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Structural effects on compressive strength enhancement of concrete-like materials in a split Hopkinson pressure bar test

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Abstract: Many researches have confirmed that the dynamic increase factor (DIF) of concrete-like materials in compression measured by split Hopkinson pressure bar (SHPB) includes considerable structural effects, which do not belong to strain-rate effect. It has been found that the factors responsible for structural effects include material parameters (i.e. hydrostatic dependence, dilation parameter), specimen geometry (i.e. diameter), end interface friction and material inertia. However, their intrinsic relations have never been fully clarified. Based on two well-established material models (extended Drucker-Prager model in Abaqus and the Concrete Damage Model Release III in LS-DYNA), this paper uses numerical SHPB tests to investigate the interactive relations among these structural factors. It was found that the lateral confinement in a SHPB specimen is responsible for all structural effects in a SHPB test of concrete-like material. Two independent mechanisms can produce the lateral confinement, i.e. (i) friction on the interface of SHPB specimen and pressure bars, which prevents the expansion of the SHPB specimen during compression, and (ii) lateral inertia in SHPB specimen, which generates reactive radial confinement stress. Dilation can further enhance DIF, but it has to interact with either or both mechanisms. The ways that various structural factors contribute to structural effects through these mechanisms are clarified.

Keywords: Split Hopkinson pressure bar; structural effects; concrete-like materials; dynamic increase factor; finite-element modelling.

1. Introduction

Further studies [1-4] support the quantitative findings in Li and Meng [5], which showed that the dynamic uniaxial compressive strength of concrete-like materials measured by split Hopkinson pressure bar (SHPB) includes considerable pseudo strain-rate effects (or structural effects), introduced by various structural factors (e.g. end interface friction, specimen geometry, lateral inertia etc.). Clearly, the apparent strain-rate enhancement of material strength due to structural effects cannot be considered as a genuine strain-rate effect. Numerical split Hopkinson pressure bar (NSHPB) tests [6] based on the “reconstitution method” [7] have been used to study the structural effects [1-4, 6]. In a NSHPB test, a phenomenological strain-rate-independent strength model is employed for the SHPB sample material. Therefore, the apparent strain-rate effect on the dynamic compressive strength from a NSHPB test represents the structural effect.

It has been shown [1-4, 6] that when the compressive strength of a material is sensitive to hydrostatic pressure, the lateral confinement developed in the SHPB sample can enhance its axial compressive strength. Two conditions are necessary for the existence of structural effect in a SHPB test, i.e. (i) the compressive strength of the tested material is hydrostatic sensitive, and (ii) lateral confinement can be developed in the SHPB sample. Since the strength of concrete-like materials is much more sensitive to hydrostatic pressure (e.g. Drucker-Prager model) than that of metallic materials (e.g. von-Mises model), structural effects in a SHPB test are normally associated with concrete-like materials rather than metallic materials.

Structural effects due to end interface friction, specimen diameter and lateral inertia have been studied before [1-4, 6, 8-10]; however, shear dilation was either ignored or considered implicitly. A recent numerical study [11] investigated the effect of the shear dilation on the compressive strength of concrete in a SHPB test and found that the dynamic increase factor (DIF) of concrete obtained from SHPB tests increases with material dilation. Dilation (or dilatancy) of concrete-like materials refers to the volume increase resulting from the formation and growth of micro-cracks around aggregate in shear and compression [12, 13], which can directly contribute to the increase of lateral confinement, and thus, the increase of measured compressive strength in a SHPB test. When shear dilation has been included in the strength model of a material, its apparent strain-rate effect on the compressive strength in a SHPB test should be considered partly as a structural effect. However, the way that dilation contributes to the increase of compressive strength may be different from other structural factors. Furthermore, it is still unclear how individual structural factors interact and produce the overall structural effect. Therefore, it is necessary to perform further studies to clarify the above issues.

In this paper, we perform finite-element (FE) simulations using two well-known concrete constitutive material models that take into account material dilation, i.e. the extended Drucker-

Prager model (D-P model) in Abaqus and the Concrete Damage Model Release III (Material 072R3 or K&C model) in LS-DYNA, to study the interactions among structural factors and their influence on the apparent compressive strength measured in a SHPB test. The mechanisms responsible for the DIF enhancement through material dilation are discussed to generalise the conclusions from numerical results. FE models are described in Section 2. Numerical results are presented in Section 3 followed by a discussion about the interactions among structural factors and their influence on DIF in Section 4. Conclusion remarks are presented in Section 5.

2 Numerical simulations of SHPB tests

The FE simulations of SHPB tests in this work were performed with a similar setup as that presented by Grote et al. [14]. The geometrical details of the specimen and the incident and transmission bars are given in Table 1. The numerical simulations were performed using the commercial (FE) software Abaqus/Explicit [15] and LS-DYNA [16]. To reduce computational costs, a representative quarter-model was constructed using 8-node brick elements with reduced integration (Fig. 1). The elements of the specimen and the pressure bars around the specimen-bar interfaces (Fig. 1) have an average size of $0.2 \times 0.2 \times 0.2 \text{ mm}^3$. The total numbers of elements are 17250 for the SHPB specimen and 43365 for each pressure bar, respectively. A mesh sensitivity analysis showed that smaller element size did not further change the numerical results. A transition mesh was used in the radial direction of the specimen and in the axial direction of the bars to reduce the total number of elements (Fig. 1).

The input and output bars of the SHPB were modelled using an isotropic elastic material model with input parameters shown in Table 1. The specimen was modelled in Abaqus using the D-P model with linear pressure-dependence and material parameters shown in Table 1 as well as quasi-static hardening-softening input curve shown in Fig. 2, which has an initial yield stress of 18 MPa and peak unconfined uniaxial compressive strength f'_c of 40 MPa. For LS-DYNA numerical simulations, the Material 072R3 was employed using the parameter generation capability, which is based only on the unconfined compressive strength ($f'_c=40 \text{ MPa}$) [17]. An overview of the dilation parameters in each model is given in Section 2.1. A full description of the constitutive relations for both models is given in the Appendix.

Table 1 Geometrical details, material properties and material model parameters

Material properties and geometrical details	Specimen [14]	Input/output bar
Density ρ (kg/m ³)	2000	7800
Young's modulus E (GPa)	30	210
Poisson's ratio ν	0.19	0.3
Diameter D (mm)	12, 20	21

Length L (mm)	6	1000
Material model parameters	Specimen	
Unconfined uniaxial compressive strength f'_c (MPa) (D-P and Material 072R3 models)	40	-
Friction angle β (°) (D-P model)	0, 30, 50	-
Dilation angle ψ (°) (D-P model)	0, 30, 50	-
Parameter K (D-P model)	1	
Dilation parameter ω (Material 072R3)	0, 0.5, 1	-

Fig. 1 FE model of the split Hopkinson pressure bar test.

Fig. 2 Stress-strain curves for different input parameters and lateral confinements at quasi-static loading ($\dot{\epsilon}=8 \times 10^{-2} \text{ s}^{-1}$) for the D-P model in Abaqus using the material properties in Table 1.

2.1 Shear dilation parameters in D-P and Material 072R3 models

A full description of the constitutive relations for the Extended Drucker-Prager (D-P) model in Abaqus and the Concrete Damage Model Release III (Material 072R3) in LS-DYNA can be found in the Appendix.

In the (D-P) model, the plastic flow potential G is defined as [18],

$$G = t - p \tan \psi \quad (1)$$

where t is the deviatoric stress measure, p is the equivalent pressure stress and ψ is the dilation angle (Appendix). If $\psi = 0$, the plastic deformation is incompressible and when $\psi > 0$ the material dilates.

In the Material 072R3, the plastic flow potential g is defined as [19],

$$g = \sqrt{3J_2} - \omega F(I_1, J_2, J_3) \quad (2)$$

where F is a loading surface depending on the stress invariants I_1, J_2 and J_3 (Appendix). Dilatancy is taken into account via the parameter ω , which defines the amount of dilation that can occur in the material. When $\omega = 0$, there is no volume change during plastic flow and when $\omega > 0$ the material dilates.

By comparing the full description of both models in the Appendix, it is unlikely to establish a direct relationship between D-P model and Material 072R3 model due to the difference between their respective plastic potential functions and strength surfaces. Consequently, the dilation parameters in these two models have no general relationship. However, dilation parameters ω in Material 072R3 model has a similar function to that of the dilation angle ($\tan\psi$) in the D-P model under certain conditions. For example, when comparing the plastic flow potentials for the D-P model (Eq.(1)) and Material 072R3 (Eq.(2)), it can be seen that ω in the latter model plays a similar role to that of the dilation angle ($\tan\psi$) in the D-P model when $K=1$ (where K is the ratio of the yield stress in triaxial tension to the yield stress in triaxial compression in D-P model, see Appendix) which leads to $t=q/2=(\sqrt{3J_2})/2$ (where q is the Mises equivalent stress). In this case, both material models can be reduced to the same non-associative flow rule (i.e. $g=2G=\sqrt{3J_2}$) when $\omega=\tan\psi=0$, and to the fully associative flow rule when $\omega=1$ and $\tan\psi=\tan\beta$ (where β is the friction angle in D-P model). This implies that zero value of dilation parameters in both models is associated with plastic incompressibility (or zero plastic dilation).

For concrete, a partial associative flow rule should be used according to experimental observation. In general, the variation range of dilation parameters in Material 072R3 and the D-P model are $0 < \omega < 1$ and $0 < \tan\psi < 3$, respectively, which control the amount of associativity, and therefore, the change in volume (dilation) during plastic flow.

2.2 Numerical analyses of SHPB test

A parametric study of the effects of structural factors (i.e. the friction angle, dilation, friction coefficient, specimen diameter, inertia and Poisson's ratio) on the DIF of concrete at strain-rates in the range of 10^1 - 10^3 s⁻¹ is performed. For the numerical simulations performed in Abaqus/Explicit, three friction angle values $\beta = (0^\circ, 30^\circ, 50^\circ)$, three dilation angle values $\psi = (0^\circ, 30^\circ, 50^\circ)$ and three friction coefficient values $\mu = (0, 0.1, 0.2)$ are used. The effect of inertia is studied using reduced values of density ρ (20 and 200 kg/m³). A larger specimen diameter of 20 mm and zero Poisson's ratio are also investigated. For the numerical simulations performed in LS-DYNA, three dilation parameter values $\omega = (0, 0.5, 1)$ and two friction coefficient values $\mu = (0, 0.2)$ are used.

To achieve different strain-rates, trapezoidal stress pulses with different rising time and intensity are applied to the end of the incident bar. The rising time varies from 70 to 100 μs while the duration and unloading times are fixed to 20 μs as they do not have an effect on the strain-rate because the specimen reached peak strength during the loading time. The intensity of the pulse varies from 50 to 1000 MPa.

Strain-rate effect is ignored in both material models in order to study structural effects. To measure the axial stress and strain-rate in the specimen in numerical simulations, several methods were compared: (i) the formulae described in [20, 21] (Eq.(3) and Eq.(4)), which use the stresses and velocities at the specimen interfaces to obtain the time histories of the average engineering stress $\sigma(t)$ and average strain-rate $\dot{\epsilon}(t)$ over the specimen, i.e.,

$$\sigma(t) = \frac{\sigma_I(t) + \sigma_T(t)}{2} \quad (3)$$

$$\dot{\epsilon}(t) = \frac{V_I(t) - V_T(t)}{L_S} \quad (4)$$

where $(\sigma_I(t), V_I(t))$ and $(\sigma_T(t), V_T(t))$ are the stress and velocity time histories at the interfaces between the incident bar and the specimen, and between the transmitter bar and the specimen, respectively, and the constant L_S is the specimen length; (ii) an average of the direct measure of axial stress and axial strain-rate in all the elements of the specimen; and (iii) an average of the direct measure of the axial stress and axial strain-rate in a layer of elements located at the transverse mid-plane in the specimen. It was found that the maximum relative differences for the results obtained with the three methods are within 2%. Thus, we used the latter method to save data processing time.

It is known that if the peak stress in the specimen is reached before the stress equilibrium is achieved, the test may not be valid. The dynamic stress equilibrium was verified using the parameter $R(t)$ defined as [20, 21],

$$R(t) = 2 \left| \frac{\sigma_I(t) - \sigma_T(t)}{\sigma_I(t) + \sigma_T(t)} \right| \quad (5)$$

It is generally accepted that if the stress difference across the SHPB specimen is no more than 5% of the average stress, i.e., $R(t) \leq 0.05$, the stress equilibrium condition is satisfied [22]. Numerical simulations performed in Abaqus/Explicit (Fig. 3) show the variation of $R(t)$ with the normalised time t/t_0 where $t_0 = L_S/c_S$ is the transit time duration for the longitudinal incident pulse

travelling one specimen length. The specimen length L_S in Fig. 3 is 6 mm and the elastic wave speed is 3105 m/s ($t_0=1.93\mu\text{s}$); the stress equilibrium is achieved after 6 times of stress wave reflections in the SHPB specimen before the peak strength is reached. Therefore, the measured strength can be considered valid [20]. It can also be seen in Fig. 3 that a large oscillation occurs after the peak stress is reached. This is due to the loss of stress equilibrium when the specimen is largely damaged. However, these large oscillations do not affect the numerical results in this investigation since only the peak strength is of interest. It can also be seen in Fig. 3 that for $\dot{\varepsilon}(t)>640 \text{ s}^{-1}$ the stress equilibrium is not achieved before the peak strength is reached and therefore the simulation may not be valid. For this reason, the results for $\log(\dot{\varepsilon})>2.8$ will not be used in this study.

Fig. 3 Dynamic stress equilibrium $R(t)$ versus normalised time t/t_0 for a 12-mm mortar specimen subjected to different strain-rates in SHPB test.

3 Numerical results and discussion

In this section, the effects of dilation and several other structural factors such as coefficient of friction, Poisson's ratio, specimen diameter and material density on DIF are presented. Numerical simulations are performed using D-P model in Abaqus and Material 072R3 model in LS-DYNA. The default specimen diameter is 12 mm unless otherwise specified.

3.1 Numerical simulations using D-P model in Abaqus

Figure 2 shows quasi-static compression stress-strain curves performed at low strain-rate ($\dot{\epsilon}=8\times10^{-2} \text{ s}^{-1}$) with different radial confinement pressure P , friction coefficient and material parameters, using D-P model in Abaqus. It is noted that $\dot{\epsilon}=8\times10^{-2} \text{ s}^{-1}$ is used for quasi-static simulation in order to save simulation time because the results at this strain-rate are almost identical to the results obtained at strain-rate of $\dot{\epsilon}=10^{-3} \text{ s}^{-1}$. It can be seen in Fig. 2 that without radial confinement pressure and interface friction, no enhancement of the compressive stress-strain curves is observed even though the material friction angle is not zero. This implies that under quasi-static loading condition, the uniaxial compressive stress state in the specimen will not be changed if the specimen is not subjected to lateral constrain (either by applying radial confinement pressure or being constrained by interface friction).

The influences of both radial confinement pressure and friction coefficient on the compressive strength are affected by the friction angle significantly. It is interesting to note that when the friction angle is zero, radial confinement pressure and interface friction can still influence the compressive strength, but not significantly when compared to the cases with large friction angle. This is because even when the friction angle is zero the D-P model is not exactly the same as the metal plasticity model whose compressive stress-strain curve is not influenced by lateral confinements. For given values of P and β , dilation parameter ψ has a slight influence on the maximum quasi-static compressive strength for high values of P . However, it is shown in Fig. 4 that ψ has considerable influence on the apparent dynamic compressive strength of mortar as indicated by the DIF- $\log(\dot{\epsilon})$ curves, particularly for strain-rates higher than 100 1/s. These results show that numerical modelling results are sensitive to parameters μ , β and ψ in certain regions, and therefore, they should be carefully calibrated in numerical modelling.

The effect of friction on the dynamic response of SHPB specimen is well known, which may lead to the over-prediction of DIF [3, 6, 23]. This issue is important for concrete-like sample because its end surface is usually rougher than the surface of a metallic sample [1]. Therefore, friction should not be completely neglected even though lubricant is used. Figure 4 also shows the effect of the friction coefficient μ on the dynamic strength of mortar for various β and ψ . It can be seen that when $\beta=\psi=0$, the increase of μ has some influence on the measured DIF; however when β and ψ increase, the DIF increases largely with the increase of μ . This can be explained by the fact that when β increases the material becomes more pressure dependant, and therefore, more sensitive to the lateral confinement. Experimental results from Grote et al. [14] are included in Fig. 4 as a reference. The value of the coefficient of friction was not given in

Grote et al. [14] neither it was mentioned if lubricant was used. It is believed that the increase of DIF shown in Grote et al. [14] is only partially due to genuine strain-rate effects.

Fig. 4 Numerical results at different strain-rates using various material parameter sets in the D-P model in Abaqus/explicit.

Figure 5 shows the contour plot of radial stress and equivalent plastic strain (PEEQ) for specimens at strain-rate of $\log(\dot{\epsilon}) \approx 2.7$, for various sets of β , ψ and μ . As expected, the lateral confinement, which can be represented by the compressive radial stresses, increases when the interface friction coefficient is increased. It can also be seen that for specimens with same friction coefficient, the lateral confinement increases when the dilation parameter is increased.

Fig. 5 Contour plots of radial stress and PEEQ for $\log(\dot{\epsilon}) \approx 2.7$ for various material parameters sets.

It is noted that the dependence of the peak stress on the average radial stress in Fig. 5 is somewhat comparable with the peak stress dependence on the lateral confinement P in Fig. 2. It is also observed that PEEQ increases when the dilation parameter is increased. The radial stress (or dynamic confinement) is produced in the hourglass-shaped region [11], which is delimited

by the fault regions with high PEEQ and volumetric expansion or dilation (In Fig. 5, see dashed black line in the PEEQ contour of $\beta=50^\circ$, $\psi=50^\circ$, $\mu=0.2$). This phenomenon has been observed in [11] from numerical simulations using the Material 072R3 model. It is worth to address again the importance of the dilation parameter in the numerical prediction of the compressive strength of the concrete sample, particularly for strain-rates higher than 10^2 s^{-1} . Therefore, a good calibration of this parameter should be performed to ensure accurate predictions.

To understand the inertia effects on the DIF, numerical simulations were carried out using one-tenth and one-hundredth of the material nominal density, i.e., $\rho=200 \text{ kg/m}^3$ and $\rho=20 \text{ kg/m}^3$, respectively. Figure 6 shows a comparison of numerical SHPB tests for three different material densities. It can be seen that when the density is reduced to $\rho=200 \text{ kg/m}^3$, the DIF does not increase with the increase of strain-rate at the same rate as it does for the nominal density ($\rho=2000 \text{ kg/m}^3$) (note: the reduction of material density will increase the stress wave speed and reduce the stress equilibrium time, and thus, will not affect the validity of SHPB test simulation). Since axial inertia affects the additional axial stress only for soft materials, the additional stress due to axial inertia is normally not concerned for solid materials [24, 25]. Thus, the observed inertia effect can only be attributed to the lateral inertia, which generates lateral confinement leading to the increase of DIF. When density is further reduced to 20 kg/m^3 , the DIF is almost independent of strain-rate. The correlation between strain-rate and strain acceleration in a SHPB test was noted in [2]. With the increase of strain-rate, axial strain acceleration is also increased, which leads to the increase of lateral confinement as a result of the increase of radial inertia. Therefore, inertia effect is an important mechanism contributing to the structural effects in a SHPB test. However, Fig. 6 also shows the different DIF values obtained for different coefficients of friction when other parameters are the same, which again showed that interface friction is an independent mechanism for structural effect.

Previous researches have stated that Poisson's ratio has an effect on the radial inertia in SHPB [26, 27]. In order to investigate this effect, numerical simulations with zero Poisson's ratio were performed and results are shown in Fig. 6. It can be seen that the predictions of DIF when using either $\nu=0$ or $\nu=0.19$ are the same for a friction coefficient $\mu=0$; however, when $\mu=0.2$, DIF increases with Poisson's ratio when it varies from $\nu=0.0$ to $\nu=0.19$. It transpires that interface friction may restrain the lateral elastic deformation of the sample due to Poisson's effect, which generates lateral confinement and causes the increase of DIF.

Fig. 6 Finite element model of SHPB test using different material densities and Poisson's ratios.

Figure 7 shows numerical simulations for specimens with diameters of 12 mm and 20 mm. It can be seen that an increase in the specimen diameter while keeping the same length (6 mm) leads to an increase of DIF which agrees with previous research [1, 2]. Larger sample diameter restricts the lateral deformation through radial inertia and interface friction, and thus, increases the lateral confinement causing the increase of DIF. When the material density is artificially reduced, the influence of specimen diameter on DIF still exists. Therefore, larger specimen diameter promotes the lateral confinement in a dynamic compressive test, leading to the increase of DIF.

Fig. 7 Finite element model of SHPB test using different material densities and friction coefficients for diameters of 12 mm and 20 mm.

3.2 Numerical simulations using Material 072R3 in LS-DYNA

Figure 8 depicts quasi-static compression stress-strain curves at low strain-rate ($\dot{\epsilon}=8 \times 10^{-2} \text{ s}^{-1}$) with different lateral confinement P using Material 072R3 model in LS-DYNA. It can be observed that the dilation parameter ω does not affect the shape of the softening part of the

curves for $\omega=(0.0, 0.5)$. However, the stress-strain curves degrade at higher rates with the increase of ω . It can also be seen that ω does not affect the compressive strength of the sample for a given value of P .

Figure 9 shows the effects of the dilation parameter ω and the coefficient of friction μ on DIF using the Material 072R3 model in LS-DYNA. It is observed that when μ increases the DIF increases as expected due to the increase of lateral confinement. For simulations with the same friction coefficient, DIF increases when ω increases from 0.5 to 1; however, there is no significant effect when ω increases from 0 to 0.5. These observations agree generally with Abaqus simulation results.

Figure 10 shows contour plots of radial stress and effective plastic strain for specimens subjected to strain-rate of $\log(\dot{\epsilon}) \approx 2.7$, respectively, for various sets of ω and μ . Unlike Abaqus, LS-DYNA does not have the option to display results in cylindrical coordinate system, and thus, the radial stress in Fig. 10 is displayed by cutting sections of the specimen using planes parallel to the faces with symmetrical boundary conditions at a distance of 0.05 mm. The stresses in the directions perpendicular to the loading direction represent the radial stress at these cutting planes. Similar to the results observed in Fig. 5, the lateral confinement represented by the radial stress increases when the friction coefficient is increased and/or the dilation parameter is increased. The peak stresses observed in Fig. 10 are somewhat comparable to the quasi-static peak stresses in Fig. 8 due to the lateral confinement P . The effective plastic strain also increases when the dilation parameter is increased.

Fig. 8 Stress-strain curves for different input parameters and lateral confinements at $\dot{\epsilon}=8\times10^{-2} \text{ s}^{-1}$ for the Material 072R3 model in LS-DYNA using the material properties in Table 1.

Fig. 9 Numerical results at different strain-rates using various material parameter sets in the Material 072R3 model in LS-DYNA.

Fig. 10 Contour plots of radial stress and effective strain for $\log(\dot{\epsilon}) \approx 2.7$ for various material parameters sets.

4 Further discussions about the interactions among structural factors and their influence on DIF

While it is clear that dilation affects DIF according to the results shown in previous sections and in [11], it remains unclear what mechanisms are responsible for the DIF enhancement through the material dilation. To understand how dilation affects DIF and interacts with other factors, nine cases shown in Table 3 are carefully selected and evaluated. Time histories of compressive stress, radial stress, volume change and plastic strain for each case are shown in Fig. 11 where the normalized time starts from the beginning of the loading in the specimen until the maximum compressive strength is reached. In all cases, $\log(\dot{\epsilon}) \approx 2.7$ is used.

Table 3 Nine selected cases to demonstrate the effect of dilation and other factors on DIF

CASE	A	B	C	D	E	F	G	H	I
Friction angle β (°)	0	50	50	50	50	50	50	50	50
Dilation angle ψ (°)	0	50	0	0	50	50	0	0	50
Density ρ (kg/m ³)	2000	2000	20	2000	20	2000	2000	20	20
Poisson's ratio ν	0.19	0.19	0	0	0.19	0	0.19	0	0.19
μ	0.2	0	0.2	0	0	0	0	0	0.2
D (mm)	20	20	12	12	20	12	12	20	20
DIF	1.22	1.64	1.29	1.17	0.99	1.35	1.15	0.99	2.10

For Case A (Fig.11(a)), dilation parameter ψ and friction angle β are set to zero to have a hydrostatic independent material. It can be seen, however, that some DIF enhancement (DIF $\approx$ 1.22) still exists. Similar observation is shown in Fig. 4 for $\psi=\beta=0$ (smaller specimen diameter of 12 mm is used). This may be due to the flow rule of the D-P model, which creates the multi-axial stress state resulting in some DIF enhancement. For Case B (Fig.11(b)), despite of coefficient of friction being zero ($\mu=0$), large increase of DIF ($\sim$ 1.64) is observed, which demonstrates that other structural factors (dilation and inertia) can produce DIF enhancement. Case C (Fig.11(c)) demonstrates that for a specimen with $\psi=0$, low density and small diameter, interface friction can independently contribute to structural effects for a hydrostatic-dependent material (DIF=1.29). For Case D with $\mu=0$, $\psi=0$ and small diameter, an enhancement of DIF ($\sim$ 1.17) shows that inertia ($\rho=2000$ kg/m³) can independently contribute to the structural effect for a hydrostatic-dependent material. For Case E, with low density, no interface friction and large dilation parameter (Fig.11(e)), there is large volume change in the plastic region but the radial stress is very small resulting in negligible DIF enhancement (DIF $\approx$ 0.99). For Case F (Fig.11(f)) with no interface friction, DIF $\approx$ 1.35, which demonstrates that dilation can enhance inertia effects. For Case G in Fig.11(g) with no dilation effect and no interface friction, DIF enhancement is marginal (DIF $\approx$ 1.15) when $\nu=0.2$, which can be compared to Case D with the

exactly same parameters except $\nu=0$. This shows that Poisson's ratio has little structural effect on DIF enhancement when there is no dilation effect or interface friction (See Fig. 6); however, results in Fig. 6 for large dilation and large interface friction show that Poisson's ratio has some small influence on DIF. A comparison of Fig. 11(d) with Fig. 11(g) shows that the reduction of volume before peak strength is larger in Case D ($\nu=0$) than in Case G ($\nu=0.19$). This suggests that the increase of Poisson's ratio may enhance DIF through dilation or interface friction by facilitating lateral expansion of the specimen. For Case H (Fig.11(h)) with $\mu=0$, $\psi=0$, large diameter and low density, there is no DIF enhancement ($DIF\approx 0.99$). This shows that the material inertia (density) is necessary for the DIF enhancement in a SHPB specimen with large diameter. Finally for Case I (Fig.11(i)) with large diameter, dilation and interface friction, DIF is enhanced significantly ($DIF=2.10$). This large enhancement suggests that despite of low density, dilation can also enhance DIF through interface friction when Case I is compared with Case E. This indicates that dilation can enhance DIF through either inertia or friction, or both of them. This means that radial expansion alone (without radial inertia due to material density or radial confinement due to friction) results in no DIF enhancement in a material that is sensitive to hydrostatic pressure. This point can be further illustrated if we focus on Cases B, E, F, I with large dilation parameter ψ . For these cases, there is large volumetric change in the plastic regime when compared to the rest of the cases (Fig. 11). The material expansion leads to the increase of the radial stress when density is large (Case B, F) or interface friction is large (Case I), which in turn produces lateral confinement that results in an increase of DIF.

When the strain-rate is high enough ($\log(\dot{\epsilon}) > 2$), there is a correlation between the axial strain-rate and the axial strain acceleration [2]. The accelerated axial motion produces an accelerated radial motion leading to an inertia-induced radial confinement. When the strain-rate is below the transition strain-rate, axial acceleration is significantly small. This can be seen in Fig.12 (a) in which the axial acceleration is compared for some of the cases in Table 3. Two extra cases similar to Case B (i.e. Case B with $\log(\dot{\epsilon})\approx 2$ and Case B with $\psi=30^\circ$) are also included in Fig. 12. It can be seen that the axial acceleration for Case B with $\log(\dot{\epsilon})\approx 2$ is much smaller than those in other cases in Fig. 12 with higher strain-rate, and its corresponding DIF is marginal (~ 1.04). It can also be seen in Fig. 12 (a), that the axial acceleration is somewhat similar in all cases in which $\log(\dot{\epsilon})\approx 2.7$. This supports the experimental finding in [2] that axial acceleration is correlated with strain-rate. By comparing Case B (large density, DIF enhancement) with Case E (low density, no DIF enhancement), which have similar axial accelerations, it can be seen that the axial acceleration can only enhance DIF through material inertia.

Fig. 11 Time histories of compressive stress, radial stress, volume change and plastic strain for 9 different loading cases (Cases A-I in Table 3) using D-P model in Abaqus.

Figure 12 (b) shows the time history of the plastic Poisson's ratio (ppr), defined as the ratio between lateral strain and axial strain, for the same cases shown in Fig. 12 (a). It can be seen that ppr increases with the increase of dilation. For Case B with $\psi=30^\circ$, DIF=1.42. This DIF enhancement is smaller than that of Case B with $\psi=50^\circ$ (DIF=1.64). This shows that the reduction of dilation results in a reduction of ppr and this in turn results in less DIF enhancement. Since ppr affects the relationship between axial and radial motion, large ppr enhances the radial motion and the lateral expansion. Thus, it plays similar roles as Poisson's ratio in elastic regime. With inertia, the lateral expansion will cause lateral confinement. Inertia starts to play its role when strain-rate is larger than a critical value.

Fig.12 Time histories of (a) axial acceleration and (b) plastic Poisson's ratio (ppr) for different loading cases (Table 3) using D-P model in Abaqus

Based on the above analyses of the numerical results, a flow chart (Fig. 13) is constructed to show the interactive mechanisms that produce structural parameters that influence DIF. DIF can be enhanced by structural effects for hydrostatic-dependent materials when lateral confinement is introduced due to structurally-produced radial stresses. The two main mechanisms that can produce lateral confinement are interface friction and radial inertia. These two mechanisms can either individually or collaboratively exist to produce lateral confinement. Other structural factors (e.g. Poisson's ratio, dilation and specimen diameter) can further enhance DIF, but they have to interact with either or both mechanisms. Finally, large axial acceleration, which is correlated with strain rate in SHPB test, is necessary for the radial acceleration to generate radial inertia in a SHPB specimen.

Fig.13 Flow chart of parameters affecting the enhancement of DIF

5 Conclusions and Remarks

The numerical study conducted in this paper shows that the increase of the apparent dynamic increase factor (DIF) with strain-rate in concrete-like materials in a SHPB test is a complex phenomenon related not only to material strain-rate effects but also to structural effects. Strain-rate independent material model is employed in this study in order to isolate the structural effects for a hydrostatic-dependent material. It is found that two mechanisms can produce lateral confinement, i.e. interface friction and radial inertia, which can enhance DIF individually or collaboratively for hydrostatic-dependent materials. Dilation, Poisson's ratio (or material flow rule in a more general sense) and specimen diameter can further enhance DIF, but they have to interact with either or both lateral confinement mechanisms. It was also found that radial inertia (radial acceleration) starts to enhance DIF when strain-rate reaches a critical strain-rate (transition strain-rate) in the order of 10^2 s^{-1} . The correlation between axial strain acceleration and strain-rate observed numerically in the present study supports the reported experimental observation. However, there is still a lack of explanation for the existence of such correlation. Although, the existence of genuine strain-rate effects have been suggested by meso-scale

simulations and micro-crack models [28-30], the findings in this study show that structural effects have a significant contribution to the measured DIF, and therefore, one needs to be cautious about interpreting SHPB data for concrete-like materials. Otherwise, the material model with considering strain-rate effect based directly on SHPB measurement may over-predict material strength and lead to non-conservative design and assessment of protective structures.

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Appendix

A.1 Extended Drucker-Prager constitutive model in Abaqus

The D-P model with linear yield criterion is given by [15],

$$F = t - p \tan \beta - d = 0 \quad (\text{A1})$$

where β is the friction angle of the material, which is the slope of the linear yield surface in the t - p stress plane (Fig. A1(a)). The cohesion of the material d is given by,

$$d = \sigma_c \left(1 - \frac{1}{3} \tan \beta \right) \quad (\text{A2})$$

where σ_c is the unconfined uniaxial compressive yield stress. The deviatoric stress measure t is defined as,

$$t = \frac{1}{2} q \left[\frac{1}{K} - \left(1 - \frac{1}{K} \right) \left(\frac{r}{q} \right)^3 \right] \quad (\text{A3})$$

where $q = \sqrt{\frac{3}{2} \mathbf{S} : \mathbf{S}} = \sqrt{3J_2}$ is the Mises equivalent stress; r is the third invariant of the deviatoric stress given by $r = \left(\frac{9}{2} \mathbf{S} : \mathbf{S} : \mathbf{S} \right)^{1/3} = 3 \left(\frac{1}{2} J_3 \right)^{1/3}$ where $\mathbf{S}$ is the deviatoric stress defined as $\mathbf{S} = \boldsymbol{\sigma} + p\mathbf{I}$, being $\boldsymbol{\sigma}$ and $p = -\frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) = -\frac{1}{3}I_1$ the stress tensor and equivalent pressure stress, respectively. The first invariant of the stress tensor and the second and third invariants of the deviatoric stress tensor are I_1, J_2, J_3 , respectively. The parameter K is the ratio of the yield stress in triaxial tension to the yield stress in triaxial compression [15]; K has to be within the range $0.788 \leq K \leq 1$ to ensure the convexity of the yield surface. The flow rule of the D-P model is given by [18],

$$d\epsilon^p = \frac{d\epsilon^p}{c} \frac{\partial G}{\partial \sigma} \quad (\text{A4})$$

where ϵ^p is the equivalent plastic strain; c is a constant given by $(1 - 1/3 \tan \psi)$ if hardening is defined in uniaxial compression, which is the concern of the present study. The plastic flow potential G is defined as,

$$G = t - p \tan \psi \quad (\text{A5})$$

where ψ is the dilation angle. If $\psi = 0$, the plastic deformation is incompressible (a non-associative flow rule) and when $\psi > 0$ the material dilates (Fig. A1(a)). When $\psi = \beta$ and $K=1$, a fully-associative flow rule is obtained and the original Drucker-Prager model is reinstated [15]. It can be seen in Fig. A1(a) that $\psi = \theta - \theta_n$, in which θ varies from θ_n to $\theta_n + \beta$ (i.e. $0 \leq \psi \leq \beta$). For concrete-like materials, it has been found that ψ and β are in the ranges of $20-50^\circ$ and $20-60^\circ$, respectively [31-33]. When the hardening is defined in uniaxial compression, the flow rule precludes dilation angles $\psi > 71.5^\circ$ (or $\tan \psi > 3$). It is unnecessary to discuss this restriction since it unlikely occurs for the studied materials in this paper [15].

Fig. A1 Yield surface and flow direction in (a) D-P model in Abaqus [15]; (b) Material 072R3 model in LS-DYNA [12].

A.2 Concrete Damage Model Release III (Material 072R3) in LS-DYNA

Material 072R3 implemented in LS-DYNA was employed to study the effect of dilation on SHPB testing of concrete. The model has linear behaviour until the yield stress point is reached. The plastic response of the material is predicted via three independent strength surfaces, i.e., the yield failure surface $\Delta\sigma_y$, the maximum failure surface $\Delta\sigma_m$ and the residual failure surface $\Delta\sigma_r$ [12, 34]:

$$\Delta\sigma_y = a_{0y} + \frac{p}{a_{1y} + a_{2y}p} \quad (\text{A6a})$$

$$\Delta\sigma_m = a_0 + \frac{p}{a_1 + a_2p} \quad (\text{A6b})$$

$$\Delta\sigma_r = \frac{p}{a_{1f} + a_{2f}p} \quad (\text{A6c})$$

where a_{0y} , a_{1y} , a_{2y} , a_0 , a_1 , a_2 , a_{1f} , a_{2f} are user-defined parameters that control the shape of the strength surfaces. The loading surfaces, $\Delta\sigma_L$ representing strain hardening after yield and the post-failure surfaces $\Delta\sigma_{pf}$ representing strain softening after maximum strength, are defined as [19],

$$\Delta\sigma_L(I_1, J_2, J_3) = r(J_3)[\eta(\lambda)(\Delta\sigma_m(p) - \Delta\sigma_y(p)) + \Delta\sigma_y(p)] \quad (\text{A7a})$$

$$\Delta\sigma_{pf}(I_1, J_2, J_3) = r(J_3)[\eta(\lambda)(\Delta\sigma_m(p) - \Delta\sigma_f(p)) + \Delta\sigma_f(p)] \quad (\text{A7b})$$

where $\eta(\lambda)$ is a function of the internal damage parameter λ and is calibrated experimentally; $r(J_3)$ is a scale factor in the form of William-Warnke equation [19, 34], which introduces the dependence of J_3 in order to properly model material behaviour under confinement [19].

The plastic flow potential g in the Material 072R3 in LS-DYNA is defined as [19],

$$g = \sqrt{3J_2} - \omega F(I_1, J_2, J_3) \quad (\text{A8})$$

where F is defined as the loading surfaces in Eqs.(A7a, A7b). The plastic flow rule is given by an expression of the plastic strain increment $d\varepsilon_{ij}^p$ as [35],

$$d\varepsilon_{ij}^p = \frac{\partial g}{\partial \sigma_{ij}} d\tilde{\mu} \quad (\text{A9})$$

where σ_{ij} is the stress tensor and $d\tilde{\mu}$ is the plasticity consistency parameter [35, 36]. Dilation is taken into account by the Material 072R3 via the parameter ω , which defines the amount of dilation that can occur in the material. When $\omega = 1$, a fully-associative flow rule applies, i.e., plastic flow develops along the normal to the yield surface. When $\omega = 0$, there is no volume change during plastic flow (Fig. A1(b)), which is a non-associative flow rule, e.g. Prandtl-Reuss flow rule ($\theta = \theta_n$; Fig. A1(b)), assuming complete plastic incompressibility [12]. Typical

ω values for concrete have been found to be between 0.5 and 0.7 [12], which indicates that partially-associative flow rule is used.

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