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DOI:

[10.1049/iet-cta.2017.0407](https://doi.org/10.1049/iet-cta.2017.0407)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Song, J., Zuo, Z., & Ding, Z. (2017). Backstepping Control of Sandwich-like Nonlinear Systems with Deadzone Nonlinearity. *IET Control Theory and Applications*. <https://doi.org/10.1049/iet-cta.2017.0407>

Published in:

IET Control Theory and Applications

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Backstepping Control of Sandwich-like Nonlinear Systems with Deadzone Nonlinearity

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Abstract: This paper studies the output tracking control of a class of sandwich-like systems with state-dependent deadzone nonlinearity between two cascaded subsystems (e.g., the gear transmission systems). An exact differentiator-based backstepping control for such a sandwich-like nonlinear system is proposed. The proposed controller utilizes the high-order sliding mode robust exact differentiator to compensate the impact of non-strict feedback coupling term due to the sandwiched deadzone. The stability of the closed-loop system is guaranteed to be uniform-ultimately bounded and the output tracking error converges to a residual set. Finally, two simulation examples are provided to demonstrate the tracking performance and effectiveness of the proposed controller.

1 Introduction

Deadzone, as a static ‘memoryless’ nonlinearity which means the insensitiveness to small signals, limits the static and dynamic performance of feedback control systems. It is ubiquitous in industrial systems, such as mechanical systems, servo valves and DC motors. The existence of deadzone is inclined to cause instability and oscillation in system responses. Hence, the passion for study on the controlled systems with deadzone has never been reduced in control community.

To address the control problem for systems with deadzone, various methods have been reported in the past decades. Generally, the methods for dealing with deadzone may be divided into two classes. The first one is to construct an inverse deadzone model to cancel the deadzone nonlinearity in control design, which can be traced back to the work due to Tao and Kokotovic [1] where an adaptive inverse deadzone model was constructed. Cho and Bai [2] presented an analytical result on constructing an inverse deadzone to eliminate the effect of deadzone asymptotically based on the assumption that the input and output of the deadzone are available for measurement. Besides, by using fuzzy rules based on state errors, a fuzzy inverse deadzone model was proposed in [3]. Recently, to overcome the non-smoothness of the inverse deadzone, Zhou *et al.* [4]–[6] present a new deadzone inverse model by using a smooth indicator function to approximate the classical inverse deadzone, and furthermore, the accuracy of approximation can be improved by adjusting the characteristic parameters of the function. The other class of methods are initiated in [7], Wang *et al.* put forward a new description for deadzone which is composed by a linear part and a bounded disturbance part. This method avoids constructing an inverse deadzone model. However, this paper only considers the symmetric case. In [8], Ibrir *et al.* extended this method to a non-symmetric deadzone model and an adaptive controller was designed to solve the output tracking problem. These new deadzone descriptions in [7] and [8] have a common assumption that the slope in the linear part is precisely known. To eliminate this requirement, Zhang and Ge [9] introduced an adaptive compensation term for disturbance part and used the integral Lyapunov function to prove the semi-globally uniform ultimate boundedness of the closed-loop system. Following the same line, Zuo *et al.* proposed a novel differentiable deadzone model with a bounded approximation error in [10, 11]. And then the result was expanded in [12] to approximate the asymmetric deadzone. The

new differentiable deadzone model significantly facilitates the controller design [10]–[12]. These two classes of methods have been proved to be effective in the overwhelming majority of cases when the deadzone is located at the input.

Only a few results, however, consider the deadzone linearity appears in sandwich-like nonlinear systems. The sandwich-like nonlinear systems are referred to a class of systems with non-smooth nonlinearities (e.g. deadzone, backlash) sandwiched by interconnected subsystems. Many two-mass or master-slave systems can be categorized to these systems, such as the aircraft elevator control system driven by a hydraulic actuator and the gear transmission systems. Related work may be initiated in [13] where Saberi *et al.* prove the global stability of the cascade system by using smooth dynamic feedback with two constraint conditions and further results can be found in [14, 15]. To address the tracking problem for sandwich-like systems, Taware *et al.* [16] first proposed an inner-outer loop structured controller with a nonlinearity inverse and in [17], Taware and Tao applied the adaptive technique to the unknown deadzone case. Besides, Zhang *et al.* [18] studies a class of MIMO nonlinear systems in triangular control structure with input deadzones.

Different from the existing results, this paper focuses on applying the backstepping control approach to a class of sandwich-like nonlinear systems with deadzone. The basic idea of this paper follows the second class method as discussed previously and formulates the cascade subsystems into a compact high-order nonlinear system for controller design. However, the whole system is no longer in strict-feedback or lower-triangular form, which makes the direct application of backstepping [19, 20] inhibitive. Recently, various modified backstepping controllers have been proposed for different systems with deadzone nonlinearity [21]–[23], such as stochastic nonlinear systems, switched nonlinear large-scale systems. In particular, for non-strict feedback systems, the most common existing results usually integrate the fuzzy or neural network techniques to overcome the non-strict feedback terms. For example, Wang *et al.* [25] decompose the non-strict feedback term into a sum of smooth functions of the error which are approximated by using neural networks. Similarly, the approximation-based adaptive fuzzy tracking controller is proposed in [24]. However, in general, the neural network and fuzzy networks are restrictive in practice due to their heavy computation burden.

Motivated by the above discussions, we put forward an exact differentiator-based backstepping control design for a class of sandwich-like systems with deadzone nonlinearity. The key idea in

this paper lies in the use of a high-order sliding mode exact differentiator to counteract the impact of non-strict feedback term. The introduction of such an exact differentiator is perfectly integrated into the recursive steps and makes the backstepping control applicable for a class of systems in non-strict feedback form. The uniform-ultimately boundedness of the closed-loop system is proved rigorously within the Lyapunov framework. To validate the proposed design, a gear transmission system with elastic deadzone is employed and the simulation results show the efficacy of the proposed controller. The main contributions of this paper are twofold. (i) A new backstepping controller is firstly proposed for a class of sandwich-like systems with deadzone nonlinearity; (ii) An exact high-order sliding mode differentiator is applied to estimate the non-strict feedback term with fast convergence. The proposed controller is validated by two simulation examples including an academic system and a practical gear transmission system with *elastic* deadzone.

This paper is organized as follows. Section 2 presents the preliminary results on the higher-order sliding mode differentiator. Section 3 formulates a class of sandwich-like nonlinear systems. In Section 4, an exact differentiator-based backstepping controller is designed and the stability analysis is given. Then, two simulation examples are presented to show the effectiveness of the controller proposed in Section 5. Finally, the paper is ended with a conclusion in Section 6.

2 Preliminaries

In this section, we recall some preliminary results about the exact robust high-order sliding mode differentiator which is useful for the subsequent controller design.

The main objective of the tracking differentiator is to obtain arbitrary-order derivatives of input signals. In this paper, the higher-order sliding mode differentiator will be employed in backstepping procedures, which is proposed in [26] as

$$\begin{aligned} \dot{z}_0 &= v_0, \\ v_0 &= -\lambda_n L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t)) + z_1 \\ \dot{z}_1 &= v_1, \\ v_1 &= -\lambda_{n-1} L^{1/n} |z_1 - \dot{z}_0|^{(n-1)/n} \text{sign}(z_1 - \dot{z}_0) + z_2 \\ &\vdots \\ \dot{z}_{n-1} &= v_{n-1}, \\ v_{n-1} &= -\lambda_1 L^{1/2} |z_{n-1} - \dot{z}_{n-2}|^{1/2} \text{sign}(z_{n-1} - \dot{z}_{n-2}) + z_n \\ \dot{z}_n &= -\lambda_0 L \text{sign}(z_n - \dot{z}_{n-1}) \end{aligned} \quad (1)$$

where $f(t)$ is the input signal; z_i , $i = 0, 1, \dots, n$ are the outputs which represent the tracking results for the arbitrary-order derivative $f^{(i)}(t)$ respectively; λ_i , $i = 0, 1, \dots, n$ is positive parameter which provide for the finite-time convergence of the differentiator; L is a known Lipschitz constant of the n th derivative.

Remark 1. L is the Lipschitz constant which is usually found by computer simulation [31]. In particular, a common choice of the differentiator parameters, $\lambda_0 = 1.1$, $\lambda_1 = 1.5$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 5$, $\lambda_5 = 8$, is sufficient for $n \leq 5$ [26].

Lemma 1. [26] The parameters being properly chosen, the following equalities are true after a finite time of a transient process

$$z_0(t) = f(t); \quad z_i(t) = v_{i-1}(t) = f^{(i)}(t), \quad i = 1, \dots, n$$

Remark 2. Lemma 1 guarantees that all estimation errors $\sigma_i(t) \triangleq z_i(t) - f^{(i)}(t)$, $i = 0, 1, \dots, n$, and $\varsigma_i(t) \triangleq z_i(t) - v_{i-1}(t)$, $i = 1, 2, \dots, n$ are uniformly bounded, i.e., there exist constants $\bar{\sigma}_i$ and $\bar{\varsigma}_i$ such that $|\sigma_i| \leq \bar{\sigma}_i$ and $|\varsigma_i| \leq \bar{\varsigma}_i$ for all $t > 0$. Furthermore, the errors σ_i and ς_i converge to zero after a finite time of a transient process.

Fig. 1: The sandwich-like system with deadzone

3 Problem formulation

Consider a sandwich-like nonlinear system with deadzone which consists of two subsystems:

$$\Sigma_1 : \begin{cases} \dot{\bar{\varepsilon}}_i = p_i(\bar{\varepsilon}_i) + \varepsilon_{i+1}, & i = 1, 2, \dots, r-1 \\ \dot{\bar{\varepsilon}}_r = p_r(\bar{\varepsilon}_r) + b_1 D(\bar{\varepsilon}_r, \bar{\xi}_m) \end{cases} \quad (2)$$

$$\Sigma_2 : \begin{cases} \dot{\bar{\xi}}_j = w_j(\bar{\xi}_j) + \xi_{j+1}, & j = 1, 2, \dots, m-1 \\ \dot{\bar{\xi}}_m = w_m(\bar{\xi}_m) + b_2 [u - D(\bar{\varepsilon}_r, \bar{\xi}_m)] \end{cases} \quad (3)$$

$$y = \varepsilon_1 \quad (4)$$

where $\bar{\varepsilon}_r = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_r]^T \in \mathcal{X}_1 \subset \mathbb{R}^r$ and $\bar{\xi}_m = [\xi_1, \xi_2, \dots, \xi_m]^T \in \mathcal{X}_2 \subset \mathbb{R}^m$ are the states of subsystem Σ_1 and Σ_2 , respectively, $\bar{\varepsilon}_i := [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_i]^T$, $\bar{\xi}_j := [\xi_1, \xi_2, \dots, \xi_j]^T$, $i = 1, 2, \dots, r$, $j = 1, 2, \dots, m$, $u \in \mathbb{R}$ is the control input and y is the output; $p_i(\bar{\varepsilon}_i)$, $w_j(\bar{\xi}_j)$ are smooth functions; b_1 and b_2 are constants; $D(\bar{\varepsilon}_r, \bar{\xi}_m) : \mathbb{R}^{r+m} \rightarrow \mathbb{R}^{r+m}$ represents the coupling term of deadzone nonlinearity between Σ_1 and Σ_2 , defined by:

$$D(\bar{\varepsilon}_r, \bar{\xi}_m) = \begin{cases} D_r(\ell(\bar{\varepsilon}_r, \bar{\xi}_m)), & \ell(\bar{\varepsilon}_r, \bar{\xi}_m) > b_r \\ 0, & b_l \leq \ell(\bar{\varepsilon}_r, \bar{\xi}_m) \leq b_r \\ D_l(\ell(\bar{\varepsilon}_r, \bar{\xi}_m)), & \ell(\bar{\varepsilon}_r, \bar{\xi}_m) < b_l \end{cases} \quad (5)$$

where $\ell(\bar{\varepsilon}_r, \bar{\xi}_m)$ is a state-dependent function, b_r and b_l are the breakpoints, $D_r(\ell(\bar{\varepsilon}_r, \bar{\xi}_m))$ and $D_l(\ell(\bar{\varepsilon}_r, \bar{\xi}_m))$ are dynamics outside the interval $[b_l, b_r]$.

To facilitate the controller design, the two subsystems (3) and (4) are written into a compact form. Let $\bar{x}_n = [x_1, x_2, \dots, x_r, x_{r+1}, x_{r+2}, \dots, x_n]^T = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_r, \xi_1, \xi_2, \dots, \xi_m]^T \in \mathcal{X} \subset \mathbb{R}^{r+m}$, where $n = r + m$. The sandwich-like system can be rearranged as

$$\Sigma : \begin{cases} \dot{x}_1 = f_1(\bar{x}_1) + x_2 \\ \vdots \\ \dot{x}_r = f_r(\bar{x}_r) + b_1 D(\bar{x}_n) \\ \dot{x}_{r+1} = f_{r+1}(\bar{x}_{r+1}) + x_{r+2} \\ \vdots \\ \dot{x}_n = f_n(\bar{x}_n) + b_2 [u - D(\bar{x}_n)] \\ y = x_1 \end{cases} \quad (6)$$

Assumption 1. The deadzone $D(\bar{x}_n)$ can be written as $D(\bar{x}_n) = F(\bar{x}_n) + \delta(\bar{x}_n)$ which implies that the deadzone can be approximated by a differentiable function $F(\bar{x}_n)$ with an approximation error $\delta(\bar{x}_n)$, bounded by $|\delta(\bar{x}_n)| \leq \bar{d}$, where $\bar{d}$ is a known constant.

Remark 3. Assumption 1 is quite general in dealing with the deadzone nonlinearity. To show this, consider the following deadzone model in [28]:

$$\text{Dead}[\theta] = \begin{cases} k_s(\theta - \alpha), & \text{if } \theta > \alpha \\ 0, & \text{if } |\theta| \leq \alpha \\ k_s(\theta + \alpha), & \text{if } \theta < -\alpha \end{cases} \quad (7)$$

where α is the deadzone gap, k_s is the torque coefficient, θ is the relative displacement. Then, the deadzone can be rewritten as

$Dead[\theta] = k_s \theta + \Delta d$ with $\Delta d \leq k_s \alpha$ [7]. In other words, the dead-zone is decomposed by a linear differentiable part and a bounded disturbance part, which implies Assumption 1 holds.

Assumption 2. The differential function in Assumption 1 is $F(\bar{x}_n) = F_1(\bar{x}_r) + F_2(x_{r+2}, x_{r+3}, \dots, x_n) + F_3(x_{r+1})$, where $F_3(x_{r+1}) = \rho x_{r+1}$ with a constant ρ . In particular, F_2 is an C^{n-r+1} function with its j th derivative bounded on $\mathcal{X}$, $j = 0, 1, \dots, n-r+1$.

Remark 4. Assumption 2 is made to facilitate the backstepping control design, which seems a little restrictive. This restriction on $F_3(x_{r+1})$ requires that $F(\bar{x}_n)$ is linearly dependent on the first state of Σ_2 , usually a position state of a practical system. However, it holds in general for various backlash models.

Remark 5. It is worth noting that the term F_2 extracted from $D(x_n)$ breaks the strict feedback structure in (6) and this makes the direct application of backstepping design inhibitive. The assumption on F_2 is made for implementation of the robust exact differentiator and the solution of the differentiator can be understood in the Filippov sense [26]. Due to the cascade structure as shown in Fig. 1, the boundedness of F_2 in Σ_1 and its derivatives can be guaranteed in the sense that the responses of Σ_2 can be made faster than Σ_1 by proper selection of control gains. In practical applications, the function F_2 can be unknown for the differentiator.

Let

$$\begin{aligned} a_1 &= b_1 \rho \\ \varphi_r(\bar{x}_r) &= F_1(\bar{x}_r) + f_r(\bar{x}_r) \\ \varphi_d(\bar{x}_{r+2,n}) &= F_2(\bar{x}_{r+2,n}) \\ \varphi_n(\bar{x}_n) &= f_n(\bar{x}_n) - b_2 F(\bar{x}_n) \end{aligned}$$

where $\bar{x}_{r+2,n} \triangleq [x_{r+2}, x_{r+3}, \dots, x_n]^T$. By utilizing Assumption 1, (6) becomes

$$\begin{cases} \dot{x}_1 = \varphi_1(\bar{x}_1) + x_2 \\ \vdots \\ \dot{x}_r = \varphi_r(\bar{x}_r) + a_1 x_{r+1} + \varphi_d(\bar{x}_{r+2,n}) + b_1 \delta(\bar{x}_n) \\ \dot{x}_{r+1} = \varphi_{r+1}(\bar{x}_{r+1}) + x_{r+2} \\ \vdots \\ \dot{x}_n = \varphi_n(\bar{x}_n) + b_2 u - b_2 \delta(\bar{x}_n) \\ y = x_1 \end{cases} \quad (8)$$

Based on system (8), the following assumption is made for controller design.

Assumption 3. All the states in (8) are available for measurement, and parameters a_1, b_1, b_2 and functions $\varphi_i(\bar{x}_i), i = 1, 2, \dots, n$ are precisely known.

The control objective is to design a full-state feedback controller u such that (i) all the closed-loop signals are bounded and (ii) for any given sufficiently smooth reference trajectory $y_d(t)$, the system output $y(t)$ converges asymptotically to the neighborhood of $y_d(t)$.

4 Controller design

This section presents a backstepping control design with a high-order sliding mode differentiator to eliminate the impact of the non-strict feedback term. For simplicity, we denote $\varphi_i(\bar{x}_i), \varphi_d(\bar{x}_{r+2,n})$ and $\delta(\bar{x}_n)$ as $\varphi_i, i = 1, \dots, n, \varphi_d$ and δ , respectively.

STEP 1 Introduce two error variables

$$e_1 = x_1 - y_d \quad (9)$$

$$e_2 = x_2 - \beta_1 \quad (10)$$

where β_1 is the first stabilizing function. Choose the Lyapunov function $V_1 = \frac{1}{2} e_1^2$. By taking derivative of V_1 along (8), one obtains

$$\begin{aligned} \dot{V}_1 &= e_1 \dot{e}_1 \\ &= e_1 (\dot{x}_1 - \dot{y}_d) \\ &= e_1 [\varphi_1 + e_2 + \beta_1 - \dot{y}_d] \end{aligned} \quad (11)$$

Let

$$\beta_1 = -(\varphi_1 + k_1 e_1 - \dot{y}_d) \quad (12)$$

where k_1 is a positive constant. By substituting (12) into (11), one has

$$\dot{V}_1 = -k_1 e_1^2 + e_1 e_2 \quad (13)$$

STEP r Introduce the error variable $e_{r+1} = x_{r+1} - \beta_r$, where β_r is the r th stabilizing function. Choose the Lyapunov function as $V_r = V_{r-1} + \frac{1}{2} e_r^2$. Computing the time derivative of V_r obtains

$$\begin{aligned} \dot{V}_r &= \dot{V}_{r-1} + e_r \dot{e}_r \\ &= -\sum_{i=1}^{r-1} k_i e_i^2 + e_{r-1} e_r + e_r (a_1 x_{r+1} + \varphi_r + \varphi_d + b_1 \delta \\ &\quad - \dot{\beta}_{r-1}) \\ &= \sum_{i=1}^{r-1} k_i e_i^2 + e_{r-1} e_r + e_r [a_1 (e_{r+1} + \beta_r) + \varphi_r + \varphi_d \\ &\quad + b_1 \delta - \dot{\beta}_{r-1}] \end{aligned} \quad (14)$$

where

$$\dot{\beta}_{r-1} = \sum_{i=1}^{r-1} \frac{\partial \beta_{r-1}}{\partial x_i} (\varphi_i + x_{i+1}) + \sum_{i=1}^r \frac{\partial \beta_{r-1}}{\partial y_d^{(i-1)}} y_d^{(i)} \quad (15)$$

By (1) and Assumption 2, the $(n-r)$ th order sliding mode differentiator is employed:

$$\begin{aligned} \dot{z}_0 &= v_0, \\ v_0 &= -\lambda_{n-r} L^{\frac{1}{n-r+1}} |z_0 - \varphi_d(t)|^{\frac{n-r}{n-r+1}} \text{sign}(z_0 - \varphi_d(t)) + z_1 \\ \dot{z}_1 &= v_1, \\ v_1 &= -\lambda_{n-r-1} L^{\frac{1}{n-r}} |z_1 - v_0|^{\frac{n-r-1}{n-r}} \text{sign}(z_1 - v_0) + z_2 \\ &\vdots \\ \dot{z}_{n-r-1} &= v_{n-r-1}, \\ v_{n-r-1} &= -\lambda_1 L^{\frac{1}{2}} |z_{n-r-1} - v_{n-r-2}|^{\frac{1}{2}} \text{sign}(z_{n-r-1} \\ &\quad - v_{n-r-2}) + z_n \\ \dot{z}_{n-r} &= -\lambda_0 L \text{sign}(z_{n-r} - v_{n-r-1}) \end{aligned} \quad (16)$$

where $z_i, i = 0, 1, \dots, n-r$, are the outputs of the differentiator which are used to estimate all the derivatives of the non-strict

feedback term, $\varphi_d^{(i)}$, respectively. Applying (16), one has

$$\begin{aligned} \beta_r = & -\frac{1}{a_1} \left[\varphi_r + z_0 + \left(k_r + 1 + b_1^2 \right) e_r + e_{r-1} \right. \\ & \left. - \sum_{i=1}^{r-1} \frac{\partial \beta_{r-1}}{\partial x_i} (\varphi_i + x_{i+1}) - \sum_{i=1}^r \frac{\partial \beta_{r-1}}{\partial y_d^{(i-1)}} y_d^{(i)} \right] \quad (17) \end{aligned}$$

where k_r is a positive constants, z_0 is the estimate of φ_d . Recalling Remark 2, we have that the tracking error $|\sigma_0| = |z_0 - \varphi_d| \leq \bar{\sigma}_0$. Substituting (17) into (14) and using the Young's inequality and Assumption 1, we obtain

$$\begin{aligned} \dot{V}_r = & -\sum_{i=1}^r k_i e_i^2 - b_1^2 e_r^2 - e_r^2 + b_1 e_r \delta + e_r \sigma_0 \\ & + a_1 e_r e_{r+1} \\ \leq & -\sum_{i=1}^r k_i e_i^2 + a_1 e_r e_{r+1} + \frac{\bar{d}^2}{4} + \frac{\bar{\sigma}_0^2}{4} \quad (18) \end{aligned}$$

STEP $r+1$ Define $e_{r+2} = x_{r+2} - \beta_{r+1}$, where β_{r+1} is the $(r+1)$ th stabilizing function. Choose the Lyapunov function as $V_{r+1} = V_r + \frac{1}{2} e_{r+1}^2$. Taking the time derivative of V_{r+1} , one has

$$\begin{aligned} \dot{V}_{r+1} = & \dot{V}_r + e_{r+1} \dot{e}_{r+1} \\ \leq & -\sum_{i=1}^r k_i e_i^2 + a_1 e_r e_{r+1} + \frac{\bar{d}^2}{4} + \frac{\bar{\sigma}_0^2}{4} \\ & + e_{r+1} (\varphi_{r+1} + x_{r+2} - \dot{\beta}_r) \\ \leq & -\sum_{i=1}^r k_i e_i^2 + a_1 e_r e_{r+1} + \frac{\bar{d}^2}{4} + \frac{\bar{\sigma}_0^2}{4} \\ & + e_{r+1} (\varphi_{r+1} + e_{r+2} + \beta_{r+1} - \dot{\beta}_r) \quad (19) \end{aligned}$$

where

$$\begin{aligned} \dot{\beta}_r = & \sum_{i=1}^{r-1} \frac{\partial \beta_r}{\partial x_i} (\varphi_i + x_{i+1}) + \frac{\partial \beta_r}{\partial x_r} (a_1 x_{r+1} + \varphi_r + \varphi_d \\ & + b_1 \delta) + \sum_{i=1}^{r+1} \frac{\partial \beta_{r+1}}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial \beta_r}{\partial z_0} \dot{z}_0 \\ = & \sum_{i=1}^{r-1} \frac{\partial \beta_r}{\partial x_i} (\varphi_i + x_{i+1}) + \frac{\partial \beta_r}{\partial x_r} (a_1 x_{r+1} + \varphi_r + \varphi_d \\ & + b_1 \delta) + \sum_{i=1}^{r+1} \frac{\partial \beta_{r+1}}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial \beta_r}{\partial z_0} v_0 \quad (20) \end{aligned}$$

Let

$$\begin{aligned} \beta_{r+1} = & \sum_{i=1}^{r-1} \frac{\partial \beta_r}{\partial x_i} (\varphi_i + x_{i+1}) + \frac{\partial \beta_r}{\partial x_r} (a_1 x_{r+1} + \varphi_r + z_0) \\ & + \sum_{i=1}^r \frac{\partial \beta_r}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial \beta_r}{\partial z_0} z_1 - a_1 e_r - \varphi_{r+1} \\ & - \left[k_{r+1} + \left(\frac{\partial \beta_r}{\partial x_r} \right)^2 (1 + b_1^2) + \left(\frac{\partial \beta_r}{\partial z_0} \right)^2 \right] e_{r+1} \quad (21) \end{aligned}$$

where k_{r+1} is a positive constant, z_1 is the estimate of $\dot{\varphi}_d$. Recalling Remark 2, we have $|\sigma_0| = |z_0 - \varphi_d| \leq \bar{\sigma}_0$ and $|\varsigma_1| = |z_1 - v_0| \leq$

$\bar{\varsigma}_1$. Substituting (21) into (19) and applying the Young's inequality, we have

$$\begin{aligned} \dot{V}_{r+1} \leq & -\sum_{i=1}^r k_i e_i^2 + \frac{\bar{d}^2}{4} + \frac{\bar{\sigma}_0^2}{4} - \left[k_{r+1} + \left(\frac{\partial \beta_r}{\partial x_r} \right)^2 (1 + b_1^2) \right. \\ & \left. + \left(\frac{\partial \beta_r}{\partial z_0} \right)^2 \right] e_{r+1}^2 + \frac{\partial \beta_r}{\partial z_0} \varsigma_1 e_{r+1} + e_{r+1} e_{r+2} \\ & + \frac{\partial \beta_r}{\partial x_r} (\sigma_0 - b_1 \delta) e_{r+1} \\ \leq & -\sum_{i=1}^{r+1} k_i e_i^2 + e_{r+1} e_{r+2} + 2 \frac{\bar{d}^2}{4} + 2 \frac{\bar{\sigma}_0^2}{4} + \frac{\bar{\varsigma}_1^2}{4} \quad (22) \end{aligned}$$

STEP $r+2$ Define $e_{r+3} = x_{r+3} - \beta_{r+2}$, where β_{r+2} is the $(r+2)$ th stabilizing function. Choose the Lyapunov function as $V_{r+2} = V_{r+1} + \frac{1}{2} e_{r+2}^2$. Taking the time derivative of V_{r+2} yields

$$\begin{aligned} \dot{V}_{r+2} = & \dot{V}_{r+1} + e_{r+2} \dot{e}_{r+2} \\ \leq & -\sum_{i=1}^{r+1} k_i e_i^2 + e_{r+1} e_{r+2} + 2 \frac{\bar{d}^2}{4} + 2 \frac{\bar{\sigma}_0^2}{4} + \frac{\bar{\varsigma}_1^2}{4} \\ & + e_{r+2} (\varphi_{r+2} + x_{r+3} - \dot{\beta}_{r+1}) \\ \leq & -\sum_{i=1}^{r+1} k_i e_i^2 + e_{r+1} e_{r+2} + 2 \frac{\bar{d}^2}{4} + 2 \frac{\bar{\sigma}_0^2}{4} + \frac{\bar{\varsigma}_1^2}{4} \\ & + e_{r+2} (\varphi_{r+2} + e_{r+3} + \beta_{r+2} - \dot{\beta}_{r+1}) \quad (23) \end{aligned}$$

where

$$\begin{aligned} \dot{\beta}_{r+1} = & \sum_{i=1}^{r-1} \frac{\partial \beta_{r+1}}{\partial x_i} (\varphi_i + x_{i+1}) + \frac{\partial \beta_{r+1}}{\partial x_r} (a_1 x_{r+1} + \varphi_r + \varphi_d \\ & + b_1 \delta) + \frac{\partial \beta_{r+1}}{\partial x_{r+1}} (\varphi_{r+1} + x_{r+2}) + \sum_{i=1}^{r+1} \frac{\partial \beta_{r+1}}{\partial y_d^{(i-1)}} y_d^{(i)} \\ & + \frac{\partial \beta_{r+1}}{\partial z_0} \dot{z}_0 + \frac{\partial \beta_{r+1}}{\partial z_1} \dot{z}_1 \\ = & \sum_{i=1}^{r-1} \frac{\partial \beta_{r+1}}{\partial x_i} (\varphi_i + x_{i+1}) + \frac{\partial \beta_{r+1}}{\partial x_r} (a_1 x_{r+1} + \varphi_r + \varphi_d \\ & + b_1 \delta) + \frac{\partial \beta_{r+1}}{\partial x_{r+1}} (\varphi_{r+1} + x_{r+2}) + \sum_{i=1}^{r+1} \frac{\partial \beta_{r+1}}{\partial y_d^{(i-1)}} y_d^{(i)} \\ & + \frac{\partial \beta_{r+1}}{\partial z_0} v_0 + \frac{\partial \beta_{r+1}}{\partial z_1} v_1 \quad (24) \end{aligned}$$

Let

$$\begin{aligned} \beta_{r+2} = & \sum_{i=1}^{r-1} \frac{\partial \beta_{r+1}}{\partial x_i} (\varphi_i + x_{i+1}) + \frac{\partial \beta_{r+1}}{\partial x_r} (a_1 x_{r+1} + \varphi_r + z_0) \\ & + \frac{\partial \beta_{r+1}}{\partial x_{r+1}} (\varphi_{r+1} + x_{r+2}) + \sum_{i=1}^{r+1} \frac{\partial \beta_{r+1}}{\partial y_d^{(i-1)}} y_d^{(i)} \\ & + \sum_{i=0}^1 \frac{\partial \beta_{r+1}}{\partial z_i} z_{i+1} - e_{r+1} - \varphi_{r+2} \\ & - \left[k_{r+2} + \left(\frac{\partial \beta_{r+1}}{\partial x_r} \right)^2 (1 + b_1^2) + \sum_{i=0}^1 \left(\frac{\partial \beta_{r+1}}{\partial z_i} \right)^2 \right] e_{r+2} \quad (25) \end{aligned}$$

where k_{r+2} is a positive constant, z_2 is the estimate of $\dot{\varphi}_d$. In view of $|\varsigma_2| = |z_2 - v_1| \leq \bar{\varsigma}_2$, substituting (25) into (23) and applying

the Young's inequality, one obtains

$$\begin{aligned}\dot{V}_{r+2} &\leq -\sum_{i=1}^{r+1} k_i e_i^2 + 2\frac{\bar{d}^2}{4} + 2\frac{\bar{\sigma}_0^2}{4} + \frac{\bar{\zeta}_1^2}{4} \\ &\quad - \left[k_{r+2} + \left(\frac{\partial \beta_{r+1}}{\partial x_r} \right)^2 (1 + b_1^2) \right. \\ &\quad \left. + \sum_{i=0}^1 \left(\frac{\partial \beta_{r+1}}{\partial z_i} \right)^2 \right] e_{r+2}^2 + \frac{\partial \beta_r}{\partial x_r} (\sigma_0 - b_1 \delta) e_{r+2} \\ &\quad + \sum_{i=0}^1 \frac{\partial \beta_{r+1}}{\partial z_i} \zeta_{i+1} e_{r+2} + e_{r+2} e_{r+3} \\ &\leq -\sum_{i=1}^{r+2} k_i e_i^2 + 3\frac{\bar{d}^2}{4} + 3\frac{\bar{\sigma}_0^2}{4} + 2\frac{\bar{\zeta}_1^2}{4} + \frac{\bar{\zeta}_2^2}{4}\end{aligned}\quad (26)$$

STEP n Choose the Lyapunov function $V_n = V_{n-1} + \frac{1}{2}e_n^2$, where $e_n = x_n - \beta_{n-1}$ with β_{n-1} the $(n-1)$ th stabilizing function. Computing the derivative of V_n yields

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + e_n \dot{e}_n \\ &\leq -\sum_{i=1}^{n-1} k_i e_i^2 + \frac{n-r}{4} (\bar{d}^2 + \bar{\sigma}_0^2) + \sum_{i=1}^{n-r-1} \frac{(n-r-i)\bar{\zeta}_i^2}{4} \\ &\quad + e_{n-1} e_n + e_n (\varphi_n + b_2 u - b_2 \delta - \dot{\beta}_{n-1})\end{aligned}\quad (27)$$

where

$$\begin{aligned}\dot{\beta}_{n-1} &= \sum_{i=1}^{r-1} \frac{\partial \beta_{n-1}}{\partial x_i} (\varphi_i + x_{i+1}) + \sum_{i=r+1}^{n-1} \frac{\partial \beta_{n-1}}{\partial x_i} (\varphi_i + x_{i+1}) \\ &\quad + \frac{\partial \beta_{n-1}}{\partial x_r} (a_1 x_{r+1} + \varphi_r + \varphi_d + b_1 \delta) \\ &\quad + \sum_{i=0}^{n-r-1} \frac{\partial \beta_{n-1}}{\partial z_i} v_i + \sum_{i=1}^{n-1} \frac{\partial \beta_{n-1}}{\partial y_d^{(i-1)}} y_d^{(i)}\end{aligned}\quad (28)$$

Let

$$\begin{aligned}\beta_n &= \sum_{i=1}^{r-1} \frac{\partial \beta_{n-1}}{\partial x_i} (\varphi_i + x_{i+1}) + \sum_{i=r+1}^{n-1} \frac{\partial \beta_{n-1}}{\partial x_i} (\varphi_i + x_{i+1}) \\ &\quad + \frac{\partial \beta_{n-1}}{\partial x_r} (a_1 x_{r+1} + \varphi_r + z_0) - \varphi_n \\ &\quad + \sum_{i=0}^{n-r-1} \frac{\partial \beta_{n-1}}{\partial z_i} z_{i+1} + \sum_{i=1}^{n-1} \frac{\partial \beta_{n-1}}{\partial y_d^{(i-1)}} y_d^{(i)} - e_{n-1} \\ &\quad - \left[k_n + \sum_{i=0}^{n-r-1} \left(\frac{\partial \beta_{n-1}}{\partial z_i} \right)^2 + \left(\frac{\partial \beta_{n-1}}{\partial x_r} \right)^2 (1 + b_1^2) \right. \\ &\quad \left. + b_2^2 \right] e_n\end{aligned}\quad (29)$$

Then, the control input is

$$u = \frac{\beta_n}{b_2}\quad (30)$$

where k_n is a positive constant, z_{n-r} is the estimate of the $\varphi_d^{(n-r)}$. Recalling Remark 2 $|\zeta_{n-r}| = |z_{n-r} - v_{n-r-1}| \leq \bar{\zeta}_{n-r}$ and substituting (30) into (27), the derivative of V_n can be derived as

$$\begin{aligned}\dot{V}_n &\leq -\sum_{i=1}^n k_i e_i^2 + \frac{n-r+2}{4} \bar{d}^2 + \frac{n-r+1}{4} \bar{\sigma}^2 \\ &\quad + \sum_{i=1}^{n-r} \frac{n-r-i+1}{4} \bar{\zeta}_i^2 \\ &\leq -\mu V_n + M^*\end{aligned}\quad (31)$$

where $\mu = \min \{2k_1, \dots, 2k_n\}$, $M^* = \frac{n-r+2}{4} \bar{d}^2 + \frac{n-r+1}{4} \bar{\sigma}^2 + \sum_{i=1}^{n-r} \frac{n-r-i+1}{4} \bar{\zeta}_i^2$. Integrating both sides of (31) obtains

$$V_n(t) \leq V_n(0) \exp(-\mu t) + \frac{M^*}{\mu} (1 - \exp(-\mu t))\quad (32)$$

From the definition of V_n , we have

$$\lim_{t \rightarrow \infty} \frac{1}{2} |e_i(t)|^2 \leq \lim_{t \rightarrow \infty} V_n(t) \leq \frac{M^*}{\mu}\quad (33)$$

which implies that the tracking error e_i , $i = 1, 2, \dots, n$ exponentially converges to a compact set $E_{r_i} = \{e_i \mid |e_i| \leq \sqrt{2M^*/\mu}\}$, respectively.

Remark 6. From the ultimate bound of tracking error e_1 in (33), it can be observed that the bound can be decreased by increasing μ . In other words, the tracking performance could be improved by selecting sufficiently large control gains k_i .

To this end, the main result of this paper can be summarised in the following theorem.

Theorem 1. Consider the sandwich-like system with deadzone nonlinearity in (2) and (3). For any given sufficiently smooth desired trajectory $y_d(t)$, the controller (30) ensures that all the closed-loop signals are bounded and the tracking error $y(t) - y_d(t)$ exponentially converges to a compact set $E_{r_1} = \{e_1 \mid |e_1| \leq \sqrt{2M^*/\mu}\}$.

Proof: It has been shown from (33) that all the tracking errors are bounded. By Remark 2 and Assumption 2, the boundedness of outputs of the differentiator $z_0, z_1, \dots, z_{n-r}$ can be proved. Since $x_1 = e_1 + y_d$ and y_d is bounded, x_1 is also bounded. The boundedness of x_2 follows from the boundedness of e_2 and β_1 in (12). Similarly, the boundedness of x_i , $i = 3, \dots, r$ can be guaranteed from the boundedness of e_i and β_{i-1} . The boundedness of β_r can be ensured from the boundedness of z_0 , x_j , $j = 1, \dots, r$ and derivatives of y_d . Thus, x_{r+1} is bounded from the fact that $e_{r+1} = x_{r+1} + \beta_r$. Similarly, the boundedness of x_i , $i = r+2, \dots, n$ can be ensured recursively. From (30), the control input is bounded. Therefore, the boundedness of all the closed-loop signal follows. $\square$

Remark 7. It is worth noting that the robust backstepping control methodology [30] is also applicable if the unmatched term $\varphi_d(\bar{x}_{r+2,n})$ in (8) is bounded, i.e., $|\varphi_d| \leq \bar{w}$. Following the same line as shown in [30], the error vector $e = [e_1, e_2, \dots, e_n]^T$ of the closed-loop system is also ultimately bounded:

$$\Omega = \left\{ e \mid e^T e \leq \frac{1}{c-1} \left[\bar{w}^2 + 2|b_1| \bar{w} \bar{d} + (b_1^2 + b_2^2) \bar{d}^2 \right] \right\}$$

where $c > 1$ is the parameter. In this paper, however, the sliding mode differentiator is used to estimate and counteract exactly in finite time the unmatched term $\varphi_d(\bar{x}_{r+2,n})$ if the Lipschitz constant L of its n th derivative is known. Invoking Lemma 1, the ultimate

bound E_r due to our proposed differentiator after a finite time of a transient process can be further reduced to

$$E_r = \left\{ e | e^T e \leq \frac{1}{\mu} \left(\frac{n-r+2}{4} d^2 \right) \right\}$$

Obviously, Ω is more conservative due to the existence of $\bar{w}$.

5 Simulation

5.1 Example 1: a fourth-order nonlinear sandwich-like system

To validate the effectiveness of the proposed controller, consider the following nonlinear sandwich-like system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.3\sin(x_1) - 3x_2 + x_2^2 + D \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \sin(x_1)x_2 + x_3^2 + \cos(x_4) - 0.1x_4 + (u - D) \end{cases} \quad (34)$$

where

$$D = \begin{cases} \ell - 0.15 & \ell > 0.15 \\ 0 & |\ell| \leq 0.15 \\ \ell + 0.15 & \ell < -0.15 \end{cases}$$

is the deadzone nonlinearity with $\ell = 3(x_3 - x_1) + (x_4 - x_2)$. Note that the non-strict feedback term $\varphi_d = x_4$ appears in the second line of the system dynamics and ruins the strict-feedback form. Thus, a second-order sliding mode differentiator with $L = 1$ is adopted

$$\begin{aligned} \dot{z}_0 &= -3|z_0 - \varphi_d(t)|^{\frac{2}{3}} \text{sign}(z_0 - \varphi_d(t)) + z_1 \\ \dot{z}_1 &= -1.5|z_1 - \dot{z}_0|^{\frac{1}{2}} \text{sign}(z_1 - \dot{z}_0) + z_2 \\ \dot{z}_2 &= -1.1 \text{sign}(z_2 - \dot{z}_1) \end{aligned} \quad (35)$$

The desired trajectory is taken as $y_d = \sin(0.4t) + \sin(0.2t)$, and the proposed controller is employed with $k_1 = 2.5$, $k_2 = 1.5$, $k_3 = 1$, $k_4 = 1$. The initial conditions are $x(0) = [2, 0, 0, 0]^T$ and $z(0) = [0, 1, 0]^T$ are set. The sampling time is 0.005s.

Figs. 2–5 show the simulation results. Fig. 2 shows that a fairly satisfactory tracking performance is obtained. The bounded tracking error is presented in Fig. 3. It is worth noting that a series of small jerks can be observed from Fig. 3 which are caused by the nonsmoothness of the deadzone. The control input is provided in Fig. 4. The the outputs of the differentiator for the compensation of the non-strict feedback term are illustrated in Fig. 5. The simulation results demonstrate the efficacy of the proposed controller for the sandwich-like nonlinear systems with deadzone.

5.2 Example 2: a gear transmission system with elastic deadzone

In this section, a gear transmission system with *elastic deadzone* is employed to verify the effectiveness of the proposed controller in Section 4.

As shown in [28], the dynamics of this gear transmission system can be described by

$$\begin{cases} J_l \dot{\omega}_l = -c_l \omega_l + T_s \\ J_m \dot{\omega}_m = -c_m \omega_m - T_s + T_m \\ \omega_d = \omega_m - \omega_l \\ \text{with} \\ \dot{\theta}_m = \omega_m, \dot{\theta}_l = \omega_l, \dot{\theta}_d = \omega_d \\ T_s = k_s \theta_d + c_s \dot{\theta}_d \end{cases} \quad (36)$$

where J_l and J_m are the moment inertia of the load side and the motor side, respectively, T_s represents the transmitted shaft torque,

Fig. 2: Output tracking response: reference signal y_d , solid line; output x_1 , dash line

Fig. 3: Output tracking errors

Fig. 4: Control input

Fig. 5: Differentiator outputs

T_m is the motor torque which can be treated as an input, c_l and c_m are the viscous load friction and the viscous motor friction, respectively, k_s is the elasticity of the shaft, c_s is the inner damping of the shaft, θ_l , θ_m and θ_d are the load, motor and difference angle, respectively, ω_l , ω_m and ω_d are the corresponding angular speed of the load, motor and difference angle. Taken the deadzone in transmission shaft into account, we define θ_b and $\dot{\theta}_b$ as the deadzone angle and its derivative, respectively. Then, the real transmitted shaft torque can be written as $T_s = k_s(\theta_d - \theta_b) + c_s(\dot{\theta}_d - \dot{\theta}_b)$. In this section, we focus on the exact model for T_s proposed in [29]:

$$\begin{cases} T_s = 0 \text{ or } T_s = k_s(\theta_d - \alpha) + c_s\dot{\theta}_d, & \theta_d + \frac{c_s}{k_s}\dot{\theta}_d > \alpha \\ T_s = 0, & |\theta_d + \frac{c_s}{k_s}\dot{\theta}_d| \leq \alpha \\ T_s = 0 \text{ or } T_s = k_s(\theta_d + \alpha) + c_s\dot{\theta}_d, & \theta_d + \frac{c_s}{k_s}\dot{\theta}_d < -\alpha \end{cases} \quad (37)$$

where α is the maximal deadzone angle, implying $\theta_b = \alpha$ and $\dot{\theta}_d = 0$ when the shaft contacts with load. Therefore, $T_s = 0$ are valid only if there is no contact. Different from the widely used classical deadzone model mentioned in Remark 3, model (37) takes into account the inner damping coefficient of the shaft.

As shown in (36), the gear transmission system consists of a driven part and a load part, which is a typical sandwich-like system. Let $T_s = T_r + \delta$ with $T_r = k_s\theta_d + c_s\dot{\theta}_d$ and

$$\delta = \begin{cases} -k_s\alpha & \theta_d + \frac{c_s}{k_s}\dot{\theta}_d > \alpha \\ -k_s\theta_d - c_s\dot{\theta}_d & |\theta_d + \frac{c_s}{k_s}\dot{\theta}_d| \leq \alpha \\ k_s\alpha & \theta_d + \frac{c_s}{k_s}\dot{\theta}_d < -\alpha \end{cases}$$

Clearly, Assumption 1 holds since $|\delta| \leq k_s\alpha$.

Let $[x_1, x_2, x_3, x_4]^T = [\theta_l, \omega_l, \theta_m, \omega_m]^T$, system (36) can be written in the form of (8) with

$$\begin{cases} n = 4; r = 2; a_1 = \frac{k_s}{J_l}; b_1 = \frac{1}{J_l}; b_2 = \frac{1}{J_m} \\ \varphi_1 = 0; \\ \varphi_2 = -\frac{k_s}{J_l}x_1 - \frac{c_l+c_s}{J_l}x_2; \\ \varphi_3 = 0; \\ \varphi_4 = \frac{k_s}{J_m}x_1 + \frac{c_s}{J_m}x_2 - \frac{k_s}{J_m}x_3 - \frac{c_m+c_s}{J_m}x_4; \\ \varphi_d = \frac{c_s}{J_l}x_4; \end{cases} \quad (38)$$

To estimate the non-strict feedback term φ_d , the same differentiator (35) is adopted here too.

The system parameters are shown in TABLE 1. The desired trajectory $y_d = \sin(0.2t)$ is set. The controller gains are chosen as $k_1 = 0.55$, $k_2 = 0.1$, $k_3 = 0.1$, $k_4 = 0.1$. The initial condition of

the system state and the differentiator (35) are specified as $x(0) = [2, 0, 0, 0]^T$ and $z(0) = [0, 0, 0]^T$, respectively. The sampling time is set to be 0.002s.

The simulation results are illustrated by Figs. 6–9. The trajectory tracking response is presented in Fig. 6 and the trajectory tracking error is shown in Fig. 7, from which it can be seen that the tracking error converges a neighborhood of the origin. Fig. 8 shows the time evolution of the input. Fig. 9 presents the outputs of the differentiator which are estimates of the derivatives of the non-strict feedback term. The small jerks in the curve of z_2 can be observed due to the nonsmoothness of the deadzone. The simulation results demonstrate that the proposed controller solves the tracking control problem for the gear transmission system with sandwiched elastic deadzone.

Table 1 Parameters of the gear transmission system

Parameters	Value
Inertia of the load	$J_l = 3[\text{kg} \cdot \text{m}^2]$
Inertia of the driving side	$J_m = 0.25[\text{kg} \cdot \text{m}^2]$
Coefficient	$k_s = 3$
Damping coefficient	$c_s = 0.2$
Viscous friction coefficient of load side	$c_l = 5[\text{Nm/rad}]$
Viscous friction coefficient of motor side	$c_m = 0.7[\text{Nm/rad}]$
Backlash gap	$\alpha = 0.01[\text{rad}]$

Fig. 6: Load angle tracking response: reference signal y_d , solid line; output θ_l , dash line

6 Conclusion

In this paper, we have presented a backstepping control for a class of sandwich-like systems with deadzone. A high-order sliding mode differentiator is introduced in the backstepping design to overcome the non-strict feedback property of the system due to the existence of sandwiched deadzone nonlinearity. The closed-loop system is proven to be ultimately bounded within the Lyapunov framework. Future work includes the extension of the proposed method to the system cascaded by more than two subsystems.

7 Acknowledgments

This work was supported by the National Natural Science Foundation of China (No.61673034).

Fig. 7: Load angle tracking error

Fig. 8: Control torque

Fig. 9: Differentiator outputs for the gear transmission system

8 References

- 1 Tao, G., Kokotovic, P.V.: 'Adaptive control of plants with unknown dead-zones,' *IEEE Trans. Autom. Control*, 1994, **39**, (1), pp. 59–68
- 2 Cho, H., and Bai, E.W.: 'Convergence results for an adaptive dead zone inverse,' *Int. J. Adaptive Control Signal Processing*, 1998, **12**, (5), pp. 451–466
- 3 Oh, S.Y., Park, D.J.: 'Design of new adaptive fuzzy logic controller for nonlinear plants with unknown or time-varying dead zones,' *IEEE Trans. Fuzzy Syst.*, 1998, **6**, (4), pp. 482–491
- 4 Zhou, J., Wen, C., Zhang, C.: 'Adaptive output control of nonlinear systems with uncertain dead-zone nonlinearity,' *IEEE Trans. Autom. Control*, 2006, **51**, (3), pp. 504–511
- 5 Zhou, J., Shen, X.Z.: 'Robust adaptive control of nonlinear uncertain plants with unknown dead-zone,' *IET Control Theory Appl.*, 2007, **1**, (1), pp. 25–3
- 6 Zhou, J.: 'Decentralized adaptive control for large-scale time-delay systems with deadzone input,' *Automatica*, 2008, **44**, (7), pp. 1790–1799
- 7 Wang, X.S., Su, C.Y., Hong, H.: 'Robust adaptive control of a class of nonlinear systems with unknown dead-zone,' *Automatica*, 2004, **40**, (3), pp. 407–413
- 8 Ibrir, S., Xie, W.F., Su, C.Y.: 'Adaptive tracking of nonlinear systems with non-symmetric dead-zone input,' *Automatica*, 2007, **43**, (3), pp. 522–530
- 9 Zhang, T.P., Ge, S.S.: 'Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs,' *Automatica*, 2007, **43**, (6), pp. 1021–1033
- 10 Zuo, Z., Li, X., Shi, Z.: ' L_1 adaptive control of uncertain gear transmission servo systems with deadzone nonlinearity,' *ISA Trans.*, 2015, **58**, pp. 67–75
- 11 Shi, Z., Zuo, Z.: 'Backstepping control for gear transmission servo systems With backlash nonlinearity,' *IEEE Trans. Autom. Sci. Eng.*, 2015, **12**, 2, pp. 752–757
- 12 Zuo, Z., Ju, X., Ding, Z.: 'Control of gear transmission servo systems with asymmetric deadzone nonlinearity,' *IEEE Trans. control Syst. Tech.*, 2016, **24**, 4, pp. 1472–1479
- 13 Saberi, A., Kokotovic, P.V., Sussmann, H.J.: 'Global stabilization of partially linear composite systems,' *SIAM J. Control Optim.*, 1990, **28**, (6), pp. 1491–1503
- 14 Wang, X., Stoorvogel, A.A., Saberi, A., Grip, H. F., Roy, S., Sannuti, P.: 'Stabilization of a class of sandwich systems via state feedback,' *IEEE Trans. Autom. Control*, 2010, **55**, (6), pp. 2156–2160
- 15 Stoorvogel, A.A., Wang, X., Saberi, A., Sannuti, P.: 'Stabilization of sandwich non-linear systems with low-and-high gain feedback design,' *American Control Conference*, 2010, pp. 4217–4222
- 16 Taware, A., Tao, G., Teolis, C.: 'Design and analysis of a hybrid control scheme for sandwich nonsmooth nonlinear systems,' *IEEE Trans. Autom. Control*, 2002, **47**, (1), pp. 145–150
- 17 Taware, A., Tao, G.: 'An adaptive dead-zone inverse controller for systems with sandwiched dead-zones,' *Int. J. Control*, 2003, **76**, (8), pp. 755–769
- 18 Zhang, T., Ge, S.S.: 'Adaptive neural network tracking control of MIMO nonlinear systems with unknown dead zones and control directions,' *IEEE Trans. Neural Netw.*, 2009, **20**, (3), pp. 483–497
- 19 Krstic, M., Kanellakopoulos, I., Kokotovic, P.: *Nonlinear and Adaptive Control Design*. New York: Wiley, 1995.

- 20 Ding, Z.: *Nonlinear and Adaptive Control Systems*. London, U.K.: IET, 2013.
- 21 Li, Y., Tong, S., Liu, Y., Li, T.: 'Adaptive fuzzy robust output feedback control of nonlinear systems with unknown dead zones based on a small-gain approach,' *IEEE Trans. Fuzzy Syst.*, 2014, **22**, (1), pp. 164–176
- 22 Li, Y., Tong, S., Li, T.: 'Observer-based adaptive fuzzy tracking control of MIMO stochastic nonlinear systems with unknown control directions and unknown dead zones,' *IEEE Trans. Syst. Man. Cybern. A*, 2015, **23**, (4), pp. 164–176
- 23 Tong, S., Zhang, L., Liu, Y.: 'Observed-based adaptive fuzzy decentralized tracking control for switched uncertain nonlinear large-scale systems with dead zones,' *IEEE Trans. Syst. Man. Cybern. A*, 2016, **46**, (1), pp. 37–47
- 24 Wang, H.Q., Liu, X.P., Liu, K.F., Karimi, R.H.: 'Approximation-based adaptive fuzzy tracking control for a class of nonstrict-feedback stochastic nonlinear time-delay systems,' *IEEE Trans. Fuzzy Syst.*, 2015, **23**, (5), pp. 1746–1760
- 25 Wang, H.Q., Chen, B., Liu, K.F., Lin, C.: 'Adaptive neural tracking control for a class of nonstrict-feedback stochastic nonlinear systems with unknown backlash-like hysteresis,' *IEEE Trans. Neural Netw. Learning Syst.*, 2014, **25**, (5), pp. 947–958
- 26 Levant, A.: 'Higher-order sliding modes, differentiation and output-feedback control,' *Int. J. Control*, 2003, **75**, (9/10), pp. 924–941
- 27 Bacciotti, A., Ceragioli, F.: 'Stability and stabilization of discontinuous systems and nonsmooth lyapunov functions,' *ESAIM, Control Optim. Calculus Variat.*, 1999, **4**, pp. 361–376
- 28 Nordin, M., Gutman, P.O.: 'Controlling mechanical systems with backlash-a survey,' *Automatica*, 2002, **38**, (10), pp. 1633–1649
- 29 Nordin, M., Galic, J., Gutman, P.O.: 'New models for backlash and gearplay,' *Int. J. Adaptive Control and Signal Processing*, 1997, **11**, (1), pp. 49–63
- 30 Freeman R. A., Kokotović P.: 'Robust nonlinear control design,' Boston, USA: Birkhauser, 2008, pp. 101–136
- 31 Levant A.: 'Quasi-continuous high-order sliding-mode controllers,' *IEEE Trans. Autom. Control*, 2005, **50**, (11), pp. 1812–1816