ON THE ORIGINS OF ORDER: NON-SYMMETRIC OR ONLY SYMMETRIC RELATIONS?

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1. Introduction

Non-symmetric relations abound, arranging things so that one is above another, arranging events so that one precedes another, and so on. Our manifest and scientific images of the world and their respective domains of thought and talk are thick with commitment to them and descriptions of them - spatial, temporal, causal, mechanical, mathematical, cognitive, the list is difficult to close off. It was the recognition of the reality of such relations that inaugurated the era of analytic philosophy; recognition that they exist and aren’t reducible is what enabled Russell to decide against monism and idealism in favour of pluralism and realism (Russell 1925: 371).

Recognising that non-symmetric relations exist and aren’t reducible doesn’t explain how relations pull off the feat of arranging things, events, etc. one way rather than another. Russell originally proposed to account for how non-symmetric relations do so by attributing the feature of “direction” to them (Russell 1903: §218). Although Russell’s commitment to this view subsequently wavered, it would be fair to say that many twentieth century philosophers either took the view that non-symmetric relations have direction more or less unreflectively on board, or else simply took for granted the capacity of relations to arrange things one way rather than another, or, in fact, vacillated between these alternatives. Against the backdrop of this rather unsatisfactory state of affairs Fine has offered us a radically different account of how non-symmetric relations arrange things one way rather than another in terms of the interrelationships that obtain between the different states to which the application of non-symmetric relations give rise (Fine 2000). But really there is no need to embrace the consequence of Russell’s appeal to direction or to undergo the intellectual sumersaults that Fine’s account requires of us. All we need to do is to embrace what might be described as a form of Ostrich Realism: the view that how a non-symmetric relation applies to its relata - one way rather than
another—is ultimate and irreducible and that more substantive accounts of how relations apply to their relata yield no real explanatory benefits (MacBride 2013).

Of course if there are no non-symmetric relations in the first place then this deflationary realism goes by the board; without such relations there can no justification for enriching the ideaology of our world-theory with the primitive vocabulary required to describe the application of relations. It has been suggested, or argued, by a number of recent philosophers, including Armstrong and Dorr, that there are neither non-symmetric nor asymmetric relations but only symmetric ones.

It seems unlikely that they’re right about this. From a general methodological point of view it appears far more likely that an error is somewhere concealed in what are often labyrinthine and abstract arguments offered for the claim that there are only symmetric relations than that our cognitive systems, science and mathematics should have portrayed to us a world of non-symmetrical relations when really there are none (James 1904). But, of course, acknowledging that Armstrong’s and Dorr’s arguments must be wrong doesn’t relieve us of the philosophical task of locating where errors are to be found. So here I’m going to roll up my sleeves and take them to task. After laying out the basic motivation for adopting deflationary realism, I will argue that neither Armstrong nor Dorr’s arguments give us anything like a good reason to say anything less about relations.

2. Non-Symmetric Relations: For Deflationary Realism

Relations such that $xRy$ whenever $yRx$ are symmetric. Relations that fail to be symmetric are non-symmetric - if $xRy$ fails to guarantee that $yRx$. Asymmetric relations are a species of non-symmetric relation such that $xRy$ excludes its being the case that $yRx$. But it can only be the case that $xRy$ fails to guarantee that $yRx$, or excludes its being the case that $yRx$, if its being the case that $xRy$ is genuinely different from its being the case that $yRx$. Otherwise $xRy$ will guarantee $yRx$ after all. So it is a basic requirement upon a relation’s being non-symmetric, whether asymmetric or otherwise, that there are different ways in which the relation is capable of applying to the things that it relates. There are two ways in which a binary non-symmetric relation may hold between two things, six ways in which a ternary non-symmetric relation may hold between three things, twenty four ways
in which a quaternary relation may hold between four things, and so on. This basic requirement is the least we must allow if we are to make sense of the distinction between symmetric relations on the one hand and non-symmetric and asymmetric relations on the other.

Many philosophers, following Russell, have discerned a need to impose further requirements upon non-symmetric relations in order to make sense of their satisfying this basic one. Presupposing that the capacity of a non-symmetric relation to apply in a plethora of different ways isn’t the kind of capacity that should be taken as primitive, they have set out to explain how it is possible for a relation to be endowed with such a capacity. Many of them, also following Russell, have done so by attributing to each non-symmetric relation a “direction” or “order” whereby it proceeds from one thing it relates to another. The difference between a non-symmetric relation applying one way rather than another thus arises from its proceeding in one direction or order rather than another: the relation such that \( xRy \) rather than \( yRx \) is the relation that proceeds one way rather than another between \( x \) and \( y \). But it also appears to be a consequence of this explanation that each non-symmetric relation has a distinct converse too. For any relation such that \( xRy \), its converse may be defined as the relation such that \( yR*x \), relations which differ only with respect to the direction in which they proceed from one thing they relate to another. Since it would be arbitrary to admit that one of these relations exist but not the other we appear beholden to admit both of them.\(^1\) So now it appears that we are forced to recognise a further requirement upon non-symmetric relations: that each such relation has an existential partner, a converse that’s distinct.

If the admission of distinct converses is an inevitable existential consequence of employing direction to explain how it is possible for a relation to fulfil the basic requirement upon its being non-symmetric, this provides a reason to be doubtful of the theoretical appeal of explaining what it is to be a non-symmetric relation in such terms. This is because admitting converses appears to have as a corollary a commitment to including additional states in our ontology to house them. But this commitment clashes

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\(^1\) This line of thought, advanced by Fine (2000: 2-3), may be traced back to Russell 1903: §§218-9. For an account of the development of Russell’s engagement with the issue of how relations apply see MacBride 2013a.
with the established, common sense beliefs about how many states there are. Consider that if we accept this commitment there will not only be the state arising from a given binary non-symmetric relation such that \( xRy \) to concern us. There will also be the state that arises from the converse of the given relation such that \( yRx \). So there won’t just be the state of the cat’s being on top of the table to worry us but also the further state of the table’s being underneath the cat. But surely there’s only one state here, albeit a state that falls under two descriptions.\(^2\)

To avoid our being overwhelmed by the superfluity of states that appears to result from the admission of converses, Fine has argued that we need to reject the assumption that led to the admission of converses in the first place (Fine 2000: 16-32). This was the assumption that the capacity of a non-symmetric relation to hold in a plethora of different ways is to be explained by the direction whereby it proceeds from one thing it relates to another. What makes this explanation initially appear compelling is that it answers to the (apparently) naïve preconception that relations apply directly to the things they relate. What distinguishes a non-symmetric relation being such that \( xRy \) rather than \( yRx \) depends solely upon its proceeding from \( x \) to \( y \) rather than from \( y \) to \( x \) - nothing else intervenes in the mechanism whereby a relation applies the things it relates. Fine argues that once we give up the naïve preconception that makes this explanation appealing to us, an alternative explanation comes into view of how it is possible for a non-symmetric relation to apply in a plethora of different ways. According to Fine what distinguishes a non-symmetric relation being such that \( xRy \) rather than \( yRx \) depends upon how \( xRy \) is interconnected with \( zRw \). But this gives rise to a problem for Fine’s account. Surely it’s possible that a relation \( R \) be such that \( xRy \) even though there is no \( z \) and \( w \) such that \( zRw \). Fine’s explanation of what distinguishes \( R \) being such that \( xRy \) rather than \( yRx \) precludes this possibility, thereby ruling itself out. The preconception that relations apply directly to the things they relate it turns out isn’t really naïve at all it because it enables us to make

\(^2\) Williamson (1985) offers a related semantic argument that if we admit converse relations then it will be irredeemably inscrutable which predicates expresses which relation.
ready sense of a non-symmetric relation being such that \( xRy \) even in the absence of some \( z \) and \( w \) such that \( zRw \).\(^3\)

It is also questionable whether, as Fine claims, an explanation in terms of direction of the capacity of an asymmetric relation to apply in a plethora of different ways really does require us to over commit to converses and states to house them. It doesn’t follow from the fact that such an explanation furnishes us with the ideological wherewithal to define the notion of a converse of a non-symmetric relation that we are compelled to admit something that answers to the definition. It doesn’t follow either that because it would be arbitrary for us to select a non-symmetric relation at the expense of its converse that we have reason to affirm a theory of relations that incorporates a commitment to both of them. It may exceed the evidence we have for our theory to suppose that there whenever there is one non-symmetric relation there is more than one - its converse partner or partners too - because a commitment to one of them may suffice to satisfy the demands of the theory. We may only need to believe that one of them exists to account for things’ being arranged thus-and-so rather than so-and-thus.

But in fact we should reject any explanation of what it is to be a non-symmetric relation in terms of direction because of another seemingly innocuous consequence that Fine doesn’t make out. If a non-symmetric relation is such that \( xRy \) rather than \( yRx \) because it proceeds in one direction rather than another between \( x \) and \( y \) then there must be a fact of the matter about whether \( R \) proceeds from \( x \) to \( y \) or from \( y \) to \( x \). But we can’t logically wring out of our ordinary and scientific descriptions of the application of non-symmetric relations anything to settle whether these relations proceed one way rather than another amongst the things they relate — such facts of the matter, if there are any, do not admit of detection by our logical radar but must somehow sneak underneath. Take the state \( S_0 \colon \) Jeanette is to the left of Melanie. Does this state consist in a relation proceeding from Jeanette to Melanie or from Melanie to Jeanette? There is nothing in our description of \( S_0 \) to determine an answer one way or another. So even with regard to the application of a single relation to give rise to a given state, an explanation in terms of direction commits us to an unpalatable choice amongst unfathomable facts of the matter.

\(^3\) Further objections to Fine’s approach are raised and developed in MacBride 2007: 44-53.
concerning how that relations proceeds between the things it relates to give rise to that state. The problem is only exacerbated when we compare $S_0$ with another state $T_0$: Jeanette loves Melanie. Does $T_0$ consist in a relation proceeding in the same or a different way between Jeanette and Melanie than the relation in whose application $S_0$ consists? If the application of non-symmetric relations is really to be explained in terms of direction there must be a fact of the matter about how these relations proceed between the things they relate. Because the descriptions of $S_0$ and $T_0$ provide no basis whatsoever for answering such questions we have reason to be doubtful that these states consist of non-symmetric relations that proceed from one of the things they relate to the other.

It is easy to be misled at this point if we don’t take care to distinguish between the different degrees to which we may allow our metaphysics to embrace the idea of relatedness. The requirement that non-symmetric relations be capable of applying in a plethora of different ways is the least or first degree to which we must accept the idea of relatedness if non-symmetric relations are to be distinguished from symmetric ones. It is a further logical step beyond the first to embrace the second degree: the requirement that every non-symmetric has a distinct converse. And it is another distinct step to embrace the third degree: the requirement that there be a fact of the matter about whether the non-symmetric relation such that $xRy$ rather than $yRx$ proceeds from $x$ to $y$ or from $y$ to $x$.

We cannot avoid embracing the first degree if we admit non-symmetric relations at all. But it is not logical succession that leads us ineluctably from embracing the first to the second and third. The second degree may be avoided altogether because the notion of a converse may be definable even though nothing answers to it; whilst the necessity to embrace the third is only conditional upon an explanatory hypothesis we don’t have to adopt, i.e. a repercussion of explaining how the first degree is possible in terms of direction. The execrable consequences of embracing the third degree provide us with a reason to reject such explanations of the first (even if the consequence of the second didn’t already do for direction).

Appreciating this leaves us with a choice. Either we can reach out again into the darkness for an alternative explanation of the first degree or we can recognise that the capacity of a non-symmetric relation to relate in a plethora of different ways isn’t the kind of fact that admits of a discursive explanation but must be everywhere presupposed.
But if we don’t already think that an account of relations in non-relational terms is needed why suppose that an explanation of the first degree in other terms is needed in the first place? The failure of earlier attempts, such as Russell’s or Fine’s, to provide any credible discursive explanations provides corroborative evidence in favour of the view that the first degree doesn’t admit of further explanation, i.e. deflationary realism. It’s time that we take seriously the neglected possibility that the first degree should be embraced as primitive without need of any discursive explanation, as capturing the very idea of a non-symmetric relation once the extraneous trappings of the second and third degrees have been stripped away.

3. Armstrong: Non-Symmetric and Unwanted Necessities

Descriptions of non-symmetric relations and their applications appear throughout the scientific and mathematical theories we routinely endorse. But Armstrong has argued that a significant proportion of these descriptions, the ones that appear to pick out asymmetric relations, are inherently misleading. Such descriptions mislead us, if Armstrong’s argument has it right, because there are no asymmetric relations out there to describe.

Dorr has gone even further and argued that any description that appears to pick out a non-symmetric relation misleads us because there are no non-symmetric relations whatsoever; all our descriptions of non-symmetric relations are empty. Obviously if Armstrong and Dorr are right there really is no need to take the first degree as primitive - because only non-symmetric relations admit relatedness in the first degree. Both their arguments officially rely upon the Humean principle that there are no ‘brute necessities’ to be found in nature.

The brute necessities against which Armstrong inveighs link “distinct existences”, necessities that cannot be rendered transparent to the intellect by appealing to overlap between the items linked. If we embrace the idea of relatedness in the second degree then we will find that Armstrong’s view is straightaway compromised. This is because a non-symmetric relation’s being such that \( xRy \) will entail that it has a distinct converse such that \( yR^{*}x \). To safeguard the Humean principle that there are no brute necessities

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4 For additional arguments in favour of taking the first degree of relatedness as primitive see MacBride 2013: 8-14.
Armstrong responds by rejecting the second degree. He insists that the appearance of two states here, arising from the application of two distinct (converse) relations, is merely linguistic. Consider a’s being before b and b’s being after a. “Fairly obviously”, Armstrong reflects, “this is just one state of affairs”, a state that arises from the application of a single relation, albeit a state that may be “described in two different ways” (Armstrong 1978: 42, 1989: 85). But even if the second degree is rejected, asymmetric relations, if there are any, present a further challenge to the denial of brute necessities. An asymmetric relation such that \( xRy \) excludes its being the case that \( yRx \).

But, Armstrong asks, how could this be without a brute necessity obtaining whereby one state excludes another? Armstrong saves the Humean denial of brute necessities by simply denying that there are any such asymmetric relations to be found out there. We might have naively thought that an event A’s being before an event B excludes B’s being before A. But, Armstrong maintains, it’s not a necessary truth at all. It’s possible that time is circular—its being so is compatible with the equations of General Relativity - so A’s being before B doesn’t exclude B’s being before A. In a similar spirit Armstrong dismisses any other candidate for being an asymmetric relation (Armstrong 1989: 85, 1997: 143-4).

We shouldn’t allow ourselves to be lured by Armstrong’s choice of scientific example into generalising hastily from the fact that some candidates for being asymmetric relations have turned out to be non-symmetric that no candidates are ever fitting. Nor should we allow ourselves be lured into thinking that Armstrong’s denial of brute necessities is genuinely thoroughgoing. It appears to be so because he relies upon this Humean denial to discredit converse relations and asymmetric ones. It seems that Armstrong has the courage of his convictions, following his argument where it leads. But closer inspection reveals that Armstrong’ theory of relations remains riddled with brute necessities - even when converses and asymmetric relations are sent packing.

According to Armstrong, the world is the totality of existing states of affairs where “A state of affairs exists if and only if a particular (at a later point to be dubbed a thin particular) has a property, or, instead, a relation holds between two or more particulars” (Armstrong 1997:1). Even this most general and abstract characterisation of Armstrong’s ontology incorporates commitment to a battery of brute necessities. Were
necessary connexions fully absent from his ontology then its pieces ought to be, in Hume’s phrase, “entirely loose and separate”; it ought to be possible to throw them up into the air and let them fall wherever the breeze takes them. But the pieces aren’t entirely loose and separate; they can’t float down entirely independently from one another. There’s a rigid network of necessary connexions that controls their relative placement. It isn’t possible for a state of affairs to exist in which a monadic property is such that none or more than one particular has it. It isn’t possible for a state of affairs to exist in which a dyadic relation is such that one or none or more than two particulars are related by it. It isn’t possible for a state of affairs to exist in which there isn’t a property or a relation that’s had by one particular or holds between two or more particulars. It isn’t possible for a state of affairs to exist in which there is more than one property or relation. And this doesn’t exhaust the list of necessary connexions that exert a controlling influence on the pieces of Armstrong’s ontology. (Exercise for the reader: find more examples.)

Armstrong urges us to renounce asymmetric relations and the converses of non-symmetric ones in order to avoid brute necessities. But his ontology of states of affairs, particulars, properties and relations incorporates commitment to plenty of brute necessities. The bottom line: this undermines his argument for renouncing asymmetric relations and converses in the first place.

4. Dorr: Non-Symmetric Relations and Unwanted Necessities II
Dorr goes further than Armstrong, urging us to renounce not only asymmetric but all non-symmetric relations in order to avoid brute necessities. But his argument is also undermined by the fact that his ontology doesn’t avoid commitment to brute necessities either.

Dorr provides a more exacting account than Armstrong of what it is to be a brute necessity. Call a sentence $S$ a brute necessity iff (i) $S$ is not logically true, (ii) the only non-logical vocabulary in $S$ consists of primitive predicates, (iii) all quantifiers in $S$ are

\footnote{For further examples of necessary connexions to which the theory of universals is committed see MacBride 1999: 484-93.}
restricted to fundamental entities, (iv) $S$ is metaphysically necessary.⁶ According to Dorr the thought behind the principle that there are no brute necessities is just that “metaphysical necessity is never ‘brute’: when a logically contingent sentence is metaphysically necessary, there is always some explanation for this fact” (Dorr 2005: 161). But such explanations are typically only possible when a sentence contains some non-primitive expressions that admit of analysis or expressions that are rigidly referring. Since the only non-logical vocabulary of a brute necessity $S$ consists of primitive predicates—that are neither analysable nor referential - such explanations of $S$’s being metaphysically necessary are ruled out. Dorr surmises that unless an alternative explanation of $S$’s being metaphysical necessary can be provided we should look askance at the claim of $S$ to be a brute necessity. If Dorr is right that there can be no brute necessities, then to demonstrate that there are no non-symmetric relations Dorr need merely show that there is no admitting non-symmetric relations without also acknowledging brute necessities about them.

But is it really at all plausible that there are no brute necessities in the very exacting sense that Dorr prescribes? (Of course Wittgenstein held in the *Tractatus* that the only necessity is logical necessity (6.37) but, famously, things didn’t work out well for him there.) If we think that the world is fundamentally composed of different categories of entity but we don’t think that this is just a cosmological accident then it is difficult to see how brute necessities are ultimately to be avoided. Suppose, for example, that the world is fundamentally composed of particulars and universals, that behave in the coeval but nevertheless quite different manners suited to their respective categories. If this isn’t just how the world happens to be then there must be brute necessities that describe how particulars and universals behave differently.

⁶ In fact Dorr adds the stronger condition that a brute necessity $S$ can be known for certain a priori to be true. He argues that there are no brute necessities in this stronger sense because sentences that are known for certain a priori require to be built up from non-primitive predicates or referring expressions and, ex hypothesi, the non-logical vocabulary of $S$ consists only of primitive predicates. Since this stronger condition performs no additional role in Dorr’s argument that isn’t already performed by the weaker condition that a brute necessity is metaphysically necessary I will omit this complication.
Armstrong didn’t succeed in putting brute necessities behind him, never could have done, because his world is fundamentally composed of three different categories, states of affairs, particulars and universals, entities that essentially exhibit quite different forms of behaviour. But Dorr doesn’t succeed in avoiding brute necessities either because he still admits two categories. It’s a brute necessity that falls out of Armstrong’s system that no state of affairs has more than one universal constituent. To avoid this brute necessity Dorr recommends that we jettison states of affairs and favour instead a theory in which there is just one primitive predicate “…. holds among….” that takes one singular term and one plural term as it arguments (Dorr 2005: 189-91). Introducing this predicate has the consequence that we don’t need to report upon the application of a symmetric relation $r$ by saying that there is a state of affairs in which $r$ is borne by $x$ to $y$. We need merely report that $r$ holds amongst $x$ and $y$, a report which doesn’t commit us to a state of affairs. Never mind the (extremely important) question whether a theory with just this primitive has the capacity to account for everything that a theory that posits states of affairs explains. If particulars cannot do what is typical of symmetric relations, nor symmetric relations what is typical of particulars, then brute necessities must still lurk within his system - the particulars and universals that compose Dorr’s world just can’t be entirely loose and separate.

What are examples of such brute necessities? If we forbid particulars and universals from existing outside of their permitted combinations, there can be neither ‘bare’ particulars nor relations that don’t relate. Then it will be necessary that ($S_1$) it’s not the case that there is an $x$ such that there is no $r$ which holds among $x$ and some other things. Whilst relations can hold among the things they relate, presumably particulars are incapable of doing so. So it’s also necessary that ($S_2$), if there is anything at all, there are some things (the particulars) such that they do not hold among other things. But even though they’re necessary, $S_1$ and $S_2$ aren’t logical truths and their non-logical vocabulary doesn’t admit of further analyses; so they’re brute necessities of just the kind that Dorr is committed to denying because he holds that the only intelligible necessities are analysable. We don’t need any more examples to appreciate that the denial of brute
necessities fits ill with a fundamental ontology of particulars and universals.⁷ Dorr bases his case against non-symmetric in favour of symmetric relations on the grounds that the former but not the latter give rise to brute necessities. Even before we negotiate the details of Dorr’s argument it is apparent that his case cannot be effective.

5. Dorr: Do non-symmetric relations generate spurious possibilities?

The verb “bears” provides a significant linguistic resource for describing how relations apply to the things they relate. Using this verb we can distinguish a relation \( r \) which is symmetric (because \( x \) bears \( r \) to \( y \) whenever \( y \) bears \( r \) to \( x \)) from a relation \( s \) which is non-symmetric (because \( x \) bears \( s \) to \( y \) even though \( y \) doesn’t bear \( s \) to \( x \)). Dorr draws upon this resource to articulate what he takes to be a plausible sounding principle about relations:

CONVERSES: For every \( r \), there is an \( r^* \) such that for any \( x \) and \( y \), \( x \) bears \( r^* \) to \( y \) iff \( y \) bears \( r \) to \( x \).

Evidently this principle isn’t a logical truth. But suppose that CONVERSES is metaphysical necessary, that the only non-logical predicate that appears in CONVERSES (“bears”) is primitive and that its quantifiers are restricted to fundamental entities. Then CONVERSES qualifies to be a brute necessity. But if there are no brute necessities then this principle must somehow fail to meet the standard. If “bears” is primitive then something else must be responsible for CONVERSES falling short. Suppose that its quantifiers are restricted to fundamental entities. Since CONVERSES isn’t a logical truth, the only remaining explanation is the failure of CONVERSES to be metaphysically necessary (Dorr 2005: 159-61). So if we grant Dorr that there are no brute necessities this establishes the first premise of his argument:

(1) If “bears” is primitive then CONVERSES isn’t metaphysically necessary.

⁷ Of course a ‘way out’ here for the Humean to take would be to deny that the categories of particular and universal are fundamental. See MacBride 1999: 497-9 and 2005: 124-6. One option would be to explain away the necessary connexions found between particulars and universals by using counterpart theory. See MacBride 2005: 139-40.
The second premise of Dorr’s argument is a disjunction:

(2) CONVERSES is metaphysically necessary or “bears” is not primitive.

To establish this key premise Dorr relies upon a thought experiment about alien relations.

Before we head into the laboratory it will help prevent future disorientation if we first get a firm strategic fix upon where Dorr is heading with his argument. If (2) is true then one of its disjuncts must be true. If the former disjunct is true, i.e. if CONVERSES is metaphysically necessary, then the consequent of (1) is false. So by *modus tollens*, the antecedent of (1) is also false, in other words, “bears” is not primitive. If the latter disjunct is true, i.e. if “bears” is not primitive then, of course, “bears” is not primitive. Either way it follows from (1) and (2) that:

(3) “bears” is not primitive.

If “bears” isn’t primitive then it must be analysable in more fundamental terms. So Dorr surveys a variety of different candidate analyses of “bears” in such terms - that appeal to such notions as *state of affairs* and *argument position*. But Dorr finds himself unable to find a credible analysis of “bears”, that isn’t afflicted by such ailments as committing us to further brute necessities or that doesn’t also have the consequence that “if *x* bears *r* to *y* then *y* bears *r* to *x*” is equivalent to a logical truth. This leads Dorr to conclude:

(4) There are no non-symmetric relations.

We needn’t dwell upon the intricate sub-structure of the reasoning that leads Dorr from (3) to (4) because the thought experiment Dorr provides earlier in his argument fails to support (2).

Dorr doesn’t argue directly for (2). Instead he uses the aforementioned thought experiment to establish that the following claim is false:
(5) “bears” is primitive and CONVERSES is not metaphysical necessary.

Since the negation of (5) is truth functionally equivalent to (2), Dorr’s thought experiment, if it’s robust, indirectly supports (2). The problem that Dorr purports to find with (5) is that this combination of views “forces us to draw spurious distinctions between the possibilities (metaphysical or epistemic possibilities - it doesn’t matter which) in which CONVERSES fails” (Dorr 2005: 164). The design brief of the thought experiment Dorr constructs is to convince us that the distinctions between possibilities that (5) requires us to draw really are spurious.

Suppose for the sake of reductio that “bear” is primitive and that CONVERSES is not metaphysically necessary. Now consider a possible world in which there is a series of simple particulars, linearly ordered by exactly two independent non-symmetric relations $r_1$ and $r_2$. Dorr asks: do the relations arrange this series in the same or in opposite directions? Philosophers reflecting upon relations, such as Russell or Fine, have used the expression “direction” to mean a variety of different things but Dorr means something else quite specific by this question. He wants to know which of the following hypotheses is the case:

(i) For any distinct $x$ and $y$ in the series, $x$ bears either $r_1$ or $r_2$, but not both, to $y$.
(ii) For any distinct $x$ and $y$ in the series, $x$ bears both $r_1$ and $r_2$ to $y$, or bears neither $r_1$ and $r_2$ to $y$.

Prima facie each world that features such a series ought to provide us with the materials to furnish a determinately right answer to the question, which hypothesis is true at that world? But Dorr denies this to be the case, “I say there can be no determinately right answer, because the question is not a legitimate one. There is nothing for us to be ignorant about; no genuine respect in which two possible worlds might be dissimilar” (Dorr 2005: 164).

Dorr offers us the now long awaited thought experiment to persuade us that the distinction between these hypotheses is spurious. He invites us to imagine that our talk of charge turns out to concern two different magnitudes: charge and charge*. Our scientists
assign numbers to charge and charge* in such a manner that the charge and charge* of a particle in our region of the universe are always the same whilst the charge and charge* of a particle in a distant region are of different signs. When scientists assign these numbers they are really coding the application of two fundamental physical relations, one for the comparison of charge, the other for comparison of charge*. Because we are supposing that (5) is true, i.e. CONVERSES is false, these comparative relations lack converses.

Now does this correlation of charge and charge* amongst particles hereabouts ultimately consist in the fact that these relations “point in the same direction”, i.e. for any \( x \) and \( y \) in our region, \( x \) bears both relations or neither to \( y \), or do they “point in the opposite direction”, i.e. \( x \) bears exactly one of these relations to \( y \)? Dorr endeavours to persuade us that this question has no answer by adding a twist to the plot. It turns out alien scientists that inhabit this distant region assign numbers to charge and charge* differently to our scientists. They assign numbers in such a manner that the charge and charge* of particles in their region are equal whilst the charge and charge* of particles in our region are opposite. Dorr reflects “One need hardly be a verificationist to feel that this difference is purely a matter of convention: neither system of notation is in any way ‘better’ than the other, as far as the metaphysics of the situation is concerned” (Dorr 2005: 165).

Dorr is surely right that the difference between the terrestrial and alien scientific communities is purely a matter of convention. The different communities have simply adopted different conventions for assigning numbers to particles to code the application of two fundamental relations. But it doesn’t follow from the fact that these communities code the application of these relations differently that there is no fact of the matter concerning whether these relations whose application they code “point in the same direction or, in opposite directions”. Imagine two different gaming communities that differ with respect to whether they adopt the convention of assigning 0 to a winner and 1 to a loser or the convention of assigning 1 to a winner and 0 to a loser. It doesn’t follow from the fact that it’s arbitrary what code they use that there isn’t a fact of the matter about which players are winners and which players are losers.
Keep at the forefront of your attention that Dorr’s thought experiment is intended to convince us that (5) is incredible: that “bears” can’t be primitive and CONVERSES false because this combination of views requires us to draw spurious distinctions between possibilities. If Dorr’s science fiction indeed convinced us that (5) was incredible then the perplexity that the contemplation of (5) is supposed to occasion ought to be relieved by restoring the metaphysical necessity of CONVERSES. But perplexity with regard to the question of how to code with numbers but without arbitrariness the application of comparative relations of charge and charge* certainly isn’t restored by doing so.

Imagine that a community of super-philosophers occupying an even more remote region of the universe make contact to tell us that CONVERSES is metaphysically necessary after all. Suppose the terrestrial and alien scientific communities are alike convinced by the arguments of the super-philosophers. This leads everyone to conclude that there aren’t just two fundamental physical relations responsible for the correlation of charge and charge*, there are two others, converses of the relations originally recognised. Should the discovery of the super-philosophers that CONVERSES is metaphysically necessary lead terrestrial and alien scientific communities to resolve or dismiss their former differences with regard to the question that Dorr describes as originally dividing them: whether it is correct to assign numbers in such a manner that the charge and charge* of particles in our region are equal whilst the charge and charge* of particles in the region of the alien scientists are opposite or the other way around? The story couldn’t intelligibly climax with a terrestrial (or an alien) scientist winning a Nobel prize for discovering that only one assignment of numbers to charge and charge* is scientifically respectable.

What this shows is that the arbitrariness of numerical coding isn’t really the issue; because it’s always arbitrary how we code with numbers. What is germane is whether the comparative relations of charge and charge* point in the same direction or in opposite directions. But note that restoring CONVERSES and insisting upon four non-symmetric relations, instead of two comparative relations doesn’t provide the scientists from either community with any more purchase upon whether the relations point in the same or opposite directions than they had when they started. The super-philosophers have persuaded them to recognise the existence of two additional relations whose behaviour
depends upon the behaviour of the two mutually independent relations from which we began. They’re dependent, rather than independent, because they’re introduced as the converses of the two original relations. But adding more dependent relations doesn’t subtract from the number of possible ways that mutually independent relations can arrange the things they relate in a series. If CONVERSES is metaphysically necessary, there are still two possibilities to be distinguished: the possibility in which two mutually independent relations relate some things in the same direction whilst their converses both relate the same things in the opposite direction; and the possibility in which they relate things in opposite directions whilst the converse of each relates things in the opposite direction to it. So restoring CONVERSES does nothing to settle which direction independent non-symmetric relations point; we only know that the converses we now recognise will point in the opposite direction to them. Nor does restoring CONVERSES enable us to avoid distinguishing between the possibility in which two independent relations point in the same direction and the possibility in which they point in the opposite direction.

Dorr’s thought experiment doesn’t establish that admitting non-symmetric relations whilst denying CONVERSES is an untenable combination of views—not unless admitting non-symmetric relations was already untenable by itself, in which case adding CONVERSES doesn’t help. It follows that CONVERSES can’t really be to the point either. What we need to know is whether we can make sense of non-symmetric relations applying in the same or opposite directions in the first place.

“Direction” is a term of art but remember that Dorr intends to mean something quite specific when he uses it. The expression is defined in terms of the verb “bears”: two relations point in the same direction if for any distinct x and y in a series, either x bears both or neither to y; whilst two relations point in opposite directions if x bears one of these relations, but not both, to y. These definitions will not enable us to settle whether two relations point in the same direction or opposite directions unless we already understand the verb “bears”. But we won’t be able to understand what this verb means if we dwell solely upon the significance of contexts in which “bears” features whilst ignoring the relevant local holism. If we think of the significance of “bears” in isolation then it is unclear that any substantial constraints discipline its use. But the verb does have
a disciplined use because the contexts in which it features don’t occur in isolation. Contexts of the form “x bears r to y” typically have a travelling companion of the form “x rs y”. To move from one context to the other we grammatically transform a noun into a verb or a verb into a noun, where the arrangement of the proper names that flank the “bears…to” construction and the verb “rs” bear a co-ordinated significance. For example, we are can move from an ordinary predicative construction such as “Thetis is the parent of Achilles” to the rarefied “bears” construction “Thetis bears parenthood to Achilles” by transforming the verb “is the parent of” into the noun phrase “parenthood” and preserving the right-left orientation of the flanking proper names. The predicative context constrains the proper use of the corresponding “bears” constructions. We are entitled to say “Thetis bears parenthood to Achilles” only if we’re already entitled to say “Thetis is the parent of Achilles”.

It is because the uses of “bears” constructions are disciplined by corresponding uses of ordinary predicative constructions that their employment bears empirical significance. This enables us to distinguish between hypotheses about non-symmetric relations that Dorr finds spurious - o make verifiable claims concerning whether non-symmetric relations point in the same direction or different directions. Recall that what ultimately drives Dorr’s argument is the concern that if “bears” is primitive and two independent non-symmetric relations arrange a linearly ordered series of simple particulars then we are forced to distinguish between two hypotheses about how these relations apply.

(i) For any distinct x and y in the series, x bears either \( r_1 \) or \( r_2 \), but not both, to y.
(ii) For any distinct x and y in the series, x bears both \( r_1 \) and \( r_2 \) to y, or bears neither \( r_1 \) and \( r_2 \) to y.

Dorr denies that we can make any sense of the difference between these hypotheses. But, relying upon their transformational equivalences, we can derive from the “bears” construction whereby \( r_1 \) and \( r_2 \) point in the same direction, hypothesis (i), and the “bears” construction whereby \( r_1 \) and \( r_2 \) point in the opposite direction, hypothesis (ii), two
corresponding predicative constructions, where relation-nouns are transformed into relational-verbs,

(iii) For any distinct \(x\) and \(y\) in the series, \((x \, r_1 \, s \, y \land \lnot x \, r_2 \, s \, y) \lor (\lnot x \, r_1 \, s \, y \land x \, r_2 \, s \, y)\)

(iv) For any distinct \(x\) and \(y\) in the series, \((x \, r_1 \, s \, y \land x \, r_2 \, s \, y) \lor (\lnot x \, r_1 \, s \, y \land \lnot x \, r_2 \, s \, y)\).

Suppose that \(r_1\) is the relation being taller than and \(r_2\) is the relation being heavier than. Then by tracking these transformations it’s easy to see that a world where hypothesis (i) comes out true is very different from a world where hypothesis (ii) comes out true. In worlds where (iii), and therefore (i), is true, if someone is taller than someone else they will also be heavier than them (and vice versa). Whereas in worlds where (iv), and therefore (ii), is true, if someone is taller than someone else they won’t be heavier or if they’re heavier they won’t be taller. Different scenarios indeed!

Dorr argues that because we can’t distinguish between hypotheses (i) and (ii) we have to reject the assumption that “bears” is primitive. But we can distinguish between these hypotheses even if “bears” is primitive. This is because, even though “bears” cannot be analysed, “bears” constructions may be transformed into equivalent empirical claims that are expressed using ordinary verbs, claims that are typically testable. So Dorr fails to establish that “bears” isn’t primitive, because admitting “bears” is primitive doesn’t require us to admit spurious possibilities. Since Dorr’s case against non-symmetric relations relies upon the claim that “bears” isn’t primitive, his case collapses.

6. Dorr: Is life possible without non-symmetric relations?

The arguments against non-symmetric in favour of symmetric relations that we have considered so far aim to establish that we should avoid commitment to non-symmetric relations because of the unpalatable consequences of undertaking such a commitment - our having to admit brute necessities or draw spurious distinctions between possibilities. These arguments have been found to be wanting in one or other respect. Ultimately we cannot avoid brute necessities and the distinctions between possibilities that we are forced to draw if we admit non-symmetric relations don’t turn out to be spurious after all. But if all statements putatively about non-symmetric relations could be paraphrased away
in favour of statements about symmetric relations then it would appear that we could dispense straightaway with commitment to non-symmetric or asymmetric relations without the detour via brute necessities and spurious possibilities.

In *The Structure of Appearance* Goodman took an important logical step towards legitimating such a paraphrase strategy when he sprung upon the following equivalence between contexts in which the non-symmetric predicate “is part of” occurs and contexts in which the symmetric predicate “overlaps” occurs:

\[(>) \text{ } x \text{ is a part of } y \text{ iff whatever overlaps } x \text{ overlaps } y \text{ (Goodman 1966: 47-9).}\]

In the light of this equivalence Goodman proposed to define “is part of” in terms of “overlap” Dorr draws upon this proposal to suggest that the *prima facie* commitment of mereology to a non-symmetric relation of parthood can be paraphrased away. Dorr goes onto outline a number of other piecemeal paraphrases for obviating commitment to non-symmetric relations. Dorr suggests that the three-place non-symmetric predicate “between”, found in formalisations of Euclidean geometry, may be paraphrased away in favour of a binary symmetric relation “overlap” and quantification over line segments using the following equivalence:

\[(L) \text{ } x \text{ is between } y \text{ and } z \text{ iff every line segment that overlaps both } y \text{ and } z \text{ overlaps } x.\]

Drawing upon unpublished work of Hazen’s, Dorr also suggests that the non-symmetric set-theoretic predicate “is a member of” may be paraphrased away in terms of two binary symmetric predicates. According to orthodox set theory there are sets, called ranks, such that whenever \(x\) is a member of \(y\), there is some rank that contains \(x\) and not \(y\), but no rank that contains \(y\) and not \(x\). Relying upon the established idiom of set theory, Dorr introduces two symmetric predicates “overlaps set-theoretically” and “are of the same rank” by the following definitions:

\[(D1) \text{ Two things overlap set-theoretically iff one of them is a member of the other.}\]
\[(D2) \text{ Two things are of the same rank iff they are members of exactly the same ranks.}\]
Appealing to Hazen’s unpublished result, Dorr tells us that the following equivalence is a consequence of orthodox set theory:

\[(\in)\ a \text{ is a member of } b \iff a \text{ overlaps } b \text{ and there is something of the same rank as } b \text{ that overlaps everything of the same rank as } a.\]

Dorr concludes that, “we can adopt this biconditional as an analysis of ‘is a member of’ in terms of ‘overlap’ and ‘is the same rank as’, and posit a symmetric binary relation corresponding to each of these two predicates” (Dorr 2005: 181-2).

Don’t forget that the single primitive of Dorr’s own preferred system is a non-symmetric predicate, “---holds among…”. It is possible to permute singular terms that occur in the plural argument of this predicate without disturbing the truth value of a sentence in which it occurs: “\(r\) holds among \(x\) and \(y\)” is analytically equivalent in Dorr’s theory to “\(r\) holds among \(y\) and \(x\)”.

But the predicate is non-symmetric because permuting the singular term that occupies the singular argument position of the predicate with its plural term or any singular terms that occurs in its plural argument position isn’t guaranteed to preserve truth-value: it doesn’t follow from \(r\)’s holding among \(x\) and \(y\) that \(x\) holds among \(r\) and \(y\). Since Dorr provides no strategy for paraphrasing away this non-symmetric predicate, this undermines his claim to have shown that the view according to which “there are relations all of which are necessarily symmetric could be a defensible one” (Dorr 2005: 180). Once it is admitted that one non-symmetric predicate cannot be eliminated it is difficult to see what motivation there could be for adopting recherché paraphrases Dorr recommends to avoid using other non-symmetric predicates—to privilege a metaphysical predicate we scarcely understand at the expense of geometrical and set-theoretic predicates that have an established usage.

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\[8\] An anonymous referee suggests the following response on Dorr’s behalf. Define a symmetric predicate, “proto-hold each other”, in the following terms:

\[(P-H)\ x_1, x_2, \ldots x_n \text{ proto-hold each other iff any of them hold among the others}.\]

Next make the following stipulation:

\[(S)\ r \text{ holds among } x_1, x_2, \ldots x_n \iff r, x_1, x_2, \ldots x_n \text{ proto-hold, } r \text{ is a relation, and } x_1, x_2, \ldots x_n \text{ are not}.\]
Nonetheless there is doubtless scientific interest in establishing how far a programme for paraphrasing away commitment to non-symmetric relations can extend. Certainly the equivalences $(>), (L)$ and $(\in)$ provide necessary and sufficient conditions for $a$’s being a part of $b$, for $a$’s being between $b$ and $c$, for $a$’s being a member of $b$. But it doesn’t follow that these equivalences provide analyses of $a, b$ and $c$ being thus-and-so. Don’t forget that $(>)$, $(L)$ and $(\in)$ also provide necessary and sufficient conditions for everything that overlaps $a$ to overlap $b$, for every line segment that overlaps both $y$ and $z$ to overlap $x$, and for $a$ to overlap $b$ and there be something of the same rank as $b$ that overlaps everything of the same rank as $a$. Obviously it’s not enough to be entitled to adopt an equivalence as an analysis merely to establish that the equivalence allow us to state necessary and sufficient conditions. It needs to be established that one side of the equivalence, the one intended to serve as the analysis, is logically prior to the other. In order to paraphrase away commitment to the non-symmetric relations of part-whole, between and set-theoretic membership it needs to be established that the right-hand-sides of $(>)$, $(L)$ and $(\in)$ have logical priority over their left-hand-sides.

If Armstrong and Dorr had indeed demonstrated that commitment to non-symmetric relations is inherently suspect that would indeed provide us with a credible motivation for assigning priority to the left-hand-sides of these equivalences—because if there are no non-symmetric relations then the predicates on the right-hand-side can hardly pick them out and must bear some other significance, if Dorr is right delineated by their right-hand-sides. But Armstrong and Dorr’s arguments have failed to demonstrate that commitment to non-symmetric relations is inherently toxic. It might be suggested that the right-hand-sides of $(>)$, $(L)$ and $(\in)$ have priority over their left-hand-sides because

Now ‘reverse’ the direction of the definition so that the non-symmetric “hold amongst” is defined in terms of the symmetric “proto-holds” plus stipulation (S). As the referee also points out this definition of “hold amongst” won’t work if there are relations that hold among other relations. This is a serious problem, but there is another more immediate difficulty. (P-H) fails to guarantee that when $x_1, x_2, \ldots x_n$ proto-hold each other, one of them holds amongst all the others. As a consequence (S) allows $r$ to hold amongst $x_1, x_2, \ldots x_n$ even in circumstances where $x_n$ fails to stand in any significant relation to $x_1, x_2, \ldots x_{n-1}$. Another difficulty concerns the fact that it may only be possible to explain what it means for $r$ to be a relation in terms of $r$’s being an item that holds among other things. So even if (P-H) is emended the proposed reduction may end up being circular.
theories that employ only the vocabulary employed on their right-hand-sides are more economical or simpler than theories that employ only vocabulary that appears on their left-hand-sides. But it’s far from evident that this is the case. What is certain is that this cannot be established by isolated inspection of (>, (L) and (∈) but only by appreciation of whole theories and how they compare. And even if it were successfully shown that the right-hand-side of these equivalences enjoy priority over their left hand sides it would be difficult to avoid the suspicion that a non-symmetric, so to speak, meta-relation had been presupposed, the non-symmetric relation expressed by the predicate “has priority over”. More generally, non-symmetric logical relations are the elephant in the room in contemporary discussions of relations (MacBride 2011: 273-5).

Dorr’s suggestion about how to use (∈) to paraphrase away the membership relation confronts especial difficulties. Dorr introduces the symmetric predicates “overlap set-theoretically” and “are of the same rank” on the basis of our prior understanding of established non-symmetric vocabulary of set theory that appears on the right-hand-side of (D1) and (D2), including the non-symmetric predicate “is a member of”. It is only because “overlap set-theoretically” and “are of the same rank” are so understood in terms of the established language of set theory that we can add these definitions to set theory and get (∈) to follow - otherwise the novel vocabulary introduced will hang aloof from the established vocabulary of set theory and nothing significant will result from their union. But when Dorr proposes that we read (∈) as an analysis of “is a member of” he requires us to take a logical summersault: to understand “is a member of” in terms of “overlap set-theoretically” and “are the same rank”. But we only understand these symmetric predicates because they were introduced in terms of established non-symmetric vocabulary including “is a member of”. It follows that we cannot rely upon (∈) to provide a basis for paraphrasing away commitment to the membership relation because the symmetric vocabulary that appears on the right-hand-side of (∈) presupposes the non-symmetric vocabulary that appears on the left-hand-side.

The foreseeable response: avoid the alleged circularity by interpreting (D1) and (D2) as respective analyses of a’s being a member of b, and, a and b’s being members of exactly the same ranks. So even though epistemologically or cognitively we only achieve an understanding of “overlap set-theoretically” and “are of the same rank” via the
established use of “is a member of”, the ontological situation is the reverse. When we
describe the world using statement of the forms that appear on the right-hand-sides of
(D1) and (D2) our descriptions are grammatically misleading. The same content is
perspicuously captured by statements of the forms that appear on the left-hand-sides of
(D1) and (D2). But always remember that it’s never enough to have some equivalence
before us—the privilege has to be earned to read the equivalence as an analysis, assigning
priority to one side rather than another.

There’s another difficulty that besets this proposal to avoid circularity. Let’s focus
on (D2): Two things are of the same rank iff they are members of exactly the same ranks.
It’s important to bear in mind here that the symmetric predicate introduced here that
features on the left-hand-side of (D1) has no internal structure. It is simply introduced en
bloc as having the same significance as the predicates that feature on the right-hand-side.
So the logical structure of the left-hand-side is simply aRb. It follows that if the left-hand-
side of (D2) has priority over the right-hand-side then the grammatical structure of what
appears on the right-hand-side must be logically misleading. Instances of the right-hand-
side appear to require quantification over ranks but if (D2) is taken an analysis that
proceeds from right to left, then the appearance of quantification on the right-hand-side
must be logically misleading. In fact statements of the left-hand-side form (“x and y are
members of exactly the same ranks”) cannot involve genuine quantification over ranks at
all because, the analysis leads us to appreciate, we are only saying that x and y satisfy the
un analysable predicate “are of the same rank”—we are not saying that there is some rank
that they both share.

It’s already been remarked that (D2) cannot be harnessed up to established set
theory to help yield (∈) unless the vocabulary of set theory bears the same significance as
the right-hand-side vocabulary of (D2). But this means, if priority is assigned to the left-
hand side of (D2), that quantification over ranks in established set theory must be a
logical sham too, at least where it assumes the form of the right-hand-side of (D2) - no
matter how mathematicians may have thought they understood the structure of such
statements, i.e. as involving quantification over ranks. There may be the grammatical
appearance of quantification over ranks in such set-theoretic contexts but if (D2) is to be
understood as providing the basis of a reductive paraphrase, the grammar of set theory
must, at least in such contexts, be deemed logically misleading as well. But bear in mind that there are statements about ranks in set theory that don’t assume the form of the right-hand-side of (D2). Obviously these statements cannot be reduced by assigning priority to the left-hand-side of (D2) because they don’t assume the form of its right-hand-side. But now the difficulty is that the reduced contexts have nothing semantically in common with the contexts that cannot be reduced this way and this threatens to block the derivation of (\(\in\)) from (D1) and (D2) via set theory when the routine logical relations between set-theoretic statements of different forms about ranks have been disrupted. And, of course, the general problem remains that it seems far more probable that there is something awry with this analysis for paraphrasing away commitment to non-symmetric relations in set theory than that the scientific and mathematical communities should have been so fundamentally misled concerning the basic quantificational structure of their own lingua franca.\(^9\)

REFERENCES

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