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Horizontal Dynamic Stiffness and Interaction Factors of Inclined Piles

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Abstract:

A Timoshenko beam-on-Pasternak-foundation model (T-P) model is developed to estimate the horizontal dynamic impedance and interaction factors for inclined piles subjected to a horizontal harmonic load. The inclined pile with a fixed pile-to-cap connection is modeled as a Timoshenko beams embedded in the homogeneous Pasternak foundation. The reactions of the soil on the pile tip are simulated by three springs in parallel with corresponding dashpots. The differential equations of normal and axial vibrations of the inclined pile are solved by means of the initial parameter method. The model and the theoretical derivation are validated through comparisons with results from other theoretical models for some large-diameter end-bearing piles and inclined piles. The effects of shear deformations of the soil and inclined piles, during the vibration, are highlighted by comparing the results between the T-P model and the Euler beam-on-dynamic-Winkler-foundation (E-W) model. It is indicated that the T-P Model is more accurate for analyzing the dynamic performance of the inclined piles with small slenderness ratios and large inclined angles. The effects of the inclined angle, slenderness ratio and distance-diameter ratio on the dynamic impedance and interaction factors are studied by numerical examples. It is demonstrated that the increased angle of the pile results in an increase of horizontal impedance and decreases of interaction factors.

Keywords:

Soil-structure interaction; Inclined pile; Pile impedance; Interaction factor; Timoshenko beam; Pasternak foundation.

Introduction

Inclined piles were once considered to be detrimental for their seismic behavior (Deng et al. 2007;

31 Poulos 2006) and even recommended being avoided by some codes (AFPS 1990; CEN 2004) due to
32 their poor performances in a series of earthquakes occurred in the 1990s. However, numerical
33 (Gerolymos et al. 2008; Turan et al. 2013; Sadek and Isam 2004) and experimental studies (Okawa et
34 al. 2002; Gotman 2013) showed the advantages of the inclined piles. The ultimate reasons of failures
35 may be due to the incorrect design and construction techniques rather than the pile inclination itself.
36 As inclined piles can transmit the applied horizontal loads partly through axial compression, they
37 provide larger horizontal stiffness and bearing capacity than conventional vertical piles. Therefore,
38 employing inclined piles in foundations for buildings, bridges and marginal wharfs is an efficient
39 way to resist horizontal loads such as seismic waves, vehicle impacts and ocean waves (Gazetas and
40 Mylonakis 1998; Padrón et al. 2010). Inclined angles commonly used in practice are between 5° - 15° ,
41 in addition to the less usual cases between 20° and 25° .

42 Padron et al. (2010, 2012, 2014) investigated dynamic impedances of both single inclined pile
43 and group inclined piles, as well as the kinematic interaction factors between adjacent inclined piles,
44 using a boundary element-finite element coupling model (BEM-FEM). In their study, the pile is
45 modeled as an elastic compressible Euler-Bernoulli beam. Their investigation contributed an
46 understanding of the effect of inclined piles on the dynamic impedances of deep foundations.
47 Recently, they further studied the response of the structure supported by an inclined pile foundation
48 (Medina et al. 2015). In addition to numerical methods, some simplified analytical models are also
49 applied in the analysis of soil-pile dynamic interaction. Makris and Gazetas (1992) presented a Euler
50 Beam-on-dynamic-Winkler-foundation (E-W) model to account for the impedance of a single
51 vertical pile and interaction factors between adjacent vertical piles, which can be extended to
52 evaluate the effect of dynamic pile groups using the superposition method (Kaynia and Kausel 1982).
53 Mylonakis and Gazetas (1998, 1999) used the E-W model to determine the dynamic interaction
54 factors for axial and lateral vibrations of vertical piles in layered soil by considering the presence of
55 receiver piles. Subsequently, the E-W model has received a widespread application due to low
56 computational complexity and reasonable agreement with the rigorous solutions (Liu et al. 2014;
57 Sica et al. 2011). Gerolymos and Gazetas (2006) developed a multi-Winkler model to analyse the
58 static and dynamic stiffness of rigid caisson foundations with small slenderness ratio. In recent years,
59 some researchers have managed to extend the E-W model to the dynamics of inclined piles.
60 Ghasemzadeh and Alibeikloo (2011) used the Euler beam-on-dynamic-Winkler-foundation (E-W)

model to analyse the interaction factor between two infinitely long inclined piles. As the limitation of the theoretical assumptions, the E-W model only valid for those piles with large slenderness ratio (Yokoyama, 1996; Wang et al., 2014a). Ghazavi et al. (2013) further considered the dynamic interaction factor of adjacent inclined piles under oblique harmonic loads.

However, the E-W model neglects the shear deformations of both soil medium and the pile, which is only valid for the piles with a large slenderness ratio (i.e. piles with large diameter and/or short length). Actually, piles with small slenderness ratios are commonly used on the firm stratum to support superstructures (Mylonakis 2001). Kampitsis et al. (2013) demonstrated the advantages of Timoshenko theory on the kinematic and inertial interaction between soil and column-pile with a small slenderness ratio. Recently, Wang et al. (2014a, 2014b) presented a Timoshenko beam-on-Pasternak-foundation (T-P) model to determine the dynamic interaction factors between vertical short piles with hinged-head connection in the multilayered soil medium. The parametric studies confirmed that the T-P model subjected to high frequency excitation provided the improved results for the dynamic interaction of piles with small slenderness ratio.

As an extension of previous works (Wang et al. 2014a, 2014b), the T-P model is further applied to analyzing the impedance of single inclined pile and dynamic interaction factors between adjacent inclined piles. The shear deformation and rotational inertia of inclined piles, as well as the shear deformation of soil, have been simultaneously considered to overcome the limitation of conventional E-W model (Ghasemzadeh and Aleikloo, 2011). Transformation between the local and global coordinate systems is used in the analysis as the inclined piles can transmit the horizontal loads not only through the bending deformation but also through the axial deformation. Therefore, the horizontal and vertical interaction factors for inclined piles are coupled. The effects of slenderness ratios, pile-soil modulus ratios, the inclined angles of single pile on adjacent piles have been studied in detail.

Model description

Fig. 1 shows a cylindrical elastic pile embedded in a homogeneous half-space with an inclined angle φ_1 . The head of the pile is fixed to the cap. To conveniently evaluate the dynamic impedance atop the head of the inclined pile, a horizontal harmonic excitation $He^{i\omega t}$ in the global coordinate system (x - z)

is decomposed into an axial component $Qe^{i\omega t}$ and a normal component $Pe^{i\omega t}$ in a local coordinate system ($x'-z'$). Here, H , Q and P are the load amplitudes, ω denotes the frequency of the external load, t represents the time and $i = \sqrt{-1}$. The T-P model is introduced to analyse the normal vibration of the inclined pile, which is also a further extension of the previous work (Wang et al. 2014a). The Timoshenko beam theory is used to simulate the transverse vibration of the pile in which analyse the effect of shear deformation and the rotational inertia are considered. The Pasternak foundation is used to represent the elastic half-space. The normal reaction of soil against the pile-shaft is simulated by a system of uniformly distributed linear springs and dashpots, which are connected through an incompressible shear layer. It is confirmed by Poulos and Davis (1980) that load distribution along the inclined pile with angle less than 30° is reasonably assumed to be similar to that along the vertical pile. Therefore, the analytical expressions of springs, dashpots and the shear layer for the vertical pile, which based on some simplified physical models calibrated with results from numerical methods and experimental dates (Fwa et al. 1996; Gerolymos and Gazetas 2006; Wang et al. 2014), are introduced in the present model as follows:

$$k_x \cong \begin{cases} 1.75(L/d)^{-0.13} E_s & \text{for } L/d < 10 \\ 1.2E_s & \text{for } L/d \geq 10 \end{cases}; \quad (1a)$$

$$c_x \cong 6a_0^{-1/4} \rho_s d V_s + 2\beta_s k_x / \omega; \quad (1b)$$

$$g_x \cong 0.5k_x. \quad (1c)$$

in which, E_s , G_s , ν_s , ρ_s , β_s and V_s are the Young's modulus, shear modulus, Poisson's ratio, mass density, damping and shear wave velocity of the soil medium, respectively; L and d are the length and sectional diameter of the inclined pile, respectively. $a_0 = \omega d / V_s$ is the dimensionless frequency.

As axial vibration of the pile does not induce the shear deformation and rotational inertia, the coefficients of springs and dashpots based on the Novak's plain strain model (Novak and Aboul-Ella 1978) are given as follows:

$$k_z \cong 2\pi a_0 G_s \frac{J_1(a_0)J_0(a_0) + Y_1(a_0)Y_0(a_0)}{J_0^2(a_0) + Y_0^2(a_0)}; \quad (2a)$$

$$c_z \cong G_s \frac{4}{[J_0^2(a_0) + Y_0^2(a_0)]\omega}. \quad (2b)$$

in which, J_0 and J_1 are the first kind of Bessel function of the zero-order and the first-order,

116 respectively; Y_0 and Y_1 are the second kind of Bessel function of the zero-order and the first-order,
 117 respectively.

118 It is well known that the mechanical properties at the pile head is unaffected by the boundary
 119 condition at the pile tip for the case with a large slenderness ratio L/d . However, different from the
 120 infinite long pile, the reaction of the soil on the pile tip should be considered when it has a small
 121 slenderness ratio ($L/d < 10$) (Gerolymos and Gazetas 2006). The analytical expressions for normal,
 122 axial and rotational springs in parallel with dashpots at the pile tip are suggested by Novak and Sacks
 123 (1973) as follows:

$$124 \quad \left[\mathbf{K}_{\text{tip}}^x \right] = \begin{bmatrix} (4.3 + i2.7a_0)G_{s,\text{tip}}(d/2) & 0 \\ 0 & (2.5 + i0.43a_0)G_{s,\text{tip}}(d/2)^3 \end{bmatrix}; \quad (3a)$$

$$125 \quad K_{\text{tip}}^z = (7.5 + i3.4)G_{s,\text{tip}}(d/2). \quad (3b)$$

126 Formulation

127 Normal vibration of an inclined pile

128 For simplifying the analysis, the displacement of the inclined pile subjected to horizontal harmonic
 129 excitation is decomposed into a normal component and an axial component in a local coordinate
 130 system. The horizontal impedance of the inclined pile can be finally obtained using the superposition
 131 method. As expected, the formulation in all steps are valid for vertical piles when $\varphi_1 = 0$.

132 By using the Hamilton principle and the Timoshenko beam theory, the governing differential
 133 equations for the translational and the rotational vibrations of a loaded pile are given by

$$134 \quad \rho_p A_p \frac{\partial^2 u_a(z', t)}{\partial t^2} = -\frac{\partial}{\partial z'} \left[\kappa G_p A_p \left(\theta_a(z', t) - \frac{\partial u_a(z', t)}{\partial z'} \right) \right] - \left(k_x u_a(z', t) - g_x \frac{\partial^2 u_a(z', t)}{\partial z'^2} + c_x \frac{\partial u_a(z', t)}{\partial t} \right) \quad (4)$$

$$135 \quad \rho_p I_p \frac{\partial^2 \theta_a(z', t)}{\partial t^2} = E_p I_p \frac{\partial^2 \theta_a(z', t)}{\partial z'^2} - \kappa G_p A_p \left(\theta_a(z', t) - \frac{\partial u_a(z', t)}{\partial z'} \right) \quad (5)$$

136 where ν_p , ρ_p , E_p and G_p are the Poisson's ratio, mass density, Young's modulus, shear modulus of the
 137 pile, respectively; A_p , I_p and $\kappa = 6(1 + \nu_p)/(7 + 6\nu_p)$ are the area of cross section, inertia moment and
 138 shear coefficient of the pile, respectively. u_a and θ_a , the normal displacement and bending rotation of
 139 the pile, should experience harmonic movements, $u_a(z', t) = U_a(z')e^{i\omega t}$ and $\theta_a(z', t) = \Theta_a(z')e^{i\omega t}$ as
 140 the system is subjected to a harmonic action. Applying the differentiation chain rule on Eq. (4) and

Eq. (5), the differential equations of normal vibration and the flexural rotational vibration can be expressed as

$$\frac{d^4 U_a}{dz'^4} + \frac{E_p I_p K_p - J_p g_x + S_p (J_p + g_x)}{E_p I_p (J_p + g_x)} \frac{d^2 U_a}{dz'^2} - \frac{K_p (J_p - S_p)}{E_p I_p (J_p + g_x)} U_a = 0 \quad (6)$$

$$\Theta_a = \frac{E_p I_p (J_p + g_x)}{J_p (J_p - S_p)} \frac{d^3 U_a}{dz'^3} + \frac{E_p I_p K_p + J_p^2}{J_p (J_p - S_p)} \frac{d U_a}{dz'} \quad (7)$$

where J_p , S_p and K_p are defined as $J_p = \kappa G_p A_p$, $S_p = \rho_p I_p \omega^2$, $K_p = \rho_p A_p \omega^2 - k_x - i \omega c_x$ for brevity. It is noted from Eqs. (6-7) that 1) when $1/J_p \rightarrow 0$ and $S_p \rightarrow 0$, it simplifies to the equation of an Euler pile on elastic soil; 2) when $g_x = 0$, it reduces to the Winkler model. The general solution of Eq. (6) is

$$U_a(z') = A_1 \cosh \alpha z' + B_1 \sinh \alpha z' + C_1 \cos \beta z' + D_1 \sin \beta z' \quad (8)$$

$$\text{where } \alpha = \sqrt{\sqrt{\left[\frac{E_p I_p K_p - J_p g_x + S_p (J_p + g_x)}{2 E_p I_p (J_p + g_x)} \right]^2 + \frac{K_p (J_p - S_p)}{E_p I_p (J_p + g_x)}} - \frac{E_p I_p K_p - J_p g_x + S_p (J_p + g_x)}{2 E_p I_p (J_p + g_x)}}$$

$$\text{and } \beta = \sqrt{\frac{E_p I_p K_p - J_p g_x + S_p (J_p + g_x)}{2 E_p I_p (J_p + g_x)}} + \sqrt{\left[\frac{E_p I_p K_p - J_p g_x + S_p (J_p + g_x)}{2 E_p I_p (J_p + g_x)} \right]^2 + \frac{K_p (J_p - S_p)}{E_p I_p (J_p + g_x)}} \cdot A_1,$$

B_1 , C_1 , D_1 are unknown initial coefficients, which can be determined based on the known boundary conditions. For a linear elastic Timoshenko pile, the bending moment $M_a(z')$ and shear force $Q_a(z')$ of the pile are related to the displacement $U_a(z')$ and rotation $\Theta_a(z')$ as follows:

$$Q_a(z') = J_p \left(\frac{d U_a(z')}{dz'} - \Theta_a(z') \right) \quad (9)$$

$$M_a(z') = E_p I_p \frac{d \Theta_a(z')}{dz'}. \quad (10)$$

Using the initial parameter method (Wang et al. 2014b) with consideration of Eqs. (7-10), the relationship of both deflections and internal forces between the pile head and the pile tip can be expressed by a transfer matrix $[TA]$

$$\{U_a(L), \Theta_a(L), Q_a(L), M_a(L)\}^T = [TA] \{U_a(0), \Theta_a(0), Q_a(0), M_a(0)\}^T \quad (11)$$

in which, $[TA] = [ta(L)][ta(0)]^{-1}$. The algebraic expression for each element in $[ta(z')]$ is given in

162 Appendix A. It is convenient to divide $[TA]$ into four 2×2 sub matrices as $\begin{bmatrix} [TA_{11}] & [TA_{12}] \\ [TA_{21}] & [TA_{22}] \end{bmatrix}$ for the

163 following formula derivation. Substituting the tip boundary condition $\begin{Bmatrix} Q_a(L) \\ M_a(L) \end{Bmatrix} = [K_{\text{tip}}^x] \begin{Bmatrix} U_a(L) \\ \Theta_a(L) \end{Bmatrix}$

164 into Eq. (11), the force-displacement relationship in the normal direction at the pile head can be
165 obtained

$$166 \quad \begin{Bmatrix} Q_a(0) \\ M_a(0) \end{Bmatrix} = [\mathfrak{R}_{x'}] \begin{Bmatrix} U_a(0) \\ \Theta_a(0) \end{Bmatrix} \quad (12)$$

167 where the normal impedance matrix $[\mathfrak{R}_{x'}] = \left([K_{\text{tip}}^x][TA_{12}] - [TA_{22}] \right)^{-1} \left([TA_{21}] - [K_{\text{tip}}^x][TA_{11}] \right)$.

168 During the axial steady-state harmonic vibration, the axial displacement of the pile can be
169 characterized by $w_a = W_a e^{i\omega t}$. As the axial vibration is not involved with the shear deformation, the
170 dynamic equilibrium yields the following governing equation

$$171 \quad E_p A_p \frac{d^2 W_a}{dz'^2} - (k_z + i\omega c_z - \rho_p A_p \omega^2) W_a = 0 \quad (13)$$

172 The general solution of above equation is

$$173 \quad W_a(z') = A_2 \cosh(\eta z') + B_2 \sinh(\eta z') \quad (14)$$

$$174 \quad \text{with } \eta = \sqrt{\frac{k_z + i\omega c_z - \rho_p A_p \omega^2}{E_p A_p}}.$$

175 The axial force in the pile can be obtained from the differential relationship

$$176 \quad N_a(z') = E_p A_p \frac{dW_a}{dz'}.$$

177 The mechanical property between the pile head and tip can be expressed by the
transfer matrix $[TB]$ as follows

$$178 \quad \begin{Bmatrix} W_a(L) \\ N_a(L) \end{Bmatrix} = [TB] \begin{Bmatrix} W_a(0) \\ N_a(0) \end{Bmatrix} \quad (15)$$

179 in which, $[TB] = [tb(L)][tb(0)]^{-1}$. The algebraic expression for $[tb(z')]$ is given in Appendix B.

180 Substituting the tip boundary condition $N_a(L) = K_{\text{tip}}^z W_a(L)$ into Eq. (15), the force-displacement

181 relationship in the axial direction at the pile head can be obtained

$$182 \quad N_a(0) = \mathfrak{R}_z W_a(0) \quad (16)$$

183 where the axial impedance $\Re_{z'} = \frac{TB_{2,1} - K_{\text{tip}}^z TB_{1,1}}{K_{\text{tip}}^z TB_{1,2} - TB_{2,2}}$. Here, $TB_{i,j}$ represents the element in the i -th row
 184 and j -th column of matrix $[\mathbf{TB}]$.

185 Considering the transformation of the coordinate system between the local and global axes of
 186 the inclined pile, the horizontal impedance of the inclined pile $\Re_h$ at the pile head is given by :

$$187 \quad \Re_h = \frac{\Re_{z'} \Re_{x'1,1}}{\Re_{z'} \cos^2(\varphi_1) + \Re_{x'1,1} \sin^2(\varphi_1)} \quad (17)$$

188 where $\Re_{x'1,1}$ is the element in the first row and first column of the matrix $[\Re_{x'}]$. $\Re_h$ is a complex
 189 value. The real part K_h represents the dynamic stiffness, while the imaginary part C_h represents the
 190 damping ratio.

191 ***Dynamic interaction of adjacent inclined piles***

192 In addition to the loads transmitted from the pile cap, each inclined pile in a group experiences the
 193 action from both the normal and axial vibrations of surrounding piles. To better understand the
 194 dynamic interaction between adjacent inclined piles, a pair of inclined piles with inclined angles φ_1
 195 and φ_2 to the vertical global axis respectively is considered as shown in Fig. 2. The T-P model is
 196 developed for evaluating the normal cross interaction between two piles. The normal-normal
 197 interaction factor λ_{nn} and the axial-normal interaction factor λ_{an} are defined as the ratios of the
 198 normal displacement at the head of the receiver pile, which is caused by the horizontal vibration of
 199 the source pile, to the normal and axial displacements at the head of the source pile respectively. The
 200 axial cross dynamic interaction, in which the shear effect of both soil and piles can be neglected
 201 (Ghasemzadeh and Alibeikloo 2011), is not studied in this paper as it has been properly solved by the
 202 classical E-W model. The derivations of λ_{nn} and λ_{an} are given in following sections.

203 The kinetic source pile could excite the surrounding soil and transfer the waves to receiver piles.
 204 Based on the coordinate transformation, the normal displacement of the soil around the receiver pile,
 205 induced by the normal vibration of the source pile can be obtained by the attenuation factor $f_x(s, \phi)$:

$$206 \quad U_s^{(1)} = U_a f_x(s, \phi) \cos(\varphi_1 + \varphi_2) \quad (18)$$

207 where ϕ is the angle between the center-line direction of two piles and the direction of the applied
 208 horizontal force, as shown in Fig. 2b. s is the center distance between two piles.

209 According to the model developed by Dobry and Gazetas (1988), the attenuation factor

210 $f_x(s, \phi)$ for an arbitrary ϕ could be approximately evaluated by the values at $\phi=0$ and $\phi=\pi/2$ as
 211 follows:

$$212 \quad f_x(s, \phi) = f_x(s, 0) \cos^2 \phi + f_x(s, \frac{\pi}{2}) \sin^2 \phi \quad (19)$$

$$213 \quad \text{in which, } f_x(s, 0) = \sqrt{\frac{d}{2s}} \exp\left[\frac{-\omega(\beta_s + i)(s - d/2)}{V_{La}}\right] \text{ and } f_x(s, \frac{\pi}{2}) = \sqrt{\frac{d}{2s}} \exp\left[\frac{-\omega(\beta_s + i)(s - d/2)}{V_s}\right].$$

214 The vertical displacement field around the receiver pile also can be approximately evaluated by
 215 the factor $f_z(s)$ as follows (Dobry and Gazetas 1988)

$$216 \quad f_z(s) = \sqrt{\frac{d}{2s}} \exp\left[\frac{-\omega s(\beta_s + i)}{V_s}\right] \quad (20)$$

217 Similarly, the normal displacement of the soil around the receiver pile induced by the axial
 218 vibration of the source pile can be obtained:

$$219 \quad U_s^{(2)} = W_a f_z(s) \sin(\phi_1 + \phi_2) \quad (21)$$

220 Normal-normal interaction factor

221 Considering the dynamic interaction between the soil and the receiver pile, the normal dynamic
 222 equilibrium equation of the receiver pile, which is induced by the normal vibration of the source pile,
 223 is expressed as

$$224 \quad \frac{d^4 U_b^{(1)}}{dz^4} + \frac{E_p I_p K_p - J_p g_x + S_p (J_p + g_x)}{E_p I_p (J_p + g_x)} \frac{d^2 U_b^{(1)}}{dz^2} - \frac{K_p (J_p - S_p)}{E_p I_p (J_p + g_x)} U_b^{(1)} = f_{x1} U_s^{(1)} + f_{x2} \frac{d^2 U_s^{(1)}}{dz^2} + f_{x3} \frac{d^4 U_s^{(1)}}{dz^4} \quad (22)$$

$$225 \quad \text{in which, } f_{x1} = \frac{(J_p - S_p)(k_x + i\omega c_x)}{E_p I_p (J_p + g_x)}; \quad f_{x2} = \frac{E_p I_p (k_x + i\omega c_x) - S_p g_x}{E_p I_p (J_p + g_x)}; \quad f_{x3} = \frac{g_x}{J_p + g_p}. \quad U_b^{(1)} \text{ is the}$$

226 normal displacement of the receiver pile induced by the normal vibration of the source pile.

227 The complete solution for Eq. (22) is

$$228 \quad U_b^{(1)}(z') = A_3 \cosh \alpha z' + B_3 \sinh \alpha z' + C_3 \cos \beta z' + D_3 \sin \beta z' + \\ z F_a (A_1 \sinh \alpha z' + B_1 \cosh \alpha z') + z F_b (-C_1 \sin \beta z' + D_1 \cos \beta z') \quad (23)$$

$$229 \quad \text{where } F_a = \frac{f_{x1} + f_{x2} \alpha^2 + f_{x3} \alpha^4}{2\alpha(2\alpha^2 + \delta^2)} f_x(s, \phi) \cos(\phi_1 + \phi_2), \quad F_b = \frac{f_{x1} - f_{x2} \beta^2 + f_{x3} \beta^4}{2\beta(2\beta^2 - \delta^2)} f_x(s, \phi) \cos(\phi_1 + \phi_2).$$

230 The relationship between deflections and internal forces at the tip of the receiver pile can be
 231 written in a matrix form:

$$\begin{Bmatrix} U_b^{(1)}(L) \\ \Theta_b^{(1)}(L) \\ Q_b^{(1)}(L) \\ M_b^{(1)}(L) \end{Bmatrix} = [TA] \begin{Bmatrix} U_b^{(1)}(0) \\ \Theta_b^{(1)}(0) \\ Q_b^{(1)}(0) \\ M_b^{(1)}(0) \end{Bmatrix} + [TC] \begin{Bmatrix} U_a(0) \\ \Theta_a(0) \\ Q_a(0) \\ M_a(0) \end{Bmatrix} \quad (24)$$

where $[TC] = -[ta(L)][ta(0)]^{-1}[tc(0)][ta(0)]^{-1} + [tc(L)][ta(0)]^{-1}$. The algebraic expression for $[tc(z')]$ is given in Appendix C. By substituting the boundary conditions at the tip of the receiver

pile $\begin{Bmatrix} Q_b^{(1)}(L) \\ M_b^{(1)}(L) \end{Bmatrix} = [K_{\text{tip}}^x] \begin{Bmatrix} U_b^{(1)}(L) \\ \Theta_b^{(1)}(L) \end{Bmatrix}$ into Eq. (23), we have

$$[T1] \begin{Bmatrix} U_b^{(1)}(0) \\ \Theta_b^{(1)}(0) \end{Bmatrix} + [T2] \begin{Bmatrix} Q_b^{(1)}(0) \\ M_b^{(1)}(0) \end{Bmatrix} + [T3] \begin{Bmatrix} U_b^{(1)}(0) \\ \Theta_b^{(1)}(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (25)$$

in which, $[T1] = [K_{\text{tip}}^x][TA_{11}] - [TA_{21}]$; $[T2] = [K_{\text{tip}}^x][TA_{12}] - [TA_{22}]$;

$[T3] = [K_{\text{tip}}^x][TC_{11}] - [TC_{21}] + ([K_{\text{tip}}^x][TC_{12}] - [TC_{22}])[R_{x'}]$.

For the case of a fixed pile-to-cap connection, the boundary conditions at the head of the receiver pile are satisfied as $Q_b^{(1)}(0) = 0$ and $\Theta_b^{(1)}(0) = 0$. Submitting Eq. (12) and above boundary conditions into Eq. (25), the normal-normal interaction factor λ_{nn} is given by

$$\lambda_{nn} = \frac{U_b^{(1)}(0)}{U_a(0)} = \frac{T3_{2,1}T2_{1,2} - T3_{1,1}T2_{2,2}}{T1_{1,1}T2_{2,2} - T1_{2,1}T2_{1,2}} \quad (26)$$

Axial-normal interaction factor

Considering the dynamic interaction between the soil and the receiver pile, the normal dynamic equilibrium equation of the receiver pile, which is induced by the axial vibration of the source pile, is expressed as:

$$\frac{d^4 U_b^{(2)}}{dz'^4} + \frac{E_p I_p K_p - J_p g_x + S_p (J_p + g_x)}{E_p I_p (J_p + g_x)} \frac{d^2 U_b^{(2)}}{dz'^2} - \frac{K_p (J_p - S_p)}{E_p I_p (J_p + g_x)} U_b^{(2)} = f_{z1} U_s^{(2)} + f_{z2} \frac{d^2 U_s^{(2)}}{dz'^2} + f_{z3} \frac{d^4 U_s^{(2)}}{dz'^4} \quad (27)$$

where $f_{z1} = \frac{(J_p - S_p)(k_x + i\omega c_x)}{E_p I_p (J_p + g_x)}$; $f_{z2} = \frac{E_p I_p (k_x + i\omega c_x) - S_p g_x}{E_p I_p (J_p + g_x)}$; $f_{z3} = \frac{g_x}{J_p + g_x}$. $U_b^{(2)}$ is the

normal displacement of the receiver pile induced by the axial vibration of the source pile.

The solution of equation (27) is:

$$U_b^{(2)}(z') = A_4 \cosh \alpha z' + B_4 \sinh \alpha z' + C_4 \cos \beta z' + D_4 \sin \beta z' + F_c (A_2 \cosh \eta z' + B_2 \sinh \eta z') \quad (28)$$

$$\text{in which, } F_c = \frac{f_{z1} + f_{z2}\eta^2 + f_{z3}\eta^4}{\eta^4 + \delta^2\eta^2 - \lambda^4} f_z(s) \sin(\varphi_1 + \varphi_2).$$

The relationship of deflections and internal forces at the tip of the receiver pile can be written in

the following matrix form

$$\begin{Bmatrix} U_b^{(2)}(L) \\ \Theta_b^{(2)}(L) \\ Q_b^{(2)}(L) \\ M_b^{(2)}(L) \end{Bmatrix} = [TA] \begin{Bmatrix} U_b^{(2)}(0) \\ \Theta_b^{(2)}(0) \\ Q_b^{(2)}(0) \\ M_b^{(2)}(0) \end{Bmatrix} + [TD] \begin{Bmatrix} W_a(0) \\ N_a(0) \end{Bmatrix} \quad (29)$$

where $[TD] = [td(L)][tb(0)]^{-1} - [ta(L)][ta(0)]^{-1}[td(0)][tb(0)]^{-1}$. The algebraic expression for

$[td(z')]$ is given in Appendix D. By applying the boundary conditions at the receiver pile tip

$$\begin{Bmatrix} Q_b^{(2)}(L) \\ M_b^{(2)}(L) \end{Bmatrix} = [K_{\text{tip}}^x] \begin{Bmatrix} U_b^{(2)}(L) \\ \Theta_b^{(2)}(L) \end{Bmatrix} \text{ to Eq. (29), We have}$$

$$[T1] \begin{Bmatrix} U_b^{(2)}(0) \\ \Theta_b^{(2)}(0) \end{Bmatrix} + [T2] \begin{Bmatrix} Q_b^{(2)}(0) \\ M_b^{(2)}(0) \end{Bmatrix} + [T4] \begin{Bmatrix} W_a(0) \\ N_a(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (30)$$

$$\text{in which, } [T4] = [K_{\text{tip}}^x] \begin{bmatrix} TD_{1,1} & TD_{1,2} \\ TD_{2,1} & TD_{2,2} \end{bmatrix} - \begin{bmatrix} TD_{3,1} & TD_{3,2} \\ TD_{4,1} & TD_{4,2} \end{bmatrix}.$$

Substituting Eq. (16) and the boundary conditions at the head of the receiver pile, ($Q_b^{(2)}(0) = 0$

and $\Theta_b^{(2)}(0) = 0$), into Eq. (30), the axial-normal interaction factor λ_{an} can be obtained as:

$$\lambda_{an} = \frac{U_b^{(2)}(0)}{W_a(0)} = \frac{(T4_{2,1} + T4_{2,2}\Re_{z'})T2_{1,2} - (T4_{1,1} + T4_{1,2}\Re_{z'})T2_{2,2}}{T1_{1,1}T2_{2,2} - T1_{2,1}T2_{1,2}} \quad (31)$$

Comparison with Numerical models

Verification of T-P model for a single inclined pile

To verify the effectiveness of the present T-P model, the static stiffness and the horizontal impedance of the inclined pile are compared with those obtained by numerical models, as shown in

268 Figs. 3 and 4. The results given by Giannakou et al. (2006) are obtained by developing a simplified
269 equation based on the 3-D FEM model. The results given by Padron et al. (2010) are obtained by
270 using the FEM-BEM model. It can be seen from Figs. 3 and 4 that the analytical results obtained by
271 the present T-P model are in agreement with those obtained by BEM-FEM model. The maximum
272 difference is less than 15%. In addition, with the increasing of the inclined angle the static stiffness
273 obtained from the estimating equations (Giannakou et al., 2006) gradually deviates from those
274 obtained by the present model and the FEM-BEM model (Padron et al., 2010) as seen in Fig. 3.

275 ***Verification of T-P model for adjacent inclined piles***

276 The normal-normal interaction factor and axial-normal interaction factor obtained from the T-P
277 model are compared with those from Ghasemzadeh and Alibeikloo (2011) for different inclined
278 angles ($\varphi = 10^\circ, 20^\circ, 30^\circ$) in Figs. 5-6. Moreover, the comparisons for different distance-diameter
279 ratios ($s/d=3, 5, 10$) are given in Figs. 7-8. It can be seen that normal-normal interaction factors from
280 the present T-P model is near to the results of W-E model from Ghasemzadeh and Alibeikloo (2011).
281 A minor difference between them can be due to the shear effect of the system. However, the
282 axial-normal interaction factors have a good agreement with those from the Ghasemzadeh and
283 Alibeikloo's model.

284 Ghazavi et al. (2013) based on the elasto-dynamic theory and Saitoh et al. (2016) based on the
285 boundary elements-finite elements coupling method studied the horizontal interaction factor of a pair
286 of inclined piles with $L/d=15$, respectively. The present results obtained by the T-P model are
287 compared with those obtained by Ghazavi et al. (2013) for $s/d=5$, as well as with those obtained by
288 Saitoh et al. (2016) for $s/d=5\sqrt{2}$, as shown in Fig. 9. It can be seen from Fig. 9 that the present
289 results agree with the Ghazavi et al.'s solutions well. However, there is an acceptable difference

290 between the present solutions and the rigorous FE-BE solutions from Saitoh et al (2016) because the
291 present model utilizes an attenuation function with approximately-assumed wave propagation given
292 by Dobry and Gazetas (1988) to simulate the pile-to-pile interaction effect. If one wants to obtain a
293 better accuracy of the interaction factors, the present attenuation function used for inclined piles
294 should be further improved.

295 **Numerical examples and discussions**

296 ***Shear effect of the inclined pile on the horizontal impedance***

297 Based on an improved plane-strain foundation model and the Euler beam theory, Mylonakis
298 (2001) studied the stiffness at the head of a vertical large-diameter end-bearing pile for different
299 pile-soil modulus ratios E_p/E_s and different slenderness ratios L/d . A comparison of the results
300 between the present model and Mylonakis' model is shown in Table 1. The Young's modulus in $[K_{tip}]$
301 is given a large enough value $E_{tip}=10E_p$ in the calculation to simulate the rigid base considered by
302 Mylonakis (2001). As show in Table 1, the maximum relative error between the results from the two
303 models is up to 30% for $L/d=5$ while it drops below 3.0% for $L/d=10$, due to the salient shear effect
304 for the piles with small slenderness ratios. This example is also used to check the influence of the
305 inclined angle on the horizontal stiffness. It can be seen from Table 1 that the horizontal stiffness at
306 the pile head increases with the increase of the inclined angle. It is confirmed that the use of inclined
307 piles is advantageous in supporting horizontal loadings.

308 Moreover, T-P model can be reduced to an Euler beam-on-Pasternak-foundation model (E-P)
309 model when $1/J_p \rightarrow 0$ and $S_p \rightarrow 0$. Fig. 10 shows the stiffness variation with respect to slenderness
310 ratio of the T-P model and the E-P model for $E_p/E_s=100, 300$ and 1000 , respectively. It can be seen
311 from Fig. 10 that the results from the E-P model approach to those from the T-P model as the

312 slenderness ratio L/d increases. It can be seen from Fig. 10 that the results from T-P model are
313 smaller than those from E-P model, especially for low slenderness ratios because the E-P model
314 ignores the effects of shear deformation and rotational inertia of piles.

315 ***Shear effect of the soil on horizontal impedance***

316 Fig. 11 and Fig. 12 show the comparison of horizontal impedances obtained from the T-P model
317 and the E-W model for different pile-soil elastic modulus ratios E_p/E_s and different inclined angles φ .
318 It can be seen from Fig.11 that the results from T-P model are larger than those from the E-W model,
319 especially for small pile-soil elastic modulus ratios. As a large slenderness ratio $L/d=15$ is considered
320 in the calculation, this relative difference between the T-P model and the E-W model in Fig. 11 is
321 mainly caused by the shear effect of soil. It can be seen from Fig. 12 that the dynamic stiffness of an
322 inclined pile obtained from T-P model is also higher than that from the E-W model while the two
323 models provide similar damping. In addition, it is indicated from Fig. 12 that the shear deformation
324 of soil has a greater impact on piles with larger inclined angles by comparing the relative differences
325 between two models for the three inclined angles ($\varphi=10^\circ, 20^\circ, 30^\circ$).

326 ***Influence of the shear effects on interaction factors***

327 Now, we study a soil medium with average shear wave velocity $V_s=200m/s$ and the piles with
328 diameter $d=1m$. A typical seismic excitation with dominant period $T=0.15s$ is considered in the
329 following analysis. In such a case, $a_0=0.2$. Fig.13 and Fig.14 show the variations of normal-normal
330 and axial-normal interaction factors with the slenderness ratio for different models. It can be seen
331 from Fig. 13 that the normal-normal interaction factor obtained from the Timoshenko pile model is
332 lower than those obtained from the Euler pile model for a small slenderness ratio. However, the
333 difference decreases with the increase of the slenderness ratio. Fig. 14 indicates that the shear effect

of the inclined pile has almost no influence on axial-normal interaction factors. This is because that the vibration of the source pile does not induce the shear deformation in the axial direction. Moreover, with the increase of the slenderness ratio, the interaction factors approach to constant values which are close to the results obtained by Ghasemzadeh and Alibeikloo (2011), as seen from Fig. 13 and Fig. 14.

Influence of the inclined angle on interaction factors

The normal-normal interaction factor λ_{nn} and axial-normal interaction factor λ_{an} , as functions of inclined angles for a small slenderness ratios $L/d=5$ and a large slenderness ratio $L/d=25$, are presented in Fig. 15 and Fig. 16, respectively. It is assumed that inclined angles for the source pile and receiver pile are the same ($\varphi_1=\varphi_2$). It can be seen from Fig. 15 that λ_{nn} decreases with the increase of the inclined angle. In contrast, λ_{an} increases with the increase of inclined angle as shown in Fig. 16. Moreover, comparing the slopes of two curves in Fig. 15 (or in Fig. 16) shows that the interaction factor of inclined pile with smaller slenderness ratio is more susceptible to the inclined angle. As a result, choosing an optimum inclined angle in the pile group needs further considerations when the slenderness ratio of inclined piles is small.

Conclusion

Closed-form expressions of dynamic impedance and interaction factors for inclined piles embedded in homogeneous elastic half-space are developed based on the T-P model. The normal reaction of the soil against the pile shaft is simulated by the Pasternak's foundation modulus with springs, dashpots and the shear layer. The reaction of the soil against the pile tip is simulated by three springs in parallel with dashpots. The vibration of the inclined pile is analyzed by the Timoshenko beam theory. The correctness of the derivation is verified by some comparative studies. The effects

356 of shear deformations on both soil and inclined piles are highlighted by comparing the results from
357 the T-P model and the E-W model. The influence of the inclined angle on the horizontal impedance
358 and interaction factors are presented by numerical parametric studies. The following main aspects
359 can be emphasized:

- 360 • The relative difference of the horizontal dynamic stiffness between the T-P model and E-W
361 model increases with the increase of the inclined angle and the decrease of the slenderness ratio
362 (L/d). Therefore, it is important to take the shear effect of both soil and piles into account for
363 those piles with large inclined angle and small slenderness ratio.
- 364 • The inclined angle has a significant influence on both horizontal impedance for a single pile and
365 interaction factors for adjacent piles. It is demonstrated that increasing the inclined angle results
366 in increases of horizontal impedance and axial-normal interaction factor, while results in a
367 decrease of normal-normal interaction factor. Therefore an optimum inclined angle for the
368 inclined piles in a group needs to be studied.

369 **Appendix A**

370 The algebraic expression for $[ta(z')]$ is given as follows:

$$371 \quad [ta(z')] = \{a_1, a_2, a_3, a_4\}^T \quad (32)$$

372 in which,

$$373 \quad \{a_1\} = \{\cosh(\alpha z') \quad \sinh(\alpha z') \quad \cos(\beta z') \quad \sin(\beta z')\} \quad (33)$$

$$374 \quad \{a_2\} = \begin{Bmatrix} (\Phi\alpha + \Upsilon\alpha^3)\sinh(\alpha z') \\ (\Phi\alpha + \Upsilon\alpha^3)\cosh(\alpha z') \\ (-\Phi\beta + \Upsilon\beta^3)\sin(\beta z') \\ (\Phi\beta - \Upsilon\beta^3)\cos(\beta z') \end{Bmatrix}^T \quad (34)$$

$$375 \quad \{a_3\} = \begin{Bmatrix} (\Psi\alpha + J_p\Upsilon\alpha^3)\sinh(\alpha z') \\ (\Psi\alpha + J_p\Upsilon\alpha^3)\cosh(\alpha z') \\ (-\Psi\beta + J_p\Upsilon\beta^3)\sin(\beta z') \\ (\Psi\beta - J_p\Upsilon\beta^3)\cos(\beta z') \end{Bmatrix}^T \quad (35)$$

$$376 \quad \{a_4\} = \begin{Bmatrix} -(\Phi\alpha^2 + \Upsilon\alpha^4)E_p I_p \cosh(\alpha z') \\ -(\Phi\alpha^2 + \Upsilon\alpha^4)E_p I_p \sinh(\alpha z') \\ (\Phi\beta^2 - \Upsilon\beta^4)E_p I_p \cos(\beta z') \\ (\Phi\beta^2 - \Upsilon\beta^4)E_p I_p \sin(\beta z') \end{Bmatrix}^T \quad (36)$$

$$377 \quad \text{with } \Phi = \frac{J_p^2 + K_p E_p I_p}{J_p (J_p - S_p)}, \quad \Psi = \frac{K_p E_p I_p + J_p S_p}{J_p - W_p}, \quad \Upsilon = \frac{E_p I_p (J_p + g_x)}{J_p (J_p - S_p)}.$$

378

379 **Appendix B**

380 The algebraic expression for $[tb(z')]$ is given as follows:

$$381 \quad [tb(z')] = \begin{bmatrix} \cosh(\eta z') & \sinh(\eta z') \\ \eta E_p A_p \sinh(\eta z') & \eta E_p A_p \cosh(\eta z') \end{bmatrix} \quad (37)$$

382

383 **Appendix C**

384 The algebraic expression for $[tc(z')]$ is given as follows:

$$[tc(z')] = \{c_1, c_2, c_3, c_4\}^T \quad (38)$$

in which,

$$\{c_1\} = \{z' F_a \sinh(\alpha z') \quad z' F_a \cosh(\alpha z') \quad -z' F_b \sin(\beta z') \quad z' F_b \cos(\beta z')\} \quad (39)$$

$$\{c_2\} = \begin{Bmatrix} F_a \left[(\Phi \alpha + \Upsilon \alpha^3) z' \cosh(\alpha z') + (\Phi + 3\Upsilon \alpha^2) \sinh(\alpha z') \right] \\ F_a \left[(\Phi + 3\Upsilon \alpha^2) \cosh(\alpha z') + (\Phi \alpha + \Upsilon \alpha^3) z' \sinh(\alpha z') \right] \\ -F_b \left[(\Phi \beta - \Upsilon \beta^3) z' \cos(\beta z') + (\Phi - 3\Upsilon \beta^2) \sin(\beta z') \right] \\ F_b \left[(\Phi - 3\Upsilon \beta^2) \cos(\beta z') - (\Phi \beta - \Upsilon \beta^3) z' \sin(\beta z') \right] \end{Bmatrix}^T \quad (40)$$

$$\{c_3\} = \begin{Bmatrix} F_a \left\{ \alpha (\Psi + J_p \Upsilon \alpha^2) z' \cosh(\alpha z') + (\Psi + 3J_p \Upsilon \alpha^2) \sinh(\alpha z') \right\} \\ F_a \left\{ (\Psi + 3J_p \Upsilon \alpha^2) \cosh(\alpha z') + \alpha (\Psi + J_p \Upsilon \alpha^2) z' \sinh(\alpha z') \right\} \\ -F_b \left\{ \beta (\Psi - J_p \Upsilon \beta^2) z' \cos(\beta z') + (\Psi - 3J_p \Upsilon \beta^2) \sin(\beta z') \right\} \\ F_b \left\{ (\Psi - 3J_p \Upsilon \beta^2) \cos(\beta z') - \beta (\Psi - J_p \Upsilon \beta^2) z' \sin(\beta z') \right\} \end{Bmatrix}^T \quad (41)$$

$$\{c_4\} = \begin{Bmatrix} -F_a E_p I_p \left[2\alpha (\Phi + 2\alpha^2 \Upsilon) \cosh(\alpha z') + \alpha^2 (\Phi + \alpha^2 \Upsilon) z' \sinh(\alpha z') \right] \\ -F_a E_p I_p \left[\alpha^2 (\Phi + \alpha^2 \Upsilon) z' \cosh(\alpha z') + 2\alpha (\Phi + 2\alpha^2 \Upsilon) \sinh(\alpha z') \right] \\ F_b E_p I_p \left[2\beta (\Phi - 2\beta^2 \Upsilon) \cos(\beta z') - \beta^2 (\Phi - \beta^2 \Upsilon) z' \sin(\beta z') \right] \\ F_b E_p I_p \left[\beta^2 (\Phi - \beta^2 \Upsilon) z' \cos(\beta z') + 2\beta (\Phi - 2\beta^2 \Upsilon) \sin(\beta z') \right] \end{Bmatrix}^T \quad (42)$$

391

392 Appendix D

393 The algebraic expression for $[td(z')]$ is given as follows:

$$[td(z')] = \begin{bmatrix} F_c \cosh(\eta z') & F_c \sinh(\eta z') \\ F_c (\Phi \eta + \Upsilon \eta^3) \sinh(\eta z') & F_c (\Phi \eta + \Upsilon \eta^3) \cosh(\eta z') \\ F_c (\Psi \eta + J_p \Upsilon \eta^3) \sinh(\eta z') & F_c (\Psi \eta + J_p \Upsilon \eta^3) \cosh(\eta z') \\ -F_c (\Phi \eta^2 + \Upsilon \eta^4) E_p I_p \cosh(\eta z') & -F_c (\Phi \eta^2 + \Upsilon \eta^4) E_p I_p \sinh(\eta z') \end{bmatrix} \quad (43)$$

395

396

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Notation list

The following symbols are used in this paper:

A_p = cross-section area of the pile;

a_0 = dimensionless frequency of the excitation;

C_h = horizontal damping ratio at the pile head;

c_x, c_z = normal and axial damping along the pile shaft;

d = sectional diameter of the inclined pile;

E_p, E_s = Young's modulus of the pile and the soil along the pile shaft, respectively;

$G_s, G_{s,tip}$ = Shear modulus of the soil along the pile shaft and under the pile tip, respectively;

g_x = shear stiffness of the soil along the pile shaft;

I_p = inertia moment of the pile;

K_h = horizontal dynamic stiffness at the pile head;

K_{tip}^z = axial base impedance of the pile;

$[K_{tip}^x]$ = normal base impedance of the pile;

k_x, k_z = normal and axial stiffness along the pile shaft;

L = length of the inclined pile-shaft;

$M_a, M_b^{(j)}$ = bending moment of a source pile; rotation of a receiver pile induced by the vibration of the source pile in the j direction;

N_a = axial force of a source pile;

$Q_a, Q_b^{(j)}$ = shear force of a source pile; rotation of a receiver pile induced by the vibration of the source pile in the j direction;

s = center to center distance between two piles;

$U_a, U_b^{(j)}$ = normal displacement of a source pile; normal displacement of a receiver pile induced by the vibration of the source pile in j direction. $j=1$ for normal direction, $j=2$ for axial direction;

V_s = shear wave velocity of the soil;

W_a = axial displacement of a source pile;

ϕ = angle between the center-line direction of two piles;

430 ω = frequency of the harmonic excitation;
 431 φ_1, φ_2 = inclined angle of the source pile and the receiver pile, respectively;
 432 $\lambda_{nn}, \lambda_{an}$ = dynamic interaction factors in the normal-normal and axial-normal directions;
 433 ν_p, ν_s = Poisson's ratio of the pile and the soil, respectively;
 434 ρ_p, ρ_s = mass density of the pile and the soil, respectively;
 435 β_s = radiation damping of the soil;
 436 $\mathfrak{R}_z$ = axial impedance matrix at the pile head;
 437 $\mathfrak{R}_h$ = horizontal impedance at the pile head;
 438 $[\mathfrak{R}_{x'}]$ = normal impedance matrix at the pile head;
 439 $\Theta_a, \Theta_b^{(j)}$ = rotation angle of a source pile; rotation of a receiver pile induced by the vibration of the
 440 source pile in the j direction;
 441

442

443

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505

Fig. 1 T-P model for inclined pile-soil dynamic interaction

Fig. 2 Dynamic interaction between two adjacent inclined piles subjected to a horizontal harmonic force (a: Sectional view of adjacent inclined piles; b: Plane view of adjacent inclined piles)

Fig. 3 Comparison of T-P model and numerical models for static stiffness of inclined pile
 ($L/d=15$, $\rho_s/\rho_p=0.7$, $\nu_s=0.4$, $\beta_s=0.05$)

Fig. 4 Comparison of T-P model and BEM-FEM model for horizontal impedance of inclined pile
 ($E_p/E_s=1000$, $L/d=15$, $\rho_s/\rho_p=0.7$, $\nu_s=0.4$, $\beta_s=0.05$)

Fig. 5 Comparison of the normal-normal interaction factors for different inclined angles
 $(E_p/E_s=1000, L/d=25, \rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05, s/d=3)$

Fig. 6 Comparison of the axial-normal interaction factors for different inclined angles
 $(E_p/E_s=1000, L/d=25, \rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05, s/d=3)$

Fig. 7 Comparison of normal-normal interaction factors for different distance-diameter ratios
 $(E_p/E_s=1000, L/d=25, \rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05, \varphi_1=\varphi_2=30^\circ)$

Fig. 8 Comparison of axial-normal interaction factors for different distance-diameter ratios
 ($E_p/E_s=1000$, $L/d=25$, $\rho_s/\rho_p=0.7$, $\nu_s=0.4$, $\beta_s=0.05$, $\varphi_1=\varphi_2=30^\circ$)

Fig. 9 Comparison of normal -normal interaction factors for different models
($E_p/E_s=1000$, $L/d=15$, $\rho_s/\rho_p=0.7$, $\nu_s=0.4$, $\beta_s=0.05$, $\varphi_1=\varphi_2=20^\circ$)

Fig. 10 Variation of stiffness with slenderness ratio for different models

$$(\rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05, \varphi=20^\circ)$$

Fig. 11 Shear effect of soil on horizontal impedance of an inclined pile for different pile-soil elastic modulus ratios E_p/E_s
 ($L/d=15$, $\rho_s/\rho_p=0.7$, $\nu_s=0.4$, $\beta_s=0.05$, $\varphi=15^\circ$)

Fig. 12 Shear effect of soil on the impedance of an inclined pile for different inclined angles
 $(E_p/E_s=100, L/d=15, \rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05)$

Fig.13 Variation of normal-normal interaction factor with the slenderness ratio for different models
 $(E_p/E_s=1000, \rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05, s/d=3, a_0=0.2)$

Fig.14 Variation of axial-normal interaction factor with the slenderness ratio for different models
 $(E_p/E_s=1000, \rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05, s/d=3, a_0=0.2)$

Fig. 15 Variation of normal-normal interaction factor with inclined angle for different slenderness ratios
 ($E_p/E_s=1000$, $\rho_s/\rho_p=0.7$, $\nu_s=0.4$, $\beta_s=0.05$, $s/d=3$, $a_0=0.2$)

Fig. 16 Variation of axial-normal interaction factor with inclined angle for different slenderness ratios
 $(E_p/E_s=1000, \rho_s/\rho_p=0.7, \nu_s=0.4, \beta_s=0.05, s/d=3, a_0=0.2)$

Table 1. Effects of shear deformation and inclined angles on the normalized stiffness $K_h/(E_s d)$ at the head

Inclined angle		$E_p/E_s=100$			$E_p/E_s=300$			$E_p/E_s=1000$		
		$L/d=5$	$L/d=7$	$L/d=10$	$L/d=5$	$L/d=7$	$L/d=10$	$L/d=5$	$L/d=7$	$L/d=10$
0°	Mylonakis (2001)	2.85	2.59	2.35	3.58	3.40	3.10	4.69	4.38	4.18
	Prestent	2.594	2.512	2.296	3.634	3.391	3.095	6.274	4.829	4.244
	Error	(8.97%)	(3.00%)	(2.28%)	(1.50%)	(0.25%)	(0.15%)	(33.78%)	(10.24%)	(1.54%)
10°	Prestent	2.696	2.608	2.387	3.877	3.513	3.205	7.146	5.102	4.387
20°	Prestent	3.010	2.927	2.693	4.292	3.897	3.565	7.884	5.628	4.844
30°	Prestent	3.663	3.603	3.351	5.135	4.682	4.308	9.365	6.685	5.765

Note: the calculation parameters: $\nu_s=0.40$; $\nu_p=0.30$; $\rho_s/\rho_p=0.8$; $a_0=0.001$.