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Non-uniformity quantification of temperature and concentration fields by statistical measure and image analysis

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Abstract

The purpose of this study is to introduce a new technique for quantifying the non-uniformity parameters of temperature and concentration fields. This technique, non-uniformity coefficient (NUC), based on uniform design theory is illustrated by using numerical modeling of microchannel heat sinks with complex structure and computer experiments combined with image analysis. Based on the statistical analogy of star discrepancy, it takes into account the time space and position information (TSPI) of objective. Some of the methods reported in the literature are not found to be able to better characterize and rank very general class of temperature fields correctly. Effectiveness of the new method is

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also demonstrated on the open imaging data of concentration fields. Furthermore, this novel method could be applied to studying a variety of temperature distribution problems in which temperatures are measured with different scales.

Keywords: temperature distribution; microchannel heat sinks with complex structure; characterization of field uniformity; star discrepancy; image analysis.

1. Introduction

Temperature field is one of the most convenient parameters commonly used to study the flow characteristics in the research of heat transfer enhancement [1, 2], especially in terms of micro heat sinks with complex structure [3, 4]. Characterization of temperature field uniformity has received increasing interest in recent years [5, 6, 7]. However, no single universally and formal quantitative definition of temperature uniformity is hitherto available.

One main drawback of classical estimation of temperature field distribution in the most heat transfer investigations is the subjectivity of the measurement interpretation. Indeed, the distribution of temperature field is typically determined with the naked eyes or the researchers' sensory experiences and may yield different results if the same temperature field is in front of different individuals or observed many times by the same operator. It is important to have a quantitative measure of temperature uniformity by computers without human intervention. Besides, rapidly advancing flow imaging technologies provide powerful tools to help address the above question. Studies in flow visualization, vision measuring and image processing are greatly facilitated by a variety of flow imaging technologies, including electrical resistance tomography [8], posi-

tion emission tomography [9], magnetic resonance imaging [10], and electrical capacitance tomography [11, 12, 13], among other tomographic techniques. During the past many decades, considerable attention has been devoted to apply the technologies to the physical fields [14], such as temperature and concentration fields.

A literature survey showed that image analysis has been used to circumvent the subjectivity problem of the concentration field and developed with various degrees of success [15, 16]. For instance, Cabaret et al. (2007) proposed a new technique relying on image analysis to characterize the macro mixing of an impeller in a transparent stirred tank; on the basis of their experimental validations, this method was highly reproducible and robust [17]. Moreover, statistical image analysis is more complicated [18, 19]. For instance, Zhou et al. (2013) developed a general statistical framework for image analysis [20]. They noted an increasing trends and demands of using image processing technology for characterization of subjective human experience [21, 22]. Recently, Fei et al. (2105) and Xu et al. (2016) presented two straightforward methods (Uniformity Coefficient method and Modified Uniformity Coefficient method based on Uniform Experimental Design) for assessing the bubbles uniformity and mixing quality in a direct-contact heat exchanger. The space-time features of the mixing process have been successfully derived by the former method [23], whereas the later exhibits some advantages such as permutation invariance, rotation invariance (reflection invariance) and the ability to measure projection uniformity [24]. The three existing image analysis methods including the standard devia-

tion method [25], the coefficient of variation method [26] and the image entropy method [27, 28] have the inherent advantage of non-intrusive for measuring field uniformity. On the other hand, the approaches remain sensitive to the location of objects in temperature fields. That is to say the time space and position information (TSPI) of objective has not been taken into account in these methods. Hence, further investigations of the engineering characteristics relevant to distribution uniformity of temperature field by means of image analysis are still lacking in a way.

The uniform design proposed by Fang and Wang scatter points uniformly over the experimental domain [29, 30]. Without loss of generality, suppose there were s factors of interest (dimension of experiment region) over a standard domain $C^s = [0, 1]$. The key issue was to choose a set of N points $\mathcal{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$ such that these points were uniformly scattered on C^s . Indeed, a measure of uniformity called star discrepancy was adopted, and the goal was to choose N points with the smallest discrepancy [31, 32]. Hence, the resulting design (uniform design) and has been popularly used during the past more than three decades [33, 34]. Inspired and motivated by Fang et al. (1980, 1995, 2000), Cabaret et al. (2007), Xia et al. (2013), Rodriguez et al. (2014), and Fei et al. (2016), mathematical knowledge and image processing techniques are applied to measure quantitatively for the distribution uniformity of temperature fields. In this work, the main objective is to explore the quantitative characterization of temperature distribution using image analysis.

This article is organized as follows. In Section 2, a novel approach of quanti-

65 flying temperature uniformity is presented; then, temperature data is described
 66 in order to corroborate the validity of the new approach. In Section 3, compar-
 67 isons among various uniformity measures for temperature field are performed
 68 using real data and computer experiments. In Section 4, applications to open
 69 literature are presented. Finally, summary of findings and concluding remarks
 70 are given in Section 5.

71 2. Methodology and temperature dataset

72 2.1. Proposed method

73 There are a number of existing methods to define the sample points equidis-
 74 tribution, including discrepancy measures, point-to-point measures, and vol-
 75 umetric measures. Results and discussions to be presented in Section 4 will
 76 highlight the advantages and deficiency of these existing uniformity measures.
 77 This paper proposes a new pixel measure based on the local discrepancy func-
 78 tion and image analysis. Mathematically, a digital image is a matrix. A color
 79 image can be represented by three gray images. A gray scale image is a matrix
 80 with intensity represented as integer values from 0 to 255. Each matrix element
 81 corresponds to a pixel on the image.

Let $F_u(\mathbf{x}) = x_1 x_2 \cdots x_s$ be the uniform distribution function on $C^s = [0, 1]^s$,
 where $\mathbf{x} = (x_1, x_2, \cdots, x_s)$. Let $F_{\mathcal{X}}(\mathbf{x})$ be the empirical distribution function of
 $\mathcal{X} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\}$:

$$F_{\mathcal{X}}(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^N \mathbf{1}_{[x_i, \infty]}(\mathbf{x}) \quad (1)$$

where $I_{[x_i, \infty]}(\mathbf{x})$ is the indicator function and $\mathbf{x} \in \mathcal{X}$. Then the L_p -discrepancy
($1 \leq p < \infty$), which can be defined as the L_p -norm ($1 \leq p < \infty$) of difference
between $F_u(\mathbf{x})$ and $F_{\mathcal{X}}(\mathbf{x})$, is defined by:

$$D_p^*(\mathcal{X}) = \left[\int_{C^s} |F_u(\mathbf{x}) - F_{\mathcal{X}}(\mathbf{x})|^p d\mathbf{x} \right]^{\frac{1}{p}} \quad (2)$$

For some rectangle region $[0, \mathbf{x}] = [0, x_1] \times [0, x_2] \times \cdots \times [0, x_s] \subset C^s$, the
number of points N encapsulated can be counted. Besides, $\text{card}(\mathcal{X} \cap [0, \mathbf{x}])$
denotes the number of points in the intersection between $\mathcal{X}$ and $[0, \mathbf{x}]$. The
local discrepancy function $d_N^*(\mathbf{x})$ at the point $\mathbf{x} = (x_1, x_2, \cdots, x_s)$, therefore, is
given by:

$$\begin{aligned} d_N^*(\mathbf{x}) &= F_{\mathcal{X}}(\mathbf{x}) - F_u(\mathbf{x}) \\ &= \frac{1}{N} \text{card}(\mathcal{X} \cap [0, \mathbf{x}]) - x_1 x_2 \cdots x_s \end{aligned} \quad (3)$$

The following technique is on the basis of image analysis. The color (RGB)
image of temperature field distribution can be converted into a gray-scale image
by free ImageJ software. Next, the gray image can be converted into the digital
matrix using in-house function of MATLAB software, representing the pixel
value of RGB image of mixture.

Definition 1. In this current research, using the mathematical analogy of
local discrepancy function, the temperature local discrepancy function (TLDF)
of the local rectangular region $\boldsymbol{\theta} = [0, \theta_1] \times [0, \theta_2]$ in a real temperature field is

given by:

$$d_{PQ}^*(\theta) = \frac{\sum_{j=1}^{\theta_2} \sum_{i=1}^{\theta_1} T_{i,j}}{\sum_{j=1}^Q \sum_{i=1}^P T_{i,j}} - \frac{\theta_1 \theta_2}{PQ} \quad (4)$$

where T_{ij} denotes the temperature value at the position (i, j) , $\theta_1 \in \{1, P\}$, $\theta_2 \in \{1, Q\}$ and $\frac{\theta_1}{\theta_2} = \frac{P}{Q}$. Specifically, for the digital temperature image, temperature measurement T_{ij} in the TLDF described in Eq. 4 can be replaced by gray-scale intensity (pixel) value C_{ij} .

Here, the TLDF is deemed superior over local discrepancy function due to the fact that TLDF is adopted to evaluate temperature field uniformity of the whole object surface by infrared thermal imaging video camera; where local discrepancy function is point to restricted to a point set, namely temperature measurements at specific point, by thermocouple or pyrometer. This is the main merit of the proposed method. Nevertheless, it is also immediately clear from Eq. 4 that $d_{PQ}^*(\theta)$ is dependent on the orientation of the first location for calculation. To address this shortcoming, four TLDFs from top-left, bottom-left, bottom-right and top-right orientations, correspond to $q = 1, 2, 3, 4$, respectively, are calculated in this work.

Fig. 1 presents a brief illustration of geometric sense of TLDF with above four different start positions. Fig. 1a shows the RGB temperature field and Fig. 1b shows the gray-scale temperature field that both are demonstrated by two digital images with $100 \times 100 = 10^4$ values. Gray-scale images later are converted from the former color file which can consist of real temperature measurements or be RGB temperature image. From Fig. 1c, the two dashed lines (red and pink) represent the local and global rectangle regions, respectively.

116 The TLDF is obtained easily using the gray intensity level matrix converted from
117 real temperature field.

Definition 2. The single non-uniformity coefficient (NUC_q , $q = 1, 2, 3, 4$) of temperature field distribution is given by:

$$NUC_q = \sup_{\substack{\theta_1 \in [1, P] \\ \theta_2 \in [1, Q]}} |d_{PQ}^*(\theta, q)| \quad (5)$$

118 where $|d_{PQ}^*(\theta, q)|$ denotes the absolute value of $d_{PQ}^*(\theta, q)$. In detail, NUC_1 ,
119 NUC_2 , NUC_3 and NUC_4 denote the inhomogeneous degree of temperature
120 distribution in the whole rectangle surface of object, corresponding to top-left,
121 bottom-left, bottom-right and top-right orientations, respectively.

Definition 3. The new index, non-uniformity coefficient (NUC), employed for uniformity evaluation of temperature field distribution is given by:

$$NUC = \max \{NUC_1, NUC_2, NUC_3, NUC_4\} \quad (6)$$

122 where $\max \{\cdot\}$ denotes the largest value in the set $\{\cdot\}$. In general, if the temper-
123 ature field is reasonably equidistributed, it is expected that the proportion of
124 the sum of temperature measurements within smaller region is proportionate-
125 ly smaller. Simply, a smaller NUC denotes uniformly distributed temperature
126 field; while a large NUC indicates poorly distributed temperature field.

127 2.2. Temperature field description

128 Heat transfer enhancement of single phase in micro scale laminar region
 129 is one of the most intriguing scientific questions [3, 35]. Increasing flow area
 130 in microchannels is considered to be an effectively method to enhance heat
 131 transfer performance, because it is simple, rational and relatively economic [4].
 132 It is generally known that the temperature uniformity of the micro heat sink
 133 surface has obvious implications for the lifespan of electronic devices. Special
 134 attention also was given to the investigations of flow characteristics and heat
 135 transfer [36]. Effectively, besides investigating the liquid flow and heat-transfer
 136 characteristics inside the microchannel heat sink, another important thing is
 137 to consider the temperature uniformity of the micro heat sink surface. The
 138 average fluid temperature and heat transfer coefficient can be used to quantify
 139 the fluid flow and temperature distributions [37]. Hence, temperature field is
 140 one of the most critical parameters in studies of flow characteristic and heat
 141 transfer performance.

142 On the basis of the Refs. [3], [4], [5] and [6], Fig. 2 presents the geometries
 143 of three microchannels: a rectangular micro heat sink, a micro heat sink with
 144 triangular reentrant cavities and triangular internal ribs, and a micro heat sink
 145 with circular reentrant cavities and circular internal ribs. Moreover, the cor-
 146 responding conventional rectangular microchannels (smooth micro heat sinks)
 147 are considered as reference channels. The geometric characteristics of single
 148 microchannel with above three micro heat sinks were described in more detail
 149 by Xia et al. and Zhai et al. [3, 4, 5, 6]. Temperature fields of fluid of the micro

150 heat sinks with different structures were investigated and obtained numerically
151 [38].

152 It can be clearly seen that the contours of the three kinds of temperature
153 fields are unevenly distributed. They are, moreover, difficult to identify accu-
154 rately and quantitatively the difference between them. In this current work,
155 non-uniformity coefficient method was proposed to evaluate temperature field
156 distribution uniformity. Ultimately, our aim is to characterize and rank the
157 temperature fields in the whole surface of different microchannels with complex
158 structure.

159 **3. Results and discussion**

160 *3.1. Appraisal of different methodologies*

161 Temperature uniformity measurements are often made visually by the naked
162 eye. This has an inherent degree of subjectivity which inevitably increase the
163 error. Direct imaging analysis routines can remove the subjectivity of the es-
164 timation of temperature field uniformity. In this paper, three existing image
165 processing methods, namely, the standard deviation method [25], the coefficient
166 of variation method [26] and the image entropy method [27], are used to verify
167 the feasibility of our method. In the following discussions, the number of rows
168 is $P = 720$ and the number of columns is $Q = 1280$ in the recorded images.

In probability theory and statistics, the standard deviation (SD, also repre-
sented by the Greek letter sigma σ) is a measure that is used to quantify the
amount of variation or dispersion of a set of data values [39]. It is calculated

as:

$$SD = \sqrt{\frac{1}{PQ-1} \sum_{k=1}^{PQ} (T_k - \bar{T})^2} \quad (7)$$

where T_k denotes the temperature value in an any-shape region, $\bar{T} = \frac{1}{PQ} \sum_{k=1}^{PQ} T_k$ denotes the mean (also called the expected value) of all the temperature measurement. As a result, the smaller the value SD is, the closer the temperature values are to the perfectly equidistribution.

The coefficient of variance (CoV), also known as relative standard deviation (RSD), is a standardized measure of dispersion of probability distribution or frequency distribution. It is often expressed as a percentage, and it is defined as:

$$CoV = \frac{SD}{\bar{T}} \quad (8)$$

where SD and $\bar{T}$ are demonstrated by Eq. 7. General speaking, one should use CoV instead of SD for comparison between temperature fields with different scales (e.g., Celsius, Fahrenheit etc.) or widely different means. Nevertheless, when the average temperature $\bar{T}$ is close to 0, the CoV will approach infinity and is therefore sensitive to small changes in the mean $\bar{T}$.

The notion of entropy was used to measure the amount of temperature field information. Assuming that levels of a two-dimensional transient temperature field follow a multinomial distribution, the entropy of the rectangular PQ temperature field is defined as

$$IE = -PQ \sum_{g=0}^{255} \pi_g \log_2 \pi_g \quad (9)$$

178 where the theoretical probability π_g is estimated by the histogram value h_g .
 179 As a result, the valuation criteria of temperature distribution is as follows: if
 180 the temperature distribution presents more uniformity, the quantitative value
 181 is less. Although IE is available for quantifying the information content of a
 182 temperature field, this index cannot be applied to more accurately quantify the
 183 field uniformity due to absence of location information of objects.

184 Some of the notable contributions of this proposed approach (namely, NUC)
 185 over the above existing methods (namely, SD , CoV and IE) are summarized
 186 as follows:

187 (1) The proposed method, non-uniformity coefficient (NUC) method, gen-
 188 erally provides a single number that define the overall quality of a temperature
 189 distribution field. A large NUC indicates deviations from uniformity such as
 190 local heterogeneity or lopsidedness, while small discrepancy indicates that the
 191 temperature is more evenly.

192 (2) The actual value of the NUC is independent of the unit in which the
 193 temperature measurements have been taken, so it is a dimensionless number
 194 and can be used for comparison between temperature fields with different scales
 195 (e.g., Celsius, Fahrenheit etc.) or widely different means.

196 (3) The non-uniformity coefficient method takes into account the TSPI of
 197 objective. Maybe in some cases, uniformity determined by SD , CoV and IE
 198 have a space-time limitation. Particularly, the uniformity with the same SD ,
 199 CoV or IE are very different.

200 *3.2. Corroborating the validity of new approach*

201 As mentioned above, the micro heat sinks with different reentrant cavities
202 and internal ribs have well performance in laminar convective flow and heat
203 transfer. Fig. 3 demonstrates the validity of our method. The image processing
204 illustrated to test the new technique was performed with Matlab using the Image
205 Processing Toolbox (IPT) in this section. Furthermore, the gray-scale images
206 were obtained by transforming the RGB values of each pixel over the whole
207 image to the gray value of it.

208 As shown in Fig. 3a, it can be clearly seen that the normalized evaluation
209 values (namely, SD , CoV , IE and NUC) indicate different distributions of
210 fluid temperature in heat sink surface for three microchannels. The smaller of
211 the index, the more uniformity of temperature field in this microchannel heat
212 sink surface. In comparison of three bars of NUC , the normalized evaluation
213 value in the microchannels Tri.C-Tri.R and C.C-C.R are smaller than that in
214 the microchannel Rec. (the smooth microchannel without reentrant cavities
215 and internal ribs). So the conclusion can be drawn that the temperature in
216 the main flow zone of the microchannels Tri.C-Tri.R and C.C-C.R tend to be
217 more uniform. To some extent, this is due to the existence of reentrant cavities
218 and internal ribs because they result in a smaller thermal resistance between
219 channel wall and fluid. Furthermore, the microchannel Tri.C-Tri.R has the
220 best heat transfer rate based around one simple fact that small difference in
221 temperature is helpful heat transfer for constant heat flux. Indeed, the present
222 results show some analogies with those obtained by Liu et al. (2012) [40] and

223 Zhai et al. (2014) [5]. Closer inspection on other bars in Fig. 3a shows that the
 224 proposed index NUC is capable of ranking the temperature pictures in an order
 225 that is more agreeable to the mechanism of the heat transfer enhancement in the
 226 microchannels; while indexes SD , CoV and IE generate entirely unacceptable
 227 ranking orders, respectively.

228 Fig. 3b shows the distributions of the normalized evaluation values (namely,
 229 SD , CoV , IE and NUC) with velocity for the microchannel heat sink with
 230 circular reentrant cavities and circular internal ribs, depending on the range
 231 of velocity. Clearly, one can see that NUC are generally reduced as velocity
 232 increased in case C.C-C.R. This means that, although the Nusselt number in-
 233 creases with increasing velocity, the distribution uniformity of temperature field
 234 becomes better. Compared SD , CoV and IE with NUC , it is obviously found
 235 that the trend of SD and CoV are similar to that of NUC , whereas IE not.
 236 Furthermore, there is more difference in terms of temperature distribution u-
 237 niformity where high speeds are involved. The results were therefore robust to
 238 sensitivity analysis.

239 Those show that our method has the more accuracy and reliability than the
 240 other three existing methodologies. The present approach provides an alterna-
 241 tive way to evaluate the distribution uniformity of temperature field.

242 3.3. Design for computer experiments

In above section, we have compared the proposed approach with three ex-
 isting methods for verifying its feasibility. The comparison was made through
 temperature distribution analysis of the microchannel data. To provide more

evidence of the good performance of the proposed method, we carry out intensive computer experiments for accurate estimations of temperature distribution uniformity using MATLAB software. To generate color images for simulating temperature field, the following mathematical formulas are given by:

$$a + b, -a + b, -a - b, a - b \quad (10)$$

where a (from 0 to 16 by 0.0125) denotes abscissa value while b (from 0 to 9 by 0.0125) denotes ordinate value. Each mathematical formula could generate an RGB image using in-house function `pcolor` in IPT. Combined these four synthetic RGB images, Fig. 4a and b are respectively employed to investigate uniformity measure of temperature field in two special cases, including locations near the edge and the centre that have the higher temperatures.

Obviously, the simulated temperature fields of Fig. 4a and b have the same RGB components, but locations of those pixel points are different. Comparing the variation of resulting values in Table 1, it is found that there is little difference between the two temperature fields for quantifying their uniformity using the four methods. However, the relative differences 1 (to divide minuend by difference) and 2 (to divide subtrahend by difference) of NUC for these two special cases are both greater, whereas the relative differences of the two uniformity measurements are all close to 0 using other methods, as shown in Table 1. In addition, Fig. 4c depicts the variation with serial-number of local region (or θ in Eq. 4) of absolute value of TLDF (namely, $|d_{PQ}^*(\theta)|$) from the

orientation of top left (or northwest) within the temperature field. It is noticed that the trajectories of $|d_{PQ}^*(\theta)|$ present different behavior which is typical for distinguishing different temperature fields uniformity. So based on those results it is concluded that the new method could outperform existing methods and perform more sensitive for practical engineering application in some sense.

4. Applications of our method

4.1. Open temperature data

Guo et al. (2011) simulated temperature field under different conditions in the wellhead room of a new auxiliary shaft to find the proper layout and number of radiators that meet wellhead anti-frost requirements, in order to provide guidelines for a practical engineering design [7]. The proposed method offers three corresponding resultant values $NUC = 0.0174$, $NUC = 0.1281$, and $NUC = 0.0668$ in terms of Fig. 5a (Fig. 4a in Ref. [7]), Fig. 5b (Fig. 6 in Ref. [7]), and Fig. 5c (Fig. 10 in Ref. [7]), respectively. Clearly, the contours of the second one are unevenly distributed while the other two are uniform, stable and feel comfortable. The $|d_{PQ}^*(\theta)|$ of three temperature fields versus θ from the top-left orientation are illustrated in Fig. 5d. It indicates that $|d_{PQ}^*(\theta)|$ frequently fluctuates for small non-uniformity coefficient than great. Those results demonstrate that the proposed NUC is well agreeable to the human perception of temperature uniformity and has a high degree of reliability. Moreover, the mathematical evaluation approach, bringing new insights to quantitative study and compare the temperature field under different conditions, was employed to

test whether the control temperature meets the control requirement.

4.2. Other applications of NUC

Accurate determination of the mixing time in reactor is essential for the optimization of mixing processes and minimization of concentration gradients [25]. Fig. 6a clearly shows the 10 volume snapshots of a specific area in the mixing process reported by Rodriguez et al. (2014). They obtained the normalized green index, G_{ij}^* , and red, R_{ij}^* , for the point of the reactor at the centre of the specific area. It should be noted that, however, the proposed method was used to obtain a characteristic variation of the pixel intensity in the whole specific area, resulting in a more effective composite index. The variation of the NUC values with horizontal coordinate, N_t , at the same specific area as in reference [25] was shown in Fig. 6b. The Spearman's rank correlation coefficient ρ between NUC and R_{ij}^* is -0.7599 while the ρ between NUC and G_{ij}^* is 0.1277. The former good correlations indicate that NUC represents a meaningful parameter to study the mixing transient as R_{ij}^* . Furthermore, from a quantifying the mixture distribution point of view, our proposal develops a non-uniformity measure for characterization of subjective human perception but R_{ij}^* not and for understanding association between mixed regions and reactor geometry or operating conditions which will be the merit of the proposed approach.

Surface pressure with high spatial resolution is desired in many research scenarios due to that the pressure loads is critical parameters in studies of real-world applications [41]. Fig. 6c shows the pressure field obtained from Ref. [41]. According to Peng and Liu (2016), the jet caused a high pressure region near

the impingement point and the pressure in the surrounding area was close to ambient pressure. It can be seen that $NUC = 0.1081$ and $NUC = 0.0966$ were respectively obtained and correspond to above two pressure fields, as shown in Fig. 6c. It indicates that there is little difference of distribution uniformity between them, which is demonstrated that our method was able to extract the quantified information from the experimental data.

5. Conclusions

The new conclusions drawn in this current work are summarized as follows:

(1) A novel approach, non-uniformity coefficient method, based on uniform design theory (namely star discrepancy measure) was proposed to measure the uniformity of temperature field. The proposed measure NUC illustrated by image analysis has several notable properties which make it suitable for uniformity characterization in a wide variety of temperature fields with different scales (e.g., Celsius, Fahrenheit etc.) or widely different means. The evaluation value can be used as a benchmark for automatic decision marking.

(2) Effectiveness of the new method is demonstrated on both the presented temperature field data and the real open imaging data. For temperature field, NUC is closely related to Nusselt numbers; For pressure field, NUC distinguishes between relatively more uneven and uniform; For concentration field, NUC is an effective tool to study mixing transient. Considering the location information of objects, it can be seen that the proposed method has the more accurate estimation, with compared with the existing methods previously introduced.

(3) It is shown that this technique can be expanded and adapted to analysis measurements gradients in real-world applications, including the temperature field, pressure field, concentration field and velocity field. In general, it is immediately clear that the presented method is only applicable to rectangular region. However, this work presents an alternative route to exploring the field uniformity characterization.

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Nomenclature

a Abscissa value

b Ordinate value

347	C	Standard domain
348	$card(A)$	Number of points in area A
349	C_{ij}	Gray-scale intensity (pixel)
350	CoV	Coefficient of variance
351	$d_N^*(\mathbf{x})$	Local discrepancy function
352	$D_p^*(\mathcal{X})$	L_p -discrepancy
353	$d_{PQ}^*(\boldsymbol{\theta})$	Temperature local discrepancy
354	$F_u(\mathbf{x})$	Uniform distribution function
355	$F_{\mathcal{X}}(\mathbf{x})$	Empirical distributions function
356	g	Level of pixel
357	G_{ij}^*	Normalized green index
358	h_g	Histogram value of pixel g
359	IE	Entropy of the rectangular field
360	N	Number of sample points
361	N_t	Horizontal coordinate
362	NUC	Non-uniformity coefficient
363	p	Real number
364	P	Number of rows
365	Q	Number of columns
366	q	Positive integer
367	R_{ij}^*	Normalized red index
368	s	Dimension of experiment region
369	SD	Standard deviation

370	$\bar{T}$	Mean of all temperature
371	$T_{i,j}$	Temperature value at position (i, j)
372	T_k	Temperature in an any-shape region
373	$x_1 \sim x_s$	Coordinates of sample point
374	$\mathbf{x}_1 \sim \mathbf{x}_s$	Sample point
375	θ	Local rectangle region in 2D map
376	θ_1	Positive abscissa value
377	θ_2	Positive ordinate value
378	π_g	Theoretical probability
379	ρ	Spearman's rank correlation coefficient
380	$\mathcal{X}$	Sample points set

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Table 1: Performance comparison among the four uniformity measures of temperature distribution in the conducted computer experiments.

Measures	SD	CoV	IE	NUC
Special case 1 (Fig. 4a)	55.4648	0.3509	7.0781	0.0406
Special case 2 (Fig. 4b)	55.4602	0.3509	7.0825	0.0365
Relative difference 1 (%)	0.0083	-0.0622	0	10.10
Relative difference 2 (%)	0.0083	-0.0621	0	11.23

Figure 1: Brief illustration of TLDF with different start position selection: (a) RGB temperature image; (b) gray-scale temperature image; (c) gray intensity level matrix (top-left, bottom-left, bottom-right and top-right orientations correspond to $q = 1, 2, 3, 4$, respectively).

Figure 2: Cross sections of three micro heat sinks (upper) and there corresponding temperature fields in the surface (lower): (a) rectangular microchannel; (b) microchannel with triangular reentrant cavities and triangular internal ribs; (c) microchannel with circular reentrant cavities and circular internal ribs.

Figure 3: Validation with microchannel structure (a: Rec. denotes the reference microchannel; Tri.C-Tri.R and C.C-C.R denote the two new microchannels) and velocity (b: in terms of a special case C.C-C.R) of different evaluation values of temperature distribution.

Figure 4: Computer experiments: (a) locations near the edge have the higher temperatures; (b) locations near the centre have the higher temperatures; (c) absolute value of TLDF versus serial-number of local region from the orientation of top left (or northwest).

Figure 5: Variation of the NUC values of three reported temperature fields: (a) Fig. 4a in Ref. [7]; (b) Fig. 6 in Ref. [7]; (c) Fig. 10 in Ref. [7] and (d) variation of evaluation values.

Figure 6: Other real-world applications: (a) concentration field reported by Rodriguez et al. (2014); (b) comparison result for above fields; (c) pressure fields from Peng and Liu (2016) and their evaluation values of distribution uniformity.

Highlights

- A novel approach based on statistical measure which could efficiently and quantitatively measure the distribution uniformity of temperature was proposed.
- Effectiveness of the new method is demonstrated on both the presented temperature field data and the real open data in literature.
- It is shown that this technique can be expanded and adapted to analysis other temperature fields or physical fields.