Growth and Interaction of Normal Faults and Fault Network Evolution in Riffs: Insights from Three Dimensional Discrete Element Modelling

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ABSTRACT

The initiation, growth and interaction of faults within an extensional rift is an inherently four dimensional process where connectivity with time and depth are difficult to constrain. A 3D discrete element model is employed that represents the crust as a two-layered brittle-ductile system in which faults nucleate, propagate and interact in response to local heterogeneities and resulting stresses. Faults nucleate in conjugate sets throughout the model brittle crust; they grow through a combination of tip propagation and interaction of co-linear segments to form larger normal faults. Segment linkage occurs by merging of adjacent fault segments located along-strike, down-dip or oblique to one another. Finally, deformation localises onto the largest faults. Displacement distribution on faults is highly variable with marked along-strike and temporal variations in displacements rates. Displacement maxima continuously migrate as smaller fault segments interact and link to form the final fault plane. As a result, displacement maxima associated with fault nucleation sites are not coincident with the location of the maximum finite displacement on a fault where segment linkage overprints the record. The observed style of fault growth is consistent with the isolated growth model in the earliest stages which then gives way to a coherent (constant-length) fault growth model at greater strains.

Keywords
Discrete element modelling, rifting, normal fault, segmentation, fault growth and linkage
Understanding fault evolution in three dimensions in rift basin settings is generally informed by interpretation and analysis of the current or final static fault geometry. The evolution and interaction of faults in extensional rifts, however, is an essentially four-dimensional problem, where the initiation, growth and interaction of faults in three dimensions through time modify the nature of the fault network. The aim of this paper is to apply a numerical model of rifting to better understand the nucleation, interaction and evolution of faults in four dimensions in a rift basin subjected to a single phase of extension. Using this model the propagation and interaction of faults, scaling relationships, progressive strain localisation and dip domain generation are addressed.

It is known that faults grow by lateral propagation and linkage where the slip on isolated and later linked faults accumulates at varying rates (McLeod et al. 2000; Cowie & Roberts 2001; Walsh et al. 2003a). The processes involved are determined from interpretation of observable and extractable data at the end of either a single or multi-phase extension event. In particular, earthquake slip or displacement patterns along selected faults are used to infer the growth history and interaction of selected structures (Contreras et al. 2000; Sharp et al. 2000; Morley 2002; Manighetti et al. 2005; Bull et al. 2006; Jackson et al. 2006; Morley et al. 2007; Nicol et al. 2010; Reeve et al. 2015) with specific focus on relay structures (Acocella et al. 2000; Conneally et al. 2014; Fossen & Rotevatn 2016) and the nature of fault tip interactions (Nixon et al. 2014b; Duffy et al. 2015; Whipp et al. this volume).

It is not straightforward to determine how faults in rift settings interact within the crust. For example, investigations of fault branching with depth in 3D seismic and analogue modelling have confirmed that what appear as isolated faults at the surface are, in fact, coupled along strike through structures at depth (Kornsawan & Morley 2002; Soliva et al. 2008; Long & Imber 2011, 2012; Giba et al. 2012). Failure to recognise this fact will lead to fault network statistics that do not accurately represent the largest faults or fault connectivity within the rift. A consequence of this will be an underestimate of earthquake rupture capabilities. (Walsh et al. 2003a; Manighetti et al. 2007; Soliva et al. 2008; Nicol et al. 2010).

Several lines of research suggest that faults originate in conjugate or polymodal orientations within an extensional rift and that the earliest forming faults modify the local stress field so that later neighbouring faults dip in the same orientation.
Dip domains evolve dependent on these self-organising incipient faults. Domain boundaries (also known as transfer/accommodation zones or graben shifts) are characterised by narrow zones of overlapping fault tips where a change in polarity is marked by interlocking arrays of conjugate faults (Schlische & Withjack 2009; Kornsawan & Morley 2002; McClay et al. 2002; McClay et al. 2005; Fossen & Rotevatn 2016). Not all domain boundaries recorded between opposed dipping faults are considered to evolve from a self-organised state, however, for example where an underlying basement structure is present it has been shown to be a key influence on the location of domain boundaries (Accocella et al. 1999; Fossen & Rotevatn 2016).

The mechanism required for faults within an evolving system to interact is generally discussed in terms of two fault models – the isolated model and coherent model (Walsh et al. 2003b; Giba et al. 2012; Fossen & Rotevatn 2016). The isolated model suggests that a segmented fault array develops from random overlap and linkage of previously unrelated faults which initiate in a self-organised manner from natural heterogeneities and that strain is distributed homogenously (Cowie et al. 2000; Wu et al. 2015). According to the isolated fault model, faults develop during rifting from isolated heterogeneities within a rock volume by radial propagation and, as a result, individual segments of isolated structures expand laterally in three dimensions. As rifting progresses, these small, isolated, faults propagate rapidly to become larger structures or smaller faults link along strike and/or down dip through relays or tip propagation into larger structures (Peacock & Sanderson 1991; Cartwright et al. 1995; Dawers & Anders 1995; Wojtal 1996; Nicol et al. 1996; Gupta et al. 1998; McCleod et al. 2000; Cowie & Roberts 2001; Walsh et al. 2003b). In the coherent model, a segmented fault array develops within an organised system in which segments are kinematically linked from the start. This has been demonstrated where some degree of strain concentration is present due to reactivation of a buried fault, influencing fault propagation in the overburden (Accocella et al. 1999; Walsh et al. 2003a; Giba et al. 2012). Fossen & Rotevatn (2016) see these models as representative end members and not mutually exclusive. Jackson et al. (this volume) suggest that it is conceivable that pre-linkage faults propagate in accordance with the isolated model in their early stages, but that evidence for this is difficult to resolve from subsurface data sets and so the coherent model is more readily observed.
Natural fault networks have been shown to exhibit specific scaling properties that are now key features in interpreting and modelling fault growth and interaction. These relationships are statistical descriptions of the distribution of the frequency-size of fault attributes, which include the correlation between the displacement on faults and their lengths, and the spatial patterns of faulting. Attention has focussed on evaluation and discussion of empirical relationships such as length, width, displacement and gouge thickness (e.g. Hull 1988; Dawers et al. 1993; Gross et al. 1997; Cowie & Scholz 1992; Childs et al. 1993; Gillespie et al. 1992; Yielding et al. 1992; Torabi & Støren Berg 2011; Xu et al. 2014). In natural and physical analogue fault networks, displacement-distance profiles are commonly employed to determine the timing of interaction of faults by along-strike linkage (Cowie & Scholz 1992; Contreras et al. 2000; Ackermann et al. 2001; Cowie & Roberts 2001; Faure Walker et al. 2009; Nicol et al. 2010; Nixon et al. 2011; Xu et al. 2014; Reeve et al. 2015; Jackson & Rotevatn, 2013; Whipp et al. this volume). Displacement profiles for individual isolated faults show either triangular or elliptical profiles with displacement maxima at their centre (Marrett & Allmendinger 1990). Symmetric displacement-distance profiles become asymmetric towards the interaction point where one tip is restricted, or form a double tip restricted, ‘mesa’-style profile with steep edges when both tips are constrained (Dawers et al. 1993; Manighetti et al. 2005; Nixon et al. 2014b). Deviation of along strike profiles from hypothetical symmetric curves is thought to indicate the nature and timing of linkage of small fault segments into larger structures, where displacement maxima indicate centres of original smaller segments and minima denote points of linkage (Huggins et al. 1995; Faure Walker et al. 2009; Nixon et al. 2011; Nixon et al. 2014b; Xu et al. 2014; Reeve et al. 2015; Khalil & McClay 2016). To maintain fault displacement-length scaling, it is expected that observed along-strike deficits in throw associated with fault linkage will become less significant as the fault grows (Nixon et al. 2014b; Jackson et al. this volume). An isolated type profile will result with a throw maximum at the centre of the new, larger structure (Contreras et al. 2000; Cowie et al. 2000; Faure Walker et al. 2009; Schlagenhauf et al. 2008; Xu et al. 2014).

Outcrop and subsurface (seismic) data provides a static, final image of fault networks. In order to assess how similar geometries evolve, physical and numerical analogues are used. Physical analogues examine the upper surface topographic and
fault trace evolution through time and the final static fault geometry in cross section
These methods have been important in understanding fault growth and interaction
through time, but have not fully addressed the three-dimensional development of
fault networks.

Numerical methods use both two- and three-dimensional approximations to
investigate the interaction of either a large number of faults in a two dimensional
plane (Cowie et al. 2000) or focussed assessment of the growth and interaction of
selected pre-defined isolated structures (Walsh et al. 2001; Imber et al. 2004; Soliva
et al. 2008; Lovely et al. 2012; Allken et al. 2013). Discrete element models (DEMs)
are numerical models that use physically realistic inter-element interactions and have
been applied to model mechanical rock behaviour in scenarios where the evolution
of discontinuities can be tested (e.g., Cundall, 1971; Mora & Place, 1993; Donzé et
al. 1996; Kuhn, 1999; Camborde et al. 2000; Toomey & Bean 2000; Place et al.
2002; Imber et al. 2004; Hardy & Finch, 2005 2006; Schöpfer et al. 2007a,b, 2009;

In this study, a discrete element model is applied to investigate the initiation, growth
and interaction of faults within a normal fault network in a rift basin. The crust is
represented as a two-layer model of passive crustal extension with an upper, 15 km
thick layer representing the brittle upper crust, and a lower 15 km thick, firno-viscous
layer representing the ductile lower crust. The effects of thermal variation within the
crust during the evolution of the model are not included. This approach allows
investigation of fault nucleation and the subsequent organisation of a fault network in
4D, the 3D geometry and interaction between faults and the distribution of fault
activity and displacement in time and space. Results presented have implications for
analysis of natural fault systems, particularly the limitations of traditional methods for
reconstructing fault growth histories from final fault geometry coupled with
measurement of variations in displacement measured on pre- and syn-rift
stratigraphic horizons along fault systems.

METHODOLOGY
Multi-layer rheologies in DEM techniques have been employed to examine the influence of mechanical stratigraphy on the propagation of blind faults (Hardy & Finch 2007; Schöpfer et al. 2007a,b), boudinage (Komorócz et al. 2013) and compressional wedges (Wenk & Kuhn 2013). The crust in our model is represented as a two layer system where elements in the upper crust interact through linear elastic repulsive-attractive forces and those within the lower crust interact through linear viscous (Newtonian fluid) forces (Fig 1a) (Ranalli 1995). Elements in the upper crust are treated as an assembly of spheres that interact in pairs \((ij)\) as though connected by breakable elastic springs following

\[
F_{ij}^{\text{elastic}} = \begin{cases} 
K(r - R), r < r_b, \text{intact bond} \\
K(r - R), r < R, \text{broken bond} \\
0, & r \geq R, \text{broken bond}
\end{cases}
\]

Where \(K\) is the bond stiffness, \(R\) is the equilibrium separation between an element pair (element \(i\) and neighbour \(j\)) and \(r\) is the inter-element separation. Elements are bonded until their separation exceeds a breaking distance, \(r_b\), beyond which the bond is broken and experiences no further attractive force but will experience a repulsive force if the elements return to a compressive contact (i.e. \(r < R\)).

In the lower crust, a firmoviscous (Kelvin) body is applied to determine inter-element forces where elastic and linearly viscous forces are set in parallel (Fig. 1b). When loaded, the elastic response of the spring is delayed by the viscous response of the dashpot resulting in a non-instantaneous response. The force due to the spring in the lower crust, \(F_{ijl}^{\text{elastic}}\), is obtained from

\[
F_{ijl}^{\text{elastic}} = \begin{cases} 
K_c(r - R), r < R \\
K_t(r - R), r > R
\end{cases}
\]

where \(K_c\) is the spring stiffness in compression, consistent with upper crustal elements, and \(K_t\) is the spring stiffness in extension, set to zero. This relationship assumes that links between elements in the lower crust retain elastic properties in compression only. The viscous force is determined through

\[
F_{ijl}^{\text{viscous}} = -\eta \Delta \dot{x}
\]

where \(\Delta \dot{x}\) represents the relative velocity between an element pair and \(\eta\) is the Kelvin viscosity, chosen empirically within the experiment. The greater the relative
velocity between an element pair, the greater the force acts to return them to their equilibrium position.

The total force, $F_i$, exerted on an element is obtained by summing the forces exerted on it by its $n$ neighbours.

$$F_i = \sum_{j=1,n} F_{iju}^{\text{elastic}} + F_{ijL}^{\text{elastic}} + F_{ijL}^{\text{viscous}}$$

The interface between the upper and lower crust is defined as a step function, positioned at a depth appropriate to the thickness of the crust under investigation. Bonds between elements that bisect this interface are treated as viscous. To attenuate kinetic energy in the system and constrain the elastic nature of the springs, a damping force, $F_{ID}$, is included

$$F_{ID} = -\nu \dot{x}$$

where $\nu$ represents the damping term and $\dot{x}$, the element velocity. The damping term in this experiment is 7.0 (cf. 0.7; Potyondy & Cundall, 2004) and permits the investigation of quasi-static deformation (Donzé et al. 1994).

Finite element schemes have approximated the crust as a gravitating plate effectively floating hydrostatically on the mantle (e.g. King & Ellis 1990; Hassani & Chéry 1996). A similar method is employed here, where the crust is considered an elastic-brittle-plastic plate floating hydrostatically on a fluid mantle held in equilibrium around a specified depth (cf. King et al. 1988). This depth is determined from the ratio between crust and mantle densities where the density of the brittle crust is described as a lower estimate of crustal density based on the saturated bulk densities of rock (King et al. 1988). The force experienced due to gravity and floatation, $F_{GF}$, is added to the interaction force in the vertical, $z$-component direction where

$$F_{iGF} = g[(\rho_m - \rho_c)V_B - \rho_cV_A].$$

Here $\rho_c$ and $\rho_m$ are crust and mantle densities respectively, $g$ is the acceleration due to gravity, and $V_A$ and $V_B$ are the volume of the element that exist above and below the hydrostatic equilibrium. When an element exists completely above the equilibrium depth, a resultant downward force is experienced whereas an element
entirely below the hydrostatic equilibrium experiences a resultant upward force simulating buoyancy.

With increasing time, the loading due to gravity will cause viscous flow in the lower crust. If the medium is not constrained in the x- and y-component directions, this will cause a ‘forcing out’ of elements at boundaries. To negate this, the media is constrained by bounding walls which simulate it existing within a larger system of elements with similar mechanical properties. At each time step, if an element’s interaction force oversteps the bounding limit for the wall, the element experiences an additional repulsive force from the wall. This wall force \( F_{iw} \) assumes that element \( i \) has come into a compressive contact with an element \( w \); within the wall so that,

\[
F_{iw} = -K_w r_w
\]

where \( r_w \) is equivalent to the amount by which the element exceeds the boundary and \( K_w \) is the elastic stiffness of the wall (cf. Wenk & Huhn 2013).

There are no shear forces determined within this technique, so the behaviour of the rock mass is considered as frictionless (see Mora & Place 1994, Donzé et al. 1994; Hardy & Finch 2007). This methodology has been previously used to successfully simulate the frictional stick-slip instability in a rock assemblage without shear forces (Mora & Place 1994) and for biaxial compression tests and faulting in sedimentary successions above basement structures (Finch et al. 2003, 2004, Hardy & Finch 2006). The success of these methodologies suggests that the assumption of a frictionless rock mass is not at odds with reproducing realistic rock mechanics behaviour. Other DEM techniques incorporate frictional forces but their addition greatly increases computational time (e.g. 45 days with 12000 elements, Wenk & Huhn 2013).

The total force exerted on an element within the crust in the x- and y-component directions is given through

\[
F_{iT}^{TOTAL} = F_i + F_{ID} + F_{iw}
\]

The gravity and floatation term is included in the z-component direction; therefore the total force exerted vertically is

\[
F_{iT}^{TOTAL} = F_i + F_{ID} + F_{IGF}.
\]
Extension is implemented on all elements in small increments to simulate movement of a rigid boundary wall to the north while the southern boundary is static (Fig 1c). The boundary condition is implemented so that

\[ y_i^e(t) = y_i(t) + \Delta y \left( \frac{y_i(t)}{y_{\text{max}}(t)} \right). \]

Here, \( y_i^e(t) \) is the new element location, \( y_i(t) \) is the current element location, \( \Delta y \) is the extension increment per timestep and \( y_{\text{max}}(t) \) is the maximum length of the model in the extension direction (similar to Donzé et al. 1994). Elements are advanced to new locations within the model by integration of their equations of motion using Newtonian physics (see Hardy & Finch 2006). Discrete element models can be run in model units but for comparison with geological data, model units are often scaled to real world parameters (e.g. Place & Mora 2001; Hardy & Finch 2006).

Numerical modelling of the effect of temperature variations on fault initiation and activity during rifting are common (Behn et al. 2002; Huismans & Beaumont, 2007; Wright et al. 2012). From these continuum mechanics based models, which employ thermo-mechanical equations to investigate fault localisation, it is known that extension results in crustal thinning and horizontal fluctuations in the temperature field culminating in focussed faulting around zones of thinned crust (Behn et al. 2002; Cowie et al. 2005; Huismans & Beaumont, 2007). Deformation is distributed between sets of conjugate normal faults, however, in the absence of a regional temperature gradient (Behn et al. 2002). The purpose of this paper is to examine the initiation, growth and interaction of faults during rifting, and as such, localisation effects associated with thermal variations through time are not considered.

**Experimental set-up and data analysis**

*Discrete Element Model*

The experimental media consists of 1,080,000 elements with a regular hexagonal packing where element radii are unity (Fig 1). The initial dimensions are 213 x 200 x 102 (x,y,z) model units (m.u.). Horizons are defined as an integer value of their height in model units: there are 70 in total. Data from six horizons are extracted for discussion purposes, numbered from Horizon 52 (immediately above the upper-lower crust boundary) to Horizon 102 (the upper surface) in ten-unit increments (Fig.
1c). One model unit is equivalent to 292 m and the model represents real-world dimensions of 64 x 60 x 30 km. The upper and lower crust layers are each 15 km thick at the start of the experiment. Experiments are run for 60,000 time steps with data output at intervals of 1000, providing 60 data files. A time step represents 100 years, so the total run time is 6 Myr with an output interval of 100,000 years. The southern end of the model is fixed. Extension is incremented at 0.001 unit per time step towards the north (Fig. 1) and thus represents of rate of 3 mm yr\(^{-1}\) with each output correlating to 0.5% extension to a total of 30%. The natural strain rate determined for these experiments decreases from 1.6 x 10\(^{-15}\) s\(^{-1}\) to 1.24 x 10\(^{-15}\) s\(^{-1}\) during extension, consistent with strain rates recorded from rifted basins, which can range from 1.0 x 10\(^{-16}\) to 4.0 x 10\(^{-14}\) s\(^{-1}\) (Kusznir & Park 1987; Nicol et al. 1997).

The data presented here are from a single experiment but are representative of many experiments which evaluated scaling parameters appropriate for investigation of the development of faults in a rifted basin. Rock densities are defined as 2800 kgm\(^{-3}\) and 3300 kgm\(^{-3}\) for the crust and mantle respectively. The elastic spring constant in the upper crust (\(K\)) is 8.6 x 10\(^{10}\) Nm\(^{-1}\) and 9.7 x 10\(^{11}\) Nm\(^{-1}\) in the lower crust (\(K_c\)). For this scaling of the model, the Poisson’s ratio is 0.25 and the Young’s modulus (\(E\)) approximates to 90 GPa and 105 GPa in the upper and lower crust respectively (Mora & Place, 1994). Elements within the upper crust are randomly assigned breaking thresholds (\(r_b\)) between 0.025 m.u. and 0.1 m.u. at the start of the experiment. The breaking threshold between element pairs is determined from the average of the threshold assigned to the two elements, providing a distribution of weak and strong bonds in varying orientations around each element. These breaking thresholds scale to bond strengths between 2.1 and 8.6 GPa in the upper crust.

Analysis of fault network development

Faults are defined by analysing the separation between an element and its immediate neighbour in the extension direction at the end of the experiment. They are identified by filtering data for each element relative to heaves exceeding 50m and recording the throw (vertical displacement between element pairs). In previous methodologies (e.g. Finch et al. 2004; Hardy & Finch 2006), faults have been defined using continuous alignments of broken bonds. In this experiment, however, displacement propagates into the lower crust where bonds do not break. Therefore a filter using fault heave was deemed the most appropriate method for determining
fault locations. The heave value used has been chosen through testing and prevents fault definition including elements displaced by flexural rotation of horizons in hangingwalls and footwalls. A fault in this model is defined as a continuous alignment of elements with similar dip orientation along strike (north or south). The chosen elements are then assigned a fault number. Data associated with these faults can be extracted from earlier outputs within the experiment to determine the growth and interaction of selected structures. For example, the topographic evolution associated with a fault at the surface can be analysed by outputting displacement on its constituent elements to produce displacement-distance plots for throw at times throughout the experiment.

For each horizon in the model, 120 1D transects perpendicular to the extension direction can be extracted. The throw and heave of faults that intersect these transects can be assessed through time and employed to examine fault growth, strain accommodation and fault polarity. This is carried out for the upper surface (Horizon 102) to compare with data from fault analysis techniques. Fault displacement is output for five selected transects to demonstrate fault growth at the upper surface. The amount of strain accommodated on north- and south-dipping structures is determined from the summation of fault heaves along these transects. It is recorded against time and the relationship between dip direction and strain accommodation is plotted.

The polarity of the five selected 1D transects is calculated using

\[ P = \frac{\varepsilon_d - \varepsilon_n}{\varepsilon_t} \]

where \( \varepsilon_d \) and \( \varepsilon_n \) are the strain accommodated by the dominant and non-dominant strain orientation and \( \varepsilon_t \) is the total strain accommodated along the transect. Similar to Moriya et al. (2005), the dominant fault direction is defined as the direction in which the largest displacement fault dips. In this respect, \(-1.0<P<1.0\), where a population that is strongly polarised opposite to the dip of the largest fault will return polarities approaching \(-1.0\) and a population that dips consistently with the largest fault will return polarities near \(1.0\). As a consequence, the orientation that contains the fault with the largest strain should be consistent with the dominant direction of strain accommodation, indicating that a positive \(P\) value should be expected.
To illustrate fault evolution and linkage in three dimensions, the displacement (throw) on elements that constitute six selected faults is plotted in strike sections at 15%, 20%, 25% and 30% extension. Displacement-distance plots for these faults is presented where data is generated by sampling within 500 m intervals along the length of a fault, recording the maximum displacement regardless of depth for each interval. This is done to negate the dominance of one horizon in assessment of fault growth and highlights the location of displacement maxima through time along the fault length.

**FAULT NETWORK ORGANISATION**

In this section, a number of features of the organisation and growth of the fault network are presented. We firstly look at fault nucleation and then evaluate the entire fault network in relation to the spatial distribution of faults, their growth and interaction. Displacement accrual along selected transects, fault polarity and strain accommodation across dip domains and domain boundaries is then considered. Fault network statistics are presented in relation to the variability with depth of the frequency-size relationship, displacement on selected horizons and displacement on the largest faults within the system. The final section discusses the along-strike three-dimensional interaction of seven faults.

**Fault nucleation**

In order to assess whether lattice geometry affects the nucleation and growth of faults, the location of the earliest bond failures are output relative to depth and orientation (Fig. 2). These nucleation sites are distributed throughout the brittle layer and demonstrate that there is no dominance or focus in the distribution of initial failures with depth (Fig. 2a) or their orientation (Fig. 2b). The location of further failure in the brittle layer continues with increasing extension and is distributed throughout the crust, influenced by the stress fields surrounding existing faults.

**Evaluation of the entire fault network**

**Final fault network**
The consequence of faults initiating in a conjugate distribution is shown in Fig. 3 where the elements that constitute the final faults in the model are shown. Faults are connected along strike through a curvilinear geometry and spaced at regular intervals. By the end of the experiment, larger faults fill the brittle crust with maximum displacement at their centres (~H72, Fig. 3b). The depth to which faults project into the lower crust varies according to their along-strike length. Upper surface topography forms a series of grabens and half-grabens where basins have a maximum along-strike length < 30 km and footwall crest to footwall crest separations of around 10-15 km (Fig. 3c). Hangingwall depocentres are located at fault centres with maximum relief on the order of a kilometre. Conjugate fault interactions across relays are common and numbered circles highlight three locations where relays result in topographic lows between neighbouring basins (Fig. 3c and 4).

Fault network growth and organisation

The evolution of the fault network on the uppermost horizon (H102) together with topography and dip domains is shown in Fig. 4. Faults initiate as a large number of small, isolated structures, striking perpendicular to the extension direction (+/- 20°), and by 10% extension have lengths and separations on the order of 10-15 km (10%, Fig. 4a). Displacement on these faults generates small footwall crests and hangingwall depocentres with relief <100 m (10%, Fig. 4b) and faults form conjugate sets (10% extension, Fig. 4c). At the end of rifting (6 Myr, 30% extension), there are no faults at the upper surface that rupture the entire width of the model; the largest faults are ~30 km long (30% extension, Fig. 4c).

Fault growth and interaction is indicated by the timing of bond failure (Fig. 4a). Following initial fault growth (grey colouring, 10%, Fig. 4a), further bonds break as the early formed faults propagate laterally (green colouring, 20%). Bond failure in the final 10% extension is focussed on the propagation and linkage of faults, either along strike, or by breaching existing relays (yellow-red colouring, 20-30%). Topography evolves from small isolated basins with relief ranging from ≤ 100 m (10% extension, Fig. 4b) to large 20-30 km long basins with up to a kilometre of displacement on individual faults and hangingwall depocentres (purple) focused mainly at fault centres (30% extension, Fig. 4b). The conjugate pattern established during fault nucleation (Fig. 2) results in the surface being divided into dip domains whose boundaries trend sub-parallel to the extension direction (30%, Fig. 4c). Faults in the
centre of the model dip predominantly southwards (red, Fig. 4c), whereas those
either side of this central area dip mainly northwards (blue, Fig. 4c). Conjugate fault
interactions shown in Fig. 3 coincide with two of these domain boundaries (circles 2
and 3, 30% extension, Fig. 4).

The growth and interaction of faults at this horizon is presented through the
development of structures in three regions (A-C, Fig. 4a). Region A highlights the
along-strike linkage of two south-dipping faults from a relay ramp (10% extension) to
a single-breached relay on its southern boundary (20% extension) through to a
double breached relay at 30% extension (Fig. 4). As a consequence, there is
increased hangingwall subsidence and bed rotation at the point of linkage (20-30%
extension, Fig. 4b) and increased dip on the fault (intense red colour, 20-30%
extension, Fig. 4c).

Region B contains one north-dipping fault and eight south dipping faults that by 10%
extension, have lengths < 5 km (Fig. 4a). At 20% extension, the south-dipping faults
(red, Fig. 4c) have soft-linked and comprise four segments separated by relay ramps
(Fig. 4a-c). These then breach and form one structure, with the eastern tip being
constrained by a conjugate, north-dipping fault (circle 3, 30% extension, Fig. 4a-c)
and the western tip interacting with another north-dipping fault (30% extension, Fig.
4c). The growth of these and neighbouring faults are later used to illustrate three-
dimensional fault evolution and interaction (Figs. 11-16). Region C (10% extension,
Fig. 4c) highlights four faults which are used to demonstrate the along-strike linkage
and interaction of faults relative to surface topography (Figs. 5, 7 and 8).

Five representative transects across the upper surface are used to demonstrate
variability in the growth of faults (1-5, Fig. 5a). Selected faults (B-F) presented in
later figures are coloured to illustrate their growth. Bold dotted lines indicate faults
that rapidly accumulate displacement which then decrease or plateau (Fig. 5b). This
shows that the largest fault on any transect at the start of rifting does not necessarily
continue to dominate with time and other factors may control which faults in the
network are dominant with increasing strain. Solid bold lines indicate selected
structures where the displacement is initially small and accelerates with time. The
remaining faults encountered (coloured grey) have low displacements (<200 m) and
become inactive during rifting, shown by little/no increase in displacement with time.
Faults E (blue) and F (green) show a general trend of increasing displacement with
time, similar to faults represented by the bold solid lines (Transects 1-5, Fig. 5). Fault C (purple) on Transect 2 is one of the largest faults until 4.5 Myr, at which point the displacement becomes fixed around 400 m and its displacement rate slows. This correlates to the growth of neighbouring Faults B (red) and D (yellow) shown in Transects 1-3 (displacement increases after 4 Myr, Fig. 5b) and is discussed later (Figs. 7 & 8).

As rifting progresses, a series of dip domains (i.e. regions of similar fault dip direction) develop without any pre-existing fabrics or lineaments (Fig. 4c). The strain accommodated relative to the dip direction of faults is shown for Transects 1-5 in Fig. 6a. The variability and conjugate nature of faults during early rifting is again highlighted (< 2Myr, 10% extension, Fig. 4), with all transects displaying changes in the dominant orientation of strain accommodation before 2 Myr (Fig. 6a). Three transects have a dominant dip direction (Transects 1, 2 and 4, Fig. 6b). In Transects 1 and 2 up to 80% of the strain is accommodated by north-dipping faults, consistent with the western, north-dipping domain shown in Fig. 4c. Transect 4 represents the central south-dipping domain from Fig. 4c where, from an early stage of rifting, the greater amount of strain is accommodated by south-dipping faults. Strain accommodation along Transects 3 and 5 is more complicated, however, since they intersect domain boundaries. The percentage of strain accommodated fluctuates around 50% for up to 4.5 Myr (Transect 3) and 3.5 Myr (Transect 5) and implies that the relative growth and interaction of faults at domain boundaries directly affects strain accommodation in these zones. Later in rifting they localise to a 60:40 relationship dipping north (Transect 3) and south (Transect 5) suggesting the fixing of dip domains is not yet complete.

The polarity of faults on representative transects is shown in Fig. 6b. Transects 1, 2 and 4 (Fig. 5a) which are contained within a strong dip domain (Fig. 4c) show a mainly positive correlation between the dip direction of the dominant fault and main strain accommodation direction. The initial 3 Myr show an increasing linear relationship between polarity ($P$) with time which levels off where $0.5<P<0.6$ as rifting continues. In Transects 2 and 4, polarity switches to negative in the final 0.5 Myr (25-30% extension) suggesting that the dominant fault has changed to one that dips in an opposite direction. This implies that within a strong dip domain, there is the possibility for faults antithetic to the dominant strain accommodation direction to
continue to accumulate displacement. The polarity of Transects 3 and 5, as expected from the strain accommodation data and their location at domain boundaries, fluctuates between positive and negative as faults interact.

**Fault interaction and displacement rate evolution**

The preceding results showed that some of the largest faults at the initiation of rifting become inactive as neighbouring faults grow around them (e.g. Fault C, Fig. 5a). Fig. 7 focuses on the growth of fault segments within Region C in Fig. 4. Faults C and D are the largest faults with lengths ≥ 10 km at 15% extension and maximum displacement of 150 m, all other faults segments have lengths < 3 km. From 15 to 20% extension (3-4 Myr, white band, Fig. 7b) Faults C and D have the greatest displacement, with two segments of Fault D propagating laterally and increasing displacement up to 400 m, overtaking Fault C (250 m). Fault B comprises a number of small soft-linked segments at this stage (displacement < 200 m and < 8 km long). Fault A is a conjugate fault that interacts with Fault B at the surface and is ~ 3 km long and has displacement of < 150 m.

Displacement on Fault C markedly slows after 4.0 Myr as the initial segments of Faults A, B and D propagate laterally (25% extension, Fig. 7a) and link (thickening of 4-5 Myr band, Fig. 7b). From 5-6 Myr (25-30% extension), Fault A continues to grow, the soft-linked segments of Fault B link, and the two segments of Fault D propagate along-strike. The profiles of these faults are consistent with profiles of natural examples. Faults A and D have triangular displacement profiles where their lateral tips are restricted (cf. Fig. 8. Nixon et al. 2014b), Fault B has an asymmetric profile where its eastern tip is restricted by interaction with Fault A (point 2, Figs. 4c & 7a) and Fault C represents a traditional symmetric displacement profile consistent with an isolated fault. Before 20% extension Faults A, B and D grow laterally while accruing displacement exhibiting a growth pattern resembling the isolated fault model. Fault C however, attains its length rapidly and then accrues displacement, suggestive of the coherent growth model, although growth of this fault slows as neighbouring faults dominate at greater extension.

The displacement rates of Faults A, B, C and D at intervals of 1 Myr are shown as strike projections in Fig. 8a, and summarised as a displacement versus time plot for
selected profiles in Fig. 8b. The main period of fault growth differs for each fault and along-strike rates vary dramatically. Faults A and B have a maximum displacement rate between 4 and 6 Myr, when their lateral propagation and interaction causes an increase from < 0.1 mm yr\(^{-1}\) up to 0.37 mm yr\(^{-1}\). The hard-linkage of segments that constitute Fault B at 4 Myr is shown by the pink profile (21,000 m, Fig. 8a), with displacement rates increasing from 0 to 0.15 mm yr\(^{-1}\) at 4 Myr to 0.3 mm yr\(^{-1}\) between 5 and 6 Myr. Further linkage on Fault B between 5 and 6 Myr is shown by the red profile (16,000 m, Fig. 8a) where the displacement rate increases rapidly from 0.08 mm yr\(^{-1}\) - 0.37 mm yr\(^{-1}\). By comparison, the neighbouring parts of the fault propagate from 0.15 mm yr\(^{-1}\) - 0.3 mm yr\(^{-1}\).

The displacement rate on Fault C peaks at 4 Myr (dark grey, Fig. 8a) at 0.15 mm yr\(^{-1}\) and then decreases in the last 2 Myr of extension to < 0.03 mm yr\(^{-1}\) showing that this fault is almost inactive at the end of rifting despite initially being the largest (Figs. 7 & 8). The most complicated pattern of growth is associated with Fault D. Up to 4 Myr, there are two distinct segments where each has a displacement rate maximum at its centre (Fig. 7b). The blue, green, and purple profiles demonstrate an increasing displacement rate before 4 Myr, which then stays constant or decreases. The along-strike growth and linkage of this fault (seen at 20-30% extension, Fig. 7 a,b) is shown by increased displacement rate in the shaded region (Fault D, Fig. 8a) and increases in displacement rate for the orange, pink and black profiles between 5 and 6 Myr.

**Fault network statistics**

The maximum recorded displacement for six selected horizons in the crust (H52-H102) is shown Fig 9 at 500 kyr (2.5% extension) intervals from 2 to 6 Myr. Before 4 Myr (20% extension) the maximum displacement recorded on these horizons is depth invariant and ~ 600m. As rifting continues, the range of maximum recorded displacement increases to between 750 and 1500m, where the largest are focused around Horizon 72 (the depth of the centre of the largest faults). This suggests that small faults (displacement < 600 m) in the fault network accommodate the majority of the strain prior to 20% extension (4 Myr) distributed throughout the brittle layer and no large displacement (> 800 m) faults exist. The coalescence of these small, distributed, isolated faults into larger, crustal scale structures occurs between 20 and
25% extension where the maximum displacements on horizons start to localise around mid-crustal depths (H62-H82). The last million years (5% extension) is characterised by localisation of strain (and displacement) onto these large faults that bisect the upper crust.

The maximum displacement recorded on the 30 largest faults is plotted in log-log scale relative to elapsed time in Fig. 10 together with data from natural examples presented by Nicol et al. 1997. Similar to natural faults, data from the model results have a slope of 0.3138 ($r^2$=0.91). While the faults are still relatively small prior to 4 Myr, the data plots between 0.01 mm yr$^{-1}$ and 0.1 mm yr$^{-1}$, consistent with results shown in Fig. 9. After 4 Myr (dotted horizontal line, Fig. 10), the data plot mainly between rates of 0.1 mm yr$^{-1}$ and 1.0 mm yr$^{-1}$, indicating that the displacement rate has increased on these structures as they have linked to become larger faults.

**Distribution of linked conjugate faults in three dimensions**

To demonstrate the complexity of interactions between faults in three dimensions, a representative group of seven faults centred on Faults E and F (Region B in Fig. 4c) have been extracted (Fig. 11). This region contains three major faults >40 km long (Faults E, F and K) and four minor faults <20 km long (Faults G, H, I and J). The south-dipping faults (G, H and I) are antithetic to and intersect the larger north-dipping Fault E. Each of these faults is influenced by and interacts at depth with either of the north-dipping Faults J and K (Fig. 11 b,c). Few faults are linear along strike; most follow a curvilinear path from east to west controlled by their constituent fault segments. Intersections are at varying depth and not focussed in one horizon (white dotted lines, Fig. 11 a, d & e). At the end of rifting, displacement maxima for the two largest faults (E and F) are focussed towards their centre with depth. The relative position and nucleation time of neighbouring faults controls their down-dip and along-strike continuity (Fig. 11 d,e).

The spatial distribution with depth at the end of the experiment for Faults E to K is shown in Fig. 12. The major faults (E and F) are present on the upper horizon, where Fault E is represented by two separate ~20 km long segments to the west and east, divided along strike by Fault F. This is consistent with the gap in the upper elevation of Fault E in strike projection (Fig. 11 d,e). Fault F terminates against the
hanging wall of Fault E at its eastern tip (white circle, Fig. 12a, circle 3, Fig. 4). Depocentres associated with these faults are 750 m below the mean elevation, and prominent footwall crests have developed. The other major fault (K), to the south is represented by four segments at this elevation.

At 2.5 km depth, Fault E remains as two separate segments to the west and east with a number of smaller branches at its tip with the eastern segment terminating against Fault F (white star, Fig. 12b). At a depth of 5.1 km, Fault E is 60 km long and connects the two segments at higher horizons (Figs. 11d & 12c). It has a pronounced depocentres associated with the upper segments and a number of branches along its length (white stars, Fig. 12c). White circles along its length show where the tips of the south-dipping Faults G, H and I are in contact with it forming small grabens. Lateral continuity of Fault E is most pronounced at a depth of 7.8 km where antithetic Faults G, H and I are coincident with the strike of Fault E (white circles, Fig. 12d). At depths of 10-12 km, Fault E bifurcates again into two segments – one to the west (~15 km long) and the other to the east (~40 km long) – separated by a south-dipping fault indicated by an intermediate crest in the topography between segments (Fig. 12 e,f).

Fault F dips southward and is most laterally continuous in its upper region (Fig. 11e), with Fault H tracking it at a constant distance of around 8 km to the north (Fig. 12). With depth, Fault F interacts with Fault K to the south, which inhibits its propagation, and it branches into two segments (dotted lines and central gap, Figs. 11e and 12d-f). The mid-crustal section of Fault F is constrained to the east by the conjugate, north-dipping, Fault J (Fig. 12 c,d).

At the deepest horizon shown on Fig. 12 (-12.1 km), all seven faults (E-K) are present. The minor conjugate Faults H and J are in contact, Fault I cuts the eastern margin of the half-graben between Faults E and J and Fault G is at its maximum length (uninhibited by neighbouring faults), forming a ~20 km long graben with Fault K to the south (Fig. 12f).

Fault growth and interaction in three dimensions

The ability to extract displacement on elements with time means that the growth and interaction of faults can be investigated. Strike projections of displacement at 5%
intervals of extension from 15 to 30% are shown in Figs. 13-15, allowing fault nucleation and growth to be assessed. In the following section, we focus on Faults E and F (Figs. 7a, 11, 12), describing their evolution and interaction with adjacent faults in the network.

At 15% extension Fault E is composed of six small isolated fault patches (1-6; Fig. 13a) that have nucleated at varying depths in the upper brittle crust. Only two patches intersect the upper surface (2 and 5). At this stage in their evolution these patches have displacement maxima of 200-300 m occurring near their centres, and displacement decreases towards their tips (Fig. 16a). These patches range in strike length from 3.5 to 8 km, have aspect ratios (vertical:horizontal) from 1:1 to 2:1, and are separated along strike by relatively unfaulted regions 2 to 10 km wide. No faults initiate in the lower crust, although patches 3, 4 and 6 are located close to the upper-lower crust interface.

Between 15 and 20% extension, displacement maxima on these patches have increased to between 300 and 750 m. All fault patches have grown outward from their nucleation sites, and despite varying degrees of interaction, displacement maxima are still located at the centre of the six initial patches (light blue, Fig. 13b). Patches 1 to 4 show largely radial growth, and maintain similar aspect ratios, although downward propagation of fault patch 2 is inhibited by interaction with Fault G (cf. Figs. 13b & 14b). In contrast, patches 5 and 6 begin to hard link into a single 20 km fault segment. Patch 5 in particular shows preferential growth downward and eastward towards patch 6, and its displacement maximum has migrated deeper into the crust. In contrast, the western side of patch 5 remains relatively fixed in position due to interaction with Fault F (cf. Figs. 5a, 11a, 13b & 15b).

After 25% extension, Fault E comprises three major fault segments, 20–25 km long, with up to 1500 m displacement, separated by segment boundaries associated with marked displacement deficits (Figs. 13c, 16a). The three major fault segments have evolved by hard-linkage of earlier patches 1 and 2 (western segment), 3 and 4 (central segment) and 5 and 6 (eastern segment) (Fig. 13c). The western segment still has a significant displacement deficit at the former segment boundary between the two precursor patches (1 and 2). In contrast, displacement deficits associated with boundaries between the precursor segments of the central and eastern segments have largely been removed (Fig. 16a). The eastern and western segments
rupture the full thickness of the upper crust and extend into the lower crust, although downward propagation of the western segment is inhibited by Fault G (Fig. 14c). A more striking interaction that affects growth of Fault E is the upward propagation of its central segment. This segment of the fault remains blind. It only ruptures the lower half of the brittle crust, and its upward propagation is inhibited by the presence of Fault F above it to the south (cf. Figs. 11, 12a, 13c, 15c & 16).

At the end of rifting (30% extension), Fault E extends across the entire width of the model (Fig. 13d), and shows an overall increase in displacement rate compared to earlier in its evolution (Fig. 16a). Although it is hard-linked at depth, it is still blind along its central section, where it is overlain by Fault F and thus appears as two distinct faults separated along strike by over 20 km (cf. Fault E, Figs. 5a & 12a). Displacement maxima up to 2500 m occur at three locations along its length, coinciding with map-view locations of precursor patches 1, 3 and 5 but now localised near the base of the upper crust (Fig. 13c). Displacement minima are still evident at segment boundaries between the three segments present at 25% extension, although the displacement deficit between patches 1 and 2 in the western segment has been removed by this time (Fig. 16a).

The evolution of Fault F shares many similar characteristics with Fault E (Fig. 13). It initiates as five patches (7-11; Fig. 15a) that occur at a range of depths within the brittle crust and initially propagate outward and then link to form major fault segments. Patches 7 and 10 occur at mid- to deep-crustal levels, whereas patches 8, 9 and 11 are shallower (Fig. 15a). At mid- to deep-crustal levels, Fault F is cross-cut along its length by Fault K which accounts for gaps in its lateral continuity (e.g. above patch 7 and below patch 9, Figs. 11a & 12). Patches 8 and 9 become hard-linked by 20% extension and the boundary between them loses its displacement deficit by 25% extension (Figs. 15 b,c & 16c). In contrast, linkage between patches 7 and 8 occurs later, with rapid accrual of displacement on patches 7 and 8 between 27.5 and 30% extension (Fig. 16c). On the eastern side of Fault F, patches 10 and 11 link between 15 and 20% extension, and the displacement deficit between them is lost by 22.5%, giving rise to a single prominent displacement maximum (Figs. 15c & 16c). This growth and linkage history creates two major fault segments for Fault F by 25% extension with a major displacement low between precursor patches 9 and 10 that exists throughout the evolution of the model (Fig. 16c). This long-lived
displacement low is due to interaction with the underlying Fault K, dipping from the north. As with Fault E, at 30% extension, displacement is localised towards the middle of the fault at the base of the brittle layer (e.g. Fig. 15d).

The evolution of other faults in the region surrounding Faults E and F, i.e. Faults G, H, I and J, is shown in Figs. 14, 15 and 16b,c. The main difference between these faults and Faults E and F is that their lengths are fixed relatively early during extension. Fault G initiates at a mid-crustal depth, similar in depth to patches 3 and 4 on Fault E (cf. Figs. 13a & 14a). It propagates down dip with a small patch of slip propagating up dip toward Fault E between patches 2 and 3, but it does not propagate upward and break the surface because it lives in the stress shadow of neighbouring faults to the north. The displacement maximum for Fault G (1400 m) is on the upper, western part of its surface, rather than lying centrally, a function of it abutting and interacting with the blue fault. Similar to Faults E and F, Fault G shows an increase in displacement rate between 20% and 30% extension (Fig. 16b). Fault H initiates in the upper part of the brittle layer and propagates largely down dip until 20% extension. Its length then increases from 15 to 24 km during the final 10% extension by near-surface westward propagation. In contrast, Faults I and J show a different style of evolution marked by an overall decrease in displacement rate through time, particularly during the last 5–10% extension associated with their tip restriction against the major faults, E and F. This is the time when the major faults display an increase in displacement rate (Fig. 16b).

The displacement profiles of Faults E to J through time illustrate a number of features of the development of the fault network (Fig. 16). The final along-strike profile for Fault E shows two displacement minima at distances of 20 km (~ 500 m, between patches 2 and 3, Fig. 16a) and 40 km (~1 km, between patches 4 and 5, Fig. 16a). The westernmost minimum is associated with interaction and displacement relative to the antithetic Fault G (Figs. 11d, 12d & 16b). The reduction around 40 km (patches 4 and 5, Fig. 16a) is coincident with the surface expression of Fault F and Fault H to the north (patches 10 and 11, Fig. 16c). The final profile of Fault G represents an isolated fault with maximum displacement at the centre diminishing to the tips. The minor faults (H, I and J) show asymmetry associated with single tip restrictions. The displacement profiles of Fault E (patches 3-6) and Fault F (patches 7-9) have steep shoulders associated with faults restricting their
propagation to both the east and west despite hard-linking to neighbouring sections through displacement minima along strike (Fig. 16a & c).

**DISCUSSION**

In this model of a single phase of extensional rifting we have observed four stages of fault growth and interaction. We term these stages: (1) nucleation, (2) propagation and interaction, (3) linkage and domain fixing, and (4) localisation (Fig. 17).

In Stage 1 (Fig. 17a), a large number of small conjugate faults (< 5 km in length) nucleate rapidly, at varying depths in the crust. These faults develop as isolated fault segments that have elliptical displacement contours with displacement maxima at their centres, striking sub-perpendicular to the extension direction. In Stage 2 (Fig. 17b), these isolated fault segments propagate and interact with one another. Individual segments that are co-linear begin to link along strike or down dip (lengths ~ 15 km), and relay ramps develop at fault tips as a result of soft-linking of fault segments. Where faults have opposing dip, lateral propagation is inhibited. By Stage 3 (Fig. 17c), distinct grabens and half-grabens are developed in the hangingwalls of larger faults. These larger faults (lengths ~ 20 km) formed by the linkage of co-linear fault segments. At the fault network scale, dip domains become well established and their boundaries become fixed. The boundaries between the dip domains form narrow zones where conjugate faults interact and lateral propagation is inhibited. In the final stage of fault network evolution, Stage 4 (Fig. 17d), activity is localised on a small number of large faults (lengths > 40 km) that cut across the entire upper, brittle crust. Some faults that were initially dominant become inactive at this time. These are generally located in the strain shadow of faults that grow more rapidly during this stage of fault network evolution. In this final stage, a pronounced increase in displacement rate on the large active faults is observed and displacement maxima migrate to their centres.

In the following sections we discuss the results of our 3D numerical modelling in comparison to natural examples and other analogue and numerical models of fault growth. We specifically discuss: i) comparison to natural and analogue fault systems, ii) fault growth, propagation and linkage, and iii) fault network and dip domain development.
Comparison to natural and analogue fault systems

Many of the characteristics of the growth and linkage of the normal fault segments reported here are comparable to those from other modelling approaches (e.g. Cowie et al. 2000) and from studies of natural fault systems in the subsurface (e.g. Young et al. 2001) and outcrop datasets (e.g. Gawthorpe et al. 2003). The strain rate in the model decreases with time from $1.6 \times 10^{-15}$ s$^{-1}$ to $1.24 \times 10^{-15}$ s$^{-1}$ and is comparable with the natural strain rate observed in rifts such as the Gulf Coast and Aegean (Fig. 18). The displacement rates for the 30 largest faults in the model plot are similar to the natural data. Nicol et al., 1997 demonstrated that there is a broad correlation between regional strain rate and fault displacement rates suggesting increased strain rate is accommodated by increased displacement rate rather than an increased number of faults. This is observed in our model, where strain localises onto the largest faults as rifting progresses.

In addition to the growth and linkage of fault segments, a number of other features of the model replicate features described in other studies of normal fault geometry and growth. These include the spacing of faults and fault interactions, the form of displacement profiles along the faults, and fault scaling relationships.

Fault spacing parallel to the extension direction is on the order of 5-20 km after 30% extension, and relay breaching occurs at 10-20% extension where fault tip separations are <3 km (Fig. 4b). Fossen & Rotevatn (2016) suggest that where the thickness of the brittle upper crust is 10-15 km, as in this model, spacing between major extensional faults is expected to be around 5 km. Similar thickness to spacing ratios are observed in a range of natural examples and in scaled sandbox experiments (Morley 2002; Hus et al. 2005; Conneally et al. 2014; Nixon et al. 2014b; Whipp et al. this volume). This shows that although the methodology used for the underlying physics in this model is a simplified representation, the spatial distribution of faults is consistent with natural datasets and scaled analogue experiments.

The faults developed in the model display a variety of displacement profiles that are related to the degree of interaction between faults. Where faults are isolated and fault tips are unrestricted, symmetrical (e.g. Fault C, Fig. 7b) or triangular profiles
Profiles representing single-tip restricted (e.g. Fault B, Fig. 7b) and double-tip restricted cases (e.g. Fault A, Fig. 7b) have asymmetric and mesa-style profiles respectively. These profiles are consistent with those interpreted in natural data sets and physical analogue models (e.g. Manighetti et al. 2005; Schlagenhauf et al. 2008, fig. 2; Nixon et al. 2011, Fig. 7; Nixon et al. 2014b, Fig. 17; Fossen & Rotevatn 2016).

Fault growth, propagation and linkage
During the initial stages of model extension, faults initiate in conjugate orientations throughout the upper brittle crust. With increasing extension these small, isolated faults grow and coalesce into larger faults through along-strike and down-dip linkage (Figs 13-16). One major advantage of the numerical modelling approach is that the three-dimensional distribution of displacement during the growth and linkage process can be assessed. We see multiple transient displacement maxima that continuously migrate as different fault segments progressively interact and link to create the final fault plane. Fossilised examples of this transient behaviour are recorded in nature from the Funan Field, Pattani Basin, Gulf of Thailand, where Kornsawan & Morley (2002) observed two displacement maxima on a fault plane and inferred this to indicate vertical linkage of two faults in three dimensions. In the model, many of the early displacement maxima associated with initially isolated fault segments are lost by overprinting during the growth and linkage process. For example, initial small fault patches are distributed at various depths in the crust. As these patches propagate laterally and link, the information associated with their nucleation site is removed from the record as displacement maxima migrate each time linkage occurs to the centre of the new, larger fault segment (Figs. 12-15). Capturing evidence for these small displacement features at depth within the crust is difficult when presented with only the final distribution. These observations suggest caution should be used when employing the final displacement-distance distribution, either from displacement-distance (D-x) plots or strike projections of fault displacement, to reconstruct the initial segmentation or the progressive linkage history of a fault. An assumption that the final displacement maximum on a fault plane is coincident with the nucleation point of the fault may be misleading. A further implication of the 3D linkage of segments observed in the model is that the fault network in any one horizon should
not be considered representative of the total network, either in spatial distribution and interaction of faults or displacement profiles.

The style of fault initiation and growth developed in the model in the early stages (up to 20% extension) is consistent with the model of isolated fault growth (Cowie et al. 2000; Schlagenhauf et al. 2008) as opposed to the coherent fault growth model (Walsh et al. 2003b). The observed fault networks develop from fault segments that are initially kinematically independent and propagate radially before linking with adjacent segments to become larger fault zones (Stages 1-3, Fig. 17). In the final stage of the model, however, faults are kinematically linked at depth and there is a heterogeneous distribution of fault activity focussed on the largest faults which is more consistent with the coherent fault growth model (Stage 4, Fig. 17). Examination of fault growth for selected faults at the surface (Figs. 7 & 8) and the displacement history on strike sections of selected faults (Figs. 13-16) shows that in this final stage, when strain is localising onto the largest faults and relays are being breached, the majority of faults are mainly accruing displacement without further increasing their length. This is consistent with the notion that the two fault growth models are end-members of one system (Fossen & Rotevatn, 2016), where initially the isolated fault growth model dominates at small strains which is then replaced by the coherent fault growth model at greater strains (Jackson et al., this volume).

**Fault network organisation and dip domain development**

Kinematic coherency is maintained in the model by a combination of variation in both the number of active faults and displacement on those faults. Some transects parallel to the extension direction contain many active faults, whereas other have only two or three, but with larger displacements (Fig. 5). This is similar to natural examples, as shown by linkage of the Rangitaiki Fault (Nixon et al. 2014a) and palaeo-earthquake data (Nicol et al. 2010). Analysis of the model evolution, however, highlights the dynamic nature of fault activity across the fault network. Nicol et al. (1997) suggested that the largest faults at the early stages of extension within a network maintain their relative size advantage during rifting and are the largest faults during rift climax. The results documented here suggest this is not necessarily the case. Many of the faults with highest displacement rates during the onset of rifting
(e.g. Stages 1 and 2) do not continue to dominate but may decelerate or become inactive (e.g. Fault C, Figs. 5 & 6). The largest faults at the end of rifting (i.e. Stage 4) are the ones that are in the most favourable locations with respect to stress interactions during later stages of rifting, which is not necessarily the same as during rift initiation.

Observations from natural systems and data from physical analogue experiments indicate that a characteristic feature of rifted fault networks is the occurrence of dip domains within which the dominant fault dip is similar, separated by a rift-wide accommodation zone across which the dominant dip direction changes (e.g. Reches 1978; Morley 2002, McClay et al. 2005; Schlagenhauf et al. 2008; Schlische & Withjack 2009; Healy et al. 2015). Dip domains are a characteristic feature of the fault network developed in this model and are typically of the order of 20-30 km wide. In contrast, dip domain boundaries are narrow regions characterised by interlocking arrays of oppositely dipping faults with slightly overlapping tips (Fig. 4c). In many studies, zones of weakness such as basement lineaments are cited as the control on the location of dip domain boundaries, for example the accommodation zones in the Suez rift (e.g. Patton et al. 1994) and the Northern North Sea rift (e.g. Fossen et al. this volume). However, in the model presented here, dip domains and their boundaries develop spontaneously as a result of the interaction in three-dimensions between evolving conjugate fault sets. There are no pre-existing lineaments or weaknesses controlling the location of dip domain boundaries in this model but they are still present.

**CONCLUSIONS**

A three-dimensional discrete element model of fault growth has been used to investigate the nucleation, growth and interaction of normal fault segments and fault network evolution during a single phase of extension. The modelled fault geometry, fault spacing, displacement rates and scaling relationships are consistent with natural examples. In addition, the ability to investigate the evolution of the fault network in three dimensions has demonstrated a number of points that should be taken into consideration when examining the final static fault segment and fault network geometry in outcrop or subsurface (e.g. seismic) datasets.
1. Faults nucleate at minor heterogeneities throughout the brittle, upper crust to form a large number of small faults in conjugate arrangements during the initial 10% of extension. These initial faults are kinematically isolated and grow by radial propagation.

2. Faults dominantly grow through a combination of tip propagation and linkage of co-linear fault segments to form larger normal faults. Segment linkage occurs progressively with increasing extension and involves linkage between adjacent segments that are located along strike, down dip or oblique to one another. Segment propagation and linkage is inhibited by either the presence of neighbouring faults with an opposing dip, or tip propagation into the stress shadow of a neighbouring, more dominant, fault.

3. The observed style of fault growth is consistent with the isolated fault growth model in the early stages of rifting (before 20% extension). This is replaced by a style akin to the coherent fault growth model once strain localises onto the largest faults (after 20% extension in this case). Further increase in fault length is limited by neighbouring faults and dip domain boundaries, at which point the faults continue to accrue displacement without lengthening.

4. Displacement distribution on the faults is highly dynamic, with displacement maxima that continuously migrate as fault segments interact and link to generate the final fault plane. Displacement maxima associated with initially isolated fault segments are generally lost by overprinting during the growth and linkage process. These observations suggest caution when using the final displacement-distance distribution alone to reconstruct either the earliest fault network or the progressive linkage history.

5. Fault scaling statistics for the evolving fault network demonstrate the effect of localisation within the system. Small faults coalesce to become larger faults and the relative importance of small faults in accommodating extensional strain decreases with time. This is achieved by progressive localisation and accelerated displacement rates on larger faults and is not associated with an increase in the applied strain rate.

6. Faults show marked fluctuations in activity related to their relative position within the evolving network and the resultant changes in stress interactions. Faults that dominate the later stages of rifting are those that have optimal locations within
the late stage fault network. These dominant late-stage faults are not necessarily the faults with highest displacement rates during the early stages of rifting.

7. Dip domains, 20-30 km wide and comprising faults that have a dominantly similar dip, are a characteristic feature of the fault network developed in the model. Boundaries between dip domains are narrow regions (< 5 km wide) characterised by interlocking arrays of oppositely dipping faults with slightly overlapping tips. Dip domains and their boundaries are a natural result of the growth of the fault network and are not related to underlying basement lineaments or heterogeneity.

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FIGURE CAPTIONS

Fig. 1. (a) Example of the discrete element media consisting of 1,080,000 elements in a regular hexagonal distribution used to simulate extension in a rift setting. The crust is represented as a two-layer system of brittle (upper crust) and firmoviscous (lower crust) forces. Elements are coloured according to their horizon at the start of the experiment. (b) Sketch of the forces within the upper and lower crust and their rheological response. In the upper crust, the bond fails when a breaking threshold is exceeded. In the lower crust, strain increases when a load is applied. At t1 the load is removed and strain reduces slowly over time. (c) View of model from eastern aspect illustrating horizons used in analysis (H52-102) and the viscous response to load in the lower crust (wavy nature of dark grey marker lines).

Fig. 2. (a) Depth of broken bonds in the brittle crust in the experiment at the start of the experiment. (b) Magnitude and direction of displacement (m) for failures shown in (a).

Fig. 3. (a) Oblique three-dimensional view of along-strike complexity relative to elevation for the fault network at 30% extension relative to initially horizontal beds within the crust (Horizon 52 (H52), Horizon 72 (H72) and Horizon 92 (H92)). Contours on horizons denote 100m intervals. The extension direction is towards the north. Displacement on elements that constitute south dipping faults are coloured yellow to red (2 km) and north dipping faults are coloured pale blue to purple (2 km).
Interactions between selected conjugate faults are shown by numbers 1-3. (b) View of the fault network looking east (along-strike). (c) Topography of Horizon 102 at 30% extension. Contours are at 100m intervals, with an elevation range of 1 km scaled from blue (deep) to orange (high). The extracted fault network is included and marked by dark grey patches. Interaction between conjugate faults marked in (a) are also shown on this surface.

**Fig. 4.** Map views of Horizon 102 output at increasing extension of 10%, 20% and 30% to the north. Three regions (A-C) are shown which are used for discussion. (a) Location of faults relative to fault activity and failure time of bonds. (b) Relative topography from blue (minimum) to orange (maximum) overlain with fault locations from (a). $R$ denotes the relief range for each output; 100 m (10% extension), 500 m (20% extension) and 1 km (30% extension). Footwall crests are orange and hangingwall deeps are purple. (c) Dip orientation for faults. South-dipping planes are coloured yellow to red and north-dipping pale to dark blue. Strong colours indicate fault scarps and pale colours between them represent hangingwall and footwall surfaces dipping in the opposite direction. The dominant direction of fault dip shows alignments of both south-dipping faults (red) and north-dipping faults (blue) into dip domains with boundaries indicated by dashed black lines. Black arrows indicate dominant dip directions. Regions B and C marked on this figure are referred to later in Figs. 7 & 8 (Region C) and Figs. 11-16 (Region B).

**Fig. 5.** (a) Map of fault locations on Horizon 102 at 30% extension (6 Myr). Dotted vertical lines indicate the location of transects through the data at 6000 m intervals. Red dots indicate the location of maximum displacement on each transect. Specific faults are coloured to highlight their evolution; Fault B (yellow), Fault C (purple), Fault D (brown), Fault E (blue) and Fault F (green) (b) Evolution of displacement on faults through time along selected transects chosen to highlight the evolution of the fault network and dip domains, numbered 1 to 5. Bold dashed lines represent faults that show periods of retardation within their history and bold solid lines show growing faults.

**Fig. 6.** Data for Transects 1-5 from Fig. 5a plotted relative to (a) Percentage strain accommodated by faults against time by north-dipping (open circles) and south-dipping faults (closed squares). (b) Polarity plotted against time for the selected transects.

**Fig. 7.** (a) Map view of Region A highlighted in Fig. 9 through time from 15% to 30% extension (3-6 Myr). Traces of Faults A-D are highlighted and correlate to those in Fig. 8a. Number 2 indicates the spill point shown in Fig. 4 in this region. (b) Displacement-distance plots on Faults A-D at the upper surface. Grayscale fill indicates the time of each recording in 1 Myr (5% extension) intervals.

**Fig 8.** (a) Displacement rate in 1 Myr intervals plotted against distance for Faults A-D from Fig. 7. Selected profiles are marked by vertical lines and coloured with dots. (b) Displacement rate against time for profiles marked along the faults in (a), where colours of inter-section correlate between (a) and (b).
**Fig. 9.** Maximum displacement (throw) recorded for selected horizons (H52-102) plotted relative to extension. Intervals between pairs of solid or dashed lines represent 5.0% extension (1 Myr).

**Fig. 10.** Log-log plot of elapsed time against maximum displacement for the largest thirty faults from the model (stars) plotted relative to data from three offshore and three onshore regions presented in Nicol et al. 1997 (fig. 2). Diagonal lines represent expected fault growth rates. The dotted horizontal line corresponds to 4 Myr.

**Fig. 11.** Three-dimensional views representing seven faults in Region B, Fig. 4. Faults are coloured to aid differentiation and labelled E to K. White arrows indicate the dip direction of faults from their upper surfaces. Dotted white lines indicate intersections between faults. (a) Oblique map view including Horizon 62 with views looking (b) east and (c) west. Views of selected faults from the south (including Horizon 62) with colour-coded displacement on (d) Fault E and (e) Fault F. Displacement ranges from purple (50 m) to red (2 km). Minor Faults G to J are included and dotted white lines demonstrate intersections between faults. The white arrows denote faults dipping towards the viewer.

**Fig. 12.** Map view of elements that constitute faults shown in Fig. 12 relative to depth for (a) Horizon 102, (b) Horizon 92, (c) Horizon 82, (d) Horizon 72, (e) Horizon 62 and (f) Horizon 52. Maps are coloured from brown to cream ±750 m about their respective mean elevation. Footwall crests are cream and hangingwall depocentres are dark brown. Faults E to K at each horizon are included as their colour defined in the key (see also Fig. 11). Northern dipping faults are drawn in the immediate hangingwall and southern dipping faults are drawn on the fault crest. The dip direction of fault segments is indicated by white arrows. White circles highlight points of intersection between faults and white stars show fault bifurcation.

**Fig. 13.** Strike projection viewed from the south for Fault E at (a) 15%, (b) 20%, (c) 25% and (d) 30% extension coloured relative to displacement from 50 m (purple) to 2 km (red). Elements coloured grey represent regions where the fault has not, as yet, failed. The fault evolved from six distinct isolated segments (numbered 1-6) into one structure. The location of the change between the upper and lower crust is indicated.

**Fig. 14.** Strike projection viewed from the south for Faults G, H and I at (a) 15%, (b) 20%, (c) 25% and (d) 30% extension coloured relative to displacement from 50 m (purple) to 2 km (red). Elements coloured grey represent regions where a fault has not, as yet, failed. The location of the change between the upper and lower crust is indicated. Gaps between elements indicate where Fault E (Figs. 11 & 13) intersects these faults.

**Fig. 15.** Strike projection viewed from the south for Faults F and J which interact as a conjugate pair at (a) 15%, (b) 20%, (c) 25% and (d) 30% extension coloured relative to displacement from 50 m (purple) to 2 km (red). Elements coloured grey represent regions where the fault has not, as yet, failed. Fault F evolved from five distinct
isolated patches (numbered 7-11) into one fault. The faults interact at the yellow
dashed line.

**Fig. 16.** Maximum displacement recorded in 500m vertical swaths for elements that
represent a fault versus length. These are extracted at extension increments of 2.5%
from 10% to 30% extension for (a) Fault E, (b) Faults G, H and I and (c) Faults F and
J. Faults are coloured relative to earlier definitions in Fig. 11. Alternating light and
dark patches indicate bands of 2.5% extension between outputs. Fault segments
introduced in Figs. 12 & 15 are marked for comparison.

**Fig. 17.** Sketch representation of fault network and fault segment evolution in a
schematic basin through four stages. Fault network evolution is shown in relation to
a network of conjugate faults dipping towards (black) and away from (grey) the
extension direction in three dimensions. The strike projection of fault segment
evolution shows an idealised development of a large fault through time from a
number of smaller isolated segments at varying depths into a single through-going
structure. Its' lateral propagation is constrained at the furthest edge by a conjugate
fault that initiates at a similar time dipping in the opposite direction.

**Fig. 18.** Log-log plots of displacement rate plotted against regional strain following
figure 4 of Nicol *et al.* (1997) to compare with this experiment. The circle represents
the mean displacement rate plotted for large faults with displacement >500 m in this
model with error bars (see Fig. 6).

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Fig. 1

(a) 20 km 20 km
SOUTH NORTH
Upper crust
Lower crust

Gravity
Fault Scarp
Extension

Gravity
Fault Scarp
Extension

(b) BRITTLE
Upper crust
\[ \mu_k \]
Strain

FIRMOMVISCOUS
Lower crust
\[ \eta_k, \mu_k \]
Strain

(c) Extension
Horizons

Fig. 1
Fig. 2

(a) Upper surface
Upper-lower crust boundary

(b) North
South
Throw (m)

Percentage Extension
Depth

Depth
Percentage Extension

2.125 2.25 2.25 2.375 2.5 2.625

7.5

2.125 2.25 2.375 2.5 2.625 2.75
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9

Horizon vs. Maximum Displacement (m) for different times: 10, 15, 20, 25, and 30% of 6 Myr.
This data (N=300)

Fig. 10
Fig. 11
Fig. 12

(a) **Upper Surface (Horizon 102)**

(b) **Horizon 92 (-2.5 km)**

(c) **Horizon 82 (-5.1 km)**

(d) **Horizon 72 (-7.8 km)**

(e) **Horizon 62 (-10.2 km)**

(f) **Horizon 52 (-12.1 km)**
Fig. 13
Fig. 14
Fig. 15
Fig. 16
**Fault Network Evolution**

**a) Stage 1: Nucleation (<10%)**
Large number of small faults initiate at varying depth within the crust orientated in a conjugate system

**b) Stage 2: Propagation (10 - 15%)**
Isolated faults growing and interacting dip domains fixing due to fault nucleation in opposing orientations

**c) Stage 3: Domain fixing and linkage (15 - 20%)**
Grabens and tilted half-grabens form Relays between similar and opposite dipping faults Along-strike linkage through relay zones and initial breaching Similar dipping faults coalesce down dip planes

**d) Stage 4: Localisation (20 - 30%)**
Extension localises onto larger faults Some faults inactive in strain shadow of larger, more dominant neighbours Along-strike linkage of dip-orientated faults through breached relays Faults linked along strike and at depth

Oppositely dipping faults inhibit lateral propagation

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**Fig. 17**
Fig. 18

Strain rate ($10^{-16}$s$^{-1}$)

Displacement rate (mm yr$^{-1}$)

Aegean

Timor Sea

North Sea

Gulf Coast

Basin & Range

Kenya Rift

Model data