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Identification of Weak Areas of Network based on Exposure to Voltage Sags — Part II: Assessment of Network Performance using Sag Severity Index

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Abstract—This paper presents a new stochastic approach to comprehensive assessment of the impact of voltage sags in large scale power networks. The approach takes into account the stochastic nature of power system operation including load variation, uncertainty of fault clearing time by protection relays, fault rates of network components and the variation/uncertainty in equipment sensitivity to voltage sags. A new duration zone division method is used to derive sag duration and occurrence frequency based on stochastic distribution of clearing time required by specific protection systems. The adopted probabilistic representation facilitates estimation of network performance based on a single-event characteristic, sag severity index (SSI), developed in the companion paper (Part I). Based on SSI, a new single-site index with respect to voltage sags (Bus Performance Index, BPI³) is developed to comprehensively represent the overall bus performance with respect to voltage sags. The index is used to identify the areas of the network, named weak areas of the network in this paper, containing buses that are most exposed to potentially disruptive voltage sags. The application, robustness and sensitivity of BPI³ are thoroughly analysed and discussed in the paper.

Index Terms—Voltage sags, equipment susceptibility, voltage tolerance curves, sag severity index.

I. INTRODUCTION

Voltage sags cause frequent disruptions to modern industrial processes and malfunction of electronic equipment, resulting in substantial financial losses [1]. This issue, as one of the most critical power quality problems, has become a major focal point for many utilities and industries [2]. To reduce overall financial consequences of voltage sags, it is necessary to assess voltage sag performance of network buses as accurately as possible and consequently to identify the buses that are most affected by voltage sags. Assessing voltage sag performance at certain location requires two steps: calculating single-event characteristic for each sag event; and then calculating single-site indices based on the single-event characteristics of all sag events occurring at the location [3].

Various single-site indices have been proposed in literature to assess voltage sag performance. Sag severity at certain location can be presented through probability density and distribution functions [3]. Furthermore, the sag information can be compressed into a sag table which groups the sags based on the interval of residual voltage and duration [4-9], or by using modified reliability based SAIFI index [10-12]. These single-site indices simply give the number of events per year within a certain range of magnitude and duration, resulting in discrete representation of sag characteristics. The aforementioned indices are not suitable for comparing the actual voltage sag performance of different buses as they typically do not include multiple sag characteristics at the same time nor, and more importantly, sensitivity of equipment connected at these buses to voltage sags. A single numerical index that can take into account sag frequency, magnitude and duration at the same time is required. With such index voltage sag performance across the network can be assessed, and the weak areas of the network, i.e., areas comprising buses which are most exposed to potential disruptive voltage sags, can be identified for the purpose of network planning. An alternative approach is using single-index methods, which calculate the sum or average of the single-event characteristics of all events occurring at the location. Widely used single-site indices include sag energy index (or average sag energy index) and total/average voltage sag severity, which are based on single-event characteristics of voltage sag energy and voltage sag severity respectively [3, 13, 14]. Most of the single-site indices (except for total/average voltage sag severity) as mentioned above, do not take into account equipment sensitivity to voltage sags in sag severity assessment. The inclusion of equipment sensitivity to voltage sags is essential however, as the ultimate objective of developing severity indices is to reflect the potential impact of voltages sags on equipment/system operation.

Voltage sag performance of network buses can be assessed either by lengthy monitoring or through various types of computer simulation studies. Detailed discussion of advantages and disadvantages of these approaches in general and related to specific types of simulation studies is provided in many past papers including [3, 4, 15-19]. Similarly, a number of approaches have been proposed to assess single-site
indices, including fault positions-based stochastic method [20-24], critical distances [22, 23] and Monte Carlo (MC)-based stochastic approaches [15, 16, 20, 25]. A stochastic assessment of voltage sags is used in [26] to get the expected number and characteristics of sags using deterministic short circuit simulations and the stochastic data of the faults. Furthermore, an analytical method which does not consider the stochastic nature of network parameters and calculates sag duration based on mathematical model is used to produce the voltage sag density table in [4]. Typically, fault position and analytical methods provide long-term mean values of sag performance at a bus, while MC simulation methods allow for uncertainty to be taken into account during the assessment and provide complete distribution function that describes sag performance at the bus. For MC simulation methods, however, the number of simulations required to achieve desired accuracy of results is influenced by the number and range of uncertainties considered and it may become prohibitively high for large networks and large number of uncertain parameters.

Voltage sag characteristics can be estimated if enough data and proper models are provided [27, 28]. Considering that voltage sags are mainly caused by the faults in the network [29], the expected number of voltage sags (voltage sag frequency) for each site can be calculated from the fault statistics data kept by utilities [3], and sag duration can be estimated/derived based on protection system operation [17, 27, 28]. Protective actions are coordinated by two types of protection relays, i.e. primary and backup protection systems. The former is to remove the faults immediately once it detects the occurrence of faults. Its clearing time depends on the actual operating time required by the protection systems. In practice it varies according to the nature and location of the system faults, the age and condition of the equipment. The latter is to perform the protection actions if the primary relays fail to operate. Its clearing time depends on the severity of the fault and an intentional time delay, which is indicated by the actual design and setting of the protection systems. Considering the probabilistic nature of protection systems’ reliability and uncertainty of the clearing time, the probabilistic model is the best method to represent these stochastic variations. In an existing power system, the distribution of clearing time can be obtained using reliability modelling of the protection systems [30].

This paper proposes a comprehensive stochastic method for assessing voltage sag performance across the network taking into account a number of uncertain factors including fault rates, load variation, fault clearing time and equipment sensitivity to voltage sags. Based on failure probabilities of protection relays and the distribution of fault clearing time by corresponding protection relays, a new duration zone division method is developed to derive sag frequency and duration. Then a new index BPI is proposed to take into account sag frequency, magnitude and duration, and form a single numerical value to represent the sag performance of a site. With this index, the weak areas of the network that are exposed to disruptive voltage sags can be easily identified. The identification of weak areas facilitates efficient network planning for mitigation of voltage sags. In this paper, Section II introduces the proposed stochastic approach and the derivation of the new single-site index. The simulation results and related analysis are given in Section III. The identification of weak areas with respect to voltage sags is presented in Section IV. Section V concludes the paper.

II. METHODOLOGY

A. Estimation of Sag Magnitude and Frequency

In this study, voltage sags are identified using a stochastic approach based on simulating faults in the network. Commercially available DigSILENT/PowerFactory software is used in all sag simulations and the methodology discussed in the sequel is developed to facilitate its use. The fault rates of bus i and line j are denoted as \( f_{bi} \) and \( f_{lij} \), respectively. Both symmetrical and asymmetrical faults are considered, i.e., single line to ground fault (SLGF), line to line fault (LLF) and three phase fault (LLLF).

Evolving faults and simultaneous faults, which can be analysed using probabilistic approaches [31-33], are not considered in the study due to their infrequent occurrence and lack of credible probability model for them. The distributions of the four types of faults occurring on bus \( i \) are denoted by \( P_{SLGF,Bi} \), \( P_{LLGF,Bl} \), \( P_{LLF,Bl} \) and \( P_{LLLF,Bi} \), respectively. Similarly, the distributions of the four types of faults occurring on line \( j \) are denoted by \( P_{SLGF,Lj} \), \( P_{LLGF,Lj} \), \( P_{LLF,Lj} \) and \( P_{LLLF,Lj} \), respectively. Fault distributions are assumed to be known based on historical data.

Symmetrical faults may affect any phase with equal probability while there are three possibilities for any asymmetrical fault in terms of affected phase. In total there are 10 possible fault cases for any fault location. Corresponding fault frequencies can be obtained as follows: (taking bus faults as an example)

\[
f_{bi} = \begin{cases} 
  f_{bl} \times P_{SLGF,Bi} & \text{if } k = 1, ..., 3 \\
  f_{bl} \times P_{LLGF,Bl} & \text{if } k = 4, ..., 6 \\
  f_{bl} \times P_{LLF,Bl} & \text{if } k = 7, ..., 9 \\
  f_{bl} \times P_{LLLF,Bi} & \text{if } k = 10 
\end{cases}
\]

where \( k \in \{1, ..., 10\} \). The same rule applies to lines. When \( k = 1, ..., 9 \), the obtained fault rates \( f_{bl} \) are for asymmetrical faults, and when \( k = 10 \), the obtained \( f_{bl} \) is for symmetrical fault. \( f_{bl}, f_{bl} \) and \( f_{bl} \) represent the frequency of SLGF occurring on phases A, B and C respectively. In a similar way fault frequencies for other types of faults are calculated. For each possible fault case (10 per fault location), a short circuit simulation is run to calculate voltage sags in each phase for all buses across the network. After completing the simulation of all possible fault cases (i.e., 10 simulations per location \times number of fault locations), a number of sags occurring at each bus can be obtained. The individual sag can be defined as \( \langle \psi_{bl}, f_{bl} \rangle \), where \( \varphi \in \{1, ..., M\} \) is the index of the sags occurring at bus \( i \), \( \psi_{bl} \) is the sag magnitude of the \( m \)-th sag at bus \( i \), and \( f_{bl} \) is the occurrence frequency of the sag event. For later use, each sag event is associated with another variable, \( \epsilon_{bl} \), which represents the type of faulted component.
that results in the sag event. \( e_{Bis} \in [0, 1] \), where 0 represents the sag caused by a fault at a bus, and 1 denotes the sag caused by a fault at a line.

**B. Estimation of Sag Duration and Frequency**

Once faults occur, protection relays respond by taking action to remove the faulted components. The clearing time required by protection systems determines the duration of the voltage sags caused by the faults. The clearing time was assumed to be normally distributed, with a given mean and standard deviation [30]. The mean and standard deviation of clearing time required by the primary bus protection relays are denoted by \( \mu_1 \) and \( \sigma_1 \) respectively, while those of the backup bus protection relays are denoted by \( \mu_2 \) and \( \sigma_2 \) respectively. As for lines, the distributions of clearing time required by primary and backup line protection relays are denoted by PDF(\( \mu_3, \sigma_3 \)) and PDF(\( \mu_4, \sigma_4 \)) respectively. Since bus and line protection systems perform differently in terms of reliability, the failure probability of primary bus protection systems (denoted by \( Q_1 \)) is different from the failure probability of primary line protection (denoted by \( Q_L \)). Therefore, for a bus fault, there is a probability of \((1 - Q_b)\) that the fault clearing time will be determined by the operation speed of the primary bus protection relay. If this fails, the clearing time will be determined by the backup bus protection relay. The same rule applies to line protection relays.

Each sag \( \langle v_{Bis}, f'_{Bis} \rangle \) obtained in Section II. A is further divided into a set of sub-sag events according to its associated \( e_{Bis} \), and the distribution of clearing time performed by protection relays, according to the following two steps:

**Step 1** For sag \( \langle v_{Bis}, f'_{Bis} \rangle \), the division begins by subdividing \( f_{Bis} \) according to:

\[
f'_{Bis} = \begin{cases} 
  f_{Bis} \times (1 - Q_b) & \text{if } e_{Bis} = 0 \text{ and } r_{Bis} = 0 \\
  f_{Bis} \times Q_b & \text{if } e_{Bis} = 0 \text{ and } r_{Bis} = 1 \\
  f_{Bis} \times (1 - Q_L) & \text{if } e_{Bis} = 1 \text{ and } r_{Bis} = 0 \\
  f_{Bis} \times Q_L & \text{if } e_{Bis} = 1 \text{ and } r_{Bis} = 1
\end{cases}
\]  

(2)

where \( g = 2 \times e_{Bis} + r_{Bis} + 1 \), and \( e_{Bis} \) denotes the type of protection reply that takes protection action. \( r_{Bis} \in [0,1] \), where 0 represents the case when primary protection relay determines the duration of corresponding sag, and 1 the case that the backup protection relay determines the duration of corresponding sag. Using (2), each sag event \( \langle v_{Bis}, f_{Bis} \rangle \) is subdivided into a set of four sag, denoted by \( \langle v_{Bis}, f''_{Bis} \rangle \), where \( g \in \{1, \ldots, A\} \)

**Step 2** The distribution of clearing time is determined by protection systems. \( \langle v_{Bis}, f''_{Bis} \rangle \) is further divided based on the corresponding protection relay that takes action. The distribution of clearing time (i.e. sag duration) is presented by a normal probability density function PDF(\( \mu_g, \sigma_g \)) where \( g \in \{1, \ldots, A\} \). The area covered by PDF curve is divided into a number of duration zones (denoted by \( N \)) each covering equal area. For instance, a normal distribution function PDF(\( \mu_g, \sigma_g \)) is plotted in Fig. 1, where the area covered by PDF curve is equally divided into \( N \) duration zones. Each zone is corresponding to one sub-sag event, denoted by triplet \( \langle v_{Bis}, f''_{Bis} \rangle \), where \( t_{Bis} \) denotes the duration of the corresponding sub-sag. Duration \( t_{Bis} \) is determined by the median point (i.e. the \( t \)-axis projection of the centre of the corresponding duration zone), e.g. the red solid dot in Fig. 1. The occurrence frequency of the corresponding sub-sag is defined as:

\[
f''_{Bis} = \frac{f''_{Bis} \times \text{AREA}(\text{zone } n)}{\text{AREA}(\text{zone } j) + \text{AREA}(\text{zone } j)}
\]

(3)

where \( \text{AREA}(\text{zone } i) \) denotes the area of duration zone \( i \).

In this way, \( \langle v_{Bis}, f''_{Bis} \rangle \) is further divided into \( N \) sags \( \langle v_{Bis}, f''_{Bis} \rangle \).

It can be seen that the sag \( \langle v_{Bis}, f_{Bis} \rangle \) is divided into \( 4 \times N \) sub-sags \( \langle v_{Bis}, f''_{Bis} \rangle \), where \( g \in \{1, \ldots, A\} \) and \( n \in \{1, \ldots, N\} \) with sag duration incorporated. During this process the sag magnitude remains the same, and the sag occurrence frequency is upgraded twice, e.g. steps 1 and 2. For bus \( i \), the total number of sub-sag events becomes \( 4 \times M \times N \). For simplicity, the notation of sub-sag events is changed to \( \langle v_{Bij}, f_{Bij}, t_{Bij} \rangle \), where \( j \) is the index of sub-sag event occurring at bus \( i \) and \( j \in \{1, \ldots, A\} \).

**C. Derivation of Bus Performance Index with Respect to Voltage Sag (BPI\(^5\))**

In order to assess comprehensively bus performance with respect to voltage sags, all three key aspects of sag performance (occurrence frequency, magnitude and duration) are taken into account and represented by a single numerical index, namely Bus Performance Index (BPI\(^5\)). For each sub-sag event \( \langle f_{Bij}, v_{Bij}, t_{Bij} \rangle \) at bus \( i \), the sag severity index \( S_{Sij}(B) \), which is developed in the companion paper (Part I), is calculated using sag magnitude \( V_{Bij} \) and duration \( t_{Bij} \), taking into account the variation/uncertainty of voltage tolerance curve. The new single-site index BPI\(^5\) is then defined as:

\[
\begin{align*}
\text{BPI}\(^5\)_{Bij} = \sum_{i=1}^{AMN} f_{Bij} \times \text{SSI}_{Bij}(B)
\end{align*}
\]

(4)

**III. RESULTS OF SIMULATION AND ANALYSIS**

**A. Test System Modeling**

A 295-bus generic distribution network (GDN) [34], as shown in Fig. 2, is used in the study. It comprises 275 kV transmission in-feeds, 132 kV and 33 kV predominantly meshed sub-transmission networks, and 11 kV predominantly radial distribution network. The network consists of 276 lines including overhead lines and underground cables, 37 transformers with various winding connections, 297 loads (including 10 unbalance loads) representing industrial, commercial and domestic loads, and 12 distributed generators (including 2 wind turbines, 5 fuel cells and 5 photovoltaic) connected to 11 kV distribution network. The locations of unbalanced loads and distributed generators are marked by different labels in Fig. 2. Among the distributed generators, fuel cells and photovoltaic are single-phase connected.
The variation of operating conditions in the network is modeled by selecting 11 operating points from yearly load duration curve (LDC), see Fig. 3, obtained by considering appropriate yearly variation of the three types of load (i.e. industrial, commercial and domestic loads) obtained from a real network. The first two segments of LDC account for 2% and 8% of the yearly load duration, respectively, while the remaining 9 segments cover 10% each. The median point of each segment is selected to represent the whole segment, and the corresponding load setting is taken as the representative operating point.

TABLE I
System Fault Statistic for Components in GDS network

<table>
<thead>
<tr>
<th>Components</th>
<th>Buses</th>
<th>Lines 11 kV</th>
<th>Lines 33 kV</th>
<th>Lines 132 kV</th>
<th>Cables 11 kV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault rate (No. of events/year*100km)</td>
<td>0.08</td>
<td>8.6</td>
<td>3.7</td>
<td>0.6</td>
<td>4.9</td>
</tr>
<tr>
<td>Type of faults</td>
<td>SLGF</td>
<td>LLGF</td>
<td>LLF</td>
<td>LLLF</td>
<td></td>
</tr>
<tr>
<td>Percentages</td>
<td>73%</td>
<td>17%</td>
<td>6%</td>
<td>4%</td>
<td></td>
</tr>
</tbody>
</table>

The components at different voltage levels have different fault rates, and the detailed system fault statistics in the distribution network are given in TABLE I, adopted from [17]. The mean and standard deviation of the distribution of fault clearing time is given in TABLE II, together with the failure probability of primary protection relays. The parameter settings of the normal distribution adopted to represent the variation of the three knee points of voltage tolerance curves (as introduced in the companion paper) are given in TABLE III. All simulations are carried out using DlgsILENT/PowerFactory software. The during-fault voltage profiles of all buses are generated using PowerFactory, and the calculation of SSI and BPI³ is completed in Matlab.

TABLE II
Fault Clearing Time for Primary and Back-up Bus and Line Protection Relays

<table>
<thead>
<tr>
<th>Components</th>
<th>Relays</th>
<th>Mean (ms)</th>
<th>Std (ms)</th>
<th>Failure probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>Primary</td>
<td>60</td>
<td>3</td>
<td>0.09%</td>
</tr>
<tr>
<td></td>
<td>Back-up</td>
<td>800</td>
<td>5</td>
<td>N/A</td>
</tr>
<tr>
<td>Lines</td>
<td>Primary</td>
<td>800</td>
<td>300</td>
<td>1.09%</td>
</tr>
<tr>
<td></td>
<td>Back-up</td>
<td>800</td>
<td>300</td>
<td>2.22%</td>
</tr>
</tbody>
</table>

TABLE III
Mean and Standard Deviation of the Three Points on the Voltage tolerance Curve

<table>
<thead>
<tr>
<th>t-axis (ms)</th>
<th>Mean</th>
<th>Std</th>
<th>t-axis (p.u.)</th>
<th>Mean</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>t₁</td>
<td>20</td>
<td>0.667</td>
<td>t₁</td>
<td>0.5</td>
<td>0.023</td>
</tr>
<tr>
<td>t₂</td>
<td>200</td>
<td>9.670</td>
<td>t₂</td>
<td>0.7</td>
<td>0.0267</td>
</tr>
<tr>
<td>t₃</td>
<td>500</td>
<td>16.670</td>
<td>t₃</td>
<td>0.8</td>
<td>0.0300</td>
</tr>
</tbody>
</table>

B. Study Procedure

A comprehensive stochastic approach is developed to assess the performance of all buses in distribution networks with respect to sags. It includes probabilistic nature of faults in the network, variation of network operation condition, different reliability performance and settings of protection system and variation in voltage tolerance curves of equipment. The step by step assessment procedure is given as follow:

1) Load duration curve is obtained using realistic load data, and 11 operating points are selected according to Section III. A.
2) The fault rates of asymmetrical faults are distributed among three phases, using (1). Short circuit simulation is run for each possible fault case. After simulating all possible faults in the network (symmetrical and asymmetrical) voltage sags are recorded for each bus, and their occurrence frequency is derived from the fault rate of the component that caused the sag.
3) Sag frequency is updated according to the type of component that caused the sag and the possible protection device that takes protection action, using (2), and further updated according to the distribution of clearing time required by the corresponding protection relay and the division of duration zones, using (3). Sag duration is derived from the median point of the corresponding duration zone.
4) Probabilistic model of voltage sag tolerance curves are obtained by randomly selecting 50 curves according to a normal distribution, whose mean is the standard voltage tolerance curve. For each sag event, 50 SSI values are obtained from 50 curves respectively. PDF of the 50 SSI values is derived, and the mean of the 50 values is taken as the final sag severity index, denoted as SSI²²
5) BPI³ is obtained using (4) for each bus in 11 kV distribution network.
6) After completing steps 2-5 for each of the 11 operating points separately, 11 BPI³ values are obtained for each bus of distribution network. These 11 BPI³ values are weighted by the percentages of yearly load duration of the corresponding operation points (given in Section III. A) and summed together in order to determine the final mean BPI³.

The flowchart illustrating the proposed approach is given in Fig. 4, where Nfp denotes the number of fault locations, lop and lfc denote the indices of operating point and fault case, respectively. The procedure described above can be substantially simplified and easily applied in cases when voltage sag monitoring results are directly available. In such cases voltage sag performance of the network can be assessed by following steps 4-6 only.
The proposed stochastic approach uses the deterministic results of the short circuit simulation and the stochastic data about the faults. This is similar to existing prediction methods, e.g., fault position methods and the stochastic method presented in [26]. Similar to MC simulation methods, the proposed approach includes numerous uncertainty factors in the network but avoids a large number of simulations required by MC to achieve convergence. This is one of its advantages compared to conventional MC simulations. Furthermore, different from existing stochastic methods, the proposed method derives the sag frequency and sag duration based on the stochastic distribution of clearing time by protection system. The total required computation time of 31min consists of 30min and 8s spent on performing fault simulations in DIgSILENT/PowerFactory and 52s spent to calculate BPI\textsuperscript{5} for all buses in the network. The computation time, however, is provided here for reference only as it depends on the computer processing speed and programming efficiency.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Input power network, load duration curve, N\textsubscript{S} fault locations and their fault rates; Select 11 operating points</td>
</tr>
<tr>
<td>2.</td>
<td>Calculate frequencies of 10 faults for all fault locations using (1)</td>
</tr>
<tr>
<td>3.</td>
<td>For each bus, record voltage sag occurring on this bus, occurrence frequency of the corresponding fault, type of component that caused the sag</td>
</tr>
<tr>
<td>4.</td>
<td>Update sag frequencies for all sags in record using (a) and (b)</td>
</tr>
<tr>
<td>5.</td>
<td>Derive sag duration according to distribution of clearing time of corresponding protection relay and division of duration zones</td>
</tr>
<tr>
<td>6.</td>
<td>Calculate 50 SSI for each sag based on probabilistic model of voltage tolerance curves; Calculate SSI\textsuperscript{5}, the mean of 50 SSI</td>
</tr>
<tr>
<td>7.</td>
<td>Complete all 10×N\textsubscript{S} fault cases?</td>
</tr>
<tr>
<td>8.</td>
<td>Yes</td>
</tr>
<tr>
<td>9.</td>
<td>No</td>
</tr>
<tr>
<td>10.</td>
<td>Update sag frequencies for all sags in record using (a) and (b)</td>
</tr>
<tr>
<td>11.</td>
<td>Derive sag duration according to distribution of clearing time of corresponding protection relay and division of duration zones</td>
</tr>
<tr>
<td>12.</td>
<td>Calculate 50 SSI for each sag based on probabilistic model of voltage tolerance curves; Calculate SSI\textsuperscript{5}, the mean of 50 SSI</td>
</tr>
<tr>
<td>13.</td>
<td>Complete 11 operating points?</td>
</tr>
<tr>
<td>14.</td>
<td>Yes</td>
</tr>
<tr>
<td>15.</td>
<td>No</td>
</tr>
<tr>
<td>16.</td>
<td>Obtain final index by summing up 11 BPI\textsuperscript{5} weighted by their percentages of representation in yearly load duration</td>
</tr>
</tbody>
</table>

Fig. 4. Flowchart of proposed stochastic approach and calculation of BPI\textsuperscript{5}.

C. The Influence of Duration Zone Division on BPI\textsuperscript{5}

To prove the accuracy of BPI\textsuperscript{5}, an error is introduced to the process of sag duration derivation. Different from the method of duration zone division introduced in Section II, B, the space covered by PDF curve is divided according to \( t \)-axis using the following method: the line between points \((\mu_a-3\sigma_a, 0)\) and \((\mu_a + 3\sigma_a, 0)\) are equally divided into \( N \) segments, and the area above each segment is taken as one duration zone. The duration zone division approach introduced in Section II, B (denoted by division approach 1) and the approach introduced here (denoted by division approach 2) are applied to calculate BPI\textsuperscript{5}, respectively. For division approach 2, it is expected to underestimate BPI\textsuperscript{5}, as it only considers the sags whose duration is within the range of \([\mu_a - 3\sigma_a, \mu_a + 3\sigma_a]\). Approximately, 99.73\% of the total area covered by PDF is taken into account, while the rest 0.27\% of the area is neglected. The two division approaches are applied respectively to derive the sag duration and occurrence frequency. The number of duration zones, \( N \), will affect the accuracy of assessed BPI\textsuperscript{5}. The larger \( N \) is, the more accurate the derived BPI\textsuperscript{5} is. However, large number of duration zones increases the computation load. To observe the influence of duration zone division on BPI\textsuperscript{5}, the number of duration zones \( N \) is set to different values, and the corresponding BPI\textsuperscript{5} is calculated. In this case, only the standard voltage tolerance curve is applied, in order to observe the relationship between \( N \) and BPI\textsuperscript{5}. The results are presented in Fig. 5, where the BPI\textsuperscript{5} of Bus 1 for operating point 1 is calculated with \( N \) increasing from 5 to 49. It can be seen that the derived BPI\textsuperscript{5} converges towards a certain value as \( N \) increases. For division approach 2, when \( N>15 \), the assessed BPI\textsuperscript{5} curve is flat, which indicates that the BPI\textsuperscript{5} converged to final value for \( N=15 \). Fig. 5 not only presents the calculated BPI\textsuperscript{5}, but also its convergence characteristic with respect to varying \( N \). This convergence feature can be used to determine the setting of \( N \) when considering the trade-off between assessment accuracy and computation load.

D. Effect of Number of Voltage Tolerance Curves on BPI\textsuperscript{5}

A gap between the two lines in Fig. 5 indicates different final values of BPI\textsuperscript{5} calculated using different division approaches. Division approach 1 takes into account the whole area covered by PDF curve, and consequently derived BPI\textsuperscript{5} is expected to be more accurate than BPI\textsuperscript{5} derived from division approach 2. The assessed BPI\textsuperscript{5} values obtained from the two approaches for \( \alpha=49 \) are 418.6044 and 417.4738, respectively, and the difference between them can be calculated by \((418.6044-417.4738)/418.6044=0.27\%\), which is exactly the same as the percentage of the area neglected in division approach 2. This confirms, as expected, slightly higher accuracy of the BPI\textsuperscript{5} derived using division approach 1. The small difference (0.27\%) between calculated BPI\textsuperscript{5} using the two different approaches does not really favour one over the other and it can be concluded that the influence of duration zone division on BPI\textsuperscript{5} is negligible.
more than 50 curves should be used in modeling the variation of voltage tolerance curves in order to calculate BPI\textsuperscript{5} more accurately. When the variation/uncertainty range of voltage tolerance curves is increased (setting the standard deviations of \(t_1\), \(t_2\), \(t_3\), \(v_1\), \(v_2\), \(v_3\) to 0.010667, 0.01967, 0.02667, 0.07333, 0.077667 and 0.08 respectively), the calculated BPI\textsuperscript{5} corresponding to different \(W\) is given in Fig. 6(b). By comparing the results it can be seen that BPI\textsuperscript{5} of Fig. 6(b) is larger than that of Fig. 6(a). The calculated value of BPI\textsuperscript{5} increases with the increase in variation/uncertainty range of voltage tolerance curve due to the presence of sags which are within the variation/uncertainty area of the voltage tolerance curves, i.e. the sags that are similar to sub-sag (\(s=0.2\), \(t=0.02\)), as discussed in the companion paper. It was observed that for this type of sub-sags, the corresponding SSIM\textsuperscript{M} increases with the increase in the variation/uncertainty range of voltage tolerance curves.

The variation of BPI\textsuperscript{5} along the \(W\)-axis of Fig. 6, is further analysed in TABLE IV. The calculated BPI\textsuperscript{5} values are divided into 5 sets based on the selection of \(W\). The standard deviation of the five sets of BPI\textsuperscript{5} is given in TABLE IV, where the last two rows are corresponding to the two different variation/uncertainty ranges of voltage tolerance curves respectively. For both variation levels, the derived standard deviation of BPI\textsuperscript{5} is reduced when \(W\) is increased. This suggests, as previously observed, that the calculated BPI\textsuperscript{5} is more stable when larger number of voltage tolerance curves (\(W\)) is used and that when the variation/uncertainty range of voltage tolerance curve is increased, the uncertainty of BPI\textsuperscript{5} also increases.

<table>
<thead>
<tr>
<th>(W)</th>
<th>5-100</th>
<th>100-200</th>
<th>200-300</th>
<th>300-400</th>
<th>400-500</th>
</tr>
</thead>
<tbody>
<tr>
<td>small variation</td>
<td>0.1137</td>
<td>0.0623</td>
<td>0.0376</td>
<td>0.0413</td>
<td>0.0351</td>
</tr>
<tr>
<td>large variation</td>
<td>0.4472</td>
<td>0.2027</td>
<td>0.1325</td>
<td>0.1062</td>
<td>0.1010</td>
</tr>
</tbody>
</table>

IV. IDENTIFYING CRITICAL AREAS OF THE NETWORK WITH RESPECT TO VOLTAGE SAGS

To observe the sag performance of various buses visually, i.e., to identify areas of the network which are most vulnerable to voltage sags, a heat map is used. The final mean BPI\textsuperscript{5}, obtained from step 6 of the study process given in Section III. B, is calculated for each bus of the 11 kV section of test distribution network. The calculated mean BPI\textsuperscript{5} is used here to generate heat maps of the network, as shown in Fig. 7(a), though they can be equally used to rank buses in the network with respect to exposure to voltage sags. With the help of heat maps, it is much easier, compared to simple numerical bus ranking, to identify the weak areas of the network with respect to exposure to voltage sags. The bus performance is poor in the area marked in red. The heat map using BPI\textsuperscript{5} obtained for operating point 1 only is presented in Fig. 7(b). It can be seen that there are very small differences between the two heat maps shown in Fig. 7, though they have been produced using different number of operating conditions of the network.

A. The Influence of Operating Condition on Bus Ranking Using BPI\textsuperscript{5}

To investigate the influence of operating conditions considered in deriving BPI\textsuperscript{5}, and consequently heat maps of the network, buses are ranked according to BPI\textsuperscript{5} derived from OP1 (operating point 1), OP6 and OP11, which represent heavy, medium and low load demand respectively. The nine buses which are most affected by the voltage sags for each operating condition (based on calculated BPI\textsuperscript{5}) are listed in TABLE V. The bottom row of the table “Overall” lists the buses ranked based on the final mean BPI\textsuperscript{5} determined from 11 operating conditions. It can be seen that the bus ranking obtained based on OP6 (medium loading of the network) is the same as the final ranking. As for OP1 and OP11, although their ranks are different to the last row, the difference is small. This suggests that the influence of the operating condition on the final ranking of network buses is minor.

To illustrate the effect of operating condition on BPI\textsuperscript{5}, the worst performing bus (Bus 196) and the best performing bus (Bus 61) are selected for further analysis, as well as Bus 1. The BPI\textsuperscript{5} derived from 11 operating points respectively are given in Fig. 8. It can be seen that the variation of BPI\textsuperscript{5} for all three buses obtained for operating points 2-11 is quite small, while the BPI\textsuperscript{5} obtained for OP1 (extreme loading of the network accounting for about 2% of the time of the year) is noticeably different from the others (though very fine scale is used).

TABLE V

<table>
<thead>
<tr>
<th>Ranking</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP1</td>
<td>196</td>
<td>195</td>
<td>193</td>
<td>194</td>
<td>210</td>
<td>200</td>
<td>197</td>
<td>198</td>
<td>199</td>
</tr>
<tr>
<td>OP6</td>
<td>196</td>
<td>194</td>
<td>193</td>
<td>195</td>
<td>210</td>
<td>200</td>
<td>197</td>
<td>199</td>
<td>198</td>
</tr>
<tr>
<td>OP11</td>
<td>196</td>
<td>194</td>
<td>193</td>
<td>195</td>
<td>210</td>
<td>200</td>
<td>197</td>
<td>198</td>
<td>199</td>
</tr>
<tr>
<td>Overall</td>
<td>196</td>
<td>194</td>
<td>193</td>
<td>195</td>
<td>210</td>
<td>200</td>
<td>197</td>
<td>199</td>
<td>198</td>
</tr>
</tbody>
</table>

To be specific, at Bus 1, the BPI\textsuperscript{5} obtained for OP1 is
different from the mean of BPI\textsuperscript{S} obtained from other OPs by 1.14%; at Bus 61, the BPI\textsuperscript{S} obtained for OP1 is different from the mean of others by 0.82%; while at Bus 196, the difference is only 0.46%. It can be seen that the influence of different OPs on the variation of BPI\textsuperscript{S} is very small.

![Normalized BPI\textsuperscript{S} for different operating points.](image)

**B. Robustness of Bus Ranking Using BPI\textsuperscript{S}**

The sensitivity of BPI\textsuperscript{S}, and consequently area of vulnerability of the network, to various parameters, including load demand, number of considered voltage tolerance curves, number of considered duration zones, fault rates of various components and distribution of the clearing time required by various protection relays, is analysed here based on Bus 1.

Each parameter is set in turn to a number of different values with all other parameters fixed, and a corresponding set of BPI\textsuperscript{S} values is calculated. The standard deviation of calculated BPI\textsuperscript{S} values is used to present the sensitivity of BPI\textsuperscript{S} to the variation of that parameter. The setting range of various parameters and the derived standard deviation of BPI\textsuperscript{S} are given in TABLE VI. It should be noticed that the range of parameter settings would impact the standard deviation of BPI\textsuperscript{S}. For simplicity, parameters 1-8 follow the same rule of setting: the parameter is set to a number of values which are uniformly distributed between −10% and +10% of its original setting (given in Section III. A). The settings of parameters 9 and 10 are selected based on previous analysis in Sections III. D and III. C, respectively.

**TABLE VI**

Sensitivity of \( BPI^S \) to Various Parameters

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Settings</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fault rate of buses</td>
<td>[0.0720, 0.0880]</td>
<td>0.1308</td>
</tr>
<tr>
<td>2</td>
<td>Fault rate of lines</td>
<td>[6.1200, 7.4800]</td>
<td>27.9004</td>
</tr>
<tr>
<td>3</td>
<td>Fault rate of primary protection relays on buses</td>
<td>[0.0107, 0.0131]</td>
<td>0.0979</td>
</tr>
<tr>
<td>4</td>
<td>Fault rate of primary protection relays on lines</td>
<td>[0.0200, 0.0244]</td>
<td>1.6154</td>
</tr>
<tr>
<td>5</td>
<td>Mean of clearing time of primary protection relays on buses</td>
<td>[54, 66]</td>
<td>0.1068</td>
</tr>
<tr>
<td>6</td>
<td>Mean of clearing time of backup protection relays on buses</td>
<td>[720, 880]</td>
<td>0.1029</td>
</tr>
<tr>
<td>7</td>
<td>Mean of clearing time of primary protection relays on lines</td>
<td>[270, 330]</td>
<td>28.7946</td>
</tr>
<tr>
<td>8</td>
<td>Mean of clearing time of backup protection relays on lines</td>
<td>[720, 880]</td>
<td>0.9508</td>
</tr>
<tr>
<td>9</td>
<td>Number of voltage tolerance curves</td>
<td>[50, 500]</td>
<td>0.0527</td>
</tr>
<tr>
<td>10</td>
<td>Number of divided duration zones</td>
<td>[15, 49]</td>
<td>0.0110</td>
</tr>
<tr>
<td>11</td>
<td>Load demand</td>
<td>N/A</td>
<td>1.3776</td>
</tr>
<tr>
<td>12</td>
<td>Selection of operating points</td>
<td>N/A</td>
<td>0.6471</td>
</tr>
</tbody>
</table>

As for parameter 11, the standard deviation of the 11 BPI\textsuperscript{S} values, which are obtained from 11 operating points respectively, is used to represent the sensitivity of BPI\textsuperscript{S} to the variation of load demand. For parameter 12, the percentages of the yearly load duration of the 11 operating points, introduced in Section III. A, are taken into account as discrete probability distribution while calculating the standard deviation.

It can be seen from the last column of TABLE VI that BPI\textsuperscript{S} is by far the most affected by the fault rate of lines and the mean value of clearing time required by primary line protection relays. These two parameters impact the frequency and duration of sags, respectively. Lines and their associated primary protection relays have larger influence on BPI\textsuperscript{S} compared to buses, as the fault rate of lines is much larger than that of buses. It can be also seen that BPI\textsuperscript{S} is not very sensitive to variation in other parameters for the settings given in TABLE VI.

**Fig. 9. Tornado diagram of eight parameters in sensitivity analysis of BPI\textsuperscript{S}.**

The sensitivity analysis of the first eight parameters, which are related to the network components, is represented using a Tornado diagram, which depicts the most sensitive precedent parameter along with the impact on the overall result [35]. Tornado analysis determines the effect on the overall result by changing one variable at a time. The Tornado diagram is shown in Fig. 9, and it indicates the range (both minimum and maximum) of the obtained BPI\textsuperscript{S} when the corresponding parameter setting varies. The vertical solid line marks the base value of BPI\textsuperscript{S}, i.e., the BPI\textsuperscript{S} obtained using the default parameter settings given in Section III. A. It can be seen that BPI\textsuperscript{S} is mostly affected by parameters 2 and 7, i.e., the fault rate of lines and the mean value of clearing time required by primary line protection relays, which is in line with the standard deviation analysis above.

**V. CONCLUSIONS**

This paper presents a methodology for assessment of voltage sag performance in distribution networks and identification of the most affected buses. It takes into account a number of probabilistic phenomena affecting sag performance of individual bus and generic equipment sensitivity curve. The assessment incorporates both network performance and potential customers’ plant sensitivity to voltage sags. Though the methodology is illustrated on distribution network case study in the paper, it can be equally well applied to any type of power network, distribution or transmission, meshed or radial.

Detailed voltage profiles of buses are derived based on simulating various faults in the network. Since the majority of sags are caused by faults and cleared by protection relays, the sag duration and frequency are derived based on protection relay reliability and fault rates of different components in the network, using a newly developed duration zone division method which decomposes the probabilistic distribution of
clearing time by primary and secondary protection relays. The resulting sag magnitude and duration are used to derive the single-event characteristic SSIs developed in the companion paper, for each sag at each bus in the network.

Following this new single-event characteristic, bus performance index with respect to voltage sags (BPI\textsuperscript{S}), is defined to quantify bus sag performance by combining the sag occurrence frequency and SSI. Even though the calculation and application of BPI\textsuperscript{S} are illustrated using the results of sag simulations in the paper, i.e., sag characteristics obtained from fault simulations in the network, they can be equally well calculated if relevant sag characteristics are obtained from sag monitoring results which makes the BPI\textsuperscript{S} suitable for post-processing of sag monitoring results as well. Using BPI\textsuperscript{S} and by incorporating yearly load variation of the network a robust assessment of voltage sag performance at buses in the network is achieved. Heat maps are used to identify the critical areas of the network exposed to potentially severe consequences of voltage sags. A number of factors affecting accuracy of the assessment are analysed and sensitivity of the results to different parameters is established.

REFERENCES