Planning Low-Carbon Electricity Systems under Uncertainty Considering Operational Flexibility and Smart Grid Technologies

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Summary

Electricity grids’ operators and planners need to deal with both the rapidly increasing integration of renewables and an unprecedented level of uncertainty originated by unknown generation outputs, changing commercial and regulatory frameworks aimed to foster low-carbon technologies, evolving availability of market information on feasibility and costs of various technologies, etc. In this context, there is a significant risk to lock into inefficient investment planning solutions determined by current deterministic engineering practices that neither capture uncertainty nor represent the actual operation of the planned infrastructure under high penetration of renewables. We therefore present an alternative optimisation framework to plan electricity grids that deals with uncertain scenarios and represents increased operational details. The presented framework is able to model the effects of an array of flexible, smart grid technologies that can efficiently displace the need for conventional solutions. We then argue, and demonstrate via the proposed framework and an illustrative example, that proper modelling of uncertainty and operational constraints in planning is key to valuing operationally flexible solutions leading to optimal investment in a smart grid context. Finally, we review the most used practices in power system planning under uncertainty, highlight the challenges of incorporating operational aspects, and advocate the need for new and computationally-effective optimisation tools to properly value the benefits of flexible, smart grid solutions in planning. Such tools are essential to accelerate the development of a low-carbon energy system and investment into the most appropriate portfolio of renewable energy sources and complementary enabling smart technologies.

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1. Introduction

The evolving landscape: from conventional electricity systems to low-carbon smart grids

Current electricity grids comprise large generating units that produce power generally far from load centres, high-voltage transmission networks that ship power from production centres to load centres (up to the so-called primary substations), and lower-voltage distribution networks that transfer power from primary substations to commercial, industrial and residential consumers. In this electricity system, referred to as conventional, (i) generation is typically carbon intensive, powered by fossil fuels, and in some cases supported by hydro plants that are usually large so as to take advantage of economies of scale; (ii) transfer capability of networks (that are passive and inflexible) is mainly delivered through more investment in asset-heavy infrastructure (e.g., lines and transformers); and (iii) demand is mostly dominated by consumers’ end-use requirements only and thus unresponsive to system/market conditions. In this context, operators aim to run electricity grids in an economically efficient and reliable manner by mainly controlling generation outputs. Such a control allows maintaining the instantaneous production-demand balance and transfers within network capacity limits. In addition, sufficient security margins in electricity infrastructure (generation and network) are kept to withstand credible outages (e.g., sudden failure of a generating unit or network circuit). In planning timescales, network design decisions attempt to eliminate congestions through investment in asset-heavy infrastructure up to the point where the marginal benefit of more network capacity (i.e., savings in congestion costs and reliability costs driven by the installation of further lines and transformers\(^1\)) is equal to the marginal cost of network investment [1]. While network infrastructure is centrally planned, generation investment is generally market driven and hence decentralised; historically, network planners have thus responded through more network investment to new connection requirements from generation, which engenders system expansion.

In the coming years, the above-mentioned conventional electricity network will evolve towards the so-called smart grid. Firstly, generation will increasingly be low-carbon and distributed across the electricity system, being even located at the point of power consumption (e.g., rooftop photovoltaic – PV – panels). Low-carbon generation from renewables such as wind and solar power will create the need to counteract their variable and partly unpredictable power injections through operational measures so as to maintain the instantaneous production-demand balance (i.e., system frequency), generation outputs and network transfers within capacity limits, and the required security margins in generation and transfer capability (i.e., reserves and network redundancy). Secondly, distribution networks will transition from mostly passive networks, where the control problem is resolved at the planning stage by designing for the worst-case peak-demand scenarios, to active systems, where new Information and Communications Technologies (ICT) and controllable distributed energy resources, including storage and flexible demand, will provide real-time supply/demand balance to efficiently integrate local renewables while also interacting with the transmission system [2][3]. Thirdly, the transmission system itself will become much more active through (i) new controllable and flexible technologies such as Flexible AC Transmission Systems (FACTS) [4] and High-Voltage DC (HVDC) systems [5] that can control power flows through the network without changing power injections/withdrawals, (ii) System Integrity Protection Schemes (SIPS) that can enforce rapid increase/reduction of power in importing/exporting areas after a network outage occurs by, for instance, curtailing generation and/or demand [6], and (iii) various Wide Area Monitoring and Control (WAMC) equipment supported by ICT technologies that will increase the capability of system operators to monitor and control electricity assets in real time and throughout wider areas through increased communication [7][8]. Last and more importantly, demand will be controllable and end users will become active participants in system and market operation, thereby opening up opportunities for aggregating and coordinating consumers and system needs (e.g., for system balancing and congestion management), taking advantage of flexibility from smart appliances (e.g., dishwashers, tumble dryers, etc.), electric heaters, batteries, etc. [9][10]. Also, electrification of other energy demands from the heating/cooling (e.g., electric heat pumps and heating ventilation and air conditioning equipment) and transport (e.g., electric vehicles) sectors can further expand the opportunities to control electricity loads and coordinate them with system needs owing to their intrinsic virtual storage capabilities [11][12].

In the low-carbon smart grid context outlined above, and focusing on the system as a whole and the transmission level, a change in system frequency due to system level supply/demand imbalance can be resolved through the combined action of various fast generating units and loads (e.g., disconnection of non-critical load from dishwashers and/or refrigerators [10], contribution from distributed battery systems including those from parked electric vehicles, connection of distributed backup generation, etc.), while network congestions can be resolved through changes in topology and/or impedances (e.g., changing FACTS setpoints) rather than through costly changes in the output of generation. This increase in operational flexibility can be used to address both real-time operation and time-ahead scheduling, where

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\(^1\) Congestion cost is driven by network transfer limits and is equal to the extra cost of operation due to network capacity constraints, while reliability cost is driven by the demand curtailment associated with network capacity and is equal to the expected unsupplied demand times the Value of Lost Load, VoLL.

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strategic decisions are made in advance (e.g., a few hours ahead relative to real time) to deal with the uncertain evolution of wind and solar power generation, and so adapt to different realisations that may happen in real time. In this context, generation reserves can be coordinated with demand response, charge/discharge actions from storage, and even network topology reconfiguration so as to deliver the needed balancing services requested in real time due to wind/solar forecast errors and their potentially high variability [9][13]. In particular, such coordination of multiple, flexible operational measures can increase network utilisation and decrease holding levels of generation reserves that may be significantly costly due to the high levels of wind and solar power expected in the near future [14][15].

In smart grid planning, it remains unclear whether new asset-heavy investment (e.g., a new line and/or transformer) will be needed even in the presence of increased ICT infrastructure and fast control of network devices, storage and demand, which can rapidly instigate reduction of transfers and even eliminate network congestion [14]. The availability of distributed storage, for instance, could locally provide power to consumers while battery charging and transfer actions take place during hours where the network is not congested. Likewise, fast control of network devices could eliminate post-contingency, real-time congestion by changing network topology (e.g., through line switching), impedances (e.g., through the adjustment of FACTS setpoints and Soft Normally Open Points [16], i.e., relying on power electronics-based technology) and power flows (e.g., through dispatchable HVDC links), thereby reducing the need for network redundancy. Furthermore, the reliability role of redundant transmission and generation infrastructure could be substituted by the deployment of flexible devices that can carry out advanced operational measures, creating a strong relation between planning and operation that should be carefully studied when expanding the system [17]. In addition, economics and reliability will not be the only drivers of network planning and thus new investment solutions should also facilitate the achievement of environmentally-driven energy policy targets. Furthermore, due to the increased levels of requests for new renewable generation connections featuring extremely shorter construction times (shorter than those needed to build network infrastructure and definitely shorter than those needed to build large conventional power plants), network planners will need to undertake proactive investment, in anticipation of connection requirements from generation, taking real advantage of the economies of scale of transmission investment. In the smart grid paradigm, network planners should thus proactively drive system expansion rather than react to generation proposals, and support network decisions through the deployment of flexible, smart network technologies in order to more effectively adapt to the unfolding future scenarios.

Opportunities for advanced optimisation models to plan electricity grids

The increase in automation and flexibility due to new generation, storage, network, demand and ICT technologies is paramount to face intermittent and uncertain power outputs from renewable generation and defer the need for conventional network infrastructure. In fact, any attempt to face the increase in renewables through current practices and without the adequate upgrade in emerging network technologies could be extremely costly and even infeasible [14]. To truly unleash flexibility, however, operators will need to control a much wider array of setpoints to ensure that economics and reliability are maintained at acceptable levels. For example, it is expected that in the UK generating units providing fast frequency control (i.e., fast balancing) could increase from about ten (large generating units providing frequency control) to some 600,000 (if 10% of small PV and wind units support frequency control); network automatic controls such as voltage regulation devices could increase from 10,000 to 900,000; and automatic controls in homes (energy management systems) could increase from virtually zero to 15 million (if half of the installed smart meters were to link with energy management devices) [18]. Along with the increase in control setpoints (and thus optimisation variables), the amounts of data available in operational timescales will escalate at all levels due to technologies such as smart meters –at the consumer level– and phasor measurement units (PMUs) [19] –at both transmission and distribution network levels– that will allow operators to significantly improve their visibility and thus state estimation of the system. With a clearer picture of system conditions in real time through increased volumes of data and with a higher level of controllability of network equipment, operators will thus be able to run the system in a more secure fashion, utilising network infrastructure at higher levels (i.e., with less redundancy and less congestion) and therefore improving the overall economic performance of the system operation activity as well as deferring the need for conventional electricity infrastructure investment.

In addition, there is an increasing need to capture coordination actions across various energy vectors, including all electricity sectors (from local energy management systems, distribution and transmission networks to generation), gas, heat/cooling and transport [12]. In fact, flexibility from electricity demand can be coupled with heat and/or mobility needs from consumers, and this may have important impacts on electricity network operation and investment beyond distribution networks, affecting also the transmission and generation infrastructure. For example, flexible demand from electric vehicles (i.e., battery charging/discharging actions) or heat pumps [11] can be controlled so as to reduce electricity demand during peak hours, provide frequency control services, provide congestion management services (in

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2 Building a 1000-MW line in anticipation of connection requirements is likely to be more efficient than building 10 lines of 100 MW as a result of 10 different connection requirements at different points in time, albeit there is a risk associated with the potential stranded network infrastructure if generation projects do not materialise as anticipated.
distribution and transmission networks), etc., ultimately driving both less costly operation and less investment in network and generation assets [9][20]. Coupling with other pieces of infrastructure, for example the gas network, can then further increase the power system’s flexibility and support integration of larger volumes of renewables [21], as well as even provide new forms of storage [22].

Recognition of power system dynamics and stability phenomena (that are usually in the timescale of a few seconds) is also becoming increasingly important in both the operational and planning decision-making processes [23][24], and this is also crucially linked with the penetration of advanced ICT infrastructure in power networks. Indeed, it is envisaged that current redundancy levels (that are utilised to provide security under the current operational doctrine) will be replaced with increased automation, monitoring, communication, control, etc. (i.e., via corrective control actions) [24]. However, these might fail to properly operate in real time, and such malfunctions, for example in the form of delays in communication, could originate network stability problems and even a cascading load disconnection [25][26]. Hence, adequate balance between network redundancy and advanced corrective control actions will need to be recognised so as to properly determine (i) transfer limits and congestion levels in operational timescales, and (ii) the need for new network reinforcements in the long term, considering both power and ICT network reliability [27][28].

Finally, an adequate treatment of uncertainty in both operational and planning timescales will be essential in the presence of renewables in order to minimise the risk to lock into inefficient solutions which ignore the multiple possible evolutions that networks may experience. In operational timescales, prediction of renewable generation production from wind and solar power plants will give rise to certain levels of forecast error against which operators hedge through balancing services [29][30]. Thus, if wind power outputs, for example, are less than expected, there will be a mix of generation and demand-based services that can counteract the lack of wind energy production. It will also be important that network congestion be efficiently managed in real time, even under those conditions where large forecast errors realise. In this context, the location of reserve services and the possibility to undertake corrective actions over flexible network components will be paramount. In planning timescales, network design will need to anticipate renewables connection and thus face uncertainty levels associated with the location and volume of new generation. Hence, planners will require a strategic approach and determine a balanced portfolio of present and future network investments in such a way that present, “here-and-now” decisions (using stochastic programming terminology) are sufficiently flexible to be adapted by further investment in the future when information regarding new generation capacity becomes more certain. Under this paradigm, flexible investment options and “wait-and-see” investment strategies may be valuable to avoid the implementation of network solutions that cannot be efficiently adapted to the unfolding future [31]-[33].

Overall, the comparison among alternative solutions when planning electricity grids will become increasingly complex since an extremely large set of options need to be objectively assessed before implementation, including calculation of their performance in operational timescales. In fact, as we will also demonstrate in this work, neglecting operational aspects, as customarily done in traditional planning tools, will provide inefficient solutions in a low-carbon smart grid context. Moreover, there will be a large volume of information that will become available due to new monitoring technology and a large number of control variables across voltage levels, electricity sectors/markets, and energy vectors, that need to be coordinated even across borders (in the so-called whole-system approach). In this regard, time resolution will also be critical to capture the effect of variable generation, the need for ancillary services (e.g., reserves) and even the occurrence of complex stability phenomena that can jeopardise system security. In addition, uncertainty will need to be recognised at various timescales in order to minimise the risk to lock into inefficient solutions [34]. Furthermore, there are also multiple objectives, beyond economics, that need to be balanced including security of supply, carbon emissions, and energy policy targets that should be met in the long term. Clearly, this creates a challenging environment for modellers who attempt to identify optimal decisions in planning.

**Paper scope and contributions**

On the above premises, the goal of this paper is threefold:

- to explain the new level of complexity associated with the low-carbon, smart grid context (especially when compared with that of conventional electricity systems), highlighting how this affects infrastructure planning concepts and practices;
- to propose an alternative optimisation framework to plan infrastructure in smart grids, exposing the modelling and computational challenges related to its implementation in actual power systems; and
- to demonstrate the importance of considering uncertainty along with operational flexibility when planning investment in innovative, smart grid technologies.

Note that there are a range of studies in the context of power system expansion planning under uncertainty [17][31][33]-[64] and power system expansion planning with representation of increased operational details, including unit commitment constraints [58][60][65]-[68]. Expanding on this, we propose a unified framework that couples infrastructure investment with market clearing/unit commitment and real-time decisions, while considering the uncertainties in both planning and operational timescales. We argue that the proposed framework is of utmost importance in the smart grid context, where the value of new, innovative technologies is fundamentally based on the provision of flexibility services that allow planners and operators to efficiently deal with various sources of uncertainty and constraints affecting both investment decisions and system operation.
The remainder of this paper is organised as follows. Section 2 presents the proposed modelling framework to value smart grid technologies. Section 3 describes the formulation of an instance of the proposed framework and its application to a small, textbook-like case study. Section 4 reviews the state of the art in power system planning under uncertainty and discusses the challenges, at the computational and algorithmic levels, of embedding operational aspects and scaling the optimisation models to real-size power networks. Section 5 concludes. Finally, the main notation used throughout the paper is described in the Appendix.

2. The Modelling Framework to Value Smart Grid Technologies

As argued in the previous section, the true value of smart grid technologies lies in their flexibility and thus ability to cope with uncertainty and variability\(^3\), and this will need to be properly recognised when planning electricity system infrastructure. In this context, we propose a modelling framework based on mathematical programming concepts, which can capture the effects of relevant operational constraints (so as to recognise the necessary flexibility levels to deal with a number of variable conditions in operational timescales) as well as the importance of uncertainty when planning new investment. To do so, the proposed framework considers decision variables in two different timescales (investment and operation), and the presence of long-term uncertainty to reflect the changing landscape faced by system planners, especially in terms of available technologies, costs and market conditions, energy policy and incentives, etc. (that may affect, for example, the future development of generation capacity that will in turn affect network expansion). In this framework, optimal investment decisions are made before the realisation of uncertain scenarios (e.g., investment cost of a new technology), whereas operational decisions aim to run the resulting infrastructure after each possible uncertain scenario realises and for a number of operating conditions (e.g., demand levels, wind power outputs, etc.). The optimal investment decisions minimise a given metric or objective function (e.g., expected total cost, maximum cost, maximum regret among all scenarios) and comply with a set of constraints that ensures feasibility under an array of physical laws and market rules as well as certain reliability levels (e.g., system reserves requirements). The mathematical framework is shown in Eqs. (1)-(3), where functional \(f_x(\cdot)\) represents the chosen utility function to select the optimal investment vector \(x(\cdot)\), \(C^I(\cdot,\cdot)\) and \(C^O(\cdot,\cdot)\) respectively represent the investment and operational cost functions, \(y(\cdot)\) is the matrix of operational decisions (per component and per operating condition or time period), and \(\varepsilon\) represents a given scenario of the uncertainty set \(E\). \(X(\cdot)\) and \(Y(\cdot,\cdot)\) are the feasibility sets of investment and operational decisions, respectively, where the latter depends on the optimal investment vector \(x(\cdot)\).

\[
\begin{align*}
\min_{x(\cdot),y(\cdot)} f_x(\{C^I(x(e),\varepsilon) + C^O(y(e),\varepsilon)\}) \\
\text{Subject to:} \\
x(e) \in X(e); \forall e \in E \\
y(e) \in Y(x(e),\varepsilon); \forall e \in E
\end{align*}
\]

The proposed framework is general enough to accommodate both two- and multi-stage optimisation programs, being \(x(\cdot)\) not dependent on \(\varepsilon\) in the former case. Functional \(f_x(\cdot)\) defines whether the model corresponds to a stochastic [69][70] or robust optimisation program [71]. Set \(Y(\cdot,\cdot)\), that aims to capture how the resulting infrastructure is operated and its corresponding costs, contains the physical (and market) laws associated with power system operation and may include: first and second Kirchoff’s laws and power flow equations, generation and network capacity constraints (including minimum and maximum limits), inter-temporal constraints including minimum up and down times and ramping limitations, system reserves requirements, etc. [72]. Note that operational constraints characterise the system flexibility levels from existing and prospective generation and network infrastructure. Therefore, these constraints are critical to determine the need for flexible, smart grid technologies when increasing levels of renewable generation are integrated into the electricity system.

Furthermore, we can add another level of complexity to our framework if we consider that system operation has to be planned ahead of real time (as well as investment has to be planned ahead of system operation). Under this new consideration, network infrastructure has to be also flexible to deal with the forecast errors (or short-term uncertainty) related to actual system conditions that will realise in the short-term future (e.g., next hours, next day) with respect to the expected conditions forecasted in advance by system operators. Clearly, poor forecast methods used, for example, to

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\(^3\) Uncertainty refers to the set of alternative scenarios that may potentially occur (in the long or short term) following a known or unknown probability distribution, while variability is certain and refers to the time-varying operating conditions that the electricity system will face in operational timescales (i.e., combination of demand levels and renewable generation outputs).

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predict future wind and solar power outputs will need higher flexibility levels from the electricity system infrastructure so as to deal with unforeseen events. In this context, new investments should be able to cope not only with various long-term uncertainties (e.g., trends in the investment cost of a new technology that may affect the development of generation capacity in the future), but also uncertain changes that may occur more rapidly in real time (e.g., due to wind forecast errors). To that end, the above-mentioned framework can be extended as shown in Eqs. (4)-(7), where \( f_{x,e,\xi} \) models the objective function being minimised, whereas \( y(\cdot) \) and \( z(\cdot,\cdot) \) represent the operational decisions undertaken in the scheduling or operations planning timescale (e.g., day-ahead) and in real time, respectively. \( \xi \) represents a given scenario of the uncertainty set \( \mathcal{E} \) that contains a number of realisations of operational parameters such as outputs of wind and solar power generation, generation and network outages, etc. Set \( Z(\cdot,\cdot,\cdot) \) represents the operational decisions that can be made in real time (e.g., actual deployment of generation reserves), which clearly are constrained by prior decisions made during the investment timescale (vector \( x(\cdot) \)) and operations planning timescale (matrix \( y(\cdot) \)).

\[
\min_{x(\cdot),y(\cdot),z(\cdot,\cdot)} \mathcal{C}^I(x(\cdot),e) + \mathcal{C}^0(y(\cdot),z(e,\xi),e,\xi) \tag{4}
\]

Subject to:
\[
x(\cdot) \in X(e); \forall e \in \mathcal{E} \tag{5}
\]
\[
y(\cdot) \in Y(x(\cdot),e,\xi); \forall e,\xi \in \mathcal{E} \tag{6}
\]
\[
z(e,\xi) \in Z(x(\cdot),y(\cdot),e,\xi); \forall e,\xi \in \mathcal{E} \tag{7}
\]

As discussed in subsequent sections, both frameworks pose important challenges at the computational and algorithmic levels that will need to be carefully addressed in the future to work with scalable optimisation models suitable for large-scale power networks. For illustration purposes, the next section is devoted to the description of an instance of Eqs. (1)-(3) and its application to a small, textbook-like case study.

3. Effects of Long-Term Uncertainty and Short-Term Operational Constraints on System Investment Portfolios

Overview
In order to illustrate the effects of relevant system operational constraints as well as the importance of uncertainty when planning new investment, we formulate an integrated generation and network planning model jointly capturing both operational aspects and long-term uncertainty. The resulting optimisation program is a particular instance of Eqs. (1)-(3) that is subsequently applied to a 3-node test system. The proposed model allows the system operator/planner to manage (i) short-term constraints associated with the (in)flexibility from conventional plants to cope with uncertainty and fast variability of renewables’ outputs, and (ii) long-term uncertainty associated with the evolution of investment costs of new technologies. In this context, the proposed framework provides the system operator/planner with efficient investment solutions (which we consider here as portfolios of generation, transmission, storage and flexible transmission equipment) in order to facilitate the operational management of the system (especially under high penetration of renewables) and permit a cost-effective adaptation of the infrastructure even under the realisation of unfavourable scenarios in the long term. Importantly, the model also recognises the prolonged construction times of investments such as conventional plants and long transmission lines (that create a lag between the investment decision and commissioning time\(^4\)), as well as the faster installation process of infrastructure such as wind and solar power units, batteries, and FACTS devices (that are assumed to be installed right after investment decisions have been made).

Problem formulation
Using the notation presented in the Appendix, we formulate our integrated generation and network planning problem as a stochastic, mixed-integer linear program. Note that, for unit consistency, all variables and parameters are expressed in per unit.

Long-term uncertainty
Long-term uncertainty is modelled through a multi-stage scenario tree (see Figure 1), which describes potential evolutions of future investment costs (especially in renewable technologies such as solar power generation), albeit further uncertainties can be included in a straightforward manner. Hence, as formulated in Eq. (8), the model minimises the

\(^4\) Commissioning time refers to when an asset starts its operation (e.g., when a line starts transferring power). The commissioning time minus the decision time is equal to the construction time of an asset.
expected value of the total cost, which includes the costs of investments and system operation as shown in Eqs. (9) and (10), respectively.

\[
\begin{align*}
\text{min} \left\{ \sum_{m \in M} \rho_m r_m (T_m + \pi^Q O_m) \right\} \\
I_m = \sum_{g \in G} \pi^G_{g,m} \hat{P}^G_{g,m} + \sum_{l \in L} \pi^L_{l,m} \mu^L_{l,m} + \sum_{b \in B} \pi^B_{b,m} \hat{P}^B_{b,m} + \sum_{l \in L} \pi^L_{l,m} \mu^L_{l,m} ; \quad \forall m \in M \\
O_m = \sum_{l \in L} \sum_{g \in G} \pi^G_{g,m} \hat{P}^G_{g,m,k} ; \quad \forall m \in M
\end{align*}
\]

As formulated in Eq. (9), investment costs include terms related to new conventional and renewable generation, new lines, new storage plants and new FACTS devices (shown in this order in Eq. (9)). Eq. (10) models the fuel costs of thermal power units across all time periods.

As per Eqs. (11) and (12), we assume that investments are irreversible and that some of them present a lag of one epoch (i.e., several years) between decision and commissioning times, respectively. Notice that Eq. (12) constrains some investment decisions to be equal among sibling nodes, which is caused by the above-mentioned one-epoch lag. Due to their prolonged construction times, investment in conventional assets cannot be commissioned in epoch 1 as imposed in Eq. (13).

\[
\begin{align*}
\hat{p}^G_{g,m}, \mu^L_{l,m}, \hat{p}^B_{b,m}, \mu^L_{l,m} \geq \hat{p}^G_{g,p(m)}, \mu^L_{l,p(m)}, \hat{p}^B_{b,p(m)}, \mu^L_{l,p(m)} ; \quad \forall g \in \bar{G}, \forall l \in \bar{L}, \forall b \in \bar{B}, \forall m \in \{M \setminus m = 1\} \\
\hat{p}^G_{g,m} = \hat{P}^G_{g,k} ; \quad \forall g \in \bar{G}, \forall m \in \bar{M}, \forall k \in S_m \\
\mu^L_{l,m} = \mu^L_{l,k} ; \quad \forall l \in \bar{L}, \forall m \in \bar{M}, \forall k \in S_m
\end{align*}
\]

Notice that Eq. (12) constrains some investment decisions to be equal among sibling nodes, which is caused by the above-mentioned one-epoch lag. Due to their prolonged construction times, investment in conventional assets cannot be commissioned in epoch 1 as imposed in Eq. (13).

\[
\begin{align*}
\hat{p}^G_{g,1} = 0 ; \quad \forall g \in \bar{G} \\
\mu^L_{l,1} = 0 ; \quad \forall l \in \bar{L}
\end{align*}
\]

System operation

The effect of the transmission network is characterised by a linearised power flow model. Eq. (14) models the first Kirchhoff’s law for nodal injections and withdrawals, whereby the sum of power inputs is equal to the sum of power outputs in a node. Power flows through a line can be decomposed into a transfer contribution from the line itself and another from its FACTS device (if installed) as shown in Eq. (15) (this is known as the power injection model of a FACTS device [73]), and the total transfer is limited by the capacity of the line (see Eq. (16)). Eq. (17) shows that FACTS devices (in this case a quad-booster phase-shifting transformer [73]) also have limited capability to shift power. In addition, Eq. (18) represents the second Kirchhoff’s law, where a disjunctive, big-M-based model [74] is used for new lines, being \( \Psi \) a sufficiently large positive constant. For existing network infrastructure, we can assume that \( \mu^L_{l,m} = \).

\[
\begin{align*}
\hat{p}^G_{g,m} = P^G_{g} ; \quad \forall g \in \bar{G}, \forall m \in \bar{M} \\
\mu^L_{l,m} = L^L_{l} ; \quad \forall l \in \bar{L}, \forall m \in \bar{M}
\end{align*}
\]
\( \mu_{l,m}^Q = 1 \) in all equations. Similarly, for those lines where there are no candidate FACTS devices \((i.e., l \notin \mathcal{L})\), we assume \( \mu_{l,m}^Q = 0 \) and therefore \( \hat{f}_{l,m,t}^Q = 0 \).

\[
\sum_{g \in \mathcal{G}} P_{g,m,t}^G + \sum_{b \in \mathcal{B}} P_{b,m,t}^B - D_{n,m,t} = \sum_{l \in \text{from}_n} f_{l,m,t}^L - \sum_{l \in \text{to}_n} f_{l,m,t}^T; \quad \forall n \in N
\]

\[
f_{l,m,t} = f_{l,m,t}^L + f_{l,m,t}^Q; \quad \forall l \in L, \forall m \in M, \forall t \in T
\]

\[
-\hat{f}_{l,m,t}^L \leq f_{l,m,t}^L \leq \bar{f}_{l,m,t}^L; \quad \forall l \in L, \forall m \in M, \forall t \in T
\]

\[
-\hat{f}_{l,m,t}^Q \leq f_{l,m,t}^Q \leq \bar{f}_{l,m,t}^Q; \quad \forall l \in L
\]

\[
-\psi(1 - \mu_{l,m}^L) + \frac{\theta_{\text{from}}^L - \theta_{\text{to}}^T}{X_l} f_{l,m,t}^L \leq \frac{\theta_{\text{from}}^L - \theta_{\text{to}}^T}{X_l} + \psi(1 - \mu_{l,m}^L); \quad \forall l \in L, \forall m \in M, \forall t \in T
\]

Generating units are also constrained by both their available power capacity (driven by the availability of renewable resources, the maximum generation capacity for all units and outages for conventional units) and their ramp rate limits (Eqs. (19) and (20), respectively, where we assume that only conventional generation features ramping-related limitations). Furthermore, energy storage plants, which are modelled through Eqs. (21)-(24), can provide services associated with energy arbitrage \((i.e., \text{charge/discharge actions to improve the system load factor, charging during off-peak periods and discharging during peak periods following the minimisation of the total cost in Eq. (8)})\) and flexibility \((\text{since storage plants are not affected by ramp rate constraints as those specified in Eq. (20) for generating units})\).

\[
P_{g,m,t}^G \leq \bar{P}_{g,m,1}^G; \quad \forall g \in \mathcal{G}, \forall m \in M, \forall t \in T
\]

\[
-R_g \bar{P}_{g,m}^D \leq P_{g,m,t}^G - \bar{P}_{g,m,1}^G \leq R_g \bar{P}_{g,m}^U; \quad \forall g \in \mathcal{G}, \forall m \in M, \forall t \in T
\]

\[
p_{b,m,t}^B \leq \bar{P}_{b,m}^B; \quad \forall b \in \mathcal{B}, \forall m \in M, \forall t \in T
\]

\[
p_{b,m,t}^{BE} = p_{b,m,t}^{BE,1} - p_{b,m,t}^{BE,-1} - p_{b,m,t}^B \leq \bar{P}_{b,m}^B; \quad \forall b \in \mathcal{B}, \forall m \in M, \forall t \in T
\]

Finally, all decision variables are continuous and non-negative, except for (i) power transfers, voltage angles and net outputs of storage plants which may be negative, and (ii) investment variables associated with new network infrastructure that are binary, indicating whether a candidate asset has been chosen for investment \((\text{binary variable equal to 1})\) or not \((\text{binary variable equal to zero})\).

Note that further details can be included in a straightforward manner in the operational model by adding more variables, constraints and cost functions related to units’ minimum stable generations, start-up and shut-down costs, minimum up and down times, etc. \((\text{see for instance [65][67][68][75])}, \text{albeit the current level of detail (Eqs. (8)-(24)}\) suffices to illustrate the importance of operational constraints \((\text{through ramp rate and time-coupling constraints, besides power flow and capacity constraints})\) when planning future infrastructure. Likewise, further details associated with the necessary security margins \((\text{e.g., generation and network reserves})\) to face random faults of system components can also be critical in investment planning models \([28]\). In the context of stochastic programming, these considerations will significantly increase the number of variables and constraints and thus the computational burden, as discussed in Section 4.

Risk aversion

While the previous objective function in Eq. (8) corresponds to a risk-neutral planner, the decision-making process of a risk-averse, conservative planner can also be accommodated in the proposed optimisation framework. To that end, Eq. (8) is replaced with Eqs. (25)-(27) where a weighted average of the expected total cost and the maximum total cost across scenarios is minimised.

\[
\min \{ \lambda \bar{C} + (1 - \lambda) \bar{C} \}
\]

\[
\bar{C} = \sum_{m \in M} \rho_m r_m (\tau^L m + \tau^Q m)
\]
\[ \tilde{c} \geq \sum_{m \in SC} r_m (\tau^1 I_m + \tau^0 O_m); \quad \forall j \in SC \]  

Planning under perfect (deterministic) information

Note that we can also minimise each scenario’s total cost as shown in Eq. (28). Here, the planner assumes perfect information of the future and thus the associated (deterministic) solution may represent an investment option that cannot be efficiently adapted if a different scenario (which was, in fact, ignored) realises. This case can also be considered as a base, reference case.

\[ \min \left\{ \sum_{m \in SC} r_m (\tau^1 I_m + \tau^0 O_m) \right\}; \quad \forall j \in SC \]  

Illustrative example

Input data

We study the electricity network depicted in Figure 2 with 3 nodes (N1, N2 and N3), 3 existing conventional generators (G1, G2 and G3), 3 existing lines (L12, L13 and L23) and 1 load (D). In order to face the forecasted demand growth along a planning horizon comprising three epochs spanning five years each, the system can be expanded through wind power generation (W), solar power generation (S), a line (L’23), a FACTS device, namely a quad-booster phase shifting transformer (QB) [73], and a battery storage system (B). The input data used to run our stochastic program are shown in Table 1, where base values for power and voltage are 100 MVA and 66 kV, respectively.

![Figure 2: 3-node system topology, where candidate assets for investment are shown in dashed lines.](image)

<table>
<thead>
<tr>
<th>Investment cost</th>
<th>Operational cost [$/MWh]</th>
<th>Capacity [MW]</th>
<th>Storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind power generation [$/MW/yr]</td>
<td>150,000</td>
<td>G1</td>
<td>60</td>
</tr>
<tr>
<td>Solar power generation [$/MW/yr]</td>
<td>250,000</td>
<td>G2</td>
<td>60</td>
</tr>
<tr>
<td>Battery [$/MW/yr]</td>
<td>130,000</td>
<td>G3</td>
<td>100</td>
</tr>
<tr>
<td>QB [$/yr]</td>
<td>8,000</td>
<td>L’23</td>
<td>57</td>
</tr>
<tr>
<td>L’23 [$/yr]</td>
<td>10,000</td>
<td>L13</td>
<td>57</td>
</tr>
<tr>
<td>Demand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Relevant input data.

For illustration purposes, uncertainty is solely associated with future uncertainty of investment costs of solar power generation. The problem’s scenario tree is illustrated in Figure 3, which describes both the transition probabilities between nodes and PV investment costs in each node (relative to that listed in Table 1). Note that investment costs of solar power generation might significantly fall in the future in scenario 1. Under such a scenario, solar power plants might be installed at the consumers’ location, leaving conventional generation and network assets stranded (at least partially). On the other hand, if future investment costs remain at today’s levels, as modelled in scenario 4, wind power generation (which is assumed to be significantly less costly at present) might likely be installed. Given that the location of candidate wind power generation is far from the load centre, investment in this technology would also require further transmission investment that needs to be planned significantly ahead of actual commissioning. In fact, we assume that while the transmission line features a lag of 1 epoch (e.g., 5 years), the remaining assets can be commissioned faster (i.e., with negligible lag). Further to the need for dealing with investment uncertainty, the planner may also face important

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challenges at the operational timescale when managing increased amounts of renewable generation. In this outlook, demand, wind and solar power profiles, as shown in Figure 4, are used to clearly illustrate the need for ramping, which will be used here as an example of operational constraint to be considered in planning. Due to the illustrative purpose of this example, days are assumed equal across a year and thus weekly and seasonal variations are ignored. While this simplification does not affect our general conclusions, further details like those neglected in this example may need to be captured for real applications.

The proposed model has been implemented using FICO® Xpress [76]. For the sake of reproducibility, the code and associated data files can be downloaded from [77].

Figure 3: 3-stage scenario tree indicating investment cost evolution of solar power generation (as a percentage of the cost listed in Table 1) and transition probabilities. We define set $M=\{2,4,6\}$, which contains one child node per parent node.

Figure 4: Demand, wind and solar power profiles (in per unit) in the representative day.

**Results**

**Effects of long-term uncertainty on system planning**

Table 2 shows the results per epoch and scenario attained by the deterministic model (Eq. (28)). While in scenarios 1 and 2 the significant drop in investment cost of solar power generation encourages the adoption of this technology in the future (producing power right at the point of consumption), the still relatively high investment cost of solar power generation in later epochs of scenarios 3 and 4 encourages investment in wind power, far from the load centre. Expectedly, conventional network reinforcement in the form of a new transmission line is needed in scenarios 3 and 4 (unlike in scenarios 1 and 2) in order to transfer power from remote areas to the load centre. In the first epoch when the investment cost of solar power generation remains relatively high, wind power plants are built in all scenarios.

Table 2: Results from the deterministic model: Expansion plan per epoch and scenario, and costs per scenario (W, S, L’23, B and QB mean investments in wind power generation, solar power generation, line, battery and quad-booster transformer, respectively, and the additional capacity in MW is indicated within < >).

<table>
<thead>
<tr>
<th>Expansion plan per epoch and scenario</th>
<th>Scenario 1 low</th>
<th>Scenario 2 mid-low</th>
<th>Scenario 3 mid-high</th>
<th>Scenario 4 high</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoch 1 W&lt;16&gt;</td>
<td>W&lt;16&gt;</td>
<td>W&lt;19&gt;</td>
<td>W&lt;19&gt;</td>
<td></td>
</tr>
<tr>
<td>Epoch 2 S&lt;39&gt;</td>
<td>S&lt;39&gt;</td>
<td>W&lt;7&gt;, L’23</td>
<td>W&lt;7&gt;, L’23</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 presents the results per epoch and scenario determined by the risk-neutral stochastic model. In contrast to the above deterministic case, this table demonstrates that in no scenario there is any conventional network investment needed to cope with the uncertainty levels faced by the planner in the first epoch. Indeed, the model determines that epoch 1 is too early to decide whether to invest in a long power line (commissioned at the beginning of epoch 2) that may end up stranded under the realisation of scenarios 1 and 2 where solar power emerges right at the location of consumption. Rather, the stochastic approach chooses to “wait and see” and thus potential network congestions (under scenarios 3 and 4) are managed in the operational timescale through smart technologies such as FACTS/QB devices that can be chosen upon and quickly installed right in epoch 2. This solution is complemented later on with a battery storage plant to face larger amounts of wind power that materialise towards the end of the analysed planning horizon, also considering the drop in cost of batteries in the meantime.

It is important to point out that Table 2 shows the costs associated with the best expansion plan for each scenario considered by the planner to characterise the uncertain future. In contrast, the solution summarised in Table 3 corresponds to the investment plan performing best on average for the scenario set under consideration. In other words, the costs reported in Table 2 will only be incurred if the corresponding scenario actually materialises. Otherwise, the costs associated with such expansion plans may significantly differ should other scenarios occur. Moreover, the expected total costs for the deterministic solutions summarised in Table 2 will be greater than or equal to that of the optimal stochastic solution. Table 4 presents the total costs per scenario and the expected total costs for the deterministic solutions reported in Table 2 and the stochastic solution given in Table 3. As can be observed, the stochastic solution is identical to the best expansion plans for scenarios 1 and 2, whereas, for scenarios 3 and 4, a slight 0.03% cost increase is incurred over the best expansion plans for such scenarios. On the other hand, cost increases as high as 8.2% can be experienced if the planner adopts a deterministic solution and a different than forecasted scenario realises. Moreover, Table 4 also shows that the stochastic solution outperforms all deterministic solutions in terms of the expected total cost with considerable improvement factors ranging between 3.7% and 1.1%.

Interestingly, although the stochastic model constrains the final volume of wind capacity connected in scenarios 3 and 4, it is possible to demonstrate that, overall, this is better than building a line that ends up stranded in scenarios 1 and 2. In fact, a sensitivity analysis where we force the stochastic model to commission a new transmission line in epoch 2 (shown in Table 5) reveals that costs under scenarios 1 and 2 escalate with respect to those observed under the truly stochastic solution and this cannot be compensated for by the cost reduction under scenarios 3 and 4. Furthermore, the early investment in a transmission line is not only economically inefficient but also prevents the utilisation of smart grid technologies (FACTS/QB and battery plant) that have been displaced by the line.

Table 5: Results from the risk-neutral stochastic model with forced commissioning of conventional network infrastructure: Expansion plan per epoch and scenario, costs per scenario and expected costs (W, S, L’23, B and QB mean investments in wind power generation, solar power generation, line, battery and quad-booster transformer, respectively, and the additional capacity in MW is indicated within < >).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Deterministic solution for scenario 1</th>
<th>Deterministic solution for scenario 2</th>
<th>Deterministic solution for scenario 3</th>
<th>Deterministic solution for scenario 4</th>
<th>Stochastic solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1 low</td>
<td>300,870</td>
<td>301,287</td>
<td>309,289</td>
<td>309,289</td>
<td>300,870</td>
</tr>
<tr>
<td>Scenario 2 mid-low</td>
<td>304,853</td>
<td>304,368</td>
<td>309,289</td>
<td>309,289</td>
<td>304,368</td>
</tr>
<tr>
<td>Scenario 3 mid-high</td>
<td>330,777</td>
<td>327,587</td>
<td>309,289</td>
<td>309,289</td>
<td>309,370</td>
</tr>
<tr>
<td>Scenario 4 high</td>
<td>334,760</td>
<td>330,668</td>
<td>309,289</td>
<td>309,289</td>
<td>309,370</td>
</tr>
</tbody>
</table>

| Expected total cost [k$] | 317,815 | 315,977 | 309,289 | 309,289 | 305,994 |
work investment is necessary to minimise costs. It is important to mention that this extreme case corresponds to the minimum total costs that are slightly higher than those featured by the optimal stochastic solution and well below those featured by the deterministic solutions. Thus, Tables 3, 5 and 6 show the inefficiencies (advantages) of implementing deterministic (stochastic) solutions when planning under uncertainty. Note that, in this illustrative example (that demonstrates the need to make investment plans more flexible through smart grid technologies and thus face uncertain scenarios more efficiently), uncertainty is associated with investment costs of solar power generation only. Hence, the consequences of following deterministic planning can be much more significant in real systems subject to additional sources of uncertainty such as energy policy incentives, development of new technologies, demand growth, various decentralised decisions from market participants, etc.

Finally, risk aversion is analysed through the minimisation of a weighted average of the expected and the maximum total costs across scenarios (see Table 7), with parameter \( \lambda \) in Eq. (25) taken as equal to 0.5 for the low-aversion case, and equal to zero in the high-aversion case (corresponding to the min-max cost optimisation). As compared with the risk-neutral solution shown in Table 3, the optimal risk-averse investment plans feature larger volumes of wind power generation in epoch 1. Although both expansion plans are less efficient solutions in terms of the expected total cost, increasing the levels of wind power generation commissioned in epoch 1 is an effective measure to reduce the costs in the more costly scenarios, i.e., scenarios 3 and 4. Interestingly, for low levels of risk aversion, the planner prefers to install larger amounts of wind power generation capacity in epoch 1 rather than investing in conventional network infrastructure that can allow the planner to install further wind power generation capacity towards the last epoch. This is so because conventional network investment would unnecessarily increase the total costs in scenarios 1 and 2 (and thus the expected total cost). For higher levels of risk aversion and, in the extreme case, for the fully averse min-max cost solution, conventional network investment is necessary to minimise the total costs under the worst conditions, which also eliminates the need for smart grid solutions. It is important to mention that this extreme solution focuses on total cost minimisation under the worst conditions (in this case, scenarios 3 and 4) irrespective of the inefficiencies caused in terms of the expected total cost and, in particular, under those scenarios where solar power generation realises in the future (when the new transmission line becomes unnecessary and stranded).

<table>
<thead>
<tr>
<th>Expansion plan per epoch and scenario</th>
<th>Epoch 1</th>
<th>Epoch 2</th>
<th>Epoch 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>W&lt;19&gt;</td>
<td>S&lt;19&gt;</td>
<td>S&lt;39&gt;, L’23</td>
<td></td>
</tr>
<tr>
<td>S&lt;18&gt;</td>
<td>S&lt;8&gt;</td>
<td>S&lt;5&gt;, W&lt;2&gt;</td>
<td></td>
</tr>
<tr>
<td>W&lt;10&gt;, L’23</td>
<td>W&lt;1&gt;</td>
<td>W&lt;1&gt;</td>
<td></td>
</tr>
<tr>
<td>Costs per scenario [k$]</td>
<td>Investment</td>
<td>Operation</td>
<td>Total</td>
</tr>
<tr>
<td>W&lt;19&gt;</td>
<td>59,959</td>
<td>60,110</td>
<td>36,487</td>
</tr>
<tr>
<td>S&lt;18&gt;</td>
<td>240,975</td>
<td>244,321</td>
<td>272,859</td>
</tr>
<tr>
<td>W&lt;10&gt;, L’23</td>
<td>300,934</td>
<td>304,431</td>
<td>309,346</td>
</tr>
<tr>
<td>Expected costs [k$]</td>
<td>Investment</td>
<td>Operation</td>
<td>Total</td>
</tr>
<tr>
<td>W&lt;19&gt;</td>
<td>48,261</td>
<td>257,753</td>
<td>306,014</td>
</tr>
</tbody>
</table>

Table 6: Results from the risk-neutral stochastic model with forced commissioning of higher levels of wind power generation in the first epoch: Expansion plan per epoch and scenario, costs per scenario and expected costs (W, S, L’23, B and QB mean investments in wind power generation, solar power generation, line, battery and quad-booster transformer, respectively, and the additional capacity in MW is indicated within ‘ < > ’).

<table>
<thead>
<tr>
<th>Expansion plan per epoch and scenario</th>
<th>Epoch 1</th>
<th>Epoch 2</th>
<th>Epoch 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&lt;19&gt;</td>
<td>S&lt;38&gt;</td>
<td>S&lt;5&gt;</td>
<td></td>
</tr>
<tr>
<td>W&lt;19&gt;</td>
<td>64,533</td>
<td>63,856</td>
<td>18,030</td>
</tr>
<tr>
<td>B&lt;1&gt;</td>
<td>236,500</td>
<td>240,593</td>
<td>271,283</td>
</tr>
<tr>
<td>W&lt;7&gt;, QB</td>
<td>301,033</td>
<td>304,449</td>
<td>309,313</td>
</tr>
<tr>
<td>Costs per scenario [k$]</td>
<td>Investment</td>
<td>Operation</td>
<td>Total</td>
</tr>
<tr>
<td>W&lt;19&gt;</td>
<td>51,113</td>
<td>254,914</td>
<td>306,027</td>
</tr>
<tr>
<td>S&lt;18&gt;</td>
<td>240,975</td>
<td>244,321</td>
<td>272,859</td>
</tr>
<tr>
<td>W&lt;10&gt;, L’23</td>
<td>309,313</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Results from the risk-averse model: Expansion plan per epoch and scenario, costs per scenario and expected costs (W, S, L’23, B and QB mean investments in wind power generation, solar power generation, line, battery and quad-booster transformer, respectively, and the additional capacity in MW is indicated within ‘ < > ’).
Effects of short-term operational constraints on system planning

In the previous examples, the results discussed in Table 2 to Table 7 account for ramp rate constraints in all scenarios as indicated in Table 1. Table 8 shows the resulting infrastructure investment and the total costs for the solutions to the following three different deterministic cases for scenario 1, which corresponds to a low investment cost of solar power generation:

(i) when the infrastructure is optimised by ignoring ramp rate constraints: although the total cost is the lowest, it underestimates the actual cost of operating large volumes of renewables;

(ii) when the infrastructure is optimised by ignoring ramp rate constraints and the actual cost of operation is adequately quantified in a second stage when considering actual ramp rate limitations: the total cost is the highest since the planner ignores important operational constraints (when the system infrastructure was determined); and

(iii) when the infrastructure is adequately planned and optimised by accounting for ramp rate constraints (G1 and G2’s ramp rates equal to 10%/h): this corresponds to the true cost of the optimal solution.

The results shown in Table 8 demonstrate that modelling detailed operational constraints can be critical when planning electrical infrastructure. For example, if this electricity system were planned with fully flexible ramp rates from conventional plants (in other words, ignoring ramp rate constraints as in solution (i), which is a widespread implicit assumption in most planning exercises and available planning tools), larger volumes of renewables would be installed with no need for storage plants. This would clearly increase the cost of operation in reality, when considering the actual system’s flexibility limitations (see solution (ii)), and would probably result in investment into suboptimal technologies. In contrast, if ramp rates are adequately considered in the planning model as in solution (iii), then less renewable energy capacity is built because of the diminished ability of the system to cope with intermittency. Further, storage capacity becomes necessary to provide operational flexibility.
Table 8: Results from the deterministic model under three different assumptions on system’s flexibility for scenario 1 corresponding to a low investment cost of solar power generation: Expansion plan per epoch and solution, and costs per solution (W, S, L’23, B and QB mean investments in wind power generation, solar power generation, line, battery and quad-booster transformer, respectively, and the additional capacity in MW is indicated within < >). Roman numerals correspond to the cases described in the main text.

<table>
<thead>
<tr>
<th>Expansion plan per epoch and solution</th>
<th>Epoch 1</th>
<th>Epoch 2</th>
<th>Epoch 3</th>
<th>Solution (i)</th>
<th>Solution (ii)</th>
<th>Solution (iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment</strong></td>
<td>W&lt;16&gt;</td>
<td>W&lt;16&gt;</td>
<td>W&lt;14&gt;</td>
<td>s&lt;39&gt;</td>
<td>s&lt;39&gt;</td>
<td>s&lt;40&gt;</td>
</tr>
<tr>
<td><strong>Operation</strong></td>
<td>s&lt;18&gt;</td>
<td>s&lt;18&gt;</td>
<td>s&lt;11&gt;</td>
<td>s&lt;11&gt;</td>
<td>s&lt;11&gt;</td>
<td>s&lt;40&gt;</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>59,896</td>
<td>59,896</td>
<td>55,507</td>
<td>240,974</td>
<td>242,437</td>
<td>246,467</td>
</tr>
<tr>
<td><strong>Costs per solution [k$]</strong></td>
<td>300,870</td>
<td>302,333</td>
<td>301,974</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Furthermore, Table 9 shows that if the system is planned through the risk-neutral stochastic model while ignoring ramp rate constraints, storage infrastructure (originally installed in scenarios 3 and 4 as shown in Table 3) is not needed. This illustrates the importance of considering both system’s flexibility limitations (Table 8) and uncertainty (Table 9) in order to properly capture the value of smart grid technologies and invest into the most appropriate portfolio in first place.

Table 9: Results from the risk-neutral stochastic model without ramp rate constraints on G1 and G2: Expansion plan per epoch and scenario, costs per scenario and expected costs (W, S, L’23, B and QB) mean investments in wind power generation, solar power generation, line, battery and quad-booster transformer, respectively, and the additional capacity in MW is indicated within < > and N/I refers to “no investment”).

<table>
<thead>
<tr>
<th>Expansion plan per epoch and scenario</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment</strong></td>
<td>Scenario 1</td>
<td>Scenario 2</td>
<td>Scenario 3</td>
<td>Scenario 4</td>
</tr>
<tr>
<td><strong>Operation</strong></td>
<td>low</td>
<td>mid-low</td>
<td>mid-high</td>
<td>high</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>S&lt;18&gt;</td>
<td>S&lt;5&gt;</td>
<td>S&lt;2&gt;</td>
<td>S&lt;10&gt;, QB</td>
</tr>
<tr>
<td><strong>Costs per scenario [k$]</strong></td>
<td>59,896</td>
<td>60,047</td>
<td>35,827</td>
<td>35,827</td>
</tr>
<tr>
<td><strong>Expected costs [k$]</strong></td>
<td>240,974</td>
<td>244,321</td>
<td>273,525</td>
<td>273,525</td>
</tr>
<tr>
<td><strong>Operation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>300,870</td>
<td>304,368</td>
<td>309,352</td>
<td>309,352</td>
</tr>
<tr>
<td><strong>Operation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>47,999</td>
<td>258,087</td>
<td>305,986</td>
<td></td>
</tr>
</tbody>
</table>

Discussion

As mentioned earlier, it is important to emphasise that there are a range of studies in the context of power system expansion planning under uncertainty (see for instance [17][31][33]-[64]) and power system expansion planning with representation of increased operational data, including unit commitment constraints (see for instance [58][60][65]-[68]). Extending on this, we argue that the proposed unified framework evaluated above is of utmost importance in the smart grid context, where the value of new, innovative technologies is fundamentally based on the provision of flexibility services that allow planners and operators to efficiently deal with various sources of uncertainty and constraints affecting both investment decisions and system operation.

Furthermore, notice that the above results correspond to an instance of Eqs. (1)-(3), where operational uncertainty has been neglected and, instead, replaced with the modelling of variability in operational parameters such as wind and solar power outputs. Another level of complexity will be to run an instance of Eqs. (4)-(7) where uncertainty is considered in both planning and operational timescales. Under this consideration, generation and network infrastructure have to be also flexible to deal with equipment outages and forecast errors that may happen in real time and thus new investments should be able to cope with both long-term uncertainties and also uncertain changes that may occur rapidly in real time. There are various studies that demonstrate the importance of short-term uncertainty and security for power system planning (see for instance [28]), and we believe that, with the advent of new technology, such importance will be exacerbated since innovative technologies can successfully provide (at least partially) the flexibility and security needed in real time and hence displace (or at least defer) part of the new conventional infrastructure. Therefore, modelling both long- and short-term uncertainty will be key to properly quantify the benefits of smart grid technologies.

4. Modelling and Computational Challenges

Following up on our multistage stochastic programming modelling framework presented above and demonstrated on a small test system, and with reference to state of the art in power system planning under uncertainty, in this section we aim to: (i) present in more general terms the main families of problems dealing with power system planning under uncertainty (of which our framework is a sub-set); (ii) discuss the main challenges, at the computational and algorithmic levels, of embedding detailed operational aspects in planning under uncertainty problems, especially when dealing with real-size networks; and (iii) advocate the need for new optimisation tools that could address these computational challenges in order to properly value the benefits of flexible, smart grid solutions in planning.

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Current models for power system planning under uncertainty lie within one of the four classes of problems resulting from the combination of: (i) the temporal framework adopted for decisions with uncertainty about future evolution, namely, dynamic (or multistage), whereby in each period decisions are made under uncertainty of successive ones, versus static (or single stage), whereby decisions are made in a single period under the same information level for all future periods (see e.g. [48]); and (ii) the modelling of uncertainty characterisation, namely, stochastic programming, which uses probabilistic models to characterise uncertainty [70], versus robust optimisation, which ensures feasibility for a user-defined set of uncertainty realisations or scenarios and is particularly useful when the definition of a probability distribution is not an easy task, e.g., occurrence of contingencies, strategic actions of market participants, evolution of fuel prices, etc. [71]).

**Multistage stochastic planning**

The class of multistage stochastic optimisation models [70] is of great importance in power system planning and encompasses many applications in many subareas (e.g., hydrothermal planning [35], transmission expansion planning [42][49] and generation expansion planning [41][50][51], and aims to emulate the way decisions are made in real word while uncertainty is revealed over time. For computational tractability, this logic is applied only to investment decisions and long-term uncertainties, while short-term operational uncertainties and decisions are generally simplified or even disregarded, on the basis of the (traditionally) weak time dependence between short-term and investment-related decisions. However, as we argued through our multistage optimisation textbook-like examples, it is key that operational aspects that come with smart grid technologies and flexible devices are suitably taken into account in planning too.

The requirement for more detailed modelling of operational flexibility introduces manifold computational and methodological challenges in multistage stochastic planning problems, which may be intractable for real-life benchmarks. Decomposition techniques can therefore be most useful. In this outlook, a natural decomposition scheme arises from the temporal and conditional nature of the uncertainty-revelation structure, whereby the problem can be cast as a discrete-time control problem that follows Bellman’s principle, thus being suitable for dynamic programming. Benders-type decomposition procedures and dynamic programming (see [70]), as well as sampling techniques such as Monte Carlo simulation are well-known approaches used to find high-quality solutions (see [35][78][79]) due to their scalability and computational efficiency to solve realistic problems from industry. For instance, the stochastic dual dynamic programming (SDDP) approach (see [35]) is used in most hydrothermal-based power systems worldwide. However, the discrete nature of state variables in planning problems introduces loss of convexity that prevents the direct use of the SDDP approach. This is also the case when considering nonlinear relations or discrete actions at the market or operational levels, such as those arising when modelling network flexibility. In this sense, recent advances in decomposition methods provide algorithms capable to deal with some of these non-convexities in different ways [80]-[82]. In general, developing computationally efficient methods to address non-convexities is an important research avenue to be able to fully capture the benefits of smart grid technologies in realistic planning problems.

**Static stochastic and robust planning**

The class of static stochastic and robust planning models arises when, roughly speaking, all the investment decisions $x(\cdot)$ in the model comprising Eqs. (4)-(7) are in the first stage, and the uncertainty realisations are in the second stage (two-stage stochastic or two-stage robust problems). Static planning is largely used in industry applications, particularly for grid reinforcements, whereby the decision and uncertainty hierarchy is typically the following: “investment decisions for the whole time horizon ($x(\cdot)$) $\Rightarrow$ “long-term uncertainty revelation ($\epsilon(\cdot)$) $\Rightarrow$ “unit commitment or market scheduling decisions ($y(\cdot)$) $\Rightarrow$ “short-term uncertainty revelation ($\xi(\cdot)$) $\Rightarrow$ “short-term operational decisions ($z(\cdot,\epsilon,\xi)$)”. Depending on the application, one of the three decision levels are simplified and, in general, two-stage decision models are used to address static problems. Such two-stage models use a simplified (or “myopic”) and in general more conservative approach for investment dynamics. Thus, much more detailed operational aspects and short-term uncertainties can be considered relative to the multistage approach (which uses simplified operational models to avoid tractability issues). As a result, the two-stage framework is largely adopted by industry. However, two-stage models require the consideration of many epochs and snapshots of the system operation. Therefore, again Benders decomposition [80], Dantzig Wolfe [41], progressive hedging [54], and column-and-constraint generation methods [17][83] are common approaches used to decompose the original problems into two stages: the planning stage, where the set of decision variables comprises variables $x(\cdot)$, and the operational stage, where decision variables $y(\cdot)$ are scenario dependent.

While in the stochastic framework, with a probabilistic model being used to characterise uncertainty, the decision maker risk-attitude is expressed through utility functions and risk measures [64], in robust optimisation the decision maker imposes feasibility for all scenarios within a given set, denoted by uncertainty set, thereby giving rise to a worst-case setting. In this case, the risk level (or “conservativeness” level) as commonly referred to in the literature on robust optimisation [71][84][85]), is accounted for by means of the comprehensiveness (or “broadness”, in the sense of cardinality) of the set of scenarios. Available approaches to uncertainty modelling (e.g., ellipsoidal uncertainty sets [85][86] and polyhedral uncertainty sets [84]) define convex sets relying on nominal values for the uncertain coefficients of linear inequality constraints and on distance norms limiting the size of the uncertainty sets. Therefore, differently from the stochastic optimisation framework, where the explicit evaluation of the short-term operational cost for all the scenarios is needed, in the robust optimisation framework the worst-case scenario is found through parameterised optimisation problems. Hence, robust optimisation implicitly accounts for all the scenarios within the uncertainty set through efficient search algorithms, thereby addressing some of the tractability issues associated with the consideration of many scenarios in the stochastic optimisation approach. This is a salient virtue of the robust approach that has recently

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triggered an increasing number of publications in robust models for power system operation and planning (see e.g., [17][39][46][48][53][62][63][87][88]). This virtue notwithstanding, there are still several challenges that should be addressed in the field of robust optimisation models for power system planning. In particular, two relevant avenues of research are devoted to (i) the characterisation of uncertainty sets that preserve relevant statistical properties of the uncertainties and (ii) addressing the tractability issues associated with the presence of non-convexities in the second-stage problem. Recently, promising results propose new perspectives for merging stochastic and robust optimisation, thereby giving rise to the notion of distributionally-robust optimisation [71][89].

5. Conclusions

In the context of the transition from conventional power systems to low-carbon energy systems, whereby a control-based paradigm relying on flexible smart grid technologies replaces the classic asset-based paradigm, this paper has discussed some of the main challenges associated with planning future energy systems to securely and cost effectively integrate renewable energy sources. In particular, we have presented an optimisation framework to plan electricity grids that deals with uncertainty and properly accounts for relevant operational constraints, so that truly optimal solutions, based on flexible technologies, can be determined (although this is limited, in practice, by computational burden and the ability to deal with very large optimisation problems through suitable algorithms). In fact, the resulting investment options contain an array of smart grid technologies that can effectively provide the needed flexibility in operation to cope with renewables’ fluctuations and the necessary adaptation capacity in the long term to adjust system expansion more cost effectively even under the occurrence of unfavourable scenarios. We thus demonstrated that and how smart grid technologies can displace the need for conventional network assets that are instead preferable when uncertainty is ignored and operational constraints are neglected.

Two key conceptual results of our work, with important policy implications, are (i) investment in specific low-carbon generation technologies over time may strongly depend on the (deterministic, stochastic or robust) approach and risk aversion of the planner, and (ii) optimal generation investment decisions need to be complemented by decisions on investment into smart grid technologies and/or network assets. In these regards, the proposed model allows fully capturing synergies and competition among different generation, smart and transmission technologies so that the truly optimal solution is attained. In contrast, the deterministic planning approaches (often in the form of “roadmaps”) and relevant planning tools that are currently used are bound to delivering solutions that are not only more expensive, but also misleading in terms of optimal technology mix (generation, storage, etc.). This is particularly important in the light of, for example, setting policy incentives for certain technologies, for which enabling smart grid technologies such as FACTS, batteries, etc., should also be considered as optimal complements to renewable generation.

Following up on the findings highlighted above, we then also presented the state of the art in power system planning under uncertainty and discussed the need for new optimisation tools that can properly value the benefits of flexible, smart grid solutions by embedding detailed operational aspects in the planning problem. In particular, important progresses are necessary in terms of sampling techniques, decomposition methods, parallel and cloud computing, among others, to fully unlock the application of mathematical programming to plan investments in electricity grids when considering a large number of potential future scenarios in the long term, and a detailed representation of system operation in the short term.
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Appendix. Nomenclature

Variables

\( \hat{\mathcal{C}} \) Expected total cost across all scenarios [\$]
\( \hat{\mathcal{C}} \) Maximum total cost across all scenarios [\$]
\( f_{l,m,t} \) Power flow of line \( l \) in scenario tree node \( m \) in time period (hour) \( t \) [MW]
\( f_{l,m,t}^0 \) Power flow of line \( l \) in scenario tree node \( m \) in time period (hour) \( t \) (without the contribution from the FACTS device) [MW]
\( f_{l,m,t}^Q \) Power flow contribution of the FACTS device in line \( l \) in scenario tree node \( m \) in time period (hour) \( t \) [MW]
\( l_m \) Investment cost of scenario tree node \( m \) [\$]
\( O_m \) Operational cost of scenario tree node \( m \) [\$]
\( p_{b,m}^B \) Capacity of battery storage plant \( b \) in scenario tree node \( m \) [MW]
\( p_{b,m,t}^B \) Power output of battery storage plant \( b \) in scenario tree node \( m \) in time period (hour) \( t \) [MW]
\( p_{b,m,t}^B \) Discharge power of battery storage plant \( b \) in scenario tree node \( m \) in time period (hour) \( t \) [MW]
\( p_{b,m}^\text{From/To} \) Charge power of battery storage plant \( b \) in scenario tree node \( m \) in time period (hour) \( t \) [MW]
\( p_{b,m,t}^G \) Energy stored in battery storage plant \( b \) in scenario tree node \( m \) in time period (hour) \( t \) [MWh]
\( p_{g,m}^G \) Capacity of generator \( g \) in scenario tree node \( m \) [MW]
\( p_{g,m,t}^G \) Power output of generator \( g \) in scenario tree node \( m \) in time period (hour) \( t \) [MW]
\( \theta_{l,m,t} \) Voltage angle in From/To node of line \( l \) in scenario tree node \( m \) in time period (hour) \( t \) [rad]
\( \mu_{L,m} \) Installation of line \( l \) in scenario tree node \( m \) (0,1)
\( \mu_{F,m}^Q \) Installation of the FACTS device in line \( l \) in scenario tree node \( m \) (0,1)

Parameters

\( A_{g,t}^{\text{av}} \) Availability factor of generator \( g \) in time period (hour) \( t \) [p.u.]
\( D_{n,m,t} \) Demand in node \( n \) in scenario tree node \( m \) in time period (hour) \( t \) [MW]
\( f_l \) Capacity of line \( l \) [MW]
\( f_{l,m,t}^Q \) Maximum capability to shift power of the FACTS device in line \( l \) [MW]
\( f_{m} \) Discount factor of scenario tree node \( m \) [p.u.]
\( R_{g}^{\text{down}} \) Downwards ramp rate limit of generator \( g \) as a percentage of its capacity per hour [%/h]
\( R_{g}^{\text{up}} \) Upwards ramp rate limit of generator \( g \) as a percentage of its capacity per hour [%/h]
\( X_l \) Reactance of line \( l \) [\( \Omega \)]
\( \eta_b \) Round-trip efficiency of battery storage plant \( b \) [p.u.]
\( \lambda \) Risk-aversion factor [p.u.]
\( \pi_{b}^{\text{inv}} \) Unitary investment cost of battery storage plant \( b \) (annuitized) [$/MW/yr]
\( \pi_{g}^{\text{inv}} \) Unitary investment cost of generator \( g \) (annuitized) [$/MW/yr]
\( \pi_{l}^{\text{inv}} \) Investment cost of line \( l \) (annuitized) [$/yr]
\( \pi_{Q}^{\text{inv}} \) Investment cost of the FACTS device in line \( l \) (annuitized) [$/yr]
\( \pi_{g}^{\text{op}} \) Variable cost of generator \( g \) [$/MWh]
\( p_m \) Energy capacity of battery storage plant \( b \) measured in hours at full power capacity [h]
\( \tau^I \) Scaling factor to transform annuitized investment cost into investment cost in an epoch [p.u]
\( \tau^0 \) Scaling factor to transform operational cost in representative days into operational cost in an epoch [p.u]
\( \psi \) Sufficiently large positive constant [MW]

Set-related symbols

\( B \) Set of battery storage plants
\( \hat{B} \) Set of candidate battery storage plants
\( B_n \) Set of battery storage plants in node \( n \)
\( \text{From}_n \) Set of lines that start from node \( n \)
\( G \) Set of generators
\( \tilde{G} \) Set of candidate generators
\( G^c \) Set of conventional generators
\( \tilde{G}^c \) Set of candidate conventional generators
\( G_n \) Set of generators in node \( n \)
\( L \) Set of lines
\( \tilde{L} \) Set of candidate lines
\( L^o \) Set of candidate FACTS devices
\( M \) Set of scenario tree nodes
\( M^S \) Set of the scenario tree nodes that contains one child node of each parent node in the scenario tree
\( N \) Set of nodes
\( p(m) \) Parent node of scenario tree node \( m \)
\( S_m \) Set of sibling nodes of scenario tree node \( m \)
\( SC \) Set of uncertain scenarios
\( Sc_j \) Set of scenario tree nodes in scenario \( j \)
\( T \) Set of time periods (hours)
\( T_{on} \) Set of lines that go to node \( n \)


