal{G}_{RF}$
\end{itemize}
Chapter 1

Introduction

Wireless communications give our work, education, and entertainment the freedom of transferring data without the limits of a physical connection. The opportunities provided by wireless communications inspire the engineering community to push the boundaries of innovation. Transmission over millimeter wave (mmWave) represents the latest milestone in the advancement of communication systems. Although the research on mmWave still taking its first steps, it is being encouraged by the consumer desire for higher data rates at constant connectivity and lower latency. The spectrum currently provided by the unlicensed 60 GHz band provides tenfold the current bandwidth provided by WiFi and Fourth Generation (4G) cellular networks [1].

Experts anticipate that mmWave will be the basis for the next generation cellular network. In a scheme where the cloud is expected to be the centre of operation, transmission from and into the cloud is essential. MmWave brings new capabilities to the designers of the next emerging architecture. This exciting future allows implementing applications like the data centres, information showers, and Internet of Things (IoT) [2,3].

However, many challenges need to be addressed for this technological revolution to happen. In this work, the impact of system architecture and signal processing techniques on improving the mmWave link connectivity is investigated. Still, developing the future communication systems will also require advancement in such areas as circuit design and antenna models. Plus, a better understanding of the mmWave frequencies electromagnetics.
1.1 Motivation

In many aspects, the current wireless network will not be capable of handling the demands over the next few years. The rapid development of the wireless devices (tablets and smartphones as an example) changed the distribution of the needs for wireless data geographically and time-wise (for instance, reports indicate that smartphones represented only 45 percent of total mobile devices and connections in 2016, but represented 81 percent of total mobile traffic). The leap in technology also increased the demand for wireless data at a rate that was not anticipated a few decades ago. It is stated that the Global mobile data traffic grew 63 percent last year reaching 7.2 exabytes per month at the end of 2016. It is also expected that Global mobile data traffic will increase sevenfold between 2016 and 2021 reaching 49.0 exabytes per month by 2021 [1, 4]. Other emerging trends include the growth in the number of connected devices other than smartphones, such as wearable technologies and Machine-Type-Communication (MTC) devices. This development urges the need to re-think the cellular network architecture. One of the proposed schemes involve dynamic dense networks where the cell size is much smaller than the current micro-cell and serve less number of users. The new proposed architecture, however, requires higher synchronisation between the cells. In such architecture, using wired backhaul links (e.g. fibre optics) will be a challenging and costly solution [5–8].

Millimeter wave band, 3GHz to 300 GHz, represent a perfect transmission media for the next generation cellular network. The past two decades have witnessed an accelerated progress of licensing mmWave in many countries. MmWave features have attracted considerable attention from research groups to investigate the possibilities and challenges associated with employing this band. MmWave enormous bandwidth makes it suitable for both access and backhaul links. Wireless backhaul has been recently proposed as one of the solutions to avoid the cost and complications of a new wired infrastructure, e.g. see [9–12].

The mmWave band has been successfully utilised as high data rate media in several indoor applications. For instance, 60 GHz band was employed to produce the early mass commercial applications of mmWave in Wireless Personal Area Network (WPAN) such as Wireless HD [13] and Wireless Gigabit Alliance (WiGig) [14,15]. This led to publishing the first multi-GB IEEE standard, IEEE 802.15.3c, which defines Physical (PHY) and Medium Access Control (MAC) layers for mmWave WPANs [16,17]. One the other hand, utilising the mmWave
in the outdoor environments is still a challenge. Outdoor measurements indicate that the additional propagation losses due to rain and oxygen observation plus the shadowing and blockage effects resulted in a short mmWave link. Additionally, the channel is expected to have a sparse nature due to reflection. For more details on outdoor measurements, see e.g. [18–22].

As the antenna size at the mmWave frequency is very small, more elements could be fitted into the antenna array. Thus, several research groups propose increasing the directivity as a way to extend the mmWave link coverage [2, 23]. Analogue Beamforming (ABF) is one of the proposed solutions to add gain to the antenna array without the need to acquire channel knowledge. Analogue beamforming involves the use of phase shifters to direct (or steer) the signal towards the required direction, see e.g. [24–27]. Although ABF is considered simple and cost effective, it has several drawbacks such as lacking the ability to multi-stream transmission. As a result, Hybrid Beamforming (HBF) precoding techniques have been proposed [28]. Hybrid beamformers use less RF chains than the number of antenna elements which makes it both cost and energy efficient considering the massive antenna arrays, see e.g. [29–31].

Cooperative networks have been used during the last few decades in wireless communication networks to provide a better connection with lower power requirements. Furthermore, in the light of the dense network proposal and the short mmWave links, relays will be essential for the next generation architecture functionality [8]. Relays could be used to extend the Point-to-Point (P2P) link in the backhaul or as a micro base station between the main base station and the user [32]. Recently, some research works investigated the benefits of using relays in the mmWave based networks. The studies explained how the connectivity of mmWave networks depends on the main system parameters such as the density and size of obstacles. It was shown that multi-hop relaying can significantly improve the connectivity and that even a single relay device positioned at the height of other nodes can improve 50% of the links in substantial levels in a 60 GHz indoor network [33,34]. However, very little research has been published on the design of relays that could be used in such architecture. Besides, no previous study has addressed the problem of designing Full-Duplex (FD) relay in the mmWave environment. Published literature on multiple relays in mmWave is limited to investigating the routing protocols, selecting the best appropriate relay hops for a traffic flow, and how relay networks help minimising the blockage.
CHAPTER 1. INTRODUCTION

in mmWave environment [35–37].

The work in this thesis investigate utilising mmWave as a media for communication in the Fifth Generation (5G) cellular networks. Specifically, an improved gain using various beamforming techniques have been produced. In addition, employing relay systems to extend the mmWave link is investigated. Both single and multi-relay processors are designed and discussed in details.

1.2 Aims and Objectives

This work aims to extend the coverage of the mmWave link in the outdoor environments. The thesis investigates the effect of beamforming and relay systems on extending the point-to-point mmWave link. In order to reach the purposes of the work, the main objectives were:

- To develop algorithms that are suitable to design the analogue and hybrid processors at the relay. The algorithms use the greedy and split principles to produce the analogue and digital processors.

- To investigate several channel sounding techniques to be used when the channel information is not available. The variety of training duration and codebook size for each method leads to different performance measures.

- To analytically derive the efficiency of the beamforming systems in the outdoor mmWave environment. This includes analysing the performance of the analogue beamforming as well as the hybrid systems. The study of the performance aids to assess how much beamforming is needed.

- To analyse the energy efficiency of the beamforming systems. The study of the energy explores the possibility of having green communications and extending the battery life in some applications.

- To examine the impact of using digital phase shifters in implementing the analogue beamforming systems. The phase quantization error associated with digital phase shifters might cause an alignment diffraction in addition to the appearance of sidelobes.

- To study the benefits of using single and multiple relay nodes in the mmWave network and to examine the impact of analogue and hybrid beamforming on relay systems.
Overall, this work presents a detailed investigation of different network layouts to extend the coverage of point-to-point link in the mmWave environment. This work helps reach a comprehensive view of the characteristic of variable beamforming techniques in various network scenarios.

1.3 Key Contributions

In this thesis, various network topologies and beamforming systems to extend the length of the mmWave link have been investigated. A detailed analysis of the analogue beamforming systems including the effect of channel sounding overhead and phased quantization error has been derived. Then, the required algorithms to utilise relay systems in mmWave networks are developed. Some key contributions of this work are summarised as follows.

- Exact analysis of the analogue beamforming gain in the presence of phase quantization error has been derived. The error resulting from phase correlation between the array elements due to utilising digital phase shifters leads to direction misalignment and gain loss. The previous studies have ignored the impact of this error in mmWave systems. The beamforming gain analysis highlights the system performance under these scenarios is important for better understanding the behaviour under more general circumstances.

- Determining the minimum number of quantization bits required to achieve a reasonable spectral efficiency. It is essential to reduce the number of quantization bits due to the overhead associated with each additional bit.

- The investigation of channel sounding techniques to avoid acquiring channel knowledge at the transmitter. The work includes investigating the energy consumption and search duration of each method. Three different methods are considered: Exhaustive search, Side-to-Side search, and $n$-tier search. All the three techniques require predefined codebooks and no channel knowledge. For a better understanding of the system behaviour, a new performance measure factor that compares the spectral efficiency of the system to the energy efficiency was proposed. The factor could be used for a trade-off between the spectral and energy efficiency of the system when the training time is in question. The study of the performance measure factor help in determining an optimum channel sounding duration that produces
satisfactory spectral efficiency results with minimal energy consumption. The analysis and part of the results are under submission to the IEEE Transactions on Vehicular Technology (TVT).

- An algorithm to jointly design the hybrid processors in a full-duplex relay has been proposed based on the Orthogonal Matching Pursuit (OMP) principle. The proposed algorithm differs from the existing work in the concurrent design feature in which each algorithm iteration uses the information extracted from the previous iterations. To date, full-duplex relay design has not been extensively studied. Therefore, a detailed analysis of the proposed system model using a mmWave channel with a limited number of paths has been derived in this work. The results highlight the effect of the channel and the array size on the performance of the system. Part of the analysis and the results have been accepted and presented on the IEEE WCNC 2016.

- Develop a split design algorithm to determine the hybrid processors at the multi-relay network. So far in the open literature, there have been no attempts to design the hybrid processors in multi-relay systems under the mmWave channels. Therefore, two different beamforming systems were proposed to increase the gain:
  - Analogue beamforming relays with hybrid beamforming at the terminal points.
  - Hybrid beamforming relays with hybrid beamforming at the destination and no processing at the source.

The results indicate the benefits of adding relays to the network. Part of the analysis and the results have been accepted and presented on the IEEE WCNC 2017.

1.4 Thesis Structure

The rest of this thesis is organised as follows.

Chapter 2 provides background information on key concepts in mmWave transmission and systems. The chapter begins by giving a brief overview of
the mmWave licensing history in the last century. Then, the propagation characteristics of the mmWave channels are described. The chapter also discusses the indoor and outdoor channel models describing the related measurement studies and literature. The second part of the chapter highlights the key theoretical concepts of antenna arrays, and the error resulted from using digital phase shifters to steer the signal.

Chapter 3 is sub-divided into two parts. The first part presents a literature review on state-of-the-art beamforming techniques in the mmWave systems. Analogue beamforming where single RF chain is used to minimise the power consumption is studied. Then, the new hybrid beamforming system with more than a single RF chain was discussed. The second part gives an overview of relay systems. The full-duplex relay and the multi-relay system models are mathematically described and explained. The final section summarises the latest researches and literature on relays in mmWave environments.

In Chapter 4, the use of analogue beamforming systems to extend the point-to-point link coverage in mmWave systems is investigate. The system spectral efficiency is analysed under certain assumptions in the presence of phase quantization error. The work examines three sounding techniques used to avoid the need for channel knowledge at the transmitter or the receiver. The three methods are compared by calculating the spectral and power efficiency of each technique. The chapter includes a derivation of the beamforming gain for two different scenarios. Exact expressions for the spectral efficiency, the SNR, and the capacity of the system were also derived. The new results help determine the most efficient search method, the duration of the search, and the optimum number of quantization bits to use.

In chapter 5, full-duplex relay with hybrid beamforming processors is used to extend the link between two points. An algorithm is developed for jointly designing the analogue and baseband matrices of the relay. In this chapter, the performance analysis of the system is derived under certain assumptions to help understand the system behaviour under more realistic conditions.

Chapter 6 investigates the use of the multi-relay network with analogue beamforming to prolong the link between two points in the mmWave environment. The results compare the system performance with the upper bound and the FD relay system.

Chapter 7 proposes the use of hybrid beamforming systems in multi-relay
networks. The performance of the system was compared to the unconstrained systems where full complexity RF chains are utilised.

Chapter 8 concludes the thesis and prospects for the future extension of the work are outlined.

1.5 Publications


Chapter 2

Background

MMWAVE is the latest piece of the puzzle of building the next cellular networks. Its vast spectrum allows the growing data demand associated with new applications. However, there is a lot of work needed in channel modelling and exploring the possibilities and challenges for mmWave in both indoor and outdoor environments. This chapter provides a background for the research on mmWave and some related topics. Section 2.1 reviews the history of mmWave systems development and frequency licensing. Section 2.2 discuss the propagation characteristics in mmWave while Section 2.3 address the indoor and outdoor channel models at the high frequency bands. Section 2.4 describe the antenna arrays and the error introduced by utilising digital phase shifters.

2.1 Millimeter Wave History

J. C. Bose pioneered the mmWave communications in the late 1890s when he was experimenting the transmission of electromagnetic waves at 60 GHz over a distance of 23 meters [38]. However, the actual use of this band was not until half a century later with the introduction of mmWave based radio astronomy in the 1960s, followed by the first military applications in the 1970s. The production of the mmWave integrated circuits in the 1980s led to the mass production of mmWave devices [39].

The 1990s was the real revolution era of the mmWave frequency band. Early in that decade, the Collision Avoidance Radar (CAR) was introduced as the first consumer-oriented use of mmWave above 40 GHz, operating at 77 GHz. In 1995, the Federal Communications Commission (FCC) authorized 57 GHz - 64 GHz for
unlicensed commercial use [40]. This action was followed by the authorization of the 59 GHz - 66 GHz band in Japan and the 57 GHz - 66 GHz band in Europe. During the late 1990s, Japan initiated research on 60 GHz communication which led to the development of the point-to-point base station-user using Monolithic Microwave Integrated Circuit (MMIC) with speed up to 156 Mbps. The end of the 20 century also witnessed the introduction of Complementary Metal-Oxide-Semiconductor (CMOS) technology in manufacturing mmWave devices which helped the expansion of using mmWave frequency band.

It was not until the late 1970s that the E-band was established for the first time as the band that include the frequencies 71-76 GHz and 81-86 GHz [41]. In 2002, the E-band was open for exclusive federal government use. Later, in October 2003, the FCC announced the E-band frequencies (71-76 GHz, 81-86 GHz, and 92-95 GHz) become available for licensed communication use. In 2005, the first commercial radios were installed. Later that year, European Telecommunications Standards Institute (ETSI) released a fix E-band, and followed that by the release of technical rules to use the band.

The interest of IEEE 802 group to develop a mmWave PHY layer start growing in 2003, which resulted in the formation of the 802.15 group on wireless personal area network. After the success of the interest group, a study group for mmWave PHY was formed in March 2004. They agree to develop PHY for 1 Gbps with the existing MAC IEEE 802.15.3b, and a group was established to create an indoor model. In September 2009, the IEEE-SA Standards Board approved IEEE 802.15.3c-2009. It took four and a half years for the task group to complete the first IEEE standard for mmWave frequencies. Such a duration has been common for many IEEE standards that provide new PHYs [17].

New York University wireless research group started working on measurements in the office environment in 2014. Later that year, the FCC released a notice of inquiry: use of spectrum band above 24 GHz for a mobile radio system [42]. Only to be followed by its UK counterpart, Ofcom, in 2015 releasing a call for input on spectrum above 6 GHz for future mobile communication.

2.2 Propagation in Millimeter Waves

Learning the characteristics of the radio wave propagation leads to understanding the design of the communication systems, its power requirements and the length
of the wireless links. Both small-scale and large-scale propagation conditions must be properly characterised for a particular operating frequency band, and little is known at frequencies above 60 GHz [43].

The propagation of the signal is directly affected by the frequency band of operation. The short wavelength of the mmWaves makes ordinary objects like trees and light posts obstacles that might likely block the signal. The short distance after which the signal dropped below the thermal noise level allowed the short range communications, which is also known as “whisper radio” communications. For longer range communications, rain and other weather factors are principal components in determining the cell size. Dense network architectures, where highest spectral frequency reuse is needed, use bands with peaks in atmospheric absorption while frequencies with low-attenuation are best suited for longer distance backhaul or cellular radio applications. Penetration, reflection, and diffraction characteristics are also important to understand to formulate a full image of the channel behaviour [2,18].

2.2.1 Large-Scale Propagation

Large-scale propagation properties characterise the variations due to path loss and shadowing as the transmitter and the receiver becomes separated over long distances, from meters to hundreds or thousand of meters. Small-scale propagation effects, on the other hand, occurs over very short distances as a result of receiving multiple paths at the receiver. Fig. 2.1 illustrates an example of the slow large-scale variations and the faster small-scale fading. The figure shows the rapid change in the signal as a result of movement and the much slower change in the average of the signal with distance [44,45]. The results of wideband Non-Line-Of-Sight (NLOS) measurements in the 9.6, 28.8, and 57.6 GHz bands showed significant signal attenuation (as great as 100 dB) due to large building obstructions [46].

2.2.1.1 Free Space Path Loss

The loss in signal strength of an electromagnetic wave that would result from a Line-Of-Sight (LOS) path through free space (usually air), with no obstacles nearby to cause reflection or diffraction is known as the free space path loss. According to Harald T. Friis [47], the free space received power at a receiver
CHAPTER 2. BACKGROUND

Figure 2.1: An example of small-scale and large-scale fading [44].

separated from the transmitter by a distance $d_x$ is defined by the Friis free space equation [43]

$$\rho_r(d_x) = \frac{\rho \Omega_t \Omega_r}{\mathcal{A}} \left( \frac{\lambda}{4\pi d_x} \right)^2$$

(2.1)

where $\rho_r$ and $\rho$ are the received and transmitted power, respectively, $\Omega_r$ is the receiver antenna gain, $\Omega_t$ transmitter antenna gain, and $\lambda$ is the operating wavelength of transmission. The unitless factor $\mathcal{A}$ is greater than unity and account for the antenna and components losses (losses not related to propagation). The free space model anticipates that the received power declines as a function of the link length. Additionally, for a fixed transmitter-receiver distance and fixed antenna gains, the free space path loss is proportional to the square of the operational frequency [44].

To demonstrate the effect of frequency on the path loss, we compare the free space path loss for the cellular systems (operate at 460 MHz), 2.4 GHz WiFi system, and mmWave system operating at 60 GHz. Assuming no additional losses ($\mathcal{A} = 1$), omnidirectional antennas, and equal transmission power level, the results for 1 meter distance were: $-25.7$ dB for 460 MHz, $-40$ dB for 2.4 GHz, and $-68$ dB for 60 GHz. For 100 meter separation between transmitter and receiver,
the free space losses are: $-65.7$ dB for cellular systems, $-80$ dB for WiFi, and $-108$ dB for mmWave systems. These estimations indicate that omnidirectional antennas will not be favourable at mmWave frequencies [43].

As the free space model in (2.1) does not hold for $d_x = 0$, large-scale model usually use a close-in free space path loss reference distance $d_0$ ($\gg \lambda$) in the far field. The received power $\rho_r(d_x)$ at any distance ($d_x > d_0$) may be related to the received power at $d_0$. It is customary to represent the propagation path losses in decibel values as the propagation models indicate that the received power decreases logarithmically with the distance. Furthermore, decibel values are much easier to handle mathematically since the multiplication of linear values results in a simple addition in decibel values. Now, the average large-scale path loss for a distance $d_x$ is expressed using a Path Loss Exponent (PLE), $\kappa$, [43,44]

$$PL(d_x)[dB] = PL(d_0) + 10\zeta \log_{10} \left( \frac{f}{f_0} \right) + 10\kappa \log_{10} \left( \frac{d_x}{d_0} \right)$$

where $\zeta$ is the frequency-loss factor. The path loss model in (2.2) is frequency-dependent due to the wide bandwidth of the channel. The value of the PLE depends on the particular environment, with $\kappa = 2$ being the PLE of free space. In certain situations when constructive interference and antenna directivity exist, the value of the PLE might get under 2. Yet, typically, $\kappa > 2$ because free space is the optimistic environment [43].

LOS measurements at 28 GHz indicate a PLE of 1.98 (virtually identical to the theoretical free space value of 2) at distances up to 100 m [48]. Nevertheless, at 38 GHz, [19,49] found that relatively long-range links (>$750m$) could be established. However, the setup where these measurements were performed has much lower building density and greater opportunities for LOS connectivity than would be found in a typical urban deployment. Measurements of the large-scale properties of the 28, 38, and 73 GHz wireless channels showed that mmWave systems could double the capacity over long-term evolution systems [2,50].

2.2.1.2 Weather (Atmospheric) Losses

Atmospheric effects constitute an addition to the signal degradation at mmWave frequencies, particularly in the 60 GHz band [43]. The molecular constituency of air and water take part in determining the coverage distance in high-frequency bands due to the small wavelength. Fig. 2.2 shows some of the atmospheric
attenuation characteristics of mmWave propagation. The oxygen absorption at 183, 325, and 380 GHz makes these band a perfect fit for the short range dense communications in indoor environments [51].

Rain significantly affects the signal at some mmWave bands, as shown in Fig. 2.3. For instance, heavy rain could cause up to 18 dB/km additional attenuation at the E-band [2]. Since the atmosphere attenuation peaks at 60 GHz, these frequency bands are considered for indoors and excluded from being used in outdoor communications. It will be necessary for outdoor mmWave links to overcome the rain effect by increasing the antenna gain through beamforming. Furthermore, as the physical dimensions of the snow flex and hailstones are on
2.2.1.3 Diffraction and Penetration

In cellular networks, the indoor users connect to the network through outdoor base station which requires the signal to travel through walls and windows causing high penetration loss which is worse in mmWave bands [54]. Research efforts were active in measuring the penetration characteristics of the mmWave channels for the next generation mobile communication. Initial tests by Samsung at the 28 GHz and the 40 GHz of obstructions such as wood, water, hands, and leaves...
showed that metal and water could attenuate the signal by 30 to 40 dB when very close to the receiver. Penetration tests for glass with metallized layers showed that attenuation increased by 25 to 50 dB per layer [46, 48]. Therefore, outdoor and indoor scenarios will probably be separated in 5G cellular architecture [54]. Diffraction happens when the signal bends around the objects in the path. At lower frequency bands, diffraction occurs as a result of things like the curved surface of the earth, hilly or irregular terrain, building edges, or obstructions blocking the LOS path between the transmitter and receiver [45]. However, due to the short wavelength at higher frequency bands, objects like the human body might cause diffraction at mmWave. Although diffraction is a useful mechanism for microwave communications, it is considered an ineffective and least reliable propagation mechanism in mmWave communications as it becomes very lossy with a movement of few centimetres [55]. Furthermore, many small objects in the environment can be considered as scatterers, thus exhibiting a highly reflective and scattering nature to create alternative viable links, except for the direct link [56].

2.2.2 Small-Scale Propagation

Small-scale fading describes the rapid fluctuation of the signal at short distances (divisions of wavelength) as a result of interference between multiple versions of the transmitted signal due arriving at the receiver with marginally different times. The multipath waves produce additional small fading effects such as time dispersion due to multipath delays and random frequency modulation caused by the Doppler shifts on different signal versions [44].

The small-scale fading of the wireless channel can be directly affiliated to the Channel Impulse Response (CIR). CIR contains all the information necessary to simulate or analyse any radio transmission through the channel [44].

If a single pulse is transmitted over a multipath channel, the received signal will appear as a pulse train, with each pulse in the train corresponding to the LOS component or a different multipath wave associated with a distinct scatterer or cluster of scatterers [45]. Fig. 2.4 illustrates an example of multipath channel’s CIR. In the following subsections, we discuss two important characteristics of the multipath channel: the time delay spread it causes to the received signal, and the Doppler spread of the channel as a result of movement and/or time-varying nature of the channel.
2.2.2.1 Delay Spread

The time delay spread of a wireless channel features the arrival times and energy of the received propagation paths. If the delay spread is small compared to the inverse of the signal bandwidth, then there is little time spreading in the received signal. However, when the delay spread is relatively large, there is a significant time spreading of the received signal which can lead to substantial signal distortion [45].

Some of the parameters to describe the delay spread of the signal includes the minimum excess delay time \( \tau_0 \) which is the time associated with the first path arrival; the maximum excess delay indicates the largest delay between initial reception and the last measurable multipath component of particular amplitude. The Root Mean Square (RMS) delay spread \( \tau_{RMS} \) is a parameter that characterises the propagation delays of a channel. \( \tau_{RMS} \) is useful in estimating if the channel will cause inter-symbol interference without the use of an equaliser. Generally, given \( \tau_{RMS} \), it is require \( \frac{\tau_{RMS}}{T_s} \) symbols to be equalised to remove the inter-symbol interference effect for symbol rate \( \frac{1}{T_s} \) [44].

Several measurement studies have been conducted to understand the delay spread behaviour of the mmWave channel. In the indoor environment, it is found that
the delay spread behaviour is relatively independent of the receiver location [57]. However, the delay spread is proved to be proportional to the room dimensions and the wall reflection coefficient [43]. In the outdoor channel, measurements show that using wider beamwidth steerable antennas or smaller gain antennas resulted an improved signal coverage with higher multipath delay spread due to lower path loss component values. That is over short distances and compared to narrow beam antennas. However, over long distances, arrays with large gain are preferred to obtain a LOS link.

2.2.2.2 Doppler Effect

The Doppler effect describes the change in the frequency as a result of movement. For a receiver moving at a velocity \( v \) towards a transmitter travelling with a velocity \( v_0 \) into the receiver direction, the frequency change at the receiver, also known as Doppler frequency, is given by [43]

\[
f_d = \tilde{f} - f = f \left( \frac{c + v}{c - v_0} - 1 \right)
\]

where \( f \) is the operating frequency of the signal, \( \tilde{f} = \frac{c + v}{c - v_0} f \) is the perceived frequency at the receiver, and \( c \) is the speed of light. As a result of (2.3), the Doppler effect might be 15-30 times higher at 28-60 GHz compared with microwave communication systems. It should be mentioned that the Doppler effect is classically derived under the assumption of omnidirectional antennas. Using highly directive antenna arrays in the system will introduce changes in the fading characteristics of the channel that depends on the angles of arrival [43].

Research shows that the wireless channel changes much faster at higher frequencies. Increasing the carrier frequency by a factor of ten found to result 10 times faster change. This develops a more rapid time-varying nature on the mmWave channel requiring shorter frame times or packet durations. Research studies still investigating the scope of utilising the directional antenna arrays on changing the frame time in future mmWave systems [19, 43].

2.3 Millimeter Wave Channel Modelling

The propagation characteristics at mmWave bands are quite different from those of frequency bands below 6 GHz. Smaller wavelengths at mmWave frequencies
have often been thought to result in higher attenuation (due to oxygen absorption and precipitation) through air, than that observed at today's cellular bands. Therefore, the deployment of mmWave systems is considered very challenging, particularly for outdoor communications [18, 56].

The diffraction and penetration characteristics of the mmWave channel, as explained in Section 2.2.1.3, lead to the expectation that the mmWave channels will exhibit a sparse multipath nature, instead of the rich-scattering nature demonstrated in conventional microwave channels. As a result, it is not possible to directly use microwave propagation models for mmWave systems. An in-depth understanding of the mmWave propagation characteristics is essential for the design and analysis of future mmWave wireless networks [55].

2.3.1 Indoor Channel Models

Modelling the indoor wireless channel at mmWave frequencies serve in determining design parameters such as link budget, coverage layout, and power management [43]. Over the last decade, with the increased interest in higher data rates for wireless communications, extensive measurements at mmWave band have been conducted in indoor environments [57–59]. However, one of the most investigated bands is the 60 GHz for its use in the unlicensed WirelessHD, WiGig Wireless Local Area Network (WLAN) devices [15], and the development of 60-GHz LAN and Personal Area Network (PAN) systems [51, 60].

Measurements indicate that large-scale PLE, $\kappa$, varies from 0.40 to 2.10 for LOS indoor scenario. Similarly, it varies from 1.97 to 5.40 for NLOS scenarios [56, 59, 61]. It is also found that the propagation properties are affected by the dimensions of the room, the polarization and beamwidth of the antenna array, and the wall reflection coefficient. Thus, in a typical LOS office environment, $\kappa$ takes values in the ranges from 1.16 to 2.17, and with NLOS the values range 2.83 to 4.00 [57, 62].

The IEEE published two standards for the PAN that operates at the mmWave frequencies. IEEE 802.15.3c, a single-input multiple-output channel model and only characterizes the Angle of Arrival (AoA) in azimuth domain [17], and IEEE 802.11ad a Multiple-Input Multiple-Output (MIMO) channel model and characterizes the double-directional angle properties [14]. The small-scale channel model used in the two standards is based on a modified version of the standard Saleh-Valenzuela (S-V) propagation model, [63], where the channel is assumed
Figure 2.5: Graphical representation of the indoor channel model [17].

to be a sum of $P$ scattering clusters, each of which contributes $L$ propagation paths to the channel. Fig. 2.5 illustrates the nature of the clustered model. The existence a direct LOS link between the transmitter and the receiver is considered as the main difference to S-V models. Mathematically, the CIR model can be represented as follows [64,65]

\[
\mathbf{h}(t, \theta^t, \theta^r) = \alpha_{\text{LOS}} \delta(t, \theta^{\text{LOS}}) + \\
\sum_{p=0}^{P} \sum_{l=0}^{L} \alpha_{p,l} \delta(t - T_p - \tau_{p,l}) \delta(\theta^r - \Phi^r_p - \epsilon^r_{p,l}) \delta(\theta^t - \Phi^t_p - \epsilon^t_{p,l})
\] (2.4)

where the LOS component is assumed to arrive at zero delay with angle $\theta^{\text{LOS}}$ and gain $\alpha_{\text{LOS}}$. Each of the rays in (2.4) is arriving with an individual delay of $(T_p + \tau_{p,l})$. The model also indicates that the rays are clustered in angles which mean they arrive in small deviations from a few key angles given by $\theta^r$ for angle of arrival and $\theta^t$ for departure. The impinging departure direction of each ray is given by $\theta^t - \Phi^t_p - \epsilon^t_{p,l}$. Similarly, the arrival angle of the ray is given by $\theta^r - \Phi^r_p - \epsilon^r_{p,l}$. Given no additional directivity gain, the multipath components
within each cluster are exponentially deteriorating [43]

\[ E \{ |a_{p,l}|^2 \} = \mathcal{U}_0 e^{-T_p/\chi c} e^{-\tau_{p,l}/\chi} \]  

(2.5)

where \( \mathcal{U}_0 \) is the mean energy of the first path of the first cluster, \( \chi_c \) is the cluster decay factor, and \( \chi \) is the ray decay factor. \( p, l \) refers to the \( l \)th path of the \( p \)th cluster [43,66].

It should be noted that the time of arrival, azimuth-angle, and elevation angle distributions are all assumed independent [65]. For a summary of the values for different distributions, the reader might refer to [43].

Large-scale fading at the 60 GHz band is based on a log distance dependence of path loss with log-normal shadowing. Log-normal shadowing means the path loss is normally distributed over the decibel scale. The path loss model is given by

\[ PL(d_x, f) = PL(d_0) + 10\zeta \log_{10} \left( \frac{f}{f_0} \right) + 10\kappa \log_{10} \left( \frac{d_x}{d_0} \right) + X_\sigma \]  

(2.6)

where \( X_\sigma \) is the shadowing log-normally distributed random variable [65].

The IEEE 802.15.3c channel model includes nine different models for various scenarios and environments, as described in Table 2.1. From the list of settings, it is illustrated that the IEEE 802.15.3c model can be suitable for both LOS and NLOS indoor environments. Measurements showed that the mean number of clusters ranges most typically from 3 to 4. It was also found that the number of clusters distribution depends greatly on the environment. IEEE 802.11ad is similar to IEEE 802.15.3c with more focus on the 60 GHz band for commercial use. The IEEE 802.11ad channel model uses two different situations to categorise the clusters: Station-to-Station (STA-STA) and Station-to-Access Point (STA-AP), then, the number of direct and reflected paths for each model is determined. Table 2.2 show the average number of clusters for each scenario in small conference room environment as found using ray tracing method. A detailed comparison of the channel models in two standards can be found in [67].
Table 2.1: IEEE 802.15.3c channel models for various indoor environments where a human is holding a portable device [43].

<table>
<thead>
<tr>
<th>Channel Model</th>
<th>Scenario</th>
<th>Environment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CM1</td>
<td>LOS</td>
<td>Residential</td>
<td>Home with wood or concrete walls and floors. Carpet, furniture, doors and windows exist</td>
</tr>
<tr>
<td>CM2</td>
<td>NLOS</td>
<td>Residential</td>
<td></td>
</tr>
<tr>
<td>CM3</td>
<td>LOS</td>
<td>Office</td>
<td>Concrete or metal walls and floor. Multiple office desks, chairs and computers. Long corridors connect offices together</td>
</tr>
<tr>
<td>CM4</td>
<td>NLOS</td>
<td>Office</td>
<td></td>
</tr>
<tr>
<td>CM5</td>
<td>LOS</td>
<td>Library</td>
<td>Large doorways and large windows. Multiple desks and book-filled shelves.</td>
</tr>
<tr>
<td>CM6</td>
<td>NLOS</td>
<td>Library</td>
<td></td>
</tr>
<tr>
<td>CM7</td>
<td>LOS</td>
<td>Desktop</td>
<td>Office and computer clutter. Often enclosed by cubicle.</td>
</tr>
<tr>
<td>CM8</td>
<td>NLOS</td>
<td>Desktop</td>
<td></td>
</tr>
<tr>
<td>CM9</td>
<td>LOS</td>
<td>Kiosk</td>
<td>Station in a public venue. User stand in front of the kiosk.</td>
</tr>
</tbody>
</table>
CHAPTER 2. BACKGROUND

Table 2.2: The average number of clusters for the IEEE 802.11ad channel model for a conference room environment [43].

<table>
<thead>
<tr>
<th>Type of clusters</th>
<th>Number of clusters for STA-STA sub-scenario</th>
<th>Number of clusters for STA-PA sub-scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS path</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>First order reflection from walls</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Second order reflection from two walls</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>First order reflection from ceiling</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Second order reflection from the walls and ceiling</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

2.3.2 Outdoor Channel Models

There has been relatively limited study of the mmWave outdoor channels as compared to indoor environments. With the increased interest of 5G communication groups and industry in the large spectrum provided by the mmWave, the knowledge of outdoor propagation characteristics will be more essential. While mmWave indoor channels have been well modelled, there are not yet a satisfactory outdoor mmWave channel model [43].

Measurements have been performed in different urban cities, such as Daejeon, Korea [68] and Manhattan, New York, USA [2, 18, 69], at frequency bands of 10, 18, 28, 38, 60, 72 and 81-86 GHz [68, 70–73]. Propagation through a canopy of orchard trees at 9.6, 28.8, and 57.6 GHz bands indicated that through the first 30 meters of foliage depth, signal attenuation over distance is linear with approximately 1.3-2.0 dB/m of loss, and beyond this distance attenuation was only about 0.05 dB/m [74]. Other outdoor measurements in a city street environment at 55 GHz showed that power decreased much more rapidly with distance through narrower streets compared to a direct path or through wide city streets [75].
CHAPTER 2. BACKGROUND

E-band channel measurements band from 81-86 GHz for point-to-point communications in a long street canyon environment in Helsinki, Finland showed very little multipath delay spread, and yielded excellent agreement between measured and modelled delay spreads [76, 77]. The study proved that multipath exists in the E-band channel, but extensive channel measurements for mobile or backhaul were not reported [22].

2.3.3 MIMO Channel Model

MIMO systems will be used in the thesis to steer the signal beam and provide additional gain. We adopt a geometric channel model based on the extended Saleh-Valenzuela model in which the channel is assumed to be a sum of $P$ scattering clusters, each of which contributes $L$ propagation paths to the channel [66, 78]. With half-wave spaced ULAs used at the transmitter and the receiver, the channel matrix, $H$, can be expressed as

$$H = \sqrt{\frac{N_r N_t}{PL}} \sum_{p=1}^{P} \sum_{\ell=1}^{L} \alpha_{p\ell} a_r(\varphi_{p\ell}^r, \theta_{p\ell}^r) a_t^*(\varphi_{p\ell}^t, \theta_{p\ell}^t)$$

(2.7)

where $N_t, N_r$ are the array size at the transmitter and receiver, $\alpha_{p\ell}$ is the complex small-scale fading gain of the $\ell$-th sub-path in the $p$-th cluster, $\theta_{p\ell}^r(\varphi_{p\ell}^r)$ and $\theta_{p\ell}^t(\varphi_{p\ell}^t)$ are the azimuth (elevation) angles of arrival and departure, respectively. The gains $\alpha_{p\ell}$ are complex Gaussian random variables with zero mean and variance $\sigma^2_{\alpha}$. The mean angle associated with each cluster is uniformly distributed over $[0, 2\pi]$. The vectors $a_r(\varphi_{p\ell}^r, \theta_{p\ell}^r)$ and $a_t(\varphi_{p\ell}^t, \theta_{p\ell}^t)$ are the array response vectors at the receiver and transmitter, respectively.

The array response vectors $a_t$ and $a_r$ corresponding to Angle of Departure (AoD) and Angle of Arrival (AoA) in the azimuth of the frame are given as

$$a_r = \frac{1}{\sqrt{N_r}} \begin{bmatrix} e^{j2\pi \frac{\varphi_{p\ell}}{\lambda} \sin(\theta^r)} & \ldots & e^{j2\pi (N_r - 1) \frac{\varphi_{p\ell}}{\lambda} \sin(\theta^r)} \end{bmatrix}^T$$

(2.8)

$$a_t = \frac{1}{\sqrt{N_t}} \begin{bmatrix} e^{j2\pi \frac{\varphi_{p\ell}}{\lambda} \sin(\theta^t)} & \ldots & e^{j2\pi (N_t - 1) \frac{\varphi_{p\ell}}{\lambda} \sin(\theta^t)} \end{bmatrix}^T$$

(2.9)

where $d$ is the space between the antennas, $\lambda$ is the wavelength, and the array is pointing along the x-axis.
2.3.4 Ray Tracing and Stochastic Modelling

One of the traditional approaches for modelling physical channels is propagation prediction using ray tracing simulations. Ray tracing techniques approximate the propagation of electromagnetic waves by representing the wave-fronts as simple particles. Thus, the reflection, diffraction, and scattering effects on the wave-front are approximated using simple geometric equations instead of the more complicated Maxwell’s wave equations [45]. Although ray tracing simulations often suffer some geometry database errors and do not always include all of the relevant propagation mechanisms, ray tracing results have been successfully used to model wireless radio propagation [71]. Ray tracing could be simply implemented using a simple two-ray model that predicts signal variation resulting from a ground reflection interfering with the LOS path. More sophisticated models could include a ten-ray reflection model that predicts the variation of a signal propagating along a straight street or hallway and a general model that predicts signal propagation for any propagation environment. Fig. 2.6 illustrates the two-ray and second order reflection ray tracing models.

Another approach to model the channel is stochastic modelling, which characterizes the channel behaviour using the probability distribution functions of the propagation parameters [55]. This approach is considered more analytically tractable as it avoids the need for physical modelling of the environment. A Geometric Based Stochastic Channel Model (GSCM) was proposed in Wireless World Initiative New Radio Project (WINNER) and the International Telecommunication Union - Radiocommunication Sector (ITU-R), which is a very popular platform for 4G channel simulation [79]. More details on the mmWave geometric models can be found in [80] and references within.

2.4 Antenna Arrays

In mmWave communication systems, it is necessary to utilise antennas with highly directive characteristics (enormous gains) to meet the demands of long distance communication. The additional gain can be accomplished by increasing the electrical size of the antenna by setting an arrangement of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multi-elements, is referred to as an array. In an array of identical elements, several ways can be used to shape the overall pattern of the antenna. Such methods
Figure 2.6: Ray tracing channel modelling (a) Side view of two ray model with LOS and ground reflection paths [45] (b) Top view of outdoor propagation geometry illustrating LOS and wall reflected paths up to the second order [81].
include the geometrical configuration of the overall array (linear, circular, rectangular, spherical), the relative displacement between the elements, the signal amplitude and excitation phase of the separate antennas, and the corresponding pattern of the individual elements [82].

2.4.1 Uniform Linear Array

The simplest and one of the most practical arrays is formed by placing the antennas along a line. The total field of the array is equal to the field of a single element positioned at the origin multiplied by a factor which is broadly referred to as the array factor. That is [82],

\[
E_{\text{total}} = E_{\text{single antenna element at reference point}} \times \text{Array Factor.} \tag{2.10}
\]

The array factor is a function of the geometry of the array and the excitation phase. By varying the separation and/or the phase between the elements, the characteristics of the array factor, consequently of the total field, of the array can be controlled. Each array has its array factor. The array factor, in general, is a function of the number of elements, their geometrical arrangement, the elements’ relative magnitudes and relative phases, and the spacings between the elements.

Since the array factor does not depend on the directional characteristics of the radiating elements themselves, it can be formulated by replacing the actual elements with isotropic (point) sources. An array of identical elements with equal elements’ magnitude and each with a progressive phase is referred to as a uniform array [82].

Referring to the geometry of Fig. 2.7, let us assume that all the elements have equal amplitudes, but each element has a progressive phase, \( \varepsilon \), lead current excitation relative to the preceding one (\( \varepsilon \) represents the phase by which the current in each antenna leads the current of the preceding element). The array factor can be obtained by considering the elements to be point sources. If the real elements are not isotropic sources, the total field can be formed by multiplying the array factor of the isotropic sources by the field of a single antenna. The array factor, \( \nabla \), for \( N \)-elements array is given by [82]

\[
\nabla = 1 + e^{j(kd \cos \phi + \varepsilon)} + e^{j2(kd \cos \phi + \varepsilon)} + \cdots + e^{j(N-1)(kd \cos \phi + \varepsilon)} \tag{2.11}
\]
Figure 2.7: Far field geometry of N-element array of isotropic antennas.

\[ F = \sum_{n=1}^{N} e^{j(n-1)(kd \cos \phi + \varepsilon)} \]  

where \( k = \frac{2\pi}{\lambda} \) is a scaler quantity known as the wave-number, and \( d \) is the spacing between the array elements. The array factor can be also written as

\[ \nabla = \sum_{n=1}^{N} e^{j(n-1)\varpi} \]  

where \( \varpi = kd \cos \phi + \varepsilon \).

The array factor could be used to determine the direction of the main lobe, the sidelobes and the nulls in the beam. Further details on using the array factor equation to find the first and second maxima in addition to further mathematical derivations could be found in [82].
2.4.2 Array Steering

In many applications, it is desirable to have the maximum radiation of an array directed towards a specific direction. To optimise the design, the maxima of the single element and of the array factor should both be steered toward the required direction. The requirements of the separate elements can be accomplished by the judicious choice of the radiators, and those of the array factor by the proper separation and excitation of the individual radiators. Referring to (2.11)-(2.13), the first maximum of the array factor occurs when

\[ \varpi = kd \cos \phi + \varepsilon = 0 \]  

(2.14)

Let us assume that the maximum radiation of the array is required to be oriented at an angle \( \phi_0 \) \((0 \leq \phi_0 \leq 180)\) [83]. To get this direction, the phase excitation, \( \varepsilon \), between the elements must be adjusted so that

\[ \varpi = kd \cos \phi + \varepsilon |_{\phi=\phi_0} \]
\[ = kd \cos \phi_0 + \varepsilon = 0 \]
\[ \Rightarrow \varepsilon = -kd \cos \phi_0 \]  

(2.15)

Thus by controlling the progressive phase difference between the elements, the maximum radiation can be squinted in any desired direction to form a scanning array. This is the basic principle of electronic scanning phased array operation. Since in phased array technology the scanning must be continuous, the system should be capable of continuously varying the progressive phase between the elements. In practice, this is accomplished electronically by the use of ferrite or diode phase shifters.

2.4.3 Phase Quantization

Digital phase shifters are known to have lower power consumption, higher integration, and occupy a smaller size as compared to analogue ones. Thus, it provides more reliability and cost efficiency. When the beam is to be directed into a given direction, the individual phases of the antenna elements are calculated as discussed in Section 2.4.2. However, using \( q \)-bit phase shifters, each antenna
element phase is to be chosen from a size $2^q$ phases set

$$\vartheta_m \in \left\{ 0, 2\pi \left( \frac{1}{2^q} \right), 2\pi \left( \frac{2}{2^q} \right), \ldots, 2\pi \left( \frac{2^q - 1}{2^q} \right) \right\}. \quad (2.16)$$

The use of these quantized phases introduces the phase quantization error which directly causes beam steering error and the appearance of parasitic sidelobes \[84,85\].

Over the past century, several literature studies have been published to address the issue on the phase quantization, trying to minimise the negative impact associated with the digital phase shifter utilisation. These works include the well-known phase-added method by Miller, \[86\], where a random start (physically) introduced in the feed section of each element which should be phased. Then, the added phases subtracted from the phases which are to be set up to steer the beam and round off the remaining phases. This is a random phasing which makes the mean pointing deviation equal to zero. The physical significance of this method is that the average pointing deviation is equal to zero. There are no parasitic sidelobes \[87\]. Two probable value method purpose having no pointing deviation in the mean power pattern by assuming that the imaginary part of the mean exponential function of the phase errors is equal to zero. In three Probable Value Method, \[84\], the digital phase shifters are set up to approximate with the three nearest quantization phases instead of one or two as in the previous methods. This method has similar results to that for the phase-added method but does not require start phases at the elements. Other methods include the mean phase error equal-to-zero method by Aronov, \[88\], as well as the random phasing method \[89\].

Table 2.3 summarizes the highlights of these methods. For each method, \[87\] derived a distinct probability distribution of phase errors. The probability distribution of the phase errors also relates to the other important properties of the random phased array, such as the variance of the field pattern, which usually equals the average far out sidelobe level, the gain reduction due to random phasing, and the variance of the beam pointing deviation. A detailed comparison of these methods has been introduced in \[84,87\].
Table 2.3: Comparison of random phasing methods [87].

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean pointing deviation</th>
<th>Mean Maximum Parasitic sidelobe level (dB)</th>
<th>Variance of Phase Error</th>
<th>Beamsteering Unit function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round off</td>
<td>≠ 0</td>
<td>−6$q$†</td>
<td>$δ^2/3$†</td>
<td>Rounding off</td>
</tr>
<tr>
<td>Mean Phase error equal zero</td>
<td>≠ 0</td>
<td>−12$q$</td>
<td>$2δ^2/3$</td>
<td>Random number generation, test for rounding up or down</td>
</tr>
<tr>
<td>Phase added</td>
<td>0</td>
<td>Not present</td>
<td>$δ^2/3$</td>
<td>Memory of start phases, rounding off</td>
</tr>
<tr>
<td>Two probable value</td>
<td>0</td>
<td>−12$q$</td>
<td>$≈ 2δ^2/3$</td>
<td>Random number generation, test for rounding up or down</td>
</tr>
<tr>
<td>Three probable value</td>
<td>0</td>
<td>Not present</td>
<td>$δ^2$</td>
<td>Random number generation, test to choose one of three values</td>
</tr>
</tbody>
</table>

† $q$: number of bits

‡ $δ = π/2^q$
2.5 Chapter Summary

In this chapter, key propagation characteristics of the mmWave channel are discussed. The large-scale and small-scale propagation effects were studied showing how the environment is affecting communications at the high-frequency bands. The chapter also reviews the measurements performed at the mmWave frequencies in the indoor and outdoor environments which led to the first standard at the mmWave band, IEEE 802.15.3c. Finally, we summarise the principle of antenna arrays and how they are used to steer the signal towards a particular direction. In addition, the chapter includes explaining the error resulted from using digital phase shifters in the arrays.
Chapter 3

Millimeter Wave Relay Networks: A Literature Review

In addition to the increased free space losses, there are relatively higher atmospheric losses introduced at the mmWave band. These losses limit the length of the mmWave link in the outdoor environments. Various proposals have been made to overcome these challenges. One of the main solutions proposed by several research groups is the use of smaller, dense networks where the cell size is in the few meters zone. Other recommend extending the link distance using massive MIMO systems and increasing the directivity of the array.

Although the small wavelength of the mmWave band brings some challenges, it also provides solutions. The antenna size at such high-frequency bands is very small which allows packing a large number of antennas into the transmitting and receiving arrays. Thus, beamforming is very necessary to increase the directivity and to extend the link coverage at mmWave systems. During the last decade, several research centres have been investigating beamforming design and analysing the impact on the network performance. Various forms of analogue beamformers are proposed where phase shifters or switches are used to steer the beam [24, 26, 90]. More recent attention has focused on the use of hybrid (analogue and digital) beamforming processors to overcome the limitations usually associated with Analogue Beamforming (ABF) [28, 29, 91]. Design algorithms for various scenarios have been proposed using single and multiple users settings [31, 92].

Furthermore, an important and growing body of literature has investigated the use of relays to build mmWave based networks. Relays that could be fixed on
trees and lampposts is a sensible solution that enables the architecture of the small cells. A major advantage of using cooperative networks is the minimal power requirements for both the user and the base station. Additionally, relays help avoid the shadowing area that is considered a major setback for mmWave systems.

In this chapter, some key papers in literature have been selected to establish for the schemes and architectures proposed in this thesis to improve the connectivity of mmWave link. To introduce for analogue beamforming systems and study its impact on the mmWave communication we consider the work published in [24]. With regards to hybrid beamforming systems, the models, and the design algorithms we discuss the works published in [29,93]. In the case of full-duplex relays with multiple antenna we refer to the work published in [94], and regarding the design of relays in mmWave communications, we refer to [95]. Finally, to study the multi-relay network, we refer to the works in [96,97].

There is a relatively limited number of research involving the design of relay systems in the mmWave environment, yet, the interest is growing in this topic. It is considered one of the focus points of research in wireless communications.

### 3.1 Beamforming in mmWave Systems

With the propagation losses introduced at the mmWave frequencies, an additional gain needs to be added to the system in order to be suitable for adopting at the next generation wireless communications. One important approach is to utilise the privilege introduced by the small size of the antenna elements at the mmWave frequencies to increase the directivity gain by adding more antenna elements to the system. The fundamental principle of beamforming is to transmit identical information on each element of the array while varying the amplitude and/or the phase of the signal at each antenna. The direction and shape of the array’s effective beam are controlled by the change of the phases and amplitudes of the individual antennas in the array, as explained in Section 2.4.

In conventional MIMO systems, the beamforming is performed at the baseband level, also known as Digital Beamforming (DBF). Digital signal processing is performed where each antenna element is connected to a separate RF chain\(^1\) (hence

\(^1\)RF chain is typically referred to the combination of LNA, ADC (or DAC) and antenna power amplifier.)
it is considered as full complexity RF chain system), as illustrated in Fig. 3.1. However, at higher frequency bands, it is practically difficult to implement the power and low noise amplifiers for all the antennas. Additionally, using the same number of transceivers as the large antenna number will be both costly and power consuming. The only reliable solution is to use less RF chains than the antenna array size [93, 98].

3.1.1 Analogue Beamforming

One of the simplest approaches to improve the directivity gain in mmWave systems is analogue beamforming where a single RF chain is used. A network of phase shifters is used to control the phase of the signal fed to the antenna array, as shown in Fig. 3.2. Analogue beamforming system is used to provide the narrow beam required in wireless backhaul links in [24]. The authors employed beam alignment techniques using adaptive subspace sampling and hierarchical beam codebooks. Then, the performance was compared with other search and alignment methods.

To further demonstrate the principle of analogue beamforming, consider a MIMO
communication system with transmit beamforming vector $\mathbf{f}$ and a receive beamforming vector, also known as the combining vector, $\mathbf{w}$. Using $s$ as the information to be transmitted, the system input-output relationship can be written as

$$y = \mathbf{w}^* \mathbf{H} s + \mathbf{w}^* \mathbf{n}$$

(3.1)

where $\mathbf{H}$ is the mmWave channel, and $\mathbf{n}$ is the noise vector. The effective channel captured at the receiver will be equal to $\mathbf{w}^* \mathbf{H} \mathbf{f}$. Now, the accurate selection of the beamforming and combining vectors results in an increased channel gain $|\mathbf{w}^* \mathbf{H} \mathbf{f}|$, with the maximum gain is achieved by setting $\mathbf{f}$ to the dominant left singular vector of $\mathbf{H}$ and $\mathbf{w}$ to the right singular vector.

The small separation between the elements in the array (usually are much less than the coherence distance of the channel) leads to a relation among the different versions of the signal arriving at each particular antenna. Which is a function of the signal of arrival. Considering an example with similar conditions to that used in [24] with a single path and a uniform linear array (where the antenna elements are uniformly half-wavelength spaced) are assumed. The channel model in (2.7) can be re-written as $\mathbf{H} = \alpha \mathbf{a}_r(\theta^r) \mathbf{a}_t^*(\theta^t)$, where $\alpha$ is the complex gain of the single path. This channel is sparse and has unity rank since there is only one propagation path. In this example, the optimum beamforming vectors can
be implemented using phase shifters by setting $f = a(\theta^p)$ and $w = a(\theta^r)$. The performance of the analogue beamforming in this case is limited by the use of quantized phase shifts and the resulted phase quantization error, as explained in Section 2.4.3. This makes it more challenging to finely tune the beams and steer nulls.

The vast number of antenna elements in the system also make it harder to obtain channel information using the traditional estimation techniques. Several studies advocate employing channel sounding to determine the best weight vectors for the phased arrays using a codebook which includes beam patterns at different resolutions [24, 25, 27, 99–105]. The ABF system sounds the channel specific number of times to find the optimum performance by judging the beamforming gain. For packet transmission, where the channel will be fixed for a specific time duration, the sounding (or beam alignment) returns:

$$(w_{\text{opt}}, f_{\text{opt}}) = (w_{v_{\text{opt}}}, f_{v_{\text{opt}}}) \quad \text{with} \quad v_{\text{opt}} = \arg \max_v |y_{\text{train}}[v]|^2$$ (3.2)

where $1 \leq v \leq L_T$ is the number of training packets, $(w_{\text{opt}}, f_{\text{opt}})$ is the optimum beamformer-combiner pair that maximises the gain, and $y_{\text{train}}$ is the received signal during the sounding period. During the sounding, sampling and searching the space could be performed adaptively or non-adaptively. The most straightforward method is the comprehensive search, in which the two sides examine all the possible vector combination in order to reach a decision of which provide the maximum gain. As shown in Fig. 4.2, the search process involves a rotary check of all the direction vectors in the codebook. A detailed comparison between various channel sounding techniques is discussed in Section 4.2.2.

To illustrate the beam training concept, we describe the codebook-based beamforming protocol in IEEE 802.15.3.c [16]. The protocol uses sets of antenna weights known as beamforming codebooks. Each vector of the codebook controls the phases and amplitudes of the array for a certain beam pattern. The protocol assumes that all devices support three kinds of beam patterns, as shown in Fig. 3.3: quasi-omni pattern, sector, and beam. Quasi-omni pattern is the lowest resolution pattern specified in the codebooks. It is used to refer to an antenna pattern that covers a broad region of interest space around Devices (DEVs). Sector is the second level resolution pattern and is used to refer to a direction of an array pattern that covers a relatively broad area of consecutive or non-consecutive
beams. Beam is the highest resolution pattern specified in the codebooks [106]. The patterns are used as part of a multi-stage protocol: DEV-to-DEV linking, sector-level searching, beam-level searching and an optional beam-tracking stage. The objective of the first stage is to identify the best quasi-omni patterns for both transmission and receiving by each device. The second stage determines the best transmission and receiving sectors for each device. Control information is fed back over the quasi-omni patterns. The final stage proceeds using the results of the previous search to find the sharpest beam for transmission and reception. Beam-tracking is an optional stage that is used to track the changes in the transmit and receive weight vector due to channel variation over time. With the help of the beam-tracking stage, the beamformer does not have to re-conduct immediately even if the optimal beam pair is lost. Instead, a backup beam pair found during tracking may be used to continue the already started data streaming [43, 106].

There are several implications of using analogue beamforming for mmWave MIMO. Analogue beamforming with a single RF chain only supports single-user and single-stream transmission. This means it is not possible to realise multi-stream or multi-user benefits associated with MIMO. In addition, steering the beams is not trivial, especially when a communication link has not yet been established.
3.1.2 Hybrid Beamforming

The analogue system has several limitations in controlling the signal amplitude and signal phase control resolution and several other constraints. Hybrid architectures are proposed to improve beamforming gain and enhance the MIMO communication benefits by enabling multiple stream precoding [107, 108]. As illustrated in Fig. 3.4, the hybrid system separate the MIMO optimisation process into analogue and digital domains. The principal design criterion is limiting the number of RF chains and make it less than the number of antennas is the system. The analogue beamforming is considered as a special case of hybrid systems when the number of RF chains and the number of streams equals one. Using multiple streams, the hybrid approach allows spatial multiplexing and multi-user MIMO to be implemented. Mathematically, the input-output relation of the MIMO system with hybrid precoding at the transmitter and hybrid combining at the receiver is represented as

\[
y = \sqrt{\rho} W_{BB}^* W_{RF}^* H F_{RF} F_{BB} s + W_{BB}^* W_{RF}^* n
\]  

(3.3)

where \( F_{RF} \) is a matrix corresponding to the analogue beamforming and \( F_{BB} \) is a baseband precoder matrix. Similar definitions apply to \( W_{RF} \) and \( W_{BB} \).
Figure 3.5: Analogue processing for hybrid beamforming based on phase shifters: (a) each RF chain is connected to all the antennas; (b) each RF chain is connected to a subset of antennas [28].

As illustrated in Fig. 3.5, two hybrid architectures are proposed in literature. In the first structure, each transceiver is connected to all the antennas while in the second architecture, the array is divided into sub-arrays, each of which is connected to a separate RF chain. The first structure is considered as a natural combination of analogue and digital precoding and has high structural complexity. The second structure, on the other hand, has lower beamforming gain with a much reduced complexity [93].

Furthermore, when it comes to implementing the phase shifters in hybrid systems, some works propose using digitally controlled phase shifters. The digital precoders help in correcting the analogue lack of precision. Another realisation makes use of switching networks to further complexity reduction and better energy efficiency. This architecture, illustrated in Fig. 3.6, exploits the sparse nature of the mmWave channel by implementing a compressed spatial sampling of the received signal. The analogue combiner design is performed by a subset antenna selection algorithm instead of an optimisation over all quantized phase
 values. Every switch can be connected to all the antennas if the array size is small or to a subset of antennas for larger arrays.

Now, a general approach for hybrid precoding in (3.3) would be to maximize the mutual information given by [90]

\[ I(\rho, FRF, FBB, WRF, WBB) = \log |I + \rho R_w^{-1} W^* H F F^* H^* W| \] (3.4)

where \( W = W_{RF} W_{BB}, \ F = F_{RF} F_{BB}, \) and \( R_w = W W^*. \) Optimizing (3.4) directly is challenging due to the constraint sets. An alternative proposed in [32] by assuming that the receiver performs ideal decoding, neglecting the receiver hybrid constraint. Effectively this removes the terms that depend on \( W \) from (3.4). With some approximations, this leads to a new problem where the hybrid precoders are found by approximating the unconstrained precoder \( F_u \), given by

---

**Figure 3.6:** Analogue processing for hybrid beamforming based on switches: (a) each RF chain can be connected to all the antennas; (b) each RF chain can be connected to a subset of antennas. [28].
the channel Singular Value Decomposition (SVD)\(^2\) solution \[29\]

\[
(F_{RF}^{\text{opt}}, F_{BB}^{\text{opt}}) = \arg\min_{F_{BB}, F_{RF}} ||F_u - F_{RF}F_{BB}||_F, \\
\text{s.t. } F_{RF} \in \mathcal{F}_{RF}, \\
||F_{RF}F_{BB}||_F^2 = N_s
\] (3.5)

where \(\mathcal{F}_{RF}\) is the set of feasible RF precoders which correspond to a hybrid architecture based on phase shifters, i.e., the set of matrices with constant magnitude entries. \[29\] proposed an approximate solution for this problem by using the sparse channel model and produced a related problem that involves configuring the RF beamforming vectors from a dictionary of steering vectors based on channel angles. This solution was found to be close to the unconstrained digital answer and offer substantial gains over the case of single-stream analogue beamforming. The hybrid precoding design problem based on the dictionary approach is extended to an architecture based on subarrays in \[110\]; the sparsity of the channel is also used to define an efficient way to find the near-optimal precoder.

Most work on hybrid precoding, like \[29, 91\], requires the availability of channel knowledge, at least at the receiver. To relax this assumption, \[108\] develops a hybrid precoding algorithm for mmWave systems based on partial channel knowledge. With a two-stage algorithm, \[108\] showed that the hybrid precoding performance with perfect channel knowledge could be approached when each of the transmitter and the receiver knows only its AoDs (or AoAs). Other variations of hybrid precoding with arrays of sub-arrays of phase shifters were considered in \[93, 111\]. An algorithm that adaptively estimates the channel parameters was developed in \[30\] using a hierarchy multi-resolution codebook for training. Multi-user hybrid beamforming presented in \[92\], where the base station has hybrid beamforming system and serves multiple users with ABF and a single RF chain. The work introduces the performance analysis for two extreme cases to get an insight of the proposed algorithm: with single path channel, and with large antenna systems.

\(^2\)For more details on the SVD, see Appendix C.
3.2 Relay Networks

Recently, numerous research studies consider relay assisted links as the base for various new architectures of the next generation cellular communications. At low-frequency bands, relays are used to overcome the effect of signal fading due to multipath propagation and strong shadowing. As the propagation characteristics get worse at high frequencies, the implementation of relay networks becomes more essential. The cooperative network is considered as a fundamental architecture that provides reliability and higher spectral efficiency over direct links. One of the leading motives behind the use of cooperative communications lies in utilising the spatial diversity provided by the network nodes.

This section discusses the use of single and multi-relays to provide better point-to-point links. In order to fully understand the system model and performance, we selected few key papers from the literature of relay networks both in MIMO and Single Input, Single Output (SISO) systems. Namely, we discuss the single full-duplex relay system presented in [112], the multi-relay networks in [96, 97], and the design of the half-duplex relay processors at mmWave frequencies presented in [113].

3.2.1 Full-Duplex Relay System

A relay node act as a bridge for the signal between two nodes. There are two schemes when it comes to communicating the signal between two nodes. The relay might use two different time slots, one for receiving and the other for transmission, which is known as Half-Duplex (HD) relaying. Alternatively, the relay can transmit and receive the signal for the two points simultaneously using one time slot, which is known as Full-Duplex (FD) relaying. While full-duplex relaying provides a better use of the resources available, it encounters the unfavourable effect of self-interference where the receiving array receives the relay transmission.

Recently, there has been tremendous progress in the self-interference mitigation techniques that made full-duplex relaying a reliable scheme. [114] compared several mitigation schemes: natural isolation, time-domain cancellation, and spatial suppression. [114] also proposes spatial suppression by antenna and beam selection, null-space projection, and Minimum Mean Square Error (MMSE) filtering. The results confirm that self-interference can be mitigated effectively. Full-duplex relaying and a wide range of self-interference mitigation techniques were reviewed
Consider using a relay to connect a receiver to a transmitter in a point-to-point link. If the source applies a DBF processing matrix, \( F \), to the transmission data, \( s \), the relay received signal would be

\[
y_r = H_1 F s + n_1
\]  

(3.6)

where \( H_1 \) is the source-to-relay channel, and \( n_1 \) is noise at the relay.

The FD relay transmits and receives during the same time slot, this causes the relay to receive its own transmission via the self-interference channel \( H_s \). Assuming no processing delay at the relay, the relay transmitted signal is given by

\[
x_r = G (y_r + H_s x_r)
\]  

(3.7)

where \( G \) is the relay processing matrix, \( (H_s x_r) \) represents the self-interference signal of the FD relay. One could solve (3.7) for \( x_r \) as [94]

\[
x_r = (I - GH_s)^{-1} G (H_1 x + n_1).
\]  

(3.8)

where the destination will receive the signal through DBF matrix \( W \). The processed signal is given by

\[
y = \sqrt{\rho} W^* H_2 x_r + W^* n_2
\]  

(3.9)

where \( H_2 \) is the relay-to-destination channel matrix, \( n_2 \) is the noise at the receiver, and \( \rho \) is the transmitted power. The instantaneous achievable rate assuming Gaussian signalling is

\[
R = \log_2 \left[ I_{N_d} + \frac{\rho}{N_0} R_n^{-1} \left| WH_2 (I - GH_s)^{-1} G H_1 F \right|^2 \right]
\]  

(3.10)

The effective noise covariance matrix of the system could be written as

\[
R_n = \sigma_n^2 \left[ \left| WH_2 (I - GH_s)^{-1} G \right|^2 + |W|^2 \right]
\]

Using the relay transmitted signal in (3.7), and defining \( \bar{G} = (I - GH_s)^{-1} G \), the
destination received signal (before processing) could be written as

\[ y = H_2 G H_1 x + \tilde{G} n_1 + n_2 \]  

(3.11)

which can be seen as the signal model for the full-duplex relay channel without self-interference. Thus, the signal model (3.11) becomes equivalent to the half-duplex model, keeping in mind the additional time slot for FD system. The optimum transfer matrix that maximizes the mutual information for the received signal in (3.11) is given by [94]

\[ \tilde{G}^o = V_2 U_1^* \]  

(3.12)

where \( U_1 \) is a unitary matrix of \( H_1 \) SVD, \( H_1 = U_1 \Sigma_1 V_1^* \), and \( V_2 \) is similarly defined. Here, \( \tilde{G}^o \) could be referred to as the optimum transfer matrix for the FD relay without the self interference. Solving for \( G \), the transfer matrix of the relay could be written as [94]

\[ G = \tilde{G}^o (I + H_s \tilde{G}^o)^{-1}. \]  

(3.13)

A comprehensive analysis of the performance of full-duplex relay system has been presented in [94]. There are relatively few studies in the area of relay design at the mmWave environment. The greater part of the literature on relays in mmWave focusses on higher layers like routeing, as explained in Section 3.2.3. The first systematic study of designing relay processors in mmWave communications was reported in [113]. The authors proposed an iterative algorithm to design the hybrid processors of a half-duplex relay based on the orthogonal matching pursuit algorithm for sparse approximation while assuming Orthogonal Frequency Division Multiplexing (OFDM) signalling. The proposed method requires the design of optimal precoder/combiner and relaying matrices for all subchannels which are computationally expensive.

### 3.2.2 Multi-Relay Network

Transmitting the signal through one or more relays is one way to compensate for the effect of signal fading due to multipath propagation and strong shadowing. This can be accomplished via a wireless network consisting of geographically separated nodes. Thus, providing a reliable wireless network that meets the
users’ demands. Recently, there has been renewed interest in relay networks. The underlying motivation behind the use of cooperative communications lies in the exploitation of spatial diversity provided by the network nodes, as well as the efficient use of power resources [96]. A recent review on several aspects of cooperative communications can be found, for instance in [116], and in the references therein.

This subsection gives a brief review of the multi-relay network in MIMO systems. Consider a single source is communicating with a single destination through \( K \) relays, As illustrated in Fig. 3.7. Amplify-and-Forward Relay scheme is used with two time slots to perform the process. During the first time slot, the source transmits the signal \( s \) and the \( k \)th relay received signal will be

\[
y_{r,k} = H_{1,k} s + n_{1,k}
\]

where \( H_{1,k} \) is the \( k \)th source-to-relay MIMO channel, \( n_{1,k} \) has a covariance matrix \( \mathcal{R}_{n_{1,k}} = \sigma^2_{n_{1,k}} I_{M_r} \). During the second time slot, the relays transmit their signals to the destination. The transmitted signal of the \( k \)th relay can be written as

\[
x_{r,k} = G_k y_{r,k}
\]

where \( G_k \) is the processing matrix of the \( k \)th relay. Now, the destination received signal can be expressed as

\[
y = \sum_{k=1}^{K} (H_{2,k} G_k H_{1,k} s + G_k n_{1,k}) + n_2
\]

where \( H_{2,k} \) is the \( k \)th relay-to-destination MIMO channel, and \( n_2 \) is the additive white noise at the destination with covariance matrix \( \mathcal{R}_{n_2} = \sigma^2_{n_2} I_{N_r} \). The destination receives the signal through baseband matrix \( W \), the processed signal is given by [97]

\[
\hat{s} = W^* H_2 G H_1 s + W^* G n_1 + W^* n_2
\]

where:

- \( H_1 = [H_{1,1}^T, H_{1,2}^T, \cdots, H_{1,K}^T]^T \) is the source-to-relay channel matrix,
- \( n_1 = [n_{11}^T, n_{12}^T, \cdots, n_{1K}^T]^T \) is the noise matrix at the relays,
- \( G = \text{bd}[G_1, G_2, \cdots, G_K] \) is the equivalent block diagonal relay amplifying matrix (bd[·] stands for a block-diagonal matrix), and
Figure 3.7: Parallel MIMO multi-relay system.

$H_2 = [H_{2,1}, H_{2,2}, \cdots, H_{2,K}]$ is the relay-to-destination channel matrix. The relay amplifying matrices ($G_k$) alongside the relay-to-destination channels $H_{2,k}$ are used to cancel the effect of the source-to-relay channels $H_{1,k}$, and the noise vectors at the relays $n_{1,k}$. Minimizing $\mathbb{E} || s - \tilde{G}y_r ||$, where $\tilde{G} = [H_{2,1}G_1, H_{2,2}G_2, \cdots, H_{2,K}G_K]$, and $y_r = [y_{r,1}, y_{r,2}, \cdots, y_{r,K}]$. The MMSE solution is given by

$$G_k = H_{2,k}^* \left( H_{2,k}H_{2,k}^* \right)^{-1} \left( R_s^{-1} + H_1^* R_{n_{1,k}}^{-1} H_1 \right) H_{1,k}^* R_{n_{1,k}}. \quad (3.18)$$

Given $\tilde{G}$, the MMSE solution for destination unconstrained processor could be described as

$$W = \left( R_s^{-1} + H_1^* G H_2^* R_{n_{eff}}^{-1} H_2 H_1 \right)^{-1} H_1^* G H_2^* R_{n_{eff}} \quad (3.19)$$

where $R_s$ is the covariance matrix of the transmitted signal, and

$$R_{n_{eff}} = H_2 G R_{n_1} G^* H_2^* + R_{n_2}. \quad (3.20)$$
CHAPTER 3. LITERATURE REVIEW

More substantial details to the design of relay networks and the performance analysis of various configuration criteria can be found in [96,97].

3.2.3 Relay Networks in Millimeter Waves

Although the large sized antenna arrays and the directivity gain improved the quality of mmWave systems, these solutions could not be effective by their own. Employing relays in a mmWave link is a promising solution to alleviate further the losses resulted from blockage and propagation losses [117,118]. With the proposed small cells architecture for mmWave communications and the use of dense access points, relaying is of particular importance. Relays benefits include: providing coverage extension, linking indoor and outdoor users, and improving the cellular handover experience [119]. An improvement of the mmWave link quality by 100 percent was indicated due to the added paths provided by relays in [120]. Additionally, [121] showed that the high-quality live video streaming range could be extended over 300 meters using relays.

Most of the literature of relays in mmWave consider the protocols and the improvement of the connectivity. In [33], multi-hop relaying is used to improve connectivity of mmWave networks. In [122], the relay selection problem was investigated in a two-hop relay-assisted mmWave cellular network based on obstacle analysis. [33] studied the relation between the size of the obstacle and the range of distances in which routeing relays are selected. The secrecy beamforming designs for mmWave two-way amplify-and-forward MIMO relaying networks was investigated in [123]. Cooperative networks in mmWave were discussed in [37,124].

In the area of relay design, [113] propose an OMP-based sparse approximation algorithm for jointly designing the Rx/Tx hybrid MIMO processors of a half-duplex AF relay in mmWave communications. Recently, [125] introduced the joint source and relay precoding design for mmWave systems under the assumption that the source, relay and destination nodes are equipped with multiple antennas. With recent advance in self-interference cancellation and hardware designs, FD is also considered in mmWave communications for high-speed and low-latency transmission. A 60 GHz transceiver with FD fibre optic transmit and receive chains was developed for short-range broadband application in [126].
3.3 Chapter Summary

This chapter has provided a summary of the literature relating to the methods and architectures proposed to extend the coverage of mmWave link. Section 3.1 discussed the beamforming techniques used to increase the directivity of the mmWave array. Channel sounding has been used to determine the best weight vectors for the phased arrays, [24] compares the joint search with the ping-pong based search to show that although the joint search is more straightforward, it is slower in finding the best weight vectors. [99] considered two scenarios to analyse the overhead associated with the channel search, full control pilot reuse, and near-orthogonal pilots. It also introduces effective, reliable rate metric which captures the resources overhead and SINR penalty of the channel search process. However, this metric was not considering the energy consumption of the process and how that will affect the general consideration of the system.

Analogue beamforming is a convenient and cost-effective when it comes to a mmWave point to point link, however, having multiple users (or multiple points) which might also require multiple data streams is not feasible. Accordingly, a lot of the research is directed towards using hybrid beamforming, where the number of RF chains is more than one and less than the number of antennas in the system [29, 31, 91, 93]. [29] used Orthogonal Match Pursuit (OMP) algorithm to design the hybrid processors with low complexity, [91, 93] introduces the hybrid beamforming design challenges in large scale antenna systems. [30, 130, 131] relied on the channel estimation to provide the basis for precoding matrix design. As compared to HBF, ABF has lower design complexity, lower energy consumption ratings, and it does not require channel knowledge. [132] introduces a study of the optimum number of RF chains to be used in the hybrid system based on a trade-off criteria. Recently, some research groups proposed a mixed ABF and HBF network scenarios where the base station is equipped with HBF system while the users have ABF [92].

Relay network is an effective solution to avoid the losses resulted from shadowing and other atmospheric effects at mmWave frequencies. However, there is a lack of work on the design of relay processors at the mmWave outdoor environment. Section 3.2 review the technologies of relay networks at lower frequencies to guide the work on designing a relay with hybrid processor working at the mmWave band.
Chapter 4

The Efficiency of Analogue Beamforming in the Presence of Phase Quantization Errors

Recently, increasing the directivity gain of the receiving and transmitting arrays is suggested to overcome the relatively higher losses in mmWave systems [24, 127]. The vast number of antenna elements in the system make it harder to obtain channel information using the conventional estimation techniques. Therefore, several studies recommend using channel sounding to select the best weight vectors for the phased arrays. [24] compares the joint search with the ping-pong based search to show that although the joint search is simpler to implement, it is slower in finding the best weight vectors. [99] considered two scenarios to analyse the overhead associated with the channel search: full control pilot reuse and near-orthogonal pilots, in addition to introducing a rate metric that captures the resources overhead. However, this metric does not consider the energy consumption of the process and how that will affect the general consideration of the system. [27] proved that the computational complexity of the search could be made independent of the number of antennas.

In this chapter, we investigate the use of analogue beamforming as one of the simplest, cost-effective solutions to extend the mmWave point-to-point links. We derive the detailed expression for the beamforming gain under certain channel conditions. We examine the effect of Phase Quantisation Error (PQE) resulted from utilising digital phase shifters on the system performance. Furthermore, we analytically derive an exact expression for the spectral efficiency of the system
where the beamforming gain and channel training duration is taken into consideration. This chapter also investigates three different channel sounding techniques, namely: exhaustive search, side-to-side search, and $n$-tier search. The comparison includes calculating the channel sounding time overhead and how it affects the performance as well as comparing the energy consumption of each search technique. Additionally, we propose a metric to determine the minimum search duration required to achieve better spectral efficiency with minimal power consumption. The numerical results assist in determining the optimum search period to produce an acceptable performance while improving the energy efficiency. The results also help identify the minimum number of quantization bits required to produce about ninety percent of the optimistic results.

The rest of the chapter is organised as follows. The system model and the main assumptions are presented in Section 4.1. The section also introduces to the design of the array weight vectors and the phase quantization error. In Section 4.2 we describe the principle of channel sounding and compare the different search techniques used in this work. In Section 4.3, the analysis of the beamforming gain is presented. The system spectral efficiency analysis is presented in Section 4.4. The analytical results are compared with the simulations in Section 4.5, and the chapter summary is presented in Section 4.6.

### 4.1 System Configuration

In this section, we present the system model for the ABF point-to-point link with multiple antenna arrays. Additionally, we will briefly discuss the array weight vectors construction and the phase quantization error.

#### 4.1.1 System Model

Consider the point-to-point (P2P) mmWave link in Fig. 4.1. We study the performance of ABF system employing half-wave spaced Uniform Linear Array (ULA) of $N_t$, $N_r$ elements at the transmitter and the receiver, respectively. In order to improve the gain of the received signal, the directivity of the beam is managed by introducing phase controlling on the signal fed to the array elements. The Antenna Weight Vectors (AWVs) are used to control the phase at the transmitter and the receiver.

Given the transmitting weight vector ($\mathbf{f}$) and the receiving weight vector ($\mathbf{w}$), the
received signal is written as

\[ y = \sqrt{\rho} \ w^* H f s + w^* n \]  

(4.1)

where \( s \) is the information signal with \( \mathbb{E} |s|^2 \leq 1 \), \( \rho \) is average transmit power, \( H \) is the \( N_r \times N_t \) mmWave channel matrix, and \( n \) is the Gaussian noise vector with power \( N_0 \), i.e. \( \mathbb{E} [nn^*] = N_0 I \). Defining the transmission SNR as \( \gamma_t = \rho/N_0 \), the received SNR can be described as

\[ \gamma_r = \gamma_t |w^* H f|^2 \]  

(4.2)

and the power gain of the model is

\[ \Omega = \frac{\gamma_r}{\gamma_t} = |w^* H f|^2 \]  

(4.3)

which is equal to the array gain.
4.1.2 Array Weight Vectors Design

Motivated by its practicality, we use the phase shifters to generate the beam patterns instead of adjusting both the phase and amplitude of the all the elements. However, it is worth mentioning that producing multiple beams using phase shifters only might be challenging. Multi-beam could become useful in applications where more than one direction is required.

We define the codebooks $F, W$ as the $N_t \times C$ and $N_r \times C$ set of AWVs at the transmitter and the receiver, respectively. Each column of the codebook corresponds to the phase weights of the array elements to steer the beam towards a certain direction.

Using ULAs, the column of the codebook can be written as

$$f_i = \frac{1}{\sqrt{N_t}} \left[ e^{-j2\pi(d/\lambda)\sin(\phi_i)} \ldots e^{-j(N_t-1)2\pi(d/\lambda)\sin(\phi_i)} \right]^T \quad (4.4)$$

where $\phi_i$ is the required angle of rotation of the beam.

4.1.3 Phase Quantization Error

The digital phase shifters consist of a fixed phase shifters whose values belong to the set $\{\theta \in 0, 2\pi \left(\frac{1}{2^q}\right), 2\pi \left(\frac{2}{2^q}\right), \ldots, 2\pi \left(\frac{2^q-1}{2^q}\right)\}$, where $q$ is the number of quantization bits. As shown in (4.4), in order to steer the beam towards ($\phi_i$) direction, the phases of the antennas are calculated based on this particular direction. However, the digital phase shifter can not achieve the exact values. Therefore, the AWVs in the analogue beamforming is defined with quantized phases and written as

$$f = [e^{j\theta_1} \ldots e^{j\theta_{N_t}}]^T \quad (4.5)$$

This quantization might cause two or more elements having the same phase which leads to phase correlation that causes phase quantization error.

4.2 Channel Training

This section describes the process of preparing the system for signal transmission. The process includes preparing the set of vectors to direct the signal, also known as codebooks, repeatedly transmitting a training signal, then choosing the transmit-receive vector pair that produces the best beamforming gain.
4.2.1 Channel Sounding

The process of repeatedly transmitting a training signal to track the channel behaviour is known as Channel Sounding (CS). The pair of the AWVs that maximizes the received signal gain is considered as optimum beamforming vector pair. Mathematically this could be expressed as

$$\text{maximize } \Omega = |w^*Hf|^2$$
subject to $f \in F, w \in W$. \hspace{1cm} (4.6)

It is well known that the pair of vectors solving (4.6) can be easily determined using SVD, yet that requires channel knowledge. However, acquiring CSI is highly challenging in ABF due to the fact that the receiver observes the noisy version of the channel ($w^*Hf$) rather than the channel $H$. That is in addition to the high cost of the channel estimation due to the large antenna arrays which makes the SVD approach not feasible in mmWave communication.

With the lack of channel knowledge, the transmitter and the receiver collaborate to determine the best weight vectors by observing the system behaviour using channel sounding.

We learn from (2.7) that the randomness of the channel is mainly related to the AoA/AoD on both sides. Then, the maximization problem is reduced finding the angles of the strongest path and set the weight vectors to steer the beam towards that direction. However, with the predefined codebooks, this performed by observing the beamforming gain to choose the optimum AWVs, this process is also known as hard-alignment.

The duration of the channel sounding depends on the size of the codebooks at both the transmitter and the receiver, and the search technique used in the process. The following subsection lists the three techniques we compare in this work. The techniques differ in the level of complexity, the length of the training period, and the energy consumption.

4.2.2 Search Techniques

We study the performance of three different search techniques, in which every side of the link can take one beam direction per slot. Comparing the performance of the three methods is performed via simulations in Section 4.5.
4.2.2.1 Comprehensive Search

In comprehensive search, the two sides examine all the possible vector combinations in order to reach a decision of which pair provides the maximum gain. As shown in Fig. 4.2, the search process involves a rotary check of all the direction vectors in the codebook. Although the principle of the search is straightforward and guarantees to find the best vector pair, the decision is reached slowly. Comprehensive search has a relatively high time complexity of $O(C^2)$, where $C$ is the size of the codebook on both sides. Given a certain channel training period $T_L$, the codebook size will be $C = \sqrt{T_L}$ which is considered small as compared to other methods, as will be explained shortly. Both sides use full antenna array capabilities during each iteration of the search which in turn maximizes the energy consumption. The total energy consumed for the antenna arrays during the channel sounding using this method is $(2NC^2)$, where $N$ is the array size on each side.

4.2.2.2 Side-to-Side (S2S) Search

Similar to the comprehensive search, the domain is evenly sampled over the $C$ codebook vectors. The search process is performed over two phases. During the first phase, one side uses all the antenna in the array to search for the direction that provides the best gain while the other side uses one antenna for omni-directional transmission. After finding the best AWV, the sides switch roles with the first side uses the antenna array to transmit toward the selected direction and the other side searches for the best gain direction using full array elements, as shown in Fig. 4.3. This technique has a lower time overhead compared to the comprehensive search with $O(2C)$. Additionally, it is obvious that the energy consumption of this technique is lower due to the omni-directional transmission.
during the first phase which makes the total energy units used during the process equals to \((3N + 1)C\). Given a certain channel training period \(T_L\), the codebook size will be \(C = T_L/2\).

### 4.2.2.3 \(n\)-tier Search

This technique uses \(n\) tiers of search, wherein each tier the number of antennas and the size of the codebook used is different. The process consists of two phases, one phase for each side. Each of the phases includes \(n\) iterations, where choosing \(n\) depends on the complexity level of the system. The design of the codebooks and the number of antennas used in each tier is based on two criteria:

- After deciding the number of tiers \((n)\), \(n\) different codebooks is generated. The first codebook design criteria will be based on the size of the codebook where \(C = C_1 \times C_2 \times \cdots \times C_n\), the other codebook design condition is to ensure that \(C_1 \leq C_2 \leq \cdots \leq C_n\).

- As the size of the codebook varies between the tiers, the number of antennas used in each iteration varies as well. The criteria of choosing the number of antennas in each tier is \(N_1 < N_2 < \cdots < N_n = N\).

Similar to the S2S search, the \(n\)-tier search process is performed on one side of the system at a time. This will provide energy and time saving. The procedure of the 3-tier search is shown in Fig. 4.4 and could be summarized as follows:

- During the first tier of the first phase, the transmitting side search a smaller size codebook where the domain is sampled over few vectors (2 in this example) while the receiving side uses a single antenna for omni-directional transmission. The time complexity of this tier is \(O(C_1)\), \((O(2))\) in this
Figure 4.4: $n$-tier channel sounding process. 3-tier example, with $C_1 = 2$, $C_2 = 4$, $C_3 = 4$, and $C = 32$. 
Figure 4.5: The total communication block length consist of channel training and data transmission blocks.

example, afterwards the best direction is chosen. The energy consumption of this tier equals to $(N_1 + 1)C_1$.

• The second tier of the first phase initiated by designing the codebook for this iteration. The space covered by the chosen direction of the previous tier is samples into $C_2$ directional vectors, 4 vectors in this example. The equivalent codebook size for this iteration is equal to $(C_1 \times C_2)$, or 8 in this example. This tier of the process concluded by choosing the direction that provides the higher gain while the receiver still using omni-directional mode. This tier requires signals with narrower beamwidth than the previous tier; thus, larger arrays are needed. This explains the second design criteria. The time complexity of this step is $O(C_2)$, $(O(4))$ in this example. The energy consumption of this tier equals to $(N_2 + 1)C_2$.

• The process in the previous step is repeated for the remaining tiers. The example in Fig. 4.4 uses three tiers with $C_3 = 4$, making tier-3 equivalent codebook size equals 32. The energy consumption of this tier equals to $(N_3 + 1)C_3$.

• During the tiers of the second phase, the receiver follow similar procedure searching for the best direction while the transmitter uses full antenna arrays to transmit on the direction chosen in the previous phase.

• The overall time complexity of the process equals to $(2 \sum_{i=1}^{n} C_i)$ as compared to $(\prod_{i=1}^{n} C_i)$ for a comprehensive search with equivalent codebook size.

Although this technique has a relatively more complicated process, it provides the highest energy and time effectiveness.

For all the methods mentioned above, increasing the search time results in a
Table 4.1: Summery of the total search time and the energy consumption of the channel sounding techniques.

<table>
<thead>
<tr>
<th></th>
<th>Phase 1 duration</th>
<th>Phase 2 duration</th>
<th>Total duration</th>
<th>Energy consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehensive search</td>
<td>-</td>
<td>-</td>
<td>$C^2$</td>
<td>$2NC^2$</td>
</tr>
<tr>
<td>S2S search</td>
<td>$C$</td>
<td>$C$</td>
<td>$2N$</td>
<td>$(3N + 1)C$</td>
</tr>
<tr>
<td>$n$-tier search</td>
<td>$\sum_{i=1}^{n} C_i^\dagger$</td>
<td>$\sum_{i=1}^{n} C_i$</td>
<td>$2 \sum_{i=1}^{n} C_i$</td>
<td>$\sum_{i=1}^{n} (2N_i^\ddagger + N + 1)C_i$</td>
</tr>
</tbody>
</table>

$\dagger C = \prod_{i=1}^{n} C_i$

$\ddagger N_1 < N_2 < \cdots < N_n = N$

larger codebook, thus a higher accuracy and better array gain. This comes at the cost of the time assigned for data transmission, as shown in Fig. 4.5. In Section 4.4, we study the spectral efficiency of the system taking into consideration the total search time and its effect on the performance.

Table 4.1 summarizes the three techniques time overhead and energy consumption\(^1\). The $n$-tier search has the lowest time complexity. Additionally, with the optimum number of antennas used in each tier, the $n$-tier outperforms the other techniques in the energy saving considerations.

### 4.3 Beamforming Gain Analysis

Studying the performance of the beamforming system requires defining the statistical characteristics of the signal-to-noise ratio at the receiver. In the mmWave outdoor environment, with the multiple clusters, the complicated nature of the channel, and the angles of arrival and departure, finding an exact description to

\(^1\)Note that all the assumptions are based on linear power consumption relationship where effects such as heat and initiation consumption are neglected.
the SNR becomes increasingly complicated. Therefore, the studies of the different system performances considered several assumptions to simplify the analysis [66,92]. Since the aim of the channel sounding is to search for the beamformer-combiner pair that maximizes the beamforming gain \( \Omega = |w^H f|^2 \), we will present in this section the analysis of the beamforming gain assuming single path channels. The study of this particular case will give a good insight into the system performance under more general circumstances.

We start by presenting the mathematical derivation for the system under general conditions. We then study the system behaviour under some specific asymptotic conditions.

In the special case of single path channel and assuming that all scattering happens in azimuth which means the two nodes implement horizontal beamforming only (considering only the azimuth, and neglecting elevation) [30], the channel model in (2.7) becomes

\[
H = \sqrt{N_r N_t} \alpha a_r(\theta^r)a_t^*(\theta^t) \tag{4.7}
\]

where \( \alpha \) is the complex gain of the single path, \( \theta^t \) is a uniformly distributed random variable represents the AoD, or \( \theta^t \sim U[0, 2\pi] \), and \( \theta^r \) is the AoA which is distributed as \( \theta^r \sim U[0, 2\pi] \). Using (4.7), we can re-write the beamforming gain in (4.3) as

\[
\Omega = |w^H f|^2 = N_r N_t \alpha a_r(\theta^r)a_t^*(\theta^t) f^*a_t(\theta^t)a_r^*(\theta^r)w. \tag{4.8}
\]

Using the definitions in (2.8), (2.9) and (4.4), we can simplify the term \( (a_t^*(\theta^t)f) \) in (4.8) as:

\[
a_t^*(\theta^t)f = \frac{1}{N_t} \left[ e^{j2\pi \frac{d}{\lambda} \sin(\theta^t)} \ldots e^{j(N_t-1)2\pi \frac{d}{\lambda} \sin(\theta^t)} \right].
\]

\[
e^{-j\psi^t_0} e^{-j(2\pi \frac{d}{\lambda} \sin(\theta^t) + \psi^t_1)} \ldots e^{-j((N_t-1)2\pi \frac{d}{\lambda} \sin(\theta^t) + \psi^t_{N_t-1})}
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where $\phi^t$ is the direction angle of the vector $f$, and $\psi^t_i$ is the quantization error for the $(i+1)$th antenna of the transmitting array. The distribution of the quantization error depends on the randomness techniques used, for more details see [87].

Now, the term $[a_i^t(\theta^t)f f^* a_i(\theta^t)]$ is reduced to

$$a_i^t(\theta^t)f f^* a_i(\theta^t) = \frac{1}{N_t^2} \sum_{m=0}^{N_t-1} e^{j(2\pi \frac{m}{N_t} (\sin(\theta^t) - \sin(\phi^t)) - \psi^t_m)} e^{-j(2\pi \frac{m}{N_t} (\sin(\theta^t) - \sin(\phi^t)) - \psi^t_m)}.$$  (4.10)

Defining $c = 2\pi (d/\lambda)$, $\mu = [\sin(\theta^t) - \sin(\phi^t)]$, and $\xi = (\sin(\theta^t) - \sin(\phi^t))$, where $\phi^r$ is the direction angle of the vector $w$, the gain in (4.8) will be

$$\Omega = \frac{\alpha \alpha^*}{N_t N_r} \sum_{m=0}^{N_r-1} e^{j(c m \mu - \psi^r_m)} \sum_{n=0}^{N_t-1} e^{-j(c m \mu - \psi^t_n)} \sum_{m=0}^{N_r-1} e^{j(c m \xi - \psi^r_m)} \sum_{n=0}^{N_t-1} e^{-j(c m \xi - \psi^t_n)}$$

by defining $\mathcal{Z} = \sin(\theta^t)$, $\mathcal{Y} = \sin(\theta^t)$, and substituting $\xi = (\sin(\theta^t) - \sin(\phi^t)) = \mathcal{Z} - \sin(\phi^t)$, $\mu = \mathcal{Y} - \sin(\phi^t)$, we get

$$\Omega = \frac{\alpha \alpha^*}{N_t N_r} \sum_{m=0}^{N_r-1} \sum_{n=0}^{N_t-1} e^{j(2\pi \frac{m}{N_r} (\mathcal{Y} - \sin(\phi^t)) - (\psi^r_m - \psi^t_n))} \cdot \sum_{m=0}^{N_r-1} \sum_{n=0}^{N_t-1} e^{j(2\pi \frac{m}{N_t} (\mathcal{Z} - \sin(\phi^t)) - (\psi^r_m - \psi^t_n))}.$$  (4.12)

with $\alpha \sim \mathcal{CN}(0,1)$. $\mathcal{Y}, \mathcal{Z}$ distribution is derived in Appendix A.

Eq. (4.12) describes the exact beamforming gain for the ABF system in the presence of PQE. However, the expression is very complicated to solve. Therefore, an approximation approach was considered using two simplified scenarios as described in the following subsections.

Analogue phase shifters have been employed to simplify the analysis, in other words, we assume zero phase quantization error. This assumption also helps to
study the behaviour of the system when a large number of quantization bits are used which yields a negligible error.

Assuming no quantization error conditions means that $\psi^t = \psi^r = 0$. The beamforming gain in (4.12) becomes:

$$
\Omega = \frac{\alpha \alpha^*}{N_r N_t} \sum_{m=0}^{N_t-1} \sum_{n=0}^{N_t-1} e^{j(c(m\mu - n\mu))} = \frac{\alpha \alpha^*}{N_r N_t} \frac{1 - \cos(cN_t \mu)}{1 - \cos(c\mu)} \cdot \frac{1 - \cos(cN_t \xi)}{1 - \cos(c\xi)}
$$

(4.13)

where we used the identity $[\cos x = 1/2(e^{ix} + e^{-ix})]$.

Now, substituting $Z = \sin(\theta^r)$, $Y = \sin(\theta^t)$, we get

$$
\Omega = \frac{\alpha \alpha^*}{N_r N_t} \frac{1 - \cos[2\pi N_t (d/\lambda)(Y - \sin \phi^t)]}{1 - \cos[2\pi (d/\lambda)(Y - \sin \phi^t)]} \cdot \frac{1 - \cos[2\pi N_t (d/\lambda)(Z - \sin \phi^r)]}{1 - \cos[2\pi (d/\lambda)(Z - \sin \phi^r)]}
$$

(4.14)

Knowing $|\alpha|^2$ has an exponential distribution with Complementary Cumulative Distribution Function (CCDF) $P(|\alpha|^2 > x) = e^{-x}$ and conditioning on both $(Y, Z)$, form (4.14) we get the CCDF of the gain as

$$
P(\Omega > x | Y = y, Z = z) = \exp \left[ -N_r N_t x \frac{1 - \cos[2\pi (d/\lambda)(Y - \sin \phi^t)]}{1 - \cos[2\pi N_t (d/\lambda)(Y - \sin \phi^t)]} \cdot \frac{1 - \cos[2\pi (d/\lambda)(Z - \sin \phi^r)]}{1 - \cos[2\pi N_t (d/\lambda)(Z - \sin \phi^r)]} \right]
$$

(4.15)

then,

$$
P(\Omega > x) = \int_{-1}^{1} \int_{-1}^{1} \exp \left[ -N_r N_t x \frac{1 - \cos[2\pi (d/\lambda)(Y - \sin \phi^t)]}{1 - \cos[2\pi N_t (d/\lambda)(Y - \sin \phi^t)]} \cdot \frac{1 - \cos[2\pi (d/\lambda)(Z - \sin \phi^r)]}{1 - \cos[2\pi N_t (d/\lambda)(Z - \sin \phi^r)]} \right] \cdot \frac{1}{\pi^2 \sqrt{1 - y^2} \sqrt{1 - z^2}} dy dz
$$

(4.16)

Eq. (4.16) above describes the CCDF of the beamforming gain without subspace sampling, i.e. $F$ and $W$ have only one vector each. Fig. 4.6 shows the gain distribution for different number of antennas for the direction of $0^\circ$, i.e. $\phi^t = \phi^r = 0^\circ$. Simulations using Monte Carlo methods are used to confirm the analytical results obtained earlier. The results indicate an improved beamforming gain as a result of increasing the number of antennas in the system. Fig. 4.7 shows the theory and simulation of the beamforming gain Cumulative Distribution Function.
Figure 4.6: Beamforming gain CDF for $L = 1$ with $\phi_t = \phi_r = 0^\circ$, and $N = 4, 8, 16, 32$. (CDF) for a different direction.

### 4.4 Spectral Efficiency

As described earlier, the Signal-to-Noise Ratio (SNR) of the system in Fig. 4.1 can be written as

$$\gamma_r = \mathbb{E}\left[\frac{|\sqrt{\rho} w^H H s|^2}{|w^H n|^2}\right]$$

(4.17)

since the combiner vector is a unit norm, and with defining $\Omega_{max}$ as the maximum beamforming gain obtained using the optimum beamformer-combiner pair, (4.17) could be written as

$$\gamma_r(f_{opt}, w_{opt}) = \gamma_l \Omega_{max}.$$  

(4.18)

Finding the optimum pair depends on the channel sounding method used and the duration of the training period. As will be shown in Section 4.5, the longer the training period the closest the performance to reach the optimistic results. However, this comes at the cost of the transmission time, as demonstrated in Fig. 4.5. Although a better beamforming gain is produced, a longer duration will
negatively affect the overall system performance. As a result, the achievable rate and the beamforming gain does not provide the complete picture.

Now, subject to the assumption that the system transmits Gaussian signal, the achievable rate at the receiver equal to the logarithm of one plus the signal-to-noise ratio. Using (4.18), we define the spectral efficiency as follows

\[
\eta = \left( \frac{T - T_L}{T} \right) \mathbb{E} \left[ \log_2 (1 + \gamma_t \Omega_{\max}) \right]
\]

Using integration by parts, with \( u = \ln(1 + \gamma_t x) \) and \( dv = f(x) \), then, \( du = \frac{\gamma_t}{1 + \gamma_t x} dx \) and \( v = -P(\gamma_{\max} > x) \), the efficiency is solved as

\[
\eta = \left( \frac{T - T_L}{T} \right) \frac{1}{\ln 2} \left[ \ln(1 + ax)P(\gamma_{\max} > x) \bigg|_0^\infty + \int_0^\infty P(\gamma_{\max} > x) \frac{\gamma_t}{1 + \gamma_t x} dx \right]
\]

\[
= \left( \frac{T - T_L}{T} \right) \frac{1}{\ln 2} \int_0^\infty (1 - P(\gamma_{\max} \leq x)) \frac{\gamma_t}{1 + \gamma_t x} dx
\]

Figure 4.7: Beamforming gain CDF for \( L = 1 \), with \( \psi = 0.7\pi \), \( \phi = 0 \), and \( N = 4, 8, 16, 32 \).
where $T$ is the slot duration, $T_L$ is the length of the channel training, and $\mathbb{P}(\Omega > x)$ is as described in (4.16).

Eq. (4.19) above indicates that studying the characteristics of the spectral efficiency in mmWave is too complicated. In order to get a more simplified version of the spectral efficiency, we study the performance of analogue beamforming system with continuous beamsteering capabilities. Such assumption alongside the assumptions of single path channel is used to simplify the analysis of other systems in mmWave environments [92].

As explained earlier, using continuous beamsteering means the codebook is infinite in size, and then, the beamforming and combiner vectors will be selected optimally. The following lemma uses this assumption to simplify the signal-to-noise ratio expression.

**Lemma 1.** If the analogue beamforming system in Fig. 4.1 has a continuous beamsteering capability, then, if we assume single path channel, the SNR will be distributed as an exponential random variable.

**Proof.** As the channel $\mathbf{H}$ has only one path, and given the continuous beamsteering capability assumption, the optimal beamformer and combiner vectors will be $\mathbf{w}^{\text{opt}} = \mathbf{a}_r(\theta^r)$, and $\mathbf{f}^{\text{opt}} = \mathbf{a}_t(\theta^t)$ [92], then, the numerator of the signal-to-noise ratio in (4.17) is simplified as:

$$|\sqrt{\rho} \mathbf{w}^* \mathbf{H} \mathbf{s}|^2 = |\sqrt{\rho} N_r N_t \mathbf{a}_r^* \mathbf{a}_r \mathbf{a}_t^* \mathbf{a}_t \mathbf{s}|^2 \overset{(a)}{=} |\sqrt{\rho} N_r N_t \alpha \mathbf{s}|^2$$

where $(a)$ is because the beamforming vector is unit norm. Similarly, since the combiner vector is unit norm, we could re-write (4.17) as

$$\gamma_r = N_r N_t \frac{\rho}{N_0} |\alpha|^2. \quad (4.20)$$

Now, as $\alpha \sim \mathcal{CN}(0, \sigma_\alpha^2)$, then $|\alpha|^2$ has a Chi-squared distribution of the second degree (or exponential distribution).

The following theorem uses the SNR lemma above to provide an expression of the system capacity. The assumptions used helped to derive a closed form and
upper bound for the capacity of the system. The study of more general conditions is left for future research.

**Theorem 4.1.** Let the beamforming and combiner vectors of the system shown in Fig. 4.1 are designed with continuous angles, then, the capacity of the system over a single path channel is upper-bounded by the capacity of AWGN channel, and it is defined as

\[
\text{Capacity} = \frac{1}{\ln 2} e^{N_r N_t N_0 / \rho} \Gamma(0, N_r N_t N_0 / \rho)
\]

(4.21)

where \( \Gamma(a, b) \) denotes the complementary gamma function.

**Proof.** The capacity of the system, assuming Gaussian signal is

\[
\text{Capacity} = \mathbb{E} \left[ \log_2 \left( 1 + N_r N_t \frac{\rho N_0}{\mathbb{E}[|\alpha|^2]} \right) \right]
\]

(4.22)

where \( x = |\alpha|^2 \). Using Jensen’s inequality

\[
\text{Capacity} \leq \log_2 \left( 1 + N_r N_t \frac{\rho}{N_0} \mathbb{E}[|\alpha|^2] \right)
\]

\[
= \log_2 \left( 1 + N_r N_t \frac{\rho}{N_0} \sigma^2_\alpha \right)
\]

(4.23)

and with \( \alpha \sim \mathcal{CN}(0, 1) \), the system capacity will be upper-bounded by the capacity of the Additive White Gaussian Noise (AWGN).

Now, we can use Lemma 1 to get the capacity expression directly from solving

\[
\text{Capacity} = \int_0^\infty \log_2 \left( 1 + x N_r N_t \frac{\rho}{N_0} \right) e^{-x} dx.
\]

\[\square\]

4.5 Simulation and Numerical Results

In this section, we compare the performance of the three search techniques both from the gain and the energy consumption aspects analytically and using the Monte Carlo simulations. The results are compared with the optimistic system performance where the system had the channel information and was able to direct
the signal towards the channel’s AoA and AoD. The channel is modelled with $L$ clusters, and each cluster contributes a single ray with uniformly random AoAs and AoDs. All the clusters are assumed to have equal power. We also assume a uniform linear arrays with inter-elements spacing $d = \lambda/2$.

Firstly, the analytical results of theorem 4.1 are examined using single path channel and array size $N \in \{8, 16\}$ at both sides of the transmission link. The performance of the comprehensive search technique is compared with the optimistic results and the upper bound as shown in Fig. 4.8. The results highlight the effect of the codebook size on the overall performance of the search technique by comparing three different codebooks with $C \in \{4, 5, 6\}$. Larger codebook means that the space is sampled into more vectors which increases the probability of finding a better match with the channel AoA/AoD. However, having large codebook size increases the search time which will cause less time for data transmission as explained earlier in Fig. 4.5. Additionally, as the total energy consumption of the comprehensive technique is $NC^2$, using more vectors in the codebook will result in higher energy consumption. The figure also demonstrates a perfect match between the analytical and the simulation results.

The performance of the exhaustive sounding technique also examined under
more realistic conditions where the channel has a sum of $L = 10$ paths. The results in Fig. 4.9 show that the ABF system will have a similar behaviour to that with single path channel. Additionally, the results indicate that the efficiency of the exhaustive search is better with single path channels. This results from the existence of a single AoA (or AoD) that the phase shifters network is trying to find instead of 10 paths. This increase the chance of getting a better match.

Secondly, the effect of the quantization and the number of quantization bits is considered. Fig 4.10 compares the performance of the comprehensive search technique with the optimistic results for a different number of quantization bits. Although having more quantization bits provide better spectral results, it provides higher overhead. The results indicate that more than 90% of the optimistic performance is achievable with three quantization bits. About 98% of the performance could be achieved by adding one more bit. These numbers considered acceptable considering the minimal effect gained by adding one or more bits.

Additionally, we study the effect of the search duration on the system performance for the three techniques. The results shown in Fig. 4.11 compare the achieved beamforming gain for the three techniques with the optimistic and the upper bound results. The curves show that eventually, all the techniques will be able to find the best direction to match the channel’s AoA/AoD. However, $n$-tier
search technique is the fastest to reach the optimistic results, while the comprehensive search is the slowest. That is due to the size of the effective codebook in \( n \)-tier search is much larger than the size of the codebook in the comprehensive and S2S search, as listed in Table 4.1.

As explained earlier, increasing the search time comes on the cost of the data transmission time and the energy consumption as it is directly related to the size of the codebook. In order to decide the optimum search time, a trade-off needs to be performed between the spectral efficiency and the energy consumption. For this sake, we introduce a system performance measure factor defined as \( \Delta = \frac{\text{Spectral efficiency}}{\text{Energy consumption}} \). This means that lower energy consumption and higher spectral efficiency provides higher performance measure factor. Fig. 4.12 compares \( \Delta \) for different search durations. The optimum search time for the given parameter is found to be around 36 time units. This search time provides more than 78% spectral efficiency of the optimistic results using the \( n \)-tier search, with the S2S and comprehensive search achieving around 55% and 20%, respectively.

Figure 4.10: Spectral efficiency versus number of quantization bits as compared to the system with continuous angles capabilities. \( N = 16 \), and \( L = 1 \).
Figure 4.11: Beamforming gain versus the training time length, with $N = 16$ and $SNR = 0 dB$.

Figure 4.12: Performance measure factor versus search time. Selecting the optimum search duration which produce good spectral efficiency with minimal energy consumption.
4.6 Chapter Summery

Millimeter wave is on the road-map to be an essential element in building the next architecture of wireless communication. In this chapter, we design an analogue beamforming system to overcome some of the challenges naturally presented at such high frequencies. We derive a detailed analysis for the beamforming gain in the existence of the phase quantization error resulting from using digital phase shifters. We were able to determine the minimum number of quantization bits required to achieve about 90% of the optimistic results (where the system assumes to have no PQE and phase shifters with continuous angle capabilities).

We investigate using channel sounding techniques to determine the best beamformer-combiner pair that maximises the beamforming gain without acquiring channel knowledge. Different sounding techniques have been compared, namely, exhaustive search, side-to-side search, and \( n \)-tier search. The comparison includes the duration and the power consumption of each technique. The results indicate the \( n \)-tier search proved to have superior time and energy efficiency as compared to exhaustive and side-to-side search.

However, finding the optimum pair is highly dependent on the duration of the training. Longer training period resulted in a larger codebook and better vectors selection. That comes at the cost of the transmission time which makes the beamforming gain and SNR not very accurate measure of the system performance. We propose the analysis of the spectral efficiency of the system taking into consideration the beamforming gain and the duration of the transmission. We derive the spectral efficiency equation of the analogue beamforming system under certain assumptions such as using channels with a limited number of paths. The study gives useful insights into the behaviour of the system under more general conditions.

Finally, we propose the performance measure factor as a metric for spectral-energy efficiency trade-off. We were able to estimate the minimum search duration to produce the best spectral efficiency with minimal energy consumption.
Chapter 5

Hybrid Beamforming for Full-Duplex Relay Systems

Wireless point-to-point links will be essential in the small dense cells scenario recently proposed for 5G. Using mmWave in the backhaul wireless link is an attractive proposition. A typical scenario is illustrated in Fig. 5.1 where relays are used to connect the small cell basestation with the macro-cell basestation in dense networks via mmWave communication links. However, there is still a lack of studies of the Full-Duplex (FD) relays in the mmWave environment. The performance of both Half-Duplex (HD) and amplify-and-forward FD relays was compared in the Rayleigh channels for single antenna system [129], and it was shown that the FD relaying could provide higher performance than the HD, even with the existence of self-interference (SI). The same studies applied for MIMO systems providing similar results [94].

In this chapter, we propose employing hybrid beamforming to utilise Amplify-and-Forward (AF) full-duplex relays in mmWave communication for wireless links between the small cell base station and the macro-cell base station. We build an algorithm to jointly design the hybrid matrices at the relay using the greedy approach. The performance of the proposed system is analysed using a channel with a limited number of paths. The analysis highlighted the influence of the array size at the source, relay, and destination on the system performance. The results emphasise the importance of hybrid beamforming in mmWave communications. A significant performance improvement for the full-duplex links over HD was demonstrated. Finally, the results show the effect of increasing the number of antennas in the system. Specifically, this chapter establishes that the
array size at the source and the relay receiver have a prominent impact on the system performance than the size of the array at the destination and the relay transmitter.

The rest of the chapter organised as follows. The system model with the main assumptions used in the chapter is presented in Section 5.1. The design of the hybrid processors using the sparsity algorithm is presented in Section 5.2, and analysed in Section 5.3. Numerical and simulation results are presented in Section 5.4 before summarising the chapter in Section 5.5.

5.1 System Model

In this chapter, we consider using relays to connect a small cell base station with \( N_t \) antennas and \( N_{RF} \) RF chains to the macro-cell base station which has \( N_t \) antennas and \( N_{RF}^{t} \) RF chains. The relay has \( M \) antennas, assigns \( M_r = \beta M \) for receiving and \( M_t = (1 - \beta)M \) for transmitting, with \( \beta \) being the relay Tx-to-Rx antenna ratio. Similarly, the relay is equipped with \( M_{RF} \) RF chains, where
\( M_{RF}^r = \beta_{RF} M_{RF} \) of them for receiving, and \( M_{RF}^t = (1-\beta_{RF}) M_{RF} \) for transmitting, with \( \beta_{RF} \) being the relay Tx-to-Rx RF chain ratio, as depicted in Fig. 5.2.

The use of multiple RF chains allows multi-stream communication using \( N_s \) data streams with \( N_s \leq \min(N_{RF}^t, N_{RF}^r, M_{RF}^t, M_{RF}^r) \leq \min(N_t, N_r, M_t, M_r) \). We also define the ratio \( \omega = \frac{N}{N_{RF}} \) to be the hybridization ratio, with \( N \) the number of antennas in the array and \( N_{RF} \) the number of RF chains used. \( 1 \leq \omega \leq N \), with \( \omega = 1 \) being a fully digital system, and \( \omega = N \) being an analogue beamformer.

The source applies an \( N_{RF}^t \times N_s \) baseband precoder, \( F_{BB} \), using its \( N_{RF}^t \) transmit chains, followed by an \( N_t \times N_{RF}^t \) RF beamformer, \( F_{RF} \), using analogue circuitry. The sampled transmitted signal takes the form:

\[
x = F_{RF} F_{BB} s \tag{5.1}
\]

where \( s \) is the \( N_s \times 1 \) transmitted symbol vector such that \( \mathbb{E}[ss^*] = \frac{1}{N_s} I_{N_s} \). \( F_{RF} \) is implemented using analogue phase shifters, therefore, its entries have equal norm, i.e. \( \left( F_{RF}^{(i)} F_{RF}^{(i)^*}\right)_{(\ell,\ell)} = N_t^{-1} \), while \( F_{BB} \) has no hardware constraints except the total power constraint. The \( M_r \times 1 \) relay received signal would be:

\[
y_r = \sqrt{\rho} H_1 F_{RF} F_{BB} s + n_1 \tag{5.2}
\]

where \( n_1 \) is \( M_r \times 1 \) vector of i.i.d. \( \mathcal{CN}(0, \sigma_n^2) \) noise at the relay, \( H_1 \) is the \( M_r \times N_t \) source-to-relay mmWave channel, and \( \rho \) is the transmitted power. The relay applies an \( M_r \times M_{RF}^r \) RF combiner, \( G_R \), at the receiving side, an \( M_{RF}^r \times M_{RF}^r \) baseband precoder, \( G_{BB} \), and \( M_t \times M_{RF}^r \) RF precoder, \( G_T \), at the transmitting side on the signal.

The FD relay transmits and receives during the same time slot, this causes the relay to receive its own transmission via the \( M_r \times M_r \) self-interference channel \( H_s \).

There has been no recognised model to describe the self-interference channel in mmWave environment yet. In this work, we will be modelling the self-interference channel similar to the mmWave channel described in Section 2.3.3. Assuming no processing delay at the relay, the relay transmit signal is given by

\[
x_r = G_T G_{BB} G_R^* (y_r + H_s x_r) \tag{5.3}
\]

where \( (H_s x_r) \) represents the self-interference signal of the FD relay. Now, using
Figure 5.2: Full-duplex relay system model with hybrid beamforming.
(5.1), and defining $G = G_T G_{BB} G_R^*$, one could solve (5.3) for $x_r$

$$x_r = (I_{M_t} - G H_s)^{-1} G (\sqrt{\rho} H_1 x + n_1).$$  \hspace{1cm} (5.4)$$

The destination will receive the signal through $N_r \times N_r$ RF combiner $W_{RF}$, and process it using the $N_{RF} \times N_s$ baseband matrix $W_{BB}$. The processed signal is given by

$$y = W_{BB}^* W_{RF}^* H_2 x_r + W_{BB}^* W_{RF}^* n_2$$  \hspace{1cm} (5.5)$$

where $H_2$ is the $N_r \times M_t$ relay-to-destination channel matrix, and $n_2$ is the $N_r \times 1$ noise vector similar to $n_1$ with $CN(0, \sigma_n^2)$. Substituting (5.1) and (5.4) into (5.5), the useful signal could be written as

$$x_u = \sqrt{\rho} W_{BB}^* W_{RF}^* H_2 (I_{M_t} - G H_s)^{-1} G H_1 F_{RF} F_{BB} s$$  \hspace{1cm} (5.6)$$

and the effective noise signal could be written as

$$x_n = W_{BB}^* W_{RF}^* H_2 (I_{M_t} - G H_s)^{-1} G n_1 + W_{BB}^* W_{RF}^* n_2.$$  \hspace{1cm} (5.7)$$

The instantaneous achievable rate assuming Gaussian signalling is

$$R = \log_2 \left| I_{N_s} + \frac{\rho}{N_s} R_n^{-1} \left( W H_2 (I_{M_t} - G H_s)^{-1} G H_1 F \right) \right|$$  \hspace{1cm} (5.8)$$

with $W = W_{BB}^* W_{RF}^*$, $F = F_{RF} F_{BB}$, and

$$R_n = \sigma_n^2 \left| \left( W H_2 (I_{M_t} - G H_s)^{-1} G \right) \left( W H_2 (I_{M_t} - G H_s)^{-1} G \right)^* + WW^* \right|$$  \hspace{1cm} (5.9)$$

is the effective noise covariance matrix. The achievable rate (5.8) will be used in designing the hybrid beamformers in Section 5.2.

5.2 Hybrid Beamformers Design

In this section, we introduce the design problem for the hybrid beamforming processors at the relay. We also discuss in details the proposed algorithm to design both the digital and the analogue matrices.
5.2.1 Problem Formulation

A general approach for designing the hybrid mmWave precoders for the system shown in Fig. 5.2 would be to maximize the achieved rate. Directly solving this problem requires joint optimization over the seven precoding matrices \((F_{RF}, F_{BB}, G_T, G_{BB}, G_R, W_{RF}, W_{BB})\), which can be mathematically expressed as

\[
\arg\max_{F_{RF}, F_{BB}, G_T, G_{BB}, G_R, W_{RF}, W_{BB}} \left\| \left( \log_2 I_N + \frac{\rho}{N_s} R_n^{-1} (WH_2(I_{N_t} - GH_s)GH_1 F) \right) (WH_2(I_{N_t} - GH_s)GH_1 F)^* \right\|_F,
\]

subject to \(F_{RF} \in \mathcal{F}_{RF}, G_T \in \mathcal{G}_T^{RF}, W_{RF} \in \mathcal{W}_{RF}, G_R \in \mathcal{G}_R^{RFC}\)

\[
\|F_{RF}F_{BB}\|_F^2 = N_s
\]

\[
\|G_{MM}(G_1(\mathbf{x} + \mathbf{n}_1)\|_F^2 = N_s
\]

where \(\mathcal{F}_{RF}, \mathcal{G}_T^{RF}, \mathcal{G}_R^{RFC}, \mathcal{W}_{RF}\) are sets of feasible RF vectors used to design the analogue beamformers, and \(R_n\) is as described in (5.9). (\(\|F_{RF}F_{BB}\|_F^2 = N_s\)) and (\(\|G_{MM}(G_1(\mathbf{x} + \mathbf{n}_1)\|_F^2 = N_s\)) are the power constraints at the source and the relay, respectively.

Several sets that satisfy the design of the analogue beamformers were proposed in literature, see e.g. [31]. In this work, we use the array response beamformers by utilizing analogue phase shifters to reduce the complexity. Unfortunately, the non-convex nature of the RF constraints makes it hard (if not impossible) to solve the problem in a reasonable time [29, 133]. In order to overcome this obstacle, we propose an approximate solution by decoupling the source, relay and destination. Accordingly, designing the hybrid processors in each of them will not be constrained by other nodes’ constrains. Afterwards, the hybrid processor design problem is approximated in order to minimize the Frobenius norm of the difference between the hybrid processors and the unconstrained processors \((F_u, G_u, W_u)\).

The unconstrained processors are obtained by conventional MIMO system where each antenna is connected to a separate RF chain (full complex RF chains). At the source, the unconstrained precoder represents the channels’ right singular vectors, i.e. defining the channel’s ordered singular value decomposition (SVD) to be \(H_1 = U_1\Sigma_1 V_1^*\) where \(U_1\) is an \(N_r \times \text{rank}(H_1)\) unitary matrix, \(\Sigma_1\) is a
rank($H_1$) × rank($H_1$) diagonal matrix of singular values arranged in decreasing order, and $V_1$ is a $M_t \times \text{rank}(H_1)$ unitary matrix. The unconstrained precoder will be $F_u = \tilde{V}_1$, where $\tilde{V}_1$ consists of the $N_s$ columns of $V_1$ corresponding to the largest singular values of $H_1$. Similarly, at the destination, $W_u = \tilde{U}_2$, with $\tilde{U}_2$ is defined similarly to $\tilde{V}_1$ [45,134]. The design of the relay unconstrained precoder is described in Section 3.2.

Given the feasible RF sets, the sparse approximation could be used to design the hybrid beamformer. The optimum hybrid processor will be a linear combination of selected vectors of the feasible set. Now, in order to formulate the design problem at the relay, we define $M_t \times N_\Lambda$ matrix $\Lambda_T = [A_T|A]$ (with $N_\Lambda > P \times L$), where $[A|B]$ denotes horizontal concatenation. $A_T$ columns consist of the transmitter array response vectors of the relay-to-destination channel, i.e. $A_T = [a_t(\phi_t^1, \theta_t^1) a_t(\phi_t^2, \theta_t^2) \cdots a_t(\phi_t^P, \theta_t^P)]$. $A$ is to be used when $P \times L \leq N_{RF}$ and its columns are chosen to satisfy arbitrary analogue beamforming constraints. Similarly, we define $\Lambda_R = [A_R|A]$, with $A_R = [a_r(\phi_r^1, \theta_r^1) a_r(\phi_r^2, \theta_r^2) \cdots a_r(\phi_r^P, \theta_r^P)]$.

The sparsity nature of the mmWave channel is exploited to compose an OMP algorithm to search for the best vectors. Therefore, the problem of the difference between the unconstrained processor and the hybrid processors is re-written in a form that serves this purpose

$$\tilde{G}_{BB}^\text{opt} = \arg\min_{G_{BB}} \|G_u - \Lambda_T \tilde{G}_{BB} \Lambda_R^*\|_F,$$

s.t. $\|\text{diag}(\tilde{G}_{BB} \tilde{G}_{BB}^*)\|_0 = \beta_{RF} M_{RF},$

$$\|\Lambda_T \tilde{G}_{BB} \Lambda_R^*\|_F^2 = N_s \quad (5.11)$$

where $\tilde{G}_{BB}$ is an $N_\Lambda \times N_s$ sparse matrix having $\beta_{RF} M_{RF}$ nonzero rows (which constitute of $G_{BB}$), $\|\text{diag}(\tilde{G}_{BB} \tilde{G}_{BB}^*)\|_0 = \beta_{RF} M_{RF}$ ensures that $\tilde{G}_{BB}$ is $\beta_{RF} M_{RF}$ sparse. As only $\beta_{RF} M_{RF}$ rows of $\tilde{G}_{BB}$ are nonzero, only $\beta_{RF} M_{RF}$ vectors of $\Lambda_T$ and $\Lambda_R$ are selected, and they represent the vectors of $G_{T}^\text{opt}$ and $G_{R}^\text{opt}$, respectively. In other words, in (5.11), the columns of $G_u$ are approximated by a linear combination of $\Lambda_T$ and $\Lambda_R$ with the nonzero elements of $G_{BB}$ specify the weights.
5.2.2 Design Algorithm

In this section, we propose a new suboptimal design algorithm to create the hybrid beamforming matrices that minimise the difference between the hybrid and the unconstrained processors. The design algorithm is based on the greedy OMP\(^1\) principle and will jointly design the the RF beamformers \(G_T^{\text{opt}}\) and \(G_R^{\text{opt}}\) and the baseband processor \(G_{\text{BB}}^{\text{opt}}\). The algorithm procedures are outlined in Algorithm 5.1 and can be explained as follows.

- The algorithm begins by initiating the identification matrices \((\Psi_R, \Psi_T)\) as empty sets to be used later in the search for the most correlated vectors. The algorithm also set up the residual matrix, \(G_{\text{res}}\), initially to be equal to the unconstrained processor matrix.

- The second part involves choosing the columns of \(\Lambda_T\) and \(\Lambda_R\) that correlate the most with \(G_u\). The algorithm search for the best vectors (steps 2a, 2b, and 2c), then, add each selected column to \(G_T\) and \(G_R\) (steps 2d, 2e).

- After selecting the first column of \(G_T\) and \(G_R\), the least squares problem \((G_u = G_T G_{\text{BB}} G_R^*)\) is solved to find the coefficients of \(G_{\text{BB}}\) (step 3).

- The residual matrix is then updated by subtracting the selected columns of \(G_T\), \(G_R\), and \(G_{\text{BB}}\) from the unconstrained processor matrix (step 4).

- The process is repeated \(\beta_{RF} M_{RF}\) iterations to select the remaining columns of \(G_T\), \(G_R\), and calculating the rest of \(G_{\text{BB}}\) coefficients.

- It is important to note that during the successive iterations, the columns correlate the most with the residual \(G_{\text{res}}\) and the identified columns of \(G_R\) or \(G_T\) (steps 2a-2e), then added to the RF beamformers.

- Finally, during the last part of the algorithm, the power constraint is satisfied (step 5) and the beamforming matrices are provided.

Similar algorithms can be used to design the RF processors at both the source and the destination, for the detailed algorithms, see Appendix B.

To sum up, Algorithm 5.1 searches for a linear combination of the analogue

\(^{1}\text{OMP is one of the earliest methods for sparse approximation. It is an iterative greedy algorithm that selects at each step the column, which is most correlated with the current residuals, for a detailed description see [109,135].}\)
Algorithm 5.1: Relay hybrid beamformers design algorithm.

**Input:** $\Lambda_T, \Lambda_R, G_u$.

1. **Initialize.** Set the index sets $\Psi_R = \Psi_T = \emptyset$, the residual $G_{\text{res}} = G_u$, and set the counter $i = 1$.

2. **Identify.** Find columns of $\Lambda_R, \Lambda_T$ that most strongly correlate with the residual.

   (a) if $i = 1$ then
   $\Psi_R = G_u \Lambda_R$
   $\Psi_T = \Lambda_T^* G_u$
   else
   $\Psi_R = G_T^* G_{\text{res}} \Lambda_R$
   $\Psi_T = \Lambda_T^* G_{\text{res}} G_R$
   end

   (b) $u = \arg\max_{n=1,\ldots,P \times L} (\Psi_R \Psi_R^*)_{n,n}$
   (c) $v = \arg\max_{n=1,\ldots,P \times L} (\Psi_T \Psi_T^*)_{n,n}$
   (d) $G_R^{(i)} = \Lambda_R^{(u)}$
   (e) $G_T^{(i)} = \Lambda_T^{(v)}$

3. **Estimate.** Find the coefficients of $G_{BB}$ by solving least squares problem ($G_u = G_T G_{BB} G_R^*$):

$$G_{BB} = (G_T^* G_T)^{-1} G_T^* G_u G_R (G_R^* G_R)^{-1}$$

4. **Iterate.** Update the residual:

$$G_{\text{res}} = G_u - G_T G_{BB} G_R$$

Increment $i$. Repeat (1)-(4) for $i \leq \beta_{RF} M_{RF}$

5. **Output.** Satisfy the transmission power constraint

$$G_{BB} = \sqrt{N_s} \frac{G_{BB}}{||G_u - G_T G_{BB} G_R^*||_F}$$

**Output:** $G_T, G_{BB}, G_R$. 
beamforming matrices columns such that the difference with the unconstrained processor is kept to minimum. The algorithm find the optimum solution in a definite number of steps (which equals the number of RF chains used in the system). In a typical implementation of OMP, the identification step is the most expensive part of the computation [109].

5.3 Performance Analysis

This section presents the performance analysis of the proposed hybrid beamforming relay system using channels with limited number of paths. For simplicity, we assume no self-interference in the system. The analysis of this special case will provide valuable insights into the system performance in more general settings. The following theorem characterise the achievable rate by the destination when the algorithm presented in Section 5.2 is used to design the hybrid processors at the source, relay, and destination of the system.

**Theorem 5.1.** Using relay system with number of antennas at the destination equals the number of relay receive antennas. Then, under channels with limited number of paths ($L_1 = M_{RF}^s$, $L_2 = M_{RF}^r$), and no self-interference channel, i.e. $H_s = 0$, the achievable rate of the system is given by

$$R = \log_2 \left[ 1 + \gamma \left( \frac{\sigma_{\max}^1 \cdot \sigma_{\max}^2}{(\sigma_{\max}^2)^2 + 1} \right)^2 \right]$$

(5.12)

where $\sigma_{\max}^1$ and $\sigma_{\max}^2$ are the maximum singular values for the source-to-relay and relay-to-destination channels, respectively, and $\gamma = \frac{\rho}{N_0}$.

**Proof.** Since zero self-interference is assumed, the received signal in (5.5) could be re-written as:

$$y = \sqrt{\rho} W_{BB}^* W_{RF}^* H_2 G H_1 x + W_{BB}^* W_{RF}^* H_2 G n_1 + W_{BB}^* W_{RF}^* n_2$$

(5.13)

The useful signal in (5.6) can be re-written as

$$x_u = \sqrt{\rho} W_{BB}^* W_{RF}^* H_2 G H_1 x$$

$$= \sqrt{\rho} W_{BB}^* W_{RF}^* H_2 G_T G_{BB} G_{RF}^* H_1 F_{RF} F_{BB} s$$

(5.14)
The channel model in (2.7) could be written in the compact form $H_v = A_{R_v} Z_v A_{T_v}^*$, where $v \in \{1, 2\}$, $Z$ is a diagonal matrix containing the multi-path gains, and the matrices $A_{T_v}$, $A_{R_v}$ are as defined earlier in section 5.2.

Now, since $N_{RF} = L$, Algorithm 5.1 will result $F_{RF} = A_{T1}, G_{R} = A_{R1}, G_{T} = A_{T2}$, and $W_{RF} = A_{R2}$. Then, (5.14) can be simplified as

$$x_u = \sqrt{\rho} W_{BB}^* A_{R2}^* A_{R2} Z_{2} A_{T2} A_{T2} G_{BB} A_{R1}^* A_{R1} Z_{1} A_{T1}^* A_{T1} F_{BB} S$$

$$= (a) \sqrt{\rho} W_u^* A_{R2} (A_{R2}^* A_{R2})^{-1} A_{R2}^* A_{R2} Z_{2} A_{T2}^* A_{T2} (A_{T2}^* A_{T2})^{-1} A_{T2}^* G_{u} A_{R1}^* A_{R1}$$

$$= (A_{R1}^* A_{R1})^{-1} A_{R1}^* A_{R1} Z_{1} A_{T1}^* A_{T1} (A_{T1}^* A_{T1})^{-1} A_{T1}^* F_u S$$

$$= \sqrt{\rho} W_u^* H_{2} G_{u} H_{1} F_u S$$

$$= (b) \sqrt{\rho} \tilde{U}_{1}^* U_{2} \Sigma_{2} \tilde{V}_{1}^* V_{2} \tilde{U}_{1}^* U_{1} \Sigma_{1} \tilde{V}_{1}^* V_{1} S$$

$$= \sqrt{\rho} \tilde{\Sigma}_{2} \tilde{\Sigma}_{1} S$$

(5.15)

where $(a)$ is from substituting $X_{BB} = (X_{RF}^* X_{RF})^{-1} X_{RF}^* X_u$, with $X \in \{ F, W \}$, and $G_{BB} = (G_{T}^* G_{T})^{-1} G_{T}^* G_{u} G_{R} (G_{R}^* G_{R})^{-1}$. While $(b)$ is directly by using the channel’s SVD to be $H_v = U_v \Sigma_v V_v^*$. On the other hand, the active noise in (5.7) will be

$$x_n = W_{BB}^* W_{RF}^* H_{2} G_{n1} + W_{BB}^* W_{RF}^* n_2.$$ (5.16)

Without loss of generality, we assume $N_s = 1$. The first part of the left side could be simplified as

$$W_{H_{2} G_{n1}} = W_u^* A_{R2} (A_{R2}^* A_{R2})^{-1} A_{R2}^* A_{R2} Z_{2} A_{T2}^* A_{T2} (A_{T2}^* A_{T2})^{-1} A_{T2}^* G_{u} A_{R1}^* A_{R1}$$

$$(A_{R1}^* A_{R1})^{-1} A_{R1}^* n_1$$

$$= \sigma_{2}^{\max} \tilde{U}_{1}^* A_{R1} (A_{R1}^* A_{R1})^{-1} A_{R1}^* n_1$$

and

$$(W_{H_{2} G_{n1}})^2 = (W_{H_{2} G_{n1}})(W_{H_{2} G_{n1}})^*$$
\[
= \sigma_{n_1}^2 (\sigma_{n_2}^2)^2 \tilde{U}_1^* A_{R1} (A_{R1}^* A_{R1})^{-1} A_{R1}^* \tilde{U}_1 \\
= (\sigma_{n_2}^2 \sigma_{n_1}^2)^2 \tilde{U}_1^* A_{R1} (A_{R1}^* A_{R1})^{-1} A_{R1}^* \tilde{U}_1 \\
= \sigma_{n_1}^2 (\sigma_{n_2}^2 \sigma_{n_1}^2)^2 \tilde{U}_1^* \tilde{U}_1 \\
= \sigma_{n_2}^2 (\sigma_{n_1}^2)^2 \quad (5.17)
\]

Now, the second part of (5.16) is simplified as

\[
Wn_2 n_2^* W^* = \sigma_{n_2}^2 W_{opt}^* A_{R2} (A_{R2}^* A_{R2})^{-1} A_{R2}^* A_{R2} (A_{R2}^* A_{R2})^{-1} A_{R2}^* W_{opt} \\
= \sigma_{n_2}^2 \tilde{U}_2^* A_{R2} (A_{R2}^* A_{R2})^{-1} A_{R2}^* \tilde{U}_2 \\
= (\sigma_{n_2}^2 \sigma_{n_2}^2)^2 \tilde{U}_2^* \tilde{U}_2 \\
= \sigma_{n_2}^2 \tilde{U}_2^* \tilde{U}_2 \\
= \sigma_{n_2}^2 \quad (5.18)
\]

where (a) is by assuming \( N_r = M_r = L \), and using the property \((AB)^{-1} = B^{-1}A^{-1}\).

Now, assuming \( \sigma_{n_1}^2 = \sigma_{n_2}^2 = \sigma_n^2 \), and defining \( \gamma = \frac{\rho}{N_0} \), we can use (5.15), (5.17), and (5.18) to show that the rate will be simplified the form shown in (5.12).

Theorem 5.1 indicates that there is a direct relationship between the maximum singular values of the channels and the spectral efficiency. It also highlights the fact that the singular values of the source-to-relay channel have a superior impact than those of the relay-to-destination channel. This theorem could also be used to characterize the effect of array size on the achievable rate. It is well known that increasing the array size will increase the singular values of the channel, then, considering (5.12), we expect that the source and relay receive side arrays have a higher influence on the rate than the destination and relay transmission side.
Figure 5.3: Spectral efficiency versus SNR for the special when $L = N_{RF} = 4$, $N_s \in \{1, 3\}$, $N_{\Lambda} = 20$, and zero self-interference. The array size of the source relay, and destination are $N_t = 16$, $M_r = 4$, $M_t = 16$, $N_r = 4$.

5.4 Simulation and Numerical Results

In this section, we examine the efficiency of the FD relay system using the proposed algorithm. The system performance is compared the known results for the unconstrained system (which uses full complexity RF chains) similar to that used in [112] and the half-duplex relay system in [113]. The channel is modelled with $L$ clusters and each cluster contributes a single ray with uniformly random AoAs and AoDs. All the clusters are assumed to have equal power. We also assume a uniform linear arrays with inter-elements spacing $d = \lambda/2$. The noise variance $\sigma_{n_1}^2 = \sigma_{n_2}^2 = \sigma_n^2$, and the signal to noise ratio is defined as $SNR = \frac{\rho}{N_s\sigma_n^2}$. Furthermore, we tested the validity of the assumptions made in Theorem 5.1 (namely, the channel with limited paths and the absence of self-interference) by adapting a general simulation environment where the channel has unconstrained number of paths and the existence of self-interference. Other simulation parameters include: the number of antennas $N_t$, $N_r$, $M \in \{16, 32, 64\}$, the number of RF chains $N_{RF}^t$, $N_{RF}^r$, $M_{RF}^t$, $M_{RF}^r \in \{1, 3, 6\}$, the number of data streams $N_s \in \{1, 3\}$, and the relays’ Tx-to-Rx RF chain ratios are $\beta_{RF} = 0.5$.

Firstly, we examine the performance of the proposed algorithm under the as-
sumptions used in Theorem 5.1. The channel has a limited number of paths 
\((L = 4)\) which equals the number of RF chains (namely, \(N_{RF}^t = N_{RF}^r = 4\), and \(M_{RF} = 8\)), and no self-interference. The number of the antennas used in the arrays are as follows: \(N_t = 16, M_t = 4, M_r = 16, N_r = 4\). Furthermore, the performance is compared under two data stream values \(N_s \in \{1, 3\}\). The results in Fig. 5.3 indicate an improved performance for the FD relay over the HD relay. The results also state about 15% spectral efficiency reduction as compared to the full complexity matrices. However, with the use of less RF chains, the hybrid system also introduces about 70% power saving over the digital system. Note that since \(N_{RF} = L\), the algorithm used the columns of the optional matrix \(A\) with \(N_A = 20\), as explained earlier in section 5.2.

In Fig. 5.4 a more general channel environment is used to examine the behaviour of Theorem 5.1. We used a channel with \(L = 20\) paths, and a self-interference channel with \(\sigma^2_{SI} = 0\) dB to simulate the results in Fig. 5.4a. The results indicate a slightly higher performance reduction from the digital system, with comparable energy efficiency improvement. The similarity in the behaviour of the system under the general and the asymptotic conditions encourage the use of Theorem 5.1 outcomes to explain the behaviour of the system using the proposed algorithm. Fig. 5.4b compares the performance of the three system when the array of the source, relay, and destination are equipped with 64 antennas (the relay uses \(\beta = 0.5\)). The results also compare the performance for different number of data streams.

One of the primary outcomes of Theorem 5.1 is the impact of the two channel matrices \((H_1, H_2)\) and the array size on the achievable rate. In order to highlight this, we examined the performance of the system against the number of antennas at the source, relay, and destination. The array size changed for the pair that effect each channel, i.e. to highlight the impact of \(H_1\), the array size of the source \((N_t)\) and the receive side of the relay \((M_t)\) are changed while fixing the other pair with 32 antennas while \(M_t\) and \(N_r\) are changed to assess the impact of \(H_2\). Fig. 5.5 results justify the outcomes of Theorem 5.1. As seen in (5.12), the singular values of the source-to-relay channel, which directly related to the size of the channel matrix, has a greater influence on the performance than those of the relay-to-destination channel. The results also show the positive impact of increasing the number of RF chains on the system.

For further understanding of the number of RF chains effect on the performance.
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Figure 5.4: Spectral efficiency against SNR for a channel with $L = 20$ paths, and a self-interference channel with $\sigma_{SI} = 0$ dB.

(a) Number of RF chains is 4, $N_s \in \{1, 3\}$, and $N_t = 16$, $M_r = 4$, $M_t = 16$, $N_r = 4$.

(b) Number of RF chains is 4, $N_s \in \{1, 3\}$, and $N_t = 16$, $N_r = 16$, $M_r = 16$, and $\beta = 0.5$. 

Figure 5.4: Spectral efficiency against SNR for a channel with $L = 20$ paths, and a self-interference channel with $\sigma_{SI} = 0$ dB.
Figure 5.5: Spectral efficiency for different array sizes. Changing the array size at one side of the system while fixing the other with 32 antenna arrays. \( L = 20, \ N_{RF} \in \{3, 4, 6\}, \ N_s = 1, \text{SNR}=20 \text{ dB}, \text{ and } \sigma_{SI} = 5 \text{ dB}. \)

Fig 5.6 compares the hybrid beamforming relays with the unconstrained relays for different hybridization ratio values. Small \( \omega \) represent a hybrid system that is closer to digital as the number of RF chains is close to the number of antennas. The results highlight the advantage of digital systems over the analogue processors. However, that comes at the cost of energy consumption. A larger number of RF chains means lower power saving. Fig 5.7 illustrate that effect by showing system performance measure factor, \( \Delta \), for different hybridization ratios. From this point of view, hybrid systems outperform the relay with digital processors.

### 5.5 Chapter Summery

In this chapter, we propose using a full-duplex relay system to extend the point-to-point mmWave links. Hybrid beamforming is set-up at the relays to provide power saving over digital processors. We develop a new sparsity-based algorithm for designing the hybrid (analogue/digital) precoders to be used at the source, destination, and the relay. A detailed analysis of the proposed algorithm performance was derived using a channel with a limited number of paths. In such circumstances, the spectral efficiency of the system demonstrated direct relation...
Figure 5.6: Spectral efficiency versus the hybridization ratio ($\omega$). The number of antennas in each array in the system = 32, SNR=5 dB, and $N_s = 2$.

Figure 5.7: system performance measure factor $\Delta$ versus the hybridization ratio ($\omega$). The number of antennas in each array in the system = 32, SNR=5 dB, and $N_s = 2$. 
to the maximum singular values of the source-to-relay channel and the relay-to-destination channel. However, the source-to-relay channel proved to have higher impact on the rate than the other channel.

Studying the results show that a trade-off between improving the power efficiency and reducing spectral efficiency could be reached to choose between hybrid and digital systems.
Chapter 6

Analogue Beamforming over Multi-Relay Networks

Relay networks attract a great deal of research interest in recent years as an architecture to increase the system performance [97]. The study of two-hop multiple single-antenna relay systems was introduced in [136], and the optimisation of the MMSE solution for MIMO relay networks has been investigated in [96]. In mmWaves, there is a shortage of the research in single relay networks, rather than multi-relays. Combining the benefits of the broad mmWave spectrum with the flexibility provided by the multi-relay networks is a key solution in designing small dense cells architecture that secures the high throughput needed.

In this chapter, we investigate the performance of hybrid beamformers in the two-hop multi-relay networks in mmWave as a solution to extend the point-to-point link. Hybrid precoding is used at the two terminal nodes with analogue beamformers/combiners at the relays. A split domain algorithm is proposed inspired by the performance the channel sounding techniques for ABF discussed in Chapter 4. The performance of the system is examined under different channel and system layouts. The results highlight the positive impact of employing relays in the mmWave link as well as the effect of array size on the overall system performance.

The first contribution in this chapter is the introduction of multi-relay systems in the mmWave environment. Secondly, we propose a developed algorithm that takes the split approach (analogue and digital) to design both the analogue beamformers at the relays and the hybrid processors at the source and destination.
The chapter also study the impact of having more relays and larger arrays on the overall performance.

The rest of the chapter organised as follows. The system and network models with the main assumptions used in the chapter are presented in Section 6.1. The design of the hybrid and analogue beamformers is presented in Section 6.2. Simulation results are presented in Section 6.3 before sum up the chapter in Section 6.4.

6.1 System Model

The network structure studied in this chapter consist of a single source connecting to a single destination through multi-relay systems. Both the transmitter and receiver use hybrid beamformers while the relays are equipped with analogue beamformers only. This section presents the network model and the details of the beamformers setup.

6.1.1 Network Model

Consider a multi-relay network consists of one source, one destination, and \( K \) relays, as illustrated in Fig. 6.1. The source is equipped with \( N_t \) antennas and \( N_{t_{\text{RF}}} \) RF chains. Similarly, The destination has \( N_r \) antennas and \( N_{r_{\text{RF}}} \) RF chains. On the other hand, each relay has \( M_r \) receive antennas, \( M_t \) transmit antennas and a single RF chain. In this chapter, we consider source-destination connection via one stream through each relay (i.e. \( N_s = K \), with \( N_s \) represent the number of streams). We assume a limited number of relays where \( K \leq \min\{N_{r_{\text{RF}}}, N_{t_{\text{RF}}}\} \). It is also assumed the source uses only \( K \) out of the available RF chains.

6.1.2 Beamforming System Setup

Using the available multiple RF chains, both the source and the destination process the signal in the analogue and digital domains. During the transmission, the source applies the \( N_s \times N_{t_{\text{RF}}} \) digital precoder \( \mathbf{F}_{\text{BB}} = [f_{1_{\text{BB}}}, f_{2_{\text{BB}}}, \cdots, f_{N_{t_{\text{RF}}}_{\text{BB}}}] \) followed by an \( N_t \times N_{r_{\text{RF}}} \) analogue precoder \( \mathbf{F}_{\text{RF}} = [f_{1_{\text{RF}}}, f_{2_{\text{RF}}}, \cdots, f_{N_{r_{\text{RF}}}_{\text{RF}}}] \) on the transmit
data. Therefore, the $k$-th relay received signal will be

$$y_{r,k} = H_{1,k} \sum_{i=1}^{K} F_{RF} f_i^{BB} s_i + n_{1,k}$$  \hspace{1cm} (6.1)$$

where $s = [s_1 \cdots s_K]$ is the $N_s \times 1$ transmit signal with $\mathbb{E}[ss^*] = \frac{\rho}{N_s} I_{N_s}$, $H_{1,k}$ is the $M_r \times N_t$ $k$th source-to-relay mmWave channel, which is modelled as described in Section 2.3.3. $n_{1,k}$ has a covariance matrix $R_{n_{1,k}} = \sigma_n^2 n_{1,k} I_{M_r}$.

At the $k$th relay, the signal is received through an analogue combiner, $c_k$, to be transmitted through an analogue beamformer, $b_k$. The $k$th relay transmitted signal will be

$$x_{r,k} = b_k c_k^* y_{r,k}.$$  \hspace{1cm} (6.2)$$

During the second time slot, each of the relays transmits the signal in (6.2) and the destination will observe the signal

$$y = \sum_{k=1}^{K} \left( H_{2,k} b_k c_k^* H_{1,k} \sum_{i=1}^{K} F_{RF} f_i^{BB} s_i + H_{2,k} b_k c_k^* n_{1,k} \right) + n_2$$  \hspace{1cm} (6.3)$$

where $H_{2,k}$ is the $N_r \times M_t$ $k$th relay-to-destination mmWave channel, and $n_2$ is the additive white noise at the destination with covariance matrix $R_{n_2} = \sigma_n^2 n_2 I_{N_r}$.

Afterwards, the destination will process the signal through $N_r \times N_{RF}^r$ analogue combiner $W_{RF}$, and the digital processor $W_{BB}$. The processed signal will be given by

$$\hat{s} = W^* H_2 BC^* H_1 F_{RF} F_{BB} s + W^* H_2 BC^* n_1 + W^* n_2$$  \hspace{1cm} (6.4)$$

where $W = W_{RF} W_{BB}$,

$H_1 = [H_{1,1}^T, H_{1,2}^T, \cdots, H_{1,K}^T]^T$ is the $KM_r \times N_t$ source-to-relay channel matrix,

$n_1 = [n_{1,1}^T, n_{1,2}^T, \cdots, n_{1,K}^T]^T$ is the $KM_r \times 1$ noise matrix at the relays,

$H_2 = [H_{2,1}, H_{2,2}, \cdots, H_{2,K}]$ is the $N_r \times KM_r$ relay-to-destination channel matrix,

$C = [c_1^T, c_2^T, \cdots, c_K^T]^T$ is the $KM_r \times 1$ equivalent all relays combiner matrix, and

$B = [b_1^T, b_2^T, \cdots, b_K^T]^T$ is the $KM_r \times 1$ equivalent all relays beamformer matrix.

We then can write the useful as

$$x_u = \sqrt{\rho} W_{BB}^* W_{RF}^* \sum_{k=1}^{K} H_{2,k} b_k c_k^* H_{1,k} F_{RF} f_k^{BB}$$  \hspace{1cm} (6.5)$$

\footnote{An additional subscript is added to indicate the intended relay.}
the sum interference signal at the relays is described as

\[ x_i = \sqrt{\rho} W_{BB}^* W_{RF}^* \sum_{k=1}^{K} \sum_{j \neq k}^{K} H_{2,k} b_k c_k^* H_{1,k} F_{RF_j} f_{BB} \]  

(6.6)

and the effective noise signal could be written as

\[ x_n = W_{BB}^* W_{RF}^* \sum_{k=1}^{K} (H_{2,k} b_k c_k^* n_{1,k}) + W_{BB}^* W_{RF}^* n_2 \]  

(6.7)

using (6.5), (6.6), and (6.7) the instantaneous achievable rate assuming Gaussian signalling is

\[
R = \log_2 \left| 1 + \frac{\rho \left| W^* \sum_{k=1}^{K} H_{2,k} b_k c_k^* H_{1,k} F_{RF_j} f_{BB} \right|^2}{\rho \left| W^* \sum_{k=1}^{K} \sum_{j \neq k}^{K} H_{2,k} b_k c_k^* H_{1,k} F_{RF_j} f_{BB} \right| + \left| W^* H_{2,BC}^* \right|^2 + \left| W^* \right|^2} \right|.
\]  

(6.8)
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The analogue beamformers are implemented using analogue phase shifters, therefore, their entries have equal norm, i.e. at the source \( \left( \mathbf{F}_{\text{RF}}^{(i)} \mathbf{F}_{\text{RF}}^{(i)^*} \right) \) = \( N_i^{-1} \). On the other hand, digital beamformer at the source, \( \mathbf{F}_{\text{BB}} \), has no hardware constraints except the total power constraint. With the limitation on the analogue beamformers hardware, such as choosing the phase shifters angles out of quantized sets, the analogue beamformer vectors need to be chosen from finite size codebooks. The codebook size determines the duration of the channel training as explained in Chapter 4.

6.2 Beamformers Design

In this section, we propose an algorithm to design the hybrid beamformer at the source and the destination as well as the analogue beamformers at the relays to maximise the achievable rate at the destination. With two matrices in each relay and an additional two at each terminal node, maximising (6.8) requires the joint design of the \((2K + 4)\) beamforming matrices. The design problem could be written as

\[
\text{argmax} \quad \left( \log_2 \left( 1 + \frac{\mathbf{W}_{\text{BB}}^* \mathbf{W}_{\text{RF}}^* \sum_{k=1}^{K} H_{2,k} b_k c_k^* H_{1,k} \mathbf{F}_{\text{RF}} f_{\text{BB}}^k}{\mathcal{R}_{\text{eff}}} \right) \right) \quad \mid \mathbf{F}_{\text{RF}} \in \mathcal{F}, \mathbf{W}_{\text{RF}} \in \mathcal{W}, b_i \in \mathcal{B}, c_i \in \mathcal{C} \quad (i = 1, \ldots, K)
\]

subject to \( \| \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \| _F^2 = K \)

where

\[
\mathcal{R}_{\text{eff}} = \gamma \left| \mathbf{W}_{\text{BB}}^* \mathbf{W}_{\text{RF}}^* \sum_{k=1}^{K} \sum_{j \neq k} H_{2,k} b_k c_k^* H_{1,k} \mathbf{F}_{\text{RF}} f_{\text{BB}}^j \right|^2 + \left| \mathbf{W}_{\text{BB}}^* \mathbf{W}_{\text{RF}}^* \sum_{k=1}^{K} (H_{2,k} b_k c_k^* n_{1,k}) \right|^2 + \| \mathbf{W}_{\text{BB}}^* \mathbf{W}_{\text{RF}}^* \|^2
\]

is the effective noise covariance matrix, \( \gamma = \frac{\rho_n}{\sigma_n^2} \), and \( \mathcal{F}, \mathcal{W}, \mathcal{B}, \mathcal{C} \) are sets of feasible RF vectors used to design the analogue beamformers.
Unfortunately, as explained in Chapter 5, the constraints on the analogue beamformers make finding the joint solution untraceable. In this chapter, we propose splitting the design into analogue and digital domains. The design will be performed over two phases. The first phase involves producing the analogue beamforming matrices using channel search techniques similar to those discussed in Chapter 4. Then, during the second stage, the digital processors are designed using conventional digital design methods such as Zero Forcing (ZF). The design procedure is outlined in Algorithm 6.1, and explained in details in the two subsequent subsections.

### 6.2.1 Phase One: Analogue Domain

The analogue beamforming matrices are designed by choosing the best beamforming vectors from sets of predefined vectors known as codebooks. The process includes repeatedly transmitting training signal and search for the vector pair that maximises a measuring factor (we use beamforming gain as the measuring factor in this work). As explained in Chapter 4, this process is known as channel sounding (CS), and it is performed without the need for CSI using relatively small overhead. In this chapter, Exhaustive Search (ES) where the all the possible beamforming/combining vectors combinations are tested is used to find the best pair. ES is considered the most straightforward search technique, yet it has higher overhead as compared to other search methods. Optimistically, if the analogue beamformers have continuous angles capabilities and the infinite codebook size, the selected vectors will have the same angles as the AoA/AoD.

Each node of the system has its codebook to use during the channel sounding. In Algorithm 6.1, $B, C$ represent the transmitting and receiving codebooks at the relay, respectively. $F, W$ represent source, destination codebooks. We also propose using sectored codebooks where the source and destination sectorize the available spectrum over the possible relays, assuming the relays are distributed uniformly and equally distanced. $F_k$ and $W_k$ represent the sectored codebook assigned to the $k$th relay at the source and destination, respectively. The duration of the sounding period depends on the size of the codebook, for instance, the length of the sounding between the source and the $k$th relay will be $T_L = N_{F_k} \times N_C$, where $N_X$ is the number of columns in $X$. 
At each iteration of phase one, a relay combiner \( c_k \), and the related source beamformer \( f_k^{RF} \) are found. Simultaneously, the relay beamformer \( b_k \), and the destination combiner \( w_k^{RF} \) are also determined. After \( K \) iterations, \( F_{RF}, W_{RF}, b_i \) and \( c_i \) (\( 1 \leq i \leq K \)) are all constructed. The first phase is concluded by calculating the effective channels

\[
\tilde{h}_{1,k} = c_k^* H_{1,k} F_{RF}
\]

and

\[
\tilde{h}_{2,k} = W_{RF}^* H_{2,k} b_k
\]

the effective channels will be used later in the digital processor design.

### 6.2.2 Phase Two: Digital Domain

The second phase is dedicated entirely to designing the digital processors at the source and the destination. As shown in Algorithm 6.1, the source and destination utilize Zero-Forcing (ZF) to design the digital processor using the effective channel calculated in the previous stage. Due to the sparse nature of the mmWave channel and the sectored beamforming, the effective channels are expected to be well-conditioned to be used in designing the digital processors using ZF technique.

Finally, the design is concluded by satisfying the power condition for the digital processor at the source.

### 6.3 Simulation and Numerical Results

The performance of the network model is examined by measuring the achievable rate through computer simulations. All the relays are assumed to have the same distance from the source and destination so that all the channels have the same statistics. Each of the propagation channels is modelled with \( L \) clusters channel, and each cluster contributes with single ray with uniformly distributed AoAs and AoDs. The complex path gains are assumed to be Gaussian distributed with equal variances. We assume a ULA with inter-elements spacing \( d = \lambda/2 \). The noise variance \( \sigma_{n_{1,k}}^2 = \sigma_{n_{2}}^2 = \sigma_n^2 \), and the signal to noise ratio is defined as

\[
SNR = \frac{\rho}{N_0 \sigma_n^2}.
\]

Firstly, the performance of the system under single path channel is compared to the optimistic results where the CS uses continuous angles, as shown in Fig. 6.2.
Algorithm 6.1: Analogue and digital beamformers split design algorithm.

**Input:** analogue sets:
\[ \mathcal{F} = [f_1, f_2, \cdots, f_{N_F}], \quad \mathcal{W} = [w_1, w_2, \cdots, w_{N_W}], \]
\[ \mathcal{C} = [u_1, u_2, \cdots, u_{N_C}], \quad \mathcal{B} = [v_1, v_2, \cdots, v_{N_B}] \]

**Initializing:** Sectorize \( \mathcal{F} \) and \( \mathcal{W} \) based on the number of relays

- **Phase One:** Design the analogue processors.
  - Set the analogue sets for each relay.
  - Search for the optimum pair that maximize the gain:
    * source-relay
    \[
    (f_{RF}^k, c_k) = \arg \max_{f \in \mathcal{F}_k} \forall u \in \mathcal{C} \quad \left| u^*H_{1,k}f \right|^2
    \]
    * relay-destination
    \[
    (w_{RF}^k, b_k) = \arg \max_{w \in \mathcal{W}_k} \forall v \in \mathcal{B} \quad \left| w^*H_{2,k}v \right|^2
    \]
  - Compute the effective channels at both sides as \( \tilde{h}_{1,k} = c_k^*H_{1,k}F_{RF} \), and \( \tilde{h}_{2,k} = W_{RF}^*H_{2,k}b_k \).

- **Phase Two:** Design the digital processors at the source and destination.
  - Define the effective first hop and second hop channels as
    \[
    \tilde{\mathbf{H}}_1 = \left[ \tilde{h}_{1,1}^T, \tilde{h}_{1,2}^T, \cdots, \tilde{h}_{1,K}^T \right]^T \quad \text{and} \quad \tilde{\mathbf{H}}_2 = \left[ \tilde{h}_{2,1}, \tilde{h}_{2,2}, \cdots, \tilde{h}_{2,K} \right],
    \]
    respectively.
  - Design \( F_{BB} \) and \( W_{BB} \) using ZF:
    \[
    F_{BB} = \tilde{\mathbf{H}}_1^* \left( \tilde{\mathbf{H}}_1 \tilde{\mathbf{H}}_1^* \right)^{-1}
    \]
    \[
    W_{BB} = \tilde{\mathbf{H}}_2^* \left( \tilde{\mathbf{H}}_2 \tilde{\mathbf{H}}_2^* \right)^{-1}.
    \]
  - Compute the final digital processors that satisfy the power constraints.
    \[
    f_{BB}^k = \frac{f_{BB}^k}{\|F_{RF}f_{BB}^k\|_F} \quad \forall k
    \]

**Output:** \( F_{RF}, W_{RF}, b_i, c_i \) (\( 1 \leq i \leq K \))
Two different size codebooks are used, one with seven vectors and the other with 11 vectors. Adding vectors to the codebook improve the performance and eventually, the system performance will approach the optimistic results. However, this comes at the price of the time as the sounding duration for the larger codebook is three times the duration of the other. ULAs with 16 antennas are used at the source, destination, and each of the relays.

The number of relays effect to the network performance is demonstrated in Fig. 6.3 as compared to the full-duplex relay system in Chapter 5 and the HD rely in [95]. The results indicate that adding more relays to the network improves the multi-relay performance. Increasing the number of relays from 2 to 12 improve the performance from about 70 % of the FD relay efficiency to about 90 %. Changing the number of relays from 4 to 12 relays almost double the spectral efficiency. However, this comes at the cost of energy efficiency. As interference is considered the main drawback of the multi-relay system, adding interference mitigation capabilities will aid to improve the performance in spite of the added complexity.

Fig. 6.4 compares the system performance for different array size at source/destination with fixed relay arrays at eight antennas on each side. Increasing the
number of antennas in the array will improve the directivity, which leads to better performance. However, the capacity to improve the performance is limited by the size of the relays arrays. We also notice the effect of the hybridization ratio (Δ) on the performance. For a fixed number of RF chains, as the array size get larger, Δ becomes greater. Thus, the system acts more as analogue beamforming than as a hybrid or a digital one. This explains the flat performance as the array size tend to increase.

Finally, the performance of the proposed algorithm is examined under realistic conditions using mmWave channels with $L = 10$ paths and compared to the FD relay system, as shown in Fig. 6.5. The results indicate that, considering the same number of streams, using four analogue relays as compared to a single hybrid relay causes a performance reduction of about 40%, without significant improvement in the power efficiency. The benefit of using multiple analogue relays will be in the redundancy provided for the link in case of blockage.
Figure 6.4: Spectral efficiency versus array size at the source and destination. The number of antennas at each relay fixed to \( M_r = M_t = 8 \). \( K \in \{2, 4\} \), SNR = 5dB, and \( L = 10 \).

Figure 6.5: Spectral efficiency versus SNR with \( L = 10 \), \( K \in \{2, 4, 8\} \) relays, \( N_{RF} = 4 \) and \( N_r = N_t = M = 16 \).
6.4 Chapter Summary

In this chapter, we propose a two phase split algorithm that increases the achievable rate of multi-relay systems in mmWaves. The source and the destination are equipped with hybrid beamformers and multiple RF chains, while each of the relays is equipped with single RF chain and an analogue beamformer. The beamformers work on directing the signal to obtain higher beamforming gains help overcoming the path losses in outdoors mmWave environments. The performance of the system is compared to the optimistic results where the analogue beamformers have continuous angles capabilities. The analogue beamformers/-combiners are designed using the modified joint channel sounding by sectoring the coverage spectrum based on the number of relays.

The results showed that the increasing the array size increases the spectral efficiency of the system. In addition, larger codebooks produce higher beamforming gains, but at the cost of the spectral efficiency of the system as the transmission time will be reduced. Finally, increasing the number of the relays in the system have a positive impact on the system performance.
Chapter 7

Hybrid Beamforming over Multi-Relay Networks

This chapter investigates using multiple relays to link two points in the outdoor mmWave environments. The system model employs hybrid beamformers in the AF half-duplex relays to overcome the propagation losses usually arise in such environments. We develop hybrid beamformers design algorithm based on the sparsity approach discussed in Chapter 5. The performance of the system is compared with the conventional multi-relay [96] and single relay systems. The chapter explores the behaviour of the proposed algorithm under different assumptions and real life scenarios.

The main contribution of this chapter is developing a design algorithm to jointly create the analogue and digital matrices of the hybrid beamformers in multi-relay mmWave systems. The modified design algorithm takes into consideration the limitations associated with adding more relays to the network. Results show significant performance improvement for multi-relay links as compared to those with a single relay. The results also highlight the influence of the array size and the number of RF chains on the overall performance.

The rest of the chapter organised as follows. The system and network models with the main assumptions used in the chapter are presented in Section 7.1. The design of the hybrid processors using the sparsity algorithm is presented in Section 7.2. Simulation results are presented in Section 7.3 before reviewing the chapter in Section 7.4.
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7.1 System Model

As illustrated in Fig. 7.1, the network model considered in this chapter consists of a single source communicating with a single destination through $K$ relays. The source is equipped with $N_t$ antennas, and the destination has $N_r$ antennas. Each relay has $M$ antennas, assigns $M_r = \beta M$ for receiving and $M_t = (1 - \beta)M$ for transmitting, with $\beta$ being the relay Tx-to-Rx antenna ratio. Amplify-and-Forward relay scheme is adopted with two time slots to perform the process. During the first time slot, the source transmits the $N_t \times 1$ signal $s$ with $\mathbb{E}[ss^*] = \rho I$. The $k$th relay received signal could be written as

$$y_{r,k} = \mathbf{H}_{1,k}s + \mathbf{n}_{1,k}$$

(7.1)

where $\mathbf{H}_{1,k}$ is the $k$th source-to-relay mmWave channel, which is modelled as described in Section 2.3.3. $\mathbf{n}_{1,k}$ is the additive noise vector with covariance matrix $\mathcal{R}_{n_{1,k}} = \sigma^2_{n_{1,k}} I_{M_t}$. During the second time slot, the relays transmit their signals
to the destination, and the transmitted signal of the $k$th relay can be written as

$$x_{r,k} = G_{T,k}G_{BB,k}G_{R,k}^*y_{r,k}$$

(7.2)

where $G_{T,k}, G_{BB,k}, G_{R,k}$ are the $M_t \times M_{RF}$ analogue transmit matrix, the $M_{RF} \times M_{RF}$ baseband matrix, and the $M_t \times M_{RF}$ analogue receive matrix of the hybrid beamformer of the $k$th relay, respectively. $M_{RF}$ is the number of RF chains at each side of the relay. $G_{T,k}, G_{R,k}$ are implemented using analogue phase shifters, therefore, their entries have equal norm, i.e. for the transmit side 

$$(G_{T,k}(i,j))^T(G_{T,k}(i,j))^* = M_{t}^{-1},$$

while $G_{BB,k}$ has no hardware restrictions except the total power constraint.

Now, defining $G_k = G_{T,k}G_{BB,k}G_{R,k}^*$, the destination received signal can be expressed as

$$y = \sum_{k=1}^{K} (H_{2,k}G_k H_{1,k}s + G_k n_{1,k}) + n_2$$

(7.3)

where $H_{2,k}$ is the $k$th relay-to-destination mmWave channel, and $n_2$ is the additive white noise at the destination with covariance matrix $R_{n_2} = \sigma^2_{n_2} I_{N_r}$. The destination receives the signal through $N_r \times N_{RF}$ RF combiner $W_{RF}$, and processes it using the $N_{RF} \times N_s$ baseband matrix $W_{BB}$. The processed signal is given by

$$\hat{s} = W^*H_2G_1s + W^*G_1n_1 + W^*n_2$$

(7.4)

where $N_{RF}$ is the number of RF chains at the destination, $W = W_{RF}W_{BB}$, $H_1 = [H_{1,1}^T, H_{1,2}^T, \cdots, H_{1,K}^T]^T$ is the $KM_t \times N_t$ source-to-relay channel matrix, $n_1 = [n_{1,1}^T, n_{1,2}^T, \cdots, n_{1,K}^T]^T$ is the $KM_t \times 1$ noise matrix at the relays, $G = [G_1, G_2, \cdots, G_K]$ is the $KM_t \times KM_t$ equivalent block diagonal relay amplifying matrix, and $H_2 = [H_{2,1}, H_{2,2}, \cdots, H_{2,K}]$ is the $M_t \times KM_t$ relay-to-destination channel matrix.

### 7.2 Hybrid Beamformer Design

The hybrid beamformer at the relay consists of one analogue beamformer at the receive side, a digital baseband processor, and another analogue beamformer at the transmitter side of each relay. Similarly, the hybrid processor at the destination consists of an analogue processor and a digital baseband matrix.
The design of the hybrid processors that minimize the error between the estimated signal and the transmitted signal requires a joint optimisation of all the matrices in the system, as described in (7.5)-(7.7) below

\[
(G_{opt}^{R}, W_{opt}^{B}) = \arg\min_{G^{R}, k, G^{B}, k, G^{T}, k, W^{RF}, W^{BB}} \| s - W_{BB}^* W_{RF}^* W_{BB} y \|_F, \tag{7.5}
\]

subject to \( G_{T,k} \in G_{RF}^{T,k}, G_{R,k} \in G_{RF}^{r,k}, W \in W_{RF} \) \( \tag{7.6} \)

\[
\| G_{T,k} G_{BB,k}^* (H_{1,k} s + n_{1,k}) \|_F^2 = \rho_k \tag{7.7}
\]

where \( G_{RF}^{T,k}, G_{RF}^{r,k} \), and \( W_{RF} \) in (7.6) represent the sets of the feasible RF processors at the relay and the destination. While (7.7) describe the power constraint with \( \rho_k \) is the transmitted power of the \( k \)th relay.

As explained earlier in Chapter 5, jointly optimizing the problem in (7.5) with the constrains in (7.6) is known to be intractable. We propose a near optimal solution by firstly decoupling the relay and destination, then minimise the Frobenius norm of the difference between the hybrid processors and the unconstrained precoder \( G_{u,k}, W_u \). The unconstrained processors are designed using full complex RF chains as described in Section 3.2.

### 7.2.1 Design Problem Formulation

In this section, the hybrid beamformer matrices are designed based on the sparse approximation. The process begins by writing the problem of minimizing the error between the hybrid beamformer and the unconstrained precoder. For instance, the design problem at the \( k \)th relay will be written as:

\[
(G_{opt}^{R,k}, G_{BB,k}^{opt}, G_{T,k}^{opt}) = \arg\min_{G^{R,k}, G^{BB,k}, G^{T,k}} \| G_{u,k} - G_{T,k} G_{BB,k} G_{R,k}^* \|_F, \tag{7.8}
\]

subject to \( G_{T,k} \in G_{RF}^{T,k}, G_{R,k} \in G_{RF}^{r,k}, \)

\[
\| G_{T,k} G_{BB,k} G_{R,k}^* (H_{1,k} s + n_{1,k}) \|_F^2 = \rho_k
\]

which sum up to using the array vectors of the channels to select the best representation of \( G_{R,k}^{opt} \) and \( G_{T,k}^{opt} \). The problem in (7.8) can be solved using the greedy OMP principle. Now, the RF vectors sets, \( G_{RF}^{T,k} \) and \( G_{RF}^{r,k} \), can be embedded into
the design problem which could be re-written as

\[
\hat{G}_{BB,k}^{opt} = \arg\min_{\hat{G}_{BB,k}} \|G_{u,k} - \Lambda_{T,k} \hat{G}_{BB,k} \Lambda_{R,k}^*\|_F,
\]

s.t. \( \|\text{diag}(\hat{G}_{BB,k} \hat{G}_{BB,k}^H)\|_0 = M_{RF} \),

\[
\|\Lambda_{T,k} \hat{G}_{BB,k} \Lambda_{R,k}^*\|_F^2 = \rho_k \tag{7.9}
\]

below are some definitions related to the problem above:

- \( \Lambda_{T,k} \) is an \( M_t \times N_A \) matrix defined as \( \Lambda_{T,k} = [A_{T_2,k}|A] \) (with \( N_A > P \times L \)), where
  - \( A_{T_2,k} \) columns consist of the transmitter array response vectors of the relay-to-destination channel, i.e. \( A_{T_2,k} = \begin{bmatrix} a_t(\theta_{1,2}^k) & a_t(\theta_{2,2}^k) & \cdots & a_t(\theta_{P \times L}^k) \end{bmatrix} \).
  - \( A \) is used when \( P \times L \leq N_{RF} \) and its columns are chosen to satisfy arbitrary analogue beamforming constraints.
- \( \Lambda_{R,k} \) is similar to \( \Lambda_{T,k} \) and defined as \( \Lambda_{R,k} = [A_{R_1,k}|A] \), with \( A_{R_1,k} = \begin{bmatrix} a_r(\theta_{1,1}^k) & a_r(\theta_{2,1}^k) & \cdots & a_r(\theta_{P \times L}^k) \end{bmatrix} \).
- \( \hat{G}_{BB,k} \) is an \( N_A \times N_s \) sparse matrix having \( M_{RF} \) nonzero rows (which constitute of \( G_{BB,k} \))
- \( \|\text{diag}(\hat{G}_{BB,k} \hat{G}_{BB,k}^H)\|_0 = M_{RF} \) ensures that \( \hat{G}_{BB,k} \) is \( M_{RF} \) sparse.

Now, as only \( M_{RF} \) rows of \( \hat{G}_{BB,k} \) are nonzero, only \( M_{RF} \) vectors of \( \Lambda_{T,k} \) and \( \Lambda_{R,k} \) are selected, and they represent the vectors of \( G_{T,k}^{opt} \) and \( G_{R,k}^{opt} \), respectively. In other words, in (7.9), the columns of \( G_{u,k} \) are approximated by a linear combination of \( \Lambda_{T,k} \) and \( \Lambda_{R,k} \) with the nonzero elements of \( \hat{G}_{BB,k} \) specify the weights.

### 7.2.2 Design Algorithm

In this section, we develop an algorithm to design the hybrid processors at each of the relay nodes. The design algorithm is based on the OMP principle to jointly design the three matrices of the relay, namely \( G_{T,k}^{opt}, G_{R,k}^{opt} \) and \( G_{BB,k}^{opt} \). The procedure of the algorithm is outlined in Algorithm 7.1, and could be explained as follows.
• The algorithm is initiated by defining the analogue vector sets that will be used in the process, namely $\Lambda_{T,k}$ and $\Lambda_{R,k}$. The algorithm will also set-up the identification matrices ($\Psi_R$, $\Psi_T$) as empty sets to be used later to search for the most correlated vectors. In the final stage of the initiating step, the residual matrix $G_{\text{res},k}$ initially to be equal to the unconstrained processor matrix, $G_{u,k}$.

• During the first iteration, the columns of $\Lambda_{T,k}$ and $\Lambda_{R,k}$ that correlate the most with $G_{u,k}$ are identified (step 1a).

• After determining the most correlated vectors, the selected columns are added to $G_{T,k}$ and $G_{R,k}$, (step 1b).

• Afterwards, $G_{\text{BB},k}$ coefficient are obtained by solving the least squares problem ($G_{u,k} = G_{T,k}G_{\text{BB},k}G_{R,k}$), (step 1c).

• The residual is updated by subtracting the selected columns of $G_{T,k}$, $G_{R,k}$, and $G_{\text{BB},k}$ (step 1d).

• On each of the successive iterations, unlike the first iteration, the algorithm identifies the columns of $\Lambda_{T,k}$ and $\Lambda_{R,k}$ that not only correlate the most with the residual $G_{\text{res},k}$ but with the already selected $G_{R,k}$ or $G_{T,k}$ columns as well (step 2).

• Then, the new identified columns are added to the RF beamformers.

• Finally, the baseband processing matrix is produced by stratifying the power condition (step 2).

The hybrid processors at the destination are designed similarly, see Appendix B for detailed algorithm.

7.3 Simulation and Numerical Results

The performance of the proposed algorithm is examined by measuring the achievable rate through computer simulations. All the channels are assumed have the same statistics and each propagation channel is modelled with $L$ clusters and each cluster contributes with single ray with uniformly random AoAs and AoDs. The complex path gains are assumed to be Gaussian distributed with equal variances.
Algorithm 7.1: Relay hybrid processors design algorithm.

Input:
- Define the analogue sets: $\Lambda_{T,k} = [A_{T_2,k} | A], \Lambda_{R,k} = [A_{R_1,k} | A]$
- Define the unconstrained processors $G_{u,k}$

Initialization: Set up the initial residual matrix $G_{res,k} = G_{u,k}$

1. First Iteration ($i = 1$).
   
   (a) Identify the columns of $\Lambda_{T,k}, \Lambda_{R,k}$ that correlate the most with $G_{u,k}$:
   
   \[
   \Psi_R = G_{u,k}A_{R,k} \\
   \Psi_T = \Lambda_{T,k}^* G_{u,k}
   \]
   
   \[
   u = \arg\max_{n=1,\ldots,P \times L} (\Psi^*_R \Psi_R)_{n,n} \\
   v = \arg\max_{n=1,\ldots,P \times L} (\Psi^*_T \Psi_T)_{n,n}
   \]
   
   (b) Add the identified columns to $G_{T,k}, G_{R,k}$:
   
   \[
   G^{(i)}_{R,k} = \Lambda_{R,k}^{(u)} \\
   G^{(i)}_{T,k} = \Lambda_{T,k}^{(v)}
   \]
   
   (c) Calculate $G_{BB,k}$ by solving the least squares problem:
   
   \[
   G_{BB,k} = (G^*_{T,k}G_{T,k})^{-1}G^*_{T,k}G_{u,k}G_{R,k}(G^*_{R,k}G_{R,k})^{-1}
   \]
   
   (d) Calculate the residual:
   
   \[
   G_{res,k} = \frac{G_{u,k} - G_{T,k}G_{BB,k}G^*_{R,k}}{||G_{u,k} - G_{T,k}G_{BB,k}G^*_{R,k}||_F}
   \]

2. The remaining ($M_{RF} - 1$) Iterations ($i = 2, \ldots, M_{RF}$).

   (a) Identify the columns of $\Lambda_{T,k}, \Lambda_{R,k}$ that correlate the most with the residual $G_{res,k}$:
   
   \[
   \Psi_R = G^*_{T,k}G_{res,k}A_{R,k} \\
   \Psi_T = \Lambda^*_{T,k}G_{res,k}G_{R,k}
   \]
   
   (b) Repeat steps (1b) to (1d) above.

3. Satisfy the power condition:

   \[
   ||G_{T,k}G_{BB,k}G^*_{R,k}(H_{1,k}S + n_{1,k})||^2_F = \rho_k
   \]

Output: $G_{T,k}, G_{BB,k}, G_{R,k}$ ($1 \leq k \leq K$)
CHAPTER 7. HBF MULTI-RELAY NETWORKS

Figure 7.2: Spectral efficiency versus SNR with $L = 20$, $N_{RF}^r = 3$, $N_A = 20$, and $K \in \{1, 2, 4, 6, 8\}$ relays. The array size of the source relay, and destination are $N_t = 6$, $M_r = 8$, $M_t = 8$, $N_r = 6$.

We assume a ULA with inter-elements spacing $d = \lambda/2$. The noise variance $\sigma^2_{n_1, k} = \sigma^2_{n_2} = \sigma^2_{n}$, and the signal to noise ratio is defined as $SNR = \frac{\rho}{N_r \sigma^2_n}$. Other simulation parameters as follows: the number of RF chains $N_{RF}^r$, $M_{RF} \in \{3, 4, 5\}$, and the number of data streams equals the number of antennas at the source.

Firstly, the performance of the proposed hybrid multi-relay system is compared with the unconstrained system in [96] and the single relay system. The results in Fig. 7.2 show an improvement in the performance associated with adding more relays to the network. As expected, due to the use of more RF chains, the unconstrained processors outperforms the hybrid ones. Closer inspection of the results shows that while HBF relays provide only 40 % of the unconstrained relays spectral efficiency, yet they provide power saving of about 60 %. It is also shown that 4 hybrid relays are required to match the performance of one unconstrained relay. However, the argument for HBF system is made based on the energy and cost saving due to the reduced number of RF chains.

The effect of the number of the relays is further examined in Fig. 7.3. The results are obtained using channels with $L = 20$ paths and array size of 6 antennas at the source/destination, and 8 antennas at the relay receive/transmit sides. The results confirm what is previously obtained using different number of
Figure 7.3: Spectral efficiency versus number of relays when $L = 20$, $SNR = 5dB$, and $N_{RF} = 4$. The array size of the source relay, and destination are $N_t = 6$, $M_r = 8$, $M_t = 8$, $N_r = 6$.

Finally, the system performance was examined for different array sizes at each of the relays for fixed source/destination arrays as shown in Fig. 7.4. The number of antennas at the relays could be adjusted to reach the peak performance. For a fixed number of RF chains, having larger arrays means higher hybridization ratio, $\omega$, which results a system that is more analogue than digital as the value of $\omega$ increases from about 1 to higher values. Obviously, low $\omega$ values produce better performance as the system acts more as the unconstrained one. The results also highlight the positive impact of adding RF chains to the system. However, this comes on the cost of energy consumption and manufacturing cost.

7.4 Chapter Summery

This chapter proposed the design of the hybrid processors in a mmWave-based multi-relay network. The relays and the destination are equipped with multiple antennas and hybrid beamformers to direct the signal and obtain higher beamforming gains to overcome the path losses in outdoors mmWave environments. A modified design algorithm was developed to jointly find the analogue and digital
Figure 7.4: Spectral efficiency versus array size at the relay transmit and receive sides. Number of antennas at the source and destination fixed to $N_t = N_r = 6$. $K \in \{6, 8, 10\}$, $M_{RF}, N_{RF} \in \{4, 5\}$, SNR= 5dB, and $L = 20$. 
matrices of the hybrid processor. The results highlight the benefits of adding more relays to the network on the overall performance. It was also shown that unconstrained processors outperform the hybrid ones and it requires 4 hybrid relays to match the performance of single unconstrained relay. However, hybrid relays provide energy and cost saving which is considered crucial in the next generation wireless communications. The chapter also discussed the effect of increasing the array size for a fixed number of RF chains. Having larger arrays means higher hybridization ratio which leads to an analogue-like system behaviour. Yet, the performance could be boosted by adding more RF chains to the system. MMSE is used to design the unconstrained processors at the relays and the destination with no processing at the source.
Chapter 8

Conclusions and Future Directions

8.1 Conclusions

The work presented in this thesis focused on the challenges of adopting mmWave frequencies in the next generation of cellular networks. The study examines various topologies and techniques to extend the point-to-point link at mmWave outdoor environments. This work has investigated the difference between single and multiple-relay networks, full-duplex and half-duplex relaying, and analogue beamforming as compared to hybrid beamforming processors.

Overall, it was concluded that adding relays to the mmWave network increases the efficiency and provide a more reliable connection. It was shown that different channel sounding techniques in ABF systems provide a range of spectral and energy efficiencies which could be used to a trade-off between the techniques. Although ABF proved to be a power efficient architecture, it has several drawbacks. One of which is the restriction to single stream. Chapter 6 provide a solution by using hybrid systems at the base stations and analogue systems at the relays. Chapters 5 and 7, however, investigated full hybrid systems.

The analysis of various scenarios was presented to give a perspective of how the system behave under more general conditions. The specific conclusions of each chapter were as follows.

Chapter 2 was started by tracking the history of mmWave licensing and the development of the related systems. Next, both large-scale and small-scale properties of the mmWave channels were discussed. Then, a detailed review of the
Channel measurement studies at the indoor and outdoor environments were presented. The chapter also some key principles of antenna arrays and beamsteering by controlling the phases of the signals.

Chapter 3 presented a review of the analogue and hybrid beamforming systems at mmWave communications and relay networks in conventional MIMO channels. Chapter 3 also include a literature review of beamforming and employing relays in mmWave based networks.

Chapter 4 investigated the use of analogue beamformers to increase the gain of the array at the transmitter and the receiver. The proposed system uses single RF chain and a series of digital phase shifters to control the phase of the signal fed to the array elements which in turn change the direction of the beam. The analysis of the analogue beamforming system was presented under certain conditions, namely, single path channels and continuous angle capabilities at the phase shifters. As digital phase shifters are adopted, the phase quantisation error (PQE) is developed as a result of the correlation between the array elements is inevitable. Chapter 4 study the effect of PQE of the beamforming gain of the system.

ABF uses channel sounding techniques to avoid the need for channel knowledge at the transmitter or the receiver. Besides, the large number of antennas and the nature of the mmWave channel make acquiring the channel knowledge rather difficult. Channel sounding techniques send a pilot signal a particular number of times to find the best combining and beamforming vectors. This chapter compares three different channel sounding techniques, namely exhaustive search, side-to-side search, and $n$-tier search. Both energy and spectral efficiency of the three techniques was examined in details. The results help to determine the minimum search duration required to achieve better spectral efficiency with minimum power consumption. We were also able to identify the lowest number of quantization bits needed to produce accepted results.

In Chapter 5, the full-duplex (FD) amplify-and-forward (AF) relaying is used to improve the mmWave connectivity. Hybrid (analogue and digital) beamforming where more than one RF chain and less than the number of array antennas is used to improve the gain. The spectral efficiency of the system was analysed using a channel with a limited number of paths. It is found that the efficiency is directly affected by the channels singular values. Also, it was proven that the singular values of the source-to-relay channel have a superior impact than those
of the relay-to-destination channel. The design algorithm uses the greedy principle of the Orthogonal Matching Pursuit (OMP) method. The results emphasise the importance of hybrid beamforming in mmWave communications. The results also demonstrate a significant performance improvement for the full-duplex links over half-duplex. Finally, the results show the effect of increasing the number of antennas in the system.

Chapters 6 and 7 studies multi-relay networks in mmWave. In chapter 6, the efficiency of the multi-relay with analogue beamforming systems is investigated while Chapter 7 presented the network with hybrid relays. In Chapter 6, the proposed algorithm split the design into two domains: analogue and digital. The analogue phase requires no channel information to find the best beamforming vectors where channel sounding techniques similar to those used in Chapter 4 are adopted. The algorithm has relatively limited training overhead as compared to Chapter 7 where the algorithm was based on OMP and require full channel knowledge. The performance of the systems is examined under different channel and system layouts. Results show significant performance improvement for multi-relay links as compared to links with a single relay. The results highlight the positive impact of employing relays in the mmWave link as well as the effect of array size on the overall system performance.

8.2 Future Directions

One of the main challenges in designing mmWave systems is sensitivity to blockages. It was shown in Chapter 2, for instance, that a path loss exponent of 2 for line-of-sight propagation but 4 (plus additional power loss) for non-line-of-sight. MmWave cellular research will need to incorporate sensitivity to blockages and more complex channel models in the analysis. In order to apply the mmWave technique to 5G systems while guaranteeing effective coverage, anti-blockage mechanisms are required through which the mmWave system can adaptively switch from LOS transmission mode to NLOS transmission mode, such that seamless network connectivity can be maintained. The most obvious extension of the work in this thesis is to take it into the practical field and adopt the proposed algorithms in a real case scenarios in the outdoors environment. In what follows, possible research directions that might relate to the work in this thesis taking in consideration the blockage effect are suggested.
8.2.1 Adaptive (Smart) Antenna Arrays

One of the scenarios for the shadowing and blockage effect is illustrated in Fig. 8.2. In this scenario, the multipath environment is considered to be time invariant. Each moving obstacle causes a change (or blockage) of the path between the transmitter and the receiver. This causes overall performance of the state-of-the-art techniques to depend on the randomness level of the wireless channel. One solution is to employ antenna systems where the antenna array position change depending on the angle of the incoming signal [141]. The main principle relies on the fact that the phase of each multipath component is affected by the position of the antenna array. The design objective is therefore to determine the best array position that will provide a constructive combination of the individual components for maximising the received signal power and reduce fading. Due to the sparse nature of the mmWave channels, small alterations in the antenna location (in the range of few wavelengths) is expected to vary the phase of each tap coefficient as a result of change in total propagation distance. This sparse nature of the mmWave multipath channel is the key enabling factor that makes the control of the overall channel response via adaptation possible.

8.2.2 Adaptive Beamforming

The relatively lower main-lobe directivity gain for the low-resolution codebooks adds to the challenges of the channel time changing behaviour. This makes the probability of successfully estimating the AoA/AoD ranges of the previous level becomes lower and lower. Adaptive beamforming can be used to support the flexible switch between LOS and NLOS transmissions, as well as to establish a suitable multi-hop NLOS link to the intended user. To achieve this, adaptive array processing algorithms are required that can act quickly when the antenna beams are blocked. This requires faster algorithms to cope with the changing nature of the mmWave channel and larger codebooks with more vectors to select from. Backup beamforming vectors, adaptive antenna selection, and multi-beam (or multi-peak beam) transmission are possible scenarios that could assist building the new algorithm.
8.2.3 Adaptive Relay Networks (Relay Selection)

Blockages in the network are usually caused by concrete buildings which cannot be penetrated by mmWaves. Fig. 8.2 demonstrate a blockage scenario in mmWave based network where relays are used. Several researchers have tried to model blockages with varied level of success based on different geographical scenarios. One way is to use the Poisson point process (PPP) based random blockage model, where $e^{-\beta d_x}$ is considered to be the probability of LOS with $\beta$ being the blockage density and $d_x$ the distance between the transmitting and receiving nodes. Another model that has been considered in literature is a fixed LOS probability model, see e.g. [120,139,140].

We discussed in this thesis employing multiple relays to extend the mmWave link coverage. However, due to the impact of blockage, some of the relay nodes may not be available or capable of supporting the transmission from source node to the destination node and only a subset of the relay nodes may participate in the communication. One of the possible extensions of this work is investigating the distribution of the active relays taking in consideration the blockage effect.
Building a model to select the best relay to avoid the blockage is an interesting extension of the work. Some of the literature already studying relay selection algorithm that could prove to be useful [138].
Bibliography


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Appendix A

Distribution of $\sin(AoA/AoD)$

This section is to find the distribution of $\sin(\theta^t)$ with $\theta^t \sim U[0, 2\pi]$.

Let $X = \theta^t$ and $Y = \sin X$, then the density of the random variable $X$ is

$$f_X(x) = \begin{cases} 
1/2\pi & \text{if } x \in [0, 2\pi] \\
0 & \text{if } x \notin [0, 2\pi] 
\end{cases}$$

The distribution function is

$$F_X(x) = \begin{cases} 
0 & x < 0 \\
x/2\pi & x \in [0, 2\pi] \\
1 & x > 2\pi 
\end{cases}$$

The random variable $Y$ takes values on $[-1, 1]$. Hence, $P(Y \leq y) = 0$ for $y \leq -1$ and $P(Y \leq y) = 1$ for $y \geq 1$. Let now $y \in (-1, 1)$. We have

$$F_Y(y) = P(Y \leq y) = P(\sin(X) \leq y)$$

The equation $\sin(x) = y$ has two solutions in the interval $[0, 2\pi]$: $x = \arcsin(y)$, $\pi - \arcsin(y)$ for $y > 0$ and $x = \pi - \arcsin(y)$, $2\pi + \arcsin(y)$ for $y < 0$. 162
Hence,

\[ F_Y(y) = \frac{\pi + 2 \arcsin(y)}{2\pi}, \quad y \in (-1, 1) \]

The distribution function of \( Y \) is

\[
F_Y(y) = \begin{cases} 
0 & y \leq 0 \\
\frac{\pi + 2 \arcsin(y)}{2\pi}, & y \in (-1, 1) \\
1 & y > 1.
\end{cases}
\]

We differentiate the above expression to obtain the probability density:

\[
f_Y(y) = \begin{cases} 
\frac{1}{\pi \sqrt{1-y^2}}, & y \in (-1, 1) \\
0, & y \notin (-1, 1)
\end{cases}
\]  \hspace{1cm} (A.1)
Appendix B

Hybrid Beamforming Design Algorithms

This appendix introduces the design algorithms of the hybrid beamforming matrix at the source and the hybrid beamforming matrix at the destination for the relay systems. The algorithms are based on the sparsity principle presented in [29]. Algorithm B.1 designs the hybrid processors at the transmitter. The initial step include defining the analogue sets and predefined matrices required for selecting the best vectors. Then, $N_{RF}^i$ iterations are performed to select the columns of $(A_T^1)$ that correlate the most with the unconstrained processors. The baseband matrix coefficients are then calculated using the least squares methods and the algorithm ends by satisfying the power condition.

Similarly, the destination hybrid combiner could be designed. The optimal combiner will be defined as $W_u = \tilde{U}$, with $\tilde{U}$ consist of the $N_s$ largest singular values of $H$. Algorithm B.2 will be used to design both $W_{RF}$ and $W_{BB}$. 
Algorithm B.1: Hybrid processors design algorithm (transmitter side)

1. Define the inputs to the algorithm:
   (a) $A_{T1} = [a_{t1}(\theta_{t1}^1) a_{t1}(\theta_{t1}^2) \cdots a_{t1}(\theta_{t1}^L)]$ where $a_{t1}(\theta_{t1}^l)$ is the $l$-th array response vector of the transmitting side for the channel between the source and the relay ($H_1$).
   (b) $F_u = \tilde{V}_1$ is the $N_t \times N_s$ optimum precoder unconstrained precoder, and $\tilde{V}_1$ is the $N_s$ largest channel’s right singular vectors.

2. Define a residual error matrix $F_{res}$. Initially defined as $F_{res} = F_u$.

3. Start a loop: $1 \leq i \leq N_{RF}^t$

4. Find the column in ($A_T$) that most strongly correlate with the optimum precoder ($F_u$):
   (a) Define a matrix $\Psi = A_{T1}^* F_{res}$,
   (b) Find the index that indicates the maximum as:
   $$u = \text{argmax}_{n=1,\ldots,L} (\Psi \Psi^*)_{n,n}$$
   (c) Add this column to the RF precoding matrix $F_{RF}$ as:
   $$F_{RF} = [F_{RF} | A_{T1}(:,u)]$$

5. Solve $F_u = F_{RF} F_{BB}$ for $F_{BB}$ using the least squares method as:
   $$F_{BB} = (F_{RF}^* F_{RF})^{-1} F_{RF}^* F_u$$

6. Remove the effect of the chosen column from the residual error as:
   $$F_{res} = \frac{F_u - F_{RF} F_{BB}}{||F_u - F_{RF} F_{BB}||_F}$$

7. End the loop.

8. Ensure the power constraint is exactly satisfied as:
   $$F_{BB} = \sqrt{N_s} \frac{F_{BB}}{||F_{RF} F_{BB}||_F}$$
Algorithm B.2: Hybrid processors design algorithm (receiver side)

1. Define the inputs to the algorithm:
   
   (a) $\mathbf{A}_{R2} = [\mathbf{a}_{r2}(\theta_r^1) \mathbf{a}_{r2}(\theta_r^2) \cdots \mathbf{a}_{r2}(\theta_r^L)]$ where $\mathbf{a}_{r2}(\theta_r^\ell)$ is the $\ell$-th array response vector of the receiving side for the channel between the relay and the destination ($\mathbf{H}_2$).
   
   (b) $\mathbf{W}_u = \tilde{\mathbf{U}}_2^*$ is the $N_s \times N_r$ optimum unconstrained combiner, and $\tilde{\mathbf{U}}_2$ is the $N_s$ largest channel’s left singular vectors.

2. Define a residual error matrix $\mathbf{W}_{res}$. Initially defined as $\mathbf{W}_{res} = \mathbf{W}_u$.

3. start a loop: $1 \leq i \leq N_{RF}^r$

4. Find the column in ($\mathbf{A}_{R2}$) that most strongly correlate with the optimum combiner ($\mathbf{W}_u$):
   
   (a) Define a matrix $\mathbf{\Psi} = \mathbf{W}_{res} \mathbf{A}_{R2}^*$,
   
   (b) Find the index that indicates the maximum as:
   
   $$u = \operatorname{argmax}_{n=1,\ldots,L} (\mathbf{\Psi} \mathbf{\Psi}^*)_{n,n}$$
   
   (c) add this column to the RF combiner matrix $\mathbf{W}_{RF}$ as:
   
   $$\mathbf{W}_{RF} = [\mathbf{W}_{RF}|\mathbf{A}_{R2}(;u)]^*$$

5. Solve $\mathbf{W}_u = \mathbf{W}_{BB}^* \mathbf{W}_{RF}^*$ for $\mathbf{W}_{BB}$ using the least squares method as:
   
   $$\mathbf{W}_{BB} = (\mathbf{W}_{RF}^* \mathbf{W}_{RF})^{-1} \mathbf{W}_{RF}^* \mathbf{W}_u$$

6. Remove the effect of the chosen column from the residual error as:
   
   $$\mathbf{W}_{res} = \frac{\mathbf{W}_u - \mathbf{W}_{BB}^* \mathbf{W}_{RF}^*}{||\mathbf{W}_u - \mathbf{W}_{BB}^* \mathbf{W}_{RF}^*||_F}$$

7. end the loop.
Appendix C

Singular-Value Decomposition

The singular-value decomposition (SVD) is an orthogonal decomposition of a matrix. Let us assume that \( A \) is an \( n \times m \) matrix of rank \( r \). Then there exist an \( n \times n \) orthogonal matrix \( U \) and an \( m \times m \) orthogonal matrix \( V \) such that \( U^*U = I_n \), \( V^*V = I_m \) and

\[
A = U \Sigma V^*
\]  

where \( \Sigma = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix} \), \( S = \text{diag}(\sigma_1, \cdots, \sigma_r) \) is an \( r \times r \) diagonal matrix. The \( r \) diagonal elements of \( S \) are strictly positive and are called the singular values of matrix \( A \). For convenience, it is assumed that \( \sigma_1 \geq \cdots \geq \sigma_r \). More specifically, we have

\[
A = \begin{bmatrix} U_1 U_2 \\ \text{S0} \\ \text{00} \end{bmatrix} \begin{bmatrix} V_1 \\ \text{00} \end{bmatrix}
\]  

\[
= U_1 S V_1^*
\]

The sub-matrix sizes are all determined by \( r \) (which must be \( \leq \min\{m, n\} \)), i.e., \( U_1 \) is an \( n \times r \) matrix, \( U_2 \) is an \( n \times (n - r) \) matrix, \( V_1 \) is an \( m \times r \) matrix, \( V_2 \) is an \( m \times (m - r) \) matrix, and the \( 0 \) sub-blocks in \( \Sigma \) are compatibly dimensioned. The columns of \( U \) are called the left singular vectors of \( A \) (and are the orthonormal eigenvectors of \( AA^* \)). Similarly, the columns of \( V \) are called the right singular vectors of \( A \) (and are the orthonormal eigenvectors of \( A^*A \)) \([142,143]\).
Appendix D

Production Notes

The main body of this thesis was typeset using \LaTeX and Texstudio document preparation software and the bibliography was done using bibtex. The two main numerical tools used for diseases are MATLAB R2013a and Mathcad 15.0. The block diagrams and system models were drawn using Microsoft PowerPoint 2010. The latex output was converted to portable document format (PDF) the size of which is about 6 megabytes. This is takes approximately 15 seconds to comply.

The work was done at diversity of Manchester on a Dell Optiplex 980 workstation running 64-bit Windows 7 Enterprise operating system with a 3.2 gigahertz microprocessor and 8 Gigabytes of memory.