Evaluating the performance of aggregate production planning strategies under uncertainty

A thesis submitted to The University of Manchester for the degree of Doctor of Philosophy in the Faculty of Humanities

2017

Aboozar Jamalnia

Alliance Manchester Business School
The University of Manchester

Aboozar Jamalnia

Degree: Doctor of Philosophy

Thesis title: Evaluating the performance of aggregate production planning strategies under uncertainty

Date: 10/05/2017

Abstract: The thesis is presented in three papers format. Paper 1 presents the first bibliometric literature survey of its kind on aggregate production planning (APP) in presence of uncertainty. It surveys a wide range of the literatures which employ operations research/management science methodologies to deal with APP in presence of uncertainty by classifying them into six main categories such as stochastic mathematical programming, fuzzy mathematical programming and simulation. After a preliminary literature analysis, e.g. with regard to number of publications by journal and publication frequency by country, the literature about each of these categories is shortly reviewed. Then, a more detailed statistical analysis of the surveyed research, with respect to the source of uncertainty, number of publications trend over time, adopted APP strategies, applied management science methodologies and their sub-categories, and so on, is presented. Finally, possible future research paths are discussed on the basis of identified research trends and research gaps.

The second paper proposes a novel decision model to APP decision making problem based on mixed chase and level strategy under uncertainty where the market demand acts as the main source of uncertainty. By taking into account the novel features, the constructed model turns out to be stochastic, nonlinear, multi-stage and multi-objective. APP in practice entails multiple-objectivity. Therefore, the model involves multiple objectives such as total revenue, total production costs, total labour productivity costs, optimum utilisation of production resources and capacity and customer satisfaction, and is validated on the basis of real world data from beverage manufacturing industry. Applying the recourse approach in stochastic programming leads to empty feasible space, and therefore the wait and see approach is used instead. After solving the model using the real-world industrial data, sensitivity analysis and several forms of trade-off analysis are conducted by changing different parameters/coefficients of the constructed model, and by analysing the compromise between objectives respectively. Finally, possible future research directions, with regard to the limitations of present study, are discussed.

The third paper is to appraise the performance of different APP strategies in presence of uncertainty. The relevant models for various APP strategies including the pure chase, the pure level, the modified chase and the modified level strategies are derived from the fundamental model developed for the mixed chase and level strategy in paper 2. The same procedure, which is used in paper 2, follows to solve the models constructed for these strategies with respect to the aforementioned objectives/criteria in order to provide business and managerial insights to operations managers about the effectiveness and practicality of these APP policies under uncertainty. Multiple criteria decision making (MCDM) methods such as additive value function (AVF), the technique for order of preference by similarity to ideal solution (TOPSIS) and VIKOR are also used besides multi-objective optimisation to assess the overall performance of each APP strategy.
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Preface

Aggregate production planning (APP) is a medium term production and employment planning that typically covers a time horizon which ranges from 3 to 18 months, and is concerned with determining the optimum production volumes, hiring and lay off rates, work force and inventory levels, backordering and subcontracting quantities, and so on for each time period within the planning horizon with respect to the limitation of production resources.

The present research proposes a novel decision model to APP under uncertainty. By taking into account the novel features, the constructed model turns out to be stochastic, nonlinear, multi-stage and multi-objective. The model evaluates the performance of five APP strategies with regard to 7 objectives/criteria. The research gaps and novel features of the proposed APP model are discussed in detail in relevant parts of the thesis. The present study is the first attempt of its kind that systematically appraise the performance of a comprehensive range of APP strategies after detailed analysis of existing literature and by building upon the author’s previous experience on developing APP decision models.

The thesis is presented in three papers style instead of the traditional thesis format for several reasons. From the personal evaluation of many PhD theses, the author found out that only about one-third of the average 80000-85000 words content of the traditional thesis format is original contribution. That is, great portion of their content is about reviewing existing concepts and methodologies. As such, the novel parts of a thesis could be presented in the papers structure, which makes it much easier for other researchers to read the research results and findings presented in a more concise form. Accordingly, it also facilitates searching among published research outputs. Furthermore, many PhD graduates normally extract papers from their traditional thesis framework to submit to the relevant journals for publication. Therefore, by providing the thesis in three papers style directly, many redundancies will be eliminated. Finally, the author has already published several papers in different journals. Hence, regarding this experience, he believes it would be more convenient for him to present his research in alternative format, three papers format, efficiently.

As already detailed in Abstract section, the paper 1, as the first literature survey paper on APP models under uncertainty, conducts an in-depth bibliometric literature survey on quantitative APP models in presence of uncertainty, which is accompanied by detailed statistical analysis of the surveyed literature and recommendations on possible future research paths. In paper 2, a new, stochastic, nonlinear, multi-objective optimisation model with novel features including elaborated pricing, advertising, demand management mechanisms and workforce
productivity measurement is developed to deal with APP subject to uncertainty regarding the mixed chase and level strategy to give a holistic picture of APP. A comprehensive set of seven objectives such as total profit, customer satisfaction, utilisation of production resources and workforce productivity costs are considered. The model is then implemented in a beverage manufacturing company.

In paper 3, four extra mathematical models are derived from the model developed in paper 2, for the mixed chase and level policy, to model other APP strategies including the pure chase, the pure level, the modified chase and the modified level strategies. The performance of APP strategies is compared with each other’s on the basis of abovementioned objectives/criteria.
Title: Aggregate production planning under uncertainty: a bibliometric literature survey and future research directions

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Aggregate production planning under uncertainty: a bibliometric literature survey and future research directions

Abstract: The last literature survey on aggregate production planning (APP) was performed in 1992. Therefore, there is a reasonable need to conduct an up to date literature survey in this area. This is the first bibliometric literature survey of its kind on APP in presence of uncertainty. Different types of uncertainty including stochasticity, fuzziness, possibilistic forms, etc. have been incorporated into many management science techniques to study APP decision problem under uncertainty. In current research, a wide range of the literatures which employ operations research/management science methodologies to deal with APP in presence of uncertainty are surveyed by classifying them into six main categories such as stochastic mathematical programming, fuzzy mathematical programming and simulation. After a preliminary literature analysis, e.g. with regard to number of publications by journal and publication frequency by country, the literature about each of these categories is shortly reviewed. Then, a more detailed statistical analysis of the surveyed research, with respect to the source of uncertainty, number of publications trend over time, adopted APP strategies, applied management science methodologies and their sub-categories, and so on, is presented. Finally, possible future research paths are discussed on the basis of identified research trends and research gaps.

Keywords: Aggregate production planning (APP); Uncertainty; Literature; Model.

1.1. Introduction

Aggregate production planning (APP) is a type of medium term capacity planning that usually covers a time horizon of 3 to 18 months and its aim is to determine optimal level of production, inventory and human resources regarding the limitations of production resources and other constraints. The purpose of APP is (I) determining overall level of each product category to meet fluctuating and uncertain demand in near future, (II) adopting decisions and policies in regard to hiring, lay off, overtime, backorder, subcontracting, inventory level and available production resources.

APP has attracted considerable attention from both practitioners and academia (Shi & Haase, 1996). Since the pioneering studies by Holt, Modigliani and Simon (1955) and Holt, Modigliani and Muth (1956) proposed linear decision rule, and Bowman (1956) suggested transportation method to deal with APP, researchers have developed different methodologies to handle the APP problem.
Fig. 1.1 outlines the APP position among other types of production planning and control techniques and their interconnected relationships from a holistic perspective. As can be seen from the Fig. 1.1, in the hierarchy of production planning activities, APP falls between long-term strategic planning decisions such as new product development and short term shop floor scheduling practices.
Uncertainty is described by Funtowicz and Ravetz (1990) as a situation of inadequate information, which can be present in three forms: inexactness, unreliability, and border with ignorance. Walker et al. (2003) adopt a general definition of uncertainty as being any departure from the unachievable ideal of complete determinism.

A large portion of the existing research studies the deterministic state of APP and ignores its inherent uncertain nature. This assumption may be valid in several APP decision making problems where product demand exhibits a smooth pattern, i.e. demand has low coefficient of variation and workforce market, materials price and availability and other related factors show a rather consistent state.

However, in practical business environments, products usually have shorter life cycles, demand is uncertain and variable, customers’ preferences are changing, production capacity is limited, workforce market condition is unstable, subcontracting may impose higher costs and has its own difficulties, raw materials supply is uncertain and increase in backorders leads to customers’ dissatisfaction and makes them change their purchasing source. These all display the dynamic and uncertain characteristics of APP, and the need to incorporate these uncertainties into the APP decision models. Therefore, the utilisation of traditional deterministic methodologies may lead to considerable errors and imprecise decisions.

A significant number of studies have been devoted to APP subject to uncertainty by considering different forms of uncertainty including stochasticity, possibilistic forms, fuzziness and randomness.

Fig. 1.2 indicates APP literature map (diagram). The early approaches applied to study APP are categorised as: (1) linear programming (Hanssmann and Hess, 1960; Charnes and Cooper, 1961), (2) linear decision rule (Holt, Modigliani and Simon, 1955; Holt, Modigliani and Muth, 1956), (3) transportation method (Bowman, 1956), (4) management coefficient method (Bowman, 1963), (5) parametric production planning (Jones, 1967), (6) search decision rule (Taubert, 1968), (7) simulation (Vergin, 1966) and (8) tabular/graphical methods (Peterson and Silver, 1979; Tersine, 1980).

Then, subsequent studies used different methods to deal with various kinds of APP problems, which can be divided into three general categories: (I) studies that apply deterministic management science techniques to APP decision making problem, (II) research which incorporates uncertainty in management science methods to study APP problem, (III) and finally qualitative research on APP that apply qualitative research approaches such as surveys, reviews and case studies. Instead of operations research methodologies.
The abovementioned classic approaches to deal with APP such as linear decision rule, transportation method and management coefficient method, which were popular in 1950’s, 1960’s and early 1970’s, are outdated and are no longer in use. Therefore, in present research the literature which has applied the subsequently developed approaches to handle APP are surveyed from about mid 1970’s until March 2017. As Fig. 1.2 shows, the operations research/management science methods that have been adopted in literature to study APP under uncertainty are generally classifiable into six main categories: stochastic mathematical programming, possibilistic programming, fuzzy mathematical programming, simulation, metaheuristics and evidential reasoning. Each of these categories could be divided into smaller sub-categories, which will be described in detail in subsequent parts.
Qualitative approaches

Management science models under uncertainty

Early management science approaches

Deterministic management science methods

Possibilistic programming
Stochastic mathematical programming
Fuzzy mathematical programming
Simulation
Metaheuristics
Evidential reasoning

Deterministic linear programming
Deterministic nonlinear/quadratic programming
Metaheuristics
Deterministic multi-objective optimisation

Literature on APP

Qualitative approaches

Surveys
Case studies
Reviews

Fig. 1.2: APP literature map
The paper is further organised as follows. The need for a bibliometric literature survey on APP under uncertainty is justified in the next part. Section 1.3 gives a preliminary literature analysis. The classification plan is presented in Section 1.4. In Section 1.5, the literature on APP under uncertainty is reviewed elaborately. Section 1.6 goes through a more detailed statistical analysis of the surveyed literature. In Section 1.7, conclusions are drawn and possible future research directions are discussed.

1.2. The need for bibliometric literature survey on APP under uncertainty

The last literature survey on APP was conducted by Nam and Logendran (1992). Therefore, an up to date literature survey in this area is required. This is the first bibliometric-based literature survey of its kind on APP under uncertainty. The authors decided to consider APP, as a central activity in production planning and control which was depicted in Fig. 1.1, instead of general production planning in order to provide an in-depth and focused literature analysis. The researchers have been incorporating uncertainty in APP to make decision models which better represent the present day turbulent industrial environments. The research on APP in presence of uncertainty has been growing constantly over the recent decades. Current study considers the existing research on APP under uncertainty as a crucial and constantly growing part of the research about APP. The research on deterministic APP decision models would require a separate literature survey, again, to present another in-depth and specialised literature analysis. The detailed statistical/numerical analysis of literature regarding journal contributions, publication frequencies over time, methodologies applied to study APP under uncertainty, etc. provides research insights about recent research trends and research gaps for interested researchers. The recommendations on future research directions which are drawn based on recent research trends and existing research gaps will provide a basis for other researchers to make their own research agenda.
1.3. Preliminary analysis of the literature

Uncertainty has been incorporated in operations research/management science-based models of APP in different shapes including stochasticity, randomness, possibility, fuzziness and vagueness of the information.

Search for the term “aggregate production planning (APP)” found a large number of results but they were filtered by adding the words “uncertain/uncertainty”, “stochastic/stochasticity”, “possibility/possibilistic”, “random/randomness”, “fuzzy/fuzziness”, “probability/probabilistic” and “chance constrained”. As of 19th March 2017, a total of 82 publications were surveyed, which include 69 journal articles (84.15%), 5 conference/proceedings papers (6.10%), 6 book chapters (7.32%), one PhD thesis (1.22%) and one paper from Social Science Research Network (1.22%). The Table 1.1 shows the details of the reviewed literature.

As can be seen from Table 1.1, journal papers comprise the largest proportion of the surveyed literature, i.e. 84.15%, and two journals *International Journal of Production Research* and *Production Planning & Control* make the highest contributions among the reviewed publications, which are 9.76% and 8.54% respectively. The next four equal contribution levels, 6.10%, belong to *European Journal of Operational Research, International Journal of Production Economics, Computers & Industrial Engineering* and *International Journal of Advanced Manufacturing Technology*.

Table 1.1: The details of the surveyed literature

<table>
<thead>
<tr>
<th>Surveyed literature</th>
<th>Publication frequency</th>
<th>Percentage of total</th>
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<tbody>
<tr>
<td>International Journal of Production Research</td>
<td>8</td>
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<tr>
<td>Production Planning &amp; Control</td>
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<td>8.54</td>
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<td>European Journal of Operational Research</td>
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<td>International Journal of Production Economics</td>
<td>5</td>
<td>6.10</td>
</tr>
<tr>
<td>Computers &amp; Industrial Engineering</td>
<td>5</td>
<td>6.10</td>
</tr>
<tr>
<td>International Journal of Advanced Manufacturing Technology</td>
<td>5</td>
<td>6.10</td>
</tr>
<tr>
<td>Management Science</td>
<td>3</td>
<td>3.66</td>
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<tr>
<td>Journal of the Operational Research Society</td>
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<td>Applied Mathematical Modelling</td>
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<td>International Journal of Systems Science</td>
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<tr>
<td>Journal of Statistics &amp; Management Systems</td>
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</tr>
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<td>Mathematical Problems in Engineering</td>
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<td>Conferences /Proceedings</td>
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<td>6.10</td>
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<tr>
<td>Book Chapters</td>
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<td>PhD Theses</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>82</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Table 1.2 shows top ten countries in terms of number of publications in the area of APP subject to uncertainty based on the affiliation of the first author. As Table 1.2 indicates, Iran, Taiwan and United States top the list with publication frequencies of 14 (17.07%), 11 (13.41%) and 10 (12.20%) out 82 (100%).
Table 1.2: Top ten countries in terms of number of publications on APP under uncertainty

<table>
<thead>
<tr>
<th>Order</th>
<th>Country</th>
<th>Number of publications</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Iran</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>Taiwan</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>US</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>China</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Brazil</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>Canada</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Hong Kong</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>Italy</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>Malaysia</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>Turkey</td>
<td>3</td>
</tr>
</tbody>
</table>

1.4. Classification scheme

As already mentioned in Section 1.1, the methodologies applied in the literature to handle APP under uncertainty can be classified into six main categories: stochastic mathematical programming, possibilistic programming, fuzzy mathematical programming, simulation, metaheuristics and evidential reasoning. Each of these categories is divided into sub-categories, which have been shown in Table 1.3.

Table 1.3: Classification of the methodologies applied to study APP subject to uncertainty

<table>
<thead>
<tr>
<th>Categories</th>
<th>Sub-Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic mathematical programming</td>
<td>Stochastic linear programming; stochastic nonlinear programming; stochastic multi-objective optimisation; robust optimisation; stochastic control; stochastic queuing; stochastic process</td>
</tr>
<tr>
<td>Possibilistic programming</td>
<td>Possibilistic linear programming; possibilistic linear multi-objective optimisation; interactive possibilistic linear programming</td>
</tr>
<tr>
<td>Fuzzy mathematical programming</td>
<td>Fuzzy multi-objective optimisation; fuzzy linear programming; fuzzy nonlinear programming; fuzzy logic control; fuzzy robust optimisation; approximate reasoning</td>
</tr>
<tr>
<td>Simulation</td>
<td>Discrete-event simulation; system dynamics; Monte Carlo simulation; fuzzy random simulation</td>
</tr>
<tr>
<td>Metaheuristics</td>
<td>Genetic algorithm; tabu-search; harmony search algorithm; particle swarm optimisation; hunting search algorithm; firefly algorithm</td>
</tr>
<tr>
<td>Evidential reasoning</td>
<td>Belief-rule-based inference method</td>
</tr>
</tbody>
</table>
In short, these categories and sub-categories are described as follows:

**Stochastic mathematical programming:** It includes mathematical models for APP under uncertainty that apply stochastic linear programming, stochastic nonlinear programming, stochastic multi-objective optimisation, and so on where demand for products, constants and coefficients of the mathematical programming models and decision variables are of stochastic/random nature. This group also includes mathematical programming models with probabilistic constraints or chance constrained models.

**Possibilistic programming:** Possibilistic linear programming and possibilistic linear multi-objective optimisation methods belong to this category. In general, the possibilistic programming models to deal with APP is recommended when the information about the forecasted demand, parameters and coefficients of the constructed mathematical programming models and objective function/goal values are imprecise in essence.

**Fuzzy mathematical programming:** This class of models for APP in presence of uncertainty covers a wide range of mathematical programming models in fuzzy environment such as fuzzy linear programming, fuzzy nonlinear programming and fuzzy multi-objective optimisation. In this set of models, uncertainty is present in the form of fuzziness, which involves market demand, objective/goal values, constants, coefficients and constraints of the developed management science models.

**Simulation:** Discrete-event simulation, system dynamics, Monte Carlo simulation, etc. are among the simulation methodologies that have been proposed to run APP decision problem so that forecasted demand, objective/goal values, parameters/coefficients and constraints are supposed to be uncertain in their nature.

**Metaheuristics:** Due to the nonlinearity, combinatorial and large scale nature of APP problems, metaheuristics have proved to be efficient techniques to solve APP problems with uncertain characteristics. In this group of APP models, uncertainty is present in decision variables, customer demand, objective function/goal values, constraints, constants and coefficients of the constructed operations research models.

**Evidential reasoning:** At present, to the best of the authors’ knowledge, just one paper on evidential reasoning to APP has been published, which employs a belief-rule-based inference method to handle APP decision making problem with uncertain demand.

The APP literature that applies each of the abovementioned methodologies will be reviewed in the next part.
1.5. Elaborate review of the literature on APP subject to uncertainty

In following sections the literature about quantitative APP models under uncertainty is reviewed regarding different methodologies that it employs. The studies have been reviewed in chronological order within each category.

1.5.1. Fuzzy mathematical programming

The literature on application of fuzzy mathematical programming approaches in the APP context can be classified into studies which apply I) fuzzy multi-objective optimisation, II) fuzzy goal programming, III) fuzzy linear programming, IV) fuzzy nonlinear programming, V) fuzzy logic control, VI) fuzzy robust optimisation and VII) approximate reasoning techniques. As an explanation, although the fuzzy goal programming could be considered as a subset of fuzzy multi-objective optimisation but due to the significant number of publications that apply fuzzy goal programming to APP, it has been presented as a separate sub-division to show a clearer picture of the literature.

1.5.1.1. Fuzzy multi-objective optimisation

Since APP problem always involves several criteria (objectives) and due to the vagueness of the acquired information, fuzzy multi-objective programming has been widely used in this area. Lee (1990), Gen, Tsujimura and Ida (1992), Wang and Fang (2001), Wang and Liang (2004), Ghasemy Yaghin, Torabi and Fatemi Ghomi (2012), Madadi and Wong (2014), Gholamian et al. (2015), Gholamian, Mahdavi and Tavakkoli-Moghaddam (2016), Kalaf et al. (2015), Sisca, Fiasché and Taisch (2015) and Fiasché et al. (2016) utilised various kinds of fuzzy multi-objective optimisation models to study APP under uncertainty.

Lee (1990) recommended fuzzy linear programming and fuzzy multi-objective linear programming approaches to handle APP problem under fuzziness with fuzzy objective values, fuzzy demand, etc. Gen, Tsujimura and Ida (1992) presented an interactive fuzzy linear multi-objective programming method for APP such that all coefficients/parameters are regarded as triangular fuzzy numbers. Wang and Fang (2001) proposed a fuzzy linear multi-objective optimisation approach to APP decision making problem where product price, subcontracting cost, production capacity, and so forth are all characterised as fuzzy variables.

Wang and Liang (2004) developed a fuzzy linear multi-objective optimisation model to deal with APP decision problem, which tries to minimise total production costs, inventory holding and backordering costs and costs of changes in the workforce level. In their model, objective
functions are of fuzzy nature. A hybrid fuzzy multi-objective APP decision model in a two echelon supply chain with both quantitative and qualitative objectives and constraints was recommended by Ghasemy Yaghin, Torabi and Fatemi Ghomi (2012) where cost parameters, warehouse space, etc. are assumed to be fuzzy variables. A multi-objective fuzzy APP model with qualitative and quantitative objectives was proposed by Madadi and Wong (2014). In their model, forecasted demand, production costs, and so on are regarded as fuzzy numbers. Gholamian et al. (2015) and Gholamian, Mahdavi and Tavakkoli-Moghaddam (2016) developed a fuzzy multi-site multi-objective mixed integer nonlinear APP model in a supply chain under uncertainty with fuzzy demand, fuzzy cost parameters, etc. A modified fuzzy multi-objective linear programming method to APP that minimises total production costs and total labour costs is proposed by Kalaf et al. (2015), which involves fuzzy aspiration levels of the objectives and fuzzy tolerance levels. Sisca, Fiasché and Taisch (2015) constructed a fuzzy multi-objective linear programming model for APP in a reconfigurable assembly unit for optoelectronics where product price, inventory cost, etc. are supposed to be fuzzy variables. Fiasché et al. (2016) developed a fuzzy linear multi-objective optimisation model of APP in fuzzy environment where the product price, unit cost of not utilising the resources, etc. are of fuzzy nature.

1.5.1.2. Fuzzy goal programming

Da Silva and Marins (2004), Wang and Liang (2005b), Tavakkoli-Moghaddam et al. (2007), Jamalnia and Soukhakian (2009), Belmokaddem, Mekidiche and Sahed (2009), and Sadeghi, Razavi Hajiağa and Hashemi (2013) developed a variety of fuzzy goal programming models to tackle APP problem in presence of uncertainty.

Da Silva and Marins (2004) developed a fuzzy goal programming model for APP in a Brazilian sugar mill. In their study, the goal values are presented as triangular and trapezoidal fuzzy numbers. Wang and Liang (2005b) presented an interactive fuzzy multi-objective linear programming approach for APP decision problem with fuzzy goal values. Tavakkoli-Moghaddam et al. (2007) suggested a fuzzy mixed-integer goal programming model to run APP problem, which includes fuzzy goal values, fuzzy technological coefficients, fuzzy constraints upper bounds and fuzzy demand. Jamalnia and Soukhakian (2009) proposed a hybrid fuzzy goal programming approach that includes both quantitative and qualitative objectives with fuzzy aspiration levels.

Belmokaddem, Mekidiche and Sahed (2009) applied a fuzzy goal programming method with different goal priorities to APP where the goal values are of fuzzy nature. Sadeghi, Razavi
Hajiagha and Hashemi (2013) proposed a fuzzy goal programming model of APP with fuzzy aspiration levels where coefficients and parameters of the model are assumed to be grey numbers.

1.5.1.3. Fuzzy linear programming


Dai et al. (2003) presented a fuzzy linear programming methodology to deal with APP in condition of imprecise information and fuzzy constraints. Liang et al. (2011) constructed a fuzzy linear programming model of APP, which attempts to minimise total production cost subject to constraints on inventory levels, workforce levels, etc. where objective function and its coefficients and constraints’ upper/lower bounds are assumed to be fuzzy variables. A fuzzy mixed-integer linear programming model for APP with fuzzy demand, fuzzy warehouse space, fuzzy cost parameters, and so forth in a multi-echelon multi item supply chain network was developed by Pathak and Sarkar (2011).

Omar, Jusoh and Omar (2012) investigated the benefits of applying fuzzy mathematical programming in APP context by developing a fuzzy mixed-integer linear programming model to APP with fuzzy demand, fuzzy cost parameters, etc. in a resin manufacturing plant, which considers both fuzzy and possibilistic uncertainties. Wang and Zheng (2013) proposed a fuzzy linear programming method to APP in a refinery industry in Taiwan, which aims at maximising total profit so that market demand and cost items are characterised as fuzzy numbers.

A fuzzy linear programming model of APP with imprecise data which involves fuzzy demand and fuzzy cost items was suggested by Iris and Cevikcan (2014). Chen and Huang (2014) proposed the extension principle to solve the developed fuzzy linear programming model for APP where the forecasted demand, maximum available labour, and so on are of fuzzy nature.

1.5.1.4. Fuzzy nonlinear programming


Tang, Wang and Fung (2000) proposed a fuzzy nonlinear programming model of APP with quadratic objective function, which is to minimise total production and inventory costs where

Chen and Huang (2010) constructed a fuzzy nonlinear programming to APP using the membership function of the fuzzy minimal total cost so that maximum workforce level and forecasted demand adopt fuzzy nature. An APP problem with considering learning effects and demand under uncertainty was studied by Chen and Sarker (2015). Then, their fuzzy nonlinear programming model was compared to two other models which had not considered learning effects and uncertain demand.

1.5.1.5. Other fuzzy mathematical programming approaches

Turksen and Zhong (1988) proposed an approximate reasoning schema to implement an expert system in APP where independent variables, which are sales forecast, inventory level and workforce level at the end of current period and decision variables, which are production rate and change in workforce level for next period, could be of uncertain nature. Ward, Ralston and Davis (1992) utilised C language fuzzy logic controller to study APP such that Inventory level, labour level, etc. are represented by fuzzy sets. A robust fuzzy model for APP was developed by Rahmani, Yousei and Ramezanian (2014), which includes fuzzy customer demand, fuzzy cost items, etc.

1.5.2. Stochastic mathematical programming

The literature on stochastic mathematical programming approaches for APP in presence of uncertainty includes stochastic linear programming, stochastic nonlinear programming, robust optimisation, stochastic control, etc., which are reviewed concisely in this section.

1.5.2.1. Stochastic linear programming

The research on stochastic linear programming to APP subject to uncertainty includes the studies carried out by Lockett and Muhlemann (1978), Kleindorfer and Kunreuther (1978), Günther (1982), Thompson, Wantanabe and Davis (1993) and Leung, Wu and Lai (2006). Lockett and Muhlemann (1978) developed a stochastic linear programming model of APP with zero-one variables, which involves uncertainties about whether the outcome of a job is Ok, rework or scrap. Kleindorfer and Kunreuther (1978) proposed a methodology to show how
forecast horizons for stochastic aggregate planning problems with uncertain demand relate to the planning procedures and the information system within the organisation. Günther (1982) presented a stochastic linear programming approach to deal with APP problem under demand uncertainty. Thompson, Wantanabe and Davis (1993) developed linear programming frameworks to evaluate several APP policies where customer demand, most of the coefficients of the linear programming model and some parameters were presented with probability distributions to reflect the uncertainty in APP environment. A stochastic linear programming method to handle APP with stochastic demand and stochastic cost parameters was proposed by Leung, Wu and Lai (2006).

1.5.2.2. Stochastic multi-objective optimisation

Rakes, Franz and Wynne (1984), Chen and Liao (2003) and Nowak (2013) utilised stochastic multi-objective optimisation techniques to consider APP under uncertainty. Rakes, Franz and Wynne (1984) applied a chance-constrained goal programming approach to APP. In their model, the product demands, time required for inspection and products testing and so forth are random variables. Chen and Liao (2003) adopted a multi-attribute decision making approach to select the most efficient APP strategy such that selling price, market demand, cost coefficients, etc. are assumed to be stochastic variables. Nowak (2013) presented a procedure which combines linear multi-objective programming, simulation and an interactive approach to model APP with uncertain demand.

1.5.2.3. Stochastic nonlinear programming

Lieckens and Vandaele (2014) developed a multi-product, multi routing model where a routing consists of a sequence of operations on different resources so that the uncertainty is associated with the stochastic nature of both demand patterns and production lead times.

1.5.2.4. Robust optimisation


Leung and Wu (2004) proposed a robust optimisation model for APP to minimise summation of costs related to production, labour, inventory, hiring and lay off where the forecasted demand, hiring and lay off costs and labour costs are random variables under different economic growth scenarios. Kanyalkar and Adil (2010) proposed a robust optimisation approach which integrates APP with a detailed plan in a multi-site procurement-production-distribution system under demand uncertainty.

Mirzapour Al-e-hashem, Malekly and Aryanezhad (2011) and Mirzapour Al-e-hashem, Aryanezhad and Sadjadi (2012) suggested robust multi-objective optimisation models to deal with APP problem with two objective functions that aims at minimisation of total costs and maximisation of the customer services with cost parameters, demand, etc. under uncertainty. The former is solved using LP-metrics method, and the latter with a combination of an augmented ε-constraint method and genetic algorithm.

Niknamfar et al. (2015) developed a robust optimisation model for aggregate production-distribution planning so that unit production and fixed costs for production units, unit storage and fixed costs for distribution centres, selling prices, and so forth adopt uncertain nature in a three-level supply chain. Modarres and Izadpanahi (2016) proposed a linear multi-objective optimisation model to APP with uncertain product demand which tries to minimise operational costs, energy costs and carbon emission. To deal with uncertain input data, a robust optimisation approach is also applied.

Entezaminia, Heidari and Rahmani (2016) suggested a robust optimisation approach to handle a multi-site APP problem in green supply chain with regard to potential collection and cycling centres under uncertainty where customer demand and cost parameters are supposed to be of uncertain nature. Makui et al. (2016) implemented APP for products with very limited
expiration dates. A robust optimisation method is also used due to inherent uncertainty of parameters of the constructed APP model.

1.5.2.5. Stochastic control

Love and Turner (1993), Shen (1994), Silva Filho (2005) and Silva Filho (2014) recommended different types of stochastic control approaches to handle APP problem under uncertainty. Love and Turner (1993) suggested a stochastic optimal control methodology to APP with uncertain demand, and the performance was compared to that of deterministic approaches to the problem. Shen (1994) applied three stochastic control methods (certainty equivalence, passive learning and active learning) to study the classical APP problem which was considered by Holt et al. (1955) where the system error vector, the parameter error vector, etc. are stochastic variables.

Silva Filho (2005) formulated APP problem as a chance-constrained stochastic control problem under imperfect information of states (i.e. the inventory levels). A linear-quadratic Gaussian (LQG) optimal control model with chance constraints on state and control variables was proposed by Silva Filho (2014) for APP. In the constructed model demand adopts stochastic nature.

1.5.2.6. Other stochastic mathematical programming methodologies

Silva Filho (1999) modelled APP by using a stochastic process approach with a chance constraint on inventory where cumulative demand is represented as random variable. In the proposed model, production, inventory and workforce costs are supposed to be quadratic functions.

An aggregate stochastic queuing (ASQ) model was introduced by Hahn et al. (2012) to anticipate capacity buffers and lead time offsets for each time bucket of the APP model where set up times and processing times in the ASQ model are of stochastic form.

Gongbing and Kun (2014) constructed a data envelopment analysis (DEA)-based model to APP with stochastic demand.

1.5.3. Simulation

Simulation modelling of APP problem under uncertainty covers a spectrum from discrete-event simulation and system dynamics to fuzzy random simulation.

1.5.3.1. Common discrete-event simulation

Lee and Khumawala (1974) assessed the performance of four different APP policies under demand uncertainty by simulating the activities of an operating firm. McClain and Thomas (1977) utilised both simulation and linear programming techniques to evaluate the horizon effects in APP with seasonal demand where in simulation case, the demand was supposed to be random normal variable. Lee, Steinberg and Khumawala (1983) compared the effectiveness of the aggregate-disaggregate and material requirements planning approaches to production planning in a simulation environment such that demand was generated by using stochastic functions. Their research applied linear decision rule as the optimal aggregate technique in the aggregate-disaggregate approach.

Tang, Fung and Yung (2003) conducted a simulation analysis for multi-product APP problem under fuzziness of demand and capacities. Tian, Mohamed and AbouRizk (2010) applied a simulation-based approach to aggregate planning of a batch plant which produces concrete and asphalts so that fluctuating demand could be generated by using a statistical distribution, e.g. uniform, normal, etc. Gansterer (2015) investigated the impact of APP with demand under uncertainty in a make-to-order environment utilising a discrete-event simulation method within a comprehensive hierarchical production planning framework.

Altendorfer, Felberbauer and Jodlbauer (2016) evaluated the effect of long term forecast error on optimal planned utilisation factor for a production system with stochastic customer demand. Simulation is used to determine overall costs like capacity, backorder and inventory costs.

1.5.3.2. Other simulation modelling techniques

Khouja (1998) developed an APP framework to evaluate volume flexibility using Monte Carlo simulation with normally distributed demand.

Ning, Wansheng and Zhao (2006) constructed a fuzzy random model for APP in which market demand, production cost, etc. are all assumed to be fuzzy random variables. Then, the proposed model is solved employing hybrid optimization algorithm combining fuzzy random
simulation, genetic algorithm, neural network and simultaneous perturbation stochastic approximation algorithm.

By employing an integrated system dynamics and discrete-event simulation, Jamalnia and Feili (2013) evaluated effectiveness and practicality of different APP strategies regarding total profit criterion where the forecasted demand was represented as random normal distribution.

1.5.4. Metaheuristics


1.5.4.1. Genetic algorithms

Wang and Fang (1997) presented an inexact approach which imitates the human decision making process by generating a family of inexact solutions to a fuzzy linear programming with fuzzy objective values and fuzzy constraints using a genetics-based algorithm within an acceptable level as candidates for a decision maker to consider further. Fichera et al. (1999) suggested possibilistic linear programming and genetic algorithm as a decision support system for APP to assist decision makers in APP decisions in a vague environment where the constraint on balance equation for production, inventory and demand and total production capacity are of possibilistic form. An interactive fuzzy-genetic methodology to solve aggregate production-distribution planning in supply chain subject to the fuzziness of total profit, total expenses, etc. was developed by Aliev et al. (2007).

1.5.4.2. Tabu search

Baykasoğlu and Göçken (2006) proposed a tabu-search method to solve a fuzzy goal programming model of APP with fuzzy goal values. Baykasoglu and Gocken (2010) proposed a multi-objective APP with fuzzy parameters, and solved the model by employing fuzzy numbers ranking methods and tabu search.

1.5.4.3. Other metaheuristic approaches

Aungkulanon, Phruksaphanrat and Luangpaiboon (2012) applied a harmony search algorithm with different evolutionary elements to solve a fuzzy multi-objective linear programming
model for APP with fuzzy objectives. Luangpaiboon and Aungkulanon (2013) presented a multi-objective linear programming decision making model for APP with inventory under uncertainty. Their proposed model was solved by applying hybrid metaheuristics of the hunting search (HuSIHSA) and firefly (FAIHSA) mechanisms on the improved harmony search algorithm. Chakrabortty et al. (2015) solved an integer linear programming model of APP with imprecise operating costs, demand and capacity related data by employing a particle swarm optimisation approach.

1.5.5. Possibilistic programming

The literature on possibilistic programming approaches that utilised to study APP subject to uncertainty ranges from regular possibilistic programming methods to interactive possibilistic programming approaches.

1.5.5.1. Ordinary possibilistic programming

Hsieh and Wu (2000) and Sakallı et al. (2010) proposed various forms of possibilistic programming approaches to study APP with imprecise information. Hsieh and Wu (2000) proposed a possibilistic linear multi-objective optimisation approach to consider APP decision making problem with imprecise demand and cost coefficients, which take triangular possibility distribution functions. Sakallı, Baykoç and Birgören (2010) presented a possibilistic linear programming model for APP in brass casting industry. In the constructed model, demand quantities, percentages of the ingredient in some raw materials, etc. have imprecise nature, and adopt triangular possibility distributions.

1.5.5.2. Interactive possibilistic programming

Wang and Liang (2005a), Liang (2007) and Liang (2007) developed interactive possibilistic programming models for APP problem with imprecise information. Wang and Liang (2005a) presented a novel interactive possibilistic linear programming (i-PLP) approach, which considers APP with imprecise forecasted demand, related operating costs, and capacity. Their model tries to minimise total costs regarding the constraints on inventory levels, labour levels, overtime, etc. A multi-objective APP problem with imprecise demand, cost coefficients, available resources and capacity was studied by Liang (2007) with applying an
interactive linear multi-objective possibilistic programming model. The proposed model minimises total production costs and oscillations in work-force level.

Liang (2007) presented an i-PLP method to solve APP problems where the objective function, forecasted demand, related capacities and operating costs adopt imprecise nature. The study aims at minimising total manufacturing costs subject to bounds on inventory, labour, overtime, and so on for each operating cost category.

1.5.6. Evidential reasoning

Li et al. (2013) presented a belief-rule-based inference methodology for APP under demand uncertainty. The proposed model was implemented by using a paint factory example to conduct a comparative study and sensitivity analysis.

1.6. Further detailed analysis of the literature

1.6.1. Source of uncertainty in APP models

As has been indicated in Table 1.4, the literature was analysed based on which type of the six main methodologies described in Section 1.4 they use, and which elements are subject to uncertainty. Uncertainty could be present in different elements of the developed quantitative models for APP such as forecasted demand, objective function values, goals aspiration levels, constants/coefficients, constraints and decision variables. Although, forecasted demand would also be represented under the coefficients/parameters category but because of the great number of the occasions it has been under uncertainty in the studied APP models, we represent it as a separate part to make the literature analysis more informative.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Forecasted demand</th>
<th>Objective/Goal values</th>
<th>Coefficients/Parameters</th>
<th>Constraints</th>
<th>Decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic mathematical programming</td>
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<td>9</td>
<td>25</td>
<td>20</td>
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<tr>
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<td>5</td>
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<td>5</td>
<td>5</td>
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<tr>
<td>Fuzzy mathematical programming</td>
<td>24</td>
<td>26</td>
<td>25</td>
<td>25</td>
<td>3</td>
</tr>
<tr>
<td>Simulation</td>
<td>10</td>
<td>2</td>
<td>9</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Metaheuristics</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Evidential reasoning</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>66</td>
<td>49</td>
<td>70</td>
<td>61</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 1.4: The source of uncertainty in APP models in presence of uncertainty

Table 1.4 shows that coefficients/parameters account for the highest frequency of the uncertainty in total, 70 (27.45%) out of 255 (100%), among the components of the APP models under study. That is, in total in 70 studies (publications), coefficients/parameters were subject
to a form of uncertainty. Two of the equally highest frequencies of the coefficients/parameters under uncertainty, i.e. 25 (35.71%) out of 70 (100%), belong to the studies that apply fuzzy mathematical programming and stochastic mathematical programming methods. The third tier is represented by the literature that employs simulation techniques which contributes to 9 (12.86%) out of 70 (100%) occasions of the coefficients and parameters uncertainty that is a sharp decrease compared to the first two highest frequencies.

Forecasted market demand comes in the second place among the elements of the surveyed APP models under uncertainty in terms of frequency of being uncertain, which adds up to 66 (25.88%) out of 255 (100%) in total. Similar to the coefficients/parameters case, fuzzy mathematical programming, stochastic mathematical programming and simulation modelling methodologies top the list for the number of occasions that forecasted demand characterised as uncertain in the reviewed literature with corresponding frequencies 24 (36.36%), 22 (33.33%) and 10 (15.15%) out of 66 (100%) respectively.

Constraints represent the third level of the uncertainty frequencies among the elements of the reviewed APP models in presence of uncertainty with total frequency of 61 (23.92%) out of 255 (100%). Again, similar to the two previously analysed APP model components, fuzzy mathematical programming and stochastic mathematical programming techniques make the highest contributions in terms of number of occasions that the surveyed research studies include uncertain constraints, which are 25 (40.98%) and 20 (32.79%) out of 61 (100%) respectively. But, unlike the two previously analysed elements of the APP models, now metaheuristics come in the third place with respect to the number of occasions that the surveyed literature includes uncertain constraints, i.e. 6 (9.84%) out of 61 (100%).

Finally, objective/goal values and decision variables come in the fourth and fifth places in terms of the total occasions that these APP model components are subject to uncertainty with corresponding frequencies of 49 (19.22%) and 9 (3.53%) out of 255 (100%).

More details about the frequencies regarding each of these components and the relevant methodologies are presented in Table 1.4.

1.6.2. Trends for frequency of published research regarding each main category of the methodologies

Table 1.5 shows the number of publications in each decade regarding the respective methodologies applied in the literature to study APP in presence of uncertainty.

As is evident from Table 1.5, the two highest frequencies of the published research on APP under uncertainty belong to 2010s and 2000s with total frequencies of 42 (51.22%) and 21
(25.61%) out of 82 (100%) respectively. 1990s come in the third place with total number of 11 (13.41%) publications out of 82 (100%). The number of the studies in other decades with respect to relevant methodologies has been presented in Table 1.5. Generally, the total number of literature about APP subject to uncertainty has been increasing constantly from 1970s until 2010s.

In 2010s, the research that applies fuzzy mathematical programming and stochastic mathematical programming techniques accounts for 18 (42.86%) and 14 (33.33%) out of total publications number, i.e. 42 (100%), which put them in the first and second orders respectively. The studies that utilise simulation and metaheuristics methods jointly come in the third place with equal frequencies of 4 (9.52%) out of 42 (100%).

For the decade starting in 2000, of 21 studies (100%) the highest number, 9 (42.86%), goes to the literature which applies fuzzy mathematical programming to APP. Possibilistic programming and stochastic mathematical programming methods come in the next place with equal contribution of 4 (19.05%) out of 21 (100%).

In the time period 1990-1999, three of the highest frequencies of the studies about APP under uncertainty, belong to stochastic mathematical programming, fuzzy mathematical programming and metaheuristics-based methodologies with corresponding frequencies of 5 (45%), 3 (27.27%) and 2 (18.18%) out of 11 (100%).

Total number of the literature on fuzzy mathematical programming to APP in presence of uncertainty for all decades, 31 (37.80%) out of 82 (100%), stays in the first place. The second and third levels of the frequencies, 27 (32.93%) and 10 (12.20%) out of 82 (100%), are represented by stochastic mathematical programming and simulation methods respectively, which has been shown in Fig. 1.3.

Table 1.5: The number of publications on APP under uncertainty over time

<table>
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<tr>
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<tbody>
<tr>
<td>Stochastic mathematical Programming</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>14</td>
<td>27</td>
</tr>
<tr>
<td>Possibilistic programming</td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Fuzzy mathematical Programming</td>
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<td>3</td>
<td>9</td>
<td>18</td>
<td></td>
<td>31</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Metaheuristics</td>
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<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Evidential reasoning</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>4</td>
<td>4</td>
<td>11</td>
<td>21</td>
<td>42</td>
<td>82</td>
</tr>
</tbody>
</table>
Fig. 1.4-a: Trend analysis plot for the number of studies on fuzzy mathematical programming to APP

Fig. 1.4-b: Trend analysis plot for the number of studies on stochastic mathematical programming to APP

The regression lines for the number of studies on stochastic mathematical programming and metaheuristics-based models to APP decision problem under uncertainty show the second and the third steepest lines with the respective slopes of 0.0365 and 0.01495 during the last 43 years.
The least steep trend line belongs to the frequencies of studies on the basis of simulation modelling techniques to APP subject to uncertainty, with the slope 0.00680, which is indicator of the lowest growth rate in the amount of literature in this area.

1.6.3. Frequency of published research with regard to each sub-category of the methodologies

Table 1.6 shows the number of publications when each category of the methodologies for APP in presence of uncertainty are split into sub-categories. As can be seen from the Table 1.6, under the fuzzy mathematical programming category, methodologies such as fuzzy multi-objective optimisation, fuzzy linear programming and fuzzy goal programming respectively represent the three highest numbers of studies: 11 (35.48%), 7 (22.58%) and 6 (19.35%) out of 31 (100%).

Of 27 publications about stochastic mathematical programming for APP, robust optimisation technique represents the highest share on the number of published research among others, i.e. 8 (29.63%). Stochastic linear programming stays in the second order with total number of publication 5 (18.52%) out 27 (100%). With the equal frequencies of 4 (14.81%) out of 27 (100%), stochastic nonlinear programming and stochastic control methods come in the third
place. Frequency of 3 (11.11%) out of 27 (100%) puts stochastic multi-objective optimisation in the fourth place.

Common discrete-event simulation as a subset of the simulation methodology in general, has been utilised in research on APP in presence of uncertainty in 7 occasions (70%) out of 10 (100%), which is the greatest contribution among other simulation methods. Monte Carlo simulation, system dynamics and fuzzy random simulation all with equal frequencies, 1 (10%) out of 10 (100%), stay in the second place.

The number of published studies for other main categories of the methodologies and the corresponding sub-categories can be seen from Table 1.6.
Table 1.6: The frequencies of studies regarding each sub-category of the methodologies applied to APP under uncertainty

<table>
<thead>
<tr>
<th>Methodologies</th>
<th>Number of publications</th>
<th>Related references</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fuzzy mathematical programming</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuzzy linear programming</td>
<td>7</td>
<td>Dai, Fan and Sun (2003); Liang et al. (2011); Pathak and Sarkar (2011); Omair, Jusoh and Omar (2012); Wang and Zheng (2013); Iris and Cevikcan (2014); Chen and Huang (2014)</td>
</tr>
<tr>
<td>Fuzzy nonlinear programming</td>
<td>4</td>
<td>Tang, Wang and Fung (2000); Fung, Tang and Wang (2009); Chen and Huang (2010); Chen and Sarkar (2015)</td>
</tr>
<tr>
<td>Fuzzy multi-objective optimisation</td>
<td>11</td>
<td>Lee (1990); Gen, Tsujimura and Ida (1992); Wang and Fang (2001); Wang and Liang (2004); Ghasemy Yaghin, Torabi and Fatemi Ghomi (2012); Madadi and Wong (2014); Gholamian et al. (2015); Gholamian, Mahdavi and Tavakkoli-Moghaddam (2016); Kalaf et al. (2015); Sisca, Fiasché and Taisch (2015); Fiasché et al. (2016)</td>
</tr>
<tr>
<td>Fuzzy goal programming</td>
<td>6</td>
<td>Wang and Liang (2005b); Jamalnia and Soukhakian (2009); Belmokaddem, Mekidiche and Sahed (2009); Tavakkoli-Moghaddam et al. (2007); Da Silva and Marins (2004); Sadeghi, Razavi Hajajaga and Hashemi (2013)</td>
</tr>
<tr>
<td>Fuzzy logic control</td>
<td>1</td>
<td>Ward, Ralston and Davis (1992)</td>
</tr>
<tr>
<td>Approximate reasoning</td>
<td>1</td>
<td>Turksen and Zhong (1988)</td>
</tr>
<tr>
<td>Fuzzy robust optimisation</td>
<td>1</td>
<td>Rahmani, Yousei and Yousei (2014)</td>
</tr>
<tr>
<td><strong>Stochastic mathematical programming</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stochastic linear programming</td>
<td>5</td>
<td>Muhlemann (1978); Kleindorfer and Kunreuther (1978); Günther (1982); Thompson, Wantanabe and Davis (1993); Leung, Wu and Lai (2006)</td>
</tr>
<tr>
<td>Stochastic nonlinear programming</td>
<td>4</td>
<td>Vörös (1999); Ning, Liu and Yan (2013); Mirzapour Al-e-hesham, Baboli and Sazvar (2013); Lieckens and Vandaele (2014)</td>
</tr>
<tr>
<td>Stochastic multi-objective optimisation</td>
<td>3</td>
<td>Rakes, Franz and Wynne (1984); Chen and Liao (2003); Nowak (2013)</td>
</tr>
<tr>
<td>Robust optimisation</td>
<td>8</td>
<td>Wu (2004); Kanyalkar and Adil (2010); Mirzapour Al-e-hesham, Malekly and Aryanezhad (2011); Mirzapour Al-e-hesham, Aryanezhad and Sadjadi (2012); Niknamfar, Akhavan Niaki and Pasandideh (2015); Modarres and Izadpanahi (2016); Entezamiria, Heidari and Rahmani (2016); Makui et al. (2016)</td>
</tr>
<tr>
<td>Stochastic control</td>
<td>4</td>
<td>Love and Turner (1993); Shen (1994); Silva Filho (2005); Silva Filho (2014)</td>
</tr>
<tr>
<td>Aggregate stochastic queueing</td>
<td>1</td>
<td>Hahn et al. (2012)</td>
</tr>
<tr>
<td>Stochastic data envelopment analysis</td>
<td>1</td>
<td>Gongbing and Kun (2014)</td>
</tr>
<tr>
<td>Stochastic process</td>
<td>1</td>
<td>Silva Filho (1999)</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular discrete event simulation</td>
<td>7</td>
<td>Lee and Khumawala (1974); McClain and Thomas (1977); Lee, Steinberg and Khumawala (1983); Tang, Fung and Yung (2003); Tian, Mohamed and AbouRizk (2010); Gansterer (2015); Attendorfer, Felberbauer and Jodlbauer (2016)</td>
</tr>
</tbody>
</table>
Table 1.6: (Continued)

<table>
<thead>
<tr>
<th>Methodologies</th>
<th>Number of publications</th>
<th>Related references</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo simulation</td>
<td>1</td>
<td>Khouja (1998)</td>
</tr>
<tr>
<td>System dynamics</td>
<td>1</td>
<td>Jamalnia and Feili (2013)</td>
</tr>
<tr>
<td>Fuzzy random simulation</td>
<td>1</td>
<td>Ning, Wansheng and Zhao (2006)</td>
</tr>
</tbody>
</table>

**Metaheuristics**

| Genetic algorithms                         | 3                      | Wang and Fang (1997); Fichera et al. (1999); Aliev et al. (2007) |
| Tabu-search                                | 2                      | Baykasoğlu and Göçken (2006); Baykasoglu and Gocken (2010) |
| Harmony search algorithm                   | 1                      | Aungkulanon, Phruksaphanrat and Luangpaiboon (2012) |
| Hunting search (HuSIHSA) and firefly (FAIHSA) | 1                      | Luangpaiboon and Aungkulanon (2013) |
| Particle swarm optimisation                | 1                      | Chakrabortty et al. (2015)                              |
| **Possibilistic programming**              |                        |                                                        |
| Possibilistic linear programming           | 3                      | Sakalli, Baykoç and Birgören (2010); Wang and Liang (2005a); Liang (2007) |
| Possibilistic linear multi-objective optimisation | 2                      | Hsieh and Wu (2000); Liang (2007)                     |
| **Evidential reasoning**                   |                        |                                                        |
| Belief-rule-based inference method         | 1                      | Li et al. (2013)                                        |

1.6.4. The most cited research on APP under uncertainty

Table 1.7 shows the most cited research on APP in presence of uncertainty according to Google Scholar. As is evident from Table 1.7, the studies performed by Wang and Liang (2004), Aliev et al. (2007) and Wang and Liang (2005a) have the highest citation numbers, i.e. 228, 218 and 200 respectively. Their research was published in *Computers & Industrial Engineering*, *Information Sciences* and *International Journal of Production Economics* correspondingly. The author Reay-Chen Wang is present in three of the ten most cited papers as the first author. The author Dingwei Wang contributes to one of the ten most cited papers as the first author and to two of them as the second and third author. Authors Jiafu Tang and Richard Y.K. Fung each have authored one of the ten most cited papers as the first author and one another as the second or the third author.
Table 1.7: The most cited research on APP under uncertainty

<table>
<thead>
<tr>
<th>Order</th>
<th>Publication</th>
<th>Number of citations</th>
<th>Journal</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Aliev et al. (2007)</td>
<td>218</td>
<td>Information Sciences</td>
</tr>
<tr>
<td>6</td>
<td>Jamalnia and Soukhakian (2009)</td>
<td>112</td>
<td>Computers &amp; Industrial Engineering</td>
</tr>
<tr>
<td>8</td>
<td>Leung and Wu (2004)</td>
<td>82</td>
<td>Production Planning &amp; Control</td>
</tr>
<tr>
<td>9</td>
<td>Tang, Wang and Fung (2000)</td>
<td>79</td>
<td>Production Planning &amp; Control</td>
</tr>
</tbody>
</table>

1.6.5. Literature with respect to the applied APP strategy

Table 1.8 presents the number of the published studies on APP under uncertainty with respect to different APP strategies, i.e. mixed chase and level, pure chase, pure level, modified chase, modified level and demand management strategies. As Table 1.8 shows, the vast majority of the surveyed literature applies the mixed chase and level strategy, i.e. 82 out of 82. Modified chase and modified level strategies with equal frequencies of 3 (3.66%) out of 82 (100%), and pure chase, pure level and demand management strategies with equal frequencies of 1 (1.22%) out of 82 (100%) come in the second and third orders respectively. However, studies performed by Thompson et al. (1993) and Chen and Liao (2003) compared various APP policies in presence of uncertainty, and found out that chase strategies family are the most effective strategies, or are among the most effective strategies.

Table 1.8: Comparing the research on APP in presence of uncertainty with respect to the utilised APP strategy

<table>
<thead>
<tr>
<th></th>
<th>Mixed chase and level strategy</th>
<th>Pure chase strategy</th>
<th>Pure level strategy</th>
<th>Modified chase strategy</th>
<th>Modified level strategy</th>
<th>Demand management strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic mathematical programming</td>
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<td>0</td>
<td>2</td>
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<td>0</td>
</tr>
<tr>
<td>Possibilistic programming</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fuzzy mathematical programming</td>
<td>31</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Simulation</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Metaheuristics</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Evidential reasoning</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>82</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
1.7. Conclusions and future research directions

In this research, a wide scope of literature on APP under uncertainty was analysed. This literature includes journal papers, book chapters, conference/proceedings papers and PhD theses which were classified into six main categories on the basis of applied methodologies. The uncertainties present in the constructed management science models are of sorts like stochasticity, fuzziness and impreciseness of the information. First, the relevant literature about each of abovementioned methods was reviewed concisely, and then a more detailed statistical analysis of the surveyed research was followed.

Two journals, *International Journal of Production Research* and *Production Planning & Control*, make the highest contributions in terms of number of publications among the reviewed literature. Total numbers of studies which apply fuzzy mathematical programming and stochastic mathematical programming to APP in presence of uncertainty come in the first and second places respectively. The trend lines for the frequency of studies on fuzzy mathematical programming and stochastic mathematical programming to APP under uncertainty show the highest slopes.

Possible future research directions according to in-depth literature survey in present study are recommended as follows:

- As already was shown in Table 1.8, the absolutely prevalent APP strategy in the literature about APP in presence of uncertainty (and even in the literature on deterministic APP models) is the mixed chase and level strategy. However, the surveys conducted by Buxey (1990, 1995, 2003, 2005) revealed that the most popular APP policy among operations managers is the chase strategy, which shows a gap between APP in academia and APP in practice. This also indicates an intense gap related to the lack of the studies about quantitative APP models under uncertainty based on other APP strategies such as chase strategy, level strategy and the demand management strategy.

- Several relative advantages of the simulation techniques over mathematical programming methods, e.g. coping with dynamic or transient effects, addressing interactions between different components, the ability of providing a sufficient basis for developing explanatory and predictive models of operational processes, and so forth have been stated in the literature. Therefore, the relatively low share of the literature which apply simulation modelling to study APP subject to uncertainty (12.20%), and the least steep trend line of the frequency of the number of published
research in this area over recent decades recommend the need to do extra research in this field to compensate the unfairly narrow share of the simulation methods.

- To the best of our knowledge, only a single journal paper has been published on evidential reasoning (ER) to APP in both uncertain and deterministic manners. So, regarding the capabilities of the ER in handling the uncertainty, this could act as a foundation stone to drive more research in this area.

- Regarding the dynamic nature of APP in practice which is highly dependent on initial conditions, e.g. finished product inventory level and workforce level in the beginning, and considering the nonlinear and complex feature of many APP models, the chaos theory would be a suitable methodology to study APP system’s behaviour in unstable industrial environments.

- APP under uncertainty in manufacturing environments that follow the principles of the productions systems like just in time (JIT), lean production and flexible manufacturing system (FMS) with regard to specific requirements for each of these production philosophies can be fertile areas of the future research.
References


PAPER 2

**Title:** A novel decision model based on mixed chase and level strategy to aggregate production planning in presence of uncertainty

PAGE: 50
A novel decision model based on mixed chase and level strategy to aggregate production planning in presence of uncertainty

Abstract: The present study proposes a novel decision model to aggregate production planning (APP) decision making problem based on mixed chase and level strategy under uncertainty where the market demand acts as the main source of uncertainty. By taking into account the novel features, the constructed model turns out to be stochastic, nonlinear, multi-stage and multi-objective. APP in practice entails multiple-objectivity. Therefore, the model involves multiple objectives such as total revenue, total production costs, total labour productivity costs, optimum utilisation of production resources and capacity and customer satisfaction, and is validated on the basis of real world data from beverage manufacturing industry. Applying the recourse approach in stochastic programming leads to empty feasible space, and therefore the wait and see approach is used instead. After solving the model using the real-world industrial data, sensitivity analysis and several forms of trade-off analysis are conducted by changing different parameters/coefficients of the constructed model, and by analysing the compromise between objectives respectively. Finally, possible future research directions, with regard to the limitations of current study, are discussed.

Keywords: Aggregate production planning (APP); Uncertainty; Stochastic nonlinear multi-objective optimisation.

2.1. Introduction

2.1.1. General overview

Aggregate production planning (APP) is a medium term production and employment planning that typically covers a time horizon which ranges from 3 to 18 months, and is concerned with determining optimal production volumes, hiring and lay off rates, workforce and inventory levels, backordering and subcontracting quantities, and so on with respect to the limitations of production resources for each time period within the planning horizon. This planning technique usually involves one product or a family of similar products, i.e. with similarities in production process, skills required, raw materials needed, etc. despite small differences so that considering the problem from an aggregated viewpoint is still credible.

In the hierarchy of production planning, APP falls between long-term strategic planning decisions such as new product development and short term production scheduling activities.
Similar to other production planning family members, APP also involves several objectives/criteria in practice. Due to the dynamic nature of APP and instable state of real world industrial environments, deterministic models for APP would lead to un-robust decisions. This implies that uncertainties need to be incorporated into the APP decision models.

Current study proposes a novel decision making model to APP which takes into account dynamicity (stochasticity), nonlinearity and multiple objectivity simultaneously. The model turns out to be stochastic, nonlinear, multi-stage and multi-objective, and includes objectives such as total revenue, total production costs, total labour productivity costs, total costs of the changes in workforce level and customer satisfaction subject to bounds on inventory, backorder, subcontracting, workforce level, and so forth under uncertainty. The proposed approach models APP problem under the primary mixed strategy which integrates chase and level strategies to provide a holistic view of the APP.

The WWW-NIMBUS software (Miettinen and Mäkelä, 2006) will be used to solve the constructed stochastic, nonlinear, non-smooth, nonconvex, non-differentiable multi-objective optimisation model for the APP problem.

The paper is further organised as follows. The regular APP policies and options are explained in the next subsection, and then the problem under study is described in Section 2.1.3. The relevant literature is reviewed in Section 2.2. The proposed APP model is presented in Section 2.3 with comprehensive details. The constructed model is implemented by utilising real world industrial data in Section 2.4. Further experiments with the model are performed by trade-off analysis and sensitivity analysis in Section 2.5. Conclusion is drawn, and recommendations on possible future research directions are provided in final section.

2.1.2. Fundamental APP policies and options

Aggregate production planners are concerned with matching production capacity and demand when total expected demand and the available production capacity show significant discrepancy. Even if the cumulative demand and the cumulative supply/production are roughly equal over the planning horizon, dealing with unsmooth demand would still be a challenging task. The forecasted demand may exceed or fall below the planned capacity. In APP process, operations managers’ task is achieving an approximate equality (balance) between production capacity and projected demand within the planning horizon.

The basic demand management options are as follows (Heizer and Render, 2001; Reid and Sanders, 2002; Schroeder, 2003; Stevenson, 2005; Slack et al. 2007):
(1) pricing;
(2) promotion (advertising);
(3) backorders or reservations; and
(4) development of complementary products.

The options listed below are the basic capacity management alternatives (Heizer and Render, 2001; Reid and Sanders, 2002; Schroeder, 2003; Stevenson, 2005; Slack et al. 2007):

(1) varying workforce size by hiring and lay-offs;
(2) overtime/slack time and extra shifts;
(3) using part time or temporary labour;
(4) subcontracting; and
(5) changing inventory levels.

Three basic operations strategies can be used in APP, along with many combinations in between, to meet the fluctuating demand over time. One basic strategy is to level the workforce; the other is to chase demand with the workforce. With a perfectly level strategy, the rate of regular time output will be constant. Any variations in demand must then be absorbed using inventories, overtime, temporary workers, subcontracting, backorders or any of the demand-influencing options. With the pure chase strategy, the workforce level is changed to meet, or chase, demand. In this case, it is not necessary to carry inventory or to use any of the other variables available for APP; the workforce absorbs all the changes in demand (Reid and Sanders, 2002; Schroeder, 2003). The third strategy, the pure demand management strategy, is an approach that attempts to change or influence demand to fit available capacity by employing options such as pricing, advertising and developing alternative products and services (Slack et al., 2007).

Normally, the pure demand management policy is always considered as part of the level strategy. The present research also regards the demand management policy as a subset of the level strategy.

Each of the two pure plans is applied only where its advantages strongly outweigh its disadvantages. For many organisations, however, these pure approaches do not match their required combination of competitive and operational objectives. Most operations managers are required simultaneously to reduce costs and inventory, to minimise capital investment and yet to provide a responsive and customer-oriented approach at all times. For this reason, most organisations choose to follow a mixture of the two approaches (Slack et al., 2007).
2.1.3. Problem statement

The operations/manufacturing data was collected from ZamZam Group as a major soft drink producing company in West Asia. The company used to be the former subsidiary of PepsiCo in Iran but then it changed its brand name to ZamZam, and was extended from one plant to seventeen plants throughout the country and abroad. Over one hundred products which range from beverages to beers and mineral waters are produced by ZamZam Group. Normally, the company is operating in two 8 hour shifts. The first shift basically ends at 4pm every day, and then the second shift begins which are respectively called regular shift and extra shift by the operations management department of the company. The shifts are considered separately because of costs differences, e.g. wage costs in extra shift are higher. The subcontracting is usually produced by ZamZam Isfahan, another branch of the ZamZam Group in city of Isfahan, Iran. Backorders should be met by the next time period at the latest. The product demand follows a seasonal pattern, i.e. in spring and summer demand rises, and in autumn and winter demand declines. The company hires and lays off the workers, mostly lower skilled workers, according to changes in demand level. The newly hired workers go through a short training process. The production planners implement APP mainly by using linear programming and simulation methods alongside their experience. The seasonality of the demand for drink products and co-production by the plants in ZamZam Group (which makes options such as subcontracting practically possible) renders the Company a suitable case study for present research. As already mentioned in Section 2.1.1, the APP is done for a family of similar products. As such, this study considers carbonated soft drinks in 300ml bottles in three flavours cola, orange and lemonade as a family of products in order to conduct APP process. This multiproduct APP decision making problem covers a time horizon of 12 months which includes 4 time periods, i.e. 4 seasons to reflect the seasonal oscillations of the product demands. The customer demand is supposed to be the main source of uncertainty, and is presented in terms of three demand level scenarios: high demand, average demand and low demand with associated probabilities which are abbreviated as H, A and L respectively throughout the study. The forecasted demand acts as the driving force of the APP system. Seasonal demand patterns and unpredictability inherent in quantity and timing of received orders makes the whole APP system uncertain which in turn recommends utilising a decision modelling tool that takes account of these uncertainties.
Based on our discussion with marketing and sales managers of the company, we found out that when demand grows or declines, it will normally endure for several consecutive time periods while maintaining the seasonality pattern. Therefore, in this study, we assumed that the demand volume scenarios will be the same in all consecutive time periods in planning horizon.

The primary objective of the current research is to find the optimal values of the production in day shift and night shift, backorder, inventory, subcontracting, workforce hired and laid off in day shift and night shift, product prices, etc. over the planning horizon for the company under study in presence of uncertainty.

2.2. Literature review

The methodologies applied in the literature to deal with APP under uncertainty can be classified into six main categories: stochastic mathematical programming, possibilistic programming, fuzzy mathematical programming, simulation modelling, metaheuristics, and evidential reasoning. In present study, the most relevant category, i.e. the existing research on stochastic mathematical programming to APP which in turn comprises sub-categories such as stochastic linear programming, stochastic nonlinear programming and stochastic multi-objective optimisation models of APP, is concisely reviewed in following subsections.

2.2.1. Stochastic multi-objective optimisation

Rakes et al. (1984), Chen and Liao (2003) and Nowak (2013) utilised stochastic multi-objective optimisation techniques to consider APP under uncertainty. Rakes et al. (1984) applied a chance-constrained goal programming approach to APP. In their model, the product demands, time required for inspection and products testing and so forth are random variables. Chen and Liao (2003) adopted a multi-attribute decision making approach to select the most efficient APP strategy such that selling price, market demand, cost coefficients, etc. are assumed to be stochastic variables. Nowak (2013) presented a procedure which combines linear multi-objective programming, simulation and an interactive approach with uncertain demand.
2.2.2. Stochastic nonlinear programming

Various types of stochastic nonlinear programming models for APP subject to uncertainty were developed by Vörös (1999), Ning et al. (2013), Mirzapour Al-e-hesham et al. (2013) and Lieckens and Vandaele (2014).

Vörös (1999) studied a risk-based APP for seasonal products by proposing forward and backward procedures for determining the production quantities and sequencing of the products in an aggregate planning horizon with uncertain demand. Ning et al. (2013) presented a multi-product, nonlinear APP model by applying uncertainty theory where the market demand, production cost, and so on are characterised as uncertain variables.

Mirzapour Al-e-hesham et al. (2013) and Lieckens and Vandaele (2014) both suggested nonlinear, mixed integer programming methodologies to study APP decision problem in presence of uncertainty. Mirzapour Al-e-hesham et al. (2013) considered a multi-site APP problem in green supply chain with uncertain demand. Lieckens and Vandaele (2014) developed a multi-product, multi routing model where a routing consists of a sequence of operations on different resources. The uncertainty is associated with the stochastic nature of the both demand patterns and production lead times.

2.2.3. Stochastic linear programming

The research on stochastic linear programming to APP subject to uncertainty includes the studies carried out by Lockett and Muhlemann (1978), Kleindorfer and Kunreuther (1978), Günther (1982), Thompson et al. (1993) and Leung et al. (2006).

Lockett and Muhlemann (1978) developed a stochastic linear programming model of APP with zero-one variables, which involves uncertainties about whether the outcome of a job is Ok, rework or scrap. Kleindorfer and Kunreuther (1978) proposed a methodology to show how forecast horizons for stochastic aggregate planning problems with uncertain demand relate to the planning procedures and the information system within the organisation.

Günther (1982) presented a stochastic linear programming approach to deal with APP problem under demand uncertainty. Thompson et al. (1993) developed linear programming frameworks to evaluate several APP policies where customer demand, most of the coefficients of the linear programming model and some parameters were presented with probability distributions to reflect the uncertainty in APP environment. A stochastic linear programming method to handle APP with stochastic demand and stochastic cost parameters was proposed by Leung et al. (2006).
2.3. Model development

2.3.1. Methodological remarks

There are main features that distinguish the proposed decision making model in present study from the existing analytical models for APP in the literature, notably:

- Despite the popularity of various chase strategies of aggregate planning among operations managers (Buxey, 1990, 1995, 2003, 2005), the options included in these strategies such as the frequent hiring and lay-offs, working overtime and multiple shift operations are always major causes of productivity losses. Some literature treats the hiring process as a source of productivity loss. It is generally accepted that new workers need a certain period to adapt and to reach the same productivity as experienced workers. It is well known that lay-offs affect labour productivity not only in the short term, but also over longer horizons. Even seasonal fluctuations in employment can have an impact on productivity (Piper and Vachon, 2001). Hayes and Clark (1985) demonstrated that frequent lay-offs are associated with instability and confusion, which in turn have a negative impact on labour productivity. Frequent lay-offs and rehiring contributes to the depreciation of knowledge (Li and Rajagopalan, 1998) and increase the likelihood of forgetting prior learning when rehiring (Kher et al., 1999). Lay-offs can also have a negative effect on the workforce’s morale (Thomas and McClain, 1993) and affect the motivation level of the employees, which has also been proven to impact negatively on productivity (Huselin, 1998).

For the first time, the proposed APP decision making model in current research takes into account the productivity declines associated with chase policies of APP due to I) the frequent hiring and, thus the time required to learn the necessary skills and reach the same productivity as experienced labour, and II) workers’ motivation decline as a result of frequent lay-offs.

- In their previous research, the authors modelled demand management strategy of APP for the first time through a system dynamics simulation technique by considering the pricing and advertising options (see Jamalnia and Feili, 2013). Current study improves the previous study’s demand management strategy framework fundamentally to take into account the stochastic nature of APP problem. Therefore, this is the first study of its kind that systematically considers the demand management policy in a mathematical programming model developed for APP by utilising pricing and
advertising alternatives, which act based on precise mechanisms, that will be detailed in subsequent sections.

- In the hierarchy of production planning activities, capacity planning, as a long-range decision, is regarded as input to APP as a medium-range decision. The existing APP literature usually ignores the production capacity decisions or at best assumes the production capacity as a subjectively estimated fixed amount. Since the workforce level, as a major factor in determining the production capacity, oscillates constantly because of regular hiring and firing, supposing the production capacity for each product as a fixed value could lead to inaccurate decisions. Perhaps, an effective way in assigning the production capacity for each product would be determining the long-term/strategic capacity decisions in interaction with the medium-term APP decisions. In the proposed multi-stage, stochastic mathematical programming model of APP, the production capacities for different products are regarded as the first stage decision variables, i.e. deterministic decision variables that their values need to be determined dynamically in interaction with the stochastic part of the model at the beginning of planning horizon, and before the uncertainty is revealed. Furthermore, a novel rational objective function is developed to minimise unutilised available production resources and manufacturing capacities.

- APP in practice involves stochasticity, nonlinearity and multi-objectivity. Stochasticity may arise from uncertainty present in constants/parameters of APP models such as demand, cost coefficients and product price. In addition to the constants/parameters, decision variables in APP models also can be of stochastic nature. Several factors from quadratic cost functions and stepwise product price function (if product price is no longer supposed to be a constant) to taking into account the learning curve effect can make the APP decision models nonlinear. The need for taking into account the multiple criteria associated with APP in real world, e.g. total revenue, total production costs, customer satisfaction, and so on and the need for trade-off analysis between these objectives necessitates multi-objective optimisation. The existing APP literature includes one or two of these features for the sake of simplicity. This study represents the first APP model that deals with these three attribute all together under the same framework.

In present research, since the demand volume is hardly predictable, it makes the demand uncertain, and thus a stochastic variable. Mathematical operations related to embedding the demand management mechanism in the developed model via pricing
and advertising policies which have been shown in equations (12)-(17), taking into account the labour productivity costs through constraints (22)-(31) and objective function (3), and several novel features cause the nonlinearity of the proposed model that have been detailed in relevant sections. Multiple objectivity of the proposed model better reflects the multi-objective nature of APP, and facilitates the trade-off analysis between the objectives.

In addition, the extensive set of objectives/criteria, i.e. 7 objective functions that has been presented in current study instead of simplification gives a comprehensive picture of APP.

Table 2.1 compares the main characteristics of the proposed APP model in current study to the existing stochastic mathematical programming models of APP.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Stochastic nonlinear multi-objective optimisation</th>
<th>Stochastic linear multi-objective optimisation</th>
<th>Stochastic nonlinear programming</th>
<th>Stochastic linear programming</th>
</tr>
</thead>
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<tr>
<td>Source of uncertainty</td>
<td>Product demand</td>
<td>Product demand</td>
<td>Product demand</td>
<td>Product demand</td>
</tr>
<tr>
<td>Primary APP strategy considered</td>
<td>Mixed chase and level strategy</td>
<td>Mixed chase and level strategy</td>
<td>Mixed chase and level strategy</td>
<td>Mixed chase and level strategy</td>
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<tr>
<td>Demand management policy</td>
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<td>Not considered</td>
<td>Not considered</td>
<td>Not considered</td>
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<tr>
<td>Pricing option</td>
<td>Considered</td>
<td>Not considered</td>
<td>Not considered</td>
<td>Not considered</td>
</tr>
<tr>
<td>Advertising option</td>
<td>Considered</td>
<td>Not considered</td>
<td>Not considered</td>
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<tr>
<td>Productivity measures</td>
<td>Considered</td>
<td>Not considered</td>
<td>Not considered</td>
<td>Not considered</td>
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<tr>
<td>Capacity decisions</td>
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<td>Not considered</td>
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<tr>
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<td>Not considered</td>
<td>Not considered</td>
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<tr>
<td>Learning effect</td>
<td>Considered</td>
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<td>Not considered</td>
<td>Not considered</td>
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<tr>
<td>Customer services</td>
<td>Considered</td>
<td>Not considered</td>
<td>Not considered</td>
<td>Not considered</td>
</tr>
</tbody>
</table>
2.3.2. Stochastic nonlinear multi-objective optimisation model of APP under uncertainty for the mixed chase and level strategy

2.3.2.1. Notations

2.3.2.1.1. Decision variables

\( D_{ns}^t \): Adjusted demand for product \( n \) in period \( t \) under scenario \( s \) (unit)

\( Q_{rns}^t \): Regular shift production for product \( n \) in period \( t \) under scenario \( s \) (unit)

\( Q_{ens}^t \): Extra shift production for product \( n \) in period \( t \) under scenario \( s \) (unit)

\( B_{ns}^t \): Backorder level for product \( n \) in period \( t \) under scenario \( s \) (unit)

\( I_{ns}^t \): Inventory level of product \( n \) in period \( t \) under scenario \( s \) (unit)

\( S_{ns}^t \): Subcontracting volume for product \( n \) in period \( t \) under scenario \( s \) (units)

\( PR_{ns}^t \): Price of product \( n \) in period \( t \) under scenario \( s \) (£/unit)

\( AC_{ns}^t \): Advertising costs for product \( n \) in period \( t \) under scenario \( s \) (£)

\( H_{rs}^t \): Workers hired in regular shift in period \( t \) under scenario \( s \) (man-hour)

\( L_{rs}^t \): Workers laid off in regular shift in period \( t \) under scenario \( s \) (man-hour)

\( H_{es}^t \): Workers hired in extra shift in period \( t \) under scenario \( s \) (man-hour)

\( L_{es}^t \): Workers laid off in extra shift in period \( t \) under scenario \( s \) (man-hour)

\( PT_{rns}^t \): Time required to produce each unit of product \( n \) in regular shift by current workforce in period \( t \) under scenario \( s \) (man-hour/unit)

\( PT_{ens}^t \): Time required to produce each unit of product \( n \) in extra shift by current workforce in period \( t \) under scenario \( s \) (man-hour/unit)

\( PC_{rn}^t \): Production capacity allocated to product \( n \) in regular shift in period \( t \) (unit)

\( PC_{en}^t \): Production capacity allocated to product \( n \) in extra shift in period \( t \) (unit)

2.3.2.1.2. Parameters//constants

\( P_s \): Probability associated with scenario \( s \)

\( D_{uns}^t \): Unadjusted demand for product \( n \) in period \( t \) under scenario \( s \) (unit)

\( \sigma_{Dns} \): The regulator parameter of the demand adjustment equation under scenario \( s \)
\( CPR_n \): The coefficient in the equation which stands for the price of product \( n \) (£/unit*unit)

\( PPR_n \): The parameter/constant term in the equation which stands for the price of product \( n \) (£/unit)

\( FPR_{ns} \): Fixed price for product \( n \) in period \( t \) under scenario \( s \) (£/unit)

\( FAC_{ns} \): Fixed advertising costs for product \( n \) in period \( t \) under scenario \( s \) (£)

\( a_{AC_n} \), \( b_{AC_n} \) and \( c_{AC_n} \): The constant terms of the equation which represents the advertising costs of product \( n \) (the unit for the first and the third terms is £ and the unit for the second term is £*£)

\( \sigma_{AC_n} \): The regulator parameter of the equation which represents the advertising costs of product \( n \) (£/unit)

\( C_{jn} \): All production costs including materials cost, machines operations cost, salary paid to managers, professionals, administrative staff, etc. except costs incurred as salary paid to workers to produce product \( n \) in period \( t \) (£/unit)

\( SPT_{rn} \): Standard time required to produce each unit of product \( n \) in regular shift (man-hour/unit)

\( SPT_{en} \): Standard time required to produce each unit of product \( n \) in extra shift (man-hour/unit)

\( a_{PT_{rn}} \), \( b_{PT_{rn}} \) and \( c_{PT_{rn}} \): The constant terms of the equation which stands for the production time of product \( n \) in regular shift (the unit for the first and the third terms is man-hour/unit and the unit for the second term is 1/unit)

\( a_{PT_{en}} \), \( b_{PT_{en}} \) and \( c_{PT_{en}} \): The constant terms of the equation which stands for the production time of product \( n \) in extra shift (the unit for the first and the third terms is man-hour/unit and the unit for the second term is 1/unit)

\( F_{rn} \): Time required to produce the first unit of product \( n \) in regular shift by newly hired workforce (man-hour/unit)

\( F_{en} \): Time required to produce the first unit of product \( n \) in extra shift by newly hired workforce (man-hour/unit)

\( C_{rwn} \): Regular shift worker salary cost for product \( n \) in period \( t \) (£/man-hour)

\( C_{ewn} \): Extra shift worker salary cost for product \( n \) in period \( t \) (£/man-hour)
Factor used to calculate cumulative average time to produce units in regular shift and extra shift respectively. The value of $b_r$ or $b_e$ is calculated as:

$$b_r \text{ or } b_e = \frac{\ln(\text{learning curve } \% \text{ in decimal form})}{\ln 2}$$

- $C_{sn}^t$: Subcontracting cost per unit of product $n$ in period $t$ (£/unit)
- $C_{in}^t$: Inventory carrying cost per unit of product $n$ in period $t$ (£/unit)
- $C_{bn}^t$: Backorder cost per unit of product $n$ in period $t$ (£/unit)
- $C_{rh}^t$: Cost to hire one worker in regular shift in period $t$ (£/man-hour)
- $C_{rt}^t$: Cost to lay off one worker in regular shift in period $t$ (£/man-hour)
- $C_{eh}^t$: Cost to hire one worker in extra shift in period $t$ (£/man-hour)
- $C_{el}^t$: Cost to lay off one worker in extra shift in period $t$ (£/man-hour)

$TPC$: Total production capacity allocated to all products over the planning horizon $T$ (unit)

$MU_n$: Hours of machine usage per unit of product $n$ (machine-hour/unit)

$\nu_n$: Warehouse space needed per unit of product $n$ ($m^3$/unit)

$W_{ts max}^t$: Maximum workers level available in regular shift in period $t$ under scenario $s$ (man-hour)

$W_{es max}^t$: Maximum workers level available in extra shift in period $t$ under scenario $s$ (man-hour)

$MC_{max}^t$: Maximum machine capacity available in period $t$ (machine-hour)

$V_{max}^t$: Maximum warehouse space available in period $t$ ($m^3$)

$S_{ns max}^t$: Maximum subcontracting allowed for product $n$ in period $t$ under scenario $s$ (unit)

$B_{ns max}^t$: Maximum backorder allowed for product $n$ in period $t$ under scenario $s$ (unit)

$AC_{ns max}^t$: Maximum advertising costs for product $n$ in period $t$ under scenario $s$ (£)

### 2.3.2.2. Objectives

Since the wait and see method of stochastic programming is applied in present research, the objective functions need to be optimised for each demand quantity scenario subject to the constraints related to that specific scenario. Then, the expected value for each objective will be
calculated by multiplying the obtained values for each objective by the probabilities assigned to each scenario, and adding up the products together.

A comprehensive set of the most pertinent objectives/criteria to APP in industrial settings are taken into account.

I) Maximise total revenue

The total revenue is maximised by multiplying the sales quantity, which is the expression inside the parenthesis, by product prices.

The total revenue is used instead of total profit since in computing the total profit amount the cost items have already been considered, and thus the resulting overlaps between different objectives would undermine the trade-offs.

\[ \text{Max } Z_1^s = \sum_{n=1}^{N} \sum_{t=1}^{T} (I_{ns}^t - 1 + Q_{rns}^t + Q_{ens}^t + S_{ns}^t - I_{ns}^t)P_{rns}^t \quad \forall s \tag{1} \]

II) Minimise total production costs

Total production costs include three items: all production costs except worker salaries in regular shift and extra shift, worker salaries in regular shift and extra shift and subcontracting costs.

\[ \text{Min } Z_2^s = \sum_{n=1}^{N} \sum_{t=1}^{T} (C_{pn}^t (Q_{rns}^t + Q_{ens}^t) + \sum_{n=1}^{N} \sum_{t=1}^{T} (Q_{rns}^{t-1} P_{rns}^t + F_{rn} (|Q_{rns}^t - Q_{rns}^{t-1}| + \varepsilon) b_{rn} \max((Q_{rns}^t - Q_{rns}^{t-1}, 0) - \max((Q_{rns}^{t-1} - Q_{rns}^t), 0) P_{rns}^{t-1} C_{rwn}^t + \sum_{n=1}^{N} \sum_{t=1}^{T} (Q_{ens}^{t-1} P_{ens}^t + F_{en} (|Q_{ens}^t - Q_{ens}^{t-1}| + \varepsilon) b_{en} \max((Q_{ens}^t - Q_{ens}^{t-1}), 0) - \max((Q_{ens}^{t-1} - Q_{ens}^t), 0) P_{ens}^{t-1} C_{ewn}^t + \sum_{n=1}^{N} \sum_{t=1}^{T} C_{sn}^t S_{ns}^t \quad \forall s \tag{2} \]

III) Minimise total labour productivity costs

This objective function tries to minimise the positive deviations from the standard production time for both existing workforce, due to frequent lay-offs, and therefore workers motivation
decline, and newly hired labour force because of the learning time required to reach a normal productivity level equivalent to the productivity of experienced labour.

\[
\begin{align*}
M & i \in Z_3^s \\
& = \sum_{n=1}^{N} \sum_{t=1}^{T} \left( \max(PT_{rns}^t - SPT_{rn}, 0)Q_{rns}^{t-1}c_{rwn}^t + (\max(PT_{ens}^t - SPT_{en}, 0)Q_{ens}^{t-1})c_{ewn}^t \right) \\
& + \sum_{n=1}^{N} \sum_{t=1}^{T} \left( \max(F_{rn}(|Q_{rns}^t - Q_{rns}^{t-1}| + \epsilon)^{br} - SPT_{rn}, 0) \max(Q_{rns}^t - Q_{rns}^{t-1}, 0)C_{rwn}^t \right) \\
& + \sum_{n=1}^{N} \sum_{t=1}^{T} \left( \max(F_{en}(|Q_{ens}^t - Q_{ens}^{t-1}| + \epsilon)^{be} - SPT_{en}, 0) \max(Q_{ens}^t - Q_{ens}^{t-1}, 0)C_{ewn}^t \right) \\
& \forall s
\end{align*}
\]

IV) Minimise the changes in workforce level

Having a smoother workforce level so as to minimise the negative side effects of regular hiring and firing which were detailed in Section 2.3.1 is pursued by objective function \(Z_4^s\).

\[
\begin{align*}
M & i \in Z_4^s \\
& = \sum_{t=1}^{T} \left( C_{rh}^t H_{rs}^t + C_{rl}^t L_{rs}^t + C_{eh}^t H_{es}^t + C_{el}^t L_{es}^t \right) \\
& \forall s
\end{align*}
\]

V) Maximise customer satisfaction

The fifth objective seeks to maximise the customer satisfaction by keeping the ratio of backorders to forecasted demands as low as possible.

\[
\begin{align*}
M & a \ x Z_5^s = \sum_{n=1}^{N} \sum_{t=1}^{T} \left( 1 - \frac{B_{ns}^t}{D_{ns}^t} \right) / NT \\
& \forall s
\end{align*}
\]

VI) Minimise total inventory holding, backordering and advertising costs

\[
\begin{align*}
M & i \in Z_6^s = \sum_{n=1}^{N} \sum_{t=1}^{T} (C_{in}^t r_{ns} + C_{bm}^t B_{ns}^t) + \sum_{n=1}^{N} \sum_{t=1}^{T} AC_{ns}^t \\
& \forall s
\end{align*}
\]
VII) Minimise unutilised production resources and capacity

Maximum utilisation of the company’s resources, i.e. less subcontracting and more production in regular shift and extra shift is desired.

\[ \text{Min } Z_7 = \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ \left( \frac{S_{ns}^t}{Q_{rns}^t + Q_{ens}^t} \right) + (1 - \frac{Q_{rns}^t}{PC_{rns}^t}) + (1 - \frac{Q_{ens}^t}{PC_{ens}^t}) \right] \bigg/ 3NT \quad \forall s \]  

(7)

The expected value for objective \( k \), \( E(Z_k) \), is then obtained as follows:

\[ E(Z_k) = \sum_{s=1}^{S} P_s Z_k^s \]  

(8)

Where, \( S \) is the total number of scenarios.

2.3.2.3. Constraints

Objective functions need to be optimised with respect to a set of constraints on capacity, advertising costs, price, subcontracting, inventory, and so on.

I) Capacity constraints

Production capacities allocated to each category of the products are regarded as first stage decision variables, i.e. decisions that need to be made before the uncertain outcomes are revealed.

Constraints (9) and (10) prevent the production quantity for each product in both regular shift and extra shift from going beyond their corresponding allocated capacities in each time period.

As can be seen from constraint (11), the accumulation of the assigned production capacities to every single product in each time period should not violate the upper bound, or total production capacity.

\[ Q_{rns}^t \leq PC_{rns}^t \quad \forall n, \forall t, \forall s \]  

(9)

\[ Q_{ens}^t \leq PC_{ens}^t \quad \forall n, \forall t, \forall s \]  

(10)

\[ \sum_{n=1}^{N} \sum_{t=1}^{T} (PC_{rns}^t + PC_{ens}^t) \leq TPC \]  

(11)
II) Constraints for the demand management options

As it was already mentioned in Section 2.1.2, the common demand management options are pricing, advertising, backordering and introducing complementary products. Since launching new products requires establishing new production facilities/production lines, only the first three options are considered in present study.

By applying pricing and advertising policies, business managers can shift the demand from peak periods to off-peak periods. The mechanism through which these policies are implemented will be explained in the current section by introducing relevant constraints.

In equality constraint (12), when the low and average demand scenarios are regarded, if the backorder quantity for a product in previous time period surpasses the threshold backorder quantity $B_{n,t-1}^{t-1}$, then the coefficient of the fixed price $FPR_{n,t}$ becomes 1, and the coefficient of the expression $(CPR_{n}B_{n,t-1} + PPR_{n})$ equals zero. This means that even though the volume of the backorder/unsatisfied demand is high, but regarding the low or average quantity of the demand, the company should normally be able to meet the demand plus backorder.

Therefore, the company under study should stick to a fixed price alternative rather than increasing the price through a higher backorder volume in price equation, equation 12, to finally decrease the demand. Note that the backordered orders must be met by the next time period.

After consulting the operations managers of the company, the threshold backorder quantity of each product was intuitively determined as 55-60 percent of the maximum backorder allowed for that product in a given time period.

On the other side, in equation (12), again in case that the low and average demand scenarios are considered, if the backorder amount in previous time period is less than the backorder threshold level, the coefficient of the fixed price $FPR_{n,t}$ switches to 0, and the coefficient of the term $(CPR_{n}B_{n,t-1} + PPR_{n})$ becomes 1, which means the firm under study is going to follow the price regulation alternative, i.e. to reduce the price as a result of decrease in backorder quantity in previous planning period in the equation $PR_{n,s} = (CPR_{n}B_{n,t-1} + PPR_{n})$ to finally cause a rise in demand.

As an explanation, the linear equation of the product prices in terms of the backorders was provided using the linear regression method to estimate the parameters of the equation. A very similar mechanism is applied in equation (13), for the high demand scenario condition, to implement price regulation, and in equations (14) and (15) in order to execute the advertisement policy.
For example, in equation (14), in condition of low and average demand scenarios for a product, if the backlogged order quantity for a product in previous time period falls below its predetermined backorder threshold limit \( B_{n\delta t-1} \), the company managers will decide to increase advertising costs via relatively lower backorder quantity in advertising costs equation

\[
AC_{n\delta t}^t = \left( a_{AC_n} + \frac{b_{AC_n}}{\sigma_{AC_n}B_{n\delta t-1}^t + c_{AC_n}} \right),
\]

since its coefficient equals 1, and the coefficient of \( FAC_{n\delta t}^t \), fixed advertising costs, turns out to be 0. This will in turn lead to an increase in demand. The opposite occurs when the previous time period backordered order amount goes beyond the threshold level.

The special features of the hyperbolic function \( AC_{n\delta t}^t = \left( a_{AC_n} + \frac{b_{AC_n}}{\sigma_{AC_n}B_{n\delta t-1}^t + c_{AC_n}} \right) \) make it ideal to represent the relationship between advertising costs and backorder quantity in practice, i.e. when backorder level increases, the advertisement costs do not fall to zero with a fixed slope but tends to the constant \( a_{AC_n} \) with a decreasing slope, and when backorder level decreases, the advertising costs do not tend to infinity but approaches \( \left( a_{AC_n} + \frac{b_{AC_n}}{c_{AC_n}} \right) \) with a rising slope.

The regulator parameter \( \sigma_{AC_n} \), which assumes values between zero and one, and is determined intuitively, helps prevent out of control changes in advertising costs in terms of backorder level.

At one point, when \( B_{n\delta t}^t - B_{n\delta t-1}^t = 0 \), equations (12)-(15) shut down, since both sides of the equations become 0. However, as the variables are supposed to be continuous in general, the likelihood that this will be the case is absolutely narrow, and the formulas work out for the amounts of backlogged orders which are very close to the threshold backorder level, i.e. \( B_{n\delta t}^t \pm \varepsilon \) where \( \varepsilon \) is assumed to be a very small positive number.

Note that both \( FPR_{n\delta t}^t \) and \( FAC_{n\delta t}^t \) are calculated by plugging \( B_{n\delta t}^t \) into \( PR_{n\delta t}^t = \left( CPR_n B_{n\delta t}^t + PPR_n \right) \) and \( AC_{n\delta t}^t = \left( a_{AC_n} + \frac{b_{AC_n}}{\sigma_{AC_n}B_{n\delta t-1}^t + c_{AC_n}} \right) \) respectively.

In practice, demand for each product would have opposite relationship with that product price and direct relationship with incurred advertising costs of the product, which is embedded in equations (16) and (17). As such, it can be seen from equation (16) that under low/average demand circumstances, if backorder level in previous planning period falls below the threshold level, advertising costs will increase, and product price will drop compared to that of threshold limit, which in turn will make the expression

\[
\max \left( \frac{AC_{n\delta t}^t}{PR_{n\delta t}^t}, \frac{AC_{n\delta t}^t}{PR_{n\delta t}^t} < 0, \frac{AC_{n\delta t}^t}{PR_{n\delta t}^t} < 0 \right),
\]

that indicates the amount of growth or decline in demand, a positive value. Consequently, the adjusted/managed
demand will grow accordingly. $AC^t_{ns}$ and $PR^t_{ns}$ represent the advertisement costs and product price in terms of threshold backorder level. The regulator parameter in this equation, $\sigma_{D_{nLA'}}$, which takes on values in the interval $[0,1]$, is determined intuitively, and its role is to prevent rampant increase or decrease in the adjusted demand quantity.

In situations that backorder volume surpasses the specified threshold limit, no change will occur in unadjusted demand.

A quite similar justification could be provided for the high demand condition in equation (17).

\[
(B^t_{ns} - B^{t-1}_{n\delta})PR^t_{ns} = (\max(B^t_{ns} - B^{t-1}_{n\delta}, 0))FPR^t_{ns} + (\min(B^t_{ns} - B^{t-1}_{n\delta}, 0))(CPR_nB^t_{ns-1} + PPR_n) \quad \forall n, \forall t, \forall s \in \{L, A\} (12)
\]

\[
(B^t_{ns} - B^{t-1}_{n\delta})PR^t_{nH} = (\min(B^t_{ns} - B^{t-1}_{n\delta}, 0))FPR^t_{ns} + (\max(B^t_{ns} - B^{t-1}_{n\delta}, 0))(CPR_nB^t_{ns-1} + PPR_n) \quad \forall n, \forall t (13)
\]

\[
(B^t_{ns} - B^{t-1}_{n\delta})AC^t_{ns} = (\max(B^t_{ns} - B^{t-1}_{n\delta}, 0))FAC^t_{ns} + (\min(B^t_{ns} - B^{t-1}_{n\delta}, 0))\left(a_{AC_n} + \frac{b_{AC_n}}{\sigma_{AC_n}B^t_{ns-1} + c_{AC_n}}\right) \quad \forall n, \forall t, \forall s \in \{L, A\} (14)
\]

\[
(B^t_{ns} - B^{t-1}_{n\delta})AC^t_{nH} = (\min(B^t_{ns} - B^{t-1}_{n\delta}, 0))FAC^t_{ns} + (\max(B^t_{ns} - B^{t-1}_{n\delta}, 0))\left(a_{AC_n} + \frac{b_{AC_n}}{\sigma_{AC_n}B^t_{ns-1} + c_{AC_n}}\right) \quad \forall n, \forall t (15)
\]

\[
D^t_{ns} = D^t_{uns} \left(1 + \sigma_{D_{nLA'}} \max\left(\frac{AC^t_{ns}}{PR^t_{ns}} - \frac{AC^t_{n\delta}}{PR^t_{n\delta}}, 0\right)\right) \quad \forall n, \forall t, \forall s \in \{L, A\} (16)
\]
\[ D_{uNH}^t = D_{NH}^t \left( 1 + \sigma_{uNH} \min \left( \frac{AC_{nH}^t}{PR_{nH}^t}, \frac{AC_{nH-1}^t}{PR_{nH-1}^t}, 0 \right) \right) \] \quad \forall n, \forall t \quad (17)

\[ AC_{ns}^t \leq AC_{ns}^{max} \] \quad \forall n, \forall t, \forall s \quad (18)

### III) Production and inventory balance

Equation (19) shows that if remaining inventory from previous time period plus production in both regular shift and extra shift and subcontracting exceed the sum of backorder from previous time period and forecasted demand in present time period, the inventory level would be positive; otherwise a portion of the received orders has to be backordered. Therefore, backorder and inventory cannot exist simultaneously for the same product in a given time period.

As is evident from equation (19), in case that the sum of inventory quantity from previous time period and capacity of regular shift production in current time period do not suffice the backorder from previous time period besides product demand in current time period, the subcontracting and extra shift production is allowed. Note that in model implementation process in Section 2.4, the wage costs for production in regular shift are assumed to be lower than that of production in extra shift. Additionally, total production costs (including wage costs) for each unit of products in regular shift and extra shift are supposed to be significantly lower than subcontracting costs. These factors will also encourage producing as much as possible in regular shift before using extra shift and, producing as much as possible in regular shift and extra shift before turning toward subcontracting.

The very small quantity \( \varepsilon \) in the denominator of the fraction which acts as the coefficient of the expression \( Q_{ens}^t + S_{ns}^t \) is to make sure that the opposite side is also possible by avoiding the undefined operation, i.e. zero divided by zero.

The equations (20) and (21) prevent backorders and subcontracting from exceeding their upper limits.

\[ I_{ns}^{t-1} - B_{ns}^{t-1} + Q_{rns}^t + \frac{\max (D_{ns}^t + B_{ns}^{t-1} - PC_{rns}^t - I_{ns}^{t-1}, 0)}{\max (D_{ns}^t + B_{ns}^{t-1} - PC_{rns}^t - I_{ns}^{t-1}, 0) + \varepsilon} (Q_{ens}^t + S_{ns}^t) - D_{ns}^t = I_{ns}^t - B_{ns}^t \] \quad \forall n, \forall t, \forall s \quad (19)

\[ B_{ns}^t \leq B_{ns}^{max} \] \quad \forall n, \forall t, \forall s \quad (20)

\[ S_{ns}^t \leq S_{ns}^{max} \] \quad \forall n, \forall t, \forall s \quad (21)
IV) Recruitment constraints

Hiring, lay off and regulating the workforce level are central activities in APP process, which are modelled through precise and innovative mathematical equations/constraints in current study as follows.

The learning curve effect is considered in the constructed APP model to better reflect the worker experience factor in computing the required labour-hours. Simply stated, suppose $F$ is the time required to produce the first unit of a product, $Q$ is the cumulative quantity of production, and $b$ is the learning index which is calculated as natural logarithm (Ln) of learning curve percentage ÷ Ln 2. The cumulative average production time per unit and total cumulative production time will be $FQ^b$ and $(FQ^b)Q$ respectively.

The company needs to hire new workers if the production quantity in present time period is going to increase compared to the production quantity in previous time period in both regular shift and extra shift, and lay off otherwise, which is indicated by equations (22)-(24) and (26)-(28). As newly hired labour needs more time to learn necessary skills, and reach the productivity equivalent to the productivity level of experienced workers, it would be quite normal to utilise the learning effect in computing the man-hours required to be employed. An absolute value function is used in equations (22) and (26) to avoid computational errors due to negative bases with fractional exponents. The very small value $\varepsilon$ is added to $|Q_{rns}^t - Q_{rns}^{t-1}|$ to make sure that a computation error will not happen when the base is zero and power is decimal.

But, in making lay off decisions, the productivity of the existing experienced workforce, which is supposed to be mainly reflected through production time, is considered. Regular hiring and firing are regarded as fundamental alternatives for the chase strategies of APP. Frequent firing/lay off is expected to have an intense negative effect on the productivity of the existing workers due to declined motivation.

Among different mathematical functions, the logarithmic function would effectively represent changes in production time with regard to variations in the number of man-hours laid off. That is, given that all parameters in equation (25) are nonnegative, when there is no lay off, the production time of product $n$ would reduce to a constant, i.e. $a_{Pr_n} + Ln(c_{Pr_n})$, and when the lay off increases, the production time rises, not with fixed slope, but with decreasing slope. Equations (26)-(29) which are related to recruitments, lay-offs and production times in extra shift can be described similarly. Constraints (30) and (31) ensure that workforce levels in both regular shift and extra shift will not exceed the maximum allowable workforce level.
\[ H_{rs}^t = \sum_{n=1}^{N} F_{rn} \left( |Q_{rns}^t - Q_{rns}^{t-1}| + \varepsilon \right)^{br_{rn}} \max \left( (Q_{rns}^t - Q_{rns}^{t-1}), 0 \right) \quad \forall t, \forall s \quad (22) \]

\[ L_{rs}^t = \sum_{n=1}^{N} \max \left( (Q_{rns}^{t-1} - Q_{rns}^{t}), 0 \right) PT_{rns}^{t-1} \quad \forall t, \forall s \quad (23) \]

\[ I_{rns}^t = \max \left( (Q_{rns}^{t-1} - Q_{rns}^{t}), 0 \right) PT_{rns}^{t-1} \quad \forall n, \forall t, \forall s \quad (24) \]

\[ PT_{rns}^t = a_{pr_{rn}} + \ln \left( b_{pr_{rn}} L_{rns}^t + c_{pr_{rn}} \right) \quad \forall n, \forall t, \forall s \quad (25) \]

\[ H_{es}^t = \sum_{n=1}^{N} F_{en} \left( |Q_{ens}^t - Q_{ens}^{t-1}| + \varepsilon \right)^{be_{en}} \max \left( (Q_{ens}^t - Q_{ens}^{t-1}), 0 \right) \quad \forall t, \forall s \quad (26) \]

\[ L_{es}^t = \sum_{n=1}^{N} \max \left( (Q_{ens}^{t-1} - Q_{ens}^{t}), 0 \right) PT_{ens}^{t-1} \quad \forall t, \forall s \quad (27) \]

\[ I_{ens}^t = \max \left( (Q_{ens}^{t-1} - Q_{ens}^{t}), 0 \right) PT_{ens}^{t-1} \quad \forall n, \forall t, \forall s \quad (28) \]

\[ PT_{ens}^t = a_{pr_{en}} + \ln \left( b_{pr_{en}} L_{ens}^t + c_{pr_{en}} \right) \quad \forall n, \forall t, \forall s \quad (29) \]

\[ \sum_{n=1}^{N} Q_{rns}^{t-1} PT_{rns}^t + H_{rs}^t - L_{rs}^t \leq W_{rns}^{t} \max \quad \forall t, \forall s \quad (30) \]

\[ \sum_{n=1}^{N} Q_{ens}^{t-1} PT_{ens}^t + H_{es}^t - L_{es}^t \leq W_{ens}^{t} \max \quad \forall t, \forall s \quad (31) \]

V) Machine capacity and warehouse space

Constraint (32) is to make sure that total machine usage in regular shift and extra shift will not violate the upper bound on machine capacity available.

The company should not store finished product inventory which is more than the quantity that the available warehouse space allows, which is guaranteed by constraint (33).

\[ \sum_{n=1}^{N} MU_n (Q_{rns}^t + Q_{ens}^t) \leq MC_{rns}^{t} \max \quad \forall t, \forall s \quad (32) \]

\[ \sum_{n=1}^{N} v_{r_{ns}}^t \leq V_{r_{ns}}^{t \max} \quad \forall t, \forall s \quad (33) \]
\[ Q_{\text{rns}}^t, Q_{\text{ens}}^t, S_{\text{ns}}^t, B_{\text{ns}}^t, I_{\text{ns}}^t, P C_{\text{rns}}^t, P C_{\text{ens}}^t, P T_{\text{rns}}^t, P T_{\text{ens}}^t, H_{\text{rs}}^t, L_{\text{rs}}^t, L_{\text{rns}}^t, H_{\text{es}}^t, L_{\text{es}}^t, L_{\text{ens}}^t, D_{\text{ns}}^t, P R_{\text{ns}}^t, A C_{\text{ns}}^t \geq 0 \]

2.4. Case study

The constructed stochastic, nonlinear, nonconvex, non-differentiable, multi-stage, multi-objective optimisation model of APP under uncertainty is validated by implementing it in ZamZam Group, which was described in problem statement section, based on the following conditions:

- As already stated in Section 2.1.3, carbonated soft drinks in 300ml bottles in three flavours Cola, Orange and Lemonade are considered as a family of products in order to run APP process. The multiproduct APP decision making problem is conducted over a span of 12 months which includes 4 time periods, i.e. 4 seasons to reflect the seasonality of the forecasted demand.
- The unadjusted forecasted demand for each product under different scenarios, cost figures and other input data are presented in Table 2.2-Table 2.4.
- Maximum total capacity allocated to all products over the planning horizon \( T \) is 138,670,042 bottles.
- The initial inventories for products 1, 2 and 3 are 208,796, 102,698 and 38,113 bottles respectively.
- There is no initial backorder.
- Previous time-period production in regular shift at the beginning of the planning horizon for products 1, 2 and 3 is 10,278,331, 2,944,435 and 907,620 bottles respectively.
- Previous time-period production in extra shift at the beginning of the planning horizon for all products is zero.
- The time required to produce first unit of all products in the automated production line in all time periods by newly employed workers is 0.001088 man-hour/bottle in both regular shift and extra shift, and the standard production time for all products in both regular shift and extra shift is estimated as 0.00075 man-hour/bottle.
- The upper bound on workforce level in all time periods for both regular shift and extra shift is 17021 man-hour.
- The learning rate in both regular shift and extra shift is supposed to be 0.95.
(\(CP_{n}, PPR_{n}\)) for products 1, 2 and 3 are computed through linear regression method by using past data as (0.000000002028, 0.0894), (0.00000004092, 0.0897) and (0.0000001287, 0.0908) correspondingly.

The approximation for \(a_{AC_{n}}b_{AC_{n}}c_{AC_{n}}\), are respectively (-37,749, 178,519,000,000, 2,513,079), (-21,987, 56,251,340,000, 1,459,053) and (-6,469, 5,122,434,000, 427,396) for products 1, 2 and 3 by applying curve fitting methods.

The approximated values for \(a_{PT_{n}}, b_{PT_{n}}, c_{PT_{n}}\) in both regular shift and extra shift for products 1, 2 and 3 are (4.550524, 0.00000001442927, 0.01056907), (4.546664, 0.00000002897013, 0.01060994) and (4.564944, 0.00000008527615, 0.01041776) respectively by using curve fitting methods.

The probability of low, average and high demand scenarios are estimated by marketing managers as 0.30, 0.50 and 0.20 respectively using the past demand data.

\(\sigma_{D_{n}}\) and \(\sigma_{AC_{n}}\) are intuitively determined as 0.20 and 1 respectively through a trial and error process by running the model several times with regard to different values of these parameters.

All monetary values, e.g. costs, revenues, profits, etc. are supposed to be in British Pound (GBP).

### Table 2.2: Unadjusted forecasted product demand (in bottles)

<table>
<thead>
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<th>Product</th>
<th>Scenario</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
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### Table 2.3: Cost figures (in GBP)

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<th>(C_{t_{mn}})</th>
<th>(C_{t_{ren}})</th>
<th>(C_{t_{cm}})</th>
<th>(C_{t_{rn}})</th>
<th>(C_{t_{sn}})</th>
<th>(C_{t_{en}})</th>
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### Table 2.4: The input data

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<th>$A_{nh}^{max}$</th>
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### Table 2.4: (Continued)

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<th>Time period</th>
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The core model for the mixed chase and level strategy by exerting the recourse approach for the industrial case under study has 504 variables, over 1050 constraints and 7 objective functions. Besides the deficiencies of the recourse approach (which do not lie in present study’s scope), this model has no feasible space mostly because of the presence of large number of highly inconsistent constraints related to different demand scenarios. As such, we resort to the wait and see approach as another stochastic programming methodology. By adopting the wait and see method, rather than putting all of the constraints related to different demand scenarios together and calculating the expected values in the objective functions, we will have a separated problem for each scenario. After getting the solutions of these problems, then the expected values for each objective function could be calculated regarding different scenarios.

Consequently, employing the wait and see approach creates three equal size problems with 184 variables, 205 constraints and 7 objectives, where each problem represents one of the three demand volume scenarios, i.e. low, average and high.

These nonlinear, multi-objective optimisation problems are non-smooth due to the presence of the max/min operators, and non-differentiable because of the existence of absolute value functions and rational functions in the model. They are nonconvex as well, which diminishes the existing algorithms and software capabilities to deal with them efficiently. However, the WWW-NIMBUS software has the capability to run these kinds of problems.

During an interactive process, and by several classifications, the decision markers, i.e. operations managers, selected the most satisfactory solutions among the set of Pareto optimal solutions, which were presented in Table 2.5.

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<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$Z_7$</th>
<th>Total profit (GBP)</th>
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</tbody>
</table>
The Total profit column in Table 2.5 is simply calculated by deducting the cost items, \(Z_2, Z_3, Z_4\) and \(Z_6\), from the total revenue \(Z_1\) for each scenario. Because of the relatively low quantity of backorder compared to demand volume for all products, the customer satisfaction level, \(Z_5\), is quite high for all scenarios. Since most of the received orders are met by manufacturing in regular shift and extra shift instead of subcontracting, the unutilised production capacity and resources, \(Z_7\), is significantly low, i.e. lower than 10% in all scenarios. Total production costs and total labour productivity costs have the highest and lowest amounts respectively among the cost items. The row \(E(Z_k)\) is the expected value of each column.

The detailed solutions of the model for decision variables with respect to the average demand scenario (the most likely scenario) have been provided in Table 2.6.

As Table 2.6 indicates, since the backorder quantities of all products are lower than their threshold level, the adjusted demand shows some increase for all products. As already stated, demand follows a seasonal pattern, i.e. rises in spring and summer, time periods 1 and 2 respectively, and declines in autumn and winter, time periods 3 and 4 respectively. Consequently, production in both regular shift and extra shift, subcontracting, backordering, etc. depend on this seasonal demand pattern. As can be seen from Table 2.6, production time in regular shift and extra shift depend on lay off rate. That is, when lay off rate increases, production time also increases. Since we cannot have backorder and inventory simultaneously in a given time period, the values of these two decision variables are not positive at the same time within a specific time period. This is also true about hiring and lay off decision variables.

Table 2.6: The solutions for decision variables with regard to the average demand scenario

<table>
<thead>
<tr>
<th>Product</th>
<th>Time period</th>
<th>(Q^t_{ns})</th>
<th>(Q^t_{ens})</th>
<th>(S^t_{ns})</th>
<th>(B^t_{ns})</th>
<th>(I^t_{ns})</th>
<th>(D^t_{ns})</th>
<th>(PR^t_{ns})</th>
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<td>9764414</td>
<td>5128044</td>
<td>1352371</td>
<td>479112.5</td>
<td>0</td>
<td>18421850</td>
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<td></td>
<td>2</td>
<td>12251580</td>
<td>6893643</td>
<td>2250134</td>
<td>576873.3</td>
<td>0</td>
<td>22096150</td>
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<td>7025935</td>
<td>1991727</td>
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<td>0.09843751</td>
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<td>2789918</td>
<td>0</td>
<td>0</td>
<td>1140000</td>
<td>12804550</td>
<td>0.1023844</td>
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<td>5357878</td>
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<td>1268123</td>
<td>209000.0</td>
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<td>550708.5</td>
<td>0</td>
<td>0</td>
<td>209000</td>
<td>2163666</td>
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Table 2.6: (Continued)

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<th>$PT^t_{ens}$</th>
<th>$PT^t_{en}$</th>
<th>$PC^t_{en}$</th>
<th>$PC^t_{en}$</th>
<th>$L^t_{ens}$</th>
<th>$L^t_{ens}$</th>
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<td>2</td>
<td>20343.33</td>
<td>0.0006855588</td>
<td>0.0006907746</td>
<td>1322335</td>
<td>7167241</td>
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<tr>
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<td>18489.73</td>
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<td>0.007262385</td>
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<td>4</td>
<td>20286.31</td>
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<td>0.01121918</td>
<td>11300020</td>
<td>3042345</td>
<td>955.4841</td>
<td>304.1474</td>
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<td>1</td>
<td>16566</td>
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<td>0.0006930337</td>
<td>6718734</td>
<td>3397951</td>
<td>0</td>
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<td>0.007111014</td>
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<td>0.01093747</td>
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<td>1914078</td>
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<td>0.0006997249</td>
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<td>0.0008168638</td>
<td>0.01037631</td>
<td>1719795</td>
<td>589705.5</td>
<td>143.0723</td>
<td>536.5157</td>
</tr>
</tbody>
</table>

Table 2.6: (Continued)

<table>
<thead>
<tr>
<th>Time period</th>
<th>$H^t_{rs}$</th>
<th>$H^t_{en}$</th>
<th>$L^t_{ens}$</th>
<th>$L^t_{ens}$</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>1160.107</td>
<td>2913.414</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1664.042</td>
<td>1243.059</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>56.53672</td>
<td>318.0996</td>
<td>487.7772</td>
<td>25.245</td>
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<tr>
<td>4</td>
<td>48.63386</td>
<td>27.94257</td>
<td>1443.121</td>
<td>5031.432</td>
</tr>
</tbody>
</table>

2.5. Further experiments with the model

The sensitivity of the obtained solutions to the changes in objective functions and the parameters are examined via the following scenarios.

To avoid excessive elaboration, the following experiments are conducted based on the most probable scenario, i.e. the average demand scenario, as the outcomes for other demand scenarios would normally be similar.

2.5.1. Scenario 1: construct pay-off table

The APP model is run when only one objective is considered each time, and then the values of other objective functions are computed using the solutions obtained for decision variables. As such, the result would be a pay-off table which has been shown in Table 2.7.

Table 2.7: Pay-off table with regard to scenario 1

<table>
<thead>
<tr>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$Z_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>12425495.01</td>
<td>8384408.86</td>
<td>6046.43</td>
<td>29918.14</td>
<td>0.98370698</td>
<td>329513.45</td>
</tr>
<tr>
<td>Run 2</td>
<td>11418666.13</td>
<td>7850200.567</td>
<td>5377.46</td>
<td>29352.49</td>
<td>0.9830239</td>
<td>339814.18</td>
</tr>
<tr>
<td>Run 3</td>
<td>11344458.39</td>
<td>7877253.001</td>
<td>4836.006</td>
<td>29400.36</td>
<td>0.984167</td>
<td>316203.26</td>
</tr>
<tr>
<td>Run 4</td>
<td>11555656.12</td>
<td>8474569.806</td>
<td>6580.23</td>
<td>27991.53</td>
<td>0.9840565</td>
<td>355382.44</td>
</tr>
<tr>
<td>Run 5</td>
<td>11454465.8</td>
<td>8323891.045</td>
<td>6383.770</td>
<td>2846.05</td>
<td>0.9846838</td>
<td>314964.49</td>
</tr>
<tr>
<td>Run 6</td>
<td>11308298.45</td>
<td>8699349.84</td>
<td>6347.672</td>
<td>29469.35</td>
<td>0.9845023</td>
<td>310925.20</td>
</tr>
<tr>
<td>Run 7</td>
<td>11372436.24</td>
<td>8158420.857</td>
<td>6298.19</td>
<td>29195.34</td>
<td>0.9838712</td>
<td>333014.64</td>
</tr>
</tbody>
</table>
The results are also shown in Fig. 2.1. Because the values of objectives range from numbers between zero and one to eight digit numbers, it would be hard to show them on the same figure simultaneously. Thus, the current values of objectives, i.e. their values with respect to the average demand scenario in Table 2.5 are assumed to be 1 (regardless of whether they are of minimisation or maximisation type), and the percentages of increase or decrease in their values regarding different runs are depicted in Fig. 2.1. For example, 1.2 as the value of an objective means 20% increase in the value of that objective. This will apply to Fig. 2.2 and Fig. 2.3-a-Fig. 2.3-d as well.

2.5.2. Scenario 2: consider minimisation/maximisation objectives separately

Consider the maximisation objectives, $Z_1$ and $Z_5$, and minimisation objectives, $Z_2$, $Z_3$, $Z_4$, $Z_6$ and $Z_7$, together each time and run the model. The results are presented in Table 2.8.

<table>
<thead>
<tr>
<th></th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$Z_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>12248948.63</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.984265</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Run 2</td>
<td>-</td>
<td>8106467.071</td>
<td>5667.46</td>
<td>29381.39</td>
<td>-</td>
<td>319662.95</td>
<td>0.036432214</td>
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</tbody>
</table>

2.5.3. Scenario 3: conduct trade-off analysis

Conduct trade-off analysis on the basis of objective values for the average demand scenario, which were presented in Table 2.5, with respect to conditions that are stated in Table 2.9. The signs + or – before the percentages show the corresponding increase or decrease in objective values. By this scenario, the amount(s) that the given objective(s) need(s) to be sacrificed to
improve specific objective(s) is/are determined. The trade-off analysis results have been presented in Table 2.10 and Fig. 2.2.

Table 2.9: The implementation data of scenario 3

<table>
<thead>
<tr>
<th>Run</th>
<th>Z_1</th>
<th>Z_2</th>
<th>Z_3</th>
<th>Z_4</th>
<th>Z_5</th>
<th>Z_6</th>
<th>Z_7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>13125431</td>
<td>8791267.525</td>
<td>6289.567</td>
<td>32921.591</td>
<td>0.99338763</td>
<td>363893.052</td>
<td>0.01497789</td>
</tr>
<tr>
<td>Run 2</td>
<td>11232743.85</td>
<td>7381200.813</td>
<td>6020.572</td>
<td>27850.096</td>
<td>0.9527377</td>
<td>354071.929</td>
<td>0.1061323</td>
</tr>
<tr>
<td>Run 3</td>
<td>10738989</td>
<td>7222915.0625</td>
<td>6171.63</td>
<td>28636.936</td>
<td>0.9144639</td>
<td>356595.412</td>
<td>0.093945</td>
</tr>
<tr>
<td>Run 4</td>
<td>12689786.013</td>
<td>8798227.343</td>
<td>5832.96</td>
<td>32466.189</td>
<td>0.9917634</td>
<td>306910.08</td>
<td>0.01497789</td>
</tr>
<tr>
<td>Run 5</td>
<td>12860535.938</td>
<td>9021467.661</td>
<td>5624.229</td>
<td>31739.32</td>
<td>0.992675</td>
<td>323960.64</td>
<td>0.0204766</td>
</tr>
<tr>
<td>Run 6</td>
<td>12520706.597</td>
<td>8652489.63</td>
<td>5407.524</td>
<td>31905.44</td>
<td>0.9527377</td>
<td>329689.628</td>
<td>0.0312309</td>
</tr>
</tbody>
</table>

Table 2.10: Trade-off analysis with regard to the conditions of scenario 3

<table>
<thead>
<tr>
<th>Run</th>
<th>Z_1</th>
<th>Z_2</th>
<th>Z_3</th>
<th>Z_4</th>
<th>Z_5</th>
<th>Z_6</th>
<th>Z_7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>0.99338763</td>
<td>363893.052</td>
<td>0.01497789</td>
<td>13125431</td>
<td>8791267.525</td>
<td>6289.567</td>
<td>32921.591</td>
</tr>
<tr>
<td>Run 2</td>
<td>0.9527377</td>
<td>354071.929</td>
<td>0.1061323</td>
<td>11232743.85</td>
<td>7381200.813</td>
<td>6020.572</td>
<td>27850.096</td>
</tr>
<tr>
<td>Run 3</td>
<td>0.9144639</td>
<td>356595.412</td>
<td>0.093945</td>
<td>10738989</td>
<td>7222915.0625</td>
<td>6171.63</td>
<td>28636.936</td>
</tr>
<tr>
<td>Run 4</td>
<td>0.9917634</td>
<td>306910.08</td>
<td>0.01497789</td>
<td>12689786.013</td>
<td>8798227.343</td>
<td>5832.96</td>
<td>32466.189</td>
</tr>
<tr>
<td>Run 5</td>
<td>0.992675</td>
<td>323960.64</td>
<td>0.0204766</td>
<td>12860535.938</td>
<td>9021467.661</td>
<td>5624.229</td>
<td>31739.32</td>
</tr>
<tr>
<td>Run 6</td>
<td>0.9527377</td>
<td>329689.628</td>
<td>0.0312309</td>
<td>12520706.597</td>
<td>8652489.63</td>
<td>5407.524</td>
<td>31905.44</td>
</tr>
</tbody>
</table>

Fig. 2.2: Trade-off analysis with respect to the conditions of scenario 3

2.5.4. Scenario 4: conduct sensitivity analysis

Analyse the sensitivity of the model to changes in production and subcontracting costs, wage costs and hiring and lay off costs. The implementation data is provided in Table 2.11. The relevant parameter values are modified on an interval of 30% decreases to 30% increases in order to study the resulting effects on the related parts of the model.
The results of the sensitivity analysis in regard to scenario 4 have also been presented in Fig. 2.3.

Table 2.11: The implementation data of scenario 4

<table>
<thead>
<tr>
<th>Item</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
<th>Run 6</th>
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</thead>
<tbody>
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<td>m}$</td>
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<td>-20%</td>
<td>-10%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>$C_{i</td>
<td>n}$</td>
<td>-30%</td>
<td>-20%</td>
<td>-10%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>$C_{i</td>
<td>m}$ and $C_{i</td>
<td>n}$</td>
<td>-30%</td>
<td>-20%</td>
<td>-10%</td>
<td>10%</td>
</tr>
<tr>
<td>$C_{i</td>
<td>m}$, $C_{i</td>
<td>n}$, $C_{i</td>
<td>m}$ and $C_{i</td>
<td>n}$</td>
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<td>-20%</td>
</tr>
</tbody>
</table>

Table 2.12: Sensitivity analysis with regard to scenario 4

<table>
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<th>Objectives</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
<th>Run 6</th>
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</thead>
<tbody>
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<td>11978977.7</td>
<td>11777170.1</td>
<td>1175961.5</td>
<td>12094228.6</td>
<td>11890390.3</td>
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<td>$Z_2$</td>
<td>6127763.775</td>
<td>6779257.59</td>
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<td>9557748.92</td>
<td>10374096.8</td>
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<tr>
<td>$Z_3$</td>
<td>5909.88</td>
<td>5585.55</td>
<td>5961.17</td>
<td>6177.67</td>
<td>5883.13</td>
<td>5919.53</td>
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<tr>
<td>$Z_4$</td>
<td>29574.81</td>
<td>29546.98</td>
<td>29397.9</td>
<td>28999.25</td>
<td>29178.99</td>
<td>29502.16</td>
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<tr>
<td>$Z_5$</td>
<td>0.9820683</td>
<td>0.9838229</td>
<td>0.983443</td>
<td>0.9833418</td>
<td>0.983048</td>
<td>0.9826378</td>
</tr>
<tr>
<td>$Z_6$</td>
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<td>333689.69</td>
<td>336293.26</td>
<td>337249.46</td>
<td>340409.47</td>
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<td>1188339.5</td>
<td>11398687.2</td>
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<td>1199482.6</td>
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<td>7942253.08</td>
<td>8073017.26</td>
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<td>5878.16</td>
<td>5919.53</td>
<td>5936.35</td>
<td>5875.96</td>
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<td>29713.52</td>
<td>29860.24</td>
<td>29384.69</td>
<td>29339.40</td>
<td>29866.37</td>
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<td>0.9835545</td>
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<td>4652.56</td>
<td>5233.92</td>
<td>6233.93</td>
<td>7070.12</td>
<td>7785.24</td>
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<td>0.9832633</td>
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<tr>
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<td>0.07176568</td>
<td>0.07351944</td>
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<td>12030562.3</td>
<td>12160324.8</td>
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<td>$Z_{23}$</td>
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<td>8206919.24</td>
<td>8238423.27</td>
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<td>8200127.49</td>
<td>8171065.31</td>
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<td>$Z_{24}$</td>
<td>5981.86</td>
<td>5981.86</td>
<td>5901.60</td>
<td>5636.85</td>
<td>5676.84</td>
<td>5776.95</td>
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<td>$Z_{25}$</td>
<td>20635.48</td>
<td>23583.41</td>
<td>26795.11</td>
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<td>$Z_{26}$</td>
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<td>0.9827001</td>
<td>0.9827545</td>
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<td>$Z_{27}$</td>
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<td>347517.87</td>
<td>350388.53</td>
<td>334401.2</td>
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<td>$Z_{28}$</td>
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<td>0.06751612</td>
<td>0.0648144</td>
<td>0.0535023</td>
<td>0.0783841</td>
</tr>
</tbody>
</table>

Table 2.12 indicates the amounts of changes for all seven objectives with respect to changes in relevant coefficients. In each round, the objective values which are directly affected by the modifications in the given parameters are bolded.

The results of the sensitivity analysis in regard to scenario 4 have also been presented in Fig. 2.3-a-Fig. 2.3-d.
Several managerial and business insights to operations managers could be drawn here from the abovementioned scenarios:

- It can be seen from Table 2.7 and Table 2.8 that there exist trade-offs between objectives. For instance, in scenario 1 when the model is solved for a single objective in each run, each objective reaches its most optimum value as there is no need to sacrifice an objective in favour of other objectives.

- Compared to the objective values presented in Table 2.5, for the situation that the whole set of objective functions are considered simultaneously in the solution process, the quantities of objectives obtained with regard to the conditions of scenario 2 are more satisfactory. Apart from the smaller number of objectives, 2 for run 1 and 5 for run 2 compared to the 7 as the total number of objectives, this would probably be due to the homogeneity of the nature of the objective functions as well. That is, they are all of maximisation or minimisation type which may have reduced the inconsistency between the objectives.

- According to trade-off analysis in Table 2.10 and Fig. 2.2, to decision makers, i.e. the company managers, the objectives $Z_1$ and $Z_2$ have the highest importance. That is, they prefer to sacrifice less from $Z_1$ and $Z_2$ to get more from other objectives. They value $Z_6$ most in the next level of importance.
• As is evident from Table 2.12, the developed APP model is most sensitive to changes in production costs $C_{pn}$, since the total production costs shows the biggest relative changes from each run to another run accordingly.

• Changes in subcontracting cost are also expected to have an impact on total production costs with similar intensity compared to that of production costs. Due to the relatively small quantity of the subcontracted orders, however, the magnitude of the respective fluctuations in total production costs, as result of increase/decrease in subcontracting costs, turns out to be much lower.

• Variations in wage rates paid to workers in both regular shift and extra shift, $C_{rwn}$ and $C_{ewn}$, correspondingly affect $Z_2$ and $Z_3$, i.e. total production costs and total labour productivity costs. Although the values of the objective functions are chosen among a set of Pareto-optimal solutions by the decision makers during several classifications, a proportionate increase or decrease in their quantities are observed in response to increase/decrease in wage rates.

A similar trend is seen for round 4 where fluctuations in costs of changes in workforce level occur in accordance with variations in hiring and lay off costs, which have been provided in detail in Table 2.12.

2.6. Conclusions and future research directions

This study proposed a novel stochastic, nonlinear, multi-stage, multi-objective decision making model to APP based on mixed chase and level strategy which considers multiple objectives such as total revenue, total production costs, total labour productivity costs, total costs of the changes in workforce level and customer satisfaction subject to constraints on inventories, backorders, subcontracting, workforce level, and so forth where the forecasted demand acts as the main source of uncertainty.

The recourse approach of stochastic programming led to infeasible space due to the large number of highly inconsistent constraints related to different demand scenarios. The wait and see method, as another stochastic programming approach, was used instead.

The constructed stochastic, nonlinear, nonconvex, non-differentiable, multi-objective optimisation model for the APP problem was solved using WWW-NIMBUS software. The solution determined the optimal values of the production in regular shift and extra shift, backorder, inventory, subcontracting, workforce hired and laid off in regular shift and extra shift, product price, etc. over the planning horizon for the company under study in presence of uncertainty.
Further experiments with the model were performed by sensitivity analysis via changing different parameters of the model, and by different forms of trade-off analysis. Several future research directions are recommended for APP, especially by utilising the management science techniques:

i. Recourse approach as one of major stochastic programming approaches has a serious drawback. The main shortcoming of this approach is that it considers all constraints relating to different scenarios with equal probability (with certainty or P=1, where P stands for the probability of associated scenarios) in constraints section of the stochastic mathematical programming. In other words, it puts all the constraints relating to different scenarios together, and solves the problem where the objective function is the expected value regarding different scenarios. This does not sound true, since when all constraints relating to different scenarios are put together in one mathematical programming problem, it looks like all scenarios happen at the same time with equal probabilities (with P=1 or certainty). A suitable methodology needs to be developed to resolve this shortcoming.

ii. In this study, we assumed that the demand volume scenarios will be the same in all consecutive time periods. That is, if, for example, demand quantity is high in first time period, it will also be high in all future time periods. We made this assumption based on our discussion with the company’s managers, where according to their long term experience with customer demand in the company under study, they approved that demand normally has similar mood in several consecutive time periods while maintaining the seasonality pattern. However, this assumption could be invalid in different cases, and needs to be taken into account in relevant APP models.

iii. APP problems modelled by multi-stage stochastic programming techniques, e.g. Markov decision process would normally need to deal with the curse of dimensionality due to rather large scale of real world APP decision problems. One of the efficient methods to handle this issue can be Monte Carlo methods employed within the reinforcement learning structure to find the optimal actions in each stage by maximising the reward (profit).

iv. Aggregate production planners have traditionally been doing APP for the sake of maximising profit without properly taking into account the sustainability aspects, e.g. workers mental and physical health regarding workloads as the result of operating in overtime and multiple shifts, increased workplace incidents/injuries due to excessive overtime and nightshifts, negative psychological effects of frequent hiring and lay off
on employees, greenhouse gas emissions, declined customer satisfaction level because of regular backordering, and so forth. Very little research has been conducted on APP with regard to sustainability factors. Therefore, these sustainability dimensions need to be systematically incorporated into analytical models of APP, e.g. simulation models, mathematical models, etc. in order to develop a decision making framework which is more consistent with contemporary operations management requirements.

v. The outlook for Big Data driven approaches to APP particularly in larger manufacturing corporations, e.g. big car producing companies, airplane manufacturing corporations, etc. in order to provide decision support systems which have specific utilities in production planning and control activities, would be a promising area of research.
References


PAPER 3

Title: Evaluating the performance of aggregate production planning strategies under uncertainty

PAGE: 87
Evaluating the performance of aggregate production planning strategies under uncertainty

Abstract: The current study builds upon the paper 2 by developing four extra aggregate production planning (APP) models to different APP strategies other than the mixed chase and level strategy. These models are derived from the fundamental APP model that was proposed for the mixed chase and level strategy in paper 2. Therefore, the relevant models for APP strategies including the pure chase, the pure level, the modified chase and the modified level strategies are taken from the basic model developed for the mixed chase and level strategy. The same procedure, as described in paper 2, follows to solve the models constructed for these strategies with respect to the corresponding objectives/criteria in order to provide business insights to operations managers about the effectiveness and practicality of various APP policies in presence of uncertainty.

Multiple criteria decision making (MCDM) methods such as additive value function (AVF), the technique for order of preference by similarity to ideal solution (TOPSIS) and VIKOR are also used besides multi-objective optimisation to assess the overall performance of each APP strategy. The pure chase and the modified chase strategies show the best performance in general, followed by the pure level strategy.

A detailed sensitivity analysis is also conducted by changing the weights assigned to different criteria in abovementioned MCDM methods to evaluate the impacts that these weight changes can have on the final rank of each APP strategy.

Keywords: Aggregate production planning (APP); Strategy; Uncertainty; Model.

3.1. Introduction

3.1.1. General overview

Aggregate production planning (APP) is a medium range production and employment planning that normally spans a time horizon which ranges from 3 to 18 months and is about determining the optimum production quantities, hiring and lay off rates, work force and inventory levels, backordering and subcontracting volumes, and so on for each time period within the planning horizon subject to the limitations of available production resources. Such planning technique typically involves one product or a family of similar products, i.e. products with similarities in production process, skills required, materials needed, etc. despite minor differences so that considering the problem from an aggregated viewpoint is still valid.
APP has attracted considerable attention from both practitioners and academia (Shi & Haase, 1996). Since 1955 that the pioneering studies by Holt et al. (1955) and Holt et al. (1956) proposed linear decision rule, and Bowman (1956) suggested transportation method to deal with APP, researchers have developed different methodologies to handle the APP problem. Fig. 3.1, as already presented in paper 1, outlines the APP position among other types of production planning and control techniques, and their interconnected relationships from a holistic perspective. As can be seen from the Fig. 3.1, in the hierarchy of production planning activities, APP falls between long-term strategic planning decisions such as new product development and short term shop floor scheduling practices.

The forecasted demand acts as the driving force of the APP system. Seasonal demand patterns together with unpredictability inherent in quantity and timing of received orders makes the whole APP system uncertain, which in turn recommends utilising a decision modelling tool that takes account of these uncertainties.

As such, due to the dynamic nature of APP and instable states of real world industrial environments, the deterministic models for APP would lead to unrobust decisions. Moreover, similar to other production planning family members, APP also involves several objectives/criteria in practice. Therefore, the present study utilises a novel stochastic, nonlinear, multi-stage, multi-objective decision making model of APP which considers multiple objectives such as total revenue, total production costs, total labour productivity costs, total costs of the changes in workforce level and customer satisfaction subject to bounds on inventory, backorder, subcontracting, workforce level, and so forth under uncertainty.
In the first phase, the comprehensive stochastic, nonlinear, multi-objective mathematical programming model of APP which was developed in paper 2, under the primary mixed chase and level strategy subject to demand uncertainty, is reconsidered. WWW-NIMBUS software (Miettinen and Mäkelä, 2006) will be used to solve this stochastic, nonlinear, nonconvex, non-smooth, non-differentiable, multi-objective optimisation model of the APP problem.
Then, the relevant models for other APP strategies including the pure chase, the pure level, the modified chase and the modified level are derived from the fundamental model developed for the mixed chase and level strategy. The same procedure, as described above, follows to solve the models constructed for these strategies with respect to the aforementioned objectives/criteria to provide managerial and business insights for operations managers about the effectiveness and usefulness of various APP policies.

Additive value function (AVF), the technique for order of preference by similarity to ideal solution (TOPSIS) and VIKOR, as multiple criteria decision making (MCDM) methods, are also utilised in addition to multi-objective optimisation to evaluate the overall performance of each APP plan.

The paper is further organised as follows. In next two parts the regular APP options and policies are explained, and then the problem under study is described. Section 3.2 briefly reviews the most relevant literature; the research gaps are also considered in this section. Model development together with the notations is presented in Section 3.3. In Section 3.4, models are developed, and run for various APP strategies. In Section 3.5, APP strategies are analysed in detail by applying MCDM methods, and finally the conclusions and possible future research extensions are presented in Section 3.6.

3.1.2. Common APP options and strategies

The common APP options and policies were already detailed in Section 2.1.2 in paper 2.

3.1.3. Problem statement

The problem under study was fully described in Section 2.1.3 in paper 2.

The present research objective is to appraise different APP strategies’ performance under uncertainty. Main research questions are:

- Which APP strategies are more effective and useful with regard to relevant criteria under uncertainty?
- What are the advantages/disadvantages of different kinds of APP policies? In which conditions these strategies perform optimally?
- With regard to each APP strategy, what are the optimal values of the production in regular shift and extra shift, backorder and inventory level, subcontracting, workforce hired and laid off in regular shift and extra shift, product price, and so forth over the planning horizon for the company under study in presence of uncertainty?
3.2. Literature review

The literature on stochastic mathematical programming methods to APP was reviewed in detail in part 2.2 of paper 2, and thus the research about appraising APP policies is shortly overviewed in this section.

3.2.1. Literature on appraising APP policies


Through simulating the activities of an operating firm, Lee and Khumawala (1974) assessed the performance of four different APP policies under demand uncertainty. Dubois and Oliff (1991) surveyed a cross section of manufacturers about present practices of APP. Using input from practitioners and academicians a questionnaire was developed to examine strategies that the firm uses to deal with short range and long range demand fluctuations, major inputs to APP decisions, etc.

Buxey (1990, 1995, 2003, 2005) conducted surveys in groups of Australian firms in different industries to find out which APP policies are the most widely used in practice.

Thompson et al. (1993) developed linear programming frameworks to evaluate several APP policies where customer demand, most of the coefficients of the linear programming model and some parameters were presented with probability distributions to reflect the uncertainty in APP environment. Chen and Liao (2003) adopted a multi-attribute decision making approach to select the most efficient APP strategy such that selling price, market demand, cost coefficients, etc. are assumed to be stochastic variables.

Gulsun et al. (2009) developed a deterministic multi-objective optimisation model for APP which is used as a basis to select the most appropriate APP strategy. Jamalnia and Feili (2013) employed an integrated system dynamics and discrete-event simulation approach in order to evaluate the effectiveness and practicality of different APP strategies on the basis of total profit measure, where the market demand was regarded as random variable.
3.2.3. Research gaps in the literature

The research gaps in existing literature on examining the effectiveness and usefulness of different APP strategies are discussed chronologically in this section, together with the motives behind doing the present research:

a) The APP methods examined in the study performed by Lee and Khumawala (1974), used to be among the very early methodologies proposed to deal with APP, but they are currently regarded as conventional approaches to handle APP decision process that are no longer in use. The newer trends in the past decades on developing novel approaches and methodologies to deal with APP demand utilising corresponding state of the art APP policies.

b) Studies conducted by Buxey (1990, 1995, 2003, 2005) and Dubois and Oliff (1991) have drawbacks in common. First, these studies were surveys in specific geographical areas, and their results only represent the popularity of using relevant APP policies/strategies from the respondents’ viewpoints, e.g. from operations managers viewpoints in those geographical locations, while the APP strategies employed by the industry managers may not necessarily be the optimal ones. Second, these categories of studies provide results that can hardly be generalised for other geographical locations. And third, the results have not been achieved by using an efficient analytical method based on suitable criteria, and only presents managers’ responses on most widely used APP policies.

c) Researches performed by Thompson et al. (1993) and Chen and Liao (2003) have several major shortcomings in common that cast serious doubts on the accuracy of the obtained results. The option of varying the workforce level to meet the fluctuating demand is not considered in their constructed mathematical programming model at all, and using the overtime is considered as the only way to make changes in production capacity, whereas changing the labour force level has always been regarded as a fundamental alternative to the chase strategy in operations management literature.
Furthermore, the subcontracting option, as one of key options for level strategy, is totally ignored in their proposed mathematical programming model.

In addition, the six APP strategies which were assessed in their study, to some extent, fall into the pure chase and the pure level strategies scopes in current research, and therefore do not match other three popular APP strategies.

Finally, the descriptions and characteristics of the APP strategies examined in aforementioned studies seem quite inconsistent with common definitions and natures of those APP strategies, and are conceptually and technically inaccurate. For example, in mathematical equations defined for the chase strategies, they put the production volume (in regular time and overtime) equal to demand volume but as mentioned above, the hiring and lay off, as fundamental options in chase strategies, are not allowed in their models. Apart from conceptual inaccuracy, regarding the limitations of production in overtime by current workforce, this would make the model infeasible since by common definition of chase strategies, backorder and inventory, as options to meet the demand, are not permitted, or very limited quantities of backorder and inventory are permitted.

d) The research conducted by Gulsun et al. (2009) which is extension of the research performed by Chen and Liao (2003) also contains similar flaws which were detailed above in the last two paragraphs of section c. Furthermore, they define an objective function to represent the workers motivation decline due to hiring and lay off but that objective does not connect the hiring and lay off to workers motivation decline at all, and more resembles total costs of hiring and lay off.

e) Jamalnia and Feili (2013) previously compared the effectiveness of various APP strategies by applying a simulation modelling approach. But, to avoid the excessive complexity of the model, they made the comparison between different APP strategies on the basis of a single criterion, i.e. total profit.

Taking into account other criteria like workforce productivity costs, costs of changes in workforce level, utilisation of production resources and capacity and customer satisfaction besides total profit would present a wide-ranging appraisal of APP strategies.

The current study will provide a comprehensive evaluation of APP strategies performance by overcoming the limitations discussed above.

Table 3.1 presents the comparison between existing researches on appraising the APP policies.
Table 3.1: Comparison between the studies on evaluation of the APP strategies

<table>
<thead>
<tr>
<th>Study</th>
<th>Type of study</th>
<th>Changing workforce level</th>
<th>Changing production capacity</th>
<th>Subcontracting</th>
<th>APP strategies modelled (surveyed)</th>
<th>APP strategies evaluation method</th>
<th>The most preferred APP strategy (method)</th>
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<tr>
<td>Buxey (1990, 1995, 2003, 2005)</td>
<td>Empirical research by using survey</td>
<td>Not applicable</td>
<td>Not applicable</td>
<td>Not applicable</td>
<td>All common APP strategies</td>
<td>Multiple criteria</td>
<td>Chase strategy</td>
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<td>Dubois and Oliff (1991)</td>
<td>Empirical research by using survey</td>
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<td>Performed by using overtime</td>
<td>Not considered</td>
<td>Some forms of chase and level strategies</td>
<td>Multiple criteria</td>
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<td>Performed by using overtime</td>
<td>Some forms of chase and level strategies</td>
<td>Not considered</td>
<td>Some forms of chase and level strategies</td>
<td>Single criterion</td>
<td>Level strategy</td>
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<tr>
<td>Chen and Liao (2003)</td>
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<td>Some forms of chase and level strategies</td>
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<td>Some forms of chase and level strategies</td>
<td>Multiple criteria</td>
<td>Chase strategy</td>
</tr>
<tr>
<td>Gulsun et al. (2009)</td>
<td>Analytical Modelling</td>
<td>Performed by changing workforce level, overtime and subcontracting</td>
<td>All common APP strategies</td>
<td>Considered</td>
<td>Linear decision rule, search decision rule, etc.</td>
<td>Single criterion</td>
<td>Chase strategy</td>
</tr>
<tr>
<td>Lee and Khumawala (1974)</td>
<td>Analytical modelling</td>
<td>Performed by changing workforce level, extra shift/overtime and subcontracting</td>
<td></td>
<td>Not considered</td>
<td></td>
<td>Multiple criteria</td>
<td></td>
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<tr>
<td>The present study</td>
<td>Analytical modelling</td>
<td></td>
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</tr>
</tbody>
</table>

3.3. Model development for the fundamental mixed chase and level strategy

The underlying model for the mixed chase and level strategy was elaborately developed and solved using the real world data from ZamZam Group in Sections 2.3.2 and 2.4 in paper 2.

3.4. Examining the performance of other APP strategies

The mathematical models for other APP strategies including the pure chase, the pure level, the modified chase and the modified level strategies are constructed on the basis of the fundamental model developed for the mixed chase and level strategy, and their performance is compared to each other’s.
3.4.1. The pure chase strategy

At first sight, a chase plan looks the optimum policy. It positively impacts a wide category of costs, and thus improves the company’s earnings, and reduces its financial risks. Instead of excessive reliance on distant forecasted sales, the management seeks to adjust production capacity in a flexible way on the basis of near future demand predictions. It also gives a firm the opportunity to recruit wide range necessary skills on temporary basis.

In practice, a chase strategy could be realistic choice provided that the production fluctuations are effectively handled. The rationale behind the chase policy is very similar to that of just in time (JIT) production. Conditions which require dealing with valuable, bulky or hard to store, perishable and under the risk of obsolescence products makes the chase plan ideal.

To implement the pure chase strategy, the subcontracting, inventory stock, backorder and demand management strategy components including pricing ad advertising options are disregarded in the model developed for the mixed chase and level strategy. The company is going to follow the JIT philosophy, that is, it receives the orders, and then produces accordingly.

As already stated, all quantitative models for relevant APP strategies are derived from the model developed for the basic mixed chase and level strategy. Objective functions (3) and (4) remain unchanged. In practice, ignoring the possibility of backordering, keeping inventory, subcontracting, and so on means assigning the value 0 to them. Thus, by plugging 0 into objective functions (5) and (6), their value will be 1 and 0 respectively.

Objective functions (1), (2) and (7) are modified as follows:

I) Maximise total revenue

\[
Max Z_1^s = \sum_{n=1}^{N} \sum_{t=1}^{T} (Q_{rns}^t + Q_{ens}^t) F P R_{ns}^t \\
\forall s \quad (1)
\]

Note that since the pricing option has been disregarded, the price in this equation is a fixed price.
II) Minimise total production costs

\[
\begin{align*}
\text{Min } Z_2^s & = \sum_{n=1}^{N} \sum_{t=1}^{T} (c_{rns}^t (Q_{rns}^t + Q_{ens}^t)) \\
& + \sum_{n=1}^{N} \sum_{t=1}^{T} (Q_{rns}^{t-1} PT_{rns}^t + F_{rn} (|Q_{rns}^t - Q_{rns}^{t-1}| + \varepsilon) b_{r} \max((Q_{rns}^t - Q_{rns}^{t-1}), 0) - \max((Q_{rns}^{t-1} - Q_{rns}^t), 0) PT_{rns}^{t-1}) C_{rwn}^t \\
& + \sum_{n=1}^{N} \sum_{t=1}^{T} (Q_{ens}^{t-1} PT_{ens}^t + F_{en} (|Q_{ens}^t - Q_{ens}^{t-1}| + \varepsilon) b_{e} \max((Q_{ens}^t - Q_{ens}^{t-1}), 0) - \max((Q_{ens}^{t-1} - Q_{ens}^t), 0) PT_{ens}^{t-1}) C_{ewn}^t \\
& - Q_{rns}^t, 0) PT_{ens}^{t-1} C_{ewn}^t \quad \forall s \quad (2)
\end{align*}
\]

III) Maximise utilisation of production resources and capacity

\[
\begin{align*}
\text{Min } Z_7^s & = \sum_{n=1}^{N} \sum_{t=1}^{T} [(1 - Q_{rns}^t/P_{rns}] + (1 - Q_{ens}^t/P_{ens})] / 2NT \quad \forall s \quad (3)
\end{align*}
\]

The constraints (9)-(11) and (22)-(32) remain unchanged, and the constraints (12)-(18), (20)-(21) and (33) are removed from consideration. The upper bounds for constraints (30) and (31) are increased up to 50%. The constraint (19) is transformed into the following constraint:

\[
Q_{rns}^t + \left( \frac{\max(D_{ns}^t - P_{rns}^t, 0)}{\max(D_{ns}^t - P_{rns}^t, 0) + \varepsilon}\right) Q_{ens}^t = D_{ns}^t \quad \forall n, \forall t, \forall s \quad (4)
\]

Table 3.2 shows the solutions for the pure chase strategy.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$Z_7$</th>
<th>Profit (GBP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>9626285.974</td>
<td>5826362.256</td>
<td>6023.292</td>
<td>28220</td>
<td>1</td>
<td>0</td>
<td>0.0249374</td>
<td>3765680.43</td>
</tr>
<tr>
<td>A</td>
<td>1158519</td>
<td>6956084.359</td>
<td>7056.344</td>
<td>32779.65</td>
<td>1</td>
<td>0</td>
<td>0.01822924</td>
<td>4589270.601</td>
</tr>
<tr>
<td>H</td>
<td>13845461.764</td>
<td>8266610.225</td>
<td>7993.426</td>
<td>36962.333</td>
<td>1</td>
<td>0</td>
<td>0.0133922</td>
<td>5533895.784</td>
</tr>
<tr>
<td>E($Z_4$)</td>
<td>11449573.645</td>
<td>6879272.901</td>
<td>6933.845</td>
<td>32248.292</td>
<td>1</td>
<td>0</td>
<td>0.01927428</td>
<td>4531118.586</td>
</tr>
</tbody>
</table>

97
As can be seen from Table 3.2, total expected revenue and total expected production costs, $E(Z_1)$ and $E(Z_2)$ respectively, are lower than that of mixed chase and level strategy because the demand management policy embedded in the mixed chase and level strategy causes an increase in adjusted demand quantity, and therefore increase in the sales quantity and total production costs accordingly. But, the lesser decrease in total expected revenue, 2.57%, compared to the expected proportionate decrease which is seen in total expected production costs, 14.30%, is due to the lowered prices in the mixed chase and level strategy case that are lower than the fixed prices, $FP_{1n_t}$, as result of applying the pricing policy. A part of the decrease in total production costs would also be attributed to the lower costs of producing in regular shift and extra shift compared to subcontracting costs which has been discarded in current operations strategy. As is expected, the pure chase strategy has resulted in higher costs of changes in labour force level and costs related to human resource productivity and their corresponding expected values, i.e. $E(Z_3)$ and $E(Z_4)$. However, the amount of this rise might not be as much as expected because the demand adjustment mechanism applied with the mixed chase and level strategy would have a contribution in unsmoothing the demand level which in turn will cause higher rates of hiring and lay off.

Pure chase strategy tries to meet the market demand solely by producing in regular shift and extra shift, and adjusting the manufacturing capacity by varying the workforce level. Therefore, it has much better performance in utilising the company’s production capacity and resources, which is approved by very small percentage of unutilised production resources/capacities in $E(Z_7)$. The pure chase plan presents a significantly higher expected profit in comparison with the basic mixed strategy.

Although, the mixed chase and level strategy contributes to revenue/sales growth but as stated above, at the same time the price reductions due to the lower volumes of the backorders neutralise a portion of sales growth impact on total revenue. On the other side, the increase in demand causes a corresponding increase in production, and thus total production costs. In addition to these factors, ignoring the $Z_6$, total inventory carrying, backordering and advertising costs, explains the higher total profit for the pure chase strategy despite the slight increases in $Z_3$ and $Z_4$. 
3.4.2. The modified chase strategy

The limited production resources available for companies make it hard or even impossible to chase the customer demand closely. Furthermore, regarding the lengthy training periods for the newly hired workforce, sharp and instant ramp up in workforce level would not be an easy task. These reasons urge the operations managers to choose a modified chase policy. The modified chase strategy necessitates keeping given quantity of inventories.

To apply the modified chase plan, stockpiling option is allowed in all time periods. In time periods 3 and 4, i.e. autumn and winter when the demand for soft drink products fall, the firm will store a previously determined volume of products, e.g. 10-15% of demand volume in order to be used in upcoming periods especially in time periods 1 and 2.

Hence, in the model developed for the pure chase strategy, the objective $Z_6$ turns out to be positive but it only includes the inventory holding costs. Therefore, we must have:

I) Minimise total inventory holding costs

$$
\text{Min } Z^6_s = \sum_{n=1}^{N} \sum_{t=1}^{T} C^t_{ns} I^t_{ns} \quad \forall s \quad (5)
$$

And, the constraint (19) is modified as follows:

$$
I_{ns}^{t-1} + Q_{rns}^t + \left( \frac{\max(D_{ns}^t - PC_{rn}^t - I_{ns}^{t-1}, 0)}{\max(D_{ns}^t - PC_{rn}^t - I_{ns}^{t-1}, 0)} \right)Q_{ens}^t - D_{ns}^t \\
= I_{ns}^t \quad \forall n, \forall t, \forall s \quad (6)
$$

The constraint (33) is also added to the list of constraints.

Table 3.3 indicates the solutions for the modified chase strategy. According to the modified chase strategy, no subcontracting and backorder is allowed, inventory stocked in all time periods are procured by production level beyond the demand volume, pricing and demand management policies are not applied, and the demand is met fully. Therefore, the sales volume and thus revenue would remain the same.

Since the inventory keeping plan, especially in time periods 3 and 4, mandates higher production amounts in these time periods in comparison with the same time periods in the pure chase strategy case, the total production quantity and consequently the total production costs show the proportionate increase.
Although, when the modified chase plan is used, lower $Z_3$ and $Z_4$, total costs of changes in workforce level and total labour productivity costs, are generally expected but because this strategy is applied for the first time over the planning horizon, i.e. very limited amount of inventory is available at the beginning of the planning horizon to be used particularly in time periods 1 and 2, the company has to hire higher levels of labour force in time periods 1 and 2 (to meet the risen demand and stock some inventory), and then lay off higher levels of workforce in time periods 3 and 4, due to significant decline in the market demand in autumn and winter (Even though, keeping a minimum volume of inventory is still considered but the sharp decrease in demand compensates the effect of increase in production for inventory stocking purpose).

Compared to the pure chase plan, the $Z_7$, optimum utilisation of the production capacity and company’s resources, shows a very slight improvement normally as result of higher levels of production in both regular shift and extra shift which would mean better utilisation of the company’s manufacturing capacity.

However, the total expected profit declines slightly, i.e. 3.824%.

### 3.4.3. The pure level strategy

In spite of numerous advantages that already mentioned for the chase policy, there are conditions which limit its applicability, e.g. situations where the recruited workforce needs intensive and continuous training. Additionally, as already stated the frequent hiring and firing would lead to productivity losses and the workers motivation decline.

To put the pure level strategy into practice, hiring and lay off in both regular shift and extra shift are ignored, and any variation in the customer demand must be met by applying all other available options such as inventory, overtime, subcontracting, backorder or any of the demand influencing policies.
The maximum 3 hour overtime besides the normal 8 hour regular shift is performed by the current workforce. The upper bound of the subcontracting is increased to 30% of the adjusted demand in each time period.

Ignoring hiring and lay-off options means their corresponding decision variables take on the value 0, and therefore the objectives $Z_3$ and $Z_4$ also assume the value 0. All other objective functions except objective function (2) remain unchanged.

The new objective function (2) will be as follows:

I) Total production costs

$$
\text{Min } Z_2^s = \sum_{n=1}^{N} \sum_{t=1}^{T} C_{pn} (Q_{rns}^t + Q_{ens}^t) + \sum_{n=1}^{N} \sum_{t=1}^{T} C_{rwn}^t (Q_{rns}^t PT_n) + C_{ewn}^t (Q_{ens}^t PT_n) + \sum_{n=1}^{N} \sum_{t=1}^{T} C_{sn}^{t} S_{ns}^t \quad \forall s (7)
$$

Where $PT_n$ represents the normal production time, when there is no hiring and lay off, regardless of demand scenarios, operating in regular shift or overtime, etc. $Q_{ens}^t$ shows production quantity in overtime.

Constraints (9)-(21), (32) and (33) remain unchanged. Constraints (22)-(29) are lifted. Constraints (30) and (31), workforce level constraints, are transformed into a single constraint as below:

$$
\sum_{n=1}^{N} (Q_{rns}^t + Q_{ens}^t) PT_n \leq W_{s}^{t \max} \quad \forall t, \forall s (8)
$$

Because the existing workforce carries out the overtime as well, the upper limit of the labour force, $W_{s}^{t \max}$, has no notation of operating in regular shift or overtime but the maximum 3 hour overtime is added to the 8 hour regular shift working hours to calculate this upper bound.

As can be seen from Table 3.4, in comparison with the mixed chase and level plan, total expected revenue, $E(Z_1)$, has declined about 23.044% mainly for two reasons: I) the backorder quantities for all products in all time periods go beyond the threshold level, and in two occasions of the three occasions, i.e. when the product demand turns out to be low and average, the demand adjustment mechanism, which would have led to an increase in demand, is turned off. Moreover, in case high demand scenario occurs, again because backorder volume has exceeded the threshold level, the demand management policy causes a proportionate decrease in the demand.
Consequently, the demand volume and therefore the sales amount is reduced, II) the higher level of backorder means a portion of the demand is unsatisfied within the planning horizon which correspondingly has a negative impact on total revenue. The production decline as result of abovementioned reasons explains the proportional decrease in total production costs, \( Z_1 \), as well.

As is expected, the rampant increase in backorder and subcontracting volumes would lead to a significant drop in customer satisfaction, \( Z_5 \), and sharp rises in total inventory holding, backordering and advertising costs, \( Z_6 \), and unutilised production resources and capacity, \( Z_7 \). The huge increase in \( Z_6 \) has the highest contribution in turning the total profit into a considerable loss.

Table 3.4: The solutions obtained for the pure level strategy

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( Z_1 )</th>
<th>( Z_2 )</th>
<th>( Z_3 )</th>
<th>( Z_4 )</th>
<th>( Z_5 )</th>
<th>( Z_6 )</th>
<th>( Z_7 )</th>
<th>Profit (GBP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>7336299.07</td>
<td>4766707.774</td>
<td>0</td>
<td>0</td>
<td>0.217391</td>
<td>4729758.25</td>
<td>0.2139374</td>
<td>-2160166.954</td>
</tr>
<tr>
<td>A</td>
<td>9279759.243</td>
<td>5891370.38</td>
<td>0</td>
<td>0</td>
<td>0.2075576</td>
<td>5822674.20</td>
<td>0.2011018</td>
<td>-2434285.337</td>
</tr>
<tr>
<td>H</td>
<td>11014145.954</td>
<td>8266610.225</td>
<td>0</td>
<td>0</td>
<td>0.186823</td>
<td>6979639.563</td>
<td>0.1919792</td>
<td>-4232103.834</td>
</tr>
<tr>
<td>( E(Z_4) )</td>
<td>9043598.533</td>
<td>6029019.567</td>
<td>0</td>
<td>0</td>
<td>0.2063607</td>
<td>5726192.488</td>
<td>0.2031279</td>
<td>-2711613.522</td>
</tr>
</tbody>
</table>

3.4.4. The modified level strategy

Normally, there are limits on storage capacity available. In addition, increase in accumulated backlogged orders would have serious impact on customer satisfaction level. Moreover, skilled workforce may need several months to master certain tasks, and several years to achieve complete job rotation. This means regular hiring and lay off when dealing with the skilled workers would be waste of time and money. These are instances which call for the modified level strategy.

To execute this strategy, the company keeps its core skilled workers, and performs hiring and firing for the lower skilled workforce. The subcontracting upper limit is lowered to 25% of the adjusted demand. Hiring and laying off costs are reduced 35%, and workers learning rate is increased to 0.975, because of dealing with lower skilled manpower. Hiring and lay off will have an upper bound which is supposed to be 40% of the hiring and lay off in the mixed chase and level strategy. All other objectives and constraints do not change.

Table 3.5 shows the solutions to the modified level plan.

Even though, similar to the pure level strategy case, the backorder volume still surpasses the threshold level in all time periods for all products and for all demand scenarios, but partial hiring and lay off helps reduce the overwhelmingly high quantity of backorders through higher
production rates, that in turn helps increase the satisfied demand volume, and then the sales amount, \( Z_1 \). Compared to the pure level strategy, the higher production rates in regular shift and extra shift leads to higher total production costs, \( Z_2 \). As previously stated, the objective function \( Z_3 \) is to minimise the positive deviations from standard production time. The learning effect contributes to a significant improvement in production times of all products by newly hired workforce after producing a significant volume of products so that they even fall below the standard production times of those products. Moreover, the restricted lay off levels (alongside the restricted hiring levels) have had a similar effect to that of learning effect through hiring, and have caused the production times of different products become shorter than their standard production times. Thus, total positive deviations from the standard production times, \( Z_3 \), turns out to be almost zero.

The reduced costs of hiring and firing together with constrained hiring and firing levels result in decreased \( Z_4 \), or total costs of changes in workforce level. \( Z_5 \), customer satisfaction, improves as result of reduction in huge quantity of the backlogged orders. Decrease in backorder volumes directly causes a fall in total inventory, backorder and advertising costs, \( Z_6 \). As is expected, diminish in backorder volumes and growth in production rates lead to more effective use of manufacturing capacity and production resources, which is reflected in \( Z_7 \).

Finally, the tangible improvement in cash flow, through sales rise and a significant decrease in \( Z_6 \), makes the total profit positive.

Table 3.5: The solutions obtained for the modified level strategy

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( Z_1 )</th>
<th>( Z_2 )</th>
<th>( Z_3 )</th>
<th>( Z_4 )</th>
<th>( Z_5 )</th>
<th>( Z_6 )</th>
<th>( Z_7 )</th>
<th>Profit (GBP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>8500039.514</td>
<td>5414264.186</td>
<td>0.5032583</td>
<td>10190.163</td>
<td>0.5769716</td>
<td>2706279.94</td>
<td>0.179374</td>
<td>369304.722</td>
</tr>
<tr>
<td>A</td>
<td>10508146.272</td>
<td>6681390</td>
<td>0.5610319</td>
<td>11536.47</td>
<td>0.5545092</td>
<td>3338061.491</td>
<td>0.160921</td>
<td>477157.746</td>
</tr>
<tr>
<td>H</td>
<td>12557234.795</td>
<td>7976243.382</td>
<td>0.698327</td>
<td>12792.791</td>
<td>0.539787</td>
<td>3978301.684</td>
<td>0.1519792</td>
<td>589896.24</td>
</tr>
<tr>
<td>( E(Z_\epsilon) )</td>
<td>10315531.949</td>
<td>6560222.932</td>
<td>0.5711588</td>
<td>11383.842</td>
<td>0.5583035</td>
<td>3276575.064</td>
<td>0.16465909</td>
<td>467349.538</td>
</tr>
</tbody>
</table>
3.5. Further analysis of the results

3.5.1. Assessing the performance of APP strategies based on single criterion

The aforementioned APP strategies are ranked based on each of the eight criteria mentioned above as follows:

I) Based on total expected revenue, $E(Z_1)$

Mixed chase and level strategy $> \text{the pure chase strategy} \approx \text{the modified chase strategy} > \text{the modified level strategy} > \text{the pure level strategy}$

Where $>$ means an alternative is preferred over another and $\approx$ means the alternatives are equivalent.

II) Based on total expected production costs, $E(Z_2)$

The pure level strategy $> \text{the modified level strategy} > \text{the pure chase strategy} > \text{the modified chase strategy} > \text{mixed chase and level strategy}$

III) Based on total expected labour productivity costs, $E(Z_3)$

The pure level strategy $> \text{the modified level strategy} > \text{mixed chase and level strategy} > \text{the pure chase strategy} > \text{the modified chase strategy}$

IV) Based on total expected costs of changes in workforce level, $E(Z_4)$

The pure level strategy $> \text{the modified level strategy} > \text{mixed chase and level strategy} > \text{the pure chase strategy} > \text{the modified chase strategy}$

V) Based on expected customer satisfaction degree, $E(Z_5)$

The pure chase strategy $\approx \text{the modified chase strategy} > \text{mixed chase and level strategy} > \text{the modified level strategy} > \text{the pure level strategy}$
VI) Based on total expected inventory holding, backordering and advertising costs, $E(Z_6)$

The pure chase strategy $\succ$ the modified chase strategy $\succ$ mixed chase and level strategy $\succ$ the modified level strategy $\succ$ the pure level strategy

VII) Based on expected unutilised production resources and capacity, $E(Z_7)$

The modified chase strategy $\succ$ the pure chase strategy $\succ$ mixed chase and level strategy $\succ$ the modified level strategy $\succ$ the pure level strategy

VII) Based on total expected profit

The pure chase strategy $\succ$ the modified chase strategy $\succ$ mixed chase and level strategy $\succ$ the modified level strategy $\succ$ the pure level strategy.

3.5.2. Assessing the performance of APP strategies using MCDM methods

The MCDM techniques, AVF, TOPSIS and VIKOR, are used to assess the overall performance of different APP strategies by taking all of the criteria mentioned above into account except total profit, again to avoid the significant overlaps between criteria.

First, the criteria are assigned weights by employing the analytic hierarchy process (AHP), and then these weights are utilised in the process of aggregation used by the aforementioned methods. The weights of the attributes $Z_1- Z_7$ are correspondingly determined as: 0.3203, 0.2219, 0.0624, 0.0781, 0.1339, 0.1379 and 0.0453. Table 3.6 shows the overall rankings of the APP strategies together with total aggregated scores regarding each strategy.

As Table 3.6 indicates, the APP strategies from the chase family dominate in rankings performed by AVF and VIKOR but the level strategies top the list of TOPSIS ranking. In AVF ranking, the pure level strategy stays just behind chase strategies, and outperforms both mixed chase and level and the modified level polices with slightly higher overall score. In the ranking conducted by the VIKOR method, the modified level plan stays next to the mixed chase and level strategy, and the pure level strategy is at the bottom of the list.

The TOPSIS ranking puts the pure chase plan in third place, which is followed by the mixed chase and level and the modified chase strategies respectively with very close scores.
The results of these three rankings are now aggregated by calculating the rank averages and by Borda and Copeland methods which are presented in Table 3.7.

According to Table 3.7, the chase strategies top the list when the rankings are aggregated through computing the average of the ranks and Borda and Copeland methods, followed by the level strategies and the mixed chase and level strategy. The ranking results of the both Borda and Copeland methods are exactly the same.

However, considering all the above, the following remarks would be insightful from both academic and business/managerial viewpoints:

I) In current study, in the mixed chase and level strategy condition, in all time periods, regarding all demand scenarios, the backorder volumes fell below the threshold levels according to the real world data gathered from the company under study. If the backorder quantities had exceeded the threshold levels, the results would have probably been different considering the degree to which the backorders have surpassed the threshold points.
II) With regard to the extent to which a production process is capital-intensive or labour-intensive, it might be questionable that to what degree increasing the workforce level by recruiting the workers would necessarily mean a corresponding increase in production rate. However, ZamZam Company has already been operating with significant idle production capacity as it lost a portion of its market share to the newly established Pepsi and Coca Cola companies’ branches throughout the country in recent years. This implies that regular hiring could effectively help increase the production volume in the corporation under study.

III) A moderate rate of hiring and lay off and wage costs were considered in present research. However, higher rates of these cost items besides the higher weights for $Z_3$ and $Z_4$, total labour productivity costs and total costs of changes in workforce level, might have considerable effect on APP strategy ranks.

3.5.3. Sensitivity analysis of the rankings

In this part, the sensitivity of the ranking results to changes in the weights assigned to each criterion is examined. The sensitivity analysis is conducted by changing the present weights of the criteria, $(W_1, W_2, W_3, W_4, W_5, W_6, W_7) = (0.3203, 0.2219, 0.0624, 0.0781, 0.1339, 0.1379, 0.0453)$, as follows:

**Scenario 1:** Put new weights $= (0.3803, 0.1819, 0.0824, 0.0981, 0.1239, 0.0879, 0.0453)$. The outcome is presented in Table 3.8. As can be seen from Table 3.8, the rankings of APP strategies according to AVF and TOPSIS have not changed. However, in AVF ranking, the pure level strategy stays in the third place with slightly lower score compared to that of the pure chase strategy which is most probably because of reducing the weight of $Z_7$ where the pure level strategy has the worst performance. VIKOR ranking shows a slight change, i.e. now, the mixed chase and level and the modified level strategies are in the third and fourth orders respectively instead of being in the same level. This is normally due to increasing the weight of $Z_1$ in which the mixed chase and level strategy has the best performance, and reducing the weight of $Z_2$ where the mixed chase and level strategy does much worse in comparison with the modified level strategy.

**Scenario 2:** Put new weights $= (0.3503, 0.1019, 0.0624, 0.0781, 0.1339, 0.2579, 0.0153)$. Based on this scenario, in all rankings the pure chase strategy is still the best. The modified chase strategy still stays in the second order in AVF ranking but it is ranked number one together with the pure chase strategy in VIKOR ranking. Although lowering the weights of $Z_2$ and $Z_7$ where the chase
strategies perform pretty well has a negative impact on their ranks, increasing the weights of $Z_1$ and $Z_6$ compensates these negative effects. The mixed chase and level strategy is ranked best by VIKOR, and the second best by TOPSIS as result of increasing the weight of $Z_1$ in which the mixed chase and level strategy has the best performance, and decreasing the weights of $Z_2$ and $Z_7$ where the mixed chase and level strategy proves to be relatively inefficient.

**Scenario 3:** Put new weights $= (0.3203, 0.0419, 0.1624, 0.1781, 0.0339, 0.0379, 0.2253)$. According to the new set of weights, the pure level strategy gets the highest scores as the best policy in AVF and TOPSIS rankings. The modified level strategy is ranked as the second best by TOPSIS. This is mostly because of assigning higher weights to $Z_3$ and $Z_4$ where the chase strategies have poor performance, and lower weights to $Z_5$ and $Z_6$ where the level strategies prove to be relatively weak. These results are obtained despite assigning lower weight to $Z_2$ in which the level strategies present the best performance, and higher weight to $Z_7$ in which the level strategies present the poorest performance.

As different methods use different procedures in their ranking process, the ranking obtained by VIKOR is completely different. It puts the mixed chase and level, the pure chase and the modified chase strategies in the first level. In mixed chase and level case, the reason for the improved VIKOR ranking is a sharp decrease in weight of $Z_2$ where the mixed chase and level strategy is the least efficient, and increase in the weights of $Z_3$ and $Z_4$ where this strategy performs relatively better. Likewise, a sharp increase in the weight assigned to $Z_7$ where the chase strategies proved to be the most effective, and a significant reduction in the weight assigned to $Z_2$ where the level strategies are more effective than pure strategies explain the improvement in rankings of the pure chase and the modified chase strategies.

**Scenario 4:** Put new weights $= (0.0703, 0.0519, 0.1724, 0.2281, 0.3039, 0.0979, 0.0753)$. On the basis of scenario 4 conditions, all of the three ranking methods rank a strategy from the level strategies family as the most efficient, i.e. the pure level strategy is ranked number one by AVF and TOPSIS; and in VIKOR ranking, the modified level strategy comes in the first place. Despite assigning significantly higher weight to $Z_2$ in which the chase strategies present the best performance, a sharp decrease in the weight of $Z_1$ in which the level strategies are the least efficient, and a sharp increase in the weights of $Z_3$ and $Z_4$ where the chase strategies have the poorest performance put the pure or the modified level strategies ahead of chase strategies.
Note that the abovementioned weight changes could lead to different rankings with regard to specific procedures that are used by each ranking technique. This justifies the significant difference between the ranking obtained by VIKOR and two other methods in spite of similar changes in criteria weights, e.g. unlike AVF and TOPSIS rankings, in VIKOR ranking the mixed chase and level and the pure chase strategies are ranked number one.

However, these rankings can also be explained by the degree of increase or decrease in criteria weights, and the final impact this can have on ultimate APP strategy rankings. For example, in the mixed chase and level strategy case, a significant reduction in the weight of $Z_2$ where this strategy is the weakest together with significant increase in $Z_3$ and $Z_4$ where the chase strategies proved to be the weakest, and a sharp increase in $Z_5$ where the level strategies have the poorest performance explain the reason that the mixed chase and level strategy in comparison with other APP strategies has the best score under VIKOR ranking with regard to scenario 4.

Table 3.8: The sensitivity analysis results

<table>
<thead>
<tr>
<th>New weights</th>
<th>APP strategy</th>
<th>Overall score</th>
<th>Rank</th>
<th>Overall score</th>
<th>Rank</th>
<th>Overall score</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1</strong>: (0.3803, 0.1819, 0.0824, 0.0981, 0.1239, 0.0879, 0.0453)</td>
<td>The mixed chase and level</td>
<td>0.64753</td>
<td>4</td>
<td>0.39470</td>
<td>4</td>
<td>0.24785</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>The pure chase</td>
<td>0.77230</td>
<td>1</td>
<td>0.42941</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The modified chase</td>
<td>0.69550</td>
<td>2</td>
<td>0.38743</td>
<td>5</td>
<td>0.09439</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The pure level</td>
<td>0.69062</td>
<td>3</td>
<td>0.61257</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>The modified level</td>
<td>0.57375</td>
<td>5</td>
<td>0.59429</td>
<td>2</td>
<td>0.43845</td>
<td>4</td>
</tr>
<tr>
<td><strong>Scenario 2</strong>: (0.3503, 0.1019, 0.0624, 0.0781, 0.1339, 0.2579, 0.0153)</td>
<td>The mixed chase and level</td>
<td>0.56163</td>
<td>3</td>
<td>0.58071</td>
<td>2</td>
<td>0.08240</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The pure chase</td>
<td>0.83272</td>
<td>1</td>
<td>0.61257</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The modified chase</td>
<td>0.57776</td>
<td>2</td>
<td>0.53225</td>
<td>3</td>
<td>0.05136</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The pure level</td>
<td>0.54825</td>
<td>4</td>
<td>0.38743</td>
<td>5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>The modified level</td>
<td>0.47710</td>
<td>5</td>
<td>0.52835</td>
<td>4</td>
<td>0.46048</td>
<td>4</td>
</tr>
<tr>
<td><strong>Scenario 3</strong>: (0.3203, 0.0419, 0.1624, 0.1781, 0.0339, 0.0379, 0.2253)</td>
<td>The mixed chase and level</td>
<td>0.42774</td>
<td>4</td>
<td>0.44384</td>
<td>4</td>
<td>0.03880</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The pure chase</td>
<td>0.57259</td>
<td>3</td>
<td>0.56661</td>
<td>3</td>
<td>0.03743</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The modified chase</td>
<td>0.60715</td>
<td>2</td>
<td>0.38743</td>
<td>5</td>
<td>0.22757</td>
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</tr>
<tr>
<td></td>
<td>The pure level</td>
<td>0.65224</td>
<td>1</td>
<td>0.61257</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>The modified level</td>
<td>0.35638</td>
<td>5</td>
<td>0.60727</td>
<td>2</td>
<td>0.33639</td>
<td>4</td>
</tr>
<tr>
<td><strong>Scenario 4</strong>: (0.0703, 0.0519, 0.1724, 0.2281, 0.3039, 0.0979, 0.0753)</td>
<td>The mixed chase and level</td>
<td>0.42287</td>
<td>4</td>
<td>0.41950</td>
<td>3</td>
<td>0.08893</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The pure chase</td>
<td>0.56658</td>
<td>2</td>
<td>0.41890</td>
<td>4</td>
<td>0.10291</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The modified chase</td>
<td>0.49214</td>
<td>3</td>
<td>0.38743</td>
<td>5</td>
<td>0.42696</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>The pure level</td>
<td>0.59143</td>
<td>1</td>
<td>0.61257</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>The modified level</td>
<td>0.28502</td>
<td>5</td>
<td>0.59983</td>
<td>2</td>
<td>0.21072</td>
<td>1</td>
</tr>
</tbody>
</table>
3.5.4. Comparing the results of different studies on evaluating the APP policies

The present study by taking into account multiple criteria concludes that the most effective APP strategies are from chase family, which approves the results of existing research about evaluating APP policies which were presented in Table 3.1. The current research considered criteria that take into account the labour productivity costs and costs of changes in workforce level due to frequent hiring and lay off, which is common in chase strategies, to have a comprehensive evaluation of various APP strategies.

The findings of the present research are also consistent with the findings of surveys conducted by Buxey (1990, 1995, 2003, 2005) where they all found out that the most prevalent APP strategies in Australian industries are different versions of chase strategies. However, modelling APP problems solely on the basis of mixed chase and level strategy is quite dominant in APP literature. This does not only show the gap between APP literature and APP in practice but also indicates the need to do more research on APP by utilising strategies other than the mixed chase and level.

3.6. Conclusions and directions for future research

The present study proposed a framework based on a set of stochastic, nonlinear, multi-objective optimisation models to evaluate the performance of various APP strategies under uncertainty, where the customer demand was regarded as the main source of uncertainty. In the first stage, a novel, precise, stochastic, nonlinear, multi-objective optimisation decision making model was constructed for APP based on the mixed chase and level strategy to consider APP from a holistic view. Then, the relevant models for other APP strategies such as pure chase, pure level, modified chase and modified level were derived from the model that was developed for the mixed chase and level strategy. Different APP strategies were compared with regard to criteria/objectives like total profit, total revenue, total production costs and total labour productivity costs.

The MCDM methods AVF, TOPSIS and VIKOR were used to rank various APP strategies regarding different criteria, and then the rankings of different methods were aggregated by computing the average of the ranks and by Borda and Copeland methods. On the whole, the pure chase and the modified chase strategies had the best performance after aggregating the rankings of different methods, followed by the pure level strategy.

The research can be extended in several directions in the future, mainly:
I. The present study was conducted in an industry which combines both capital-intensiveness and labour-intensiveness features. Case studies from more labour-intensive industries, especially in chase strategy cases, would produce more informative results/findings.

II. The interface between operations management and marketing management has been a growing area of research. To the best of the authors’ knowledge, there is no published work on evaluating APP strategies from marketing management viewpoint. This is especially true regarding the fact that the demand management mechanisms embedded in level strategies act by applying pricing and advertising options that normally are considered as marketing mechanisms as well.

III. The existing research on assessing APP strategies with respect to productivity measures, although very few, has always done this using analytical management science-based methodologies. As already stated throughout the paper, frequent hiring and lay off, excessive usage of overtime, extra shifts, etc., which are regarded as the integral parts of chase strategies, impact the workers productivity via psychological factors. Therefore, there is a need for qualitative research to address these issues from behavioural perspective too.

IV. As discussed in Section 3.5.4, multiple consecutive surveys by Buxey (1990, 1995, 2003, 2005) revealed that chase strategies are the most popular APP strategies in Australian firms while as can be seen from Table 1.8 (in paper 1), the mixed chase and level strategy is the most frequently used strategy in research which utilises analytical models of APP. This does not only show the big gap between APP literature and APP in real world industrial environments but also indicates the need for more research on analytical decision models of APP based on strategies other than the mixed chase and level strategy.

V. APP includes a chain of interrelated decisions in different time periods, and these decisions are influenced by/influencing other decisions and policies adopted in previous and next time periods within the planned time horizon. This means APP problem can be formulated as a dynamic programming model. For instance, regular shift production, extra shift production and subcontracting are impacted by inventory level in previous time period, and impact the inventory level in the next time period. As such, in each stage (time period), the state would be inventory level remaining from previous time period, and the decision variables are production in regular shift, production in extra shift,
subcontracting and backordering at current time period. As detailed in Section 3.1, APP is inherently uncertain decision making problem. Thus, integrating uncertainty into the developed dynamic programming model of APP to build a stochastic dynamic programming model could help improve the quality of the decision model. However, APP problems in practice tend to be of large scale which arises the curse of dimensionality that needs to be managed effectively. Fortunately, there exists numerous ways to handle this issue, including the decomposition/partitioning techniques and scenario reduction methods.


Conclusions and future research paths

The thesis was presented in three papers format. In first paper, a wide scope of literature on APP under uncertainty was surveyed from bibliometric viewpoint. The uncertainties present in the constructed management science models of APP in the literature are of sorts like stochasticity, fuzziness and impreciseness of the information. This literature includes journal papers, book chapters, conference/proceedings papers and PhD theses which were classified into six main categories on the basis of the methodologies applied, e.g. stochastic mathematical programming, possibilistic programming, etc. First, the relevant literature about each of abovementioned methods was reviewed concisely, and then a more detailed statistical analysis of the surveyed research was followed.

Paper 2 proposed a novel decision model to aggregate production planning (APP) decision making problem based on mixed chase and level strategy under uncertainty where the market demand acts as the main source of uncertainty. By taking into account the novel features, the constructed model turned out to be stochastic, nonlinear, multi-stage and multi-objective. APP in practice entails multiple-objectivity. Therefore, the model involves multiple objectives such as total revenue, total production costs, total labour productivity costs, optimum utilisation of production resources and capacity and customer satisfaction, and was validated on the basis of real world data from beverage manufacturing industry. Applying the recourse approach in stochastic programming led to empty feasible space, and therefore the wait and see approach was used instead. After solving the model using the real-world industrial data, sensitivity analysis and several forms of trade-off analysis were conducted by changing different parameters/coefficients of the constructed model, and by analysing the compromise between objectives respectively.

The paper 3 built upon the paper 2 by developing four extra aggregate production planning (APP) models to different APP strategies other than the mixed chase and level strategy. These models were derived from the fundamental APP model that was proposed for the mixed chase and level strategy in paper 2. Therefore, the relevant models for APP strategies including the pure chase, the pure level, the modified chase and the modified level strategies were taken from the basic model developed for the mixed chase and level strategy.

The same procedure, as described in paper 2, was followed to solve the models constructed for these strategies with respect to the corresponding objectives/criteria in order to provide business insights to operations managers about the effectiveness and practicality of various APP policies in presence of uncertainty.
Multiple criteria decision making (MCDM) methods such as additive value function (AVF), the technique for order of preference by similarity to ideal solution (TOPSIS) and VIKOR were also used besides multi-objective optimisation to assess the overall performance of each APP strategy. The pure chase and the modified chase strategies showed the best performance in general, followed by the pure level strategy.

A detailed sensitivity analysis was also conducted by changing the weights assigned to different criteria in abovementioned MCDM methods to evaluate the impacts that these weight changes can have on the final rank of each APP strategy.

The present research could be extended in several directions as follows:

- As already was shown in Table 1.8, the absolutely prevalent APP strategy in the literature about APP in presence of uncertainty (and even in the literature on deterministic APP models) is the mixed chase and level strategy. However, the surveys conducted by Buxey (1990, 1995, 2003, 2005) revealed that the most popular APP policy among operations managers is the chase strategy, which shows a gap between APP in academia and APP in practice. This also indicates an intense gap related to the lack of the studies about quantitative APP models under uncertainty based on other APP strategies such as chase strategy, level strategy and the demand management strategy.

- Several relative advantages of the simulation techniques over mathematical programming methods, e.g. coping with dynamic or transient effects, addressing interactions between different components, the ability of providing a sufficient basis for developing explanatory and predictive models of operational processes, and so forth have been stated in the literature. Therefore, the relatively low share of the literature which apply simulation modelling to study APP subject to uncertainty (12.20%), and the least steep trend line of the frequency of the number of published research in this area over recent decades recommend the need to do extra research in this field to compensate the unfairly narrow share of the simulation methods.

- Recourse approach as one of major stochastic programming approaches has a serious drawback. The main shortcoming of this approach is that it considers all constraints relating to different scenarios with equal probability (with certainty or P=1, where P stands for the probability of associated scenarios) in constraints section of the stochastic mathematical programming. In other words, it puts all the constraints relating to different
scenarios together, and solves the problem where the objective function is the expected value regarding different scenarios. This does not sound true, since when all constraints relating to different scenarios are put together in one mathematical programming problem, it looks like all scenarios happen at the same time with equal probabilities (with P=1 or certainty). A suitable methodology needs to be developed to resolve this shortcoming.

- In this study, we assumed that the demand volume scenarios will be the same in all consecutive time periods. That is, if, for example, demand quantity is high in first time period, it will also be high in all future time periods. We made this assumption based on our discussion with the company’s managers, where according to their long term experience with customer demand in the company under study, they approved that demand normally has similar mood in several consecutive time periods while maintaining the seasonality pattern. However, this assumption could be invalid in different cases, and needs to be taken into account in relevant APP models.

- The present study was conducted in an industry which combines both capital-intensiveness and labour-intensiveness features. Case studies from more labour-intensive industries, especially in chase strategy cases, would produce more informative results/findings.

- The interface between operations management and marketing management has been a growing area of research. To the best of the authors’ knowledge, there is no published work on evaluating APP strategies from marketing management viewpoint. This is especially true regarding the fact that the demand management mechanisms embedded in level strategies act by applying pricing and advertising options that normally are considered as marketing mechanisms as well.

- The existing research on assessing APP strategies with respect to productivity measures, although very few, has always done this using analytical management science-based methodologies. As already stated throughout the paper, frequent hiring and lay off, excessive usage of overtime, extra shifts, etc., which are regarded as the integral parts of chase strategies, impact the workers productivity via psychological factors. Therefore, there is a need for qualitative research to address these issues from behavioural perspective too.