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DOI:

[10.1016/S0308-0161\(02\)00152-7](https://doi.org/10.1016/S0308-0161(02)00152-7)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Tafreshi, A., & Oswald, T. (2003). Global buckling behaviour and local damage propagation in composite plates with embedded delaminations. *International Journal of Pressure Vessels and Piping*, 80(1), 9-20.
[https://doi.org/10.1016/S0308-0161\(02\)00152-7](https://doi.org/10.1016/S0308-0161(02)00152-7)

Published in:

International Journal of Pressure Vessels and Piping

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GLOBAL BUCKLING BEHAVIOUR AND LOCAL DAMAGE PROPAGATION OF COMPOSITE PLATES WITH EMBEDDED DELAMINATIONS

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Abstract

Finite element models were developed to study global, local and mixed mode buckling behaviour of composite plates with embedded delaminations under compression. The global modelling results were compared with corresponding experimental results. It is shown that the numerical results for embedded delaminations agree very well with the experimental results, whereas the difference between the results was high for delaminations located at the edge of the plates. It is also shown that at lower loading levels the interaction of global and local buckling is negligible. At higher loading levels the strain energy release rate distribution and the delamination growth potential at the delamination front strongly depend on the shape of the debonded region and the local buckling mode. It was observed that the local buckling mode was highly influenced by the laminate stacking sequence. In the course of global buckling a parametric study was carried out to investigate the influence of the delamination size, shape and alignment of a series of composite plates.

Keywords: Delamination, composites, finite element method, buckling analysis, laminated plates

1. Introduction

The utilisation of composites in aerospace applications is well established today due to the known benefits, such as high specific stiffness or strength and the material's tailoring facilities for creating high performance structures. During use, composite structures in aerospace applications are exposed to changing climatic conditions that affect the moisture content and temperature related expansion within the material and also to the impact of foreign objects. These can produce delamination. Delamination is the dominant mode of failure in composites at high-cycle fatigue, initiated by crack tips that arise in low-cycle fatigue.

The phenomenon of progressive failure in laminated composite structures is yet to be understood, and as a result, reliable strategies for designing optimal laminated composite structures for desired life and strength are not yet available [1]

For the past two decades analytical and numerical analysis have been carried out by many researchers[2-14] to analyse delaminated composite structures, considering their buckling and post-buckling behaviour. The early work belongs to Chai et al [2] who characterized the delamination buckling models by the delamination thickness and number of delaminations through the laminate thickness.

In a study by Klug et al. [8], Mindlin plate finite element was used to model the response of delaminated composite plates under in-plane compression. The crack closure method was used to calculate the strain energy release rate at the delamination front. Comparison of their results with the existing three-dimensional result [12] indicated that their method was sufficient and accurate.

In the present study, a different modelling approach [15] is introduced for investigation of the global buckling behaviour and local damage propagation of composite plates containing embedded delaminations. This requires less computing time and space for the same level of accuracy. A parametric study has also been carried out to investigate the influence of the delamination size, shape and alignment of a series of composite plates. The global buckling results are compared with the corresponding experimental results obtained by Chai et al. [16].

Here, the ABAQUS package [17] was used for the analysis. For the best accuracy in each case the convergence test was performed and a mesh with the adequate number of elements was employed.

Nomenclature

A^*	delamination area fraction
A_{del}	delamination area
A_{pl}	plate area
E	Young's modulus
G_{ij}	shear modulus
G	strain energy release rate
h_1, h_2	thickness of upper and lower sublaminates
M_x, M_y	reaction nodal moments
N_x, N_y, N_z	reaction nodal forces
P_{cr}	critical load
P_{cr}^*	normalised critical load
r_{del}	delamination aspect ratio (delamination length/delamination width)
r_{pl}	plate aspect ratio (plate length/plate width)
t	thickness
$u_i (i=x,y,z)$	displacements in the i direction
U	strain energy
x, y	Cartesian coordinates
ψ_x, ψ_y	rotation about the y and x directions, respectively

ϕ_x	fibre volume fraction
ϕ_m	matrix volume fraction
λ	incremental load factor
ν	Poisson's ratio

2. Finite element modelling and verification

Compared to intact ones, laminate plates that contain delaminations show significantly more complex buckling behaviour. When a delaminated composite plate is subjected to uniaxial in-plane compression, local buckling of the delaminated region or mixed buckling mode, which is a combination of local and global buckling, may occur before global buckling (without local buckling), as shown in Fig. 1.

The investigation of global and local effects has been performed using two different FE models. Fig. 2a shows the global model and the boundary conditions used for the analysis. Based on the approach of Klug et al. [8], the whole laminate can be split into upper and lower sublaminae. The corresponding nodes of the upper and lower sublaminae in the areas that are assumed to be intact are connected by displacement equations that arise from Mindlin/Kirchhoff plate theory [18]. The displacement field according to this theory is as follows:

$$\begin{aligned} u_x &= u_x^0 + z\psi_x \\ u_y &= u_y^0 + z\psi_y \\ u_z &= u_z^0 \end{aligned} \tag{1}$$

where u_x , u_y and u_z are the translational and ψ_x and ψ_y are the rotations at arbitrary locations within the upper or lower sublaminae with the distance z to the midplane of the respective sublaminate and u_x^0 , u_y^0 and u_z^0 are the translational midplane displacements in the respective directions. In particular, the degrees of freedom of the nodes of both sublaminae are tied

together in both displacements and rotations along the interface so that both sublaminates together deform like an intact single laminate. Therefore,

$$\begin{aligned} u_x^{1,0} - \left(\frac{h_1}{2}\right) \psi_x^1 &= u_x^{2,0} + \left(\frac{h_2}{2}\right) \psi_x^2 \\ u_y^{1,0} - \left(\frac{h_1}{2}\right) \psi_y^1 &= u_y^{2,0} + \left(\frac{h_2}{2}\right) \psi_y^2 \\ u_z^{1,0} &= u_z^{2,0} \end{aligned} \quad (2)$$

where the first superscripts 1 and 2 of the displacements refer to the upper and lower sublaminates, respectively, and the second superscript 0 refers to the sublaminates midplane. In areas of delamination Klug et al. [8] employed GAP elements between the corresponding nodes of the upper and lower sublaminates to avoid the interpenetration. Interpenetration would neglect the behaviour of real laminates and thus, result in wrong estimations of the critical load and the average total strain energy release rate G . They used eight-noded shell element, designated S8R in ABAQUS, to model the laminated plate. The method employed for calculation of the strain energy release rate G will be discussed in Section 5.

In the present study Fig. 3a shows the proposed model for investigation of effects of delaminations on the global critical buckling load. The intact regions will be represented by a single layer of shell elements, whereas the delaminated sections will be modelled by upper and lower sublaminates that are connected by GAP elements. For the interface region a modified version of the sublaminates connection method by Mindlin plate theory will be employed. Therefore, also the rotational displacement components (ψ_j) of all nodes of the stacked layers at the transition border are coupled in addition to the translational displacements. Thus, the coupling between the midplanes of the sublaminates of the damaged and the midplane of the laminate of the intact area is described by the following set of equations.

$$\begin{aligned}
 \text{Displacements: } & u_x^{1,0} - \left(\frac{h_0}{4}\right) \psi_x^1 = u_x^{2,0} + \left(\frac{h_0}{4}\right) \psi_x^2 = u_x^{0,0} \\
 & u_y^{1,0} - \left(\frac{h_0}{4}\right) \psi_y^1 = u_y^{2,0} + \left(\frac{h_0}{4}\right) \psi_y^2 = u_y^{0,0} \\
 & u_z^{1,0} = u_z^{2,0} = u_z^{0,0} \\
 \text{Rotations: } & \psi_x^1 = \psi_x^2 = \psi_x^0, \quad \psi_y^1 = \psi_y^2 = \psi_y^0, \quad \psi_z^1 = \psi_z^2 = \psi_z^0 \quad (4)
 \end{aligned}$$

This is for the case of a delamination at the midplane of the laminate where $h_1=h_2$ and $h_0=h_1+h_2$. It should be noted that all the delaminations considered in this paper are at midplane unless stated otherwise.

For investigation of local effects of delamination a different model has been selected. The local model is a quarter of the global model but contains four times as many elements as the corresponding section of the global model. Fig. 2b Shows the geometries and boundary conditions of the local model. Fig. 3b shows a close-up view of the FE model used for the investigation of local effects. The local model is similar to the global model except that there is a small transition zone, around the delamination region, which is necessary for calculation of G. The intact region is modelled by a single layer of laminated shell elements which is connected to the double layer by use of the Mindlin/Kirchhoff equations and an additional coupling of the rotational degrees of freedom similar to the global model.

The transition zone is introduced for calculation of G according to the approach of Klug et al. [8] because without having two stacked layers of elements it cannot be applicable.

3. Comparison of numerical and experimental results for a series of delaminated plates

Chai et al [13] carried out an experimental study to investigate the effect of prescribed interplay delaminations on the post-buckling strength of carbon fibre composite panels. The

main objective of their work was to consider the possible structural exploitation of the post-buckling strength of carbon fibre composite panels and to investigate the possible structural degradation due to the adverse effect of interply delaminations.

For the present study three of their specimens were selected. The dimensions of the selected panels are given in Table 1. To aid comparisons, the numbers in Table 1 correspond to those of Chai et al [16]. All test panels had the detailed sequence of $[+45/0/0/90/0/-45/0]_s$. The reinforcing material was Grafil XA-S unidirectional carbon fibre in 914C Fibredux epoxy resin. The engineering constants of the unidirectional plies forming the composite are given in Table 2.

A comparison has been made between the critical buckling load obtained by FE modelling and the corresponding experimental results of the specimen of Table 1 having single and multiple circular delaminations. The delaminations are at various in-plane locations and at different positions through the thickness. Positions and sizes of the delaminated regions are illustrated in Tables 3-5. The delamination shape was described by Chai et al [16] being penny shaped, therefore, a circular shape with a radius of 0.01m is assumed. The numerical and experimental values of the critical buckling loads of batches 1 and 2 are presented in Tables 3-5. All batch 3 specimens contained no delamination [16]. Tables 3-5 also show the numerical and experimental results of the critical buckling loads for these delaminated plates which are in good agreement. It is shown that the proposed model could produce the results for embedded delaminations successfully, whereas different results may be obtained for delaminations located at the edge of the plate. Therefore, using the present numerical method, embedded delaminations can be modelled and analysed effectively, requiring neither a great deal of computing time and memory nor the expense of experimental testing.

4. Examination of global effects of delamination

A delaminated composite plate has a lower ability to resist compressive loads. The reduction in this ability depends on the shape, size and position of the delamination. Here the influence of the delamination size, shape and alignment on the critical buckling load is studied.

For the models, the plate thickness, in-plane geometry and stacking sequence are the same according to the plates examined by Chai et al [16]. See Tables 1 and 2 and Fig. 2a. However, a different geometry is introduced with length of $a=1.0\text{m}$, width of $b=0.5\text{m}$ and thickness $t_{\text{lam}}=0.00175\text{m}$ ($b/t_{\text{lam}}=286$). The resulting aspect ratio of the plate is ($r_{\text{pl}}=a/b=0.5$) is convenient because for the buckling mode, corresponding to the critical load, the number of half waves in each x and y direction is equal to one.

4.1 Effect of size

For the purpose of investigating the influence of the delamination size on the global critical load, central rectangular delaminations were introduced that had the same aspect ratio as the plate. Their fraction of the total available area of the plate is expressed by the parameter

$$A^* = \frac{A_{\text{del}}}{A_{\text{pl}}} \quad (5)$$

where A^* was varied in a range of $0.02 < A^* < 0.9$. Two more data sets were identified for the case of the intact plate ($A^*=0.0$) and the case of the plate being debonded at the whole area ($A^*=1.0$). The resulting normalised critical load

$$P_{cr}^* = \frac{P_{cr, \text{del}}}{P_{cr, \text{int}}} \quad (6)$$

was then plotted against the corresponding delamination area A^* . Fig. 4 shows that the normalised critical load decreases in an exponential manner with increasing A^* and is equal to 0.197 when the entire plate is delaminated ($A^*=1.0$).

4.2. Effect of shape and alignment

The effect of delamination shape and orientation is also studied. Two shape types were investigated, rectangular and elliptical delaminations. In the first model series, the delaminations were located in the longitudinal direction of the plate. Simultaneously with the investigation of the shape influence, it was also possible to consider the influence of orientation. The rectangular and elliptical delaminations contained in the second model series were obtained in the transverse direction of the plate. In order to compare the results, equal delamination areas were chosen for all four cases (Fig. 5).

Fig. 5 also shows the variation of P_{cr}^* with the delamination area fraction for all four cases. It can be seen that in the range of $A^* < 0.07$ for all the delamination shapes there is a small decrease of the critical load in comparison with the intact plate. In the range of $A^* > 0.07$, the plates with delaminations in the transverse direction have higher decrease in the buckling loads than the longitudinal ones.

5. Local effects of delamination

5.1 Method and examination schedule

The FE model used for examining local effects has been explained above. Three delamination shapes are considered, circular, elliptic aligned longitudinal and elliptic aligned transverse each having the same area fraction $A^* = 0.05$ and being located at the plate midplane. The main interest of this local investigation will be the growth potential at the delamination border.

The modified crack closure technique [19, 20] is used to calculate the strain energy release rate. In this technique, the strain energy release during crack extension is assumed to be

equal to the work needed to close the opened surfaces. In the finite element model, the crack closure energy is obtained by summing up the energies associated with the discrete nodes within the considered area. Fig. 6a shows part of a typical mesh and corresponding cells for calculation of the strain energy. Fig. 6b shows a model of a pair of four elements in the upper and lower sublaminates at the delamination front. The crack front is located beneath nodes c_j ($j=1-5$). The upper nodes (c_j , d_j and e_j) are tied to the lower nodes (c_j' , d_j' and e_j') as explained before. These constraints result in reaction forces N_x , N_y and N_z and moments M_x and M_y at these nodes. The reaction forces and moments at a node are equal to the respective sums of the nodal forces and moments of all of the elements sharing this node. Assume that the crack front extends from the current location c_j - c_j' to e_j - e_j' . Since the extension Δa is very small, the crack opening displacements at c_j and c_j' are assumed to be equal to those at a_j and a_j' before the crack extension. A similar assumption can be made for the crack opening displacements at d_j and d_j' .

The crack closure energy associated with a pair of nodes (c_1 , c_1') can be expressed in the form

$$U = N_x \Delta u_x + N_y \Delta u_y + N_z \Delta u_z + M_x \Delta \psi_x + M_y \Delta \psi_y + N'_x \Delta u'_x + N'_y \Delta u'_y + N'_z \Delta u'_z + M'_x \Delta \psi'_x + M'_y \Delta \psi'_y \quad (7)$$

where N_i, M_i and N'_i, M'_i are nodal forces and moments before crack extension at nodes c_1 and c_1' , respectively, and u_i, ψ_i and u'_i, ψ'_i are the nodal displacements and rotations at nodes a_1 and a_1' , respectively. Δu_i denotes the difference in the nodal displacements between nodes a_j and c_j , and $\Delta u'_i$ denotes the difference of the nodal displacements between nodes a_j' and c_j' .

This calculation is now done for all the corresponding pairs of nodes in the region considered, resulting in a total strain energy U , that is released in that area. The average strain energy release rate is then obtained by dividing the total strain energy released by the considered

area. For example, the total strain energy released over the cell c_2 , c_4 , e_4 and e_2 is calculated as

$$U = U_{c_3c_3'} + U_{d_3d_3'} + \frac{1}{2}(U_{c_2c_2'} + U_{c_4c_4'}) \quad (8)$$

The strain energy release rate at point c_3 is taken as

$$G = U / A \quad (9)$$

where

$$A = \Delta a \times c_2 c_4 \quad (10)$$

where Δa is the virtual crack extension (Fig. 6). Therefore, the size of the transition zone (local model) does not affect the G calculation but the accuracy of such a calculation obviously depends on the finite element mesh, especially at the crack front.

5.2. Verification of the local

The model used for the local investigation is a plate ($a=b=0.1\text{m}$) having a circular delamination with various loading strain levels ε_0 . The loading direction is parallel to the x -axis. A quarter of the plate is employed for the analysis which is simply supported (z -direction) at the edge $y=b/2$ and clamped (except for the loading direction) at the edge $x = \pm \frac{a}{2}$. The model is symmetric about the x and y axes (Fig. 2b). The laminate is quasi-

isotropic $[\pm 45/0/90]_s$ with the equivalent engineering constants of

$$E_1 = E_2 = 52.58\text{GPa}, E_3 = 12.66\text{GPa}, G_{12} = 20.14\text{GPa}, G_{13} = G_{23} = 4.48\text{GPa}, \nu_{12} = 0.31,$$

$$\nu_{13} = \nu_{23} = 0.33, \text{ similar to those employed by Klug et al. [8].}$$

In contrast to the loading procedure employed by Klug et al. [8], the initial deflection that is necessary to make the structure buckle was established by an imperfection to the original mesh. The applied imperfection rested on an eigenmode buckling analysis of the structure,

similar to the method employed in Ref. [21]. The maximum initial perturbation was 3% of the thickness of the upper sublamine. For this sake the FORTRAN program FPERT, in ABAQUS, was employed.

It should be noted that in Klug et al's method [8], a small transverse perturbation load was applied at the centre of the delamination of the upper sublamine to initiate a transverse deflection.

Next, a nonlinear load-displacement analysis, with the RIKS option in ABAQUS, was performed. The RIKS method is particularly useful for problems of highly nonlinear nature, especially for buckling and collapse studies. The method obtains equilibrium solutions by controlling the path length along the load-displacement curve within each increment (rather than controlling the load-displacement increment), so that the load magnitude becomes an unknown of the system. The consequence and simultaneously the drawback with this method is that the numerical problem cannot be solved for given strain levels, so that comparisons between different models or configurations are not straightforward. As the loading increments are different for each case, therefore, the computed curves must be interpolated between the computed points to find the corresponding values for a given strain level, provided the increments are sufficiently small. Fig. 7 shows the strain energy distribution around the delamination front at different strain levels using RIKS method. Also for strain level of 0.5%, the results of Klug et al[7] are presented.

Next, a second analysis was carried out without the RIKS option. ABAQUS by default uses Newton's method as a numerical technique for solving the nonlinear equilibrium equations. Using this method it was possible to get results at distinct strain levels. The results of the

second method are also presented in Fig. 7. It can be seen that the results using the Newton's method agree with those obtained by Klug et al [8]. It is believed that RIKS option in ABAQUS provides stable but not always correct numerical solutions. Therefore, the Newton's method seems to be more reliable.

It should be pointed out that, in the present study, the intact region is modelled by a single layer of shell elements, whereas Klug et al. [8] used two layers of shell elements for both intact and delaminated regions of the plate. Therefore, for the same level of accuracy, the current modelling requires less computing time and space.

6. Mixed mode buckling

Fig. 8 shows the model configuration and selected delamination shapes and sizes for a global-local (Fig. 1) investigation. For the material properties and stacking sequence of the models, refer to Table 2. A compressive load at the edge $x=a/2$ was increased gradually ($0.5 < \lambda < 2.0$) until the critical load computed for the corresponding complete model was reached. Therefore, local buckling at the delaminated region occurred before global buckling for all three cases. However, the corresponding lateral displacements were very small. The size of the gap at the delamination centre for each case, when $\lambda=1.0$, is given in Fig. 8.

As the applied load exceeded the critical level, the same basic global buckling shape occurred as was observed with the global investigations. In contrast, the lateral displacement field was locally different at the delaminated regions. No significant gap occurred between the upper and lower sublaminates in the delaminated region for the circular and longitudinal elliptical cases. However, the whole delamination area exhibited a common buckling shape that was different from the one observed in the case of the intact laminate. Only the model with a

transverse aligned elliptical delamination showed a typical “mixed mode” buckling shape.

The gap at the debonded region was enforced only for the model containing a transverse elliptical delamination, whereas global buckling displacements were increasing for all models. The distribution of the average strain energy release rate G depends strongly on the above-discussed local buckling shape. See Table 6.

7. Local buckling at high strain level and effect of stacking sequence

A different model was created to investigate the delamination growth potential at higher loading. The intact area and the lower sublamine were restrained against z -displacements to avoid global buckling of the plate, similar to the model proposed by Klug et al [8]. The dimensions used for the analysis are $a/4=b/2=0.05\text{m}$. However, the laminate thickness ($t_{\text{lam}}=0.00175\text{m}$), the stacking sequence $([+45/0/0/90/0/-45/0]_s)$ and the location of the delamination remained the same as in Section 6 (Table 2). In this case the loading was applied by a gradual increase of displacement along the edge $x=a/2$. The axial loading strain ε_0 was varied in the range of 0.1-0.6%. Fig. 9 shows the G distribution along the delamination front for the three investigated shapes for $\varepsilon_0=0.4\%$. The transverse aligned elliptical delamination shows significant higher G values along the debonding front than the circular and longitudinal elliptical delamination. The maximum G value is occurring in the region of $s \rightarrow 1$ for the transverse elliptical delamination.

From Fig.9 it also appears that the stacking sequence $[0/-45/0/90/0/0/45]$ of the upper sublamine in connection with the applied loading strain parallel to the x -axis leads to a strong resistance to the development of a gap between the debonded surfaces. Obviously, the compression in x -direction produces moments in the upper sublamine that act against the

arise of a gap. Following this analysis, the behaviour of an upper sublamine with reversed stacking sequence was studied. The corresponding results can also be found in Fig. 9. Contrary to the results received for the original laminate, now the models containing circular and longitudinal elliptical delaminations showed a strongly developed buckling shape having a high peak deflection at the delamination centre, whilst the transversely aligned elliptical delamination is not subjected to buckling of equal strength and shape.

8. Conclusions

Two different FE models were developed for global and local investigations considering the effects of delaminations on the critical load and the delamination growth potential.

The results of the global modelling approach were compared with experimental results. It was observed that the FE results for the embedded delaminations were in good agreement with the experimental results, whereas different results were obtained for delaminations located at the edge of the plate. In comparison with the other numerical model, it was shown that embedded delaminations can be modelled and analysed effectively without requiring a great deal of computing time and memory.

The global modelling approach established in this work offers high potential for further development. So far, the material properties were assumed to be linear. However, the structure of the model offers convenient extension to nonlinear behaviour such as friction between the crack surfaces.

An investigation of the laminate at higher loading was performed in order to gain knowledge about the behaviour in the post-buckling stage. It was observed that the G distribution and,

corresponding to that, the delamination growth potential at the delamination front strongly depended on the shape of the debonded region and the local buckling mode. It was also seen that the local buckling mode was highly influenced by the laminate stacking sequence that determined the deflection response to the applied loading. From a comparison of two stacking sequences, it was seen that at a defined loading there are stacking sequences that favour delamination growth and others that exhibit high resistance against the crack extension. Consequently, a laminate can be tailored to delamination growth resistance.

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Table 1 Dimensions of the test specimens

Batch No.	Specimen[16]	Width (b) (mm)	Length (a) (mm)	Thickness(t_{lam}) (mm)
1	SCB1	120	458	1.75
2	SCB2	90	458	1.75
3	SCB3	90	458	1.83

Table 2 Material specification of XA-S 60%/914C and stacking sequence of [+45/0/0/90/0/-45/0]_s

Mass density (kg/m^3)	1570.0
Young's modulus (GPa)	$E_1=130.0$, $E_2=9.0$, $E_3=9.0$
Shear modulus (GPa)	$G_{12}=4.8$, $G_{23}=3.5$, $G_{31}=4.8$
Poisson's ratio	$\nu_{12}=0.28$, $\nu_{23}=0.35$, $\nu_{31}=0.28$
Fibre volume fraction	$\phi_f=0.6$
Matrix volume fraction	$\phi_m=0.4$

Table 3 Experimental [16] and numerical results for the delaminated plates of batch No. 1 in Table 1

Test panel specification and delamination pattern	z-location plies	Exp. Results (kN)	FE solution (kN)
SCB1-1	-	61.64± 1.62	61.78
SCB1-2	7/8	57.88± 1.523	61.81
SCB1-3	3/4 and 11/12	56.66± 1.49	61.56
SCB1-4	3/4 and 11/12	61.54± 1.61	61.79
SCB1-5	7/8	61.54± 1.61	61.77
SCB1-6	3/4 and 11/12	56.66± 1.49	61.70

Circular delamination with a radius of 0.01m

Table 4 Experimental [16] and Numerical results for the delaminated plates of batch No. 2 in Table 1

Test panel specification and delamination pattern	z-location plies	Exp. Results (kN)	FE solution (kN)
SCB2-1	-	112.16± 6.78	106.5
SCB2-3	3/4 and 11/12	106.9± 6.46	105.89
SCB2-4	3/4 and 11/12	116.00± 7.02	106.07

Circular delamination with a radius of 0.01m

Table 5 Experimental [16] and Numerical results for the delaminated plates of batch No. 3 in Table 1

Test panel specification and delamination pattern	z-location plies	Exp. Results (kN)	FE solution (kN)
SCB3-2	6/7	124.00	121.55
SCB3-2	6/7	119.75	121.45
SCB3-3	6/7	133.00	120.32
SCB3-4	6/7	116.25	121.19

Circular delamination with a radius of 0.01m

Table 6 Maximum strain energy release rate (G) and its position(s) around the crack front for the mixed mode and local buckling of plates with circular and Elliptical delaminations at critical load level ($\lambda=1$)

Model	Circular		Elliptical longitudinal		Elliptical transverse	
	Gmax J/m ²	Position(s)	Gmax J/m ²	Position(s)	Gmax J/m ²	Position(s)
Mixed mode	0.000316	0.92	0.000423	0.92	0.02020	0.92
Local	0.000271	0.92	0.000411	0.92	0.02755	0.92

Fig. 1. Buckling modes for a delaminated composite plate.

Fig. 2 Geometry of the plate with one delamination a) Global model b) Local model

Fig. 3 Close-up views of the models used for study of effects of delamination on global and local buckling a) Global model (Without the transition zone) b) Local model (With the transition zone)

Fig. 4 Effect of delamination size on normalised critical buckling load (P_{cr}^*) for rectangular delamination ($a=1.0\text{m}$, $b=0.5\text{m}$, $t=1.75\text{mm}$)

Fig. 5 Effect of delamination shape and alignment on normalised critical buckling load (P_{cr}^*) over a range of ($0 < A^* < 0.2$) for rectangular (longitudinal and transverse) and elliptical (longitudinal and transverse) delaminations, ($a=1.0\text{m}$, $b=0.5\text{m}$, $t=0.00175\text{m}$)

Fig. 6 Schematic of delamination front region

Fig. 7 Strain energy release rate(G) distribution along the crack front for different loading strain using RIKS and standard nonlinear methods and also the results for loading strain of 0.5% extracted from Ref[8], circular delamination ($a=b=0.1$, $R=0.015$)

Fig. 8. Development of buckling shape with increasing compressive load ($0.5 < \lambda < 2.0$) for plates with circular, elliptical (longitudinal) and elliptical (transverse) delaminations, $a=1.0m$, $b=0.5m$, $t=1.75mm$, $h_1=h_2=0.875mm$.

Fig. 9 Strain energy rate(G) distribution around the delamination front for the strain loading level of 0.4% for circular, elliptical longitudinal and elliptical transverse delaminations, stacking sequence of [0/-45/0/90/0/0/45]_s and reversed