Polarisation MIMO Indoor Wireless Communications Using Highly Compact Antennas and Platforms

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By

Joseph A Burge

School of Electrical and Electronic Engineering
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<th>Description</th>
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<tbody>
<tr>
<td>2D</td>
<td>Two dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three dimensional</td>
</tr>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of arrival</td>
</tr>
<tr>
<td>AoD</td>
<td>Angle of departure</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
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<tr>
<td>BER</td>
<td>Bit error rate</td>
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<tr>
<td>CDF</td>
<td>Cumulative distribution function</td>
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<tr>
<td>CP</td>
<td>Circular polarisation</td>
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<tr>
<td>CST</td>
<td>Computer Simulation Technology</td>
</tr>
<tr>
<td>CST-MWS</td>
<td>Computer Simulation Technology – Microwave Studio</td>
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<tr>
<td>E-field</td>
<td>Electric field</td>
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<tr>
<td>IE</td>
<td>Integral equation</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse fast Fourier transform</td>
</tr>
<tr>
<td>LHC</td>
<td>Left hand circular</td>
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<tr>
<td>LOS</td>
<td>Line-of-sight</td>
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<tr>
<td>LP</td>
<td>Linear polarisation</td>
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<tr>
<td>MIMO</td>
<td>Multiple-in multiple-out</td>
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<tr>
<td>MISO</td>
<td>Multiple-in single-out</td>
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<tr>
<td>MLD</td>
<td>Maximum likelihood detection</td>
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<tr>
<td>MMSE</td>
<td>Minimum mean squared error</td>
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<tr>
<td>MRC</td>
<td>Maximal ratio combining</td>
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<tr>
<td>NLOS</td>
<td>None-line-of-sight</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal frequency division multiplex</td>
</tr>
<tr>
<td>OSIC</td>
<td>Ordered successive interference cancellation</td>
</tr>
<tr>
<td>OSTBC</td>
<td>Orthogonal space-time block coding</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability distribution function</td>
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<tr>
<td>PEC</td>
<td>Perfect electric conductor</td>
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<tr>
<td>PSK</td>
<td>Phase shift keying</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature amplitude modulation</td>
</tr>
<tr>
<td>QHA</td>
<td>Quadrifilar helical antenna</td>
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<tr>
<td>QPSK</td>
<td>Quadrature phase shift keying</td>
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<td>Acronym</td>
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<tr>
<td>RHC</td>
<td>Right hand circular</td>
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<td>RL</td>
<td>Ray launcher</td>
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<td>SD</td>
<td>Spatial diversity</td>
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<td>SER</td>
<td>Symbol error rate</td>
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<tr>
<td>SIC</td>
<td>Successive interference cancellation</td>
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<tr>
<td>SIMO</td>
<td>Single-in multiple-out</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to interference and noise ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-in single-out</td>
</tr>
<tr>
<td>SM</td>
<td>Spatial multiplexing</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to noise ratio</td>
</tr>
<tr>
<td>STBC</td>
<td>Space-time block code</td>
</tr>
<tr>
<td>TE</td>
<td>Transverse electric</td>
</tr>
<tr>
<td>TM</td>
<td>Transverse magnetic</td>
</tr>
<tr>
<td>V-BLAST</td>
<td>Vertical Bell Laboratories Layered Space-time</td>
</tr>
<tr>
<td>VSWR</td>
<td>Voltage standing wave ratio</td>
</tr>
<tr>
<td>WiFEED</td>
<td>Wireless Friendly and Energy Efficient Buildings</td>
</tr>
<tr>
<td>XPD</td>
<td>Cross-polar discrimination</td>
</tr>
<tr>
<td>ZF</td>
<td>Zero forcing</td>
</tr>
</tbody>
</table>
**List of Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Angular separation between rays at launch</td>
</tr>
<tr>
<td>$\alpha_{fn}$</td>
<td>Angular separation between ray and its furthest neighbour</td>
</tr>
<tr>
<td>$\alpha_{i,j}$</td>
<td>Expected gain of MIMO sub-channel between antenna $j$ and antenna $i$</td>
</tr>
<tr>
<td>$\alpha_{nn}$</td>
<td>Angular separation between ray and its nearest neighbour</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Propagation constant in dielectric slab</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>Propagation constant in free space</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Voltage reflection coefficient</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Spatial resolution of ray launcher</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>$\Delta f_{3dB}$</td>
<td>Half power bandwidth of antenna</td>
</tr>
<tr>
<td>$\delta f$</td>
<td>Frequency increment</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity of free space</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative permittivity</td>
</tr>
<tr>
<td>$\varepsilon_{rc}$</td>
<td>Complex dielectric constant of conductive material</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency</td>
</tr>
<tr>
<td>$\eta_{ohmic}$</td>
<td>Efficiency when losses are entirely ohmic</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Ray incident angle from normal</td>
</tr>
<tr>
<td>$\theta_r$</td>
<td>Ray reflected angle from normal</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>Ray transmitted angle from normal</td>
</tr>
<tr>
<td>$\theta_r, \phi_r$</td>
<td>Angle of arrival</td>
</tr>
<tr>
<td>$\theta_t, \phi_t$</td>
<td>Angle of departure</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Relative permeability</td>
</tr>
<tr>
<td>$\rho_{env}$</td>
<td>Envelope correlation coefficient</td>
</tr>
<tr>
<td>$\rho_{i,j}$</td>
<td>Correlation coefficient in the $i, j$th element of $\mathbf{R}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Conductivity</td>
</tr>
<tr>
<td>$\sigma_0^2$</td>
<td>Noise signal variance</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>Scale parameter of Rician distribution</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Polarisation ellipse tilt angle</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Resonant angular frequency</td>
</tr>
<tr>
<td>$a$</td>
<td>Radius of smallest sphere which accommodates antenna</td>
</tr>
<tr>
<td>$C$</td>
<td>Capacity</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Instantaneous capacity</td>
</tr>
<tr>
<td>$D$</td>
<td>Absolute antenna directivity</td>
</tr>
<tr>
<td>$D_{max}$</td>
<td>Maximum dimension of antenna</td>
</tr>
<tr>
<td>$d$</td>
<td>Shortest distance from origin to plane</td>
</tr>
<tr>
<td>$d$</td>
<td>Thickness</td>
</tr>
<tr>
<td>$d\Omega$</td>
<td>Differential solid angle</td>
</tr>
<tr>
<td>$d_r$</td>
<td>Distance between ray and receiver centre</td>
</tr>
<tr>
<td>$E_{\theta,\phi}$</td>
<td>Spherical components of electric field vector</td>
</tr>
<tr>
<td>$E_{\theta j}$</td>
<td>$\theta$ component of electric field radiated by antenna $j$</td>
</tr>
<tr>
<td>$E_{\phi j}$</td>
<td>$\phi$ component of electric field radiated by antenna $j$</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Incident electric field</td>
</tr>
<tr>
<td>$E^r_k$</td>
<td>$k$th contribution to reflected electric field</td>
</tr>
<tr>
<td>$E^t_k$</td>
<td>$k$th contribution to transmitted electric field</td>
</tr>
<tr>
<td>$E_{max}$</td>
<td>Major axis of polarisation ellipse</td>
</tr>
<tr>
<td>$E_{min}$</td>
<td>Minor axis of polarisation ellipse</td>
</tr>
<tr>
<td>$E^r_{total}$</td>
<td>Total reflected electric field</td>
</tr>
<tr>
<td>$E^t_{total}$</td>
<td>Total transmitted electric field</td>
</tr>
<tr>
<td>$E_{x,y,z}$</td>
<td>Cartesian components of electric field vector</td>
</tr>
<tr>
<td>$E_{\perp,\parallel}$</td>
<td>Ray fixed components of electric field vector</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Resonant frequency</td>
</tr>
<tr>
<td>$G$</td>
<td>Absolute antenna gain</td>
</tr>
<tr>
<td>$h_{i,j}$</td>
<td>Narrowband path gain between antenna $j$ and antenna $i$</td>
</tr>
<tr>
<td>$h_{r,i,j}$</td>
<td>$i,j$th sub-channel gain following correlation and inclusion of fixed part of channel</td>
</tr>
<tr>
<td>$I_j$</td>
<td>Current supplied to antenna $j$</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Polarisation orthogonality</td>
</tr>
</tbody>
</table>
$K$ Rician K-factor
$K_{i,j}$ K-factor of sub-channel $i,j$
$k_d$ Non-centrality parameter of Rician distribution
$k_e$ Maximum number of events considered by ray launcher
$L$ Length
$l^k_i$ Locally estimated received signal at antenna $i$ in $k$th timeslot
$M$ Modulation constellation order
$M_r$ Minimum number of rays required by ray launcher
$m_{i,j}$ Element of $M$ used to scale gain of sub-channel $i,j$
$N_0$ Noise power spectral density
$N_r$ Number of received rays
$N_{tess}$ Tessellation frequency
$n_{bits}$ Number of transmitted bits
$n_{err}$ Number of bits received in error
$n_i$ AWGN noise sample at antenna $i$
$n_r$ Number of receive antennas
$n_r^H$ Number of horizontal receive antennas
$n_r^V$ Number of vertical receive antennas
$n_t$ Number of transmit antennas
$n_t^H$ Number of horizontal transmit antennas
$n_t^V$ Number of vertical transmit antennas
$\hat{P}_e$ Bit error rate estimate
$P_{in}$ Input power
$\hat{P}_k$ Normalised power weighting of $k$th ray
$P_{ohmic}$ Power lost through ohmic losses
$P_{rad}$ Radiated power
$P_t$ Transmit power
$p$ Polarisation efficiency
$Q$ Q factor
$R$ Complex sub-channel frequency response
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>Reflection coefficient exterior to dielectric</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Reflection coefficient interior to dielectric</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Radial distance to outer boundary of reactive near-field region</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Radial distance to outer boundary of radiating near-field region</td>
</tr>
<tr>
<td>$R_{coat}$</td>
<td>Radius of dielectric coat</td>
</tr>
<tr>
<td>$R_{i,j}$</td>
<td>Complex frequency response of sub-channel $i,j$</td>
</tr>
<tr>
<td>$R_{in}$</td>
<td>Input resistance</td>
</tr>
<tr>
<td>$R_{loss}$</td>
<td>Loss resistance</td>
</tr>
<tr>
<td>$R_{ohmic}$</td>
<td>Ohmic resistance</td>
</tr>
<tr>
<td>$R_{rad}$</td>
<td>Radiation resistance</td>
</tr>
<tr>
<td>$R_{RX}$</td>
<td>Receiver sphere radius</td>
</tr>
<tr>
<td>$R_{TE}$</td>
<td>Final reflection coefficient of TE mode wave</td>
</tr>
<tr>
<td>$R_{TM}$</td>
<td>Final reflection coefficient of TM mode wave</td>
</tr>
<tr>
<td>$r_1$</td>
<td>Receive correlation coefficient between $h_{1,1}$ and $h_{1,2}$</td>
</tr>
<tr>
<td>$r_2$</td>
<td>Receive correlation coefficient between $h_{2,2}$ and $h_{2,2}$</td>
</tr>
<tr>
<td>$S_{21,i}$</td>
<td>Path gain between a farfield transmit antenna and receive antenna $i$</td>
</tr>
<tr>
<td>$S'_{21,i}$</td>
<td>Path gain between a farfield transmit antenna and receive antenna $i$ in isolation</td>
</tr>
<tr>
<td>$S_{i,j}$</td>
<td>Scattering parameter between port $j$ and port $i$</td>
</tr>
<tr>
<td>$S_k$</td>
<td>The $k$th Stokes parameter</td>
</tr>
<tr>
<td>$\hat{S}_k$</td>
<td>Normalised Stokes parameter, $S_k$</td>
</tr>
<tr>
<td>$s_j$</td>
<td>Transmitted symbol value by antenna $j$</td>
</tr>
<tr>
<td>$s_{max}$</td>
<td>Maximum length of straight line in ray launcher environment model</td>
</tr>
<tr>
<td>$s_r$</td>
<td>Size of smallest reflector in ray launcher environment</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Transmission coefficient for single crossing into dielectric</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Transmission coefficient for single crossing out of dielectric</td>
</tr>
<tr>
<td>$T_{TE}$</td>
<td>Final transmission coefficient of TE mode wave</td>
</tr>
<tr>
<td>$T_{TM}$</td>
<td>Final transmission coefficient of TM mode wave</td>
</tr>
<tr>
<td>$t_1$</td>
<td>Transmit correlation coefficient between $h_{1,1}$ and $h_{2,1}$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>Transmit correlation coefficient between $h_{1,2}$ and $h_{2,2}$</td>
</tr>
<tr>
<td>$u_1$</td>
<td>Cross correlation coefficient between $h_{1,1}$ and $h_{2,2}$</td>
</tr>
</tbody>
</table>
Cross correlation coefficient between $h_{1,2}$ and $h_{2,1}$

Uncoupled voltage at the terminals of receive antenna $i$

Uncoupled voltage at the terminals of transmit antenna $j$

Voltage across the terminals of antenna $i$

Voltage across the terminals of antenna $i$ as a result of coupling only

Coupled voltage at the terminals of receive antenna $i$

Coupled voltage at the terminals of transmit antenna $j$

Time averaged stored electric and magnetic energy by an antenna

Input reactance

Normalised ray separation

Received signal by antenna $i$

Received signal by antenna $i$ in $k$th time slot

Source impedance

Input impedance of antenna $i$

Receive mutual impedance between antenna $j$ and antenna $i$

Weighted average receive mutual impedance between antenna $i$ and antenna $j$

Transmit mutual impedance between antenna $j$ and antenna $i$

Input impedance

Receiver load impedance

Ray direction

Electric field vector

Incident electric field vector

Reflected electric field vector

Transmitted electric field vector

Antenna power gain vector

Transmit antenna complex field gain vector

Receive antenna complex field gain vector

Vector containing noise samples

Vector of distorted noise terms
\( \hat{n} \) Plane normal  
\( R_j \) Vector of reflection coefficients for \( j \)th reflection  
\( s \) Transmit signal vector  
\( T_k \) Vector of transmission coefficients for \( k \)th transmission  
\( u_R \) Uncoupled receive antenna voltage vector  
\( u_T \) Uncoupled transmit antenna voltage vector  
\( \hat{u}_\theta \) Unit vector in the \( \theta \) direction  
\( \hat{u}_\phi \) Unit vector in the \( \phi \) direction  
\( \hat{u}_\perp \) Perpendicular base of ray fixed coordinate system  
\( \hat{u}_\parallel^i \) Parallel base of ray fixed coordinate system for incident ray  
\( \hat{u}_\parallel^r \) Parallel base of ray fixed coordinate system for reflected ray  
\( v_R \) Coupled receive antenna voltage vector  
\( v_T \) Coupled transmit antenna voltage vector  
\( y \) Receive signal vector  
\( \bar{y} \) Estimate of transmitted signal vector  
\( 1 \) Matrix of 1s  
\( H \) MIMO channel matrix  
\( H_c \) Correlated Rayleigh fading channel matrix  
\( H_f \) Matrix containing the fixed parts of MIMO channel  
\( H_r \) Rician fading MIMO channel matrix  
\( H_w \) Rayleigh fading channel matrix  
\( H_X \) Power weighted, correlated Rician fading channel matrix  
\( I_{n_r} \) \( n_r \times n_r \) identity matrix  
\( K \) Matrix of sub-channel K-factors  
\( M \) MIMO sub-channel power weighting matrix  
\( Q \) Change of basis matrix  
\( R \) MIMO channel correlation matrix  
\( R_r \) Kronecker receive correlation matrix  
\( R_t \) Kronecker transmit correlation matrix  
\( Z^R \) Receive mutual impedance matrix
$Z^T$  Transmit mutual impedance matrix

$X$  Channel power imbalance matrix
Abstract

Polarisation MIMO Indoor Wireless Communications Using Highly Compact Antennas and Platforms

Joseph Andrew Burge

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy, 2016

In the indoor environment, multipath fading causes the received signal amplitude to fluctuate rapidly over space and frequency. Multiple-in multiple-out (MIMO) systems overcome this phenomenon through the use of multiple antennas on transmitters and receivers. This establishes multiple independent MIMO sub-channels between antenna pairs, which allows a theoretical increase in capacity which is linear with the number of antennas, while requiring no additional power or bandwidth expenditure.

The capacity increase is reliant upon MIMO sub-channels being well decorrelated. Decorrelation may be achieved by separating antennas in space. On devices where space is limited, an alternative approach is to use antennas with orthogonal polarisations, which may be positioned closer together. Existing literature states that the performance of polarisation MIMO systems is typically inferior to that of spatial MIMO systems under diversity applications, but can be superior in multiplexing applications. These statements are based on the analysis of a statistical channel model, using channel conditions assumed to be typical of an ideal polarisation MIMO system. There is little existing literature which examines how close these assumptions are to a practical polarisation MIMO channel, or whether the above statements remain true of practical systems.

This thesis presents a novel end-to-end, predominantly deterministic approach to the modelling of polarisation MIMO systems. A bespoke MIMO channel model is used to estimate capacity and error rate under diversity and spatial multiplexing applications in the indoor environment. The parameters of the channel model are obtained deterministically from a ray launching propagation model, using antenna patterns of orthogonally polarised small antenna systems positioned in the indoor environment. The individual differences in the channel gains and K-factors of each sub-channel are accounted for. Correlation is accounted for using a full correlation matrix, rather than the Kronecker model. Particular attention is paid to mutual coupling of closely spaced antennas. Using this analysis, it is shown that for practical antennas and systems conditions of the polarisation MIMO channel may differ from those assumed in literature. The effect of this in terms of channel capacity and system bit error rate is directly determined and presented.

Performance of polarisation MIMO systems, using co-located and spatially separate orthogonally polarised antennas, is compared to that of spatial MIMO systems, which use co-polar antennas with limited spatial separation. Additionally, comparison is made between compact polarisation MIMO systems which use orthogonal linear polarised antennas and those using orthogonal circular polarised antennas. Further analysis examines the significant effect of objects in the antenna near-field regions. The effects of the presence of a metal case on antenna performance are presented, before its impact on the channel conditions and ultimately the resultant MIMO performance is shown.
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1. Introduction

This thesis investigates the multiple-in multiple-out (MIMO) benefit to wireless communication systems in the indoor environment, through the use of orthogonally polarised antennas, on compact platforms.

There is an ever increasing demand for high quality, high data rate services in modern wireless communication systems. One of the fundamental difficulties in wireless communications is the effect of multipath fading \cite{1, 2}. This is the phenomenon where, in a multipath propagation environment, many transmission paths exist between transmitter and receiver. The different paths are a result of objects in the environment reflecting, diffracting and scattering waves. At the receiver, the signal associated with each path arrives with a different amplitude, phase and group delay. These signals may combine constructively, resulting in a higher signal to noise ratio (SNR) or destructively, resulting in a lower SNR. When they combine destructively and the SNR becomes exceptionally low it is known as a deep fade. A technique which is used to overcome this problem without any additional power or bandwidth requirement is MIMO. MIMO systems exploit diversity by installing multiple antennas at the transmitter and the receiver. This results in separate MIMO sub-channels between antenna pairs. In a rich scattering environment, each sub channel experiences a different fade. In theory, MIMO can provide a linear increase in system capacity with number of antennas \cite{2}. This increase relies on the signals arriving at each antenna being decorrelated.

Traditionally, MIMO sub-channel are decorrelated through the use of antennas which are separated in space. Literature suggests that this separation must be several wavelengths for sufficient decorrelation \cite{2-4}. At cellular base stations this tends not to be a problem, however on portable devices, space is very limited. An alternative solution is to use polarisation diversity, where the signals are decorrelated through the use of orthogonally polarised antennas. Polarisation diversity using cross-polarised linear antennas has been implemented at cellular base stations for quite some time \cite{2}. The use of polarisation diversity on portable devices, where space is limited is less well researched. Even less studied is the use of circular polarised (CP) antennas in polarisation MIMO systems.

The main objective of this work is to research the use of polarisation MIMO in the indoor environment, using antennas on small sized portable devices. The use of circular as well as linear polarisation is considered.
1.1. Overview of MIMO Systems

Overcoming the effect of multipath fading is one of the fundamental difficulties in wireless communications [1, 2]. One approach, which requires no additional transmit power or bandwidth is through the introduction of diversity to the system. This is achieved by transmitting multiple copies of the same data over different sub-channels. These sub-channels may be established in a number of ways, such as by transmitting simultaneously at different frequencies (frequency diversity), transmitting again at a different time (time diversity), through different antenna patterns (pattern diversity), from different positions in space (spatial diversity), by transmitting using different polarisations (polarisation diversity), or through a combination of these.

Spatial diversity may be implemented through the use of multiple antennas on a receiver and/or a transmitter. The use of multiple receive antennas allows a system to receive multiple copies of the same transmitted signal, each having experienced a different propagation environment. If the signal arriving at one antenna has experienced a deep fade, there is a chance that the signal at other antenna(s) may not have. Through the combination of the signals received by all antennas, diversity receivers can minimise the effect of deep fades and reduce the error rate of the system. The 1959 paper by Brennan [5] analyses three of the most popular reception diversity combining schemes; selection diversity, maximal-ratio diversity and equal-gain diversity. The use of multiple receive antennas for diversity in fact dates back to Marconi’s work in the very early days of radio [6]. Diversity reception systems, with one transmit antenna and multiple receive antennas are sometimes referred to as single-in-multiple-out (SIMO) systems.

Their analogue is multiple-in-single-out (MISO) systems, which make use of transmit diversity. The appeal of these systems was recognised by Alamouti in 1998 [7]. He acknowledged a desire for diversity in the downlink of mobile communication systems, but appreciated that adding multiple receive antennas to many mobile terminals was far less economic than adding additional transmit antennas to the base stations, which serve many users. The transmit diversity scheme that he developed in [7] enabled transmit diversity in the downlink, with similar diversity performance to maximal ratio combining (MRC) receive diversity. It was in fact the first transmit diversity scheme which achieved full diversity, whilst requiring only linear processing at the receiver. Furthermore, this scheme could be generalised for systems which also have multiple receive antennas. This marked
the start of the revolution in wireless communications caused by multiple-in-multiple-out (MIMO) technology.

MIMO is a special case of diversity, where multiple antennas exist on both sides of the transmission channel. If \( n_t \) transmit antennas and \( n_r \) receive antennas are used, then \( n_t n_r \) separate MIMO sub-channels are established, between transmitter and receiver. These systems experience a performance increase in terms of reduced error rate, due to the diversity gain. At high SNR levels, error rate decreases at a rate proportional to \( 1/SNR^{DG} \) [8], where \( DG \) is the diversity gain. Providing each sub-channel fades independently, systems with suitable transmit coding, achieve a diversity gain equal to the diversity order, \( n_t n_r \). Hence, MIMO systems offer a significant error rate improvement over single in single out (SISO) systems, for no additional expense in transmit power or bandwidth.

The MIMO sub-channels may also be used to exploit a gain in throughput or transmission rate, through the use of spatial multiplexing (SM) schemes as opposed to diversity schemes. The maximum possible MIMO throughput gain is \( \min(n_t, n_r) \) [6]. This is achieved by demultiplexing the transmission bit stream into \( n_t \) separate lower rate streams and modulating and transmitting these independently from the \( n_t \) transmit antennas. Under suitable channel conditions, the spatial signatures of these signals are adequately separated, so that the receiver, with knowledge of the channel can extract both transmitted datastreams and recombine them to restore the original bit stream. The trade-off between diversity gain and multiplexing gain is discussed in [8] and [9].

One way of characterising a MIMO channel is through its capacity. The capacity of a communications channel, in bits per second per Hz, is the theoretical maximum error-free data rate that the channel can support. It may be determined through information theory, as first done by Shannon in 1948 for a SISO link over an additive white Gaussian noise (AWGN) channel in [10]. The capacity of the MIMO channel under various conditions is discussed in detail by Foschini and Gans, as well as Teltar, in [11, 12]. The instantaneous capacity, \( C_i \) of a MIMO channel with no channel state information at the transmitter, assuming frequency flat fading conditions is as follows.

\[
C_i = \log_2 \left( \det \left( I_{n_r} + \left( \frac{P_t}{n_t N_0} \right) HH^H \right) \right) \text{bps/Hz} \tag{1.1}
\]

\(^1\) Under the frequency flat fading assumption, signal bandwidth is sufficiently small that it is assumed the channel gains are constant over the band.
Here, $I_{n_r}$ is the $n_r \times n_r$ identity matrix, $P_t$ is the total transmit power from all transmitters, $N_0$ is the noise power spectral density at each receive antenna, $H$ is the $n_t \times n_r$ MIMO channel matrix and $H^H$ is the conjugate transpose of the matrix, $H$.

1.1.1. Diversity Schemes

Alamouti’s transmit diversity scheme [7] may be generalised to MIMO systems which use two transmit antennas and $n_r$ receive antennas, to give a diversity order of $2n_r$. The scheme achieves diversity by spreading the transmit symbol sequence over space and time. The symbol stream is broken down into blocks of two symbols, which are transmitted simultaneously from two antennas, in two symbol periods. In the first period, the first symbol, $s_1$ is transmitted from the first antenna and the second symbol, $s_2$ is transmitted from the second antenna. In the second period, $-s_2^*$ is transmitted from the first antenna and $s_1^*$ is transmitted from the second antenna, where $s_1^*$ is the complex conjugate of $s_1$. This is often represented in matrix form as below, where the two rows represent two symbol periods and the columns represent the two transmit antennas.

$$
\begin{bmatrix}
    s_1 & s_2 \\
    -s_2^* & s_1^*
\end{bmatrix}
$$

(1.2)

Because symbols are transmitted at a rate equivalent to one symbol per time period, this is a rate 1 code. The code is fully orthogonal, in the sense that the dot products of the two vectors forming its columns, as well as the two vectors forming its rows are both zero. This enables the two symbols which have combined in the channel to be decoupled and detected individually at the receiver, using maximum likelihood detection (MLD) [7, 13], which requires only linear processing.

Alamouti’s code was considered a breakthrough and inspired a search for similar codes, which could be used with more transmit antennas to reduce BER over fading channels. Tarokh et al [14], in 1999 generalised the concept of Alamouti’s orthogonal space time code for any $n_t > 2$, using the solution to the Hurwitz-Radon problem [15]. Similar to the Alamouti scheme, these codes extract full $n_t n_r$ diversity, requiring only linear processing at the receiver and achieve rate one. The problem with them is that they are only valid for entirely real symbols, meaning that they are not compatible with higher rate complex modulation schemes such as phase shift keying (PSK) and quadrature amplitude modulation (QAM). In fact, complex orthogonal designs which achieve rate 1 for systems with more than two transmit antennas do not exist [14, 16, 17]. Lower rate complex
designs for \( n_t > 2 \) are also identified in [14] though and their performance is evaluated in [13, 18]. These codes are only orthogonal in the temporal sense, but they may still be detected using simple linear techniques, such as MLD. Two such codes, \( G_3 \) and \( G_4 \) transmit four symbols in eight time periods using three or four transmit antennas respectively, achieving rate 1/2. Additional codes, \( H_3 \) and \( H_4 \) transmit three symbols in four time periods using three and four transmit antennas respectively, achieving rate 3/4. Diversity codes of this nature, including Alamouti’s code are referred to as space-time block codes (STBCs).

1.1.2. Multiplexing Schemes

An alternative to diversity schemes which exploit the multipath channel to increase reliability are spatial multiplexing (SM) schemes [6, 19, 20]. These schemes split the transmit data stream into separate independent sub streams, which are transmitted simultaneously. This increases throughput. A simple SM scheme involves demultiplexing the transmission data stream into \( n_t \) sub streams, which are separately modulated and transmitted simultaneously, with equal power by the \( n_t \) transmit antennas. Schemes of this type achieve a transmission rate increase which is linear with \( \min(n_t, n_r) \), with no additional power or bandwidth expenditure [6, 21, 22]. Because each symbol is transmitted from one transmit antenna only, the maximum diversity order achievable under SM is \( n_r \).

The optimal detection scheme for SM, in terms of error rate is MLD, as is commonly used for diversity schemes. Unlike the STBCs though, the code blocks used for SM are not inherently orthogonal. This means that MLD metrics may not be expanded and separated into parts which are only a function of one symbol. Instead, the decision metric requires comparison over \( M^{n_t} \) symbol combinations, where \( M \) is the constellation alphabet size for the modulation scheme in use. For this reason, MLD requires a lot of processing at the receiver and frequently, sub-optimal detection techniques such as minimum mean squared error (MMSE) or zero forcing (ZF) are used instead.

Under ZF, the received signal vector is multiplied by the inverse (or pseudo-inverse if \( n_r \neq n_t \)) of the channel matrix. This cancels the effect of the channel, forcing the interference terms on each received symbol to zero. However, it has the effect of
potentially enhancing and introducing correlation to the noise terms [6, 19, 23]. This means that despite the multiple receive antennas, diversity gain remains at unity.

Diversity performance under SM of zero forcing (and other detection schemes) may be slightly improved through the use of successive interference cancellation (SIC) and further through the use of ordered successive interference cancellation (OSIC), which are described in [21, 22, 24].

SIC detection involves detecting one symbol at a time, usually using ZF or MMSE, before subtracting the interference caused by the already detected symbol from the remaining symbols(s) and then detecting the next symbol. This process is repeated until all symbols are detected. When symbols are detected correctly, SIC reduces the MIMO channel into a set of SISO channels, with diversity gain increasing after each interference cancellation stage [11].

In practice, due to the presence of noise, symbols are not always detected correctly. In the event that they are not, the error will propagate to the detection of the next symbol. The chances of detection error can be reduced however, by choosing the order that symbols are detected according to their signal to interference and noise ratio (SINR), where the effect of other undetected symbols is treated as interference. The symbol with the highest SINR at each stage is chosen for detection, to reduce the chances of error propagation. SM, using this form of OSIC detection was first introduced by Wolniansky, Foschini et al in [24] and is frequently referred to as V-BLAST. Its performance is investigated under various channel conditions in [21, 23, 25, 26].

1.2. Overview of Polarisation MIMO Systems

The diversity and multiplexing MIMO gains are dependent upon the signals arriving via each MIMO sub-channel being sufficiently decorrelated. The decorrelation is a result of the multipath scattering environment and the choice of antennas. Assuming the multipath is suitably rich, the decorrelation is traditionally ensured by separating antenna elements on each terminal in space. However sufficient decorrelation is reported to require separation of several wavelengths [2-4]. On small platforms, size and form constraints mean that it is often not possible to achieve this separation. Similarly, environmental considerations and tower loads may also make this difficult at cellular base stations [2]. Further, the narrow angular spread of incident fields at base stations necessitates even larger separations for
diversity [27, 28]. In these situations, the use of orthogonally polarised antennas, with little to no separation offers a potentially attractive alternative for decorrelation of the sub-channels.

1.2.1. Polarisation Diversity

The use of receive polarisation diversity at base stations has been a topic of interest for some time [2, 28-32]. Collins, in [30] gives a very good theoretical discussion of this subject. He states that the polarisation from a mobile user is dependent upon the transmitted polarisation and the scattering environment, and that throughout the environment, this polarisation will seemingly vary with a random nature, due to the summation of the field contributions which have undergone different reflections and transmissions. He goes on to discuss the effect of the orientation of linearly polarised antennas in polarisation diversity systems in the urban and sub-urban environment. If the transmitter is vertically polarised, it is recognised that reflections from the ground and from walls will not decouple a great deal of energy into the horizontal polarisation. If a receiver uses horizontal and vertical antennas, then the vertical element will receive much higher power than the horizontal and, even if the two branches are well decorrelated, the diversity gain will be low. On the other hand, if the receive antennas are at ±45° to the vertical, then they will receive similar power levels, which is desirable, but they will be highly correlated. With reference to an extensive measurement campaign in [33], he concludes that in these environments, with a vertically polarised transmitter, receive antennas at ±45° offer superior diversity gain to horizontal and vertical, however neither system performs quite as well as spatial diversity. Similar observations have been made in [34]. While these references are concerned with only two branch diversity, in the outdoor environment, the observations certainly have relevance to polarisation MIMO systems. The fact that diversity gain is dependent upon not only sub-channel decorrelation, but also the balance of received signal levels is very important.

Vaughan, in [35] provides further measurement results which support Collins’ observations. He reports on the received signal levels by horizontally and vertically polarised antennas at a base station. The transmitter is a sloped monopole antenna on a car, which radiates with predominantly vertical polarisation. The base station is located in rural Denmark and the car transmitter was driven around an urban and a sub-urban environment, approximately 20 km away, with no line-of-sight to the base station. It is
reported that the signal level received by the horizontal antenna is Rayleigh distributed over space. This is to be expected, as it is entirely a result of multipath scattering. The vertically polarised signal is closer to a Rician distribution, with a mean level 12 dB higher than the horizontal in the sub-urban environment and 7 dB higher in the urban environment. The Rician distribution suggests the presence of a dominant, specular component. The lower difference in received signal level in the urban environment is quite possibly because, in the urban environment, there is greater multipath scattering which couples energy from the vertical to the horizontal polarisation. The envelope correlation coefficient between the two polarisations is very low, at 0.019 and -0.003 in the sub-urban and urban environments respectively. The paper also reports the combined horizontal and vertical signal levels under MRC for both environments. This is compared to the vertically received SISO channel to observe the diversity gain. In the sub-urban environment, up to 2 dB diversity gain is recorded, however in the urban environment, where the difference in received power levels and the correlation are both lower, the diversity gain approaches 7 dB. These results demonstrate the significant influence that the propagation environment has on the properties of received signals and thereby, the performance of polarisation diversity systems.

In [31], the effect of sub-channel correlation on polarisation diversity systems is investigated. This is done by modelling two sub-channel signal streams as initially uncorrelated pseudorandom number sequences. Two mixing processes are then performed sequentially. The first is to model correlation introduced to the signals in the transmission path, while the second is to model correlation introduced at the receive antennas to the already partially correlated signals. The correlation added at the receive antennas is a result of the imperfect orthogonality of the antennas. The model shows that if signals are well decorrelated after the transmission path, then the use of highly orthogonal receive antennas will result in well decorrelated signals at the output and hence high diversity gain. If however the signals after the transmission path are highly correlated, then even with very highly orthogonal antennas, the output signals will still be highly correlated. This is a useful way of thinking, which is equally relevant to MIMO systems. It separates the correlation due to the environment from the correlation due to the antennas and shows that the overall correlation of received signals is of course dependent upon both. This demonstrates the importance of a thorough understanding of both of these parts of the channel and how they contribute to the correlation, or decorrelation, of signals over MIMO.
sub-channels. A very useful discussion on polarisation and orthogonality is also given in [31].

1.2.2. Polarisation MIMO

In MIMO systems, at least two antennas exist at both the transmitter and the receiver. In a typical $2 \times 2$ polarisation MIMO system, these are orthogonally polarised and may be co-located. As with any $2 \times 2$ MIMO system, this establishes four separate MIMO sub-channels, between the antennas. Assuming the same two polarisations are used at both transmitter and receiver, this results in two co-polar channels and two cross-polar channels.

An early assumption in MIMO literature was that all sub-channels experience “rich” scattering and therefore may be considered independent and Rayleigh distributed [36]. In the real world, this is rarely the case and the properties of MIMO sub-channels vary significantly from a Rayleigh distribution for many reasons [22, 36, 37]. Dominant propagation paths may introduce a fixed component, resulting in a Rician distribution. This possibility is particularly applicable to the indoor environment. In addition, insufficient multi-path or insufficient antenna separation in spatial systems could result in correlation between sub-channels. Similarly, a lack of antenna polarisation orthogonality, or physically similar environmental propagation conditions for orthogonal polarisations, could result in correlation between sub-channels of polarisation systems. Polarisation systems are also likely to experience differences in received power levels between the co-polar and cross polar channels. The difference in received power level is determined by the amount of energy which is coupled between polarisations by the environment, as well as the polarisation purity of the antennas [4]. This difference in power level is often referred to as the cross-polar discrimination (XPD) of the channel.

The polarisation MIMO channel is studied by Nabar et al in [3, 4]. A channel model is described, based on XPD, Rician K-factor and sub-channel correlation. The model is intended to represent the channel between dual polar $\pm 45^\circ$ antennas. It assumes that received power in both co-polar channels is equal, as is the received power in both cross polar channels. The difference is given by the XPD. Based on this model, analytical equations are derived to estimate error rate under SM and Alamouti MIMO schemes. These are verified against MIMO simulation results over the channel model.

From the model, the authors claim that symbol error rate (SER) improvements of up to an order of magnitude are possible under SM through the use of polarisation MIMO,
compared to spatial MIMO, for a wide variety of channel conditions. Under pure Rayleigh fading, it is reported that there is a benefit to the use of dual polar systems over spatial systems for SM, when the transmitter cannot afford large element spacing and transmit correlation is high. The benefit is due to the XPD, even if the two co-polar channels are highly correlated. Conversely, it claims that if the antennas are well separated and therefore the transmit correlation coefficient is low (below 0.85), dual polar systems will always incur a performance loss when compared to spatial, under Rayleigh fading. Similar effects are observed for receive correlation only, although the performance improvement under polarisation MIMO is less pronounced, to the extent that, under receive correlation only, the use of polarisation MIMO is not advised. In a Rician fading environment, with high K-factor, it is stated that as XPD increases through the use of dual-polarised antennas, SM performance improves by an order of magnitude in terms of SER, at SNR of 15 dB.

Under Alamouti, it is concluded that polarisation systems generally experience a performance degradation compared to spatial. The analysis is based on SER estimates using analytical equations and simulations, given assumed channel parameters intended to represent spatial and dual polar systems. For example, it is assumed that a spatial system will experience equal gain on all sub-channels, whereas a dual polar system will experience equal gain on both co-polar channels and equal (lower) gain on both cross-polar channels, as well as very low transmit and receive correlation. As a rule of thumb, this is true, however these assumptions should certainly be used with caution when applied to real antennas in real environments.

Another polarisation MIMO channel model is given by Shafi et al in [2]. This is a three dimensional (3D) extension to the two dimensional (2D) 3rd Generation Partnership Project (3GPP) polarised channel model given in [38]. The 3GPP model was intended to suit an outdoor cellular environment, where most energy arrives at the user in the same plane. The 3D extension is more suited to indoor or in-vehicle propagation, where it is stated that the radiation distribution to the user is nearly uniform spherical. It is a statistical model, where the environment is represented by clusters of scatterers and is dependent upon channel statistics such as number of scatterer clusters and angular distribution of departure and arrival paths. The paper also provides measurements of XPD and sub-channel gains in the indoor environment, for a dual polar system with horizontally and vertically polarised antennas. Contrary to the assumptions in [3, 4], it is observed that at any point in space, the horizontal co-polar gain is not equal to the vertical. Similarly, the two cross-polar
gains are not equal. Further, a general trend for higher average co-polar gain in the horizontal channel than the vertical channel was recorded. This is perfectly reasonable, given the difference in indoor propagation characteristics between horizontally and vertically polarised waves [39].

The MIMO channel is inherently complex and as such, it is natural that when modelling it, various simplifications and assumptions may be made. Shafi takes a statistical approach which simplifies the problem to give a generic model which is broadly suited to many scenarios. It is less useful however if the effects of a specific system or a particular environment are to be studied. The model used by Nabar [3, 4] also makes many assumptions. One, as previously stated, is that the co-polar gains are both equal, as are the cross-polar gains. Another is that the correlation of the sub-channels may be described by the Kronecker model [40-42]. This makes assumptions which are detailed in Section 4.5.2, however [43] states that the Kroenker model is not valid for polarisation MIMO systems. In addition, [40] and [41] describe further MIMO channels where it is not valid. In fact, [44] even identifies the possibility of correlation having a beneficial impact on channel capacity when properly accounted for, which the Kronecker model does not allow. Another common omission from channel models such as that used in [3, 4] is the effect of mutual coupling. In [45], it is found that mutual coupling results in a performance degradation for MIMO systems, while in [46] it is concluded that mutual coupling may have a positive or negative effect on MIMO capacity, depending on the correlation properties of the propagation environment and the mutual coupling properties of the antennas. From [47], it is clear that mutual coupling increases as antennas are positioned closer to each other. As such, it is reasonable that the effect of this should be properly accounted for, particularly when considering MIMO systems on small platforms.

Anreddy and Ingram [37] document an interesting study, where polarisation MIMO channel measurements are taken in a line-of-sight (LOS) and non-line-of-sight (NLOS) indoor environment. Spatial MIMO systems, co-located dual polarised systems and “hybrid” MIMO systems are implemented. The hybrid system uses orthogonally polarised antennas, which are separated in space to utilise spatial as well as polarisation diversity. Vertical and horizontal linearly polarised antennas are used. The antenna separation of the spatial and hybrid systems is varied between $0.5\lambda$ and $2\lambda$. From the channel response measurements, K-factor, correlation and XPD are inspected and MIMO channel capacity is calculated. A “virtual array” measurement approach is used, where sub-channels between
spatially separate antennas are measured individually, using the same transmit and receive antennas, which are moved in-between measurements. This approach enables the assessment of spatial systems with separation as low as 0.5\(\lambda\), however any mutual coupling effects caused by the proximity of the antennas in real spatial and hybrid systems are not accounted for. It should also be noted that the antennas were not small. Dual polar horizontal and vertical patch antennas were used at 2.4 GHz, with dimensions of 14.5 x 8.5 x 1 inches.

In this study, it is observed that the gain of the vertical co-polar channel is higher than the horizontal. This is again contrary to the assumption in [3, 4] and is the opposite of the results in [39]. This demonstrates that the properties of the MIMO channel are highly dependent upon their specific physical environment. It is also observed that in polarisation MIMO systems, the potential diversity gain is reduced when the cross-polar sub-channel gains are low. In the presence of high XPD, it is stated that the maximum diversity order reduces from \(n_t n_r\) to \(n^V_t n^V_r + n^H_t n^H_r\) in a system with \(n^V_t\) vertical transmitters, \(n^V_r\) vertical receivers, \(n^H_t\) horizontal transmitters and \(n^H_r\) horizontal receivers.

The level of sub-channel correlation is also studied. For the spatial system in LOS, the sub-channels are considered “sufficiently” decorrelated with antenna separation of 3\(\lambda/2\). Any additional separation provides no further reduction in correlation. In NLOS, correlation is significantly lower throughout, although there is still a slight downward trend as separation increases. The polarisation and hybrid systems experience correlation far lower than the spatial systems.

In terms of capacity, in LOS the spatial system with 0.5\(\lambda\) separation (and high correlation) has similar capacity to the dual polar and hybrid systems, which have lower correlation but lower total received power. As separation increases for the spatial and hybrid systems, capacity increases. In NLOS, correlation is sufficiently low for the spatial system that even at 0.5\(\lambda\) separation, it achieves greater capacity than the hybrid or polar system. No results are presented in terms of actual error rate for these systems under a particular MIMO scheme and no investigation into alternative polarisations is made.

Further indoor measurements on the polarised MIMO channel are documented in [48]. Here, the correlation of the received signal envelope over space is examined in the indoor NLOS environment. Dual-polar horizontal/vertical patch antennas are used and it is found
that signals received on the two co-polar channels show no significant envelope correlation. Measurements of the outdoor polarisation MIMO channel are documented in references such as [49, 50], however the behaviour of the channel in the outdoor environment is significantly different to that in the indoor environment.

Literature describing indoor MIMO systems using polarisations other than linear is extremely limited. One experiment is described in [51] however, where Pei-Yuan Qin et al present a study of orthogonal frequency division multiplex (OFDM) MIMO channel capacity, using reconfigurable dual circular polarised (CP) patch antennas. A hybrid system, where the transmitter and receiver each use two orthogonally polarised antennas, separated by one wavelength, is compared to a spatial system, where the same antennas have been reconfigured to all be of the same (circular) polarisation. Capacity is evaluated from channel measurements in a LOS and NLOS indoor environment. In LOS, a significant capacity increase of 25 to 35% is reported for the hybrid system over the spatial system. This is attributed to lower sub-channel correlation on the orthogonally polarised channels of the hybrid system. In the NLOS environment, the correlation of the spatial sub-channels was lower than in LOS and as such a capacity increase of up to only 6.7% was reported for the hybrid system. No comparison was made to linearly polarised systems and no analysis of error rate performance under particular MIMO schemes was performed.

1.3. Aims of Thesis

The polarisation MIMO channel models in literature generally rely on knowledge of various parameters of the channel, such as K-factor, XPD and correlation. These parameters are dependent upon both the antenna systems and the environment. A general insight into polarisation MIMO performance may be gained by setting these to known values which are typically experienced in a certain environment, as in [4]. For a more in depth evaluation of polarisation MIMO performance, using practical antenna configurations in specific environments, these parameters should be obtained using a deterministic approach. One such approach is through channel measurements. A full comparison between several different antenna configurations in multiple different environments however, would require many extensive measurement campaigns. Further, it would be difficult to separate the effects of the propagation environment from those of the antenna and platform arrangements, without performing further measurements. Very little
existing literature has been identified which studies polarisation MIMO performance in this detail.

As such, a primary aim of the thesis is to present a novel end-to-end modelling approach to predict MIMO performance for a wide range of physical antenna arrangements and polarisations, in a realistic indoor environment. A predominantly deterministic approach is preferred, in order that observations on MIMO performance may be related to specific antenna or environmental attributes and to produce results that are as close an approximation as possible to those expected by real systems.

The purpose of the model is to investigate the potential MIMO benefit through the use of orthogonally polarised antennas, compared to spatially separate co-polar antennas, for devices where space is limited. To reduce the size of antennas, a dielectric coat is used with high relative permittivity.

It is shown in [37, 39] that the propagation environment affects horizontally and vertically polarised fields differently. This work aims to expand on this observation by investigating any differences in the propagation behaviour between orthogonal linear and orthogonal circular polarisations. The effects on MIMO performance of any differences may then be directly examined using the end-to-end modelling approach.

The distribution and polarisation of radiated far-field energy from an antenna is affected by not only the geometry of the antenna conductors, but also objects positioned close to the antennas, such as a ground plane, or other device components [52]. As such, this work also aims to provide a method to account for the effects of such objects on antenna behaviour and the resultant effect upon MIMO performance. Specifically, a metal case of similar dimensions to a mobile phone is modelled in the presence of the antennas. Its effect on antenna behaviour and MIMO performance is investigated.

The behaviour of an antenna is also affected by the presence of other nearby antennas, by way of mutual coupling. Mutual coupling is dependent upon the current distribution on each antenna and its effect alters the radiation pattern, impedance and voltages at the terminals of an antenna. As such, it is considered an important process which must be understood and accounted for in the modelling of MIMO systems with limited space. Another aim of the thesis is to present the differences in the mutual coupling mechanism when antennas are in transmit mode, compared to when they are in receive mode, and to
provide a method to fully account for the mutual coupling at the antennas of polarisation and spatial MIMO systems.

From [4], it is clear that depending on channel conditions, the MIMO channel capacity benefit may be better exploited under a diversity scheme than a multiplexing scheme, or vice-versa. A certain channel may be highly beneficial under one scheme but provide no benefit under the other. To enable a complete evaluation of the ability of MIMO systems, this thesis aims to present the MIMO performance of modelled systems in terms of channel capacity, as well as error rate performance under both diversity and multiplexing schemes.

1.4. Overview of Approach

A flow chart is given in Figure 1-1 which shows an overview of the end-to-end polarisation MIMO modelling approach which has been developed.
Figure 1-1 – Flow chart of polarisation MIMO end-to-end system model

**Antenna and Platform Modelling**

The first step is the antenna modelling. The commercial software package, CST-MWS (Computer Simulation Technology – Microwave Studio) is used to design and simulate compact antennas for use in a $2 \times 2$ MIMO system. Dielectric loaded dipoles which are separated in space have been simulated for the spatial MIMO systems. For the polarisation MIMO systems, dielectric loaded co-located crossed-dipoles have been simulated. The crossed dipole elements are fed individually for use in the dual linear polarised systems and a feed network has been modelled for the dual circular polarised systems to apply the necessary phase shifts. To examine the impact on antenna behaviour and MIMO performance of objects positioned in the antenna near-field regions, two sets of antenna models are created. In the first set, antennas are modelled with no other nearby objects,
while in the second set, they are positioned around a metal case. Particular attention is paid to antenna radiation patterns, polarisation and its orthogonality, mutual coupling and the antenna envelope correlation coefficients of closely positioned antennas. These attributes are all critical to polarisation MIMO systems.

As identified by Chu [53] (and discussed in Section 2.5.1), small antennas are inherently narrow band. The MIMO performance of the narrowband compact antennas is assessed over a 20 MHz band, centred at 2.4 GHz. For reference, full size uncoated broadband antennas have also been modelled, over a 400 MHz band around 2.4 GHz, with no case. This provides an interesting comparison, as the MIMO channel parameters in the indoor environment are different over a narrow band, compared to a broad band, which results in different MIMO system performance.

**Propagation Modelling**

The simulated antenna patterns are used in the propagation modelling stage to apply the correct gains and phase shift to fields radiated by transmit antennas and captured by receivers. The propagation model is a bespoke three-dimensional (3D) ray launcher which has been written using C and MATLAB. This is a deterministic model, where “rays” are launched in many directions from a transmitter and traced through the environment using geometric optics. Upon intersections with objects in the environment, transmission and reflection coefficients are applied to the fields associated with rays. Rays that reach the receiver are recorded and their fields contribute towards the received signal.

A bespoke ray launcher is favoured as opposed to a commercially available package to enable greater flexibility in terms of the inputs and output results and their formats. This is particularly useful when studying polarisation specific channels and when using antennas with non-standard radiation patterns. The ray launcher is used to determine the frequency response of each sub-channel over the signal bandwidth. From these, the MIMO “channel parameters”, $X$, $K$ and $R$ are determined. $X$ is an $n_t \times n_r$ matrix, the elements of which are the average sub-channel gains, taken over the signal frequency band. $K$ is also an $n_t \times n_r$ matrix. Its elements are the Rician K-factors of each sub-channel frequency response over the signal bandwidth. $R$ is an $n_t n_r \times n_t n_r$ correlation matrix, the elements of which are the correlation coefficients between the sub-channel responses. An inverse fast Fourier transform is performed on the frequency responses obtained by the ray launcher and the sub-channel correlation coefficients are taken over the resultant impulse responses.
**Mutual Coupling**

The remaining sub-channel parameter required by the MIMO channel model is the receive mutual impedance, contained in the matrix, $Z^R$. When antennas are positioned close together, the current on one antenna results in the production of electric and magnetic fields in the near field regions of that antenna. These fields cause the radiation into the far field, but will also induce a current in nearby antennas. This interaction is known as mutual coupling and is fully discussed in Section 2.4. The process is dependent upon the current distribution on the antennas. When the excitation source is at the terminals of a transmit antenna, the current distribution is different to that when antennas are used as receivers and the excitation source is in the far-field. As such, the mutual coupling mechanism is different for transmit mode antennas when compared to receivers [54-58].

At the transmitter, mutual coupling is accounted for in the CST-MWS antenna simulations. In these, transmit antennas are simulated in the presence of one another. Each transmit antenna is excited individually, while the other is loaded with its source impedance. The simulation produces the antenna patterns, which are the result of the direct radiation from each antenna, as well as the mutual coupling.

At a receiver, the current distribution on the antenna elements is dependent upon the angle of arrival of the energy from the excitation source. As such, the receive mutual impedance, which relates the uncoupled voltages at the receive antenna terminals to the coupled voltages, is also dependent on angle of arrival. To account for the receive mutual coupling in the end-to-end model, a weighted average approach, which is described in Section 2.4.3 is used to estimate receive mutual impedance from the angle of arrival and the power contribution of received rays, as determined by the ray launcher. The effect of the receive mutual impedance is then accounted for in the MIMO simulations, through multiplication with the receive mutual impedance matrix. To ensure that the effects of mutual coupling at the receiver are not accounted for twice, the receive antenna patterns used by the ray launcher, when obtaining the sub-channel frequency responses, are those of the receive antennas simulated “in isolation”. That is to say, the models contain only the antenna whose pattern is desired. The other is removed, although if a case is included in the model, this remains.

**Polarisation MIMO Channel Model**

To estimate the MIMO performance in the environment modelled by the ray launcher, a bespoke polarisation MIMO channel model has been developed. This generates the
$n_t \times n_r$ MIMO channel matrix, $\mathbf{H}$. The elements of $\mathbf{H}$ represent the MIMO sub-channel responses, which are all modelled as narrowband quasi-random complex gains. Each gain comprises a random Rayleigh distributed component, which represents the multipath scattering, as well as a component with fixed amplitude, which represents the dominant, specular paths. The ratio of these is set by the Rician $K$-factors, contained in $\mathbf{K}$. The Rayleigh parts of the sub-channels are correlated through multiplication with the correlation matrix, $\mathbf{R}$. Each sub-channel is then weighted, such that the expected magnitude of its gain over many instances of the channel is equal to that given by its corresponding element of $\mathbf{X}$. Finally, the receive mutual coupling is accounted for by multiplication of the channel with the receive mutual impedance matrix, $\mathbf{Z}_R$. The transmit mutual impedance is already inherently accounted for in the values of $\mathbf{X}$, $\mathbf{K}$ and $\mathbf{R}$, which are obtained from the ray launcher results using coupled transmit antenna patterns simulated using CST-MWS.

**MIMO Capacity Estimation and System Simulation**

The performance of the modelled MIMO systems is analysed in terms of estimated capacity and bit error rate (BER) performance under the Alamouti and V-BLAST MIMO schemes. Capacity is estimated, in bits per second per Hz (bps/Hz), by taking the expectation of (1.1) over many instances of the channel, $\mathbf{H}$. The error rate performance is estimated using Monte-Carlo MIMO system simulations, which have been written in MATLAB. These simulations model the modulation, coding and transmission of a large number of bits over the bespoke MIMO channel model, $\mathbf{H}$. The received signals are then decoded and detected, before the number of bits received in error are counted. The Alamouti scheme is used to give an indication of the performance of each system in a diversity MIMO application, while the V-BLAST scheme is used to give an indication of performance under a spatial multiplexing scheme.

1.5. **Relation to Existing Literature**

The following points summarise the relevance of this work in relation to existing literature and highlight its novel contributions.

- In existing literature, channel models have been identified, which have been used to estimate performance of polarisation MIMO systems. The performance estimates are based on a number of simplifications and assumptions regarding the conditions
of the polarisation MIMO channel. This work provides a novel end-to-end, predominantly deterministic approach to the modelling of polarisation MIMO systems in the indoor environment. A bespoke MIMO channel model is presented, based on deterministically obtained channel parameters for antenna models, in a propagation environment which is modelled through ray launching. It enables a full analysis of system performance, directly relating the physical antenna and environment models to channel behaviour and MIMO performance, in terms of capacity and bit error rate. The flexibility of this approach means that it may be applied to any antenna or building model.

- This work presents a thorough evaluation of MIMO performance, comparing spatial MIMO systems, with limited antenna separation, to polarisation MIMO systems, using co-located dual-polar antennas. The parameters of the MIMO channel are obtained through deterministic modelling of physical antenna systems and environments, as opposed to using assumed channel conditions which do not fully reflect the behaviour of physical systems. This is the first study which has been identified where the performance of such systems is compared in terms of capacity as well as bit error rate, under diversity and multiplexing applications, using compact antennas, where all relevant aspects of the channel, including mutual coupling are considered.

- Existing literature identifies that the propagation conditions in the indoor environment are different for waves of different polarisation. Despite this, no existing work has been identified which makes comparisons between the performance of dual-linear polarised MIMO systems and dual-circular polarised MIMO systems. This work provides that comparison, where the differences in propagation conditions dependent upon polarisation are accounted for by the 3D ray launching propagation model and the bespoke polarisation MIMO channel model.

- This work further extends existing literature, through the examination of the effect of objects close to the antennas of polarisation and spatial MIMO systems. This is done through the analysis of MIMO systems where antennas are positioned around a metal case. The end-to-end analysis allows the effect of the case on antenna behaviour to be studied in isolation, before examining the resultant effect on the channel and ultimately, upon MIMO performance.
1.6. Organisation of Thesis

The remainder of the thesis is organised as follows:

Chapter 2 is primarily concerned with the modelling of compact antennas. Fundamental antenna properties such as radiation patterns, impedance and bandwidth are introduced. Polarisation and orthogonality are then discussed and defined. The Poincaré sphere is introduced as a useful method to visually present polarisation states. Mutual coupling is then fully discussed. The differences between the mutual coupling mechanism at transmitters and receivers are addressed and the approach for estimating receive mutual impedance is described. Following this, the antenna models which have been used to obtain the end-to-end MIMO system results are introduced, with and without a metal case. The antennas are dipoles and crossed dipoles, miniaturised using a spherical coat of high relative permittivity.

Chapter 3 describes the bespoke ray launching propagation model which has been developed and presents its validation against other propagation models. An overview of ray techniques is provided, before the key components and considerations of the model are discussed. The validation of the model is built up from comparisons against directly calculated results for very simple cases, such as single reflections, to comparisons against results obtained using CST-MWS and other commercially available propagation models, for more complex environments such as a terraced house and a floor of an office building.

Chapter 4 discusses the work related to MIMO systems and the MIMO channel modelling. The MIMO flat fading transmission model and the Rayleigh fading channel model are introduced. These are the starting point for the bespoke polarisation MIMO channel model. The two MIMO transmission schemes, Alamouti and V-BLAST, which have been simulated are discussed alongside their detection schemes. The Monte-Carlo method for estimating BER under these schemes is then reviewed, before the extended polarisation MIMO channel model is introduced. This is constructed from the Rayleigh fading model, which is correlated according to the correlation matrix. Fixed components are then introduced, with their magnitude relative to the Rayleigh components set by the sub-channel K-factors. The sub-channels are then weighted according to the average gains over the signal bandwidth, before the receive mutual impedance matrix is applied. Lastly, the channel model and MIMO system simulations are validated against published results with known channel parameters.
In Chapter 5, the end-to-end MIMO system results are presented for the antenna systems described in Chapter 2, using the ray launching propagation model of Chapter 3 and the extended MIMO channel model of Chapter 4. The modelled office floor environment is described and any assumptions which are not explained elsewhere are stated. A summary of the channel parameter results is given. These are the correlation, K-factors and average sub-channel gains obtained from the results of the ray launcher. These parameters form a very large set of results, which are given in full in Appendices A and B. MIMO performance results are then presented in terms of channel capacity, BER under the Alamouti scheme and BER under V-BLAST. Performance is compared between spatial MIMO systems with antenna separation between $0.25\lambda$ and $\lambda$, co-located dual polarised systems and a hybrid system using dual-polar antennas which are separated by $0.25\lambda$. Results are presented using compact dielectric loaded antennas, with and without a case, as well as large, uncoated antennas, over a greater bandwidth.

Chapter 6 provides a conclusion. This gives a summary of the work which has been undertaken and the important results. It also reviews the aims of the thesis and discusses their fulfilment. Finally, it summarises the main contributions that the thesis makes to the field of research and provides some suggestions for further work.
2. Orthogonally Polarised Antennas on Small Platforms

2.1. Introduction

Size and form constraints on modern portable wireless devices mean that small antennas are often required on many platforms. In MIMO systems, space constraints mean that it is often difficult to separate antennas sufficiently to achieve adequate spatial diversity. In this work, polarisation diversity is implemented, where compact orthogonally polarised antennas are used to provide diversity on small platforms where space is limited.

This chapter describes the antenna design and modelling process which has been used throughout the work. Particular attention is paid to antenna gain, radiation patterns, impedance, bandwidth, efficiency, mutual coupling and of course polarisation. These are very important antenna parameters for polarisation MIMO systems.

When considering compact antennas, a fundamental limit exists with respect to size, efficiency and bandwidth, which results in a trade-off between these three properties [53, 59, 60]. The technique for antenna miniaturisation which has been used is dielectric loading, where conductors are surrounded by a medium of high relative permittivity.

The following different structures have been modelled:

- Dielectric loaded co-polar dipoles, separated in space, which are used for spatial MIMO systems
- Dielectric loaded crossed dipoles, which are used for linear polarised (LP) and circular polarised (CP) co-located dual-polar MIMO systems
- A spatial hybrid system, where two orthogonally circular polarised antennas are separated in space.

Additionally, some uncoated broadband dipole models are presented, as a reference for comparison to the compact models. These are the full size equivalents of the co-polar dipoles separated in space and the crossed dipoles used for LP and CP. Due to their larger size, these antennas may be operated over a larger bandwidth than the compact antennas. The dielectric loaded antennas are considered in free-space, then in the presence of a metal box which resembles the case of a small device. The antenna models were designed and simulated using Computer Simulation Technology - Microwave Studio (CST-MWS).
The simulated antenna patterns are used by the ray launching propagation model (as described in Chapter 3) to determine the magnitude, phase and polarisation of the field radiated from each transmit location and to apply the relevant antenna gain and phase shift to fields incident at each receive location.

A design requirement of the antennas modelled in this chapter is the ability to transmit and receive decorrelated signals along the MIMO sub-channels, in an indoor environment. To do this, pattern diversity must be maximised, whilst using as little space as possible for the antennas. Attention must be paid to mutual coupling, which results in signals at the terminals of one antenna coupling to the terminals of nearby antennas, but can also result in greater pattern diversity. For the polarisation MIMO systems, pattern diversity is implied through the use of antennas with orthogonal radiation patterns. A significant challenge is to maintain this orthogonality over as much of the farfield pattern as possible. This is particularly cumbersome when antennas are placed around a case, which often has a severe effect on the radiation patterns and their polarisation. Another challenge is therefore to produce radiation patterns which have sufficient gain throughout the farfield, to capture as much incident energy as possible. This is not only to increase the total power transferred by the system, but also because the energy arriving from each direction travels a different path through the environment and will therefore arrive with a level of decorrelation.

The remainder of this chapter is organised as follows. Section 2.2. briefly introduces fundamental antenna properties, with reference to polarisation MIMO systems. Section 2.3. fully describes polarisation. Reference is made to the Stokes Parameters and the Poincaré sphere representation of polarisation, before the concept of orthogonality is introduced. Section 2.4. discusses mutual coupling and mutual impedance in MIMO systems. The difference between transmit and receive mutual impedance is outlined and a technique for determining receive mutual impedance is given. Section 2.5. gives some background on compact antennas, the fundamental limits when designing compact antennas and an introduction to antenna miniaturisation through dielectric loading. Section 2.6. gives more detail on the modelling approach, using CST-MWS. Section 2.7. describes all of the antenna models used in this work and presents their relevant simulation results. To begin with, broadband uncoated antennas, which are used as a reference to compare the compact antennas to, are introduced in Section 2.7.1. Section 2.7.2 introduces compact co-polar antennas used for spatial diversity, while Section 2.7.3 introduces the orthogonally
polarised compact antennas which are used for polarisation diversity. Finally, Section 2.8 summarises the chapter.

2.2. Antenna Properties

The basic properties of an antenna are its gain, radiation pattern, impedance, bandwidth and efficiency, which are defined below. Of particular importance to a polarisation MIMO system is the polarisation throughout the radiation pattern, as this is used as a mechanism to decorrelate the signals between different antennas. Polarisation is discussed in detail in Section 2.3. For linear passive antennas, these basic properties are the same whether the antenna is used for transmission or receiving, due to the reciprocity theorem [61]. Another important consideration in any MIMO systems, when antennas are positioned close to each other is mutual coupling. This results in a mutual impedance which is dependent upon the current distribution on the antenna elements. As a result, it is often different when antennas operate as transmitters, which are excited by a feed at their terminals, compared to when they operate as receivers, which are excited by sources in their farfield [54-56]. This is discussed in further detail in Section 2.4.

Antenna design is of course a huge subject and an introduction as well as much more information can be found in resources such as [52, 61-69].

2.2.1. Near-field vs Far-field

Gain, directivity and radiation patterns are defined according to the radiated fields of an antenna, which exist in the far-field region. This is the region where the electric and magnetic fields from the antenna behave in the classical sense, where radiation intensity decreases with the square of distance from the source, angular distribution of field intensity is independent of radial distance and the magnetic and electric field components are transverse [64, 70]. This is in contrast to the reactive near-field region which is defined as “the portion of the near-field region immediately surrounding the antenna, wherein the reactive field predominates” [64]. Within this region, energy is stored in the form of reactive fields which are not radiated into the far-field. Objects within this region such as other antennas, terminal structure or human tissue interact with the reactive fields resulting in changes to the antennas input impedance and radiation characteristics. In between the reactive near-field region and the far-field region is the radiating near field region, sometimes referred to as the Fresnel region. Here, radiation fields predominate, but
angular distribution of field intensity is still dependent upon radial distance. Figure 2-1 shows the three regions around a large antenna of maximum dimension, $D_{\text{max}}$.

The boundaries between these three regions are not distinct and are dependent upon the nature of the antenna, however there are common reference distances given in [64] which may be used for guidance. For regular antennas, the reactive near-field region may be considered to exist up to $R_a = 0.62\sqrt{D_{\text{max}}^2 \lambda}$, where $R_a$ is the radial distance from the centre of the antenna, $D_{\text{max}}$ is the largest dimension of the antenna and $\lambda$ is the wavelength. The radiating near-field region may then be considered from $R_a$ to $R_b = 2D_{\text{max}}^2 / \lambda$. For compact antennas however, where $D_{\text{max}} \ll \lambda$, the radiating near-field region may not exist and, $R_a = R_b$ is often taken to be $\lambda / 2\pi$.

![Diagram of field regions of an antenna](image)

**Figure 2-1 – Field regions of an antenna**

### 2.2.2. Gain and Directivity

Antenna gain is a measure of far-field radiation intensity in a specified direction (often the direction of maximum radiation intensity). In a given direction, it is defined $4\pi$ times the ratio of power radiated per unit solid angle in that direction, to the total power accepted by the antenna [68]. This is equivalent to the ratio of power radiated per unit solid angle in the given direction to power radiated per unit solid angle in any direction from a hypothetical isotropic antenna with the same input power. An isotropic antenna is a lossless antenna with uniform radiation in all directions. Mathematically, antenna gain, $G(\theta, \phi)$ in the direction $\theta, \phi$ is described as follows [68, 69].
\[ G(\theta, \phi) = 4\pi \frac{\text{power radiated per unit solid angle in } \theta, \phi \text{ direction}}{\text{total power delivered to antenna}} \] (2.1)

Directivity is similar to gain, except the ratio is relative to total power radiated by the antenna, rather than delivered to it. It is defined as follows, where \( D(\theta, \phi) \) is the directivity in the direction \( \theta, \phi \).

\[ D(\theta, \phi) = 4\pi \frac{\text{power radiated per unit solid angle in } \theta, \phi \text{ direction}}{\text{total power radiated by antenna}} \] (2.2)

For an antenna of very high efficiency, the gain and directivity are very close. On the other hand, for an antenna of low efficiency and high losses, the gain is much less than the directivity.

### 2.2.3. Radiation Pattern

The radiation pattern describes the variation of the farfield radiation properties of an antenna with direction. It is often presented in a visual form of either a 3D or polar plot, where gain, directivity or electric field (E-field) strength at a given distance from the antenna is plotted against angle of a spherical coordinate system. One common pattern is the omnidirectional pattern, typically used for broadcast type applications where equal coverage is desired in all directions. The pattern of a vertical dipole is omnidirectional in the horizontal plane but contains some directivity in the vertical plane.

A radiation pattern may be specified with a mathematical description of the variation of its parameters with direction, or with a record of the values of its parameters at discreet directions. To properly account for the polarisation of the fields radiated by an antenna, it is essential to have knowledge of the field in two components which are orthogonal to themselves and the direction of propagation, as well as the relative phases of these components. The radiation patterns used by the ray launcher in this work are represented by discrete values of complex electric field strength at a reference distance of 1 m, in \( \theta \) and \( \phi \) components of the Institute of Electrical and Electronics Engineers (IEEE) standard spherical coordinate system used for antenna measurements [68]. These are recorded at increments of 5° in the \( \theta \) and \( \phi \) directions.

Figure 2-2 shows the IEEE standard spherical coordinate system which is used throughout this work. The antenna under consideration is assumed to be at the centre of the sphere and field components are specified at a constant distance from the centre, with position...
defined by the angles $\theta$ and $\phi$. $\theta$ is the polar angle, measured from the $Z$ axis towards the $XY$ plane, while $\phi$ is the azimuthal angle, measured from the $X$ axis. The figure also shows the spherical components of the radiated electric field vector, denoted $\vec{E}_\theta$ and $\vec{E}_\phi$, which are perpendicular to one another and the direction of propagation. The electric field of a wave propagating in the direction on the diagram may be fully specified in these two components.

![IEEE Standard Spherical Coordinate system used in Antenna Measurements](image)

Figure 2-2 – IEEE Standard Spherical Coordinate system used in Antenna Measurements [68]

2.2.4. Impedance

The input impedance of an antenna relates the voltage to the current at the input to the antenna. The impedance is dependent upon all of the radiating elements of an antenna, the transmission line components which are used to connect them, as well as other antennas or objects which are close to the antenna. It is also dependent upon frequency. The input impedance of an antenna, $Z_{in}$ consists of a real part, $R_{in}$ and an imaginary part, $X_{in}$.

$$Z_{in} = R_{in} + jX_{in}$$

(2.3)

The imaginary part of the impedance is known as the reactance. This is a result of the reactive energy stored in the near field of the antenna, which is not radiated. Therefore, for a perfect match, the input impedance of an antenna must be entirely real throughout the antennas frequency band of operation. The real part comprises the radiation resistance, $R_{rad}$ and a loss resistance, $R_{loss}$ which is caused by ohmic losses in the antenna conductors.
\[ R_{in} = R_{rad} + R_{loss} \]  (2.4)

The radiation resistance accounts for the resistance due to power being radiated into the environment, while the loss resistance is caused by ohmic losses in the structure. For electrically large antennas, the resistance is usually almost entirely composed of the radiation resistance, however for electrically small antennas, the loss resistance can become significant [70].

For maximum transfer of energy between a system and an antenna, the antenna input impedance must be matched to its feed impedance. If it is not then some of the power will be reflected where the antenna is connected to its feed. This can be measured in terms of voltage reflection coefficient (\( \Gamma \)) voltage standing wave ratio (VSWR) or the scattering parameter [71], \( S_{11} \) when the antenna is considered as a one port network. The voltage reflection coefficient, \( \Gamma \) when an antenna with input impedance \( Z_{in} \) is mismatched with a source impedance \( Z_g \) is given by equation (2.5). The VSWR is related to this coefficient according to equation (2.6) [65, 70].

\[
\Gamma = \frac{Z_g - Z_{in}}{Z_g + Z_{in}} \]  (2.5)

\[
VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \]  (2.6)

2.2.5. Bandwidth

Most properties of an antenna are frequency dependant and as such there must be a specified frequency of operation for which their values are true and a range of frequencies around it for which they remain true, or at least true to some extent. This range of frequencies is the bandwidth. Depending upon the operational requirements of the antenna, bandwidth may be specified in terms of any of these properties, or even more than one. It is common that one property will be more frequency sensitive than the others and so it is appropriate that it should be used to specify the bandwidth. For small antennas where linear dimensions are of the order \( \lambda/2 \) or less it is usually impedance which is the limiting factor, whereas for CP antennas it is possible that polarisation characteristics may be the limiting factor [65].
When considering gain or directivity, bandwidth is specified as the frequency range for which gain or directivity is within certain limits. If radiation pattern is used then bandwidth may be the frequency range for which a certain pattern shape is maintained within given limits. For polarisation, it may be the frequency range where the ratio of co-polar to cross-polar gain, axial ratio or polarised components of the gain are within specified limits. Similarly, if impedance, return loss or VSWR are used, then bandwidth is the frequency range where at least one of these properties is within a certain limit. Many different definitions for bandwidth are discussed in [72].

2.2.6. Efficiency

The radiation efficiency, \( \eta \) of an antenna is defined as the ratio of radiated power, \( P_{\text{rad}} \) to the amount of power accepted at the antennas input terminals, \( P_{\text{in}} \), [70].

\[
\eta = \frac{P_{\text{rad}}}{P_{\text{in}}} \quad (2.7)
\]

Efficiency is often given as a percentage or in dB. It may also be classified as the ratio of power gain in any specified direction, to the directivity in the same direction. This definition is useful for calculating efficiency from measured results. Efficiency is considered an intrinsic property of an antenna and does not consider its feed network. For this reason, efficiency does not usually account for losses due to imperfect impedance matching.

In a wire antenna, where losses are entirely ohmic, caused by the resistance of the antenna elements, the radiated power is proportional to the radiation resistance, \( R_{\text{rad}} \) and the lost power, \( P_{\text{ohmic}} \) is proportional to the ohmic resistance, \( R_{\text{ohmic}} \). In this case, efficiency, \( \eta_{\text{ohmic}} \) may also be written according to these resistances [70].

\[
\eta_{\text{ohmic}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{ohmic}}} = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{ohmic}}} \quad (2.8)
\]

For more complicated antennas however, such as those with dielectric loading, there may be other mechanisms which result in further losses, such as the excitation of trapped modes within the dielectric [68].
2.2.7. Envelope Correlation Coefficient

Envelope correlation coefficient, $\rho_{\text{env}}$ is a useful property of antennas used for MIMO systems. It gives a measure of the degree of correlation between the radiated fields from two sources. It may be calculated from the radiated field components from the two sources as follows [73, 74], where $E_{\theta_1}$ and $E_{\phi_1}$ are the complex $\theta$ and $\phi$ field components from source 1, $E_{\theta_2}$ and $E_{\phi_2}$ are the complex $\theta$ and $\phi$ field components from source 2, $\hat{\Phi}$ denotes spherical integration and $d\Omega = \sin \theta \, d\theta \, d\phi$ is the differential solid angle.

$$
\rho_{\text{env}} = \frac{\left| \hat{\Phi}\left\{ E_{\theta_1}(\theta, \phi)E_{\theta_2}^*(\theta, \phi) + E_{\phi_1}(\theta, \phi)E_{\phi_2}^*(\theta, \phi) \right\}d\Omega \right|^2}{\hat{\Phi}\left\{ E_{\theta_1}(\theta, \phi)E_{\theta_1}^*(\theta, \phi) + E_{\phi_1}(\theta, \phi)E_{\phi_1}^*(\theta, \phi) \right\}d\Omega \times \hat{\Phi}\left\{ E_{\theta_2}(\theta, \phi)E_{\theta_2}^*(\theta, \phi) + E_{\phi_2}(\theta, \phi)E_{\phi_2}^*(\theta, \phi) \right\}d\Omega}
$$

(2.9)

A value of $\rho_{\text{env}}$ close to zero suggests that the signals received by the two antennas, in an environment with sufficiently rich multipath scattering, should be well decorrelated, while a value of $\rho_{\text{env}}$ close to one suggests that they will be correlated. It should be noted however that (2.9) is independent of the effects of the environment, and assumes the antennas are illuminated by uniformly distributed incident energy. Nevertheless, this is still a useful indicator when designing antenna systems for MIMO applications. It is stated in [51] that antennas with $\rho_{\text{env}} < 0.5$ satisfy the criterion to enable a good level of diversity. An alternate method for calculating $\rho_{\text{env}}$ from scattering parameters is given in [73]. This method is not valid for antennas with high losses though and as such is of less use when studying compact antennas.

2.3. Polarisation

Polarisation describes the shape and orientation of a propagating electric field. If a wave is travelling in the $Z$ direction, then the most simple polarisation states to consider are where the electric field is linear and parallel to the $X$ axis or parallel to the $Y$ axis. These may be considered $X$-polarised and $Y$-polarised respectively. Waves of these polarisations may be created using a wire antenna such as a dipole, orientated in these directions. Alternatively, if a wave is travelling across the surface of the earth it may be considered horizontally or vertically polarised, according to its orientation relative to the surface of the earth. Other common linear polarisation states are $\pm 45^\circ$, where the electric field vector is at $\pm 45^\circ$ relative to a given direction. More generally, any polarisation state may be defined by the complex field components of the electric field in two directions which are perpendicular to
the direction of propagation. When considering propagation from an antenna, it is convenient for these two directions to be the $\mathbf{\hat{u}}_\theta$ and $\mathbf{\hat{u}}_\phi$ directions of the IEEE standard spherical coordinate system [68] used for antenna measurements. Because radiated field intensity and phase vary with angle, one should be aware that the polarisation of fields radiated by an antenna is also therefore dependent upon angle. For instance, crossed dipoles which are fed with a 90° phase offset will radiate CP in the direction which is perpendicular to both dipoles, but will radiate linear polarisation in all directions on the plane containing the dipoles.

It is quite likely that a wave will have field components in both directions perpendicular to the direction of propagation. For example, if a wave has equal and in phase field components, $E_\theta$ and $E_\phi$ in the $\mathbf{\hat{u}}_\theta$ and $\mathbf{\hat{u}}_\phi$ directions respectively, then the wave is 45° polarised relative to these directions. Supposing $E_\theta$ and $E_\phi$ are out of phase by 90°, the electric field vector will now rotate in a circle. If the field vector is rotating clockwise from the point of view of the transmitter it is considered right-hand circular (RHC) polarised. Alternatively if it is rotating anti-clockwise from the same point of view, it is considered left-hand circular (LHC) polarised. Should the $\theta$ and $\phi$ components be of unequal magnitude, or of equal magnitude but with a phase difference not equal to 90°, then the electric field vector will trace out an ellipse. This is known as elliptical polarisation.

![General case of elliptical polarisation, showing right hand rotation sense, where direction of propagation is out of the page](image)

**Figure 2-3** – General case of elliptical polarisation, showing right hand rotation sense, where direction of propagation is out of the page

Figure 2-3 shows the polarisation ellipse for a wave propagating in the direction out of the page. The properties of this ellipse, axial ratio and tilt angle, may be used to fully describe
its polarisation [31, 62]. Axial ratio is the ratio of the major \( E_{\text{max}} \) and minor \( E_{\text{min}} \) axes of the polarisation ellipse. It is defined as positive for left hand rotating ellipses and negative for right hand [63]. Tilt angle, \( \psi \) is the clockwise angle between a reference direction and the major axis of the ellipse when viewed in the direction of propagation. The reference direction is arbitrary, although when describing polarisation from an antenna, it is common to use the \( \hat{u}_\theta \) direction of the spherical coordinate system. The rotation sense is indicated by the arrow, which in this case indicates right handed rotation. Further introduction to polarisation may be gained from many references such as [31, 61-63, 68, 75]. It should be noted that throughout this work, only completely polarised waves are considered. This means that they can be described using field components in two orthogonal directions which have constant amplitudes over time.

2.3.1.1. Stokes Parameters and the Poincaré Sphere

There are many ways to represent a polarisation state. In [75], Rumsey presents a method to represent elliptical polarisations on a Carter or Smith impedance chart. Another convenient way is on the Poincaré sphere (Figure 2-4). This is a 3D graphical method where any polarisation may be represented by a position on a sphere. RHC and LHC occupy the poles and linear states are around the equator. All other states are elliptical and may be uniquely represented by a position between these points, where latitude is indicative of axial ratio and longitude is indicative of tilt angle. Positions in the upper hemisphere have right hand rotation sense, while positions in the lower hemisphere have left hand rotation sense. The Poincaré representation was first introduced in [75] and is well described in [68] and [76].

Stokes parameters may also be used. These are four values which fully describe a wave. \( S_0 \) gives the intensity of the wave, \( S_1 \) gives the degree of linear polarisation with respect to two arbitrary orthogonal reference axes (such as \( \theta/\phi \) or H/V), \( S_2 \) gives the degree of linear polarisation with respect to two axes at 45° to the reference axes and \( S_3 \) gives the degree of circular polarisation. Stokes parameters are identified in [77] and well documented in [76]. It should be noted however, that [76] discusses polarisation from an optics point of view, where polarisation states are considered from the point of view of a person looking at a source, rather than from a transmitter, as in the IEEE standard [68].
From the spherical components, $E_\theta$ and $E_\phi$ of the electric field radiating in a given direction from an antenna and their complex conjugates, $E_\theta^*$ and $E_\phi^*$, the Stokes parameters are defined as follows [76].

\[
S_0 = E_\theta E_\theta^* + E_\phi E_\phi^* \quad (2.10)
\]
\[
S_1 = E_\theta E_\theta^* - E_\phi E_\phi^* \quad (2.11)
\]
\[
S_2 = E_\theta E_\phi^* + E_\phi E_\theta^* \quad (2.12)
\]
\[
S_3 = j(E_\phi E_\theta^* - E_\theta E_\phi^*) \quad (2.13)
\]

Here, the superscript The parameters $S_1$ to $S_3$ are typically normalised through division by the field intensity, $S_0$ to give $\hat{S}_1$, $\hat{S}_2$ and $\hat{S}_3$ with values which range between -1 and 1. As these are taken from spherical components, $S_1$ gives the degree of $\theta$ over $\phi$ polarisation. $\hat{S}_1 = 1$ indicates pure $\theta$ polarisation, while $\hat{S}_1 = -1$ indicates pure $\phi$ polarisation. $\hat{S}_2$ gives the degree of $\pm 45^\circ$ polarisation. $\hat{S}_2 = 1$ indicates $45^\circ$ polarisation relative to $\vec{u}_\theta$, while $\hat{S}_2 = -1$ indicates $-45^\circ$ polarisation relative to $\vec{u}_\theta$. $\hat{S}_3$ gives the degree of circular polarisation. $\hat{S}_3 = 1$ indicates pure RHC polarisation, while $\hat{S}_3 = -1$ indicates pure LHC polarisation.

The normalised Stokes parameters may be used to conveniently plot the polarisation state on a Poincaré sphere of radius 1, using the Cartesian coordinates $(x, y, z) = (\hat{S}_1, \hat{S}_2, \hat{S}_3)$. Figure 2-4 shows an example where the polarisation of the fields radiated by two antennas are plotted. The field from antenna 1 is RHC polarised, with $E_\theta_1 = 1$ V/m, $E_\phi_1 = -j$ V/m. This results in the Stokes parameters, $(\hat{S}_1, \hat{S}_2, \hat{S}_3) = (0, 0, 1)$. The field from antenna 2 is $\phi$ polarised, with $E_\theta_2 = 0$, $E_\phi_2 = 1$ V/m, giving the Stokes parameters, $(\hat{S}_1, \hat{S}_2, \hat{S}_3) = (-1, 0, 0)$. 
2.3.1.2. Polarisation Orthogonality

If two linearly polarised waves are travelling along the same path and their electric field vectors are at exactly 90° to each other then the polarisation states are considered orthogonal and no interaction between them will occur. If these waves were incident on two receive antennas which are also orthogonal and perfectly aligned to the incident waves, then each antenna would only receive energy from its co-polar wave and would be perfectly isolated from its cross polar wave. The same is true for circular polarisation states with opposite senses of rotation. For elliptical polarisation, the rotation sense must be opposite and the tilt angles at 90° separation for the states to be orthogonal. This scenario is of course ideal for polarisation MIMO systems, where transmission channels are decorrelated through the use of orthogonal polarisations. In a multipath propagation environment however, even if the waves were perfectly orthogonal at transmission, it is likely that by the time they reach the receiver their polarisations will have degenerated into different unpredictable states which are less orthogonal. Furthermore, when real antennas are positioned orthogonally, it is likely that some near field interaction between them and objects in their environment may cause their radiated fields to be less than perfectly orthogonal. This is observed in Section 2.7.3.2.
If two polarisation states are plotted on the Poincaré sphere then they are only perfectly orthogonal if they are positioned diametrically opposite each other. If they are not, then the degree of orthogonality may be quantified from the angular distance between the points. This method is given in [68] and [76]. In [68], polarisation efficiency, $p$ is defined as the ratio of power in an incident wave that is received by the antenna under consideration, to the power which would be received if the antenna was perfectly matched to the polarisation of the incident wave. Polarisation efficiency is also sometimes called polarisation mismatch loss. If the polarisation of the transmitted wave and the polarisation of the receive antenna are both plotted on the Poincaré sphere then $p$ is calculated as follows,

$$p = \cos^2 \zeta$$

(2.14)

where $2\zeta$ is the angle between the two points from the centre of the sphere. If the polarisation states are identical then $p = 1$ and the receive antenna will receive all of the power of the wave. In this situation, the incident wave and the receive antenna are said to be co-polarised. If they are orthogonal then $2\zeta = 180^\circ$ and so $p = 0$ and no power is received by the receive antenna. They are now said to be cross-polarised.

Another mathematically equivalent method to calculate polarisation efficiency is given in [61] and [31] as follows.

$$p = \frac{|E_{\phi_1}E_{\phi_2} + E_{\theta_1}E_{\theta_2}|^2}{\left(|E_{\phi_1}|^2 + |E_{\theta_1}|^2\right)\left(|E_{\phi_2}|^2 + |E_{\theta_2}|^2\right)}$$

(2.15)

$E_{\theta_1}$ and $E_{\phi_1}$ are the complex $\theta$ and $\phi$ field components which are radiated from antenna 1 towards antenna 2 when antenna 1 is excited, while $E_{\theta_2}$ and $E_{\phi_2}$ are the complex $\theta$ and $\phi$ field components which antenna 2 would radiate towards antenna 1 if antenna 2 was excited. This equation is equally valid when E-field $\theta$ and $\phi$ components are substituted for any pair of orthogonal components.

Again, this method gives the amount power transferred between antenna 1 and antenna 2, which are not perfectly polarisation matched, relative to the power which would be transferred if they were perfectly polarisation matched.

Perhaps more useful to this project is the examination of the orthogonality between fields from two co-located sources, such as those in a dual polarised antenna structure. To do
this, equation (2.15) may be adapted by conjugating the fields from one of the ports. This is because in (2.15), the fields from each source are considered from opposite directions, as a wave from one antenna is travelling towards the other. It also seems appropriate to subtract the equation from one so that it is equal to one when polarisation states are perfectly orthogonal and zero when they are identical. In this way, polarisation orthogonality, \( I_p \) is defined linearly as follows.

\[
I_p = 1 - \frac{|E_1^\phi E_2^\phi + E_1^\theta E_2^\theta|^2}{(|E_1^\phi|^2 + |E_1^\theta|^2)(|E_2^\phi|^2 + |E_2^\theta|^2)}
\]  

Equation (2.16) has been used alongside the Poincaré sphere to examine the far field polarisation and orthogonality of the simulated antenna structures which are described further in this chapter. Upon doing so, it became apparent that polarisation states vary in a largely unpredictable manner, often over small angular distances, in terms of both \( \theta \) and \( \phi \). This is particularly true when objects, such as a metal case, are introduced into the near field regions of the antennas. Because of this, it is useful to visually present the orthogonality of fields from two antennas in all directions.

Figure 2-5 and Figure 2-6 show an example of this, where the farfields of a co-located dual polar coated crossed dipole CP antenna are studied. The dipoles are positioned on the \( X \) and \( Z \) axes. In these figures, orthogonality is calculated using (2.16) and plotted as the colour of the 3D plot. The radius is proportional to the addition of the absolute power gains from both sources. In Figure 2-5 the antenna was simulated in free-space (Section 2.7.3.1.), while in Figure 2-6, the antenna was in the presence of a metal case (Section 2.7.3.2.). As can be seen, in free-space, the fields are radiated with high orthogonality for all angles, other than those close to the plane containing the dipoles, and have fairly uniform total gain. Upon the introduction of the case however, the radiation patterns are distorted, resulting in the non-uniform radius of the plot and the orthogonality in most directions is far less. This structure is studied in more detail in Section 2.7.3.
Figure 2-5 – Farfield orthogonality for co-located coated crossed dipole CP antennas in free-space

Figure 2-6 – Farfield orthogonality for co-located coated crossed dipole CP antennas in presence of metal case
2.4. Mutual Coupling

When antennas are positioned close together, the current on one antenna results in the production of electric and magnetic fields in the near field regions of that antenna. These fields cause the radiation into the far field, but will also induce a current in nearby antennas. This interaction is known as mutual coupling and is well documented in references such as [46, 47, 54, 55, 70, 78-80]. The result is threefold. The antenna radiation patterns are altered relative to their free-space patterns, the antenna impedances change and voltages at the terminals of the antennas change.

The change in terminal voltages due to the coupling must be taken into account in a MIMO system simulation. This may be done through the use of the transmit and receive mutual impedance matrices, $\mathbf{Z}^T$ and $\mathbf{Z}^R$ respectively [79]. In a $2 \times 2$ MIMO system, the terminal voltage vector at the transmit antennas with coupling, $\mathbf{v}_T = [v^T_1, v^T_2]^T$ is determined from the uncoupled vector, $\mathbf{u}_T = [u^T_1, u^T_2]^T$ using (2.17), while the coupled vector at the receiver, $\mathbf{v}_R = [v^R_1, v^R_2]^T$ is determined from the uncoupled vector, $\mathbf{u}_R = [u^R_1, u^R_2]^T$ using (2.18).

$$\mathbf{v}_T = \mathbf{Z}^{-1}^T \mathbf{u}_T$$ (2.17)

$$\mathbf{v}_R = \mathbf{Z}^{-1}^R \mathbf{u}_R$$ (2.18)

The mutual impedance matrices in a $2 \times 2$ system are as follows.

$$\mathbf{Z}^T = \begin{bmatrix} 1 & -\frac{Z^T_{1,2}}{Z_{1,1}} \\ \frac{Z^T_{2,1}}{Z_{2,2}} & 1 \end{bmatrix}$$ (2.19)

$$\mathbf{Z}^R = \begin{bmatrix} 1 & -\frac{Z^R_{1,2}}{Z_{1,1}} \\ \frac{Z^R_{2,1}}{Z_{2,2}} & 1 \end{bmatrix}$$ (2.20)

$Z_{1,1}$ and $Z_{2,2}$ are the input impedances of antenna 1 and 2 respectively, $Z^T_{i,j}$ is the transmit mutual impedance between $j$th and $i$th transmit antennas and $Z^R_{i,j}$ is the receive mutual impedance between $j$th and $i$th receive antennas.
2.4.1. Transmit Mutual Impedance

Conventionally, mutual impedance between antennas $i$ and $j$, $Z_{i,j}$ is defined in the transmit sense as the ratio of the voltage, $V_i$ induced across the terminals of antenna $i$, to the current, $I_j$ supplied to antenna $j$ whilst antenna $i$ is open circuit [79, 80], as in

$$Z_{i,j} = \frac{V_i}{I_j} \bigg|_{I_i=0} \quad (2.21)$$

In a MIMO system, neither antenna’s terminals are open circuit, so perhaps a more suitable definition for transmit mutual impedance is given in [79], as the ratio of voltage induced at the terminals of (transmit) antenna $i$, which is loaded by the transmitter source impedance, $Z_g$, to the exciting current through the load of (transmit) antenna $j$, as follows.

$$Z_{i,j}^T = \frac{V_i}{I_j} \quad (2.22)$$

The only difference between (2.21) and (2.22) is that in (2.22), $Z_{i,j}^T$ accounts for the load attached to antenna $i$. $Z_{i,j}^T$ according to (2.22) may be used with (2.17) and (2.19) to determine the coupled voltages at the terminals of transmit antennas in a MIMO system.

2.4.2. Receive Mutual Impedance

In the case of receive mutual coupling, the problem is a little more complicated. At the receiver, the excitation source comes from a wavefront in the farfield, rather than either of the considered antennas. Once this strikes an antenna, a current is induced. This current results in a small amount of radiation back into space and a large amount of energy transfer to the load. Some of the radiation back into space will undoubtedly be captured by the second antenna, resulting in a contribution to the voltage at its terminals and further radiation again back in to space. This is a different coupling path which results in a different mutual impedance, $Z^R$ when antennas are used as receivers rather than transmitters [54-58]. To account for this difference, Hui et al introduced the receive mutual impedance between antennas $i$ and $j$, $Z_{i,j}^R$ [55-58]. This is defined under ‘receive’ conditions, where excitation is from an external farfield source and both antennas are loaded with the receiver load impedance, $Z_L$.

$$Z_{i,j}^R = \frac{V_i^C}{I_j} \quad (2.23)$$
Here, \( I_j \) is the current induced on antenna \( j \) from the farfield source. This current causes some re-radiation which couples into antenna \( i \), producing the coupled voltage \( V_i^C \) across its load. Note that in (2.23), \( V_i^C \) is the voltage caused by the coupling only and does not include the voltage directly induced by the farfield source.

A measurement procedure for obtaining \( Z_{i,j}^R \) is given in [55]. It involves placing the receive antenna array in an anechoic chamber, with a transmitting antenna in the farfield. The receive antennas are both loaded with impedance \( Z_L \). The path gain between the transmit antenna and the terminals of receive antenna \( i \) is recorded as \( S_{21,i} \). The gain between the transmit antenna and the terminals of receive antenna \( j \) is recorded as \( S_{21,j} \). Finally, the isolated path gain between the transmit antenna and the terminals of receive antenna \( i \), with the second antenna removed is recorded as \( S'_{21,i} \).

From these, the receive mutual impedance between the two antennas, \( Z_{i,j}^R \) is calculated using,

\[
Z_{i,j}^R = \frac{(S'_{21,i} - S_{21,i})}{S_{21,j}} Z_L
\]  

(2.24)

2.4.3. Weighted Average Receive Mutual Impedance

When directional antennas are used at the receiver, the current distribution on the elements is dependent on the angle of arrival from the excitation source. Because the mutual coupling is a result of this current distribution, the receive mutual impedance becomes dependent upon the angular distribution of incident energy. To account for this, a novel weighted average approach to determining \( Z_{i,j}^R \) in an environment modelled using the ray launcher has been implemented. This consists of three steps:

1. \( Z_{i,j}^R(\theta, \phi) \) is determined vs angle of arrival (AoA). This involves simulating the measurement procedure above, with the farfield excitation source rotated around the receiver. For receivers with two antennas, \( Z_{1,2}^R \) and \( Z_{2,1}^R \) are found this way at \( \theta \) and \( \phi \) intervals of 5° for \( 0 \leq \theta < 180° \) and \( 0 \leq \phi < 360° \). For dual polar receivers, when determining \( Z_{i,j}^R(\theta, \phi) \), a far-field source is used which is co-polar to antenna \( j \). This is because energy which is co-polar to antenna \( j \) causes the greatest current distribution on antenna \( j \) and hence is most responsible for the mutual impedance, \( Z_{i,j}^R \). As an example, Figure 2-7 shows the magnitude of the
simulated $Z_{2,1}^R$ vs AoA for two co-polar coated dipoles separated by $0.5\lambda$, positioned above a metal case (as described in Section 2.7.2.2.). As can be seen, the receive mutual impedance varies between around 6 and 15 Ω dependent upon AoA.

Figure 2-7 – Magnitude of receive mutual coupling, $Z_{12}^R$ vs AoA for coated dipoles separated by $0.5\lambda$ around metal case. Also marked are AoAs of rays containing 80% of power received by antenna 2 in a NLOS office environment (see Section 5.2.)
2. The ray launcher is used to obtain the AoAs and associated power weighting of all energy incident on antenna \( j \). (It is the current induced on the “coupler” antenna, \( j \) which results in the mutual coupling to antenna \( i \), according to \( Z_{i,j}^R \).) The AoAs are the arrival angles of the \( N_r \) received rays incident on the coupler antenna, while the power weightings are the power transferred by each ray, normalised against the total power received by the coupler.

In the example of Figure 2-7, the AoAs of the strongest rays containing 80% of received power at antenna 2 are marked with a red ‘x’. The environment was the indoor NLOS office environment described in Section 5.2, at a distance of one metre along the measurement route.

3. \( Z_{i,j}^R(\theta, \phi) \) is interpolated at the AoAs determined by the ray launcher and weighted by the power weighting. This gives the weighted average receive mutual coupling, \( \overline{Z_{i,j}^R} \) as follows, where \( Z_{i,j}^R(\theta_k, \phi_k) \) is the receive mutual impedance interpolated at the AoA of the \( k \)th ray and \( \hat{P}_k \) is the normalised power transferred by the \( k \)th ray.

\[
\overline{Z_{i,j}^R} = \sum_{k=1}^{N_{suc}} Z_{i,j}^R(\theta_k, \phi_k) \cdot \hat{P}_k \tag{2.25}
\]

Figure 2-8 demonstrates the small change in receive mutual impedance due to the change in received AoAs as the example receiver moves along a measurement route in the simulated NLOS environment.

![Weighted Average Receive Mutual Impedance](image)

Figure 2-8 – Real (Re) and imaginary (Im) parts of the weighted average receive mutual impedance vs distance in NLOS office environment
When considering polarisation MIMO systems with coated antennas there are some further considerations which must be taken into account during the determination of the receive mutual impedance:

- Considered antennas are orthogonally polarised. This raises the question of which polarisation should the farfield source have when determining $Z_{i,j}^R$ vs AoA? In this work, the farfield source is always co-polar to antenna $j$. This is the antenna which the energy is coupling from when considering $Z_{i,j}^R$.

- Some of the dual polar antennas considered in this work, such as the linear crossed dipoles, are co-located and share the same dielectric coat. When recording the isolated $S_{2,1}$ between one element of this antenna and the farfield source, it is therefore not clear how to ‘remove’ one element. In this work, the element is removed and replaced by the medium surrounding it. This means that for LP crossed dipoles in air, the element is simply removed, while for LP crossed dipoles with a dielectric coat, the element is removed and its volume replace by the dielectric.

- Finally, in the case of crossed dipoles used for CP, the RHC antenna and the LHC antenna both use both of the crossed dipoles and share parts of the feed network. This again leaves confusion over how to record the isolated path gain to one antenna. In this work, as no components can be removed to obtain the isolated result, one load is simply removed from the feed network when modelling the isolated path gain to the other.

The weighted average receive mutual impedance, $\overline{Z}_{i,j}^R$ found in this way may be used in (2.20) and (2.18) to determine the coupled voltages at the terminals of receive antennas in a MIMO system.

### 2.5. Compact Antennas

This work focuses on MIMO systems using highly compact antennas and platforms. In 1947, Wheeler [59] identified that antennas with dimensions much smaller than the wavelength they operate at are subject to further limitations than larger antennas. He suggested that an electrically small antenna is one whose maximum dimension is less than one radianlength, where a radianlength is defined as $\lambda/2\pi$. He also made the important observation that as antenna size is decreased, radiation resistance decreases relative to the
ohmic resistance of an antenna plus matching network. This results in lower efficiency. He did however, demonstrate that an efficient match was still achievable for small antennas, at the expense of bandwidth. This trade-off between efficiency, bandwidth and size is the basis of the fundamental limits on small antennas. The basics of antenna miniaturisation is covered in many antenna textbooks [64, 65, 69] and discussed in further detail in references such as [52, 59, 66, 67, 72, 81, 82].

2.5.1. Fundamental Limits

2.5.1.1. Q Factor

Q factor is a property of a resonating structure. It gives an indication of the ratio of energy stored to energy lost through radiation. For a resonator, a high value is desirable as this means most of the energy is stored and alternates between electric and magnetic fields. When considering an antenna, a low value is desired as this means that little energy remains stored in the near fields and most of the energy is radiated. A general and exact expression for Q factor in terms of antenna resistance, $R$ and the gradient of reactance $X$ versus frequency, $f$ is as follows [83, 84].

$$Q = \frac{f \partial X}{2R \partial f}$$  \hspace{1cm} (2.26)

This shows that $Q$ is a frequency dependent parameter. At resonant angular frequency $\omega_0$ of an unloaded antenna, $Q$ relates an antennas total average stored energy, $W$ to the total power accepted by the antenna, $P_{in}$ as follows, where $W$ consists of the time averaged stored electric and magnetic energy [52, 72].

$$Q(\omega_0) = \omega_0 \frac{|W(\omega_0)|}{P_{in}(\omega_0)}$$  \hspace{1cm} (2.27)

Although the exact relation between $Q$ and bandwidth is not rigidly defined, with different definitions given to suit different situations, it is widely accepted that they are inversely proportional to one another. If the Q factor is high, as is generally the case with small antennas, then the half power bandwidth, $\Delta f_{3dB}$ of a lossless matched antenna is twice the inverse of the Q factor, as in [52, 83],

$$\frac{\Delta f_{3dB}}{f_0} = \frac{2}{Q}$$  \hspace{1cm} (2.28)
Q factor may also be related to the bandwidth, $\Delta f$ around resonant frequency, $f_0$ over which VSWR is below a given level as follows [60].

$$ Q = \frac{f_0}{\Delta f} \frac{VSWR - 1}{\sqrt{VSWR}} $$  \hspace{1cm} (2.29)

2.5.1.2. Chu's Limit on Q Factor

Due to the relation between bandwidth and Q factor, finding the upper bound for bandwidth of an antenna necessitates finding the lower bound of $Q$. To do this, referring back to (2.27), it is necessary to have knowledge of the stored energy and the radiated energy from the antenna. Chu (1948) in [53], does this by enclosing the antenna in the smallest sphere of radius, $a$ which will accommodate it. It is assumed that the reactive near field is zero within this sphere. While this is not always true, the assumption will only lead to an underestimation of the minimum possible $Q$ and so is considered acceptable. Chu then calculates the impedance of all of the spherical waves which leave the surface of the sphere and then estimates the reactive field and the radiated fields using equivalent lumped element circuit theory. According to this method, the minimum possible $Q$ for a small electric dipole is as follows, where $\beta_0 = 2\pi/\lambda$ is the propagation constant in free space.

$$ Q > \frac{1}{\beta_0 a} + \left( \frac{1}{\beta_0 a} \right)^3 $$  \hspace{1cm} (2.30)

Using this bound and the relationship between $Q$ and bandwidth in (2.28) or (2.29), the theoretical maximum possible bandwidth, given a specified maximum allowable VSWR may be calculated for a lossless electrically small antenna, based on its size. It should be noted that the sphere of radius $a$ should contain the antenna as well as any other components which can be used to contribute to resonance, for example the ground plane.

Chu’s value is an absolute limit and practical antennas do not achieve this. More recently, further work has been carried out to try to refine the limit. Thal (2006) [85] took in to account the fields inside the sphere containing the antenna, which had previously been assumed to be zero. He concluded that Chu’s limit should be increased by a factor of 1.5 and 3 for antennas radiating the TM and TE modes, respectively. Since then, Best (2009) [86] has in fact presented an electric dipole antenna, radiating in the TM mode and a
magnetic dipole antenna, radiating in the TE mode which achieve Q factors of approximately 1.53 and 3.18 times Chu’s limit. These values are very close to the stricter lower bounds given by Thal.

2.5.1.3. Efficiency

The limits above assume a lossless antenna and result in narrow limits on bandwidth. In the case of small antennas, ohmic losses are usually high and therefore efficiency is low [66]. When considering lossy antennas, Chu’s limit must be multiplied by the radiation efficiency, $\eta$, as in the example below [60].

$$Q > \eta \left( \frac{1}{\beta_0 a} + \left( \frac{1}{\beta_0 a} \right)^3 \right)$$  \hspace{1cm} (2.31)

This means that antennas operating at lower efficiencies can in fact achieve a wider impedance bandwidth, demonstrating the trade off in small antenna design between size, bandwidth and efficiency.

2.5.2. Dielectric loading

The resonant length of an antenna is related to the wavelength within the medium which surrounds it. By enclosing an antenna in material of high permittivity or permeability it will behave as though it is electrically larger than the same antenna in air. If an antenna of length, $L_0$ in air, is surrounded by an infinite coat of dielectric with relative permittivity, $\epsilon_r$ and relative permeability, $\mu_r$ then to maintain the same electrical length, its new length in the dielectric, $L$ is as follows [52].

$$L = \frac{L_0}{\sqrt{\epsilon_r \mu_r}}$$  \hspace{1cm} (2.32)

Of course an infinite coat of dielectric is an unreal scenario, and so $L$ will in actual fact take a value between $L_0$ and that given in (2.32), depending on the size and shape of the dielectric medium.

This sort of loading, in general increases the energy stored in the nearfields and hence raises $Q$ and decreases bandwidth. Furthermore, a high permittivity often results in dielectric losses, which decrease efficiency and gain. Materials of high permeability tend to have even higher losses and for this reason are rarely used. Dielectric loading is the main miniaturisation technique used in this work. In Sections 2.7.2 and 2.7.3, a ceramic
coat of high dielectric constant is used to reduce the resonant length of dipole antennas and co-located dual-polar crossed dipole antennas at 2.4 GHz.

2.6. CST Microwave Studio Simulation of Orthogonally Polarised Compact Antennas

The antenna and platform models were all designed and simulated using Computer Simulation Technology - Microwave Studio (CST-MWS). The transient solver was used to obtain the (conventional) impedance characteristics, S-parameters and the farfield radiation patterns which were utilised in the ray launching propagation models.

The transient solver uses a finite integration technique in the time domain. This involves discretising the calculation domain into discrete locations and calculating the fields in these locations at discrete time samples. The fields are calculated vs time using an integral form of Maxwell’s equations [70], applied to the grid space [87], which was a hexahedral mesh. The Discrete Fourier transform (DFT) is used to extract broadband frequency domain results from only one solver run. Perfect Boundary Approximation [88] increases accuracy by allowing curves to be accurately represented within a coarse mesh. The solver uses an automatic 3D mesh generation algorithm with expert based adaptive mesh refinement. All models were simulated using the expert based adaptive mesh refinement until their S-parameters converged. To instil further confidence in the results, they were then also compared to results of the same models obtained using the Integral Equation (IE) solver.

The IE solver is a frequency domain solver which is in fact best suited to electrically large structures. It uses a surface mesh, where only surface boundaries are discretised. This leads to a linear equation system with far less unknowns than a volume approach. The equations are solved using Method of Moments discretisation and the Multilevel Fast Multi-pole Method, as described in [89].

When determining the receive mutual impedance, as described in Section 2.4.2, it is necessary to know the voltages and currents at the antenna terminals, when excited by a farfield source. The transient solver is not suited to large problems like this as it requires calculation of the fields within every mesh-cell in the calculation volume. This would require extremely long calculation times and result in low accuracy. Instead, the IE solver was used when farfield excitation was required.
Figure 2-9 shows a cross section of a coated crossed dipole antenna, which has been divided into hexahedral mesh cells for the transient solver. As can be seen, the surrounding volume as well as the structure is meshed and when the simulation is performed, the fields in all mesh cells are calculated. Figure 2-10 shows the same antenna with a surface mesh for the IE solver. It should be noted that for the purpose of illustration the mesh density in both of these figures is significantly reduced compared to the density used for simulation.

2.7. Antenna Models on Small Platforms

The remainder of this chapter serves to introduce the antenna models used throughout the core research. To begin with, a brief overview of the uncoated broadband antennas is given in Section 2.7.1. These have a bandwidth of 400MHz and are used as a reference to compare the coated antennas to. Section 2.7.2 introduces the narrowband coated antennas used for spatial diversity systems, while Section 2.7.3 introduces the narrowband coated antennas used for the polarisation diversity systems. The bandwidth of the coated antennas is 20 MHz. In Sections 2.7.2 and 2.7.3, the antennas are first discussed on their own, without the presence of a case, before their behaviour when positioned around a small metal case is examined. To distinguish between the models with and without the case, the antennas modelled on their own, without the presence of the metal case are referred to as
“without case”, while the antennas modelled around the case are referred to as models “with case”. Each model has two antenna elements, which may be used for transmitting or receiving in a $2 \times 2$ MIMO system, at 2.4 GHz.

2.7.1. Uncoated Broadband Antennas

To give a reference with which to compare the compact narrowband antenna systems, a number of uncoated, broadband antennas have been modelled. These consist of regular half-wavelength dipoles, used for spatial diversity, and co-located crossed dipoles, used for polarisation diversity. For spatial diversity, the antennas are both vertically polarised, separated by 0.25, 0.5 and 1 wavelengths. Their full length is 56.6 mm, with a feed gap of 1.5 mm and the cross section of the elements is $1 \text{ mm} \times 1 \text{ mm}$ square.

The polarisation systems use similar dipoles, re-tuned to a full length of 57.13 mm to maintain resonance at 2.4 GHz. One is vertically polarised while the other is horizontal, with co-located centres. The antennas for the spatial diversity systems and the LP system are fed using CST discrete ports which are impedance matched to the antennas. These antennas all achieve a bandwidth of 2.2 to 2.6 GHz, over which $S_{1,1} < -9 \text{ dB}$.

The CP systems require a feed network so that the horizontal elements are fed with a $\pm 90^\circ$ phase difference relative to the vertical element. The sign of the phase difference determines whether the antenna is RHC or LHC polarised. A schematic of the feed network, which is also used for the compact CP crossed dipole antenna, is shown in Figure 2-11. When the RHC feed is excited (port 1), its power is split evenly by the hybrid coupler. Half the power is phase shifted by $-90^\circ$ and delivered via the top 3 dB splitter to the terminals of the horizontal antenna. The other half is not phase shifted and is delivered via the bottom 3 dB splitter to the terminals of the vertical antenna element. This results in signals of equal amplitude at both antenna elements, with the horizontal element lagging the vertical by $90^\circ$. When the horizontal and vertical elements are aligned with the $X$ and $Z$ axes respectively of a Cartesian coordinate system, the fields combine to create RHC polarisation in the $+Y$ direction. The 3 dB splitters are used to combine the signals with those from the LHC feed (port 2), which are treated in a similar way, but with the phase delay on the vertical rather than horizontal element, to create LHC polarisation in the $+Y$ direction. The remaining ports of the hybrid couplers are connected to theoretical perfect absorbers, which absorb some of the energy which is reflected at the antenna element.
terminals as well as that which may have coupled from one polarisation to the other. All components of the feed network are impedance matched at 2.4 GHz to the impedance of the antenna elements.

The network makes use of theoretical lossless components and perfect absorbers. In reality, these ‘ideal’ components cannot be realised. In this work however, the coupling caused by the feed network is of far less interest than that caused by the fields around the antennas and the environment, so the ideal components have been used. An alternate feed network which applies a 90° phase shift to one antenna element is used in [90] and [91], which consists of only a single hybrid coupler. The RHC feed is connected to port 1 and the LHC feed connected to port 3 of a hybrid coupler, similar to those in Figure 2-11. Port 2 and port 4 are then connected to the terminals of the horizontal and vertical elements respectively of the crossed dipoles. This approach reduces complexity but was found to result in higher coupling between the signals at the RHC and LHC feeds, so was not favoured in this work.

**Figure 2-11 – Feed network for dual polar crossed-dipole CP antennas**

**Table 2-1 – Simulated uncoated broadband antenna properties at 2.4 GHz**

<table>
<thead>
<tr>
<th>Diversity</th>
<th>Spatial</th>
<th>Polarisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarisations</td>
<td>Co-polar, V</td>
<td>H/V</td>
</tr>
<tr>
<td>Antenna Separation (λ)</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>Impedance, $Z_{1,1} = Z_{2,2}$ (Ω)</td>
<td>66.2+j4.0</td>
<td>73.2+j6</td>
</tr>
<tr>
<td>$S_{2,1}$ (dB)</td>
<td>-9.0</td>
<td>-13.2</td>
</tr>
<tr>
<td>$\rho_{env}$</td>
<td>0.09</td>
<td>8.0e-7</td>
</tr>
</tbody>
</table>
Table 2-1 shows some simulated properties of the uncoated broadband antenna models at 2.4 GHz. Firstly, antenna impedance is shown. As can be seen, this changes with the presence and proximity of another antenna. The impedance of the dipole used for the spatial systems in free-space is \(68.3 + j1.32\ \Omega\), while the impedance of a single dipole as used in the polarisation systems is \(74.8 + j3.1\ \Omega\).

Next, an indication of the coupling between antennas is given, through the \(S_{2,1}\) scattering parameter, when the antenna ports are loaded with a resistive load, \(Z_L\) which matches the real part of the antenna impedances, as in \(Z_L = Re[Z_{1,1}]\). As can be seen, for the co-polar spatial systems, \(S_{2,1}\) decreases with separation, but the orthogonally positioned elements of the co-located polarisation systems achieve a far lower \(S_{2,1}\).

Also shown is the farfield pattern envelope correlation coefficient, \(\rho_{env}\), calculated using (2.9). This is very low throughout, which suggests that the antennas may perform well in MIMO systems, given the correct environment. \(\rho_{env}\) is highest for the spatial system with the smallest separation and lowest for the orthogonally polarised antennas.

![Figure 2-12](image1.png)  
**Figure 2-12** – Farfield directivity for uncoated broadband co-polar dipoles, separated by 0.25\(\lambda\)  

![Figure 2-13](image2.png)  
**Figure 2-13** – Farfield directivity for uncoated broadband co-polar dipoles, separated by \(\lambda\)

Figure 2-12 and Figure 2-13 give an idea of the effect the mutual coupling has on the antenna patterns. They both show the directivity in the azimuth plane of one antenna in a spatial diversity system. In Figure 2-12, the other antenna is positioned 0.25\(\lambda\) away, in the
\( \phi = 0 \) direction, while in Figure 2-13, the other antenna is positioned \( \lambda \) away, in the \( \phi = 0 \) direction. In both cases, the antenna patterns of the other antennas are symmetrical copies of those in the figures, due to the symmetry of the models. The proximity of the antennas results in greater directivity. This usually results in a decrease in total received power, but can also increase the pattern diversity, which could improve MIMO performance. It is also stated in [92] that it may in fact result in an increase in total received power, if the following conditions are met:

i. Element spacing is between 0.4\( \lambda \) and 0.9\( \lambda \)

ii. The environment is one of directional scattering conditions

iii. The antenna array is orientated orthogonally to the main direction of arrival.

Figure 2-14 shows the directivity in the azimuth plane of one antenna positioned with 0.5\( \lambda \) separation from another. The other antenna is located in the direction of \( \phi = 0 \). The directivity in the direction orthogonal to the array orientation (at \( \phi = 90,270 \)) is 3.1 dBi. In comparison, the directivity on the azimuth plane of a similar antenna in isolation is approximately 2.15 dBi. This demonstrates how such separation may result in an increase in total transferred power due to mutual coupling, when the transmit and receive arrays are both orthogonal to the direction of the direct path between them.

![Figure 2-14](image)

**Figure 2-14** – Farfield directivity for uncoated broadband co-polar dipoles, separated by 0.5\( \lambda \)

As the antenna separation is increased, the pattern tends to that of a dipole in free-space, which is omni-directional in the azimuth plane.
The antenna patterns for the orthogonal crossed dipoles, used for the LP system are unchanged from the free-space dipole patterns, while the pattern of the crossed dipole CP system is shown in Figure 2-15. This shows the absolute directivity in the azimuth plane from the excitation of one polarisation. The polarisation is CP, with opposite sense for $0 < \phi < 180^\circ$ compared to $180^\circ < \phi < 360^\circ$. The rotation senses are determined by which port of the feed network is excited.

![Farfield Directivity Abs (Theta=90)](image)

**Figure 2-15 – Directivity of CP crossed dipoles in the azimuth plane**

Figure 2-16 and Figure 2-17 show the farfield polarisation orthogonality for the crossed dipoles used for LP and CP respectively. The dipoles are positioned on the $X$ and $Z$ axes, giving orthogonal fields in the $\pm Y$ direction. In both cases, orthogonality tends to zero on the $XZ$ plane, although it appears that for LP, the orthogonality extends a little further. The co-polar dipoles for spatial diversity radiate fields with effectively zero orthogonality.
2.7.2. Compact Narrowband Spatial Diversity Antennas

The compact antennas used for spatial diversity consist of small dipoles, surrounded by a spherical coat of high relative permittivity, $\varepsilon_r = 80$. These have been modelled in copolar pairs, without a case present (without case), as well as in position at the top of a metal case (with case). The models without the case are a useful reference from which to examine the effects of the case, on both the antenna properties, and in Chapter 5, on MIMO performance. Figure 2-18 shows a diagram of this type of antenna.
2.7.2.1. Without Case

In the models without the case, the dipoles are of full length, \( L = 4.22 \) mm, with a square cross section of width, \( W = 0.35 \) mm. They are fed using discrete ports across a feed gap of 0.35 mm. The dielectric coat has radius, \( R_{coat} = 9.54 \) mm. They have been modelled in co-polar pairs, with separation of 0.25\( \lambda \), 0.5\( \lambda \) and \( \lambda \) between antenna centres. The maximum possible bandwidth for a return loss below -10 dB according to Chu’s limit (using (2.6), (2.29) and (2.30)), for an antenna of this size is 143.5 MHz. The simulated antenna in isolation in fact has a bandwidth of only 25.5 MHz around 2.4 GHz, over which the return loss is below -10 dB.

Table 2-2 – Simulated narrowband coated dipole antenna properties without case at 2.4 GHz

| Diversity Polarisation | Spatial Impedance, \( Z_{1,1} = Z_{2,2} \) (\( \Omega \)) | \( S_{2,1} \) (dB) | \( \rho_{env} \) | LOS Receive Mutual Impedance, \( |Z_{2,1}^R| = |Z_{1,2}^R| \) (\( \Omega \)) | NLOS Receive Mutual Impedance, \( |Z_{2,1}^R| = |Z_{1,2}^R| \) (\( \Omega \)) |
|------------------------|-------------------------------|---------------|-------------|---------------------------------|---------------------------------|
| Antenna Separation (\( \lambda \)) | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 | 0.25 | 0.5 | 1 |
| Impedance, \( Z_{1,1} = Z_{2,2} \) (\( \Omega \)) | 71.7+13.7 | 73.6+5.9 | 76.5-3.8 | -22.1 | -27.4 | -33.1 | 7.22 | 4.18 | 2.35 | 8.03 | 4.54 | 2.45
| \( S_{2,1} \) (dB) | -22.1 | -27.4 | -33.1 | 7.22 | 4.18 | 2.35 | 8.03 | 4.54 | 2.45
| \( \rho_{env} \) | 0.19 | 0.033 | 0.0038 | 7.22 | 4.18 | 2.35 | 8.03 | 4.54 | 2.45
| LOS Receive Mutual Impedance, \( |Z_{2,1}^R| = |Z_{1,2}^R| \) (\( \Omega \)) | 7.22 | 4.18 | 2.35 | 8.03 | 4.54 | 2.45
| NLOS Receive Mutual Impedance, \( |Z_{2,1}^R| = |Z_{1,2}^R| \) (\( \Omega \)) | 8.03 | 4.54 | 2.45
Table 2-2 shows some of the simulated antenna properties for the narrowband coated dipoles without the case. As can be seen, impedance is again dependent upon the proximity of the antennas. Coupling effects are greatly reduced when compared to the uncoated dipoles at the same separation. This can be seen by the $S_{2,1}$ decrease of over 13 dB at separation $= 0.25\lambda$ and even more at greater separation. It is also evident from the antenna pattern cuts in Figure 2-19 at $0.25\lambda$ and Figure 2-20 at $\lambda$ separation. These are much closer than the uncoated equivalent patterns (Figure 2-12 and Figure 2-13) to the omni-directional patterns in azimuth that these antennas have when simulated in isolation. This shows that the dielectric coat has the effect of reducing the mutual coupling effects caused by the interaction of the antennas’ near-fields. Interestingly, because the antenna patterns are closer to the omni-directional pattern, they exhibit a lower degree of pattern diversity, resulting in higher $\rho_{env}$ than when the antennas are uncoated. This is particularly apparent at $0.25\lambda$ separation, where $\rho_{env}$ is 0.09 when the antennas are uncoated, but 0.19 when they are coated. This is a very good example of a situation where a decrease in mutual coupling does not necessarily mean greater MIMO capabilities. The polarisation from both of the antennas is linear in all cases, resulting in zero orthogonality in all directions. The patterns do not change significantly over the band 2.39 to 2.41 GHz.
Figure 2-19 – Farfield directivity for narrowband coated co-polar dipoles, separated by $0.25\lambda$

Figure 2-20 – Farfield directivity for narrowband coated co-polar dipoles, separated by $\lambda$

Also, included in Table 2-2 is the receive mutual impedance of the antenna in a LOS and NLOS environment. This is the weighted average receive mutual impedance, calculated as described in Section 2.4.3. It was calculated along the two metre simulation routes described for the LOS and NLOS environments described in Section 5.2. It was found that the receive mutual impedance changes with the difference in incident energy in the two locations, as reflected in the table. The change over the length of each individual route is negligible, so the values in the table are the averages over each route.

2.7.2.2. With Case

Figure 2-21 shows an example of the narrowband coated dipoles positioned around the case. The case is modelled as a perfect electric conductor\(^2\) (PEC), to represent the ‘worst case’ in terms of anticipated interaction with the antennas. Its dimensions are $120 \times 60 \times 10$ mm, approximately resembling the size and form of a compact handheld device. The antennas are positioned with a clearance of 5 mm above the top face of the case. Their centres are aligned with the centre of the case in the $Y$ direction. Antenna 1 is positioned with its centre also aligned with the edge of the case, and antenna separation is measured

\(^2\) A PEC is a theoretical material with zero electrical resistivity (infinite conductivity). It is used to represent metal of finite but negligible resistivity to reduce simulation time.
from its centre, to the centre of antenna 2, in the \((X)\) direction extending over the width of the case. This model has been simulated with antenna separation of \(0.3\lambda\) and \(0.5\lambda\). In both cases, the antenna coating has been resized to maintain resonance at 2.4 GHz. At \(0.5\lambda\), \(R_{\text{coat}} = 9.5267\) mm, while at \(0.3\lambda\), \(R_{\text{coat}} = 9.4440\) mm. At \(0.3\lambda\), the model is no longer symmetrical, meaning that the impedances of the two antennas with the same dimensions are no longer equal. In this case, the size of both antennas is chosen as a compromise which achieves resonance close to 2.4 GHz for both antennas. Similarly, the load impedance used is the same for both antennas, chosen as the mean of the real parts of the two antenna impedances, which was 60.1 \(\Omega\). In all cases, the bandwidth in terms of \(S_{1,1} < -10\) dB exceeds 2.39 to 2.41 GHz.

Figure 2-21 – Spatial diversity antennas with PEC case

A parameter sweep of antenna separation was performed when deciding the two values to model. \(S_{2,1}\) and \(\rho_{\text{env}}\) were recorded in increments of \(0.05\lambda\) between \(0.2\lambda\) and \(0.5\lambda\). Figure 2-22 shows \(\rho_{\text{env}}\), where a minimum of approximately zero is observed at \(0.3\lambda\) separation, before an increase as separation is increased beyond \(0.3\lambda\). Figure 2-23 shows the variation of \(S_{2,1}\) with separation. This decreases by 4 dB between \(0.2\lambda\) and \(0.3\lambda\).
where it approximately levels off at around -20.5 dB. These two results suggest that from at antenna point of view at least, separation of 0.3\(\lambda\) should give optimum MIMO performance, so this situation was chosen to compare to the 0.5\(\lambda\) case. These results of course, do not account for the fact that incident energy at a receiver may well be better decorrelated by the environment at two positions in space with greater separation. A further parameter sweep was performed to determine the optimum position for the antennas separated by 0.3\(\lambda\), along the top of the case. It was in fact found that this position made little difference to \(\rho_{env}\) and \(S_{2,1}\), with \(\rho_{env}\) only varying by < 0.02 and \(S_{2,1}\) varying by 0.7 dB at a level of -20 dB. Both were at a minimum with the outermost antenna aligned with the edge of the case (as in Figure 2-21).

![Envelope Correlation Coefficient](image1.png)  
![\(S_{2,1}\)](image2.png)

**Figure 2-22** – \(\rho_{env}\) vs separation of coated narrowband dipoles above PEC case  
**Figure 2-23** – \(S_{2,1}\) vs separation of coated narrowband dipoles above PEC case
Table 2-3 shows the simulated impedance, $S_{2,1}$, $\rho_{env}$ and receive mutual impedance for these antennas. The presence of the case in the antennas near-field region has increased the mutual coupling between the antennas, as shown by $S_{2,1}$ which, at $0.5\lambda$ separation has increased from -27.4 to -20.8 dB with the addition of the case and at $0.3\lambda$, is higher with the case than at $0.25\lambda$ without it.

When antenna separation is $0.3\lambda$, it was observed that the receive mutual impedance, $Z_{1,2}$ is slightly different to $Z_{2,1}$, as reflected in the table. It is likely that this is predominantly due to the lack of symmetry in the model causing the antenna patterns to be different. This results in a different angular distribution of received power by each antenna, which is used to weight the mutual impedance versus AoA result.
Table 2-3 – Simulated narrowband coated dipole antenna properties with case at 2.4 GHz

<table>
<thead>
<tr>
<th>Diversity</th>
<th>Spatial</th>
<th>Co-polar, V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarisation</td>
<td>Antenna Separation ((\lambda))</td>
<td>0.3</td>
</tr>
<tr>
<td>Antenna 1 Impedance, (Z_{1,1}) ((\Omega))</td>
<td>56.7+j5.0</td>
<td>64.6-j0.3</td>
</tr>
<tr>
<td>Antenna 2 Impedance, (Z_{2,2}) ((\Omega))</td>
<td>63.5-j6.2</td>
<td>64.6-j0.3</td>
</tr>
<tr>
<td>(S_{2,1}) (dB)</td>
<td>-20.6</td>
<td>-20.8</td>
</tr>
<tr>
<td>(\rho_{env})</td>
<td>0.0007</td>
<td>0.127</td>
</tr>
<tr>
<td>LOS Receive Mutual Impedance, (</td>
<td>Z_{1,2}^R</td>
<td>) ((\Omega))</td>
</tr>
<tr>
<td>LOS Receive Mutual Impedance, (</td>
<td>Z_{2,1}^R</td>
<td>) ((\Omega))</td>
</tr>
<tr>
<td>NLOS Receive Mutual Impedance, (</td>
<td>Z_{1,2}^R</td>
<td>) ((\Omega))</td>
</tr>
<tr>
<td>NLOS Receive Mutual Impedance, (</td>
<td>Z_{2,1}^R</td>
<td>) ((\Omega))</td>
</tr>
</tbody>
</table>

The differences in the antenna patterns at 0.3\(\lambda\) are demonstrated in Figure 2-24, which shows the directivity of antenna 1, and Figure 2-25, which shows the directivity of antenna 2. These are both cuts in the XZ plane, which is the plane with which the case is aligned. To aid with visualisation of these patterns, in both diagrams, the null is pointed roughly towards the centre of the antenna case and the antennas are positioned in the polar direction. As can be seen, there is a significant difference in the patterns which is caused by the presence of the case and the antenna positions relative to it. Figure 2-26 shows the directivity of antenna 1 of the model with 0.5\(\lambda\) separation. The similarity between this and Figure 2-24, where antenna 2 was closer, suggests that the distortion of the antenna patterns is more a result of the proximity of the case than the other antenna. This may be expected, considering the comparative sizes of the case and the antennas, as well as the material of the case and its proximity to the antennas. The directivity of antenna 2 at 0.5\(\lambda\) separation is a symmetrical copy of Figure 2-26, reflected in the polar direction.
Figure 2-24 – Directivity of coated antenna 1 with case at $0.3\lambda$ separation. Cut in plane aligned with case

Figure 2-25 – Directivity of coated antenna 2 with case at $0.3\lambda$ separation. Cut in plane aligned with case

Figure 2-26 – Directivity of coated antenna 1 with case at $0.5\lambda$ separation. Cut in plane aligned with case

Figure 2-27 shows the orthogonality of the farfields radiated by the antennas separated by $0.3\lambda$. Most remarkably, these antennas which are both orientated in the same direction, now radiate orthogonally polarised fields over a considerable angle around the $\pm Y$ directions (in front and behind the model). This is because of the near-field interactions with the case, and also happens at $0.5\lambda$ separation, although over a slightly narrower solid
angle. Upon inspection of the polarisations in this region from both antennas on the Poincaré sphere, it becomes apparent that they are in fact radiating elliptical polarisation with opposite rotation sense, approximately orthogonal tilt angle and similar axial ratios. Figure 2-28 shows the polarisation of the fields radiated by the $0.3\lambda$ model in the $\theta = 100, \phi = 90$ direction (just below the $Y$ axis). As can be seen, these are elliptical with fairly wide axial ratio, rotating with opposite sense and approximately $90^\circ$ between their tilt angles. Orthogonality here is 0.9991. This means that despite the fact that these antennas appear to be co-polar, they could exhibit some polarisation diversity (as well as pattern diversity) when used in a MIMO system, due to the distortion of the radiation patterns caused by the presence of the case.

![Figure 2-27 – Farfield orthogonality of co-polar narrowband coated dipoles separated by $0.3\lambda$, with case](image1)

![Figure 2-28 – Poincaré representation of polarisation at $\theta = 100, \phi = 90$ from narrowband coated co-polar dipoles separated by $0.3\lambda$, with case](image2)

### 2.7.3. Compact Narrowband Polarisation Diversity Antennas

The compact antennas used for polarisation diversity consist of two crossed dipoles with co-located centres (as with the uncoated models), which share a spherical coat of $\epsilon_r = 80$. The two dipoles are fed individually for the LP system, or using a feed network similar to that used by the uncoated CP antenna (Figure 2-11), for the CP system. With no case present, a co-located LP and CP model has been created and evaluated, as well as a hybrid system, where two coated crossed dipole antennas are positioned with $0.25\lambda$ separation. These are fed using hybrid couplers to radiate orthogonal circular polarisation. The hybrid model is used to provide comparisons between a model which utilises both spatial and
polarisation diversity to those which utilise only one or the other. The co-located LP and CP antennas have also been modelled with the case.

2.7.3.1. Without Case

The models with no case use dipoles of the same length and width as the compact spatial diversity systems, but with the feed gap increased to 0.45 mm, so that the crossed elements do not intersect each other. The coat radius is also changed to, $R_{coat} = 9.475$ mm, to maintain resonance at 2.4 GHz.

When used for the co-located LP system, the elements are fed using discrete ports with load resistance equal to the real part of their impedance. For the co-located CP system, a feed network the same as used by the uncoated CP dipoles (Figure 2-11) is used to provide the necessary phase shifted signals at the element terminals. The components of this network are all matched to the real part of the impedance of the antenna elements at 2.4 GHz. For the polarisation-spatial hybrid system, the elements of both crossed dipole antennas are fed using hybrid couplers, as in Figure 2-29.

![Diagram of antenna feed networks for hybrid polarisation-spatial diversity system](image)

Table 2-4 shows the simulated properties of these antenna systems. The LP system achieves very low coupling, as shown by its $S_{2,1}$ result and its receive mutual impedance. The orthogonality of the model also results in very low $\rho_{env}$. These results do not change significantly over the 2.39 to 2.41 GHz band.
The coupling of the co-located CP model is slightly higher. This coupling is mainly a result of imperfect match between the feed network and the antenna element terminals. This means that although $S_{2,1}$ is approximately -50 dB at 2.4 GHz (as in the table), at either extreme of the band, it increases to around -18 dB, due to the narrowband nature of the compact dipole elements. The receive mutual impedance for the CP systems was found to vary in a similar fashion over the band. The values given in the table are those at 2.4 GHz. However, to take account of this variance over the band, when performing the end-to-end MIMO system modelling using these antennas (as described in Chapter 5) the mean of $Z_{i,j}^R$ at 2.39 and 2.4 GHz was taken. For these antennas, this was found to be 6.5Ω in the LOS environment and 6.4Ω in NLOS.

The properties of the hybrid system do not vary significantly over the band. The coupling on this model is significantly higher than the other two. This is because, when the orthogonal antennas are no longer co-located, their electric fields are no longer perfectly orthogonal, meaning that the near-field of one polarisation interacts with the other, resulting in mutual coupling, as demonstrated by the $S_{2,1}$ of -23 dB and the higher receive mutual impedance values. It was found that in the modelled environments, $Z_{1,2}^R \neq Z_{2,1}^R$ for the hybrid system. This is due to the difference in angular distribution of the received power by each orthogonally polarised antenna, which is used to weight the AoA dependent receive mutual impedance. This effect is not noticeable for the co-located CP antennas because their coupling is dominated by reflections at the antenna element terminals.

<table>
<thead>
<tr>
<th>Diversity</th>
<th>Polarisation</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarity</td>
<td>H/V LP</td>
<td>RH/LH CP</td>
</tr>
<tr>
<td>Antenna Separation ($\lambda$)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Antenna 1 Impedance, $Z_{1,1} = Z_{2,2}$ ($\Omega$)</td>
<td>53.5-j0.3</td>
<td>54.0-j0.1</td>
</tr>
<tr>
<td>$S_{2,1}$ (dB)</td>
<td>-77.2</td>
<td>-49.9</td>
</tr>
<tr>
<td>$\rho_{env}$</td>
<td>8.2e-11</td>
<td>2.0e-7</td>
</tr>
<tr>
<td>LOS Receive Mutual Impedance, $</td>
<td>Z_{1,2}^R</td>
<td>$ ($\Omega$)</td>
</tr>
<tr>
<td>LOS Receive Mutual Impedance, $</td>
<td>Z_{2,1}^R</td>
<td>$ ($\Omega$)</td>
</tr>
<tr>
<td>NLOS Receive Mutual Impedance, $</td>
<td>Z_{1,2}^R</td>
<td>$ ($\Omega$)</td>
</tr>
<tr>
<td>NLOS Receive Mutual Impedance, $</td>
<td>Z_{2,1}^R</td>
<td>$ ($\Omega$)</td>
</tr>
</tbody>
</table>
Without the case, the radiation patterns in terms of directivity for the co-located systems are approximately the same as their full size counterparts. Each element of the LP system has a directivity pattern similar to that of a free-space dipole, and the directivity of the CP system is similar to that in Figure 2-15. Naturally, this leads to similar plots of orthogonality to the uncoated results in Figure 2-16 for LP and Figure 2-17 for CP. The radiation patterns of the antennas in the hybrid system are also very similar to the co-located crossed dipole CP patterns; the only difference being a slightly lower gain (<1 dB difference) in the direction of the other antenna. The orthogonality plot for this system is indistinguishable from that of the uncoated co-located antenna in Figure 2-17.

2.7.3.2. With Case
The co-located LP and CP antennas have been modelled in the presence of a PEC case, of similar dimensions to that used for the spatial systems. In both cases, a parameter sweep was performed, moving the co-located antennas along the top of the case to find the optimum positions in terms of $S_{2,1}$ and $\rho_{env}$. The clearance between the top of the case and the radius of the antenna coat was again fixed at 5 mm. For the LP antenna $\rho_{env}$ remained between 0 and 0.02 throughout the sweep, but $S_{2,1}$ found a minimum at -56 dB in the centre of the face, where it was 20 dB below that at the edges. Therefore this was the position chosen for the LP antenna. $S_{2,1}$ of the CP antenna on the other hand was not significantly affected by the position. This is because the coupling in this model is dominated by the effect of the reflections at the antenna element terminals in the feed network. A minimum of $\rho_{env} = 0.1$ was found at the edges of the face though, with maximums of 0.2 found around one-fifth and four-fifths of the way along the face. As such, the antenna was positioned at the edge of the face, with its centre aligned with the side of the case.

The addition of the case removes the symmetry of the model, meaning that now $Z_{1,1} \neq Z_{2,2}$ and both elements are resonant at slightly different frequencies. As with the spatial diversity systems, the dimensions of the antennas and the characteristic impedance of their feeds are chosen as a compromise between both elements. The dipole lengths are the same as without the case, but the radius of the cladding is changed to 9.449 mm. The port impedance and feed network characteristic impedance is set to 55Ω.
Table 2-5 – Simulated antenna properties of narrowband co-located crossed dipoles for LP and CP, with case

<table>
<thead>
<tr>
<th>Diversity</th>
<th>Polarisation</th>
<th>H/V LP</th>
<th>RH/LH CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarisation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antenna Separation ((\lambda))</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Antenna 1 Impedance, (Z_{1,1}) ((\Omega))</td>
<td>58.4-j0.5</td>
<td>51.8-j3.2</td>
<td></td>
</tr>
<tr>
<td>Antenna 2 Impedance, (Z_{2,2}) ((\Omega))</td>
<td>47.6-j5.3</td>
<td>56.6-j0.7</td>
<td></td>
</tr>
<tr>
<td>(S_{2,1}) (dB)</td>
<td>-55.8</td>
<td>-28.1</td>
<td></td>
</tr>
<tr>
<td>(\rho_{env})</td>
<td>3.2e-6</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>LOS Receive Mutual Impedance, (</td>
<td>Z_{1,2}^R</td>
<td>) ((\Omega))</td>
<td>0.30</td>
</tr>
<tr>
<td>LOS Receive Mutual Impedance, (</td>
<td>Z_{2,1}^R</td>
<td>) ((\Omega))</td>
<td>0.27</td>
</tr>
<tr>
<td>NLOS Receive Mutual Impedance, (</td>
<td>Z_{1,2}^R</td>
<td>) ((\Omega))</td>
<td>0.47</td>
</tr>
<tr>
<td>NLOS Receive Mutual Impedance, (</td>
<td>Z_{2,1}^R</td>
<td>) ((\Omega))</td>
<td>1.92</td>
</tr>
</tbody>
</table>

Table 2-5 shows the simulated antenna properties for the co-located LP and CP crossed dipoles with the case. The LP system maintains very low coupling, as shown by the \(S_{2,1}\) result and the receive mutual coupling. It also maintains a very low \(\rho_{env}\).

It can be seen that \(Z_{1,2}^R\) and \(Z_{2,1}^R\) in NLOS differ by almost 1.5\(\Omega\) in the presence of the case. The effect of mutual impedance at this magnitude is insignificant to MIMO performance, however an investigation into this seeming large difference was carried out nonetheless. Upon doing so, it was observed that in the NLOS environment, the rays which contribute to the received signal by the vertical antenna (antenna 2) are mainly contained in the azimuth plane. The receive mutual impedance, \(Z_{1,2}^R\) under excitation from a farfield source on the azimuth plane (calculated according to (2.24)) has a magnitude of approximately 0.5\(\Omega\), and does not vary significantly as the source is rotated around the array in this plane. As such, the weighted average of \(Z_{1,2}^R\) according to the arrival angles and contribution of the incident rays is approximately 0.5\(\Omega\). In comparison, the rays which contribute most to the received signal by the horizontal antenna (antenna 1) have a greater angular spread in the \(\theta\) and \(\phi\) directions. As such a substantial contribution to the received signal by this antenna is made by rays which arrive at angles from which \(Z_{2,1}^R\) (calculated according to (2.24)) is higher than 0.5\(\Omega\). It is the contribution to the weighted average of \(Z_{2,1}^R\) caused by these rays which causes it to be higher than the weighted average of \(Z_{2,1}^R\).

\(S_{2,1}\) of the CP antenna increases significantly with the addition of the case, from -49.9 dB to -28 dB. This is predominantly because the case changes the input impedance of the
crossed dipole elements of this antenna, increasing the mismatch loss at their terminals. Again, this causes $S_{2,1}$ to vary with frequency, due to the low bandwidth of the compact dipole elements. At 2.39 GHz it is -16 dB and at 2.41 GHz it is -18 dB. The means of $Z_{1,2}^R$ and $Z_{2,1}^R$ (which are approximately equal for the CP antenna) taken at 2.39 and 2.4 GHz are again used when modelling the MIMO performance of these antennas in Chapter 5. The mean is 8.0 Ω in both cases. $\rho_{env}$ also increases with the addition of the case to 0.11.

Figure 2-30 to Figure 2-33 show the effect that the metal case has on the radiation patterns of the crossed dipoles. These figures show the absolute directivity at 2.4 GHz. Because the case has a particularly distorting effect on these patterns, they are presented as 3D plots so that their features may be studied in more detail than possible with polar plots. The radiation patterns were observed to not vary significantly within the 2.39 to 2.41 GHz band. Figure 2-30 shows that of the X orientated dipole, while Figure 2-31 shows that of the Z polarised dipole. The farfield orthogonality of the fields from these two antennas is shown as the colour on Figure 2-34, with the radius proportional to radiated field strength of both polarisations. As can be seen, the fields remain orthogonal for a wide angle in the azimuth plane and also in the XY plane. These are the directions where orthogonality is highest for the uncoated reference antennas too. Through inspection of the polarisations in these directions on the Poincaré sphere, it was observed that they remain approximately linearly polarised with E-field vectors orientated close to the $\theta$ and $\phi$ directions. In the regions where orthogonality decreases, away from these planes, the polarisations appeared to remain approximately linear, but with the E-field vectors pointing in similar directions.

Figure 2-32 and Figure 2-33 show the absolute directivity of the crossed dipole CP antennas with the case. In Figure 2-32, the RHC feed is excited and in Figure 2-33, the LHC feed is excited. Again, the presence of the case has significantly affected the radiation patterns of these antennas. Figure 2-35 shows the far-field orthogonality (colour) and total field strength (radius) of these patterns. The polarisations of the fields from these antennas are severely affected by the presence of the case. As can be seen, only very small angular regions exist where orthogonality is greater than 0.6. This is very different to the equivalent plot for an antenna like this without the case (Figure 2-17). The polarisation of the fields from this antenna vary significantly with angle. They are generally elliptical, at times with very narrow ellipses and often both with the same rotation sense. The axial ratio is in fact rarely below 10 dB from either antenna. Figure 2-36 shows the two radiated
polarisations in the $Y$ direction. With the addition of the case, the polarisation from port one and two are RHC and LHC respectively in this direction, whereas in the presence of the case it can be seen that they are both elliptical, with left hand rotation sense and differing tilt angles and axial ratios. Their orthogonality is 0.56. The low orthogonality throughout most of the far-field means that MIMO systems using these antennas are likely to suffer from lack of polarisation diversity and high cross polar interference.
Figure 2-30 – Farfield absolute directivity of $X$ orientated narrowband coated dipole with case

Figure 2-31 – Farfield absolute directivity of $Z$ orientated narrowband coated dipole with case

Figure 2-32 – Farfield absolute directivity of RHC (in $Y$ direction) excitation of narrowband coated crossed dipoles with case

Figure 2-33 – Farfield absolute directivity of LHC (in $Y$ direction) excitation of narrowband coated crossed dipoles with case

Figure 2-34 – Farfield orthogonality of narrowband coated crossed dipoles used for LP with case

Figure 2-35 – Farfield orthogonality of narrowband coated crossed dipoles used for CP with case
Figure 2.36 – Radiated polarisations in Y direction by co-located CP antenna in the presence of a metal case

2.8. Summary

In this chapter, the compact antenna models used for the core research are described. Many of their properties which are relevant to polarisation MIMO systems are discussed in detail. General antenna properties such as gain, directivity, radiation patterns, field regions, impedance, bandwidth and efficiency are introduced in Section 2.2. Envelope correlation coefficient which is of particular interest to MIMO systems is also introduced in Section 2.2.

Section 2.3. describes the concept of polarisation and polarisation orthogonality. Polarisation describes the shape and orientation of a propagating electric field. The Poincaré sphere and Stokes parameters are introduced as a useful way of visualising polarisation states. An equation is given to quantify the polarisation orthogonality of fields radiating from multiple antennas. This is particularly useful because in a polarisation MIMO system, signals on MIMO sub-channels are decorrelated through the use of antennas designed to radiate (or receive) fields with orthogonal polarisations.

Antenna mutual coupling is discussed in Section 2.4. Mutual coupling is a result of the near field interaction between closely spaced antennas and it affects MIMO systems in a multitude of ways. It distorts the radiation patterns of antennas, which may affect the total transferred power by a system, as well as its degree of pattern diversity. It also introduces correlation between signals at nearby antennas. This is because the current on one antenna produces a field which induces a voltage at the terminals of its neighbour. This effect is
quantified by the antenna system mutual impedance. Reference is made to literature which identifies the fact that ‘conventional’ mutual impedance, defined with one antenna transmitting and others open circuit, is less applicable to MIMO systems. Instead, other definitions are given for mutual impedance; one where the antennas are in transmit mode and one where they are in receive mode. These are studied and a novel procedure is described to calculate receive mutual impedance, using a weighted average approach, based on the angular distribution of received power and knowledge of antenna receive mutual impedance versus AoA. Using this approach, it is demonstrated that the receive mutual impedance varies depending on the position of the antennas in a modelled environment. This being said, the variation is very little. In fact, it is suggested that at the low levels of mutual impedance experienced with the dielectric coated antennas, the differences to the MIMO performance results between an approach which accounts for the receive mutual impedance in the novel way, compared to an approach which assumes the receive mutual impedance is simply the reciprocal of transmit mutual impedance, are very little.

Some of the early work on compact antennas is reviewed in Section 2.5. Here, it is identified that a fundamental trade-off exists between antenna size, efficiency and bandwidth. The size reduction approach of dielectric loading is then introduced, where materials of high dielectric permittivity are used to make antennas behave electrically larger than in air. This approach has been used to design the highly compact dipole and crossed dipole antennas in Section 2.7.

Section 2.6. reviews the use of CST-MWS to design and simulate the antenna models, before 2.7. discusses the antennas in detail. Many compact antenna models have been created, for use in both spatial diversity MIMO systems and polarisation diversity MIMO systems. As a reference, simple uncoated, broadband antennas have also been modelled in free-space. These are dipoles for use in spatial MIMO systems, and co-located crossed dipoles, for use in LP and CP polarisation MIMO systems. The crossed elements in the CP systems are fed using hybrid couplers so that there is a 90° phase offset between the two linear elements. Owing to their large size, these have a bandwidth of 400 MHz, which is much greater than that of the compact antennas.

The compact narrowband antenna models are dipoles and crossed dipoles, but with a spherical coat of $\varepsilon_r = 80$. This reduces their size significantly, but also reduces their
bandwidth to approximately 20 MHz. It is observed that for the spatial, co-polar antenna pairs without the case, the dielectric coat also reduces mutual coupling. This is demonstrated by lower $S_{2,1}$, lower receive mutual impedance and antenna patterns which are closer to the antennas isolated patterns. This in fact reduces pattern diversity, which is evident from increased envelope correlation coefficient.

The co-located crossed dipole LP and CP antennas, with no case, radiate with similar directivity to their full-size counterparts. They maintain very low coupling, low envelope correlation coefficient and high orthogonality. This is because their elements are positioned perfectly orthogonally and hence the fields they produce remain orthogonal. A compact narrowband polarisation-spatial hybrid system has also been modelled, where two compact crossed dipole CP antennas, with opposite senses are separated in space by 0.25$\lambda$. This system exploits both spatial and polarisation diversity. The radiation patterns from these antennas are largely the same as their isolated or co-located counterparts, but coupling is significantly higher. This is because when the elements are separated in space, their fields are no longer perfectly orthogonal and as such, significant near field interaction takes place.

The narrowband dipoles for spatial diversity and the LP and CP co-located antennas have also been modelled in the presence of a metal case, of similar dimensions to a phone. They are positioned on the top of the case and a parameter sweep was performed to determine their optimum positions, in terms of $S_{2,1}$ and envelope correlation coefficient. The presence of the case severely affects the behaviour of the antennas. Generally, coupling increases and for all of the models, the radiation patterns are extremely distorted. This has particularly interesting effects on the polarisation of the radiated fields. The radiated polarisations from the co-located CP antenna deteriorate to elliptical in all directions, often with the same rotation sense and unpredictable tilt angles. This results in very poor orthogonality, which would be likely to lead to poor polarisation diversity and low cross polar isolation in a MIMO system.

The radiated polarisations from the co-located LP antenna remain linear and, for considerably wide angles, maintain high orthogonality. This suggests much better MIMO performance than the CP antenna in the presence of the case.
Perhaps most interestingly, the polarisation from the co-polar spatial diversity antennas deteriorates to elliptical, with opposite senses and close to orthogonal tilt angles, in the presence of the case. This is true for separation of both 0.3 and 0.5 $\lambda$ and produces fields with high orthogonality in front of the antennas. These antennas should be able to exploit spatial diversity as well as an element of polarisation diversity and one would expect them to perform well in a MIMO system. They do of course require more space on a device than a co-located antenna, however.

The simulated antenna patterns for the broadband and the narrowband antennas, with and without the case, have been used in conjunction with the ray launching propagation model (described in Chapter 3), to determine the MIMO sub-channel responses between the antenna systems, located in the indoor environment. The receive mutual impedances have been used to account for coupling between receive antennas in the extended MIMO channel model, as described in Section 4.5.5.
3. Propagation Modelling

3.1. Introduction

MIMO communications are dependent upon the multipath nature of the propagation environment \[93\]. To understand how the propagation environment affects the performance of a MIMO system, a propagation channel model is needed. An insight, based on measurable statistics of the environment may be obtained through the use of a stochastic channel model, such as those in \[2, 38\]. Stochastic models like this are relatively simple and quick to implement but they are also very general. To accurately predict MIMO performance for a particular system in a particular environment, it is desirable to directly obtain the multipath propagation channel characteristics in a deterministic way.

Deterministic models are usually much more computationally intensive and time consuming than stochastic models. The exact deterministic modelling of a propagation environment would directly solve Maxwell’s equations in the presence of the correct boundary conditions. However, for large complex environments, or for spatial accuracy smaller than one wavelength, specifying the boundary conditions of these presents huge difficulties and an unfeasible amount of computational complexity would be required \[94\]. Instead, an approximation can be achieved through the use of ray techniques, such as ray tracing and ray launching. These techniques use a detailed description of the propagation environment and apply geometric optics to determine every significant path between a transmitter and a receiver. The field present at the receiver is calculated from the combination of field contributions from all of these paths.

A bespoke ray launcher has been developed which fully tracks the polarisation and phase as rays propagate through a three-dimensional (3D) environment, between antennas with known patterns, as determined in Chapter 2. A bespoke model, as opposed to a commercially available package, was chosen to allow greater freedom in terms of input and output results and their format. To obtain the parameters of the polarisation MIMO channel model, which is developed in Chapter 4, it is essential to have knowledge of the complex channel frequency responses as opposed to simply their magnitude. It is equally important that the responses may be conveniently obtained at a frequency resolution sufficient to capture all of multipath fading detail. When using a commercial ray launcher,
this is not always possible. For example, Remcom’s Wireless InSite can only output the magnitude of the channel gain at a single frequency per run. Another advantage of the bespoke model is that it is possible to extract intermediate results to aid with the understanding of specific problems. For instance, it may be desirable to examine the arrival angles or polarisation of individual rays at a certain location, to aide with the understanding of an end-to-end path gain result, which only gives the combined contribution of all rays, including the effects of the antenna gains.

The inputs to the bespoke ray launcher are a database which contains descriptions of all objects in the environment, the antenna patterns and the locations of the transmitters and receivers. The model launches many rays, with uniform angular distribution and traces them through the environment. Reflection and transmission coefficients are applied to the rays’ fields upon interactions with objects and the angle dependent antenna gains are applied at the transmitters and receivers. At the receiver, rays are filtered and combined to give each complex sub-channel gain between the antennas at the specified locations, with the given antenna patterns. This procedure is repeated at many frequencies over the signal bandwidth, to obtain the sub-channel frequency responses. These frequency responses are used to set the parameters of the polarisation MIMO channel model which is developed in the following chapter. A channel power imbalance matrix $X$ and a K-factor matrix, $K$ are produced, as well as correlation matrix, $R$ which is obtained from the sub-channel impulse responses, after taking an inverse fast Fourier transform (IFFT) of the frequency responses.

The ray launcher can also produce 3D plots of the ray paths through the environment. This is another useful aide for understanding the behaviour of the propagation environment. [93-97] are among many useful resources on ray techniques.

The remainder of this chapter consists of the following. Section 3.2. gives an overview of ray techniques. Section 3.3. discusses the approach used to describe the propagation environment. Section 3.4. describes the geodesic launching technique which ensures that the rays are launched with approximately uniform angular distribution. Section 3.5. presents techniques to mitigate the effects of duplicate received rays which have taken effectively the same path through the environment. Reflection and transmission of rays upon intersection with objects in the environment is discussed in Section 3.6. Section 3.7. describes the necessary field transformations, between Cartesian, spherical and ray-fixed bases. Section 3.8. accounts for the effect of ray length through using a propagation factor.
Section 3.9. introduces the ability of the model to produce 3D visual plots of ray paths through the environment. Section 3.10. is concerned with the validation and testing of the reflection, transmission and antenna gain functionality of the model. End-to-end validation of the model is provided in Section 3.11. Finally, Section 3.12. gives a summary of the chapter.

3.2. Overview of Ray Techniques

There are two closely related ray techniques which may be used for propagation modelling. While the terminology for these sometimes varies, for the purpose of this report these are referred to as ray tracing and ray launching. In both, the transmitter is initially considered as a point source. Ray tracing then uses the image method [98]. This is where objects in a ray’s path are identified and virtual sources or “images” are created by reflecting the ray’s source about the surface of the object, where the intersection occurs. Further images are then created to account for more objects in the ray’s path, up to a limit for the number of interactions. Thus, the exact ray paths between transmitter and receiver are found, but the computational complexity increases exponentially with the number of events considered [94].

Alternatively, ray launching (sometimes referred to as the “brute force” or shooting and bouncing rays method) breaks down the angular space around the source into discreet rays. Each ray is then traced for a given number of reflections or transmissions (sometimes referred to as events), until it either intercepts a receiver, which is modelled as a sphere, and is considered received, or its field strength has dropped below a given threshold and it is disregarded. With this technique, computational complexity increases linearly with number of events, hence it is more suited to environments of higher complexity [94]. This is the approach which has been used in this work.

It is assumed that spherical waves propagate from a point source transmitter. When ray launching, the sphere around the transmitter is split into many “ray tubes”. The field associated locally with each ray tube may be considered a plane wave for the purpose of interaction with objects, provided a divergence (or spreading) factor is later applied to account for the ray’s spreading with distance from transmitter.
The resolution of the brute force approach is limited by the number of ray tubes launched from the transmitter. It is stated in [94] that the minimum number of required rays for a “complete” ray search using this method, $M_r$ is

$$M_r \geq 4\pi \cdot (k_e + 1)^2 \cdot \left(\frac{s_{\text{max}}}{\Delta}\right)^2,$$  \hspace{1cm} (3.1)

where, $k_e$ is the maximum number of events considered, $s_{\text{max}}$ is the maximum length of a straight line which lies within the considered geometry and $\Delta$ is the spatial resolution of the method, which is limited by $\lambda < \Delta < s_r$, where $s_r$ is the size of the smallest reflector in the environment and $\lambda$ is wavelength.

The core of the 3D ray launching model is primarily written in C. C is an object orientated language, meaning that features of the environment and rays may be conveniently declared as objects, each with certain properties, such as the dielectric properties of environment structures and the direction and electric field contained by a ray. Functions can then be written to perform tasks such as intersection tests between rays and surfaces, or reflections and transmissions of rays by surfaces.

Rays are launched from the transmitter with specific direction before standard vector techniques [99] are performed to check for interception with a receiver sphere. If a ray does not intercept a receiver an intersection check is performed for each object in the environment. If a ray does not intercept any object, it is disregarded. If a ray hits an object other than a receiver, it is split into a reflected and a transmitted ray, with fields modified according to reflection and transmission coefficients. The reflected and transmitted rays are traced in the same way until they reach a receiver or exceed their field strength or event count threshold. When a ray does reach a receiver sphere, its divergence factor and a phase factor are applied as a function of its total unfolded path length.

A post processing script, written in MATLAB is then used, which takes the raw results from the core ray launcher (RL) and applies the transmit and receive antenna patterns, before filtering the rays to ensure that each unique ray path is only counted once. A failure to do this could result in the field from duplicate rays, which essentially represent the same path through the environment, being accounted for more than once.
Equation (3.2) shows how the total electric field associated with one ray at the receiver is calculated.

\[
E = \sqrt{\frac{\lambda^2}{4\pi}} g_r(\theta_r, \phi_r) \left( \prod_{j} R_j \prod_{k} T_k \right) \frac{e^{-j\beta_0s}}{s} \frac{P_{\text{rad}}}{4\pi} g_t(\theta_t, \phi_t) \tag{3.2}
\]

\(E\) is the electric field vector of the ray. This is most generally expressed using the Cartesian basis, however it is in fact transformed between spherical, Cartesian and ray-fixed bases throughout the ray launching process, as discussed in Section 3.7. For the vector multiplications in (3.2), it is important that vectors are manipulated under the same bases and the multiplications are performed element-wise.

\(g_t(\theta_t, \phi_t)\) is the complex transmit field gain at the angle of departure (AoD), \((\theta_t, \phi_t)\). This, multiplied by \(\sqrt{Z_f}\) times the preceding radical term gives the E-field at a reference distance of 1 meter, under total transmit power, \(P_{\text{rad}}\) [83]. \(Z_f \approx 377\Omega\) is the free-space impedance. The \(e^{-j\beta_0s}/s\) term accounts for the phase difference and divergence factor associated with the distance, \(s\) travelled by the ray. \(R_j\) and \(T_k\) contain the reflection coefficients for the \(j\)th reflection and the transmission coefficients for the \(k\)th transmission, respectively. The remaining terms, to the left of the brackets, when multiplied by \(1/\sqrt{Z_f}\), account for the effective area of the receive antenna [83], which captures the incident rays field, with complex field gain, \(g_r(\theta_r, \phi_r)\) at the ray angle of arrival (AoA), \((\theta_r, \phi_r)\). The \(Z_f\) terms in (3.2) are omitted for clarity as they cancel. The antenna gain patterns, \(g_t\) and \(g_r\) are imported from an external file which is simulated using CST-MWS. They are both normalised such that the average power radiated per elemental solid angle when the antenna is transmitting is equal to 1 [83], as in (3.3), where \(G(\theta, \phi) = g(\theta, \phi)^2\) is the antenna power gain in the direction \(\theta, \phi\). The values for \(G_t\) and \(G_r\) are then interpolated at the ray’s AoD and AoA respectively.

\[
\frac{1}{4\pi} \int_{0}^{\pi} \int_{0}^{2\pi} G(\theta, \phi) \sin \theta \, d\theta \, d\phi = 1. \tag{3.3}
\]

The total field received by an antenna is the complex addition of the field given by equation (3.2) for all rays after filtering to remove rays which have travelled similar paths. This process is repeated for all frequencies of interest and at all locations of interest.
The model accounts for propagation over a direct path, via transmission through objects and via reflection from objects. Another propagation mechanism is diffraction, which occurs at surface edges. Diffraction causes a portion of energy which is incident on the edge of an object to redirect into the region of space which is otherwise shadowed by the object [100]. In the outdoor environment, propagation in this manner allows communication over hills, where there is no direct line of sight. It is also responsible for waves diffracting from rooftops to street level in the urban environment. The simplified indoor environments which are modelled in this thesis, contain few surface edges from which diffracted waves would contribute significantly to the total received signal. As such, to improve the computational efficiency of the ray launching process in complex multiple room environments, diffracted rays are omitted from the model. If desired however, it would be possible to account for these rays using the uniform theory of diffraction, as described in [96, 100, 101].

The terms in (3.2) and the ray launching process are described in further detail in the remaining sections of this chapter. Figure 3-1 shows a flow chart of the model.
Figure 3-1– Flow cart of the ray launching model
3.3. Environment Description

The model requires a database of every object within the considered environment so that the ray intersection tests can be performed. The model allows for spheres such as the receiver sphere to be entered, as well as cuboid objects, represented by bounded quadrilateral planes with a given thickness.

A plane is described through linear algebra using the equation,

\[ ax + by + cz + d = 0. \] (3.4)

Here, the vector \( \mathbf{n} = [a, b, c] \) is the normal vector to the plane, which points outward from the plane perpendicularly. \((x, y, z)\) describes any point on the plane. \(d\) is the shortest distance from the origin of the coordinate system to the plane.

Using \(d\) and \(\mathbf{n}\), the RL performs line-plane intersection tests [99] for each ray with each plane. Equation (3.4) is unbounded and so the intersection test may return a positive result for an intersection which is on the plane, but in fact outside the object boundaries. For this reason, the model also requires the coordinates of the four corners of each plane. It calculates the intersection point between a ray and a plane and then determines whether it lies within the bounds. Shapes with more than four sides may be constructed from multiple quadrilateral planes. Every plane has a thickness which is used to calculate its reflection and transmission coefficients. Linear equations of planes are further described in [99]. As a check, to reduce the chance of errors when creating the environment database, the model automatically checks that the coordinates of all four corners of the planes do indeed lie on the plane specified, using (3.4).

3.4. Geodesic Ray Launching

To successfully record all paths from the source to the receiver, all possible angles of departure and arrival must be considered. To do this, rays are launched in many directions from the transmitter location, while the receiver is modelled as a sphere, the radius of which increases with unfolded ray path length. Intersection checks are performed between this sphere and the ray centres. The size of the receiver sphere is critical. If it is too small, rays could be missed, but if it is too large, multiple rays may be captured which have travelled essentially the same path, resulting in the field associated with these rays being counted multiple times.
A receiver sphere of radius, \( R_{RX} = \frac{\alpha s}{\sqrt{3}} \) is often reported \([102-105]\) as being the most suitable size. Here, \( s \) is total unfolded ray length and \( \alpha \) is the separation between adjacent ray launch angles. This size works well, however in three dimensions it becomes difficult to maintain uniform angular separation of rays, \( \alpha \) in all directions. Common methods of dividing the space around the transmitter are insufficient. For example, the use of a spherical coordinate system results in a much smaller \( \alpha \) in the phi direction when close to the poles than in the theta direction. Selecting \( \alpha \) according to its mean value throughout the sphere, or its constant value in the theta direction would lead to the possibility of many duplicate rays being recorded for rays which were launched near to the poles. In addition, selecting \( \alpha \) according to the closest ray (in the phi direction) around the poles will result in a very small receiver sphere which could easily lead to rays being missed.

In \([102, 104-107]\), the use of a geodesic polyhedron to approximate the spherical space around the transmitter is suggested, where rays are launched in the directions of the face normals or the vertices of a geodesic polyhedron. The reason for selecting this geometry is to maximise the uniformity of the ray launch directions. A regular icosahedron is capable of doing exactly this. It has 20 triangular faces and 12 vertices. Angular separation, is constant between all adjacent face normals as well as all adjacent vertices. This would be the perfect geometry in terms of uniformity of launch angles, however no regular polyhedron has more than 20 sides. Using a regular polyhedron would therefore result in an unacceptably low resolution of the model.

Instead, it is necessary to create a geodesic geometry by splitting the triangular faces of the regular icosahedron into multiple triangular segments, through the process of tessellation \([108]\). This is where \( N_{tess} - 1 \) lines, parallel to each side of the face, of equal separation are drawn between each side and the vertex opposite it. The intersections of these sets of lines form the new vertices, where \( N_{tess} \) is the tessellation frequency.

Figure 3-2 shows an example of the tessellation of the triangular face of an icosahedron where \( N_{tess} = 4 \). The interior vertices are the points where the \( N_{tess} - 1 \) new lines intersect, while the edge vertices are those where a new line intersects with an edge of the original triangular face. The figure also shows two of the hexagonal wavefronts which are associated with rays launched in the directions of the interior vertices. The wavefronts launched at the interior and the edge vertices are hexagonal, while those at the original
vertices are always pentagonal. The total number of rays created using this method is \(10N_{tess}^2 + 2\).

![Tessellation of icosahedron face for \(N_{tess} = 4\). Wavefronts are hexagonal for interior and edge vertices but pentagonal for the original vertices.](image)

3.5. Double Ray Count Mitigation

The geodesic launch geometry gives great improvement over spherical coordinates at the poles, however it still presents difficulties when determining the ideal reception sphere size. To guarantee the capture of at least one ray, the sphere radius must be at least \(\alpha s/\sqrt{3}\) [102, 104, 106]. Even when local \(\alpha\) is known and used however, this can still result in double counting of rays, with a probability of 20.9% [106], as demonstrated by Figure 3-3. This probability is independent of tessellation frequency.
In the event of a double count, the adjacent rays will have travelled approximately the same path, arriving with very similar amplitude and phase. This will result in up to a doubling of field for that ray path and hence a 6 dB increase in power. For a randomly placed receiver on a geodesic wavefront, the double counting of rays will account for a mean power increase of at least 1.25 dB over the wavefront, or potentially more if localised $\alpha$ is not used [106, 109]. This is clearly undesirable and is an effect which needs to be mitigated.

### 3.5.1. Weighting Function Approach

One mitigation approach, used in [106], applies a weighting function to the field of rays which pass near to the receiver. The weighting function is dependent upon the distance between the rays and the receiver. A tabulated weighting function was determined by the authors of [106] using a Monte-Carlo approach with the goal of finding a radially symmetric function, which when applied to all nearby ray points on Figure 3-3 and summed equals a constant, independent of position on the wavefront. Brute force iterations increased or decreased the tabulated function values until the curve converged to a useful function, as shown in Figure 3-4. The tabulated values of the function are given in [106].
Figure 3-4 - Plot of the weighting function which is applied to rays near the receiver

The normalised separation distance, $x_r$, for any ray is the ratio of the distance, $d_r$, between that ray and the receiver centre, and the distance between that ray and its nearest neighbour, $\alpha_{nn}s$, where $\alpha_{nn}$ is the angular distance between the ray under consideration and its nearest neighbour.

$$x_r = \frac{d_r}{\alpha_{nn}s}$$ (3.6)

For the purpose of the calculation of $x$, the wavefront may be considered plane and so $d$ is simply the perpendicular distance from the ray to the receiver centre. A ray’s contribution is zero if its distance from the receiver is greater than the distance to its nearest neighbour, so the receiver sphere radius is set to the distance from the ray to its nearest neighbour, $\alpha_{nn}s$ for the purpose of the intersection tests, where $\alpha_{nn}$ is the angular distance between the considered ray and its nearest neighbour, while $s$ is the total unfolded ray path length. The weighting function, $f(x_r)$ may be applied in a post processing stage, to the ray contribution as determined by the RL.

This approach was investigated and works well in most situations. However, problems occur if there is an abrupt change in the field magnitude of adjacent rays. For instance, in the case of cross polar LOS transmission between two aligned dipoles, the most direct ray experiences an extremely low path gain (essentially nothing). The weighting function for this ray is one, as it intersects the centre of the receiver sphere. Adjacent rays will also intersect the receiver sphere, albeit off centre, with a low weighting. The problem is that
for a small (< 1°) change in AoD, antenna gain can increase by several orders of magnitude. This is demonstrated in Figure 3-5, which shows the phi component of antenna gain at a cut of theta = 90° for a dipole on the X axis simulated using CST-MWS. The weighting function is smooth and therefore, although a low weighting is applied to these rays, their contribution is very large, compared to the direct ray, as the decreased weighting is far outweighed by the increase in antenna gain. When the weighted rays’ contributions are summed, the result is a path gain which is still relatively low, but much higher than would be expected for perfectly aligned cross polar transmission. For these reasons, for the modelling of propagation for polarisation MIMO systems, where the cross-polar channel gain is of importance, a ray filtering approach was preferred to the weighting function.

![Figure 3-5](image)

Figure 3-5 – phi component gain of x polarised dipole at theta = 90° cut, with very steep increase away from phi = 0, 180°

3.5.2. Ray Filtering Approach

The ray filtering approach, as used in [102] and [95], removes double count or ‘duplicate’ rays using a filter in a post processing stage. Duplicate rays are expected to have travelled almost identical paths; therefore they will have very similar lengths. This being said, filtering according to ray length alone is not sufficient as of course there may be many non-identical paths of similar length. The filter may also consider AoA and AoD. This is a better approach but still leaves some uncertainty as to suitable filter threshold values for all parameters. It was also found that for a large number of rays, filtering in this way became fairly time consuming.

Because duplicate rays travel the same path, they encounter the same interactions with the same objects of the environment. A reception sphere of radius, $R_{RX} = \alpha_{fn} s / \sqrt{3}$ [102, 104, 106] is used to ensure that at least one ray from each wavefront is captured, where $\alpha_{fn}$
is the angular distance from the considered ray to its furthest neighbour and \( s \) is the total ray unfolded path length. Captured rays are recorded along with the sequence of objects with which they interact. Rays which have experienced the same interactions, with the same surfaces, in the same order are considered duplicates. In the event of duplicates existing, only the ray which passes closest to the receiver centre is kept.

3.6. Reflection and Transmission

When a wave hits the boundary between free-space and an object in the environment, such as a wall, a portion of its energy is reflected at the boundary and a portion is transmitted across the boundary, into the object. When the transmitted portion reaches the far boundary with free-space, a portion of the energy is reflected back inside the object, towards the original boundary and a portion is transmitted out of the object. This process repeats indefinitely, with the reflected energy inside the object progressively decreasing due to losses in the material and upon each reflection, as some energy is transmitted across the boundary and out of the object. The combination of the initial reflected energy from the incident wave and the energy which is transmitted back out of the object on the same side as the incident wave combines to create the reflected wave. Similarly, all of the energy which is transmitted through the object, both directly and following internal reflections, which leaves from the opposite side to the incident wave, combines to create the transmitted wave.

\[
E_{\text{total}}^r = E_1^r + E_2^r + \cdots
\]

\[
E_{\text{total}}^t = E_1^t + E_2^t + \cdots
\]

Figure 3-6 - Reflection and transmission of plane wave incident on thin dielectric layer in free space
The process is illustrated by Figure 3-6. This shows a plane wave, denoted by its electric field, \( E_i \), which is incident on the left hand side of a dielectric slab of relative permittivity, \( \varepsilon_r \) and thickness, \( d \). The wave intersects the boundary of the slab with air at an angle of \( \theta_i \) to the surface normal. At the boundary, a reflected wave is created at the angle \( \theta_r = \theta_i \) to the normal, with the field \( E_1^r \). The electric field of the reflected wave can be calculated from the incident field using the reflection coefficient for a single reflection exterior to the surface, \( R_1 \). A transmitted wave is also created which progresses into the slab at \( \theta_t \) from the normal, where \( \sin \theta_t = \sin(\theta_i/\sqrt{\varepsilon_r}) \). The field of this ray can be calculated using the transmission coefficient for a single crossing from free-space into the dielectric, \( T_1 \). Upon intersection with the right hand boundary with air, a transmitted wave is created which leaves the slab at the angle \( \theta_i \) to the normal, with electric field denoted, \( E_1^t \). \( E_1^t \) may be calculated according to the transmission coefficient for a single crossing into air, \( T_2 \). A reflected wave is also created which remains inside the dielectric and travels back toward the left hand boundary. The field of this wave is a result of the reflection coefficient for a single reflection interior to the slab, \( R_2 \). As can be seen, this process continues and the combined electric field of the transmitted waves is the complex summation of the contributions, \( E_1^t + E_2^t + \cdots \), while the combined electric field of the reflected waves is the complex summation of \( E_1^r + E_2^r + \cdots \).

The reflection and transmission coefficients for single crossings and single reflections at the boundary of a dielectric are given in [96, 97]. They are dependent upon properties of the wave, such as its frequency, angle with the dielectric surface normal, \( \theta_i \) and polarisation, as well as properties of the dielectric, such as its relative permittivity, \( \varepsilon_r \). If the surface is conductive, with conductivity, \( \sigma \) (S/m) then \( \varepsilon_{rc} \) replaces \( \varepsilon_r \) [97, 110], where,

\[
\varepsilon_{rc} = \varepsilon_r - j \frac{\sigma}{\varepsilon_0 \omega}, \tag{3.7}
\]

In (3.7), \( \varepsilon_0 \) is the dielectric permittivity of a vacuum and \( \omega = 2\pi f \) is the angular frequency of the incident wave. The reflection coefficients for a single reflection exterior to the dielectric for transverse electric (TE) and transverse magnetic (TM) mode waves, \( R_{1(TE)} \) and \( R_{1(TM)} \) respectively, are as follows.

\[
R_{1(TE)} = \frac{\cos \theta_i - \sqrt{\varepsilon_{rc} - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_{rc} - \sin^2 \theta_i}} \tag{3.8}
\]
The reflection coefficient for a single reflection interior to the dielectric, \( R_2 = -R_1 \). The transmission coefficients for a single crossing from free space into the dielectric and vice-versa, \( T_1 \) and \( T_2 \) respectively, are given by \( T_1 = 1 + R_1 \) and \( T_2 = 1 + R_2 \).

The final reflection and transmission coefficients for a slab of finite thickness, which account for all of the internal reflections and their subsequent transmissions out of the dielectric result in geometric series which reduce to \( R_{(TM,TE)} \) and \( T_{(TM,TE)} \) as follows [96].

\[
R_{(TM,TE)} = \frac{R_{1(TM,TE)}(1 - P_d^2 P_a)}{1 - R_{1(TM,TE)}^2 P_d^2 P_a} \quad (3.10)
\]

\[
T_{(TM,TE)} = \frac{(1 - R_{1(TM,TE)}^2) P_d P_t}{1 - R_{1(TM,TE)}^2 P_d^2 P_a} \quad (3.11)
\]

Here, \( P_d \) is a term to account for the phase delay to the field due to one crossing of the slab, while \( P_a \) accounts for the difference in path length to an observer between each of the reflected waves (\( E_r^1, E_r^2, E_r^3 \ldots \)) in Figure 3-6. \( P_t \) is a phase factor which is required to relate the phase of the transmitted wave to the point of incidence with the dielectric.

\[
P_d = e^{-j\beta l} \quad (3.12)
\]

\[
P_a = e^{j\beta_0 2l \sin \theta t \sin \theta i} \quad (3.13)
\]

\[
P_t = e^{j\beta_0 t} \quad (3.14)
\]

The slab is of thickness, \( d \) and so \( l = d / \cos \theta t \) is the length that a ray travels for one crossing of the slab at oblique incidence. \( \beta_0 = 2\pi/\lambda \) is the propagation constant in free space, while \( \beta = \beta_0 \sqrt{\epsilon_{rc}} \) is the propagation constant in the dielectric slab and \( t = l \cos(\theta i - \theta t) \).

(3.10) and (3.11) show the reflection and transmission coefficients used by the RL. When a ray is passed to the reflect and transmit functions, its field is first transformed into components of the ray fixed coordinate system, which are perpendicular and parallel to the plane of incidence. The plane of incidence is a plane perpendicular to the dielectric surface, on which the surface normal, the incident ray and the reflected ray lie. The perpendicular component is multiplied by \( R_{(TE)} \) while the parallel component is multiplied...
by $R_{(TM)}$ to give the field of the reflected ray. The transmitted ray has its perpendicular component multiplied by $T_{(TE)}$ and the parallel component by $T_{(TM)}$. The methods for transforming the field between spherical, Cartesian and ray-fixed components are described in the following section of this chapter.

Because objects in the environment are represented by planes, the intersection between a ray and a wall is always considered to be at the centre point of a cross section of the wall. The reflected ray departs from this point, at an angle of $\theta_r = \theta_i$ to the normal, in the normal plane of incidence. The transmitted ray departs from the same point, with the same direction as the incident ray.

Figure 3-7 to Figure 3-10 show an example of the final reflection and transmission coefficients, calculated by the functions used in the ray launching model. Figure 3-7 and Figure 3-8 show the magnitude of the final reflection coefficient for TM and TE mode plane waves, respectively. The frequency is 6 GHz and the wall thickness is 0.5 m. Results are shown for $\varepsilon_r = 5$, with $\sigma = 0.02$ S/m. These are values similar to those expected for a concrete wall [111]. Results are also shown for $\varepsilon_r = 5$ and $\sigma = 0$. The effect of the multiple internal reflections in the slab combining constructively and destructively is evident as the coefficients fluctuate with $\theta_i$. The amplitude of the fluctuation is reduced as conductivity is increased, introducing losses in the slab. Figure 3-9 and Figure 3-10 show the magnitude of the final transmission coefficients for TM and TE mode plane waves respectively. These are plotted for $f = 60$ GHz while $d$ remains at 0.5 m, $\varepsilon_r$ remains at 5 and $\sigma = 0.02$. As can be seen, the spacing between ripples in these plots is much closer at increased frequency. The results in Figure 3-7 to Figure 3-10 are supported by results in [111].
Figure 3-7 – Magnitude of reflection coefficient for TM mode wave vs angle of incidence. 
$\varepsilon_r = 5, f = 6$ GHz, thickness = 0.5 m

Figure 3-8 – Magnitude of reflection coefficient for TE mode wave vs angle of incidence. 
$\varepsilon_r = 5, f = 6$ GHz, thickness = 0.5 m

Figure 3-9 – Magnitude of transmission coefficient for TM mode wave vs angle of incidence. 
$\varepsilon_r = 5, \sigma = 0.02$ S/m $f = 60$ GHz, thickness = 0.5 m
3.7. E-Field Component Transformations

To fully track the polarisation of each ray it is necessary to know the complex field components in at least two directions perpendicular to the ray direction. The model launches each ray with a field vector which is polarised in the theta direction and another which is polarised in the phi direction. Both of these fields are then transformed into global Cartesian components. The two fields are treated independently at each interaction, where they are transformed into components of the “ray-fixed” coordinate system, $E_{\perp,\parallel}$, before the reflect and transmit coefficients are applied and they are transformed back to the Cartesian components. When the transmit antenna gain is applied, the theta component of the gain is applied to the field launched in the theta polarisation and the phi component to the field launched in the phi polarisation. At the receiver, the fields are transformed into spherical components of the receive antenna and the receive antenna gain is applied. The theta component of the gain is applied to the components from both fields which arrive in the theta polarisation and the phi component of the gain is applied to the components from each field which arrive in the phi polarisation.
The model uses the IEEE standard spherical coordinate system [68], as shown in Figure 3-11. Under this system, a unit vector describing the launch direction of the ray in the Cartesian coordinate system, $\hat{d}$, is calculated as follows [112].

$$\hat{d} = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{bmatrix}$$ \hspace{1cm} (3.15)

The electric field is then transformed into the global Cartesian components, using the following matrix multiplication [112].

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \phi & -\sin \phi \\ \cos \theta \sin \phi & \cos \phi \\ -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix}$$ \hspace{1cm} (3.16)

When a ray intercepts an object and reflection and transmission occur, it is necessary to split the field into components perpendicular and parallel to the ordinary plane of incidence, according to the “ray-fixed coordinate system” [96]. The ordinary plane of incidence is the plane which is normal to the wall and contains the incident and the reflected ray. This is illustrated in Figure 3-12 which shows a ray incident upon a plane, the transmitted ray through the plane and a reflected ray. The unit vector, $\hat{d}$, describes the direction of the incident and transmitted ray while $\hat{r}$ is the direction of the reflected ray. $E^i$ is the electric field of the incident ray. This may be described by a component in the $\hat{u}_\perp$ direction, which is perpendicular to the plane of incidence and the ray direction, and a component in the $\hat{u}_\parallel$ direction, which is perpendicular to the ray direction but parallel to
the plane of incidence. $\mathbf{E}^t$ is the transmitted electric field, with components in the same directions, while $\mathbf{E}^r$ is the reflected field, with components in the $\hat{u}_\parallel$ direction and the $\hat{u}_\perp^r$ direction. $\hat{u}_\perp^r$ is the reflection of $\hat{u}_\parallel^i$ about the normal, $\hat{n}$.

\[
\hat{u}_\perp = \hat{n} \times \hat{d} / |\hat{n} \times \hat{d}|
\]  
\[
\hat{u}_\parallel^i = \hat{d} \times \hat{u}_\perp
\]  
\[
\hat{u}_\perp^r = \hat{R} \times \hat{u}_\perp
\]  

(3.17)  
(3.18)  
(3.19)

The incident field, $\mathbf{E}^i$ is transformed into components in these directions using a change of basis matrix, $\mathbf{Q}^T$, where $\mathbf{Q}^T$ is the transpose of $\mathbf{Q}$, a matrix whose columns are the new vector bases. Change of bases in vector spaces are described in [99].

\[
\mathbf{E}^i_{(\parallel\perp)} = \mathbf{E}^i_{(x,y,z)} \mathbf{Q}^T
\]  
\[
\begin{bmatrix}
E^i_x \\
E^i_y \\
E^i_z
\end{bmatrix} = \begin{bmatrix}
\hat{u}_{\parallel(x)}^i \\
\hat{u}_{\parallel(y)}^i \\
\hat{u}_{\parallel(z)}^i
\end{bmatrix} \begin{bmatrix}
\hat{u}_{\parallel(x)} \\
\hat{u}_{\parallel(y)} \\
\hat{u}_{\parallel(z)}
\end{bmatrix}
\]  

(3.20)  
(3.21)

Figure 3-12 – Ray-fixed coordinate system

The three unit vectors, $\hat{u}_\perp$, $\hat{u}_\parallel$ and $\hat{u}_\parallel^r$ form the orthogonal bases for the local ray-fixed coordinate system and are calculated as follows [96], where $\times$ denotes the vector cross product.
When the incident field is considered in the ray-fixed components, the reflect and transmit coefficients for TE and TM mode waves are applied to the perpendicular and parallel components of the field respectively to account for the interaction with the surface, as follows.

\[
\begin{bmatrix}
E_\parallel^r \\
E_\perp^r
\end{bmatrix} =
\begin{bmatrix}
R_{TM} & 0 \\
0 & R_{TE}
\end{bmatrix}
\begin{bmatrix}
E_\parallel^i \\
E_\perp^i
\end{bmatrix} \tag{3.22}
\]

\[
\begin{bmatrix}
E_\parallel^t \\
E_\perp^t
\end{bmatrix} =
\begin{bmatrix}
T_{TM} & 0 \\
0 & T_{TE}
\end{bmatrix}
\begin{bmatrix}
E_\parallel^i \\
E_\perp^i
\end{bmatrix} \tag{3.23}
\]

The field of the transmitted ray, \(E^t_{(\parallel,\perp)}\) may now be transformed back to its Cartesian components, by multiplication with \(Q\). The field for the reflected ray, \(E^r_{(\parallel,\perp)}\) is transformed back in a similar way, however it is important that the vector \(\hat{u}_\parallel^i\) in \(Q\) is replaced by \(\hat{u}_\parallel^r\), as the orientation of the parallel component has now been reflected by the surface.

The reflected and transmitted rays are then traced to their next interaction. If this is with an object, their fields are again transformed into the new local ray-fixed components and the reflect and transmit coefficients applied. If a ray instead reaches a receiver, the effect of the ray length is applied, then angle of arrival is determined and the fields are transformed into the spherical components of the receiver.

3.8. Effect of Ray Length

For each ray-object interaction, the wavefront of the ray is locally considered to be plane. This is a valid assumption for the purpose of the interaction, but in reality as a field propagates from a point source, wavefronts are created as spherical surfaces which encase the source and are perpendicular to the direction of propagation. Naturally, as distance increases from a source, the area of a spherical wavefront increases proportionally with the square of distance. This results in a decrease in power density between wavefronts at increased distance. To account for this, a propagation factor must be applied to the modelled rays. This is done once a ray is considered to have reached a receiver. The propagation factor consists of a divergence factor, to account for the decrease in power density with distance, and a phase factor to account for changing phase with distance. The divergence factor for a spherical wave is \(\frac{\rho_0}{\rho_0 + s_{excess}}\), while the phase factor is \(e^{-j\beta_0 s_{excess}}\). \(\rho_0\) is a reference distance where the field is known, while \(s_{excess}\) is the additional distance.
to the point under consideration. $\beta_0$ is the propagation constant for free space. The propagation factor is applied to the field of each ray using the following expression.

$$E(s) = E(1) \frac{e^{-j\beta_0 s_{\text{excess}}}}{s}$$  \hspace{1cm} (3.24)

$E(1)$ is the reference field vector (at $s = \rho_0 = 1$), which has had all of the reflect and transmit coefficients applied to it, while $s = \rho_0 + s_{\text{excess}}$ is the total unfolded path length a ray has travelled between the transmitter and receiver. This is calculated by the RL as a summation of the distance from the point source transmitter, to the intersection point of the first interaction, plus the distances between the remaining intersection points, plus the distance between the final intersection and the receiver. This is the distance between the intersection with the environment and the intersection with the receiver sphere, plus the additional distance from the intersection with the sphere, to the plane on which the receiver centre lies which is orthogonal to the ray direction. Figure 3-13 illustrates this additional distance. Accounting for this improves the accuracy of the relative phases of multiple received rays which pass through the sphere at different distances from its centre, and hence results in a more accurate representation of the E-field polarisation at the receiver.

It should be noted that the fields of all rays also contain an $e^{j\omega t}$ time dependence term, which is omitted for clarity.

![Figure 3-13 – Illustration of additional distance between ray intersections with receiver sphere and plane orthogonal to ray direction, containing receiver centre](image-url)
3.9. 3D Visual Representation of Rays

The core RL process records the locations of all the points where rays interact with objects in the environment. This allows a post processing script to create a 3D plot of the paths taken by the rays through the environment. As an example, Figure 3-14 shows received rays in the office floor environment, which is described in Section 5.2. The transmitter is at the end of the corridor, while the receiver is in the fourth room on the right, with no LOS. They are both vertically polarised, uncoated dipoles. The 20 rays which contribute the most power to the sub-channel gain at 2.4 GHz are shown, with their colour corresponding to their power contribution, normalised against the contribution of the strongest ray. It can be seen that for this sub-channel, the most significant rays remain on the horizontal plane which contains both of the antennas. The most significant ray takes the direct path between transmitter and receiver, through the office wall and the corridor wall. The AoAs are well distributed in the azimuth plane only.

Figure 3-15 shows the same result with both antennas horizontally polarised (in the X direction of the figure). Now, the significant rays are not contained in the horizontal plane, they reflect off the floor and ceiling as well as the walls. The AoA distribution in the azimuth plane is smaller, but in the elevation plane is significantly larger. These plots are a useful visual aide when evaluating the propagation behaviour in a multipath environment.

Figure 3-14 – 20 Most powerful rays for vertically polarised transmission and reception in office floor environment
Figure 3-15 - 20 Most powerful rays for horizontally polarised transmission and reception in office floor environment

3.10. Model Validation

The RL is a key component of the end-to-end MIMO system modelling discussed in this thesis. As such, it is crucial that its results are accurate and reliable. To ensure that they are, an extensive validation procedure has been performed. The following sub-sections describe this validation.

As a preliminary, the transmit and reflect functions are tested in isolation. This is to validate the application of the transmission and reflection coefficients, as well as the relevant E-field transformations between Cartesian and ray-fixed bases. A similar approach is taken with regard to the antennas, to validate the application of the antenna gain patterns, the field transformations between spherical and Cartesian bases and the capturing of received rays.

Following this, complete end-to-end validation of the RL is carried out, where the calculated frequency response for given environments is compared to the frequency response determined by other methods. The complexity of the environments is gradually increased, starting with the reflection from a single object, then propagation in an empty room, propagation in a room where LOS is obscured by a partial wall, before the modelling of a complex office floor. Results from the simplest cases are compared to
frequency responses calculated directly and also to results obtained using CST-MWS. Results for larger and more complex models are compared to results obtained using a commercially available ray launcher called Remcom Wireless InSite. Finally, the E-field intensity throughout a floor of a modelled Victorian house is compared to results obtained using the electro-magnetic simulator, FEKO as part of the Wireless Friendly and Energy Efficient Buildings (WiFEEB) project by Sheffield University [113].

3.10.1. Reflection and Transmission

The reflection and transmission functions are called when a ray intersects a surface in the environment. They are passed the ray object and the surface object. From the surface normal and the ray direction, the ray-fixed coordinate system bases are determined and the E-field of the ray is transformed to these bases. The transmission or reflection coefficient is calculated and applied to the fields, before they are transformed back to the Cartesian bases and the new ray is returned.

As a preliminary check of these functions, a test script has been written, which calls them in isolation. They are passed a surface with dielectric properties, \( \varepsilon_r = 5, \sigma = 0.02 \) and thickness of 0.5 m, and a ray with a perpendicular polarised and a parallel polarised field (relative to the normal plane of incidence), both equal to one. This is repeated with the angle of incidence between the ray and the surface normal swept from 0 to 90°. As the angle of incidence changes, the Cartesian components of the parallel field of the ray are changed to maintain \( E_\parallel = 1 \). Similarly, the fields of the returned rays are converted from global Cartesian, back to the ray-fixed bases. These transformations are done externally to the functions which are under test. The magnitude and phase of the resultant fields are compared to the magnitude and phase of the coefficients, calculated directly using equations (3.10) and (3.11). Upon comparison, the two results were in exact agreement. This procedure validates the application of correct transmit and reflection coefficients, as well as the field transformations between Cartesian and ray-fixed bases. This check has been carried out using the RL for interactions with a surface in the \( YZ \) plane as well as the \( XZ \) plane, with the same results.

3.10.2. Antennas

To check the behaviour associated with the antennas, the full RL is used at a single frequency and the end-to-end path gain recorded. Initially, the transmitter is at a fixed
location and a unity gain receiver is swept around it at a distance of 1 m, at cuts of theta = 50, 90 and 130°. Path loss and phase change in the theta and phi polarisations are compared to the transmit antenna gain and phase patterns, at the same cuts. This has been done using a Z polarised dipole and an X polarised dipole as transmit antennas. Furthermore, the same has been done with the receive antenna fixed, using a Z, then X polarised dipole, with a unity gain transmitter swept around it.

In this test, the path gain and phase change using the RL are not expected to be the same as the antenna gain patterns, due to the propagation factor caused by the path length and the many other constants in (3.2). However, it is crucial that the relative path gain and phase variation with angle observed using the RL are in agreement with the antenna gain pattern.

This procedure ensures that the antenna patterns are being applied correctly at both ends of the link, as well as checking the component transformations between spherical and Cartesian bases. It also provides initial validation of the capturing and combination of received rays.

3.11. End-to-end Ray Launcher Validation

A number of more complex end-to-end test cases have been modelled using the RL and compared with directly calculated results, results obtained using CST-Microwave Studio (CST), commercial ray launching software (Remcom Wireless InSite) and to results simulated using FEKO, as part of the Wireless Friendly and Energy Efficient Buildings (WiFEEB) project by Sheffield University [113].

Comparison is made to results obtained using CST for simple cases, however due to the size of modelled environments, obtaining accurate results using CST transient solver requires many meshcells and becomes extremely time consuming, even for very simple models. For this reason, results obtained using Remcom Wireless InSite are used to validate larger and more complex environments. Wireless InSite is an indoor and outdoor 3D commercial ray launching package which works in the frequency domain. Unfortunately, it does not allow for easy calculation of a broadband channel response, nor does it conveniently allow phase and polarisation of field at the receiver to be extracted. It is for these reasons that the bespoke RL is the preferred propagation model for the core work. FEKO is an EM simulator which uses uniform theory of diffraction and geometric
optics. It has been used by the University of Sheffield as part of the WiFEEB project to model field intensity from a dipole transmitter in a typical Victorian house.

The first test cases all involve dipole to dipole transmission, where path gain is examined. The frequency range studied is 0.8 to 1.2 GHz. The RL uses dipole antenna patterns, obtained through CST simulation of a dipole in free space. The case of a single reflection off a perfect electric conductor (PEC) surface is first examined. Then a similar single reflection off a lossy dielectric surface is studied. Two very simple rooms are modelled, then a single floor of an office building is studied. Finally, E-field intensity throughout a plane inside a Victorian house is examined. For the single reflections off a surface, polarisations both perpendicular and parallel to the plane of incidence are checked. In the case of the simple rooms, co polar transmission is examined. For the office floor, the full four element dual polar channel matrix is recorded, while for the Victorian house, absolute field intensity is compared.

3.11.1. Single Reflection off PEC Surface

![Diagram](image)

**Figure 3-16 – Single reflection off PEC surface, where antennas are perpendicular to the normal plane of incidence**

In the first model, dipole antennas are centred 1.5 m above the top of a PEC surface which lies on the $XZ$ plane. Separation between the antenna centres in the LOS direction is 3 m. Path gain is recorded for perpendicular polarisation, where the antennas were perpendicular to the normal plane of incidence and parallel polarisation, when they are positioned in line with the $Y$ axis, which is parallel to the normal plane of incidence. Figure 3-16 shows this situation, with the antennas perpendicularly polarised (dipoles positioned in the $X$ direction, out of the page).
Comparison is made between results obtained using the RL, CST and a direct calculation, where path gain is calculated as the summation of the fields contained by a LOS ray and a reflected ray, using (3.2). For the direct calculation and the RL, results are plotted for 41 frequency points between 0.8 and 1.2 GHz with frequency increment, $\Delta f = 0.01$ GHz. The RL and the direct calculation assume that the antennas patterns are constant over the frequency band. To ensure like-for-like comparison with the CST result, antenna mismatch is accounted for through multiplication of the signal at each antenna by $\sqrt{1 - |\Gamma|^2}$, where $\Gamma$ is the voltage reflection coefficient at the terminals of the antenna due to imperfect antenna match.

Figure 3-17 - Path gain for single reflection off PEC, parallel polarisation

Figure 3-18 - Path gain for single reflection off PEC, perpendicular polarisation

Figure 3-17 and Figure 3-18 show the results for the parallel and perpendicularly polarised antennas respectively, using the RL and CST. The results calculated directly are in exact agreement with the RL results as so are omitted. The CST results are close to the RL and direct results however they do deviate from them slightly, particularly at either end of the band, in the parallel case. This is likely to be a result of inaccuracies in the CST model. It is anticipated that the result would approach the RL and direct result if further mesh refinement was performed, however the results presented typically required in excess of 140 million meshcells and so further refinement would have required run times which would be unworkable.
3.11.1. Single Reflection off Lossy Dielectric

Figure 3-19 - Single reflection off lossy dielectric surface, where antennas are perpendicular to the normal plane of incidence

A similar scenario has been modelled, where the PEC surface is replaced by a lossy dielectric slab resembling concrete, with $\varepsilon_r = 4.718$, $\sigma = 0.0213$ S/m. The slab thickness is 0.4 m and the antennas are centred at 1.5 m above the centre of the slab. Separation of the antennas in the LOS direction remains at 3 m. This scenario is shown in Figure 3-19, where the polarisation is perpendicular to the plane of incidence.

The RL results for reflection off the dielectric slab are compared to results obtained using Wireless InSite. Results are plotted for 41 frequency values, where frequency increment, $\delta f = 0.01$ GHz and the antennas are assumed to be perfectly matched throughout the band.

Figure 3-20 - Path gain for single reflection off lossy dielectric surface, parallel polarisation

Figure 3-21 - Path gain for single reflection off lossy dielectric surface, perpendicular polarisation
Figure 3-20 and Figure 3-21 show the path gains determined using the RL and Wireless InSite for the parallel and perpendicular polarisations respectively. Results using the two methods are in strong agreement with each other. It can be seen that the effect of the multipath in the parallel polarised case is much less significant than for the perpendicular case. This is because the reflected ray in this case is launched and received at angles away from the direction of maximum gain and as such it contributes much less power than the direct ray. In fact, the path gain here is very close to the free space path when the dielectric is not present.

3.11.2. Room A

![Room A diagram](image)

**Figure 3-22 – Room A**

Two very simple test rooms have also been modelled, with results obtaining using the RL compared to those obtained using Wireless InSite. Room A is an empty cuboid room of dimensions \((x \times y \times z) = (3 \times 4.5 \times 3)\) m. Walls, floor and ceiling are modelled as concrete slabs, again of \(\varepsilon_r = 4.718, \sigma = 0.0213\) S/m, with thickness of 0.4 m, centred on the \(x, y, z\) dimensions stated. Co-polar dipole antennas are used, with the transmitter centred at \((1.5, 1, 1.5)\) m and the receiver at \((1.5, 3, 1.5)\) m. Both antennas are polarised in the \(X\) direction. Figure 3-22 shows this model.

Results are obtained using the RL for 101 frequency points, with separation, \(\delta f = 0.004\) GHz. Due to the time consuming nature of the Wireless InSite runs, results are obtained for only 21 frequency values, with \(\delta f = 0.02\) GHz. The results are shown in Figure 3-23. It can be seen that InSite results are of a similar magnitude to the RL plot and
follow a similar trend. The slight differences are attributed to the mechanics of the reception sphere. The RL uses a sphere of radius proportional to ray length (according to $R_{RX} = \alpha f n s/\sqrt{3}$) to ensure that at least one ray from each wavefront is captured, before filtering out double counts. When consulting the InSite reference manual it states, “Experience has shown that a ray separation of $0.2^\circ$ and a collection surface 2.5 m in radius works well in most situations”. Presumably this is more suited to the modelling of large outdoor environments, however further explanation of how the model behaves for smaller environments is not provided. It was observed in the development of the RL that adjusting the receiver sphere size results in a slightly different placement of the multipath peaks and nulls, which may explain the null in the RL response at around 1.15 GHz. Furthermore, it was observed in preliminary work that if multiple receivers are positioned in a small grid around the desired receiver location, with their received fields averaged, the response for this room is closer to the InSite result, with the null around 1.15 GHz vastly reduced. For the core results, discussed in Chapter 5, rather than averaging over a grid, end-to-end results are studied over a measurement route, to ensure that the effects of similar signal variations over space are considered.

Figure 3-23 – Path gain vs frequency, Room A
3.11.3. Room B

Room B is the same as room A, except a concrete partial wall exists, as can be seen on Figure 3-24. The wall is in the \(xz\) plane, with a height of 1 m and a width of 1 m. It is centred at \((1.5, 2, 1.5)\) m and is 0.2 m thick. Furthermore, the receiver has been moved in the \(-x\) direction to \((1, 3, 1.5)\) m. The effect is that the model is less symmetrical and importantly, the LOS path no longer exists. Both antennas are polarised in the \(Z\) direction. Again, the RL is run for 101 frequency points while the InSite model is run for 21.

![Figure 3-24 – Room B](image)

Room B

Figure 3-25 shows the path gain results for Room B. The two responses are in strong agreement throughout most of the band, although an exceptional minimum was recorded on the InSite model at around 1.04 GHz which does not exist to such an extent on the RL result. This could again be a result of slightly different receiver sphere mechanics or
alternatively it may simply be down to the frequency resolution of the RL response, with
the model not having been run at the exact frequency of that null.

3.11.4. Office Floor

To test the RL in a more complex environment, a floor of a modern office building has
been modelled using both the RL and InSite. A floorplan of the space is shown in Figure 3-26, while a 3D representation of it is in Figure 3-27. It consists of a floor and ceiling as well as two end walls, all made of concrete of 0.3 m thickness. The two outer walls are made from solid tempered glass of thickness 0.03 m. The entire floor is 24 x 12 m and the ceiling is at a height of 3.5 m. The space inside is divided by a central corridor with five rooms on either side. The corridor is 3 m wide, while the rooms are all 4.5 m wide. The left hand side of the corridor consists of an 8 m long room, followed by four 4 m long rooms. On the right hand side of the corridor are four 4 m long rooms, one 3 m long room and an open space of length 5 m, which joins onto the corridor. “Length” is measured in the y direction of Figure 3-26, while “width” is measured in the x direction. (0.03 × 1 × 2) m doors are positioned along the corridor with separation of one, two or three metres between them and end walls. The partial walls which break the floor into offices are made from the same concrete as the end walls which has $\varepsilon_r = 7$ and $\sigma = 0.015$ S/m. The doors are made from wood with $\varepsilon_r = 5$ and $\sigma = 0$, while the tempered glass has $\varepsilon_r = 7.3$ and $\sigma = 0$. 

Figure 3-26 – Floorplan of Office
Figure 3-27 – 3D representation of Office Floor
A transmitter is positioned at the top of the corridor at \((x, y, z) = (0, 23, 1.5)\) m, where the origin of the coordinate system is the centre of the edge where the floor and bottom wall meet. A receiver is positioned in the second room on the right, at \((3, 6, 1.5)\) m. Initially the “shell” of the office is modelled, where only the floor, ceiling and outer walls and windows are included in the model. Co-polar transmission is examined, where both antennas are \(x\) polarised and considered horizontal. The RL is run at 101 frequency points between 0.8 and 1.2 GHz and the InSite model run for 21 points. The results are shown in Figure 3-28. As can be seen there is far more multipath interference over the same frequency band as a result of the larger distances (relative to \(\lambda\)) involved. The results obtained using InSite appear to be in very good agreement with those using the RL.

![Figure 3-28 – Path gain for “shell” of Office Floor model](image)

Next, the full model is considered. The four sub-channels of a dual polarised system are modelled where transmitter and receiver both have dipole elements polarised in the \(x\) and \(z\) direction. These are referred to as horizontal and vertical respectively. The RL is run at 101 frequency points between 0.8 and 1.2 GHz and the InSite model run at 41 points. The results are presented in Figure 3-29 to Figure 3-32, where \(h_{hh}\) is the sub-channel gain between horizontally polarised dipoles, \(h_{hv}\) is the channel between a vertically polarised transmit antenna and horizontally polarised receive antenna and so forth.

Again, the results from the RL are broadly in agreement with the InSite results. The average levels are very similar. There are differences at times in the fast fading behaviour, which could be a result of different receiver sphere mechanisms, or other small differences in the construction of the models, given the relative complexity of the environment.
Figure 3-29 – Path gain for $h_{h,h}$ sub-channel over Office Floor

Figure 3-30 – Path gain for $h_{h,v}$ sub-channel over Office Floor

Figure 3-31 – Path gain for $h_{v,h}$ sub-channel over Office Floor

Figure 3-32 – Path gain for $h_{v,v}$ sub-channel over Office Floor
3.11.5. Field Intensity in Victorian House

As a further validation step, a Victorian terraced house has been modelled, with E-field intensity studied on a horizontal plane through the ground floor. Results are compared to those obtained using FEKO, as part of the work performed at the University of Sheffield, towards the Wireless Friendly and Energy Efficient Buildings (WiFEEB) project [113]. FEKO is an electromagnetic simulator, which utilises geometric optics and the uniform theory of diffraction.

Figure 3-33 – [113] Floorplan of Victorian house which has been modelled

Figure 3-33 shows a floorplan of the house which has been modelled. The floor to ceiling height for all floors is 2.4 m. Many simplifications have been made in the environment modelling for both the bespoke RL and the FEKO simulations. Figure 3-34 shows a 3D representation of the building model used by the RL. As can be seen, other than the walls between the two bedrooms and the two ground floor rooms, no internal walls or partitions are included. All internal doors, other than the door between the front room and kitchen on the ground floor are omitted. All external doors and windows have been replaced with external walls. No furniture or staircases are modelled and the room behind the kitchen is also omitted.

The internal and external walls which are included are modelled as brick, of different thicknesses. The floor of the basement is modelled as external wall, as is the floor of the kitchen. The ceiling and floor (combined) in between the rest of the floors is modelled as a single 10 cm layer of wood. The ceiling of the upper floor is also modelled as a single
layer of wood, with no further materials above it. The relative permittivity, $\varepsilon_r$, conductivity, $\sigma$ and the thickness of the materials are shown in Table 3-1.

A vertically polarised dipole transmitter is positioned in the kitchen, 1.2 m above the ground, 1.3 m from the middle internal wall and 2.1 m from the external wall. The E-field intensity throughout a horizontal plane, 1.3 m above the ground floor is studied. To do so, the plane is discretised into a grid, with a receiver positioned at each location on the grid. At each position, the E-field vectors from all received rays are summed, then the norm of the total is taken to give the E-field intensity. The transmit frequency is 2.4 GHz, with a transmit power of 50 mW.
The E-field intensity with the door closed, in Figure 3-35, is compared to that with the door open, in Figure 3-36. Figures (a) show the result using FEKO, while figures (b) show the result using the bespoke RL. The door attenuates less than the internal wall, so in all cases a “beam” of higher intensity can be seen propagating into the front room, through the door way. When the door is opened, the beam intensity is higher still. Using the RL, the shadowing by the internal wall appears to be slightly more significant than in the results using FEKO, with slightly lower intensity in the areas in the front room which are not directly covered by the “beam” through the door. It is possible that this is because the RL
does not account for fields diffracted around the door frame. Other than the slight
difference in these regions, the two sets of results show very good agreement.

3.12. Summary

The propagation environment plays a major role in the correlation or decorrelation and the
amplitude of signals received over MIMO sub-channels. In order to understand and to
estimate the effect that the propagation environment has on the capacity and performance
of polarisation MIMO systems, it is necessary to use a propagation channel model. To
fully account for the effects of specific environments and to enable the end-to-end system
modelling for compact MIMO systems, a deterministic approach to propagation channel
modelling is favoured.

As such, a bespoke 3D ray launcher has been developed which fully tracks the polarisation
and phase of rays between a transmitter and a receiver. By creating a bespoke model,
greater flexibility in terms of the input and output results and their formats is possible than
would be if a commercial package was used.

The model launches many rays according to a geodesic launch geometry, to maintain as
close to uniform angular separation between rays as possible. This is described in Section
3.4. Rays are traced through the environment according to geometric optics. Rays which
intersect a receiver, which is modelled as a sphere, are recorded and contribute to the
received field. A filter is used in the post processing stage to remove any received rays
which have travelled essentially the same path as other rays. These ‘duplicate’ rays are
identified as rays which have interacted with the same sequence of objects, in the same
order as other rays. Only the ray which passes closest to the receiver centre is kept. This
procedure is described in Section 3.5.2.

The field associated with each ray is launched in the spherical components of the transmit
antenna. It is then transformed into global Cartesian components, before being
transformed again into components of the ray-fixed coordinate system, before application
of transmission and reflection coefficients when rays hit objects. The field is then returned
to Cartesian components ready for the next object intersection or arrival at the receiver,
where it is transformed to spherical components, with respect to the receive antenna. The
procedure for changing the components or “bases” of the electric field is outlined in
Section 3.7. The reflection and transmission coefficients are described in Section 3.6.
After rays are received and filtered, the effect of the transmit and receive antenna gain patterns are applied, based on their AoDs and AoAs. A propagation factor is also applied to account for the total distance a ray travels. At a single frequency, the total received signal at a given location is the complex addition of the contributions by all the unique received rays.

Every intersection a ray makes with the environment is recorded, which allows a post processing script to plot a 3D figure which shows all of the successful ray paths between transmitter and receiver. An example of this is given in Section 3.9. This is a useful tool for gaining a better understanding of the propagation environment behaviour.

The ray launcher is used to estimate the frequency responses of the MIMO sub-channels for the MIMO systems studied in this work. As such it is crucial that the model reliably produces accurate results. To this end, a thorough validation process has been completed and is document in Sections 3.10. and 3.11. Initially, the reflection and transmission functions, then the application of the antenna patterns are tested in isolation. At this stage, results are compared to those calculated directly, those obtained using CST and those obtained using the commercial ray launcher, Wireless InSite. Following this, results are validated for environments of gradually increasing complexity, by comparison to equivalent results obtained using Wireless InSite and an additional electromagnetic simulator, FEKO. The environments are built up from a single reflection off a PEC, then dielectric slab, to a terraced house and an office floor.

The ray launcher is used to estimate the frequency responses of the MIMO sub-channels between the compact antenna models presented in Chapter 2. From the frequency responses, the parameters of the extended polarisation MIMO channel model, described in chapter four are determined.
4. Polarisation MIMO Systems

4.1. Introduction

In a multipath scattering environment, many different paths exist between a transmitter and a receiver. The combination of energy from all these paths results in multipath fading, where constructive and destructive interference causes received signal level to fluctuate rapidly over space, frequency and, if the environment is mobile, time. MIMO systems overcome this problem by placing multiple antennas on both ends of a wireless transmission link. This establishes several separate MIMO ‘sub-channels’, each of which experiences a different fade. In a system with $n_t$ transmit and $n_r$ receive antennas, $n_t n_r$ separate sub-channels exist. Under suitably rich fading conditions, MIMO systems can exploit a theoretical capacity increase which is linear with $n_t n_r$ [1, 2].

Many different MIMO schemes exist which may be used to exploit the capacity increase. Spatial diversity schemes [6, 7, 13, 14, 18, 114-116], such as space-time block codes (STBCs) send multiple copies of the same data over the different sub-channels to increase reliability. Alternatively, spatial multiplexing schemes may be used, which demultiplex the transmission data stream into multiple sub-streams, which are transmitted simultaneously to increase throughput.

This work considers MIMO systems with two transmit and two receive antennas, often referred to as $2 \times 2$ systems. The ray launcher is used to determine the frequency responses of the four MIMO sub-channels over the system bandwidth. These responses are a result of the antenna gain patterns and the propagation through the environment. Properties of these responses are used to construct a novel polarisation MIMO extended channel model. This channel model is used to assess capacity of the MIMO systems, as well as their error rate performance under the Alamouti spatial diversity scheme and the V-BLAST spatial multiplexing scheme. Error rate is estimated using Monte-Carlo system simulation, where the transmission of a large number of symbols over the extended channel model is simulated and the number of errors is counted.

The bespoke polarisation MIMO channel model is an extension of the popular Rayleigh fading model. The sub-channels are correlated through the use of an $n_t n_r \times n_t n_r$ correlation matrix, the elements of which are determined from the correlation between the channel responses obtained by the ray launcher. The presence of any dominant specular
paths in the channel is accounted for through the inclusion of a component with a fixed magnitude, as well as the Rayleigh distributed component. The ratio between the two components is set by the sub-channel’s Rician K-factor. This is the K-factor associated with the Rician distribution of the amplitude of the frequency response obtained for the sub-channel by the ray launcher. The expected gains of the sub-channels are weighted according to the mean sub-channel gains determined by the ray launcher. Each sub-channel has its own K-factor and expected gain. The model also accounts for antenna mutual coupling at the transmitter and the receiver. Transmit mutual coupling is inherently included in the results from the ray launcher. These are obtained using the coupled transmit antenna patterns, which are simulated with both transmit antennas present and loaded with their generator impedance, $Z_g$. At the receiver, the mutual coupling mechanism is slightly different from that at the transmitter, as discussed in Section 2.4. This is because the excitation is from a source in the far-field, rather than at the terminals of either antenna. To allow for this, isolated receive antenna patterns are used by the ray launcher, then receive mutual coupling is accounted for by the extended MIMO channel model, through the use of a receive mutual impedance matrix.

The remainder of this chapter is organised as follows. Section 4.2. introduces the basic MIMO flat fading transmission model which mathematically relates the signals at the terminals of receive antennas to the signals at the terminals of the transmit antennas. Section 4.2.1 describes the Rayleigh fading channel, which accurately models transmission in a rich scattering environment, with well separated co-polar antennas. This is the starting point, from which the extended polarisation MIMO channel model is constructed. Section 4.3. discusses the diversity and multiplexing MIMO schemes which have been simulated over the extended MIMO channel model, to obtain system error rate estimates. Section 4.4. discusses the Monte-Carlo simulation approach. The extended channel model is described in detail in Section 4.5. Section 4.6. discusses the validation of the channel model and the MIMO system simulations. Finally, Section 4.7. summarises the chapter.

4.2. MIMO Flat-fading Transmission Model

The general MIMO transmission model mathematically describes the signals at the terminals of the receive antennas of a MIMO system, in relation to the transmitted symbols, the MIMO channel and the noise.
Figure 4-1 shows a general $n_t \times n_r$ MIMO system, where $n_t$ is the number of transmit antennas and $n_r$ is the number of receive antennas. This system establishes $n_t n_r$ separate MIMO sub-channels between transmitter and receiver. The response of each sub-channel relates the voltage at its receive antenna to the voltage at its transmit antenna. Under narrowband, flat-fading conditions, the response of each sub-channel may be represented by a complex path gain. The gain between transmit antenna $j$ and receive antenna $i$ is denoted $h_{i,j}$. It includes the effects of the antenna patterns, the propagation along all of the scattered paths through the environment as well as any correlation with other sub-channel gains, caused by the environment and the antennas.

$$y_1 = h_{1,1} s_1 + h_{1,2} s_2 + n_1$$
$$y_2 = h_{2,1} s_1 + h_{2,2} s_2 + n_2$$

Here, $s_1$ and $s_2$ are the symbols transmitted from transmit antenna one and two respectively. These are picked from a complex constellation and have the form $s = I + jQ$, where $I$ is the amplitude of the in-phase component and $Q$ is the amplitude of the quadrature-phase component. The time and frequency dependence, $e^{j\omega t}$ is assumed but omitted for clarity. $n_1$ and $n_2$ are the noise samples present at receive antennas one and
two. Assuming this to be additive white Gaussian noise (AWGN), these may be modelled as zero-mean complex Gaussian random variables with variance, $\sigma^2_0$ equal to the noise power.

This may be represented for any MIMO system in matrix and vector form, to give the general MIMO transmission model, as follows.

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (4.3)$$

Here, $\mathbf{y} = (y_1, y_2, ..., y_{n_r})^T$ is the receive signal vector, $\mathbf{s} = (s_1, s_2, ..., s_{n_t})^T$ is the transmit signal vector, $\mathbf{n} = (n_1, n_2, ..., n_{n_r})^T$ is a vector containing the noise samples, where the superscript $^T$ denotes transposition. $\mathbf{H}$ is the $n_r \times n_t$ MIMO channel matrix containing the elements $h_{i,j}$, where $i = (1, 2, ..., n_r)$ and $j = (1, 2, ..., n_t)$, representing the sub-channel gains.

### 4.2.1. Rayleigh Fading Channel Model

The Rayleigh Fading channel model [13, 19, 117] is a statistical channel model, which is frequently used to represent the effects of propagation through a rich scattering environment. It assumes that each sub-channel gain is an independent combination of a large number of statistically independent reflected and scattered paths with random amplitudes and phase [19]. In this way, each element of $\mathbf{H}$ is modelled as an independent zero-mean complex Gaussian random variable, with variance of 0.5 per complex dimension. This gives an expected sub-channel gain, $E[|h|^2] = 1$. In this situation, if transmit power is normalised such that the average symbol energy, $E[|s|^2] = 1$, the signal to noise ratio may be adjusted by adjusting the variance of the noise samples, $\sigma^2_0$ as follows; where SNR is described by the ratio of the energy per symbol to the noise power spectral density (the noise power in 1 Hz bandwidth), $E_s/N_0$.

$$\sigma^2_0 = 1/(E_s/N_0) \quad (4.4)$$

The Rayleigh fading channel matrix is often denoted $\mathbf{H}_w$. It is known to produce accurate results in NLOS environments with rich scattering and sufficient antenna spacing, when all antennas are identically polarised [22]. As such it is useful for comparing the performance of different MIMO schemes. It is also the basis of the bespoke polarisation MIMO channel model which is described in Section 4.5.
4.3. MIMO Schemes

Many schemes exist to exploit the increased capacity of MIMO channels. The schemes fall broadly into two categories. These are: spatial diversity, which improves reliability by sending multiple copies of the same data over the separate MIMO channels and spatial multiplexing, which increases throughput by simultaneously sending multiple different data streams over the separate MIMO sub-channels. These types of schemes are used in many indoor applications, such as WiMAX [118], Wi-Fi [119], LTE and LTE-A [120].

In this work, MIMO performance under the Alamouti spatial diversity scheme and the Vertical-Bell Laboratories Layered Space-Time (V-BLAST) spatial multiplexing scheme is assessed. Many alternatives to these schemes exist, but they are usually essentially derivatives of these two. The evaluation of performance under these two fundamental schemes, may be used as a general indication as to how a system may perform in other spatial diversity or multiplexing schemes. The Alamouti and V-BLAST transmission and detection schemes are described below.

These are “open-loop” schemes, where it is assumed that the receiver has knowledge of the channel, through transmission of a known pilot sequence, but no channel state information is available to the transmitter. In “closed-loop” systems, information describing the channel is fed back from the receiver to the transmitter. This allows the transmitter to apply some pre-filtering to the transmit signals, which exploits the channel knowledge to optimise capacity [6, 121]. While these schemes are not considered in this thesis, the MIMO channel model described in Section 4.5. would be equally valid for their study.

4.3.1. Spatial Diversity

4.3.1.1. Alamouti Code Matrix

The Alamouti scheme uses orthogonal space-time block coding (OSBTC), where transmit symbols are encoded into blocks which spread the symbols over space and time. The ‘block’ is represented by the transmit code matrix. In the case of Alamouti code, a complex orthogonal transmit code matrix is used, as shown below [7, 14]. This consists of the two symbols to be transmitted as well as conjugated and negated copies of them, achieving diversity.

\[
\begin{bmatrix}
s_1 & s_2^* \\
-s_2^* & s_1^*
\end{bmatrix}
\]  

(4.5)
The rows represent transmission time slots, while the columns represent transmit antennas. $s_1$ and $s_2$ are the first and second symbols respectively from the sequence to be transmitted. In the first time slot, $s_1$ is transmitted from the first transmit antenna while $s_2$ is transmitted from the second transmit antenna. This scheme requires two time slots to transmit two symbols, making it a rate one scheme.

Referring to the MIMO transmission model (Section 4.2.), in a system with an arbitrary number of receive antennas, the signals at the $i$th receive antenna during the first and second time slots, $y_i^1$ and $y_i^2$ respectively may be deduced as follows,

$$y_i^1 = h_{i,1} s_1 + h_{i,2} s_2 + n_i^1$$
$$y_i^2 = -h_{i,1} s_2^* + h_{i,2} s_1^* + n_i^2$$

where $n_i^1$ is the noise at the $i$th receiver during the first symbol period and $n_i^2$ is the noise at the $j$th receiver during the second symbol period.

### 4.3.1.2. Maximum Likelihood Detection

Maximum likelihood detection (MLD) [7, 122] is used to estimate the transmitted symbols, $s_1$ and $s_2$ from the received signals $y_i^1$ and $y_i^2$. It is a strategy which requires only linear processing, making it a very attractive approach for use on mobile terminals. It is assumed that the channel is known at the receiver through the use of a training sequence. From this, local values for $y_i^1$ and $y_i^2$ are estimated in the presence of only the channel and no noise, for all possible values of $s_1$ and $s_2$. Once this is done, received symbols are decided based on the shortest squared Euclidean distance between the actual received signals and the estimated ones. That is to say, if the locally estimated values for $y_i^1$ and $y_i^2$ in the presence of no noise are $l_i^1$ and $l_i^2$, then received symbols are decided according to the combination of $s_1$ and $s_2$ that minimises the metric,

$$\sum_{i=1}^{n_r} |y_i^1 - l_i^1|^2 + |y_i^2 - l_i^2|^2.$$  

This is equivalent to minimising the following metric [13], which is dependent on the received signals, the complex channel gains and $s_1$ and $s_2$.

$$\sum_{i=1}^{n_r} \left( |y_i^1 - h_{i,1} s_1 - h_{i,2} s_2|^2 + |y_i^2 + h_{i,1} s_2^* - h_{i,2} s_1^*|^2 \right)$$

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To detect using the above metrics requires comparison of over 16 combinations of the symbols \( s_1 \) and \( s_2 \) for QPSK, or \( M^2 \) combinations for \( M \)-order constellations. Due to the orthogonal nature of the Alamouti code (and other OSTBCs) however, the metric in (4.9) may be expanded and then separated into multiple parts, each being a function of only one symbol. This allows each symbol to be detected individually and reduces the number of values which must be compared for each symbol to the number of symbols in the constellation. The two simplified decoding metrics for the Alamouti scheme are given by (4.10) for the detection of \( s_1 \) and (4.11) for the detection of \( s_2 \), where \( i = 1, 2 \) transmit antennas.

\[
\left[ \sum_{i=1}^{n_r} \left( y_i^1 h_{i,1}^* + (y_i^2)^* h_{i,2} \right) \right] - s_1 \right]^2 + \left( -1 + \sum_{i=1}^{n_r} \sum_{j=1}^{2} |h_{i,j}|^2 \right) |s_1|^2 \tag{4.10}
\]

\[
\left[ \sum_{i=1}^{n_r} \left( y_i^1 h_{i,2}^* - (y_i^2)^* h_{i,1} \right) \right] - s_2 \right]^2 + \left( -1 + \sum_{i=1}^{n_r} \sum_{j=1}^{2} |h_{i,j}|^2 \right) |s_2|^2 \tag{4.11}
\]

The Alamouti scheme and other OSTBCs and their detection schemes are further described in references such as [6, 7, 13, 22]. Their BER performance in a Rayleigh fading channel is given in [13].

4.3.2. Spatial Multiplexing (SM)

SM schemes increase throughput by demultiplexing a single data stream into \( n_t \) independent substreams, which are modulated and transmitted simultaneously, with equal power, by the \( n_t \) transmit antennas. This enables a transmission rate improvement which is linear with \( \min(n_t, n_r) \), requiring no additional power or bandwidth [6].

In a 2x2 MIMO system, a frequently used SM scheme simultaneously transmits the first symbol, \( s_1 \) from transmit antenna one and the second symbol, \( s_2 \) from transmit antenna two. This achieves a transmission rate of one symbol per transmit antenna, per timeslot. When transmission in this way is combined with ordered successive interference cancellation (discussed below), the scheme is referred to as V-BLAST.

For a 2x2 SM system, referring to the MIMO transmission model of equation (4.3), the received signals at the two receive antennas in any one timeslot are as follows.
\[ y_1 = h_{1,1}s_1 + h_{1,2}s_2 + n_1 \]  \hspace{1cm} (4.12)
\[ y_2 = h_{2,1}s_1 + h_{2,2}s_2 + n_2 \]  \hspace{1cm} (4.13)

The optimal detection technique, in terms of minimised error probability, for this system is in fact MLD, as in Section 4.3.1.2. The performance of MLD for SM transmission is analysed in [123]. Under MLD, unlike the OSTBCs used for diversity, orthogonality is not inherent to the transmit signals in SM. This means that the MLD decision metric may not be expanded and separated into orthogonal parts which are only a function of one symbol. Hence, the decision metric requires comparison over \( M^{nt} \) symbol combinations, where \( M \) is the modulation constellation order. As a result of this, MLD requires a lot of processing at the receiver and frequently, sub-optimal detection techniques such as minimum mean squared error (MMSE) or zero forcing (ZF) are used.

4.3.2.1. Zero-forcing Detection

Under ZF (when \( n_t = n_r \)), an estimate of the transmitted signal vector, \( \hat{y} \) is obtained by multiplying the received signal vector, \( y \) by the inverse of the channel matrix, \( H^{-1} \), as follows, before the receive bit stream is then decided through slicing the elements of \( \hat{y} \).

\[ \hat{y} = H^{-1}y = s + H^{-1}n = s + \tilde{n}, \]  \hspace{1cm} (4.14)

If \( n_r \neq n_t \) then \( H^{-1} \) is replaced by the pseudo inverse of \( H \), \( H^+ = (H^H H)^{-1} H^H \), where superscript \( H \) denotes the conjugate transpose.

As can be seen, this cancels out the effect of the channel, which mixes the symbols and thereby forces the interference term(s) on each received symbol to zero. However, it also has the effect of potentially enhancing and introducing correlation to the new noise terms, \( \tilde{n} \) [6, 19, 23]. This means that despite the multiple receive antennas, diversity gain remains at unity.

4.3.2.2. Ordered Successive Interference Cancelling Detection (V-BLAST)

Diversity gain under SM may be slightly improved through the use of successive interference cancellation (SIC) and further through the use of ordered successive interference cancellation (OSIC), which are described in [21, 22, 24].

SIC detection involves detecting one symbol at a time, usually using ZF or MMSE, before subtracting the interference caused by the already detected symbol from the remaining
symbols(s) and then detecting the next symbol. This process is repeated until all symbols are detected. When symbols are detected correctly, SIC reduces the MIMO channel into a set of SISO channels, with diversity gain increasing after each interference cancellation stage [11].

In practice, symbols are not always detected correctly and if they are not, the error will propagate to the detection of the next symbol. The chances of detection error can be reduced however, by choosing the order in which the symbols are detected, according to their signal to interference and noise ratio (SINR), where the effect of other undetected symbols is treated as interference. The symbol with the highest SINR at each stage is chosen for detection, to reduce the chances of error propagation. This form of OSIC detection for SM is commonly referred to as V-BLAST [6, 21, 22]. The recursive algorithm for ZF V-BLAST for an arbitrary number of transmit antennas is given in [24].

4.4. Monte-Carlo MIMO System Simulations

MIMO system error rate performance in this work is evaluated through the use of Monte-Carlo error rate simulations. Monte-Carlo simulations [124] are a particularly useful technique for estimating error rate in communication systems which are too complex for an analytical approach to be feasible [125]. They work by simulating the modulation, coding and transmission of a very large number of data bits over a stochastic channel model. The received signals are then decoded and detected, before the number of bits received in error are counted.

The signals at the receiver are calculated using the MIMO transmission equation (4.3) from Section 4.2. The elements of $\mathbf{H}$ are determined according to the channel model which is introduced in Section 4.5. The elements of the transmitted symbol vector, $\mathbf{s}$ are created from bits chosen at random, then modulated according to a zero-mean quadrature amplitude modulation (QAM) constellation. The elements of the AWGN noise vector, $\mathbf{n}$ are modelled as zero-mean complex Gaussian random variables, with variance equal to the noise power.

The channel elements are modelled as quasi-static, flat fading, where the gain of the elements is constant over the bandwidth of the signal and remains constant for the transmission of one 128 bit frame. A new channel is then generated for the following frame.
The simulations are written in MATLAB, which is a matrix orientated environment. This means the frames may be represented by arrays while the channel may be represented as a matrix, allowing matrix operations to be used to efficiently process the effects of the channel.

4.4.1. Accuracy of Monte-Carlo Error Rate Simulations

The Monte-Carlo approach gives an estimate of BER, $\hat{P}_e$ based on $n_{bits}$ total transmitted bits and $n_{err}$ counted bit errors.

$$\hat{P}_e = \frac{n_{err}}{n_{bits}}.$$  \hfill (4.15)

To find the exact value of $P_e$ would require an analytical approach or infinite $n_{bits}$. Given the complexity of the channel model, an analytical solution would be very difficult and of course it is not possible to perform the simulation for an infinite number of bits. Instead, the estimate, $\hat{P}_e$ is determined, but it is important to understand the accuracy of this approximation. This can be done using its confidence interval. Equations for the confidence interval in a Monte-Carlo BER simulation are given in [126] and further discussed in [124, 125, 127, 128]. The confidence interval gives the difference between $p_1$ and $p_2$, where there is a given (high) probability that, $p_2 \leq \hat{P}_e \leq p_1$. This probability is known as the confidence level, $1 - \alpha_c$. The relationship between confidence level and confidence interval is as follows.

$$\text{Prob}[p_2 \leq \hat{P}_e \leq p_1] = 1 - \alpha_c$$  \hfill (4.16)

As a rule of thumb, $n_{bits}$ should be of the order $10/P_e$. This produces a 95% confidence interval of approximately $(0.5\hat{P}_e, 2\hat{P}_e)$ and is generally considered a reasonable uncertainty [126, 128]. The simulated BER results presented in this thesis were obtained by transmitting a minimum of 5000 blocks (of 128 bits) and until $n_{bits} > 20/\hat{P}_e$. This was done for each BER estimate, at each location along the simulated routes. According to [126, 128], running until $n_{bits} > 10/\hat{P}_e$ should suffice, however to further reduce the confidence interval and improve accuracy, $n_{bits} > 20/\hat{P}_e$ was chosen.

Figure 4-2 shows a typical BER result obtained in this way for QPSK transmission over a Rayleigh fading channel, under 2x2 Alamouti coding. In this example, the worst case 95% confidence interval was calculated as $(0.62\hat{P}_e, 1.53\hat{P}_e)$, at $E_s/N_0 = 20 \text{ dB}$. This is of
course within the suggested reasonable bound of \((0.5\hat{P}_e, 2\hat{P}_e)\) and can be seen to correspond to an \(E_s/N_0\) interval of approximately \(\pm 0.5\) dB.

![Estimated BER and 95% Confidence Interval for 2x2 Alamouti Coded QPSK Transmission over Rayleigh Fading Channel](image)

**Figure 4-2** - Estimated BER and 95% confidence interval, \((p_1, p_2)\) for Alamouti coded QPSK transmission over Rayleigh fading channel vs \(E_s/N_0\)

### 4.5. Polarisation MIMO Extended Channel Model

To assess the capability of the MIMO systems which have been modelled, a novel polarisation MIMO channel model has been developed. This is a statistical model, which has been extended from the basic Rayleigh fading model, introduced in Section 4.2.1. The parameters of the model are the sub-channel power weightings, the correlation between sub-channels, the Rician K-factors of each sub-channel and the receive mutual impedance. These parameters are obtained deterministically, from the antenna models and from the ray launching procedure. This allows the convenient end-to-end channel modelling of any antenna system in any environment, from which MIMO capacity or error rate performance may be determined.

Some existing channel models are discussed in Section 1.2.2. These typically account for some of the above parameters, but not all of them. They also make a number of assumptions which are not necessarily correct for polarisation MIMO systems. For
example, the channel model in [4] assumes that the correlation between sub-channels may be represented by the Kronecker model. However, in the course of this work, as well as in [43], it has been observed that this is not always the case. It also assumes that the power gains of the co-polar sub-channels are equal, as are the cross-polar gains. Furthermore, it does not allow for the possibility of different K-factors on each sub-channel.

The bespoke channel model, described in the remainder of this chapter, accounts for sub-channel correlation, using a full \( n_t n_r \times n_t n_r \) correlation matrix. It allows for differences in power gain as well as K-factor in all sub-channels, using \( n_t \times n_r \) K-factor and sub-channel power weighting matrices. It also optionally allows for transmit and receive mutual impedance to be accounted for through the use of transmit and receive mutual impedance matrices. The model is of a \( 2 \times 2 \) system, however it could be easily extended for systems using more antennas.

The following sub-sections provide an overview of the channel model, before a description of how it is constructed, alongside a discussion of the parameters it requires and how they are obtained from the sub-channel responses generated by the ray launcher.

### 4.5.1. Overview of Model

A MIMO system transmits \( n_t \) signals of a given bandwidth from the transmit antennas. These signals travel through the \( n_t n_r \) MIMO sub-channels, where they become mixed, before they are received by the \( n_r \) receive antennas. Each MIMO sub-channel has its own complex response, which varies over the bandwidth of the signal. The response relates the signal at the terminals of its receive antenna to the signal at the terminals of its transmit antenna. Each response is a result of the combination of many signals from scattered paths between antennas, with each signal having travelled a path of different length and having undergone different interactions with the environment. In the presence of one or more dominant paths, the distribution of the response over the bandwidth is approximately Rician. Supposing there are no dominant paths, the distribution of the response would be approximately Rayleigh. It is quite likely that some correlation will exist between the sub-channel responses, due to similarities in their propagation paths, or as a result of antenna mutual coupling.

The bespoke MIMO channel model represents each sub-channel response as a narrowband complex gain. The gain comprises a random, Rayleigh distributed component which
represents the multipath scattering, and a fixed amplitude component, which represents the dominant, specular paths. The ratio of these is set by the Rician K-factor. This is the K-factor of the distribution of the sub-channel frequency response, generated by the ray launcher, over the signal bandwidth. The Rayleigh parts of the sub-channels are correlated through multiplication with a correlation matrix. The values in the correlation matrix describe the level of correlation between the sub-channel responses, as determined by the ray launcher, over the signal bandwidth. This accounts for the correlation resulting from the propagation environment and the transmit mutual coupling. The receive mutual coupling is not accounted for in the ray launching process, which uses the isolated receive antenna patterns. Instead, the receive mutual impedance is determined using the procedure in 2.4.3 and accounted for in the MIMO channel model, through multiplication with the receive mutual impedance matrix. The magnitudes of the sub-channel gains are weighted according to their average gain over the signal bandwidth, as determined from the channel responses from the ray launcher.

4.5.2. Correlation

The first step in the construction of the channel model is to take a Rayleigh fading channel matrix and apply the correlation. The Rayleigh fading channel matrix, $H_w$ consists of $n_t \times n_r$ independent and identically distributed complex Gaussian random variables with zero mean and variance of 0.5 per complex dimension. This represents transmission through a rich fading environment, with perfect decorrelation of signals by the antenna elements at either end of the transmission. In reality, it is likely that there will be some correlation between the channel elements. This may be due to insufficient separation of antennas at either terminal, a lack of multipath in the environment, or through correlation of the antenna patterns, particularly if they are not perfectly orthogonal in a polarisation MIMO system.

One way of accounting for this correlation in a $2 \times 2$ system is through the use of the Kronecker correlation model [40-42]. This model involves multiplying the Rayleigh channel matrix by the square roots of the transmit and receive correlation matrices, $R_t = [1 \; t^* \; 1]$ and $R_r = [1 \; r \; r^* \; 1]$ respectively, where $t$ and $r$ are the transmit and receive correlation coefficients. This gives the following expression for the correlated Rayleigh fading channel matrix, $H_c$. 

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\[ H_c = R_r^{1/2} H_w R_t^{1/2} \]  \hspace{1cm} (4.17)

The convenience and simplicity of this expression makes this a very popular model, however it relies on the following “Kronecker assumptions”;

i. Correlation, \( t_1 \) between the sub-channels \( h_{1,1} \) and \( h_{2,1} \) from transmit antenna 1 is the same as the correlation, \( t_2 \) between the sub-channels \( h_{1,2} \) and \( h_{2,2} \) from transmit antenna 2. That is, \( t_1 = t_2 = t \).

ii. Correlation, \( r_1 \) between the paths \( h_{1,1} \) and \( h_{1,2} \), which arrive at receive antenna 1 is the same as correlation, \( r_2 \) between the paths \( h_{2,1} \) and \( h_{2,2} \) which arrive at receive antenna 2. That is, \( r_1 = r_2 = r \).

iii. The cross correlation, \( u_1 \) between \( h_{1,1} \) and \( h_{2,2} \) is equal to the product of the transmit and receive correlations, while the cross correlation, \( u_2 \) between \( h_{1,2} \) and \( h_{2,1} \) is equal to the product of the conjugate of the receive correlation with the transmit correlation. That is, \( u_1 = rt \) and \( u_2 = r^*t \).

It was found over the course of the work and also by others [40, 41], that these assumptions are not always correct. For this reason, in the extended MIMO channel model, the full \( n_t n_r \times n_t n_r \) correlation matrix, \( R \) is used. This specifies the unique correlation between all possible sub-channel pairs. The correlation matrix \( R \) is given below. As a visual aide to demonstrate the association between each element of \( R \) and the sub-channels which it correlates, Figure 4-3 shows all of the sub-channels in a \( 2 \times 2 \) system, with each element of \( R \) labelled against a dashed ellipsis around the sub-channels which it correlates.

\[
R = \begin{bmatrix}
1 & r_1 & t_1 & u_1 \\
r_1 & 1 & u_2 & t_2 \\
t_1 & u_2 & 1 & r_2 \\
u_1 & t_2 & r_2 & 1
\end{bmatrix}
\]  \hspace{1cm} (4.18)
Using $\mathbf{R}$, the correlated Rayleigh fading matrix, $\mathbf{H}_c$ is obtained as follows, where $\text{vec}(\mathbf{H}_c^T)$ denotes the vectorisation of the transpose of $\mathbf{H}_c$.

$$\text{vec}(\mathbf{H}_c^T) = \frac{1}{R} \text{vec}(\mathbf{H}_w^T)$$  \hspace{1cm} (4.19)

The transposition is necessary to ensure that the correlation described by the elements of $\mathbf{R}$ is applied to the correct sub-channels. To clarify the vectorisation process, if,

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix},$$  \hspace{1cm} (4.20)

then,

$$\text{vec}(\mathbf{H}^T) = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{2,1} & h_{2,2} \end{bmatrix}^T.$$  \hspace{1cm} (4.21)

In the case that the correlation obeys the Kronecker assumptions, $\mathbf{R} = \mathbf{R}_t \otimes \mathbf{R}_r$, where $\otimes$ denotes the Kronecker product, and (4.17) is equivalent to (4.19).

### 4.5.2.1. Correlation Matrix from Channel Responses

The elements of $\mathbf{R}$ describe the correlations between the sub-channel impulse responses which are obtained by the ray launcher. The ray launcher determines the frequency response of the four MIMO sub-channels. The inverse fast Fourier transform (IFFT) [129] is then used to obtain the channel impulse responses from these (this approach is also used in [130]). From here, the correlation between impulse responses is found according to

![Figure 4-3 – Illustration of correlation in 2 × 2 MIMO system](image-url)
Pearson’s linear correlation coefficient [37, 131]. For instance, let the vectors $\mathbf{x}$ and $\mathbf{y}$ each contain $N$ time domain samples which represent the two impulse responses of which the correlation is to be determined. First, the covariance is found as follows, where $\bar{x}$ denotes the mean of $\mathbf{x}$ and $x_i$ denotes the $i$th element of $\mathbf{x}$, with similar notation used for $\mathbf{y}$.

$$\text{cov}(\mathbf{x}, \mathbf{y}) = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})$$  \hspace{1cm} (4.22)

The correlation, $\rho_{x,y}$ between $\mathbf{x}$ and $\mathbf{y}$ is then found by normalising the covariance, $\text{cov}(\mathbf{x}, \mathbf{y})$ by $\sigma_x\sigma_y$, where $\sigma_x$ and $\sigma_y$ are the standard deviations of $\mathbf{x}$ and $\mathbf{y}$ respectively, as in,

$$\rho_{x,y} = \frac{\text{cov}(X,Y)}{\sigma_x\sigma_y}. \hspace{1cm} (4.23)$$

The correlation between all channel elements is found in this way, giving values which vary between -1 and 1, where -1 indicates a perfect inverse linear relationship, 1 indicates perfect direct linear relationship and 0 indicates that there is no linear relationship. Values in between indicate the degree of correlation.

4.5.3. Rician K-Factor

It is quite likely that the propagation environment may contain random (potentially correlated) multipath scattering, as well as one or more dominant specular, perhaps LOS, paths. In this case, the MIMO sub-channel frequency responses are approximately Rician distributed [6, 132, 133]. The ratio of the received power from the specular paths to the power from the scattered paths is known as the Rician K-factor.

In the extended MIMO channel model, each instance of the channel contains the correlated Rayleigh components in the matrix $\mathbf{H}_c$ as defined in the previous sub-section, while the fixed components are contained in $\mathbf{H}_f$. These are combined as follows.

$$\mathbf{H}_r = \sqrt{\frac{\mathbf{K}}{1 + \mathbf{K}}} \mathbf{H}_f + \sqrt{\frac{1}{1 + \mathbf{K}}} \mathbf{H}_c$$  \hspace{1cm} (4.24)

$\mathbf{H}_r$ is the $n_t \times n_r$ Rician fading channel matrix with correlation, $\mathbf{K}$ is an $n_t \times n_r$ matrix containing the K-factors of each sub-channel, $\mathbf{1}$ is an $n_t \times n_r$ matrix of ones and all operations are performed elementwise. $\mathbf{H}_f$ is the $n_t \times n_r$ matrix containing the fixed
components. These are circularly symmetric complex random variables, with amplitude of one and uniformly random phase. The reason for this is that the magnitude of the fixed components may be estimated from the ray launcher results with a reasonable degree of certainty, however their phase, relative to the phases of the fixed components of the other subchannels is less certain. This is because their phase is dependent upon exact knowledge of the building dimensions and dielectric properties, which for real buildings would be exceedingly difficult to determine.

4.5.3.1. K-factors from Channel Responses

The RL process determines the frequency response over the signal bandwidth for each sub-channel, at each position of interest in space. These frequency responses are a result of the combination of many specular paths of different lengths. Over the frequency band, the combination of these path gains results in a mean component, plus a component which varies randomly over frequency, caused by the change in ratio between wavelength and the physical lengths of the many different paths as \( \lambda \) changes. This effect means that the amplitude of the sub-channel frequency response, \( |R| \) may be characterised by a Rician distribution, with the following probability density function (PDF) [132], where \( I_0 \) is the zero-th order modified Bessel function of the first kind.

\[
 f(R) = \frac{R}{\sigma_d^2} \exp\left\{-\frac{R^2 + k_d^2}{2\sigma_d^2}\right\} I_0\left(\frac{Rk_d}{\sigma_d^2}\right)
\] (4.25)

The distribution is characterised by the non-centrality parameter, \( k_d \) and the scale parameter, \( \sigma_d \). The distribution fitting tool within MATLAB is used to fit the channel gain amplitude response to a Rician distribution and determine these parameters. \( k_d \) is in fact equal to the amplitude of the fixed component of the response, while \( \sigma_d^2 \) is equal to the variance of the response, due to the random components. This means that the K-factor can be determined as [132],

\[
 K(dB) = 10 \log_{10}\left(\frac{k_d^2}{2\sigma_d^2}\right). \tag{4.26}
\]

\( \alpha^2 = E[R^2] \) is the expected receive signal power and may be determined from the distribution using the equation below. This is the power gain to be used in the sub-channel power weighting matrix, \( \mathbf{X} \).
\[ \alpha^2 = k_d^2 + 2\sigma^2 \]  

(4.27)

4.5.3.2. Alternate Approach for Estimating K-factor

Other approaches for estimating K-factor were examined during the development of the channel model. One, which is given in [130] takes the IFFT of the frequency response to get the impulse response. The amplitude of the impulse response plots the magnitudes of the rays vs their delay. The highest peak is considered to be the contribution of the fixed component and the K-factor is the ratio of power from this ray to the sum of the powers of the remaining rays. This approach is limited by the ability to resolve rays with very close delays as well as the creation of undesired side lobes during the IFFT.

4.5.4. Sub-channel Power Weighting

The received power through each MIMO sub-channel, is in general different. This is a result of the different paths taken through the environment by each sub-channel, as well as the different antenna patterns of each antenna in the system and their cross polar discrimination (XPD). The expectation of the power gain of each sub-channel according to the ray launcher is given by \( \mathbf{X} \), where \( \mathbf{X} \) is as follows and its elements, \( \alpha_{i,j} = \sqrt{E[|R_{i,j}|^2]} \) are determined from the parameters of its Rician distribution, according to equation (4.27).

\[
\mathbf{X} = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix}
\]

(4.28)

The elements of the extended MIMO channel model must be weighted using \( \mathbf{M} \), such that their expected gain, \( E[|h_{i,j}|^2] = \alpha_{i,j}^2 \). The elements of \( \mathbf{M} \), \( m_{i,j} \) are determined as follows.

\[
m_{i,j}^2 = \frac{\alpha_{i,j}^2}{E[|h_{r,i,j}|^2]}
\]

(4.29)

Here, \( m_{i,j} \) is the weighting factor for the \( i,j \)th sub-channel, \( \alpha_{i,j}^2 \) is the desired power gain of the \( i,j \)th sub-channel, as determined from the RL results and \( E[|h_{r,i,j}|^2] \) is the expectation of the \( i,j \)th element of the MIMO channel matrix, \( \mathbf{H}_r \) following correlation and application of the K-factors. While the expected power gain of the elements of \( \mathbf{H}_w \) is of course 1, this is no longer guaranteed following the correlation and application of K-factor. Hence, \( m_{i,j}^2 \neq \alpha_{i,j}^2 \).
The power weighted MIMO channel matrix, $H_X$ is obtained from $H_r$ through element-wise multiplication with $M$ as follows, where $\odot$ denotes matrix element-wise multiplication.

$$H_X = M \odot H_r$$ (4.30)

4.5.5. Mutual Coupling

The final stage of the channel construction is the application of the receive mutual coupling. As discussed in Section 2.4, the coupling at the receiver occurs by means of a slightly different mechanism to that at the transmitter. At the transmitter, the excitation is at the antenna terminals, whereas at the receiver, the excitation comes from sources in the far-field. This results in slightly different current distributions on the antennas, which causes different mutual coupling behaviour.

The effects of the transmit mutual coupling are inherently included in the simulated transmit antenna patterns, assuming the antennas are simulated in the presence of each other and are both loaded.

The receive mutual coupling cannot be simulated as easily, because it is dependent upon the spatial distribution of the energy incident on the antennas. To properly account for the receive mutual coupling in the MIMO channel model, in the RL, the ‘isolated’ antenna patterns are used at the receiver. These are the antenna patterns simulated individually, with the other antenna removed (although the case remains present). This obtains the antenna patterns with no mutual coupling. Using these in the RL results produces the channel responses with mutual coupling accounted for at the transmitter, but not the receiver.

The mutual coupling at the receiver is then applied to the MIMO channel model through multiplication with the inverse of the receive mutual impedance matrix, $Z^R$ recalled from Section 2.4, as,

$$Z^R = \begin{bmatrix} 1 & -\frac{Z^R_{1,2}}{Z_{1,1}} \\ \frac{Z^R_{2,1}}{Z_{2,2}} & 1 \end{bmatrix}. \quad (4.31)$$

In (4.31), $Z_{1,1}$ and $Z_{2,2}$ are the input impedances of receive antennas one and two, respectively. $Z^R_{1,2}$ and $Z^R_{2,1}$ are the receive mutual impedances which are determined for the
antennas, at each location within the modelled environment, using the procedure described in Section 2.4.3.

The extended channel model, with coupling accounted for in this way, \( \mathbf{H} \) is obtained from \( \mathbf{H}_X \) as follows [79].

\[
\mathbf{H} = \mathbf{Z}^{R^{-1}} \mathbf{H}_X
\]  
(4.32)

If the transmit mutual impedance was not already accounted for in the RL process, then the transmit mutual impedance matrix, \( \mathbf{Z}_T \) (2.19) could also be included in (4.32), as follows.

\[
\mathbf{H} = \mathbf{Z}^{R^{-1}} \mathbf{H}_X \mathbf{Z}_T^{-1}
\]  
(4.33)

4.5.6. Total Channel Gain

The total power gain of the MIMO channel, \( \mathbf{H} \) is defined as the sum of the power gains of all of its sub-channels [134]. This is equivalent to the squared Frobenius norm of the channel matrix, \( \| \mathbf{H} \|_F^2 \). As a result of the sub-channel power weighting applied in Section 4.5.4, the expectation of the total MIMO channel power gain, \( E[\| \mathbf{H} \|_F^2] \) is equal to the total MIMO channel power gain as determined by the RL, \( \| \mathbf{X} \|_F^2 \).

\[
E[\| \mathbf{H} \|_F^2] = \| \mathbf{X} \|_F^2
\]  
(4.34)

In each MIMO simulation, the total transmit power, \( P_t \) and the noise signal variance at each receive antenna, \( \sigma_0^2 \) are pre-set and remain constant throughout the environment. Signal to noise ratio then varies directly according to \( \mathbf{X} \), throughout the environment. Alternate methods for setting the signal to noise ratio, using different channel normalisation are given in [134].

4.5.7. Channel Capacity

The capacity, \( C \) of the modelled MIMO channel, \( \mathbf{H} \) may be estimated by taking the expectation of its instantaneous capacity given by (1.1), as follows [11, 12].

\[
C = E \left[ \log_2 \left( \det \left[ \mathbf{I}_{n_r} + \left( \frac{P_t}{n_t N_0} \right) \mathbf{HH}^H \right] \right) \right] \text{bps/Hz}
\]  
(4.35)

This is the theoretical maximum error-free data rate, in bits per second per Hz, that the channel can support, assuming there is no knowledge of the channel at the transmitter. \( \mathbf{I}_{n_r} \) is the \( n_r \times n_r \) (\( = 2 \times 2 \)) identity matrix, \( P_t \) is the total transmit power from all
transmitters, \( N_0 \) is the noise power spectral density at each receive antenna and \( H^H \) is the conjugate transpose of the MIMO channel, \( H \).

4.6. Validation

The MIMO system modelling described in this chapter forms the final stage of the end-to-end modelling approach, used to predict the performance of polarisation MIMO systems. As such, it is of critical importance that the approach produces accurate and reliable results. The Monte-Carlo technique of error rate estimation for MIMO transmission and reception is widely used. As a result, there are many references containing published results which show error rate under different MIMO schemes, simulated using this approach. During the development of the MIMO system simulations, some of these were used to validate the implementation of the modulation, encoding, detection and error rate calculation. The simulation of systems using the Alamouti scheme, over a pure Rayleigh and correlated Rayleigh fading channel, under the Kronecker assumptions, was validated against plots of BER versus signal to noise ratio in [13] and [135]. The simulation using V-BLAST was validated against BER versus signal to noise ratio results given in [21] and [6]. Validation in this way provides confidence in the implementation of the MIMO transmission model, the simulated modulation, encoding and detection under each scheme and the error rate estimation using the Monte-Carlo approach.

Of course, there is no existing literature which provides results using the novel polarisation MIMO channel model which is presented in this chapter. For validation, it was therefore necessary to force the model to imitate a channel with conditions under which results have been published. For example, results are presented in [4] which show symbol error rate (SER) under different degrees of transmit and receive correlation in a 2 × 2 MIMO system. The K-factor is 0 for all sub-channels, but the effect of XPD is shown by adjusting the expected magnitude of both cross-polar sub-channel gains between 0 and 1. The expected amplitude of both co-polar gains is fixed at 1. The signal to noise ratio remains constant, according to the ratio \( E_s/\sigma_0^2 = 17 \) dB, where \( E_s \) is the total energy required to transmit one symbol and \( \sigma_0^2 \) is the noise signal variance at each receive antenna. These conditions have been replicated using the novel channel model by adjusting \( t_1, t_2, r_1 \) and \( r_2 \) in the correlation matrix, setting \( \alpha_{1,1} \) and \( \alpha_{2,2} \) to 1 and adjusting \( \alpha_{1,2} \) and \( \alpha_{2,1} \) together to represent the varying XPD. It was necessary to set the receive mutual impedance matrix to the identity matrix and the cross correlation elements of \( R, u_1 \) and \( u_2 \) were set to 0.
Figure 4-4 – Simulated SER vs expected cross-polar sub-channel gain using novel channel model and as published in [4]

Figure 4-4 shows the simulated SER using the novel channel model under these conditions, as well as the published result from [4]. 16-QAM modulation was used with Alamouti coding. Results are plotted against the magnitude of the sub-channel gains, $\alpha_{1,2}^2 = \alpha_{2,1}^2$, for increasing transmit correlation, $t_1 = t_2 = 0.1, 0.7$ and 1. The receive correlation coefficients, $r_1$ and $r_2$ were both set to 0 in the example shown, although the effect of receive correlation with no transmit correlation was also simulated by setting $t_1$ and $t_2$ to 0, while $r_1$ and $r_2$ were set to 0.1, 0.7 and 1. It was found that varying the receive correlation coefficient in this way had a similar effect on SER, producing the same results as shown for the transmit correlation. These results demonstrate the detrimental effect of transmit and receive correlation under Rayleigh fading, when using the Alamouti scheme. Furthermore, they show that high XPD (which corresponds to low $\alpha_{1,2}$ and $\alpha_{2,1}$), as can be the case for dual polar MIMO systems, results in an increase in BER under the Alamouti scheme.

As can be seen, the results obtained using the novel channel model are in strong agreement with those published in [4]. This was also the case when receive correlation coefficients, $r_1$ and $r_2$ were varied and transmit correlation was 0. This provides validation of the model.
in terms of its application of correlation and the treatment of expected sub-channel gain levels.

To validate the application of K-factor and the fixed part of the channel, the transmit correlation coefficients, \( t_1 = t_2 \) were set to 0.5, while the receive correlation coefficients, \( r_1 = r_2 \) were set to 0.3. The K-factor of all sub-channels was varied between 0 and 20, while the XPD remained constant. The magnitude of the expected co-polar sub-channels, \( \alpha_{1,1} = \alpha_{2,2} \) was set to 1 and the magnitude of the expected cross-polar sub-channels, \( \alpha_{1,2} = \alpha_{2,1} \) was set to \( \sqrt{0.6} \). To replicate channel conditions which are equivalent to those modelled in [4], it was necessary to set the cross correlation elements of \( R, u_1 \) and \( u_2 \) to 0, the receive mutual impedance matrix to the identity matrix and to assign an identical phase to all elements of the fixed part of the channel, \( H_f \). The signal to noise ratio remained constant, with \( E_s/\sigma_0^2 = 17 \) dB. Figure 4-5 shows the simulated SER over the extended channel under these conditions. Alamouti coding is used, with 16-QAM modulation. As can be seen, the high K-factors are highly beneficial to error rate under the Alamouti scheme in these conditions. Also shown on Figure 4-5 is the simulated SER versus K-factor under these conditions, as presented in [4]. Again, the results simulated using the novel channel model are in strong agreement with those published.
Summary

This chapter discussed the MIMO system modelling which has been used to obtain the core results of this thesis. A bespoke extended polarisation MIMO channel model has been developed and used to obtain MIMO capacity and BER results under the Alamouti and V-BLAST MIMO schemes. The model is dependent upon deterministic results from the RL propagation model and the antenna simulation, meaning that is capable of estimating MIMO performance for specific antennas and environments.

In Section 4.2, the general narrowband MIMO transmission model is introduced. This relates the signals at the terminals of the receive antennas to those at the transmit antennas, according to the channel matrix with the addition of noise. The Rayleigh fading channel model is also introduced, which accurately models transmission in a rich scattering environment, with well separated co-polar antennas. This model is the starting point for the bespoke polarisation MIMO channel model.

Section 4.3 discusses the MIMO coding and detection schemes which have been simulated over the polarisation MIMO channel, to obtain the end-to-end MIMO BER estimates. The Alamouti and V-BLAST schemes have been used to give an indication of the performance
of the MIMO systems when used for spatial diversity and spatial multiplexing respectively. The Alamouti scheme is an OSTBC which achieves rate one, with a diversity order of four. It has been implemented with maximum likelihood detection. V-BLAST is a spatial multiplexing scheme, capable of increasing throughput by a factor of two. It uses ZF-OSIC detection which allows it to also achieve a small diversity gain.

The error rate estimates are obtained through Monte-Carlo simulation. This is where the simulated transmission of a very large number of bits is performed over a stochastic channel model, and the number of bits received in error is counted. This approach is described in Section 4.4, alongside some considerations regarding the accuracy of such a method.

The novel polarisation MIMO channel model is fully described in Section 4.5. This model completely accounts for sub-channel correlation, specular as well as scattered components, differing expected channel gains across all sub-channels, as well antenna mutual coupling.

This model has many differences when compared to other channel models in existing literature. Correlation is described fully in an $n_t n_r \times n_t n_r$ matrix, rather than through the use of the Kronecker model. All sub channels have independent expected gains and K-factors, which are described in $n_t \times n_r$ matrices. The elements of these matrices are determined from the outcome of the RL, based on a database which describes the environment and the antenna patterns. Mutual coupling is accounted for at the transmitter through the use of antenna patterns simulated with the transmit antennas loaded and in the presence of each other. As discussed in Section 2.4 and Section 4.5.5, the mutual coupling at the receiver is a result of a slightly different mechanism because the antennas are excited by a source in the far-field. The channel model accounts for this using a receive mutual impedance matrix, the elements of which are determined according to the procedure in Section 2.4.3. The expected total power gain of the channel is equal to the channel total power gain determined by the RL.

The MIMO system simulations were validated against published error rate results under the Alamouti and V-BLAST transmission schemes, as described in Section 4.6. To validate the extended channel model, it was necessary to force the channel conditions to replicate those of another channel model, for which error rate results are available in literature. This is also described in Section 4.6.
This extended channel model has been used to obtain the MIMO capacity and error rate results for the modelled MIMO systems, which are presented in the following chapter.
5. Polarisation MIMO in Complex Indoor Environments

5.1. Introduction

In this chapter, MIMO system results are presented for the antenna models which have been developed in Chapter 2. These results are based on the antenna patterns and mutual impedance, with the antennas positioned in an office floor environment. Transmission in line of sight (LOS) conditions is compared to that in non-line of sight (NLOS). The transmitter is in a fixed location, while the receiver is moved along a 2 m measurement route. Propagation between the antennas is modelled using the ray launching propagation model introduced in Chapter 3. The ray launcher estimates the frequency responses of the four MIMO sub-channels. From these, the channel parameters; correlation, K-factor and sub-channel power weightings are determined for use in the polarisation MIMO channel model, alongside the receive mutual impedance, as in Chapter 4. This channel model represents the MIMO channel matrix, $\mathbf{H}$.

Using the model of $\mathbf{H}$, MIMO system capacity is estimated. Furthermore, MIMO system error rate is estimated through Monte-Carlo system simulation of MIMO transmission over $\mathbf{H}$. Diversity performance is evaluated under the Alamouti scheme, as well as multiplexing performance, under V-BLAST.

From the results in Chapter 2, it is clear that the addition of a metal case to the compact antenna models has a significant effect on the behaviour of the antennas. To examine how this affects the MIMO system performance and capacity, results are presented with and without the case present. The transmission bandwidth for the compact narrowband antenna systems is 20 MHz, around the centre frequency of 2.4 GHz.

Without the case present, spatial MIMO systems with antenna separation of $0.25\lambda$, $0.5\lambda$ and $\lambda$ are modelled, as well as co-located dual polar systems, using crossed dipole LP and CP antennas. Additionally, a hybrid system is presented, using orthogonally polarised crossed dipole CP antennas, with separation of $0.25\lambda$. Finally, a system is modelled where the transmitter uses LP and the receiver uses CP antennas.
With the case present, spatial MIMO systems with separation of $0.3\lambda$ and $0.5\lambda$ are modelled. These are compared to the polarisation MIMO systems, using LP and CP co-located coated crossed dipoles.

As a reference, results are also presented for broadband systems, which use full-size, uncoated dipoles and crossed dipoles. The bandwidth of these models is 400 MHz. Broadband spatial MIMO systems have been modelled, where antenna separation is $0.25\lambda$, $0.5\lambda$ and $\lambda$, as well as polarisation MIMO systems, using co-located crossed dipoles for LP and CP. No case is included with the broadband models and mutual coupling is treated in the conventional way at both transmitter and receiver. This means that the antenna patterns used by the ray launcher for both transmitters and receivers are those obtained through the simulation of each antenna as a transmitter, in the presence of the other antenna, which is loaded with $Z_0$. As the coupling is accounted for in this way, $Z_R$ in the MIMO channel model (4.32) for the broadband systems is set to the identity matrix.

This work extends existing literature through the presentation of an end-to-end, predominantly deterministic MIMO modelling approach for compact dual polarised and spatial systems in the indoor environment. Novel analysis directly relates properties of the antenna models, such as their separation and the presence of a case, to the MIMO capabilities of systems utilising these antennas. MIMO capabilities are examined not only in terms of theoretical maximum capacity, but also performance under the Alamouti diversity scheme and the V-BLAST spatial multiplexing scheme.

Existing literature [3, 4] identifies channel conditions under which polarisation MIMO systems are capable of providing a MIMO benefit over spatial systems. The work described in this chapter however, ascertains the channel conditions of practical systems, using deterministic antenna and propagation models. With knowledge of these conditions, MIMO channel capacity and performance is estimated with direct relation to the considered antennas and environment.

The remainder of this chapter is organised as follows. Section 5.2. fully describes the 3D office floor environment model which has been used, as well as the relevant parameters of all objects in the environment. Section 5.3. details any assumptions that have been made in the end-to-end system modelling which have not already been discussed in earlier chapters. Section 5.4. discusses the channel parameter results. These are the intermediate results, obtained from the ray launcher and input to the MIMO channel model. These results are
extensive and as such are presented in Appendices A and B, with a summary of them provided in Section 5.4.1. The MIMO channel capacity results are presented in Section 5.5. Section 5.6. gives the MIMO diversity results, using the Alamouti scheme, while Section 5.7. provides the MIMO spatial multiplexing results, using the V-BLAST scheme. Finally, Section 5.8. provides a summary of the chapter.

5.2. The Office Floor Environment

The capacity and error rate results are evaluated in an environment modelled to resemble a floor of an office building. The transmitter is in a fixed location, while the receiver is moved along two measurement routes. Figure 5-1 shows a floorplan of the environment, while Figure 5-2 presents a 3D visualisation.

![Figure 5-1 – Floor plan of the modelled office floor environment](image-url)
The space consists of multiple small rooms positioned either side of a corridor which runs parallel to the $Y$ axis. To the left of the corridor is an 8 m long room, followed by four 4 m long rooms, where length is measured in the $Y$ direction. To the right of the corridor are four 4 m long rooms, followed by a 3 m long room, then an opening which joins the corridor. The rooms on both sides are 4.5 m wide and the corridor is 3 m wide, where width is measured in the $X$ direction. The floor to ceiling height is 3.5 m throughout, which is measured in the $Z$ direction. The entire environment measures $12 \times 24 \times 3.5$ m.

The external walls, floor and ceiling are made of concrete block of 0.2 m thickness. The internal walls are made of plasterboard with 0.1 m thickness. 11 wooden doors of $2 \times 1 \times 0.03$ m are included between the rooms and the corridor. Table 3-1 shows the material, relative permittivity, $\varepsilon_r$, conductivity, $\sigma$ and thickness of the objects in the environment. Permittivity and conductivity are chosen according to [136], at 2.4 GHz.

**Table 5-1 – Office floor environment building material properties**

<table>
<thead>
<tr>
<th>Object</th>
<th>Material</th>
<th>$\varepsilon_r$</th>
<th>$\sigma$ (S/m)</th>
<th>Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>External walls</td>
<td>Concrete block</td>
<td>5.31</td>
<td>0.066</td>
<td>0.2</td>
</tr>
<tr>
<td>Floor and ceiling</td>
<td>Concrete block</td>
<td>5.31</td>
<td>0.066</td>
<td>0.2</td>
</tr>
<tr>
<td>Internal walls</td>
<td>Plasterboard</td>
<td>2.94</td>
<td>0.02155</td>
<td>0.1</td>
</tr>
<tr>
<td>Doors</td>
<td>Wood</td>
<td>1.99</td>
<td>0.01201</td>
<td>0.03</td>
</tr>
</tbody>
</table>
The environment is represented using Cartesian coordinates, with the origin at floor level, at the start of the corridor as can be seen in Figure 5-1 and Figure 5-2. The transmitter location is specified as \((x, y, z) = (-0.1, 23, 1.35)\) m. This is the position of centre of the first transmit antenna. For the co-located systems, it is also the centre of the second antenna. For the spatial systems and the hybrid system, antenna separation between the centre of antenna one and antenna two is measured in the \(+X\) direction, from antenna one. When considering the orthogonally polarised systems, antenna one and two are horizontally and vertically polarised, respectively in LP systems and RHC and LHC, respectively in the CP systems. The polarisation sense is specified in the \(+Y\) direction.

The start positions for the LOS and NLOS receiver routes are \((x, y, z) = (-0.1, 2, 1.35)\) m and \((x, y, z) = (3, 14, 1.35)\) m, respectively. Similarly, these are the positions that receive antenna one is centred on at the start of the measurement route. The separation between the receive antennas is also measured in the \(+X\) direction. The receivers are both moved from these start positions, in the \(+X\) direction, along the 2 m measurement route in 5 cm increments. This results in 41 spatial samples. At each sample location, the ray launcher obtains the frequency responses of the four MIMO sub-channels, between the antenna locations, using the simulated antenna patterns. For the compact antenna systems, the frequency responses are taken over the 20 MHz signal bandwidth, using 51 samples, separated by \(\delta f = 400\) kHz. For the full size antennas, the signal bandwidth is 400 MHz, over which the frequency responses are taken using 201 samples with \(\delta f = 2\) MHz. It is crucial that the frequency resolution is sufficient to fully capture all of the multipath fading detail on the responses.

### 5.3. Assumptions

The results presented in this chapter are obtained through the end-to-end modelling of an inherently complex system. As such, a number of simplifications and assumptions are made. Those that are specifically relevant to the antenna modelling, propagation modelling or MIMO channel modelling are identified in the previous chapters. The following points however are intended to clarify any additional assumptions which apply to the overall approach.
• All antennas are assumed perfectly matched over the signal bandwidth. In reality this is not the case, the antennas have a return loss caused by the differences in the impedance between the antennas and their feed. The return losses of all of the antennas considered in this chapter are below -10 dB throughout the signal bandwidth. As such, for simplicity they are assumed to be zero.

• Antenna efficiency is assumed to be 100%. While this is not the case for practical antennas, particularly with regards to compact antennas [60, 66], the emphasis of the thesis is on the polarisation of the systems, rather than antenna design. The assumption of 100% efficiency is the “best case”, which is used to examine the viability of the systems without requiring extensive antenna design work to increase efficiency.

• All antenna patterns are assumed to be constant over the signal bandwidth. In reality, this is not the case, however the changes over the bandwidths of the modelled systems are small and their shapes remain very similar.

5.3.1. Signal to Noise Ratio

The signal to noise ratio (SNR) of the modelled systems, defined as the ratio between the total power received by all antennas and the noise power per receive antenna is given as follows, where \( P_t \) is the total transmit power from all antennas, \( \sigma_0^2 \) is the variance of the noise signal at each receiver and \( \|H\|_F^2 \) is the squared Frobenius norm of the channel.

\[
SNR = \frac{P_t}{n_t \sigma_0^2}
\]  

(5.1)

The expectation of the squared Frobenius norm of the channel is equal to the total channel gain, as determined from the ray launcher results, \( E[\|H\|_F^2] = \|X\|_F^2 \). This shows how the system signal to noise ratio is set according to the propagation conditions of the environment and the antenna gains. As a result, SNR varies over the measurement route and is different over the same route, when different antennas are used. In the end-to-end system simulations, \( P_t \) and the noise power, \( \sigma_0^2 \) are held constant over the measurement route for all like-for-like comparisons. \( \sigma_0^2 \) is set to a value for each MIMO scheme which results in comparable BERs, given \( \|X\|_F^2 \). Table 5-2 shows the values which \( \sigma_0^2 \) is set to for each system, under each MIMO transmission scheme, as well as for the capacity evaluation.
Table 5-2 – Noise power for each MIMO system

<table>
<thead>
<tr>
<th>Antenna System</th>
<th>$P_t/\sigma_0^2$ (dB)</th>
<th>Alamouti</th>
<th>V-BLAST</th>
<th>Capacity Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LOS</td>
<td>NLOS</td>
<td>LOS</td>
<td>NLOS</td>
</tr>
<tr>
<td>Broadband</td>
<td>-57</td>
<td>-67</td>
<td>-67</td>
<td>-77</td>
</tr>
<tr>
<td>Narrowband, without case</td>
<td>-57</td>
<td>-67</td>
<td>-61</td>
<td>-74</td>
</tr>
<tr>
<td>Narrowband, with case</td>
<td>-63</td>
<td>-73</td>
<td>-67</td>
<td>-80</td>
</tr>
</tbody>
</table>

5.4. Channel Parameter Results

In this chapter, the end-to-end MIMO system capacity and error rate results are presented. These are obtained using the MIMO channel model, presented in Chapter 4. The inputs to the channel model are the channel parameters; correlation, K-factors, sub-channel expected power gains and the receive mutual impedance. The correlation, K-factors and power gains are contained in the matrices, $R$, $K$ and $X$, respectively. These are determined from the sub-channel responses using the procedures in Section 4.5, for each position on the receiver routes. These intermediate results are included in Appendix A. Plots against distance along the measurement routes are included in both the LOS and NLOS environments, for the uncoated broadband antennas, the coated narrowband antennas with no case and the coated narrowband antennas with the case. All elements of these matrices are plotted, to give a full insight into the behaviour of the channel. In addition, a summary of the key observations is given at the end of this sub-section.

In the “Elements of $X$” plots, the average sub-channel gains taken over the frequency band at each location in space, $\alpha_{i,1}, \alpha_{i,2}, \alpha_{2,1},$ and $\alpha_{2,2}$ are plotted against distance, where $\alpha_{i,j}$ is the average gain over the frequency band of the sub-channel impulse response, $R_{i,j}$ which is calculated using (4.27). This becomes the expected magnitude of the narrowband sub-channel gain, $h_{i,j}$ in the MIMO channel model. In the “Elements of $K$” plots, the sub-channel K-factors are similarly plotted over distance, where $K_{i,j}$ is the K-factor of the frequency response, $R_{i,j}$, calculated according to (4.26) and applied to sub-channel $h_{i,j}$ of the MIMO channel model. The elements of $R$ are also plotted individually, labelled according to,
\[ \mathbf{R} = \begin{bmatrix} 1 & r_1 & t_1 & u_1 \\ r_1 & 1 & u_2 & t_2 \\ t_1 & u_2 & 1 & r_2 \\ u_1 & t_2 & r_2 & 1 \end{bmatrix} \]  

(5.2)

These individual sub-channel parameter results are very useful when studying the behaviour of each sub-channel. For more of an overview, Appendix B presents the same channel parameter results, averaged over the four sub-channels. The “Average Sub-channel Gain” plots show the average sub-channel gains vs distance for all systems, where average sub-channel gain, \( \text{mean}(\alpha) \) is calculated as follows.

\[
\text{mean}(\alpha) = \frac{1}{n_t n_r} \sum_{i=1}^{n_r} \sum_{j=1}^{n_t} \alpha_{i,j}^2 = \frac{1}{n_t n_r} \| \mathbf{X} \|_F^2
\]

(5.3)

This result is \(1/n_t n_r\) times the total received power by the system, making it a useful metric when considering the differences in total received power between the systems with different antenna configurations.

The “Average Sub-channel K-factor” plots are produced in a similar way, where average K-factor, \( \text{mean}(K) \) is presented against distance and calculated as follows.

\[
\text{mean}(K) = \frac{1}{n_t n_r} \sum_{i=1}^{n_r} \sum_{j=1}^{n_t} K_{i,j}
\]

(5.4)

The “Average Sub-channel Correlation” plots show the average of the elements of \( \mathbf{R} \), excluding the diagonal elements which are all equal to 1. This is calculated as follows, where \( \rho_{i,j} \) is the \( i, j \)th element of \( \mathbf{R} \).

\[
\text{mean}(\mathbf{R}) = \frac{1}{(n_t n_r)^2 - n_t n_r} \sum_{i=1}^{n_r} \sum_{j=1, i\neq j}^{n_t} \rho_{i,j}
\]

(5.5)

Appendix B includes plots of these average parameters against distance, as well as their cumulative distribution functions (CDFs) over the distance, which are useful for gaining a general overview of their behaviour in the two environments.

The remaining channel parameter which is required by the MIMO channel model is the receive mutual impedance. As stated in Chapter 2, this does not change significantly over the measurement route. As such, its average values over the route are used in the MIMO channel model. These are given for all of the coated antennas in Chapter 2. It should be
noted that the receive mutual impedance of the co-located CP antenna is the same when the co-located LP transmitter is used as when the co-located CP transmitter is used. This is because in both cases the incident angular power distribution at the receiver is identical, which results in the same weightings being applied in the weighted average receive mutual impedance calculation of equation (2.25).

5.4.1. Overview of Channel Parameter Results

5.4.1.1. Broadband Uncoated Antennas

The full size, uncoated antennas are studied as a reference case for comparison with the compact, coated antennas. Owing to their larger size, they may be operated with a significantly greater bandwidth than their compact counterparts. The full size antenna systems are modelled with a bandwidth of 400 MHz, while the compact antennas are modelled with a bandwidth of 20 MHz.

The difference in bandwidth leads to significantly different observations on the channel parameters. Importantly, the K-factors for the broadband systems are markedly lower than those over the narrow band. The reason for this is clear from the sub-channel frequency responses. Figure 5-3 shows frequency responses of the four MIMO sub-channels of the broadband uncoated crossed dipole CP antennas, at the start of the LOS measurement route. Figure 5-4 shows the responses of the narrowband coated crossed dipole CP antennas in the same location. The effects of considerable multipath fading over frequency are evident on the broadband plot, with signal fluctuations about the mean of up to 25 dB. Over the narrowband responses on the other hand, the signal only fluctuates with amplitude of around one dB on the cross-polar channels and no more than 8 dB on the co-polar channels. As a result, the K-factors of the broadband systems are typically below 6 dB in LOS and below 3 dB in NLOS, whereas the narrowband compact antenna systems have K-factors frequently in excess of 15 dB.

From the CDFs in Appendix A, it can be seen that the average K-factors of the dual polar broadband systems in LOS are generally lower than those of the spatial systems. This is because the cross polar sub-channels have a higher dependence on multipath propagation than co-polar sub-channels. In NLOS, the lack of a direct path results in lower K-factors for all of the broadband systems, with little difference between the dual polar and spatial configurations.
When studying the sub-channel gains, it can be seen that the dual polar systems typically receive less total power than the spatial systems. This is to be expected as two of their sub-channels are cross-polar and, even in the NLOS environment, leakage between polarisations due to scattering is not sufficient to result in similar gains across all four sub-channels.

The average (and therefore total) received power for both of the dual-polar systems is exactly the same. This is because they both use the same physical antennas, transmitting with the same power levels. The feed arrangement is the only difference between the LP and CP systems.

In LOS, the dual-polar average sub-channel gain is less than that of the spatial systems with separation greater than $0.25\lambda$ and of similar level to the $0.25\lambda$ system. The reason that the average gain of the $0.25\lambda$ spatial system is lower than the other spatial systems is due the proximity of its antennas. The close proximity results in lower spatial diversity, increasing the likelihood both antennas experience deep fades simultaneously. Furthermore, it can be seen from the pattern of these antennas at $0.25\lambda$ separation (Figure 2-12) that the coupling at this proximity results in decreased gain in the direction of the direct path between transmitter and receiver, when compared to the patterns at greater separation.

The average gain of the spatial system at $\lambda$ separation in LOS shows the least fluctuation over distance. This is because the greater separation reduces the chances of both antennas being in a deep fade at the same time.
The spatial system with $0.5\lambda$ separation achieves the highest average gain over the LOS route. This is in agreement with the observation in [92], recalled from Section 2.7.1, that mutual coupling may result in an increase in total received power, if the following conditions are met:

i. Element spacing is between $0.4\lambda$ and $0.9\lambda$

ii. The environment is one of directional scatting conditions

iii. The antenna array is orientated orthogonally to the main direction of arrival.

In the NLOS environment, condition iii. is not met and the average gains of the spatial systems with $0.5\lambda$ and $\lambda$ separation are similar. The $0.25\lambda$ system receives less power than these, but not as little as the dual polar systems.

The average sub-channel correlation of the dual polar systems is generally lower than that of the spatial systems, in both environments. The spatial systems have correlation across all sub-channels, which tends to increase as separation is decreased. In LOS, average correlation over the measurement route is highest on the system with separation at $0.25\lambda$, closely followed the $0.5\lambda$ system, both of which experience average correlation greater than 0.9 over considerable portions of the measurement route. This level is to be expected, particularly in the LOS environment, where closely spaced co-polar antennas result in sub-channels that are physically very similar. When separation is increased to $\lambda$, the average correlation decreases and is below 0.6 over 60% of the route.

The average correlation of the LP system is the lowest. It should be noted however that this is brought down by the elements, $r_1$, $r_2$, $t_1$ and $t_2$ which are particularly low. These are the correlations between co-polar and the (lower gain) cross-polar channels. $u_1$, which is the correlation between the two co-polar channels is generally higher than these, as is $u_2$, which is the correlation between the two cross-polar channels. In the case of the CP dual polar system, the correlations, $u_1$ and $u_2$ between the co-polar and the cross-polar sub-channels respectively are approximately one over the entire distance. This results in a higher average correlation for the CP system than the LP, in both LOS and NLOS. The high correlations, $u_1$ and $u_2$ are also evident from the CP frequency responses in Figure 5.3. Furthermore, it can be seen that these sub-channels are well correlated over space, as well as frequency, from their sub-channel gain (elements of $X$) plots in Appendix A. The sub-channel gain, $\alpha_{1,1}$ is very close to $\alpha_{2,2}$ throughout both measurement routes.
In NLOS, the spatial systems all experience lower average correlation than in LOS. This is due to the increased dependence upon scattered contributions, in the absence of a direct path. The average correlation is highest with $0.5\lambda$ separation, where average correlation is greater than 0.7 for around 50% of the simulated distance. Interestingly, with $0.25\lambda$ separation, average correlation is significantly lower than this, below 0.6 for 80% of the route. This lower correlation could be a result of increased pattern diversity due to mutual coupling.

The LP system has the lowest average correlation in NLOS, where it is below 0.3 over 80% of the route. Again, this is brought down to an extent by the correlations between cross and co-polar channels, however the correlation between the co-polar channels is significantly lower in NLOS than in LOS, remaining below 0.5 over most of the distance.

The average correlation of the CP system in NLOS is comparable to that of the spatial system with $\lambda$ separation, however it is dominated by the elements $u_1$ and $u_2$, the correlation between the co-polar sub-channels and the cross-polar sub-channels respectively, which are close to one throughout.

The final observation of the broadband channel parameters is the difference between the levels of the co-polar gains and the cross-polar gains of the dual polar systems. This is an indication of the channel cross polar discrimination (XPD). For the LP system, the difference is close to 40 dB in LOS and 20 dB in NLOS. XPD of this level may be beneficial to SM systems under high transmit correlation or high K-factors, where the cross-polar sub-channels present interference. For diversity systems however, high XPD equates to a reduction in diversity branches. The XPD of the CP system is lower than that of the LP system, at approximately 20 dB in LOS and only around 4 dB in NLOS.

5.4.1.2. Compact Narrowband Antennas without Case
The main difference between the channel parameters of the broadband systems and those of the narrowband systems using compact antennas is in the sub-channel K-factors. Over the significantly narrower band, the field combination of many scattered paths results in far less amplitude variation around the mean level than is apparent on the broadband responses, as illustrated by Figure 5-4. This results in substantially higher K-factors, meaning that the MIMO channel is dominated by the fixed components in $H_f$, weighted by
the sub-channel power levels in $X$. The average K-factors over the narrow band are predominantly in the range of 15 to 20 dB in LOS and 10 to 15 dB in NLOS.

As with the broadband parameters, there is a tendency for the average sub-channel gains of the dual polar systems to be lower than those of the spatial systems, due to the low gain cross-polar channels. The difference is less striking though, because the sub-channel gains fluctuate with a greater amplitude over the distance, as can be seen from the spread of the average sub-channel gain CDFs in Appendix B. This happens because the expected sub-channel gains, $\alpha_{i,j}$ are taken over the smaller bandwidth. The narrow bandwidth means that the multipath fading is less apparent over frequency in each individual spatial location, but is more apparent over space. A challenge therefore for diversity systems in these conditions is to maintain high average received power, in spite of the rich multipath fading over space. This is analogous to minimising the sub-channel correlation over space, so that multiple sub-channels do not experience deep fades in the same place.

It can be seen that in LOS, the average sub-channel gain fluctuations over distance for the system with $\lambda$ separation are of lower amplitude than when separation is $0.25\lambda$, or a co-located dual polar system is used. This is because the sub-channel gains, using spatially separate antennas, vary more independently than those using co-located orthogonally polarised antennas or spatially separate antennas with limited separation. In NLOS, the average sub-channel gains of all systems fluctuate to a similar extent (which is lower than in LOS). This is because the richer multipath in NLOS reduces the depth of multipath nulls on individual sub-channel gains. Furthermore, it results in lower correlation between sub-channel gains over distance, reducing the probability of experiencing nulls on multiple sub-channels simultaneously.

Consultation of the average sub-channel correlation plots show that in LOS, there is a trend for lower correlation of impulse responses with increased antenna separation. Although this correlation is of less relevance when K-factor is high, with reference to the plots of individual elements of $X$ over distance, it appears that the trend is also true with respect to the correlation over space. In NLOS, as observed for the broadband systems, the sub-channels of the spatial systems are all well decorrelated due to the increased dependence on multipath scattering. This is true of the impulse responses as well as over space.
The average sub-channel gains of the co-located dual polar systems using crossed dipoles are again the same at each location in space. This is true for the LP and the CP systems, as well as the LP to CP system, which uses a dual polar LP transmitter to a dual polar CP receiver. Their differences lie in the individual sub-channel expected power gains. Figure 5-5 to Figure 5-8 show the expected sub-channel gains (contained in the matrix, \( \mathbf{X} \)) versus distance in the LOS environment for the co-located LP, co-located CP, CP spatial hybrid and the LP to CP systems respectively.

**Figure 5-5** – Expected sub-channel gains vs distance for co-located compact narrowband LP system in LOS

**Figure 5-6** – Expected sub-channel gains vs distance for co-located compact narrowband CP system in LOS
The co-located LP system has very high XPD of around 40 dB, which reduces the number of effective diversity branches to two. The correlation between the expected co-polar gains over space is high, although they are not identical. In NLOS, this correlation reduces, as does the XPD, to around 20 dB.

For the co-located CP system, the XPD is much lower and in certain locations in LOS (around 0.4, 0.8 1.3 and 1.8 m), reduces to zero. It can also be seen that the correlation between the co-polar channels as well as between the cross polar channels is approximately one. This is true over the impulse responses and over space. Furthermore, it is true of the co-located CP system (without the case) in LOS and NLOS.

The hybrid system uses orthogonally polarised crossed dipole CP antennas, separated by 0.25λ. This adds an element of spatial diversity to the polarisation diversity and results in XPD which is comparable to the co-located CP system, but with lower correlation of all sub-channels over space and over their impulse responses. The cross-polar gains are still
predominantly lower than the co-polar gains though, which is disadvantageous under diversity schemes, when compared to the purely spatial MIMO systems.

The co-located LP to CP system was modelled with the intention of achieving similar gains over all sub-channels as well as low correlation. Because all sub-channels are between antennas of different polarisations, this system is most dependent upon multi-path propagation. As a result, in the NLOS environment it has the lowest average K-factor when presented on the CDF and in LOS it has the joint lowest, with the 0.25\(\lambda\) system. That being said, the average K-factor remains above 5 dB over 80% of the route, in NLOS and above 13 dB over 80% of the route in LOS, so the channel is still dominated by the fixed components in \(\mathbf{H}_f\). This system also achieves the lowest average sub-channel correlation between impulse responses, which is below 0.6 throughout the entire NLOS route and below 0.5 throughout the LOS route. It can be seen from Figure 5-8 however, that the sub-channel gains of \(h_{1,1}\) and \(h_{2,1}\) as well as \(h_{1,2}\) and \(h_{2,2}\) in LOS are very highly correlated over space. This is also the case in NLOS, although not quite to the same extent. The result of this is that, for at least half of the distance simulated over both routes, there is a significant (in excess of 5dB) difference between the two sub-channels departing from antenna one and those departing from antenna two. Unlike in the case of high XPD (where two sub-channels are also high gain and two are low gain), this reduces the determinant of \(\mathbf{HH}^H\), which is detrimental to capacity.

5.4.1.3. Compact Narrowband Antennas with Case

Spatial MIMO systems have been modelled in the presence of a case, with antenna separation of 0.5\(\lambda\) and 0.3\(\lambda\). The 0.3\(\lambda\) system was modelled because this distance is optimum in terms of \(\rho_{env}\) and \(S_{2,1}\). These antenna models are described in Section 2.7.2.2. The co-located dual-polar LP and CP antennas have also been modelled in the presence of a case, as described in Section 2.7.3.2. The position of these antennas along the top of the case was also chosen to minimise \(\rho_{env}\) and \(S_{2,1}\). As observed throughout Section 2.7.2 and 2.7.3, the presence of a metal case has a substantial effect upon the radiation behaviour of nearby antennas, particularly in terms of their directivity, polarisation and the orthogonality of fields radiated by supposedly orthogonally polarised antennas. In this chapter, the resultant effects of the proximity of the metal case, in terms of MIMO capacity and error rate performance are examined.
The MIMO channels between the compact antennas in the presence of the case again experience very high K-factors, meaning that they are dominated by the specular components in $H_f$. The K-factors are typically between 10 and 25 dB in LOS and between 5 and 15 dB in NLOS. This means that the correlation of the sub-channel impulse responses, described by $R$ has little influence. The MIMO capabilities are therefore predominantly governed by the total received power levels and the relative sub-channel power levels.

Recalling Section 2.7.2.2, the antennas of the spatial system with $0.3\lambda$ separation, in the presence of the case, radiate with elliptical polarisations which are approximately orthogonal in the direction perpendicular to the antenna array orientation. As such, in LOS this system has similarities to a deliberately dual-polar system, where $\alpha_{1,1}$ and $\alpha_{2,2}$ are typically at higher levels than the “cross-polar” $\alpha_{1,2}$ and $\alpha_{2,1}$. This is an unexpected effect, which increases the determinant of $HH^H$, but also decreases the total received power by the system. With $0.3\lambda$ separation, $\alpha_{1,1}$ and $\alpha_{2,2}$ are typically around 5 dB higher than $\alpha_{1,2}$ and $\alpha_{2,1}$, in LOS. When separation is increased to $0.5\lambda$, the antennas still radiate orthogonal fields in the same direction, but this extends over a narrower solid angle. Also, the increased separation results in more independent fading of all elements of $X$, over space. This reduces XPD and also reduces the probability of both experiencing a deep fade simultaneously, resulting in higher total received power by the $0.5\lambda$ system than the $0.3\lambda$ system over the majority of the LOS route.

In NLOS, the multipath is richer, meaning the additional separation of the $0.5\lambda$ system has less of an impact on the average sub-channel gains than in LOS, as the elements of $X$ fade more independently on both systems. Furthermore, the spread of arrival angles is broader in NLOS, which means antenna directivity has less of an effect on received power levels. As such, the average sub-channel gains of all systems fluctuate with a lower amplitude in NLOS, and overall the differences in total received power by each system are less.

When the co-located dual LP antenna is modelled with the case, as described in Section 2.7.3.2, the case has a significant impact on its directivity, however the polarisations of its radiated fields remain approximately orthogonal over considerable angles around the $\pm Y$ direction (as shown in Figure 2-34). As such, in LOS this system has considerable XPD, with $\alpha_{1,1}$ and $\alpha_{2,2}$ typically in the region of 20 dB above the cross-polar sub-channel gains. As a result of these low gain cross-polar sub-channels, this system receives the lowest
overall total power along the LOS route, typically receiving around 3 dB less than the CP or $0.3\lambda$ spatial systems.

The CP antenna on the other hand does not maintain its orthogonality in the presence of the case, over most directions, as shown in Fig 2-35. This results in low XPD in the LOS environment and total received power which is typically around 3 dB higher than that of the LP system and similar to that of the $0.3\lambda$ spatial system.

In NLOS, the differences in total received power are less significant, with the LP and CP systems very similar over most of the distance, and the spatial systems receiving between 0 and 2 dB additional total power.

5.5. MIMO System Capacity

In this section, MIMO channel capacity results are presented for the broadband uncoated antennas, the coated antennas with no case and the coated antennas with the case. This is the expected theoretical maximum channel capacity, with no channel state information at the transmitter, as below [134].

$$C = E \left[ \log_2 \left( \det \left( I_{nr} + \left( \frac{P_t}{n_t\sigma_0^2} \right) HH^H \right) \right) \right] \text{bps/Hz} \quad (5.6)$$

$H$ is the MIMO channel model, as described in Chapter 4, while $P_t$ and $\sigma_0^2$ are set according to Table 5-2.

5.5.1. Broadband Uncoated Antennas

5.5.1.1. LOS

![Figure 5-9 – MIMO channel capacity of broadband systems using uncoated antennas in LOS](image)
Figure 5-10 – CDFs of MIMO channel capacity of broadband systems using uncoated antennas in LOS

Figure 5-9 shows the estimated capacity of the broadband systems over the LOS route, while Figure 5-10 shows a CDF of these results. The capacity has a high dependence upon total received power, hence the features of Figure 5-9 are very similar to those of the average sub-channel gain plot for these systems in LOS, in Appendix B.

Importantly though, it can be seen that the capacity of the dual polar systems exceeds that of the 0.25λ spatial system throughout over 90% of the route. These systems all have very similar total received power throughout the distance, so this is a clear example of greater capacity as a result of lower correlation, despite similar received power levels, for the dual polar broadband systems, compared to the spatial system with low antenna separation.

The greatest capacity is achieved by the spatial system with 0.5λ separation. This system suffers from correlation levels close to those of the 0.25λ system, however it benefits from the exceptionally high average sub-channel gain, caused by mutual coupling, given the antenna separation and orientation relative to the direct path.

The spatial system with λ antenna separation, which experiences significantly lower correlation than the other two spatial systems, achieves the second highest capacity over most of the distance. It also suffers least from capacity variation due to fast fading over space, as can be seen from the width of its CDF. This is because its greater antenna separation reduces correlation of the sub-channel gains over space as well as frequency.

The dual-polar systems have almost identical capacity throughout. This is partly because they experience the same total received power at all times, however they do experience different levels of correlation, and K-factors and individual sub-channel expected gains.

The LP system generally has lower correlation than the CP system, particularly over the high gain co-polar sub-channels. It has very high XPD though, meaning that effectively it
has a maximum diversity order of around two because the cross polar gains are so low. The CP system has slightly lower XPD, suggesting that it could achieve a diversity order a little higher than two, however the two co-polar channels, as well as the two cross-polar channels are fully correlated. This again reduces the maximum diversity order to two, which explains why the capacity of these two systems is so similar.

5.5.1.2. NLOS

![MIMO Channel Capacity](image)

Figure 5-11 – MIMO channel capacity of broadband systems using uncoated antennas in NLOS

![CDF of MIMO Channel Capacity](image)

Figure 5-12 – CDFs of MIMO channel capacity of broadband systems using uncoated antennas in LOS

In the NLOS environment, the dual polar systems are particularly disadvantaged compared to their spatial counterparts as a result of their lower total received power. The average sub-channel gain of the dual polar systems is typically in excess of 2 dB below that of the 0.25λ spatial system in this environment and even further below the spatial systems with greater antenna separation. In NLOS, the angular spread of arrival and departure directions is greater than in LOS, so the disadvantage of the increased antenna directivity of the 0.25λ system, in terms of total received power, is less significant.
Furthermore, the richer multipath in the NLOS environment results in lower correlation in all cases, but particularly for the spatial systems. As such, the dual polar antennas provide little advantage over the spatial systems, even with low separation.

This is reflected in the capacity of the systems, shown against distance in Figure 5-11 and as CDFs in Figure 5-12. As can be seen, the capacity of the dual polar systems is lower than that of the spatial systems over almost the entire route. The capacity of the spatial systems clearly increases as separation is increased. This is not surprising, as the systems with $0.5\lambda$ and $\lambda$ separation both receive higher power than the $0.25\lambda$ system. The capacity of the $\lambda$ separation system is highest as it receives similar average power level to the $0.5\lambda$ system, but experiences far lower correlation.

The capacity of the LP system is slightly higher than that of the CP system, although they both have approximately the same total received power. This could be because the correlation between the CP co-polar channels is higher than that between the LP co-polar channels, or because the XPD of the CP system is lower than that of the LP system.

5.5.2. Compact Narrowband Antennas without Case

5.5.2.1. LOS

![MIMO Channel Capacity](image)

Figure 5-13 – MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with no case in LOS
Figure 5-13 shows the capacity versus distance of the spatial MIMO systems and the co-located dual-polar systems, using the compact coated antennas, with 20 MHz bandwidth. Figure 5-14 presents the CDFs of these results. The average capacities, taken over this route are very similar. The highest average capacity is 4.58 bps/Hz, achieved by the spatial system with $\lambda$ separation. The average capacity of the spatial system with $0.25\lambda$ separation is 4.29 bps/Hz. The average capacities of the co-located LP and CP systems are only slightly below that of the $0.25\lambda$ system, at 4.10 and 4.28 bps/Hz respectively.

It can be seen that the dual polar systems do suffer from greater fluctuations in capacity with distance though. Over the simulated distance, their capacity varies between 1 and 8.6 bps/Hz. This is similar to the $0.25\lambda$ spatial system, although from the CDF, it can be seen that the capacity distribution of the $0.25\lambda$ system is more concentrated around its mean. The capacity of the spatial systems with $0.5\lambda$ and $\lambda$ separation fluctuates between 2 and around 6.5 bps/Hz. The capacity fluctuations reflect those of the average received power shown in Appendix B.

Figure 5-15 – MIMO channel capacity of $0.25\lambda$ spatially separated and dual polar compact narrowband antenna systems with no case in LOS

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Figure 5-16 – CDFs of MIMO channel capacity of $0.25\lambda$ spatially separated and dual polar compact narrowband antenna systems with no case in LOS

In Figure 5-15 and Figure 5-16, the capacities of all of the narrowband dual polar systems, as well as the $0.25\lambda$ spatial system in LOS are compared. Figure 5-15 shows the capacity versus distance, while Figure 5-16 shows CDFs of the capacity over the distance. Recalling the equivalent results for the uncoated antennas in Figure 5-9 and Figure 5-10, it was observed that over the broad band, the co-located dual-polar antennas had a capacity advantage over the $0.25\lambda$ spatial system, due to their lower sub-channel correlation. Over the narrow band however, where sub-channels are dominated by their fixed components, it can be seen that this is not the case.

These results again show a very high dependency on the average sub-channel gains. The co-located LP, CP and the LP to CP system all receive the same total power throughout the simulated distance. As is clear from Figure 5-15, this results in very similar capacities for all three systems. The only significant difference is observable on the LP to CP system, which has slightly lower capacity than the pure LP and CP systems, around the locations where total received power peaks. These locations coincide with the peaks in capacity, for example at around 0.2, 1.1 and 2 m, where the LP to CP capacity is approximately 1 bps/Hz lower than the pure co-located LP and CP systems. The reason for the lower capacity could be the lower XPD, reducing the rank of $HH^H$ for the LP to CP system.

The CP hybrid system (denoted “CP $0.25\lambda$” in the result plots) exploits a small amount of spatial diversity as well as polarisation diversity. This reduces the amplitude of the fluctuations in total received power over distance, compared to the pure polarisation diversity systems and as such, similarly reduces the amplitude of the fluctuations in capacity. This system has slightly higher average capacity over the LOS route than the pure LP or CP systems, at 4.47 bps/Hz.
The capacity of the dual-polar systems is overall approximately equivalent to that of the $0.25\lambda$ spatial system, over most of the simulated distance. The most notable difference is between 1.3 and 1.8 m, where the total received power by the dual polar systems experiences a minimum to a greater extent than the spatial system, resulting in around 50% lower capacity.

5.5.2.2. NLOS

![Figure 5-17](image1.png)

Figure 5-17 – MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with no case in NLOS

![Figure 5-18](image2.png)

Figure 5-18 – CDFs of MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with no case in NLOS

Figure 5-17 shows the NLOS capacity versus distance of the compact spatial systems, compared to that of the co-located LP and CP dual-polar systems, while Figure 5-18 shows the CDFs of these results. As identified in Section 5.4.1.2, the richer multipath in NLOS results in lower amplitude fluctuations of averaged sub-channel gain than in LOS. As such, the fluctuations in capacity are also of lower amplitude.

The $0.25\lambda$ spatial system receives the most power over the distance and also has the greatest capacity. This is a result of the greater directivity of the coupled antennas with
little separation. The capacity of the remaining spatial systems is only slightly lower than that of the 0.25\(\lambda\) system, due to the slightly lower average sub-channel gains. This is contrary to the observation of the broadband systems in NLOS, where capacity increases with antenna separation.

The capacities of the co-located LP and CP systems are very similar, typically between 1 and 2 bps/Hz lower than the spatial systems. The co-located LP to CP system has lower capacity than this, despite its total received power being similar. This is because it has no full cross-polar channels and therefore has a lower determinant of \(\mathbf{H}^H\). The average capacity of the CP hybrid system is similar to that of the pure CP and LP systems, though with slightly lower amplitude fluctuations over distance, due to the additional diversity.

The capacity of all of the dual polar systems is shown, with comparison to the 0.25\(\lambda\) system, versus distance in Figure 5-19 and as CDFs in Figure 5-20.

![MIMO Channel Capacity](image1)

**Figure 5-19** – MIMO channel capacity of 0.25\(\lambda\) spatially separated and dual polar compact narrowband antenna systems with no case in LOS

![CDF of MIMO Channel Capacity](image2)

**Figure 5-20** – CDFs of MIMO channel capacity of 0.25\(\lambda\) spatially separated and dual polar compact narrowband antenna systems with no case in LOS
5.5.3. Coated Antennas with Case

5.5.3.1. LOS

![Graph](image)

Figure 5-21 – MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with case in LOS

![Graph](image)

Figure 5-22 – CDFs of MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with case in LOS

Figure 5-21 shows the estimated capacity versus distance for the compact antenna systems in the presence of the case, in LOS. Figure 5-22 presents CDFs of these results. Again, as a result of the high K-factors, the capacity is heavily dependent upon total received power.

As observed without the case, the spatial system with the larger separation has the highest overall capacity in LOS. This is because the separation reduces the likelihood of all sub-channels experiencing deep fades simultaneously.

In the presence of the case, the CDF plots of BER for the CP system and the $0.3\lambda$ spatial system are very similar. These systems have very similar average sub-channel gains. This is because the spatial system experiences a level of unexpected XPD, while the CP system
has reduced XPD. Both of these effects are a result of the far-field polarisation distortions caused by the presence of the case.

The LP system achieves the lowest capacity because its cross polar sub-channels are very low gain throughout, resulting in typically around 1 bps/Hz lower capacity than the CP system.

5.5.3.2. NLOS

![Graph showing MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with case in NLOS.](image1)

**Figure 5-23** – MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with case in NLOS

![Graph showing CDFs of MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with case in NLOS.](image2)

**Figure 5-24** – CDFs of MIMO channel capacity of spatially separated and co-located dual polar compact narrowband antenna systems with case in NLOS

Figure 5-23 shows the estimated capacity versus distance for the compact antenna systems with the case in NLOS, while Figure 5-24 presents CDFs of these results.

The capacity differences in NLOS, where the multipath is richer, are less than those in the more directional LOS environment. Overall, there is a slight capacity advantage to the spatial systems, which is in line with their higher average sub-channel gains.
The LP system is at less of a capacity disadvantage in NLOS than in LOS. This is because on the whole, it receives more power and also has high XPD, increasing the determinant of $HH^H$. That being said, its capacity does drop substantially between 1.3 and 1.75 m, where the horizontal to horizontal sub-channel gain is particularly low.

### 5.6. MIMO Diversity Performance

In this section, the estimated BER of all systems is presented under the Alamouti diversity scheme (Section 4.3.1) for transmission over the extended MIMO channel model. This is simulated using the Monte-Carlo approach, as described in Section 4.4. Quadrature phase shift keying modulation is implemented, with maximum likelihood detection, as these are widely used schemes [7, 13, 137].

#### 5.6.1. Broadband

5.6.1.1. **LOS**

![Simulated BER under Alamouti coding for broadband systems using uncoated antennas in LOS](image)

*Figure 5-25 – Simulated BER under Alamouti coding for broadband systems using uncoated antennas in LOS*
Figure 5-26 – CDFs of simulated BER under Alamouti coding for broadband systems using uncoated antennas in LOS

Figure 5-25 shows the simulated BER under Alamouti coding for the broadband systems in LOS, while Figure 5-26 shows a CDF plot of these results. It can be seen that despite the higher capacity of the dual polar channels, in comparison to that of the $0.25\lambda$ spatial system, the error rate of these systems is almost identical over the entire distance. It appears that the Alamouti scheme does not exploit the additional capacity in this case. The fact that dual polar systems achieve BER performance approximately equal to that of the spatial system with low separation is of interest nonetheless.

The remaining spatial systems both achieve significantly lower BERs than the dual polar and $0.25\lambda$ systems. The lowest BER is achieved by the $0.5\lambda$ system and is attributed to its particularly high total received power. The BER benefit of the $\lambda$ system over the $0.25\lambda$ spatial system and the dual polar systems is primarily also a result of its higher total received power, although it does also experience lower correlation than the spatial systems of lower separation.
5.6.1.2. NLOS

![Figure 5-27](ber_nlos.png)

Figure 5-27 – Simulated BER under Alamouti coding for broadband systems using uncoated antennas in NLOS

![Figure 5-28](cdf_nlos.png)

Figure 5-28 – CDFs of simulated BER under Alamouti coding for broadband systems using uncoated antennas in NLOS

As with the capacity of the broadband systems in NLOS, the error rate under the Alamouti scheme, is highly dependent upon the average sub-channel gain result. The BER under Alamouti for the broadband systems in NLOS is shown versus distance in Figure 5-27, while Figure 5-28 shows its CDFs. The diversity performance of the dual polar systems is significantly worse than that of the spatial systems. This is primarily a result of their lower average sub-channel gain, however as stated in the capacity discussion, the correlation in the spatial systems is lower in NLOS than LOS, which means that further decorrelation through the use of dual-polar antennas is of less significance.

It can be seen that diversity performance of the CP system slightly exceeds that of the LP system. This is in contrast to the capacity results, where LP offered the greater capacity. Both of these systems receive the same total power throughout, so the slight benefit to the CP system is attributed to its lower XPD. The CP cross-polar channel gains are only
around 5 dB below the co-polars, so their contribution increases the effective diversity gain of the system to a greater extent than for the LP system, where the cross polar channels are around 20 dB below the co-polar channels.

The performance of the 0.5\(\lambda\) and \(\lambda\) systems, when considered over the entire distance is approximately equal. This suggests there is little additional advantage in terms of diversity performance in separating these antennas beyond 0.5\(\lambda\) in the rich multipath NLOS environment.

From the simulated broadband error rate results under the Alamouti scheme, it is clear that in NLOS, the dual polar systems do not achieve as low a BER as the spatial systems, even with low separation. In the LOS environment however, their performance is similar to that of highly correlated spatial systems with low (0.25\(\lambda\)) antenna separation.

5.6.2. Compact Narrowband Antennas without Case

5.6.2.1. LOS

![Figure 5-29 – Simulated BER under Alamouti coding for spatial MIMO and co-located dual polar MIMO systems using narrowband coated antennas in LOS](image)
Figure 5-30 – CDFs of simulated BER under Alamouti coding for spatial MIMO and co-located dual polar MIMO systems using narrowband coated antennas in LOS

Figure 5-29 and Figure 5-30 show the BER for the spatial MIMO systems and the co-located dual-polar systems under Alamouti coding in LOS, presented versus distance and as CDF plots respectively. With high K-factors throughout, these results again become heavily dependent upon the sub-channel gain levels.

With separation as low as 0.25λ, or when using co-located dual polar antennas, large fluctuations in BER exist. These correspond to the multipath fluctuations of total received power over distance. As separation is increased beyond 0.25λ, the amplitude of these fluctuations decreases as a result of the increased spatial diversity and reduced \( \rho_{env} \), meaning that the fading on each sub-channel over distance becomes more independent.

The result of the high level of fluctuation at low separation is that the dual polar systems and the 0.25λ system offer considerably lower BER than the larger separation spatial systems over around 25% of the simulated distance. In the remainder of the space however, their performance is significantly worse than that of spatial systems with large separation. Furthermore, between 0.4 and 0.8 m, as well as between 1.3 and 1.8 m, the dual polar systems receive less total power than the 0.25λ spatial system and as a result experience particularly high BERs.

Figure 5-31 shows the BER results versus distance for the 0.25λ spatial system, the co-located dual-polar systems, as well as the CP hybrid system and the LP to CP system. Figure 5-32 shows CDFs of these results. It can be seen that there is no significant performance advantage through the use of the hybrid approach or the LP to CP system. In fact, the BER of the co-located CP, LP and the LP to CP system is effectively identical. The addition of a small amount of spatial diversity to the hybrid system does slightly reduce the amplitude of the BER fluctuation, however its overall performance is still worse than any of the spatial MIMO systems.
5.6.2.2. NLOS

Figure 5-31 – Simulated BER under Alamouti coding for coated narrowband dual polar and 0.25λ spatial MIMO systems in LOS

Figure 5-32 – CDFs of simulated BER under Alamouti coding for coated narrowband dual polar and 0.25λ spatial MIMO systems in LOS

Figure 5-33 – Simulated BER under Alamouti coding for spatial MIMO and co-located dual polar MIMO systems using narrowband coated antennas in NLOS
Figure 5-34 – CDFs of simulated BER under Alamouti coding for spatial MIMO and co-located dual polar MIMO systems using narrowband coated antennas in NLOS

The NLOS BER under the Alamouti scheme for the spatial systems and the co-located dual polar systems is presented versus space in Figure 5-33 and as CDFs in Figure 5-34. As a result of higher total received power, significantly lower BER is achieved by the spatial systems than the dual polar systems. The difference in BER is typically around an order of magnitude, although between 1.2 and 1.6 m, an exceptionally large difference of up to 10 dB in total received power exists between the spatial systems and the dual polar systems. This results in a BER difference of up to 5 orders of magnitude.

Figure 5-35 shows the BER of the 0.25\(\lambda\) spatial system and all of the dual polar systems using compact antennas in NLOS against distance, while Figure 5-36 shows the CDFs of these results. The BER of all dual polar systems is significantly higher than the 0.25\(\lambda\) system. It can be seen that at low BER levels, the co-located CP system achieves a slightly lower BER than the co-located LP system, however this difference is far less significant when compared to the broadband systems. The BER of the LP to CP system is also essentially the same as the co-located LP and CP systems. This suggests that under high K-factor, these systems experience approximately the same level of diversity gain, and that error rate is primarily determined by total received power level.

The CP hybrid system reduces the distribution of total received power level about the mean when compared to the co-located systems, which in turn reduces the fluctuations in BER,
however the mean level is not significantly lower than that of the co-located systems.

Figure 5-35 – Simulated BER under Alamouti coding for narrowband coated dual polar and 0.25λ spatial MIMO systems in NLOS

Figure 5-36 – CDFs of simulated BER under Alamouti coding for narrowband coated dual polar and 0.25λ spatial MIMO systems in NLOS
5.6.3. Compact Narrowband Antennas with Case

5.6.3.1. LOS

With the inclusion of the metal case in LOS, more distinct differences between antenna configurations are apparent in the BER results. This demonstrates the importance of antenna and platform design considerations, with respect to resultant patterns and polarisation, as well as simply their spatial separation. Furthermore, it demonstrates the need for a full end-to-end modelling approach, which completely accounts for the effects of the case on radiated polarisation in all directions over which energy propagates through the environment. Figure 5-37 shows the BER versus distance in LOS of the compact antenna systems in the presence of the case, while Figure 5-38 presents CDFs of these results.
The $0.5\lambda$ spatial system clearly has the lowest BER over the majority of the LOS route. The relatively large spatial separation has the benefit of creating elements of $X$ which fade independently over space and are all of reasonable magnitude.

The BERs of the $0.3\lambda$ spatial system and the CP system fluctuate in much the same manner. That being said, the BER of the co-located CP system is predominantly below that of the $0.3\lambda$ system, at times by in excess of an order of magnitude. This is due to the deterioration of the expected polarisations of each system by the presence of the case. The polarisation of the $0.3\lambda$ system deteriorates to elliptical, which causes a low level of XPD, which decreases the total received power. The polarisation of the CP system on the other hand deteriorates to narrow ellipses in most directions, which reduces XPD. In addition, the case alters directivity, which in this situation increases antenna gain in the direction of the direct path. This is particularly apparent on antenna two of this system, the pattern of which is shown in Figure 2-33. This results in a particularly high $\alpha_{2,2}$ and increases the total received power to an extent that the system generally receives greater total power than the $0.3\lambda$ system.

The LP system is at a distinct disadvantage, due to its low gain cross polar channels, which reduce maximum effective diversity order and total received power. As such, it has the highest BER throughout.
5.6.3.2. NLOS

Figure 5-39 – Simulated BER under Alamouti coding for spatially separated and co-located dual polar compact narrowband antenna systems with case in NLOS

Figure 5-40 – CDFs of simulated BER under Alamouti coding for spatially separated and co-located dual polar compact narrowband antenna systems with case in NLOS

Figure 5-39 and Figure 5-40 show the BER versus distance and as CDF plots respectively, for the compact antenna systems with the case, in NLOS. In NLOS, the multipath is much richer and arrival angles have a greater spread. This results in the average channel gains in all cases showing less fluctuation over distance than in LOS, because they are the combination of more rays. Each sub-channel also fluctuates with greater independence. Therefore, the difference in the benefit of spatial separation of 0.5\(\lambda\) compared to 0.3\(\lambda\) is less significant and instead pattern diversity is more of a benefit. The lowest overall BER is achieved by the spatial system with 0.3\(\lambda\) separation. Recalling Section 2.7.2.2. this separation is optimum in terms of \(\rho_{env}\).

The BER of the dual polar systems is generally of a similar order to that of the spatial systems over the first 1.2 m of the NLOS route. After this, the horizontal to horizontal...
sub-channel of the LP system experiences a deep fade which increases BER. The poor horizontal propagation conditions also reduce the sub-channel gains of the CP system, similarly increasing BER in this region.

### 5.7. MIMO Multiplexing Performance

This section presents the BER estimates simulated for all systems, under the spatial multiplexing transmission scheme, V-BLAST, as described in Section 4.3.2. ZF-OSIC detection is implemented and the modulation scheme is QPSK.

#### 5.7.1. Broadband

##### 5.7.1.1. LOS

Figure 5-41 – Simulated BER under V-BLAST for broadband systems using uncoated broadband antennas in LOS

Figure 5-42 – CDFs of simulated BER under V-BLAST for broadband systems using uncoated broadband antennas in LOS
Figure 5-41 shows the simulated LOS BER under V-BLAST for the broadband systems over the simulation distance, while Figure 5-42 shows a CDF of these results. It is clear that under V-BLAST, the dual polar systems perform significantly better than the spatial system with 0.25λ separation. The BER using LP or CP is less than that of the 0.25λ system, over almost the entire route. This reflects the observation in [3] that the performance of polarisation MIMO systems exceeds that of spatial systems in situations where either K-factor is high, or transmit correlation is high. The average K-factor of the 0.25λ system is between 1 and 4 in the LOS environment, meaning that the sub-channels experience a mixture of Rayleigh fading and a specular component, however the transmit correlation is very high.

There are points, for example around 1.1 and 1.95 m on the route where the performance under CP exceeds that of LP. These coincide with positions where the CP system experiences particularly high XPD. The LP system has very high XPD throughout, however it also experiences differences between the co-polar sub-channel gains, α₁₁ ≠ α₂₂ unlike the CP system. This is detrimental to V-BLAST performance.

The error rate of the spatial system with λ separation is comparable to that of the dual polar systems, which may be explained by its higher total received power than the 0.25λ system, as it still experiences high correlation. This is similar to the 0.5λ system, which experiences the highest total received power.
5.7.1.2. NLOS

Figure 5-43 – Simulated BER under V-BLAST for broadband systems using uncoated antennas in NLOS

Figure 5-44 – CDFs of simulated BER under V-BLAST for broadband systems using uncoated antennas in NLOS

Figure 5-43 shows the simulated BER of the broadband systems under V-BLAST in the NLOS environment. As can be seen, the differences in performance between systems are very little. A greater insight can be gained from the CDFs over the distance, presented in Figure 5-44. It can be seen that the lowest BER throughout the route is achieved using the spatial system with λ antenna separation. This system receives a similar total power level to the 0.5λ system, but experiences lower transmit correlation. In contrast to the behaviour under the Alamouti scheme, this reduced correlation has a notable positive effect on error rate under V-BLAST. This is particularly evident at around 0.3 and 1.5 m along the
simulation route, where the BER of the $\lambda$ system reaches minimums that coincide with minimums of transmit correlation, $t_1$ and $t_2$.

The transmit correlations in the 0.5 and 0.25$\lambda$ systems are close to one throughout most of the route and therefore significantly higher than when separation is $\lambda$. Under Rayleigh fading conditions, [3] states that the high transmit correlation results in performance degradation under SM schemes, where lower error rate may be achieved using dual polar antennas with high XPD. This statement is supported by the performance of the dual polar systems reported in this section. As can be seen from Figure 5-44, the CDF of the BER using CP is almost identical to that of the 0.25$\lambda$ system and the BER of the LP system is lower than this. This is particularly remarkable because the dual polar systems receive significantly less power than the spatial systems. This is clear from the CDF of average sub-channel gain for the broadband systems in NLOS, shown in Figure 5-45.

![Figure 5-45 – CDFs of average sub-channel gains for broadband uncoated antenna systems in NLOS](image)

The V-BLAST BER results demonstrate that in both LOS and NLOS indoor environments, using full size dipoles and crossed dipoles over a large bandwidth, performance of co-located dual polar systems is no worse than that of a spatial system using closely spaced antennas. In fact, in LOS, due to the high transmit correlation in the spatial systems, superior performance is achieved by the dual polar systems.
5.7.2. Compact Narrowband Antennas without Case

5.7.2.1. LOS

Figure 5-46 – Simulated BER under V-BLAST for spatial MIMO and co-located dual polar MIMO systems using narrowband coated antennas in LOS

Figure 5-47 – CDFs of simulated BER under V-BLAST for spatial MIMO and co-located dual polar MIMO systems using narrowband coated antennas in LOS

Under high K-factors, the performance of SM schemes such as V-BLAST is primarily governed by the XPD. Optimum conditions are when \( h_{1,1} \) and \( h_{2,2} \) are of high magnitude and \( h_{1,2} \) and \( h_{2,1} \) are significantly lower. As such, dual polar systems may in theory offer an attractive configuration. Figure 5-46 shows the BER under V-BLAST versus distance for the spatially separate and the co-located dual polar coated antennas with no case in LOS. Figure 5-47 shows the CDFs of these results.

The spatial systems experience similar gains across all sub-channels and as such their BER is almost entirely greater than \( 10^{-2} \). The dual polar systems on the other hand often have
significantly lower gains on the cross-polar sub-channels than the co-polar, which enables far lower BERs. For example, in the space around 0.2, 1.1 and 2 m, the BER of the co-located LP system drops to within the region of $10^{-4}$ to $10^{-3}$, while in the CP system it drops as low as $10^{-5}$.

It is clear however, that these optimum conditions are certainly not guaranteed in a multipath environment, even when antennas are aligned with one another and in LOS. Away from the BER minimums of the dual-polar systems, the co-polar sub-channels experience deep fades which reduce XPD and received signal level. This results in a BER at similar levels to the spatial systems and, at times higher than the spatial systems with $0.5\lambda$ or $\lambda$ separation. In general though, the co-located dual polar systems in LOS achieve a BER performance which is either significantly better than the 0.25$\lambda$ spatial system, or at least equivalent to it.

The BER versus distance results and the CDFs in Figure 5-48 and Figure 5-49 respectively compare the V-BLAST performance of the co-located LP and CP systems, the co-located LP to CP system, the CP spatial hybrid system and the co-polar spatial MIMO system with 0.25$\lambda$ separation. The co-located CP system slightly outperforms the LP system. This is because the co-polar sub-channels of the CP system maintain approximately the same average gains throughout the distance. The co-polar gains of the LP system on the other hand fade with greater independence over distance, which means that they tend to be at slightly different levels. This results in the symbol transmitted by one antenna being received at a lower level than the other, increasing BER.

The CP hybrid system performs almost as well as the co-located CP system, as it again experiences high XPD, with similar magnitude of co-polar sub-channel gains. In certain locations however, such as around 0.2 and 1.1 m, its XPD is not as high as the co-located system, resulting in a higher BER. It maintains a lower BER than the co-located LP system though, over the entire distance.

The LP to CP system results in the highest BER. This is because it receives less total power than the spatial systems, but also generally has lower gain $h_{1,2}$ and $h_{2,2}$ than $h_{2,1}$ and $h_{1,1}$. This means that the symbol transmitted by antenna two is predominantly received by both receive antennas with low SNR.
Figure 5-48 – Simulated BER under V-BLAST for narrowband coated dual polar and 0.25\( \lambda \) spatial MIMO systems in LOS

Figure 5-49 – CDFs of simulated BER under V-BLAST for narrowband coated dual polar and 0.25\( \lambda \) spatial MIMO systems in LOS
5.7.2.2. NLOS

In NLOS, in the presence of richer multipath fading, the differences in BER between the dual polar and the spatial systems are less significant. Figure 5-50 shows the NLOS BER versus distance of the spatial systems and the co-located dual polar systems using coated antennas, while Figure 5-51 presents CDFs of these results.

As can be seen, there are certain locations, most notably around 0.4 and 0.7 m, where the dual polar systems experience favourable channel conditions and achieve BER minimums which are significantly lower than those of the spatial systems. Interestingly, at 0.4 m the CP system outperforms the LP system, however at 0.7 m, the LP system has the lower BER.
Throughout the rest of the distance, the BER of the dual polar systems tends to be considerably higher. In the case of the LP system, independent fading over distance on the co-polar sub-channels means that the probability of experiencing reasonable gain on $h_{1,1}$ and $h_{2,2}$ simultaneously is low. The CP system has XPD which is generally lower than that of the LP system. Given the multipath fading on all sub-channels over distance and the fact that the antennas are not perfectly aligned, this means that XPD often reduces to effectively unity. As a result, throughout the majority of the NLOS environment, the BER of the dual polar systems is in fact slightly higher than that of the spatial systems.

In the BER plot versus distance in Figure 5-52 and the CDFs of Figure 5-53, the NLOS BER performance of all the dual-polar systems is compared to that of the $0.25\lambda$ separated spatial system. Overall, the performance of the co-located LP system is roughly equal to that of the co-located CP system. Under favourable channel conditions, they achieve up to an order of magnitude lower BER than the $0.25\lambda$ spatial system, however for the majority of the route, their BER is slightly higher. The average BER over this route for the co-located LP and CP systems respectively is $4.9 \times 10^{-2}$ and $5.3 \times 10^{-2}$, while that of the $0.25\lambda$ spatial system is $3.0 \times 10^{-2}$.

The overall performance of the hybrid system is poorer than that of the co-located systems. This is because, when the orthogonally polarised antennas are separated, XPD is generally lower and $\alpha_{1,1}$ and $\alpha_{2,2}$ begin to fade independently over space.

As in the LOS environment, the LP to CP system has the highest overall BER. This is because it has no significant XPD. Furthermore, $\alpha_{1,1}$ and $\alpha_{2,1}$ behave very similarly over space, as do $\alpha_{1,2}$ and $\alpha_{2,2}$.
Figure 5.52 – Simulated BER under V-BLAST for narrowband coated dual polar and $0.25\lambda$ spatial MIMO systems in NLOS

Figure 5.53 – CDFs of simulated BER under V-BLAST for narrowband coated dual polar and $0.25\lambda$ spatial MIMO systems in NLOS
5.7.3. Coated Antennas with Case

5.7.3.1. LOS

![Figure 5-54 – Simulated BER under V-BLAST for spatially separated and co-located dual polar narrowband compact antenna systems with case in LOS]

![Figure 5-55 – CDFs of simulated BER under V-BLAST for spatially separated and co-located dual polar narrowband compact antenna systems with case in LOS]

In the presence of the case, there are some remarkable differences in the V-BLAST performance when compared to the results without the case in Section 5.7.2. Figure 5-54 shows the LOS BER versus distance under V-BLAST for the compact antenna systems in the presence of the case. Figure 5-55 presents the CDF plots of these results.

The unexpectedly high polarisation orthogonality of the spatially separate antennas results in sub-channels $h_{1,1}$ and $h_{2,2}$ having typically higher gain than $h_{1,2}$ and $h_{2,1}$. This effective XPD is highly beneficial to V-BLAST transmission, to the extent that the BER of the spatial systems over the LOS route is in fact up to an order of magnitude below that of the co-located dual polar systems.
Conversely, the presence of the case has a detrimental effect on the V-BLAST performance of the co-located dual polar systems in LOS. The polarisations in the $\pm Y$ direction from the CP antenna are both ellipses with the same rotation sense, as shown by Figure 2-36. Furthermore, the antenna gain patterns are extremely distorted, as shown in Figure 2-32 and Figure 2-33. The effect of this on $X$ is that $\alpha_{1,1}$, $\alpha_{1,2}$ and $\alpha_{2,1}$ are typically in excess of 5 dB below $\alpha_{2,2}$. This results in a high probability that the symbol transmitted by antenna one is detected in error.

The polarisations of the LP antenna with the case remain highly orthogonal in the $\pm Y$ direction and for considerable angles away from it, as shown in Figure 2-34. The case again has a significant effect on the directivity though. As shown in Figure 2-30 and Figure 2-31 respectively, energy from the $X$ (horizontally) orientated dipole is focussed towards the $Z$ direction, while energy from the $Z$ (vertically) orientated dipole is focused around the $-Z$ direction. This results in decreased antenna gains in the direction of the direct path, which reduces the co-polar sub-channel gains. As such, the BER of the LP system in LOS is generally higher than that of the other systems.

5.7.3.2. NLOS

![Figure 5-56 – Simulated BER under V-BLAST for spatially separated and co-located dual polar compact narrowband antenna systems with case in NLOS](image)
Figure 5-56 shows the BER versus distance under V-BLAST for the compact antennas with the case in NLOS, while Figure 5-57 shows CDF plots of these results. With the receiver moved to the NLOS route, the direct path between transmitter and receiver is no longer aligned with the direction that the spatially separate antennas radiate with high polarisation orthogonality. As such, there is no particular dominance of any elements of $\mathbf{X}$ for these systems and therefore no effective XPD. This results in relatively steady BERs, predominantly of the order $10^{-1}$ to $10^{-2}$.

The LP system however provides a very interesting result. Referring to Figure 2-34, the LP antennas in the presence of the case maintain high orthogonality over an exceptionally wide angle in the azimuth plane. As a result, high XPD is maintained over a large portion of the NLOS route and when both co-polar sub-channels experience gains of similar magnitudes, a BER much lower than that of the other systems is recorded. This is particularly apparent at around 0.35, 0.9, 1.1 and 1.95 m, where BER is up to two orders of magnitude lower than the BER of the spatial systems. This effect is a result of the near field interactions of the antennas with the case and is not observed when the same antennas are modelled without the case (Section 5.7.2.2. ) or when broadband antennas are used (Section 5.7.1.2. ). Between 1.3 and 1.8 m, it should be noted that exceptionally low horizontal to horizontal sub-channel gain results in a BER peak around an order of magnitude higher than the spatial systems’ BER, however overall the V-BLAST performance of the LP system in NLOS certainly exceeds that of the CP or spatial systems.

The CP system again experiences a trend for higher gain $h_{2,2}$ than the other sub-channels. This equates to poor channel conditions under V-BLAST. As such, in NLOS it achieves a BER of similar order to the spatial systems, although when considered over the full route, it can be seen from the CDFs that it is in fact slightly higher.
5.8. Summary

This chapter presents MIMO system results, for the compact antenna systems described in Chapter 2. Propagation between these antennas in an indoor environment has been modelled deterministically using the ray launcher described in Chapter 3. From results of the antenna and propagation modelling, the MIMO channel has been modelled using the approach described in Chapter 4. In Section 5.5. of this chapter, the capacity of the MIMO channel for each system is evaluated. The MIMO error rate performance of each system is then estimated through Monte-Carlo system simulations over the channel model. Diversity performance is evaluated in Section 5.6. while spatial multiplexing performance is evaluated in Section 5.7.

Compact narrowband spatial MIMO systems, with antenna separation between $0.25\lambda$ and $\lambda$ have been compared to dual polarised MIMO systems using co-located LP and CP antennas. Furthermore, a hybrid system, using orthogonally polarised CP antennas, separated by $0.25\lambda$ has been examined, as well as a system which transmits using co-located dual-polar LP antennas and receives using co-located dual-polar CP antennas. For reference, capacity and performance of the compact antenna systems has been compared to that of broadband systems using full-size dipole antennas. In addition, the effect of the presence of a metal case on the MIMO capabilities of compact antenna systems has been examined.

This work extends existing literature through the presentation of an end-to-end, predominantly deterministic MIMO modelling approach for compact dual polarised and spatial systems in the indoor environment. Novel analysis directly relates properties of the antenna models, such as their separation and the presence of a case, to the MIMO capabilities of systems utilising these antennas. MIMO capabilities are examined not only in terms of theoretical maximum capacity, but also performance under the Alamouti diversity scheme and the V-BLAST spatial multiplexing scheme.

Existing literature provides useful discussion on the channel conditions under which polarisation MIMO systems are capable of providing a MIMO benefit over spatial systems. The work described in this chapter however, ascertains the channel conditions of practical systems, using deterministic antenna and propagation models. With knowledge of these conditions, MIMO channel capacity and performance is estimated with direct relation to the considered antennas and environment.
Full consideration is given to the channel model. For instance, each sub-channel is assumed to have an expected gain which is likely to be different to that of the other sub-channels. When considering the polarisation MIMO channel, it is not assumed that both co-polar sub-channels, or both cross-polar sub-channels are necessarily of equal gain. Similarly, separate K-factors for each sub-channel are assumed and an $n_t n_r \times n_t n_r$ correlation matrix is used to allow for correlation which does not obey the Kronecker assumptions. Particular attention is paid to mutual coupling, which is experienced when antennas are positioned close to one another. At the transmitter, coupling is accounted for in the CST modelling of the transmit antennas. The pattern of each antenna is determined through the simulated excitation of each antenna individually, while the other antenna is loaded with its feed impedance. At the receiver, where the excitation source is in the far-field, mutual impedance is determined using a weighted average approach, based on the arrival angles and power of the rays incident on the receiver. The receive mutual impedance is then accounted for using a receive mutual impedance matrix in the MIMO channel model.

From the presented results, an important observation is that the dual polarised systems typically receive less total power than the spatial systems. This is due to the low gain cross-polar sub-channels. It is found that even in rich scattering environments, leakage between orthogonal polarisations is typically not sufficient to result in all sub-channels being of similar gain. In terms of channel capacity and diversity performance this is predominantly detrimental, although it can be beneficial under spatial multiplexing.

As such, when space is not limited, optimum all round performance is achieved by spatial MIMO systems with co-polar antennas, separated by at least $0.5\lambda$. When space is limited, the performance of spatial systems and dual polarised systems in many circumstances is at least comparable. There are however two notable exceptions to this. In NLOS, the capacity and diversity performance of the dual polar systems is notably worse than the spatial systems. This is because they receive significantly less total power than the spatial systems, which are well decorrelated. Conversely, over certain sections of the LOS route, the multiplexing performance of the narrowband dual polar systems is remarkably better than the spatial systems. This is because in these sections, the dual polar systems experience high XPD, with both co-polar sub-channel gains of similar magnitude.
It is found that when using the broadband systems, with full size antennas, the effects of multipath over the larger bandwidth are more apparent, resulting in greater fluctuation of sub-channel frequency responses and lower K-factors than when using a narrowband signal. That being said, the broadband channels still tend to be far from Rayleigh, with K-factors typically around 2 in NLOS and up to 6 in LOS.

In these conditions, it is noted that high sub-channel correlation in the broadband $0.25\lambda$ spatial system in LOS results in capacity decreasing to below that of the dual-polar systems which experience lower correlation. This result is particularly interesting because the $0.25\lambda$ system receives approximately the same total power as the dual polar systems, demonstrating a greater MIMO benefit through the use of dual-polarised antennas when compared to spatially separate antennas with limited space, which receive similar total power. Under Alamouti, the dual polar systems cannot exploit this additional capacity however, as their BER is similar to that of the $0.25\lambda$ system. Under V-BLAST on the other hand, a significant BER reduction is achieved by the dual-polar systems. This demonstrates the importance of a full system model and analysis of performance under each scheme, as opposed to simply judging the ability of a system on its theoretical maximum capacity.

Over the narrow band, K-factors are significantly higher and the channel is dominated by its fixed specular components. The expected magnitudes of these experience significantly greater multipath fading over distance when taken over the narrow band than with the broad band. The correlation of the sub-channel responses over frequency becomes of little relevance and instead, correlation of expected sub-channel gains considered over distance is more important. If these are well de-correlated then the probability of multiple sub-channels experiencing a deep fade simultaneously is reduced.

The best way to ensure this decorrelation is to separate the antennas in space. With separation as low as $0.25\lambda$ however, the fading on the sub-channel gains can become fairly similar, particularly in LOS. This is where the dual polar systems offer comparable performance under Alamouti. In the NLOS environment, richer multipath reduces the correlation between sub-channels even at low separation and also reduces the magnitude of the sub-channel gain fluctuations. This means that the problem of deep fades is less significant and the spatial systems outperform the dual polar systems under the Alamouti scheme, because they receive higher total power.
It is concluded in [4] that dual polar antennas typically yield a performance degradation under the Alamouti scheme, when compared to the performance of spatially separate antennas. The results presented in Section 5.6. are broadly in agreement with this statement. When the large antennas are used over a broad band, there is a clear disadvantage in terms of BER to the use of co-located dual polar systems, when compared to spatially separate co-polar antennas when in NLOS. In LOS, the dual-polar systems experience BERs higher than the well separated co-polar antennas, however when separation is as low as 0.25\(\lambda\), the sub-channel correlation in LOS increases to the extent that the BER is approximately the same as is achieved by the dual polar systems.

Over the narrow band, there is also a clear BER advantage to the spatially separate antennas under the Alamouti scheme in NLOS. In LOS however, the advantage is less clear cut. The BER of the dual polar systems and of the 0.25\(\lambda\) system fluctuate heavily over distance and overall, the BER using dual polar antennas is certainly comparable to that of the 0.25\(\lambda\) system.

Another observation in [4] is that the use of dual-polar antennas can be beneficial to spatial multiplexing, under a wide variety of channel conditions. It is specifically stated that when K-factor is high, the use of dual polarised antennas for SM is generally highly beneficial. Through the results in this chapter, this statement may be expanded. High K-factors are generally experienced throughout all of the results of the narrowband systems. This does result in highly beneficial V-BLAST performance, under the correct conditions. Namely, high XPD and co-polar sub-channels of similar magnitude. These conditions are experienced in certain locations in the environments modelled in this chapter, however an important observation is that they are far from guaranteed. In fact, even in a LOS environment, with ideal antennas which are orientated such that the directions of maximum orthogonality are aligned with the direct path, frequently these conditions are not met. This is due to the high level of multipath induced fluctuation on narrowband sub-channel gains over distance. If all sub-channels experience a deep fade simultaneously then signal to noise ratio decreases and BER increases. If the co-polar sub-channels both experience a deep fade simultaneously (as occurs on the CP system), then XPD is reduced, which is detrimental to V-BLAST BER. Lastly, if only one of the co-polar sub-channels experiences a deep fade, then correct detection of the symbol transmitted on that polarisation is less likely, increasing BER. As such, the BER benefit to dual polar systems
with high K-factor is only really “highly beneficial” over around 40% of the distance simulated in LOS and less than 20% in NLOS.

The co-located LP system achieves the greatest XPD. It is not necessarily guaranteed the greatest V-BLAST performance however, because the expected gains of its co-polar sub-channels fade independently, which means that often one co-polar sub-channel has gain significantly lower than the other, which is detrimental to BER. The co-polar sub-channels of the co-located CP system on the other hand experience effectively the same fading over frequency and over space. This means that it does not suffer with the same problem as the LP system because the magnitude of both co-polar gains is always the same. It does however have significantly lower XPD, which often reduces to unity when the cross-polar sub-channels are of similar gain to the co-polar sub-channels. As the cross-polar sub-channels are a source of interference under V-BLAST with high K-factor, this is particularly detrimental to the BER.

The difference in the sub-channel gain behaviour between the narrowband co-located CP and LP systems has negligible effect under the Alamouti scheme, as both systems receive the same total power. Under V-BLAST it makes a small difference. In LOS, the CP system achieves a slightly lower BER, while in NLOS, in some positions the CP system has a lower BER and in others the LP system does.

From the results of the hybrid system, it seems that there is little additional benefit in separating dual-polarised antennas in space, when compared to co-located dual-polar antennas. Under Alamouti, the hybrid system achieves a slightly better diversity performance than the co-located antennas, but its performance cannot match that of the spatially separate co-polar antennas, which receive significantly greater total power. Under V-BLAST, it performs similarly to the co-located dual polarised systems, however it does not quite achieve the lowest BER values because its XPD is lower than that of the LP system and also its co-polar sub-channels fade with some independence over distance.

When a co-located LP transmitter is used with a co-located CP receiver, the same total power is received as that of the pure LP or pure CP system. As such, error rate under the Alamouti scheme is approximately the same. Under V-BLAST, a lack of effective XPD and lower total received power than the spatial systems results in the highest BER out of all systems throughout.
When antennas are positioned near to one another, they experience a level of mutual coupling. One effect of this is that antenna directivity changes. At particularly low separation, this can result in a potential increase, or decrease in total received power, depending upon the environment and the antenna array orientation. For example, the broadband spatially separate antennas in LOS experience an increase in total received power due to mutual coupling when separated by $0.5\lambda$, but experience a decrease when separated by only $0.25\lambda$. Another effect of mutual coupling is that some of the signal at the terminals of one antenna is coupled to the terminals of the other antenna, due to the mutual impedance. The mutual impedance using the coated dipole antennas is very low and its induced correlation between sub-channels is insignificant when compared to that caused by the similarity of physical propagation routes through the environment, when antennas are closely spaced.

Of far more significance is the presence of a metal case nearby to the antennas. As observed in Section 2.7.2.2. and Section 2.7.3.2. the inclusion of the case has a severe effect on antenna gain patterns and radiated polarisation. In this chapter, it is observed that these effects result in very different MIMO performance when the case is included, compared to when it is not.

The case changes the radiated polarisations in front of the spatially separate co-polar antennas from linear to elliptical, with opposite rotation sense. This causes a level of effective XPD, which most remarkably results in lower BER under V-BLAST in LOS than is achieved by either dual-polar system.

The polarisation of the fields radiated by the CP system with the case deteriorate to elliptical, often with the same rotation sense and therefore very low orthogonality in most directions. This reduces XPD and increases error rate under V-BLAST.

The LP antenna, in the presence of the case, maintains high orthogonality over a wider angle than it does without the case. In LOS however, its V-BLAST performance is particularly poor. This is because the case also distorts the antenna directivity, such that a low antenna gain is experienced in the direction of the direct path between transmitter and receiver. In NLOS on the other hand, the high orthogonality over such a wide azimuth angle is extremely beneficial and results in high XPD and greater V-BLAST performance than any of the other systems. This did not happen when the case was not included.
Of course, these observations are only applicable to the specific antenna and case models in this thesis. It is expected that alternate antenna and platform designs will result in different antenna behaviour and therefore different MIMO performance. Importantly though, the observation of such substantial effects on MIMO performance as a result of the case demonstrates the necessity of the novel end-to-end modelling approach. The approach enables the effect of the case to be directly related to MIMO performance in terms of capacity or error rate under a diversity or multiplexing scheme.

To highlight this point, in Section 2.7.3.2, it is noted that in the presence of the case, $\rho_{env}$ of the co-located LP antenna is $3.2 \times 10^{-6}$, while $S_{2,1}$ is -56 dB. For the co-located CP antenna with the case, $\rho_{env}$ is 0.11 and $S_{2,1}$ is -28 dB. Clearly the LP antenna has significantly lower pattern envelope correlation coefficient and mutual impedance than the CP antenna. From these results alone, one may well assume that sub-channel correlation using the LP antenna may be lower than when the CP antenna is used and better diversity performance may be achieved. From the results in Section 5.6.3 however, it is clear that the relative diversity performance of these systems is in fact highly dependent upon their propagation environment. In LOS, the LP system in fact results in substantially poorer diversity performance than the CP system over the entire simulated route. In NLOS on the other hand, the diversity performance of the two systems is comparable.
6. Conclusion

The main objective of this work was to research the use of polarisation MIMO in the indoor environment, using practical antennas on small sized portable devices.

One of the primary aims of the thesis was to develop an end-to-end modelling approach to predict MIMO performance for antenna systems using closely positioned spatially separate, or orthogonally polarised antennas, in the indoor environment. Certain literature [2-4] has been discussed which makes various assumptions about the conditions of the channel, to produce statistical MIMO channel models, however no existing work has been identified which allows a direct analysis of the performance of polarisation MIMO systems, based on the physical attributes of their antenna systems or the propagation environment. This aim has been met through the development of an end-to-end, predominantly deterministic modelling approach, where physical antenna systems are modelled in the indoor environment. A bespoke ray launching propagation model has been developed, from which the MIMO “channel parameters” of these systems in physical environments are obtained. These channel parameters are used to construct a novel polarisation MIMO channel model, from which MIMO channel capacity and error rate, under the Alamouti diversity scheme and the V-BLAST multiplexing scheme, are estimated.

This modelling approach has been used to examine the MIMO capabilities of compact systems which use orthogonally polarised antennas, with comparison to those which use spatially separate co-polar antennas, where spatial constraints force low separation.

Another aim of the thesis was to compare the propagation behaviour and resultant MIMO performance of dual-polar systems which use linear polarisation, to those which use circular polarisation. This objective has been fulfilled through the end-to-end modelling of systems using co-located dielectric coated crossed dipoles which are fed separately to operate with linear polarisation, as well as through a feed network to operate with orthogonal circular polarisation. No existing literature has been identified which makes like-for-like comparisons between dual-polarisation MIMO systems using polarisations other than linear.

The thesis also aimed to examine the effects of objects positioned within the near-field regions of the antennas of MIMO systems. This objective has been met through the
modelling of compact spatially separate and dual polarised antennas, positioned around a metal case.

Another important aim of the thesis was to investigate the mutual coupling behaviour at both transmit and receive antennas, with respect to MIMO performance and to fully account for this phenomenon in the MIMO performance estimations. It was identified from literature that the mutual coupling mechanism when antennas are operated as transmitters is different to that when they are operated as receivers [54-58]. This is because the mutual coupling is dependent upon the current distribution on the antenna elements. This varies depending on whether the antenna is excited at its terminals, as in transmit mode, or from a direction in the far-field, as in receive mode.

To account for this, a novel technique for estimating the receive mutual impedance was developed, using a weighted average approach, based on the angle of arrival and power of the rays incident on a receiver, as determined by the ray launcher. The receive mutual impedance, determined in this manner has been accounted for in the end-to-end MIMO performance results, through the use of a mutual impedance matrix which is applied to the MIMO channel model. Using this approach, it was observed that the receive mutual impedance varies with the position of the antennas in the modelled environments. That being said, the variation is very small. In fact, it is suggested that at the low levels of mutual impedance experienced with the dielectric coated antennas, the differences in the MIMO performance results between an approach which accounts for the receive mutual impedance in the novel way, compared to a traditional approach, which assumes the mutual coupling at receive mode antennas is the same as that when they are in transmit mode, are minimal. Nonetheless, it was important that this was given full consideration. Furthermore, the techniques for determining receive mutual impedance and accounting for it in the MIMO system model may be applied to other antenna systems in which levels of mutual coupling may be more significant.

The model also accounts for mutual coupling at the transmitter, through the simulation of the transmit antennas in the presence of each other, while both are loaded with their transmit source impedance. A full discussion of transmit and receive mutual coupling was presented in Section 2.4.

The antenna systems which have been modelled were described in Chapter 2. These are narrowband dielectric loaded compact dipoles for use by spatial MIMO systems and
narrowband dielectric loaded crossed dipoles, used by dual polar systems. As a reference, large uncoated dipoles have also been modelled, over a greater bandwidth. This provides an interesting comparison between broadband and narrowband systems, in terms of the MIMO channel conditions and the resultant MIMO performance.

It was necessary to pay particular attention to the polarisation of fields radiated by the modelled antennas. Whilst doing so, it became apparent that the polarisation of radiated fields may vary significantly with angle from an antenna. This phenomenon is crucial to the performance of polarisation MIMO systems because it affects the power transferred by each sub-channel, as well as the ability of the antennas to provide decorrelation.

To better understand this, a technique for studying the polarisation of fields radiated by antennas versus angle was developed, where the polarisations are presented on the Poincaré sphere. Using this technique, it was possible to see exactly how the radiated polarisations change in all directions from the antenna. It also made it possible to examine how orthogonal these polarisations are, which gives a good indication of the ability of the antennas to achieve high cross polar discrimination (XPD), which is a critical parameter for polarisation MIMO systems. Furthermore, a method for quantifying polarisation orthogonality versus angle and displaying this on a 3D spherical plot was developed. It is suggested that both of these tools may be very useful to an engineer who is designing antennas for use in dual polar systems.

It was observed that the presence of the metal case has a severe effect on the radiation from the antennas positioned around it. It causes the radiation patterns of all antennas to become extremely distorted. This has a particularly interesting effect on the polarisation of the fields radiated by the antennas. From analysis using the Poincaré sphere, it was clear that the polarisations from both excitations of the CP antenna, in the presence of the case, deteriorate to elliptical in all directions, often with the same rotation sense and unpredictable tilt angles. This results in very poor orthogonality in most directions, which is detrimental to the (XPD) of a dual polar MIMO system. The radiation patterns of the LP antenna in the presence of the case also become very distorted, although the effect upon polarisation is less severe than for the CP antenna. In fact, the LP antenna radiates with high orthogonality over a wider angle in the presence of the case than without. Remarkably, in the presence of the spatially separate “co-polar” antennas, the case causes polarisation to become elliptical in the direction perpendicular to the orientation of the
case, with opposite rotation senses from each antenna. This results in an unexpected region of high orthogonality in the direction “in front” of the models. In theory, this allows the antennas to exploit an element of polarisation diversity, as well as spatial diversity.

In order to understand and to account for the effects of the indoor propagation environment, a 3D ray launching propagation model has been developed, as described in Chapter 3. This is a bespoke frequency domain model which fully accounts for the polarisation and phase of propagating waves, which are modelled as rays, using geometric optics. A bespoke model was favoured as opposed to a commercial package due to the greater flexibility it provides, in terms of input and output results and their format. The inputs to the ray launcher are a database which describes all of the objects in the environment, the transmitter and receiver locations and their antenna patterns. The output is the frequency response of each MIMO sub-channel, over the modelled signal bandwidth. Due to the flexibility of the bespoke model, it was also possible to examine any intermediate results which may be of interest, for example the properties of the received rays, such as their field strength and polarisation, angles of arrival and departure or even the full path which they have taken through the environment. The study of these results enables a greater understanding of the propagation behaviour over the polarisation MIMO channel.

Ray launching provides an efficient method for deterministically modelling the effects of the propagation environment on polarisation MIMO systems. When using ray techniques to model propagation, it is particularly important to mitigate the effects of “duplicate” received rays, which have taken essentially the same path through the environment, as these can result in an incorrect representation of the total received signal. It is suggested that the best way to do this is through the use of a geodesic launch geometry and an effective ray filter. The geodesic launch geometry ensures that rays are launched with a very close approximation to uniform angular distribution. The ray filtering approach was favoured over a weighting function as it is more compatible with antenna gain patterns which contain large changes in magnitude over very small angles.

It is important that the polarisation of the fields associated with the rays is properly traced through the environment, as well as simply the field intensity. To do so, it was necessary to convert the field vectors between multiple different bases. The model launches rays with fields described in the spherical bases of the transmit antennas. The fields are then
transformed into their “global” Cartesian bases, until the ray intersects an object in the environment. Upon intersection, the fields are transformed into components of the “ray-fixed” coordinate system. At this point, the transmission and reflection coefficients for transverse electric and transverse magnetic mode waves are applied, before the fields are transformed back to their global bases and the ray is traced to its next interaction. When a ray is received, its fields are transformed into the components of the spherical bases of the receive antenna, before the antenna gain is applied.

To ensure the validity of results which are dependent upon the ray launcher, a thorough validation procedure has been completed. Initially, individual functionality, such as reflection and transmission, then the application of antenna patterns were tested in isolation. Results were compared to those calculated directly, those obtained using CST and those obtained using the commercial ray launcher, Wireless InSite. Following this, results were validated for environments of increasing complexity, by comparison to equivalent results obtained using Wireless InSite and an additional electromagnetic simulator, FEKO. The environments were built up from a single slab, to a terraced house and eventually an office floor.

The end-to-end MIMO system results are obtained through the use of a bespoke polarisation MIMO channel model, which is described in Chapter 4. A bespoke model was used so that the conditions of the channel, as determined by the ray launcher and through the examination of mutual coupling may be fully accounted for. The model is used to relate the signals at the terminals of receive antennas, to the signals at the terminals of transmit antennas, using the narrowband flat fading transmission model. Each MIMO sub-channel is represented by a narrowband gain, contained in an \( n_t \times n_r \) matrix. Each gain contains a Rayleigh distributed random component which represents propagation via rich multipath, as well as a component of fixed magnitude, which represents propagation via any direct or specular paths. The ratio between these components is set independently for each sub-channel, according to the sub-channel K-factor, which is obtained through a Rician distribution fit, applied to the sub-channel frequency response from the ray launcher. The Rayleigh components of the channel are correlated through multiplication with a correlation matrix. The elements of this matrix are the correlation coefficients between the sub-channel impulse responses, which are obtained through inverse fast Fourier transform of the frequency responses. The gains of each sub-channel are weighted independently, such that their expected magnitude is equal to the average sub-channel gain,
taken over the signal bandwidth. The effect of transmit mutual coupling is inherently included in the ray launcher result, which is obtained using coupled antenna patterns at the transmitter, while the receive mutual coupling is accounted for from the weighted average approach, using a receive mutual impedance matrix in the channel model.

Using the channel model, MIMO system results have been determined in terms of channel capacity and estimated bit error rate, under a diversity and a multiplexing MIMO scheme. Because different MIMO schemes are better suited to different channel conditions, it is suggested that to provide a complete review of the potential performance of a polarisation MIMO system, it is important to consider the performance over two such schemes, rather than considering only the channel capacity.

The bit error rate is estimated through Monte-Carlo MIMO system simulations, where the transmission of a large number of bits is simulated over the modelled channel, and the number of bits received in error is counted. As with the ray launcher, it is important that the channel model and the MIMO system simulations are known to reliably produce accurate results. As such, a validation procedure has been performed where BER results, simulated over the modelled channel, under given channel conditions, were compared those published in existing literature.

End-to-end MIMO performance results have been obtained using the novel end-to-end modelling approach described throughout the thesis and are presented in Chapter 5. The broadband systems and the narrowband systems, with and without the case, have been modelled in a line of sight (LOS) and non-line of sight (NLOS) indoor environment. The MIMO channel parameter results are shown in the appendices, with an overview given in Section 5.4.1. The end-to-end results have been obtained over simulated routes, where the transmitter was in a fixed location and the receiver was moved over a two metre distance, in each environment.

A summary of the results and the key observations was provided at the end of Chapter 5. From these, many conclusions can be drawn with regard to polarisation MIMO systems in the indoor environment. A general summary of the core conclusions is presented in Table 6-1. This provides brief statements which indicate the expected performance of compact polarisation MIMO systems when compared to spatial MIMO systems, under diversity and multiplexing applications, in the LOS and NLOS environment. These remarks are based
on the BER results presented in chapter 5, for the narrowband antennas with no case, in the LOS and NLOS environments described in Section 5.2.

Table 6-1 – Summary of core conclusions regarding performance of compact indoor polarisation MIMO systems

<table>
<thead>
<tr>
<th></th>
<th>Diversity</th>
<th>Multiplexing</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>Polarisation MIMO comparable to spatial with low separation</td>
<td>Polarisation MIMO preferable to spatial, due to channel XPD</td>
</tr>
<tr>
<td>NLOS</td>
<td>Polarisation MIMO worse than spatial, due to lower received power</td>
<td>Polarisation MIMO slightly worse than spatial, though highly dependent on location</td>
</tr>
</tbody>
</table>

As a general rule, when space on a device is not limited, superior overall MIMO performance is achieved through the use of well separated co-polar antennas, as opposed to dual polar antennas with little separation. The main disadvantage of the use of dual-polar antennas for MIMO is the fact that they result in significantly less total received power. This is due to the low gain cross-polar sub-channels which, even in a rich multipath environment, transfer significantly lower power than co-polar sub-channels. This results in high XPD, which in terms of diversity performance is generally detrimental, although it can be an advantage under a multiplexing scheme.

When space on a platform is limited and spatial separation is reduced as low as $0.25\lambda$, sub-channel correlation over the frequency responses and over distance increases. Also, mutual coupling alters the directivity of antenna patterns. In this situation it has been observed that the performance of co-located dual-polar MIMO systems is often at least comparable to that of the spatial systems. The only exception to this is in the NLOS environment, where capacity and diversity performance of the dual-polar systems is notably worse than that of the spatial systems at any separation. This is because the rich multipath in NLOS results in low correlation on the sub-channels of the spatial systems and also reduces the gain fluctuations over distance. As such, the dual polar systems do not offer any advantage in terms of reduced correlation, but are still at a disadvantage in terms of total received power.
It was observed that the conditions of the MIMO channel over a broad band are notably different to those over a narrow band. The effects of the multipath propagation over the broad band are more apparent. This is evidenced by greater multipath induced fluctuation over each sub-channel frequency response and lower resultant K-factors. In these conditions, it was observed that in LOS, the dual polar systems offer greater capacity than the 0.25\(\lambda\) separated spatial system. This is a result of lower sub-channel correlation (over the broad frequency band) when using the orthogonally polarised antennas, as both of these systems in fact received similar total power.

The Alamouti scheme was unable to exploit the additional capacity in this instance, with error rate of the 0.25\(\lambda\) system and the dual polar systems approximately equal. Under V-BLAST however, the advantage is clearly visible, with both broadband dual polar systems achieving lower BERs than the 0.25\(\lambda\) spatial system, throughout the LOS route. This observation demonstrated the need for an end-to-end modelling approach which predicts error rate performance under both types of MIMO scheme, as well as capacity.

Using the compact antennas, K-factors are significantly higher as the frequency responses fluctuate less over the narrow band. This means that the channels are dominated by their fixed magnitude, specular components. The amplitudes of these however experience greater multipath fluctuations over distance. Under these conditions, the correlation of the sub-channels taken over frequency is of little relevance, and instead the correlation of expected sub-channel gains taken over distance becomes important. If these are well decorrelated then the probability of multiple sub-channels experiencing a deep fade simultaneously is reduced. The best way to achieve this decorrelation is through the use of well separated antennas. When separation is as low as 0.25\(\lambda\) however, particularly in LOS, the expected sub-channel gains behave similarly over distance and the dual polar systems provide comparable diversity performance. This does not extend to the NLOS environment due to its richer multipath, which reduces correlation even at low separation, and also reduces the magnitude of the gain fluctuations over distance. This means that the problem of deep fades is less significant and the spatial systems outperform the dual polar systems under the Alamouti scheme, because they receive higher total power.

It is reported in literature [3, 4] that the use of dual-polar antennas for spatial multiplexing, particularly when K-factors are high, is generally highly beneficial. When using a simple channel model and taking a very general view, this is the case. However, when using the
end-to-end modelling approach, it has been demonstrated that for practical antennas in the indoor environment, the channel is more complex than that modelled in [3, 4], hence the need for the novel extended channel model. The multiplexing benefit for dual-polar systems is dependent upon high XPD and co-polar sub-channels of similar magnitude. From the results presented in this thesis, it is demonstrated that these conditions are not guaranteed. Even in a LOS environment, with close to ideal dual polar antennas, which are positioned with the direction of maximum gain and orthogonality aligned with the direct path, often these conditions are not fully met. This results in V-BLAST performance which is not significantly better than is obtained using the spatial systems. When using LP antennas, the difficulty is that different propagation conditions for each polarisation mean that frequently one co-polar channel receives significantly lower power than the other. Under CP, this does not occur, as the co-polar sub-channels are highly correlated, however the XPD is lower and often this decreases to the extent that interference from the cross-polar channels is of equal magnitude to the co-polar channels.

When comparing the narrowband dual polar LP system to the CP system, it has been observed that the differences in the channels have little effect on diversity performance. Because these systems use the same antenna elements, fed with the same amount of power, they receive the same total amount of power which, over a high K-factor channel, results in approximately the same diversity performance. Under V-BLAST, there is a small difference in performance, due to different levels of XPD and spatial correlation of the co-polar sub-channels, however there is no clear benefit to either arrangement.

When the crossed-dipole LP transmitter is used with a crossed-dipole CP receiver, it was observed that the same antenna elements again receive the same total power as the pure LP and CP systems, resulting in similar diversity performance. V-BLAST performance with this arrangement is particularly poor, due to a lack of effective XPD, when compared to the other dual polar systems, and lower total received power than the spatial systems.

The hybrid system, where orthogonally polarised CP antennas were separated in space by 0.25\(\lambda\) demonstrated slightly better diversity performance than the co-located dual-polar systems, however its performance was significantly poorer than that of well separated spatial systems, due to its lower received power. Under V-BLAST, its performance was comparable to the co-located dual-polar systems, but it provided no advantage as it resulted
in lower XPD than the co-located LP system and at times experienced significantly lower gain on one co-polar sub-channel than the other.

It has been observed that mutual coupling between closely spaced antennas has a significant effect upon MIMO performance, which should not be overlooked. Importantly, it was observed that the change in antenna directivity due to mutual coupling has the potential to increase or decrease the total receive power of a system, depending upon antenna orientation, separation and the propagation environment. This affects the MIMO performance in terms of channel capacity and error rate. The additional correlation between sub-channel responses on the other hand, which results from mutual coupling of the signal on one antenna to nearby antennas, is considered insignificant, when compared to the correlation resulting from the similarity of physical propagation routes through the environment, when antennas are closely spaced.

A final and important conclusion is that the presence of objects in the near-field regions of the antennas has the potential to severely affect the channel behaviour and the performance of both spatial and polarisation MIMO systems. When a metal case was positioned in proximity to the compact antennas, the radiated polarisation of the dipoles became elliptical, with opposite rotation senses from each antenna, in the direction perpendicular to the orientation of the case. This caused an unexpectedly high level of XPD, which resulted in lower bit error rate under V-BLAST in LOS than was achieved by either dual polar system. In contrast, the case caused the polarisation from the CP antenna to deteriorate to ellipses, which in many directions rotated with the same sense. This reduced XPD and was detrimental to V-BLAST performance. The case also had a significant effect on the radiation patterns of the antennas. This impacted the LP system, which was able to maintain high orthogonality over a wide angle, but due to the effect that the case has on its gain, its total received power in LOS was significantly reduced, which was very detrimental to V-BLAST performance. In NLOS however, the high orthogonality achieved over such a wide angle by the LP system was extremely beneficial and resulted in better V-BLAST performance than any of the other systems in this environment.

Although the results and observations regarding the presence of the case are only applicable to these specific antenna models around the metal case, the work demonstrates how severe and unpredictable an effect objects such as this can have on MIMO performance. This further demonstrates the importance of a full end-to-end modelling
The approach which has been presented may be applied to any antenna system in any environment. It also conveniently allows the study of intermediate results, such as the antenna patterns, radiated polarisations and their orthogonality, as well as the parameters of the propagation environment. Analysis in this way enables separation of the factors which contribute towards MIMO performance, enabling more informed decisions to be made in the development of polarisation and spatial MIMO systems.

6.1. Main Contributions to the Field of Research

The thesis has provided a number of novel contributions to the research topic of polarisation MIMO in the indoor environment, which may be summarised as follows.

- A full end-to-end, predominantly deterministic, modelling approach has been presented for dual-polar and spatial MIMO systems in the indoor environment. This uses a bespoke MIMO channel model, based on deterministically obtained channel parameters for antenna models in a propagation environment which is modelled through ray launching. It enables a full analysis of system performance, directly relating the physical antenna and environment models, to the resultant channel conditions and the obtainable MIMO performance, in terms of capacity and bit error rate. The modelling approach may be applied to any environment or antenna system.

- An evaluation of polarisation MIMO performance according to these metrics has been presented, comparing spatial MIMO systems, with limited antenna separation, to polarisation MIMO systems, using co-located dual-polar antennas. The parameters of the MIMO channel were obtained through deterministic modelling of physical antenna systems and environments, as opposed to using assumed channel conditions, which do not fully reflect the behaviour of physical systems.

- A comparison has been made between polarisation MIMO systems which use orthogonal linear polarised antennas and those which use orthogonal circular polarised antennas. The differences in propagation conditions dependent upon polarisation were fully accounted for through the use of the 3D ray launching propagation model and the bespoke polarisation MIMO channel model.

- An investigation into the effects of objects close to the antennas has been carried out, for polarisation and spatial MIMO systems, through analysis where antennas are positioned around a metal case. The impact of the case on radiation patterns
and importantly, polarisation and orthogonality of energy radiated by the antennas has been addressed, as well as the resultant effects on MIMO performance.

6.2. Further Work

The thesis has introduced a useful and versatile model for predicting the behaviour of polarisation MIMO systems. To build on the results presented, some suggested avenues for further work are outlined below.

1. Each sub-system of the end-to-end model has been validated against multiple different sources of equivalent results. This enables confidence in the accuracy and reliability of the final results. That being said, as with any theoretical study, it would be useful to compare the results obtained using this model to those obtained through measurement. A measurement campaign could be built up in a similar fashion to the modelling approach. This would first involve antenna pattern measurements for simple dual polar antennas, using techniques described in [68]. These results should be compared to those obtained using CST-MWS. Next, the antennas would be positioned in a real indoor environment, which may be modelled using the ray launcher. The frequency response of the MIMO sub-channels would be measured, to give results which may be compared to those obtained by the ray launcher. From the frequency responses, the MIMO channel may be modelled using the same approach as in Chapter 4, to obtain MIMO performance results. Or, to complete a full end-to-end measurement campaign, the antennas may be connected to a MIMO transmitter and receiver, to transmit a test bit sequence using a given MIMO scheme. The decoded bit sequence at the receiver would be compared to the sequence transmitted, to produce a bit error rate result.

2. The results presented in the thesis are obtained using simple antenna models of dielectric loaded dipoles and crossed dipoles. These antennas produce familiar patterns, which are well understood. As such they were an ideal starting point for the study of polarisation MIMO systems, using the approach in this thesis. Of course, this approach may be used to model MIMO systems using more complex antenna systems, which would be a very interesting topic for further study.

In Chapter 2, it was observed that the co-located LP antenna, in the presence of the case, radiates fields which remain orthogonal over particularly wide angles. In Chapter 5, it was observed that as a result of this, in the NLOS environment, the
system using this antenna experienced high XPD and achieved better spatial multiplexing performance than any of the other systems. In LOS, the system did not perform as well, because the case distorted the antennas radiation pattern, such that the gain in the direction of the direct path decreased.

With these observations in mind, it is suggested that antennas which maintain high orthogonality over a wide angle, and have radiation patterns which are highly uniform, may be well suited to spatial multiplexing polarisation MIMO systems. One antenna which may fit this criterion is the quadrifilar helical antenna (QHA), first introduced by Kilgus in [138]. It consists of four helical elements which are intertwined. These are fed with 0, 90, 180 and 270° phase difference, which is known as phase-quadrature. The result is a cardioid shaped radiation pattern, with circular polarisation over a wide angle, with the direction of maximum gain in the axial direction. It would certainly be interesting to study the MIMO capabilities of a system which uses antennas of this type. These could be positioned close to each other and constructed such that they radiate circular polarisation with opposite rotational sense. Of course, the work in this thesis suggests that the antennas will experience a mutual coupling effect, which may result in different antenna patterns and radiated polarisation from that which is observed when the antennas are in isolation.

QHAs are resonant when the helical arm length is an integer number of quarter wavelengths [139]. They are therefore significantly larger than the dielectric loaded dipoles which have been studied in this thesis and are therefore less suited to compact devices. It has however been identified and demonstrated in [60, 140-146] that antennas of this type may be miniaturised by wrapping the helical arms around a dielectric cylinder of high relative permittivity. As such, it would be interesting to use the modelling approach in this thesis to study the potential MIMO benefit which may be possible using these miniaturised QHAs on small platforms.

3. This thesis has demonstrated the severe effect on MIMO performance of a metal case in the presence of the antennas. It is suggested that a more realistic case, constructed from less metal and more plastics may have less of an effect. This theory could be tested using the same end-to-end modelling approach, with different antenna and platform models. Another potential challenge for some
compact devices, which has not been examined in this thesis, is the problem of body-loading. In a similar way to the case, human flesh has the potential to affect the behaviour of antennas and hence the performance of MIMO systems which use them. It is stated in [147] that dielectric loaded antennas have the potential to reduce the impact of the proximity of human tissue, due to their near-field containment abilities. The effects of body-loading of this nature on MIMO systems, using antennas of different types, could be easily studied using the presented modelling approach.

4. The results given in this thesis all use a $2 \times 2$ MIMO channel. This is to give a general comparison between spatial MIMO systems, which use co-polar antennas and dual-polar systems, using orthogonally polarised antennas. The same modelling approach however, could be expanded to model systems of higher dimension, such as $4 \times 4$. This would enable a broader study into systems which may use a set of two co-located dual polar antennas, separated in space. Such systems would establish eight co-polar and eight cross polar channels and a potential diversity order of 16. A channel of this nature could be utilised by a space time block code such those described by Tarokh in [13].

5. It has been demonstrated in this thesis that certain channel conditions favour diversity schemes, while others favour multiplexing schemes. It is possible that optimum use of the channel in terms of capacity could be obtained by a system which adaptively changes between MIMO schemes, in response to channel conditions. A simple scheme could perhaps switch between Alamouti and V-BLAST, according to a threshold which is related to XPD. A more complex system could perhaps also adapt the modulation scheme to suit the channel conditions, as in [148]. Lastly, if antennas are constructed from crossed dipoles or other dual-polar elements, it would also be possible to develop a system where the antenna feed configuration adapts to the channel conditions, by changing the polarisation of the antennas. Supposing two crossed-dipole antennas are present on a device, these could be configured to only excite one element each in conditions and applications where spatial MIMO provides optimum performance. If the channel conditions change and polarisation MIMO provides optimum performance, for example because the user has stepped in to a LOS region from a NLOS region, then the antennas could be fed such that they each radiate with orthogonal circular
or linear polarisation. All of these adaptive systems could be studied using a model which is based on the end-to-end modelling approach introduced in this thesis.
Appendix A. Channel Parameter Results

This appendix shows the channel parameter results, $X$, $K$, and $R$, which were determined from the sub-channel frequency responses (according to the techniques described in Chapter 4.5) which were obtained by the ray launcher.

The “Elements of $X$” plots show the average sub-channel gains taken over the frequency band at each location in space. The elements of $X$ are $\alpha_{i,j}$ for $i, j = 1, 2$, where $\alpha_{i,j}$ is the average gain over frequency of the sub-channel response, $R_{i,j}$. This is used by the extended MIMO channel model to set the expected sub-channel gains, such that $E\left[|h_{i,j}|^2\right] = \alpha_{i,j}^2$.

The “Elements of $K$” plots show the K-factors of each sub-channel, taken over the frequency band, at each location in space. $K_{i,j}$ is the K-factor of the sub-channel $h_{i,j}$, obtained by Rician distribution fitting to the frequency response, $R_{i,j}$.

The “Transmit Correlation”, “Receive Correlation” and “Cross Correlation”, plots show the correlation coefficients in the correlation matrix, $R$. These are plotted at each location in space and taken over the frequency band. The elements $R$ are denoted as follows.

$$R = \begin{bmatrix} 1 & r_1 & t_1 & u_1 \\ r_1 & 1 & u_2 & t_2 \\ t_1 & u_2 & 1 & r_2 \\ u_1 & t_2 & r_2 & 1 \end{bmatrix}$$

Below are some important notes which should be considered when consulting the results in this appendix.

- The elements of $X$, which are denoted $\alpha_{i,j}$ in the main body of the thesis (and above) are denoted $\alpha_{xi,j}$ in the remainder of this appendix.
- The cross correlation coefficients, which are denoted $u_1$ and $u_2$ in the main body of the thesis (and above) are denoted $s_1$ and $s_2$ respectively in the remainder of this appendix.
- The K-factors experienced by the uncoated full size antenna systems, over the broad band are typically much lower than those experienced by the compact narrowband systems. To give as much detail as possible in the K-factor plots, a linear scale is used for the Y axis of the broadband results, while a decibel scale is used for the narrowband results.
- In some of the narrowband results, the K-factor drops very abruptly, which may appear somewhat non-physical. This is in fact a result of the limited spatial resolution of the plots. The results were obtained at spatial intervals of 5 cm.
Broadband Uncoated Antennas, LOS
Spatial, separation = 0.25λ
Broadband Uncoated Antennas, LOS

Spatial, separation = 0.5λ
Broadband Uncoated Antennas, LOS

Spatial, separation = $\lambda$
Broadband Uncoated Antennas, LOS

Co-located dual polar, LP
Broadband Uncoated Antennas, LOS

Co-located dual polar, CP
Broadband Uncoated Antennas, NLOS
Spatial, separation = 0.25\lambda

Elements of X

Elements of K

Transmit Correlation

Receive Correlation

Cross Correlation
Broadband Uncoated Antennas, NLOS
Spatial, separation = 0.5λ
Broadband Uncoated Antennas, NLOS
Spatial separation = \( \lambda \)
Broadband Uncoated Antennas, NLOS

Co-located dual polar, LP

![Graphs showing Elements of X, Elements of K, Transmit Correlation, Receive Correlation, and Cross Correlation over distance (m)].
Broadband Uncoated Antennas, NLOS

Co-located dual polar, CP
Compact Narrowband Antennas without Case, LOS
Spatial, separation = 0.25λ
Compact Narrowband Antennas without Case, LOS

Spatial, separation = 0.5\(\lambda\)
Compact Narrowband Antennas without Case, LOS

Spatial, separation = $\lambda$
Compact Narrowband Antennas without Case, LOS

Co-located dual polar, LP
Compact Narrowband Antennas without Case, LOS
Co-located dual polar, CP

Elements of $X$

Elements of $K$

Transmit Correlation

Receive Correlation

Cross Correlation
Compact Narrowband Antennas without Case, LOS
CP hybrid, separation = 0.25\(\lambda\)
Compact Narrowband Antennas without Case, LOS
Co-located dual polar, LP to CP

Elements of $X$

Elements of $K$

Transmit Correlation

Receive Correlation

Cross Correlation
Compact Narrowband Antennas without Case, NLOS
Spatial, separation = 0.25\lambda

Elements of X

Elements of K

Transmit Correlation

Receive Correlation

Cross Correlation
Compact Narrowband Antennas without Case, NLOS

Spatial, separation = 0.5\( \lambda \)
Compact Narrowband Antennas without Case, NLOS

Spatial, separation = \( \lambda \)
Compact Narrowband Antennas without Case, NLOS
Co-located dual polar, LP
Compact Narrowband Antennas without Case, NLOS

Co-located dual polar, CP
Compact Narrowband Antennas without Case, NLOS
CP hybrid, separation = 0.25λ
Compact Narrowband Antennas without Case, NLOS
Co-located dual polar, LP to CP
Compact Narrowband Antennas with Case, LOS
Spatial, separation = 0.3\lambda

**Elements of X**

**Elements of K**

**Transmit Correlation**

**Receive Correlation**

**Cross Correlation**
Compact Narrowband Antennas with Case, LOS

Spatial, separation = 0.5\( \lambda \)
Compact Narrowband Antennas with Case, LOS
Co-located dual polar, LP
Compact Narrowband Antennas with Case, LOS

Co-located dual polar, CP
Compact Narrowband Antennas with Case, NLOS
Spatial, separation = 0.3λ
Compact Narrowband Antennas with Case, NLOS
Spatial, separation $= 0.5\lambda$

Elements of $X$

Elements of $K$

Transmit Correlation

Receive Correlation

Cross Correlation
Compact Narrowband Antennas with Case, NLOS

Co-located dual polar, LP
Compact Narrowband Antennas with Case, NLOS

Co-located dual polar, CP

Elements of X

Expected path gain (dB)

Distance (m)

Elements of K

K-factor (dB)

Distance (m)

Transmit Correlation

|Correlation| 1 |

Distance (m)

Receive Correlation

|Correlation| 1 |

Distance (m)

Cross Correlation

|Correlation| 1 |

Distance (m)
Appendix B. Average Channel Parameter Results

The results in this appendix show the average values of the elements of $\mathbf{X}$, $\mathbf{K}$ and $\mathbf{R}$ for each system, over space. Results for each antenna configuration in a given environment are included on the same axes for comparison. CDFs are presented as well as plots versus distance. Results for the full size broad band antennas are presented first, then the compact antennas with no case, then the compact antennas with the case.

The “Average Sub-channel Gain” plots show the average sub-channel gains vs distance for all systems, where average sub-channel gain, $\text{mean}(\alpha)$ is calculated as follows.

$$\text{mean}(\alpha) = \frac{1}{n_t n_r} \sum_{i=1}^{n_r} \sum_{j=1}^{n_t} \alpha_{i,j}^2 = \frac{1}{n_t n_r} ||\mathbf{X}||_F^2$$

The “Average Sub-channel K-factor” plots are produced in a similar way, where average K-factor, $\text{mean}(K)$ is presented against distance and calculated as follows.

$$\text{mean}(K) = \frac{1}{n_t n_r} \sum_{i=1}^{n_r} \sum_{j=1}^{n_t} K_{i,j}$$

The “Average Sub-channel Correlation” plots show the average of the elements of $\mathbf{R}$, excluding the diagonal elements which are all equal to 1. This is calculated as follows, where $\rho_{i,j}$ is the $i, j$th element of $\mathbf{R}$.

$$\text{mean}(\mathbf{R}) = \frac{1}{(n_t n_r)^2 - n_t n_r} \sum_{i=1}^{n_t} \sum_{j=1,i\neq j}^{n_r} \rho_{i,j}$$

Below are two important notes which should be considered when consulting the results in this appendix.

- In the remainder of this appendix, the average sub-channel gain, $\text{mean}(\alpha)$ is in fact denoted, $\text{mean}(\alpha_x)$
- The K-factors experienced by the uncoated full size antenna systems, over the broad band are typically much lower than those experienced by the compact narrowband systems. To give as much detail as possible in the K-factor plots, linear scales are used for plotting the average K-factors of the broadband results, while decibel scales are used for the narrowband results.
Broadband Uncoated Antennas, LOS
Broadband Uncoated Antennas, LOS (continued)

CDF of Average Sub-channel Gain

CDF of Average K-factor

CDF of Average Sub-channel Correlation
Broadband Uncoated Antennas, NLOS
Broadband Uncoated Antennas, NLOS (continued)
Compact Narrowband Antennas without Case, LOS

Average Sub-channel Gain

Average Sub-channel K-factor

Average Sub-channel Correlation
Compact Narrowband Antennas without Case, LOS (continued)
Compact Narrowband Antennas without Case, NLOS

Average Sub-channel Gain

Average Sub-channel K-factor

Average Sub-channel Correlation

Distance (m)
Compact Narrowband Antennas without Case, NLOS (continued)
Compact Narrowband Antennas with Case, LOS

Average Sub-channel Gain

Average Sub-channel K-factor

Average Sub-channel Correlation
Compact Narrowband Antennas with Case, LOS (continued)
Compact Narrowband Antennas with Case, NLOS

Average Sub-channel Gain

Average Sub-channel K-factor

Average Sub-channel Correlation
Compact Narrowband Antennas with Case, NLOS (continued)

CDF of Average Sub-channel Gain

CDF of Average K-factor

CDF of Average Sub-channel Correlation
References


[23] H. Y. Fan, "MIMO detection schemes for wireless communication," M. Phil, Hong Kong University of Science and Technology, Hong Kong, 2002.


