Dielectric constant of a three-dimensional woven glass fibre composite: analysis and measurement

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Abstract

This paper presents a novel methodology for predicting the dielectric constant of three-dimensional woven glass fibre-reinforced composites. A well-established approach of deriving the effective dielectric constant is the dielectric mixing formulae (rule of mixtures based), which either provide a single value or offer upper and lower bounds. For composites with three-dimensional fibre architecture, an accurate model considering the three-dimensional effect is needed. Here, the anisotropic effect is revealed using electromagnetic simulation to extract the effective dielectric constant of a model material with unidirectional fibres, which are aligned or orthogonal to the electric field. The rule of mixtures based formulae are evaluated. The most suitable formula selected for each case is then extended to a general case with arbitrary fibre orientation and is further used to characterise the capacitor element of an electromagnetic model for 3D woven composites. The proposed method is compared to measurements to demonstrate the improved accuracy.

Keywords: Dielectric constant; glass fibre; 3D woven composites; simulation; modelling.
1. Introduction

Glass fibre-reinforced polymer (GFRP) composites are widely used in aerospace, shipping, automotive industries and wind turbine blades, due to their numerous advantages such as high stiffness and strength characteristics, a high strength-to-weight ratio and better corrosion/chemical resistance [1–3]. Recently, the woven fabric composites have received increasing interest, as manufacturing costs are reduced and the impact damage resistance is significantly improved [4]. For example, in 3D woven composites, the warp yarns, weft yarns and Z-binders are highly interlaced in three directions resulting in the through-thickness reinforcement that prevents delamination hence improves damage resistance. It is also important to understand the electromagnetic (EM) characteristics for applications (e.g., airborne radomes and wind turbine blades [5]) where radar interference [6], lightning discharge and electromagnetic testing are all of interest. For dielectrics such as glass fibre composites the primary constituent parameter is the real part of the relative permittivity (also called dielectric constant, \(\varepsilon'\)), as the dielectric loss (evaluated by loss tangent, tan\(\delta\)) is minimal and the magnetic permeability is equal to that of free space.

Previous studies have considered how \(\varepsilon'\) varies as a function of fibre volume fraction. A number of rule of mixtures based formulae have been developed, such as Wiener limits, Maxwell Garnett formula and Looyenga formula [7]. In these closed-form formulae, the effective dielectric constant is a function of the dielectric constants of the fibre and resin. The upper and lower bounds of the effective dielectric constant can be provided by Wiener upper limit and Wiener lower limit, respectively [8]. The limits were given based on an analysis of laminated structures, which corresponded to capacitors connected in series or in parallel with respect to the applied electric field. However, most practical cases are between the two limits, and the search for accurate models for these intermediate cases has drawn considerable attention in the dielectric constant research community.
An alternative equivalent lumped circuit model introduced by Chin et al. [9,10] was built for unidirectional and multidirectional laminates, where a lamina was modelled as parallel lumped resistor–capacitor circuit. This method accurately predicted the data obtained from free space measurement over X-band (8-12 GHz). Yao et al. [11] extended this circuit model to 3D orthogonal woven composites. In the modelling, the effects of the warp yarns and Z-yarns that were both orthogonal to the electric field were assumed the same. The simulation results did not agree well with experiments.

With 3D weaves, the permittivity becomes more anisotropic, so the effect of fibre direction should be thoroughly studied. In this paper, we introduce an EM pre-processing approach that investigates the permittivity when the fibre directions are orthogonal. A composite model with unidirectional fibres is built in a waveguide section. The scattering parameters obtained from simulation are used to extract the effective dielectric constant, and this is repeated for a range of fibre volume fractions. These basic models are then compared to the results produced by a set of mixing formulae, and for each fibre direction a particular formula is selected. This recommendation is then used to develop a capacitor model of a 3D woven composite structure from which the overall dielectric constant is estimated. This cycle is completed for a number of practical setups and the predicted results compare well with the experiment.

2. Evaluation of the effect of fibre directions

2.1 Numerical simulation

The dielectric constant of the fibre-resin mixture is dependent on the angle between the fibre orientation and the electric field of the incident electromagnetic wave [12]. This polarisation effect is investigated by numerical simulation using CST® software. As shown in Figure 1(a), a square unit cell made up of a circular cross-sectional fibre and resin is employed here to represent the microstructure of the yarn. The dielectric properties of both fibre and resin are listed in Table 1.
Table 1 Dielectric properties of the fibre and resin used for investigation of the polarisation effect

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<tbody>
<tr>
<td>Dielectric constant</td>
<td>6.20</td>
<td>3.00</td>
</tr>
<tr>
<td>Loss tangent</td>
<td>$1.50 \times 10^{-3}$</td>
<td>$1.67 \times 10^{-2}$</td>
</tr>
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</table>

Assuming the size of the unit cell is $L$ and the diameter of the fibre is $D$, the fibre volume fraction $v_f$ is written as

$$v_f = \frac{\pi D^2}{4L^2} \quad (1)$$

As illustrated in Figure 1 (b), the electric field of the incident EM waves with TE$_{10}$ mode is in the Y direction and the waves propagate along the Z direction. Three representative cases are considered, i.e., the fibre direction parallel to the electric field (Case 1, Figure 1 (c)), orthogonal to the electric field (Case 2, Figure 1 (d)) and parallel to the propagation direction (Case 3, Figure 1 (e)). The cuboids for the three cases are modelled within the X-band waveguide, the inner dimensions of which are 22.86 mm ($a'$) × 10.16 mm ($b'$). X-band is chosen as this range is widely used for navigation radars [14].

$L$ is set to be 2.54 mm ($1/9$ of $a'$ and $1/4$ of $b'$) for easy implementation. In each case, $v_f$ is 60 %. As seen in Figure 1(f), the thickness along the waveguide ($t$) is 2.54 mm, and Port 1 and Port 2 are electromagnetically distant from the sample with air gaps of 50 mm.

(a) unit cell of the fibre-resin mixture
The simulation results presented in Figure 2 demonstrate that the scattering parameters (reflection coefficients $S_{11}$ and transmission coefficients $S_{21}$) are significantly affected by fibre directions. And some of the assumptions made in the literature are incorrect, i.e., Case 2 and Case 3 are electromagnetically equivalent [11].
2.2 Calculation of the effective permittivity from the scattering parameters

The Tischer model [15] can be adopted for evaluation. Assuming there is only one sample layer with a thickness of $t$, the expressions for $S_{11}$ and $S_{21}$ are:

$$
[S_{11}]_{\text{free-mim}} = \frac{-2(\gamma_0^2 + \beta_0^2) \sinh \gamma_0 t}{2(\gamma_0^2 - \beta_0^2) \sinh \gamma_0 t + 4j\gamma_0 \beta_0 \cosh \gamma_0 t}
$$

(2a)

$$
[S_{21}]_{\text{free-mim}} = \frac{4j\gamma_0 \beta_0}{2(\gamma_0^2 - \beta_0^2) \sinh \gamma_0 t + 4j\gamma_0 \beta_0 \cosh \gamma_0 t}
$$

(2b)
where \( \beta_0 = 2\pi / \lambda_g \) is the phase constant of the transmission line. \( \gamma_0 \) is the complex propagation constant. \( \lambda_g \) is the guide wavelength of the empty waveguide.

\[
\gamma_0 = j2\pi \sqrt{\frac{\varepsilon_r}{\lambda^2} - \frac{1}{\lambda_c^2}} \tag{3a}
\]

\[
\lambda_g = \sqrt{\frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \tag{3b}
\]

where \( \lambda \) and \( \lambda_c = 2a' \) are the free space wavelength and cut-off wavelength, respectively. The Newton-Raphson approximation method is used to obtain the unknown complex propagation constant \( \gamma_0 \) from the complex transcendental equations:

\[
\begin{bmatrix} S_{11} \end{bmatrix}_{\text{CST}} - \begin{bmatrix} S_{11} \end{bmatrix}_{\text{fiber-resin}} e^{-j\beta_0 L_{air}} = 0 \tag{4a}
\]

\[
\begin{bmatrix} S_{21} \end{bmatrix}_{\text{CST}} - \begin{bmatrix} S_{21} \end{bmatrix}_{\text{fiber-resin}} e^{-j\beta_0 (L_{air1} + L_{air2})} = 0 \tag{4b}
\]

where \( L_{air1} \) and \( L_{air2} \) are the lengths of the air gaps shown in Figure 1 (f). Here, the S-parameters obtained from CST simulation are phase shifted due to the presence of the air gaps.

Finally, the effective permittivity \( (\varepsilon_{\text{eff}}') \) can be obtained from

\[
\varepsilon_{\text{eff}}' = \lambda^2 \left[ \left( \frac{\gamma_0}{2\pi} \right)^2 + \frac{1}{\lambda_c^2} \right] = \varepsilon_r' (1 - j \tan \delta) \tag{5}
\]

As mentioned above, only the real part of the relative effective permittivity (i.e., dielectric constant) is of interest in the following analysis. Two limiting cases with either 100% glass fibre or resin are calculated to verify the accuracy of the permittivity extraction approach. As presented in Figure 3 (a), it is indicated that there is good agreement between the simulation and the real values (Table 1). In each representative case, the dielectric constant slightly increases with increasing frequency. The highest values exist in Case 1, which is followed by Case 2 and Case 3. It is noted again that the results of Case 2 and Case 3 differ significantly. The variation of the dielectric constant with respect
to $v_f$ is shown in Figure 3 (b). At each $v_f$, the dielectric constant values are averaged over the frequency range. The curves for Case 2 and Case 3 overlap when $v_f$ is less than 20 %. When $v_f$ is above 60 %, the curve for Case 2 becomes close to that for Case 1.

(a) The simulation results for two limiting cases and the three representative cases with $v_f=60 \%$

(b) Variation of the dielectric constant with respect to $v_f$

Figure 3 Comparison of the dielectric constants of the three representative cases

2.3 Comparison between simulation results and the rule of mixtures based formulae

Mixing formulae are based on relative volumes of fibres and resin. A number of formulae have been reported in the literature and are reproduced below for completeness:

Wiener upper limit:  
$$\varepsilon' = v_f \varepsilon'_f + (1 - v_f) \varepsilon'_m$$  
(6)

Wiener lower limit:  
$$\varepsilon' = \frac{v_f \varepsilon'_m + (1 - v_f) \varepsilon'_f}{v_f \varepsilon'_m + (1 - v_f) \varepsilon'_f}$$  
(7)

Hashin Shtrikman upper bound:  
$$\varepsilon' = \varepsilon'_f + \frac{1 - v_f}{v_f - \frac{1}{3 \varepsilon'_f - \varepsilon'_f - \varepsilon'_m}}$$  
(8)

Hashin Shtrikman lower bound:  
$$\varepsilon' = \varepsilon'_m + \frac{v_f}{\varepsilon'_f - \varepsilon'_m + \frac{1 - v_f}{3 \varepsilon'_m}}$$  
(9)
Maxwell Garnett mixing rule: 
\[ \varepsilon' = \varepsilon'_m + \frac{3\nu_f\varepsilon'_f}{[(\varepsilon'_f + 2\varepsilon'_m)/(\varepsilon'_f - \varepsilon'_m)] - \nu_f} \]  
(10)

Looyenga formula: 
\[ \varepsilon'^{\frac{1}{3}} = \nu_f\varepsilon'^{\frac{1}{3}}_f + (1 - \nu_f)\varepsilon'_m^{\frac{1}{3}} \]  
(11)

where \( \varepsilon'_f \) and \( \varepsilon'_m \) are the dielectric constants of the fibre and resin, respectively.

The equation for Hashin Shtrikman lower bound is identical to the Maxwell Garnett mixing rule. In the circuit model proposed by Chin [9], the expression for Y-directed fibre (Case 1) is the same as Wiener upper limit. And the formula for X-directed fibre (Case 2) is

\[ \varepsilon' = \frac{4\varepsilon'^2_m + \pi\varepsilon'_m\sqrt{\nu_f}(\varepsilon'_f - \varepsilon'_m)}{4\varepsilon'_m + \pi(\varepsilon'_f - \varepsilon'_m)(\sqrt{\nu_f} - \nu_f)} \]  
(12)

As shown in Figure 4, the dielectric constant is underestimated by the Wiener lower limit and the circuit model for Case 2, while the other expressions are close to the CST simulation results.

Specifically, the results provided by the Wiener upper limit, Hashin upper bound and Looyenga formula agree with those by simulation for Case 1, Case 2 and Case 3, respectively. Therefore, these three expressions can be used to effectively evaluate the dielectric constants of the three elementary cases with unidirectional fibres.

![Figure 4 Comparison of the predicted dielectric constants given by the simulation and the rule of mixtures based formulae](image)
2.4 Verification of the recommendation for dielectric constant calculation

The recommendation of using the Wiener upper limit for the model with Y-directed fibres, Hashin upper bound for the model with X-directed fibres and Looyenga formula for the model with Z-directed fibres was based on one type of fibre-resin specified in [13]. It is worth investigating the effect of changing the fibre-resin type on this recommendation/result. The dielectric properties of an alternative fibre-resin set are listed in Table 2. The fibre volume fraction is 60 % as well.

Table 2 Dielectric properties of another set of glass fibre and resin used for verification

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<tr>
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<tbody>
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<td>Dielectric constant</td>
<td>7.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Loss tangent</td>
<td>3.1×10⁻³</td>
<td>2.0×10⁻²</td>
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As demonstrated in Table 3, reasonable accuracy can be obtained by the proposed strategy. The error of -5.10 % for Case 1 is more acceptable than the error of 30 % that would result if the Wiener lower limit were used.

Table 3 Comparison of the effective dielectric constants calculated from simulation and the three selected formulae for another set of glass fibre and resin

<table>
<thead>
<tr>
<th>Method</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
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<tbody>
<tr>
<td>Value</td>
<td>Error (%)</td>
<td>Value</td>
<td>Error (%)</td>
</tr>
<tr>
<td>CST simulation</td>
<td>5.48</td>
<td>-</td>
<td>5.20</td>
</tr>
<tr>
<td>Wiener upper limit</td>
<td>5.40</td>
<td>-1.56</td>
<td></td>
</tr>
<tr>
<td>Hashin upper bound</td>
<td>5.19</td>
<td>-0.23</td>
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</tr>
<tr>
<td>Looyenga formula</td>
<td></td>
<td></td>
<td>5.13</td>
</tr>
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2.5 General case with non-orthogonal fibre orientations

In practice, for actual weaves fibre directions will generally not be orthogonal. A transformation has been used in the literature for 2D generalisation [17] and here this is extended for 3D modelling. As
shown in Figure 5, if the angle between the fibre direction and X axis is $\theta$ and the angle between the fibre direction and X-Y plane is $\phi$, the effective dielectric constant tensor $[\varepsilon'](\theta, \phi)$ can be given by

$$[\varepsilon'](\theta, \phi) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \phi & 0 & 0 \\ 0 & \varepsilon'_{\text{Case 2}} & 0 \\ 0 & 0 & \varepsilon'_{\text{Case 1}} \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & 0 \\ 0 & \varepsilon'_{\text{Case 2}} & 0 \\ 0 & 0 & \varepsilon'_{\text{Case 1}} \end{bmatrix}$$

(13)

Figure 5 Schematic diagram of an arbitrary fibre direction with respect to the electric field vector and propagation direction in a 3D coordinate system

3. Electromagnetic modelling of 3D woven glass fibre composites

Here a 3D woven structure is meshed into finite elements. Different from the element types (e.g., bar, beam or shell) for mechanical analysis [18], the concept of a parallel-plate capacitor is employed to represent the dielectric characteristics of the material, which is illustrated in Figure 6. A real capacitor can be represented as a capacitance and a resistance in parallel [19]. The capacitance $C_p$ is written as:

$$C_p = \frac{\varepsilon_0 \varepsilon'_p A}{d}$$

(14)

where $A$ is the area of the plate and $d$ is the distance between the two parallel plates. $\varepsilon_0 = 8.854 \times 10^{-12}$ F·m$^{-1}$ is the permittivity of free space, and $\varepsilon'_p$ is the effective dielectric constant of the in-between medium (e.g., the resin, Case 1, Case 2, Case 3 or general cases discussed above).

The resistance $R_p$ is given by
\[ R_p = \frac{d}{\sigma A} \]  

(15)

where \( \sigma \) is the conductivity of the dielectric material. However, the resistor can be viewed as open circuit, as the conductivities of both glass fibre (~\(10^{-10} \) S/m [20]) and resin (~\(10^{-7} \) S/m [21]) are significantly low (hence the loss tangent of the material is negligible). Therefore, only the capacitance is considered here.

(a) A capacitor model used as the finite element  
(b) Equivalent circuit model of the element

Figure 6 The parallel-plate capacitor model used as the finite element in the proposed electromagnetic modelling of 3D woven glass fibre composites

Based on the capacitor model, a 3D woven orthogonal structure is built as an example. As shown in Figure 7 (a), a representative volume element (RVE) [22] is represented by a matrix of \( m \times n \times \ell \) capacitors (Figure 7 (b)). The plane perpendicular to the electric field is made up of \( n \times \ell \) small areas, namely \( A_{jk} \), \( j=1, 2, \ldots, n \) and \( k=1, 2, \ldots, \ell \). The thickness of the \( i \)th layer is denoted by \( d_i \), \( i=1, 2, \ldots, m \).

(a) RVE of a 3D orthogonal woven structure
According to Equation (14), the capacitance of each element $C_{ijk}$ can be rewritten as

$$C_{ijk} = \varepsilon_0 \varepsilon'_{ijk} \frac{A_{jk}}{d_i}$$  \hspace{1cm} (16)$$

Therefore, the total capacitance can be derived:

$$C = \sum_{j=1}^{n} \sum_{k=1}^{\ell} \left( \sum_{i=1}^{m} \frac{1}{C_{ijk}} \right)^{-1}$$  \hspace{1cm} (17)$$

The overall capacitance can also be represented by

$$C_{RVE} = \varepsilon_0 \varepsilon'_{eff} \frac{A_{RVE}}{d_{RVE}} = \varepsilon_0 \varepsilon'_{eff} \frac{\sum_{j=1}^{n} \sum_{k=1}^{\ell} A_{jk}}{\sum_{i=1}^{m} d_i}$$  \hspace{1cm} (18)$$

where $A_{RVE}$ is total area orthogonal to the electric field, and $d_{RVE}$ is the whole length along the electric field. The effective dielectric constant of the 3D woven structure $\varepsilon'_{eff}$ can be readily obtained from Equations (17) and (18):

$$\varepsilon'_{eff} = \frac{\sum_{j=1}^{n} \sum_{k=1}^{\ell} \left( \sum_{i=1}^{m} \frac{1}{C_{ijk}} \right)^{-1} \cdot \sum_{i=1}^{m} d_i}{\varepsilon_0 \sum_{j=1}^{n} \sum_{k=1}^{\ell} A_{jk}}$$  \hspace{1cm} (19)$$

Figure 7 Proposed electromagnetic modelling of a 3D orthogonal woven composites
4. Verification of the proposed 3D electromagnetic modelling methodology

4.1 Comparison with the literature

The proposed methodology is compared with the prediction presented in Reference [11], where interply and intraply hybrid composites were studied. As given in Table 4, the accuracy of the dielectric constants offered by the present work is improved with errors within 5 %.

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<tbody>
<tr>
<td></td>
<td>Value</td>
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<td>Value</td>
</tr>
<tr>
<td>Interply</td>
<td>3.95</td>
<td>-7.34</td>
<td>3.8091</td>
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<tr>
<td>Intraply</td>
<td>3.91</td>
<td>-6.14</td>
<td>3.8062</td>
</tr>
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</table>

4.2 Experiment

4.2.1 Sample preparation

A 3D woven angle interlock glass fibre sample was measured for further verification. As illustrated in Figure 8, the sample consisted of four warp layers, three weft layers and the binder inserted after every three layers of the weft yarn [23]. A twofold yarn with a 1360 Tex was used in the warp and weft yarns, while a single yarn of a 680 Tex was used for the binder. The fabric counts in the warp yarns, weft yarns and binder were 3.95 ends/cm, 2.8 picks/cm and 3.0 ends/cm, respectively. The S2 glass fibres provided by AGY were infused with the epoxy resin LY564 and hardener XB 3486 from Huntsman Advanced Materials. The infusion process was done by the vacuum-assisted resin transfer moulding (VARTM) technique, then the whole assembly was moved to an oven for curing at 80 °C for eight hours. The yarn volume fractions of the warp, weft and binder were 31.21 %, 15.83 % and 3.05 %, respectively. The thickness of the sample was 3.03 mm.

It is known that the dielectric constant of S2 glass fibre is 5.20 [20]. As described in Section 3, in order to predict the effective dielectric constant of the sample, the dielectric constant of the resin should be known in advance as well. Hence, a neat epoxy resin sample was fabricated for evaluation.
4.2.2 Dielectric constant measurement

The dielectric constant measurement over X-band was performed using the transmission line technique as schematically illustrated in Figure 9 (a). An HP8510C Vector Network Analyser (VNA) was used. It was calibrated before test using the thru-reflect-line (TRL) standard [24]. A personal computer was connected to the VNA by a GPIB cable. A MATLAB® programme was developed for data acquisition and dielectric constant computation.

The test samples with the inner dimensions of the rectangular waveguide were mounted on the waveguide flange. As shown in Figure 9 (b), the binder of the 3D woven sample was along the broad dimension of the waveguide. Hence, in the modelling the warp yarns, weft yarns and binders corresponded to Case 1, Case 2 and a general case, respectively.
4.2.3 Results and discussions

$S_{11}$ and $S_{21}$ obtained from the test of the 3D woven glass fibre sample are presented in Figure 10. From the measurement data, the dielectric constant is calculated using Equations (2-5). The experimental and predicted results are given in Figure 11. The average effective dielectric constant of the neat resin is approximately 2.74, and the dielectric constant of the 3D woven sample remains stable at 3.75. The Wiener upper limit and Wiener lower limit are employed for comparison. The dielectric constants provided by the two limits are 3.98 and 3.60 with errors of 6.13 % and 4.05 %, respectively. However, the dielectric constant predicted by Equation (19) is 3.85 with a smaller error of 2.67 %.
5. Concluding remarks

Permittivity characterisation of glass fibre composites is a prerequisite to modelling electromagnetic effects. In the three-dimensional weave, the fibre direction varies throughout the structure. The interaction between the electric field of incident EM waves and the fibre depends on the relative orientation and this leads to anisotropic permittivity effects. This 3D anisotropy is evaluated here for the first time using basic EM models corresponding to orthogonal fibre directions. From the numerical results the most appropriate existing mixing formula for each case is selected.
Experimental set-ups reported in the literature and new test results presented here demonstrate improved accuracy when the anisotropy is taken into account. As a next step, this work could be extended to other types of glass fibres and other fibre architectures such as 3D braided composites.

Acknowledgements

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References


