Scale-up of Batch Rotor-Stator Mixers. Part 1 – Power Constants

J. James\textsuperscript{a}, M. Cooke\textsuperscript{a}, L. Trinh\textsuperscript{a}, R. Hou\textsuperscript{b}, P. Martin\textsuperscript{a}, A. Kowalski\textsuperscript{b}, T. L. Rodgers\textsuperscript{a,*}

\textsuperscript{a}School of Chemical Engineering and Analytical Science, The University of Manchester, Manchester, M13 9PL, UK

\textsuperscript{b}Unilever R&D, Port Sunlight Laboratory, Quarry Road East, Bebington, Wirral, CH63 3JW, U.K

*Corresponding author. Email address: tom.rodgers@manchster.ac.uk

Abstract

Rotor-stator mixers are characterized by a set of rotors moving at high speed surrounded closely by a set of stationary stators which produces high local energy dissipation. Rotor-stator mixers are therefore widely used in the process industries including the manufacture of many food, cosmetic and health care products, fine chemicals, and pharmaceuticals. This paper presents data demonstrating scale-up rules for the key power parameters; laminar power constant, Metzner-Otto constant, and Turbulent power number; for Silverson Batch rotor-stator mixers. Part 2 of this paper explores mixing times, surface aeration, and equilibrium drop sizes. These rules will allow processes involving rotor-stator mixers to be scaled up from around 1 litre to over 600 litres directly.

Graphical abstract
Highlights

- Power constants can be calculated repeatability from torque data
- Batch rotor-stator mixer power number can be given by $P_o = \frac{K_p}{Re} + P_{o_T}$
- Removing the screens increases the turbulent power number, decreases the laminar power constant, and decreases the Metzner-Otto constant.
- The laminar power constant and the Metzner-Otto constant increase with increasing scale
- The turbulent power number is dependent on the flow area

Keywords

Batch rotor-stator mixers, power number, Metzner-Otto, scale-up
1. Introduction

Rotor-stator mixers are characterized by a set of rotors moving at high speed surrounded closely by a set of stationary stators (or screens). The rotors generally rotate at an order of magnitude higher speed than conventional impellers in a stirred tank; typical tip speeds range from 10 to 50 m s\(^{-1}\) with the gap between the rotors and stators generally ranging from 100 to 3000 μm. This design allows the generation of high shear rates and high intensities of turbulence. The energy generated by the rotor dissipates mainly inside the stator and therefore the local energy dissipation rates are much higher than conventional impellers in stirred vessels (Atiemo-Obeng and Calabrese, 2004).

Rotor-stator mixers are therefore widely used in the process industries including the manufacture of many food (Rodgers and Trinh, 2016), cosmetic and health care products (Savary et al., 2016), fine chemicals (Paton et al., 2014), and pharmaceuticals (Luciani et al., 2015). The focused delivery of energy and shear by rotor-stator devices accelerate physical processes such as mixing, dissolution, emulsification, and deagglomeration (Cooke et al., 2008). To allow reliable scale-up of processes using rotor-stator devices we need to understand the relationship between rotor speed and the energy dissipated by these devices at different scales.

Turbulent power for a batch rotor-stator device can be described by a single “tank” type impeller power number, \(P_0\), equation (1) (Atiemo-Obeng and Calabrese, 2004; Calabrese and Padron, 2008; Doucet et al., 2005), where \(D\) is the impeller diameter, \(N\) is the rotor speed, \(\rho\) is the liquid density, and \(P\) is the shaft power for the impeller.

\[
P_0 = \frac{P}{\rho N^3 D^5}
\]  

The power number is essentially constant at Reynolds numbers greater than \(10^4\) in a baffled stirred vessel; however, for rotor-stator devices, previous studies (Calabrese and Padron, 2008; Doucet et al., 2005) highlighted some uncertainty as to whether this regime was in the laminar or turbulent
region. The regime is governed by the mixing Reynolds number, \( \text{Re} \), equation (2), where \( h \) is a characteristic length and \( \mu \) is the viscosity.

\[
\text{Re} = \frac{\rho ND h}{\mu}
\]  

The uncertainty for a rotor-stator arises as there are several ways of calculating the Reynolds number depending on how \( h \) is defined, either as the impeller diameter, \( D \), the screen hole diameter, \( D_{hi} \) or as the rotor-stator gap, \( D_S - D \). The Reynolds number will change from numbers associated with laminar to fully turbulent depending on the definition of \( h \) used, so it is important to pick the value that produces the true regime. In previous studies (Calabrese and Padron, 2008; Cooke et al., 2012; Doucet et al., 2005) it was assumed the characteristic length used should be the swept impeller diameter, giving equation (3) (the same as for a stirred tank) - an assumption which was felt to be justified due to the good quality of predictions subsequently obtained. This is the definition used within this paper.

\[
\text{Re} = \frac{\rho ND^2}{\mu}
\]  

The power output by rotor-stator mixers has also been shown to be periodic as the blades move past the stator holes (Utomo et al., 2008) and of course vary with the stator geometry (Utomo et al., 2009).

2. Material and methods

2.1. Experimental equipment

Figure 1 provides photos of the three experimental rigs used for these experiments. They all consist of a circular flat bottomed vessel, with liquid height equal to the diameter of the vessel. The batch rotor-stator is positioned in the centre of the vessel at a height equal to 33% the liquid height. Each vessel was also baffled with four standard 7/10 baffles.
The smallest scale consists of a 0.128 m diameter vessel with a Silverson L5M rotor-stator mixer, this device had both the standard Emulsor screen mixing head and the 5/8” Micro tubular mixing head; this system had a TorqueSense 1 Nm torque meter attached to allow calculation of the power. The rotor speed range is 0 to 10,000 rpm, controlled by the internal bench unit system. The middle scale consists of a 0.380 m diameter vessel with a Silverson AX3 rotor-stator mixer, which was used with the standard Emulsor screens; this system had a TorqueSense 5 Nm torque meter custom installed in the motor housing by Silverson. The rotor speed range is 0 to 3000 rpm, controlled by an inverter over the range 0–50 Hz. The largest scale consists of a 0.6096 m diameter vessel with a Silverson GX10 rotor-stator mixer, this device had a 4.5” Emulsor screen mixing head and a 5.8” Emulsor screen mixing head, which also had a custom large hole screen (360 holes of 5 mm diameter); this system had a TorqueSense 40 Nm torque meter custom installed in the motor housing by Silverson. The rotor speed range is 0 to 3,000 rpm, controlled by an inverter over the range 0–60 Hz. Details of the rotor stators are provided in Table 1. In each case the temperature of the system is monitored using a Picolog P100 temperature probe placed in the exit flow from the rotor-stator device to allow the temperature in the rotor-stator device to be captured as closely as possible.

Figure 1. Photos of the 3 three systems used, (a) GX10, (b) AX3, and (c) L5M.
Table 1. Dimensions of the Silverson mixers used in this study.

<table>
<thead>
<tr>
<th>Impeller</th>
<th>Base Plate Diameter Dbp (mm)</th>
<th>Rotor Diameter (mm)</th>
<th>Inside Stator Diameter (mm)</th>
<th>Stator Height (mm)</th>
<th>Blade Height (mm)</th>
<th>Hole Diameter (mm)</th>
<th>Number of holes</th>
</tr>
</thead>
<tbody>
<tr>
<td>GX10 5.8&quot;</td>
<td>285.70</td>
<td>149.23</td>
<td>149.70</td>
<td>47.49</td>
<td>34.87</td>
<td>1.59</td>
<td>3534</td>
</tr>
<tr>
<td>GX10 4.5&quot;</td>
<td>285.70</td>
<td>114.30</td>
<td>114.80</td>
<td>37.23</td>
<td>23.82</td>
<td>1.59</td>
<td>1728</td>
</tr>
<tr>
<td>AX3 standard</td>
<td>76.00</td>
<td>50.55</td>
<td>51.05</td>
<td>21.10</td>
<td>11.10</td>
<td>1.59</td>
<td>200</td>
</tr>
<tr>
<td>L5M standard</td>
<td>50.00</td>
<td>31.71</td>
<td>32.17</td>
<td>18.70</td>
<td>12.64</td>
<td>1.59</td>
<td>304</td>
</tr>
<tr>
<td>L5M tubular</td>
<td>16.50</td>
<td>14.00</td>
<td>16.50</td>
<td></td>
<td></td>
<td>2.5 × 2</td>
<td>52</td>
</tr>
</tbody>
</table>

2.2. Materials and methods

Water and Silicone oils (polydimethylsiloxane, Dow Corning 200 fluid) were chosen as the Newtonian fluids. The silicon oil used was either used as supplied or blended to obtain a wide range of viscosities. The viscosity range of the oil supplied was 10 to 10,000 centistokes (cSt) (density of 934 kg m$^{-3}$ for 10 cSt, 960 kg m$^{-3}$ for 50 cSt, and 970 kg m$^{-3}$ for greater than 350 cSt). The rheological properties of the solutions were determined at a range of temperatures with a viscometer (Haake RV 20) by using the standard cup and bob Couette configuration using either the MV1 or MV2 bob depending upon the viscosity range. The viscosity of the Newtonian fluids ranged from 0.001 to 10 Pa s and the exact value was used in each of the calculations for the actual temperature.

Various concentrations of aqueous solution of carboxymethyl cellulose (CMC), between 1 and 1.5 wt%, HERCULES POWDER grade 7H4C were used as the non-Newtonian fluids. The rheological parameters of these solutions were found to obey a power law, Eq. (4), where $\mu_a$ is the apparent viscosity, $\alpha$ is the consistency index, $\dot{\gamma}$ is the average shear rate, and $n$ the flow behaviour index.

$$\mu_a = \alpha \dot{\gamma}^{n-1} \quad (4)$$
The viscosities of these fluids were also measured over a temperature range with $\alpha$ varying with temperature. There was only a very small dependency of the flow behaviour index on temperature, less than 1%, therefore this was averaged out and the resulting recalculated consistency indices were found to fit the collected viscosity data.

2.3. Power Measurement

The torque was measured by an in-line torque meter fitted to the drive shaft for each of the systems. There are two main sources of potential error when measuring the torque on the rotor shaft. Firstly, the torque sensor used was found to exhibit a slight zero drift when operating the Silverson continuously over a number of hours. Secondly the sensor also measures bending moments on the shaft; this measurement cancels out over the course of 1 revolution of the shaft and so does not affect average shaft torque, but does make a static zero adjustment difficult.

To minimize these sources of error, the zero is measured before and after each run and the resulting values averaged. The shaft stops in a different position after each run allowing the zero to be approximated through the slight bending moments. This zero error is also reduced by using the power analysis method given later in the paper.

The shaft torque also contain losses in the shaft bearings. These bearing losses were also measured using the same methods with the shaft rotated at different speeds in air. This data yields values of the lost torque, $M_{\text{losses}}$, which are subtracted from the measured torque at the same conditions.

2.4. Determination of the Turbulent Power Constants

As previously mentioned, the turbulent regime is considered to be applicable to Reynolds numbers above $10^4$ (though this threshold may be higher for rotor-stator mixers). Grenville and Nienow (2004) gave an equation for the transition between the turbulent and transitional regimes based on mixing times as,
which means that the transition is dependent on the power number of the agitator. If the data collected is in the turbulent regime then the power number, $P_0T$, is constant, which means that we can write,

$$P_0T = \frac{2\pi NM}{\rho N^3 D^5}$$  \hspace{1cm} (6)

which can be rearranged for a straight line expression between torque, $M$, and the square of the rotor speed, $N^2$, with the intercept being any error in the zero and bending moments,

$$M = \frac{P_0T \rho D^5}{2\pi} N^2 + M_{\text{losses}}$$  \hspace{1cm} (7)

Figure 2 shows an example of this analysis for the L5M standard mixer with both water and 10 cSt silicon oil. It can be seen that the intercept is almost zero due to the methods used to take into account bearing losses, zero error, and bending moments.

*Figure 2. Example data for L5M rotor-stator mixer operating in the turbulent regime, gradient calculated is $1.008 \times 10^{-5}$. 
2.5. Determination of the Laminar Power Constants

The laminar regime is generally considered to be applicable to Reynolds numbers below 200. Based on data from several sources (Hoogendoorn and den Hartog, 1967; Zlokarnik, 1967) an equation for the transition between the transitional and laminar regimes can also be given based on mixing times as,

\[ K_p^{1/3} \text{Re}_{TL}^{2/3} = 137 \pm 46 \]  

which means that the transition is dependent on the laminar power constant of the agitator. If the data collected is in the laminar regime then the laminar power constant, \( K_p \), is constant, which means that we can write,

\[ K_p = \text{PoRe} = \frac{2\pi NM}{\rho N^3 D^5} \cdot \frac{\rho N D^2}{\mu} = \frac{2\pi M}{\mu N D^3} \]  

which can be rearranged for a straight line expression between torque and the rotor speed times by the viscosity with the intercept being any error in the zero and bending moments,

\[ M = \frac{K_p D^3}{2\pi \mu N} + M_{\text{losses}} \]  

The viscosity has been included as a variable to allow multiple viscosity oils to be plotted on the same graph to allow better calculation of the laminar power constant. The other advantage of this is that if the temperature varies during the experiment the true viscosity can be taken (with high viscosity oils the local temperature tends to increase).

Figure 3 shows an example of this analysis for the L5M standard mixer with 1000, 2500, and 3800 cSt silicon oils. It can be seen that the intercept is almost zero due to the methods used to take into account bearing losses, zero error, and bending moments.
2.6. Determination of the Transitional Power Constants

Cooke et al. (2012) demonstrate that the total power number for an in line rotor-stator mixer is just the summation of the turbulent and laminar power contributions such that,

\[ P_0 = P_{0T} + \frac{K_p}{Re} \]  

which can be rearranged for the torque in terms of a quadratic expression of the rotor speed with the intercept being any error in the zero and bending moments,

\[ M = \frac{P_{0T} \rho D^5}{2\pi} N^2 + \frac{K_p \mu D^3}{2\pi} N + M_{\text{losses}} \]  

The temperature needs to be controlled carefully if using this equation as the viscosity of the fluid has to be assumed constant. It could be taken as a variable and multiple linear regression used for the analysis if needed; however, it is more accurate to determine the power constants separately in the turbulent (where the laminar power contribution is negligible) and the laminar (where the turbulent power contribution is negligible) regimes, which was the method used in this paper. This
expression assumes a smooth transition which as we will see shortly is the case for these rotor-

stator mixers, but may not be for other types of agitator.

Figure 4 shows an example of this analysis for the L5M standard mixer with 350 cSt silicon oil. It can

be seen that the intercept is almost zero due to the methods used to take into account bearing

losses, zero error, and bending moments; the curve shown is of the type $y = ax^2 + bx + c$. It can be

seen that there is a clear contribution from both the turbulent and laminar part of the equation to

the torque.

![Graph showing torque vs. speed for L5M rotor-stator mixer in the transitional regime]

**Figure 4.** Example data for L5M rotor-stator mixer operating in the transitional regime, calculated values of $a$ and $b$ are $9.472 \times 10^{-6}$ and $1.561 \times 10^{-4}$ respectively. The dashed lines show the contribution by the turbulent ($ax^2$) and the laminar ($bx$) parts of the equation.

### 2.7. Determination of the Metzner-Otto Constant

The same approach can be carried out for non-Newtonian fluids as was carried out for Newtonian

fluids in the laminar regime, assuming that the shear rate can be given by the Metzner-Otto
equation, \( \dot{\gamma} = K_s N \) (Metzner and Otto, 1957). For a power law shear thinning fluid, \( \mu = \alpha \dot{\gamma}^{n-1} \), such as those used in this study, the Reynolds number can be given by,

\[
Re = \frac{\rho N D^2}{\alpha \dot{\gamma}^{n-1}} = \frac{\rho N D^2}{\alpha (K_s N)^{n-1}} = \frac{\rho N^{2-n}D^2}{aK_s^{n-1}}
\]  

(13)

This means that in the laminar regime,

\[
K_p = \text{PoRe} = \frac{2\pi NM}{\rho N^3D^5}, \frac{\rho N^{2-n}D^2}{aK_s^{n-1}} = \frac{2\pi M}{K_s^{n-1}aN^nD^3}
\]  

(14)

which can be rearranged for a straight line expression between torque and the rotor speed to the power of the flow behaviour index times by the consistency index with the intercept being any error in the zero and bending moments,

\[
M = \frac{K_p K_s^{n-1}D^3}{2\pi} \alpha N^n + M_{\text{losses}}
\]  

(15)

The consistency index has been included as a variable to allow multiple viscosity oils to be plotted on the same graph to allow better calculation of the Metzner-Otto constant. The other advantage of this is that if the temperature varies during the experiment the true consistency index can be taken. Figure 5 shows an example of this analysis for the L5M standard mixer with CMC solution. It can be seen that the intercept is almost zero due to the methods used to take into account bearing losses, zero error, and bending moment.
Figure 5. Example data for L5M rotor-stator mixer with CMC solution, $\mu = 26.2 \gamma^{0.37}$, calculated gradient is $1.823 \times 10^{-4}$.

3. Results and Discussion

3.1. Rotor-Stator Power Constants

Figure 6 presents the full power curve for the Silverson L5M with standard Emulsor screens. This power curve was generated using seven Newtonian fluids and one non-Newtonian fluid. In addition the power curve without the screens was measured using water, 2500 cSt silicon oil, and CMC solution. The lines shown are generated using equation (11). It can be seen from Figure 6 that the power number is lower in turbulent regime with the screen in place, but higher in the laminar regime. This is due to two effects; the first is the screens having a small clearance to the rotor increases the power needed to move within the fluid, due to increased friction of the fluid; the second is the presence of the screens reduced the area available for flow from the rotor. In the turbulent regime the second effect dominates which means the power number is lower with the screens, the screens reduce the liquid flow through the rotor, therefore not as much power is needed as not as much fluid is pumped. In the laminar regime the first effect dominates as there is
very little pumping of the fluid due to the high viscosity of the fluid, this means that more power is needed when the screen is in place. Using the methods outlined in section 2 the laminar power constant, $K_p$, for the rotor-stator is 903, the Metzner-Otto constant, $K_S$, is 156, and the turbulent power number, $P_0T$, is 2.13. The laminar power constant, $K_p$, for the rotor only is 244, the Metzner-Otto constant, $K_S$, is 23, and the turbulent power number, $P_0T$, is 3.87. This effect is mirrored for the AX3 rotor-stator system with the laminar power constant, $K_p$, for the rotor-stator being 1060, the Metzner-Otto constant, $K_S$, is 170, and the turbulent power number, $P_0T$, is 1.56. The laminar power constant, $K_p$, for the rotor is 358, the Metzner-Otto constant, $K_S$, is 42, and the turbulent power number, $P_0T$, is 2.86.

![Graph showing Po vs. Re for Screens and No Screens](Image)
The two effects mentioned above are further emphasised by data collected for the Silverson GX10. Figure 7 shows the power number versus the inverse Reynolds number for a selection of the Silverson GX10 systems in the laminar regime. The systems with the stator attached have a much higher laminar power constant than those without the stator. The 5.88” rotor with stator has the highest laminar power constant, larger than that of the 4.5” version, this is most likely due to the larger size, blade height, and blade thickness. When the screen is removed the laminar power constant for the 5.88” rotor is lower than that for the 4.5” version. This seems counter to what would be expected, but it potentially due to the presence of the base plate effecting the flow, the ratio of the base plate to the blade diameter increase in proportion with the increase in the power number.
Figure 8 shows the power number versus the Reynolds number for a selection of the Silverson GX10 systems in the turbulent regime. The systems with the stator attached have a much lower power number than those without the stator. The 5.88” rotor with stator has a lower power number than the 4.5” version, this is most likely due to the hole in the base plate being the same size in both cases, so the flow is the most restricted, reducing the flow rate, thus the power number. Increasing the hole size for the 5.88” screen to 5mm slightly increases the power number due to increasing the area for the liquid flow.

When the screen is removed, the 5.88” rotor has a higher power number than the 4.5” rotor. This is most likely due to the fact that the 5.88” rotor is bigger and in the same size vessel, meaning that it is pumping a higher flow rate of liquid. This has also been seen for Rushton and Pitched blade turbines (Bujalski et al., 1987; Chapple et al., 2002).

The summary of the measured power constants for these two systems, and the additional AX3 and L5M tubular systems are given in Table 2.
Figure 8. Turbulent power data for the Silverson GX10.

Table 2. Summary of the power constants for the systems under investigation.
3.2. Scale-up of Rotor-Stators Mixers

Figure 9 shows the variation of the key power parameters with the change in the size of the rotor-stator blade. The laminar power constant increases as the diameter of the stator increases, the trend line shown in Figure 9 is a power law fit (this is a better fit that if the rotor diameter is used). For the standard Silverson designs most of the geometric parameters either stay constant or scale with the stator diameter so without producing custom systems it is difficult to determine the key geometric parameters which control the laminar power constant.

The same effect can be seen for the Metzner-Otto constant, in this case the smallest system, L5M tubular, is the worst fit to this trend line, this is potentially because it has a different gap between the rotor and stator compared to the other systems, Table 1.

Figure 9. Summary of the power constants for the systems under investigation; top row – rotor-stator, bottom row – rotor only. Error bars are 95% confidence limits.

The turbulent power number does not scale with the rotor or stator diameter. This seems to be because the turbulent power number has a significant contribution from the flow of the liquid. For
the systems studied the turbulent power number seems to correlate with the minimum hydrodynamic radius (the minimum of either the holes in the stator or the hole in the base of the rotor-stator) divided by the diameter of the rotor. This supports the effect of the flow on the power number. The intercept value is 1.20, which could be thought of as a turbulent power number with no flow, \( P_{0z} \), it is slightly larger than the same constant seen for the inline Silverson rotor-stator mixer (Cooke et al., 2012). The inline rotor stator equation is given by,

\[
P_o = P_{0z} + kN_Q
\]  

(16)

The flow number, \( N_Q \), is given by \( Q/ND^3 \), where \( Q \) is the flow rate, which can be given by the liquid velocity times the flow area, \( Q = uA \). It can be assumed that the liquid flow is proportional to the tip speed of the rotor due to the confined nature of the flow, and the flow area is proportional to the hydraulic radius squared, i.e. \( Q = uA \propto u_{tip} h_r^2 \propto NDh_r^2 \). This means that for the batch rotor-stator system we can write,

\[
N_Q = C_1 NDh_r^2 = C_1 \frac{h_r^2}{D^2}
\]  

(17)

Such that equation (16) becomes,

\[
P_o = P_{0z} + k \frac{h_r^2}{D^2}
\]  

(18)

The laminar power constant and the Metzner-Otto constant for the rotor only case, Figure 9, increase with the size of the rotor apart from the largest system, this might be due to the presence of the base plate and the size of this not scaling with the rotor diameter. To understand what is affecting these constants again custom designed rotor-stator systems will be needed.

The turbulent power number for the rotor only case scales linearly with the ratio of the rotor to the tank diameter. This is similar to trends that have been seen for other agitator systems (Bujalski et al., 1987; Chapple et al., 2002).
The results for the GX10 5.8” with the standard screens, large hole screens, and no screens (see values in Table 2) suggest that there must be an effect of the holes on the power constants. Increasing the turbulent power number and decreasing the laminar power constant and Metzner-Otto constant with increasing hole size.

4. Conclusions

This paper provides results allowing the scale-up for Silverson batch rotor-stator systems. The laminar power constant for the rotor-stator systems can be given by,

\[ K_p = 5891D_s^{0.56} \] (19)

while the Metzner-Otto constant can be given by,

\[ K_S = 2060D_s^{0.77} \] (20)

For a full understanding of how all the dimensions of the rotor-stator system effect the power constants further work will have to be carried out providing a bigger variation in these, e.g. rotor stator gap.

The turbulent power number for the rotor-stator system takes a similar form to an inline rotor-stator system, where there is a constant term and a term that is based on the flow,

\[ P_o_T = 1.60 \left( \frac{\min(h_r)}{D} \right)^2 + 1.20 \] (21)

The \((\min(h_r)/D)^2\) term represents the area for the flow, so is probably proportional to the flow from the rotor-stator system.

The turbulent power number for the rotor only takes a similar form to that when the screens are attached, but in this case the key parameter is the ratio of the rotor to the tank diameter.

\[ P_o_T = 9.28 \frac{D}{T} + 1.598 \] (22)

Part 2 of this paper explores mixing times, surface aeration, and equilibrium drop sizes.
Acknowledgements

The authors would like to thank the SCEAS workshop for their help with equipment modification and Silverson Machines for their help and support with equipment. They would also like to thank Innovate UK and the EPSRC (EP/L505778/1) for funding.

References


