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A control scheme for LTI systems with Lipschitz nonlinearity and unknown time-varying input delay

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Abstract: In this paper, we propose a control structure for a class of systems with Lipschitz nonlinearity and unknown time-varying input delay. This scheme considers the worst case scenario in control design with Truncated Prediction Feedback (TPF) approach, and takes into account the information of the lower bound of delay in the stability analysis. A finite-dimensional controller is constructed, requiring neither the nonlinear function nor the exact delay function. The truncated prediction deviation is minimized by employing the delay range, and then bounded by integral construction and related techniques. Within the framework of Lyapunov-Krasovskii functionals, sufficient delay-range-dependent conditions are derived for the closed-loop system to guarantee the global stability. Two numerical examples are given to validate the proposed control design.

1 Introduction

In the formulation of stabilization of time-delay systems, there are two types of feedback methods in the literature: standard (memoryless) feedback and predictor-based (memory) feedback; see, for example, [1]–[13]. Many results are related to the systems with state delays. However, it is well known that the input-delayed system is more difficult to handle in control theory [2]. For predictor-based feedback, compensation is added in the controller design to offset the adverse effect of time delay and the stabilization problems are reduced to similar problems for ordinary differential equations. The predictor-based method, also called the finite spectrum assignment technique [11] or the reduction method [12], is effective for systems with delays, especially with input delays [13]–[15]. In [16], a prediction method based on the integration of the nonlinear system function is reported for a class of nonlinear systems with time-varying input delay. However, the predictor-based controllers involve integral terms of the control input, which result in difficulties in control implementation. To avoid the use of distributed terms, asymptotic prediction or dynamic prediction method is developed in [17] for system with constant input delay and the results are then extended to time-varying delay case in [18]. An other feasible solution is to ignore the troublesome integral part, and use the prediction based on the exponential of the system matrix, which is known as the Truncated Prediction Feedback (TPF) approach [19]–[24]. The TPF approach originates from the low gain feedback technique [19]. In [21], it is shown that a TPF controller works for an arbitrarily large delay as long as the open loop linear system is not exponentially unstable. The result is then extended to general linear systems in [22], including exponentially unstable open loop poles. Within the framework of Lyapunov-Krasovskii functionals, control design for Lipschitz nonlinear systems with constant input delay is presented in [24], which generalizes the results on the truncated predictor feedback for linear systems to nonlinear systems with constant input delay.

Note that all the results above are built based on the assumption that the exact delay information is known, which may be irrealizable in many circumstances. The predictor-based feedback control methods suffer from a difficulty in practical implementation when the delay function is unknown. A Lyapunov-based adaptive control design is presented in [25] to deal with unknown constant delay for

a class of linear systems. However, few results are available on stabilization for nonlinear systems with unknown time-varying input delay. The TPF control also encounters the same barrier since the prediction is based on the exact knowledge of the delay function [19]–[24].

Inspired by the above observation and based on our previous results [26]–[30], in this paper, we propose a control scheme for a class of systems with unknown time-varying input delay and Lipschitz nonlinearity. The key contributions include: (i) in contrast to [13, 16, 21–23], we propose a new control structure by merely using the delay upper bound, which greatly reduces the computation burden and improves the practical implementation; (ii) we extend the results on the truncated predictor feedback for linear systems to a class of Lipschitz nonlinear systems with unknown time-varying input delay and the derived conditions are less conservative with respect to the existing results [24]; (iii) the information of the lower bound of delay is taken into account in the stability analysis, which benefits the derived conditions.

The remainder of this paper is organized as follows. Section 2 presents the problem formulation and a few preliminary results for the stability analysis. Section 3 presents the main results on the controller design and stability analysis. Simulation results are given in Section 4. Section 5 concludes the paper.

2 Problem statement and preliminaries

We consider the system

$$\dot{x}(t) = Ax(t) + Bu(\zeta(t)) + \phi(x(t)), \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^p$ is the input, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times p}$ are constant matrices with (A, B) being controllable, $\zeta(t)$ is a continuously differentiable function that incorporates the actuator delay, and $\phi : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $\phi(0) = 0$, is a Lipschitz nonlinear function with a Lipschitz constant γ . For any two constant vectors $a, b \in \mathbb{R}^n$,

$$\|\phi(a) - \phi(b)\| \leq \gamma \|a - b\|. \quad (2)$$

For convenience, we define $\zeta(t)$ in a more standard form $\zeta(t) = t - d(t)$, where $d(t) \geq 0$ is an unknown time-varying input delay. In

this paper, $\zeta(t)$ or $t - d(t)$ will be used whenever necessary. Additionally, we need the following assumption for controller design.

Assumption 1. The function $\zeta(t)$ is a continuously differentiable function such that $0 < \zeta_0 \leq \zeta(t) < \infty$, and the unknown delay function $d(t)$ is bounded by $0 \leq \underline{D} \leq d(t) \leq \overline{D}$.

Remark 1. As pointed out in [13], Assumption 1 guarantees that the function $\zeta(t)$ is strictly increasing and $\zeta^{-1}(t)$, the inverse function of $\zeta(t)$, always exists for all t . From Assumption 1, we also have $\frac{d}{dt}\zeta^{-1}(t) = \frac{1}{\dot{\zeta}(\zeta^{-1}(t))} \leq \frac{1}{\zeta_0}$. It is worth noting that there are two main cases of time-varying delays in the literature: (1) bounded and derivative-bounded time delay; (2) continuous uniformly bounded delays [3, 4]. The condition given in Assumption 1 belongs to case 1 and is stronger than that of case 2.

A couple of preliminary results are recalled, which are useful for the stability analysis.

Lemma 1 ([1]). For a positive definite matrix P , and a function $x : [a, b] \rightarrow \mathbb{R}^n$, with $a, b \in \mathbb{R}$ and $b > a$, the following inequality holds

$$\left(\int_a^b x^T(\tau) d\tau \right) P \left(\int_a^b x(\tau) d\tau \right) \leq (b - a) \int_a^b x^T(\tau) P x(\tau) d\tau.$$

Lemma 2 ([21, 24]). For a positive definite matrix P , the following identity holds

$$e^{A^T t} P e^{At} - e^{\omega t} P = -e^{\omega t} \int_0^t e^{-\omega \tau} e^{A^T \tau} R e^{A \tau} d\tau,$$

where $\omega \geq 0$ is a scalar and $R = -A^T P - PA + \omega P$. Furthermore, if R is positive definite, $e^{A^T t} P e^{At} \leq e^{\omega t} P$.

3 Main results

3.1 Controller design

The state of system (1) can be calculated as

$$\begin{aligned} x(t) &= e^{A(t-\zeta(t))} x(\zeta(t)) + \int_{\zeta(t)}^t e^{A(t-\tau)} \phi(x(\tau)) d\tau \\ &\quad + \int_{\zeta(t)}^t e^{A(t-\tau)} B u(\zeta(\tau)) d\tau \\ &= e^{Ad(t)} x(t-d(t)) + \int_{t-d(t)}^t e^{A(t-\tau)} \phi(x(\tau)) d\tau \\ &\quad + \int_{t-d(t)}^t e^{A(t-\tau)} B u(\tau-d(\tau)) d\tau. \end{aligned} \quad (3)$$

To compensate the input delay, a straightforward predictor-based controller $u(\zeta(t)) = Kx(t)$ has been widely used in [13–16] based on the calculation in (3). However, the right side of (3) is infinite-dimensional and involves online integration of the nonlinear function and control input $u(\cdot)$, which makes the numerical computation of the prediction challenging in practice.

To avoid these problems, a finite-dimensional controller will be designed based on the truncated predictor of the system state $e^{A(t-\zeta(t))} x(\zeta(t))$. Since $\zeta(t)$ in system (1) is unknown, the worst-case scenario is taken into account and the controller takes the

following structure:

$$u(t) = K e^{A\overline{D}} x(t), \quad (4)$$

where K is a control gain matrix to be designed later. With (4), system (1) can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BK e^{A\overline{D}} x(t-d(t)) + \phi(x) \\ &= Ax(t) + BK e^{A(\overline{D}-d(t))} e^{Ad(t)} x(t-d(t)) + \phi(x). \end{aligned}$$

By (3) and $\zeta(t) = t - d(t)$, we have

$$\dot{x}(t) = \left(A + BK e^{A(\overline{D}-d(t))} \right) x(t) + \lambda_1 + \lambda_2 + \phi(x), \quad (5)$$

where

$$\begin{aligned} \lambda_1 &= -BK e^{A(\overline{D}-d(t))} \int_{t-d(t)}^t e^{A(t-\tau)} B u(\tau-d(\tau)) d\tau, \\ \lambda_2 &= -BK e^{A(\overline{D}-d(t))} \int_{t-d(t)}^t e^{A(t-\tau)} \phi(x(\tau)) d\tau. \end{aligned}$$

Remark 2. For system (1) with $\phi(x(t)) \equiv 0$, some controllers have been proposed in the existing literature, such as

$$\begin{aligned} u(t) &= K \left[e^{A(\zeta^{-1}(t)-t)} x(t) \right. \\ &\quad \left. + \int_{\zeta(t)}^t e^{A(\zeta^{-1}(t)-\zeta^{-1}(\tau))} B \frac{u(\tau)}{\dot{\zeta}(\zeta^{-1}(\tau))} d\tau \right], \end{aligned}$$

in [13], and $u(t) = K e^{A(\zeta^{-1}(t)-t)} x(t)$ in [21]–[23]. It can be seen that the proposed controller (4) eliminates the explicit dependence on the delay function, which is essentially different from the ones given in [13, 16], [21]–[24]. This avoids the online computation of the delay inverse function and thus greatly reduces the computation burden and improves the practical implementation. Furthermore, in this paper, since the actual delay is unknown, we use the upper bound of the delay to design the controller. To a certain extent, it can be considered as worst case scenario analysis.

3.2 Stability analysis

In (4), the control gain K is chosen as

$$K = -B^T P, \quad (6)$$

where P is a positive definite matrix to be designed. The following theorem presents sufficient conditions for P to ensure that the controller (4) with (6) stabilizes the system at the origin asymptotically.

Theorem 1. For an input-delayed system (1) with Assumption 1, the global asymptotic stability can be achieved by the controller (4) with $K = -B^T P$ if there exist a positive definite matrix P and constant $\rho_1 > 0$ such that

$$\rho_1 W - BB^T \geq 0, \quad (7)$$

$$\begin{bmatrix} AW + WA^T - 2BB^T + Q & W \\ W & -\gamma_0^{-1} \end{bmatrix} < 0, \quad (8)$$

are satisfied with $W = P^{-1}$, $Q = (\kappa_0 \rho_1^2 + \kappa_1 + \kappa_2 + \kappa_3) I$, $\gamma_0 = \frac{\gamma^2}{\kappa_3} + \alpha_0 + \alpha_1 + \alpha_2$, $\alpha_0 = \kappa_0^{-1} D_0 \rho_2 \omega_0^{-1} (e^{\omega_0 D_0} - 1)$, $\alpha_1 = 2\kappa_1^{-1} \rho_1^4 \overline{D}^2 e^{2\omega_0 \overline{D}} \zeta_0^{-1}$, $\alpha_2 = \kappa_2^{-1} \rho_1^2 \overline{D}^2 e^{\omega_0 \overline{D}} \gamma^2$, where $\kappa_0, \kappa_1, \kappa_2, \kappa_3$ are any positive numbers, ω_0 and ρ_2 are positive numbers

such that $\omega_0 > \lambda_{\max}(A^T + A)$, $\rho_2 \geq \lambda_{\max}(A^T A)$, and $D_0 = \bar{D} - \underline{D}$.

Proof: To start the analysis, let us try a Lyapunov function candidate $V_0 = x^T(t)Px(t)$. The derivative of V_0 along the dynamics of (5) can be obtained as

$$\dot{V}_0(t) = x^T(t) \left(PA + A^T P - PBB^T P e^{A\bar{D}} - e^{A^T \bar{D}} PBB^T P \right) x(t) + 2x^T(t)P(\lambda_1 + \lambda_2 + \phi(x)),$$

where $\bar{D} = \bar{D} - d(t)$ is time-varying and unknown. To proceed the analysis, we construct the following integral term

$$\int_0^{\bar{D}} PBB^T PA e^{As} ds = PBB^T P e^{A\bar{D}} - PBB^T P.$$

It follows that

$$\begin{aligned} \dot{V}_0(t) &= x^T(t) \left(PA + A^T P - PBB^T P \int_0^{\bar{D}} A e^{As} ds \right. \\ &\quad \left. - \int_0^{\bar{D}} e^{A^T s} A^T ds PBB^T P - 2PBB^T P \right) x(t) \\ &\quad + 2x^T(t)P(\lambda_1(t) + \lambda_2(t) + \phi(x)). \end{aligned}$$

By Young's Inequality, we get

$$\begin{aligned} &- PBB^T P \int_0^{\bar{D}} A e^{As} ds - \int_0^{\bar{D}} e^{A^T s} A^T ds PBB^T P \\ &\leq \kappa_0 PBB^T P PBB^T P + \kappa_0^{-1} \int_0^{\bar{D}} e^{A^T s} ds A^T A \int_0^{\bar{D}} e^{As} ds \\ &\leq \kappa_0 \rho_1^2 PP + \kappa_0^{-1} \rho_2 \int_0^{\bar{D}} e^{A^T s} ds \int_0^{\bar{D}} e^{As} ds, \end{aligned} \quad (9)$$

where κ_0 is any positive number, ρ_1 is positive number such that

$$\rho_1 I \geq P^{1/2} BB^T P^{1/2}, \quad (10)$$

and $\rho_2 \geq \lambda_{\max}(A^T A)$ is defined in Theorem 1. From Lemmas 1 and 2, we have

$$\begin{aligned} \int_0^{\bar{D}} e^{A^T s} ds \int_0^{\bar{D}} e^{As} ds &\leq \bar{D} \int_0^{\bar{D}} e^{(A^T + A)s} ds \\ &\leq D_0 \int_0^{D_0} e^{\omega_0 s} ds \\ &= D_0 \omega_0^{-1} (e^{\omega_0 D_0} - 1), \end{aligned} \quad (11)$$

where $\omega_0 > \lambda_{\max}(A^T + A)$ is defined in Theorem 1. Then, from (9) and (11), we can obtain that

$$\begin{aligned} \dot{V}_0 &\leq x^T(t) \left(PA + A^T P - 2PBB^T P + \kappa_0 \rho_1^2 PP + \alpha_0 I \right) x(t) \\ &\quad + 2x^T(t)P(\lambda_1 + \lambda_2 + \phi(x)) \\ &\leq x^T(t) \left(PA + A^T P - 2PBB^T P + \tilde{\kappa} PP + \alpha_0 I \right) x(t) \\ &\quad + \kappa_1^{-1} \lambda_1^T \lambda_1 + \kappa_2^{-1} \lambda_2^T \lambda_2 + \kappa_3^{-1} \phi^T(x)\phi(x) \\ &\leq x^T(t) \left(PA + A^T P - 2PBB^T P + \left(\alpha_0 + \kappa_3^{-1} \gamma^2 \right) I \right. \\ &\quad \left. + \tilde{\kappa} PP \right) x(t) + \kappa_1^{-1} \lambda_1^T \lambda_1 + \kappa_2^{-1} \lambda_2^T \lambda_2, \end{aligned} \quad (12)$$

where $\kappa_1, \kappa_2, \kappa_3$ are positive numbers, $\tilde{\kappa} = \kappa_0 \rho_1^2 + \kappa_1 + \kappa_2 + \kappa_3$, and α_0 is defined in Theorem 1. The remaining part of the analysis

is to explore the bounds of $\kappa_1^{-1} \lambda_1^T \lambda_1$ and $\kappa_2^{-1} \lambda_2^T \lambda_2$. From Lemmas 1 and 2, we have

$$\begin{aligned} \lambda_1^T \lambda_1 &\leq \rho_1^2 d(t) \int_{t-d(t)}^t x^T(\tau - d(\tau)) e^{A^T \bar{D}} PBB^T e^{A^T(t-\tau)} \\ &\quad \times e^{(A^T + A)\bar{D}} e^{A(t-\tau)} BB^T P e^{A\bar{D}} x(\tau - d(\tau)) d\tau \\ &\leq \rho_1^4 \bar{D} \int_{\zeta(t)}^t e^{\omega_0(t-\tau+\bar{D}+\bar{D})} x^T(\zeta(\tau)) x(\zeta(\tau)) d\tau \\ &\leq \rho_1^4 \bar{D} e^{2\omega_0 \bar{D}} \int_{\zeta(t)}^t x^T(\zeta(\tau)) x(\zeta(\tau)) d\tau. \end{aligned}$$

To replace the signal $x(\zeta(\cdot))$ with the signal $x(\cdot)$, we introduce the change of the integration variable, $z = \zeta(\tau)$. By Assumption 1, we have $(d/dz) \zeta^{-1}(z) = 1/\dot{\zeta}(\zeta^{-1}(z))$, and

$$\begin{aligned} \lambda_1^T \lambda_1 &\leq \rho_1^4 \bar{D} e^{2\omega_0 \bar{D}} \int_{\zeta(\zeta(t))}^{\zeta(t)} 1/\dot{\zeta}(\zeta^{-1}(z)) x^T(z) x(z) dz \\ &\leq \rho_1^4 \bar{D} e^{2\omega_0 \bar{D}} \zeta_0^{-1} \int_{t-2\bar{D}}^t x^T(z) x(z) dz. \end{aligned}$$

In a similar way, for $\lambda_2^T \lambda_2$, we have that

$$\begin{aligned} \lambda_2^T \lambda_2 &\leq d(t) \rho_1^2 \int_{t-d(t)}^t \phi^T(x) e^{(A^T + A)(t-\tau+\bar{D}-d(t))} \phi(x) d\tau \\ &\leq d(t) \rho_1^2 \int_{t-d(t)}^t e^{\omega_0(t-\tau+\bar{D}-d(t))} \phi^T(x) \phi(x) d\tau \\ &\leq \bar{D} \rho_1^2 e^{\omega_0 \bar{D}} \gamma^2 \int_{t-\bar{D}}^t x^T(\tau) x(\tau) d\tau. \end{aligned}$$

Let $V = V_0(x(t)) + W_1(x_t) + W_2(x_t)$, where

$$\begin{aligned} W_1(x_t) &= \kappa_1^{-1} \rho_1^4 \bar{D} e^{2\omega_0 \bar{D}} \zeta_0^{-1} \int_0^{2\bar{D}} \left(\int_{t-\tau}^t x^T(z) x(z) dz \right) d\tau, \\ W_2(x_t) &= \kappa_2^{-1} \rho_1^2 \bar{D} e^{\omega_0 \bar{D}} \gamma^2 \int_0^{\bar{D}} \left(\int_{t-\tau}^t x^T(z) x(z) dz \right) d\tau, \end{aligned}$$

are two Krasovskii functionals. The derivative of $V(t)$ along the dynamics of (5) can be obtained as

$$\begin{aligned} \dot{V}(t) &= \dot{V}_0(t) + \dot{W}_1(x_t) + \dot{W}_2(x_t) \\ &\leq x^T(t) \left(PA + A^T P - 2PBB^T P \right. \\ &\quad \left. + (\kappa_0 \rho_1^2 + \kappa_1 + \kappa_2 + \kappa_3) PP + \left(\kappa_3^{-1} \gamma^2 + \alpha \right) I \right) x(t) \\ &\quad + \kappa_1^{-1} \rho_1^4 \bar{D} e^{2\omega_0 \bar{D}} \zeta_0^{-1} \int_{t-2\bar{D}}^t x^T(z) x(z) dz \\ &\quad + \kappa_2^{-1} \rho_1^2 \bar{D} e^{\omega_0 \bar{D}} \gamma^2 \int_{t-\bar{D}}^t x^T(z) x(z) dz \\ &\quad + 2\kappa_1^{-1} \rho_1^4 \bar{D}^2 e^{2\omega_0 \bar{D}} \zeta_0^{-1} x^T(t) x(t) \\ &\quad - \kappa_1^{-1} \rho_1^4 \bar{D} e^{2\omega_0 \bar{D}} \zeta_0^{-1} \int_{t-2\bar{D}}^t x^T(z) x(z) dz \\ &\quad + \kappa_2^{-1} \rho_1^2 \bar{D}^2 e^{\omega_0 \bar{D}} \gamma^2 x^T(t) x(t) \\ &\quad - \kappa_2^{-1} \rho_1^2 \bar{D} e^{\omega_0 \bar{D}} \gamma^2 \int_{t-\bar{D}}^t x^T(z) x(z) dz \\ &\leq x^T(t) \left(PA + A^T P - 2PBB^T P + \left(\kappa_3^{-1} \gamma^2 + \alpha \right) I \right. \\ &\quad \left. + (\kappa_0 \rho_1^2 + \kappa_1 + \kappa_2 + \kappa_3) PP \right) x(t), \end{aligned} \quad (13)$$

where $\alpha = \alpha_0 + \alpha_1 + \alpha_2$, α_1 and α_2 are defined in Theorem 1.

It can be seen that condition (10) is equivalent to the condition specified in (7). By Schur complement lemma, we know that condition (8) is sufficient for $\dot{V}(t) < 0$ in (13). Hence, the stability of system (1) is guaranteed by conditions (7) and (8) in Theorem 1. This completes the proof. $\square$

Remark 3. In this work, we consider the time-varying delay function $\zeta(t)$, which includes the constant input delays in [24] and [26]–[29] as a special case with $\dot{\zeta}(t) \equiv 1$. Furthermore, if $d(t) \equiv 0$, condition (28) in [24] degenerates to

$$\begin{bmatrix} WA^T + AW - BB^T + (\rho_1^2 + 1)I & W \\ W & -\frac{1}{\gamma^2}I \end{bmatrix} < 0, \quad (14)$$

and condition (8) in this paper reduces to

$$\begin{bmatrix} AW + WA^T - 2BB^T + I & W \\ W & -\frac{1}{\gamma^2}I \end{bmatrix} < 0. \quad (15)$$

Compared with (14), condition (15) is less conservative. The conditions shown in (7)–(8) can be checked by using the iterative method developed in [23]. In addition, the introduction of a set of free parameters $\kappa_0, \kappa_1, \kappa_2, \kappa_3$ provides more design degrees of freedom.

Remark 4. It is also worth noting that, the information of the delay lower bound $\underline{D}$ is taken into account in the stability analysis. The use of $\underline{D}$ will reduce the conservatism for the sufficient conditions in Theorem 1.

Remark 5. As discussed in [22], the stability of the closed-loop system (5) results from the trade-off between the magnitude of the unstable poles in the plant and the maximum delay in the input signal. When one or more open-loop poles are in the open right-half plane, the distributed term in the classical prediction feedback may not be negligible since that the acceptable prediction horizon of the TPF control is limited.

4 Numerical examples

Example 1 (constant delay case): First, the performances of the method are checked with constant delay. We consider a numerical example used in [24], where

$$\begin{aligned} \dot{x}(t) = & \begin{bmatrix} -0.09 & 1 \\ -1 & -0.09 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t - 0.03) \\ & + g \begin{bmatrix} \sin(x_1(t)) \\ 0 \end{bmatrix}. \end{aligned} \quad (16)$$

The constant delay time is 0.03 seconds, and the Lipschitz constant $\gamma = g = 0.12$. The initial condition $x(0) = [3, -5]^T$. With $\rho_1 = 0.8, \rho_2 = 0.9, \omega_0 = 0.001, \kappa_0 = \kappa_1 = \kappa_2 = 0.05, \kappa_3 = 1$, a feasible solution of the feedback gain K is found to be

$$K = \begin{bmatrix} -0.1962 & -0.6830 \end{bmatrix}. \quad (17)$$

It is worth noting that K in (17) does not satisfy the conditions of Theorem 3 in [24]. With the conditions of Theorem 3 in [24], we obtain a feasible solution with $K = \begin{bmatrix} -0.0065 & -0.1613 \end{bmatrix}$.

Fig. 1 shows the simulation results. Conditions (7)–(8) give better performance than those derived in [24], which verifies the observation in Remark 3.

Fig. 1: Closed-loop responses by Theorem 1 in this paper and Theorem 3 in [24] with $\gamma = 0.12$.

Fig. 2: The delay function $d(t)$.

Example 2 (time-varying delay case): Consider a third-order system

$$\begin{aligned} \dot{x}(t) = & \begin{bmatrix} 0.01 & 0.1 & 0 \\ 0 & -0.1 & 0.01 \\ 0 & -0.01 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1.2 & 0 \end{bmatrix} u(\zeta(t)) \\ & + \begin{bmatrix} g \sin(x_1(t)) \\ 0 \\ 0 \end{bmatrix}, \end{aligned} \quad (18)$$

The linear part of the system represents an oscillator with a positive real pole, which is open-loop unstable. The initial condition $x(0) = [10, -8, 2]^T$ is set in the simulation. The Lipschitz constant $\gamma = g = 0.005$ and the delay function $d(t) = 0.4 + 0.3 \sin(t)$, which implies that $\zeta_0 = 0.7, \underline{D} = 0.1, \bar{D} = 0.7, D_0 = 0.6$. The delay function $d(t)$ is shown as Fig. 2. With $\rho_1 = 0.05, \kappa_0 = 10, \kappa_1 = 0.01, \kappa_2 = 0.01, \kappa_3 = 0.05$, feasible solutions of W and K are found to be

$$\begin{aligned} W = & \begin{bmatrix} 28.3029 & -30.4405 & -1.1062 \\ -30.4405 & 53.1670 & -1.4043 \\ -1.1062 & -1.4043 & 68.9260 \end{bmatrix}, \\ K = & \begin{bmatrix} -0.0031 & -0.0022 & -0.0175 \\ -0.0530 & -0.0492 & -0.0019 \end{bmatrix}. \end{aligned}$$

Shown in Fig. 3 is the simulation results of the closed-loop dynamic responses. From this figure, we see that the closed-loop system is stable and the states converge to zero asymptotically. In addition, to show the advantage of the use of $\underline{D}$, a comparison has been carried out by setting $\underline{D} = 0$. Solutions of W and the corresponding

Fig. 3: Closed-loop responses with controller (4).

Fig. 4: Open-loop performance and closed-loop performance using different $\underline{D}$.

feedback gain K are found to be

$$W = \begin{bmatrix} 34.0132 & -36.1336 & -1.5963 \\ -36.1336 & 61.7054 & -0.6889 \\ -1.5963 & -0.6889 & 63.6261 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.0029 & -0.0019 & -0.0190 \\ -0.0458 & -0.0430 & -0.0016 \end{bmatrix}.$$

The comparisons of the performance are shown in Figs. 4. The yellow line represents the norm of open-loop system state which is divergent. The orange line and the blue line represent the results of two cases: (1) $\underline{D} = 0$; (2) $\underline{D} = 0.1$. The results indicate that the use of the lower bound of delay reduces the conservatism of the derived conditions.

5 Conclusion

In this paper, we have proposed a new control structure for a class of Lipschitz nonlinear systems with unknown time-varying input delay and removed the dependence on the exact delay inverse function. This greatly reduces the computation burden and thus expands the application scenarios for TPF control methodology. Sufficient conditions for the closed-loop stability independent of the delay and nonlinear functions are derived under the framework of Lyapunov-Krasovskii functionals. Simulation results show the efficiency of the proposed design.

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6 References

- Gu K., Chen J., Kharitonov V.L.: 'Stability of time-delay systems', (Springer Science & Business Media, London, 2003)
- Richard J.: 'Time-delay systems: an overview of some recent advances and open problems', *Automatica*, 2003, **39**, pp. 1667–1694
- Fridman E., Shaked U.: 'An improved stabilization method for linear time-delay systems', *IEEE Trans. Autom. Control*, 2002, **47**, pp. 1931–1937.
- Fridman E., Shaked U.: 'Delay-dependent stability and H_∞ control: constant and time-varying delays', *Int. J. Control*, 2003, **76**, pp. 48–60.
- Xia Y., Jia Y.: 'Robust sliding-mode control for uncertain time-delay systems: an LMI approach', *IEEE Trans. Autom. Control*, 2003, **48**, pp. 1086–1091
- Yue D., Han Q.L.: 'Delayed feedback control of uncertain systems with time-varying input delay', *Automatica*, 2005, **41**, pp. 233–240
- Gao H., Chen T., Lam J.: 'A new delay system approach to network-based control', *Automatica*, 2008, **44**, pp. 39–52
- Liang J., Liu X.: 'Robust stabilisation for a class of stochastic two-dimensional non-linear systems with time-varying delays', *IET Control Theory Appl.*, 2013, **7**, pp. 1699–1710
- Li T., Qian W., Wang T., et al.: 'Further results on delay-dependent absolute and robust stability for time-delay Lur'e system', *Int. J. Robust Nonlinear Control*, 2014, **24**, pp. 3303–3316
- Wang L., Wang Z., Huang T., et al.: 'An event-triggered approach to state estimation for a class of complex networks with mixed time delays and nonlinearities', *IEEE Trans. Cybern.*, 2016, **46**, pp. 2497–2508
- Manitius A., Olbrot A.: 'Finite spectrum assignment problem for systems with delays', *IEEE Trans. Autom. Control*, 1979, **24**, pp. 541–552
- Artstein Z.: 'Linear systems with delayed controls: a reduction', *IEEE Trans. Autom. Control*, 1982, **27**, pp. 869–879
- Krstic M.: 'Lyapunov stability of linear predictor feedback for time-varying input delay', *IEEE Trans. Autom. Control*, 2010, **55**, pp. 554–559
- Mazenc F., Niculescu S.I., Krstic M.: 'Lyapunov-Krasovskii functionals and applications to input delay compensation for linear-time-invariant systems', *Automatica*, 2012, **48**, pp. 1317–1323
- Léchappé V., Moulay E., Plestan F., et al.: 'New predictive scheme for the control of LTI systems with input delay and unknown disturbances', *Automatica*, 2015, **52**, pp. 179–184
- Bekiaris-Liberis N., Krstic M.: 'Compensation of time-varying input and state delays for nonlinear systems', *J.*

- 17 Besançon G., Georges D., Benayache Z.: 'Asymptotic state prediction for continuous-time systems with delayed input and application to control', in *Proceeding of 2007 European Control Conference (ECC)*, IEEE, 2007, pp. 1786–1791
- 18 Léchappé V., Moulay E., Plestan F.: 'Dynamic observation-prediction for LTI systems with a time-varying delay in the input', in *Proc. 55th IEEE conf. decis. control, CDC.*, Las Vegas, NV, 2016, pp. 2302–2307.
- 19 Lin Z., Fang H.: 'On asymptotic stability of linear systems with delayed input', *IEEE Trans. Autom. Control*, 2007, **52**, pp. 998–1013
- 20 Zhou B., Lin Z., Duan G.: 'Stabilization of linear systems with input delay and saturation—a parametric Lyapunov equation approach', *Int. J. Robust Nonlinear Control*, 2010, **20**, pp. 1502–1519
- 21 Zhou B., Lin Z., Duan G.: 'Truncated predictor feedback for linear systems with long time-varying input delay', *Automatica*, 2012, **48**, pp. 2387–2399
- 22 Yoon S.Y., Lin Z.: 'Truncated predictor feedback control for exponentially unstable linear systems with time-varying input delay', *Syst. Control Lett.*, 2013, **62**, pp. 837–844
- 23 Yoon S.Y., Anantachaisilp P., Lin Z.: 'An LMI approach to the control of exponentially unstable systems with input time delay'. In *Proc. 52nd IEEE conf. decis. control, CDC*. Firenze, Italy, 2013, pp. 312–317
- 24 Ding Z., Lin Z.: 'Truncated state prediction for control of Lipschitz nonlinear systems with input delay', In *Proc. 53rd IEEE conf. decis. control, CDC*. Los Angeles, CA, USA, 2014, pp. 1966–1971
- 25 Bresch-Pietri D., Krstic M.: 'Delay-adaptive predictor feedback for systems with unknown long actuator delay', *IEEE Trans. Autom. Control*, 2010, **55**, pp. 2106–2112
- 26 Wang C., Zuo Z., Lin Z., et al.: 'Consensus control of a Class of Lipschitz nonlinear systems with input delay', *IEEE Trans. Circuits Syst. I, Reg. Papers*, 2015, **62**, pp. 2730–2738
- 27 Wang C., Ding Z.: ' H_∞ Consensus control of multi-agent systems with input delay and directed topology', *IET Control Theory Appl.*, 2016, **10**, pp. 617–624
- 28 Zuo Z., Wang C., Ding Z.: 'Robust consensus control of uncertain multi-agent systems with input delay: a model reduction method', *Int. J. Robust Nonlinear Control*, to appear.
- 29 Wang C., Zuo Z., Lin Z., et al.: 'A truncated prediction approach to consensus control of Lipschitz nonlinear multi-agent systems with input delay', *IEEE Trans. Control Netw. Syst.*, to appear.
- 30 Zuo Z., Lin Z., Ding Z.: 'Truncated predictor control of Lipschitz nonlinear systems with time-varying input delay', *IEEE Trans. Autom. Control*, to appear.