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DOI:

[10.1061/\(ASCE\)AE.1943-5568.0000255](https://doi.org/10.1061/(ASCE)AE.1943-5568.0000255)

Document Version

Accepted author manuscript

[Link to publication record in Manchester Research Explorer](#)

Citation for published version (APA):

Riddell-Smith, L., Cunningham, L., & Mandal, P. (2017). Design of glass elements for lateral torsional buckling: A review of existing approaches. *Journal of Architectural Engineering*, 23(3), 1-9.
[https://doi.org/10.1061/\(ASCE\)AE.1943-5568.0000255](https://doi.org/10.1061/(ASCE)AE.1943-5568.0000255)

Published in:

Journal of Architectural Engineering

Citing this paper

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Design of glass elements for lateral torsional buckling: A review of existing approaches

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Abstract

Structural glass is being used in increasingly challenging applications by engineers and architects; however there is currently no universally adopted analytical method for design of monolithic slender glass elements subject to lateral-torsional buckling. As a result designers must use guidance from several different sources. In this study, a number of prevalent standards and design guides are examined and design calculations undertaken using the methods proposed therein. The design results are compared to experimental data from literature in order to assess the level of conservatism in each method. A noticeable difference is observed among the investigated approaches, with some potentially unsafe estimations of design buckling capacity. Local tensile stresses are investigated as well as global buckling behavior, as this is an important consideration when designing brittle glass elements. Finally, a parametric study is undertaken to assess the influence of key variables on the design solutions.

Introduction

In recent years the use of glass in construction has been on the rise, with 90% of all float glass produced worldwide being used in buildings and a significant growth in demand for flat glass (Feldmann 2014). The construction industry is also seeing glass moving forward from its traditional use in fenestration to use in load-bearing elements in ever more challenging forms. The Glass Lantern structure in Istanbul (Purcher 2014) is an example of the limits to which structural glass is now being pushed. The top hung facade incorporating glass fins in Fig 1 is a commonplace example of what can be achieved using structural glass.

Fig 1: Glazed Façade, Manchester, UK. Image courtesy of Alex Chow

However, the behaviour of glass in structural applications is still generally not well understood. One area of behaviour which is critical for designing load-bearing glass elements such as the fins in Fig 1 is lateral-torsional buckling (LTB). Slender glass beams and fins commonly used in facades and canopies are susceptible to LTB and accurate prediction of the resistance of the section is important for conservative yet economical design. However, no universally adopted analytical method for this exists.

In Europe, work is currently in progress to produce a pan-European standard (as part of the EN Eurocodes series) for structural design using glass. A number of standards and design guides currently exist; however, these publications are not comprehensive and have differing theoretical basis for the methods proposed therein. As a result, designers often resort to using guidance from several different sources (Honfi 2013). Designers also currently use prototype testing and numerical modelling, but these methods can be time consuming and expensive therefore a reliable analytical method is desirable.

In the present study, the methods outlined in various prevalent codes and publications to estimate the design moment of resistance to LTB for monolithic glass sections are examined and compared. The codes and publications that have been considered are given in Table 1.

Table 1: Standards and Design Guides

Publication	Type	Country/Region of Origin	Year Published	Notes
prEN 13474-1 (CEN 1999)	Draft standard	Europe	1999	Now superseded by prEN 16612
EC3 ¹ (BSI 2005)	Standard	Europe	2005	For design using steel
AS1288 (Standards Australia 2006)	Standard	Australia	2006	
Structural Use of Glass, IABSE ² (Haldimann 2008)	Design guide	Switzerland	2008	
CNR-DT-210 (CNR 2013)	Technical document	Italy	2013	
prEN 16612 (CEN 2013)	Draft standard	Europe	2013	
Structural Use of Glass in Buildings, IStructE ³ (O'Regan 2014)	Design guide	UK	2014	
Design buckling curves for glass beams and columns (Bedon 2015a)	Journal article	Italy	2015	

¹ BS EN 1993-1-1:2005

² International Association for Bridge and Structural Engineering

³ Institution of Structural Engineers

ASTM E1300 (ASTM 2012), the prevalent standard for design of structural glass in the USA, is intended for design of glass in out-of-plane loading only and as such offers no specific guidance on glass sections in LTB. It is therefore not considered here.

Note that some symbols in the following section have been changed from the original source for consistency of notation.

Calculating LTB Resistance

In this paper the term M_{cr} represents the elastic critical buckling moment – the theoretical moment at which buckling will occur in a member, neglecting any material or geometric imperfection effects and assuming fully elastic behaviour. This gives a good indication of the real buckling load of a structural member, however in reality material and geometric imperfections may be significant and therefore any design method should include some factor of safety against these effects (Mandal 2002).

The approach adopted in EC3 (BS EN 1993-1-1) is to apply a ‘buckling reduction factor’ χ_{LT} to account for geometric imperfections as well as a material partial safety factor. The buckling reduction factor is strongly dependent on the slenderness ratio $\bar{\lambda}$, which in turn is derived from the elastic critical buckling moment of the section M_{cr} (see equations (1) and(2) below). The parameter α_{LT} is a geometric imperfection factor which relates to a buckling curve – a plot of slenderness ratio against buckling reduction factor, calibrated for a specific material or section type.

The key design result is $M_{LT,Rd}$ (hereafter referred to as M_{Rd}). This is the maximum moment that a member may be subjected to without inducing LTB.

$$M_{Rd} = \chi_{LT} \frac{f_y W_y}{\gamma_m} \quad (1)$$

$$\chi_{LT} = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad \text{Where: } \phi = 0.5[1 + \alpha_{LT}(\bar{\lambda} - 0.2) + \bar{\lambda}^2] \quad (2)$$

The same procedure adopted by EC3 may be reasonably applied to monolithic glass sections, as glass is essentially linear-elastic up to the point of fracture. In fact, a number of recent studies on lateral torsional buckling of glass sections use an equation for calculating M_{cr} taken from EC3 and have found this method to give reasonably accurate results compared with experimentally determined buckling loads for laminated glass beams (Belis 2013; Bedon 2014). A key departure from EC3 design method when it is applied to glass is that the design resistance of the section is based on the tensile fracture strength of the material rather than the yield strength used for steel sections (which indicates the limit of elastic deformation). This is due to the brittle nature of glass, which dictates its mode of failure.

The 2008 IABSE design guide *Structural Use of Glass* recommends using this method. M_{cr} is calculated using equation (3), which is very similar in form to the equation for M_{cr} from EC3. In this equation M_{cr} is a function of: the bending stiffness of the section about the minor axis (EI_z); the torsional stiffness of the section (GJ); the point of load application (z_a) and loading and support conditions (C_1, C_2).

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \left[\sqrt{C_2 z_a + \frac{GJ L^2}{\pi^2 EI_z}} + C_2 z_a \right] \quad (3)$$

In the IABSE guide, the buckling reduction factor χ_{LT} is described as a function of the slenderness ratio $\bar{\lambda}$ but the relationship between the two variables is not given. The guide suggests that a conservative approach may be to use buckling curve (c) of BS EN 1993-1-1, based on a study by Luible (Luible 2004). For this approach equation (2) is used, with a value of 0.49 is used for α_{LT} .

The same approach to calculate the buckling reduction factor has also been proposed by Bedon and Amadio (Bedon 2015a), who have calibrated the method for glass elements using experimental and numerical data. They recommend a value of $\alpha_{LT} = 0.45$, based on glass elements with an initial bow imperfection of $L/400$. This magnitude of imperfection was proposed by Belis et al (Belis 2011) based on measurements taken from 312 glass specimens from various manufacturers.

CNR-DT-210 gives a complete method for calculating the design resistance to LTB for glass elements. This method gives the equation for M_{cr} in a slightly different form (see equation (4), although the main variables are the same.

$$M_{cr} = C_1 \frac{\pi}{L} \sqrt{EI_z GJ} \quad (4)$$

CNR-DT-210 also uses equation 3 from BS EN 1993-1-1 to calculate the buckling reduction factor, but uses a value of $\alpha_{LT} = 0.26$.

The Australian standard AS1288 offers a simpler, alternative method to calculate the LTB resistance of a glass element. The design moment of resistance to LTB is defined as M_{cr} divided by **one single** safety factor of 1.14. M_{cr} is calculated using equation (5).

$$M_{Rd} = \frac{M_{cr}}{1.14} \quad \text{Where: } M_{cr} = \left(\frac{g_2}{L} \right) [(EI)_z (G)]^{1/2} \left[1 - g_3 \left(\frac{z_a}{L} \right) \left(\frac{(EI)_z}{(G)} \right) \right]^{1/2} \quad (5)$$

Again, the equation for M_{cr} is in a different form, but is still dependent on EI_z , GJ and loading and support conditions (g_2 , g_3).

The AS1288 method is also recommended in the 2014 IStructE publication *Structural Use of Glass in Buildings* (2nd edition) (O'Regan 2014).

For all methods except AS1288, M_{Rd} is calculated using equation 1 in accordance with EC3 or CNR-DT-210.

Calculating Design Tensile Strength

Chapter 3 of the IABSE guide recommends a model based on fracture mechanics and statistical analysis of flaw distribution to calculate the design strength of glass f_d . Although this may give a more accurate estimate of fracture strength, this method is very complex, and therefore is not suitable for everyday design. The reader may then refer to Chapter 4 of the guide, which contains a review of existing simplified approaches. The method which is perhaps the most widely adopted of all those mentioned in the IABSE guide (as it was published in the draft standard as opposed to research papers) is that outlined in prEN 13474-1 and given in equation (6). In this method the material characteristic tensile strength f_k is combined with a number of modification or 'k' factors to account for various parameters affecting the strength of the section and then divided by a material safety factor γ_m .

$$f_d = \frac{k_{mod} f_{g,k}}{\gamma_m k_A} + \frac{(f_{b,k} - f_{g,k})}{\gamma_{m,v}} \quad (6)$$

The size effect factor k_A accounts for the increased probability of a critical flaw being present in larger pieces of glass. The factor k_{mod} accounts for environmental conditions and load duration.

prEN 13474 has been superseded by prEN 16612 since the IABSE guide was published. The general method for calculating the design tensile strength does not differ between the two versions, but prEN 16612 uses a slightly different form of the above equation with additional k factors introduced to account for the glass surface profile and strengthening treatment if applicable. The material safety factors also differ between the two versions. For the sake of using as much information from a single source as possible, the equations and safety factors suggested in prEN 13474 (1999) will be used in combination with the IABSE method here.

CNR-DT-210 also uses a similar method to calculate the design tensile strength of the glass, but a number of additional reduction factors are introduced (see equation (7).

$$f_{g,d} = \frac{k_{mod} k_{ed} k_{sf} \lambda_{gA} \lambda_{gL} f_{g,k}}{R_m \gamma_m} + \frac{k'_{ed} k_v (f_{b,k} - f_{g,k})}{R_{m,v} \gamma_{m,v}} \quad (7)$$

Here, the size effect is broken down into two factors: λ_{gA} accounts for the surface area of the glass and λ_{gL} accounts for the length of the edge of the glass section. The additional factors k_{ed} and k_{sf} account for edge finish and surface profile respectively. R_m is a partial safety factor depending on the type of structure.

Partial Safety Factors

In addition to the number of different methods for calculating LTB resistance moment, there is disparity between the partial safety factors recommended by the various publications. These should be considered within the context of each code or design guide as they contribute to, but do not solely determine, the overall factor of safety incorporated into the design resistance calculated.

Table 2: Glass Material Safety Factors

Source	γ_m (annealed glass)	$\gamma_{m,v}$ (toughened glass)
prEN 13474-1	1.8	2.3
CNR-DT-210	2.5	1.35
IStructE	1.6	1.2

Note: The AS1288 method for determining the LTB resistance of glass sections does not include a material safety factor, and instead suggests the designer multiply the elastic critical buckling moment M_{cr} by a factor of $\gamma = 1.14$, and use this as the design buckling capacity.

Table 3: Load Safety Factors – Wind Uplift Case

Origin of standard	γ_G (permanent actions)	γ_Q (variable actions)	Ψ_0 (accompanying variable)
Europe (UK National Annex) (BSI 2005)	1.35 unfavourable 1.0 favourable	1.5 leading	0.7 roof imposed 0.5 snow
Australia (Standards Australia 2002)	1.2 destabilising 0.9 stabilising	1.5 imposed 1.0 snow	0 imposed

In addition to the material safety factors, the ‘k’, ‘ λ ’ and ‘ R_m ’ factors used for calculating glass design strength can influence the calculated LTB capacity. The recommended values of these factors from IABSE or CNR-DT-210 for short term loading and consequence class 1 were used in the designs undertaken for this study. For brevity, these are not replicated here but can be found in (Haldimann 2008; CNR 2013).

Combinations of Partial Methods

For this study, complete methods are used wherever possible. Where incomplete methods exist (and no accompanying partial methods are explicitly recommended), logical combinations are suggested based on information required and information available from other sources.

The methods considered here are given in table 4.

Table 4: Methods explored

Method	M_{cr}	f_d (including γ_m)	α_{LT}	χ_{LT}	γ
AS1288	AS1288	-	-	-	AS1288
CNR-DT-210	CNR-DT-210	CNR-DT-210	CNR-DT-210	CNR-DT-210 (BS EN 1993-1-1)	-
IABSE + CNR-DT-210	IABSE	IABSE (prEN 13474-1)	CNR-DT-210	CNR-DT-210 (BS EN 1993-1-1)	-
IABSE + EC3	IABSE	IABSE (prEN 13474-1)	BS EN 1993-1-1	BS EN 1993-1-1	-

Case Study: Design Data

Design calculations were undertaken using the methods outlined in the previous section. The section sizes were chosen so that the results could be compared to experimental data published by Luible (Luible 2004). Sections of monolithic glass only were considered, as laminated glass sections require more complex design calculations and are beyond the scope of this study. Luible's experiments involved a three-point bending test with the glass beam on fork supports, therefore the glass beams considered here are modelled as simply supported under a concentrated load at mid-span. The test campaign utilised 53 specimens in total with 13 different beam geometries, as shown in Table 5.

Table 5: Experimental Data

Reference	Glass Type	L_{eff} (mm)	h (mm)	t (mm)
A	HS	600	200	10
B	HS	860	200	10
C	HS	1040	200	10
D	HS	1100	200	10
E	HS	1100	200	15
F	HS	1300	200	10
G	HS	1500	200	15
H	HS	1600	200	10
I	HS	2000	200	10
J	FT	860	200	10
K	FT	1040	200	10
L	FT	1500	200	8
M	FT	3000	300	10

Where: L_{eff} is the unrestrained buckling length between supports

HS = heat strengthened

FT = fully tempered

These support conditions (simply supported with a central point load) are fairly typical of those found in glass fins supporting facades, such as the fin shown in Fig 1. However it should be noted that any form of lateral restraint provided from the connection or the facade panels has been neglected from the design calculations here. This is discussed further in the parametric study.

Design Results

For guidance on interpreting the graphs which accompany the following sections, please refer to the footnote of the notation table.

In all cases, regardless of glass type, the IABSE method gives the highest value for elastic critical buckling moment M_{cr} and the CNR-DT-210 method gives the lowest (see Fig 2). It can be seen that there is significant range in the design solutions, with the maximum values being as much as 25% greater than the minimum.

Fig 2: Design Solutions for M_{cr} as a Percentage of Minimum Solution Value

Fig 3 shows a clear correlation between the section span/depth ratio and the range of M_{cr} values given by the design methods for heat-strengthened glass. This suggests that the disparity in results from the different methods reduces as span/depth increases (ie. As the section becomes more slender) and global behaviour dominates, reducing the influence of material imperfections. The same trend was observed for the fully-tempered glass sections, although there are fewer data points to confirm the correlation.

Fig 3: Error in M_{cr} vs Span/Depth Ratio, Sections A-I

For all heat strengthened sections described in Table 5 (sections A-I), IABSE + EC3 gave the minimum value of M_{Rd} , and AS 1288 gave the maximum value. The range in solutions is large, with results given by AS1288 being as much as four times greater than those given by IABSE + EC3. The difference between results obtained for M_{Rd} via the three separate IABSE methods is relatively small (up to 18%) when compared to the difference between IABSE and other methods. This can be seen in Fig 4 – the results from AS1288 are in some cases so much greater than those from other methods that it is difficult to compare the results graphically. For comparison of results with AS1288 omitted, see Fig 5, which more clearly shows the relative difference between the other design methods.

Fig 4: M_{Rd} Design Solutions (% of Minimum Solution Value)

Fig 5: M_{Rd} Design Solutions (% of Minimum Solution Value) Excluding AS1288

The design overall factor of safety (which incorporates all partial safety factors and any contingency and is hereby defined as M_{cr}/M_{Rd}) indicates the factor of safety between the moment of resistance used for design purposes and the theoretical moment which characterises the onset of LTB. IABSE + EC3 consistently gave the highest overall factor of safety of all the design methods considered. The magnitude of this factor of safety ranges from 5.3 to 1.8. Conversely, AS1288 gives an overall factor of safety of 1.14 regardless of any variables. For all glass sections, this was the minimum factor of safety given by any of the design methods considered here. See Fig 6 for results.

Fig 6: Overall Factor of Safety M_{cr}/M_{Rd}

Experimental Results

The experimental values of M_{EXP} , taken from Luible's work (Luible 2004), indicate the moment at which the glass began to fracture. It was noted by Luible that the load which induced fracture was usually less than the theoretical buckling load, due to assumptions in the analytical method and friction effects at the point of load application.

Contrary to Luible's findings, for four out of the thirteen glass sections M_{EXP} exceeds the upper bound of the design solutions for M_{cr} from the various design methods considered here (sections H, J, L and M, which all had M_{EXP} values less than the analytical solution in Luible's study - see Fig 7 for results. This means that for these sections, the design solutions for M_{cr} were conservative. For a further seven of the glass sections, the experimental M_{EXP} value lies within the range of design M_{cr} values. For these sections, the lower bound design solution (from CNR-DT-210) was a conservative estimate, whereas the upper bound design solution (from IABSE) over-estimated the elastic buckling capacity of the sections.

There does not appear to be any correlation between section geometry and the difference between M_{EXP} and M_{cr} for any of the design methods.

Fig 7: Design M_{cr} Values as a Percentage of M_{EXP}

For most sections, the experimental M_{EXP} values exceed all of the design M_{Rd} solutions given by the various methods - see Fig 8 for results. This indicates that the design moment of resistance to LTB for

these sections is conservative and includes an overall factor of safety. The exceptions are sections A and G. For sections A and G, only the design M_{Rd} value given by AS1288 exceeds M_{EXP} . This indicates that AS1288 overestimated the buckling capacity of the section in these cases, but all other methods were conservative. Sections A and G do not appear to have any geometric properties in common.

Fig 8: Design M_{Rd} Values as a Percentage of M_{EXP}

Local Section Capacity

Glass is a brittle material which is prone to sudden fracture when the tensile strength is exceeded. When designing glass elements, the designer must check local behaviour as well as global behaviour – in particular, tensile stress concentrations. This is especially critical for sections with small span/depth, which are usually governed by local rather than global behaviour.

The elastic bending stress corresponding to the upper bound design M_{cr} value was calculated for all sections. For seven out of thirteen of the sections considered here, the characteristic tensile strength of the material is not exceeded when the maximum design M_{cr} is applied to the beam, therefore local tensile failure is not the critical failure mode.

For the other six sections, where the tensile strength was exceeded, the maximum allowable moment M_{LIM} was back-calculated using the characteristic tensile strength of the material as the limiting stress (see Fig 9). The amount by which M_{cr} exceeds M_{LIM} ranges from 4% to 139%, indicating a wide range in the difference between local tensile capacity and global buckling capacity of the section.

M_{LIM} was also compared to the experimental data M_{EXP} . In all cases M_{LIM} was less than M_{EXP} , suggesting that the glass characteristic tensile strength used for calculation of M_{LIM} was a conservative estimate of the actual glass tensile strength.

Fig 9: Design M_{cr} and Experimental M_{EXP} Values as a Percentage of M_{LIM}

Parametric Study

The beams considered thus far have been simply supported under a central point load. However glass members designed in practice may have a variety of support and loading conditions. In addition beam span, thickness and height will all vary by application of the glass member.

A parametric study was undertaken to assess the influence of certain variables over the design solutions. The key findings were as follows:

Beam geometry

The difference between M_{Rd} values given by AS1288 and all other methods increases significantly at glass thickness $> 10\text{mm}$. The difference in M_{Rd} values given by all methods excluding AS1288 is comparatively small and relatively unaffected by glass thickness – see Fig 10.

Similarly, the difference between M_{Rd} values given by AS1288 and all other methods increases significantly at beam depth $> 300\text{mm}$ and at beam span $< 3\text{m}$.

Fig 10: Design M_{Rd} vs Glass Thickness

Load duration

AS1288 gives M_{Rd} results which are independent of load duration, and therefore gives a significantly higher value (over 6x) than other methods for long term loading.

Glass type

AS1288 is also independent of glass type. The greatest variation in M_{Rd} results given by the different methods is for annealed glass. IABSE + EC3 gives the smallest M_{Rd} results for all glass types.

Support/loading conditions

The factors C_1 , C_2 , g_2 and g_3 which account for support and loading conditions are dependent on the shape of the bending moment induced by these conditions. However the different design methods do not cover the same bending moment shapes; therefore to make a comparison between the methods, values for these factors must be taken from elsewhere.

The C_1 and C_2 factors given by the IABSE guide and CNR-DT-210 are similar to those given for the same bending moment shapes for steel doubly-symmetric sections by SCI P360 (Gardner 2011), a design guide for steel sections which is compatible with EC3. Therefore C_1 and C_2 factors were taken from SCI P360 for some bending moment shapes not covered by IABSE and CNR-DT-210. Where the bending moment shape under consideration was not available in AS1288, the closest approximation was chosen – see Fig 11.

Fig 11: Loading and Support Conditions Factors

Label	Bending Moment Shape	C_1	C_2	g_2	g_3
CST (constant)		1.0	0	3.3 ^b	1.3 ^b
TSS (triangular, simply-supported)		1.36	0.55	4.2	1.7
TFF (triangular, fixed-fixed)		1.683 ^a	1.645 ^a	6.5	5.3
CLR (cantilever)		1.77 ^a	0 ^a	4.0	2.0

^aValues from SCI P360

^bValues for a simply supported beam under 2 point loads

There is some variation (up to 52%) between the values of M_{cr} given by the different methods for bending moment TFF. Conversely there is very little range in the values of M_{cr} for sections with bending moment TSS (up to 7%).

Fig 12 shows the range in M_{Rd} solutions given by the various methods, which is much greater due to the relatively high value given by AS1288 which is up to four times greater than the minimum solution. The range between values given by all methods excluding AS1288 is up to 46%. Again, this variation is greatest for moment shape TFF. The minimum variation for M_{Rd} is for moment shape CLR.

Fig 12: M_{Rd} as a Percentage of Minimum, Variation with Bending Moment Shape

There has been recent interest in the LTB performance of glass beams with continuous lateral restraint, as this is a support condition often seen in canopies and facades where glass beams or fins are bonded to glass panels by adhesive joints. Of the codes and standards explored here, only AS 1288 gives any provisions for dealing with continuous flange restraint, therefore it was not possible to compare the design methods in this context and it is therefore considered beyond the scope of this study. However studies have shown that a significant increase in LTB capacity can be achieved by taking the benefit of such continuous restraint. The interested reader is referred to (Bedon 2015b; Bedon 2016; Luible 2016).

Discussion

Up to 25% difference was observed between the values calculated via the different methods for M_{cr} – a significant range. The overall factor of safety between M_{cr} and M_{Rd} also varies significantly with section geometry, although the difference between the overall factors of safety obtained via the different methods is fairly consistent as can be seen by the trends in Fig.6.

The method outlined in the IABSE guide may be combined with that given in EC3 for calculating the design moment of resistance to LTB. Three different values of α_{LT} were considered, and a relatively small difference in results was observed. Greater variation was observed between the design results for the IABSE method, the AS1288 method and the CNR-DT-210 method with the maximum solution being up to four times greater than the minimum.

For approximately one quarter of the sections considered, all M_{cr} values given by the various methods were less than the experimental buckling moment M_{EXP} , suggesting conservatism in the design solution. All except two of the sections considered (85%) had an M_{EXP} value greater than M_{Rd} given by any analytical method.

For approximately half of the sections considered, fracture due to local tensile capacity exceedance was not the critical mode of failure. For the sections which were governed by local tensile stresses, there was significant variation in the difference between the buckling moment M_{cr} and the limiting moment to prevent exceedance of the glass tensile strength M_{LIM} .

Finally it was noted that provisions for different loading and support conditions differ between the various methods. The range in solutions from the different method also varies with the shape of the

bending moment, with a greatest variation in M_{Rd} solutions for a triangular bending moment which is zero at one end (corresponding to a cantilever beam).

Conclusions

Clearly the different methods have differing theoretical basis regarding the structural and material behavior of glass. It is important to consider this when comparing results from these methods as limits of applicability may apply to some methods but not others (eg. AS1288 does not take glass type into account, but other methods do). The designer should always use their judgement as to whether the chosen design method is suitable for their application.

CNR-DT-210 consistently gave the minimum value for M_{cr} and the IABSE method gave the maximum which tends to suggest the IABSE method is more conservative. For slender sections with large span/depth which are usually governed by global rather than local behaviour, the variation in results for M_{cr} is less and therefore the choice of design method has less of an impact on the solution. In addition, it has been found here that the chosen value of α_{LT} (within the range considered in this study) has little impact on the final solution. In any case it should be noted that M_{cr} is a theoretical buckling capacity only, with inherent assumptions in its calculation and does not represent a complete design solution. A more important indicator of the level of conservatism in the design methods is M_{Rd} .

For all design cases considered the maximum solution for M_{Rd} was given by AS1288 and the minimum given by IABSE + EC3. The solution given by AS1288 may be more than four times greater than that given by the IABSE + EC3. This is a very large discrepancy which suggests that either one or both of these methods has a large error in the solution and great care should be taken when using either of these methods for design purposes.

AS1288 gives an overall factor of safety of 1.14 between M_{cr} and M_{Rd} , regardless of any other variables. In all design cases considered, this was the smallest overall factor of safety given by any method, with the largest overall factor of safety given by IABSE + EC3. This suggests that the AS1288 method should be used with care as it has less conservatism than other methods. Conversely, the IABSE method may be considered over-conservative in some cases, with an overall factor of safety as high as 5.5.

Comparison to experimental data showed that generally most design solutions for M_{Rd} from the methods considered here were conservative. However, M_{EXP} was exceeded in two cases by M_{Rd} given by AS1288, suggesting a potential lack of conservatism in this method.

Acknowledgements

The University of Manchester is acknowledged for financial and academic support of this research. WSP |Parsons Brinckerhoff are also gratefully acknowledged for support of this work. Alex Chow is acknowledged for the photograph in Fig 1.

Table 6: Notation

A	Cross sectional area = $t \times h$
C_1, C_2	Bending moment shape factors
E	Young's modulus
$f_{b,k}, f_{g,k}$	Characteristic strength of glass (b=annealed, g=toughened)
f_d	Design strength of glass

f_y	Yield strength (of steel)
g_2, g_3	Loading/support conditions factors
G	Shear modulus
h	Beam depth
I_z	Second moment of area about minor axis
J	Torsion constant
$k'_{ed}, k_A, k_{ed}, k_{mod}, k_{sf}, k_V$	'k' modification factors for glass design strength
L, L_{eff}	Beam span
M_{Rd}	Design moment of resistance to LTB
M_{cr}	Elastic critical buckling moment
$R_m, R_{m,V}$	Partial safety factors for importance class of structure
t	Glass thickness
W_y	Elastic section modulus about major axis
z_a	Height of load application above neutral axis of cross section
α_{LT}	Buckling curve coefficient
$\gamma, \gamma_G, \gamma_Q$	Load Safety factors (G=permanent loads, Q=variable loads)
$\gamma_m, \gamma_{m,V}$	Material Safety factors (m=annealed glass, mV=strengthened glass)
$\bar{\lambda}$	Slenderness ratio
$\lambda_{gA}, \lambda_{gL}$	Modification factors for glass design strength
φ	Buckling curve parameter
χ_{LT}	Buckling strength reduction factor
Ψ_0	Reduction factor for accompanying variable actions

Note that data in this paper is typically presented via spider charts. Each radial axis line represents one of the sections considered (as described in Table 5). The values on the radial axis represent the solution value, typically expressed as a percentage of the minimum solution value (ie. The minimum solution would be expressed as 100%). The data points and trend lines on the graph show the different design methods considered. Therefore the reader is able from these charts to compare three variables: section geometry, design method and solution value.

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