We present a Peccei-Quinn (PQ)-symmetric two-Higgs doublet model that naturally predicts a fermionic singlet dark matter in the mass range 10 keV–1 GeV. The origin of the smallness of the mass of this light singlet fermion arises predominantly at the one-loop level, upon soft or spontaneous breakdown of the PQ symmetry via a complex scalar field in a fashion similar to the so-called Dine-Fischler-Sredniki-Zhitnitsky axion model. The mass generation of this fermionic radiative light dark matter (RLDM) requires the existence of two heavy vectorlike SU(2) isodoublets, which are not charged under the PQ symmetry. We show how the RLDM can be produced via the freeze-in mechanism, thus accounting for the missing matter in the Universe. Finally, we briefly discuss possible theoretical and phenomenological implications of the RLDM model for the strong CP problem and the CERN Large Hadron Collider (LHC).

I. INTRODUCTION

Ongoing searches for the elusive missing matter component of the Universe, the so-called dark matter (DM), have offered no conclusive evidence so far. From analyses of the cosmic microwave background power spectrum and from pertinent astronomical studies, we now know that about one quarter of the energy budget of our Universe should be in the form of DM, and so many candidate theories have been put forward to address this well-known DM problem [1]. Among the suggested scenarios, those predicting weakly interactive massive particles (WIMPs) constitute one class of popular models that may not only account for the DM itself, but also leave their footprints in low-energy experiments, or even at high-energy colliders, such as the LHC [2]. In particular, for WIMPs near the electroweak scale, the WIMP-nucleon scattering cross section is estimated to be somewhat below $10^{-46}$ cm$^2$ as measured by LUX [3].

Projected experiments that lie not very far ahead in future will be capable of reaching sensitivity in the ballpark $10^{-47} - 10^{-48}$ cm$^2$ [4], and so they will be getting closer to the neutrino-nucleon background cross section, the infamous “neutrino floor,” where disentangling neutrino signals from those of WIMPs will become almost an impossible task [5]. Therefore, DM models have to be constructed (or revisited) to avoid such severe constraints, e.g. by contemplating scenarios that either sufficiently suppress the WIMP-nucleon interaction, or move the DM mass to the sub-GeV or ultra-TeV region.

Several models have been proposed featuring a light DM in the mass range $\mathcal{O}$(keV) – $\mathcal{O}$(GeV), such as sterile neutrino DM [6–10], light scalar DM [11] and milli-charged DM [12], including their possible implications for future DM searches [13,14]. However, one central problem of such models is the actual origin of the small mass for the light DM, which could be more than 6 orders of magnitude below the electroweak scale.

In this paper we address this mass hierarchy problem, by presenting a new radiative mechanism that can predominantly account for the smallness in mass for the light DM. The so-generated radiative light dark matter (RLDM) is a fermionic singlet $\Sigma$ and can naturally acquire a mass in the desired range: 10 keV–1 GeV. A minimal realization of this radiative mechanism requires the extension of the Standard Model (SM) by one extra scalar doublet, resulting in a Peccei-Quinn (PQ)-symmetric two-Higgs doublet model [15,16], augmented by two fermionic heavy vectorlike SU(2) isodoublets $D_1$ and $D_2$, which are not charged under the PQ symmetry. The mass of the RLDM is predominantly generated at the one-loop level, upon soft or spontaneous breakdown of the PQ symmetry via a complex scalar field, e.g. $\Sigma$, in close analogy to the so-called Dine-Fischler-Sredniki-Zhitnitsky (DFSZ) axion model that addresses the strong CP problem [17,18].

We analyse the production mechanisms of the RLDM in the early universe, and show that it can account for its missing matter component via the so-called freeze-in mechanism [19]. In fact, we illustrate how the freeze-in mechanism remains effective in the RLDM model, without the need to resort to suppressed Yukawa couplings. In this context, we investigate two possible scenarios of both theoretical and phenomenological interest. In the first scenario, we consider the breaking of the PQ scale $f_{\text{PQ}}$ to be comparable to the one required for the DFSZ model to solve the strong CP problem, i.e. $f_{\text{PQ}} \sim 10^9$ GeV.
We find that such a PQ scale can exist within this realization, provided an appropriate isodoublet mass $M_0$ and reheating temperature $T_{RH}$ is considered. In the second scenario, we relax the constraint of the strong CP problem on $f_{PQ}$, and investigate its possible lower limit, with the only requirement that $T_{RH}$ be larger than the critical temperature $T_c$ of the SM electroweak phase transition, thus allowing for the $B + L$-violating sphaleron processes to be in thermal equilibrium. This requirement is introduced here, so as to leave open the possibility of explaining the cosmological baryon-to-photon ratio $\eta_B$ via low-scale baryogenesis mechanisms, such as electroweak baryogenesis [20,21] and resonant leptogenesis [22–25]. In this second scenario, we find that the heavy Higgs bosons of the two-Higgs doublet model (2HDM) may have masses as low as a few TeV, which are well within reach of the LHC.

The layout of the paper is as follows. In Sec. II, we first introduce the PQ-symmetric 2HDM, augmented with a singlet fermion $S$ and a fermionic pair of vectorlike doublets $D_{1,2}$. Then, we describe the radiative mechanism for the RLDM, once the PQ symmetry is broken softly, and show that a radiative mass in the range 10 keV–1 GeV can be naturally generated. In Sec. III, we outline the relevant Boltzmann equation for computing the relic abundance of the RLDM. Utilizing the freeze-in mechanism, we present numerical estimates for the allowed parameter space of our RLDM model. Based on these results, we explore the possibility whether our model can account for the strong CP problem within a scenario similar to the DFSZ axion model. Moreover, we investigate whether an absolute lower limit exists for the heavy Higgs-boson masses in our effective 2HDM. Indeed, we find that our RLDM model may allow for heavy Higgs bosons at the TeV scale, whose existence can be probed at the LHC. Finally, Sec. V summarizes our conclusions and outlines possible new directions for further research.

II. RADIATIVE MECHANISM

In this section we present a minimal extension of the SM, in which the small mass of the light DM, in the region 10 keV–1 GeV, can have a radiative origin, generated at the one-loop level. This radiative mechanism is minimally realized within the context of a constrained 2HDM obeying a Peccei-Quinn symmetry. In addition, the model under study contains a singlet fermion $S$ charged under the PQ symmetry and a fermionic pair of massive isodoublets $D_{1,2}$ with zero PQ charges. Finally, we delineate the parameter space for which a viable scenario of radiative light dark matter can be obtained consistent with the observed relic abundance.

A. The model

In the 2HDM under consideration, we impose a global PQ symmetry $U(1)_{PQ}$, which forbids the appearance of a bare mass term for the singlet fermion $S$ at the tree level. This PQ symmetry will be broken softly or spontaneously which in turn triggers a radiative mass for $S$ at the one-loop level. The fermion $S$ is stable and receives naturally a small sub-GeV mass, leading to a RLDM scenario. On the other hand, we note that a candidate for a light DM would probably be relativistic at its freeze-out, resulting in an extremely large relic abundance (similar to [26]) for the allowed range of DM masses that are larger than about 3 keV, e.g. see [27,28]. Therefore, the DM should be produced out of thermal equilibrium in the early universe. The mechanism that we will be utilizing here is the so-called freeze-in mechanism [19], which assumes that the DM particles were absent initially and are produced only later from the plasma.

The relevant Yukawa and potential terms of our model are given by

$$-\mathcal{L}' = Y_1 e^{ab} \Phi_{1a} D_{1b} S + Y_2 \Phi_{2a}^* D_{2a} S + M_D e^{ab} \Phi_{1a} D_{2b} + \text{H.c.},$$

$$V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_{1a}^* \Phi_{1a} + m_{22}^2 \Phi_{2a}^* \Phi_{2a}$$

$$- m_{12}^2 (\Phi_{1a}^* \Phi_{2a} + \text{H.c.}) + \frac{\lambda_1}{2} (\Phi_{1a}^* \Phi_{1a})^2$$

$$+ \frac{\lambda_2}{2} (\Phi_{2a}^* \Phi_{2a})^2 + \lambda_3 \Phi_{1a}^* \Phi_{1a} \Phi_{2a}^* \Phi_{2a}$$

$$+ \lambda_4 (\Phi_{1a}^* \Phi_{2a})^2,$$

where $a, b = 1, 2$ are SU(2)$_L$-group indices (with $e^{12} = -e^{21} = +1$), $S$ is a Weyl-fermion SM singlet, $D_{1,2}$ are two Weyl-fermion SU(2)$_L$ doublets, and $\Phi_{1,2}$ are two scalar SU(2)$_L$ doublets. A complete list of the PQ and hypercharge quantum numbers of the aforementioned particles is given in Table I, including a $Z_2$ parity which excludes the mixing of dark-sector particles with those of the SM. For simplicity, we assume that the new dark-sector interactions are CP invariant and so take their respective couplings to be real in the physical mass basis.

As can be seen from (2), we have assumed that the PQ symmetry is broken by the lowest dimensionally possible mass operator in the scalar potential $V$, namely by allowing only the dimension-2 mixing term $m_{12}^2$ between $\Phi_1$ and $\Phi_2$. This dimension-2 operator breaks softly the $U(1)_{PQ}$ symmetry in the potential, but could result from spontaneous

| TABLE I. Quantum number assignments of particles pertinent to the RLDM model. |
|-------------------|-----------------|-----------------|-----------------|-----------------|
|                  | SU(2)$_L$       | U(1)$_Y$        | U(1)$_{PQ}$     | $Z_2$           |
| $\Phi_1$         | 1               | 0               | $-1$            | Odd             |
| $D_{1}$          | 2               | $-1$            | 0               | Odd             |
| $D_{2}$          | 2               | 1               | 0               | Odd             |
| $\Phi_2$         | 2               | 1               | 1               | Even            |
| $S$              | 1               | 0               | $-1$            | Odd             |
obtain singlet fermion to the radiatively induced mass level contribution turns out to be subdominant compared given by the mass parameters from the diagram shown in Fig. 1. The mass parameters \( m_{11} \) and \( m_{22} \) of the scalar potential \( V \) in Eq. (2) may be eliminated in favor of the VEVs \( v_{1,2} \) of the Higgs doublets \( \Phi_{1,2} \), by virtue of the minimization conditions on \( V \) (for a review on 2HDMs, see [29]). These VEVs are related to the SM Higgs VEV \( v \), through: \( v^2 = v_1^2 + v_2^2 \). In the kinematic region where \( m_{12}^2 \gg v^2 \), the mass parameters \( m_{11}^2 \) and \( m_{22}^2 \) are approximately given by

\[
\begin{align*}
m_{11}^2 &\approx m_{12}^2 \tan \beta + O(v^2), \\
m_{22}^2 &\approx m_{12}^2 \cot \beta + O(v^2),
\end{align*}
\]

where \( \tan \beta = v_2 / v_1 \).

### B. One-loop radiative mass

Having introduced the minimal model under investigation, we can now discuss the radiative mechanism responsible for the generation of a mass of dimension 3 for the singlet fermion \( S \). We assume that \( m_{12} \gtrsim 1 \) TeV, such that the main contribution to the mass of the \( S \) particle comes from the diagram shown in Fig. 1. In addition, there will be a tree-level mass \( M_{S}^{\text{tree}} \) generated after the SM electroweak phase transition, given by \( M_{S}^{\text{tree}} = Y_1 Y_2 v^2 / M_D \). Under the assumption that \( M_D \) is very large, i.e. \( M_D \gg v \), the tree-level contribution turns out to be subdominant compared to the radiatively induced mass \( M_{S}^{\text{rad}} \), and hence it can be ignored for most of the parameter space. We will return to this point at the end of this section.

After evaluating the relevant one-loop self-energy graph shown in Fig. 1 at zero external momentum \( (p \rightarrow 0) \), we obtain

\[
M_{S}^{\text{rad}} = -2Y_1 Y_2 M_D m_{12} I(M_D, m_{11}, m_{22}),
\]

where

\[
I(M_D, m_{11}, m_{22}) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - M_D^2)(q^2 - m_{11}^2)(q^2 - m_{22}^2)}.
\]

Employing the approximate relations given in (3) and (5), the one-loop radiative mass of \( S \) is finite and may conveniently be expressed as follows:

\[
M_{S}^{\text{rad}} = \frac{2y^2}{(4\pi)^2} M_D \quad \text{for } r \ll 1,
\]

\[
M_{S}^{\text{rad}} = \frac{y^2}{(4\pi)^2} M_D \quad \text{for } r \sim 1,
\]

\[
M_{S}^{\text{rad}} = \frac{2y^2}{(4\pi)^2} \frac{M_D \ln r^2}{r^2} \quad \text{for } r \gg 1.
\]

Note that for \( M_D \gg m_{12} \) (corresponding to \( r \gg 1 \)), the radiative mass \( M_{S}^{\text{rad}} \) of the singlet fermion \( S \) is suppressed by the square of the hierarchy factor \( r \). The latter allows for scenarios, for which the Yukawa couplings are of order 1, i.e. \( y^2 \equiv Y_1 Y_2 = O(1) \), for 10 keV \( \leq M_{S}^{\text{rad}} \leq 1 \) GeV. On the other hand, for \( r \sim 1 \) and \( r \ll 1 \), one needs either a low \( M_D \) of order TeV and \( y \approx 0.1 \), or \( M_D \approx 10^8-10^9 \) GeV and \( y \approx 10^{-3}-10^{-4} \).

In Fig. 2, we display the values of the coupling parameter \( y = \sqrt{Y_1 Y_2} \), as a function of \( M_D \), which yield a radiatively induced mass \( M_{S}^{\text{rad}} \) for the singlet fermion \( S \) in the region

\[
M_D = 10^{-1} \times m_{12}, \text{ tan} \beta = 1.
\]

![FIG. 1. One-loop diagram responsible for the mass generation of the singlet fermion \( S \).](image1.png)

![FIG. 2. Predicted values for \( M_D \) versus \( y = \sqrt{Y_1 Y_2} \) as obtained from (7), for \( M_S = M_{S}^{\text{rad}} \) ranging from 10 keV to 1 GeV, after setting \( \tan \beta = 1 \) and \( r = 10^{-2} \).](image2.png)
10 \text{ keV} \leq M_{\text{rad}}^2 \leq 1 \text{ GeV}, \text{ for } l_D = 1 \text{ and } r = 10^{-2}. \text{ In particular, we see that for every set of } M_{\text{rad}}^2, M_D, r, \text{ there is an acceptable range of perturbative values for } y. \text{ However, if } r \gg 1, \text{ the desirable value of } y \text{ may exceed } 10 \text{ according to (10), and our perturbative results do no longer apply. Such nonperturbative values of } y \text{ are excluded from our numerical estimates for the determination of the relic abundance of } S \text{ which we perform in the next section.}

In a similar context, we note that a large mass for } m_{11}, m_{12}, m_{22} \text{ and } M_D \text{ might seem to be a huge fine-tuning for generating a light sub-GeV radiative mass for } S. \text{ However, we may easily convince ourselves that this is not the case. The absence of fine-tuning can be seen in an easier way, if we rotate from the general weak basis spanned by } \Phi_1 \text{ and } \Phi_2 \text{ to the so-called Higgs basis } [29,30], \text{ which is not canonical. Moreover, in this rotated Higgs basis, one has that the new Higgs-mass parameters obey the relation: } \tilde{m}_{\Phi_1}^2 \gg \tilde{m}_{\Phi_2}^2, \tilde{m}_{12}. \text{ In addition, the analogue of the diagram in Fig. 1 is now represented by a set of two self-energy graphs, where the fields } H_1 \text{ and } H_2 \text{ are circulating in the loop. The ultraviolet (UV) infinities cancel, after the contributions from these two diagrams are added. For } l_D = 1 \text{ and } r = 1, \text{ we then obtain the same result as the one stated in (9). Hence, we observe that a small mass for the singlet fermion } S \text{ arises naturally in a SM+S effective field theory. This effective field theory results from integrating out the heavy } D_{1,2} \text{ and } H_2 \text{ fields from (1) in the Higgs basis.}

Besides the radiative mass } M_{\text{rad}}^2 \text{ of } S \text{ which violates the PQ symmetry by two units (cf. Table I), there will be a tree-level contribution to the mass of } S \text{ after the SM electroweak phase transition. For most } r \text{ values of interest here, the relative size of the two contributions can naively be estimated to be}

$$M_{\text{tree}}^2 \sim \frac{8\pi^2 v^2}{M_D^2}. \tag{11}$$

Thus, for } M_D \gg \sqrt{8\pi v} = 2.2 \text{ TeV, the tree-level contribution can be safely ignored. In our numerical estimates, the tree-level mass term } M_{\text{tree}}^2 \text{ is always less than } 10\% \text{ of the radiative mass term } M_{\text{rad}}^2. \text{ Hence, the total mass } M_S \text{ of the stable fermion } S \text{ is given predominantly by the radiative mass term, implying that } M_S = M_{\text{rad}}^2 \text{ to a very good approximation.}

We conclude this section by commenting on the possibility of considering a radiative model alternative to the one discussed here. For instance, one may envisage a scenario that instead of the single } S, \text{ one of the neutral components of the doublets } D_{1,2} \text{ becomes the RLDM. In this case, however, the charged component } D^\pm \text{ from } D_{1,2} \text{ will be almost degenerate with the light sub-GeV DM particle, which is excluded experimentally. The general SM + } D_{1,2} \text{ effective theory has been studied in [31].}

### III. DARK MATTER ABUNDANCE

In this section we first describe the relevant effective Lagrangian that governs the production of the stable fermions } S \text{ in the early universe. We then solve numerically the Boltzmann equation that determines the yield } Y_S = n_S/s \text{ of these fermions } S, \text{ where } n_S \text{ is the number density of } S \text{ particles and } s \text{ is the entropy density of the plasma. Having thus estimated the value of } Y_S, \text{ we can then use it to deduce the respective relic abundance } \Omega_S h^2 \text{ of the } S \text{ particles in the present epoch. Finally, we present approximate analytic results for } \Omega_S h^2 \text{ and compare these with the observationally favored value: } \Omega_{\text{DM}} h^2 = 0.12.

As mentioned in the previous section, the stable fermions } S \text{ will play the role of the DM, which are produced via the freeze-in mechanism [19]. The key assumption is that the DM fermions } S \text{ were absent (i.e. their number density was suppressed) in the early universe and were produced later from annihilations and decays of plasma particles, e.g. from } \Phi_{1,2} \text{ and } D_{1,2}, \text{ according to the model discussed in Sec. II. Furthermore, we will assume that } D_{1,2} \text{ were also absent in the early universe, so as to avoid overclosure of the Universe, unless the Yukawa couplings } Y_{1,2} \text{ are taken to be extremely suppressed, such that decays of the sort } D_i \to hS \text{ are made slow and inefficient. The latter results in a contrived scenario, in which obtaining a viable DM parameter space requires a good degree of fine-tuning. In order for the SU(2)_{\text{1}}\text{-doublet fermions } D_{1,2} \text{ to be absent, we take their bare mass } M_D \text{ to be above the reheating temperature } T_{\text{RH}} \text{ of the Universe. This simplifies considerably our analysis, as the heavy fermions } D_{1,2} \text{ can be integrated out.}

The effective Lagrangian that determines the production rate of } S \text{ particles after reheating is given by}

$$-\mathcal{L}_{\text{eff}} = \frac{1}{2\lambda} \left( \Phi^\dagger_1 \Phi_1 + \tilde{a} \Phi^\dagger_2 \Phi_2 + \tilde{b} \Phi^\dagger_1 \Phi_2 + \tilde{c} \Phi^\dagger_2 \Phi_1 \right) SS + \text{H.c.} \tag{12}$$

where } \tilde{a}, \tilde{b}, \text{ and } \tilde{c} \text{ denote the Wilson coefficients of the dimension-5 operators. The calculation of the relic abundance is not straightforward in this basis, since } \Phi_{1,2} \text{ mix and the identification of the physical fields is obscured, especially after spontaneous symmetry breaking where further mixing between the scalar fields is introduced. Therefore, according to our discussion at the end of Sec. II B, it would be more convenient to rotate the scalars to the so-called Higgs basis } [29], \text{ where only one doublet } H_1 \text{ develops a VEV and is identified with the SM Higgs doublet. To further simplify calculations, and without much loss of generality, we assume that the Higgs basis is also the mass eigenstate basis. This assumption is well justified for...
relative large values of $m_{12}$, as it leads to the so-called alignment limit of the 2HDM [32–36], which is favored in the light of global analyses of experimental constraints [37,38]. In the Higgs basis, the dimension-5 effective Lagrangian reads

$$\mathcal{L}_{\text{eff}}^{d=5} = \frac{y^2}{M_D} \frac{t_\beta}{1 + t_\beta^2} (H_D^{\dagger}H_1 - H_D^{\dagger}H_2 - t_\beta H_1^{\dagger}H_2$$

$$+ t_\beta H_2^{\dagger}H_1)SS + \text{H.c.},$$

(13)

where $H_1$ is the SM Higgs doublet and $H_2$ is the heavy scalar doublet with $\langle H_2 \rangle = 0$.

### A. Boltzmann equation for $Y_S$

In order to determine the relic abundance of $S$ particles, we need to solve the Boltzmann equation for their yield $Y_S$. Since we assume that the singlets $S$ remained out of equilibrium throughout the history of the Universe (at least up to the phase of reheating), our only concern will then be their production. The main production channels, depending on the plasma temperature $T$, are the following:

$$H_D^{\dagger}H_1, H_D^{\dagger}H_2, H_1^{\dagger}H_2 \to SS \quad \text{for} \quad T_C \leq T < T_{\text{RH}},$$

$$H_1^{\dagger}H_2 \to SS \quad \text{for} \quad T < T_{\text{RH}},$$

$$h \to SS \quad \text{for} \quad T < T_C,$$

(14)

where $h$ is the Higgs field with mass $m_h \approx 125$ GeV and $T_C \approx 130$ GeV is the critical temperature of the SM electroweak phase transition. For $T < T_C$, one has to add new channels, for instance $W^+W^- \to SS$, but their contribution to the production of the DM particles is negligible compared to $h \to SS$.

Following [19], the Boltzmann equation for the yield $Y_S$ becomes

$$sHdY_S/dT = -\frac{1}{512\pi^2} \sum_{i,j \neq H_1} \int_{(m_i+m_j)^2}^{\infty} d\mu_{ij} |M_{ij}|^2 K_1(\sqrt{\delta}/T)$$

$$+ \left(\frac{t_\beta}{1 + t_\beta^2}\right)^2 \frac{y^4 m_h^4}{2\pi^4 M_D^2} K_1(\frac{m_h}{T}),$$

(15)

where $T$ is the temperature of the plasma, $H$ is the Hubble parameter, $K_1$ is the first modified Bessel function of the second kind, $P_{ij} \equiv \sqrt{\delta - (m_i + m_j)^2}$ is a kinematic factor, and $|M_{ij}|^2$ is the squared matrix element, summed over internal degrees of freedom, for the $2 \to 2$ annihilation processes: $H_1^{\dagger}H_1 \to SS$. The last term on the rhs of (15) arises from the decay $h \to SS$, upon ignoring the mass of the $S$ particles. Also, upon ignoring $M_{ij}$, the squared matrix elements $|M_{ij}|^2$ for the various $2 \to 2$ processes are

The solution to the Boltzmann equation is obtained by integrating (15) over the temperature $T$. The limits of integration for the various channels are the ones shown in (14). However, before doing that, we have to make an assumption for the critical temperature and the thermal corrections to the masses of the scalar fields. In what follows, we assume that the critical temperature $T_C$ and the thermal effects on the masses (for $T > T_C$) are similar to the pure SM Higgs sector and they are given by [39]

$$T_C \sim m_h, \quad m_{12}^2 \approx \frac{1}{2} T^2, \quad m_{22}^2 \approx \frac{1}{2} \frac{y^2}{\beta} m_{12}^2 \approx \frac{1}{2} T^2.$$

(17)

Under these assumptions and restricting $T_{\text{RH}}$ to be above $T_C$, we can compute the yield $Y_S$ at $T \approx 0$, which in turn implies the relic abundance [40]

$$\Omega_S h^2 \approx 0.12 \times \left(\frac{M_S}{1 \text{ GeV}}\right) \left(\frac{Y_S(T = 0)}{4.3 \times 10^{-6}}\right).$$

(18)

### B. Approximate results for $\Omega_S h^2$

In general, the yield $Y_S$ cannot be calculated analytically, but depending on the reheating temperature $T_{\text{RH}}$, we are able to present approximate analytic results. We find that for decoupled $D_{12}$, i.e. $T_{\text{RH}} > M_D$, the relic abundance $\Omega_S h^2$ derived from $Y_S$ in (18) takes on the form

$$\Omega_S h^2 \approx 0.12 \times \left(\frac{M_S}{10^3 \text{ GeV}}\right)^2 \left(\frac{2 \times 10^4 \text{ GeV}}{M_D}\right)^2$$

$$\times \left(\frac{10^{-4}}{M_D}\right)^2 \left(\frac{T_{\text{RH}}}{10^4 \text{ GeV}}\right),$$

(19)

for $T_{\text{RH}} \gg m_{12}$, and

$$\Omega_S h^2 \approx 0.12 \times \left(\frac{M_S}{10^3 \text{ GeV}}\right)^2 \left(\frac{2 \times 10^4 \text{ GeV}}{M_D}\right)^2$$

$$\times \left(\frac{10^{-4}}{M_D}\right)^2 \left[1 + \left(\frac{T_{\text{RH}}}{10^4 \text{ GeV}}\right)\right],$$

(20)

for $T_{\text{RH}} \ll m_{12}$. Equations (19) and (20) are accurate up to 1%, except for $T_{\text{RH}} \sim m_{12}$, where the deviation from the
IV. RESULTS

In Sec. II B, we have shown that the mass of the singlet $S$ can be generated at the one-loop level, if the PQ symmetry is softly broken, and in Sec. III we have calculated the relic abundance of the $S$ particles. In this section, we will be exploring the validity of the parameter space of our minimal model. To this end, one may consider the parameters,

$$T_{\text{RH}}, \quad M_D, \quad y^2, \quad t_\beta \quad \text{and} \quad m_{12},$$

as being independent. However, we prefer to solve the mass formula $M_S^{\text{rad}}$ in (7) for $y^2$ and replace it with a physical observable, the $S$-particle mass $M_S$, which is taken in our numerical estimates to be in the region: $10 \text{ keV} \leq M_S \leq 1 \text{ GeV}$. Consequently, the parameters that we allow to vary independently are

$$T_{\text{RH}}, \quad M_D, \quad M_S, \quad t_\beta \quad \text{and} \quad m_{12}. \quad (21)$$

We perform a scan over this parameter space, while imposing the perturbativity constraint on the Yukawa couplings: $Y_{1,2} < \sqrt{4\pi}$. In this way, we find the values of these parameters that satisfy the observed DM relic abundance [42]:

$$\Omega_S h^2 = \Omega_{\text{DM}} h^2 = 0.1198 \pm 0.0026. \quad (22)$$

In Fig. 3 we present contour lines on the $T_{\text{RH}} - m_{12}$ plane for discrete values of the $S$-particle mass $M_S$ in the region: $1 \text{ keV} \leq M_S \leq 1 \text{ GeV}$, for $t_\beta = 1$ and $r = 10^{-2}$, which give the DM relic abundance (22). For $m_{12} = 10^{10} \text{ GeV}$, the reheating temperature $T_{\text{RH}}$ can vary between the critical temperature $T_c = 130 \text{ GeV}$ and $10^{8} \text{ GeV}$. This upper bound on $T_{\text{RH}}$ may be as high as $10^{14} \text{ GeV}$, if the parameter $r = M_D/m_{12}$ is increased to the value $r = 10^{5}$, as depicted in Fig. 4. Yet, at the same time, $m_{12}$ increases by 1 order of magnitude or so. On the other hand, for $m_{12} = 1-10 \text{ TeV}$, an acceptable DM relic abundance is reached only for large $r$ and for $M_S \approx 1 \text{ keV}$, as can be seen from Fig. 4. Most remarkably, we notice that the predicted values for $\Omega_S h^2$ are compatible with the observed DM relic abundance $\Omega_{\text{DM}} h^2$, for a wide range of values for the parameters $m_{12}$, $M_D$ and $T_{\text{RH}}$. Interestingly enough, the required Yukawa couplings $Y_{1,2}$ for a viable RLDM are sizeable, and always larger than the electron Yukawa coupling.

We recall here that we explore only regions where the fermion doublets $D_{1,2}$ are decoupled after the reheating of the Universe, i.e. we assume $M_D \gg T_{\text{RH}}$. As a working hypothesis, we assume the decoupling condition: $M_D > 3T_{\text{RH}}$. This condition is motivated by the fact for $T \approx M_D/3$, the $D_{1,2}$ particles become nonrelativistic and, as a consequence, its number density is exponentially suppressed by a Boltzmann factor. Correspondingly, for the scenario considered in Fig. 3, the heavy scalar $H_2$ will be also decoupled, because $m_{H_2} \gg M_D$.

Furthermore, we observe that for $T_{\text{RH}} \gtrsim 10^{9} \text{ GeV}$, $m_{12}$ becomes linearly dependent on the reheating temperature, as expected from the approximate analytic expression in (20). We also obtain a similar behavior in Fig. 4. In this case, however, the heavy scalar doublet $H_2$ is no longer constrained to be decoupled. As a result, there is an interface region at $T_{\text{RH}} \sim m_{12}$ that lies between the two
linear regimes, $T_{RH} \ll m_{12}$ and $T_{RH} \gg m_{12}$. At the interface region, there is a transition caused by the contribution of the heavy scalar doublet $H_2$ to the production of singlet fermions $S$ [cf. (14)], which can reach equilibrium with the plasma when $T_{RH} \gg m_{12}$.

### A. Solving the strong CP problem

It is known that in the SM there is an explicit breaking of $CP$ (and $P$) discrete symmetry due to the instanton-induced term

$$\mathcal{L}_\theta = \frac{\theta}{32\pi} \text{Tr}(G_{\mu
u} \tilde{G}^{\mu\nu}).$$

In the above, $\theta$ is a $CP$-odd parameter which can be absorbed into the quark masses. However, this $\theta$ parameter cannot be fully eliminated, since the combination: $\bar{\theta} = \theta - \text{ArgDet}M_q$, where $M_q$ is the quark mass matrix, becomes a physical observable. It contributes to the neutron dipole moment and experimentally, it is severely bounded to be $|\bar{\theta}| \lesssim 10^{-11}$ [43]. The problem of why $\bar{\theta}$ is much smaller than all other $CP$-violating parameters, such as the well-known parameter $\epsilon_K \sim 10^{-3}$ from the $K^0\bar{K}^0$ system, introduces another hierarchy problem in the SM known as the strong problem. A possible solution, suggested by Peccei and Quinn [15,16], is to promote the $\theta$ parameter into a dynamical field which naturally minimizes the energy. This dynamical field, called the axion [44,45], is a pseudo-Goldstone boson of the global anomalous PQ symmetry.

The SM has no global anomalous $U(1)_{PQ}$ symmetry. One possible way to realize such a symmetry is to non-trivially extend its Higgs sector by adding a second Higgs doublet, resulting in the PQ-symmetric $2HDM$. However, charging simply the field doublets $\Phi_1$ and $\Phi_2$ under the PQ symmetry as done in Table I does not lead to a healthy model. Such a model predicts a visible keV-axion with $CP$-odd parameter which can be sizeable DM component resulting in a two-component DM, consisting of the axion and the $S$ particle, and so a more careful treatment will be required.

### B. Detection of RLDM

We observe that for small enough reheating temperatures, $T_{RH} \sim 1$ TeV, the fermion doublets $P_{1,2}$, as well as the heavy scalar doublet $H_2$, can lie at the TeV scale, provided that $M_S$ is of order $\mathcal{O}(10)$ keV. This is shown in Figs. 3 and 4 for light $M_t$, where $M_{H_1}$ and $m_{12}$ lie in the vicinity of the TeV scale. As a result, the DM particle $S$ can be probed indirectly by looking for its associated “partners” of the heavy Higgs doublet $H_2$. In general, we expect that at the LHC, the heavy sector of the $2HDM$ will be efficiently explored up to the TeV scale [34,48]. For the RLDM scenario at hand, however, such exploration may be somehow challenging, when looking for charged Higgs bosons with masses larger than $\sim 1$ TeV for a wide range of $\tau_H$ values [34,48].

On the other hand, direct detection experiments for sub-GeV DM particles focus on their interactions with atomic electrons, e.g. see [49,50]. However, in the RLDM scenario, such a detection of $S$ particles is practically unattainable, because $S$ interacts feebly with the SM Higgs boson with a coupling proportional to $v/M_D \ll 1$ yielding a cross section for $Se \rightarrow Se$, which is highly suppressed by fourth powers of the electron-to-Higgs-mass ratio, i.e.

$$\tilde{\sigma}_{Se} \approx \frac{\lambda_S^4 \tau_H^2}{\pi (1 + t_H)^3} \left(\frac{m_e}{m_h}\right)^4 \frac{1}{M_D^4} \approx 10^{-50} \times \frac{\lambda_S^4 \tau_H^2}{(1 + t_H)^3} \left(\frac{1 \text{ GeV}}{M_D}\right)^2 \text{ cm}^2.$$  

Hence, a simple estimate shows that $\tilde{\sigma}_{Se}$ is much smaller than its current experimental reach: $\tilde{\sigma}_{Se}^{\text{exp}} = 10^{-38}$ cm$^2$.

Another potentially observable effect could originate from the invisible Higgs-boson decay, $h \rightarrow SS$. Current LHC analyses report the upper bound [51]: $Br(h \rightarrow \text{inv}) < 0.28$, which for the RLDM scenario translates into...
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problem. We have found that for appropriate

isodoublet masses (e.g. in Fig. 3

GeV), the RLDM

particle

in such a scenario can successfully account for the

missing matter component of the Universe. In addition, we

have investigated whether a lower mass limit exists for the

heavy Higgs scalars, within the context of a viable RLDM

scenario. We have found that the masses of the heavy scalars

can be as low as TeV, which allows for their possible detection

at the LHC in the near future.

The PQ-symmetric scenario we have studied here generates

a viable RLDM at the one-loop level. However, one

can envisage other extensions of the SM, in which the

required small mass for the light DM could be produced at

two or higher loops. For instance, if the SM is extended by

two scalar triplets, a small DM mass can be generated

through their mixing at the two-loop level, in a fashion

similar to the Zee model. In this context, it would be

interesting to explore possible models where both the tiny

mass of the SM neutrinos and the small mass of the light

DM have a common radiative origin and study their

phenomenological implications.

V. CONCLUSIONS

One central problem of most electroweak scenarios that

require the existence of very light DM particles in the keV-
to-GeV mass range is the actual origin of this sub-GeV

scale. To address the origin of such a small scale, we have

presented a novel radiative mechanism that can naturally

generate a sub-GeV mass for a light singlet fermion

is charged under the PQ symmetry. Instead, the singlet fermion

is stable and can successfully play the role of

the DM.

In order to minimally realize such a radiative light dark

matter, we have considered a Peccei-Quinn symmetric two-

Higgs doublet model, which was extended with the addition

of a singlet fermion \( S \) and a pair of massive vectorlike SU(2)
isodoublets \( D_{1,2} \) that are not charged under the PQ symmetry. Instead, the singlet fermion \( S \) is charged under the PQ symmetry and so it has no bare mass at the tree level. However, upon soft breaking of the PQ symmetry, we have shown how the singlet fermion \( S \) receives a nonzero mass at the one-loop level. The so-generated radiative mass for the singlet fermion \( S \) lies naturally in the cosmologically allowed region of \( \sim 10 \text{ keV} - 1 \text{ GeV} \).

We have computed the relic abundance of the RLDM \( S \), for different plausible heavy mass scenarios. Specifically, for all scenarios we have been studying, we have assumed that the \( S \) particles were absent in the early universe, while the fermion isodoublets \( D_{1,2} \) stay out of equilibrium through the entire thermal history of the Universe, because their gauge-invariant mass \( m_D \) is taken to be well above the reheating temperature \( T_{RH} \). Then, we have found that the observationally required relic abundance for the RLDM \( S \) can be produced via decays and annihilations of Higgs-sector particles.

We have analyzed a heavy mass scenario where the PQ-breaking scale \( f_{PQ} \) can reach values \( \sim 10^9 \text{ GeV} \) as required by the Dine-Fischler-Sredniki-Zhitnitsky axion model to explain the strong CP problem. We have found that for appropriate isodoublet masses (e.g. in Fig. 3 \( m_D \sim 10^{-2} f_{PQ} \)), the RLDM particle \( S \) in such a scenario can successfully account for the missing matter component of the Universe. In addition, we have investigated whether a lower mass limit exists for the heavy Higgs scalars.

In summary, at least for the foreseeable future, the

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RADIATIVE LIGHT DARK MATTER