ROBUSTNESS OF REINFORCED CONCRETE FRAMED BUILDING AT ELEVATED TEMPERATURES

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

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Abstract

This thesis presents the results of a research programme to investigate the behaviour and robustness of reinforced concrete (RC) frames in fire. The research was carried out through numerical simulations using the commercial finite element analysis package TNO DIANA. The main focus of the project is the large deflection behaviour of restrained reinforced concrete beams, in particular the development of catenary action, because this behaviour is the most important factor that influences the frame response under accidental loading. This research includes four main parts as follows: (1) validation of the simulation model; (2) behaviour of axially and rotationally restrained RC beams at elevated temperatures; (3) derivation of an analytical method to estimate the key quantities of restrained RC beam behaviour at elevated temperatures; (4) response and robustness of RC frame structures with different extents of damage at elevated temperatures.

The analytical method has been developed to estimate the following three quantities: when the axial compression force in the restrained beam reaches the maximum; when the RC beams reach bending limits (axial force = 0) and when the beams finally fail. To estimate the time to failure, which is initiated by the fracture of reinforcement steel at the catenary action stage, a regression equation is proposed to calculate the maximum deflections of RC beams, based on an analysis of the reinforcement steel strain distributions at failure for a large number of parametric study results. A comparison between the analytical and simulation results indicates that the analytical method gives reasonably good approximations to the numerical simulation results.

Based on the frame simulation results, it has been found that if a member is completely removed from the structure, the structure is unlikely to be able to develop an alternative load carrying mechanism to ensure robustness of the structure. This problem is particularly severe when a corner column is removed. However, it is possible for frames with partially damaged columns to achieve the required robustness in fire, provided the columns still have sufficient resistance to allow the beams to develop some catenary action. This may be possible if the columns are designed as simply supported columns, but have some reserves of strength in the frame due to continuity. Merely increasing the reinforcement steel area or ductility (which is difficult to do) would not be sufficient. However, increasing the cover thickness of the reinforcement steel to slow down the temperature increase is necessary.
Declaration

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## Notation

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<td>P</td>
<td>impact force from falling damaged members</td>
</tr>
<tr>
<td>M</td>
<td>total mass after impact</td>
</tr>
<tr>
<td>k</td>
<td>spring stiffness</td>
</tr>
<tr>
<td>v</td>
<td>velocity after impact</td>
</tr>
<tr>
<td>m₁</td>
<td>mass of lower beam</td>
</tr>
<tr>
<td>m₂</td>
<td>mass of upper beam</td>
</tr>
<tr>
<td>v₁</td>
<td>velocity when impacted damages were made</td>
</tr>
<tr>
<td>v₂</td>
<td>initial velocity</td>
</tr>
<tr>
<td>ε₉ₘₜ</td>
<td>thermal strain</td>
</tr>
<tr>
<td>εₜₜ</td>
<td>instantaneous stress-dependent strain</td>
</tr>
<tr>
<td>ε₅ₕₑᵉᵖ</td>
<td>creep strain</td>
</tr>
<tr>
<td>εₜᵣ</td>
<td>transient state strain (or Load Induced Thermal Strain – LITS)</td>
</tr>
<tr>
<td>L</td>
<td>length of beam</td>
</tr>
<tr>
<td>ΔL</td>
<td>axial extension</td>
</tr>
<tr>
<td>T</td>
<td>tensile force</td>
</tr>
<tr>
<td>C</td>
<td>compressive force</td>
</tr>
<tr>
<td>δ</td>
<td>vertical deflection</td>
</tr>
<tr>
<td>M</td>
<td>nominal bending capacity</td>
</tr>
<tr>
<td>δ₉ₜᵣ</td>
<td>extension due to geometry</td>
</tr>
<tr>
<td>h</td>
<td>height of beam</td>
</tr>
</tbody>
</table>
\[\Delta\] beam centre displacement

\[\varnothing\] angle between original location and changed location of a beam

Chapter 3

\[\varphi\] internal frictional angle

\[c\] cohesion coefficient

Chapter 4

\[T_f\] fire temperature in °C

\[T_0\] initial ambient temperature in °C

\[T_h\] Time (in hours)

\[\theta\] concrete temperature in °C

\[C_p(\theta)\] specific heat of concrete

\[\delta_m\] mechanical deflection of beam

\[\delta_{th}\] thermal deflection of beam

\[f_c\] concrete strength

\[b\] width length of a beam

\[d\] effective depth of a beam section

Chapter 5

\[F_a\] axial force in the beam (tension positive)

\[P\] applied load

\[L\] beam span

\[R'\] Reinforcement steel elongation

\[M_m\] internal bending moment capacity at beam mid-span
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>C</td>
<td>resultant force on the compressive side of the cross-section</td>
</tr>
<tr>
<td>T</td>
<td>resultant force on the tensile side of the cross-section</td>
</tr>
<tr>
<td>D</td>
<td>depth of the cross-section</td>
</tr>
<tr>
<td>a</td>
<td>depth of the compression zone</td>
</tr>
<tr>
<td>D_c</td>
<td>concrete cover to reinforcement</td>
</tr>
<tr>
<td>δ_T</td>
<td>beam total deflection</td>
</tr>
<tr>
<td>a_{1,2...n}</td>
<td>thermal expansion of each layer at elevated temperature</td>
</tr>
<tr>
<td>a'_{1,2...n}</td>
<td>modified thermal expansion of each layer at elevated temperature</td>
</tr>
<tr>
<td>A_r</td>
<td>total area for raw thermal expansion</td>
</tr>
<tr>
<td>A_m</td>
<td>total area for modified thermal expansion</td>
</tr>
<tr>
<td>k_t</td>
<td>thermal curvature</td>
</tr>
<tr>
<td>α</td>
<td>coefficient of thermal expansion of concrete</td>
</tr>
<tr>
<td>h</td>
<td>height of cross-section</td>
</tr>
<tr>
<td>ΔT</td>
<td>temperature difference between the top and bottom of beam cross-section</td>
</tr>
<tr>
<td>E_C</td>
<td>elastic modulus of concrete</td>
</tr>
<tr>
<td>α_{c/3}</td>
<td>strain at one-third of the maximum compressive strength of concrete</td>
</tr>
<tr>
<td>α_c</td>
<td>strain at the maximum compressive strength of concrete</td>
</tr>
<tr>
<td>α_u</td>
<td>ultimate strain of concrete</td>
</tr>
<tr>
<td>F</td>
<td>axial force</td>
</tr>
<tr>
<td>F_n</td>
<td>axial force due to thermal expansion on each layer</td>
</tr>
<tr>
<td>σ_n</td>
<td>thermal stress on each layer</td>
</tr>
<tr>
<td>K_{A_r(1,2...n)}</td>
<td>axial stiffness of each layer of the beam</td>
</tr>
</tbody>
</table>
A  
section area of a beam

$\alpha_{(1,2\ldots n)}$  
reduction factor for elastic modulus of each layer at elevated temperature

$K_{s(1,2\ldots n)}$  
spring stiffness of each layer

N  
number of layers

$K_{E(1,2\ldots n)}$  
effective axial restraint of each layer of the beam

$F'_n$  
alxial force for the different level of axial restraint

$M_{P_{\text{max}}}$  
maximum applied bending moment in the beam

$M_n$  
bending moment resistance of the reinforced concrete cross-section

$A_s$  
area of reinforcement steel

$F_y$  
tensile strength of reinforcement steel at ambient temperature

$F_T$  
temperature dependent tensile resistance of the reinforcement steel

e(x)  
reinforcement steel strain
Chapter 1. Introduction

1.1 Research background

Framed reinforced concrete structures are one of the most common types of construction. Concrete structures are commonly perceived to possess good resistance properties including non-combustibility and slow increase in temperature owing to their low thermal conductivity.

However, when a reinforced concrete structure is subjected to fire, its load-carrying capacity decreases due to reduction in the mechanical properties (strength and stiffness) of concrete and reinforcement, possible spalling of concrete and thermal stresses due to non-uniform temperature distribution in the cross-section.

Reinforced concrete elements can usually be easily designed to achieve high resistance periods. For example, in the case of reinforced slabs (depth 300mm), a fire resistance rating of 2 hours (based on the reinforcement attaining a critical temperature of 500°C) can be easily met (Denoël, 2007).

However, the good fire resistance is based on individual elements. When reinforced concrete members are part of a framed concrete structure, the concrete structure may perform completely differently from the individual structural elements. This is because there are strong interactions between different members of the structure. The most important interaction is caused by axial restraint and this effect is not considered in studies on individual elements. This thesis is concerned with behaviour of restrained concrete beams in fire and their performance in reinforced concrete frames.

Figure 1 - 1 sketches the behaviour of a restrained beam and an unrestrained beam at elevated temperatures. Although it was originally used to explain the behaviour of axially restrained steel beams, it applies equally to axially restrained reinforced concrete beams. As can be seen in this figure, the behaviour of a restrained beam under high temperatures is completely different compared with a beam without
restraint. The behaviour of a restrained beam has three main stages. First, compressive axial forces develop in the beam due to restrained thermal expansion.

The increase in compression force lasts until the beam reaches its maximum resistance under combined compressive force and bending. Thereafter, the beam’s compressive force decreases. At zero axial force, the applied load on the beam is resisted by the bending resistance of the beam. This indicates that the axial shortening of the beam due to lateral deflection is equal to the thermal expansion of the beam. Thereafter, the axial force in the beam changes to tension. The development of an axial force in the beam has profound implications on the design of reinforced concrete frames in fire. Firstly, it exerts forces on the joints and the rest of the structure, and failure to consider this force could lead to unsafe design; secondly, the axially restrained beam under catenary action may survive much higher temperatures than the axially unrestrained beam. This can be a very useful means of limiting progressive collapse.

In the case of an unrestrained beam, which is the basis of current design methods for beam fire resistance, there is no axial force development in the beam and total absence of interaction between the beam and the surrounding structure. The beam fails due to formation of a plastic hinge, accompanied by beam runaway deflection.
(a) Catenary action of an axially restrained beam in fire (Wang 2002)

(b) Runaway deflection of an unrestrained beam in fire (Lamont et al., 2006)

Figure 1 - 1 The behaviour of an axially restrained beam and an unrestrained beam in fire
1.2 Originality and objectives of research

The aim of this study is to understand the behaviour of axially restrained concrete beams and its effects on fire resistance of reinforced concrete frames. This topic has not been investigated thoroughly by others.

This project can broadly be classified into two parts: Part one is intended to identify methods to enhance fire resistance of axially restrained concrete beams, particularly focusing on how to enable reinforced concrete beams to maintain their integrity after losing a centre column, as illustrated in Figure 1 - 2. This will be carried out by performing extensive numerical simulations using the commercial finite element software TNO DIANA, which has a robust solution process for concrete structures. TNO DIANA, instead of other more widely used general finite element packages such as ABAQUS or ANSYS, was chosen for this research because it has superior concrete material models for modelling crack and failure of reinforced concrete structures. Based on the numerical simulation results, simplified equations for describing the entire beam behaviour will be proposed. Part two of this research will investigate the effects of structural interactions on reinforced concrete frame behaviour under several scenarios, as shown in Figure 1 - 3.

The specific objectives of this thesis are as follows:

- To numerically simulate the large deflection behaviour of reinforced concrete beams at elevated temperatures;

- To identify methods of enhancing the fire resistance time of axially restrained reinforced concrete beams;

- To assess the feasibility of reinforced concrete beams to survive fire exposure after removal of a column (Figure 1 - 2);
To develop a simplified calculation method to quantify axially restrained concrete beam behaviour;

To investigate the behaviour of frame structures by considering different levels of fire damaged columns at elevated temperatures. Figure 1 - 3 (Case 1) represents complete loss of two columns; Figure 1 - 3 (Case 2) shows that the columns can only maintain partial resistance.

To identify methods of retaining structural robustness in fire under different levels of fire damage to the columns;

Due to high demands on computational effort, this research considers only RC frame behaviour without slabs. The restraints to the modelled beam represent those from the surrounding structure. It is assumed that only the beam is exposed to fire and the surrounding structure is cool.

Figure 1 - 2 A reinforced concrete beam after removal of the central column
1.3 Thesis outline

This Ph.D thesis has the following seven chapters.

Chapter 2 of this thesis provides a thorough literature review of research studies that are relevant to understanding the behaviour of axially restrained reinforced concrete structured beams and reinforced concrete frames. Gaps in the knowledge will be identified to justify the originality and objectives of this research.

Chapter 3 provides detailed descriptions of using the general finite element software TNO DIANA to numerically model the behaviour of axially restrained reinforced concrete (RC) structures. This chapter includes the results of a mesh sensitivity study and catenary action in RC beams at normal and high temperatures. This chapter also demonstrates the validity of the simulation models by comparing relevant simulation and test results, focusing on catenary action at normal and high temperatures.

Chapter 4 presents details and results of a comprehensive parametric study on the behaviour of axially restrained RC beams, with the aim of identifying ways of improving methods of enhancing the strength and fire resistance of axially restrained RC beams through development of catenary action.

Chapter 5 presents the development of a simplified analytical method to quantify the main aspects of the behaviour of axially restrained RC beams in fire.
Chapter 6 extends the numerical simulation study to RC frames to investigate frame behaviour in fire after suffering different levels of damage to certain columns and the feasibility of the structure retaining its robustness without collapse.

The thesis concludes with Chapter 7, which presents the main conclusions from the whole research as well as recommending areas for future study.
Chapter 2. Literature review

The main research objective of this thesis is to understand the load carrying mechanisms of reinforced concrete structures at failure so as to investigate methods of improving robustness of such structures in fire. In particular, this research will focus on catenary action of reinforced concrete beams, which is one of the main strategies of providing structural robustness, in the form of the tying method. Catenary action can only be activated if there is axial restraint to the beam. Accordingly, this literature review will be focused on the following two aspects: research on robustness of reinforced concrete structures and behaviour of axially restrained RC beams. Both ambient temperature and high temperature research studies will be reviewed. Since concrete is a quasi-brittle material with complex behaviour, it is important that an appropriate constitutive relationship is used in modelling. Therefore, this chapter will also include a review of modelling concrete.

2.1 Robustness of reinforced concrete structures

2.1.1 Introduction

Interest in robustness of structures started after the Ronan Point accident and has become an important recent research topic for the structural engineering community since the September 11, 2001 attack on the World Trade Center buildings.

In 1968, the Ronan Point building (Figure 2 - 3) in London, which was constructed of precast walls and floors, collapsed in one corner of the building after one of the corner apartments was destroyed because of a gas explosion. Because of the lack of an alternative load path, failure of this one apartment removed the support to the apartments above and triggered progressive collapse of the structure. This accident led to the establishment of the first set of prescriptive rules to control disproportionate collapse (Moore 2002). The Roan Point accident also influenced other design codes, such as the “Basic building code (BOCA, 1996)”, National
The main prescriptive methods are:

*Tying*: Effective horizontal and vertical ties should be provided between primary structural members (Figure 2 - 1). The ties are intended to provide a high degree of redundancy to increase structural continuity and alternative load paths if any structural member removed because of an accidental damage. In general, the ties can be the steel members and or the reinforcement steel bars through the beam to column joint. The minimum tying forces are 75KN for steel structures and 60 KN for reinforced concrete structures. Providing tying is usually the first choice in the prescriptive method for structural robustness.

*Bridging*: when the tying method cannot be used, the bridging method can be recommended. Should any untied member be lost due to accidental loading, the resulting area of collapse should be limited and localized. If an untied element is notionally removed, the area of damaged zone should be limited by the following specific values: 15% of the floor area or 70m² (Figure 2 - 2) whichever is lower.
Figure 2 - 2 Recommended limits of acceptable damage (Approved document A, 2000)

*Key element*: if neither the tying nor the bridging method is feasible, the member should be designed as a key element to sustain a load of 34 KN/m² (5 lb/in²).
2.1.2 Existing methods of providing structural robustness

As expected, these prominent structural failures have led to intensive research studies to understand the progressive failure mechanisms and to devise methods to control progressive collapse, or in other words, to provide structural robustness.

Unlike normal structural design, provision of structural robustness deals with structural response under unknown loading conditions. As such, there is still no agreement on a quantification method for structural robustness. The following quotes give an indication of how robustness has been defined:

- “The consequences of structural failure are not disproportional to the effect causing the failure” (CEN, 1994)
- “The ability of a structure to withstand extreme events without being
damaged to an extent disproportionate to the original cause” (Agarwal et al., 2006).
- “The ability to react appropriately to abnormal circumstances (i.e. circumstances “outside of specifications”)” (Meyer, 1997)
- “The ability of a system to maintain function even with changes in internal structure or external environment” (Callaway et al., 2000)
- “…robustness is taken to imply tolerance to damage from extreme loads or accidental loads, human error and deterioration” (Baker et al., 2008)
- “…defined as insensitivity of a structure to local failure. It’s a property of the structure alone and independent of the possible causes and probabilities of the initial local failure” (Starossek, 2008)

A common strategy to control progressive, or disproportionate, collapse is to demonstrate that the remaining structure does not suffer disproportionate collapse after the initial loss of certain structural members, whether due to an impact, explosion or fire. Thus, testing the response of a structure after removal of a member, in particular a column, has become a common method of evaluating structural robustness. Economic design of structures dictates that a structure should have just enough resistance under the design loading condition for the support conditions and the adopted load carrying mechanism. Therefore, when one or more structural members are removed in consideration for structural robustness, the support conditions to the structure are changed, for the worse, and the structure would usually not have sufficient resistance under the assumed load carrying mechanism. Therefore, provision of structural robustness would normally require the partially damaged structure to develop alternative load carrying mechanisms to enable the partially damaged structure to continue supporting the applied loads under the extreme loading conditions encountered in structural design for robustness.

A review of the various current design methods below will help to identify the alternative load carrying mechanisms, often implicitly assumed, in the existing main design methods for structural robustness.
2.1.3 UK and European guidelines

Accidental loads such as those produced by fire can cause a building to experience a progressive collapse. The United Kingdom (Building Regulations Approved Document A classes, 2000) and the European Union (Eurocodes 1 Part 1.7:1998) recommend the size and occupancy of the building to reduce the risk of a progressive collapse followed by four classes and three types respectively. In the case of Euro code (EN 1991-1-7:1998), the contents are very similar to the British code and the regulations are shown in Table 2 - 2.

<table>
<thead>
<tr>
<th>Class</th>
<th>Building type and occupancy</th>
</tr>
</thead>
</table>
| 1     | - Houses not exceeding four storeys.  
      | - Agricultural buildings.    
      | - Buildings into which people rarely go, provided no part of the building is closer to another building, or area where people do go, than a distance of 1.5 times the building height. |
| 2A    | - 5 storey single occupancy houses.  
      | - Hotels not exceeding 4 storeys.  
      | - Flats, apartments and other residential buildings not exceeding 4 storeys.  
      | - Offices not exceeding 4 storeys.  
      | - Industrial buildings not exceeding 3 storeys.  
      | - Retailing premises not exceeding 3 storeys of less than 2000 m2 floor area in each storey.  
      | - Single storey Educational buildings.  
      | - All buildings not exceeding 2 storeys to which members of the public are admitted and which contain floor areas not exceeding 2000 m2 at each storey. |
| 2B    | - Hotels, flats, apartments and other residential buildings greater than 4 storeys but not exceeding 15 storeys. |
- Educational buildings greater than 1 storey but not exceeding 15 storeys.
- Retailing premises greater than 3 storeys but not exceeding 15 storeys.
- Hospitals not exceeding 3 storeys.
- Offices greater than 4 storeys but not exceeding 15 storeys.
- All buildings to which members of the public are admitted and which contain floor areas exceeding 2000 m² for the notional but less than 5000 m² at each storey.

3
- All buildings defined above as Class 2A and 2B that exceed the limits on area and/or number of storeys.
- All buildings containing hazardous substances and/or processes.
- Grandstands accommodating more than 5000 spectators.

---

**Table 2 - 2 Building classes (EN 1991-1-7, 1998)**

<table>
<thead>
<tr>
<th>Classes</th>
<th>Building type and occupancy</th>
</tr>
</thead>
</table>
| 1       | - Single occupancy houses not exceeding 4 storeys.  
- Agricultural buildings.  
- Buildings into which people rarely go, provided no part of the building is closer to another building, or area where people do go, than a distance of $1\frac{1}{2}$ times the building height. |
| 2A      | - 5 storey single occupancy houses.  
- Hotels not exceeding 4 storeys.  
- Flats, apartments and other residential buildings not exceeding 4 storeys.  
- Offices not exceeding 4 storeys.  
- Industrial buildings not exceeding 3 storeys. Retailing premises not exceeding 3 storeys of less than 1000 m² floor area in each storey.  
- Single storey educational buildings |

---
| 2B (Upper risk group) | Hotels, flats, apartments and other residential buildings greater than 4 storeys but not exceeding 15 storeys.  
| | Educational buildings greater than single storey but not exceeding 15 storeys.  
| | Retailing premises greater than 3 storeys but not exceeding 15 storeys.  
| | Hospitals not exceeding 3 storeys.  
| | Offices greater than 4 storeys but not exceeding 15 storeys.  
| | All buildings to which the public are admitted and which contain floor areas exceeding 2000 m\(^2\) but not exceeding 5000 m\(^2\) at each storey.  
| | Car parking not exceeding 6 storeys. |
| 3 | All buildings defined above as Class 2 Lower and Upper Consequences Class that exceed the limits on area and number of storeys.  
| | All buildings to which members of the public are admitted in significant numbers.  
| | Stadia accommodating more than 5 000 spectators  
| | Buildings containing hazardous substances and/or processes |

In the case of class 1 buildings, there is no need to check their robustness. For class 2A, effective horizontal ties or anchorage of suspended floor to walls for frames and load bearing walls construction should be provided. For class 2B, there should be effective vertical ties for framed and load-bearing wall construction together with effective vertical ties in all supporting columns and walls. For class 3 buildings, a systematic risk assessment should be carried out. However, at present, there is a lack of guidance on how this assessment can be performed.
2.1.4 GSA and DoD design guide

The general approach taken in the guidelines of the General Service Administration (GSA, 2003) and the Department of Defence (DoD, 2009) of the United States is the alternative load path method.

![Diagram showing tying approach to provide robustness.](image)

**Figure 2 - 4 The tying approach to provide robustness (NIST, 2007)**

Again, the tying approach, as shown in Figure 2 - 4, is specifically recommended as a method of providing structural robustness. Various analysis methods, such as linear elastic static analysis, linear elastic dynamic analysis, nonlinear static analysis and nonlinear dynamic analysis, may be used to analyse the damaged structure. When carrying out such analyses, an impact factor of 2, to relate the dynamic loading condition during progressive collapse to the static load, should be used. Also, internal ties enable acting under the right angles between the floor and roof.
To reduce the risk of progressive collapse, the investigation of connections or frame structures is necessary. However most researches about progressive collapse are limited to a single member or connections between the main columns and beams. A number of researchers have conducted many experimental and numerical studies about the robustness of steel and concrete frames in ambient temperature, and those studies will be examined in the next section (2.2.2).
2.1.5 Research studies of structural robustness

Collapse of the World Trade Center towers on September 11, 2001 (Figure 2 - 5) brought to the international structural engineering community the destructive power of disproportionate, or progressive, collapse, and the importance of providing structures with sufficient robustness. Consequently, many more recent research studies have been devoted to understanding the mechanism of progressive collapse and how structures may be made more robust.

[Image]

Figure 2 - 5 World Trade Center on fire (Eager and Musso, 2001)

Kaewkulchai and Williamson (2006) simulated behaviour of planar frame structures in their research on progressive collapse. They specifically investigated the effects of the impact force of damaged members falling on the structure below, as indicated in Figure 2 - 6.
Figure 2 - 6 The impact of a failed member in a planar framed structure
(Kaewkulchai and Williamson, 2006)

(a) Base system for an impact force on a below beam

(b) Spring mass system of an impacted beam at midspan

Figure 2 - 7 Diagram to express an impacted structure (Kaewkulchai and Williamson, 2006)
For modelling the impact force, the authors assumed an impact damage as an additional mass (Figure 2 - 7(a)) and a time-dependent analysis scheme was used. A single-degree-freedom model represented the beam with massless spring having a constant stiffness k ((Figure 2 - 7(b)). For the base system shown, k is equal to $48EI/L^3$. For calculating the impact force, the following equations were used:

$$\frac{1}{2}mv^2 = \frac{1}{2}k\delta^2 = \frac{1}{2}\frac{p^2}{k}$$

$$P = \sqrt{mk} \cdot v$$

where $P$ = impact force, $m$ = total mass after impact ($m_1 + m_2$), $k$ = spring stiffness, $v$ = velocity after impact ($v_1 + v_2$), $m_1$ = mass of lower beam, $m_2$ = mass of upper beam, $v_1$ = velocity when impacted damages were made and $v_2$ = initial velocity.

Accordingly, the midspan moment can be calculated from $M = PL/4$, and then the failure time can be calculated.

Sasani and Kropelnicki, 2008 carried out experimental and numerical research studies to investigate progressive collapse in a seven storey RC frame under a removal column scenario. The scheme of the building is shown in Figure 2 - 8.

(a) Plan of building
(b) A scenario of removal column

**Figure 2 - 8** Description of plan and scheme for a RC frame under column removal (Sasani and Kropelnicki, 2008)

The experiment was conducted on a 3/8 scaled model (Figure 2 - 9). Numerical modelling was conducted using the finite element program ANSYS (2000). A 3/8 scale model without centre column was constructed with fixed boundary condition. The experimental requirements are satisfied according to ACI 318 (2002).

**Figure 2 - 9** Reinforcement details of the beam (Sasani and Kropelnicki, 2008)
Although the tensile reinforcement steel on the bottom of the beam failed, the strength and deformational capacity remained according to the test results. The initial analytical model showed good agreement with test results however it was stopped before catenary action. The reinforcement steel indicated fracture at the centre point. The reason of terminating simulation was due to using the concrete crushing model in ANSYS (2000). The post analytical model represented good agreement with the test result of catenary action.

Using the TNO DIANA (2010) finite element software, Decan and Taerwe (2008) numerically investigated the behaviour of a two-span RC beam with column removal. They achieved significant increase in the beam load carrying capacity due to the development of catenary action. The load carrying capacity of the beam at large deflections was governed by the strength and elongation of reinforcement steel, and continuous bottom reinforcement over the central column support was important to ensure tensile resistance.

The National Institute of Standard and Technology (NIST) conducted a set of tests to investigate the behaviour of reinforced concrete assemblies under catenary action until collapse (Lew et al., 2011). These tests were designed to achieve two main goals: to find out the response of reinforced concrete structures under a column removal scenario and to obtain experimental data for validation of computational models. Figure 2 - 10 shows a test specimen after reaching test failure. They simulated the tests using the general finite element software LS – DYNA (Hallquist J, 2007)
(a) A test specimen after failure

(b) Failure model of the test

Figure 2 - 10 Description of a test specimen after failure (Lew et al., 2011)
Figure 2 - 11 Overview of detailed and reduced model (Lew et al., 2011)
(a) Relationships of applied load and vertical displacement

(b) Relationships of horizontal and vertical displacement

Figure 2 - 12 Compared outputs of test results and computational results (Lew et al., 2011)
Their simulations were carried out using both detailed and reduced models shown in Figure 2 - 11. Figure 2 - 12 compares their simulation and test results, demonstrating good agreement between test results and computational results for the relationships of vertical displacements and applied loads and horizontal displacements. The results in Figure 2 - 12 indicates a high strength reserve under catenary action (deflection > 40in) compared to bending capacity (deflection < 5in). Although there was no measurement of axial reaction forces in the tests, both the detailed model and the reduced model obtained similar results as shown in Figure 2 - 13.

![Figure 2 - 13 Comparison of the beam axial forces between using detailed model and reduced model (Lew et al., 2011)](image)

Similar research studies have been carried out by others, including Yi et al. (2008), Su et al (2009) and Yu and Tan (2010). These research studies all suggest the importance of catenary action in reinforced concrete beams to arrest progressive collapse under the column removal scenario. The next section of this thesis will provide more detailed information of the tests which will be used for validation of the numerical simulations of this research.
2.1.6 Test results of axially restrained RC beams at normal temperature

Lew, Pujol and Sozen (Lew et al., 2011) carried out full scale tests on restrained RC beams and column assemblies at Purdue University in the US. These tests examined two types of frame systems (an intermediate moment frame (IMF) and a special moment frame (SMF)). The IMF system was expected to sustain limited inelastic deformation in the members and connections and it must support an inter-story drift angle of up to 0.02 radian. This system is typically used in low/mid – seismic regions. In the case of SMF, it was expected to withstand significant inelastic deformations in the members and connections and it must support an inter-story drift angle of up to 0.04 radian. It is generally used in mid/high – seismic regions. The tests were conducted under the middle column disappearing scenario. Figure 2 - 14 shows the test arrangement.

(a) IMF specimen
Figure 2 - 14 Catenary test: up – IMF, bottom – SMF (Lew et al., 2011)
The results shown in Figure 2 - 15 and Figure 2 - 16 clearly show the same three phases of behaviour of axially restrained beams as described in section 2.2.1. This indicates the feasibility of using catenary action to improve structural robustness. In both cases, final failure was due to fracture of the bottom reinforcing bar with a major crack at the location. As shown in Figure 2 - 15 and Figure 2 - 16, the trends of the test results are similar with two frames. In this test, axial load was not evaluated but the horizontal movement was evaluated instead. In the case of the IMF test, horizontal movement showed the compressive reaction force initially. After that, the forces were realised and it changed into tensile forces. For the SMF test, this movement presented the compressive displacement, and reduced the compressive
movement but the tensile displacements were not observed. The reason for this is the different level of axial restraint and the size of the column. As shown in Figure 2 - 14, the reinforcement arrangement of SMF’s column is higher than that of IMF, meaning that the level of axial restraint and the column size of the specimen are both large. When the level of axial restraint is high, the axial force will be higher. In the case of the larger column size, high compressive arch action pushes the column out, but in the catenary stage, the horizontal movement was unable to return to the direction of the beam due to the high stiffness of the column.

Su et al (2009) reported similar test results from twelve reduced-scale specimens. Each specimen was comprised of two beams with the central column removed. The tests examined the influences of the following variables: (1) beam cross-section dimensions, (2) beam length, (3) concrete strength, (4) reinforcement ratio.

These tests revealed similar behaviour to the other tests, as indicated in Figure 2 - 17. The tests showed the advantage of the compressive arch action and the results show the extra strength enhancements ranging from 50% to 160% of the beam.

![Figure 2 - 17 Deformed shape & axial reaction force and vertical force vs. vertical deflection relationships (Su et al., 2009)](image)

Other similar studies include those of Yu and Tan (2010), as shown in Figure 2 - 18 for the test setup and results, and in Figure 2 - 19 for details of the test specimens.
In conclusion, the author stated that “flexural action develops until all plastic hinges occur. Catenary action kicks in at the moment of the applied force reversing and increasing again.”

They examined the influences of the following three variables: (1) reinforcement ratio through joints, (2) reinforcement detailing at joint regions, and (3) beam span-to-depth ratio. They concluded that “compressive arch action can significantly increase the structural resistance with a small span-to-depth ratio and a low longitudinal reinforcement ratio, and catenary action can significantly increase the structural resistance with a large span-to-depth ratio and high longitudinal reinforcement”.

Figure 2 - 18 Failure mode & axial reaction force vs vertical force relationship (Yu and Tan 2010)
In summary, substantial catenary action can develop in reinforced concrete beams provided there is sufficient axial restraint and reinforcement ductility. The complete behaviour of an axially restrained beam is as generally depicted in Figure 2 - 28. Failure in the catenary action stage is due to fracture of the tensile reinforcement. At failure, large cracks tend to spread through the entire height of the beam at the location of the removed column support. On either side of the removed column support, the beam segments tend to deform as rigid structural members. The enhancement in strength of the beam under catenary action, over that under bending, may be increased by the following methods: increasing the area of reinforcement steel, increasing the strength of concrete, decreasing the beam’s span depth ratio and the fracture strain of reinforcement steel.

Whilst the above research studies focus on mechanisms of progressive collapse of RC structures, Crawford (2002) and Orton (2007) investigated retrofit methods for mitigation against progressive collapse of RC buildings.

Orton (2007) carried out similar tests, but by using CFRP (carbon fibre reinforced polymer) sheets to increase the tension resistance as a possible method of retrofitting to enhance structural robustness. Figure 2 - 20 shows the test arrangement and Figure 2 - 21 presents the test results of the vertical load-deflection relationship. The results in Figure 2 - 21 demonstrate that the additional resistance due to catenary tension is

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**Figure 2 - 19 Detailing and boundary condition of the test specimens (Yu and Tan 2010)**
very high compared to the bending resistance. Figure 2 - 21 also shows that towards
the end of loading, the vertical load and axial load follow a similar pattern. This is
because the beam has reached catenary action under which the resistance to vertical
load comes from tensile (axial) force in the reinforcement.

(a) Schematic of test set up (Orton, 2007)

(b) Picture of the test set up

Figure 2 - 20 Test setup of Orton (2007)
Figure 2 - 21 Vertical load and axial load – displacement relationships (Orton 2007)
2.1.7 Mechanical behaviour of concrete at elevated temperatures

However, the above studies have all concentrated on reinforced concrete structures at ambient temperature. This may be considered to simulate the situation of structures under such extreme events as impact or explosion. It is also important to understand how disproportionate collapse may occur in reinforced concrete structures in fire because there are many uncertainties in understanding RC structural behaviour in fire and fire resistance design of RC structures. To do so, it is first necessary to develop understanding of the mechanical behaviour of concrete at elevated temperatures. At elevated temperatures, concrete undergoes a number of stages of behaviour. According to FIB (2007) and Bazant and Kaplan (1996), the different stages are listed in Table 2 - 3.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Key processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>100°C</td>
<td>Water evaporation</td>
</tr>
<tr>
<td>100 - 300°C</td>
<td>Thermal spalling</td>
</tr>
<tr>
<td>100 - 800°C</td>
<td>Dehydration of the ingredients of cement paste</td>
</tr>
<tr>
<td>350 - 900°C</td>
<td>Transformations in the aggregate begin, depending on aggregate type: 350°C – some river gravels, 570°C – siliceous aggregates, 650°C – calcareous aggregates, 700°C – basalt aggregates</td>
</tr>
<tr>
<td>400 -600°C</td>
<td>Dissociation of calcium hydroxide Ca(OH)$_2$, into CaO and water</td>
</tr>
<tr>
<td>374°C</td>
<td>Critical point of water when no free water is possible</td>
</tr>
<tr>
<td>573°C</td>
<td>$\alpha$ - $\beta$ transformation of quartz in aggregates and sangs; this is an endothermic process accompanied by violent increase of the material volume. It is one of the main causes for which the siliceous aggregate based concrete show the lowest resistance to high temperature</td>
</tr>
<tr>
<td>700 -800°C</td>
<td>Decarbonation of calcium carbonate CaCO$_3$ into CaO and CO$_2$</td>
</tr>
</tbody>
</table>
Although research studies on spalling of concrete at elevated temperatures date back over half a century, there is still a lack of understanding of the fundamental mechanisms of this phenomenon. As a result, design methods, such as those in EN 1992-1-2(2002), are based on empirical evidences from experiments. There is still a large amount of uncertainty in how to deal with spalling of concrete.

The strength and stiffness of concrete decrease at high temperatures (Buchanan, 2001). EN 1992-1-2 (2002) propose relationships to quantify the mechanical properties such as compressive strength, tensile strength and ultimate strain of concrete at elevated temperatures (Table 2 - 4)

### Table 2 - 4 Compressive and tensile strength at elevated temperatures (EN 1992-1-2: 2002)

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>Compressive strength f(_c)(temp)/f(_c)</th>
<th>Tensile strength f(_t)(temp)/f(_t)</th>
<th>Ultimate strain (\varepsilon_{cu})(temp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1</td>
<td>1</td>
<td>0.0200</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>1</td>
<td>0.0225</td>
</tr>
<tr>
<td>200</td>
<td>0.95</td>
<td>0.8</td>
<td>0.0250</td>
</tr>
<tr>
<td>300</td>
<td>0.85</td>
<td>0.6</td>
<td>0.0275</td>
</tr>
<tr>
<td>400</td>
<td>0.75</td>
<td>0.4</td>
<td>0.0300</td>
</tr>
<tr>
<td>500</td>
<td>0.6</td>
<td>0.2</td>
<td>0.0326</td>
</tr>
<tr>
<td>600</td>
<td>0.45</td>
<td>0</td>
<td>0.0350</td>
</tr>
<tr>
<td>700</td>
<td>0.3</td>
<td>0</td>
<td>0.0375</td>
</tr>
<tr>
<td>800</td>
<td>0.15</td>
<td>0</td>
<td>0.0400</td>
</tr>
<tr>
<td>900</td>
<td>0.08</td>
<td>0</td>
<td>0.0425</td>
</tr>
<tr>
<td>1000</td>
<td>0.04</td>
<td>0</td>
<td>0.0450</td>
</tr>
<tr>
<td>1100</td>
<td>0.01</td>
<td>0</td>
<td>0.0475</td>
</tr>
<tr>
<td>1200</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

in cement paste and aggregates

<table>
<thead>
<tr>
<th>1350°C</th>
<th>Melting of concrete</th>
</tr>
</thead>
</table>
Because of strong interactions in reinforced concrete structures in fire, it is important that the strains of concrete at elevated temperatures are accurately quantified. However, at present, this understanding is not sufficiently developed. In general, the total strain in concrete at elevated temperatures may be defined as (EN 1992-1-2:2002)

\[
\varepsilon = \varepsilon_{\text{th}} + \varepsilon_{\sigma} + \varepsilon_{\text{creep}} + \varepsilon_{\text{tr}} \\
\text{Equation 2 - 1}
\]

where

\(\varepsilon_{\text{th}}\) is the thermal strain
\(\varepsilon_{\sigma}\) is the instantaneous stress-dependent strain
\(\varepsilon_{\text{creep}}\) is the creep strain
\(\varepsilon_{\text{tr}}\) is the transient state strain (or Load Induced Thermal Strain – LITS)

Concrete heated under high stresses can be observed to shrink rather than expand (Law and Gillie 2008) as a result of LITS, as shown in Figure 2 - 22.

![Figure 2 - 22 Difference between strain when heated with and without applied stress (Law and Gillie, 2008)](image_url)
However, currently, it is still not possible to quantify LITS accurately. Therefore, in general, the creep strain and LITS are implicitly considered by being included in the thermal strain. Whilst this may be adequate when assessing the load carrying capacity of individual members without interaction with other structural members, more accurate information and explicit inclusion of these two strain terms may be necessary when dealing with interactions in reinforced concrete structures. However, in this research, because of a lack of reliable information on these two strain terms, only the thermal strain is included when modelling reinforced concrete structural behaviour in fire. This lack of accurate and reliable model for concrete strain at elevated temperatures further underlines the uncertainties that should be considered when assessing reinforced concrete structures in fire.
2.1.8 Previous test results and simulations of axially restrained RC beams in fire

Among the very few research studies on axially restrained reinforced concrete beams in fire, Dwaikat and Kodur (2009) presented 6 tests. Figure 2 - 23 shows their test setup and Table 2 - 5 summarises the main details of the tests. These tests were carried out to examine influences of the following variables: (1) fire exposure, (2) concrete type, (3) support type, and (4) applied load. All the six beams were 3962 mm span and 408 X 254 mm in cross section as shown in Figure 2 - 23.

![Figure 2 - 23 Geometric condition of test specimens (Dwaikat and Kodur, 2009)](image)

<table>
<thead>
<tr>
<th>Beam</th>
<th>Fire Exposure</th>
<th>Concrete Type</th>
<th>Support Type</th>
<th>P(KN)</th>
<th>Fire resistance(min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>ASTM E119</td>
<td>NSC</td>
<td>SS(^b)</td>
<td>50</td>
<td>180</td>
</tr>
</tbody>
</table>
The test results are presented in Figure 2 - 24. Because the tested beams (B1, B3, B5) had no axial restraint, they experienced “run-away” deflection. Test B4 also did not have an axial restraint, however, because of the low fire load (LF), the beam temperature was not sufficiently high to cause failure of the beam. Tests B2 and B6 had axial restraints. Unfortunately, the fire exposure was not sufficiently severe to cause failure of the beams and there was no observation of development of catenary action. Even though the test is one of the few fire tests of an axially restrained concrete beam, the axial load was not evaluated.

In conclusion, Dwaikat and Kodur stated that “load level and axial restraint have significant influence on the fire response of RC beams.” Lower loads leads to higher fire resistance and axially restrained beams have higher fire resistance. Thus, the failure and the fire resistance of an RC beam exposed to fire should be determined.
based on a realistic fire load, and boundary condition. Although this test dealt with axially restrained RC beams, catenary action was not experienced.

2.1.9 Numerical investigation of axially restrained beam behaviour

In addition to experimental examinations, a number of researchers have also carried out numerical simulations of restrained reinforced concrete beams at normal temperatures and under fire conditions.

Sasani and Kropelnicki (2008) simulated their test results using the commercial finite element program, ANSYS. Their model utilised eight-node solid elements for concrete and two-node truss elements for the reinforcement bar. For concrete material, a smeared cracking model and a strain-based concrete crushing model were used. Also, Drucker-Prager yield criterion was used for the plasticity of the concrete material. Figure 2 - 25 compares their simulation and test results, demonstrating that their numerical model is suitable.
AXIALLY RESTRAINED CONCRETE BEAMS AT HIGH TEMPERATURE

Lim et al (2004) simulated axially restrained RC floor slabs in fire using the specialist fire engineering software SAFIR (Franssen, 2000). The slabs were 200mm thick and had a 5m span. In this thesis, the ASTM E119 (ASTM 1999) was adopted as the fire scenario. In the research, a 2D beam finite element model was used and the loading condition was a uniform load. They examined the effects of providing different levels of axial restraint.

Figure 2 - 25 Comparison of test and simulation force-displacement relationships (Sasani and Kropelnicki, 2008)
The results do not seem to be correct. For example, when the axial restraint is 25%, the increase in compression force is greater than when the level of axial restraint is higher. Also with very high levels of axial restraint (50%, 100%, pin-pin indicating infinite axial restraint), the beams were still under compression after nearly 250 minutes of fire exposure, yet other beams experienced compression failure at smaller compression forces with much shorter fire exposure. Their results will not be discussed further.

As demonstrated above, there have been some numerical simulations of axially restrained reinforced concrete beams at ambient temperature. However, there is a severe lack of such studies at elevated temperatures and in fire. This is one of the important motivations of the research reported in this thesis.
2.2 Catenary action in beams

As concluded in the previous section, the tying force approach, utilising catenary action in beams, has been identified as the principal method of providing structural robustness. Because of this, a number of research studies, all conducted at ambient temperature, have been performed by various researchers. A review of the relevant research studies can provide an understanding of this load carrying mechanism.

2.2.1 Behaviour of axially restrained RC beams

Consider a reinforced concrete beam with axial restraint at the ends, as shown in Figure 2 - 27. Its behaviour, in terms of load-axial reaction, and load-vertical deflection relationships, is shown in Figure 2 - 28. In Figure 2 - 27, the use of simply supported ends of an axially restrained beam is to get free rotations of top and bottom ends of a beam. Free rotation can meet easily a catenary action in computational simulations.

![Figure 2 - 27 An axially restrained beam](image)
At the early stage, compressive arch action accompanies bending action of the beam and a compressive axial load is generated in the beam in addition to bending moments. This will last until the beam loses its load carrying capacity under combined bending and compression, as indicated by Point A. Thereafter, the compressive force is released in order for the beam to reach equilibrium. In a real structure, if the applied load is maintained, the structure would need to experience a period of dynamic response to enable it to find a new equilibrium while maintaining the applied load. With increasing deflection, the applied load can be increased as the compression load is further released. This will last until all the compression load is completely released, reaching point B in Figure 2 - 28. This point is termed pure bending in this thesis because there is no axial force in the beam and the beam is under bending only. After point B, it is possible for the beam to continue resisting loads by developing catenary action. During this stage, tensile forces are developed in the beam and the tensile force contributes to the resistance of the beam to the applied load. Finally the beam will collapse due to the fracture of the reinforced steel (C point).

The key objective of this research is to quantify the above behaviour at elevated temperatures and to research methods that can prolong the period from point B to point C.

**Figure 2 - 28 General behaviour of an axially restrained beam**
To understand a detailed catenary action, it is necessary to learn the mechanisms for the numerical modelling governing catenary action. Prior to conducting a numerical study, a model applied one point load at the centre position on the span and the amount of moment was considered as nought. The equation is based on the basic concepts of equilibrium and compatibility.

- **Equilibrium**

Figure 2 - 29 shows the free body diagram of a half beam. It could be assumed from the equilibrium equation that the sum of moments will reach zero at the midpoint and after catenary action, the value of the moment could be neglected: almost nought.

\[
\begin{align*}
\sum M &= 0 \\
\rightarrow -M + \frac{P}{2} \times \frac{L}{2} - T\delta &= 0 \\
\rightarrow T\delta &= \frac{PL}{4} - M \\
\rightarrow T &= \left(\frac{PL}{4} - M\right) \frac{1}{\delta} \\
\rightarrow T &= \frac{PL}{4\delta} \quad \text{; after catenary action, } M \text{ will be negligible.}
\end{align*}
\]
where $M$ is the nominal flexural capacity, $T$ is the axial tension, $\delta$ is the centre deflection, $P$ is the point load, and $L$ is the length of the modelled beam.

- Compatibility

In order to understand the catenary action response of the beam, the axial extension of the beam must also be taken into account. Figure 2 - 30 is the expression of axial extension due to geometry and support movement.

\[
(L + \Delta L)^2 = \delta^2 + L^2
\]

\[\rightarrow \delta^2 = (L + \Delta L)^2 - L^2\]

\[\rightarrow \delta = \sqrt{(L + \Delta L)^2 - L^2}\]

**Figure 2 - 30 Deflection on compatibility**
- Axial extension

The quantity of axial tension and beam deflection has an effect on the axial extension of a beam. Figure 2 - 31 shows the geometrical diagram to illustrate the axial extension of catenary model.

![Figure 2 - 31 Extension due to geometry](image)

\[(L + \delta g)^2 = L^2 + h^2\]

\[L + \delta g = \frac{L^2 + h^2}{L + \delta g}\]

\[= \frac{L^2}{L + \delta g} + \frac{h^2}{L + \delta g}\]

(where \(\frac{L}{L + \delta g} = \cos\phi\), \(\frac{h}{L + \delta g} = \sin\phi\))

\[L + \delta g = L \cdot \cos\phi + h \cdot \sin\phi\]

\[\delta g = h \cdot \sin\phi + L \cdot \cos\phi - L\]
\[ = h \cdot \sin \phi + L(\cos \phi - 1) \]

also,

\[ \phi = \tan^{-1} \left( \frac{A}{L} \right) \]

An analytical model is necessary to characterize the load and deflection relationship of reinforced concrete beam in catenary action so that the additional axial load in the beam, which have implications on safety of the rest of the structure, can be quantified and the survival time of the beam under canteary action can be assessed. The analytical equations are based on the fundamental concepts of equilibrium, compatibility and material characteristics.

\[ \text{2.2.2 Behaviour of axially restrained RC beams at high temperatures} \]

In contrast, there is a paucity of research investigating axially restrained reinforcement concrete beams at high temperatures, and this is the main motivation of this thesis.

![Description of an axially restrained beam at elevated temperature](image)

**Figure 2 - 32 Description of an axially restrained beam at elevated temperature**
Consider an axially restrained reinforced concrete beam exposed to fire from below, as illustrated in Figure 2 - 32. The Figure 2 - 33 depicts the general behaviour of the beam, presented as the axial reaction force – time and vertical deflection – time relationships. Initially, the behaviour of the axially restrained beam is dominated by the compressive axial reaction force due to restrained thermal expansion. During this stage, the vertical deflection will increase gradually. With increasing compressive force and deflection, the beam will gradually exhaust its resistance under combined bending and axial compression (point A in Figure 2 - 33). Thereafter, the beam
temporarily loses its equilibrium and deforms rapidly. At this stage, the compression force is released and the beam gradually returns to the pure bending condition (point B in Figure 2 - 33). Then, the beam transits to combined bending and catenary action. The deflection will steadily increase until fracture of the reinforcement (point C in Figure 2 - 33). Point B is the conventional fire resistance time, determined by bending of the beam. The difference in times between point B and point C is the additional fire resistance time and it is this value that helps with the robustness of the structure under fire conditions.

2.2.3 Gaps in existing knowledge and research needs

As a summary, the tying force approach is the principal method used in various current design methods to achieve structural robustness. The implicit assumption in the tying force approach is the activation of catenary action in the beams. However, there have been very few research studies to investigate whether the tying force approach may be suitable if the structure is required to have sufficient robustness in fire. This is the main aim of the current research.

It should be pointed out that this research is not concerned with conventional fire limits as embodied in design codes of practice such as Eurocode 2 Part 1.2(2002). In these conventional design codes, the fire and mechanical loads are specified. The structure is assumed to be initially intact and the aim is to ensure that the intact structure has sufficient fire resistance through utilising the conventional load carrying mechanism such as bending. In contrast, this research considers the situation that a structure may be initially damaged (such as by impact) and then subjected to fire attack, or the design fire situation may not be adequate for the real fire situation, for example, the mechanical property models of concrete used in fire resistance design may not be adequate to express the severity of the effects of high temperatures on concrete. Therefore, an important outcome of this research is to identify the extra fire resistance of reinforced concrete beams under the alternative load carrying mechanism of catenary action over that under the conventional bending mechanism. Equally important is the search for methods to increase this extra fire resistance.
2.3 Summary and objectives of research

This chapter has presented a review of the methods in a number of major standards to ensure robustness of structures, relevant research studies to investigate the mechanisms of disproportionate collapse of reinforced concrete structures, uncertainties in the mechanical properties of concrete at elevated temperatures and research studies to understand large deflection behaviour of reinforced concrete beams in fire. The various uncertainties related to reinforced concrete structures in fire necessitate investigations of their potential of disproportionate collapse, which is the aim of this project. The tying method, through activating catenary action, has been identified as a main mechanism capable of reducing the risk of disproportionate collapse. This mechanism will be the main focus of this research. At present, the experimental and numerical researches relevant to robustness of reinforced concrete structures have been concentrated on ambient temperature structures and any relevant study in fire is only concerned with connections and single members. This research will investigate reinforced concrete frame behaviour with partially damaged members.

Therefore, the main objectives of this thesis are:

- To understand catenary action and the load carrying mechanism of an axially restrained reinforced concrete beam.
- Under accidental loading conditions such as fire loads, to identify a methodology to achieve the robustness of a reinforced concrete frame.
- To discover methods of enhancing the robustness of a damaged frame structure
Chapter 3. Numerical modelling and validation for axially restrained concrete beams in ambient temperature and fire

3.1 Introduction

Concrete is a complex material and robust and accurate modelling of concrete structures, at ambient and elevated temperatures, requires careful consideration of the material used in the model simulation. This chapter will describe the concrete material behaviour, and review and assess the different concrete material simulation models. All simulations were carried out using the DIANA program (TNO-DIANA, 2010), which was chosen for its superior ability to deal with concrete behaviour. To consider catenary action, the analyses should account for both geometrical and material nonlinearities. Robust and accurate modelling of concrete structures will also require appropriate selection of the finite element mesh and the numerical solver. This chapter will then present results of a sensitivity study for this purpose. Finally, this chapter presents comparisons between numerical simulation results and a number of relevant test results on restrained reinforced concrete beams at ambient temperature and under fire, to demonstrate the validity of the simulation model.

3.2 Concrete modelling in DIANA

Reliable quantification of tensile crack behaviour is critical to successful modelling of reinforced concrete structural behaviour. Two methods may be used: discrete crack model (Ngo et al., 1967) and smeared crack model (Rashid, 1968) in DIANA.

The discrete cracking approaches suitable to represent an individualised crack for which the exact location is known in advance. However, the simulation results using the discrete crack model depend heavily on the mesh size and the experience of the
modeller. In contrast, the smeared crack model is less sensitive to these factors. Smeared crack model assumes the cracks are continuous and is more easily implemented numerically. This has the advantage of achieving numerical convergence reliably and was used in this research. Nevertheless, the modelling results are mesh dependent and mesh sensitivity analysis will be carried out to determine the appropriate mesh size. Therefore, although in RC structures, cracks are likely to be discrete, the smeared crack model is chosen in this research. In the smeared crack model, the crack propagates through the simulation element. The smeared crack model can be used with a number of nonlinear plasticity models in compressive and tensile behaviour. The compressive plasticity models are:

- Mohr-Coulomb
- Drucker-Prager

The tensile softening models are:

- Brittle cracking
- Linear tension softening
- Multilinear tension softening
- Moelands tension softening
- Hordijk tension softening
3.2.1 Compressive behaviour

3.2.1.1 Mohr-coulomb

There are two plasticity models for modelling concrete in compression. Of these two, the Mohr-coulomb plasticity model is more often used. This model is shown in Figure 3 – 1 and is expressed as:

\[
\frac{1}{2} (\sigma_1 - \sigma_3) = -\frac{1}{2} (\sigma_1 + \sigma_3) \sin \phi + c \cdot \cos \phi \quad \text{Equation 3- 1}
\]

where :

\( \phi \): the internal frictional angle (usually \( \approx 30^\circ \) for concrete (TNO-DIANA, 2010))
\( c \): the cohesion \( c = f_c (1 - \sin \phi)/2 \cos \phi \)

![Drucker-Prager and Mohr-Coulomb plasticity models, showing the failure surfaces according to the principle stresses, (TNO DIANA, 2010)](image)
3.2.1.2 Drucker-Prager

The Drucker-Prager plasticity model is mainly used for modelling soil and rock materials. As shown in Figure 3 - 1, the Drucker-Prager plasticity model is a smooth approximation of the Mohr-coulomb failure surface. The failure surface equation is:

\[ A + B(\sigma_1 + \sigma_2 + \sigma_3) = \sqrt[6]{\frac{1}{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}} \]

Equation 3- 2

where:

\[ A = \frac{6 \cdot c \cdot \cos \phi}{\sqrt{3}(3 + \sin \phi)} \]

\[ B = \frac{2 \cdot \sin \phi}{\sqrt{3}(3 + \sin \phi)} \]

In these equations, \( c \) is the internal frictional angle which is assumed to be 10° (TNO-DIANA, 2010).
3.2.2 Tensile Behaviour

Figure 3 - 2 shows different models of the tensile stress-strain relationships of concrete. The ascending branch is linear. Four models are shown for the tension softening branch: brittle, linear, multilinear and curved line (Hordijk). For this research, the brittle model would not be suitable because of the sudden drop of stress to zero, which is not representative of concrete tensile behaviour and would cause a severe convergence problem.

![Figure 3 - 2 Tensile stress-strain relationships of concrete](image)

Figure 3 - 3 compares simulation results using three different tensile models for an axially restrained beam (section size: 200 mm X 200 mm, length : 2000 mm).

The linear and Hordijk models give similar results. However, in the case of using the linear tensile model, the tensile strength is reduced more steeply than in the Hordijk model after the concrete has reached the tensile strength, making it more difficult to achieve numerical convergence when using the linear model. Therefore, the Hordijk curve will be used in this research.
<table>
<thead>
<tr>
<th>Material properties</th>
<th>Elastic modulus (MPa)</th>
<th>Yield strength (MPa)</th>
<th>Ultimate strength (MPa)</th>
<th>Fracture strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced bar (R16)</td>
<td>200,000</td>
<td>300</td>
<td>440</td>
<td>11</td>
</tr>
<tr>
<td>Concrete</td>
<td>Compressive strength : 28MPa, $G_f = 0.08$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tensile strength : 2.22MPa, Initial modulus of elasticity : 25,757MPa</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Material properties

(b) Analytical results

Figure 3 - 3 Comparison of load-deflection curves using different tensile softening models, for a simple supported reinforced concrete beam, (span length : 2000mm, section size : 200 x 200 mm, mesh size : 40 mm)
3.3 Mesh Sensitivity study

In this research, the finite element program (DIANA) is used for its relative stability in modelling concrete structures. The beam shown in Figure 3 - 4 is used for the mesh sensitivity study. The beam is axially restrained and simply supported for bending at both ends. Table 3 - 1 gives the length and cross-section dimensions of the different beams and their cross-section mesh sizes (Figure 3 - 5). In all cases, the cross-section aspect ratio is 1 and the depth to length ratio is 0.15. For 2D modelling of concrete beams in DIANA, there are three element types (beam element, plane strain element and plane stress element). The plane strain element is suitable for thin plates and the plane stress element is better for beam structures. Therefore 4-node, plain stress quadratic elements (Q8MEM) are used in this research. There are two methods for reinforcement modelling: discrete reinforcement and embedded reinforcement. The discrete reinforcement method is suitable for modelling the bond-slip relationship of single bars in detail on the other hands the embedded reinforcement is better for global structural analysis. Therefore, the embedded reinforcement method was selected for this research.

<table>
<thead>
<tr>
<th>Beam size (mm)</th>
<th>Mesh sizes (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span length = 3000 mm</td>
<td>100 x 100 mm, 66.66… x 66.66… mm</td>
</tr>
<tr>
<td>Section size = 200 x 200 mm</td>
<td>50 x 50 mm, 40 x 40 mm, 33.33… x 33.33… mm</td>
</tr>
<tr>
<td>Span length = 4500 mm</td>
<td>75 x 75 mm, 60 x 60 mm, 50 x 50 mm</td>
</tr>
<tr>
<td>Section size = 300 x 300 mm</td>
<td>42.857 x 42.857 mm, 37.5 x 37.5 mm,</td>
</tr>
<tr>
<td>Span length = 6000 mm</td>
<td>80 x 80 mm, 66.66… x 66.66… mm, 50 x 50 mm</td>
</tr>
<tr>
<td>Section size = 400 x 400 mm</td>
<td>40 x 40 mm, 33.33… x 33.33… mm,</td>
</tr>
</tbody>
</table>
Figure 3 - 4 Beam dimensions for mesh sensitivity study

Figure 3 - 5 Mesh sizes for mesh sensitivity study of cross-section dimensions 200mm x 200mm
3.3.1 Comparison of results of different section sizes

Figure 3 - 6(a) and Figure 3 - 7 compare the simulation results for beam dimensions of 3000 mm (span) x 200 mm x 200 mm (cross-section dimensions). The simulation results converge for mesh sizes of 50 x 50 mm, 40 x 40 mm and 33.33… x 33.33… mm. Therefore, the 40 x 40 mm mesh size can be used.

For the 4500 mm x 300 mm x 300 mm beam, Figure 3 - 6 (b) and Figure 3 - 7 show that a mesh size of 42 x 42 mm gives convergence in the simulation results.

For the 6000 mm x 400 mm x 400 mm beam, as shown in Figure 3 - 6 (c) and Figure 3 - 7, using the 40 x 40 mm mesh size also achieved convergence in the results.

As shown in figure 3 – 7, there is no notable difference in the first peak loads. However in the case of the final loads, using a mesh size of 40mm x 40mm gives results that are least sensitive to the mesh size for all different beam cross-section dimensions. Figure 3 – 8 shows the compared data for test results of a reinforced concrete frame and simulation results. Therefore, the 40mm x 40mm mesh size is appropriate.
(a) Load-deflection curves, beam span 3000mm, cross-section size 200x200mm

(b) Load-deflection curves, beam span 4500mm, cross-section size 300x300mm
(c) Load-deflection curves, beam span 6000mm, cross-section size 400x400mm

Figure 3 - 6 Sensitivity of vertical load – deflection relationship to mesh size for different beams
Figure 3 - 7 Summary of mesh sensitivity study results, beam first peak and final loads
Figure 3 - 8 Comparison on experimental data and simulated data for mesh sensitivity study
3.4 Comparison with test results

To date, there have been no publications in the available literature on axially restrained reinforced concrete beams that go through the entire range of behaviour until fracture during catenary action at elevated temperatures. Instead, Yu and Tan (2010) have reported ambient temperature test results of axially restrained RC beams including the development of catenary action and Dwaikat and Kodur (2009) have reported fire tests on axially restrained RC beams but without the catenary action stage. Yi et al. (2008) reported results of reinforced concrete frame behaviour after removal of a column. This section presents a comparison of the author’s numerical simulations against these test results.

3.5 Axially restrained reinforced concrete (RC) beams at ambient temperature

Yu and Tan at the Nanyang Technological University in Singapore carried out two preliminary tests on axially restrained RC beams in 2010. Figure 3 - 8 shows the test arrangement and Figure 3 - 9 shows the beam details, cross-section dimensions, span and reinforcement arrangement. The test beam was nominally two spans, with the central column removed. At the ends, the beam was attached to two column segments. The axial restraint was simulated using two rods connected to a rigid block. Table 3 - 2 gives the material properties, based on the test results for the concrete and steel reinforcement used in the test specimens. The two different reinforcement arrangements of the tests simulated two design conditions; seismic and non-seismic.
Figure 3 - 8 Description of beam set up (Yu and Tan, 2010)

Figure 3 - 9 Details of reinforcement arrangement (Yu and Tan, 2010)
Table 3 - 2 Details of the test beams

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Detailing</th>
<th>Beam size (mm)</th>
<th>Reinforcement ratio at the middle joint</th>
<th>$F_{cu}$ (MPa)</th>
<th>$F_y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$h$ $b$ $L$</td>
<td>Top</td>
<td>Bottom</td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>Seismic</td>
<td>250 150 2750</td>
<td>0.90% (1T13+2T10)</td>
<td>0.49% (2T10)</td>
<td>31.2</td>
</tr>
<tr>
<td>S2</td>
<td>No</td>
<td>250 150 2750</td>
<td>0.73% (3T10)</td>
<td>0.49% (2T10)</td>
<td></td>
</tr>
</tbody>
</table>

Concrete
- Compressive cylinder strength : 31.2MPa
- Tensile strength : 3.2MPa
- Initial modulus of elasticity : 27,663MPa

The concrete was simulated using 4-node quadratic plane stress elements, with a mesh size of 40 x 40 mm. The longitudinal reinforcement bars were simulated using embedded reinforcement elements, as were the stirrups.

3.5.1 Comparison between test and simulation results

Each of the tests was simulated twice, one assuming total restraint (end restraint stiffness = infinite) and one using the realistic restraint conditions of the tests including appropriate gaps in the support, as shown in Figure 3 - 10. Table 3 – 3 presents the properties of the boundary conditions of the tests, extracted by the author from the test paper.

Figure 3 - 13 and Figure 3 - 14 compare the test results with the simulation results for the two assumed axial restraint conditions. Assuming infinite axial restraint is clearly not appropriate, giving compressive forces in the beam that were much higher than the test results and also much higher beam resistance. Using the realistic
axial restraint properties, the simulation results are in very good agreement with the test results.

Figure 3 - 11 compares the test and simulation failure modes. In the test, the specimen collapsed due to failure (fracture) of the reinforcement after catenary action. Figure 3 - 12 shows the numerical results of the stress-strain relationship of the reinforcement steel at the bottom centre of the beam. This location is almost the same as in the test result (Figure 3 - 11).

Table 3 - 3 Boundary conditions used in the real test data

<table>
<thead>
<tr>
<th>Horizontal restraints</th>
<th>Tension stiffness (KN/m)</th>
<th>Compression stiffness (KN/m)</th>
<th>Tension Gap (mm)</th>
<th>Compression Gap (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TopAR*</td>
<td>55957.26</td>
<td>0</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>BtmAR*</td>
<td>63942.28</td>
<td>102326.82</td>
<td>1.7</td>
<td>-3.9</td>
</tr>
</tbody>
</table>

(where, TopAR : top axial restraint, BtmAR : bottom axial restraint)
(a) Detailed boundary condition

(b) Horizontal support at reaction wall side

Figure 3 - 10 Test specimen and boundary condition (Yu and Tan, 2010)
Figure 3 - 11 Failure mode of specimen (Yu and Tan, 2010)

Figure 3 - 12 Reinforcement stress-strain relationship at failure mode
Figure 3 - 13 Comparison of axial reaction force – deflection relationships between test and simulations

Figure 3 - 14 Comparison between vertical load – displacement relationships between test and simulations
3.5.2 Comparison for axially restrained RC beams at elevated temperatures

Dwaikat and Kodur (2009) reported results of a series of six fire tests on axially restrained reinforced concrete beams. The reported results only include beam behaviour during the stage when the beams were under compression. Figure 3 - 15 shows the beam setup. Table 3 - 4 presents the main features of the six tests.

![Beam setup diagram](image)

**Figure 3 - 15 Beam details and location of thermocouples in test beams**
(Dwaikat and Kodur, 2009)
### Table 3 - 4 Test specimen details and conditions (Dwaikat and Kodur, 2011)

<table>
<thead>
<tr>
<th>Beam</th>
<th>Fire Exposure</th>
<th>Concrete Type</th>
<th>Support Type</th>
<th>P(KN)</th>
<th>Fire resistance(min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>ASTM E119</td>
<td>NSC</td>
<td>SS</td>
<td>50</td>
<td>180</td>
</tr>
<tr>
<td>B2</td>
<td>SF</td>
<td>NSC</td>
<td>AR</td>
<td>50</td>
<td>NF</td>
</tr>
<tr>
<td>B3</td>
<td>ASTM E119</td>
<td>HSC</td>
<td>SS</td>
<td>50</td>
<td>160</td>
</tr>
<tr>
<td>B4</td>
<td>SF</td>
<td>HSC</td>
<td>SS</td>
<td>50</td>
<td>NF</td>
</tr>
<tr>
<td>B5</td>
<td>LF</td>
<td>HSC</td>
<td>SS</td>
<td>60</td>
<td>146</td>
</tr>
<tr>
<td>B6</td>
<td>LF</td>
<td>HSC</td>
<td>AR</td>
<td>50</td>
<td>NF</td>
</tr>
</tbody>
</table>

NSC: Normal strength concrete, HSC: High strength concrete SS: simply supported, AR: Axially restrained, NF: Not failure

For the numerical simulation, the material properties (tensile strength, fracture energy and elastic modulus) of each type of concrete can be calculated by using the CEB-FIP model code (CEB-FIP 2010), and the results are given in Table 3 – 5.

### Table 3 - 5 Calculated material properties for numerical simulations

<table>
<thead>
<tr>
<th>Material Property for concrete</th>
<th>NSC</th>
<th>HSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength (MPa)</td>
<td>55</td>
<td>99</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>4.36</td>
<td>6.455</td>
</tr>
<tr>
<td>Fracture Energy (Nmm/mm^2)</td>
<td>0.0886</td>
<td>0.1408</td>
</tr>
<tr>
<td>Elastic Modulus (N/mm^2)</td>
<td>39708</td>
<td>47376</td>
</tr>
</tbody>
</table>

- Normal strength Concrete

\[
\text{Tensile strength} = 1.4 \times \left(\frac{55}{10}\right)^{\frac{2}{5}} = 4.36 \text{ Mpa}
\]

\[
\text{Fracture Energy} = 0.03 \times \left(\frac{47}{10}\right)^{0.7} = 0.0886 \text{ Nmm/mm}^2
\]

\[
\text{Elastic Modulus} = 2.15 \times \left(\frac{55 + 8}{10}\right)^{\frac{1}{3}} = 39708.667 \text{ Mpa}
\]
In all tests, three Ø19mm bars were used for tensile reinforcement and two Ø13mm bars for compressive reinforcement. For the shear reinforcement, Ø6mm stirrups were used at spacing of 150 mm. The steel of the main reinforcing bars and stirrups had specified yield strengths of 420 and 280 MPa, respectively.

The elevated temperature mechanical properties of concrete and steel were based on the reduction factors in EN 1992-1-2 (2002). Appendix 3.1 lists the temperature dependent values for the compressive strength, tensile strength, Young’s modulus of concrete and the yield strength of steel.

The fire tests included three fire temperature-time curves: the standard ASTM E119 (ASTM, 2008) curve, a long fire (LF) and a sort fire (SF) curve. Figure 3 - 16 shows these three fire temperature-time curves.

![Fire Temperature-Time Curves](image)

**Figure 3 - 16 The three fire temperature-fire curves in the tests (Dwaikat and Kodur, 2009)**
3.5.2.1 Comparison for heat transfer analysis

In this thesis, the coupled thermo mechanical simulation was used. Therefore, it was necessary to conduct heat transfer analysis to obtain detailed temperature profiles of the structure. Figure 3 - 15 shows the locations of thermocouples in the fire tests.

Figure 3 - 18 compares the simulation and recorded temperature results for positions B at the beam centre for tests B1 and B3. It can be seen that the simulation results are reasonably accurate.

![Figure 3 - 17 Measured temperatures as a function of time of Beams B1 and B3 in test (Dwaikat and Kodur, 2009)](image)
Figure 3 - 19(a) and Figure 3 - 19(b) compare the test and simulation results for the beam deflection–time relationships for the six tests, grouped for the three different fire exposures. The simulated fire resistance time for test B3 was moderately higher than the test result. This may be attributed to spalling, which was observed in the test due to the HSC used, but not included in the first simulation. Figure 3 - 19(c) compares the test and revised simulation results assuming spalling. In this simulation, the depth of spalling was assumed to be 15 mm for three heated sides (bottom and two sides). The simulation results with spalling are in good agreement with the test results, indicating that the simulation model is satisfactory. However, quantification of spalling is necessary.

It can be seen in all cases that the simulation results are in reasonable agreement with the test results for the beam behaviour and the beam fire resistance if the beam has failed during fire testing.
(a) Comparison of test and simulation results for three beams (B1)

(b) Comparison of test and simulation results for three beams (B3)
(c) Comparison of test and simulation results for three beams (B5)

(d) Comparison of test and simulation results for three beams (B2, B4, B6)
3.5.3 Comparison for Reinforced frame at ambient temperatures

Yi et al. (2008) reported results of a reinforced concrete frame test which involved removal of a column. A four bay, eight-storey RC frame structure was designed in accordance with the concrete design code of China (Concrete code of China, 2002) which is similar to ACI 318-02 (ACI Committee 318, 2002). Figure 3 - 20 shows the test frame and Table 3 - 1 and Table 3 - 7 present details of the test frame, including steel reinforcement details for the beams and the column. The external loads were applied by displacement control. As shown in Figure 3 - 21, the reinforcement bars near the end of the first floor beam adjacent to the middle column ruptured after catenary action. Therefore, this comparison can be used to demonstrate the capability of the numerical model to simulate catenary action.
Table 3 - 6 Basic information of prototype and model frame (Yi et al., 2008)

<table>
<thead>
<tr>
<th>Items</th>
<th>Prototype frame</th>
<th>Test frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor height</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Floor</td>
<td>4,700 mm</td>
<td>1,567 mm</td>
</tr>
<tr>
<td>Other Floors</td>
<td>3,300 mm</td>
<td>1,100 mm</td>
</tr>
<tr>
<td>Span</td>
<td>8,000 mm</td>
<td>2,667 mm</td>
</tr>
<tr>
<td>Beam size</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>600 mm</td>
<td>200 mm</td>
</tr>
<tr>
<td>Width</td>
<td>300 mm</td>
<td>100 mm</td>
</tr>
<tr>
<td>Column size</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>600 mm</td>
<td>200 mm</td>
</tr>
<tr>
<td>Width</td>
<td>600 mm</td>
<td>200 mm</td>
</tr>
<tr>
<td>Axial force on top of third floor middle column</td>
<td>981 KN</td>
<td>109 KN</td>
</tr>
</tbody>
</table>

Table 3 - 7 Beam and column reinforcement details for test frame (Yi et al 2008)

<table>
<thead>
<tr>
<th>Longitudinal reinforcement</th>
<th>Lateral reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column, mm</td>
<td>Beam, mm</td>
</tr>
<tr>
<td>Top bar</td>
<td>Bottom bar</td>
</tr>
<tr>
<td>4D12</td>
<td>2D12</td>
</tr>
</tbody>
</table>
(a) Detail showing rupture of reinforcing bar in beam

(b) Frame after testing

Figure 3 - 21 Failure mode of the test frame (Yi et al., 2008)

Table 3 - 8 lists the material properties for the reinforcement steel and concrete.
Table 3 - 8 Material properties of reinforcement steel and concrete (Yi et al., 2008)

<table>
<thead>
<tr>
<th>Material</th>
<th>Measured values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal reinforcement</td>
<td></td>
</tr>
<tr>
<td>Yield strength</td>
<td>416 MPa</td>
</tr>
<tr>
<td>Ultimate tensile strength</td>
<td>526 MPa</td>
</tr>
<tr>
<td>Ratio elongation</td>
<td>Five times = 27.5%</td>
</tr>
<tr>
<td></td>
<td>Ten times = 23%</td>
</tr>
<tr>
<td>Lateral reinforcement</td>
<td></td>
</tr>
<tr>
<td>Yield strength</td>
<td>370 MPa</td>
</tr>
<tr>
<td>Concrete (C30)</td>
<td>Cubic strength of compression</td>
</tr>
</tbody>
</table>

During the test, the horizontal displacement at position A and concrete strain at position B and C at both the top and bottom surfaces of the beam were measured (Figure 3 - 22). These are compared with the simulation results. The 4-node plain stress quadratic elements (Q8MEM) were used for concrete and embedded elements were used for reinforcement steel. The load was applied by displacement control at the centre of the top column.

Figure 3 - 22 Location of horizontal displacement measurement (position A) and concrete strain measurement (Top: B, Bottom: C)
Figure 3 - 23 Comparison for load – beam vertical displacement between test and simulation results

Figure 3 - 23, Figure 3 - 24 and Figure 3 - 25 compare the test simulation results; Figure 3 - 23 for the load-beam vertical deflection relationship, Figure 3 - 24 for strain – displacement relationships and Figure 3 - 25 for frame horizontal displacement – beam vertical displacement relationships. The closeness between the test and simulation results indicates that the simulation model is suitable. Figure 3 - 23 shows clear sign of catenary action development at large beam displacements, as indicated by increasing rate of big load at large beam displacement. This is confirmed by the increase in tensile strain and decrease in compressive strain of the concrete in Figure 3 - 24 and pull-in (shortening) of the frame in Figure 3 - 25.
Figure 3 - 24 Comparison for concrete strains at top and bottom surfaces of the centre section between test and simulation results

Figure 3 - 25 Comparison for frame horizontal displacement between test and simulation results
3.6 Summary

This chapter has discussed different simulation models for concrete and assessed the sensitivity of the simulation results to mesh size. 2-D plain stress elements were used to save computational effort. It has been shown that a mesh size of about 40 x 40 mm, using plain stress elements, is suitable for simulating restrained reinforced concrete beams. The concrete can be simulated using the Mohr-Coulomb model for compression, the smeared crack model for tension, and the Hordijk model for tension softening. The reinforcement can be modelled using the embedded model approach.

Detailed comparisons, between the simulation results and the restrained beam tests of Yu and Tan (2010) at ambient temperature, and between the simulation results and the test results of Yi et al. (2008) for reinforced concrete frames with column removal, demonstrate that the simulation model is suitable for modelling axially restrained reinforced concrete beams and frames that exhibit all stages of behaviour, including development of catenary action in reinforced concrete beams. However, there was no experimental result for axially restrained RC beams under catenary action. Additional comparisons between the simulation results and the test results of Dwaikat and Kodur (2009) suggest that the author's model is also suitable for simulations of reinforced concrete beams in fire. The simulation model will be used in Chapter 4 to conduct an extensive parametric study to investigate how the fire resistance performance of reinforced concrete beams may be improved.
Chapter 4. Behaviour of axially restrained concrete beams in fire

4.1 Introduction

This chapter presents the outcome of a parametric study to investigate the behaviour and fire resistance of axially restrained concrete beams. The objectives of this parametric study are twofold: (1) to provide data for checking the analytical prediction model to be developed in Chapter 5; and (2) to discover how to change the different design parameters to improve the survival time of axially restrained reinforced concrete beams.

Figure 4-1 Beam description for parametric study

Figure 4-1 shows the general arrangement of the beam. One concentrated load is applied at the centre of the beam to simulate the load coming from the column above after removal of the column below.

The numerical simulation included both thermal analysis to obtain temperature distributions in the cross-section and mechanical modelling to obtain the structural performance of the structure. However, when examining the effects of changing different design parameters, only the structural arrangements were varied while keeping the temperature distributions unchanged.
As explained in Chapter 3, the finite element software TNO DIANA was used for the numerical simulations. Concrete was modelled by 4-node quadratic plane stress elements (Q8MEM) and embedded elements were used for the reinforcement steel. Reinforcement steels are embedded in structural elements. It implies perfect bond between the reinforcement and the surrounding material. For analysis of heat transfer, heat flow element (B2HT) which is a 2-node boundary element for general potential flow analysis was used.

The mesh size was 40 x 37.5 mm based on the sensitivity study in Chapter 3. 2-D modelling was chosen for computational efficiency. Therefore, it is assumed that the behaviour of the beam is the same in the width direction of the cross-section.

For modelling concrete, the smeared crack model was used, with Mohr-Coulomb plasticity and Hordijk tension softening. For the reinforcement steel, the von Mises plasticity model was used. Table 4 - 1 presents the ambient temperature mechanical properties of concrete and steel used in this study. At high temperatures, the mechanical properties of steel reinforcement and concrete change according to EN 1992-1-2:2002.

<table>
<thead>
<tr>
<th></th>
<th>Concrete</th>
<th>Reinf. steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$ (Young’s modulus)</td>
<td>26000 MPa</td>
<td>200000 MPa</td>
</tr>
<tr>
<td>$\nu$ (Poisson’s ratio)</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$F_c$ (Cylinder strength)</td>
<td>20 MPa</td>
<td>420 MPa</td>
</tr>
<tr>
<td>$F_t$ (Tensile strength)</td>
<td>1.5 MPa</td>
<td></td>
</tr>
<tr>
<td>$G_f$ (Fracture energy)</td>
<td>0.08 Nmm/mm²</td>
<td></td>
</tr>
</tbody>
</table>

The dimensions of the reference beam are: cross-section size 300 x 150 mm, length 6 m, and reinforcement area 600 mm². Figure 4 - 2 shows the beam dimensions.
4.2 Thermal analysis

The beam is assumed to be subjected to the standard ASTM E119 (ASTM 2008) standard fire exposure and the standard temperature-time curve is shown in Figure 4-3. The temperature-time curve is calculated using the following equation:

\[
T_f = T_0 + 750 \left(1 - \exp(-3.79553\sqrt{T_h})\right) + 170.41\sqrt{T_h} \quad \text{Equation 4 - 1}
\]

where: \(T_f\) = fire temperature in °C, \(T_0\) = initial temperature in °C, \(T_h\) = Time (hours)
In this research, heat transfer was carried out to obtain temperatures of the structure, then followed by simulation of the mechanical behavior at elevated temperatures. For heat transfer modelling, Table 4 - 2 lists the thermal properties of concrete and the thermal boundary conditions used in the modeling, according to EN 1992-1-2:2002. The steel reinforcement was not included and it is assumed that the steel reinforcement temperature is the same as that of the concrete at the same position.

In this research, 2-D modelling was used for computational efficiency. This means that the cross-sectional behaviour of beam varies only in the height direction and is the same across the width. The beam was assumed to be exposed to fire at the bottom and the ends, as shown in Figure 4 - 4. Plain heat transfer element type B2HT (TNO-DIANA 2010) was used. In order to facilitate linked modelling, 2-D heat transfer simulation was carried out throughout the span of the beam, even though the temperature distribution along the beam length was uniform. Temperature difference at the ends would have minor effect because the beam behaviour is dominated by its behaviour in the span. The mesh size was 40 x 37.5 mm as used in the mechanical simulation. The EN 1992-1-2:2002 lower thermal conductivity of
concrete was used in the simulation. The values are given below.

**Thermal conductivity** = \(1.36 - 0.136 \left(\frac{\theta}{100}\right) + 0.0057 \left(\frac{\theta}{100}\right)^2\ \text{W/mK}\)  

\[\text{Equation 4 - 2}\]

for \(20^\circ\text{C} \leq \theta \leq 1200^\circ\text{C}\)

where \(\theta\) is the concrete temperature in °C.

The specific heat of concrete is also according to EN 1992-1-2:2002, as given below:

\[\text{C}_p(\theta) = 900\ \text{(J/kg K)}\]  

for \(20^\circ\text{C} \leq \theta \leq 100^\circ\text{C}\)  

\[\text{Equation 4 - 3}\]

\[\text{C}_p(\theta) = 900 + (\theta - 100)\ \text{(J/kg K)}\]  

for \(100^\circ\text{C} \leq \theta \leq 200^\circ\text{C}\)  

\[\text{Equation 4 - 4}\]

\[\text{C}_p(\theta) = 1000 + (\theta - 200)/2\ \text{(J/kg K)}\]  

for \(200^\circ\text{C} \leq \theta \leq 400^\circ\text{C}\)  

\[\text{Equation 4 - 5}\]

\[\text{C}_p(\theta) = 1100\ \text{(J/kg K)}\]  

for \(400^\circ\text{C} \leq \theta \leq 1200^\circ\text{C}\)  

\[\text{Equation 4 - 6}\]

where \(\theta\) is the concrete temperature(°C). The \(\text{C}_p(\theta)\) (KJ/Kg K) – temperature relationship is illustrated in Appendix 4.1

**Table 4 - 2 Thermal properties of concrete and boundary conditions for heat transfer**

<table>
<thead>
<tr>
<th>Thermal property</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity</td>
<td>4 - 2</td>
</tr>
<tr>
<td>Thermal capacity (specific heat)</td>
<td>4 – 3</td>
</tr>
<tr>
<td>To</td>
<td>4 - 6</td>
</tr>
<tr>
<td>Convective heat transfer coefficient on the fire side</td>
<td>25 W/m²</td>
</tr>
<tr>
<td>Convective heat transfer coefficient on the air side</td>
<td>10 W/m²</td>
</tr>
</tbody>
</table>
The heat transfer simulation results (temperature distributions in the structure), were used as input data for the subsequent mechanical analysis of structural behaviour. Because the purpose of this study is to compare the effectiveness of changing different design parameters to prolong the beam survival time and to generate a set of data to check the analytical model, it was not considered necessary to refine the thermal analysis. Therefore, the temperature-independent thermal properties in Table 4 - 2 were used and no attempt was made to fine tune the thermal analysis results.
4.3 Structural behaviour of axially restrained beams

4.3.1 Typical behaviour of axially restrained reinforced concrete beams in fire

Figure 4 - 5 presents typical results of axially restrained reinforced concrete beams in fire, with Figure 4 - 5 (a) giving the beam axial force-time relationship and Figure 4 - 5(b) presenting the deflection-time relationship, and Figure 4 - 5 (c) and Figure 4 - 5 (d) showing the evolutions of the reinforcement strain-time relationship and stress-time relationship. The concrete stress at the top and the bottom of the cross-section in the centre are shown in Figure 4 - 5 (e). Figure 4 - 6 and Figure 4 - 7 present evolutions of the crack pattern and longitudinal stress distribution in the beam. In Figure 4 - 5 (a), the tensile capacity of reinforcement steel (calculated as the elevated tensile strength of reinforcement steel multiplied by its area) is also included because it becomes the main load sustaining component when the beam is in the catenary action stage. These results are for the reference beam case, for which the details are given in section 4.1.
(a) Beam axial force – time relationship

(b) Deflection – time relationship
(c) reinforcement steel strain – time relationship

(d) steel reinforcement stress – time relationship
(e) Concrete element axial stress – time relationship

Figure 4 - 5 Simulation results of the reference beam

(a) cracks at time 0

(b) cracks at time 87 minutes
(c) cracks at time 88 minutes

(d) cracks at time 250 minutes

Figure 4 - 6 Evolution of the crack pattern of the reference beam

Unit : N/mm$^2$

(a) Axial stress at time 87 minutes (Point A in Figure 4.5a)
(b) Axial stress at time 88 minutes

(c) Stress at time 109 minutes (Point B in Figure 4.5a)
(d) Axial stress at time 140 minutes (during catenary action)

Unit: N/mm$^2$

(e) Axial stress at time 250 minutes (Point C in Figure 4.5a)

Figure 4-7 Evolution of the distribution of axial stress in the reference beam

These figures show the following three stages of behaviour:

1. Beam in compression

Initially, the beam expands. Because the thermal expansion is restrained, a compression force is generated in the beam. This compression force increases as the time, and hence the temperature, increases. The beam deforms, initially mainly due to thermal bowing and then due to reduced mechanical properties of the materials at high temperatures. The compressive force reaches the peak value (point A in Figure 4-5(a)) when the beam reaches temporary instability. There is limited crack development in the beam (Figure 4-6(c)) under combined axial compression and bending. Afterwards, the compression force in the beam is relieved as indicated in Figure 4-5(e) which shows the stresses in the top and bottom concrete elements at the beam centre. The axial stress in the top element of the concrete is compressive (negative value) until Point A (87 minutes). Figure 4-7(a) shows that the entire beam is mainly in compression when the compression force of the beam has reached the maximum (87 minutes).
2. Transition from compression to catenary action

After reaching the peak value, the compression force in the beam decreases. When it reaches zero (point B in Figure 4 - 5 (a)), the applied load on the beam is resisted by the bending moment in the beam. The stress distribution at the same time in Figure 4 - 7 (c) clearly shows the bending stress pattern in the beam. Furthermore, with reference to Figure 4 - 5 (c), it is clear that the reinforcement steel has reached yield and the concrete has reached its compressive strength.

The current method of fire resistant design of reinforced concrete beams does not consider axial restraint, and hence the axial force in the beam is zero. Therefore, point B in Figure 4 - 5 (a) for the axially restrained beam corresponds to the limit of the axially unrestrained beam.

At this stage, the beam undergoes acceleration in the rate of deflection. If the beam has no axial restraint, runaway deflection would occur (as shown in Figure 4 - 5 (b)) and the beam would lose its equilibrium. However, because the beam is axially restrained, runaway deflection does not happen and the beam can reach equilibrium under catenary action.

3. Catenary action

Catenary action develops when the beam’s deflection is large (due to the inability of the beam to resist the applied load in bending) and the shortening of the beam overtakes the thermal expansion. During the catenary stage, the resistance of the beam is gradually transferred from bending to axial tension (Figure 4 - 5e). It is possible for the beam to completely crack through (Figure 4 - 6d) and for the load carrying capacity of the beam to come from the tensile resistance of the steel reinforcement (Figure 4 - 5 (a)). Figure 4 - 7(d) illustrates the stress distribution of the entire beam, clearly indicating tension of the reinforcement steel and near zero stress of the concrete. Failure of the beam is caused by fracture of the reinforcement, as shown in Figure 4 - 5 (d).

When assessing the structural robustness of the beam in fire, because the focus is on survival of the beam, it is acceptable to define the limit of the beam as point C in
Figure 4 - 5 (a). It is possible for the fire resistance time at point C to be much greater than that at point B which is used in conventional fire resistance design of reinforced concrete beams based on bending resistance.

The parametric study below will investigate how to improve the survival temperatures of axially restrained beams.

### 4.4 Parametric study results

To investigate how different design parameters affect the survival time of axially restrained reinforced concrete beams, and hence to suggest methods of prolonging the time, a parametric study has been conducted. Prolonged survival of the beam is enabled by development of catenary action, through tensile resistance of the reinforcement and large deflection of the beam. Therefore, the development of catenary action is likely to be affected by the following parameters:

- Size(area) of steel reinforcement
- Elongation (maximum strain) of steel reinforcement
- Span length/depth ratio
- Load ratio
- Reinforcement position
- The level of axial restraint
- The level of rotational restraint

Figure 4 - 2 shows the geometric condition of the reference beam. When conducting the parametric studies, the value of only one parameter is changed while the values of the other parameters are kept the same as the reference beam. The values of the parameters are shown in Table 4 - 3.
### Table 4 - 3 Input values for parametric study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of steel</td>
<td>400mm$^2$, 600mm$^2$</td>
</tr>
<tr>
<td>Fracture strain of steel reinforcement</td>
<td>10%, 15%, 20%</td>
</tr>
<tr>
<td>Span length/Depth ratio</td>
<td>13.33..., 20, 26.66…</td>
</tr>
<tr>
<td>Load ratio</td>
<td>70%, 50% of simply supported RC beam capacity at ambient temperature</td>
</tr>
<tr>
<td>Cover thickness</td>
<td>50mm, 75mm</td>
</tr>
<tr>
<td>Level of axial restraint</td>
<td>Full, 100%, 50%, 10%, 5%, 1% and 0.1%, of beam axial stiffness, No restrain</td>
</tr>
<tr>
<td>Level of rotational restraint</td>
<td>Full, 0.1% of beam rotational stiffness, Free rotation</td>
</tr>
</tbody>
</table>

#### 4.4.1 Reinforcement size

Figure 4 - 8 presents the effects of changing the reinforcement size. When the reinforcement area increases, the bending resistance of the beam is increased, resulting in increased time under bending resistance. Due to the rapid change in reinforcement strength at high temperatures, the increase in the reinforcement temperature at the bending failure time of the beam (beam axial force = 0) due to increasing the reinforcement area is relatively small. For example, when using a reinforcement area of 400mm$^2$, at the bending failure time, the steel reinforcement temperature is 487.13°C and the strength retention factor of the steel is 0.725. Increasing the steel reinforcement area by 50% to 600mm$^2$, the steel reinforcement temperature is 580.97°C and the steel strength reduction factor has decreased to 0.538. Nevertheless, owing to the slow rate of temperature increase in the reinforcement, the fire exposure time at the time when the beam reaches its bending resistance (beam axial force =0) shows an impressive increase, from 71 minutes to 109 minutes.

Owing to axial restraint, the beam is able to survive the fire exposure for a very long
time after bending failure. For example, the final failure time for the beam with 400mm$^2$ steel reinforcement is 214 minutes, compared to the bending failure time of 71 minutes. Increasing the steel reinforcement area to 600mm$^2$ prolongs the survival time of the beam to 250 minutes. When the tensile resistance of the steel increases, by increasing the reinforcement area of the beam, a smaller beam deflection is required to resist the same vertical load, thus requiring a smaller strain in the reinforcement. Since failure of the beam is caused by fracture of the steel reinforcement, increasing the reinforcement area allows the beam to survive longer until the beam deflection reaches a similar level as the beam with lower reinforcement area. This is clearly shown by the similar maximum deflection values shown in Figure 4 - 8(b) at failure of the beams. Figure 4 - 8 (c) plots the rebar strain-time relationships, showing that the steel reinforcement in both beams has reached the defined fracture strain of 0.15. Figure 4 - 8 (d) shows the time histories of the rebar stresses at the middle of the beams: the stresses of the reinforcement steels at beam failure are nearly zero, indicating total fracture.

However, increasing the steel reinforcement area does not seem a very effective method of enhancing the survival time of the beam (changing from 214 minutes for a 400mm$^2$ reinforcement steel area to 250 minutes for a 600mm$^2$ reinforcement area). This is a result of the steel reinforcement strength decreasing rapidly at high temperatures.
(a) Beam axial force – time relationships

(b) Deflection – time relationships
Figure 4 - 8 Effect of changing steel reinforcement area

(c) reinforcement steel strain – time relationships

(d) reinforcement steel stress – time relationships
Because the beam is completely restrained axially, there is a very high compression force. This must be considered in the design of the surrounding structure. Increasing the reinforcement size slightly increases this force because it allows the beam to achieve a higher time when failure occurs due to combined bending and compression.

The maximum deflection of the beam at failure is about 0.8 m, giving a deflection/span ratio of about 1/7.5. This may be too high and may cause integrity failure of the structure, due to large openings (cracks) in the beam. Nevertheless, if the aim of the fire resistance design is to ensure the survivability of the beam, then the axially restrained beam has the potential to greatly increase survival time of the beam (final failure) from the bending failure time (for conventional fire resistance design) of the beam.

### 4.4.2 Ductility of reinforcement

Figure 4 - 9 compares the beam axial force – time relationships and beam deflection – time relationships between different levels of strain ductility: 10%, 15% and 20%. 10% is considered as the lower bound value, given in Eurocode 2 (EN 1992-1-1: 2002). Evidences from tests (Chapter 3, section 3.5.3) indicate higher ductility of reinforcement. Increasing the reinforcement ductility from 10% to 15% resulted in a significant increase in the survival time of the beam, from 162 minutes to 250 minutes. However, further increasing the reinforcement ductility to 20% resulted in a relatively small increase in the beam survival time, from 250 minutes to 262 minutes. The reason for this is related to the reduction factor of reinforcement. For reinforcement ductility of 10%, 15% and 20%, the strengths of reinforcement are 133.29Mpa, 69.21Mpa and 60.76Mpa (reduction factor = respectively 0.317, 0.164 and 0.144), corresponding to the reinforcement temperatures (720.04°C, 754.33°C and 763.29°C) at the beam failure times of 210 minutes, 250 minutes and 262 minutes respectively. A further small increase in the reinforcement temperature would reduce the reinforcement strength quickly, requiring an accelerated increase in the beam deflection (Figure 4 - 9 (b)) to allow the beam to retain equilibrium. This
required increase in beam deflection quickly exhausts the ductility of the reinforcement (Figure 4 - 9 (c)). Because the ductility of steel reinforcement is usually lower than the minimum value used in this study, this study suggests that any increase in the reinforcement ductility would be quite effective in increasing the beam survival time.

(a) Beam axial force – time relationships

(b) Deflection – time relationships

Figure 4 - 9 Effects of changing ductility of the reinforcement
4.4.3 Span/depth ratio

For this study, the load ratio was fixed. This means that the applied load had to be changed because the beam span was changed while maintaining the beam depth. Figure 4 - 10(a) and 4 - 10 (b) compare the beam axial force-time and beam deflection-time relationships for beam span/depth ratios of 13.33…, 20 and 26.66… respectively. Because the load ratio was unchanged, the bending failure times were similar.

(a) Beam axial force – time relationships
(b) Deflection – time relationships

Figure 4 - 10 Effects of changing the beam span/depth ratio

It is interesting to notice from Figure 4 - 11 that during the catenary action stage, the beam deflections for different span/depth ratios are similar at the same time. This is because at this stage, the applied load (bending moment) is resisted by the tensile force in the beam multiplied by the vertical deflection of the beam according to the following relationship:

Figure 4 - 11 Free body diagram for a half beam
\[ \sum M = 0 \]
\[ \rightarrow -M + \frac{P}{2} \times \frac{L}{2} - T\delta = 0 \]
\[ \rightarrow T\delta = \frac{PL}{4} - M \]
\[ \rightarrow T = \left(\frac{PL}{4} - M\right)^{\frac{1}{\delta}} \]  
\text{Equation 4 - 7}
\[ \rightarrow T = \frac{PL}{4\delta} \] ; during the catenary action when M is negligible.

where M is the bending capacity of the beam, T is the axial tension force in the beam, \( \delta \) is the beam mid-span deflection, P is the applied point load, and L is the length of the modelled beam. Because the maximum bending moment of the beam (PL/4) and the reinforcement tensile force for the different beams are the same, the different beams have very similar deflections.

This also explains why increasing the beam span-depth ratio prolongs the beam survival temperature. When increasing the beam span, the reinforcement strain is lower when the beam deflection is the same for different beam spans, as shown in Figure 4 - 10(c). Therefore, the longer beam can deform further before its steel reinforcement fractures.

When dealing with robustness of structures, the assumed scenario is often the removal of a column. This would result in large span/depth ratios, meaning that catenary action is likely to be an effective means of providing survivability of the beam with increased span.
4.4.4 Load ratio

Load ratio is conventionally used to indicate the level of applied load in structure when subjected to fire. It refers to the applied load in fire to the load carrying capacity of the structure at ambient temperature. In this research, it is calculated as the ratio of the maximum bending moment in beam in fire to the bending moment capacity of the unrestrained beam at ambient temperature. Figure 4 - 12(a) and 4 - 12 (b) compare the beam axial force-time and beam deflection-time relationships for different load ratios. Obviously, increasing the load ratio reduces the bending failure time; the difference in bending failure times (when the beam axial force = 0) being 41 minutes, as shown in Figure 4 - 12 (a). The final failure time for the higher load ratio beam is lower than that for the lower load ratio beam; the difference being about 20 minutes as shown in Figure 4 - 12.

(a) Beam axial force-time relationships
To control the cost of construction, the applied load on the beam is unlikely to be changed substantially for fire resistant design. Therefore, these results suggest that reducing the applied load ratio is unlikely to be an effective method of prolonging the beam survival time.

### 4.4.5 Effect of cover depth

Figure 4 - 13 (a) and (b) compare the beam axial force – time relationships and beam deflection – time relationships for the different cover thicknesses. Increasing the cover thickness from 50mm to 75mm resulted in a reduction in the bending capacity of a beam. This is because when the cover thickness increases, the bending strength of the beam decreases due to reduction of the effective depth of the section, as shown in Equation 4 – 8.
\[ M_n = f_c \cdot a \cdot b \cdot \left( d - \frac{a}{2} \right) \]

Equation 4 - 8

where \( f_c \) = concrete strength, \( b \) = width length, \( d \) = effective depth of section

On the other hand, the final failure time for the beam with higher cover thickness is longer than that for the beam with lower cover thickness due to the reduced temperature of reinforcement steel. Nevertheless, the change in the survival time of a beam is shown in Figure 4 - 13.

(a) Beam axial force – time relationships
4.4.6 Level of axial restraint

The axial restraint to the beam in a real structure comes from the surrounding structure to the beam. Therefore, infinite axial restraint is an idealised assumption. In the next chapter, an analytical method will be derived to predict the restrained beam behaviour with infinite axial restraint, due to the relative simplicity of the problem. To demonstrate whether assuming infinite axial restraint is representative, thereby checking the applicability of the analytical method, this section investigates how the restrained beam’s behaviour is affected by the level of axial restraint.

To model flexible axial restraint, a spring is attached to the geometrical centre of the beam cross-section at the ends, as shown in Figure 4 - 14. For free rotation, other nodes on the end section did not add any axial restraint. The axial stiffness of the springs starts from 1% of the beam’s axial stiffness.
Figure 4 - 14 Axially restrained beam with flexible axial restraints

Figure 4 - 15(a) and 4 - 15 (b) compare the beam axial force-time and beam deflection-time relationships between different levels of axial restraint. As expected, the compression force in the beam reduces significantly when the restraint stiffness decreases. Therefore, the beam with a lower level of axial restraint can last longer when it reaches temporary failure under combined bending and compression. During the catenary action stage, the beam behaviour is almost unchanged, with very similar beam axial forces and deflections. This is because the tensile force in the beam, during the catenary action stage, comes from the tensile resistance of the reinforcement, and this value is not affected by the different levels of axial restraint. Because the resistance of the beam in catenary action is from the beam tension force acting on the beam deflection (Equation 4 - 7), the beam deflection is similar, as shown in Figure 4 - 15 (b). As can be seen, the maximum compressive strain diminishes to a negligible level during the catenary action stage. Failure of the beam is due to reinforcement fracture when the reinforcement strain has reached the maximum fracture strain value (15%).
Figure 4 - 15 Simulation results on different levels of axial restraint

(a) Beam axial force – time relationship

(b) Deflection – time relationship
Table 4 - 4 compares the survival times of the beams with different levels of axial restraint. When the axial restraint stiffness is reasonably high (> 50% of the beam axial stiffness), the difference between beam survival times is quite small. Therefore, for the sake of simplifying the analytical derivation, it can be considered acceptable to assume that the axial restraint is infinite.

<table>
<thead>
<tr>
<th>The level of axial restraint</th>
<th>Survival time (failure time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>250 Minutes</td>
</tr>
<tr>
<td>100 % of beam axial stiffness</td>
<td>248 Minutes</td>
</tr>
<tr>
<td>50 % of beam axial stiffness</td>
<td>247 Minutes</td>
</tr>
<tr>
<td>10 % of beam axial stiffness</td>
<td>245 Minutes</td>
</tr>
<tr>
<td>5 % of beam axial stiffness</td>
<td>243 Minutes</td>
</tr>
<tr>
<td>1 % of beam axial stiffness</td>
<td>240 Minutes</td>
</tr>
<tr>
<td>Simple support</td>
<td>114 Minutes</td>
</tr>
</tbody>
</table>

4.4.7 Level of rotational restraint

In order to check the effects of different levels of rotational restraint on beam behaviour at high temperatures, the 6M beam of section size 300 x 150mm was used. The load ratio was 0.7, calculated based on the bending moment capacity of the simply supported beam without axial restraint. The highest (full) and lowest (0) levels of rotational restraint were simulated. As shown in Figure 4 - 16, the rotational restraints were modeled by using two spring elements at the top and bottom of the beam end section. The axial restraint was fixed at 50% of the beam axial stiffness.
Figure 4 - 16 Application of rotational restraints at the beam ends

Figure 4 - 17(a) and 4 - 17 (b) compare the beam axial force-time and beam deflection-time relationships. At lower temperatures, with an increase in the beam end rotational restraint, there is a reduction in the beam deflection (Figure 4 - 17 (b)). Because the same cross-section is used and the level of axial restraint is the same, the rate of increase in the compressive force in the beam is the same for both levels of rotational restraint. At high temperatures and large deflections of the beam, the beam behaviour is controlled by catenary action and the level of rotational restraint has very little effect on the beam’s behaviour and survival time.
(a) Beam axial force – time relationships

(b) Deflection – time relationships

Figure 4 - 17Effects of different levels of rotational restraints
4.5 Summary and conclusions

This chapter has presented the results of a parametric study to investigate the effects of changing different parameters on the behaviour and survival time of axially restrained reinforced concrete beams. The key quantities of the beam behaviour are:

- The maximum compressive force in the beam: this affects the design of the surrounding structure;
- The bending failure time: this is reached when the axial force in the beam is zero. This is the conventional fire resistance time when not considering the effects of axial restraint.
- Survival time: the time when the beam fails, due to fracture of the reinforcement. This is a measure of the robustness of the beam.

This study considered the following parameters:

- Area of the steel reinforcement;
- Fracture strain of the reinforcement;
- Position of the reinforcement along the beam height;
- Span/depth ratio;
- Load ratio;
- Axial restraint stiffness.
- Rotational restraint stiffness

Based on the simulation results, the following conclusions may be drawn:

- The survival time of an axially restrained reinforced concrete beam can be much higher than the bending failure time of the beam. This suggests that the effects of axial restraint can be used for the benefit of demonstrating robustness of reinforced concrete structures in fire.
- Increasing the reinforcement area results in a corresponding increase in the beam bending failure time and beam survival time. There is a slight increase
in the beam maximum compression force. However, the increase in the beam bending failure time and beam survival time is modest. Therefore, increasing the reinforcement area is not a very effective primary method of prolonging the beam survival time.

- Increasing the reinforcement fracture strain (ductility) increases the beam survival time, but has no effect on other quantities. This effect is high when the change in the reinforcement fracture strain starts from a relatively low level (e.g. 10%). Further increases in the reinforcement fracture strain give diminishing returns. However, since the fracture strain of existing reinforcement steel is low (around 5%), it is worthwhile pursuing manufacturing techniques to increase the fracture strain of the steel reinforcement.

- Increasing the beam span/depth ratio tends to help the development of catenary action in the beam, resulting in higher survival times. It also reduces the maximum compressive force in the beam. Since the scenario of testing robustness of structures involves column removal, hence increasing beam span, this result suggests that catenary action would be a feasible solution to maintain the integrity of a structure in fire, even after partial damage (column removal) in the structure.

- Reducing the load ratio results in longer beam bending failure time and beam survival time. However, the effects are modest. Since reducing the applied load would increase the construction cost, it is not suggested as a primary method of enhancing robustness of the beam.

- Even when the level of axial restraint is modest (about 1% of the beam axial stiffness), the beam can still develop substantial levels of catenary action to increase the beam survival time. When the axial restraint stiffness increases, the change in the beam survival time is relatively small. Therefore, it is acceptable to assume that the beam has infinite axial restraint. When it is not necessary to consider a flexible axial restraint, the analytical method to be developed in the next chapter is simplified.

- The beam end rotational restraint has little effect on the large deflection behaviour and survival time of reinforced concrete beams at high temperatures.
Chapter 5. Analytical model for axially restrained reinforced concrete beams in fire

This chapter develops an analytical model to predict the behaviour of axially restrained 2D reinforced concrete beams at elevated temperatures. The analytical model will be checked against the results of the numerical simulations presented in the previous chapter. As explained in Chapter 4, there are three key events in the behaviour of an axially restrained reinforced concrete beam. Referring to the beam axial force – time and beam deflection – time relationships illustrated in Figure 5 - 1, these three key events are: (a) when the axial compression force in the beam achieves the maximum value; (b) when the beam reaches the bending resistance limit and enters into catenary action (axial force = 0) and (c) when the beam fails. The objective of this chapter is to develop an analytical method to estimate these three points in the beam axial force – time and lateral deflection – time relationship.

The analytical model will be developed for reinforced concrete beams with axial restraint but no rotational restraint. Assuming no rotational restraint causes the prediction results to be on the safe side. In addition, an important advantage of making the above assumption is to simplify the analytical development.

Other assumptions of the analytical model are:

1. The temperature distribution is non-uniform in the cross-section;
2. The temperature distribution of the beam is uniform in the horizontal direction;
3. The temperature data are provided as input data
4. In the analytical development, the reinforced concrete cross-section is divided into a number of layers and the temperature of each layer is assumed to be uniform;
5. A concentrated load is applied at the beam mid-span.
Figure 5-1 Typical axial force and beam deflection – time relationships
5.1 Development of the Analytical Model

5.1.1 Equilibrium equations

Figure 5 - 2 shows the free body diagram for half of a general reinforced concrete beam. Assuming that the beam has a point load at the beam centre, the equilibrium equation is:

\[ F_a (\delta_m + \delta_{th}) + M_m = PL/4 \]  

Equation 5 - 1

where:

\( \delta_m \) is the mechanical deflection.

\( \delta_{th} \) is the thermal deflection.

\( F_a \) is the axial force in the beam (tension positive).

\( P \) is the applied load.

\( L \) is the beam span.

\( M_m \) is the internal bending moment capacity at the beam mid-span.
Figure 5 - 3 shows the free body diagram to calculate $M_m$, with the calculations given in equation 5 - 2. The internal bending moment capacity is taken about the initial longitudinal axis of the beam cross-section, which has undergone a deflection of $\delta$.

![Free body diagram of one beam](image)

Figure 5 - 3 Free body diagram of one beam

$$M_m = C \cdot d_c + T \cdot d_t$$  \hspace{1cm} \text{Equation 5 - 2}

In Figure 5 - 3, $C$ is the resultant force on the compressive side of the cross-section, $T$ is the resultant force on the tensile side of the cross-section, $D$ is the depth of the cross-section, $a$ is the depth of the compression zone, $\delta$ is the total vertical deflection of the beam, $d_c = D/2 - a/2$, $d_t = D/2$ (half of depth) – $D_c$ and $D_c$ is the concrete cover to reinforcement. In this calculation, it is assumed that the reinforcement steel has reached its yield stress.

The temperature distribution in a reinforced concrete beam cross-section is non-linear, as shown in Figure 5 - 4. Assuming the coefficient of thermal expansion of the material is temperature independent, it is necessary to convert the non-uniform temperature profile into a linear one, with the gradient of the linear distribution being used to calculate the thermal curvature. Establishing the linear distribution is based on the principle of plane section remaining plane. The linear distribution of
temperature is therefore parallel to the deformed plane of the cross-section due to thermal strain. The linear temperature distribution would divide the cross-section into a tension zone and a compression zone. The total resultant force under thermal bowing in the cross-section should be zero. Because the temperature at the top of the cross-section is low, a small change in strain would cause a large change in stress. Therefore, it is assumed that thermal strain at this position is zero. Based on the above assumptions and on the further assumption that the elevated temperature Young’s modulus of concrete is temperature independent, the linear temperature distribution causes the total area of thermal strains in the tensile zone and in the compression zone to be equal, as illustrated in Figure 5 - 4.

The total vertical deflection of the beam is:

\[ \delta_T = \delta_{th} + \delta_m \]

where, \( \delta_T \) is the total vertical deflection, \( \delta_{th} \) is the thermal deflection and \( \delta_m \) is the mechanical deflection.

\[ \text{Curvature} \]

\[ \text{Figure 5 - 4 Equivalent temperature gradient to calculate the thermal deflection} \]

where \( a_{1,2,\ldots,n} \) = thermal expansion of each layer at elevated temperature, \( a'_{1,2,\ldots,n} \) = modified thermal expansion of each layer at elevated temperature, \( A_r \) = total area for raw thermal expansion, and \( A_m \) = total area for modified thermal expansion
In Figure 5 - 4, the equivalent temperature gradient can be calculated below process.

1) Calculate \( a_{1(1,2...n)} \) at elevated temperatures
2) Calculate \( A_t = A_{r1} + A_{r2} + ... + A_{rn} \)
3) Calculate \( a'_n \) through \( A_t = A_m \)
4) Calculate curvature

The equivalent temperature gradient is constant along the beam length. Therefore the maximum thermal deflection can be calculated as follows:

\[
\kappa_t = \frac{\alpha \Delta T}{h} = \frac{d^2 \delta}{dx^2} \\
\text{Equation 5 - 4}
\]

\[
\frac{d\delta}{dx}(0) = k_t \cdot x - 3000 \cdot k_t \quad 0 \leq x \leq L/2 \\
\text{Equation 5 - 5}
\]

Giving:

\[
\delta_{th} = \frac{\alpha \Delta T \cdot L^2}{8h} \\
\text{Equation 5 - 6}
\]

where \( k_t \) is the thermal curvature, \( \alpha \) is the coefficient of thermal expansion of concrete, \( h \) is the height of the cross-section, \( \Delta T \) is the temperature difference of the beam cross-section (figure 5 - 4), \( L \) is the length of beam and \( \delta_{th} \) is the maximum thermal deflection at the beam mid-span.

For the loading condition investigated and assuming that the flexural stiffness of the transformed cross section properties is \( EI \), the maximum mechanical deflection at the beam mid-span can be calculated as:

\[
\delta_m = \frac{PL^3}{192EI} \\
\text{Equation 5 - 7}
\]

It should be pointed out that the above equation only applies if the material is linear elastic and the beam has the same flexural stiffness along the entire span. Furthermore, \( EI \) is calculated at ambient temperature and is assumed to be
temperature independent. Therefore, the above equation does not accurately represent the beam mechanical deflection in fire. However, because the beam mechanical deflection is small compared to the beam thermal deflection during the early stage of fire exposure and compared to the beam catenary deflection during the later stage of fire exposure, the above simplifying assumptions are considered acceptable.

5.1.2 Analytical solution for point A in Figure 5.1

The “A” point marks the condition when the compression force in the axially restrained reinforced concrete beam has reached the maximum value. To determine this point, the compression force of the beam due to axial restraint is calculated as a function of temperature. Figure 5 - 5 shows the differences in the temperatures of the different layers between the free thermal strain and the linearised thermal curvature (Figure 5 - 4). They give the strains that are restrained by the axial restraint, which will induce the compression force in the beam.

The differential thermal strains of the reinforced concrete cross-section in the different layers are:

\[
\begin{align*}
\frac{\Delta L_1}{L} \text{ (thermal strain)} &= \alpha \cdot \Delta T_1 \\
\frac{\Delta L_2}{L} \text{ (thermal strain)} &= \alpha \cdot \Delta T_2 \\
&\vdots \\
\frac{\Delta L_n}{L} \text{ (thermal strain)} &= \alpha \cdot \Delta T_n
\end{align*}
\]

Equation 5 - 8

where \( \alpha \) is the coefficient of thermal expansion of concrete, \( \Delta T_n \) is the increase in temperature at each layer of the beam and \( \Delta L_n \) is the thermal expansion at each layer of the beam.
Using the restrained thermal strain, the concrete compressive stress can be estimated using the concrete stress-strain-temperature relationships, as given in Equation 5 - 9 (Feenstra, 1993). At high temperatures, the mechanical properties of steel reinforcement and concrete change according to EN 1992-1-2:2002, and the values used are in the previous chapter, in section 4.2.

\[
f = \begin{cases} 
-f_c \left( \frac{1}{3} \frac{a_j}{\alpha_c} \right) & \text{if } \alpha_c < a_j \leq 0 \\
-f_c \frac{1}{3} \left( 1 + 4 \left( \frac{a_j - \alpha_c}{\alpha_c - \alpha_u} \right) - 2 \left( \frac{a_j - \alpha_c}{\alpha_c - \alpha_u} \right)^2 \right) & \text{if } \alpha_c < a_j \leq \alpha_c \\
-f_c \left( 1 - \left( \frac{a_j - \alpha_u}{a_u - \alpha_c} \right)^2 \right) & \text{if } a_u < a_j \leq \alpha_u \\
0 & \text{if } a_j < a_u 
\end{cases} 
\]  
Equation 5 - 9

where \( \alpha_{c/3} \) is the strain at one-third of the maximum compressive strength, \( \alpha_c \) is the strain at the maximum compressive strength and \( \alpha_u \) is the ultimate strain, given below:

\[
\alpha_{c/3} = -\frac{1}{3} \frac{f_c}{E_c} 
\]  
Equation 5 - 10
The thermal axial forces on each layer can be calculated as follows:

\[ F_n = \sigma_n \times \text{Area of mesh on each layer} \]  \hspace{1cm} \text{Equation 5 - 13}

where \( F_n \) is the axial force due to thermal expansion on each layer, \( \sigma_n \) is the thermal stress on each layer.

Axial force(thermal) = \( F_1 + F_2 + \cdots + F_n \) \hspace{1cm} \text{Equation 5 - 14}

The axial force in equation 5 - 14 is for beam with complete axial restraint. For flexible axial restraints, the axial force should be estimated using another process.

In the case of flexible axial restraint, the axial forces of each layer are shown in Figure 5 - 6. In the case of a beam with different level of flexible axial restraint, the value of axial force in each layer will depend on the amount of axial restraint.

(a) Free body diagram for axial force
The axial stiffness $K_A$ of each layer of the beam is calculated as:

$$K_{A(1,2...n)} = \alpha_{(1,2...n)} \cdot \frac{E_c A}{L \cdot N}$$  \hspace{1cm} \text{Equation 5 – 15}$$

where $K_A$ is the stiffness of the layer, $L$ is the beam length, $E_c$ is the elastic modulus of concrete at ambient temperature, $A$ is the total beam cross-section area, $\alpha$ is the reduction factor for elastic modulus of the beam layer at elevated temperature, and $N$ is the total number of layers in the beam cross-section.

The spring stiffness of each layer is calculated as:

$$K_{(1,2...n)} = \frac{K}{N}$$  \hspace{1cm} \text{Equation 5 – 16}$$

where $K_{(1,2...n)}$ is the spring stiffness of each layer, $K$ is the total axial restraint stiffness, $N$ is the number of layers.

The effective axial restraint ($K_E$) of each layer is given by:

$$\frac{1}{K_{E(1,2...n)}} = \frac{1}{K_{(1,2...n)}} + \frac{1}{K_{A(1,2...n)}} + \frac{1}{K_{(1,2...n)}}$$  \hspace{1cm} \text{Equation 5 – 17}$$

The detailed calculation process can be found in Appendix 5.2.
The new temperature profile can be calculated by deduction of the linear temperature profile (Figure 5 - 7) from the free thermal expansion.

Therefore, the axial forces on each layer can be expressed as shown in equation 5 - 19

\[
\begin{pmatrix}
    F'_1 \\
    F'_2 \\
    \vdots \\
    F'_n
\end{pmatrix}
= 
\begin{pmatrix}
    K_{E1} \Delta_1' \\
    K_{E2} \Delta_2' \\
    \vdots \\
    K_{En} \Delta_n'
\end{pmatrix}
\]

Equation 5 – 18

The total axial force is given by:

Total axial force = \( F'_1 + F'_2 + \cdots + F'_n \)

Equation 5 – 19

where \( F'_n \) is the axial force for the different level of axial restraint.

5.1.3 Analytical solution for point B in Figure 5.1

At point B in Figure 5 - 1, the axial force in the beam is zero. Therefore, it is assumed that the externally applied load on the beam is resisted by the internal bending moment of the cross-section. In fact, this condition is the limit of the axially...
unrestrained beam. The condition for point B is:

\[ M_{\text{Pmax}} \approx M_n \]  \hspace{1cm} \text{Equation 5 - 20}

where \( M_{\text{Pmax}} \) is the maximum applied bending moment in the beam and \( M_n \) is the bending moment resistance of the reinforced concrete cross-section. To obtain the beam deflection at point B, it is assumed that the beam transits to full catenary action immediately after reaching point B. Therefore, the beam axial force is the same as the reinforcement steel tensile capacity and the bending moment resistance of the beam is zero. Therefore, the vertical deflection of point B is:

\[ \delta = \frac{P L}{4 A_s F_y} \]  \hspace{1cm} \text{Equation 5 - 21}

where \( A_s \) is the area of reinforcement steel, \( F_y \) is the strength of reinforcement steel, \( P \) is the applied load, \( L \) is the length of a beam and \( \delta \) is the vertical deflection.

### 5.1.4 Analytical solution for point C in Figure 5.1

During the catenary action stage, it is assumed that the reinforcement steel has reached its tensile resistance and the concrete section is completely cracked. Therefore, the bending moment resistance of the cross-section is zero. Accordingly, the equilibrium equation is:

\[ PL/4 - (F_T \delta_T) = 0 \]  \hspace{1cm} \text{Equation 5 - 22}

where \( F_T \) is the temperature dependent tensile resistance of the reinforcement steel.

This equation gives increasing beam deflection as the value of \( F_T \) decreases at increasing temperature. Point “C” marks the ultimate failure condition of the axially restrained reinforced concrete beam, due to fracture of the reinforcement after the development of catenary action. If the maximum limit of vertical deflection of the beam is known, point C can be determined.
In this research, the maximum beam vertical deflection is obtained by estimating the reinforcement steel strain distribution at the point of beam failure. Figure 5 - 8 shows the results of reinforcement steel strain distribution for a number of beams, with a different amount of strain limit for the reinforcement, from the numerical simulation parametric study reported in Chapter 4. The strain distribution can be divided into two parts: A-B and B-C.

For part B-C, the strain distribution can be assumed to be linear until the maximum strain reaches the reinforcement rupture strain at the centre of the beam. The strain value for part A-B is relatively small; about 0.015. Part A-B is about three quarters and part B-C about one quarter of the half span of the beam, based on the simulation results.

Integration of the reinforcement strain over the beam length gives the total elongation of the reinforcement, as follows:

\[ \int_0^{L/2} e(x) \, dx = R' \]

Equation 5 - 23

where \( e(x) \) is the reinforcement steel strain and the integration is over half the beam length.
length L and R’ is reinforcement steel elongation.

Once the total elongation of the reinforcement is obtained, the maximum vertical deflection of the beam can be obtained through Pythagoras’ theorem on assumption of the linear deformation shape of half of the beam, as shown in Figure 5 - 8.

![Figure 5 - 9 Linear deflection profile of the beam at failure](image)

Table 5 - 2 compares the calculated maximum deflections using the proposed method with the numerical simulation results for all the parametric study cases of Chapter 4.

**Table 5 - 1 Overall comparison of vertical deflections between simulation and analytical results of fire exposure time for the simulation cases in Chapter 4**

<table>
<thead>
<tr>
<th>Area of steel</th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>Calculation</td>
<td>Simulation</td>
<td>Calculation</td>
</tr>
<tr>
<td>400 mm²</td>
<td>105.46 mm</td>
<td>101.59 mm</td>
<td>245.86 mm</td>
</tr>
<tr>
<td>600 mm²(basic)</td>
<td>99.73 mm</td>
<td>95.37 mm</td>
<td>279.18 mm</td>
</tr>
<tr>
<td>Fracture strain</td>
<td>Simulation</td>
<td>Calculation</td>
<td>Simulation</td>
</tr>
<tr>
<td>10%</td>
<td>99.73mm</td>
<td>95.37mm</td>
<td>279.18 mm</td>
</tr>
<tr>
<td>15% (basic)</td>
<td>99.73mm</td>
<td>95.37mm</td>
<td>279.18 mm</td>
</tr>
<tr>
<td>20%</td>
<td>99.73mm</td>
<td>95.37mm</td>
<td>279.18 mm</td>
</tr>
<tr>
<td>Cover thickness</td>
<td>Simulation</td>
<td>Calculation</td>
<td>Simulation</td>
</tr>
<tr>
<td>50mm(basic)</td>
<td>99.73mm</td>
<td>95.37mm</td>
<td>279.18 mm</td>
</tr>
<tr>
<td>75mm</td>
<td>100.48 mm</td>
<td>98.16 mm</td>
<td>205.07 mm</td>
</tr>
<tr>
<td>Load ratio</td>
<td>Simulation</td>
<td>Calculation</td>
<td>Simulation</td>
</tr>
<tr>
<td>70% (basic)</td>
<td>99.73mm</td>
<td>95.37mm</td>
<td>279.18 mm</td>
</tr>
<tr>
<td>50%</td>
<td>104.32 mm</td>
<td>98.59 mm</td>
<td>321.43 mm</td>
</tr>
<tr>
<td>Span/depth ratio</td>
<td>Simulation</td>
<td>Calculation</td>
<td>Simulation</td>
</tr>
</tbody>
</table>
5.2 Comparison between the results of analytical predictions and simulations

5.2.1 Overall comparison

Table 5 - 2 and Table 5 - 3 show the overall comparison between the analytical calculation results and the simulation results for all the cases in chapter 4; Table 5 - 2 presenting the results in fire exposure time and Table 5 - 3 providing the results for the steel reinforcement temperature. Figure 5 - 10 (a), (b) and (c) show the comparison in Table 5 - 2 in graphical form for the three respective points, with the horizontal axis showing the simulation time and the vertical axis showing the analytical time. Figure 5 – 11 (a), (b) and (c) show the comparison in Table 5 - 3. It is seen that the analytical results are in reasonably good agreement with the simulation results for all the three points (points A, B and C).
(a) Comparison of the time of Point A for all cases

(b) Comparison of the time of Point B for all cases
(C) Comparison of the time of Point C for all cases

**Figure 5 - 10 Comparison of three points for all cases**

where N is the simulation result, N’ is the calculation result

A: 400mm$^2$ area of steel, B: basic beam (Table 5 -2), B1: 10% fracture strain B2: 15% fracture strain, C: 75mm cover thickness, D: 50% load ratio, E: 4M length, E1: 8M length, F: 100% axial restraint, F1: 50% axial restraint, F2: 10% axial restraint, F3: 5% axial restraint and F4: 1% axial restraint
(a) Comparison of the time of Point A for all cases

(b) Comparison of the time of Point B for all cases
Figure 5 - 11 Comparison of the times of three points for all cases

Table 5 - 2 Overall comparison between simulation and analytical results of fire exposure time for the simulation cases in Chapter 4

<table>
<thead>
<tr>
<th>Area of steel</th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Calculation</td>
<td>Simulation</td>
</tr>
<tr>
<td>400 mm²</td>
<td>53 min</td>
<td>56 min</td>
<td>73.5 min</td>
</tr>
<tr>
<td>600 mm² (basic)</td>
<td>84 min</td>
<td>86 min</td>
<td>109 min</td>
</tr>
<tr>
<td>Fracture strain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>84 min</td>
<td>86 min</td>
<td>109 min</td>
</tr>
<tr>
<td>15% (basic)</td>
<td>84 min</td>
<td>86 min</td>
<td>109 min</td>
</tr>
<tr>
<td>20%</td>
<td>84 min</td>
<td>86 min</td>
<td>109 min</td>
</tr>
<tr>
<td>Cover thickness</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50mm (basic)</td>
<td>84 min</td>
<td>86 min</td>
<td>109 min</td>
</tr>
<tr>
<td>75mm</td>
<td>71 min</td>
<td>76 min</td>
<td>82.5 min</td>
</tr>
<tr>
<td>Load ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70% (basic)</td>
<td>84 min</td>
<td>86 min</td>
<td>109 min</td>
</tr>
<tr>
<td>50%</td>
<td>108 min</td>
<td>112 min</td>
<td>146 min</td>
</tr>
<tr>
<td>Span/depth ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.33 (4M)</td>
<td>117 min</td>
<td>118 min</td>
<td>118 min</td>
</tr>
<tr>
<td>20 (6M) (basic)</td>
<td>84 min</td>
<td>86 min</td>
<td>109 min</td>
</tr>
<tr>
<td>26.66 (8M)</td>
<td>48 min</td>
<td>54 min</td>
<td>109 min</td>
</tr>
</tbody>
</table>

(C) Comparison of the time of Point C for all cases
5.2.2 Comparison of axial force – time relationships

Figure 5 - 12 compares the estimated and simulated beam axial force –time relationships for all the simulation cases. Also plotted in the figure is the tensile...
capacity of the steel reinforcement. The analytical axial force – time curve is constructed as follows:

1. Estimate the axial force -temperature relationship until Point A;
2. Estimate point B and connect point A with Point B
3. Assume the axial tensile force instantly increases to the tensile capacity of the reinforcement
4. Follow the axial resistance curve until the beam deflection reaches the limit at Point C

Appendix 5 - 2 provides an example of how the curve is constructed.

This comparison shows that the analytical method gives reasonable estimation of the beam axial force – time relationship. The agreement is very good during the compression stage. When the restrained beam enters the catenary action stage, the calculation method assumes that the reinforcement steel immediately reaches its tensile capacity. This assumption clearly overestimates the tensile force in the beam. Nevertheless, as catenary action develops further, the tensile force in the beam approaches that of the tensile capacity of the steel reinforcement. Nevertheless, this is not considered a problem for practical assessment of the robustness of axially restrained reinforced concrete beams in fire. This is because the tensile force development in the steel reinforcement before fracture is almost flat and the final tensile force in the beam gives a very close estimate of the maximum axial force in the beam. It is the maximum axial force in the beam that would influence assessment of the surrounding structure to ensure it does not fail before the beam fully develops catenary action.
(a) Comparison of simulation and analytical beam axial force (changing the area of steel)

(b) Effects of changing the fracture strain of steel reinforcement
(c) Effects of changing the concrete cover to steel reinforcement

(d) Effects of changing the beam load ratio
(e) Effects of changing the beam span/depth ratio

(f) Effects of changing the level of axial restraint (from full to 10%)
(g) Effects of changing the level of axial restraint (from 5% to 0.1%)

Figure 5 - 12 Comparisons between simulation and analytical results for beam axial force – time relationships
5.3 Summary

This chapter has presented the development and validation of a simplified method for predicting the behaviour of axially restrained reinforced concrete beams in fire. The main focus of the derivations is on the three key events of the behaviour: (a) reaching the maximum compression force in the beam, (b) reaching the limit of flexural bending resistance, and (c) beam failure due to reinforcement steel fracture. For the critical event of beam failure, the maximum beam deflection was estimated by an assumed strain distribution in the reinforcement steel. Comparisons between the analytical solutions and the numerical simulation results of Chapter 4 have demonstrated that the analytical method provides reasonably accurate predictions of the following key quantities: the maximum compression force, the time to reach the beam flexural bending moment capacity, the final failure time and the final tensile force in the beam. Because these quantities would be required for assessing the robustness of axially restrained reinforced concrete beams in fire, the proposed analytical method may be used as the basis of a design method for this purpose.
Chapter 6 Frame behaviour of reinforced concrete buildings in fire

6.1 Introduction

Chapter 5 has investigated the behaviour of axially restrained reinforced concrete (RC) beams with different levels of axial and rotational restraints at the beams’ ends. It has been found that, axially restrained RC beams can develop catenary action to achieve beam survival temperatures much higher than the beam limiting temperatures based on bending resistance. In realistic composite structures, the RC beam is part of a whole structure and interacts with the surrounding structure. Therefore, for the entire structure to achieve structural robustness, it is important that the surrounding structure is able to retain structural integrity when catenary action develops in the composite beam. This is the aim of this chapter.

When assessing the robustness of composite structures in fire, it is important to define the conditions under which the assessment is made. In investigations on structural robustness at ambient temperature, member removal is often adopted as the scenario. If this was used as the scenario for assessing structural robustness in fire, it would mean that prior to fire attack, the member had been 100% damaged by another event. Wang (2014) argued that this may not be appropriate to the fire situation as this would mean simultaneous occurrence of two extreme events (one removing a structural member, and then followed by fire). Nevertheless, this scenario will be considered in this chapter to observe the consequences and the feasibility of ensuring structural robustness under simultaneous action of member removal and fire exposure.

A more realistic situation would be that members of a structure suffer additional damages than that has been allowed for in fire resistance design. The additional damage can come from a variety of sources, for example, uncertainty in how spalling of concrete is treated. It would be very difficult to put a precise value on the extent of partial damage prior to fire attack. Wang (2014) argued that assuming 20% damage
would be suitable, as recommended by the design guidance on assessing the robustness of structures published by the Institution of Structural Engineers in the UK. According to this scenario, this chapter will investigate the responses of a number of representative composite frame structures with different levels of damage to different members of the structure prior to fire exposure, ranging from 0 (reference case) to 100% (member removal).

As has been mentioned in Chapters 3 and 4, 2-D modelling is adopted in this research for computational efficiency. Although the simulated structure is 3-D, the 2-D assumption is considered acceptable because the critical aspects of the structural behaviour are in the longitudinal direction (force change) and vertical direction (temperature and stress changes), whilst in the width direction, the structural behaviour is almost uniform (same stress and temperature).

Having defined the damage scenario, the next question is to decide the criteria that can be used to judge whether or not the structure has sufficient robustness. In this research, it is stated as follows: the RC frame possesses sufficient structural robustness if in the damaged state, it can survive the time of the undamaged RC frame based on bending resistance (which is used in current fire resistance design).

Therefore, the main objectives of this chapter are:

- To assess the potential of progressive collapse of RC frame structures after undergoing different degrees of damage to the structures;
- To assess the demands of retaining RC structural integrity in fire.
6.2 Basic data of the simulation frame

The simulation frame models are based on the test frame (Yi et al., 2008) which was designed in accordance with the concrete code of China (Concrete code of China, 2002), which is similar to the American Concrete Institute code ACI 318-02 (ACI Committee 318, 2002). Figure 6 - 1 Basic information of the simulation frame shows the layout of the frame and member designations. It should be pointed out that realistic frames would have much bigger dimensions. This frame has been selected for simulation in this chapter to reduce the computation time. Figure 6 - 1(b) shows the reinforcement arrangement for different members of the frame. Table 6-1 gives the mechanical properties of steel and concrete. The elevated temperature properties of concrete and reinforcement steel change according to the models in Eurocode 2 (EN 1992-1-2:2002).

(a) Basic layout of the simulation frame
(b) Reinforcement arrangement

Figure 6 - 1 Basic information of the simulation frame

Table 6 - 1 Mechanical properties of reinforcement steel and concrete

<table>
<thead>
<tr>
<th>Material</th>
<th>Measured values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal reinforcement</td>
<td></td>
</tr>
<tr>
<td>Yield strength</td>
<td>416 (MPa)</td>
</tr>
<tr>
<td>Ultimate tensile strength</td>
<td>526 (MPa)</td>
</tr>
<tr>
<td>Ratio elongation</td>
<td>20%</td>
</tr>
<tr>
<td>Lateral reinforcement</td>
<td></td>
</tr>
<tr>
<td>Yield strength</td>
<td>370 (MPa)</td>
</tr>
<tr>
<td>Concrete (C30)</td>
<td></td>
</tr>
<tr>
<td>Cube compression strength</td>
<td>25 (MPa)</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>2.2 (MPa)*</td>
</tr>
<tr>
<td>Fracture Energy</td>
<td>0.079 (Nmm/mm²)*</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>32009 (Mpa)*</td>
</tr>
</tbody>
</table>

*Estimated according to the CEB-FIP model (CEB-FIP 2010).
6.3 Simulation cases

To investigate the behaviour of reinforced concrete frame structures with different extents of damage prior to fire exposure, the following seven scenarios have been investigated:

1. No damage: this is the reference model and the main output of this model is the reference fire resistance time. The reference fire resistance time is based on flexural bending behaviour, as indicated by point B of the beam axial force – time relationship shown in Figure 6 - 2. This structure is shown in Figure 6 - 4 (a).

2. Central column removed: this case investigates whether the damaged structure can achieve sufficient robustness in fire. The criterion for judging this is to compare the failure time of the damaged structure with the reference fire resistance time obtained in case 1. This structure is shown in Figure 6 - 4 (b).

3. Left column removed: the scope of this investigation is the same as in Case 2 except that a different column has been removed. This structure is shown in Figure 6 - 4 (c).

4. Partially damaged central column: a 30mm layer is removed from the column to simulate partial damage, which may be caused by spalling of the concrete. The scope of this study is the same as case 2 except that the column is partially damaged. This structure is shown in Figure 6 - 4 (d).

5. Partially damaged beam: as with case 4, except that the beam is partially damaged by removing a 30mm section from the beam. The structure is shown in Figure 6 - 4 (e).

6. Partially damaged left column: as with the previous cases of partial damage, shown in Figure 6 - 4 (f).

7. Partially damaged column and beam: as with the previous cases of partial damage, but both the central column and the beam are partially damaged, shown in Figure 6 - 4 (g).
In all cases, the structural temperatures of the fire exposed members were obtained from TNO DIANA heat transfer results under the ASTM E119 (ASTM, 2008) fire temperature – time relationship. Figure 6 - 3 compares the reinforcement steel temperatures in the beams with and without damage.

**Figure 6 - 2 Beam axial force – time relationship (Case -1)**
Figure 6 - 3 Temperatures of bottom reinforcement steel of damaged (Case 5 and Case 7) and undamaged beams (other cases)
Left column (C1) removed

(c) Case3

(d) Case4

(e) Case5

(f) Case6
Figure 6 - 5 shows the general arrangement of the reinforced concrete frame and the applied loads. Four concentrated loads are applied at the centres of the beams and three concentrated loads are applied at the top of the columns. These applied loads were based on achieving 40% of the bending moment resistance of a simple supported beam. In the numerical model, concrete was modelled by 4-node quadratic plane stress elements (Q8MEM) and embedded elements were used for the reinforcement steel, as used in chapter 3. The mesh size was 40 x 40 mm based on the sensitivity study results in section 3.3. At high temperatures, the mechanical properties of steel reinforcement and concrete change according to EN 1992-1-2:2002.
Figure 6 - 5 Loading conditions on the reinforced concrete frame

Case 1 is used to illustrate the typical behaviour of RC frames in fire. When the temperatures are increased, the columns expand vertically and the beams expand horizontally (Figure 6 - 7 (b)). Since these expansions are restrained by the surrounding structures, additional compressive forces are generated in these members, as shown in Figure 6 - 6 (a). However, in general, the restraint to column expansion is relatively small, therefore, the additional compression force in the columns is small. This increase in compression force in the centre column is accompanied by a reduction in compression force in the edge columns (Figure 6 - 6 (b)). At 114 minutes, the centre column loses its load carrying capacity and its axial compression force is rapidly reduced and the applied load on the structure is shed to the adjacent edge columns (Figure 6 - 6(b)). Before this time, the compression forces in the beams generate increasing bending moments in the edge columns as shown in Figure 6 - 6 (c). After failure of the centre column at 114 minutes, the structure is still able to resist the applied loads because the beams develop catenary action. The bending moments in the edge columns rapidly decrease after release of the compression force in the beams. During the catenary action stage, the beam tensile force increases slightly and this is accompanied by a slight increase in the bending moment of the edge columns. The bending moments in the columns are significant and they should be included in design of the edge columns to ensure that they can continue to support the beams during the catenary action stage.
The results of this model suggest that catenary action in reinforced concrete beam is a viable load carrying mechanism to ensure structural integrity, as demonstrated by Yi et al (2008), provided the surrounding columns are able to resist the additional force shed by the failed column, either due to accidental damage (column removal in Yi et al 2008), or fire as in this research.

(a) Comparison of axial forces in the beam and lower column at centre (tension positive)
Figure 6 - 6 Comparison of various structural behaviour for the reference case frame

(b) Axial forces in a lower edge column

(c) Bending moment ratio of a lower edge column
(a) Time 0 min

(b) Time 20 min

(c) Time 114 min
(d) Time 207 min (failed)

(e) Reinforcement steel stress-time relationships
Figure 6 - 7 Frame deformation patterns and reinforcement steel strains at different times for Case 1 (reference frame)

For the beams, the rotational restraint afforded by the surrounding structure increases the hogging bending moment initially, as shown in Figure 6 - 8 which plots the variations of hogging and sagging bending moments in the beam as a function of the fire exposure time. On the other hand, the sagging moment decreases. Indeed, for a considerable period of time, the entire beam is under a hogging moment. Also, additional bending moments are generated in the columns due to the P-δ effect, as shown by the column bending moment – time relationships in Figure 6 - 9.

On further increases in the structural temperature as the fire exposure time increases, the additional compressive force in the beam decreases due to the reduction in mechanical properties of concrete at high temperatures. The sum of the hogging and sagging bending moments varies in the same trend as the compressive force in the beam, because this variation represents the additional internal bending resistance that the beam has to provide in order to be in equilibrium with the P-δ effect in the beam. When the compressive force in the beam decreases to zero, the applied load on the
beam is resisted purely by bending. The time this is reached is called the reference fire resistance time of the structure because this is the basis of the current fire resistance design method.

Depending on whether the supporting column has failed, the beams may continue to sustain the fire exposure by developing catenary action where the axial force in the beam becomes tensile. Because catenary action is a stabilising mechanism, the internal bending moments in the beam decrease. Pure catenary action may develop if the surrounding structure can continue to supply the axial restraint to the beam and the beam and the connections have sufficient deformation capacity. For the reference frame, the final failure time is 207 minutes. This is much higher than the reference fire resistance time of 115 minutes. For the reference frame case, the final failure mode is due to column global failure, as shown in Figure 6 - 7d. For the reference frame, this happens when the beams have developed a substantial amount of catenary action. However, if the columns cannot provide the necessary support and axial restraint to the beam, failure of the frame can happen much earlier, as will be demonstrated in the next section.
Figure 6 - 8 Bending moment diagram – time relationships of a reference beam

Figure 6 - 9 Bending moment diagram – time relationship of column in the reference frame
6.5 Robustness of RC frames with different damage

Figure 6 - 12 compares the beam behaviour of the different cases, using the beam axial force – time relationships (Figure 6 - 12 (a)) and beam deflection – time relationships. The reference fire resistance time of the structure is 115 minutes, when the beam axial force in the reference frame (case 1) is zero. Table 6 - 2 summarises the simulation results, including the failure mode, the bending resistance limit (defined as the time when the beam axial force is zero) and the final collapse time, for all the case studies. Also included in Table 6 - 2 are the survival times of the simple supported column with and without partial damage, under the applied load in the reference frame (=44 kN).

Figure 6 - 13 shows details of the failure modes of the different simulation cases. To help understand the failure modes of the structures, Figure 6 - 10 compares the beam reinforcement steel maximum strain – time relationships.

If any of the columns is removed (Case 2 for central column removal and Case 3 for left column removal), the damaged structure cannot survive the fire exposure to the reference fire resistance time of 115 minutes. Even though the beams have sufficient robustness at ambient temperature, as demonstrated by their ability to support the applied load before fire exposure, their survival times under fire exposure are low (75 minutes for central column removal (Case 2) and 24 minutes for left column removal (Case 3). The structures failed because the beams became double span and the internal forces in the beams were greatly increased. For the central column removal (Case 2), there is a small amount of catenary action, but the reinforcement fractured (see Figure 6 - 10, judged by the strain reaching the fracture strain of 20%) before the structure could reach the reference fire resistance time of 115 minutes. For the left column removal (Case 3), a lack of axial restraint to the beam prevented any development of catenary action. In fact, because of the higher loads in the left column, the bending moments in the cantilever beam are very high, leading to very early failure of the beam, which was caused by fracture of the reinforcement steel at the top of the connection to the central column (Figure 6 - 13 (case 3)).
<table>
<thead>
<tr>
<th>Case</th>
<th>Structural condition</th>
<th>Failure mode</th>
<th>Bending limit*</th>
<th>Survival time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simple supported column</td>
<td></td>
<td></td>
<td>117 min</td>
</tr>
<tr>
<td>2</td>
<td>Simple supported damaged column</td>
<td></td>
<td></td>
<td>79 min</td>
</tr>
<tr>
<td>3</td>
<td>Undamaged reference model</td>
<td>Central column</td>
<td>114 min</td>
<td>207 min</td>
</tr>
<tr>
<td>4</td>
<td>Central column removed</td>
<td>Central column</td>
<td>78 min</td>
<td>156 min</td>
</tr>
<tr>
<td>5</td>
<td>Left column removed</td>
<td>Beam failed</td>
<td>77 min</td>
<td>100 min</td>
</tr>
<tr>
<td>6</td>
<td>Damaged central column (30mm)</td>
<td>Central column</td>
<td>113 min</td>
<td>138 min</td>
</tr>
<tr>
<td>7</td>
<td>Damaged beam and column (30mm)</td>
<td>Central column</td>
<td>77 min</td>
<td>93 min</td>
</tr>
</tbody>
</table>

* defined as the time when the beam axial force is zero.
Figure 6 - 10 Comparison of the maximum stress and strain – time relationships for reinforcement steel
If the structural members (beams and columns) are partially damaged, it is possible for the damaged frame to achieve the reference fire resistance time of 115 minutes. This is the case with partially damaged columns only (156 minutes in Case 4 and 138 minutes in Case 6). As explained earlier, the simply supported column gives a very conservative (low) estimate of the column fire resistance, due to not considering the beneficial effects of the interaction between the columns and the surrounding structures (continuity). If these beneficial effects are included, as in the case of simulating the frame structure, the damaged columns could still provide sufficient support to the beams to develop catenary action to ensure that the frames survive the fire exposure longer than the reference fire resistance time of 115 minutes. Figure 6-14 compares the axial forces of the central columns for all cases except Case 2. Except for the reference frames (Case 1 and Case 4), failure of the frame occurred quickly after column failure, as indicated by the column force decreasing to nearly zero.

When the beam is partially damaged (Case 5 and Case 7), the depth of the beam is reduced and the temperature of the reinforcement steel is increased at the same time. This reduced the bending resistance of the beam, hence these beams reached their bending limits much earlier than the undamaged beams. More critically, the reduced tensile strengths of the reinforcement steel (due to increased temperatures) cause the beams to undergo very large deflections long before the reference fire resistance time of 115 minutes. Even though the damaged beams developed catenary action (Figure 6-12a), the catenary action duration is short due to the rapid increase in reinforcement steel temperature of the damaged beams (Figure 6-3).

(a) Bending limit (case7, time = 77min) (b) Bending limit (case5, time = 77min)
(c) Strain – time relationships of reinforcement steel in beams (Case 5 and Case 7)

Figure 6-11 Comparison for beam centre bottom reinforcement steel strain – time relationships for different cases of damaged beam

(a) Beam axial force – time relationships for different case studies
(b) Beam vertical deflection-time relationships for different cases

Figure 6 - 12 Comparison of beam behaviour between different cases

(a) Time 0 min  (b) Bending limit - time 114 min  (c) Failed - Time 207 min

(Case 1)
(a) Time 0 min  (b) Bending limit - time 67 min  (c) Failed - Time 75 min

(Case 2)

(a) Time 0 min  (b) Failed - Time 24 min

(Case 3)

(a) Time 0 min  (b) Bending limit - time 78 min  (c) Failed - Time 75 min

(Case 4)
Figure 6 - 13 Comparison of failure regions for all cases
6.5.1 Methods of improving robustness of damaged RC frame structures

Based on the results in the previous section, it can be concluded that when the left column (simulating a corner column) is removed, it would not be possible to ensure robustness of the structure because the survival time of the damaged structure (24 minutes) falls far short of the reference fire resistance time (115 minutes) of the structure. However, the survival times of other damaged structures (Case 2 – 75 minutes, Case 5 – 100 minutes, Case 7 – 93 minutes) are sufficiently close to the reference fire resistance time that it is worth investigating possible methods of raising the survival times to the reference fire resistance time.
In the parametric studies of Chapter 4, increasing the reinforcement steel ductility in the beam was considered a feasible method to improve the beam survival time. In addition, as in the case of a damaged beam (Case 5), the high temperature of the reinforcement steel can be a critical problem preventing the frame from reaching the reference fire resistance time. Therefore, increasing the cover thickness of reinforcement steel can also be an effective method to enhance the frame survival time.

This section will investigate whether this is still feasible, in the context of RC frames with damaged members. Table 6 - 3 lists the changes (underlined for highlighted) made to the reinforcement and

Table 6 - 4 summarises the results of additional simulations and compares them to the original simulation results.

### Table 6 - 3 Changes to the reinforcement steel

<table>
<thead>
<tr>
<th>Original longitudinal reinforcement</th>
<th>Changed longitudinal reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column, mm</td>
<td>Changed longitudinal reinforcement</td>
</tr>
<tr>
<td>Beam, mm</td>
<td></td>
</tr>
<tr>
<td>4D12</td>
<td>4D12</td>
</tr>
<tr>
<td>Ratio elongation</td>
<td>Ratio elongation</td>
</tr>
</tbody>
</table>

### Table 6 - 4 Comparison of new simulation results with the original results for bending limit and survival times

<table>
<thead>
<tr>
<th>Simulation case</th>
<th>Bending limit*</th>
<th>Survival time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central column removal, original case</td>
<td>67 min</td>
<td>75 min</td>
</tr>
<tr>
<td>Reinforcement steel fracture strain increased from 20% to 25%</td>
<td>67 min</td>
<td>159 min</td>
</tr>
<tr>
<td>Beam reinforcement steel area increased from 100.53mm² to 226.19mm² in beams</td>
<td>71 min</td>
<td>192 min</td>
</tr>
<tr>
<td>Case</td>
<td>Description</td>
<td>Time 1</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>Case 5</td>
<td>Damaged beam (30mm), original case</td>
<td>77 min</td>
</tr>
<tr>
<td>Case 5-F</td>
<td>Reinforcement steel fracture strain increased from 20% to 25%</td>
<td>77 min</td>
</tr>
<tr>
<td>Case 5-A</td>
<td>Beam reinforcement steel area increased from 100.53mm$^2$ to 226.19mm$^2$ in beams</td>
<td>85 min</td>
</tr>
<tr>
<td>Case 5-C10A</td>
<td>Cover thickness of reinforcement steel increased from 40mm to 50mm and beam reinforcement steel area increased from 100.53mm$^2$ to 226.19mm$^2$ in beams</td>
<td>101 min</td>
</tr>
<tr>
<td>Case 5-C20A</td>
<td>Cover thickness of reinforcement steel increased from 40mm to 60mm and beam reinforcement steel area increased from 100.53mm$^2$ to 226.19mm$^2$ in beams</td>
<td>100 min</td>
</tr>
</tbody>
</table>

* based on the beam axial force = 0.

In the case of central column removal (Case 2), because the other columns are able to provide the axial restraint to the beams to allow them to develop catenary action, and the beams are undamaged (so that the steel temperature increase is slow as in the reference case), either increasing the reinforcement steel fracture strain from 20% to 25% or increasing the steel reinforcement area is effective in increasing the survival time of the structure. These increased survival times (159 minutes and 192 minutes) are now sufficient to achieve robustness of the structure, based on the criterion that these times are higher than the reference fire resistance time of 115 minutes of the undamaged structure.

However, when the beams are partially damaged (Case 5 and Case 7), the temperature increase in the beam reinforcement steel is too fast, as shown in Figure 6 - 3. Therefore, even though the modified structures with increased reinforcement steel area or ductility can survive high temperatures in the reinforcement steel, the times taken to reach these high temperatures are still much shorter than the reference
fire resistance time. For example, at the fire exposure time of 103 minutes, the steel
reinforcement temperature for the partially damaged beam is 755.53 °C. In
comparison, this temperature is reached at about 190 minutes in the undamaged
beam. This time is close to the failure time of the reference structure of 207 minutes
(Table 6 - 2). This study suggests that it is vital to slow down the temperature
increase of the beam in order to ensure robustness of the structure in fire. For
example, if the concrete cover to reinforcement is increased from the original 40mm
to 50mm or 60mm, the reinforcement steel temperatures increase much more slowly
(Figure 6 - 15). As a result, the survival times of the frames are increased
significantly (see Table 6-4), and these values (123 minutes and 136 minutes) are
now higher than the reference fire resistance time of 115 minutes, meaning that the
damaged frames have sufficient robustness. It should be pointed out that if this was
adapted in practice, it would lead to increased depth of cross-section of the beam
because keeping the depth constant while increasing the reinforcement cover would
mean moving the reinforcement inwards in to the cross-section, resulting in reduced
bending resistance of the beam. Nevertheless, the purpose of this exercise is to
investigate the effect of increasing reinforcement cover. Whether or not this would
be adopted in practice would depend on comparison of the need to increase the
beam’s survival time in fire against the need to minimise the depth of the cross-
section.
Figure 6 - 15 Temperatures of reinforcement steel for different cover thicknesses

(a) Axial force – time relationships for different case studies
(b) Vertical deflection – time relationships for different case studies

Figure 6 - 16 Axial force and vertical deflection-time relationships (Case 2)

(a) Axial force – time relationships for different case studies
(b) Vertical deflection – time relationships for different case studies

Figure 6 - 17 Axial force and vertical deflection-time relationships (Case 5)
6.6 Summary and Conclusions

This chapter has presented the results of a number of case studies to investigate the load carrying mechanisms and survival times of a reinforced concrete frame with different degrees of damage prior to fire exposure and methods of improving their survival times. To judge whether a damaged structure has sufficient robustness, its survival time is compared to the reference fire resistance, defined as the bending failure time of the undamaged structure, which is quantified when the beam axial force is zero. Although the number of case studies is small, it is possible to draw the following general conclusions:

1. If a member is completely removed from the structure, the structure is unlikely to be able to develop an alternative load carrying mechanism to ensure robustness of the structure. The problem is particularly severe when a corner column is removed.

2. It is possible for frames with partially damaged columns to achieve the required robustness in fire, provided the columns still have sufficient resistance to allow the beams to develop some catenary action. This may be possible if the columns are designed as simply supported columns, but have some reserves in strength in the frame due to continuity.

3. If the reinforcement steel in the damaged beams rises in temperature very quickly (for example, due to spalling of the concrete), it would be difficult for the damaged structure to achieve the required robustness. Even though the beams can develop substantial catenary action and the reinforcement steel temperatures are very high at failure, the time taken to reach these temperatures may still be much shorter than the reference fire resistance time. Merely increasing the reinforcement steel area or ductility (which is difficult to do) would not be sufficient. However, increasing the cover thickness of the reinforcement steel to slow down the temperature increase is necessary.

4. In the case of central column removal (but without other additional damage to the structure), it may still be possible to improve the performance of the structure to survive the reference fire resistance time (hence ensuring robustness of the structure) by increasing the reinforcement steel area of the
beams (which is possible) or the ductility (which is difficult).
Chapter 7 Conclusions and recommendations for further study

The aim of this research is to develop an understanding and to improve the robustness of reinforced concrete (RC) framed structures in fire. The research was carried out using the commercial finite element package DIANA. Detailed parametric studies were performed to investigate the elevated temperature behaviour of RC beams with different axial and rotational end restraints, to develop an analytical method to predict the key quantities of restrained RC beams, and to investigate the survivability and robustness of RC framed structures with different extents of damage prior to fire exposure. This chapter presents a summary of the main conclusions and recommendations for future study.

7.1 Finite element modelling methodology

Research on the survivability and robustness of RC framed structures in fire involves very large deformations and material nonlinearity and failure. This presents enormous challenges to numerical modelling. Based on the experiences of the author, the following guidance is suggested:

1. The smeared crack modelling for concrete material can be used. The applied compressive Mohr-Coulomb plasticity and the Hordijk tensile softening models can be applied to model concrete.

2. For 2D concrete modelling, the plain stress quadratic elements (Q8MEM) are suitable. In the case of reinforcement steel, the embedded reinforcement method was chosen and was demonstrated to give good results in global structural analysis.

3. Using the mesh size of 40mm is reasonable.

4. To account for geometric nonlinearity due to large deflections, the failure criteria was used by strain based failure of reinforcement steel as one of the most important failure criteria is related to the rupture of reinforcement steel...
during catenary action.

7.2 Behaviour of axially and rotationally restrained RC beams

To investigate the effects of different design parameters on the behaviour and survival time of axially restrained reinforced concrete beams, a parametric study was carried out. It was assumed that the reinforcement steel was fully bonded with the surrounding concrete. The behaviour of an axially restrained beam goes through three stages: initial compression due to restrained thermal expansion, followed by almost pure bending when the axial force in the beam is very small, then transits to catenary action when the concrete is cracked and the tensile force in the reinforcement steel resists the applied catenary tensile action, until fracture of the tensile reinforcement steel. The main effects of the different design parameters are:

1. Area of steel reinforcement: When the area of steel reinforcement increases, the survival time shows an obvious increase. However, at high temperatures, the effects of increasing the reinforcement steel area become moderate.

2. Fracture strain of reinforcement: increasing the fracture strain of reinforcement steel can greatly benefit the restrained RC beam behaviour and may be an attractive way to enhance the beam survival time.

3. Position of reinforcement: increasing the reinforcement steel cover thickness will increase the survival time due to reduced temperature.

4. Span/depth ratio: Increasing this value gives increased survival time of the RC beam under the same load ratio. This is because as the beam becomes longer, it can deform more at the same strain level; and under catenary action, resistance to the applied load is directly proportional to the vertical deflection of the beam.

5. Load ratio: as the load ratio increases, the bending failure time and survival time are reduced.

6. Axial restraint stiffness: after reaching a certain level, the axial restraint stiffness has minor effects on the beam limiting and survival temperatures.
This level of axial restraint can be achieved in most realistic RC structures.

(7) Rotational restraint stiffness: When the level of rotational restraint increases, the bending limiting temperature of the RC beam is increased, but the beam survival time does not change.

7.3 Development of an analytical model for axially restrained RC beams in fire

The behaviour of an axially restrained beam is controlled by three key events: (i) when the axial compression force in the beam reaches the maximum; (ii) when the bending limit of the beam is reached; and (iii) when the beam finally fails due to fracture of the reinforcement steel. An analytical method has been derived to quantify these three key events. A summary of the analytical method is provided below.

To deal with non-uniform temperature in the cross-section of the RC beam, the cross-section is divided into a number of layers. Based on the assumption of the plane section remaining plane after deformation, a linear temperature distribution is obtained from the consideration of zero net axial force in the beam in the unrestrained condition. This linear temperature distribution is used to calculate the thermal deflection of the beam. The thermal strain-induced axial forces of the layers of the cross-section are calculated by converting the restrained thermal strains (the difference between the free thermal strain and the thermal strain form the linear temperature distribution) into thermal stresses and integrating these thermal stresses. In addition, a compression force is also generated in the cross-section as part of the tension-compression couple of the bending moment of the cross-section. The maximum compressive force of the beam is reached when the bending moment from the compressive force in the beam (P-δ effect) reaches the bending resistance of the beam.

The bending limit is obtained when the bending resistance of the RC beam in fire is in equilibrium with the applied load and when the axial force in the beam is zero.
To estimate the time to failure, which is initiated by the fracture of reinforcement steel at the catenary action stage, a regression equation is proposed to calculate the maximum deflections of RC beams, based on an analysis of the reinforcement steel strain distributions at failure for a large number of parametric study results.

A comparison between the analytical and simulation results indicates that the analytical method gives reasonably good approximations to the numerical simulation results.

7.4 How to evaluate the robustness of a frame structure

In this thesis, the evaluation of the robustness of a reinforced concrete frame is based on comparing the survival times of damaged RC frames with the standard fire resistance time of the frame. The standard fire resistance time of the frame is defined as the bending failure time of the undamaged frame. The damaged structure is considered to possess sufficient robustness if it is able to survive the reference fire resistance time. The accidental damages include column removal (Case 2 for centre column removal and Case 3 for edge column removal) and damaged beams and columns to simulate possible additional damages, such as unaccounted for spalling. Additional simulations were carried out for the damaged frames to investigate possible methods to prolong their survival times to achieve the reference fire resistance time.

Based on the frame simulation results, it has been found that if a member is completely removed from the structure, the structure is unlikely to be able to develop an alternative load carrying mechanism to ensure robustness of the structure. This problem is particularly severe when a corner column is removed. However, it is possible for frames with partially damaged columns to achieve the required robustness in fire, provided the columns still have sufficient resistance to allow the beams to develop some catenary action. This may be possible if the columns are designed as simply supported columns, but have some reserves of strength in the
frame because of continuity. On the other hand, if the beams are damaged and the reinforcement steel temperature in the damaged beams rises very quickly (for example, due to spalling of the concrete), it would be difficult for the damaged structure to achieve the required robustness. Even though the beams can develop substantial catenary action and the reinforcement steel temperatures are very high at failure, the time to reach these temperatures may still be much shorter than the reference fire resistance time. Merely increasing the reinforcement steel area or ductility (which is difficult to do) would not be sufficient. However, increasing the cover thickness of the reinforcement steel to slow down the temperature increase is necessary.

Whilst the above conclusions were obtained based on the limited scope of frame geometry and loading condition of this study, the observed behaviour was sufficiently generic to identify the effectiveness of changing different design parameters to achieve robustness of RC frames under partial fire damage. Nevertheless, further more detailed research, for more extensive frame geometries, loading conditions, initial damage conditions, should be conducted to enable detailed quantification rules to be developed to help design and construction to achieve enhanced robustness of RC frames in fire.

Further studies should also be conducted to incorporate the effects of slabs.

### 7.5 Recommendations for future study

As mentioned in the introductory paragraph, there are many difficult challenges in modelling the large deflection behaviour of RC structures in fire until failure. Therefore, many necessary assumptions have been made in this research to achieve some degree of progress in this topic. Further research studies are necessary to remove these assumptions and to achieve a complete understanding of RC structural behaviour in fire. Possible future research topics on numerical modelling of RC structures in fire include the following:
- Slabs should be included. This would necessitate modelling the RC structure in 3 dimensions.
- When the RC structure searches for an alternative load path (e.g. catenary action) after exhausting the load carrying capacity of the first failure mode (e.g. bending) in fire, there is a temporary loss of stability of the structure and dynamic response is involved. The dynamic effects should be included in future research.
- An efficient modelling methodology should be developed to enable large scale RC structures to be modelled reliably (without the numerical convergence problem) and quickly (simulation time in the order of minutes or hours, not days or weeks).
- Realistic and representative fire tests on RC structures with partially damaged structural members should be conducted to provide data to validate the numerical simulation models.
- In this thesis, the location of the axial restraint is at the centre of the beam cross-section. However, in real structures, the location of the axial restraint may be decided by the type of connection and the location of the reinforcement steel. A more general method of analysis should be developed. Furthermore, the effects of rotational restraint should be included in the analytical model.
- For the assessment of robustness of structures at ambient temperature, member removal is specified. However, this would not be appropriate for RC structures in fire. As an alternative to specifying member damage prior to fire attack, as adopted in this research, one possibility is to test the structure under rare fire conditions that are not used in normal fire resistance design.
- Methods of improving the survival time of RC structures should be developed. Improving the ductility of reinforcement steel could yield great benefits.
- Research on the robustness of structures is one of the most topical subjects of the structural engineering research community. However, provisions to achieve adequate robustness for RC structures in fire may incur additional costs in the construction. Cost-benefit analyses should be carried out, based on risk assessment, to ensure limited resources are used optimally.
Reference


Franssen, J. M. "SAFIR (2000); Non linear software for fire design." Univ. of Liege.


Appendix 3-1

Below tables are shown in the reduction factor of compressive strength, tensile strength elastic modulus for concrete and reinforcement steel in elevated temperatures.

Table A3 - 3 Reduction factors for stress-strain relationship of reinforcement steel at elevated temperatures (EN 1992-1-2:2002)

<table>
<thead>
<tr>
<th>Steel Temperature</th>
<th>Reduction factors at temperature θa relative to the value of fy or Ea at 20 °C</th>
<th>Reduction factor (relative to fy) for proportional limit</th>
<th>Reduction factor (relative to Ea) for the slope of the linear elastic range</th>
</tr>
</thead>
<tbody>
<tr>
<td>θa</td>
<td>k_{p,θ} = f_{p,θ} / f_y</td>
<td>k_{y,θ} = f_{y,θ} / f_y</td>
<td>k_{E,θ} = E_{a,θ} / E_a</td>
</tr>
<tr>
<td>20°C</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>100°C</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>200°C</td>
<td>1</td>
<td>0.807</td>
<td>0.9</td>
</tr>
<tr>
<td>300°C</td>
<td>1</td>
<td>0.613</td>
<td>0.8</td>
</tr>
<tr>
<td>400°C</td>
<td>1</td>
<td>0.420</td>
<td>0.7</td>
</tr>
<tr>
<td>500°C</td>
<td>0.780</td>
<td>0.36</td>
<td>0.6</td>
</tr>
<tr>
<td>600°C</td>
<td>0.470</td>
<td>0.18</td>
<td>0.31</td>
</tr>
<tr>
<td>700°C</td>
<td>0.23</td>
<td>0.075</td>
<td>0.09</td>
</tr>
<tr>
<td>800°C</td>
<td>0.11</td>
<td>0.05</td>
<td>0.0675</td>
</tr>
<tr>
<td>900°C</td>
<td>0.06</td>
<td>0.0375</td>
<td>0.045</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Concrete Temperature</th>
<th>Reduction factors at temperature $\theta$</th>
<th>$\varepsilon_{ct, \theta}$</th>
<th>$\varepsilon_{cu, \theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_{c, \theta} / F_{ck}$</td>
<td>$\varepsilon_{ct, \theta}$</td>
<td>$\varepsilon_{cu, \theta}$</td>
</tr>
<tr>
<td>1000°C</td>
<td>0.04</td>
<td>0.0125</td>
<td>0.025</td>
</tr>
<tr>
<td>1100°C</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1200°C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Concrete Temperature** | **Reduction factors at temperature $\theta$** |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a relative to the value of $f_y$ or $E_a$ at 20 °C</td>
</tr>
<tr>
<td>20°C</td>
<td>1</td>
</tr>
<tr>
<td>100°C</td>
<td>1</td>
</tr>
<tr>
<td>200°C</td>
<td>0.95</td>
</tr>
<tr>
<td>300°C</td>
<td>0.85</td>
</tr>
<tr>
<td>400°C</td>
<td>0.75</td>
</tr>
<tr>
<td>500°C</td>
<td>0.6</td>
</tr>
<tr>
<td>600°C</td>
<td>0.45</td>
</tr>
<tr>
<td>700°C</td>
<td>0.3</td>
</tr>
<tr>
<td>800°C</td>
<td>0.15</td>
</tr>
<tr>
<td>900°C</td>
<td>0.08</td>
</tr>
<tr>
<td>1000°C</td>
<td>0.04</td>
</tr>
<tr>
<td>1100°C</td>
<td>0.01</td>
</tr>
<tr>
<td>1200°C</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure A3 - 1 The reduction factor of coefficient of concrete tensile strength (EN 1992-1-2 :2002)
Appendix 4-1

In the case of the specific heat, the relationships with temperature are like Figure A4-1 and Figure A4-2.

![Graph showing specific heat as a function of temperature at different moisture contents](image)

Figure A4-1 Specific heat, as function of temperature at 3 different moisture contents (EN 1992-1-2:2002)
Figure A4 - 2 Volumetric specific heat as function of temperature at a moisture content (EN 1992-1-2:2002)
Appendix 5 - 1

This appendix presents an example to show how the curve three key quantities are calculated and how the beam axial force-time relationship and the beam deflection-time relationship are constructed. The concrete cross-section is divided into 9 layers

Input data:

Beam section : 300x150 mm
Length : 6000 mm
Fracture strain of (reinforcement steel) : 0.15
The level of axial restraint ($K_A$) : 10 percent (-19500N/mm)

Table A5 - 1 lists the ambient temperature mechanical properties of steel and concrete.

**Table A5 - 1 the ambient temperature mechanical properties of steel and concrete**

<table>
<thead>
<tr>
<th></th>
<th>Concrete</th>
<th>Reinforcement steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$ (Young’s modulus)</td>
<td>26000 MPa</td>
<td>200000 MPa</td>
</tr>
<tr>
<td>$V$ (Poisson’s ratio)</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$F_c$ (Cylinder strength)</td>
<td>20 MPa</td>
<td>420 MPa</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>$10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$F_t$ (Tensile strength)</td>
<td>1.5 MPa</td>
<td></td>
</tr>
<tr>
<td>$G_f$ (Fracture energy)</td>
<td>0.08 Nmm/mm$^2$</td>
<td></td>
</tr>
</tbody>
</table>

1) Calculate the beam stiffness of each layer at elevated temperatures $T$ ($K_{A(1,2...n)}$)

\[
K_{A(1,2...n)} = \alpha_{(1,2...n)} \cdot \frac{E \cdot A}{L \cdot N}
\]
Time 1 (1 min) : $K_{A1} = 21666.66667 \text{ N/mm (temperature : 20.38} ^\circ \text{C)}$

\cdot

\cdot

: $K_{An} = 21666.66667 \text{ N/mm (temperature : 75.88} ^\circ \text{C)}$

Time 2 (2 min) : $K_{A1} = 2166.666667 \text{ N/mm (temperature : 20.60} ^\circ \text{C)}$

\cdot

\cdot

: $K_{An} = 21666.66667 \text{ N/mm (temperature : 114.75} ^\circ \text{C)}$

2) Calculate the spring stiffness ($K_{(1,2,...,n)}$) on each layer

\[ K_{(1,2,...,n)} = \frac{K}{N} \]

$K_{(1,2,...,n)} = \frac{19500}{9} = 2166.66667$

3) Calculate effective axial restraint ($K_E$) of each layer

\[ \frac{1}{K_{E(1,2,...,n)}} = \frac{1}{K_{(1,2,...,n)}} + \frac{1}{K_{A(1,2,...,n)}} + \frac{1}{K_{(1,2,...,n)}} \]

\[ \frac{1}{K_{E1}} = \frac{1}{2166.66...} + \frac{1}{21666.6...} + \frac{1}{2166.66...} = 1031.4746... \]

Time 1 : $K_{E1} = 1031.746... \text{ N/mm}$

: $K_{E2} = 1031.746... \text{ N/mm}$
4) Calculate thermal expansion through temperature distribution on each layer

\[
\begin{align*}
\Delta L_1 (\text{thermal expansion}) &= \alpha \cdot \Delta T_1 \cdot L_1 \\
\Delta L_2 (\text{thermal expansion}) &= \alpha \cdot \Delta T_2 \cdot L_2 \\
\vdots & \\
\Delta L_n (\text{thermal expansion}) &= \alpha \cdot \Delta T_n \cdot L_n
\end{align*}
\]

- Calculate new temperature on each layer through linear temperature distribution (Figure 5 – 7)

### Table A5 - 2 Original the difference of temperature

<table>
<thead>
<tr>
<th>Time</th>
<th>1Layer</th>
<th>2Layer</th>
<th>3Layer</th>
<th>4Layer</th>
<th>5Layer</th>
<th>6Layer</th>
<th>7Layer</th>
<th>8Layer</th>
<th>9Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 min</td>
<td>0.38919</td>
<td>0.0453</td>
<td>0.005414</td>
<td>0.00174</td>
<td>0.009722</td>
<td>0.082825</td>
<td>0.71116</td>
<td>6.141155</td>
<td>55.88104</td>
</tr>
<tr>
<td>2 min</td>
<td>0.60616</td>
<td>0.146735</td>
<td>0.027431</td>
<td>0.015128</td>
<td>0.07485</td>
<td>0.5386</td>
<td>3.4438</td>
<td>19.70687</td>
<td>94.75019</td>
</tr>
<tr>
<td>300 min</td>
<td>261.819</td>
<td>320.8228</td>
<td>385.6317</td>
<td>457.6684</td>
<td>538.66</td>
<td>630.5434</td>
<td>735.4715</td>
<td>855.4004</td>
<td>991.19</td>
</tr>
</tbody>
</table>

1. Calculate bottom temperature by using the concept of linear temperature
distribution

At that time, each layer is assumed to trapezoid. Then calculate the total area and the temperature of the bottom layer of concrete.

Total area of trapezoid

Time 1 min

Total area: \((0.38919+0.0453)*37.5/2 + (0.0453+0.005414)*37.5/2 + \ldots = 1317.468319\)

The temperature of the bottom layer of concrete is

Time 1 min

The temperature = \((\text{Total area}/150) – 0.38919 = 8.759771\)

2. Calculate the linear thermal curvature and the new temperatures of the layers

Linear thermal curvature

Thermal curvature

Time 1 min

Thermal curvature = \(((0.00001*(8.759771))/300 = 2.79\times10^{-7}\)

- Calculate the angle and the temperatures of the layers

Angle

\(\tan \theta = 300/(8.759771 – 0.38919) = 35.8398\)

\(\theta = 88.40175^\circ\)

The new temperatures of the layers, after reduction of the linear temperature distribution, are

\[T_{n-1} = (300 – 37.5)/\tan \theta = 7.32458\]
\[ T_{n,2} = 12.94 \]
\[ T_{n,3} = 15.94 \]

3. Calculate the new thermal expansion of each layer after reduction of the linear temperature distribution:
\[ \Delta_n = 0.00001 \times 6000 \times 8.759771 = 0.525586 \]
\[ \Delta_{n-1} = 0.00001 \times 6000 \times 12.9499 = 0.439474 \]

4. Calculate axial forces and summarize on each layer
\[ F_1 = 1031.746\ldots \times 0.023351 = 24.09271\ldots \]
\[ F_2 = 1031.746\ldots \times 0.062779 = 64.77235\ldots \]
\[ \ldots \]
\[ F_n = 1031.746\ldots \times 0.525586 = 518.1788069\ldots \]

Time 1 min
Axial force = \( F_1 + F_2 + \ldots + F_n = -2084.76 \)

Time 2 min
Axial force = -4294.29
Figure A2-1 Comparison of axial forces on simulation and calculation results

5) Calculate three points

A point

Time 1:

\[
(23589.18\times 6000/4) + (252000(150-98.82/2) - 3.212) - 252000(3.212+100))
\]

= 33912796

Time 2:

\[
(23589.18\times 6000/4) + (252000(150-98.82/2) - 3.212) - 252000(3.212+100))
\]

= 33313371.5

Time 86:

\[
(23589.18\times 6000/4) + (16039.71(150-82.21/2) - 114.89) - 16039.71(114.89+100))
\]

= -26563.849
B point

Time : 1

\[(2358918 \times 600/4) - 252000 \times (3.212 \times 100)) = 9374165.873\]

Time : 2

\[(2358918 \times 600/4) - 252000 \times (4.36759 + 100)) = 9083136.098\]

•

•

Time : 112

\[(2358918 \times 600/4) - 157802.149 \times (122.9 + 100)) = -68062.164\]

C point

\[2250 \times 0.015 = 33.75\]

\[(0.15 - 0.015) \times 750/2 + 0.015 \times 750 = 61.875\]

\[33.75 + 61.875 = 95.625\]

Vertical deflection

\[\sqrt{(3000 + 95.625)^2 - 3000^2} = 763.475\]

\[T = ((2359.14 \times 3000)/4)/763.475 = 23437.89\]

Time = 242 minutes