Seismic Design and Performance of Hospital Structures

Equipped with Buckling-Restrained Braces

in the Lakebed Zone of Mexico City

A thesis submitted to The University of Manchester for the degree of

Doctor of Philosophy

in the Faculty of Engineering and Physical Sciences

2016

Hector Guerrero-Bobadilla

School of Mechanical, Aerospace and Civil Engineering
# List of Contents

**List of Figures** ............................................................................................................................... 7  
**List of Tables** .............................................................................................................................. 11  
**Declaration** .................................................................................................................................. 12  
**Copyright statement** .................................................................................................................. 12  
**Publications** .................................................................................................................................. 13  
**Dedication** ..................................................................................................................................... 15  
**Acknowledgements** ..................................................................................................................... 16  
**Notation** ....................................................................................................................................... 17  
**Abstract** ....................................................................................................................................... 21  

## 1. Introduction............................................................................................................................ 22  
1.1 Background ......................................................................................................................... 22  
1.2 Aim and Objectives ........................................................................................................... 24  
1.3 Development of the project ............................................................................................... 26  
1.4 Outline of the Thesis .......................................................................................................... 27  

## 2. Literature Review .................................................................................................................. 31  
2.1 Introduction ......................................................................................................................... 31  
2.2 Buckling-Restrained Braces (BRBs) .................................................................................. 32  
 \hspace{1em} 2.2.1 Parts .................................................................................................................. 33  
 \hspace{1em} 2.2.2 Experimental behaviour ................................................................................... 34  
2.3 Performance-Based Seismic Design (PBSD) ................................................................. 37  
 \hspace{1em} 2.3.1 Performance-Based Seismic Design Process .................................................. 37  
 \hspace{1em} 2.3.2 Assessment of the expected performance ....................................................... 39  
 \hspace{1em} \hspace{1em} 2.3.2.1 Seismic hazard analysis ................................................................. 40  
 \hspace{1em} \hspace{1em} 2.3.2.2 Dynamic response analysis ............................................................... 40  
 \hspace{1em} \hspace{1em} 2.3.2.3 Damage analysis ................................................................................. 41  
 \hspace{1em} \hspace{1em} 2.3.2.4 Loss analysis ....................................................................................... 42  
2.4 Seismicity of Mexico ....................................................................................................... 45  
 \hspace{1em} 2.4.1 Earthquake sources ......................................................................................... 45  
 \hspace{1em} 2.4.2 Amplifications in the lakebed zone of Mexico City .................................... 47  
 \hspace{1em} 2.4.3 Global context ................................................................................................. 49  
2.5 Hospital Structures ............................................................................................................. 50  
 \hspace{1em} 2.5.1 Importance ....................................................................................................... 52  
 \hspace{1em} 2.5.2 Configuration ................................................................................................. 53  
2.6 Summary ............................................................................................................................. 55
3. A Method for Preliminary Seismic Design and Assessment of Low-Rise Structures

Protected with Buckling-Restrained Braces

3.1 Introduction ................................................................................................................57
3.2 Previous works ...........................................................................................................58
3.2.1 Maley et al. [59] method ...................................................................................59
3.2.2 Vargas and Bruneau [60] method .......................................................................60
3.2.3 Teran-Gilmore and Virto-Cambray [1] method ..................................................60
3.2.4 Summary ...............................................................................................................61
3.3 BRBs and SDOF structures .......................................................................................62
3.3.1 The idea of designing frames with BRBs .............................................................62
3.3.1.1 Designing BRBs for known properties of the primary structure ......................64
3.3.1.2 Designing BRBs for a desired contribution to the load capacity of the system ....69
3.3.2 Conventional vs. dual SDOF oscillators ...............................................................69
3.4 BRBs and MDOF structures .......................................................................................72
3.4.1 Definition .............................................................................................................72
3.4.2 Proposed procedure for preliminary design of low-rise buildings ......................73
3.4.2.1 Estimation of maximum ductility ratios in the dual structure .........................77
3.4.2.2 Estimation of the required cross-sectional areas of the BRBs .........................78
3.4.2.3 Correction for axial deformation in columns ...................................................80
3.5 Example of design .....................................................................................................81
3.5.1 Description and requirements of design ...............................................................81
3.5.2 Design with the proposed procedure ..................................................................83
3.5.3 Validation of the design by nonlinear analyses ....................................................88
3.5.3.1 Static analysis .....................................................................................................88
3.5.3.2 Dynamic analysis ..............................................................................................90
3.6 Preliminary assessment of the performance ............................................................92
3.6.1 Seismic hazard analysis ......................................................................................93
3.6.2 Dynamic response analysis .................................................................................93
3.6.3 Damage state analysis .........................................................................................96
3.6.4 Loss analysis ........................................................................................................97
3.7 Discussion ..................................................................................................................99
3.8 Conclusions ..............................................................................................................103

4. Comparative Experimental Studies of a Steel Frame Model with and without Buckling-Restrained Braces

4.1 Introduction ..............................................................................................................105
4.2 Test setup for the frame building model .................................................................106
4.2.1 The model ...........................................................................................................106
4.2.2 BRBs used in the tests .................................................................108
4.2.3 Instrumentation and measured data ........................................109
4.2.4 Seismic Input ...........................................................................110
4.2.5 Design of the experiment ........................................................110

4.3 Experimental results .......................................................................113
4.3.1 Free vibration tests ......................................................................113
4.3.2 Shaking table tests with white noise input ......................................116
4.3.3 Shaking table tests with seismic input ...........................................118
  4.3.3.1 Response at Peak Ground Acceleration (pga) of 0.1g ..............119
  4.3.3.2 Response to higher values of pga .............................................121

4.4 Discussion ......................................................................................123
4.5 Conclusions ...................................................................................124

5. Comparative Experimental Studies of Reinforced Concrete Precast Models with and without Buckling-Restrained Braces ........................................................................126

5.1 Introduction .....................................................................................126
5.2 Models ............................................................................................128
  5.2.1 Precast system ............................................................................128
  5.2.2 Construction and design .............................................................130
  5.2.3 Theoretical capacity curves .........................................................131
  5.2.4 BRB elements ............................................................................132

5.3 Experiment ......................................................................................133
  5.3.1 Material Properties .....................................................................133
  5.3.2 Instrumentation and measurements ...........................................134
  5.3.3 Input ..........................................................................................135
  5.3.4 Test programme ........................................................................136

5.4 Experimental results .......................................................................139
  5.4.1 Behaviour of isolated BRBs .........................................................139
  5.4.2 Response to low-intensity white-noise input ...............................139
  5.4.2.1 Effects of BRBs on natural frequencies and global stiffness ......140
  5.4.2.2 Effects of BRBs on damping ratio ............................................143
  5.4.3 Response to seismic input ..........................................................145
  5.4.3.1 Effects of seismic intensity on natural frequencies and global stiffness 145
  5.4.3.2 Effects of seismic intensity on damping ratio ..........................148
  5.4.3.3 Dynamic response to seismic input ..........................................149
  5.4.4 The damage observed in the models ..........................................152
  5.4.5 Comparison of the experimental results to numerical analyses .....153

5.5 Discussion .....................................................................................155
6. Improving the Seismic Performance of Hospitals Located in the Lakebed of Mexico City using Buckling-Restrained Braces .................................................................161
6.1 Introduction ......................................................................................................161
6.2 Design of Typical Hospitals in Mexico City ....................................................162
6.3 Upgrading Typical Hospitals with BRBs ..........................................................164
6.4 Improvements due to the Inclusion of BRBs ....................................................165
6.4.1 Lateral load capacity ..................................................................................165
6.4.2 Dynamic response ......................................................................................166
6.4.3 Incremental dynamic analysis (IDA) .............................................................167
6.4.4 Probability of collapse ................................................................................169
6.4.5 Probability of loss of functionality ...............................................................170
6.5 Parametric Study on Dual SDOF Systems .......................................................172
6.5.1 Effects of BRBs on maximum displacement demand .....................................174
6.5.2 Effects of BRBs on velocity and acceleration demands ................................174
6.6 Conclusions ....................................................................................................177
7. Residual Displacements in Conventional and Dual Structures .........................179
7.1 Introduction .....................................................................................................179
7.2 Residual Displacements in Conventional SDOF Oscillators .........................180
7.2.1 Effects of post-yielding stiffness ratio ..........................................................181
7.2.2 Effects of period of vibration .......................................................................183
7.2.3 Effects of hysteretic response ......................................................................184
7.2.4 Effects of maximum displacement ductility ...............................................185
7.2.4.1 SDOF oscillators with elastic-perfectly plastic behaviour .........................185
7.2.4.2 SDOF oscillators with post-yielding stiffness ratio different of zero .............186
7.2.5 Effects of lateral strength reduction factor ...................................................187
7.2.5.1 SDOF oscillators with elastic-perfectly plastic behaviour .........................188
7.2.5.2 SDOF oscillators with post-yielding stiffness ratio different from zero ......190
7.2.6 Effects of damping ratio .............................................................................192
7.2.7 Effects of transition from elastic to plastic response ....................................193
7.3 Residual Displacements in Dual SDOF Oscillators ........................................195
7.3.1 Effects of stiffness and strength ratios .........................................................197
7.3.2 Effects of ductility of the primary and secondary parts ...............................199
7.3.2.1 Ductility demand of the secondary part ...................................................199
8. Evaluating the Economic Benefits of using Buckling-Restrained Braces in Hospital Structures

8.1 Introduction ................................................................. 211
8.2 Evaluated structures .................................................. 212
  8.2.1 Initial considerations .............................................. 212
  8.2.2 Design ................................................................. 214
  8.2.3 Dynamic response ................................................ 214
8.3 Initial cost ........................................................................... 216
8.4 Expected losses .............................................................. 218
  8.4.1 Seismic hazard analysis .......................................... 219
  8.4.2 Response analysis .................................................. 220
  8.4.3 Damage state analysis .......................................... 221
  8.4.4 Loss analysis ......................................................... 223
    8.4.4.1 Intensity-based assessment ............................... 223
    8.4.4.2 Time-based assessment ................................. 224
8.5 Cost-benefit analysis .................................................... 227
8.6 Discussion ...................................................................... 229
8.7 Conclusions ................................................................... 230

9. Conclusions and Further Work ............................................. 232
  9.1 Conclusions .............................................................. 232
  9.2 Further work ............................................................ 236

References ............................................................................. 239

Appendix A. A Proposal of a new Buckling-Restrained Brace ............................................................................. 245
Appendix B. Seismic Records for Dynamic Analysis ......................................................................................... 247
Appendix C. Typical Contents and Quantities for Healthcare Infrastructure .............................................................. 252
Appendix D. Designs using the Method Proposed in Chapter 3 ................................................................................. 255
Appendix E. Table of Properties of Concrete used in the Tests of Chapter 5 ............................................................ 265
Appendix F. Engineer Demand Parameters of Dual SDOF Oscillators of Chapter 8 ............................................................ 267
Appendix G. Subroutine to Solve the Equation of Motion of Dual Oscillators..................269

Count words: 59917
List of Figures

Figure 1-1. Organisation of the Thesis ............................................................... 30
Figure 2-1. Behaviour of conventional braces and BRBs under cyclic loading ............ 32
Figure 2-2. Typical Buckling-Restrained Brace ................................................. 34
Figure 2-3. Performance-Based Seismic Design flowchart ..................................... 38
Figure 2-4. Typical Performance Function ........................................................... 38
Figure 2-5. PEER framework for performance assessment [33] ................................. 40
Figure 2-6. Fragility functions and repair cost of a typical partition wall [31] ................ 42
Figure 2-7. Estimation of performance using the Monte Carlo procedure of [31] .......... 44
Figure 2-8. Sources of seismicity of Mexico (adapted from [37]) .............................. 46
Figure 2-9. Accelerograms of the 19/091985 Michoacán earthquake ......................... 48
Figure 2-10. Response spectra for the accelerograms recorded in Stations CU and SCT .... 49
Figure 2-11. Response spectra for the SCT and L’Aquila accelerograms .................... 50
Figure 2-12. Number of beds per each 1000 inhabitants among the countries of the OECD [50] 51
Figure 3-1. Conventional vs. Dual structures ..................................................... 63
Figure 3-2. A dual SDOF oscillator and its behaviour curves .................................. 63
Figure 3-3. Parameters affecting the response of dual oscillators and design for BRBs .... 68
Figure 3-4. Displacement demands of conventional and dual oscillators .................... 70
Figure 3-5. Responses in the time domain of a conventional and a dual SDOF oscillator ... 71
Figure 3-6. An MDOF structure equipped with BRBs represented by a dual SDOF oscillator 72
Figure 3-7. Algorithm for PBSD of structures equipped with BRBs .......................... 75
Figure 3-8. Deformation of pinned structures as contributed by columns and braces .... 79
Figure 3-9. Five-storey framed building equipped with BRBs .................................. 82
Figure 3-10. Displacement demands ...................................................................... 85
Figure 3-11. Calibration of Giuffre-Menegotto-Pinto steel model to model BRBs ........... 89
Figure 3-12. Pushover analysis of the designed structure equipped with BRBs for $b_r=60\%$ ....... 89
Figure 3-13. Maximum inter-storey drift demands from nonlinear dynamic analyses .... 92
Figure 3-14. Dynamic response analysis of the dual SDOF oscillator ....................... 95
Figure 3-15. Reparability curve of the example hospital ......................................... 97
Figure 3-16. Cumulative distribution functions of repair cost for design option (a) .......... 98
Figure 4-1. Building model tested on a shaking table ........................................... 107
Figure 4-2. Schematic configuration of the BRBs used in the tests ........................... 108
Figure 4-3. Determination of filter cut-offs .......................................................... 110
Figure 4-4. Model during the first stage of the tests ................................................. 112
Figure 4-5. Model during the second stage of the tests ........................................... 113
Figure 4-6. Measured accelerations at the top floor for the five bracing cases ............... 114
Figure 4-7. Load-deformation curves of isolated BRBs vs. deformation demands produced in test 2b . . 115
Figure 4-8. Estimated damping ratio for different BRB configurations ....................... 116
Figure 4-9. Transfer Functions (from the top floor to the base) for the test model .......... 117
Figure 4-10. Estimation of equivalent viscous damping ratio for test 2b................................. 118
Figure 4-11. Responses of the test model (with and without BRBs) for PGA = 0.1g and record SCT-2. 120
Figure 4-12. Arias intensity at the top floor, relative to the base, for tests 2c and 9c...................... 121
Figure 4-13. Maximum responses of the test model subjected to incremental seismic intensity ...... 122
Figure 5-1. Description of the experimental study ........................................................................ 127
Figure 5-2. Dimensions and view of Model 2 .............................................................................. 129
Figure 5-3. The precast system ..................................................................................................... 129
Figure 5-4. Cross-sections of the scaled models ......................................................................... 131
Figure 5-5. Theoretical capacity (pushover) curves ................................................................. 131
Figure 5-6. BRBs used in the tests .............................................................................................. 132
Figure 5-7. Typical stress-strain curve for Mexican steel bars of diameters up to 13 mm .......... 134
Figure 5-8. Instrumentation on the models .................................................................................. 135
Figure 5-9. SCT-EW record of the 19/09/1985 Michoacán, Mexico earthquake ......................... 136
Figure 5-10. Systematic inclusion of the BRBs in Model 2 ........................................................ 138
Figure 5-11. Cyclic tests in BRB elements .................................................................................... 139
Figure 5-12. Natural frequencies of Model 2 without and with BRBs .......................................... 140
Figure 5-13. Effects of BRBs on frequency and global stiffness for different brace configurations ... 141
Figure 5-14. Variations of natural frequency in Model 2 when subjected to white noise input ...... 142
Figure 5-15. Estimation of equivalent viscous damping ratio for Model 2 with, and without, BRBs... 143
Figure 5-16. Effects of BRBs on the damping ratio of Model 2 for brace configurations of Figure 5-10 144
Figure 5-17. Variations of damping ratio in Model 2 when subjected to white noise input ...... 145
Figure 5-18. Effects of seismic intensity on the fundamental natural frequency and global stiffness of Models 1 and 2 .............................................................. 147
Figure 5-19. Effects of seismic intensity on the frequency and global stiffness of Model 2 ............... 147
Figure 5-20. Effects of seismic intensity on the damping ratio of the models ............................... 148
Figure 5-21. Envelopes of the response to the SCT-EW record at 100% in both models ............ 149
Figure 5-22. Envelopes of the response to the SCT-EW record at 100% in Model 2 ...................... 150
Figure 5-23. Peak response against input intensity of the SCT-EW records for both models........... 151
Figure 5-24. Peak response against input intensity of the SCT-EW records for Model 2 ............. 152
Figure 5-25. Typical damage observed following the tests .......................................................... 153
Figure 5-26. Displacements at the top floor for SCT-EW at 100%................................................. 154
Figure 5-27. Envelopes of the response for SCT-EW at 100% ..................................................... 155
Figure 6-1. Six-storey RC frame representative of typical hospitals in Mexico City .................... 163
Figure 6-2. Cross-sections of columns and beams of the RC frame ........................................ 163
Figure 6-3. Upgrading the example hospital with BRBs .............................................................. 165
Figure 6-4. Capacity curves before and after upgrading .............................................................. 166
Figure 6-5. Inter-storey drift demands to the SCT-EW seismic record ....................................... 166
Figure 6-6. Demands at the top floor to the SCT-EW seismic record ........................................ 167
Figure 6-7. IDA curves of the maximum inter-storey drift demands ......................................... 168
Figure 6-8. Comparison of the demands before and after upgrading .......................................... 169
Figure 6-9. Collapse fragility function before and after upgrading .......................................................... 170
Figure 6-10. Residual displacements before and after upgrading ............................................................. 171
Figure 6-11. Probability of loss of functionality ......................................................................................... 171
Figure 6-12. Spectrum of lateral capacity considered in the parametric study ................................................ 172
Figure 6-13. Elastic pseudo-acceleration spectra for conventional SDOF oscillators generated by the ground motions used in the parametric study .................................................................................. 173
Figure 6-14. Spectra of maximum displacement demands ............................................................................. 175
Figure 6-15. Spectra of absolute velocity demands ....................................................................................... 176
Figure 6-16. Spectra of absolute acceleration demands ............................................................................... 177
Figure 7-1. Factors affecting residual displacements ................................................................................. 180
Figure 7-2. Response to the SCT ground motion for different post-yielding stiffness ratios ....................... 181
Figure 7-3. Mean and dispersion of residual displacements on conventional oscillators in very soft soils ......................................................................................................................... 182
Figure 7-4. Effects on residual displacements of period of vibration ........................................................... 183
Figure 7-5. Types of hysteretic response ....................................................................................................... 184
Figure 7-6. Effects of hysteretic response on residual displacements ........................................................... 185
Figure 7-7. Effect of ductility in residual displacements ................................................................................. 186
Figure 7-8. Effect of ductility and post-yielding stiffness ratio on residual displacements ............................. 187
Figure 7-9. Effect of lateral strength factors in residual displacements .......................................................... 189
Figure 7-10. Effect of reduction factor ($R_y$) in the ratio $C_r$ .................................................................... 190
Figure 7-11. Effect of strength reduction factor and post-yielding stiffness ratio .......................................... 191
Figure 7-12. Effect of damping ratio on residual displacements ................................................................. 192
Figure 7-13. Elastic to plastic transition with post-yielding stiffness ratio of 5% ............................................. 194
Figure 7-14. Effect on residual displacements of transition from elastic to plastic response ......................... 194
Figure 7-15. Influence of post-yielding stiffness ratio in the type of transition ............................................. 195
Figure 7-16. Representation of dual systems ................................................................................................. 196
Figure 7-17. Additional factors affecting residual displacements in dual systems ....................................... 197
Figure 7-18. Effect of stiffness and strength ratios on residual displacements ............................................ 198
Figure 7-19. Effect of ductility of the secondary part on residual displacements .......................................... 199
Figure 7-20. Effect of ductility of the primary part in residual displacements .................................................. 200
Figure 7-21. Effect of post-yielding stiffness ratios on residual displacements .............................................. 201
Figure 7-22. Effects of negative post-yielding stiffness ratios on residual displacements ............................. 202
Figure 7-23. Effects of type of hysteretic response of the primary part on residual displacements ............... 203
Figure 8-1. Layout of hospitals studied in this chapter .................................................................................... 212
Figure 8-2. Cases studied in this chapter ........................................................................................................ 213
Figure 8-3. Qualitative initial cost, load capacity and displacement response of the studied cases .......... 213
Figure 8-4. Response estimated for $pga=0.20g$: three-storey frame ......................................................... 215
Figure 8-5. Response estimated for $pga=0.20g$: six-storey frame ............................................................... 215
Figure 8-6. Response estimated for $pga=0.20g$: nine-storey frame ............................................................ 216
Figure 8-7. Seismic hazard curve for Mexico City estimated using CRISIS 2007 [107] ................................. 219
Figure 8-8. Results of IDA corresponding to Case 0 of the six-storey frame ........................................... 221
Figure 8-9. Collapse fragility functions ............................................................. ...................................... 221
Figure 8-10. Fragility functions and repair actions of a typical partition wall (taken from the PACT database [31])....................................................................................... ................................................... . 222
Figure 8-11. Cumulative distribution functions of repair cost: three-storey frame .......................... 223
Figure 8-12. Cumulative distribution functions of repair cost: six-storey frame ............................. 224
Figure 8-13. Cumulative distribution functions of repair cost: nine-storey frame ............................ 224
Figure 8-14. Average annual repair costs and times: three-storey frame ............................................. 225
Figure 8-15. Average annual repair costs and times: six-storey frame ...................................................... 225
Figure 8-16. Average annual repair costs and times: nine-storey frame .................................................. 225
Figure 8-17. Probabilities of collapse and of loss of functionality: three-storey frame ................. 226
Figure 8-18. Probabilities of collapse and of loss of functionality: six-storey frame ......................... 226
Figure 8-19. Probabilities of collapse and of loss of functionality: nine-storey frame ....................... 226
Figure 8-20. Initial and lifecycle costs: three-storey frame ................................................................. 228
Figure 8-21. Initial and lifecycle costs: six-storey frame ........................................................................ 228
Figure 8-22. Initial and lifecycle costs: nine-storey frame ................................................................. 229
List of Tables

Table 3-1. Maximum inter-storey drifts for each objective of design of the study example .......... 82
Table 3-2. Properties of the design options .................................................................................. 87
Table 3-3. Structural elements of the design options .................................................................. 87
Table 3-4. Response demands on a dual SDOF oscillator from IDA: design option (a) (b=60%) ...... 94
Table 4-1. Cross-sections of the structural elements ................................................................. 107
Table 4-2. Selected records for the tests ...................................................................................... 110
Table 4-3. Summary of the test programme ................................................................................ 111
Table 5-1. Summary of the test programme for Model 1 .......................................................... 137
Table 5-2. Summary of the test programme for Model 2 .......................................................... 138
Table 7-1. Factors affecting residual displacements on conventional SDOF oscillators .......... 204
Table 7-2. Additional factors affecting residual displacements in dual SDOF oscillators .......... 205
Table 8-1. Objectives of design of the hospitals studied in this chapter ................................... 214
Table 8-2. Estimation of initial cost for Case 0 ............................................................................ 217
Table 8-3. Steel weight, in kg, for Cases 1 to 3 .......................................................................... 217
Table 8-4. Estimation of initial cost for Cases 1 to 3 ................................................................. 218
Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning;

Copyright statement

i. The author of this thesis (including any appendices and/or schedules to this thesis) owns certain copyright or related rights in it (the “Copyright”) and s/he has given The University of Manchester certain rights to use such Copyright, including for administrative purposes.

ii. Copies of this thesis, either in full or in extracts and whether in hard or electronic copy, may be made only in accordance with the Copyright, Designs and Patents Act 1988 (as amended) and regulations issued under it or, where appropriate, in accordance with licensing agreements which the University has from time to time. This page must form part of any such copies made.

iii. The ownership of certain Copyright, patents, designs, trade marks and other intellectual property (the “Intellectual Property”) and any reproductions of copyright works in the thesis, for example graphs and tables (“Reproductions”), which may be described in this thesis, may not be owned by the author and may be owned by third parties. Such Intellectual Property and Reproductions cannot and must not be made available for use without the prior written permission of the owner(s) of the relevant Intellectual Property and/or Reproductions.

iv. Further information on the conditions under which disclosure, publication and commercialisation of this thesis, the Copyright and any Intellectual Property and/or Reproductions described in it may take place is available in the University IP Policy (see http://documents.manchester.ac.uk/DocuInfo.aspx?DocID=487), in any relevant Thesis restriction declarations deposited in the University Library, The University Library’s regulations (see http://www.manchester.ac.uk/library/aboutus/regulations) and in The University’s policy on Presentation of Theses
Publications

Papers submitted (or to be submitted) to peer-review journals:


Papers presented (or to be presented) in conferences:


**Awards**


**Special Presentations**

Dedication

To my lovely wife Nancy

To my daughter Karla and my son Hector

To all my relatives and friends
Acknowledgements

I would like to express my sincere gratitude to Dr. Ji for his guidance, patience and encouragement during the development of this study. His advice and feedback has been crucial to developing this research project.

I would also like to recognise Dr. Adrian Bell, who acted as internal examiner of my first and second year reports. His recommendations and commentaries are highly appreciated.

I am totally indebted to Dr. Jose A. Escobar from the Institute of Engineering at UNAM, Mexico for his constant help and support. We have developed a very productive collaboration. He has believed in me and I really appreciate that. His friendship will be remembered always.

I would like to express my thanks to Dr. Amador Teran-Gilmore from the Universidad Autonoma Metropolitana and Dr. Jorge Ruiz-García from the Universidad Michoacana de San Nicolás de Hidalgo. Their suggestions and commentaries have been invaluable during the development of this project.

A special recognition to Professor Teresa Alonso, Director of the Latin American Program of the Faculty of Engineering and Physical Sciences at The University of Manchester, for believing in me and in many Mexican students. She has facilitated enormously our enrolment to the University.

I would also like to recognise to Manuel Suarez, Guillermo Coeto, Roberto Duran, Manuel Calzeta and Felipe Bennetts for their help during the experiments conducted in Mexico in 2013 and 2014.

The kind donations from Itisa and Corebrace to conduct the experiments are highly appreciated.

The sponsorship provided by CONACyT and SEP from Mexico is also most recognised.

Finally, I am most grateful to my wife Nancy and my children Karla and Hector for their love and support. With them, life is absolute happiness.
Notation

\( a \)  Stiffness ratio of the primary to the secondary part of a dual system
\( A \)  Cross-sectional area
\( a_1 \)  Relative contribution of the primary part of a dual system to the stiffness
\( a_2 \)  Relative contribution of the secondary part of a dual system to the stiffness
\( A_{\text{assu},i} \)  Initially assumed cross-sectional area of BRBs in the \( i \)th storey of a building
\( A_{\text{BRB},i} \)  Cross-sectional area of BRBs in the \( i \)th storey of a building
\( A_p \)  Annualised losses
\( b \)  Strength ratio of the primary to the secondary part of a dual system
\( b_1 \)  Relative contribution of the primary part of a dual system to the strength
\( b_2 \)  Relative contribution of the secondary part of a dual system to the strength
\( C_0 \)  Total initial cost of hospitals for Case 0, as defined in Chapter 8
\( c_1 \)  Damping coefficient of the primary part of a dual system
\( c_2 \)  Damping coefficient of the secondary part of a dual system
\( C_n \)  Cost of non-structural elements and contents
\( C_r \)  Ratio of residual to peak elastic displacements
\( C_S \)  Cost of structural elements
\( C_T \)  Total initial cost of a hospital
\( d \)  Displacement demand
\( d_e \)  Elastic displacement demand
\( d_i \)  Displacement at the \( i \)th storey of a building
\( d_{\text{max}} \)  Displacement threshold
\( d_{y1} \)  Yielding displacement of the primary part of a dual system
\( d_{y2} \)  Yielding displacement of the secondary part of a dual system
\( d_d \)  Displacement demand on dual systems
\( d_{CP} \)  Displacement threshold for \textit{Collapse Prevention} performance level
\( d_{FO} \)  Displacement threshold for \textit{Fully Operability} performance level
\( d_{LS} \)  Displacement threshold for \textit{Life Safety} performance level
\( d_N \)  Displacement at the top floor of a building
\( d_{Op} \)  Displacement threshold for \textit{Operability} performance level
\( E \)  Modulus of elasticity
\( E_c \)  Modulus of elasticity of concrete
\( E_s \)  Modulus of elasticity of steel
\( f \)  Natural frequency
\( f_c' \)  Compression resistance of concrete
Effective stiffness factor of BRBs in the $i$th storey of a building
Safety factor higher than unity
Restoring force on the primary part of a dual system
Restoring force on the secondary part of a dual system
Ultimate tensile strength of steel
Nominal yielding stress of steel
Acceleration of the gravity
Inter-storey height of the $i$th storey of a building
Second moment of area
Arias Intensity
Interest rate
Stiffness matrix of a structure
stiffness of the primary part of a dual system
stiffness of the secondary part of a dual system
Stiffness matrix of the bracing system of a structure equipped with BRBs
Stiffness provided by the BRBs fitted in a building
Stiffness provided by the columns of a building
Stiffness matrix of the moment resisting frame of a structure with BRBs
Unbounded length of a BRB
Yielding length of a BRB
Modal mass
Exponent of the Bouc-Wen model
Number of storeys in a building
Axial loads in the $j$th structural element
Net present value of a stream of annualised losses
Buckling force of the unbounded part of a BRB
Peak ground acceleration
Seismic behaviour factor according to the Mexico City Building Code
Post-yielding stiffness factor
Post-yielding stiffness factor of the primary part of a dual system
Post-yielding stiffness factor of the secondary part of a dual system
Strength reduction factor
$Pseudo$-acceleration
Time
Period of vibration
Period of vibration of the primary part of a dual system
\( T_2 \) Period of vibration of the secondary part of a dual system
\( T_{assu} \) Initial period of the secondary substructure of a dual MDOF structure
\( T_{braces} \) Contribution of the BRBs to the period of vibration
\( T_{cols} \) Contribution of the supporting columns to the period of vibration
\( T_g \) Dominant period of the soil
\( u(t) \) Displacement of a dual SDOF system
\( \dot{u}(t) \) Velocity of a dual SDOF system
\( \ddot{u}(t) \) Acceleration of a dual SDOF system
\( \dddot{u}_g(t) \) Acceleration of the ground
\( V \) Shear at the base of a building
\( V_r \) Shear capacity required to maintain elastic a SDOF oscillator
\( V_{yT} \) Total yielding shear capacity of a dual system
\( V_{y1} \) Shear capacity of the primary part of a dual system
\( V_{y2} \) Shear capacity of the secondary part of a dual system
\( W_T \) Total weight of a building
\( \alpha \) Angle of inclination of a BRB
\( \beta \) Total dispersion of the dynamic response of a structure
\( \beta_a \) Uncertainty associated with the record-to-record variability
\( \beta_c \) Uncertainty associated with the construction quality
\( \beta_m \) Uncertainty associated with the completeness of the numerical model
\( \Gamma_1 \) First-mode participation factor
\( \Delta_{\text{ maxi}} \) Inter-storey displacement threshold
\( \Delta_{\text{ byi}} \) Inter-storey yielding displacement of BRBs in the \( i \)th storey of a building
\( \varepsilon \) Strain associated to the ultimate tensile strength of steel
\( \varepsilon_{\text{ sh}} \) Strain associated to the start of hardening of steel
\( \mu \) Ductility factor
\( \mu_1 \) Ductility factor of the primary part of a dual system
\( \mu_{\text{ max}} \) Ductility threshold of the primary part of a dual structure
\( \mu_2 \) Ductility factor of the secondary part of a dual system
\( \mu_{\text{ max}} \) Ductility threshold of the secondary part of a dual structure
\( \theta_{\text{ maxi}} \) Inter-storey drift threshold
\( \theta_{\text{ CP}} \) Inter-storey drift for \textit{Collapse Prevention} performance level
\( \theta_{\text{ FO}} \) Inter-storey drift for \textit{Fully Operability} performance level
\( \theta_{\text{ LS}} \) Inter-storey drift for \textit{Life Safety} performance level
\( \theta_{op} \) Inter-storey drift for Operability performance level

\( \omega \) Angular frequency of vibration

\( \xi \) Damping ratio

\( \xi_1 \) Damping ratio of the primary part of a dual system

\( \xi_2 \) Damping ratio of the secondary part of a dual system

**Acronyms**

- BRB  Buckling-Restrained Brace
- BS  Bracing System
- CDF  Cumulative Distribution Function
- CP  Collapse Prevention
- DM  Damage Measure
- DS  Damage State
- DV  Decision Variable
- EDP  Engineering Demand Parameter
- FE  Finite Element
- FEM  Finite Element Model
- FFT  Fast Fourier Transformation
- FO  Full Operability
- IDA  Incremental Dynamic Analysis
- IM  Intensity Measure
- LS  Life Safety
- MDOF  Multiple Degree Of Freedom
- MRF  Moment Resisting Frame
- Op  Operability
- PBSDD  Performance-Based Seismic Design
- PEER  Pacific Earthquake Engineering Research Centre
- PF  Performance Functions
- OECD  Organisation for Economic Cooperation and Development
- RC  Reinforced Concrete
- RDs  Residual displacements
- RMS  Root Mean Square
- SCT-EW  Ground motion recorded in the station SCT, Component EW
- SDOF  Single Degree Of Freedom
- TF  Transfer Functions
- UNAM  *Universidad Nacional Autónoma de México* (Spanish)
Abstract

Hospitals are regarded as some of the most important structures in society due to the service that they provide. Knowing this, governments spend large amounts of money on these facilities. Also, codes of design require to provide them more reserve capacity than that for conventional structures. However, large damages (such as collapses and permanent or temporary interruptions of their functionality) have still been observed in hospitals during strong earthquakes. Unfortunately, it is precisely after this type of event that their service is in high demand and failure in providing that service could lead to further disastrous or fatal consequences. Therefore, the use of protective technologies, combined with rational procedures of design, would help to reduce damage and probable losses of functionality in hospital structures.

In this thesis, a procedure for seismic design of structures equipped with a type of protective technology, namely, buckling-restrained braces (BRBs), is proposed. Then, the results of experimental and numerical studies are presented to understand the benefits of using BRBs in structures. This study highlights that BRBs are very effective to dissipate seismic energy and can act as structural fuses, i.e. disposable devices that may be replaced after an earthquake without interruptions in the functionality of the structure.

One of the advantages of the proposed procedure is that it takes into account explicitly the characteristics and contributions of both, the main structure and the BRBs. It is based on the assumption that a structure protected with BRBs can rationally be represented by a dual SDOF system whose parts yield at different displacement levels. Other advantages include: 1) better control of the displacement demands on the structure; 2) achievement of the fuse concept beforehand; and 3) rapid assessment of the probabilistic performance of the structure.

The experimental studies consisted of testing steel and concrete models, with and without BRBs, on a shaking table. In addition to calibrate and validate the proposed method of design, the tests have helped to find that, due to BRBs: 1) the damping ratio is increased significantly; and 2) the dynamic response, to ground motions characteristic of the lakebed zone of Mexico City, is reduced in terms of lateral displacements, inter-storey drifts, floor velocities and floor accelerations.

The numerical studies are: 1) a study of the response of typical hospitals improved with BRBs; 2) a study of residual displacements in conventional and dual systems; and 3) evaluation of the economic benefits of using BRBs in structures. On these studies, hypothetical hospitals located in the lakebed zone of Mexico City were considered. The results show that the use of BRBs is very beneficial in medium- and low-rise buildings, while adverse effects may be observed in high-rise structures.

This thesis, with title: “Seismic Design and Performance of Hospital Structures Equipped with Buckling-Restrained Braces in the Lakebed Zone of Mexico City”, is submitted to The University of Manchester by Hector Guerrero-Bobadilla for the degree of Doctor of Philosophy in February 2016.
Chapter 1

Introduction

1.1 Background

Hospitals provide invaluable services; that is why they represent one of the major investments on which governments spend their money. Failures of these facilities after extreme events such as earthquakes could be fatal and the cost due to failures can be significantly more expensive than that of the structure itself. In this regard, the seismic design of hospitals shall be consistent with their level of importance and cost. This is especially significant in Mexico, a country that has a great need of constructing more and better hospitals because it has the lowest indicator of healthcare infrastructure among the countries of the Organisation for Economical Co-operation and Development (OECD). Moreover, hospitals in Mexico City, a city with more than 20 million inhabitants, have largely been affected by earthquakes in the past. As an example, several hospitals collapsed after the M8.1 1985 Michoacan Earthquake and many others suffered significant damages.
Of particular interest is the fact that most of the collapses and damages were located in a specific zone of Mexico City, i.e. the lakebed zone – which is characterised by very soft soil deposits with shear wave velocities of less than 80 m/s and water content of up to 400%. It has been well documented that the soft soils of Mexico City amplify the movement and present dominant periods of vibration. This produces significant demands on structures with fundamental periods close to those of the soil due to resonance effects. Similarly, short-period structures with stiffness degradation (such as concrete structures) may be vulnerable to resonance effects.

Aware of the problems that earthquakes cause in structures, researchers have proposed many seismic protection systems with the intention of reducing losses. One of the most effective protection systems is buckling-restrained braces (BRBs). This system is capable of dissipating significant amounts of energy by means of stable hysteretic behaviour; which is symmetric in tension and compression. Another advantage of BRBs is that they can act as structural fuses, i.e. disposable devices that may be replaced after the occurrence of a major earthquake event without interruptions in the functionality of the structure.

Although BRBs have largely been studied in recent years, the design and behaviour of structures equipped with these devices still need to be further studied. Especially, it has been observed that BRBs are often stiffer than the main structural system and yield at small levels of lateral displacement. This leads to highly nonlinear behaviour of the devices and it needs to be considered during the design process in order to avoid having structures that do not behave as expected.

Performance-Based Seismic Design (PBSD) was proposed by the research community as a philosophy of design with clear and reliable understanding of the risk of
life and economic losses. It takes into account all possible sources of uncertainty (including seismic hazards, materials, completeness of the numerical modelling, construction quality, and variations of repair costs) in order to quantify the probability of experiencing different types of losses considering different possible earthquakes that may affect the structure during its useful life. Unfortunately, PBSD is only reserved for a reduced number of facilities due to the increased engineering involvement that requires.

Based on the reasons expressed in the previous paragraphs, it is observed that there is still necessary to propose new procedures that allow: a) designing structures equipped with protection systems (such as BRBs) in a reliable way; and b) facilitating the application of PBSD philosophy. This is especially important in the lakebed zone of Mexico City due to the characteristics of the seismic movements and the damages observed in the past. Therefore, in this study, the combination of Performance-Based Seismic Design philosophy and Buckling-Restrained Braces (as seismic protection systems) is analysed and assessed in order to achieve more efficient hospital structures, i.e. less expensive structures with better performance under earthquake actions.

1.2 Aim and Objectives

This study aims to develop a Performance-Based Seismic Design (PBSD) procedure for designing structures equipped with Buckling-Restrained Braces (BRBs) and to assess the benefits of protecting hospital buildings in the lakebed zone of Mexico City using these devices.

Therefore, this thesis is related to:

- Proposal of a PBSD method for building structures equipped with BRBs
Experimental and numerical studies of steel and concrete buildings with and without BRBs

The particular seismic response of structures at the lakebed zone of Mexico City

The objectives of this study are:

1. To conduct a literature review for understanding the effects of BRBs in the response of structures subjected to earthquake ground motions and the current PBSD methods.

2. To propose a method for seismic design of structures equipped with BRBs that takes into account the highly nonlinear behaviour of BRBs and improves the application of the PBSD philosophy in such structures.

3. To conduct comparative experimental studies of a steel frame model and assess the effects of BRBs on it when subjected to white noise and seismic input.

4. To conduct comparative experimental studies of two reinforced concrete precast models and assess the effects of BRBs on them when subjected to white noise and seismic input.

5. To evaluate the benefits of improving existing hospitals using BRBs.

6. To evaluate the parameters affecting residual displacements (RDs) in structures equipped with BRBs. This is significant because RDs play a key role in post-earthquake behaviour of structures. Small RDs could generate significant (or unaffordable) repair costs while large RDs could lead to demolition of a building - even when the structure may present small damage.

7. To evaluate expected (or probable) losses and lifecycle costs in hospitals equipped with BRBs and compare them to those of traditional hospitals (i.e. without BRBs) in order to assess the economic benefits of using these devices.
1.3 Development of the project

During the literature review, back in 2012, it was found that BRBs possess high energy dissipation capacity; which is generated by stable and symmetric (in tension and compression) hysteretic loops. These loops, in turn, are generated by plastic deformation of the material composing the core of the devices. Since they are made of conventional construction materials, BRBs represent a major invention in the field of earthquake resistant design because they dissipate large amounts of energy at low cost. Furthermore, BRBs can be used as structural fuses, i.e. disposable devices that may be replaced after the occurrence of a strong earthquake.

Later, a method for designing buildings equipped with BRBs proposed by Teran and Virto [1] served as motivation to continue with the study in the subject. In their method, these authors assumed that a structure equipped with BRBs behaves as a dual system. However, the input of design was a spectrum calculated using conventional elastoplastic oscillators; which do not consider the fact that the parts of a dual system yield at different displacement levels and may lead to biased designs. As a result, the idea of proposing a new method, that overcomes some shortcomings observed in that method, and in others available in the literature, was born.

By the beginning of 2013, shaking table experiments on a steel building model at a scale of 1/10 were being conducted by Professor Escobar of the Institute of Engineering at the National Autonomous University of Mexico (or UNAM). A proposal of including a set of tests with BRBs in the experimental programme was submitted and, in November of that year, I went to Mexico to conduct the tests. The results were very interesting because they allowed the observation of two effects due to BRBs: a) significant increase of the damping ratio; and b) reduction of the dynamic response to seismic action, in terms of lateral displacements, floor velocities and floor accelerations.
Given the success of the tests on the steel model, a new proposal was elaborated to conduct more shaking table experiments of two bigger models; which were at a scale of 1/3. The models were made of reinforced concrete precast elements. Although the cost of these experiments was high, sponsors were invited to participate. The models were donated and constructed by the company Itisa (from Mexico), while one set of BRBs was sponsored by Corebrace (from the USA). I had the fortune of coordinating the logistics of the experiments from Manchester and I went to Mexico (for a second time) between October and November of 2014 to conduct the experiments. The results confirmed that BRBs increase the damping ratio and reduce the dynamic response in building models. A significant finding of this experimental study was that BRBs also reduce and delay stiffness degradation in reinforced concrete structures.

The experience gained during the experimental tests provided the basis to develop a new type of BRB; which is capable of producing similar response to commercially available BRBs but at significantly lower cost.

Once the experiments were finished and the data processed, during 2015 numerical studies were conducted with the intention of finding the benefits of fitting BRBs in structures not only in terms of dynamic response but also in terms of probabilistic losses and lifecycle costs – which is the objective of PBSD philosophy.

1.4 Outline of the Thesis

This thesis contains nine chapters where this introduction serves as the first one. The arrangement of contents in chapters is based on the work to realise each of the research objectives. In this way, the second chapter is a literature review covering related topics, such as buckling-restrained braces, current methods for designing structures equipped
with BRBs and Performance-Based Seismic Design Philosophy. The main contents are presented through Chapters 3 to 8 as follows:

- Chapter 3 proposes a PBSD method for preliminary design and assessment structures equipped with BRBs. It is assumed that the main structure and the bracing system (i.e. the BRBs) behave as a dual system whose parts yield at different levels of displacement and loading. The advantages of this method are explained and discussed. An example of application is also provided.

- In Chapters 4, comparative experimental studies of a five-storey steel building models with and without BRBs, on a scale of 1/10, are presented. The experiments were conducted on a shaking table at the National Autonomous University of Mexico. The results show that BRBs provide significant benefits when introduced in the building models, such as increasing the damping ratio and reducing the dynamic response.

- Chapter 5 shows experimental studies of two reinforced concrete precast building models with and without BRBs conducted also on a shaking table at the National Autonomous University of Mexico. The models were of four storeys height and were on a scale of 1/3. The results also show that BRBs increase the damping ratio and reduce the dynamic response of the models. Additionally, for the concrete models, BRBs also help to reduce stiffness degradation, which is important in short-period structures subjected to ground motions with longer dominant periods of vibration to avoid resonance effects. An additional outcome of the experiments was the proposal of a new type of BRB – which presents similar behaviour to current devices but at significantly lower costs. Details of the proposed BRB are shown in Appendix A.
• In Chapter 6, the benefits of providing BRBs to existing hospitals are assessed. The performance of typical hospitals, located in the lakebed zone of Mexico City and designed according to current practices, is assessed before and after being upgraded with BRBs. A small contribution of the BRBs to the lateral load capacity is selected to avoid high demands in existent elements such as foundations and connections. The results show significant improvements in the dynamic response due to the BRBs. Moreover, a parametric study in SDOF structures is conducted to understand the period range on which structures located in the lakebed zone of Mexico City are benefitted by BRBs.

• In Chapter 7, residual displacements (RDs) in SDOF structures with and without BRBs are studied due to their importance in post-earthquake evaluations. First, the parameters affecting RDs in conventional oscillators are analysed. It is found that the post-yielding stiffness plays a key role in RDs. Then, the parameters that affect RDs in dual oscillators (as representative of structures equipped with BRBs) are studied. It is found that RDs remain small in dual systems if the primary part remains elastic and the plastic deformation is concentrated in the secondary part. On the contrary, RDs increase dramatically if the primary part exhibits plastic deformation. Recommendations are provided to minimise RDs in dual structures.

• In Chapter 8, the economic benefits of using BRBs in structures are evaluated. Probabilistic losses and lifecycle costs in three-, six- and nine-storey hospitals with and without BRBs are estimated. It is found that BRBs allow reducing them significantly. Furthermore, BRBs also reduce the probability of collapse and of loss of functionality, which is significant for hospitals because they may continue operating after being hit by strong earthquakes.

Finally, Chapter 9 presents the conclusions and further work formulated in this thesis.
The relationships between the contents in the main chapters can be summarised in the following flow chart:

**Figure 1-1. Organisation of the Thesis**
Chapter 2

2.1 Introduction

An extensive literature review was conducted in order to identify the current advances in the design and the behaviour of structures equipped with Buckling-Restrained Braces (BRBs), with particular attention to hospitals located in the lakebed zone of Mexico City. The characteristics and experimental behaviour of BRBs are reviewed in section 2.2. Methods for designing structures equipped with BRBs are reviewed in section 3.2. Section 2.3 describes the current advances in Performance-Based Seismic Design. In section 2.4, the seismicity of Mexico and the effects of earthquakes in the lakebed zone of Mexico City are investigated. Hospitals, including their architectural and functionality requirements, are reviewed in section 2.5. This literature review provides an understanding of the current situation and advances on the design and behaviour of structures equipped with BRBs and suggests where further research on this topic would be beneficial.
2.2 Buckling-Restrained Braces (BRBs)

Originally proposed by Wakabayashi et al. [2] and further developed by many others such as Watanabe et al. [3] and Black et al. [4], Buckling-Restrained Braces are a type of seismic passive dissipator. BRBs are studied in this project because they have a high potential to reduce damage produced by earthquakes. In recent years, they have gained a great reputation as lateral resisting systems because of their advantages over other systems (see [5-7]). Other types of brace systems (such as special concentrically braces or ordinary concentrically braces) depend on buckling to dissipate energy. This buckling behaviour leads to rapid degradation of strength and stiffness under cyclic deformations (as shown schematically in Figure 2-1a). On the other hand, BRBs present symmetric and stable behaviour as observed in Figure 2-1b.

![Figure 2-1. Behaviour of conventional braces and BRBs under cyclic loading](image)

Further advantages of BRBs include [5]: (1) they present neither stiffness nor strength degradation; (2) they yield at early displacements before other elements such as beams and columns – in this way they can be designed to act as structural fuses, or
disposable elements that may be replaced after the occurrence of a severe earthquake; and (3) they are easy to be replaced.

Their main disadvantages are [5]: (1) there are no defined criteria to assess whether a device is damaged or not, or when it should be replaced; (2) they are sensible to initial axial deformations so that special attention must be given to some brace configurations such as chevron; and (3) they are often proprietary products so that their prices tend to be higher than conventional braces.

2.2.1 Parts

A typical BRB is shown in Figure 2-2. In general, BRBs are composed of a core and a case [7]. The core is commonly made of commercial steel and is weaker in the middle than at the ends (Figure 2-2a). This is done with the intention of concentrating plastic deformations in the weaker zone of the core while linear-elastic behaviour is expected at the ends. On the other hand, the case commonly consists of a steel tube and a filling of mortar. Before casting the filling mortar, the steel core is located inside the steel tube and an unbonding material is located around the steel core in order to avoid direct interaction between the core and the filling mortar. Once the mortar reaches its nominal strength, buckling in the core is prevented when the BRB is subjected to compressive forces. This results in stable and symmetric hysteretic behaviour [5], as shown previously in Figure 2-1b.

As noticed by Uang et al. [5] and Wada et al. [8], stable hysteretic behaviour is expected from BRBs if the buckling force \( P_{cr} \) of the ends of the core is higher than the yielding capacity of the weaker zone, this is

\[
P_{cr} = \frac{\pi^2 EI}{L_c'} > F_y (Af_y) \tag{2-1}
\]
where $E$ is the modulus of elasticity of the core, $I$ is the second moment of area of the unbounded zone (i.e. the ends), $L_e$ is the unbounded length on the brace, $A$ is the cross-sectional area of the yielding zone, $f_y$ is the yielding stress of the material and $F_s$ is a safety factor larger than unity.

![Typical Buckling-Restrained Brace](image)

**Figure 2-2.** Typical Buckling-Restrained Brace

### 2.2.2 Experimental behaviour

Buckling-Restrained Braces (BRBs) have extensively been studied as: 1) individual members, 2) members located in sub-assemblages and 3) as bracing systems in building frames.

As individual members, they showed high energy dissipation capability, in terms of hysteretic behaviour with full, symmetric and stable loops (see [6, 9-10] among many others). Different core section shapes, cases and materials have been tested (for example, see [5, 11-12]). Good results have been observed when the case of the BRB is...
filled with mortar, the core cross-section is rectangular or cruciform, and the core is made of mild or low-yield steel. Typical modes of failure have been identified, i.e.: global buckling of the brace [13-14], local buckling in the core due to higher modes [15-17], and buckling in the unbounded zones (i.e. the ends) where the core is not effectively confined [11]. Other common sources of failure are cumulative plastic ductility [18-19] and at brace-beam-column connections [20-21]. If not well controlled, these types of failure may have a significant impact on the global response of the structures in which they are fitted [20].

As sub-assemblages, models equipped with BRBs have been tested in order to assess their lateral resistant capacity, especially that of their connection ends when subjected to rotations [7, 22-23]. Acceptable response has been observed within the levels of drift design. However, for demands beyond these limits, failures have been identified in the brace-beam-column connections due to out-of-plane deformation of the gusset plates. Therefore, special attention must be paid when designing the gusset plate connections. For example, Chou et al. [24] proposed a dual gusset plate to enhance the stability of the connection.

For bracing systems in building frames, experimental research is limited. Particularly, shaking table tests are of interest because they provide a reliable way to assess the response of structures subjected to dynamic loads [25]. In this regard, several such tests have been conducted on steel frames equipped with BRBs. For example, Vargas and Bruneau [26] tested a 1/3 scale model of a three-storey one-bay steel frame, with and without BRBs, on a shaking table in the USA. They reported that the lateral displacements and inter-storey drifts were reduced by 70%, while floor accelerations remained almost unchanged. An increase of the damping ratio, from 2% in the bare model to 5% in the fully-equipped model, was also reported. Kasai et al. [27] tested a
full-scale five-storey steel framed building equipped with four different types of energy dissipaters (including BRBs), on a shaking table in Japan. The frame was tested without, and with, the devices in four of the five storeys. It was subjected to low-intensity white noise and the Takatori motion of the 1995 Kobe earthquake, scaled to different intensities levels. The results showed that the inclusion of BRBs reduced the displacements, storey shears and absolute floor accelerations. Damping ratios (of less than 2%) were reported in the model with and without BRBs for the low-intensity tests with white noise input. This observation was different from the findings in the study by Vargas and Bruneau [26]. However, damping ratios between 4% and 9% were reported in the model with BRBs for the tests with seismic input. It was also observed that the damping ratios were intensity-dependent. In China, Hu et al. [28] tested a full-scale pin-connected steel frame equipped with BRBs. They found that the BRBs performed well under high-intensity seismic ground motion; drift demands were well controlled and no damage was encountered in the structural elements of the frame. No values of damping ratio were reported. Yamaguchi et al. [29] tested a steel sub-assemblage composed of one column, a beam and a BRB as a diagonal. Shaking table tests with, and without, the BRB, showed that lateral displacements were reduced by 65% when the BRB was included. The damping ratios with, and without, the BRB were 2.3% and 1.7%. Hikino et al. [30] tested a single-storey single-bay steel frame with two BRBs in chevron configuration on a shaking table. Good behaviour was observed as long as out-of-plane buckling of the brace-beam connection was prevented. A damping ratio of 3% was reported for the frame with BRBs.
2.3 Performance-Based Seismic Design (PBSD)

Performance-Based Seismic Design (PBSD) is a philosophy of design. It consists of a group of activities conducted to design a structure for a given probability of not exceeding certain threshold of losses, considering a potential range of future earthquakes that may affect it [31]. In PBSD, designers, owners and other stakeholders agree to accept a level of risk for each of the different levels of earthquake hazards considered ([31-32]). Since seismic risk is a probabilistic term, it has to be predicted and evaluated with quantifiable confidence including all possible sources of uncertainty. The final product of PBSD is a structure capable of achieving the agreed performance, which may be superior to code specifications. This may be the case of special structures such as nuclear power plants, hospitals or structures where the owners desire superior behaviour such as hi-tech manufactures and database stores of banks.

2.3.1 Performance-Based Seismic Design Process

The PBSD process is applied in three steps (as seen in Figure 2-3): 1) selection of objectives; 2) preliminary design; and 3) assessment of the performance. If the objectives of design are met, the PBSD process ends; otherwise, the design or the objectives are modified iteratively until expectations of designers, owners and other involved people are satisfied. Note that the preliminary design is crucial to avoid a significant number of iterations that may lead to impractical application of the PBSD philosophy.

The assessment of the performance is often conducted using Cumulative Density Functions (CDFs), which express the probable losses of a building when subjected to earthquake loading. These CDFs are also termed as Performance Functions. A typical Performance Function is shown in Figure 2-4 where the horizontal axis (Impact Quantity) may represent losses such as the number of deaths, repair cost, repair time,
etc. The vertical axis indicates the probability that the losses are going to be equal to or less than a specific value.

**Figure 2-3.** Performance-Based Seismic Design flowchart

**Figure 2-4.** Typical Performance Function

For decision makers (like owners, invertis, financial institutions, governments, etc.), Performance Functions are more meaningful than other traditional measures of the performance (e.g. base-shear, inter-storey drifts or displacements). Performance
Functions can be directly converted into Maximum Expected Losses, Scenario Expected Losses among others.

With Performance Functions, like that shown in Figure 2-4, different design options and structural systems (whether traditional or innovative) can be compared and the most convenient option can be selected. In this way, more informed and intelligent decisions are made based not only on constructional costs but also on life-cycle considerations [32].

2.3.2 Assessment of the expected performance

To assess the expected (or probable) performance of a structure, its Performance Functions must be determined. For that purpose, the framework proposed by the Pacific Earthquake Engineering Research Centre (PEER) [33] is used in this project. Shown in Figure 2-5, the PEER framework consists of the integration of four independent probabilistic analyses, which are: 1) seismic hazard, 2) dynamic response, 3) damage, and 4) loss. The integration is conducted using the total probability theorem, namely

\[
\lambda(DV) = \int \int \int G(DV \mid DM) \, dG(DM \mid EDP) \, dG(EDP \mid IM) \, d\lambda(IM) \tag{2-2}
\]

where \(\lambda(DV)\) is the mean annual frequency of exceeding a given decision variable (DV), for example, the repair cost after a given earthquake; \(\lambda(IM)\) is the mean annual frequency of exceeding a given intensity measure (IM), for example, the \(pga\); \(G(EDP\mid IM)\) is the probability of exceeding a given engineering demand parameter (EDP) such as inter-storey drift ratio, conditioned to IM equal to a given value; \(G(DM\mid EDP)\) is the probability of exceeding a damage measure (DM) such as the number of cracks in a concrete beam given a value of EDP; and \(G(DV\mid DM)\) is the probability of exceeding a value of DV given a value of DM.
Each of these analyses is described in the following subsection.

### 2.3.2.1 Seismic hazard analysis

This step consists of quantifying the seismic hazard at the location of the structure. It is normally conducted using probabilistic seismic hazard analysis (see [34]) which takes into account all the seismic sources and corresponding uncertainties of possible future earthquakes affecting the structure. The attenuation relationships of the ground motions between the probable epicentres and the location of the structure are also considered. More information can be found, for example, in [31] and [34].

### 2.3.2.2 Dynamic response analysis

In this step, incremental dynamic analysis (IDA) [35] is normally conducted in detailed finite element models to obtain engineer demand parameters, such as inter-storey drifts and stresses, for a wide range of ground motions and seismic intensities. Since this step could be significantly time consuming, a reduced number of analyses is normally used to obtain the statistics of the structural response and a Monte Carlo procedure is conducted to generate simulated demands [36]. On the other hand, the collapse fragility (i.e. the cumulative distribution function of the probability of collapse given an intensity measure) is also estimated in this step because it has a significant impact in the results.
2.3.2.3 Damage analysis

In this step, damage states ($DS$) and corresponding repair actions and costs are defined for each component (or group of components) of the building. $DS$ are normally defined in the form of fragility functions. As an example, Figure 2-6a shows fragility functions for a typical partition wall made of gypsum with metal studs. This information was taken from the database compiled in the PACT programme in [31]. Three damage states have been defined, namely: $DS_1$, $DS_2$ and $DS_3$. If, for example, the wall is subjected to an inter-storey drift demand of 0.005, the probability of the wall to have a damage state $DS_1$ or worse is 0.93, $DS_2$ or worse is 0.22, and $DS_3$ or worse is 0.03. Additionally, the repair actions and cost should be defined for every damage state. As an example, Figure 2-6b shows the unit repair cost due to $DS_1$ for the partition wall of the example. As can be observed, a tri-linear function is typically defined to consider efficiencies of scale, i.e. reduction of unit repair cost as the quantities increase. Uncertainty is also included to consider the factors affecting reparability after an earthquake, such as the availability of resources. Another important issue to consider in this step is the quantities of the components (structural, non-structural and contents) in the building. Normally, they are not precisely known at the design stage. Therefore, normative quantities, which are available in databases for typical occupancies, may be considered enough in this step because they provide a reasonable estimation [31]. In this regard, the Normative Quantity Estimation Tool of the ATC-58 Project [31] provides normative quantities for several occupancies, including healthcare infrastructure.
2.3.2.4 Loss analysis

In this step, a decision variable, such as the total repair cost is estimated using the demands of the structural response (determined in the dynamic response analysis) and the fragility data (determined in damage analysis). For this purpose, the Monte Carlo procedure developed by Yang et al. [36] and later adopted by the ATC-58 project [31] is used here. This procedure is explained with the help of Figure 2-7 and summarised as follows:

1. An intensity measure value is selected (e.g. $pga=0.20g$).
2. A uniform random number between zero and unity is generated. If the random number is higher than the probability of collapse (determined in the dynamic response analysis), a collapse is considered to be occurred; and the total repair cost is that corresponding to the replacement cost (including demolition and clearance cost).
3. If collapse has not occurred, the total repair cost is estimated as follows:
i) **Obtain a demand simulation.** A vector of statistically consistent simulated demands is generated as described previously in the dynamic response analysis (Section 2.3.2.2) using the response statistics.

ii) **Determine if the building is reparable.** Here, a reparability curve is used. This curve is a function of the residual (or permanent) inter-storey drift ratio and provides the probability that a structure is irreparable. Thus, for a given residual inter-storey drift ratio demand, a random number between zero and unity is generated. If the number is higher than the probability of irreparability, the structure is considered irreparable.

iii) **Irreparable.** If irreparability is determined, the total repair cost is determined as the replacement cost of the structure (including demolition and clearance cost).

iv) **Reparable.** If the structure is reparable, a damage state is first determined for each component. To determine the damage state, the fragility functions described previously in Section 2.3.2.3 are used (see Figure 2-6a). Then, for a given demand simulation (e.g. inter-storey drift), a random number between zero and unity is generated. The damage state is found from the fragility function. For the example of Figure 2-6a and an inter-storey drift demand of 0.005, a random number above 0.93 would result in an undamaged state, between 0.22 and 0.93 would result in $D_{S1}$, between 0.03 and 0.22 would result in $D_{S2}$ and less than 0.03 would result in $D_{S3}$. Once the damage state is determined, the associated repair cost is obtained by multiplying the unit cost and the associated quantity. The total repair cost is the summation of the repair cost of all the components of the structure. Similarly, other decision variables such as repair time and number of casualties can be estimated.
Steps 2 and 3 are repeated many times (hundreds or thousands) to generate a smooth distribution of possible damage states and repair cost. Yang et al. [36] have found that as few as 200 repetitions, termed as realisations, can provide stable cost estimates.

![Flowchart](image)

**Figure 2-7.** Estimation of performance using the Monte Carlo procedure of [31]

The process finishes here for scenario- or intensity-based assessment of the performance. However, if a time-based assessment shall be conducted, a number of intensity-based assessments needs to be conducted and, using a site-specific seismic hazard curve (determined by a seismic hazard analysis, see Section 2.3.2.1), the total
repair cost, as a function of the mean annual frequency of exceedence, is estimated. More guidance can be found in [31] and [36].

2.4 Seismicity of Mexico

2.4.1 Earthquake sources

Mexico is a seismic country highly affected by different earthquakes. According to several reports, on average every 10 years a major earthquake (M>7.5) occurs in Mexico (e.g. see [37-39]). In particular, the capital, Mexico City, is threatened by four main sources [40]. They are shown in Figure 2-8 and described as follows:

- **Local earthquakes.** These are originated close to or in the Mexico City valley, they may reach magnitudes up to four and may not generate significant affectations in the city [41];

- **Shallow crustal earthquakes in continental Mexico.** They may reach magnitudes around seven. These earthquakes have occurred in the past and have generated significant damages in populated areas. As an example, in 1912 an earthquake of magnitude seven destroyed the town of Acambay, which is located at 100 km northwest of Mexico City. Economic analyses of the damages, that this earthquake would generate in current Mexico City, reach billions of US dollars [42].

- **Normal faulting earthquakes in the subducted oceanic lithosphere.** These earthquakes are generated at depths between 40 and 80 km and are produced by the normal rupture of the plates subducted under the North American plate. These earthquakes are a very high risk for populated areas. As an example, in 1931 an earthquake destroyed the city of Oaxaca [39].
Chapter 2. Literature Review

46

a) Historic earthquakes of Mexico

b) Subduction of Cocos plate under North American Plate

Figure 2-8. Sources of seismicity of Mexico (adapted from [37])

- **Subduction zone earthquakes.** Reaching magnitudes higher than eight, these earthquakes are generated in the subduction zone of the Pacific Ocean where the Cocos and the Rivera plates go under the North American plate. They are the major earthquake hazard to Mexico City. The damages and losses generated by past events have been enormous. The most severe damages were produced by the 19/09/1985 earthquake (M 8.1), which generated more than 400 buildings with total or partial collapse, about 10,000 deaths and economic losses between 3 and 4 billions of US
dollars [43]. More details of this event can be found elsewhere [44-45] – however, it is worth pointing out that from the collapses, over 240 buildings (which included 11 hospitals) were less than 10 stories height, which is the range of “desirable” characteristics of hospitals, as will be described in the next section.

Singh et al. [38] have identified different seismic regions along the subduction zone of Mexico and have estimated the average recurrence periods of large earthquakes. The recurrence periods for those seismic regions vary from 30 to 75 years. It brings the attention the existence of some gaps with high seismic potential where no large earthquakes have occurred for a long time. In particular, the Guerrero Gap, an approximate segment of 200 km long close to the Acapulco bay, has not experienced a significant earthquake since 1911 (M 7.6). Some studies suggest that this gap has enough energy accumulated to generate a great earthquake (M 8.1-8.4) or several large events (M 7.5-7.8) during the next years or few decades [46]. According to simulations by Kanamori et al. [47], such earthquakes could generate seismic demands in structures of Mexico City larger than twice those experienced during the 19/09/1985 Michoacán Earthquake. Therefore, it is justified (and necessary) the study and use of protective systems (such as BRBs) to prevent losses like those experienced in the past.

### 2.4.2 Amplifications in the lakebed zone of Mexico City

Even when Mexico City is very far from the subduction zone (more than 300 km away), ground accelerations in the lakebed zone of the city have reached high values (peak values up to 160 cm/s² have been observed). This phenomenon has been documented by many researchers (e.g. [48]) showing that the movement is amplified in the soft soil deposits of the lakebed zone, which are composed of a clay layer of variable thickness
and water content of up to 400%. Figure 2-9 shows the East-West component of the accelerations recorded in Mexico City in the stations CU (located in hard soil) and SCT (located in the lakebed zone) during the 19/09/1985 Michoacán earthquake. It is worth remarking that the distance between these stations is less than 5 km while the epicentre of the earthquake occurred at more than 400 km away. As seen in the figure, the peak ground acceleration in the lakebed zone is of the order of four times that recorded in the hard soil.

Furthermore, the movements in the soft soils of the lakebed zone have dominant periods of vibration. This characteristic increases the demands in structures with similar periods to the dominant period of the soil due to resonance effects. As an example, Figure 2-10a shows the pseudo-acceleration spectra of the records of Figure 2-9. It is observed in the figure that the demands in structures with periods around 2 seconds are around 10 times higher for structures located in the lakebed zone than for structures located in hard soil.

![Figure 2-9. Accelerograms of the 19/091985 Michoacán earthquake](image-url)
The response of the Mexican authorities to reduce future damages was to increase the design spectrum. Figure 2-10b shows that the design spectrum before 1985 (thin line) was almost doubled to reach the current spectrum of design (thick line). For comparison, the response spectrum of Figure 2-10a, corresponding to the lakebed zone, is also included. It is observed that the design spectrum of before 1985 was easily rebased by the SCT-EW record for periods of up to 3 seconds - which corresponded to most of the structures at that time. On the other hand, the current spectrum of design is still rebased at periods close to the dominant period of the soil.

![Figure 2-10. Response spectra for the accelerograms recorded in Stations CU and SCT](image)

**Figure 2-10.** Response spectra for the accelerograms recorded in Stations CU and SCT

### 2.4.3 Global context

In order to compare the seismic demands in structures located in the lakebed zone of Mexico City against others in other seismic regions of the World, the pseudo-acceleration and displacement spectra are also determined for the 06/04/2009 L’Aquila earthquake (recorded in the station Gran Sasso in 2009). This movement was selected because it has similar peak ground acceleration ($pga=0.16g$) to that of the SCT-EW record which was recorded in the lakebed zone of Mexico City (Figure 2-10). Figure 2-11 compares the response spectra of both movements. It can be seen that the shape of
the spectra are totally different even when they have similar \textit{pga}. Pseudo-accelerations and displacement demands are significantly higher in the lakebed zone of Mexico City for periods longer than 0.5 s. Figure 2-11b shows that the lateral displacements are an issue of concern for structures located in the lakebed zone of Mexico City. Therefore, the use of lateral resisting systems that allow reduction and control of the lateral displacements (such as BRBs) may be very effective for such structures.

Deep analysis of Figure 2-11b suggests that structures with periods of vibration around 1 s and stiffness degradation problems are highly vulnerable to ground motions like the SCT-EW motion because the displacement demands increase exponentially. In this regard, typical hospital structures, which are subject of this thesis and are further studied in the next section, represent a big challenge because they typically have periods of vibration around, or less than, 1 s (i.e. they are medium- to low-rise structures).

2.5 Hospital Structures

Although the analyses presented in this thesis may apply for a broad type of structures, hospitals have been selected due to the importance of: a) the service that they provide,
and b) the investment that they represent. Furthermore, Mexico, where this project is focused, needs urgently to construct more hospitals efficiently and upgrade many existing ones. New hospitals are urgently needed because Mexico has the lowest indicator of healthcare infrastructure among the countries of the Organisation for Economical Co-operation and Development (OECD) (see Figure 2-12). On the other hand, many existing hospitals need to be upgraded because Mexico has signed an agreement with the Pan American Health Organisation, which requires that by 2015, all hospitals in the country must remain fully operational after the occurrence of any major natural disaster [49].

Figure 2-12. Number of beds per each 1000 inhabitants among the countries of the OECD [50].
2.5.1 Importance

Hospitals are regarded as one of the most important types of structure of our modern societies. In this regard, many codes of seismic design classify hospitals as “important” structures and require them to be designed with higher load capacity than conventional structures. In the pursuit of simplicity, the codes impose a factor of importance; which is higher than unity and is applied to the spectral ordinates of the design spectra. For example, the Mexico City building code [51] requires to use an importance factor of 1.5 while Eurocode 8 [52] requires a factor of 1.4. Unfortunately, it has been observed in past events that the use of an importance factor has not been enough to avoid interruptions of functionality, extensive damages or collapses of these facilities [44-45]. Therefore, a different approach, based on energy dissipation and control of the response rather than in force capacity, is necessary to achieve more efficient designs with better behaviour.

On the other hand, governments and international organisations have recognised that the repair cost of a hospital could be far more expensive than the complete replacement cost. As an example, the motto of the World Disaster Reduction Campaign (WDRC) of the United Nations is “the most costly hospital is the one that fails” [53]. This statement sounds rational because, in addition to the repairing cost, the downtime and other uncountable damages related to the non-given services are enormous. Therefore, the use of preventing and protective strategies could be regarded as an important source of savings under the occurrence of future natural disasters.

The WDRC argues that making a hospital more secure could cost only about 4% more than its initial cost. This is argued because in hospitals the cost of the structure alone could represent only about 20% of the initial cost, while non-structural elements
and contents could represent up to 80%. Similar proportions of costs have been reported in an independent study by Taghavi and Miranda [54].

Based on the arguments of the previous paragraphs, it is of interest to study the use of protective systems, such as Buckling-Restrained Braces, in hospitals. As previously introduced, BRBs are attractive to mitigate damages because they dissipate significant amounts of energy and can act as replaceable fuses while the operations and integrity of hospitals may remain unaffected.

2.5.2 Configuration

In order to be able to provide an optimal design and a precise assessment of a structure, its characteristics and requirements must be known well. In the case of hospitals, three main areas must interact in order to provide the service; these areas are nursing, clinical, and support areas. The interaction between these areas provides the architectural configuration of the building and the structural resisting system must be capable of providing the optimal response so that the functionality is not interrupted. This is especially important in situations of emergency where high demands of the service are expected (e.g. after the occurrence of a major natural hazard). Additionally, the architectural forms and configurations are strongly related to the number of beds, and the social and cultural costumes of the local society [55].

According to Cox and Groves [55], the desirable characteristics of hospitals are:

- *Ventilation and light*. They are of paramount importance. Thus, external walls should be avoided if possible to reduce costs of operating artificial environments. In this case, bracing systems like BRBs could provide a good alternative because light and ventilation are not obstructed as compared with external walls.
• *Enlarged plan.* It is valued that the building is narrow in one horizontal direction and large in the other so that it could be easily ventilated; although, this is not always possible in highly populated areas. The enlarged form of a building will additionally allow rapid horizontal communication and movement of personnel between departments having a close relationship.

• *No tall structures.* One disadvantage of very tall structures is that there are many interventions between services zones and clinical work; besides, the extension of the maintenance area could reach up to the same area as the one they provide the service. Tall buildings could be an economical solution but they are not very safe in case of emergencies because there are stairs and lifts that could fail.

• *Medium- and low-rise structures are preferred.* Since health care itself is the most important, the structure must be designed to not provoke interruptions in this important activity. In this sense, medium- and low-rise structures (normally with periods of vibration around 1 s or less) are preferred for hospital configurations because vertical communications between clinical and service areas could be optimised using small stairs, ramps and lifts. One-storey structures may not be desirable because horizontal travel distances could reduce the hospital efficiency.

Another important aspect to consider in the design process is the principle that ‘the building is permanent but the uses are constantly changing; any medical zone could be placed anywhere at any time’ [55]. Hospitals must be designed to grow and be modified in the future because unforeseen demands will increase, especially in developing countries where the requirements change more rapidly. In this case, the use of structural elements that can easily be relocated would be very attractive. BRBs provide such flexibility.
2.6 Summary

After searching the literature, the following conclusions and further work have been identified:

- BRBs provide several advantages over other lateral resisting systems. Therefore, it is of interest to further study their applicability and benefits to hospitals, whether new or existent. This is significant because hospitals are important structures and cost huge resources to governments. Besides, Mexico, where this project is focused, has a big deficit of hospitals (as seen in Figure 2-12).

- Experiments showed that BRBs are very effective to dissipate energy. When fitted in building models, BRBs improved the seismic response significantly. However, these tests have been only conducted in steel models and under specific seismic movements. Tests on concrete models and under different seismic actions, such as those recorded in Mexico City, are still needed to gain a better understanding of the behaviour under such conditions.

- A few methods for designing structures protected with BRBs have been proposed recently. The main advantage of these methods is that they are based in the control of the lateral displacement demands. However, some shortcomings have been identified such as using design spectra generated from conventional elastoplastic oscillators. A new proposal of a method for designing structures protected with BRBs should help to remove the assumptions of current methods, improve the control of lateral displacements and be able to apply the PBSD philosophy in a practical basis.

- PBSD allows the estimation, in probabilistic terms, of the performance of different structural systems. Further work should be focused on investigating the probabilistic performance of hospitals (traditional or equipped with seismic protection systems).
when subjected to seismic loading. In this way, direct comparison of the performance (for example in terms of expected losses) shall provide better understanding of the benefits of providing dissipation systems (e.g. BRBs) to these facilities.

- Mexico is a country highly affected by strong earthquakes and there is evidence of the existence of seismic gaps with high potential to generate very strong earthquakes within the short term. Therefore, the use and effectiveness of protection technologies in this region needs to be further investigated, in order to avoid catastrophic damages such as those observed in the (M8.1) 1985 Michoacán Earthquake. Furthermore, the particular characteristics of the ground motions observed in the lakebed zone Mexico City (including amplifications of the movement, resonance effects, and large displacement demands) need to be considered because they are of great concern in this region of the World.

- Reparability, or the probability that a structure is reparable after an earthquake, affects the estimation of performance of a given structure. Since maximum residual inter-story drift is the parameter that defines reparability, it should be further investigated in structures equipped with BRBs.

- Even when economic quantities are more meaningful to decision makers than dynamic response parameters, no much attention has been paid to assess the economic benefits of using BRBs in structures. Therefore, they should be further studied.

The following chapters of this thesis are strongly related to these points and contribute to a better understanding of the design and behaviour of structures equipped with Buckling-Restrained Braces (BRBs), with particular attention to hospitals located in the lakebed zone of Mexico City.
Chapter 3

A Method for Preliminary Seismic Design and Assessment of Low-Rise Structures Protected with Buckling-Restrained Braces

3.1 Introduction

Since Buckling-Restrained Braces (BRBs) present high nonlinear behaviour, structures equipped with these devices should be designed using compatible methods, which take into account their behaviour. In this chapter, a method for preliminary Performance-Based Seismic Design of structures fitted with BRBs is presented. It is based on the assumption that a structure protected with BRBs can rationally be represented by a dual SDOF oscillator whose parts yield at different displacement levels. The advantages of the proposed method are: 1) designers are able to select from a variety of parameters to control the displacement demands on the structure; for example, they can balance explicitly the relative participation of the BRBs and the main frame; 2) the maximum ductility demands on each part of the dual system are estimated at the beginning of a
design so that the fuse concept can be satisfied beforehand; and 3) additional information for preliminary assessment of the performance is generated.

The organisation of this chapter is as follows: Section 3.3 presents previous methods available in the literature to designing structures equipped with BRBs; then, Section 3.3 presents the idea of introducing BRBs in conventional structures to form a dual structures and compares the differences between conventional and dual SDOF oscillators in terms of the dynamic response; Section 3.4 presents the use of BRBs in multiple-degree-of-freedom (MDOF) structures and, most importantly, a proposed procedure to designing structures equipped with BRBs. A case study example of design is provided in Section 3.5 to show the applicability of the proposed method. Using the information generated in the application of the method, a preliminary assessment of the performance of the example structure is conducted in Section 3.6; Sections 3.7 and 3.8 present the discussion and conclusions of the results. It is worth to highlight that the proposed method is valid for low-rise, rigid floor, regular buildings whose dynamic response is dominated by their fundamental modes.

### 3.2 Previous works

In order to design structures protected with BRBs, methods based on the control of the response have been proposed recently. Most of these methods were proposed only for BRB frames, defined as systems whose lateral resistance is only provided by BRBs while the contributions of the main frames are neglected [56-58]. However, for cases where the contribution of the main structure represents a significant amount of capacity, this should be taken into account when designing and assessing the performance of structures equipped with BRBs. In this regard, Maley et al. [59], Vargas and Bruneau [60] and Teran-Gilmore and Virto-Cambray [1] have proposed methods that take into
account the contribution of BRBs. These methods are reviewed in more detail in the next subsections.

3.2.1 Maley et al. [59] method

This method is valid for steel frames and is based on the equivalent viscous damping approach – which converts the energy dissipated by plastic deformation of the materials to equivalent viscous damping. However, this approach has been criticised because there are no physical principles relating inelastic deformation to viscous damping, especially for highly inelastic response [61-62]. As a consequence, poor control of the displacement demands may be observed.

The method starts by estimating the equivalent design displacement ($\Delta_d$), mass and height of an equivalent SDOF structure (as representative of the multi-storey structure to be designed) using a linear displacement profile and a given inter-storey drift limit. Then, the equivalent viscous damping of each part (the steel frame and the BRBs) is estimated using a ductility-damping ratio relationship ($\mu$-$\xi$). The total equivalent damping ratio of the system is estimated by a weighted average. A displacement spectrum is then generated using a conventional linear-elastic SDOF oscillator and that damping – which, together with the equivalent design displacement, $\Delta_d$, enables the estimation of the “effective” period of vibration, stiffness, and design base shear. Finally, the structure is proportioned, using capacity concepts.

From some examples provided by the authors, it was observed that the average inter-storey drift demands generated by simulated earthquake ground motions were higher than their design limits; this suggests that the lateral displacements are not well controlled using this method and this may lead to unconservative designs.
3.2.2 Vargas and Bruneau [60] method

This method allows the arbitrary selection of several parameters (e.g. ratio of the main frame stiffness to the total stiffness, ratio of the main frame ductility to the ductility of the BRBs, ratio of the total load capacity to the maximum ground force applied during the motion, etcetera) in order to estimate the design base shear on the structure and the proportions to the main frame and the BRBs. However, even when the authors provide a range of admissible values for those parameters, the actual parameters estimated at the end of the design process may have results significantly different to the target values. This limitation is recognised by the authors who recommended performing nonlinear analysis at the end of the design in order to ensure that the fuse concept is achieved. Furthermore, the range of admissible design parameters was estimated from a particular type of ground motion (i.e. West Coast of the USA and soil type B) - which implies that the parameters are only applicable to that specific region and the method may not be applicable to other regions of the World.

3.2.3 Teran-Gilmore and Virto-Cambray [1] method

This method is valid for low-rise structures not significantly affected by flexural deformations and higher mode effects. It considers that the vertical loads are carried by the main frame while the lateral loads produced by earthquakes are carried by the BRBs. Although useful, this approach limits the options available to designers – who may want to compare diverse contributions (or balances) of the main frame and the BRBs to the total lateral load capacity of the dual structure in order to find the most efficient or convenient design.

The method starts with the definition of the maximum inter-storey drift limits for two performance levels: immediate operation and life safety. Then, through the use of two displacement spectra (generated using conventional oscillators), the fundamental
period of vibration is established – which defines the total stiffness of the structure and hence, the size of the BRBs. It is noted that, since conventional oscillators are used to define the displacement spectra of design, this method (and others in the literature using conventional oscillators) may lead to biased designs because the main frame and the BRBs yield at different levels of displacement - which generates significant differences in the dynamic response demands. This is further studied in the following sections.

3.2.4 Summary

From the literature, it has been observed that three methods are similar to the method proposed in this Chapter because they consider the contribution of the main structure and the BRBs to the total lateral load capacity of the system. The Maley et al. [59] method has two main limitations: a) it is only valid for steel buildings; and b) it uses the equivalent damping approach – in which the energy dissipated by plastic deformation is converted to equivalent (velocity-dependent) viscous damping. The Vargas and Bruneau [60] method has the following limitations: a) to conducting the design, it uses design parameters that are arbitrarily selected – which leads to poor control of the design; and b) the range of admissible values for the design parameters is valid only for a specific type of ground motion and a specific soil type. Finally, the Teran-Gilmore and Virto-Cambray [1] method uses design spectra generated from conventional oscillators – which do not consider the fact that the main structure and the BRBs yield at different levels of displacement.

In the following sections, a method for designing structures equipped with BRBs is proposed in which the limitations of the Maley et al. [59] method, the Vargas and Bruneau [60], and the Teran-Gilmore and Virto-Cambray [1] method are removed.
3.3 BRBs and SDOF structures

3.3.1 The idea of designing frames with BRBs

For convenience, moment resisting frames (MRFs) are referred hereafter as conventional structures while MRFs equipped with BRBs are referred as dual structures or dual systems. When a conventional structure is subjected to a major earthquake, the structure may be damaged with a large deformation as shown in Figure 3-1a. If a BRB is installed into the structure, it is expected that the BRB would absorb a good amount of energy and be damaged while the main structure remains in its elastic deformation (Figure 3-1b). This requires a rational design of the BRB or of the frame and the BRB. This subsection provides the basic idea for such a design.

While the conventional structure can be modelled using a conventional single degree-of-freedom (SDOF) oscillator with mass \( m \), damping coefficient \( c_1 \) and stiffness \( k_1 \), the dual structure can be modelled as a dual SDOF oscillator consisting of the conventional SDOF oscillator and a secondary part with damping coefficient \( c_2 \) and stiffness \( k_2 \), as shown in Figure 3-2a. Figure 3-2b illustrates the load and deformation capacities of the dual oscillator in which the primary part starts to yield at \( d_{y1} \) when subjected to a force \( V_{y1} \) and the secondary part yields at \( d_{y2} \) when experiences a force of \( V_{y2} \). The combined capacity of the dual oscillator is illustrated in a dashed line in Figure 3-2b, which is the summation of the two lower curves with the maximum capacity of \( V_{yT} \). Then, the question is how to select properly the properties of the primary part (\( d_{y1} \) and \( k_1 \), and hence \( V_{y1} \)) and the properties of the secondary part (\( d_{y2} \) and \( k_2 \), and hence \( V_{y2} \)) that allow controlling satisfactorily the displacement demands induced by earthquake actions while ensuring that the BRB yields first.
For this purpose, it is proposed that the moment resisting frame is initially designed under the condition of gravity loads. This provides an initial value of $d_{y1}$ and the lower limit of $k_1$. Then, one of the next approaches may be followed to control the displacement demands: 1) by fixing the initial values of $d_{y1}$ and $k_1$, find the values of $d_{y2}$ and $k_2$; or 2) for a desired proportion of the BRB to the load capacity, find the properties of both the primary and secondary parts. Both approaches are addressed in the following subsections.

![Figure 3-1. Conventional vs. Dual structures](image)

![Figure 3-2. A dual SDOF oscillator and its behaviour curves](image)
3.3.1.1 Designing BRBs for known properties of the primary structure

First, the yielding displacement of the BRB \((d_{y2})\) can be calculated using the following equation [1]:

\[
d_{y2} = \frac{1}{f_y} \frac{f_k}{E} \frac{h}{\cos \alpha \sin \alpha}
\]  \(3-1\)

where \(f_y\) and \(E\) are the yielding stress and modulus of elasticity of the material composing the core of the BRB; \(f_k\) is a factor that takes into account the geometry of the BRB, can be selected from catalogues of the manufacturers, and can take values between 1.5 and 2 or even higher. The other parameters \((h\) and \(\alpha\)) are defined in Figure 3-1 and are considered constant. If for selected values of \(f_k\) and the material properties \((f_y\) and \(E\)), \(d_{y2}\) were higher than \(d_{y1}\), an increment of \(f_k\) may be very effective to ensure that the secondary part yield first (i.e. to achieve the fuse concept).

By analysing equation (3-1), it can be observed that once \(f_k\) and the material properties are selected, the yielding displacement of the BRB, \(d_{y2}\), is constant and independent of \(k_2\) and \(V_{y2}\). This characteristic is convenient for designing purposes because any \(k_2\) and \(V_{y2}\) can be provided without affecting \(d_{y2}\).

Now, for the primary and secondary parts, let the load capacity factors \(b_1\) and \(b_2\) be, as defined by Figure 3-2b:

\[
b_1 = \frac{V_{y1}}{V_{yT}} \quad \text{and} \quad b_2 = \frac{V_{y2}}{V_{yT}}
\]  \(3-2\)

and the capacity ratio

\[
\frac{b_2}{b_1} = \frac{V_{y2}}{V_{y1}} = \frac{k_2 d_{y2}}{k_1 d_{y1}} \quad \text{or} \quad \frac{d_{y1}}{d_{y2}} = \frac{b_1 k_2}{b_2 k_1}
\]  \(3-3\)
where $b_1 + b_2 = 1$. Since $d_{y1}$ and $k_1$ are considered being known and $d_{y2}$ is determined using equation (3-1), the stiffness of the BRB, $k_2$ is calculated with equation (3-3) as long as the relative contribution of the BRB ($b_2$) is known. Since $b_2$ is unknown, an iterative process is required to find its value that allows controlling the lateral displacement demands of the dual oscillator.

The iterative process starts with the estimation of the ductility factors of the primary and secondary parts, when the dual oscillator experiences the maximum displacement $d_{max}$. They are:

$$\mu_1 = \frac{d_{max}}{d_{y1}} \quad \text{and} \quad \mu_2 = \frac{d_{max}}{d_{y2}}$$

(3-4)

then, using equation (3-3), the ratio of the ductility factors is

$$\frac{\mu_2}{\mu_1} = \frac{d_{y1}}{d_{y2}} = \frac{b_1 k_2}{b_2 k_1}$$

(3-5)

Equation 3-5 can be re-written as:

$$a = \frac{k_2}{k_1} = \frac{\mu_2 b_2}{\mu_1 b_1}$$

(3-6)

Equation 3-6 provides the formula to determine the stiffness ratio of the secondary part to the primary part, $a$. This equation is simple for a SDOF oscillator, but it presents the idea that the stiffness of the secondary part can be estimated from the stiffness of the primary part once the value of $a$ is known. This is possible if a value of $b_2$ is selected and the ductility factors ($\mu_1$ and $\mu_2$) are calculated based in a displacement threshold ($d_{max}$) and the yielding displacements of the primary and secondary structures ($d_{y1}$ and $d_{y2}$).
Chapter 3. A Method for Preliminary Design and Assessment of Structures with BRBs

d_{y_2}). Note that if \( \mu_2 > \mu_1 \), then the fuse concept is guaranteed because the BRB yield first as illustrated in Figure 3-2b.

For convenience, the stiffness ratio \( (a) \) is also used to relate the periods of the primary part \( (T_1) \) and the secondary part \( (T_2) \), as follows:

\[
T_2^2 = \frac{k_1}{k_2} T_1^2 = \frac{T_1^2}{a}
\] (3-7)

Therefore, once \( a \) and \( T_1 \) are known, the period \( T_2 \) can be determined. In consequence, the total period of the dual oscillator is calculated as:

\[
\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}
\] (3-8)

Then, the period \( T \) is used as input to estimate the dynamic response of dual oscillators using the dynamic equation of motion, which is as follows:

\[
\ddot{u}(t) + 2(\xi_1 + \xi_2) \frac{2\pi}{T} \dot{u}(t) + \left[ \frac{f_{s1}(u, \dot{u})}{m} + \frac{f_{s2}(u, \dot{u})}{m} \right] = -\ddot{u}_g(t)
\] (3-9)

where \( u, \dot{u}, \) and \( \ddot{u} \) are the displacement, velocity and acceleration; \( \ddot{u}_g \) is the acceleration of the ground, \( \xi_1 \) and \( \xi_2 \) are the damping ratios of the primary and secondary parts of the dual oscillator; \( T \) is the period estimated with equation (3-8); \( m \) is the modal mass; \( f_{s1}(u, \dot{u}) \) and \( f_{s2}(u, \dot{u}) \) are the restoring forces which depend on the history of displacements and the sign of the velocity, and are bounded by the load capacity of each part of the dual oscillator, which are determined using the next equations (3-10) and (3-11). In this part, it should be noted that the restoring forces do not necessarily have to follow the bilinear hysteretic model; in fact, a bilinear model is only recommended to model the behaviour of the secondary part (i.e. the BRB), while a
more suitable model is recommended to model the primary part, e.g. Takeda model for concrete frames, bilinear model for steel frames, and flag-shape model for structures with self centring capacity. Furthermore, \( P-\Delta \) effects can be taken into account in the force-displacement relationship of the primary part. This will be further described later. On the other hand, note that the restoring forces and capacity loads are divided by the modal mass; therefore, as in conventional oscillators, \( m \) does not need to be known to solve equation (3-9). Also note that for the conventional oscillator, \( \xi_2 \) and \( f_{s_2}(u, \dot{u}) \) are removed from equation (3-9).

\[
V_{d1} \quad \frac{V_{y1}}{m} = \left( \frac{2\pi}{T_1} \right)^2 d_{y1} \quad \text{(3-10)}
\]

\[
V_{d2} \quad \frac{V_{y2}}{m} = \left( \frac{V_{y1}}{m} \right) b_2 \quad \text{(3-11)}
\]

As seen in the equations of this subsection, the response of dual oscillators is highly affected by \( b_1 \) and \( b_2 \) (i.e. the relative contribution of the primary and secondary parts to the load capacity of the dual oscillator). However, \( b_1 = 1 - b_2 \). Therefore, by trying different values of \( b_2 \), the maximum displacement demands \( (u_{\text{max}}) \) estimated with equation (3-9) can effectively be controlled. Once the displacement demands are deemed satisfactory, the value of \( k_2 \) and other parameters (such as \( V_{y2}, T_2 \) and \( T \)) are readily available from equations (3-3) to (3-8).

On the other hand, note from equations (3-6) to (3-8) that not only \( b_2 \) but also other parameters (such as the ductility factor of the BRB, \( \mu_2 \)) affect the response of a dual oscillator. Therefore, \( u_{\text{max}} \) can also be controlled by providing different values of \( \mu_2 \); which, in turn, can be varied by selecting different values of \( f_k \) and properties of the materials (if commercially available), and using equations (3-1) and (3-4). Thus, there is
a significant range of options to control $u_{\text{max}}$. However, special attention should be paid to avoid very large values of $\mu_2$ that could result in unexpected failures. From experimental tests, a condition of $\mu_2 < 10$ might be deemed appropriate as summarised by Fahnestock et al. [23].

A summary of the parameters that affect the response of a dual oscillator is shown in Figure 3-3. The input parameters that can be selected to control the response are seen in Row 1, while Rows 2 and 3 show the parameters that are considered fixed in this subsection. Note that by solving equation (3-9) not only $u_{\text{max}}$ but also other response parameters (useful for preliminary assessment of the performance of the structure) are estimated. This benefit is addressed by an example in Section 3.6.

![Figure 3-3. Parameters affecting the response of dual oscillators and design for BRBs](image-url)
3.3.1.2 Designing BRBs for a desired contribution to the load capacity of the system

It can be noted from Figure 3-3 that, in order to control the maximum displacement demands ($u_{\text{max}}$), when a fixed contribution of the BRB to the load capacity ($b_2$) is required, different values of $f_k$ and (if available) $f_y$ and $E$ may be tried. However, if these parameters do not allow a satisfactory control of the response, the parameters of Row 2 ($d_{y1}$ and $T_1$) can also be modified by providing more capacity to the elements of the primary structure. An iterative process, similar to that in the previous subsection, may be required up to find a satisfactory control of $u_{\text{max}}$. Furthermore, different design scenarios can be obtained by using different combinations of the parameters of Rows 1 and 2 in Figure 3-3. Comparison between scenarios might help designers to achieve efficiency, e.g. designs with similar displacement demands and less construction cost, or designs with similar construction cost but with better behaviour under the same seismic demand.

3.3.2 Conventional vs. dual SDOF oscillators

An extensive parametric study was carried out on a conventional SDOF and a dual SDOF oscillators subjected to accelerograms of ground motions (recorded in the lakebed zone of Mexico City) using equation (3-9).

Two comparative case studies are illustrated here to show how conventional and dual oscillators behave differently; and how the proposed design method can effectively control the response to maintain the primary structure within its linear-elastic range while the BRB yields. In the first case, a BRB is designed and added to an oscillator, which changes the natural period of vibration originally equal to that of the conventional oscillator. In the second case, a BRB and an oscillator are designed as a whole to keep the same period as that of the conventional oscillator. For simplicity, only representative results are presented. In the two cases, the dual and conventional
oscillators are subjected to the SCT-EW record of the 1985 Michoacán Earthquake. Elastic-perfectly plastic behaviour is considered as defined in Figure 3-2b.

Case 1: For the conventional oscillator, $T = 0.5$ s, $\zeta = 0.05$, $V_y/m = 1.14$ N/kg and $\mu = 7$. For the dual oscillator, $T_1 = 0.5$ s, $\zeta_1 + \zeta_2 = 0.05$, $V_y/m = 1.14$ N/kg, which are the same as those of the conventional oscillator. Also, the target ductility factors are chosen as $\mu_1 = 1.0$ and $\mu_2 = 7.0$. After a few iterations, it is found that $b_2 = 0.31$ reaches the given ductility factors. Thus, $b_1 = 1 - b_2 = 0.69$. Based on equation (3-6), $a = 3.14$. Therefore, using equations (3-7) and (3-8) gives $T_2 = T_1/\sqrt{a} = 0.5/1.77 = 0.28$ s and $T = 0.24$ s.

Figure 3-4 compares the displacement response for the conventional and the dual oscillators. The maximum displacement demands for the two oscillators are 50.6 mm and 7.22 mm, respectively. It can be seen that both the maximum and permanent displacements of the dual oscillator are significantly smaller. Especially, the permanent displacement of the dual oscillator is negligible. It should also be pointed out that the dominant period of vibration of the ground motion is about 2 seconds. Therefore, the dual oscillator is additionally benefited from the fact that its period of vibration is farther away from the resonance zone than that of the conventional one in this case.

![Figure 3-4. Displacement demands of conventional and dual oscillators](image-url)
Case 2: For the conventional oscillator, the values of the parameters are the same as those in Case 1. However, the dual oscillator is redesigned to have the same period of $T=0.5$ s as the conventional one. This removes the effect of a smaller natural period as showed in Case 1. The periods of the primary and secondary parts are then $T_1=0.87$ s and $T_2=0.61$ s. The other design parameters are $b_1=0.8$, $b_2=0.2$, $a=2$, $\mu_1=0.87$ and $\mu_2=7$.

Figure 3-5 compares the displacement and force demands of the conventional and dual SDOF oscillators. It can be observed from the figure that:

- Even when they have the same period of vibration, the maximum displacement demand of the conventional oscillator is 53% larger than that of the dual oscillator.
- The conventional oscillator experiences a larger residual displacement while the dual oscillator almost returns back to the original position.
- The conventional oscillator requires 49% less force capacity than the dual oscillator to reach both $\mu_1=7$ and $\mu_2=7$, respectively.

Figure 3-5. Responses in the time domain of a conventional and a dual SDOF oscillator
The results of the comparative study indicate that: a) conventional and dual oscillators behave differently - therefore, the use of conventional oscillators to generate input design spectra may lead to biased designs; and b) through an appropriate design of the stiffness ratio \( (a) \) and resistance distributions \( (b_1 \text{ and } b_2) \) in a dual oscillator, the response can effectively be controlled to maintain the primary part elastic while the secondary part can experience large inelastic deformation when subjected to an earthquake ground motion.

### 3.4 BRBs and MDOF structures

#### 3.4.1 Definition

Similar to a dual SDOF oscillator, a MDOF structure equipped with BRBs can be modelled as a dual structure when the response of the structure is dominated by its fundamental mode - which may be the case of low rise buildings. Figure 3-6a illustrates a rigid frame structure with BRBs installed, which can be decomposed into two independent substructures: a moment resisting frame without any braces (Figure 3-6b) and a pin-connected braced frame (Figure 3-6c), in which the lateral stiffness is completely contributed by the brace members. The two decomposed substructures can be represented by the dual oscillator shown in Figure 3-6d.

**Figure 3-6.** An MDOF structure equipped with BRBs represented by a dual SDOF oscillator
The representation of a structure by two independent substructures is based on the superposition principle; in which, the stiffness matrix \((K)\) of the structure in Figure 3-6a can be estimated as the summation of the stiffness matrix of the moment resisting frame \((K_{MRF})\) in Figure 3-6b and the stiffness matrix of the braced pinned frame \((K_{BS})\) in Figure 3-6c, i.e.:

\[
K = K_{MRF} + K_{BS}
\]  

(3-12)

The fundamental periods of the system, \(T\), the moment resisting frame \(T_1\), and the braced system \(T_2\), have the following relation:

\[
\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}
\]  

(3-13)

It can be noted that the dual oscillator converted from the MDOF structure is the same as that of the previous section. However, a strategy for performance-based seismic design (PBSD) of low-rise buildings needs to be developed for designing the dual oscillator and the actual structure. Therefore, a procedure is proposed in the next subsection.

### 3.4.2 Proposed procedure for preliminary design of low-rise buildings

A procedure for preliminary PBSD of low-rise buildings is proposed and summarised in a flowchart in Figure 3-7. The key steps are explained as follows:

*Step 1*: Select the objectives of design. For each earthquake intensity level, the performance levels should be established in terms of maximum inter-storey drift (e.g. drift limit for **Fully Operability**, \(\theta_{FO}\); drift limit for **Operability**, \(\theta_{Op}\); drift limit for **Life Safety**, \(\theta_{LS}\); and drift limit for **Collapse Prevention**, \(\theta_{CP}\).*
Step 2: Design the primary structure (moment resisting frame without braces) under gravity loads. Thus, the fundamental period of vibration \( T_1 \) of the frame can be calculated. The yielding displacement \( d_{y1} \) of the frame at the top floor may be obtained from a pushover analysis. If \( P-\Delta \) effects are not negligible (as defined in FEMA-356 [63]), they have to be taken into account in the pushover analysis. \( T_1 \) and \( d_{y1} \) are to be used later in subsequent steps.

Step 3: Determine the displacement threshold. For each performance level, the displacement at the \( i \)th floor, \( d_i \), is estimated as the product of the maximum inter-storey drift limit and the height of that floor. Thus, the displacement threshold, \( d_{max} \), for an equivalent SDOF structure is estimated as follows ([64]):

\[
d_{max} = \frac{\sum_{i=1}^{N} m_i d_i^2}{\sum_{i=1}^{N} m_i d_i}
\]  

(3-14)

where \( N \) is the number of storeys; \( m_i \) is the mass in the \( i \)th storey. Equation (3-14) converts the displacement thresholds for all storeys into a single value for the dual SDOF oscillator. There should be one displacement threshold for each performance level to be designed. Note that an intensity-independent linear displacement profile has been assumed, which may be reasonably acceptable for regular, low-rise MRFs that present beam-sway mechanism [64]. For cases where the expected displacement profile may be significantly different, the floor displacements \( (d_i) \) shall be estimated accordingly. The displacement profile obtained in step 2 by pushover analysis may provide a good approximation.

Step 4: Decide the requirement for BRBs. For this purpose, the yielding capacity of the primary substructure, divided by the modal mass, is calculated from:
where $d_{\text{max}}$ and $d_{N}$ are the displacement threshold and displacement at the top floor in equation (3-14) for the maximum performance level.

A conventional SDOF oscillator with the period $T_1$ and yielding capacity of the primary structure ($V_{y1}/m$) is subjected to possible ground motions. If, from the pushover analysis of step 2, it is observed that the post-yielding stiffness ratio ($r$) is significantly different from zero, an approximate $r$ must be used here. Then, the maximum displacement demands and thresholds are compared, which determines whether BRBs are needed. If, for example, the mean plus one standard deviation of the demands is larger than the thresholds, BRBs need to be designed according to the subsequent steps; otherwise, the process is finished. It is worth to mention that the mean plus one standard
deviation demand is selected arbitrarily to illustrate the application of the methodology; however, any other demand level could be selected.

Recognising that inter-storey drifts are not uniform through the height of the building, the displacements demands should be amplified by a factor that takes into account this matter. Similar to [1] and based on observations by [65], a factor of 1.20 is recommended for structures equipped with BRBs and 1.50 is for structures without BRBs. These factors change to 1.50 and 2.0 respectively for ductility demands higher than 2.0.

**Step 5**: Estimate the threshold ductility demands of the primary ($\mu_{1_{max}}$) and secondary ($\mu_{2_{max}}$) substructures. This will be explained later in section 3.4.2.1.

**Step 6**: Select the relative participation of the secondary substructure ($0 \leq b_2 \leq 1$). Once the participation of the secondary substructure ($b_2$) is selected, the required periods of vibration of the dual system ($T_2$ and $T$) are determined using: the period of vibration of the primary substructure ($T_1$) determined in Step 1; the ductility demands estimated in the previous Step 5; and equations (3-6) to (3-8). Then, the force capacity of the secondary substructure, divided by the modal mass ($m$), is estimated from equation (3-11).

It is recognised that $T_2$ may be affected by the axial deformation in the columns of the bracing system, in particular for a high participation of the secondary substructure. This effect, estimation of the maximum error and corrections are to be discussed in section 3.4.2.3.

**Step 7**: Estimate the maximum displacement demands (e.g. $d_{50%/50y}$, $d_{20%/50y}$, $d_{10%/50y}$, $d_{2%/50y}$) of an equivalent dual SDOF oscillator using equation (3-9). In this
study, the displacement demands are estimated as the mean plus one standard deviation of the responses obtained from multiple actual or artificial hazard-compatible records. Similar to Step 4, the displacements demands should be amplified by the non-uniform inter-storey drift factor.

**Step 8**: A few iterations between Steps 6 and 7 may be required to achieve the most convenient balance between the relative participation of the protection substructure \(b_2\) and the displacement demands. Moreover, as introduced in section 3.3.1, different combinations of the input parameters of Figure 3-3 can be used to achieve efficient designs.

**Step 9**: Once the response is deemed satisfactory, determine the required cross-sectional areas of the BRBs. This is detailed in section 3.4.2.2.

Some of the steps are explained further in the following sub-sections.

### 3.4.2.1 Estimation of maximum ductility ratios in the dual structure

The maximum ductility on the primary substructure is defined as the ratio of the top-floor displacement for the maximum threshold (corresponding to the highest performance level, e.g. *Collapse Prevention*) to the yielding displacement estimated in Step 2, i.e.:

\[
\mu_{1\text{max}} = \frac{d_N}{d_{y1}}
\]  

(3-16)

The maximum ductility on the secondary substructure \(\mu_{2\text{max}}\) is estimated as the average of all inter-storey ductility ratios on the BRBs for the highest performance level, i.e.

\[
\mu_{2\text{max}} = \frac{1}{N} \sum_{i=1}^{N} \mu_i = \frac{1}{N} \sum_{i=1}^{N} \frac{\Delta_{\text{max}i}}{\Delta_{y2i}}
\]  

(3-17)
where $\mu_i$ is the inter-storey ductility ratio at the $i$th storey and

$$\Delta_{max} = \theta_{max} h_i$$  \hspace{1cm} (3-18)

$$\Delta_{2,i} = \frac{1}{f_y f_{ky}} \frac{h_i}{E \cos \alpha_i \sin \alpha_i}$$  \hspace{1cm} (3-19)

being $\theta_{max}$ the maximum allowed inter-storey drift for the highest performance level, e.g. *Collapse Prevention*.

Note that equation (3-19) is similar to equation (3-1); however, equation (3-19) takes into account the properties at each storey of the building. It is also of interest to note that the ductility at each storey cannot be supposed and assigned directly to the structure because it depends on, and shall be estimated from, the yielding displacement capacity of the BRBs.

### 3.4.2.2 Estimation of the required cross-sectional areas of the BRBs

The cross-sectional areas of the yielding zone of the BRBs are determined in two steps: 1) estimating the relative areas of all BRBs through a static analysis, and 2) calculating the absolute values through achieving the required period of vibration of the secondary substructure ($T_2$).

1. **Relative areas**: lateral loads proportional to the floor mass are applied to the braced pin-connected model (Figure 3-6c). Thus the axial loads ($N_j$) in the braces of the statically determinate frame can be easily determined.

   Then cross-sectional areas are provided to the braces proportionally to the axial loads with an initial area of 1.0 cm$^2$ assigned to the bracing members in the ground storey. For practical applications, some braces in the consecutive storeys may have the same areas.

2. **Absolute areas**: The response of the pin-connected substructure is considered being
contributed by the deformations of the beams, columns and braces. However, deformations in beams may be neglected due to diaphragm action of the floor system. Thus, the deformation of the structure may be described by two springs in series, which represent the lateral stiffnesses of the frame columns and braces, respectively (Figure 3-8). Now, let the column members become infinitively stiff. Then the response of the structure is contributed only by the BRBs. The corresponding fundamental natural period \( T_{2,\text{assu}} \) of the artificial frame with assumed areas of the BRBs \( A_{\text{assu}} \) can be determined.

As the stiffness of the artificial frame is controlled by the areas of the bracing members, the areas should be modified to reach the desired period \( T_2 \), using the following equation:

\[
A_{\text{BRB},i} = A_{\text{assu},i} \frac{K_2}{K_{2,\text{assu}}} = A_{\text{assu},i} \frac{T_{2,\text{assu}}^2}{T_2^2}
\]  

(3-20)

Now, the updated areas of BRBs will make the secondary structure have the required period of vibration, \( T_2 \). For fabrication, the above areas of the yielding zone of the BRBs and the initially-selected effective stiffness factors \( (f_{ki}) \) are provided to the manufacturer of the devices to produce the actual geometry and shapes.

![Figure 3-8. Deformation of pinned structures as contributed by columns and braces.](image-url)
3.4.2.3 Correction for axial deformation in columns

In the previous subsection, the effect of axial forces and deformation in the columns of the secondary structure is not considered. Considering that the stiffnesses or flexibilities of the columns and braces of the structure can be represented by two springs in series (Figure 3-8), the flexibility of one of the two can be evaluated by treating the other as infinitively stiff.

In other words, the total flexibility of the secondary substructure can be represented by:

\[ T_2^2 = T_{col}^2 + T_{braces}^2 \]  \hspace{1cm} (3-21)

Now, making the bracing members infinitely stiff and calculating the natural period of the revised secondary substructure \( T_{col} \), the flexibility of the columns is reflected.

Then, the following ratio of the periods can be calculated:

\[ \text{ratio} = \frac{T_{col}^2}{T_2^2} \]  \hspace{1cm} (3-22)

If the ratio is less than a given tolerance (say 5%), this test is finished and the procedure may continue. Otherwise,

1. The ductility demand of the secondary part (\( \mu_{2\text{max}} \)) is updated by multiplying it by the factor:

\[ \frac{T_{braces}^2}{T_{col}^2 + T_{braces}^2} \]  \hspace{1cm} (3-23)

which is less than unity when the flexibility of the columns is not negligible. \( T_{col}^2 \) is the fundamental period of the pin-connected substructure when the braces are
treated as infinitively stiff and represents the flexibility of the columns. 
\[ T_{braces}^2 = T_2^2 - T_{cols}^2 \] is the fundamental period of the pin-connected substructure when the columns are treated as infinitively stiff and represents the flexibility of the braces alone. Note that the contribution of beams has not been considered because they are treated as infinitively stiff due to the rigid diaphragm action of the floor system.

It is highlighted that, for safety purposes, the flexibility of the braces should be higher than that of the columns, i.e. \( T_{braces}^2 > T_{cols}^2 \). If this condition is not satisfied, redesigning of the columns is advisable considering higher cross-sectional areas. Teran-Gilmore and Coeto [66] recommend a maximum ratio \( T_{cols}^2 / T_{braces}^2 \) of 1/3.

2. The period of vibration of the bracing system (\( T_2 \)) is estimated using the updated ductility demand of the secondary part (\( \mu_{2max} \)) and equations (3-6) and (3-7). A few iterations are required between the previous step and this one up to \( T_2 \) stabilises.

3. Then, the fundamental period of vibration of the whole dual structure is found using equation (3-8). The procedure may continue normally.

3.5 Example of design

In order to verify the applicability of the proposed method, a five-storey braced building is designed with buckling-restrained braces. It is assumed that the structure is located in the lakebed zone of Mexico City and will locate a hospital.

3.5.1 Description and requirements of design

The 2D steel framed building is shown in Figure 3-9 and is designed according the following requirements: for each objective of design the maximum inter-storey drifts
are shown in Table 3-1; and the primary frame must remain elastic for the *Life Safety* performance level. The seismic masses on each floor are shown in Figure 3-9. The steel of the commercially available profiles is ASTM A992 \( (f_y = 350 \text{ MPa}) \) and the steel of the braces is ASTM A36 \( (f_y = 250 \text{ MPa}) \).

![Figure 3-9. Five-storey framed building equipped with BRBs](image)

**Table 3-1.** Maximum inter-storey drifts for each objective of design of the study example

<table>
<thead>
<tr>
<th>Objective of Design</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>Fully Operability</td>
<td>Operability</td>
<td>Life Safety</td>
<td>Collapse Prevention</td>
</tr>
<tr>
<td><strong>pga</strong></td>
<td>0.05g</td>
<td>0.10g</td>
<td>0.20g</td>
<td>0.30g</td>
</tr>
<tr>
<td><strong>Max. drift</strong></td>
<td>0.0025</td>
<td>0.005</td>
<td>0.010</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Since the proposed method requires seismic records as input, 30 ground motions, selected from the Mexican Database of Strong Motions [67] and recorded in the same zone where the building is to be located, are used here. They are shown in *Appendix B* and are *pga*-scaled in order to account for different seismic hazard levels. For a seismic
hazard with probability of 50% of being exceeded within 50 years, it is assumed that $pga = 0.05g$; for 20% in 50 years $pga = 0.10g$; for 10% in 50 years $pga = 0.20g$; and for 2% in 50 years $pga = 0.30g$. Although the $pga$ may not be considered the best parameter to indicate the seismic intensity levels, this will not affect the objective of the study.

### 3.5.2 Design with the proposed procedure

The steps proposed in section 3.4.2 are followed and summarised as follows:

**Step 1. Selection of the objectives of design.** They are already defined in Table 3-1; in which the maximum inter-storey drifts are: $\theta_{FO} = 0.0025$; $\theta_{Op} = 0.005$; $\theta_{LS} = 0.01$; and $\theta_{CP} = 0.02$, respectively.

**Step 2. Designing the primary structure under gravity loads.** The structure is initially dimensioned for gravity loads. The resultant steel profiles are: HSS16x16x1/2 in columns of storeys 1 and 2, HSS16x16x3/8 in columns of storeys 3 to 5, and W14x61 in the beams of all storeys. Then, the fundamental period of vibration of the primary frame is determined ($T_1 = 1.74$ s) by an eigenvalue analysis. The contribution of the slab is considered as recommended by [68]. A pushover analysis with mass-distributed loads is conducted using Opensees [69] to estimate the displacement at the top floor, at which the structure yields ($d_{y1} = 0.147$ m). The details of the numerical model are described later in section 3.5.3.1.

**Step 3. Calculation of displacement thresholds.** The displacement thresholds, according to equation 3-14, are $d_{FO} = 0.029$ m; $d_{Op} = 0.058$ m; $d_{LS} = 0.115$ m; and $d_{CP} = 0.230$ m.

**Step 4. Determination of requirement for BRBs.** In order to determine whether the structure requires BRBs, the yielding capacity of the primary substructure is determined...
and a conventional SDOF oscillator, with period of $T_1=1.74$ s and $V_{y1}/m=1.38$ N/kg, is subjected to the 30 records (previously described) scaled to the $pga$ values of Table 3-1. Typical damping ratios were considered, namely: $\xi=3\%$ for $pga=0.05g$ and $5\%$ for the others. Figure 3-10a shows that the displacement demands (represented by dots) are larger than the displacement thresholds (represented by dash-dot lines). Therefore, it is concluded that the structure requires BRBs.

**Step 5. Estimation of threshold ductility factors.** BRBs are introduced and the thresholds of the ductility ratios, for each objective of design, are estimated for the primary and secondary substructures according to section 3.4.2.1. An effective stiffness factor $f_k=1.50$ is chosen for all the BRBs. The ductility ratios, corresponding to the *Collapse Prevention* performance level, are: $\mu_{1\text{max}}=2.18$ and $\mu_{2\text{max}}=7.48$. Additionally, the ductility factors for all the performance levels of Table 3-1 are: $\mu_{1\text{FO}} = 0.27$; $\mu_{1\text{Op}} = 0.54$; $\mu_{1\text{LS}} = 1.09$; $\mu_{1\text{CP}} = 2.18$; $\mu_{2\text{FO}} = 0.94$; $\mu_{2\text{Op}} = 1.87$; $\mu_{2\text{LS}} = 3.74$; $\mu_{2\text{CP}} = 7.48$.

According to these ductility ratios, it is observed that the main frame would remain elastic for the first two objectives of design and may present nonlinear behaviour for the third and fourth one. On the other hand, the protection substructure may start dissipating energy from the second objective. It is important to recognise that these maximum ductility ratios are thresholds (i.e. maximum allowed), the actual ductility demands will be determined later in subsequent steps.

**Step 6. Selecting a relative participation of the secondary substructure.** $b_2 = 30\%$ is selected initially and the periods $T_2$ and $T$ are determined. They are $T_2=1.43$ s and $T=1.10$ s. However, a correction is conducted due to the effect of deformation of the columns, as described previously in section 3.4.2.3. Thus, the contribution of the columns and braces to the period of vibration are $T_{\text{col}}=0.48$ s and $T_{\text{braces}}=1.43$ s. As a
result $\mu_{2\max}=6.71$, $a=1.33$, $T_2=1.51$ s and $T=1.14$ s. Also, the strength capacity of the secondary substructure is calculated to be $V_{y}/m=0.59$ N/kg.

The maximum displacements on a dual SDOF oscillator, with the properties of the primary and secondary substructures determined in previous steps, are estimated for each record and seismic intensity level using equation (3-9). Higher values of damping ratio were considered here for dual oscillators because experimental evidence suggests that BRBs increase the damping ratio significantly due to friction produced by the interaction between the core and case of the BRBs (as will be seen in Chapters 4 and 5). They were $\xi=5, 8, 10,$ and 12% for the seismic intensities of Table 3-1. The displacements are amplified by a non-uniform inter-storey drift factor of 1.20 for intensities of $pga=0.05g$ and $0.10g$; and 1.5 for the others. Mean plus one standard deviation of the maximum displacements, for each seismic intensity level, are taken as displacement demands. They are larger than their corresponding thresholds.

Figure 3-10. Displacement demands

**Step 7. Estimation of the maximum displacement demands.** The maximum displacements on a dual SDOF oscillator, with the properties of the primary and secondary substructures determined in previous steps, are estimated for each record and seismic intensity level using equation (3-9). Higher values of damping ratio were considered here for dual oscillators because experimental evidence suggests that BRBs increase the damping ratio significantly due to friction produced by the interaction between the core and case of the BRBs (as will be seen in Chapters 4 and 5). They were $\xi=5, 8, 10,$ and 12% for the seismic intensities of Table 3-1. The displacements are amplified by a non-uniform inter-storey drift factor of 1.20 for intensities of $pga=0.05g$ and $0.10g$; and 1.5 for the others. Mean plus one standard deviation of the maximum displacements, for each seismic intensity level, are taken as displacement demands. They are larger than their corresponding thresholds.
Step 8. Determining if the design is satisfactory. Since $b_2 = 30\%$ (relative participation of the protection system) along with $T_1=1.74$ s, $d_{y1}=0.15$ m, $f_k=1.50$ and $f_y=250$ MPa are insufficient to control the displacement demands. Thus, the method allows selecting different combinations of these parameters to achieve a proper control. For simplicity, in this example $f_k$ and $f_y$ are considered fixed, therefore three options are available: a) for fixed properties of the primary structure ($T_1$ and $d_{y1}$), increase $b_2$ until find a satisfactory control of the response (as described in Section 3.3.1.1); b) for a fixed contribution of the protection substructure ($b_2$), increase the capacity of the primary structure until find a satisfactory control of the response (as described in Section 3.3.1.2); or c) a combination of them. The three options are developed for illustration purposes. The resultant properties are summarised in Table 3-2 while the profiles required for each design option are summarised in Table 3-3. It is observed that for option (a) a value of $b_2=60\%$ is required to control the displacement demands while maintaining the properties of the primary structure fixed, i.e. $T_1=1.74$ s, $d_{y1}=0.15$ m. This is illustrated in Figure 3-10b where the contribution of each part of the dual system is observed. On the other hand, for option (b) the value of $b_2=30\%$ is fixed and the capacity of the primary structure is increased to control the response. In Table 3-2 is observed that $V_{y1}/m$ increases from 1.38 N/kg to 2.62 N/kg; while $T_1$ and $d_{y1}$ reduce from 1.74 s to 1.15 s and from 0.15 m to 0.12 m, respectively. It is observed in Table 3-3 that the increase of the capacity of the primary structure results in an increase of the usage of structural steel from 35,635 kg to 57,283 kg. Finally, for option (c) a value of $b_2=40\%$ is selected and the capacity of the primary structure is found iteratively to control the displacement response. It is seen that $V_{y1}/m$, $T_1$ and the structural steel weight are between those of options (a) and (c).
Step 9. Determination of required cross-sectional areas of the BRBs. The core cross-sectional areas of each BRB in each storey are estimated according to Section 3.4.2.2. In the last column, Table 3-3 shows the cross-sectional areas of the BRBs for the first storey. Those of the other storeys are proportional to the following vector: (1.0, 0.76, 0.56, 0.36, 0.16)T. It is appreciated in the table that the largest areas correspond to the largest valued of \(b_2=60\%\) (option (a)) while the smallest areas correspond to the smallest value of \(b_2=30\%\) (option (b)).

### Table 3-2. Properties of the design options

<table>
<thead>
<tr>
<th>Option</th>
<th>(b_2)</th>
<th>(d_{y1})</th>
<th>(T_1)</th>
<th>(T_2)</th>
<th>(T_{cols})</th>
<th>(T_{brbs})</th>
<th>(T)</th>
<th>(V_{y1}/m)</th>
<th>(V_{y2}/m)</th>
<th>(V_{y3}/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[m]</td>
<td>[s]</td>
<td>[s]</td>
<td>[s]</td>
<td>[s]</td>
<td>[s]</td>
<td>[N/kg]</td>
<td>[N/kg]</td>
<td>[N/kg]</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>60%</td>
<td>0.15</td>
<td>1.74</td>
<td>0.90</td>
<td>0.48</td>
<td>0.76</td>
<td>0.80</td>
<td>1.38</td>
<td>2.07</td>
<td>3.45</td>
</tr>
<tr>
<td>b</td>
<td>30%</td>
<td>0.12</td>
<td>1.15</td>
<td>1.08</td>
<td>0.32</td>
<td>1.03</td>
<td>0.79</td>
<td>2.62</td>
<td>1.12</td>
<td>3.74</td>
</tr>
<tr>
<td>c</td>
<td>40%</td>
<td>0.13</td>
<td>1.29</td>
<td>0.97</td>
<td>0.35</td>
<td>0.90</td>
<td>0.77</td>
<td>2.22</td>
<td>1.48</td>
<td>3.70</td>
</tr>
</tbody>
</table>

### Table 3-3. Structural elements of the design options

<table>
<thead>
<tr>
<th>Option</th>
<th>(b_2)</th>
<th>Columns</th>
<th>Storey Beams Storey Steel</th>
<th>(A_{BRBs})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Location</td>
<td>Location</td>
<td>Weight, kg</td>
</tr>
<tr>
<td>a</td>
<td>60%</td>
<td>HSS16x16x1/2 1 &amp; 2</td>
<td>W14x61 1 to 5</td>
<td>35,635</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HSS16x16x3/8 3 to 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>30%</td>
<td>BOX500x25mm 1 to 3</td>
<td>W18x65 1 to 3</td>
<td>57,283</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BOX500x13mm 4 &amp; 5</td>
<td>W14x61 4 &amp; 5</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>40%</td>
<td>BOX500x19mm 1 &amp; 2</td>
<td>W16x67 1 &amp; 2</td>
<td>48,952</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BOX500x13mm 3 to 5</td>
<td>W14x61 3 to 5</td>
<td></td>
</tr>
</tbody>
</table>

By analysing the last two columns of Table 3-3, it can be observed that the larger the contribution of the BRBs to the load capacity of the structure, the smaller the steel
consumption and the larger the cross-sectional areas of the BRBs. Thus, the designer has several options to select what the most satisfactory option is. On the other hand, since equation (3-9) is solved during the application of the method for every ground motion record and intensity, additional information (useful for seismic assessment of the performance) is generated during the application of the method. This may also help the designer to select the most convenient option of design. This is addressed later in Section 3.6.

3.5.3 Validation of the design by nonlinear analyses

3.5.3.1 Static analysis

The structure designed in the last subsection was modelled with and without BRBs in OpenSees [69]. All the beam-column connections were modelled as rigid and the connections of the BRBs to the frame were modelled as pinned. The model was fully fixed to the base. The gravity loads were modelled by means of distributed loads in beams. The floors were considered as rigid diaphragms. The structural elements (columns, beams and BRBs) were modelled using distributed plasticity elements [70], i.e. elements with force-based formulation with five points of integration. These elements were used because they remove some limitations of other elements [71] and are time-efficient. Flexural and shear deformations of the elements were considered. The cross-sections were modelled with fibre elements. Although the stability coefficient, determined using the recommendations of FEMA-356 [63], suggested that $P-\Delta$ effects could be neglected, they were considered using the concept of the corotational formulation [71-72]. A concrete slab with effective thickness of 90 mm and width of 2 m was included. Regarding the materials, expected rather than nominal
resistances were modelled. The concrete of the slab possessed an expected resistance of $f'_c=31.25$ MPa and a modulus of elasticity of $E_c=21,925$ MPa. The steel was modelled using the Steel02 material (as defined in OpenSees [69]). This material corresponds to the Giuffre-Menegotto-Pinto model with strain-hardening. Similar to [72], the strain-hardening ratio was 0.3%. The other parameters controlling the transition from elastic to plastic response were $R_0=20$, $c_{R1}=0.925$, $c_{R2}=0.15$, $a_1=0$, $a_2=1$, $a_3=0$ and $a_4=1$. For the BRBs, a calibration of the parameters $a_1$ and $a_3$ was carried out to account for their behaviour observed in experiments [6]. They resulted $a_1=0.07$ and $a_3=0.05$. Comparison of the calibration is shown in Figure 3-11; where the experimental and numerical responses are very similar.

![Figure 3-11. Calibration of Giuffre-Menegotto-Pinto steel model to model BRBs](image)

**Figure 3-11.** Calibration of Giuffre-Menegotto-Pinto steel model to model BRBs

![Figure 3-12. Pushover analysis of the designed structure equipped with BRBs for $b_2=60\%$](image)

**Figure 3-12.** Pushover analysis of the designed structure equipped with BRBs for $b_2=60\%$
Then, the pushover analysis was conducted with constant gravity and incremental lateral loads proportional to the floor masses. The results are shown in Figure 3-12 for $b_2=60\%$ (i.e. design option (a) of the previous section), where the horizontal axis indicates the displacement at the top floor and the vertical axis represents the shear reaction at the base, normalised by the total weight ($W_T$) of the structure. Comparing Figures 3-12 and 3-10b shows that both are similar, i.e. the target dual SDOF oscillator and the dual MDOF structure display good agreement. Notwithstanding, some differences are observed in Figure 3-12: 1) the transition from elastic to plastic response is smooth; and 2) the post-yielding stiffness ratio is different from zero. These differences may be however considered negligible for preliminary design purposes.

### 3.5.3.2 Dynamic analysis

The model described in the previous subchapter was subjected to the same seismic records used during the design. For consistency, they were scaled to the same intensities of design (or $pga$) of Table 3-1. In total, the model was subjected to 120 analyses, in which damping ratios of $\xi=5\%, 8\%, 10\%$ and $12\%$ were considered for each seismic intensity analysed, respectively. It should be noted that, to be consistent with the design, these damping ratios are the same values used in step 7 to estimate the response of an equivalent dual SDOF oscillator. Also note that damping ratios higher than $10\%$ were considered for the larger seismic intensities because large inelastic deformations were observed during the design stage and here. These values of damping ratio are in agreement with experimental evidence, as it will be observed in the next two chapters - where experiments show that BRBs increase the damping ratio and it is intensity-dependent. As recommended by [73], viscous damping was assigned proportional to tangent stiffness in columns, beams, and braces in order to avoid artificial damping forces generated during nonlinear analyses. P-$\Delta$ effects were considered as described in
the previous section. The Newmark method of integration was used assuming constant average acceleration.

The results showed that linear-elastic response in both the frame and the BRBs was observed for intensities of $pga=0.05g$ and $0.10g$; linear-elastic response in the primary frame and nonlinear response in the BRBs was observed for $pga=0.20g$; and nonlinear response in both the frame and the BRBs was observed for $pga=0.30g$.

Figure 3-13 shows the maximum drift demands at each storey of the model at the four intensities considered. The mean and mean plus and minus one standard deviation of the demands are also shown with dark lines. It is observed that: 1) the heightwise distribution of the inter-storey drifts for linear-elastic response ($pga=0.05g$ and $0.10g$) was different to that of nonlinear response ($pga=0.20g$ and $0.30g$); and 2) as the level of the nonlinearity increased, the mean and dispersion of the drifts increased more significantly in the lower floors. These observations justify the values of the non-uniform inter-storey drift factors suggested in Step 4 of the proposed procedure (see section 3.4.2).

From Figure 3-13 is considered that the design is acceptable because the mean plus one standard deviation demands were smaller than the corresponding thresholds. Therefore, the dual SDOF oscillator represents well the behaviour of the MDOF structure with BRBs when subjected to dynamic loads.
Chapter 3. A Method for Preliminary Design and Assessment of Structures with BRBs

3.6 Preliminary assessment of the performance

One attractive characteristic of the proposed method is the additional information generated within its application; which may be useful to conduct preliminary assessments of the probabilistic performance of the building. Since the nonlinear dynamic equation of motion for the dual SDOF oscillator, i.e. equation (3-9), is solved for each ground motion and intensity considered, information such as peak and residual displacements, floor velocities and accelerations, is available and can be transformed to approximate the response of the multi-storey building to conduct the assessment without the necessity of generating a detailed and time-consuming model. Besides, the maximum and cumulative ductility demands are also available so that remaining useful life of BRBs can be determined by comparing with their expected capacity; which may be estimated as suggested by Takeuchi at al. [74]. The approximation of the resultant assessment will depend on how much the dual SDOF model represents the behaviour of the corresponding MDOF structure fitted with BRBs. It is recognised that this
assessment will be less accurate than that developed with a detailed finite element model however more appropriate than a simplified analysis (as defined in [31]).

In order to conduct a probabilistic assessment of the performance of the example structure, the PEER framework described previously in Section 2.3 is used here. An intensity-based assessment of the performance for $b_2=60\%$ (also referred as option (a)), is conducted. Other types of assessments, such as scenario- or time-based can also be developed following the recommendations in [31].

As introduced in Section 2.3, the assessment of the performance consists of four analyses, which are:

3.6.1 Seismic hazard analysis

It is normally conducted by using probabilistic seismic hazard analysis (e.g. see [34]). However, in the pursuit of simplicity, in this case study example the seismic hazard is considered given and is represented by the values of peak ground acceleration provided previously in Table 3-1.

3.6.2 Dynamic response analysis

It is normally conducted using incremental dynamic analysis (IDA) [35]. However, it is assumed in the proposed method that a dual SDOF oscillator represents rationally well the behaviour of a MDOF structure equipped with BRBs (as validated in Section 3.5.3). Therefore, the information obtained when equation (3-9) is solved is used for assessment purposes.

For preliminary assessment of the case study example, the statistics of the response, for the seismic intensities of Table 3-1, have already been determined. They are shown in Table 3-4 for the four $pga$ values. Mean demands and dispersions are shown for the response parameters useful in performance assessment. These mean
demands are converted into vectors of mean demands at each storey of the building as follows:

1. Estimation of vectors of inter-storey drift demands. Here, the peak displacements demands in Table 3-4 are multiplied by the ratio \( \frac{d_N}{d_{\text{max}}} \) (where \( d_N \) and \( d_{\text{max}} \) were defined in equation (3-14)). For the case study \( \frac{d_N}{d_{\text{max}}} = 1.39 \). Then, using these displacements, the inter-storey drift profile is selected from the pushover curve of the model fitted with BRBs (conducted previously in section 3.5.3.1).

2. Estimation of the residual drift demands. Here, the residual displacement ratios in Table 3-4, defined as the ratio of the residual to peak displacement demands, are multiplied by the drift demands estimated in the previous step. The maximum value, among all the storeys, is selected.

3. Estimation of vectors of floor velocities and accelerations. Here, the absolute velocities and accelerations of Table 3-4 are multiplied by the ratio \( \frac{d_N}{d_{\text{max}}} = 1.39 \) to obtain the values at the top floor. Then, these are linearly distributed to find the peak-ground velocity and peak-ground acceleration.

**Table 3-4.** Response demands on a dual SDOF oscillator from IDA: design option (a) \((b_2=60\%)\)

<table>
<thead>
<tr>
<th></th>
<th>( pga )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05g</td>
</tr>
<tr>
<td>Peak displacements, m</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>dispersion</td>
</tr>
<tr>
<td>Residual displacements / peak displacements</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>dispersion</td>
</tr>
<tr>
<td>Absolute velocity, m/s</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>dispersion</td>
</tr>
<tr>
<td>Absolute acceleration, g</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>dispersion</td>
</tr>
</tbody>
</table>
On the other hand, the collapse fragility (i.e. the cumulative distribution function of the probability of collapse given an intensity measure) shall be also estimated because it has a significant impact in the results of the assessment. Therefore, equation (3-9) was solved for additional seismic intensities (or $pga$). The same 30 records used in Section 3.5 were also used here. Intensities between $pga=0.02$g and $0.6$g in increments of $0.01$g were simulated. A significant number of nonlinear dynamic analyses were conducted in less than two minutes in a standard personal computer. This shows the benefits of solving equation (3-9) for preliminary assessment instead of using detailed finite element models.

Figure 3-14a shows the displacement demands vs. the incremental seismic intensity measure or $pga$. The mean and dispersion are also shown in the figure by solid and dashed dark lines. It is observed in Figure 3-14a that the record-to-record dispersion is small; which may be attributed to the fact that the period of the structure is located within the acceleration-sensitive region. This observation suggests that the $pga$ is a reasonable parameter to represent the seismic intensity for the case study example.

![Displacement demands vs. $pga$](image1)

![Collapse fragility curve](image2)

**Figure 3-14.** Dynamic response analysis of the dual SDOF oscillator
The collapse fragility curve, shown in Figure 3-14b, was determined by counting the number of ground motions that predicted collapse (for a given $p_{ga}$) divided by the total number of analysis, i.e. 30. The criteria for predicting collapse in the IDA were: 1) very large increase of displacement demands due to small increments of seismic intensity; 2) numerical instability; or 3) displacement demands higher than the collapse displacement threshold of $d_{CP}=0.23$ m. Then, a log-normal distributed function was fitted as recommended in [31], resulting in a mean $p_{ga}=0.38g$ and record-to-record dispersion of $\beta_r=0.13$. Since other sources of uncertainty have to be also considered, the total dispersion ($\beta$), was estimated as:

$$\beta = \sqrt{\beta_c^2 + \beta_t^2 + \beta_m^2}$$  \hspace{1cm} (3-24)

where $\beta_c$ is the uncertainty associated to construction quality and $\beta_m$ is the uncertainty associated with the completeness of the numerical model. Values of 0.10 and 0.40 were selected respectively as recommended in [31]. The “small” value of $\beta_c=0.10$ was selected because the construction quality of hospitals is expected to be rigorous while the “large” value of $\beta_m=0.40$ was chosen considering the assumption that a dual SDOF oscillator would not ideally represent the actual behaviour of a MDOF structure fitted with BRBs. Therefore, $\beta=0.43$. Similarly, the total dispersions of all the other response parameters of Table 3-4 were determined using equation (3-24), $\beta_c=0.10$ and $\beta_m=0.40$.

### 3.6.3 Damage state analysis

In this analysis, fragility functions of the components (structural, non-structural and contents) of the structure to be assessed are defined. Since, most of the components of the example hospital are unknown, for illustration purposes typical components and quantities were selected from the *Normative Quantity Estimation Tool* provided by the
ATC-58 project [31] for healthcare occupancy. They are listed in Appendix C. Additionally, the damage states (DS) and corresponding repair actions and costs for each component need to be defined in well detail. They were selected from the program PACT provided also by the ATC-58 Project.

On the other hand, the probability that the structure is irreparable given that collapse has not occurred shall be considered because it has a significant impact in the assessment of the performance[75]. Figure 3-15 shows the lognormal reparability curve considered here, with mean of 0.015 and dispersion of 0.30, as function of the residual inter-storey drift ratio. Additionally, it is considered that, if the primary structure remains elastic, any residual drift ratio may be removed after replacing the damaged BRBs. However, if the primary structure reaches the inelastic range, the effects of residual drifts have to be taken into account.

![Reparability curve of the example hospital](image)

**Figure 3-15.** Reparability curve of the example hospital

### 3.6.4 Loss analysis

In this analysis, a decision variable is estimated. For the example hospital, the total repair cost is selected as decision variable and it is estimated using the structural response (determined in the dynamic response analysis), the fragility data (determined
in the damage state analysis) and the Monte Carlo procedure proposed by the ATC-58 Project [31] (described previously in Section 2.3).

The total repair cost was determined for the design intensities in Table 3-1. Figure 3-16a shows the lognormal fit of the cumulative distributed function (CDF) of the total repair cost, normalised by the replacement cost. Since collapse was detected at the higher intensities, total replacement cost was required; therefore, a step in the CDF curves is observed at normalised cost of unity. The median repair costs (i.e. costs with 50% of probability of being exceeded) were 0.04, 0.11, 0.29 and 0.60 for $pga=0.05g$, $0.10g$, $0.20g$ and $0.30g$, respectively. As expected, it is observed that the higher the seismic intensity, the higher the repair cost.

On the other hand and as a matter of interest, the sensitivity of the repair cost to variations in the areas of the BRBs is assessed. For that purpose, the areas of the BRBs calculated previously in Section 3.5.2 for the design option (a), i.e. $b_2=60\%$, were scaled by 0.7 and 1.5. Figure 3-16b shows the CDF determined for $pga=0.20g$. In the figure, it is observed that repair costs change significantly as the areas of the BRBs are varied. The mean values were 0.47, 0.29 and 0.17, respectively. It is appreciated that increasing
the areas of the BRBs is an effective way of reducing the expected losses in this case study example because the structure became stiffer and farther away of the resonance zone; however, different results may be observed under different conditions.

Finally, although not shown here, similar assessments were conducted for options (b) and (c) of the case study example, i.e. \( b_2=30\% \) and \( b_2=40\% \) as defined in Section 3.5.2. For typical structural steel and BRBs cost, the probable repair cost, normalised by the replacement cost, resulted similar for the three cases; however, the constructional cost of option (a) was 10\% and 8\% smaller than the cost of options (b) and (c). Therefore, the designer may select option (a) as the most efficient in this particular case study example.

3.7 Discussion

Significance of the proposed method. Performance-Based Seismic Design (PBSD) philosophy is regarded as a realistic and reliable way of predicting and evaluating the performance of buildings (or facilities) with clear understanding of risk, e.g. see [31-32]. PBSD is superior to code provisions because it is able to predict different types of losses for different shaking intensities in a probabilistic manner, while codes are mainly intended to provide resistance to collapse without clear understanding of risk to collapse or extension of damage and repair cost [31]. However, implementation of PBSD is often reserved for critical facilities only, due to the required increase of engineering design involvement. Therefore, the proposed method for designing buildings equipped with BRBs facilitates the implementation of PBSD in such types of structures. In this way, trade-offs based not only on initial construction costs but also in life-cycle considerations can be made by decision makers at the initial stage of the design process [32]. Rapid application of PBSD is achieved by assuming that a structure equipped with BRBs can rationally be represented by a dual SDOF oscillator, as described by equation
Chapter 3. A Method for Preliminary Design and Assessment of Structures with BRBs

(3-9) and validated in Section 3.5.3. Furthermore, the same seismic ground motions are used in the design and assessment stages; therefore, the performance is better predicted.

**Advantages of the proposed method.** This method removes some of the limitations of other similar methods available in the literature for seismic design of structures equipped with BRBs, e.g.: a) it can be applied to steel and concrete structures; b) it explicitly considers the hysteretic characteristics of the main structure and the BRBs – avoiding the use of the equivalent viscous damping approach (as in [59]) or the use of design spectra generated from conventional oscillators (as in [1]); and c) important parameters such as ductility factors are not selected arbitrarily (as in [60]) but are based on the geometric and mechanical characteristic of the main structure and the BRBs.

**Justification of using equation (3-9).** It is observed in Figure 3-5 that the displacement and resistance demands of a conventional and a dual oscillator are significantly different, even when they have the same period of vibration and ductility demands. This behaviour is attributed to the fact that the parts of dual oscillators yield at different levels of displacement and resistance. Therefore, designing dual structures using spectra constructed from conventional oscillators may lead to systems that do not behave as expected. Thus, in the proposed method the dynamic equation of motion for dual oscillators (equation (3-9)) is solved for possible earthquake motions. In this way, the hysteretic characteristic of each part of the dual oscillator can be explicitly considered in the solution. If significant, \( P-\Delta \) effects are also taken into account in the force-displacement relationship of the primary part of the dual SDOF oscillator (see step 2 of section 3.4.2). Another advantage of solving equation (3-9) is that useful information is generated, so that preliminary performance-based assessments of the structure can be conducted. Besides, the time required to solve the equation is deemed negligible using current computer systems.
Efficient designs. According to Figure 3-3, designers are able to select from a significant number of combinations of parameters to control the displacement demand of dual oscillators. For example, they can balance the contribution of the BRBs and primary structure explicitly. Therefore, by applying the proposed method for various scenarios and by conducting the corresponding preliminary assessment (as seen in Section 3.6), the most convenient or efficient design option can be selected.

Estimation of ductility factors. As appreciated in equations (3-16) to (3-19), the ductility factors of a dual oscillator depend on the yielding properties of the parts. Therefore, they shall not be assigned arbitrarily. In the proposed method, they are estimated based on the mechanical and geometrical properties of the parts at the beginning of the design process. This provides the additional benefit of ensuring that the fuse concept is reached a priori. For example, if the ductility factor of the secondary part is larger than that of the primary part, the fuse concept is achieved. If the opposite is true or if the difference between both ductility factors is not deemed satisfactory, modifications to the parts can be decided before continuing with the design.

Limitations. Like many other methods, this proposed method has limitations, which are:

- The method is valid for regular, low-rise structures. Two aspects are discussed here. First, a dual SDOF oscillator with dynamic properties equivalent to those of a MDOF structure equipped with BRBs is used, thus, the method is valid for structures with response dominated by the first mode of vibration. In structures with significant participation of higher modes, such as high-rise buildings, further extension of the method is required. Second, a linear displacement profile is assumed in step 3 of section 3.4.2 to determine the displacement thresholds of dual SDOF oscillators. This may be reasonably acceptable for regular and low-rise
structures with beam-sway mechanism [64]. However, for other cases, the floor displacements \(d_i\) shall be estimated accordingly. The displacement profile obtained in step 2 by pushover analysis may provide a good approximation.

- **Additional data and a computer subroutine are required to apply the method.** Despite the advantages of the proposed method, it is recognised that more information should be collected before its application: 1) seismic records, actual or artificial, compatible with the local hazard are required - they may be obtained readily from databases most of the time available online or from professional associations of earthquake engineering; and 2) If assessment of the probabilistic performance is also being conducted, population models and information of components (structural, non-structural and contents) and their damageability are required - ATC-58 Project has facilitated this step by providing typical and normative information compiled in [31]. Moreover, a computer subroutine is required to solve equation (3-9) because without it the method would be impractical.

This subroutine has been developed in Matlab® and is available in *Appendix G*.

**Further application of the proposed method.** Another important application of the proposed method is in the retrofitting and upgrading of existing structures. In this particular case, the characteristic of the existent structures are fixed, and the designer has to select a relative participation of the BRBs to control the global response. If desired, low participation from the BRBs might be selected in order to avoid damage to existent components like connections and foundations. As previously observed, this is straightforward to do with the proposed method along with preliminary assessment of the performance to decide if retrofitting is convenient and to which level.
3.8 Conclusions

A method for preliminary design of low-rise buildings equipped with BRBs has been proposed. It is based on the assumption that a dual SDOF oscillator can represent the behaviour of an MDOF structure equipped with BRBs. An example building was designed using the proposed method to show its applicability. Taking advantage of the information that is generated in the application of the method, an intensity-based assessment of the performance of the example building was also conducted.

The following conclusions are formulated:

1. Considering the assumptions discussed in this paper, application of the PBSD philosophy can be facilitated using the proposed method. As a consequence, diverse options of design can now be compared to find efficient structures, based on trade-offs between initial costs and life-cycle considerations.

2. Figure 3-5 shows that conventional and dual oscillators behave differently, even when they have the same period of vibration and ductility demand. Therefore, the use of dual SDOF oscillators may be more rational to design structures equipped with BRBs than the use of conventional oscillators in order to avoid designs that do not behave as intended.

3. Ductility of the parts of a dual structure cannot be assigned arbitrarily because they depend on the geometric and mechanical properties of the parts. Considering this, the ductility factors are estimated at the beginning of the design process in the proposed method; which also allows reaching the structural fuse concept beforehand.

4. From nonlinear static and dynamic analyses of an example structure, it was observed that dual SDOF oscillators represent well the response of MDOF structures equipped with BRBs. As a consequence, the additional information
obtained during the solution of equation (3-9) can be transformed to approximate the response of the MDOF structure and to conduct a rapid and preliminary assessment of the performance.

5. Also from the assessment of the example structure, it was appreciated that, for this particular structure and conditions, the cross-sectional area of the BRBs is very effective to control expected losses. Since a different conclusion may be found under different conditions, rapid and preliminary assessments of the structural performance result very convenient to find efficient designs.

6. Finally, calibration of the post-elastic parameters of the steel model shows that the experimental behaviour of BRBs can be modelled with very good approximation using the Steel02 model of Opensees. Values of those parameters were provided in section 3.5.3.1.
Chapter 4

Comparative Experimental Studies of a Steel Frame Model with and without Buckling-Restrained Braces

4.1 Introduction

This chapter presents comparative experimental studies of a five-storey steel frame model at a scale of 1/10 with, and without, buckling-restrained braces (BRBs). The building model was subjected to free vibration tests and shaking table tests. The latter were conducted using low-intensity white noise and seismic input. From the free vibration tests and shaking table tests with low-intensity white noise, it was found that the BRBs contributed a significant amount of damping. This happened to the model even at low levels of vibration. The shaking table tests with seismic input were conducted using seven earthquake records, taken in the lakebed zone of Mexico City with seismic intensities from $pga=0.1g$ to $0.25g$. At an intensity of $pga=0.1g$, the results show that the model fitted with BRBs had a significantly smaller response than the bare model, in terms of displacement, inter-storey drift, floor velocity and floor acceleration.
The higher intensities were only applied to the model fitted with BRBs. The results indicate that the model with BRBs was able to withstand about 2.5 times the seismic intensity of the bare model, in terms of lateral displacement, inter-storey drift and Arias Intensity [76], as a measure of the energy contents of the movement. At the end of the tests, all BRBs were removed and the model remained in its original undamaged state.

The organisation of this chapter is as follows: the experimental setup of the building model is presented in section 4.2 – where the characteristics of the model, BRBs, instrumentation, input and testing programme are defined. The experimental results are presented in section 4.3. Sections 4.4 and 4.5 present the discussion and conclusions of the results.

4.2 Test setup for the frame building model

4.2.1 The model

The one-tenth scale model building, as shown in Figure 4-1, has a height of 1450 mm, a width of 600 mm and a depth of 300 mm. It is composed of five storeys and is made of ASTM A-36 steel with a nominal yielding stress of 250 MPa. The cross-sections of the members are given in Table 4-1. Since the main objective of the tests was to protect the main frame, the structural elements (beams and columns) were designed to remain elastic, while all nonlinear response was concentrated in the BRBs. Two mass scenarios were considered: 1) 150 kg/m$^2$ on levels one to four and 145 kg/m$^2$ on level five; 2) 417 kg/m$^2$ on levels one to four and 412 kg/m$^2$ on level five.
Figure 4-1. Building model tested on a shaking table

Table 4-1. Cross-sections of the structural elements

<table>
<thead>
<tr>
<th>Structural Element</th>
<th>Profile</th>
<th>Cross-section area</th>
<th>Second moment of area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns</td>
<td>Rectangular</td>
<td>243.84 mm$^2$</td>
<td>468.17 mm$^4$</td>
</tr>
<tr>
<td>Beams B-1 (X direction)</td>
<td>Tee</td>
<td>660.67 mm$^2$</td>
<td>207,685 mm$^4$</td>
</tr>
<tr>
<td>Beams B-2 (Y direction)</td>
<td>Rectangular</td>
<td>303.60 mm$^2$</td>
<td>64,777 mm$^4$</td>
</tr>
</tbody>
</table>
4.2.2 BRBs used in the tests

Due to the scale of the test frame model (1/10), there are no commercially available BRBs. Therefore, they had to be fabricated specifically for this study. The composition and dimensions of a typical BRB used in this study are shown in Figure 4-2.

Each BRB consists of a soft galvanised steel core, an encasing sleeve and two connection ends. The yielding stress and elasticity modulus of the core were obtained from laboratory tests. They are $f_y = 405 \text{ MPa}$ and $E_s = 118,725 \text{ MPa}$. The core was wrapped with 0.2 mm polytetrafluoroethylene (PTFE) film, as unbonding material, to separate the rod and mortar of the case. The two ends of the core were placed and fixed in connection ends – which consist of two steel tubes and an epoxy material.

![Figure 4-2. Schematic configuration of the BRBs used in the tests](image)

The cross-sections of the cores were circular with diameters of: 1.6 mm (for BRBs type 1 used in the first two storeys.); and 1.2 mm (for BRBs type 2 used in the next three storeys.).

In this experiment, the connections of the BRBs to the structure consisted of gusset plates, of ASTM A-36 steel, fastened with bolts. Two splice plates were used to connect each BRB end to each gusset plate. To join them, four 6.4 mm diameter bolts were used. In this case, the splice plates possessed large cross-sectional areas ($60 \text{ mm}^2$).
each), to avoid unexpected failures in the connections before the cores of the BRBs yielded.

### 4.2.3 Instrumentation and measured data

Three accelerometers were located on each floor: two in the longer direction ($X$) and one in the shorter direction ($Y$) (Figure 4-1d). Thus the responses in the $X$ and $Y$ direction and in the torsional direction could be monitored. Setra 141 accelerometers were used because of their high accuracy, output stability and measurement range (from static to 3000 Hz). The lateral displacements at each storey were estimated by double integration of the measured accelerations.

To avoid unintended errors during the integration process, recommendations by Boore and Bommer [77] were considered. No filtering, filtering between 0.1 and 20 Hz and between 0.5 and 20 Hz were tested. Figure 4-3a shows the 5%-damped elastic displacement spectra based on the accelerations recorded at the base of the model in one test. It is observed that none of the band-pass filters affected the response in the frequency range of interest between 1 Hz and 10 Hz. On the other hand, Figure 4-3b shows the transfer functions between the top floor and the base of the model, due to a white noise input. It shows that the band-pass filter between 0.5 Hz to 20 Hz eliminated the noise at low frequencies. Therefore, this filter was considered adequate, and errors in the estimation of the lateral displacements due to filtering may be considered negligible [78]. However, it is not possible to estimate permanent (or residual) displacements.
4.2.4 Seismic Input

Seven earthquake records were used to test the model (see Table 4-2). All seven were recorded in the lakebed zone of Mexico City, which is characterised by very soft soils, and they have dominant periods of vibration around two seconds. The time step on each record was scaled by a factor of \(\frac{1}{\sqrt{10}}\), according to the similitude laws developed for this model for the scale of 1/10.

### Table 4-2. Selected records for the tests

<table>
<thead>
<tr>
<th>Record</th>
<th>Station Code</th>
<th>Date (dd/mm/yyyy)</th>
<th>Magnitude, Ms</th>
<th>Epicentral Distance (km)</th>
<th>PGA (cm/s(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCT-1</td>
<td>SCT1</td>
<td>19/09/1985</td>
<td>8.1</td>
<td>425</td>
<td>161.63</td>
</tr>
<tr>
<td>SCT-2</td>
<td>SCT2</td>
<td>25/04/1989</td>
<td>6.5</td>
<td>311</td>
<td>39.98</td>
</tr>
<tr>
<td>SCT-3</td>
<td>SCT2</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>344</td>
<td>32.21</td>
</tr>
<tr>
<td>SCT-4</td>
<td>SCT2</td>
<td>15/06/1999</td>
<td>7.0</td>
<td>219</td>
<td>30.47</td>
</tr>
<tr>
<td>SCT-5</td>
<td>SCT1</td>
<td>15/06/1999</td>
<td>7.0</td>
<td>219</td>
<td>29.90</td>
</tr>
<tr>
<td>SCT-6</td>
<td>SCT1</td>
<td>30/09/1999</td>
<td>7.6</td>
<td>444</td>
<td>20.37</td>
</tr>
<tr>
<td>SCT-7</td>
<td>SCT2</td>
<td>20/03/2012</td>
<td>7.5</td>
<td>355</td>
<td>33.90</td>
</tr>
</tbody>
</table>

4.2.5 Design of the experiment

The input was applied in the longer direction of the model and in the orientation of the BRBs. The tests are summarised in Table 4-3 and described below:
In the first stage, the model with the first mass scenario was used for free vibration tests. An initial displacement was applied and then suddenly released to generate free vibration. The model without BRBs was first tested. Then, BRBs were introduced in the first storey and a free vibration test was conducted again. This was repeated until all five storeys of the model were fitted with BRBs. Finally the model was shaken using a (50 cm/s² RMS) white noise in order to measure its dynamic properties in an alternative manner. Reduced levels of mass were used at this stage to avoid unexpected damage to the model and elements.

**Table 4-3. Summary of the test programme**

<table>
<thead>
<tr>
<th>Test type</th>
<th>Objective</th>
<th>Test</th>
<th>Description</th>
<th>Mass scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free vibration</td>
<td>Damping</td>
<td>1a</td>
<td>Bare frame (see Figure 4-4a)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2a</td>
<td>Frame with BRBs in 1st storey (Figure 4-4b)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3a</td>
<td>Frame with BRBs up to 2nd storey (Figure 4-4c)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4a</td>
<td>Frame with BRBs up to 3rd storey (Figure 4-4d)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5a</td>
<td>Frame with BRBs up to 5th storey (Figure 4-4e)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Dynamic</td>
<td>1b</td>
<td>White noise in bare frame (Figure 4-4a)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>and damping</td>
<td>2b</td>
<td>White noise in full-equipped frame (Figure 4-4e)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3b</td>
<td>White noise in bare frame (Figure 4-5a)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4b</td>
<td>White noise in full-equipped frame (Figure 4-5b)</td>
<td>2</td>
</tr>
<tr>
<td>Shaking table tests</td>
<td>Responses</td>
<td>1c to 7c</td>
<td>Bare frame subjected to records SCT-1 to SCT-7 for PGA = 0.1g (Figure 4-5a)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>to PGA = 0.1g</td>
<td>8c to 14c</td>
<td>Full-equipped frame subjected to records SCT-1 to SCT-7 for PGA = 0.10g (Figure 4-5b)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1d to 7d</td>
<td>Full-equipped frame subjected to records SCT-1 to SCT-7 for PGA = 0.15g (Figure 4-5b)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8d to</td>
<td>Full-equipped frame subjected to records SCT-1 to SCT-7 for PGA = 0.20g (Figure 4-5b)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14d to 15d</td>
<td>Full-equipped frame subjected to records SCT-1 to SCT-3 for PGA = 0.25g (Figure 4-5b)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17d to</td>
<td>Full-equipped frame subjected to records SCT-1 to SCT-3 for PGA = 0.25g (Figure 4-5b)</td>
<td>2</td>
</tr>
</tbody>
</table>
In the second stage, the model with the second mass scenario was used. The model was subjected to (50 cm/s² RMS) white noise. Then, seven ground motion records were applied at the base, with seismic intensities of: \( pga = 0.10g, 0.15g, 0.20g \) and \( 0.25g \) respectively. For the intensity of \( 0.10g \), the model was tested with and without BRBs. For other \( pga \) values, the model was fully equipped with BRBs in all storeys. Floor accelerations were measured in all tests. Then, the effects of the BRBs in the model were analysed.

Figure 4-4 shows the model used in the first stage of the tests and how the BRBs were gradually added to the model. Figure 4-5 shows the model with added mass (Mass scenario 2) in the second stage of the tests. The purpose of having the increased mass was: 1) to provide more realistic levels of mass; 2) to produce a longer fundamental period of vibration which generates larger displacements; and 3) to ensure that the BRBs go beyond their elastic limit to dissipate energy.

![Figure 4-4. Model during the first stage of the tests](image-url)
4.3 Experimental results

4.3.1 Free vibration tests

Figure 4-6 shows the recorded accelerations at the top floor for tests 1a to 5a, as described in Table 4-3 and shown in Figure 4-4. It can be observed that the vibration of the model decayed quickly due to the contribution of BRBs. This was due to an increase in the damping and stiffness of the model structure.

The damping ratios for the first four cases were estimated by curve fitting to the first 10 oscillations of the measured data. For the last case, a white noise had to be induced in the model and the damping was estimated by balancing the damping energy equation (see next subsection).

It is significant to remark that the increase of damping ratio occurred at linear-elastic levels of response of the model and the BRBs. This can be deduced from Figure 4-7 - where the axial load-displacement curve of an isolated BRB was obtained experimentally under displacement-controlled cyclic tests. It can be appreciated that,
under a displacement level of 0.75 mm (Figure 4-7a), the BRB remained within its linear-elastic limit, whilst inelastic behaviour started at displacement levels of 1 mm or more (Figure 4-7b). When the axial displacement demands on the BRBs (as installed in the model when subjected to test 2b) are analysed, it can be seen that the BRBs remained well below the linear-elastic limit (Figure 4-7c).

![Graphs showing measured accelerations at the top floor for the five bracing cases](image)

**Figure 4-6.** Measured accelerations at the top floor for the five bracing cases
Figure 4-7. Load-deformation curves of isolated BRBs vs. deformation demands produced in test 2b

Now, Figure 4-8 shows the damping ratio estimated for each case of Figure 4-6 in which the horizontal axis shows the number of pairs of BRBs used and the vertical axis the damping ratio. It is clear that the BRBs increased the damping significantly in the building model. This finding may be significant because it is commonly assumed that BRBs do not provide damping when working in their linear range. However, it should be recognised that more experimental work is needed to validate the phenomenon shown in Figure 4-8 and the factors affecting these increments.
4.3.2 Shaking table tests with white noise input

Figure 4-9 shows the Transfer Functions (TF) of the Fourier spectra obtained for the tests 1b, 2b, 3b and 4b, which are summarised in Table 4-3. These tests included the two mass scenarios and the model with and without BRBs. The TF were obtained using the program Degtra [79] and the accelerations measured at the base and at the top floor of the model.

The fundamental frequencies and first-mode damping ratios are indicated in Figure 4-9. Two important facts are noted in this Figure: 1) for both scenarios of mass, the frequencies increased when the BRBs were introduced; in other words, the BRBs increased the stiffness of the model; and 2) the spread of TFs became wider when BRBs were included in the model, indicating that the model fitted with BRBs possessed higher damping. This is in agreement with observations by others [26-27, 80]. The damping ratio increased from 0.3% to 7.6% for the first mass scenario, and from 0.52% to 6.10% for the second mass scenario.
Figure 4-9. Transfer Functions (from the top floor to the base) for the test model

Now, it was recognised that the source of damping in the model structure was not viscous damping [73, 81]. Thus an approach similar to that proposed by Blandon and Rodriguez [82] was adopted. This approach recognises that equivalent viscous damping ratios may vary during the whole response time-history. Therefore, the equivalent damping ratios ($\xi_{eq}$) corresponding to the first mode are estimated from [83]:

\[
2\xi_{eq}\omega^2 m \int \left( \dot{u}(t) / \Gamma_1 \right)^2 dt = -\int m \ddot{u}_g(t) \left( \dot{u}(t) / \Gamma_1 \right) dt - \frac{1}{2} m \left( \ddot{u}(t) / \Gamma_1 \right)^2 - \frac{1}{2} m \dot{\omega}^2 \left( u(t) / \Gamma_1 \right)^2 \quad (4-1)
\]

where $m$ is the modal mass, $\omega$ is the angular frequency of vibration, $\ddot{u}_g$ is the acceleration at the base, $\Gamma_1$ is the first-mode participation factor, and $\dot{u}$ and $u$ are the relative velocity and displacement at the top floor of the model.
Since all parameters of equation (4-1), except $\xi$, can be readily estimated using the measured floor accelerations, an equivalent $\xi$ can be determined by minimising the difference between the left and right sides of the equation. As an example, Figure 4-10 shows the damping energy (in the time domain) estimated for the test 2b using equation (4-1). Damping ratios of $\xi = 5\%$, 7.6% and 10% were used in the left side of the equation (see the black lines). As observed, the value of $\xi = 7.6\%$ balances the damping energy estimate with the right side of the equation (see the grey line). Therefore, this value is considered as the equivalent viscous damping ratio for the model for this particular case. All other viscous damping ratios in Figure 4-9 were estimated in the same way.

![Figure 4-10. Estimation of equivalent viscous damping ratio for test 2b](image)

### 4.3.3 Shaking table tests with seismic input

The results are presented in two parts: 1) comparison of the responses on the frame model with and without BRBs for a seismic intensity of $pga=0.1g$; and 2) response of the model fully-equipped with BRBs subjected to incremental seismic intensity (i.e. for $pga=0.10g$, 0.15g, 0.20g and 0.25g).
4.3.3.1 Response at Peak Ground Acceleration (pga) of 0.1g

The response of the models with and without BRBs are compared considering five parameters: 1) lateral relative displacements, 2) inter-storey drifts; 3) absolute floor velocities; 4) absolute floor accelerations; and 5) Arias intensity \([I_a]\) [76], relative to the base.

Figure 4-11 shows only the response of the models with and without BRBs measured at the top floor of the model subjected to SCT-2. It can be observed that the response reduced when BRBs were included in the model. The average reductions for the five parameters were evaluated.

The average reduction of the maximum lateral displacement at the top floor was 58.5%; the average reduction of the maximum inter-storey drifts was 62.2%. This effect may be expected in structures located on soft soils, because the model became stiffer and had higher damping when the BRBs were included. The average reductions of the maxima values of the floor velocities and floor accelerations were 35.4 % and 26.9 %, respectively.

The reductions in floor velocities and accelerations is an important finding in this study, because this contradicts some numerical studies which suggested that BRBs generally increase these parameters (e.g. see [84]).

On the other hand, even when the reductions of the maxima values of displacements, inter-storey drifts, floor velocities and accelerations are significant, it would be of interest to compare the response in terms of energy contents in order to have an alternative measure of the expected damage. Therefore, the Arias intensity \([I_a]\) [76], as a measure of energy contents in the movement, was estimated for the acceleration at the top floor relative to the base (Figure 4-12). On average, the ratio of
the Arias intensities for the model without BRBs to that with BRBs was 7.93. This suggests that the expected damage to the structure without BRBs might be significantly larger than the damage to the structure fitted with BRBs. It can also been seen in Figure 4-12 that the ending of the intense phase (95% $I_a$) was earlier when BRBs were included.

**Figure 4-11.** Responses of the test model (with and without BRBs) for $PGA = 0.1g$ and record SCT-2
4.3.3.2 Response to higher values of $pga$

To examine the effect of the seismic intensity on the model fitted with BRBs, the intensity was increased from $pga=0.1g$ to $0.25g$, in increments of $0.05g$ for the seven records. During the tests, two local failures were observed in the BRBs. The first was observed when the SCT-1 record, scaled to $pga = 0.20g$, was applied. The BRB at the south side of the third storey broke. Both BRBs of the third storey (which were type 2) were replaced by BRBs type 1 and the tests continued. The second partial failure occurred at the north side of the third storey, when the SCT-3 record, scaled to $pga = 0.25g$, was applied. The test programme was interrupted at that stage. The two failures occurred in the opening gaps of the BRBs (see Figure 4-2).

Figure 4-13a, c, d and e show the maximum values of displacement, floor velocity, floor acceleration and 95% of the relative Arias intensity at the top floor, while Figure 4-13b shows the maximum inter-storey drift. The duration of Arias Intensity is presented in Figure 4-13f. All the parameters were estimated for the four intensity levels and the seismic records of Table 4-2. The averages of the responses are shown by a thick dark line in the figure. The maximum responses of the model without BRBs, subjected to the seven seismic records at $pga = 0.10g$, are also shown in Figure 4-13 for comparison.
It can be observed, from Figure 4-13, that the inclusion of BRBs significantly reduced the responses. The mean of the maximum displacements, maximum inter-storey drifts and the Arias Intensity of the model equipped with BRBs and subjected to an intensity of $pga = 0.25g$, were smaller than that for the model without BRBs subjected to $pga = 0.1g$. For the floor velocities and floor accelerations, the model with BRBs can
accommodate about 1.5 times more seismic intensity than the model without BRBs. In addition, by observing the dispersion of the data, it is appreciated that the model with BRBs is less sensitive to the seismic input than the bare model at the intensity level of $pga=0.1g$.

Finally, the duration of the intense phase (from $0.05I_a$ to $0.95I_a$), provided in Figure 4-13f, shows that the model fitted with BRBs had a much shorter duration of the intense phase. When the model was equipped with BRBs, the durations of the intense phases of the Arias intensities were almost constant for all four values of $pga$ and the average duration was almost 40% less than that on the bare model for $pga = 0.10g$.

4.4 Discussion

The measured results indicate that the model equipped with BRBs reduces the responses and their record-to-record variability (and hence, uncertainties). In addition, the Arias intensity (as a measure of the energy contents) showed that the model with BRBs may present significantly less damage than the model without BRBs.

Increases in damping were observed due to the inclusion of BRBs in the model. The damping increased from 0.3% to 7.6% for the first mass scenario and from 0.52% to 6.1% for the second mass scenario. However, it is difficult to quantify the contributions from the BRBs and from the connections between BRBs and the model. One limitation of the study is that the frame model is relatively small, and this might enhance the effect of the BRBs. Further larger scale tests are needed.

The permanent (or residual) deformations of the model equipped with BRBs were not possible to measure during the tests. But no permanent deformations were visually identified after each test and after all the BRBs were removed. This gives an indication
of the low permanent deformation of the tested model. The costs associated with rehabilitation of structures equipped with BRBs may be reduced to replacing the devices without interruptions to the building functionality; which may be regarded as a cost-effective solution.

Since the responses of the model equipped with BRBs were significantly improved, it may be thought that these devices would benefit structures located in the lakebed zone of Mexico City. Including BRBs in new designs could lead to better performance or lower construction costs. Also existing buildings could be upgraded to achieve enhanced levels of performance.

4.5 Conclusions

Free vibration and shaking table tests have been carried out on a five-storey model and the results are summarised in this chapter. The following conclusions have been reached:

1. The inclusion of BRBs in the model increases the stiffness and damping. As a result, all the response parameters (and record-to-record variability) were reduced for the model fitted with BRBs.

2. A significant finding of the experiment presented in this Chapter is that BRBs start dissipating energy even at linear-elastic deformation levels (as observed in Figures 4-6 to 4-8).

3. From the tests on the model, with and without BRBs, using the same $pga=0.1g$, the average values of the maximum displacement and maximum inter-storey drift were reduced by 58.5% and 62.2%, respectively. The maximum floor velocities and maximum floor accelerations were reduced by 35.4% and 26.9%, respectively. The
Arias Intensity, as a measure of the energy contents, was almost eight times less on the structure with BRBs than on the structure without BRBs.

4. These reductions suggest that structural and non-structural damage and losses of contents may be significantly less when BRBs are included in structures located in the lakebed zone of Mexico City.

5. From the tests with incremental seismic intensity, it can be concluded that the model fitted with BRBs was able to accommodate up to 2.5 times more seismic intensity in terms of lateral displacements, inter-storey drifts and Arias Intensity, and up to 1.5 times more seismic intensity in terms of floor velocity and floor acceleration.

6. Residual deformations of the model were not visually identifiable before or after all the BRBs were removed from it.
Chapter 5

Comparative Experimental Studies of Reinforced Concrete Precast Models with and without Buckling-Restrained Braces

5.1 Introduction

To examining the effects of Buckling-Restrained Braces (BRBs) on Reinforce Concrete (RC) precast models, shaking table experiments were conducted on two four-storey frame models at a scale of 1/3. One model was without BRBs and designed according to common practices in Mexico (Model 1), while the other (Model 2) was equipped with BRBs and designed according to the displacement-based methodology proposed in Chapter 3. This chapter presents three comparison groups, which are explained with the help of Figure 5-1: 1) the behaviour of the two models which were designed using different methods; 2) the behaviour of Model 2 with, and without, BRBs; and 3) the behaviour of Model 2 (intact and retrofitted) with BRBs. The fundamental natural frequencies, damping ratios and seismic response of the models were examined. The two models were initially subjected to low-intensity white noise to determine their
dynamic properties. It was found that BRBs increase the damping ratio significantly. Then, the SCT-EW accelerogram of the M8.1 Michoacán earthquake of 19/09/1985, recorded in the lakebed zone of Mexico City, scaled to 50, 100, 150 and 200%, was applied to the models. The results show that both models performed adequately, but the model with the BRBs had significantly smaller displacements, inter-storey drifts and stiffness degradation. Floor velocities and accelerations were similar for both models. Note that Model 2 was re-tested following the seismic action after replacing the BRBs with a new set. This simulates retrofitting the structure after earthquake induced damage. It is worth to highlight that numerical analyses agree well with the experimental results.

Figure 5-1. Description of the experimental study

This chapter is organised as follows: Section 5.2 describes the models (including their design, construction and theoretical capacity) and the BRBs used in the tests; Section 5.3 describes the experiment, including instrumentation, input and test programme; the
results are presented in Section 5.4. Discussion and conclusions are presented in Sections 5.5 and 5.6.

5.2 Models

A prototype structure was selected for this project. It was assumed to be located on the lakebed zone of Mexico City (zone IIIb) and to be used for residential occupancy. It had four storeys and one bay in each horizontal direction. For comparison, two test models were constructed from the same prototype. The first model (Model 1) was not equipped with BRBs and was designed according to the current design practice and code [51]. The second model (Model 2) was equipped with BRBs and designed following the methodology proposed in Chapter 3.

Because of the capacity of the shaking table (see [85]), the two test models were built at a scale of 1/3. Thus, the models had a square base of $3.30 \times 3.30$ m and a height of 4.40 m. A factor of mass per area of 1/2 was also used and similitude laws were developed accordingly. The total masses in the models were 420 kg/m$^2$ on floors 1 to 3 and 410 kg/m$^2$ on the top floor. Figure 5-2 shows the dimensions of the models and a photograph of Model 2.

5.2.1 Precast system

The system consisted of precast beams and columns joined at the nodes using a wet connection (Figure 5-3). The connection was similar to that in [86-88] but different because: a) the precast beams were not introduced into the column windows (or spaces), instead they were supported by temporary metallic supports which were removed after the cast-in-situ concrete reached its nominal resistance; and b) the floor system was supported by concrete corbels located at the bearing beams, so that the floor systems did not reduce the size of the beams and their capacity.
In addition, column-to-column connections were used in this precast system. The connection enabled erection of two or more consecutive storeys. The connections were made at the mid-height of the columns using high-strength grout and connecting steel bars.

![Dimensions and view of Model 2](image1)

**Figure 5-2.** Dimensions and view of Model 2

![The precast system](image2)

**Figure 5-3.** The precast system
5.2.2 Construction and design

For simplicity, two consecutive storeys were assembled. This means that column-to-column connections were only made in the middle of the third storey. The models were fabricated in the following order: 1) two storeys were fabricated outside the shaking table; 2) they were then mounted on the platform; and 3) the third and fourth storeys were assembled on the platform. Once finished, the models were like that shown in Figure 5-2b.

Regarding the design, the selected prototype was designed in full-scale and then scaled using a geometric factor of 1:3. It was assumed that the building would be located at the lakebed zone of Mexico City (zone IIIb) and was to be used for residential occupancy. The materials considered were: 1) concrete in the beams, columns and connections with a nominal resistance of $f'_c=50$ MPa; 2) concrete in the floor system and its topping of $f'_c=35$ MPa; 3) steel reinforcement in the structural elements with a nominal yielding stress of $f_y=420$ MPa; and 4) steel welded mesh in the floor topping with $f_y=500$ MPa. More details of the design of each model are found in Appendix D.

The resultant cross-sections of the models and their steel reinforcement are shown in Figure 5-4. The beams and columns were 150x270 mm and 200x200 mm. As expected, Model 1 had more steel reinforcement than Model 2, which compensates for the capacity provided by the BRBs. The floor system consisted of hollow slabs with a thickness of 100 mm and a 20 mm RC topping. Regarding the BRBs, they had cross-sectional areas, in the yielding zone, of 60 mm$^2$ in first two storeys and 30 mm$^2$ in storeys 3 and 4. More details of the BRBs are presented below in section 5.2.4.
5.2.3 Theoretical capacity curves

With the designed prototype (both with and without BRBs), the capacity curves were obtained by pushover analysis of 2D inelastic models using the program Opensees [69]. The Mander model was used for concrete and the Giuffre-Menegotto-Pinto material was used for steel.

The pushover analysis was conducted by applying a constant gravity load and incremental lateral loads on the model. Figure 5-5 shows the pushover curves obtained from the numerical analyses. The design base-shear, estimated from the code, is also shown in Figure 5-5. Two interesting observations are apparent: 1) the capacity of the models is about four times that required by the code; and 2) both models have a similar shear capacity with different initial stiffnesses. These observations are discussed later.

![Figure 5-4. Cross-sections of the scaled models](image)

![Figure 5-5. Theoretical capacity (pushover) curves](image)
5.2.4 BRB elements

Two types of BRBs were used in the test programme. The first type had a rectangular cross-sectional core made of steel S275 which is available in Europe. The core was cut using a laser to provide a geometric shape similar to that of commercially available BRBs. The second type of BRBs had a circular cross-sectional core and was made of steel ASTM-A36. Its core was made of rod which is commercially available in Mexico.

The parts composing the BRBs of the rectangular core are shown in Figure 5-6, which are similar to those of the BRBs with the circular core. The parts were: a) the core; b) unbonding material, 0.4 mm of polytetrafluoroethylene (PTFE); c) four plates connecting the core to the frame model (two plates on each side); d) an inner tube filled with mortar; e) two outer tubes half-filled with mortar; and f) an elastic material to allow axial deformation.

Figure 5-6. BRBs used in the tests
To fabricate a BRB, the core wrapped with the unbonding material was located inside the inner tube. The inner tube was filled with mortar. Then, the core was connected to the connecting plates using six bolts at each end. The elastic material was located at each end of the inner tube to allow free deformation in tension and compression. Then the outer tubes were positioned to cover the connecting plates and the inner tube. The outer tubes were filled with mortar from their ends to the elastic material. Finally, the connecting plates were fastened to the frame model by means of gusset plates using six bolts at each end.

5.3 Experiment

5.3.1 Material Properties

Samples of the concrete were taken and tested prior to the model tests on the shaking table. The resistances ($f'_c$) and modulus of elasticity ($E_c$) were determined and are shown in Appendix E. The modulus of elasticity of the beams and the columns was determined experimentally following the process given in [89]. For other elements, it was estimated using equation (11.3) of the specifications for concrete design of the Mexico City building code [51]; which is

$$E_c = 2700\sqrt{f'_c} + 5000$$  \hspace{1cm} (5-1)

The steel reinforcement used was commercially available in Mexico and had a nominal yield stress of 420 MPa. No samples of the steel bars were tested in this study, thus a typical stress-strain curve of the Mexican steel bars was assumed as recommended in [90] (Figure 5-7).
Figure 5-7. Typical stress-strain curve for Mexican steel bars of diameters up to 13 mm

5.3.2 Instrumentation and measurements

Accelerometers were placed on the models from the base to the top floor (Figure 5-8). At the base, one accelerometer was located in the direction of the tests (axis X). On floors 1 to 3, one accelerometer was located in the transverse direction (axis Y), and three in the X direction. On the top floor, seven accelerometers were located in the X direction and one in the Y direction. To measure possible rotation around a vertical axis, two accelerometers on each floor were located away from the centre of the floor and close to axes 1 and 2 of the model respectively (Figure 5-8).

To measure relative displacements, linear variable displacement transformers (LVDT) were placed on all storeys. They were located in axes 1 and 2, at each side of the models, oriented in the X direction. Thread LVDTs were also placed to measure absolute displacements; however, their readings were unreliable due to sudden changes in the direction of motion.

In Model 2, LVDTs were also placed on all the BRBs and two strain gauges were mounted on the BRBs in the first three levels along axis 2.
5.3.3 Input

Two types of input were used during the tests. The first type was white noise with a low-intensity of 20 gal root-mean-square. The second type was seismic ground motion. The former was used to evaluate the dynamic properties (natural frequencies and damping ratios) of the models within linear-elastic response levels. The latter was used to assess the response to seismic input and the effects of seismic intensity. The component EW of the ground motion of the 19/09/1985 Michoacán, Mexico Earthquake, recorded in the SCT station was used. This record was shown in Figure 5-8.
2-9b and is repeated here in Figure 5-9. The ground motion was selected because it was recorded in the lakebed zone of Mexico City, where the prototype structure was assumed to be located.

![Figure 5-9. SCT-EW record of the 19/09/1985 Michoacán, Mexico earthquake](image)

**5.3.4 Test programme**

Four levels of ground motion were applied in the direction parallel to the orientation of the BRBs. They were the SCT-EW record scaled to 50%, 100%, 150% and 200%. It was also scaled in time and amplitude by the factors of $\frac{1}{\sqrt{6}}$ and 2 according to similitude laws. The test programmes for Models 1 and 2 are summarised below:

For Model 1: the low-intensity white noise and the scaled SCT-EW record were applied alternatively (Table 5-1). This was to examine the dynamic behaviour and seismic response of the model, and also possible damage and variation of the dynamic properties due to the seismic action.

For Model 2, the tests were conducted in two stages (Table 5-2):

*Stage 1.* The model was first tested using white noise to assess its initial dynamic properties (test B1). In tests B2 to B5, BRBs with rectangular cores were gradually provided to assess the change in damping and natural frequency (Figure 5-10). The model was then subjected to the scaled SCT-EW record in tests B6 to B9 to assess the seismic response with BRBs in all storeys (Figure 5-10e). Tests B10 and B11 were
conducted to assess the variation of the dynamic properties of the model after severe seismic inputs. Changes in dynamic properties and stiffness degradation can be assessed by comparing the results from tests B1 and B11 for the bare model and from tests B5 and B10 for the model fully fitted with BRBs.

Stage 2. Tests of Model 2 retrofitted. All the BRBs with rectangular cores were replaced with BRBs with circular cores of the same cross-sectional area. The first test, C1, was conducted with white noise to assess the initial properties of the retrofitted model. Comparison between test results from C1 and the previous test B10 provide the variation in dynamic properties due to retrofitting. Then, in tests C2 to C5, the model was subjected to the same SCT-EW record at different intensity levels. Tests C6 and C7 were conducted to assess the dynamic properties of the retrofitted model after seismic input. Comparisons between test results from B11 and C7, or from C1 and C6, are useful for assessing the variation of the dynamic properties due to seismic input.

The test programmes provided several scenarios for comparison between the two models, between intact and retrofitted models before and after seismic actions, and the effects of different number of BRBs.

**Table 5-1. Summary of the test programme for Model 1**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Test</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic properties</td>
<td>A1</td>
<td>White noise (WN)</td>
</tr>
<tr>
<td>Response to SCT-EW</td>
<td>A2</td>
<td>SCT100%</td>
</tr>
<tr>
<td>Dynamic properties</td>
<td>A3</td>
<td>WN</td>
</tr>
<tr>
<td>Response to SCT-EW</td>
<td>A4</td>
<td>SCT100%</td>
</tr>
<tr>
<td>Response to SCT-EW</td>
<td>A5</td>
<td>SCT150%</td>
</tr>
<tr>
<td>Dynamic properties</td>
<td>A6</td>
<td>WN</td>
</tr>
<tr>
<td>Response to SCT-EW</td>
<td>A7</td>
<td>SCT200%</td>
</tr>
<tr>
<td>Dynamic properties</td>
<td>A8</td>
<td>WN</td>
</tr>
</tbody>
</table>
### Table 5-2. Summary of the test programme for Model 2

<table>
<thead>
<tr>
<th>Stage</th>
<th>Objective</th>
<th>Test</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1: Model 2 with BRBs (No initial damage)</td>
<td>Variations on dynamic properties due to BRBs</td>
<td>B1</td>
<td>WN in bare frame (Figure 5-10a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2 to B5</td>
<td>WN with BRBs as in Figure 5-10b to e, respectively</td>
</tr>
<tr>
<td></td>
<td>Response to SCT-EW with BRBs in all the storeys (Figure 5-10e)</td>
<td>B6</td>
<td>SCT50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B7</td>
<td>SCT100%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B8</td>
<td>SCT150%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B9</td>
<td>SCT200%</td>
</tr>
<tr>
<td></td>
<td>Dynamic properties after seismic input</td>
<td>B10</td>
<td>WN with BRBs in all the storeys (Figure 5-10e)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B11</td>
<td>WN in bare frame (Figure 5-10a)</td>
</tr>
<tr>
<td>Stage 2: Model 2 retrofitted with new BRBs (initial damage in structure)</td>
<td>Initial properties of the retrofitted model</td>
<td>C1</td>
<td>WN with BRBs in all the storeys (Figure 5-10e)</td>
</tr>
<tr>
<td></td>
<td>Response to SCT-EW with BRBs in all the storeys (Figure 5-10e)</td>
<td>C2</td>
<td>SCT50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C3</td>
<td>SCT100%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C4</td>
<td>SCT150%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C5</td>
<td>SCT200%</td>
</tr>
<tr>
<td></td>
<td>Dynamic properties after seismic input</td>
<td>C6</td>
<td>WN with BRBs in all the storeys (Figure 5-10e)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C7</td>
<td>WN in bare frame (Figure 5-10a)</td>
</tr>
</tbody>
</table>

**Figure 5-10.** Systematic inclusion of the BRBs in Model 2
5.4 Experimental results

5.4.1 Behaviour of isolated BRBs

Prior to the shaking table tests, cyclic load was applied to two isolated BRBs (one with a rectangular core and the other with a circular core) to assess their individual behaviour. The time-history of displacements in Figure 5-11a was applied quasi-statically. It should be noted from the figure that three levels of deformation were applied, i.e. three cycles at the yielding deformation of $\Delta_y=1.1$ mm, three cycles at $5\Delta_y$, and three cycles at $10\Delta_y$. Figure 5-11b shows the resultant stress-strain curve for the BRB with the rectangular core. Similar behaviour was observed in the BRB with the circular core. It is suggested that the BRBs exhibited stable behaviour. The load capacity of the devices was higher in compression than in tension. This may be attributed to the Poisson effect and friction interaction between the core and the casing of the BRBs, as has been noted by others (e.g. see [7]).

![Cyclic loads and Stress-strain response](image.png)

(a) Cyclic loads  
(b) Stress-strain response

**Figure 5-11.** Cyclic tests in BRB elements

5.4.2 Response to low-intensity white-noise input

Comparisons of Model 2, with and without BRBs, are presented in this section for low-intensity white-noise input. The results of tests B1 to B5 have been analysed to examine the effects of BRBs on the natural frequencies, global stiffness and damping ratios. All
the analyses were conducted in the direction of the BRBs (axis X). On the other hand, since the model was very regular in plan and elevation, no significant rotations around the vertical axis were observed. Therefore, torsional effects are not further addressed in this study.

### 5.4.2.1 Effects of BRBs on natural frequencies and global stiffness

The natural frequencies of the models were determined by means of the non-parametric system identification technique, using conventional spectral analysis [91]. Figure 5-12 shows the transfer functions (TFs) and coherence of tests B1 (bare frame) and B5 (BRBs in all the storeys); in which the first few natural frequencies are clearly shown.

![Figure 5-12. Natural frequencies of Model 2 without and with BRBs](image)

The variations of the fundamental frequencies from tests B1 to B5 are shown in Figure 5-13a, in which the horizontal axis represents the different configurations of the
braces shown in Figure 5-10. As expected, the inclusion of BRBs increased the fundamental natural frequency.

As the lateral stiffness of a frame is proportional to the square of its fundamental natural frequency, the increase of the global stiffness due to the inclusion of BRBs, when the mass remains constant, can be determined as follows:

\[
\Delta k(\%) = \left( \frac{f_i^2 - f_{NoBRBs}^2}{f_{NoBRBs}^2} \right) \times 100; \quad i = 1, 2, 3, 4
\]  

(5-2)

where \( f_{NoBRBs} \) represents the fundamental natural frequency of the bare model and \( f_i \) the model with the \( i \)th BRB fitted starting from the ground floor. The variation of the stiffness is shown in Figure 5-13b where the total increase of stiffness was 32%. It should also be noted that the increase of stiffness is higher at configurations b and c than at configurations d and e. This suggests that BRBs are more effective in the lower than in the upper storeys.

![Figure 5-13. Effects of BRBs on frequency and global stiffness for different brace configurations](image)

a) Variations of natural frequency  

b) Increase of global stiffness

Figure 5-13. Effects of BRBs on frequency and global stiffness for different brace configurations

Figure 5-14 shows the variations of the fundamental natural frequency estimated with white noise input before, and after, seismic action on Model 2, when it was a bare frame and when it was fully fitted with BRBs. For Stage 1, comparison between tests
B1 and B11 (corresponding to tests on the bare model) or between B5 and B10 (tests with BRBs) show the reduction of the fundamental natural frequency due to the action of the scaled SCT-EW ground motion indicating that the model experienced some degree of damage. Using equation (5-2), the relative stiffness degradation was 31% and 28%, respectively. In Stage 2, the model was retrofitted with a new set of BRBs. This set had circular cores with the same cross-sectional areas and factors $f_{ki}$ as those of the previous set of BRBs. Comparing tests C1 and B5, the retrofitted model (tests C1) had a lower frequency; which can be attributed to cracking and stiffness degradation of the concrete frame. Therefore, as recommended by concrete design codes (such as [51]), design methodologies based in the control of the response shall also account for accumulated stiffness degradation in concrete elements. From a comparison of tests B11 and C7 (tests in bare model) or C1 and C6 (i.e. tests with BRBs), it was observed that further stiffness degradation had occurred due to the application of the SCT-EW record in Stage 2. The relative stiffness degradation was 10% and 20%, respectively; i.e. it was smaller than that in Stage 1.

![Figure 5-14. Variations of natural frequency in Model 2 when subjected to white noise input](image-url)
5.4.2.2 Effects of BRBs on damping ratio

The damping ratio was determined under low-intensity input conditions, which generated linear-elastic response in Model 2 with, and without, BRBs. The damping ratio estimated in this section is the classic rate-dependent damping, and it is not equivalent hysteretic damping produced by inelastic deformation of BRBs as addressed in other studies, e.g. [59].

The estimation of the damping ratio for the first mode of vibration is presented in this section. However, as pointed out in Chapter 4, it is recognised that the source of damping in the model structure may not be viscous damping [73, 81]. Therefore, as traditionally addressed, an equivalent viscous damping of the model was estimated by following the approach described previously in Section 4.3.

Figure 4-10 shows the balance of energy (in the time domain) estimated for Model 2 without, and with, BRBs (i.e. tests B1 and B5). Different values of damping were evaluated. It was observed that the values of $\xi = 1.16\%$ and $5.38\%$ provided the best balances, respectively. Therefore, these values were considered as the equivalent viscous damping ratios for the model for the two particular cases.

![Figure 5-15. Estimation of equivalent viscous damping ratio for Model 2 with, and without, BRBs.](image-url)
All other viscous damping ratios were estimated in the same way. Figure 5-16 shows the increase of damping ratio of the model as the number of BRBs increased. A large increase was observed when the BRBs were added to the first and second storeys.

![Figure 5-16. Effects of BRBs on the damping ratio of Model 2 for brace configurations of Figure 5-10](image)

Figure 5-17 shows the variations of damping ratio before, and after, important changes in Model 2. This is similar to Figure 5-14 for the variations in fundamental natural frequency. It can be appreciated that tests in the model fitted with BRBs have damping ratios higher than 5%; while tests on the bare model exhibited smaller values. This observation reinforces the previous findings in Figure 5-16, i.e. that BRBs increase the damping ratio significantly. Another important observation is that the application of the SCT-EW record also increased the value of the damping ratio. For example, comparing test results from B5 and B10 (Stage 1) or C1 and C6 (Stage 2), shows that the damping ratio increased from 5.4% to 6.3% and from 5.5% to 8%, respectively. These increases due to seismic input may be attributed to: 1) the increase of cracking in the concrete elements which generated higher energy dissipation when cracks opened
and closed; 2) stiffness degradation, which generated a more flexible system and larger displacements, increasing the energy dissipation; and 3) more energy dissipation on the BRBs due to more friction contact between the core and the case after the seismic action of the SCT-EW record - which may generate irregularities in the core.

![Variations of damping ratio in Model 2 when subjected to white noise input](image)

**Figure 5-17.** Variations of damping ratio in Model 2 when subjected to white noise input

### 5.4.3 Response to seismic input

In this part, two comparisons are presented:

1. Responses of Model 1 without BRBs but designed based on code [51] (tests series A) and Model 2 with BRBs (tests series B).
2. Responses of the intact and retrofitted Model 2 (i.e. tests series B and C).

#### 5.4.3.1 Effects of seismic intensity on natural frequencies and global stiffness

The natural frequencies of the two models were also estimated for the tests with the SCT-EW ground motion. The starting and ending parts of the acceleration records were used for the estimation to avoid the large oscillations that could contain nonlinear
effects. The selection of the two time ranges was the Arias Intensity [76] values that were smaller than 3% and higher than 97%, respectively.

In Figure 5-18a, the variations of the fundamental natural frequency for tests of Model 1 and Stage 1 of Model 2 are shown at different intensities of the SCT-EW record. For Model 1, no intensity of 50% was tested while the intensity of 100% was carried out twice. The estimated fundamental natural frequencies with low-intensity white-noise input are also shown at the beginning and end in Figure 5-18a. It is seen that Model 1 had a lower fundamental natural frequency for all the test intensities and exhibited stepped changes in the natural frequency for intensities of 100%, 150% and 200%, i.e. stiffness degradation was encountered at all the tested intensities in Model 1. However, in Model 2 the fundamental frequency was consistently higher for all the tested intensities and stepped changes were only observed for the intensities of 150% and 200%, i.e. stiffness degradation occurred only at the higher intensities.

The variations of the global stiffness, due to the SCT-EW record input, were estimated using equation (5-2) and are shown in Figure 5-18b. It was observed that Model 1 exhibited higher degradation of global stiffness, up to almost 60%. Model 2 exhibited a maximum degradation of 28%. For the intensity of 100%, Model 2 did not encounter degradation; whilst Model 1 exhibited a degradation compared to that of Model 2 subjected to the intensity of 200%. Some negative values are observed for Model 2 (especially for the intensity of 50%). These negative values indicate that the natural frequency and the global stiffness are intensity-dependent and the intensity of 50% produced a reduced demand in Model 2.
Figure 5-18. Effects of seismic intensity on the fundamental natural frequency and global stiffness of Models 1 and 2

Figure 5-19 compares the variations in fundamental natural frequency and global stiffness at Stages 1 and 2 of Model 2, i.e. the intact and retrofitted Model 2. The natural frequency of the retrofitted model is roughly 10% smaller at intensities of 50% and 100% but no clear differences are observed for higher intensities. Variations of the global stiffness were similar, however with smaller values in the retrofitted model. By comparing with the final values measured when subject to white noise, Figure 5-19b shows that the total stiffness degradation for the intact model was 28%, while 20% occurred in the retrofitted model.

Figure 5-19. Effects of seismic intensity on the frequency and global stiffness of Model 2
5.4.3.2 Effects of seismic intensity on damping ratio

Figure 5-20 shows the damping ratios estimated by balancing the equation of energy (see equation (4-1) in Chapter 4, Section 4.3). Damping ratios for low-intensity white noise input are also shown, at the beginning and at the end, for comparison purposes. For the SCT-EW record, the energy was balanced using the final part of the recorded accelerograms (that corresponding to a range of time where the Arias Intensity was higher than 97%). This selection was to avoid large oscillations which may contain nonlinear effects, i.e. the damping ratio was estimated for linear-elastic response of the models and the BRBs.

The results show that the damping ratio was consistently higher in Model 2. It is also observed that damping was intensity-dependent, i.e. the higher intensity input the higher the damping ratio in the models with and without the BRBs. Figure 5-20a shows that Model 1 started with $\xi=0.8\%$ and reached 5.8% while Model 2 (for Stage 1) started at 5.4% and reached a value of 10.3%. Figure 5-20b shows that damping ratio was similar in the intact and retrofitted Model 2, except for the final white noise tests where the damping ratios were 6.3% and 8%, respectively.

Figure 5-20. Effects of seismic intensity on the damping ratio of the models
5.4.3.3 Dynamic response to seismic input

In this section, the seismic response of the models to the SCT-EW record is compared. For simplicity, only the envelopes of the response, in terms of inter-storey drifts, lateral displacements, absolute floor velocities and accelerations, are presented.

First, the envelopes of the response to the SCT-EW record scaled at 100% are shown. Figure 5-21 shows that the inter-storey drift and lateral displacements of Model 1 (without BRBs) were roughly twice those of the intact Model 2 (with BRBs, Stage 1). The absolute floor velocities and accelerations were practically the same for both models. Figure 5-22 compares the response of the intact and retrofitted Model 2. The maximum inter-storey drift and displacement of the top floor were approximately 45% higher in the retrofitted model. Floor velocity and accelerations were similar in both cases.

![Figure 5-21. Envelopes of the response to the SCT-EW record at 100% in both models](image_url)
Figure 5-22. Envelopes of the response to the SCT-EW record at 100\% in Model 2

Next, the maximum values of the response for different intensities are presented and compared. Figure 5-23 shows the results for Model 1 and Stage 1 of Model 2. It is observed that the inter-storey drifts and lateral displacements were significantly smaller in Model 2. These differences increased with the increase in intensity. For example, at the intensity of 200\% the inter-storey drift and lateral displacement were 2.4 and 2.1 times higher in Model 1 than in Model 2. On the other hand, the absolute floor velocities and accelerations were similar in both models, being slightly smaller in the Model 2.
Figure 5-23. Peak response against input intensity of the SCT-EW records for both models

Figure 5-24 shows the responses for Stages 1 and 2 of Model 2. It is observed that inter-storey drifts and lateral displacements in the retrofitted model were higher at all intensities. The maximum differences were 45% at the intensity of 100%. However, at 200% the differences reduced significantly to 10%; which may be attributed to similar levels of cracking in the concrete elements at that intensity. Therefore, at very high levels of shaking, new and retrofitted structures tend to behave similarly and both are seen as equally reliable. Regarding floor velocities and accelerations, these parameters were similar in both cases; being slightly higher in the retrofitted model.
Figure 5-24. Peak response against input intensity of the SCT-EW records for Model 2

5.4.4 The damage observed in the models

At the end of all the tests, a visual inspection of damage was conducted on the two models. No significant damage was identified which would risk the stability of the models. Neither spalling of the concrete nor exposure of the steel reinforcement was observed. Typically, damage was only seen in storeys 1 and 2. It included small cracks in the beam-to-column connections (Figure 5-25). Small cracks were also observed at the base of the columns and in the joints of new and precast concrete. The topping of the floors only showed cracks at their intersection with the columns.
5.4.5 Comparison of the experimental results to numerical analyses

In order to compare the results of the experiment to numerical analyses, numerical models of the test buildings were developed using the finite element program Opensees [69]. The numerical models used in section 5.2.3 were updated with the properties of the materials estimated in Section 5.3.1. Then they were subjected to the same acceleration motions recorded on the test platform. The damping ratios, considered in the analyses, were those shown in Figure 5-20. Figures 5-26 and 5-27 show the measured and simulated displacements and the envelopes of the response of the two models to the SCT-EW record scaled at 100%. It can be seen that the numerical and experimental results are similar.

Figure 5-25. Typical damage observed following the tests
Additionally, it is highlighted from the numerical analyses that the tensile strength capacity of the concrete \( f_t \) plays an important role to estimate the displacement demands; especially for lower levels of seismic actions. Since \( f_t \) is directly related to the level of cracking in concrete elements, variations of this parameter affect the shape of the capacity curve, i.e. while the total capacity may be similar, different initial stiffness is observed as the value of \( f_t \) is varied. Therefore, the dynamic response of the models is affected for demands below the elastic limit. By considering this, a parametric study was conducted to determine the value of this parameter which was not determined experimentally. Equation (11.4) of the specifications for concrete design of the code [51] was used and is

\[
f_t = k \sqrt{f_c'}
\]

(5-3)

where \( f_c' \) is the resistant stress of the concrete in compression and \( k \) is a factor with a suggested value in the code between 0.47 and 0.53. However, different values of \( k \) were
evaluated to reproduce the response and stiffness degradation observed in the experimental models. It was found that $k$ was between 0.15 and 0.20.

**Figure 5-27.** Envelopes of the response for SCT-EW at 100%

### 5.5 Discussion

From the literature review it was observed that steel models, rather than reinforced concrete models, equipped with BRBs have been tested on shaking tables. Therefore, this chapter focuses on tests of two RC precast models subjected to low-intensity white noise and seismic input. For comparison purposes, one of the models was designed and provided with BRBs.
5.5.1 Tests with low-intensity white noise input

BRBs increased the damping ratio of the models. From Figures 5-16 and 5-17 it is observed that the damping ratio increased significantly when BRBs were introduced in Model 2. This finding agrees with the tests of Chapter 4. It also agrees with the increase reported by Vargas and Bruneau [26] from 2 to 5%, but differs from that by Kasai et al. [27] (i.e. the same damping ratio of 2% with and without BRBs for a white noise input). The increase of damping ratio from 1.7% to 2.3% reported by Yamaguchi et al. [29] on a single beam-column sub-assemblage could also be due to the effect of the BRB. Hikino et al. [30] reported a total damping ratio of 3% in a single-storey single-bay steel frame with two BRBs in chevron configuration. Since this value might be high for such a frame, the authors attributed the source of damping to friction in the testing system. However, a part of this damping might also be attributed to the effects of BRBs in agreement with the findings of this study. As a matter of interest, experimental evidence suggests that other types of hysteretic devices do not increase the damping ratio. Benavent-Climent and Escolano-Margarit [25] conducted shaking table tests on a single-storey single-bay steel frame using hysteretic dampers composed of steel plates. They reported the same values of damping with and without hysteretic dampers. Therefore, it can be established that, while BRBs increased the damping, those used by Benavent-Climent and Escolano-Margarit [25] do not. This may be attributed to the fact that BRBs possess a casing sleeve, which may dissipate energy by means of friction interaction with the core.

5.5.2 Tests with Seismic Input

a. Comparison of damping ratios of intact and retrofitted Model 2. When Model 2 was retrofitted in Stage 2 using a new set of BRBs, the damping ratios were similar to those found in Stage 1 (i.e. the intact Model 2) for all intensities tested
(Figure 5-20b). This means that the intact and the retrofitted models benefitted from similar increases of damping when equipped with BRBs.

b. Comparison of seismic response of Model 1 and Model 2 intact: From Figures 5-21 and 5-23, it is observed that the inter-storey drifts and displacements of the model without BRBs were roughly twice those of the model with BRBs. Absolute floor velocity and acceleration were almost the same in both models, being slightly smaller in the second one. This is, BRBs have not increased floor velocities and accelerations; which is significant because losses of contents (which are sensitive to these parameters and may cost more than the structure itself in conventional buildings [54]) may not increase due to the introduction of BRBs.

c. Degradation of global stiffness in Model 1 and Model 2 intact: it was observed that the frequency and global stiffness of the models degraded as the intensity of the ground motion increased. This is especially significant in short-period structures subjected to ground motions with longer dominant period of vibration, because degradation of stiffness could bring the structure close to the resonance zone. In this regard, Model 2 (with BRBs) exhibited better behaviour, because frequency and stiffness reductions were observed only at intensities of 150% and 200% with total stiffness degradation of 28%. On the other hand, Model 1 (without BRBs) encountered reductions of frequency and stiffness at all the intensities tested (100%, 150% and 200%) and reached a total stiffness degradation of almost 60%. This means that Model 2 not only had delayed stiffness degradation but also encountered half that of Model 1. This finding is especially significant for RC precast buildings because BRBs produce an additional benefit of reducing stiffness degradation.
d. **Assessment of damage**: It was observed that both models performed adequately. No residual displacements were encountered and no instability problems were observed at the end of the tests. However, as seen in the capacity curves of Figure 5-5, an over-strength factor close to 4.0 was estimated for both models, which is higher than the over-strength factor of 2.5, commonly assumed for structures in Mexico. This may be attributed to the fact that all the beams and columns were provided with the same quantity of steel reinforcement, which was required for the most highly stressed elements. Therefore, most of the elements had a higher capacity than their expected demands. However, even with high values of over-strength, the tests were still helpful to understand the behaviour of the two RC precast buildings when they were subjected to high-intensity ground motions, especially the variations in their dynamic properties and the effects of the BRBs. Finite element models confirmed that the experimental results are reasonably valid. Future work may focus on assessing the behaviour and damage of structures with reduced lateral load capacity and reduced over-strength factors.

e. **The intact and retrofitted cases of Model 2**: It was observed that the latter had: 1) lower natural frequencies; 2) 45% and 10% higher inter-storey drifts for the SCT-EW record scaled at 100% and 200%, respectively. The differences are attributed to accumulated stiffness degradation of the main frame; which increases with the seismic intensity. More significant differences of drifts and displacements were observed at the lower levels than at the higher levels of demand. Therefore, new and retrofitted structures may behave differently or similarly depending on the level of damage.
5.5.3 Reinforced Concrete Precast Structures

Reinforced Concrete (RC) precast structures are more convenient for builders and developers because they can be assembled quickly, and they have cost savings on formwork, materials and workforce [92]. However, RC precast structures have traditionally been viewed with scepticism in seismic zones [93], which has discouraged their use in such areas. Based on the findings and results presented in this chapter, it can be established that BRBs contribute significant improvements to RC precast buildings subject to seismic loading. Therefore, the combination of BRBs and RC precast structures present an interesting alternative to traditional resisting systems in seismic zones; i.e. more efficient structures can be constructed by exploiting the advantages of the RC precast systems while benefitting from improvements due to BRBs. This may also help to reduce scepticism of RC precast structures, because their response to earthquakes and stiffness degradation were significantly enhanced when BRBs were introduced.

5.6 Conclusions

Shaking table tests were conducted on two four-storey, RC precast models. The results of the tests enabled three sets of comparisons to be made, which were: 1) behaviour of the models which were designed using different methods; 2) behaviour of Model 2 with and without BRBs; and 3) behaviour of Model 2 (intact and retrofitted) with BRBs. The fundamental natural frequencies, damping ratios and seismic response of the models were examined. The main conclusions are:

1. Tests on individual BRBs members showed their high energy dissipation capacity, with slightly higher capacity in compression than in tension; which has also been observed by other researchers (e.g. [4-7]).
2. In addition to the increase of the lateral load capacity and fundamental natural frequency, the inclusion of BRBs in the RC precast models, increased the damping significantly when the models were subjected to both low-intensity white noise input and high-intensity ground motions. This observed performance indicates that BRBs can effectively improve the behaviour of RC precast structures used in earthquake-prone zones, in particular in Mexico City.

3. BRBs helped to delay and reduce the stiffness degradation in the RC precast models; which is significant for short-period structures subjected to ground motions with longer dominant period of vibration, to avoid resonance effects.

4. Inter-storey drifts and lateral displacements in Model 1 (without BRBs) were twice those in the Model 2 (with BRBs). Absolute floor velocities and accelerations were similar for both models.

5. The Model 2 retrofitted had lower fundamental natural frequency and higher lateral displacements than the Model 2 intact due to some damage on the frame. This observation indicates that new and retrofitted structures may behave differently depending on the level of damage in the concrete elements.

6. The reductions of the inter-storey drifts and lateral displacements, when BRBs are included in RC precast structures, suggests that structural and non-structural damage may be significantly reduced. Since floor velocities and accelerations were similar, losses of contents might be similar, with and without BRBs.

7. For both models, no residual deformation and no significant damage was observed. However, a high over-strength factor of 4 (compared to the common value of 2.5 for structures in Mexico) was observed.
Chapter 6

Improving the Seismic Performance of Hospitals Located in the Lakebed of Mexico City using Buckling-Restrained Braces

6.1 Introduction

In this chapter, numerical analyses are carried out in a hypothetical existing structure, representative of hospitals located in the lakebed zone of Mexico City. First, in Section 6.2 the performance of a conventional hospital is assessed using Incremental Nonlinear Dynamic Analysis with multiple earthquake records. Then, in Section 6.3 a proposal of upgrading the hospital using Buckling-Restrained Braces (BRBs) is presented in order to explore the benefits of using these devices. The results, presented in Section 6.4, show that BRBs increase the lateral load capacity and reduce significantly the dynamic response. It is also observed that BRBs reduce the probability of collapse and of loss of functionality; which are of paramount importance in this type of facilities. Section 6.5 presents a parametric study conducted in conventional and dual SDOF systems in order to determine the period range on which existing hospitals, located in the lakebed zone of
Mexico City, may be benefitted from being upgraded using BRBs. Conclusions are formulated at the end in Section 6.6.

6.2 Design of Typical Hospitals in Mexico City

Typical hospitals in Mexico City consist of steel or reinforced concrete (RC) frames in two orthogonal directions. The majority of them are six storeys high or less; therefore, they may be considered as low-rise buildings with dynamic response dominated by the fundamental mode. Another characteristic of the hospitals is that they are typically longer in one horizontal direction than in the other [55].

For seismic design, the Mexico City Building Code [51] and its complementary specifications require providing hospitals with 50% more lateral load capacity than conventional structures, i.e. an importance factor of $I=1.5$ shall be used. The Life Loads to be considered are 1.8 kN/m$^2$ on floors and 0.7 kN/m$^2$ on roofs. These loads shall be added to the Dead Loads of the structure; which shall include the self weight of the structural elements and other permanent loads.

A significant number of hospitals have been designed and analysed in order to assess the typical response of hospitals located in the lakebed zone of Mexico City. For simplicity, this chapter presents the results of a representative six-storey six-bay RC frame in 2D (Figure 6-1).

With the spectrum provided in the code and a fundamental period of $T=0.89$ s (determined using a numerical model in SAP2000 [94] and considering cracked sections), the total base shear of design is $V=1644$ kN. This included an importance factor of $I=1.5$, a behaviour factor of $Q=2$ and the following floor masses: 80.4 ton in the roof and 83.2 ton in the other floors.
The columns and beams of the frame are designed to resist the base shear as required by the code \[51\] and its complementary specifications. The cross-sections are presented in Figure 6-2. The floor system is assumed rigid and composed of RC slabs with a thickness of 120 mm. The nominal resistance of the concrete is $f_c' = 35$ MPa and the nominal yielding stress of the steel reinforcement is $f_y = 420$ MPa.
In order to assess the seismic behaviour of the example hospital, numerical models are developed using Opensees [69]. The models are subjected to static and dynamic nonlinear analyses. The results are presented and compared later in section 6.4.

### 6.3 Upgrading Typical Hospitals with BRBs

It is recommended that BRBs provide small increase of the lateral load capacity (say less than 30%) in order to avoid damages in existent elements such as connections and foundations. Furthermore, to minimize the axial loads in the existent columns, brace configurations recommended by Ji and Bell [95] would be convenient and effective.

For the example hospital presented in this chapter, a proposal of upgrading is shown in Figure 6-3. The method proposed in Chapter 3, which is based in the control of the lateral displacements, is applied to designing the upgrading with BRBs. An increase of 15% of the lateral load capacity was decided in order to avoid large demands in the existent elements and in the new brace-to-frame connections.

A stiffness factor of \( f_k = 4.0 \) was selected for all the BRBs. Steel S275 with nominal yielding stress of 275 MPa, is considered for their core. The resulting cross-sectional areas of the BRBs, in \( \text{mm}^2 \), are 732, 616, 500, 385, 231, and 112, for storeys 1 to 6, respectively.

Similar to the bare frame, numerical analyses are conducted in Opensees [69] to assess the behaviour of the upgraded structure. The results and comparisons are presented in the next section.
6.4 Improvements due to the Inclusion of BRBs

6.4.1 Lateral load capacity

In order to assess the lateral load capacity of the example hospital before and after being upgraded, pushover analyses are conducted in Opensees [69]. Fully fixed bases are assumed. Expected rather than nominal properties of the materials are used. The Mander concrete model and the Giuffre-Menegotto-Pinto steel model are considered. For the BRBs, the parameters of the steel model, calibrated in Chapter 3, are used to represent their experimental behaviour, i.e. increase of capacity with the increase of strain deformation. The brace-to-frame connections are modelled as pinned.

Figure 6-4 shows the lateral load capacity curves obtained from pushover analyses of the example structure. It is appreciated in the figure that the BRBs increases the strength capacity by approximately 15%, as originally intended. For illustration purposes, the shear capacity required by the code is also shown in the figure. The total load capacity of the bare frame, at maximum displacement, is 2.4 times higher than that
required by the code. After being upgraded, this overstrength is close to 3. This agrees well with the overstrength factor commonly accepted for structures in Mexico City [51].

![Figure 6-4. Capacity curves before and after upgrading](image)

### 6.4.2 Dynamic response

The numerical models are subjected to the SCT-EW record of the 19/19/1985 Michoacán, Mexico Earthquake. Figure 6-5 shows the inter-storey drift demands on the example frame in Storey 2; where the maximum demand is observed. A reduction of the maximum demand of 40%, due to the BRBs, is shown.

![Figure 6-5. Inter-storey drift demands to the SCT-EW seismic record](image)

Figure 6-6 shows the displacements, floor accelerations and floor velocities at the top floor of the structure before and after been upgraded. It is appreciated that the maximum displacements reduce in similar proportions to the inter-storey drifts.
However, the floor accelerations and velocities are similar in both cases with and without BRBs.

![Figure 6-6. Demands at the top floor to the SCT-EW seismic record](image)

### 6.4.3 Incremental dynamic analysis (IDA)

The numerical models are subjected to the 30 ground motions shown in Appendix B, which were recorded in the lakebed zone of Mexico City. They are scaled to reach peak ground accelerations ($pga$) between $0.05g$ and $1.4g$ within increments of $0.05g$; where $g$ is the acceleration of the gravity. Collapse is assumed when a small increment on seismic intensity generates a large (unrealistic) increment on lateral displacement or when the computer program shows numerical instability.

Figure 6-7 shows the incremental dynamic analysis (IDA) curves of the example structure before and after being upgraded. The horizontal axis shows the maximum inter-storey drift demand, while the vertical axis gives the seismic intensity, i.e. $pga$. 

167
(normalized by $g$). The mean of the demands are shown with dark lines. Comparing Figures 6-7a and 6-7b, it is appreciated that the structure upgraded with BRBs can accommodate significantly higher levels of seismic intensity.

![Figure 6-7. IDA curves of the maximum inter-storey drift demands](image)

Figure 6-8 provides further comparison of the behaviour of the example structure with and without BRBs. The left column of the figure shows the mean of the demands, while the right column shows the reductions due to the upgrading with BRBs. It is observed that the upgrading allows reductions of drift and displacement demands between 40% and 70%. Floor velocities are reduced less than 20%. On the other hand, floor accelerations are reduced significantly for $pga < 0.5g$ and negligible for higher values of $pga$. 
Chapter 6. Improving the Seismic Performance of Hospitals with BRBs

6.4.4 Probability of collapse

The probability of collapse, conditioned to the occurrence of a $pga$ value, is estimated by dividing the number of ground motions that generated collapse by the total number
of analyses, i.e. 30. Figure 6-9 shows the cumulative distribution functions of the probability of collapse (or collapse fragility) of the example structure before and after being upgraded. These functions only contain the record-to-record variability and no other sources of uncertainty (such as quality of construction or modelling completeness) are considered. Before upgrading, the seismic intensity with 50% of probability of collapse was 0.75g. This increased to 1.0g after upgrading. In other words, for a $pga=0.75g$, collapse is likely to occur in the bare frame and unlikely after upgrading.

![Figure 6-9. Collapse fragility function before and after upgrading](image)

6.4.5 Probability of loss of functionality

For simplicity, in this chapter the loss of functionality, conditioned to a $pga$ value, is associated to the probability that a structure is deemed unsafe. For that purpose, it is determined as the number of records that generate residual drifts (RDs) higher than 0.005, divided by the total number of seismic records, i.e. 30. It is recognized that the loss of functionality is affected by different components such as non-structural elements and contents [31]; however, for simplicity only loss of functionality due to RDs is considered here.
First, the RDs were determined for the example hospital before and after it had
been upgraded. The results of the example structure are shown in Figure 6-10 along
with the corresponding reductions due to the inclusion of the BRBs. It is worth noting
that the RDs are significantly reduced for $pga > 0.1g$.

![Figure 6-10. Residual displacements before and after upgrading](image)

Figure 6-11 shows the cumulative distribution functions of the probability of
loss of functionality of the example structure. It is appreciated in the figure that the
seismic intensity at which the structure has a probability of 50% of losing its
functionality before upgrading is $pga = 0.6g$. This value changes to $pga = 0.8g$ after
upgrading. This means that, for $pga = 0.6g$, the loss of functionality is likely in the
structure without BRBs but unlikely when BRBs are introduced.

![Figure 6-11. Probability of loss of functionality](image)
6.5 Parametric Study on Dual SDOF Systems

A parametric study is conducted on SDOF structures in order to know the period range in which structures are benefited from being upgraded using BRBs. The structures were assumed to be existing hospitals located in the lakebed zone of the Mexico City and had an initial lateral capacity as required by the Mexico City Building Code [51]. The spectrum of lateral capacity \( V_{y1} \), normalized by the weight of the structure \( W \), is shown in Figure 6-12. This capacity was determined considering an importance factor of \( I=1.5 \), a seismic behaviour factor of \( Q=2 \) and an overstrength factor, \( R \), with values between 2 and 2.5. The precise value of \( R \) is determined as recommended in Appendix A of the code specifications for seismic design [51] and depends on the fundamental period of the structure.

![Figure 6-12. Spectrum of lateral capacity considered in the parametric study.](image)

Conventional and dual SDOF oscillators are subjected to the 30 seismic ground motions of Appendix B. The records are scaled at different values of \( pga \) to simulate different seismic intensity levels. For illustration purposes, the elastic pseudo-acceleration spectra of the records, along with their mean and dispersions, are shown in Figure 6-13. It is appreciated that the records have a predominant period of vibration of
Two period ranges are identified, namely: short-period range (for oscillators with period of vibration of 2 s or less) and long-period range (oscillators with period longer than 2 s). Since providing BRBs in existing structures reduces their period of vibration (i.e. the stiffness of the structure is increased), it can be observed from the figure that providing BRBs to short-period structures might be beneficial because their period moves away of the resonance zone (of 2 s). On the other hand, providing BRBs in long-period structures might be detrimental, because their period is moved towards the resonance zone. The following subsections of this chapter are developed to corroborate these statements.

**Figure 6-13.** Elastic pseudo-acceleration spectra for conventional SDOF oscillators generated by the ground motions used in the parametric study.

First, dynamic nonlinear analyses are conducted in elastic-perfectly plastic conventional SDOF oscillators. Then, they upgraded with BRBs. Three different increments of the lateral resistance by the BRBs are studied, namely: $\Delta V_{y1} = 5\%$, $\Delta V_{y1} = 15\%$, and $\Delta V_{y1} = 30\%$. Another parameter, which is studied here because affects the response, is the ductility ratio, $\mu_1 / \mu_2$, where $\mu_1$ is the ductility of the bare oscillator and $\mu_2$ is that of the BRBs.
6.5.1 Effects of BRBs on maximum displacement demand

Figure 6-14 shows the spectra of maximum displacement demands for the structures without BRBs (discontinuous line) and with BRBs (continuous lines). It is corroborated in the figure that:

- BRBs reduce the displacement demands for periods below 2 seconds; which is the predominant period of the earthquake ground motions.
- For structures with periods higher than 2 s, the displacement demands increase exponentially with the period and may become similar or even higher than those of conventional structures.
- The reductions are especially significant for $\Delta V_{y1}=15\%$ and $30\%$.
- The ductility ratio ($\mu_1 / \mu_2$) affects the displacement demands significantly, i.e. the smaller the ratio, the smaller the displacement demands.
- The effects of the BRBs on the displacement demands are more significant for the smaller values of $pga$ than for the higher values.

6.5.2 Effects of BRBs on velocity and acceleration demands

Figures 6-15 and 6-16 show the spectra of absolute velocity and acceleration demands, respectively. Again, it is corroborated that for structures with periods smaller than 2 seconds, BRBs reduce the velocity and acceleration demands significantly. For periods larger than 2 seconds, significant amplifications of the demands may occur and exceed those of conventional structures. The ductility ratio ($\mu_1 / \mu_2$) also affects the demands; the smaller the ratio, the smaller the velocity and acceleration demands. The effects of BRBs on the dynamic demands are more significant for the smaller values of $pga$. In fact, for $pga=0.70g$ the effects of the BRBs are almost negligible.
\[ \Delta V_{y1} = 5\% \quad \Delta V_{y1} = 15\% \quad \Delta V_{y1} = 30\% \]

\[ p_{ga} = 0.032g \]

\[ p_{ga} = 0.16g \]

\[ p_{ga} = 0.35g \]

\[ p_{ga} = 0.70g \]

**Figure 6-14.** Spectra of maximum displacement demands
\[ \Delta V_{y1} = 5\% \quad \Delta V_{y1} = 15\% \quad \Delta V_{y1} = 30\% \]

Figure 6-15. Spectra of absolute velocity demands
Chapter 6. Improving the Seismic Performance of Hospitals with BRBs

\[ \Delta V_{y1} = 5\% \quad \Delta V_{y1} = 15\% \quad \Delta V_{y1} = 30\% \]

<table>
<thead>
<tr>
<th>PGA</th>
<th>Spectra of absolute acceleration demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.032g</td>
<td><img src="image1" alt="Spectra" /></td>
</tr>
<tr>
<td>0.16g</td>
<td><img src="image2" alt="Spectra" /></td>
</tr>
<tr>
<td>0.35g</td>
<td><img src="image3" alt="Spectra" /></td>
</tr>
<tr>
<td>0.70g</td>
<td><img src="image4" alt="Spectra" /></td>
</tr>
</tbody>
</table>

Figure 6-16. Spectra of absolute acceleration demands

6.6 Conclusions

Numerical analyses, with and without BRBs, were conducted in hypothetical hospitals located in the lakebed zone of Mexico City. The following conclusions are formulated:

1. The dynamic responses of typical hospitals in Mexico City are effectively reduced when BRBs are introduced. In the hospital example, the lateral displacements and
inter-storey drifts were reduced between 40% and 70%. Floor velocities were reduced by 20% or less. Floor accelerations were reduced between 20% and 40% for \( pga < 0.5g \). Reductions in residual drifts were also very high (up to 90%).

2. The probabilities of collapse and of loss of functionality were also reduced significantly due to the BRBs.

3. From the parametric study on conventional and dual SDOF structures it was observed that short-period structures (as defined in Figure 6-13) are highly benefitted by BRBs, while the effects of BRBs in long-period structures may be adverse. Therefore, BRBs shall preferably be used in structures with periods smaller than the predominant period of the soil.

4. The ductility ratio \( (\mu_1 / \mu_2) \) has a significant effect on the response parameters, i.e. the smaller the ratio, the better the dynamic response.
Chapter 7

Residual Displacements in Conventional and Dual Structures

7.1 Introduction

Although overall structural damage may be insignificant, the economic impact of structures with large residual displacements (RDs) after earthquakes may be huge. This has recently been highlighted by many researchers (e.g. [75, 96-97]). Therefore, this chapter focuses in understanding the parameters affecting RDs in conventional and dual single-degree-of-freedom (SDOF) systems and recommends alternative ways to reduce or mitigate them. For that purpose, conventional and dual SDOF systems were subjected to 220 ground motions characteristic of the lakebed zone of Mexico City. Different parameters were studied. For example, for conventional systems, post-yielding stiffness ratio proved to be the most important parameter because affected RDs significantly. For dual systems, it was observed that RDs may be small when the primary part remains elastic. However, if the primary part exhibits inelastic response, RDs are increased
dramatically. In this case, the post-yielding stiffness ratio of the secondary part plays a key role. Conclusions and recommendations have been formulated and may be applicable to structures whose behaviour is not significantly affected by higher modes.

### 7.2 Residual Displacements in Conventional SDOF Oscillators

Previous studies have identified the diverse factors affecting residual displacements in SDOF oscillators, e.g. [97-99] (Figure 7-1). They include variations in period of vibration, post-yielding stiffness ratio, ductility ratio, strength reduction factor and hysteretic response. In this section these parameters are investigated in order to understand their effects on RDs of structures subjected to ground motions characteristic of the lakebed zone of Mexico City. Damping ratio and transition of the response (from elastic to plastic) are also investigated because they have not been addressed by previous studies. Since the seismic ground motions of the lakebed zone of Mexico City correspond to far-field and long-duration records, variations in source-to-site distance and duration are not investigated here.

![Figure 7-1. Factors affecting residual displacements](image-url)

Figure 7-1. Factors affecting residual displacements
7.2.1 Effects of post-yielding stiffness ratio

Post-yielding stiffness ratio \( r \), i.e. the ratio of the post-elastic stiffness to the elastic stiffness, has been regarded as one of the most important parameters affecting the amplitude of RDs [100]. Therefore, it is pertinent to study its effect on RDs of structures located on soft soils. For instance, Figure 7-2 shows the displacement time-history response of conventional oscillators subjected to the East-West component of the record gathered at SCT station during the September 19, 1985, Michoacan earthquake. Three values of \( r \) were modelled in each oscillator \((r=-5\%, 0\%, \text{and } 5\%)\). The period of vibration, viscous damping ratio, and yield strength coefficient \( C_y \) (i.e. yield lateral strength normalized with respect to the weight), of the oscillators were chosen as 0.5s (considered as representative of a short-period structure) 5\%, and 0.16, respectively. From Figure 7-2, it can be observed that \( r \) affects significantly the amplitude of RDs, which is consistent with previous studies (e.g. [98]).

![Figure 7-2. Response to the SCT ground motion for different post-yielding stiffness ratios](image)

Now, it is recognised that the response to a single motion may be misleading. Therefore, central tendency and dispersion of the residual displacements to many records could be better descriptors of the effect of post-yielding stiffness ratio. To be consistent with [99, 101], the sample mean is calculated by averaging the result data while the coefficient of variation is determined as the standard deviation divided by the
sample mean. Figure 7-3 shows the mean and coefficient of variation of the RDs, normalised by the peak transient displacements for three elastoplastic oscillators with different post-yielding stiffness ratios. The oscillators had periods of vibration of \(0.25T_g\), \(0.5T_g\) and \(0.75T_g\); where \(T_g\) is the dominant period of vibration of the ground motions and is about 2 seconds. In this way, the periods of the oscillators correspond to around 0.5, 1.0 and 1.5 seconds, respectively. In other words, they represent short-period structures. A damping ratio of 5% was considered for the three cases. The response was estimated for a target maximum ductility of 2.0 and using the 220 ground motions of Appendix B (see Table B-2). It should be noted that, instead of the 30 ground motions used in other chapters, in this chapter 220 records were used because RDs present higher variability [99]. It is apparent from Figure 7-3 that the residual displacements are highly dependent on the post-yielding stiffness ratio – and the higher the post-yielding stiffness ratio, the smaller the residual displacements. Especially, the values higher than (say) 5% or 10% seem to be appropriate to reduce RDs. On the other hand, the coefficient of variation, as a measure of dispersion, increased significantly as the post-yielding stiffness ratio increased, reaching values higher than 0.6.

![Figure 7-3](image-url)

**Figure 7-3.** Mean and dispersion of residual displacements on conventional oscillators in very soft soils
7.2.2 Effects of period of vibration

Conventional SDOF oscillators with a range of periods between $0.2T_g$ and $3T_g$ were studied using zero post-yielding stiffness (i.e. $r=0\%$). Damping ratio of 5\% and target maximum ductility demand of 2.0 were considered. The oscillators were subjected to the 220 ground motions described previously. Figure 7-4 shows the mean of the residual displacements, normalised by the maximum transient displacements, and the dispersion obtained from the analyses. It is observed that the period of vibration affected the mean of the RDs. From Figure 7-4, three spectral regions can be identified: 1) short-period range (less than about $0.7T_g$), 2) periods close to the dominant period of the ground motions (between $0.7T_g$ and $1.5T_g$), and 3) long-period range (longer than about $1.5T_g$).

In the short-period range, there is a tendency of RDs to increase as the period decreases. The minimum RDs are observed for periods close to $T_g$. On the other hand, in the long-period range the RDs tend to be constant.

Regarding the dispersion, the coefficient of variation was very sensitive to the period of vibration with most of the values between 0.5 and 0.6. No clear tendency is appreciated.

Figure 7-4. Effects on residual displacements of period of vibration
7.2.3 Effects of hysteretic response

Now, the same oscillators of the previous subchapter were studied using three different hysteretic responses (Figure 7-5): a) bilinear, b) Takeda [102], and c) Flag-shaped [64]. For the Takeda response (Figure 7-5b), the parameters $\delta=0.0$ and $\gamma=0.4$ (defined in the figure) were considered – which are commonly accepted values for reinforced concrete framed structures [103]. For the flag-shaped (Figure 7-5c), the value of parameter $\beta=0.7$ (also defined in the figure) was assumed as a representative value of self-centring systems [97]. It is important to remark that bilinear response is typically representative of steel structures, the Takeda response is representative of concrete structures and the flag-shaped response is typical of innovative structural systems with self-centring capacity (see section 5.11 in [64]). In the three cases, zero post-yielding stiffness ratio ($r=0\%$), damping ratio of $5\%$ and target ductility demand of $2.0$ were considered.

![Figure 7-5. Types of hysteretic response](image)

Figure 7-6 shows the results of the analyses. It is observed that the type of hysteretic response affects RDs significantly. The bilinear hysteretic response presented the highest residual displacements while the flag-shaped response presented very small...
values (less than 5%) - which may be regarded as negligible. The Takeda response was somehow in the middle.

There were no clear effect of the hysteretic response in the dispersion; however, it is observed that the Takeda response presented higher coefficients of variation than the bilinear response. Very high dispersion was observed for both, the bilinear and Takeda responses with most of the values between 0.5 and 0.8. The coefficients of variation of the flag-shaped response are not shown in the figure because their residual displacements were very small, and there was no point in estimating their dispersion.

![Figure 7-6. Effects of hysteretic response on residual displacements](image)

7.2.4 Effects of maximum displacement ductility

The analysis of elastic-perfectly plastic behaviour is first presented (i.e. $r=0$), followed by that with post-yielding stiffness ratio different of zero. This section is organised in this way because the trends of RDs are different when $r=0$ and when $r\neq0$.

7.2.4.1 SDOF oscillators with elastic-perfectly plastic behaviour

The effect of ductility, as a measure of the magnitude of the peak inelastic displacement demand, is analysed in this section. To this end, the same oscillators of the previous
section (with \( r = 0\% \)) were subjected to the same ground motions for ductility demands of \( \mu = 1.5, 2, 3, 4 \) and 6. Figure 7-7 shows the mean and dispersion of the residual displacements. As it is observed, the higher the ductility demands, the higher the mean residual displacements. It is also appreciated that RDs are reduced for normalised periods close to one. On the other hand, the dispersion appears to be insensitive to the level of ductility demand but very sensitive to the period of vibration. Most of the coefficients of variation are very high with values between 0.5 and 0.6.

![Figure 7-7. Effect of ductility in residual displacements](image)

### 7.2.4.2 SDOF oscillators with post-yielding stiffness ratio different of zero

Now, in order to evaluate the variations of RDs with the ductility demand when \( r \) is different from zero, one oscillator with period of vibration of \( 0.25T_g \) was analysed. This period was selected as representative of short-period structures (i.e. \( T \approx 0.5s \)), on which RDs tend to be large. The oscillator was subjected to the same 220 ground motions describer earlier considering a damping ratio of 5\% and ductility demands of 2, 4 and 6. The post-yielding stiffness ratio, \( r \), was varied from -10 to 10\%. The results are shown in Figure 7-8 where it is observed that, opposite to the case of \( r = 0\% \) (previous Section
Chapter 7. Residual Displacements in Conventional and Dual Structures

7.2.4.1), RDs do not always increase as the ductility demand increases. In fact, in this example, for post-yielding stiffness ratios greater than 2.5% the RDs decreased as the ductility demand increased. The opposite was true for $r < 2.5\%$. It is worth noting that the dispersion increased significantly as the value of $r$ increased and the RDs decreased. Although not reported here, similar results are observed for other periods of vibration. The results of Figure 7-8 show the importance of the post-yielding stiffness ratio on residual displacements because $r$ is capable of inverting the effects of the ductility, $\mu$, on RDs. This is, while RDs increase with ductility for $r<2.5\%$, they decrease with ductility for $r>2.5\%$.

Figure 7-8. Effect of ductility and post-yielding stiffness ratio on residual displacements

7.2.5 Effects of lateral strength reduction factor

The lateral strength reduction factor ($R_y$) is defined as the ratio of the strength required to maintain an SDOF oscillator elastic to its yielding load capacity. Therefore, $R_y$ is a measure of the lateral yielding load capacity of the structure relative to the ground motion intensity [104]. In mathematical terms:
\[ R_y = \frac{V_e}{V_y} = \frac{m \omega^2 d_e}{V_y} \]  (7-1)

where \( V_e \) is the lateral load capacity required to maintain an SDOF oscillator elastic, \( V_y \) is the lateral yielding load capacity of the system, \( m \) is the mass, \( \omega \) is the circular frequency of vibration and \( d_e \) is the corresponding spectral elastic displacement.

### 7.2.5.1 SDOF oscillators with elastic-perfectly plastic behaviour

Elastic-perfectly plastic SDOF oscillators (i.e. \( r=0\% \)) with period of vibration between 0.2\( T_g \) and 3\( T_g \) and 5\% damping ratio were subjected to the 220 ground motions defined previously. Mean and dispersion of the residual displacements were estimated for different lateral strength factors (\( R_y = 1.5, 2, 3, 4 \) and 6). The convention of Ruiz-Garcia and Miranda [104] for strong systems (\( R_y \leq 3.0 \)) and weak systems (\( R_y > 3.0 \)) is adopted here.

Figure 7-9 shows the mean and dispersion of the residual displacements for the different lateral strength reduction factors. The effect of the lateral strength factors in the mean of the residual displacements is evident, the higher the strength reduction factors, the higher the residual displacements. Besides, strong systems (\( R_y \leq 3 \)) were more sensitive to variations of \( R_y \) than weak systems. Also note that, for periods close to unity, RDs are significantly reduced. Regarding the dispersion, the coefficients of variation were very high with most of the values located between 0.7 and 1.2. There were no clear tendencies in the dispersions.

On the other hand and since peak displacements in the short-period range (less than 0.7\( T_g \)) tend to be very high when they are estimated using constant strength reduction factors [99], additional analyses shall be conducted to understand the
tendency in this spectral zone. The following ratio, as defined by Ruiz-Garcia and Miranda [104], is used:

\[ C_r = \frac{d_r}{d_e} \]  

(7-2)

where \( d_r \) is the residual displacement demand and \( d_e \) is the peak elastic displacement.

![Figure 7-9. Effect of lateral strength factors in residual displacements](image)

Figure 7-10 shows the mean and dispersion of \( C_r \) for different values of \( R_y \). It is observed that \( C_r \) is very sensitive to the period of vibration. Especially, it is very high in the short-period range, reaching values higher than 1.0; which means that the residual displacements were higher than the elastic displacement demands. Again, strong systems (\( R \leq 3 \)) were more sensitive to \( R_y \) than weak systems.
As a preliminary conclusion, for elastic-perfectly plastic oscillators, the higher the lateral strength factor, the higher the residual displacements. There was no clear tendency in the dispersion; however, the coefficient of variation of \( C_r \) was very high with most of the values between 0.8 and 1.2.

### 7.2.5.2 SDOF oscillators with post-yielding stiffness ratio different from zero

Now, in order to identify the effects of strength reduction factors (\( R_y \)) on RDs when \( r \) is different from zero, a 5%-damped oscillator with a period of vibration of 0.25\( T_g \) was analysed for \(-2.5% \leq r \leq 10\%\) and for \( R_y=1.5, 2, 3, 4 \) and 6. Note that the negative value of \( r \) was limited to \( r=-2.5\% \) because very high values of \( C_r \) were appreciated in the negative range. The oscillator was subjected to the same 220 ground motions described in previous sections. The mean and dispersion of parameter \( C_r \) (as defined in the previous section) were estimated and are shown in Figure 7-11. As it is observed, the effects of \( R_y \) on RDs are highly affected by \( r \). For \( r \leq 0 \), the higher the values of \( R_y \), the higher the RDs were. A transition interval is observed between \( r=0\% \) and 2\% - expect...
for $R_y=1.5$ which needs a longer transition interval (between 0% and 6%). For values of $r$ greater than (say) 2%, the higher the values of $R_y$, the smaller were the RDs. Regarding the dispersion, no clear tendency was distinguished for the coefficient of variation of $C_r$. However, for $r>0$ values around 0.9 were observed. On the other hand, for $r\leq 0$ two trends may be appreciated: a) for $R_y\geq 3$ the dispersion tends to zero – which may be attributed to the fact that, for almost all the seismic records, residual displacement demands are very large and close to the maximum inelastic displacement demands; and b) for $R_y<3$ the dispersion tends to be higher – which is attributed to higher uncertainty in the estimation of $C_r$ in that interval.

It is worth noting that small increments of $r$ have a significant impact in RDs. This agrees with observations by [99, 105], where increments of $r$ from 1% to 3% produced significant reductions of RDs. In general, it can be said that values of (say) $r>5\%$ may be convenient to ensure small amplitudes of residual displacements. Therefore, recommendations proposed by Pettinga et al. [100] to reduce RDs can also be effective in structures subjected to excitations characteristic of the lakebed zone of Mexico City.

![Figure 7-11. Effect of strength reduction factor and post-yielding stiffness ratio](image)
### 7.2.6 Effects of damping ratio

The effects of damping ratio are investigated because, according to the experiments presented in Chapters 4 and 5, it has been observed that BRBs tend to increase the damping when fitted in structures. In this context, a range of SDOF oscillators are subjected to the same 220 ground motions described in previous sections. An elastic-perfectly plastic model is used with a target ductility demand of 2. Damping ratios of 2%, 5%, 10% and 20% are simulated. On the other hand, since damping ratio also affects peak transient displacements significantly, in this section it is decided to normalise the RDs by the 5%-damped maximum displacements. In this way, RDs determined with $\xi=2\%, 5\%, 10\%$ and 20% can be compared directly.

The results, depicted in Figure 7-12, show that the mean of the residual displacements is affected by variations in the damping ratio. The mean of the RDs is reduced as the damping ratio is increased.

Regarding the dispersion, it can be said that the coefficient of variation is almost unaffected by the variations of the damping ratio. No clear tendency is appreciated however values between 0.5 and 0.6 are appreciated.

![Figure 7-12. Effect of damping ratio on residual displacements](image)

**Figure 7-12.** Effect of damping ratio on residual displacements
7.2.7 Effects of transition from elastic to plastic response

The effects of the transition from elastic to post-elastic response are analysed here. Using a simplified version of the Bouc-Wen model [106], sharp, smooth and very smooth transitions – which correspond to yielding exponents of $n=100$, $7$ and $3$ - are modelled. Figure 7-13 shows the three types of transitions modelled in this study. A target ductility demand of 2 was considered for the three cases. A post-yielding stiffness ratio of $r=5\%$ was selected intentionally. This will be discussed later.

Using the 220 ground motions described in previous sections, the mean of the residual displacements, normalised by the maximum displacements, were determined and are shown in Figure 7-14. It is apparent that:

- RDs are affected by the type of transition from elastic to plastic response in the whole period range. The very smooth transition ($n=3$) presented smaller residual displacements than the sharp transition ($n=100$). The smooth transition ($n=7$) was somehow in the middle.

- RDs are significantly reduced at normalised periods close to unity (note that these reductions are more significant for $r=5\%$ than for $r=0$ as observed in previous sections).

- The dispersion was very high in the three cases, with higher values for the smooth and very smooth transitions. Except for normalised periods close to unity (which presented the highest dispersions), the majority of the values of the coefficient of variation were around 0.8.
Figure 7-13. Elastic to plastic transition with post-yielding stiffness ratio of 5%

Figure 7-14. Effect on residual displacements of transition from elastic to plastic response

Now, in order to assess variations on the effects of the type of transition due to $r$, RDs were determined in an oscillator with period of vibration of $0.25T_g$, damping ratio of 5% and target ductility demand of 2. Figure 7-15 shows the results for post-yielding stiffness ratios between -0.1 and 0.1. As it is observed, the transition type is not significantly influenced for post-yielding stiffness ratios close to zero. However, it is highly influenced for post-yielding stiffness ratios different from zero – especially for negative values. In the positive range, the very smooth transition type presented smaller residual displacements while the opposite was true in the negative range.
As a preliminary conclusion it can be said that the type of transition from elastic to plastic response affects the RDs significantly when post-yielding stiffness ratios are different from zero.

![Graph showing influence of post-yielding stiffness ratio in the type of transition](image)

**Figure 7-15.** Influence of post-yielding stiffness ratio in the type of transition

### 7.3 Residual Displacements in Dual SDOF Oscillators

In this section, the residual displacements of dual systems are studied because, as noted in Chapter 3, the dynamic response is affected by the characteristics of the parts that constitute the system. As it is going to be seen, RDs depend highly on where of the three zones, defined by Figure 7-16, the maximum displacement demand is located. In zone 1 (defined by the square dot), RDs are null because both parts of the system present elastic deformation and they have enough restoring capacity to return the system to its un-deformed state. In zone 2 (defined by the circular dot), RDs are present because plastic deformation is present in the secondary part. Here, the magnitude of RDs depends on the elastic properties of the primary part and the elastic-plastic properties of the secondary part - their behaviour of RDs is similar to that of conventional oscillators with a high value of post-yielding stiffness ratio. Finally, in
zone 3 (defined by the triangular dot) the RDs tend to be higher than in zone 2 because plastic deformations are present in both parts of the dual system. The magnitude of RDs in zone 3 depends on the elastic-plastic properties of both, the primary and secondary parts.

On the other hand, it is important to highlight that RDs of zone 2 can be fully eliminated after damaged BRBs are removed, while those of zone 3 cannot be eliminated when the BRBs are removed because the primary part presents permanent deformations.

In this section, it is considered that period of vibration, strength reduction factor, damping ratio and type of transition (from elastic to plastic response) affect RDs in dual systems similarly as in conventional systems. However, other factors, described in Figure 7-17, need to be further studied because they affect RDs in dual systems due to the interactions between the parts that compose the systems. These factors are: strength and stiffness ratios (previously defined in Chapter 3 and represent the contribution of the primary and secondary parts to the strength and stiffness of the system); and...
ductility, post-yielding stiffness ratio and type of hysteretic response, for the primary and secondary parts of the system.

Figure 7-17. Additional factors affecting residual displacements in dual systems

7.3.1 Effects of stiffness and strength ratios

First, it is convenient to recall what the stiffness and strength ratios are. The stiffness ratio is defined as the ratio of the stiffness of the secondary part to that of the primary part, i.e. \( a = k_2/k_1 \); while the strength ratio is defined as the ratio of the strength of the secondary part to that of the primary part, i.e. \( b = V_{y2}/V_{y1} = b_2/b_1 \).

Two cases are studied varying the values of \( a \) and \( b \) for a 5%-damped oscillator with period of vibration of \( T = 0.25T_g \) (i.e. \( T \approx 0.5 \) s). In both cases, the oscillator is subjected to the same 220 ground motions described in previous sections. For the first case, the maximum ductility demands of the primary and secondary parts are controlled to be \( \mu_1 = 1.0 \) and \( \mu_2 = 4.0 \). In this way, the maximum displacement demands are located in the limit of zone 2, as defined in Figure 7-16. In other words, the primary part remains elastic while the secondary part presents plastic deformation. For the second case, the maximum ductility demands in both parts are greater than unity (i.e. \( \mu_1 = 1.5 \) and \( \mu_2 = 6.0 \)) which means that both parts present plastic deformations. This means that
the maximum displacement demands are located in zone 3, as defined in Figure 7-16. Note that in both cases, the ductility ratio, $\mu_2/\mu_1$, is four. Therefore, according to equation (3-6) the ratio $a/b$ is also four.

Figure 7-18a shows the mean RDs, normalised by the maximum displacement demands. Note, in the horizontal axes, that $a$ is proportional to $4b$ due to the constrain of $\mu_2/\mu_1=4$. It is appreciated in the figure that, for both cases, RDs increase as the stiffness and strength ratios increase; i.e. RDs increase as the contribution of the secondary part increase. It is also observed that RDs are significantly smaller in the first case (maximum displacements in zone 2) than in the second case (maximum displacements in zone 3). This observation suggests that, RDs are smaller if the primary part remains elastic. Figure 7-18b shows that the dispersion is high in both cases; however, is much higher in the first case (zone 2) than in the second one (zone 3).

![Figure 7-18. Effect of stiffness and strength ratios on residual displacements](image)

a) Mean  
b) Dispersion
7.3.2 Effects of ductility of the primary and secondary parts

7.3.2.1 Ductility demand of the secondary part

The effects of the ductility demand of the secondary part ($\mu_2$) on RDs are assessed in a 5%-damped oscillator with period of vibration of $T=0.25T_g$, subjected to the same 220 ground motions used in previous sections. The value of $\mu_2$ is varied from 2 to 7 using constant values of $a$ and $b$, namely: $a=10$ and $b=1.25$. In this way $a/b=8$ and $\mu_1 \leq 1$ according to equation (3-6). Note that $\mu_2$ varies proportional to $8\mu_1$ (see equation (3-6)), and that the primary part remains elastic (i.e. the maximum displacement demand is located in zone 2, as defined in Figure 7-16).

The results are shown in Figure 7-19. It is observed in Figure 7-19a that the residual displacements are reduced as the ductility demand of the secondary part increases. This observation is in agreement with the observations from section 7.2.4, where RDs in conventional oscillators with post-yielding stiffness ratios larger than 2.5% decrease as ductility demands increase. On the other hand, in Figure 7-19b the coefficient of variation of the RDs shows no clear tendency. High values were obtained ranging from 0.65 to 0.85.

![Figure 7-19. Effect of ductility of the secondary part on residual displacements](image)
7.3.2.2 Ductility demand of the primary part

Now, in order to assess the effects of the ductility demand of the primary part on RDs, the same oscillator of the previous section is subjected to the same records using $a=10$ and $b=2.5$. Therefore, $a/b=4$. In this way, the value of $\mu_1$ is varied from 0.5 to 3.0, i.e. the primary part reaches inelastic deformation (zone 3 of Figure 7-16). The value of $\mu_2$ is varied from 2 to 12 according to equation (3-6).

The results are presented in Figure 7-20. In Figure 7-20a, it is observed that the increase of $\mu_1$ increased the RDs significantly for values of $\mu_1>1$ (i.e. zone 3 as defined in Figure 7-16). It is important to note that, once the primary part yielded, the effect of $\mu_1$ was opposite to the effect of $\mu_2$. Regarding to dispersion, Figure 7-20b shows that the coefficient of variation was reduced significantly as $\mu_1$ increased and it is smaller in zone 3 than in zone 2.

[Figure 7-20: Effect of ductility of the primary part in residual displacements]

7.3.3 Effects of post-yielding stiffness ratio of the primary and secondary parts

In this section, the effects of post-yielding stiffness ratio of the primary part ($r_1$) and the secondary part ($r_2$) of dual systems are studied. For this purpose, two cases are
compared to the previous results of Figure 7-20; which correspond to zero post-yielding stiffness in the primary and secondary parts (i.e. \( r_1=r_2=0 \)). For the first case (Case 1), the post-yielding stiffness ratio in the primary part is considered 5% while zero is considered for the secondary part (i.e. \( r_1=5\% \) and \( r_2=0 \)). For the second case (Case 2), \( r_1=0 \) and \( r_2=5\% \) are used.

Figure 7-21a shows that the effect of \( r_1 \) on RDs is negligible (see Case 1). However, \( r_2 \) has a significant effect (see Case 2). This may be expected because the plastic deformation of the secondary part is four times larger than that of the primary part – which leads to higher participation of \( r_2 \). On the other hand, the dispersion, presented in Figure 7-21b, tends to be similar for \( \mu_1<1.5 \) and higher for Case 2 with \( \mu_1>1.5 \).

Now, as a matter of interest, the effects of negative post-yielding stiffness ratios (i.e. \( r_1<0 \) and \( r_2<0 \)) on RDs are evaluated in the same oscillator. The results, presented in Figure 7-22, are compared against those of Figure 7-20 – which correspond to \( r_1=r_2=0 \). It is observed again that the effect of \( r_1 \) on the mean RDs is negligible while the effect of \( r_2 \) is very significant. More important, the effect of \( r_2 \) is adverse this time – leading to

---

**Figure 7-21.** Effect of post-yielding stiffness ratios on residual displacements

---
very high RDs. On the contrary, the effect on the dispersion was the opposite, i.e. negative $r_2$ generates smaller dispersions.

![Figure 7-22. Effects of negative post-yielding stiffness ratios on residual displacements](image)

As a preliminary conclusion, it can be said that the effect of $r_2$ on RDs is significant while that of $r_1$ is negligible. Positive values of $r_2$ are very beneficial while negative values are highly detrimental.

### 7.3.4 Effects of hysteretic response

In this section, only the effects of the type of hysteretic response of the primary part on RDs of dual systems are assessed because that of the secondary part is considered bilinear (since this represents well the behaviour of BRBs). Three types of hysteretic response are evaluated, namely: bilinear, Takeda and flag-shaped (see Figure 7-5). It is reminded that bilinear, Takeda and flag-shaped responses typically represent the behaviour of steel, concrete and innovative (with self-centring capacity) structures.

On the other hand, it should be noted that the influence of hysteretic response of the primary part is only significant when $\mu_1>1$, i.e. when the primary part yields (see Zone 3 in Figure 7-16). Otherwise, the response of the primary part would remain
elastic and the total response of the dual system would be the same for the three types of hysteretic response.

Dual oscillators with periods of vibration between $0.2T_g$ and $3T_g$ and total damping ratio of 5% are subjected to the 220 ground motions described in previous sections. A strength ratio of $b=2$ and a stiffness ratio of $a=5$ are considered. The target ductility demands of the primary and secondary parts are considered to be $\mu_1=2$ and $\mu_2=5$. The post-yielding stiffness ratio in both parts was assumed zero (i.e. $r_1=r_2=0$). The results are presented in Figure 7-23 where it is observed that, even though the hysteretic response of the secondary part is bilinear, the hysteretic response of the primary part affects RDs significantly. The highest RDs are obtained for the bilinear response while the smallest correspond to the flag-shaped response. The Takeda response is located between them. Regarding the dispersion, the elastoplastic response presented smaller coefficient of variation than the Takeda response. The coefficient of variation of the flag-shaped response was not presented in the figure because unrealistic dispersions were observed due to mean RDs close to zero.

![Figure 7-23](image_url)

**Figure 7-23.** Effects of type of hysteretic response of the primary part on residual displacements

![Residual / Max. displacement](image_url)

![Coefficient of variation](image_url)

a) Mean  
b) Dispersion
7.4 Summary and Recommendations to Reduce Residual Displacements

7.4.1 Summary

From all the parameters studied, post-yielding stiffness ratio ($r$) affected more significantly RDs of conventional SDOF oscillators. Besides, the effects of other parameters were highly dependent on the post-yielding stiffness ratio. A summary of the factors affecting conventional SDOF oscillators are described in Table 7-1. The additional factors affecting dual systems are summarised in Table 7-2.

Table 7-1. Factors affecting residual displacements on conventional SDOF oscillators

<table>
<thead>
<tr>
<th>Factor</th>
<th>Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-yielding stiffness ratio, $r$</td>
<td>• The higher the value of $r$, the smaller the mean of RDs</td>
</tr>
<tr>
<td></td>
<td>• The higher the value of $r$, the higher the dispersion of RDs</td>
</tr>
<tr>
<td></td>
<td>• $r$ also inverted the effect of these parameters: ductility demand, strength reduction factors and type of transition of the response from elastic to plastic.</td>
</tr>
<tr>
<td></td>
<td>• The effects of period of vibration, hysteretic response and damping ratio were not inverted by $r$. However, for values of $r$ higher than (say) 5 or 10%, RDs became small enough to neglect the beneficial or detrimental effects of these parameters.</td>
</tr>
<tr>
<td>Ductility demand, $\mu$</td>
<td>• For $r &lt; 2.5%$, the higher the $\mu$, the higher the RDs</td>
</tr>
<tr>
<td></td>
<td>• For $r \geq 2.5%$, the higher the $\mu$, the smaller the RDs</td>
</tr>
<tr>
<td></td>
<td>• Dispersion was high and no clear tendency was appreciated</td>
</tr>
<tr>
<td>Strength reduction factor, $R_y$</td>
<td>• For $r \leq 0$, the higher the $R_y$, the higher the RDs. Strong systems ($R_y &lt; 3$) appear to be more sensitive to changes of $R_y$. In the short-period range, RDs were very high reaching values higher than the corresponding elastic displacement demands.</td>
</tr>
<tr>
<td></td>
<td>• For $r &gt; 5%$, the higher the $R_y$, the smaller the RDs</td>
</tr>
<tr>
<td></td>
<td>• A transition zone was observed in $0% &lt; r &lt; 5%$</td>
</tr>
<tr>
<td></td>
<td>• Dispersion was high and no clear tendency was appreciated</td>
</tr>
<tr>
<td>Transition from elastic to plastic</td>
<td>• Variations in RDs were only significant when very smooth transition was observed and $r \neq 0$.</td>
</tr>
<tr>
<td></td>
<td>• For $r &gt; 0$, RDs of systems with very smooth transitions were smaller than in systems with sharp transitions</td>
</tr>
<tr>
<td></td>
<td>• For $r &lt; 0$, the opposite was true</td>
</tr>
<tr>
<td>Period of vibration</td>
<td>• In the short-period range: RDs increased as the period decreased</td>
</tr>
<tr>
<td></td>
<td>• In the long-period range: RDs are almost constant</td>
</tr>
<tr>
<td></td>
<td>• In oscillators with periods close to the dominant period of the excitation, RDs were</td>
</tr>
</tbody>
</table>
smaller than in the short- and long-period ranges
• Regarding the dispersion, very high coefficients of variations were observed with no apparent tendency.

| Hysteretic response | • Negligible RDs were observed for flag-shaped response
• RDs were higher for bilinear response than for Takeda response
• Dispersions were smaller for bilinear response than for Takeda response |

| Damping ratio, $\xi$ | • RDs decreased as $\xi$ increased
• Very high dispersions were appreciated with no clear tendency. |

### Table 7-2. Additional factors affecting residual displacements in dual SDOF oscillators

| Stiffness ($a$) and strength ($b$) ratios | • The smaller the values of $a$ and $b$, the smaller the RDs. In other words, the smaller the contribution of the secondary system, the smaller the RDs.
• RDs were higher in Zone 3 than in Zone 2
• On the contrary, dispersions were higher in Zone 2 than in Zone 3 |

| Ductility demand on the secondary part, $\mu_2$ | • The higher the $\mu_2$, the smaller the RDs. This observation is similar to conventional oscillators with $r > 2.5%$
• The dispersion was high with no clear tendency. Values of the coefficient of variation were between 0.6 and 0.8. |

| Ductility demand on the primary part, $\mu_1$ | • The effect of $\mu_1$ is only meaningful for $\mu_1 > 1$
• When $\mu_1$ increased, the RDs increased dramatically
• The opposite was observed for the dispersion |

| Post-yielding stiffness ratio of the primary part ($r_1$) | • No significant effect on RDs and dispersion |

| Post-yielding stiffness ratio of the secondary part ($r_2$) | • Significant effects on RDs and dispersion
• A positive $r_2$ reduced the RDs while increased the dispersion
• A negative $r_2$ increased the RDs while reduced the dispersion |

| Hysteretic response of the primary part | • Bilinear response had the highest RDs with the smallest dispersion
• Takeda response had smaller RDs and higher dispersion than bilinear response
• Flag-shaped response had very small RDs. |
7.4.2 Recommendations

Once the factors affecting residual displacements have been identified, the necessity of reduce or minimise RDs emerges. While factors can be easily or economically achieved, others may be more difficult. Therefore, the following recommendations are suggested.

7.4.2.1 Conventional systems

Although dual systems may be a better way to control RDs, it is recognised that their implementation may not always be possible. Therefore, for those cases where dual systems may not be feasible, reduced RDs may be expected by providing:

1. High post-yielding stiffness ratio (say) $r > 5$ or 10%. According to Pettinga et al. [100], this parameter can be easily increased by: 1) using different steel reinforcement with beneficial features; 2) re-designing geometry and properties of primary elements; and 3) providing a secondary resisting part that should remain elastic (which is equivalent to using a dual system with maximum displacement demand within Zone 2, as defined in Figure 7-16).

2. Self-centring technology. This can be achieved by using post-tensioned beam-to-column connections as proposed by Priestley at al. [64]. This technology may be more expensive but is very effective.

The previous solutions are regarded as very effective to reduce RDs; however, if for reason they are not feasible, only small reductions can be achieved by following the next recommendations.

1. Designing for reduced levels of ductility
2. Designing for reduced strength reduction factors
3. Adding supplemental damping
4. Reinforced concrete structures instead of steel structures because they present less RDs

5. Negative values of $r$ must be avoided because they increase RDs

If, after following the previous recommendations, the RDs are still not acceptable, the use of a dual system is highly advisable.

### 7.4.2.2 Dual systems

Dual systems are very effective to reduce or eliminate residual displacements, as long as the maximum displacement demand remains located within zone 2, as defined in Figure 7-16. Besides, any residual displacements can be removed by replacing damaged BRBs after an earthquake because the primary structural part remains essentially elastic.

On the other hand, if the maximum displacement demand reaches zone 3, permanent deformations may be present in the primary structure and they will remain even if the damaged BRBs are removed. Taking this into account, the following recommendations are suggested to reduce RDs in structures protected with BRBs.

1. Designing for a maximum displacement demand within zone 2, i.e. $\mu_1 \leq 1$.
   Additionally, the smaller the contribution of the secondary part, the better. Values of the strength ratio $b < 1$ are preferable. A high ductility demand of the secondary part ($\mu_2$) is also desirable.

2. If, for a reason, the maximum displacement demands do not remain in Zone 2 and reach Zone 3, a post-yielding stiffness ratio of the secondary part $r_2 > 5\%$ or 10\% should be provided. Again, a high ductility demand of the secondary part ($\mu_2$) is preferable.

3. If the previous suggestions are not feasible, self-centring technology is advisable to provide flag-shaped response to the primary part. Pos-tensioned beam-to-
column connections may be an effective solution as suggested by Priestley at al. [64].

The previous recommendations, especially the first one, are very effective to reduce RDs; however, if they are not feasible, small reductions can be achieved by considering:

1. Primary part made of reinforced concrete
2. Providing supplemental damping
3. No negative values of post-yielding stiffness ratio of the secondary part ($r_2$)

If the RDs are still not acceptable, the design should be modified. Smaller values of $b$ and $a$, and $\mu_1 \leq 1$ may be the wise choice this time.

7.5 Conclusions

The parameters affecting residual displacements (RDs) in conventional and dual SDOF oscillators, subjected to ground motions characteristics of the lakebed zone of Mexico City, have been analysed and summarised. Recommendations to reduce RDs have also been proposed. The following conclusions are formulated.

For conventional oscillators:

1. The most significant parameter affecting RDs is the post-yielding stiffness ratio ($r$). For (say) $r > 5$ or 10%, the RDs were reduced significantly and the adverse effects of other parameters were reduced or almost eliminated. Negative values of $r$ could be disastrous.

2. High ductility factors ($\mu$) and strength reduction factors ($R_y$) increase RDs if $r < 2.5\%$, while reduce them if $r \geq 2.5\%$. 

3. RDs are very sensitive to the period of vibration. Specially, short-period structures present very high RDs - which increase as the period decreases. The smallest RDs are observed for periods close to the dominant period of the soil. In the long-period range, RDs tend to be constant.

4. The type of hysteretic response affects RDs; being the flag-shaped the most effective to reduce them. Takeda response (representative of concrete structures) presented about half RDs of those in bilinear response (representative of steel structures).

5. Damping affects RDs. The higher the damping ratio the smaller the RDs. This is significant in this study because from experiments was observed that BRBs increase the damping ratio.

6. The type of transition (from elastic to plastic response) affects RDs when \( r \neq 0 \). The effect is very significant when the transition is very smooth. RDs are smaller in sharp than in smooth transition if \( r < 0 \). The opposite is true if \( r > 0 \).

For dual oscillators:

1. RDs are small if the primary part remains elastic (i.e. \( \mu_1 < 1 \)). On the contrary, if \( \mu_1 > 1 \), RDs increase dramatically.

2. Small stiffness \( (a) \) and strength \( (b) \) ratios reduce RDs, i.e. small contribution of the secondary part is beneficial.

3. Positive or negative post-yielding stiffness ratio of the primary part \( (r_1) \) does not affect RDs significantly. On the contrary, the post-yielding stiffness ratio of the secondary part \( (r_2) \) has a significant effect. A positive \( r_2 \) reduce RDs significantly while a negative is very detrimental.

4. The type of hysteretic response in the primary part has a significant effect on RDs. While the bilinear response presents the highest RDs, the flag-shaped response
presents very small RDs. RDs of the Takeda response are between those of bilinear and flag-shaped responses.

In general, it can be said that the most effective ways to reduce RDs are: 1) providing a high value of $r$ in conventional oscillators; 2) in dual systems, controlling the displacement demand so that the primary part remains elastic or providing $r_2 > 5\%$; and 3) for both conventional and dual systems, provide self-centring capacity. The other parameters analysed in this study can also help to reduce RDs - however they are less effective.

Finally, it is highlighted that dispersion was very high in all the studied cases. Coefficients of variation higher than 0.6 were very common. These values must be taken into account when assessing the probable performance of conventional and dual structures.
Chapter 8

Evaluating the Economic Benefits of using

Buckling-Restrained Braces in Hospital Structures

8.1 Introduction

Since economic quantities are more meaningful to decision makers than dynamic response parameters, this chapter is developed to evaluate quantitatively how convenient it is to use BRBs in hospitals located in the lakebed zone of Mexico City in economic terms. The findings presented in the previous chapters are included to provide validity and robustness to these analyses. Since the cost of non-structural elements and contents are far more expensive than the cost of the structure itself, they are included in detail in the analyses.

The organisation of this chapter is as follows: Section 8.2 presents the selected structures, their design and their dynamic response; in Section 8.3 their initial cost is estimated; expected (or probabilistic) losses when subjected to earthquake ground
motions are estimated in Section 8.4; cost-benefit analysis are conducted in Section 8.5; Sections 8.6 and 8.7 present the discussion and conclusions of the results.

8.2 Evaluated structures

8.2.1 Initial considerations

Three-, six- and nine-storey framed buildings in 2D, appropriate to hospitals located in the lakebed zone of Mexico City, are designed and evaluated (Figure 8-1). These numbers of storeys are considered because they may represent structures with ±1 storey; which allows the representation of a range of hospitals between two and 10 storeys.

![Figure 8-1. Layout of hospitals studied in this chapter](image)

For comparison purposes, five cases are studied for each hospital. While these cases are schematically represented in Figure 8-2, their initial cost, load capacity, and displacement demands are qualitatively presented in Figure 8-3. They are:

- **Case 0.** The bare frame is designed to resist the full seismic loads alone (i.e. without BRBs). This case serves as reference.
- **Case 1.** The structure of Case 0 is upgraded with BRBs – which increased the initial cost and capacity but reduced the lateral displacement demands.
• **Case 2.** The main frame is re-designed under gravity loads, then BRBs are provided to match the initial cost of Case 0. The combined load capacity of the frame and BRBs is higher than that of Case 0; which allows reductions of the displacement demands.

• **Case 3.** Similar to Case 2, the main frame is designed under gravity loads, but the lateral capacity of the BRBs is smaller so that the lateral displacement demands are similar to those of Case 0 – this allows having a structure with less initial cost but similar response to that of Case 0.

• **Case 4.** Again, similar to Case 2, the main frame is designed under gravity loads. However, in this case the lateral capacity of the BRBs is larger, so that the initial cost is larger than that of Case 2 – which also increase the load capacity but reduces the displacement response.

![Figure 8-2. Cases studied in this chapter](image)

![Figure 8-3. Qualitative initial cost, load capacity and displacement response of the studied cases](image)
The five cases are designed with the same methodology; which was proposed previously in Chapter 3 and is based in the control of the lateral displacement demands. By doing this, only the effects of the BRBs are compared and effects of lack of control of the lateral displacements are avoided.

### 8.2.2 Design

The three hypothetical steel framed hospitals are designed here for cases 0 to 4, i.e. 15 designs are conducted. The hospitals are assumed to be located in the lakebed of Mexico City. The floor masses are 461 t in the top floor and 576 t in the others. A rigid floor system is assumed. The first storey has a height of 4 m and the others 3 m. The materials used in the structures are steel ASTM A992 ($f_y = 350$ MPa) in beams and columns and steel ASTM A36 ($f_y = 250$ MPa) in the core of the BRBs.

The objectives of design are defined in Table 8-1. The method proposed in Chapter 3 was applied to design each structure and case. The details of the designs and the resultant cross-sections of the steel profiles and BRBs are presented in Appendix D.

#### Table 8-1. Objectives of design of the hospitals studied in this chapter

<table>
<thead>
<tr>
<th>Objective of design</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{ga}$</td>
<td>0.05g</td>
<td>0.10g</td>
<td>0.20g</td>
<td>0.30g</td>
</tr>
<tr>
<td>Max. drift</td>
<td>0.0025</td>
<td>0.005</td>
<td>0.010</td>
<td>0.020</td>
</tr>
</tbody>
</table>

### 8.2.3 Dynamic response

As shown in Chapter 3, the application of the design method generates information useful for assessing the expected losses on structures. In this regard, statistics of engineering demand parameters (EDPs) of equivalent dual SDOF oscillators are obtained. The EDPs are shown in detail in Appendix F. They are: peak displacements,
residual displacement ratios (defined as the ratio of the residual to the peak displacements), absolute velocity and acceleration. These EDPs are then converted into vectors of demands, as described in Section 3.5, in order to be able to conduct the assessment of the performance of the structure. In other words, the responses of the SDOF dual oscillators are converted into the responses of equivalent MDOF structures equipped with BRBs. As an example, Figures 8-3 to 8-5 show, only for a \( pga=0.20g \), the inter-storey drifts, floor velocities and floor accelerations for the studied frames. It can be appreciated that the response demands are consistently larger in Case 0 (i.e. frame designed without BRBs). The inter-storey drifts of Cases 0 and 3 are similar but slightly smaller in Case 3. Cases 1, 2 and 4 have smaller inter-storey drifts consistently. On the other hand, the floor velocities and accelerations are smaller when BRBs are used (i.e. Cases 1 to 4).

![Figure 8-4. Response estimated for \( pga=0.20g \): three-storey frame](image1)

![Figure 8-5. Response estimated for \( pga=0.20g \): six-storey frame](image2)
Chapter 8. Evaluating the Economic Benefits of using BRBs in Structures

8.3 Initial cost

First, for convenience three types of cost are defined:

a) Total initial cost \( (C_T) \). It includes the total cost of structural elements, non-structural elements and contents, so that the hospital is fully functional. The total cost for Case 0 is referred hereafter as \( C_0 \) and is taken as reference for comparison purposes.

b) Structure cost \( (C_S) \). It only includes the cost of the structural elements and their connections. In this study and to be consistent with [54], the structure cost for Case 0 is considered 20\% of the total cost, i.e. \( C_S=0.2C_0 \).

c) Non-structural and contents cost \( (C_n) \). It only includes the cost of non-structural elements and contents in the building, i.e. \( C_n=C_T-C_S \). To be consistent, the five cases have the same \( C_n \) – which is given from Case 0 as \( C_n=0.8C_0 \).

In order to compare the economic benefits of using BRBs in hospitals, the total cost of the frames for Case 0, \( C_0 \), is estimated first; then, the costs corresponding to Cases 1 to 4 are determined relative to \( C_0 \).

Table 8-2 shows the estimation of the total cost for Case 0 of the three-, six- and nine-storey frames. The weight of the structural steel is shown in the second column of
the table. Then, considering a cost of (USD) $3/kg and an additional cost of 5% due to beam-to-column connections, the structure cost is determined and is shown in the third column. As previously introduced and to be consistent with [54], the total cost is estimated by dividing the structure cost by 20% (see last column of the table). The non-structural and contents cost is also shown in the fourth column for illustration purposes.

Now, the structural steel weight ($w_s$) and the weight of the BRBs ($w_{BRBs}$) for Cases 1 to 4 are shown in Table 8-3. It is appreciated in the table that, for each structure, the steel weight of Case 1 is the same as that of Case 0 (see Table 8-2). On the other hand, the steel weight of Cases 1, 2 and 4 is smaller because these cases were designed under gravity loads only. It is also noted that the weight of the BRBs of Case 3 is the smallest while that of Case 4 is the largest.

**Table 8-2.** Estimation of initial cost for Case 0

<table>
<thead>
<tr>
<th>Structure</th>
<th>Steel weight, $w_s$, kg</th>
<th>Structure cost, $C_S=3(1+0.05)w_s$</th>
<th>Non-struct. &amp; contents cost, $C_n=0.8C_0$</th>
<th>Total cost, $C_0=C_T=C_S/0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-storeys</td>
<td>38,495</td>
<td>$121,261</td>
<td>$485,042</td>
<td>$606,303</td>
</tr>
<tr>
<td>6-storeys</td>
<td>107,018</td>
<td>$337,106</td>
<td>$1,348,424</td>
<td>$1,685,531</td>
</tr>
<tr>
<td>9-storeys</td>
<td>213,063</td>
<td>$671,178</td>
<td>$2,684,591</td>
<td>$3,355,739</td>
</tr>
</tbody>
</table>

**Table 8-3.** Steel weight, in kg, for Cases 1 to 3

<table>
<thead>
<tr>
<th>Structure</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_s$</td>
<td>$w_{BRBs}$</td>
<td>$w_s$</td>
<td>$w_s$</td>
</tr>
<tr>
<td>3-storeys</td>
<td>38,495</td>
<td>1,972</td>
<td>30,406</td>
<td>2,632</td>
</tr>
<tr>
<td>6-storeys</td>
<td>107,018</td>
<td>6,290</td>
<td>64,478</td>
<td>14,076</td>
</tr>
<tr>
<td>9-storeys</td>
<td>213,063</td>
<td>12,236</td>
<td>140,375</td>
<td>20,280</td>
</tr>
</tbody>
</table>

217
Finally, the structure cost and total cost for Cases 1 to 4 are estimated and shown in Table 8-4. For convenience, they are presented in terms of $C_0$. They are estimated as follows:

a) The structure cost was $C_S = (3w_s + 9w_{BRBS})(1 + 0.05)$, i.e. $w_s$ and $w_{BRBS}$, from the previous table, are multiplied by $3/\text{kg}$ and $9/\text{kg}$, respectively. An additional cost of 5% due to connections is included. $C_S$ was then normalised by $C_0$.

b) The total initial cost was $C_T = C_S + C_n$; where $C_S$ was estimated in the previous point and $C_n = 0.8C_0$ is shown in Table 8-2. It should be noted that $C_n = 0.8C_0$ means that the five cases have the same costs due to non-structural components and contents. However, the total and structure costs are different.

**Table 8-4. Estimation of initial cost for Cases 1 to 3**

<table>
<thead>
<tr>
<th>Structure</th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
<th></th>
<th>Case 3</th>
<th></th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_S$</td>
<td>$C_T$</td>
<td>$C_S$</td>
<td>$C_T$</td>
<td>$C_S$</td>
<td>$C_T$</td>
<td>$C_S$</td>
</tr>
<tr>
<td>3-storeys</td>
<td>0.23$C_0$</td>
<td>1.03$C_0$</td>
<td>0.20$C_0$</td>
<td>1.0$C_0$</td>
<td>0.17$C_0$</td>
<td>0.97$C_0$</td>
<td>0.17$C_0$</td>
</tr>
<tr>
<td>6-storeys</td>
<td>0.24$C_0$</td>
<td>1.04$C_0$</td>
<td>0.20$C_0$</td>
<td>1.0$C_0$</td>
<td>0.17$C_0$</td>
<td>0.97$C_0$</td>
<td>0.17$C_0$</td>
</tr>
<tr>
<td>9-storeys</td>
<td>0.23$C_0$</td>
<td>1.03$C_0$</td>
<td>0.20$C_0$</td>
<td>1.0$C_0$</td>
<td>0.18$C_0$</td>
<td>0.98$C_0$</td>
<td>0.18$C_0$</td>
</tr>
</tbody>
</table>

By analysing Table 8-4, it can be observed that even when the steel weight between the different studied cases may be significantly different, the impact in the total initial cost may be very small (i.e. differences smaller than 5% are appreciated).

### 8.4 Expected losses

To evaluate the expected (or probabilistic) losses in the studied hospitals of this Chapter, the assessment methodology proposed by the FEMA P58 Project [31] and
described in Section 2.3.2 is used here. The procedure consists of four analysis; which are described in the next subsections.

8.4.1 Seismic hazard analysis

This analysis is normally conducted by a Probabilistic Seismic Hazard Analysis (PSHA) to obtain a seismic hazard curve (see [34]). In PSHA, all the possible source regions that may generate potentially damaging earthquakes are included along with their associated uncertainties. In this Chapter, a PSHA is conducted in the computer program CRISIS [107] – which includes information of the seismicity of the Mexican Republic and ground motion attenuation laws. The resulting seismic hazard curve for Mexico City is shown in Figure 8-7 and gives the Mean Annual Frequency (MAF) of exceeding certain value of $pga$. For example, for a $pga=0.10g$, the MAF is 0.01; which is equal to a return period of 100 years or a probability of 40% of being exceeded in 50 years. Now, it is recognised that this curve does not contain site effects; however this does not affect the objective of this study which is to compare, relatively, the benefits of using BRBs in hospital structures.

![Figure 8-7. Seismic hazard curve for Mexico City estimated using CRISIS 2007 [107]](image-url)
8.4.2 Response analysis

As shown in Chapter 3, the use of SDOF dual systems to estimate the response of structures equipped with BRBs provides very approximate results while reducing the time of analysis. Therefore, the vectors of the response obtained in Section 8.2.3 are used here.

On the other hand, given that the probability of collapse has a significant impact in the estimation of losses, this parameter is obtained using Incremental Dynamic Analysis (IDA) [35]. For that purpose, the dual systems and ground motions used during the design process (see Appendix D) are used here. The ground motions are increasingly scaled between $pga=0.025g$ and $1.0g$ with increments of $0.025g$. Collapse is considered to occur when: 1) a small increment of seismic intensity generates a very large (unrealistic) increase of displacement; 2) the program shows numerical instability; or 3) the displacement demands are larger than the corresponding collapse displacement threshold ($d_{CP}$).

Figure 8-8 shows the results for Case 0 of the six-storey frame. Figure 8-8a shows the IDA curve - where the horizontal axis shows the peak displacements and the vertical axis the seismic intensity, or $pga$. The mean and the mean plus and minus one standard deviation are indicated in the figure by dark lines. Figure 8-8b shows the collapse fragility function estimated similarly to those in Section 3.6.2. It is appreciated that the observed data are fitted to a log-normal distributed function; which included only the record-to-record variability. Then, the dispersion is increased to $\beta=0.45$ in order to include uncertainties due to quality of construction and completeness of the numerical model (see Section 3.6.2).
Figure 8-9 shows the log-normal fitted collapse fragility functions for each frame and case of this Chapter. It is consistently observed in the figure that for all the studied frames, Cases 1 and 4 had the smallest probability of collapse (conditioned to a given $pga$); then, in that order, Case 2, Case 0 and Case 3.

**Figure 8-8.** Results of IDA corresponding to Case 0 of the six-storey frame

**Figure 8-9.** Collapse fragility functions

### 8.4.3 Damage state analysis

This analysis requires detailed definition of the damage states ($DS$) and their corresponding consequence actions and cost for each component of the hospitals. Since
Chapter 8. Evaluating the Economic Benefits of using BRBs in Structures

hypothetical hospitals are analysed, neither the components nor their quantities are known. For comparison purposes, normative components and quantities for healthcare occupancy are selected from the *Normative Quantity Estimation Tool* provided by the FEMA P58 Project [31]. They are shown in *Appendix C*. As described in Chapter 3, this is considered enough for preliminary assessments.

Once that components and quantities are defined, the corresponding *DS* are defined in the form of fragility functions. An example of these functions was given in Chapter 3 and it is repeated in Figure 8-10 for convenience. Three damage states are defined for a typical partition wall made of gypsum with metal studs. If the wall were subjected to an inter-storey drift demand of 0.005, it would have a probability of: 0.93 of being in damage state *DS*₁ or worse; 0.2 of being in *DS*₂ or worse; and 0.03 of being in *DS*₃ or worse. Figure 8-10b shows the repair costs and actions for the damage state *DS*₁ of the example partition wall. It is appreciated that the unit repair cost may reduce as the quantity increases, i.e. the efficiency of scale is considered. Uncertainty is also considered by defining a value of dispersion and a type of distribution.

![Fragility functions](image1)

**Figure 8-10.** Fragility functions and repair actions of a typical partition wall (taken from the PACT database [31])
8.4.4 Loss analysis

In this section, the total repair cost is estimated using the statistics of the structural response and the fragility data of the previous sections. For that purpose, the Monte Carlo procedure proposed by the FEMA P58 Project [31] and introduced in Section 2.3.2 is used (see Figure 2-7 in page 44). It is important to highlight that the repair cost estimated in this chapter only includes the typical cost associated to repair the components of hospitals. No costs were associated to sophisticated equipment or compensations due to injuries or losses of human lives because of the difficulties associated to their estimation.

8.4.4.1 Intensity-based assessment

First, various intensity-based assessments are conducted for intensities between $pga=0.05g$ and $0.8g$ with intervals of $0.05g$. In the pursuit of simplicity, Figures 8-11 to 8-13 show only the cumulative distribution functions of the repair costs of the studied frames for intensities of $pga=0.10g$, $0.20g$ and $0.30g$. The repair costs have been normalised by $1.2C_0$, which includes the initial cost of Case 0, $C_0$, plus 20% for demolition and clearance of the site. It is consistently observed in the figures that the repair costs are, from the smallest to the largest, in the following order: Case 4, Case 1, Case 2, Case 3 and Case 0.

![Figure 8-11. Cumulative distribution functions of repair cost: three-storey frame](image)
8.4.4.2 Time-based assessment

In order to estimate the average annual value of the repair costs, time-based analyses are conducted by integrating the intensity-based cumulative distribution functions over all the hazard levels – which are defined by the hazard curve of Section 8.4.1. More guidance can be found in [31, 36]. Figures 8-14 to 8-16 show the average annual repair costs and times for each case and frame studied. It is consistently observed that the highest annualised losses are those of Case 0 while the smaller are those of Cases 1 to 4, i.e. with BRBs. The annualised losses of Case 4 are always the smallest; followed by Cases 1, 2 and 3, respectively.
Similarly, the probabilities of collapse and of loss of functionality during the lifetime of the hospitals (50 years) are estimated. They are shown in Figures 8-17 to 8-19. It is appreciated in the figures that the probability of collapse is very small in all the cases.
However, in relative terms, Cases 1 and 4 consistently present the smallest probabilities of collapse. Also, the probability of loss of functionality is consistently smaller for Cases 1 and 4.

**Figure 8-17.** Probabilities of collapse and of loss of functionality: *three-storey frame*

**Figure 8-18.** Probabilities of collapse and of loss of functionality: *six-storey frame*

**Figure 8-19.** Probabilities of collapse and of loss of functionality: *nine-storey frame*
8.5 Cost-benefit analysis

Although the estimation of initial costs and annualised losses help to have a good understanding of the most convenient case or option of design, cost-benefit analysis provide further comparison helpful to decide which option may be the most convenient over a period of time (e.g. 50 years). In this section, the present value of annualised losses, associated with future damages, is added to the corresponding initial cost of each studied case. In this way, not only initial cost but also lifecycle cost can be compared to decide which case or option of design is the most convenient.

First, the net present value (NPV) of the stream of annualised losses is estimated as:

\[ NPV = A_n \left[ \left(1 - \frac{1}{(1 + i_n)^t} \right) / i_n \right] \quad (8-1) \]

where \( t \) is the period of time in years – which is considered 50 years in this study; \( i_n \) is the interest rate – considered 7% in this study; and \( A_n \) is the value of the annualised losses – which shall include the repair costs and the costs due to loss of functionality. In this study, it is assumed that each day of downtime has a cost of \( 0.01C_0 \). As an example, for Case 0 of the three-storey frame, the annualised repair cost is \$6,836 and the annualised repair time is 1.27 days (see Figure 8-14); thus, \( A = \$6,836 \times (C_0/\$606,303) + 1.27 \times 0.01C_0 = 0.024C_0 \). Therefore, \( NPV = 0.024C_0[(1-1/(1+0.07)^{50})/0.07] = 0.33C_0 \).

Figures 8-20 to 8-22 show the initial and initial plus NPV of the annualised losses for all the cases and frames of this chapter. They are presented in terms of \( C_0 \). It can be appreciated in the figures that:

- In terms of initial costs, Case 3 is the cheapest case, while Cases 1 and 4 are the most expensive. However, it has to be recognised that the differences of the initial
costs are less than 5% when compared to Case 0. In this context, the differences may be regarded as insignificant.

- In terms of the total lifecycle costs (i.e. Initial cost + $NPV$), the cheapest case of all the cases is Case 4, while Case 0 (i.e. frames without BRBs) is consistently the most expensive.

- Comparison between Cases 0 and 2 shows that, even when they have the same initial cost, the lifecycle cost is significantly smaller for Case 2 (i.e. with BRBs).

- Comparison between Cases 1 and 4 shows that they have similar initial cost but Case 4 has significantly smaller lifecycle cost. Since the contribution of BRBs is higher in Case 4, it can be said that the higher the contribution of the BRBs the smaller the lifecycle costs.

Figure 8-20. Initial and lifecycle costs: three-storey frame

Figure 8-21. Initial and lifecycle costs: six-storey frame
Chapter 8. Evaluating the Economic Benefits of using BRBs in Structures

8.6 Discussion

- **Benefits of BRBs.** By analysing Figures 8-14 to 8-22, it is appreciated that BRBs may help to reduce the expected losses, lifecycle costs, probabilities of collapse and of loss of functionality on frames significantly.

- **Smaller cross-sectional profiles.** By comparing Tables 8-2 and 8-3, it is observed that the structural steel weight is smaller in Cases 2, 3 and 4, because: a) the cross-sectional depths of beams are smaller - this provides a higher inter-storey clearance which is significant from an architectural point of view; and b) beams and columns are lighter allowing further reductions of cost and time during both, mounting and demolition of the frames.

- **The best options of design.** Since Case 4 presented consistently the smaller repair costs, repair times, probabilities of collapse and of loss of functionality, it is regarded as the most convenient option. Cases 2 and 3 may be also seen as convenient options because they had better behaviour than the bare frame counterpart at similar or smaller initial cost.

- **Variability of costs.** It shall be highlighted that the study presented in this chapter is based on fixed costs of structural steel, BRBs, components and downtime. However, for significantly different costs, different conclusions may be found.
• **Limitation.** The dynamic response of the frames is estimated using equivalent dual SDOF systems. The response is then converted to vectors of response for MDOF structures. This may have an impact in the estimation of the expected losses. However, it is considered that this should affect all the studied cases proportionally; therefore, relative comparisons may still be valid.

• **Implications.** The results of this study suggest that decision makers (such as the Minister of Health of Mexico) shall consider starting constructing hospitals protected with BRBs because, with similar initial cost, better response and smaller losses due to future earthquakes are expected in such facilities when BRBs are used. On the other hand, upgrading a hospital with BRBs could cost less than 5% of its total cost; while the benefits might be substantially higher.

### 8.7 Conclusions

Three-, six-, and nine-storey framed structures, appropriated to hospitals located in the lakebed zone of Mexico City, are designed with and without BRBs. For comparison purposes, five cases are studied. The conclusions obtained from this investigation are summarised as follows:

• When BRBs are introduced in structures, appropriated to hospitals located in the lakebed zone of Mexico City, the expected losses and lifecycle costs are reduced significantly, as observed in Figures 8-14 to 8-16 and 8-20 to 8-22, respectively.

• In particular, Case 4 is regarded as the most convenient option of design because it consistently presented smaller repair costs, lifecycle costs, repair times, probability of collapse and probability of loss of functionality.

• Comparison between Cases 0 and 2 shows that, even when they had the similar initial cost, the lifecycle cost was significantly smaller for Case 2 (i.e. with BRBs).
• Comparison between Cases 0 and 3 shows that, even when they were designed for similar displacement demands, the initial and lifecycle costs of the latter were smaller.

• Comparison between Cases 1 and 4 shows that, even when they had similar initial cost and capacity, Case 4 had significantly smaller lifecycle cost than Case 1, it is therefore said that the higher the contribution of the BRBs the better.

• Comparison of Cases 0 and 1 shows that upgrading a hospital with BRBs could cost less than 5% of its total cost; while the benefits might be substantially higher

• Another benefit of BRBs is that they provide smaller cross-sectional sizes of columns and beams. This allows higher spaces, inter-storey clearances, and lighter elements that are easier to install or remove.
9.1 Conclusions

A method for designing structures equipped with BRBs has been proposed. Then, the performance of structures, appropriate to hospitals in the lakebed zone of Mexico City, has been analysed numerically and experimentally. The advantages and benefits of fitting BRBs in such structures have been identified. The conclusions from this study have been formulated as follows:

1. *The proposed method.* It is valid for regular, low-rise buildings equipped with BRBs. It has the following advantages: a) it allows designers selecting from a variety of parameters to control the displacement demands on the structure - for example, designers can select explicitly the relative contribution of the main structure and the BRBs to the load capacity; b) it also allows the rapid application of PBSD philosophy because additional information, useful for preliminary assessment of the probable performance of buildings, is generated; and c) since the ductility
factors on each part of the dual system are estimated at the beginning of the process using the mechanical and geometric properties of the parts, the fuse concept is achieved a priori.

2. *Experimental study of a steel frame model equipped with BRBs.* As observed in Figures 4-6 to 4-8, a significant finding of the experiment with the steel model is that BRBs start dissipating energy even at linear-elastic deformation levels. Increases of the damping ratio from 0.3% to 7.6% in one case and from 0.52% to 6.10% in another case were noted. The inclusion of BRBs also allowed reductions of the dynamic responses for the tests with the same seismic intensity of $p_{ga}=0.1g$ (i.e. the average amplitudes of the maximum displacement, inter-storey drift, floor velocity and floor acceleration were reduced due to the use of BRBs by 58.5%, 62.2%, 35.4% and 26.9% respectively, while the Arias Intensity was almost eight times less than that without BRBs). From the tests with incremental seismic intensity, it was appreciated that the model with BRBs was able to accommodate up to 2.5 times more seismic intensity in terms of lateral displacements, inter-storey drifts and Arias Intensity, and up to 1.5 times more seismic intensity in terms of floor velocities and accelerations than the bare model. Regarding permanent deformations, they were not visually identifiable before or after all the BRBs were removed.

3. *Experimental study of RC precast models with and without BRBs.* First, tests of individual BRBs members showed that they have high energy dissipation capacity. Then, shaking table tests allowed the observation of the following findings: a) similarly to the tests of the steel model, the inclusion of BRBs on the RC models also increases the damping significantly; b) BRBs help to delay and reduce the stiffness degradation in the RC precast models - which is significant for short-period
structures subjected to ground motions with longer dominant period of vibration to avoid resonance effects; c) the dynamic response in terms of inter-storey drifts and lateral displacements without BRBs were twice those including BRBs while floor velocities and accelerations were similar for both cases; d) After one of the models was retrofitted with BRBs, it had lower fundamental natural frequency and higher lateral displacements than the intact model due to some damages on the frame – which suggest that new and retrofitted structures may behave differently depending on the level of damage. No residual deformation and no significant damage were observed in both cases, i.e. with and without BRBs. However, a high over-strength factor of 4 (compared to the common value of 2.5 for structures in Mexico) was observed – which might be responsible for the reduced level of damage.

4. **Improving the performance of hospital structures with BRBs.** BRBs effectively reduced the dynamic response of typical hospital structures in the lakebed zone of Mexico City. In the provided example, the lateral displacements and inter-storey drifts were reduced between 40% and 80%, while floor velocities and accelerations were also reduced but in smaller proportions. Reductions in residual drifts were very high (up to 90%). The probabilities of collapse and of loss of functionality were also significantly reduced. A parametric study was conducted to find the period range on which structures are benefitted with the use of BRBs. It was observed that: a) short-period structures (as defined in Figure 6-13) are highly benefitted by BRBs, while adverse effects are observed in long-period structures; and b) The ductility ratio ($\mu_1 / \mu_2$) has a significant effect on the response parameters, i.e. the smaller the ratio, the smaller the dynamic response.

5. **Residual displacements (RDs) in conventional and dual structures.** For conventional oscillators the most significant parameter affecting RDs is the post-yielding stiffness
ratio ($r$), i.e. for $r > 5\%$ is very convenient to reduce RDs, while negative values of $r$ are disastrous. Other parameters affecting RDs were also studied. For example, RDs increase with ductility ($\mu$) and reduction factors ($R_y$) if $r<2.5\%$, while the opposite occurs if $r \geq 2.5\%$. Type of transition (from elastic to plastic response), period of vibration, damping ratio and type of hysteretic response also affect RDs significantly. Due to the interaction between the parts of dual systems, additional parameters affecting RDs were studied. It was appreciated that small contributions of the secondary part is beneficial to constrain RDs, i.e. small stiffness ($a$) and strength ($b$) ratios reduce RDs. Also, RDs are small if the primary part remains elastic (i.e. $\mu_1<1$). On the contrary, if $\mu_1>1$, RDs increase dramatically. Furthermore, for cases of $\mu_1>1$, it is noted that the post-yielding stiffness ratio of the primary part ($r_1$) does not affect RDs significantly, while that of the secondary part ($r_2$) has a significant effect, i.e. $r_2>0$ reduce RDs significantly while $r_2<0$ is very detrimental.

Another finding is that RDs remain very small when a primary part with self-centring capacity is combined with BRBs as secondary part.

6. **Evaluating the economic benefits of using BRBs in structures.** It was observed that BRBs can effectively reduce repair costs, lifecycle costs, repair times, probabilities of collapse and of loss of functionality in hospitals structures located in the lakebed zone of Mexico City. From the five studied cases, those with BRBs are significantly more efficient than the bare frame because, with small variations of the initial cost (of less than 5\%), significant improvements of the performance are observed. Besides, resulting in smaller structural profiles of columns and beams, the use of BRBs in structures provides the additional benefits of higher spaces, higher inter-storey clearances, and lighter profiles that are easier and quicker to install or remove.
As a global conclusion and base on observations in this study, it can be said that, if properly designed (e.g. using the method proposed in Chapter 3), low-rise structures (i.e. sort-period structures) located in the lakebed zone of Mexico City are highly benefitted by the use of BRBs. The experimental and numerical studies in this thesis show that their dynamic response, probability of collapse, probability of loss of functionality and probable losses due to earthquakes are significantly reduced when they are equipped with BRBs.

Although this study has achieved the objectives set in Chapter 1, there are a number of limitations. For example, the proposed method of design is basically suitable for regular structures whose responses are dominated by their fundamental modes. Additionally, BRBs may not be suitable for long-period structures (such as high-rise buildings) because they may generate adverse effects (as noted in Chapter 6). Therefore, the areas for further research have been identified and presented in the next section.

9.2 Further work

Areas of further development have been identified and are described as follows:

1. *Extension of the method proposed in Chapter 3*. Currently, the method in Chapter 3 is valid for structures (regular in plan and elevation) that are not significantly affected by higher modes. Two types of extension may be pertinent: a) for structures affected by higher modes, such as high-rise buildings; and b) for irregular structures - which may be affected by torsional effects. For structures affected by higher modes, modifications could include: a.1) more than one period of vibration shall be estimated in Step 2 (see Section 3.4.2); and a.2) the yielding displacement may be calculated including the deformations produced by higher modes. For irregular structures, modifications could include: b.1) estimation of three periods of vibration
in Step 2, two in two horizontal orthogonal directions and one around a vertical axis; and b.2) solve the nonlinear dynamic equation of motion (equation (3-9)) for three degrees of freedom, two horizontal and one around a vertical axis.

2. **Effects of BRBs in long-period structures.** According to Section 6.5, it has been observed that the effects of BRBs may be detrimental in long-period structures. A further study may help to understand how this detrimental effect occurs and in which cases can be overcome.

3. **Comparison of BRBs against other lateral resisting systems.** In this thesis, moment resisting frames (MRF) equipped with BRBs and bare MRFs have been compared. It was appreciated that more efficient structures, with less cost and better performance, can be achieved when BRBs are used. However, other lateral resisting systems need also to be compared because in poor and developing countries, such as Mexico, the use of conventional systems, such as concrete walls and masonry walls, is very popular. The main disadvantage of those systems is that they present large degradation of stiffness and strength under cyclic loading. Therefore, comparative studies not only in terms of dynamic response, but also in terms of reparability, repair times, repair costs, lifecycle costs, probability of collapse, probability of loss of functionality, and (currently very important) CO₂ footprint, may help to encourage the use of BRBs as lateral resisting system and protective technology.

4. **Different types of ground motions.** In this thesis, evaluation of the economic benefits of the use of BRBs in structures was conducted using ground motions recorded in the lakebed zone of Mexico City – which are characteristic of very soft soils, present long dominant period of vibration, and impose very large displacement demands on structures. However, ground motions from different regions of the country (and of the World) present different characteristics, energy and frequency contents. Since
their effects in lifecycle costs and other parameters may vary significantly to the results presented here, numerical studies may be required to understand better the benefits of using BRBs under different types of excitations.

5. **Useful life of BRBs.** Although BRBs have been widely studied, there are still not precise criteria that help to determine when a BRB shall be replaced. This is very important after the occurrence of a strong earthquake because conducting a replacement of a BRB when it is not needed could represent a high unnecessary expense, while not replacing a damaged BRB could be a high risk. Therefore, by conducting numerical and experimental fatigue analyses could help to decide whether a BRB shall be replaced or not after a strong earthquake.

6. **Residual displacements of multi-storey buildings.** The study of residual displacements of Chapter 7 was developed for SDOF oscillators. On the other hand, several studies have proposed factors that allow converting residual displacements of conventional SDOF oscillators to residual drifts in multi-storey buildings. Therefore, in a similar way, factors that allow converting residual displacements of dual SDOF oscillators to residual drifts in multi-storey buildings equipped with BRBs may be obtained.
References


8. Wada, A., Y. Huang, and V.V. Bertero, Innovative Strategies in Earthquake Engineering, in EARTHQUAKE ENGINEERING From Engineering Seismology to Performance-Based Engineering, Y. Bozorgnia and V.V. Bertero, Editors. 2004, CRC PRESS.


32. Krawinkler, H. and E. Miranda, Performance-Based Earthquake Engineering, in Earthquake Engineering from Seismology to Performance-Based Engineering, Y. Bozorgnia and V.V. Bertero, Editors. 2004, CRC PRESS.


References

82. BLANDÓN, J.J. and M.E. RODRÍGUEZ, *Estudio analítico-experimental y propuesta de diseño sísmico de sistema de pisos rígidos en edificios*, in *Serie Investigacion y desarrollo*. 2007, Instituto de Ingeniería - UNAM.
85. IIUNAM. *Laboratory of the shaking table*. 2015 Sep/0215; [http://www.iingen.unam.mx/es-mx/Investigacion/Laboratorios/Paginas/MesaVibradora.aspx].


Appendix A. A Proposal of a new Buckling-Restrained Brace

The purpose of developing a new Buckling-Restrained Brace (BRB) is to make a cost-effective device. In this way, this technology can be available to poor and developing countries prone to seismic hazard. This is especially important because current commercially available BRBs are property and tend to be very expensive. Access of poor communities to this technology could help them to avoid large losses (human and economic), as observed in past earthquakes.

In this context, the proposed BRB is made of common materials available in local communities and the cost of fabrication may be regarded as small because it can be made by common construction workers with the use of minor tools.

A specimen of the proposed BRB is presented in Figure A-1. It consists of three parts, namely: a core, a case and connection ends. The core is made of commercially available steel rod of 6.4 mm of diameter with yielding stress of 344 MPa (obtained from laboratory tests). The case and connection ends are made of steel tubes and conventional mortar. The tube segments of the connection ends (with diameter of 42 mm) are bigger than that of the case, which has a diameter of 30 mm. This is important because the case tube should be introduced into the connection tubes to avoid lateral deformation of the unbounded parts of the core.

It is also appreciated in the figure that full anchorage of the core must be provided within the connection ends. This is achieved by bending the core 180 degrees. Dimensions and tolerances to bend the rods may be available in concrete design codes, such as ACI-318 [108] or Eurocode 2 [109]. It is highlighted that, since the rods need to be bended, their cross-sectional area cannot be very large. Therefore, this type of BRB is only convenient for small applications (such as low-rise buildings).
The test specimen was subjected to quasi-static cyclic load and the results are presented in Figure A-2. It is observed that the specimen presented very good behaviour, i.e. stable and symmetric hysteretic loops.

![Diagram of a test specimen](image)

a) Parts and dimensions in mm of a test specimen

![Image of a specimen prepared for testing](image)

d) View of a specimen prepared for testing

**Figure A-1. Proposed Buckling-Restrained Brace**

![Image of a test specimen](image)

a) Test on the specimen

![Stress-strain curve](image)

b) Stress-strain curve

**Figure A-2. Test of a specimen of the proposed BRB**
Appendix B. Seismic Records for Dynamic Analysis

Note: Records selected from the Mexican Database of Strong Motions [67]

Table B-1. Earthquake Ground motions used in Chapters 3, 5, 6 and 8

<table>
<thead>
<tr>
<th>Record</th>
<th>Station Code</th>
<th>Date (dd/mm/yyyy)</th>
<th>Magnitude, Ms</th>
<th>Epicentral Distance (km)</th>
<th>pga (cm/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SCT1</td>
<td>19/09/1985</td>
<td>8.1</td>
<td>425</td>
<td>161.63</td>
</tr>
<tr>
<td>2</td>
<td>TLHD</td>
<td>19/09/1985</td>
<td>8.1</td>
<td>433</td>
<td>117.67</td>
</tr>
<tr>
<td>3</td>
<td>SCT1</td>
<td>30/09/1999</td>
<td>7.6</td>
<td>444</td>
<td>20.37</td>
</tr>
<tr>
<td>4</td>
<td>AL01</td>
<td>30/09/1999</td>
<td>7.6</td>
<td>448</td>
<td>26.83</td>
</tr>
<tr>
<td>5</td>
<td>GA62</td>
<td>30/09/1999</td>
<td>7.6</td>
<td>448</td>
<td>27.21</td>
</tr>
<tr>
<td>6</td>
<td>TLHD</td>
<td>13/09/1999</td>
<td>7.6</td>
<td>428</td>
<td>33.21</td>
</tr>
<tr>
<td>7</td>
<td>SCT2</td>
<td>20/03/2012</td>
<td>7.5</td>
<td>355</td>
<td>33.9</td>
</tr>
<tr>
<td>8</td>
<td>TLHD</td>
<td>21/09/1985</td>
<td>7.5</td>
<td>294</td>
<td>51.58</td>
</tr>
<tr>
<td>9</td>
<td>LI58</td>
<td>09/10/1995</td>
<td>7.5</td>
<td>584</td>
<td>16.6</td>
</tr>
<tr>
<td>10</td>
<td>SCT2</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>344</td>
<td>32.21</td>
</tr>
<tr>
<td>11</td>
<td>AL01</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>349</td>
<td>40.99</td>
</tr>
<tr>
<td>12</td>
<td>GA62</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>349</td>
<td>29.99</td>
</tr>
<tr>
<td>13</td>
<td>TLHD</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>332</td>
<td>54.55</td>
</tr>
<tr>
<td>14</td>
<td>PE10</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>344</td>
<td>32.08</td>
</tr>
<tr>
<td>15</td>
<td>LI33</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>333</td>
<td>51.53</td>
</tr>
<tr>
<td>16</td>
<td>SP51</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>341</td>
<td>40.32</td>
</tr>
<tr>
<td>17</td>
<td>TL08</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>350</td>
<td>28.91</td>
</tr>
<tr>
<td>18</td>
<td>TL55</td>
<td>14/09/1995</td>
<td>7.3</td>
<td>349</td>
<td>29.94</td>
</tr>
<tr>
<td>19</td>
<td>SCT1</td>
<td>15/06/1999</td>
<td>7.0</td>
<td>219</td>
<td>30.47</td>
</tr>
<tr>
<td>20</td>
<td>SCT2</td>
<td>15/06/1999</td>
<td>7.0</td>
<td>219</td>
<td>30.47</td>
</tr>
<tr>
<td>21</td>
<td>AL01</td>
<td>15/06/1999</td>
<td>7.0</td>
<td>222</td>
<td>33.13</td>
</tr>
<tr>
<td>22</td>
<td>GA62</td>
<td>15/06/1999</td>
<td>7.0</td>
<td>221</td>
<td>31.51</td>
</tr>
<tr>
<td>23</td>
<td>GA62</td>
<td>11/01/1997</td>
<td>6.9</td>
<td>441</td>
<td>20.22</td>
</tr>
<tr>
<td>24</td>
<td>TLHD</td>
<td>11/01/1997</td>
<td>6.9</td>
<td>449</td>
<td>24.97</td>
</tr>
<tr>
<td>25</td>
<td>SP51</td>
<td>25/04/1989</td>
<td>6.9</td>
<td>309</td>
<td>47.31</td>
</tr>
<tr>
<td>26</td>
<td>TL08</td>
<td>25/04/1989</td>
<td>6.9</td>
<td>318</td>
<td>47.55</td>
</tr>
<tr>
<td>27</td>
<td>TL55</td>
<td>25/04/1989</td>
<td>6.9</td>
<td>316</td>
<td>45.18</td>
</tr>
<tr>
<td>28</td>
<td>SCT2</td>
<td>25/04/1989</td>
<td>6.5</td>
<td>311</td>
<td>39.98</td>
</tr>
<tr>
<td>29</td>
<td>AL01</td>
<td>25/04/1989</td>
<td>6.5</td>
<td>316</td>
<td>45.95</td>
</tr>
<tr>
<td>30</td>
<td>GA62</td>
<td>25/04/1989</td>
<td>6.5</td>
<td>317</td>
<td>52.6</td>
</tr>
</tbody>
</table>
## Table B-2. Earthquake ground motions used in Chapter 7

<table>
<thead>
<tr>
<th>Date (mm/dd/yy)</th>
<th>Magnitude, Ms</th>
<th>Station Code</th>
<th>Component</th>
<th>PGA&lt;sub&gt;1&lt;/sub&gt; cm/s&lt;sup&gt;2&lt;/sup&gt;</th>
<th>Component 2</th>
<th>PGA&lt;sub&gt;2&lt;/sub&gt; cm/s&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>BA49</td>
<td>EW</td>
<td>48.3</td>
<td>NS</td>
<td>31.8</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>BL45</td>
<td>EW</td>
<td>36.5</td>
<td>NS</td>
<td>35.1</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>CJ03</td>
<td>EW</td>
<td>37.7</td>
<td>NS</td>
<td>41.6</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>CJ04</td>
<td>EW</td>
<td>38.5</td>
<td>NS</td>
<td>42.6</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>JA43</td>
<td>EW</td>
<td>33.8</td>
<td>NS</td>
<td>36.2</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>LI58</td>
<td>EW</td>
<td>34.3</td>
<td>NS</td>
<td>41.6</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>PE10</td>
<td>EW</td>
<td>31.0</td>
<td>NS</td>
<td>33.2</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>RM48</td>
<td>EW</td>
<td>20.7</td>
<td>NS</td>
<td>23.5</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>SCT2</td>
<td>EW</td>
<td>36.8</td>
<td>NS</td>
<td>31.1</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>TH35</td>
<td>EW</td>
<td>79.1</td>
<td>NS</td>
<td>49.7</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>TL08</td>
<td>EW</td>
<td>27.8</td>
<td>NS</td>
<td>35.6</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>TL55</td>
<td>EW</td>
<td>28.8</td>
<td>NS</td>
<td>27.0</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>UC44</td>
<td>EW</td>
<td>62.6</td>
<td>NS</td>
<td>42.0</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>VG09</td>
<td>EW</td>
<td>33.7</td>
<td>NS</td>
<td>34.0</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>VM29</td>
<td>EW</td>
<td>44.0</td>
<td>NS</td>
<td>41.6</td>
</tr>
<tr>
<td>03/20/12</td>
<td>7.4</td>
<td>XP06</td>
<td>EW</td>
<td>39.7</td>
<td>NS</td>
<td>31.9</td>
</tr>
<tr>
<td>01/22/03</td>
<td>7.6</td>
<td>SCT2</td>
<td>EW</td>
<td>17.1</td>
<td>NS</td>
<td>21.5</td>
</tr>
<tr>
<td>01/22/03</td>
<td>7.6</td>
<td>SCT2</td>
<td>EW</td>
<td>17.1</td>
<td>NS</td>
<td>21.5</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>AL01</td>
<td>EW</td>
<td>22.5</td>
<td>NS</td>
<td>26.9</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>BA49</td>
<td>EW</td>
<td>42.9</td>
<td>NS</td>
<td>38.9</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>BL45</td>
<td>EW</td>
<td>25.8</td>
<td>NS</td>
<td>36.8</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>CJ03</td>
<td>EW</td>
<td>27.1</td>
<td>NS</td>
<td>32.1</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>CJ04</td>
<td>EW</td>
<td>26.7</td>
<td>NS</td>
<td>32.4</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>GA62</td>
<td>EW</td>
<td>18.3</td>
<td>NS</td>
<td>27.2</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>JA43</td>
<td>EW</td>
<td>20.6</td>
<td>NS</td>
<td>24.4</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>LI58</td>
<td>EW</td>
<td>26.1</td>
<td>NS</td>
<td>31.0</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>RM48</td>
<td>EW</td>
<td>17.6</td>
<td>NS</td>
<td>21.7</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>SCT1</td>
<td>EW</td>
<td>20.7</td>
<td>NS</td>
<td>36.5</td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>SCT2</td>
<td>EW</td>
<td>20.4</td>
<td>NS</td>
<td>35.4</td>
</tr>
<tr>
<td>Date</td>
<td>Magnitude</td>
<td>Station</td>
<td>EW</td>
<td>NS</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
<td>---------</td>
<td>----</td>
<td>----</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>SI53</td>
<td>EW</td>
<td>NS</td>
<td>23.5</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>TH35</td>
<td>EW</td>
<td>NS</td>
<td>44.7</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>TL08</td>
<td>EW</td>
<td>NS</td>
<td>18.3</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>TL55</td>
<td>EW</td>
<td>NS</td>
<td>22.6</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>TLHD</td>
<td>EW</td>
<td>NS</td>
<td>29.7</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>UC44</td>
<td>EW</td>
<td>NS</td>
<td>29.1</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>VG09</td>
<td>EW</td>
<td>NS</td>
<td>29.5</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>VM29</td>
<td>EW</td>
<td>NS</td>
<td>38.3</td>
<td></td>
</tr>
<tr>
<td>09/30/99</td>
<td>7.5</td>
<td>XP06</td>
<td>EW</td>
<td>NS</td>
<td>24.4</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>AL01</td>
<td>EW</td>
<td>NS</td>
<td>28.9</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>BA49</td>
<td>EW</td>
<td>NS</td>
<td>39.4</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>BL45</td>
<td>EW</td>
<td>NS</td>
<td>20.8</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>CJ03</td>
<td>EW</td>
<td>NS</td>
<td>22.7</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>CJ04</td>
<td>EW</td>
<td>NS</td>
<td>23.4</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>DFRO</td>
<td>EW</td>
<td>NS</td>
<td>28.6</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>GA62</td>
<td>EW</td>
<td>NS</td>
<td>29.6</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>JA43</td>
<td>EW</td>
<td>NS</td>
<td>26.2</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>LI58</td>
<td>EW</td>
<td>NS</td>
<td>27.0</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>PE10</td>
<td>EW</td>
<td>NS</td>
<td>32.8</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>RM48</td>
<td>EW</td>
<td>NS</td>
<td>19.8</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>RMAS</td>
<td>EW</td>
<td>NS</td>
<td>28.8</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>RMBS</td>
<td>EW</td>
<td>NS</td>
<td>14.1</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>SCT1</td>
<td>EW</td>
<td>NS</td>
<td>31.2</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>SCT2</td>
<td>EW</td>
<td>NS</td>
<td>30.1</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>TH35</td>
<td>EW</td>
<td>NS</td>
<td>43.9</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>TL08</td>
<td>EW</td>
<td>NS</td>
<td>25.3</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>TL55</td>
<td>EW</td>
<td>NS</td>
<td>22.6</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>TLHB</td>
<td>EW</td>
<td>NS</td>
<td>26.5</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>UC44</td>
<td>EW</td>
<td>NS</td>
<td>25.9</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>VG09</td>
<td>EW</td>
<td>NS</td>
<td>45.9</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>VM29</td>
<td>EW</td>
<td>NS</td>
<td>31.3</td>
<td></td>
</tr>
<tr>
<td>06/15/99</td>
<td>6.5</td>
<td>XP06</td>
<td>EW</td>
<td>NS</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td>Magnitude</td>
<td>Station</td>
<td>Orientation</td>
<td>EW</td>
<td>NS</td>
<td>Depth</td>
</tr>
<tr>
<td>------------</td>
<td>-----------</td>
<td>---------</td>
<td>-------------</td>
<td>----</td>
<td>----</td>
<td>-------</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>AL01</td>
<td>EW</td>
<td>35.2</td>
<td>NS</td>
<td>40.9</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>CJ03</td>
<td>EW</td>
<td>26.0</td>
<td>NS</td>
<td>25.0</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>CJ04</td>
<td>EW</td>
<td>27.0</td>
<td>NS</td>
<td>24.7</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>CO56</td>
<td>EW</td>
<td>45.4</td>
<td>NS</td>
<td>44.3</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>DFRO</td>
<td>EW</td>
<td>37.4</td>
<td>NS</td>
<td>28.7</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>GA62</td>
<td>EW</td>
<td>26.0</td>
<td>NS</td>
<td>30.2</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>JA43</td>
<td>EW</td>
<td>24.6</td>
<td>NS</td>
<td>27.8</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>PE10</td>
<td>EW</td>
<td>30.1</td>
<td>NS</td>
<td>32.1</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>RMCS</td>
<td>EW</td>
<td>29.0</td>
<td>NS</td>
<td>31.2</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>TL08</td>
<td>EW</td>
<td>28.8</td>
<td>NS</td>
<td>26.6</td>
</tr>
<tr>
<td>09/14/95</td>
<td>7.1</td>
<td>TL55</td>
<td>EW</td>
<td>19.6</td>
<td>NS</td>
<td>29.8</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>BA49</td>
<td>EW</td>
<td>15.9</td>
<td>NS</td>
<td>16.6</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>BL45</td>
<td>EW</td>
<td>13.8</td>
<td>NS</td>
<td>11.3</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>CA59</td>
<td>EW</td>
<td>14.2</td>
<td>NS</td>
<td>14.2</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>CO56</td>
<td>EW</td>
<td>17.8</td>
<td>NS</td>
<td>17.2</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>DFRO</td>
<td>EW</td>
<td>12.0</td>
<td>NS</td>
<td>14.2</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>GA62</td>
<td>EW</td>
<td>15.2</td>
<td>NS</td>
<td>14.0</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>JA43</td>
<td>EW</td>
<td>10.9</td>
<td>NS</td>
<td>12.3</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>RMAS</td>
<td>EW</td>
<td>16.6</td>
<td>NS</td>
<td>19.4</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>RMBS</td>
<td>EW</td>
<td>13.7</td>
<td>NS</td>
<td>10.3</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>RMCS</td>
<td>EW</td>
<td>12.6</td>
<td>NS</td>
<td>14.3</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>SCT1</td>
<td>EW</td>
<td>15.3</td>
<td>NS</td>
<td>11.0</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>TL08</td>
<td>EW</td>
<td>14.4</td>
<td>NS</td>
<td>15.0</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>TL55</td>
<td>EW</td>
<td>13.0</td>
<td>NS</td>
<td>9.8</td>
</tr>
<tr>
<td>12/10/94</td>
<td>6.3</td>
<td>XP06</td>
<td>EW</td>
<td>15.5</td>
<td>NS</td>
<td>16.6</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>BA49</td>
<td>EW</td>
<td>17.6</td>
<td>NS</td>
<td>14.4</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>JA43</td>
<td>EW</td>
<td>8.4</td>
<td>NS</td>
<td>12.3</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>RMBS</td>
<td>EW</td>
<td>8.4</td>
<td>NS</td>
<td>6.7</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>RMCS</td>
<td>EW</td>
<td>8.0</td>
<td>NS</td>
<td>10.7</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>SCT1</td>
<td>EW</td>
<td>10.4</td>
<td>NS</td>
<td>11.1</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>TL55</td>
<td>EW</td>
<td>9.7</td>
<td>NS</td>
<td>8.3</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>UC44</td>
<td>EW</td>
<td>15.0</td>
<td>NS</td>
<td>12.3</td>
</tr>
<tr>
<td>Date</td>
<td>Magnitude</td>
<td>Station Code</td>
<td>Component</td>
<td>EW Value</td>
<td>NS Value</td>
<td>Value</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>--------------</td>
<td>-----------</td>
<td>----------</td>
<td>----------</td>
<td>-------</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>VM29</td>
<td>EW</td>
<td>11.5</td>
<td>NS</td>
<td>13.6</td>
</tr>
<tr>
<td>10/24/93</td>
<td>6.6</td>
<td>XP06</td>
<td>EW</td>
<td>9.9</td>
<td>NS</td>
<td>8.3</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>AL01</td>
<td>EW</td>
<td>37.4</td>
<td>NS</td>
<td>45.7</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>BL45</td>
<td>EW</td>
<td>52.4</td>
<td>NS</td>
<td>42.7</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>CA59</td>
<td>EW</td>
<td>46.9</td>
<td>NS</td>
<td>29.2</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>CJ03</td>
<td>EW</td>
<td>37.3</td>
<td>NS</td>
<td>40.8</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>DFRO</td>
<td>EW</td>
<td>55.4</td>
<td>NS</td>
<td>46.1</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>JA43</td>
<td>EW</td>
<td>31.4</td>
<td>NS</td>
<td>35.3</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>LI58</td>
<td>EW</td>
<td>40.5</td>
<td>NS</td>
<td>41.0</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>PC25</td>
<td>EW</td>
<td>42.5</td>
<td>NS</td>
<td>28.9</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>RM48</td>
<td>EW</td>
<td>47.9</td>
<td>NS</td>
<td>27.8</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>SI53</td>
<td>EW</td>
<td>32.9</td>
<td>NS</td>
<td>39.6</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>TL55</td>
<td>EW</td>
<td>30.7</td>
<td>NS</td>
<td>44.9</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>UC44</td>
<td>EW</td>
<td>39.8</td>
<td>NS</td>
<td>52.6</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>VG09</td>
<td>EW</td>
<td>47.1</td>
<td>NS</td>
<td>38.2</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>VM29</td>
<td>EW</td>
<td>47.2</td>
<td>NS</td>
<td>49.7</td>
</tr>
<tr>
<td>04/25/89</td>
<td>6.9</td>
<td>XP06</td>
<td>EW</td>
<td>57.0</td>
<td>NS</td>
<td>43.7</td>
</tr>
<tr>
<td>09/19/85</td>
<td>8.1</td>
<td>SCT1</td>
<td>EW</td>
<td>161.0</td>
<td>NS</td>
<td>93.3</td>
</tr>
</tbody>
</table>
Appendix C. Typical Contents and Quantities for Healthcare Infrastructure

Notes:

- Data selected from the *Normative Quantity Estimation Tool* provided by the ATC-58 Project [31]
- The quantities presented below are estimated for a floor area of 576 m².

1. Components per building

*Drift Sensitive*

<table>
<thead>
<tr>
<th>PACT Code</th>
<th>Description</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1014.011</td>
<td>Traction Elevator – Applies to most California Installations 1976 or later,</td>
<td>1 EA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>most western states installations 1982 or later and most other U.S installations 1998 or later.</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>D5012.013a</td>
<td>Motor Control Center - Capacity: all, Unanchored equipment that is not vibration isolated, equipment frag. only</td>
<td>1 EA</td>
<td>2</td>
</tr>
<tr>
<td>D5092.031a</td>
<td>Diesel generator - Capacity: 100 to &lt;350 kVA, Unanchored equipment that is not vibration isolated, equipment frag. only</td>
<td>250</td>
<td>1</td>
</tr>
</tbody>
</table>

*Acceleration Sensitive*

<table>
<thead>
<tr>
<th>PACT Code</th>
<th>Description</th>
<th>Unit</th>
<th>Quantity*</th>
</tr>
</thead>
<tbody>
<tr>
<td>D3031.011a</td>
<td>Chiller - Capacity: &lt; 100 Ton - Unanchored equipment that is not vibration isolated - Equipment fragility only</td>
<td>75 TN</td>
<td>1,2,2,3</td>
</tr>
<tr>
<td>D3031.021a</td>
<td>Cooling Tower - Capacity: &lt; 100 Ton - Unanchored equipment that is not vibration isolated, equipment frag. only</td>
<td>75 TN</td>
<td>1,2,2,3</td>
</tr>
<tr>
<td>D3052.011a</td>
<td>Air Handling Unit - Capacity: &lt; 5000 CFM - Unanchored equipment that is not vibration isolated, equipment frag. only</td>
<td>4000</td>
<td>5,8,8,14</td>
</tr>
</tbody>
</table>

* Quantity for a building of 3, 5, 6 or 9 storeys height
2. Components per typical floor

*Acceleration sensitive*

<table>
<thead>
<tr>
<th>PACT Code</th>
<th>Description</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>B3011.011</td>
<td>Concrete tile roof, tiles secured and compliant with UBC94</td>
<td>100</td>
<td>17.98</td>
</tr>
<tr>
<td>C3032.001a</td>
<td>Suspended Ceiling, SDC A,B, Area (A): A &lt; 250, Vert support only</td>
<td>250</td>
<td>19.84</td>
</tr>
<tr>
<td>D2021.011a</td>
<td>Cold Water Piping (dia &gt; 2.5 inches), SDC A or B, PIPING FRAGILITY</td>
<td>1000</td>
<td>0.25</td>
</tr>
<tr>
<td>D2022.011a</td>
<td>Hot Water Piping - Small Diameter Threaded Steel - (2.5 inches in diameter or less), SDC A or B, PIPING FRAGILITY</td>
<td>1000</td>
<td>1.36</td>
</tr>
<tr>
<td>D2022.021a</td>
<td>Hot Water Piping - Large Diameter Welded Steel - ( &gt; 2.5 inches in diameter), SDC A or B, PIPING FRAGILITY</td>
<td>1000</td>
<td>0.50</td>
</tr>
<tr>
<td>D2031.011b</td>
<td>Sanitary Waste Piping - Cast Iron w/flexible couplings, SDC A,B, BRACING FRAGILITY</td>
<td>1000</td>
<td>0.93</td>
</tr>
<tr>
<td>D3041.012a</td>
<td>HVAC Galvanized Sheet Metal Ducting - 6 sq. ft cross sectional area or greater, SDC A or B</td>
<td>1000</td>
<td>0.22</td>
</tr>
<tr>
<td>D3041.011a</td>
<td>HVAC Galvanized Sheet Metal Ducting less than 6 sq. ft in cross sectional area, SDC A or B</td>
<td>1000</td>
<td>0.47</td>
</tr>
<tr>
<td>D3041.031a</td>
<td>HVAC Drops / Diffusers in suspended ceilings - No independent safety wires, SDC A or B</td>
<td>10</td>
<td>12.40</td>
</tr>
<tr>
<td>D3041.041a</td>
<td>Variable Air Volume (VAV) box with in-line coil, SDC A or B</td>
<td>10</td>
<td>3.10</td>
</tr>
<tr>
<td>D3041.041a</td>
<td>Variable Air Volume (VAV) box with in-line coil, SDC A or B</td>
<td>10</td>
<td>3.10</td>
</tr>
<tr>
<td>D2061.011a</td>
<td>Steam Piping - Small Diameter Threaded Steel - (2.5 inches in diameter or less), SDC A or B, PIPING FRAGILITY</td>
<td>1000</td>
<td>0.12</td>
</tr>
<tr>
<td>D2061.023a</td>
<td>Steam Piping - Large Diameter Welded Steel - ( &gt; 2.5 inches in diameter), SDC D, E, or F, PIPING FRAGILITY</td>
<td>1000</td>
<td>0.19</td>
</tr>
<tr>
<td>D2022.011a</td>
<td>Hot Water Piping - Small Diameter Threaded Steel - (2.5 in. in diameter or less), SDC A or B, PIPING FRAGILITY</td>
<td>1000</td>
<td>0.50</td>
</tr>
<tr>
<td>D2022.021a</td>
<td>Hot Water Piping - Large Diameter Welded Steel - ( &gt; 2.5 inches in diameter), SDC A or B, PIPING FRAGILITY</td>
<td>1000</td>
<td>0.19</td>
</tr>
<tr>
<td>C3033.001</td>
<td>Recessed lighting in suspended ceiling - no independent support wires</td>
<td>1 EA</td>
<td>93.00</td>
</tr>
<tr>
<td>C3034.001</td>
<td>Independent Pendant Lighting - non seismic</td>
<td>1 EA</td>
<td>93.00</td>
</tr>
<tr>
<td>D4011.021a</td>
<td>Fire Sprinkler Water Piping - Horizontal Mains and Branches - Old Style Victaulic - Thin Wall Steel - No bracing, SDC A or B, PIPING FRAGILITY</td>
<td>1000</td>
<td>1.36</td>
</tr>
<tr>
<td>D4011.031a</td>
<td>Fire Sprinkler Drop Std Threaded Steel, Dropping into unbraced lay-in tile SOFT ceiling-6ft drop max, SDC A or B</td>
<td>100</td>
<td>0.74</td>
</tr>
<tr>
<td>D5012.021a</td>
<td>Low Voltage Switchgear - Capacity: 100 to &lt;350 Amp - Unanchored equipment that is not vibration isolated - Equipment fragility only</td>
<td>225</td>
<td>1.00</td>
</tr>
</tbody>
</table>
### Velocity sensitive

<table>
<thead>
<tr>
<th>PACT Code</th>
<th>Description</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>E2022.112a</td>
<td>Vertical Filing Cabinet, 2 drawer, unanchored laterally</td>
<td>1 EA</td>
<td>2.48</td>
</tr>
<tr>
<td>E2022.102a</td>
<td>Bookcase, 2 shelves, unanchored laterally</td>
<td>1 EA</td>
<td>6.20</td>
</tr>
</tbody>
</table>

### Drift sensitive

<table>
<thead>
<tr>
<th>PACT Code</th>
<th>Description</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1011.001a</td>
<td>Wall Partition, Type: Gypsum with metal studs, Full Height, Fixed Below, Fixed Above</td>
<td>100 LF</td>
<td>6.51</td>
</tr>
<tr>
<td>C3011.001a</td>
<td>Wall Partition, Type: Gypsum + Wallpaper, Full Height, Fixed Below, Fixed Above</td>
<td>100 LF</td>
<td>0.79</td>
</tr>
<tr>
<td>C2011.001a</td>
<td>Prefabricated steel stair with steel treads and landings</td>
<td>1 EA</td>
<td>1</td>
</tr>
</tbody>
</table>

with seismic joints that accommodate drift.
Appendix D. Designs using the Method Proposed in Chapter 3

This appendix is divided in two sections. In the first section (D.1), the design of the models of the experiments of Chapter 5 is presented. Section D.2 presents the design of the hospital structures of Chapter 8.

D.1. Reinforced Concrete Precast Models of Chapters 5

All the calculations were conducted in full scale and the resultant structure was scaled down by a geometric factor of 1/3. A factor of mass per area of 1/2 was also used.

D.1.1. Model 1 (without BRBs)

This model was designed for earthquake actions using the lateral equivalent forces method. A behaviour factor of $Q=2$ was considered as required in the code for precast buildings. First, the period of vibration was estimated from an elastic model of the prototype structure in SAP200 using preliminary dimensioned beams and columns. The beams and columns had cross-sections of 450x810 mm and 600x600 mm, respectively. Cracked sections were modelled as suggested by the code. The floor system was assumed rigid. The total base shear of design was estimated from the design spectrum for zone IIIb of Mexico City to be $V=665$ kN, which resulted from multiplying the weight of the building ($W_f=3,290$ kN) times the seismic coefficient ($c=0.2023$) of the structure (Figure D-1).

![Figure D-1. Spectrum for seismic design in the zone IIIb of Mexico City](image-url)
The elastic model was analysed using the estimated loads and their corresponding combinations to determine the forces of design on the elements and their corresponding quantities of steel reinforcement. For simplicity, the quantities of steel reinforcement, required for the most stressed beam and column, were provided to all the other elements. The required steel reinforcement resulted 4 bars of 38 mm plus 4 bars of 28 mm in the columns and 2 bars of 38 mm in the top and bottom sides of the beams.

D.1.2. Model 2, equipped with BRBs

The structure was designed according to the displacement-based methodology proposed in Chapter 3. The objectives of design for the prototype structure are shown in Table D-1. To start the design, the method requires sets of seismic records compatible with the local hazard (one set per each objective of design). The records used here are those presented in Appendix B. They are pga-scaled to account for the different seismic intensities of design defined in Table D-1. The steps of design, according to the method, are summarised as follows:

1. Selection of the design objectives. Four levels of objectives are considered and listed in Table D-1. The different levels of objectives are defined by the drift limits and the seismic intensities.

2. Design of the frame under gravity loads. The bare frame is initially designed for gravity loads and its equivalent period of vibration and yielding displacement are estimated as suggested in FEMA 356 [63]. The equivalent period of vibration and yielding drift are $T_1=0.49$ s and $d_y=0.039$ m. The resulting cross-sections of beams and columns of the frame are 450x810 mm and 600x600 mm. The steel
reinforcement in beams is 2 bars of 28 mm of diameter in the top and bottom sides, and that in columns is 4 bars of 38 mm.

3. *The displacement thresholds.* For each objective of design and considering the following masses: 84.5 t on floors 1 to 3 and 82 t on the top floor, the displacement limits are estimated using equation 3-14 to be $d_1 = 0.025$ m, $d_2 = 0.049$ m, $d_3 = 0.099$ m and $d_4 = 0.197$ m.

4. *Analysis of the bare frame structure.* A conventional SDOF oscillator (with period of vibration of 0.49 s and yielding displacement of 0.039 m divided by a factor of 1.34) is subjected to the records shown in *Appendix B* scaled to the different intensities shown in Table D-1. The displacement demands are amplified by a factor of 1.2 to account for torsional effects and a factor of 1.35 to account for non-uniform drifts in the height of the structure. The results show that the prototype structure does not require BRBs because the displacement demands are below their corresponding thresholds of Step 3. However, the devices are included in order to analyse their effects on Model 2.

5. *Estimation of ductility factors of the frame with BRBs.* The maximum ductility on the bare frame and the BRBs corresponding to the displacement thresholds are estimated, resulting 6.77 and 15.74, respectively. It is assumed that all the BRBs had an effective stiffness factor of 4. It is worth noting that the ductility values estimated are the upper limits - the actual ductility demands will be smaller.

6. *Contributions to the load capacity of the BRBs.* A total of 18% of the lateral load capacity is selected as the contribution from the BRBs; therefore, the frame contributes 82%. Then, the period of vibration of the model equipped with BRBs is $T = 0.39$ s. The total capacity of the model, normalised by its weight ($W_T$), is
Appendix D. Designs using the Method Proposed in Chapter 3

$V_{yT}/W_T=0.604$; which included the contribution of the frame ($V_{y1}/W=0.495$) and the BRBs ($V_{y2}/W=0.109$).

7. The displacement demands on a dual SDOF oscillator. A dual SDOF oscillator is subjected to the records shown in Appendix B scaled to the different intensities of Table D-1. The mean plus one standard deviation of the displacement demands are shown in Figure D-2. They were increased by a factor of 1.2 to account for torsional effects and a factor of 1.2 to account for non-uniform drift. The four dots represent the displacement demands while the vertical dash-dot lines represent their corresponding thresholds. The three capacity curves show the relative load capacities of the BRB, the frame and the summation of both. It is observed from Figure D-2 that the displacement demands are significantly smaller than their limits and that the main frame remains elastic while the BRBs only reach inelastic deformation for seismic intensities of $pga=0.24g$ and $0.32g$.

8. Checking acceptance. According to the numerical results, it is decided that the design is accepted.

9. Cross-sectional areas of BRBs. The cross-sectional areas of the yielding zone of the BRBs are $A_{y1} = A_{y2} = 540 \text{ mm}^2$ in storeys one and two, and $A_{y3} = A_{y4} = 270 \text{ mm}^2$ in storeys three and four.

Table D-1. Objectives of design

<table>
<thead>
<tr>
<th>Objective</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Behaviour</td>
<td>Fully operational</td>
<td>Operational</td>
<td>Life safety</td>
<td>Collapse prevention</td>
</tr>
<tr>
<td>Maximum drift, $m$</td>
<td>0.0025</td>
<td>0.005</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Seismic intensity, $PGA$</td>
<td>0.032g</td>
<td>0.16g</td>
<td>0.24g</td>
<td>0.32g</td>
</tr>
</tbody>
</table>
D.1.3. Floor system

The floor system in the prototype consisted of prestressed, hollowed, one-way slabs with a thickness of 300 mm. A 60 mm thick topping of reinforced concrete was used to stick the slabs together. In order to estimate the stress demand on the topping, a finite element analysis was conducted in an isolated slab panel. The results of the analysis suggested that the maximum stress demand on the panel was 0.77 MPa in tension and 0.66 MPa in compression. These values were significantly smaller than the capacity of the concrete. Therefore, as suggested in the building code [51] only the minimum reinforcement of $\rho_{\text{min}} = 0.0025$ (for shear stress) was provided in the topping.

D.2. Design of Hospital Structures of Chapter 8

**Step 1. Selection of the objectives of design in terms of maximum inter-storey drifts.**

They are defined in Table 8-1 and are: $\theta_{\text{FO}} = 0.0025$; $\theta_{\text{Op}} = 0.005$; $\theta_{\text{LS}} = 0.01$; and $\theta_{\text{CP}} = 0.02$.

On the other hand, the design method requires seismic records as input. Therefore, 30 records recorded in the lakebed zone of Mexico City and selected from the Mexican
Database of Strong Motions [67] are used here. For convenience, they are shown in Appendix B. The records were pga-scaled in order to account for different seismic hazard levels.

**Step 2. Designing the primary structures under gravity loads.** The main structures are initially dimensioned for gravity loads. Note that this is applicable to Cases 2 to 4 of each hospital; the design of Cases 0 and 1 will be addressed in Step 4. The resulting steel profiles of the gravity-load designs are shown in Table D-2 along with the fundamental period ($T_1$) and the yielding displacement at the top floor of each frame ($d_{y1}$). $T_1$ is determined by eigenvalue decomposition while ($d_{y1}$) is estimated from pushover analysis with mass-distributed loads in Opensees [69]. The contribution of the slab is also considered as recommended by [68].

**Table D-2. Properties of the frames designed under gravity loads**

<table>
<thead>
<tr>
<th>Structure</th>
<th>Columns (Storey: Profile name)</th>
<th>Beams (Storey: Profile name)</th>
<th>Period ($T_1$), s</th>
<th>$d_{y1}$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 storeys</td>
<td>1: HSS500x19mm</td>
<td>1 &amp; 2: W21x68</td>
<td>0.69</td>
<td>0.078</td>
</tr>
<tr>
<td></td>
<td>2 &amp; 3: HSS500x13mm</td>
<td>3: W21x62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 storeys</td>
<td>1 to 3: HSS600x16mm</td>
<td>1 to 6: W24x68</td>
<td>1.02</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>4 to 6: HSS600x13mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 storeys</td>
<td>1 to 3: HSS900x25mm</td>
<td>1 to 9: W24x68</td>
<td>1.38</td>
<td>0.162</td>
</tr>
<tr>
<td></td>
<td>4 to 6: HSS900x16mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7 to 9: HSS800x13mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table D-3. Displacement thresholds**

<table>
<thead>
<tr>
<th>Performance</th>
<th>$d_{FO}$</th>
<th>$d_{OP}$</th>
<th>$d_{LS}$</th>
<th>$d_{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 storeys</td>
<td>0.0191</td>
<td>0.0382</td>
<td>0.0763</td>
<td>0.1526</td>
</tr>
<tr>
<td>6 storeys</td>
<td>0.0337</td>
<td>0.0674</td>
<td>0.1348</td>
<td>0.2696</td>
</tr>
<tr>
<td>9 storeys</td>
<td>0.0485</td>
<td>0.0971</td>
<td>0.1942</td>
<td>0.3883</td>
</tr>
</tbody>
</table>
Step 3. Calculation of displacement thresholds. The displacement thresholds are estimated with equation 3-14 and are shown in Table D-3.

Step 4. Determination of requirement for BRBs. In order to determine whether the structures require BRBs, the yielding capacity of the primary substructures ($V_{y_1}$) is determined and conventional SDOF oscillators with the properties estimated in Step 2 are subjected to the 30 seismic records described in Step 1. Figure D-3 shows that most of the displacement demands (represented by dots) are larger than the displacement thresholds estimated in Step 3 (see dash-dot lines). Therefore, it is concluded that the structures designed under gravity loads require BRBs to reduce the lateral displacements.

![Figure D-3. Displacement demands without BRBs](image)

On the other hand, for Cases 0 and 1 (see Figure 8-3) it is considered that the main frames shall have the capacity to resist the seismic loads alone. An iterative process is conducted to find the fundamental period of the frames, $T_1$, that allows having displacement demands below the corresponding thresholds. They are shown in Table D-4 along with $d_{y_1}$ and the steel profiles of columns and beams to achieve the required periods.
Table D-4. Properties of the frames for Cases 0 and 1

<table>
<thead>
<tr>
<th>Structure</th>
<th>Columns (Storey: Profile name)</th>
<th>Beams (Storey: Profile name)</th>
<th>Period ( (T_1), \text{s} )</th>
<th>( d_{11}, \text{m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 storeys</td>
<td>1 &amp; 2: HSS500x25mm</td>
<td>1 &amp; 2: W21x68</td>
<td>0.64</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>3: HSS500x19mm</td>
<td>3: W21x62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 storeys</td>
<td>1 to 3: HSS600x38mm</td>
<td>1 to 3: W27x94</td>
<td>0.84</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>4 to 6: HSS600x19mm</td>
<td>4 to 6: W27x84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 storeys</td>
<td>1 to 3: HSS900x38mm</td>
<td>1 to 3: W27x129</td>
<td>1.04</td>
<td>0.162</td>
</tr>
<tr>
<td></td>
<td>4 to 6: HSS900x25mm</td>
<td>4 to 6: W27x102</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7 to 9: HSS800x19mm</td>
<td>7 to 9: W27x84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 5. Estimation of ductility factors.** For Cases 1 to 4, BRBs are introduced and the maximum ductility ratios, corresponding to each objective of design, are estimated for the primary and secondary substructures. The results are shown in Table D-5. An effective stiffness factor of \( f_k = 2.0 \) is chosen for the three-storey frame while \( f_k = 1.50 \) is chosen for the six- and nine-storey frames. It is worth to highlight that the ductility ratios of the table are maximum thresholds and the actual ductility demands may result smaller.

Table D-5. Maximum ductility ratios

<table>
<thead>
<tr>
<th>Structure</th>
<th>( \mu_{FO} )</th>
<th>( \mu_{Op} )</th>
<th>( \mu_{LS} )</th>
<th>( \mu_{CP} = \mu_{1\text{max}} )</th>
<th>( \mu_{2FO} )</th>
<th>( \mu_{2Op} )</th>
<th>( \mu_{2LS} )</th>
<th>( \mu_{2CP} = \mu_{2\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-storeys</td>
<td>0.32</td>
<td>0.64</td>
<td>1.28</td>
<td>2.56</td>
<td>1.28</td>
<td>2.56</td>
<td>5.13</td>
<td>10.25</td>
</tr>
<tr>
<td>6-storeys</td>
<td>0.38</td>
<td>0.76</td>
<td>1.52</td>
<td>3.04</td>
<td>0.94</td>
<td>1.87</td>
<td>3.74</td>
<td>7.48</td>
</tr>
<tr>
<td>9-storeys</td>
<td>0.43</td>
<td>0.86</td>
<td>1.73</td>
<td>3.46</td>
<td>0.94</td>
<td>1.87</td>
<td>3.74</td>
<td>7.48</td>
</tr>
</tbody>
</table>

**Step 6. Selecting the relative participation of the secondary substructure, \( b_2 \).** For Case 0, \( b_2 = 0 \) because no BRBs are used. For Case 1, \( b_2 \) is selected so that the initial cost of the core structure increases by approximately 15% as compared with Case 0; this results
in $b_2=9.1\%$, 13.0% and 14.2% for the three-, six- and nine-storey frames, respectively.

For Case 2, $b_2$ is selected so that the initial cost of the core structure is the same as in Case 0; in this way $b_2=14.3\%$, 30.8% and 37.5%, respectively. For Case 3, the values of $b_2$ of Case 2 are reduced so that the displacement demands are similar to those of Case 0; this results in $b_2=5.97\%$, 17.8% and 30.0%, respectively. Finally, for Case 4, is selected to approximate the cost of Case 1, namely: $b_2=22.9\%$, 41.5% and 48.7%, respectively.

Once the relative participation of the BRBs, $b_2$, is established, the other properties of the dual systems are estimated. They are shown in Table D-6. It is highlighted that the contribution of the columns to the fundamental period of the frames is also considered – which is $T_{cols}=0.15$ s, 0.328 s and 0.67 s for the three-, six- and nine-storey frames.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Property</th>
<th>Case 0</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-storeys</td>
<td>$T_2$, second</td>
<td>-</td>
<td>1.14</td>
<td>0.99</td>
<td>1.59</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>$T$, seconds</td>
<td>0.64</td>
<td>0.56</td>
<td>0.57</td>
<td>0.63</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>$V_{yy}/m$, N/kg</td>
<td>6.09</td>
<td>6.09</td>
<td>4.92</td>
<td>4.92</td>
<td>4.92</td>
</tr>
<tr>
<td></td>
<td>$V_{zz}/m$, N/kg</td>
<td>0</td>
<td>0.61</td>
<td>0.82</td>
<td>0.31</td>
<td>1.46</td>
</tr>
<tr>
<td>6-storeys</td>
<td>$T_2$, second</td>
<td>-</td>
<td>1.42</td>
<td>1.09</td>
<td>1.48</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>$T$, seconds</td>
<td>0.84</td>
<td>0.72</td>
<td>0.75</td>
<td>0.84</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>$V_{yy}/m$, N/kg</td>
<td>4.96</td>
<td>4.96</td>
<td>3.35</td>
<td>3.35</td>
<td>3.35</td>
</tr>
<tr>
<td></td>
<td>$V_{zz}/m$, N/kg</td>
<td>0</td>
<td>0.74</td>
<td>1.49</td>
<td>0.73</td>
<td>2.38</td>
</tr>
<tr>
<td>9-storeys</td>
<td>$T_2$, second</td>
<td>-</td>
<td>1.09</td>
<td>1.38</td>
<td>1.58</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>$T$, seconds</td>
<td>1.04</td>
<td>0.91</td>
<td>0.98</td>
<td>1.04</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>$V_{yy}/m$, N/kg</td>
<td>4.08</td>
<td>4.08</td>
<td>2.32</td>
<td>2.32</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>$V_{zz}/m$, N/kg</td>
<td>0</td>
<td>0.67</td>
<td>1.39</td>
<td>1.0</td>
<td>2.20</td>
</tr>
</tbody>
</table>
Step 7. *Estimation of the maximum displacement demands.* For Cases 1 to 4, dual SDOF oscillators with the properties of the primary and secondary substructures determined in previous steps are subjected to the seismic records and intensities described in Step 1. To be consistent with the experimental evidence observed in Chapters 4 and 5, the contribution of the BRBs to the damping ratio is also considered. Mean plus one standard deviation of the maximum displacements are taken as displacement demands. They are amplified by a non-uniform inter-storey drift factor of 1.20. The results are shown in Figures D-4 to D-6 where the displacement demands (represented by dots) are below their corresponding thresholds (represented by dash-dot lines).

Step 8. *Determining if the designs are satisfactory.* According to Figures D-4 to D-6 it is appreciated that the designs of each case and frame are satisfactory because the demands are below their corresponding thresholds.

Step 9. *Determination of required cross-sectional areas of the BRBs.* The cross-sectional areas of the cores of the BRBs of each case and frame are determined according to Section 3.2.2. For simplicity, the areas of the first storey are presented in Table D-7 while those of the other storeys are proportional to the following vectors: $(1.0, 0.614, 0.273)^T$, $(1.0, 0.791, 0.626, 0.461, 0.296, 0.132)^T$ and $(1.0, 0.847, 0.738, 0.630, 0.521, 0.412, 0.304, 0.195, 0.087)^T$ for the three-, six- and nine-storey frames, respectively. In general, it is observed that the areas of Case 4 are higher than those of the others cases. It is also appreciated that the taller the structures, the larger the cross-sectional areas of the BRBs.
Appendix D. Designs using the Method Proposed in Chapter 3

Figure D-4. Displacement demands in equivalent dual systems: three-storey frame

Figure D-5. Displacement demands in equivalent dual systems: six-storey frame

Figure D-6. Displacement demands in equivalent dual systems: nine-storey frame

Table D-7. Cross-sectional areas, in cm$^2$, of the BRB cores of the first storey

<table>
<thead>
<tr>
<th>Structure</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-storeys</td>
<td>13.5</td>
<td>18.0</td>
<td>6.84</td>
<td>32.0</td>
</tr>
<tr>
<td>6-storeys</td>
<td>46.9</td>
<td>105.0</td>
<td>60.0</td>
<td>152.0</td>
</tr>
<tr>
<td>9-storeys</td>
<td>64.0</td>
<td>127.0</td>
<td>93.0</td>
<td>190.0</td>
</tr>
</tbody>
</table>
### Appendix E. Table of Properties of Concrete used in the Tests of Chapter 5

<table>
<thead>
<tr>
<th>Element</th>
<th>Parts</th>
<th>Model</th>
<th>Storey</th>
<th>Resistance, MPa</th>
<th>Elasticity modulus, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precast beams</td>
<td>Precast parts only</td>
<td>1 &amp; 2</td>
<td>1 to 4</td>
<td>70.1</td>
<td>28,772</td>
</tr>
<tr>
<td>Precast columns</td>
<td>Precast parts only</td>
<td>1 &amp; 2</td>
<td>1 to 4</td>
<td>55.9</td>
<td>23,624</td>
</tr>
<tr>
<td>Nodes</td>
<td>Beam-column connection</td>
<td>1</td>
<td>1 &amp; 2</td>
<td>27.6</td>
<td>18,883*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3 &amp; 4</td>
<td>48.1</td>
<td>24,778*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1 &amp; 2</td>
<td>55.7</td>
<td>24,496*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>3 &amp; 4</td>
<td>53.2</td>
<td>N.A.</td>
</tr>
<tr>
<td>Floors</td>
<td>Toppings</td>
<td>1</td>
<td>1 &amp; 2</td>
<td>38.7</td>
<td>21,456*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3 &amp; 4</td>
<td>38.3</td>
<td>24,496*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1 &amp; 2</td>
<td>54.2</td>
<td>23,155*</td>
</tr>
<tr>
<td>Grout</td>
<td>Column-column connections</td>
<td>1 &amp; 2</td>
<td>3</td>
<td>60.1</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

* estimated using equation (11.3) from [51]
Appendix F. Engineer Demand Parameters of the Frames of Chapter 8

In the following tables, the following symbols are used:

\[ pga = \text{peak ground acceleration} \]
\[ g = \text{acceleration of the gravity} \]
\[ \beta = \text{Coefficient of variation} \]
\[ c_r = \text{Residual to peak displacement ratio} \]

### Three-storey hospital

<table>
<thead>
<tr>
<th>Case</th>
<th>( pga )</th>
<th>Displ., cm</th>
<th>( \beta_{\text{displ}} )</th>
<th>( c_r )</th>
<th>Standard dev. of ( c_r ), cm</th>
<th>Velocity, m/s</th>
<th>( \beta_{\text{vel}} )</th>
<th>Accel., ( g )</th>
<th>( \beta_{\text{accel}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05g</td>
<td>1.32</td>
<td>0.30</td>
<td>0.000</td>
<td>0.000</td>
<td>0.20</td>
<td>0.13</td>
<td>0.11</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>2.32</td>
<td>0.25</td>
<td>0.000</td>
<td>0.000</td>
<td>0.37</td>
<td>0.12</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>4.63</td>
<td>0.25</td>
<td>0.000</td>
<td>0.000</td>
<td>0.74</td>
<td>0.12</td>
<td>0.38</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>7.00</td>
<td>0.27</td>
<td>0.000</td>
<td>0.080</td>
<td>1.09</td>
<td>0.12</td>
<td>0.55</td>
<td>0.18</td>
</tr>
<tr>
<td>1</td>
<td>0.05g</td>
<td>0.85</td>
<td>0.31</td>
<td>0.000</td>
<td>0.000</td>
<td>0.18</td>
<td>0.14</td>
<td>0.09</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>1.53</td>
<td>0.25</td>
<td>0.000</td>
<td>0.009</td>
<td>0.34</td>
<td>0.14</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>3.01</td>
<td>0.23</td>
<td>0.000</td>
<td>0.013</td>
<td>0.67</td>
<td>0.14</td>
<td>0.30</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>4.57</td>
<td>0.18</td>
<td>0.000</td>
<td>0.013</td>
<td>0.98</td>
<td>0.15</td>
<td>0.42</td>
<td>0.15</td>
</tr>
<tr>
<td>2</td>
<td>0.05g</td>
<td>0.90</td>
<td>0.33</td>
<td>0.000</td>
<td>0.000</td>
<td>0.18</td>
<td>0.14</td>
<td>0.09</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>1.63</td>
<td>0.27</td>
<td>0.000</td>
<td>0.017</td>
<td>0.34</td>
<td>0.14</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>3.19</td>
<td>0.23</td>
<td>0.000</td>
<td>0.020</td>
<td>0.67</td>
<td>0.14</td>
<td>0.29</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>5.01</td>
<td>0.16</td>
<td>0.000</td>
<td>0.019</td>
<td>0.98</td>
<td>0.15</td>
<td>0.40</td>
<td>0.13</td>
</tr>
<tr>
<td>3</td>
<td>0.05g</td>
<td>1.15</td>
<td>0.25</td>
<td>0.000</td>
<td>0.000</td>
<td>0.18</td>
<td>0.12</td>
<td>0.10</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>2.09</td>
<td>0.21</td>
<td>0.000</td>
<td>0.008</td>
<td>0.35</td>
<td>0.12</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>4.10</td>
<td>0.17</td>
<td>0.005</td>
<td>0.006</td>
<td>0.68</td>
<td>0.12</td>
<td>0.31</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>6.05</td>
<td>0.15</td>
<td>0.004</td>
<td>0.021</td>
<td>1.01</td>
<td>0.13</td>
<td>0.44</td>
<td>0.13</td>
</tr>
<tr>
<td>4</td>
<td>0.05g</td>
<td>0.61</td>
<td>0.24</td>
<td>0.000</td>
<td>0.000</td>
<td>0.16</td>
<td>0.16</td>
<td>0.08</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>1.16</td>
<td>0.20</td>
<td>0.000</td>
<td>0.000</td>
<td>0.32</td>
<td>0.16</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>2.27</td>
<td>0.20</td>
<td>0.000</td>
<td>0.048</td>
<td>0.64</td>
<td>0.16</td>
<td>0.28</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>3.56</td>
<td>0.18</td>
<td>0.035</td>
<td>0.034</td>
<td>0.96</td>
<td>0.15</td>
<td>0.37</td>
<td>0.11</td>
</tr>
</tbody>
</table>
### Six-storey hospital

<table>
<thead>
<tr>
<th>Case</th>
<th>pga</th>
<th>Displ., cm</th>
<th>$\beta_{\text{displ}}$</th>
<th>$c_r$</th>
<th>Standard dev. of $c_r$</th>
<th>Velocity, m/s</th>
<th>$\beta_{\text{vel}}$</th>
<th>Accel., g</th>
<th>$\beta_{\text{accel}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05g</td>
<td>2.24</td>
<td>0.32</td>
<td>0.000</td>
<td>0.001</td>
<td>0.22</td>
<td>0.16</td>
<td>0.11</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>3.95</td>
<td>0.27</td>
<td>0.000</td>
<td>0.000</td>
<td>0.42</td>
<td>0.13</td>
<td>0.19</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>7.88</td>
<td>0.27</td>
<td>0.000</td>
<td>0.026</td>
<td>0.83</td>
<td>0.13</td>
<td>0.37</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>11.90</td>
<td>0.27</td>
<td>0.001</td>
<td>0.104</td>
<td>1.20</td>
<td>0.11</td>
<td>0.49</td>
<td>0.11</td>
</tr>
<tr>
<td>1</td>
<td>0.05g</td>
<td>1.48</td>
<td>0.21</td>
<td>0.000</td>
<td>0.000</td>
<td>0.19</td>
<td>0.12</td>
<td>0.10</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>2.70</td>
<td>0.18</td>
<td>0.000</td>
<td>0.000</td>
<td>0.36</td>
<td>0.11</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>5.18</td>
<td>0.16</td>
<td>0.003</td>
<td>0.013</td>
<td>0.72</td>
<td>0.12</td>
<td>0.32</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>7.46</td>
<td>0.15</td>
<td>0.010</td>
<td>0.013</td>
<td>1.05</td>
<td>0.12</td>
<td>0.43</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>0.05g</td>
<td>1.58</td>
<td>0.26</td>
<td>0.000</td>
<td>0.000</td>
<td>0.19</td>
<td>0.14</td>
<td>0.10</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>2.87</td>
<td>0.21</td>
<td>0.000</td>
<td>0.003</td>
<td>0.37</td>
<td>0.12</td>
<td>0.17</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>5.52</td>
<td>0.18</td>
<td>0.004</td>
<td>0.032</td>
<td>0.72</td>
<td>0.12</td>
<td>0.31</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>8.24</td>
<td>0.17</td>
<td>0.029</td>
<td>0.032</td>
<td>1.07</td>
<td>0.13</td>
<td>0.40</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.05g</td>
<td>1.98</td>
<td>0.27</td>
<td>0.000</td>
<td>0.000</td>
<td>0.21</td>
<td>0.13</td>
<td>0.09</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>3.62</td>
<td>0.23</td>
<td>0.000</td>
<td>0.012</td>
<td>0.40</td>
<td>0.12</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>7.10</td>
<td>0.22</td>
<td>0.019</td>
<td>0.014</td>
<td>0.77</td>
<td>0.12</td>
<td>0.30</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>10.79</td>
<td>0.21</td>
<td>0.034</td>
<td>0.066</td>
<td>1.14</td>
<td>0.12</td>
<td>0.41</td>
<td>0.13</td>
</tr>
<tr>
<td>4</td>
<td>0.05g</td>
<td>1.32</td>
<td>0.23</td>
<td>0.000</td>
<td>0.000</td>
<td>0.18</td>
<td>0.12</td>
<td>0.10</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>2.39</td>
<td>0.19</td>
<td>0.000</td>
<td>0.000</td>
<td>0.36</td>
<td>0.12</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>4.61</td>
<td>0.18</td>
<td>0.000</td>
<td>0.041</td>
<td>0.70</td>
<td>0.12</td>
<td>0.33</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>6.62</td>
<td>0.18</td>
<td>0.018</td>
<td>0.045</td>
<td>1.03</td>
<td>0.13</td>
<td>0.43</td>
<td>0.08</td>
</tr>
</tbody>
</table>

### Nine-storey hospital

<table>
<thead>
<tr>
<th>Case</th>
<th>pga</th>
<th>Displ., cm</th>
<th>$\beta_{\text{displ}}$</th>
<th>$c_r$</th>
<th>Standard dev. of $c_r$</th>
<th>Velocity, m/s</th>
<th>$\beta_{\text{vel}}$</th>
<th>Accel., g</th>
<th>$\beta_{\text{accel}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05g</td>
<td>3.67</td>
<td>0.36</td>
<td>0.000</td>
<td>0.004</td>
<td>0.26</td>
<td>0.22</td>
<td>0.11</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>6.32</td>
<td>0.32</td>
<td>0.000</td>
<td>0.001</td>
<td>0.48</td>
<td>0.20</td>
<td>0.20</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>12.40</td>
<td>0.25</td>
<td>0.000</td>
<td>0.059</td>
<td>0.92</td>
<td>0.15</td>
<td>0.37</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>19.67</td>
<td>0.30</td>
<td>0.065</td>
<td>0.148</td>
<td>1.29</td>
<td>0.13</td>
<td>0.44</td>
<td>0.05</td>
</tr>
<tr>
<td>1</td>
<td>0.05g</td>
<td>2.31</td>
<td>0.28</td>
<td>0.000</td>
<td>0.002</td>
<td>0.22</td>
<td>0.16</td>
<td>0.09</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>4.24</td>
<td>0.24</td>
<td>0.000</td>
<td>0.004</td>
<td>0.42</td>
<td>0.14</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>8.16</td>
<td>0.23</td>
<td>0.007</td>
<td>0.013</td>
<td>0.81</td>
<td>0.13</td>
<td>0.32</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>11.97</td>
<td>0.21</td>
<td>0.014</td>
<td>0.048</td>
<td>1.19</td>
<td>0.13</td>
<td>0.43</td>
<td>0.15</td>
</tr>
<tr>
<td>2</td>
<td>0.05g</td>
<td>2.65</td>
<td>0.29</td>
<td>0.000</td>
<td>0.001</td>
<td>0.23</td>
<td>0.15</td>
<td>0.09</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>4.89</td>
<td>0.26</td>
<td>0.000</td>
<td>0.017</td>
<td>0.43</td>
<td>0.14</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>9.56</td>
<td>0.23</td>
<td>0.011</td>
<td>0.043</td>
<td>0.83</td>
<td>0.12</td>
<td>0.29</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>15.15</td>
<td>0.22</td>
<td>0.045</td>
<td>0.064</td>
<td>1.20</td>
<td>0.14</td>
<td>0.38</td>
<td>0.11</td>
</tr>
<tr>
<td>3</td>
<td>0.05g</td>
<td>3.13</td>
<td>0.32</td>
<td>0.000</td>
<td>0.001</td>
<td>0.24</td>
<td>0.20</td>
<td>0.10</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>5.71</td>
<td>0.25</td>
<td>0.000</td>
<td>0.015</td>
<td>0.45</td>
<td>0.15</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>11.29</td>
<td>0.24</td>
<td>0.030</td>
<td>0.040</td>
<td>0.85</td>
<td>0.14</td>
<td>0.29</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>19.41</td>
<td>0.27</td>
<td>0.112</td>
<td>0.117</td>
<td>1.23</td>
<td>0.13</td>
<td>0.37</td>
<td>0.08</td>
</tr>
<tr>
<td>4</td>
<td>0.05g</td>
<td>2.26</td>
<td>0.28</td>
<td>0.000</td>
<td>0.000</td>
<td>0.22</td>
<td>0.16</td>
<td>0.10</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>0.10g</td>
<td>4.11</td>
<td>0.24</td>
<td>0.000</td>
<td>0.000</td>
<td>0.41</td>
<td>0.14</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.20g</td>
<td>7.86</td>
<td>0.22</td>
<td>0.000</td>
<td>0.064</td>
<td>0.81</td>
<td>0.13</td>
<td>0.32</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.30g</td>
<td>11.91</td>
<td>0.21</td>
<td>0.022</td>
<td>0.054</td>
<td>1.16</td>
<td>0.12</td>
<td>0.40</td>
<td>0.10</td>
</tr>
</tbody>
</table>
A subroutine programmed in Matlab® is presented to solve equation 3-19.

```matlab
function [u,v,a,RD,m1,m2,m1acc,m2acc,Einp] = SolveDualSystem(ag,dt,T1,dy1,b2,mu1,mu2,xhi1,xhi2)
% by HECTOR GUERRERO - November 2015
% ag = accelerogram or ground motion
% dt = time step of ag
% T1 = fundamental period of vibration of the primary structure
% dy1 = yielding displacement of the primary system already multiplied
% by the factor dmax/dN
% b2 = participation of the secondary structure or BRBs
% mu1 = maximum ductility factor of the primary part (equation (3-4a)
% mu2 = maximum ductility factor of the secondary part (equation (3-4b)
% xhi1 = damping ratio contributed by the primary part
% xhi2 = damping ratio contributed by the secondary part

% Definitions
myPi=4*atan(1);
Yes=1;
No=0;

% Preliminary calculations
b1=1-b2;
a=(b2/b1)*(mu2/mu1);
T2=T1/sqrt(a);
T=(1/T1^2+1/T2^2)^-0.5;
Vy1_m=dy1*((2*myPi/T1)^2);
Vy2_m=Vy1_m*b2/b1;
xhi=xhi1+xhi2;

% Preliminary parameters
w = 2 * myPi / T;
Kx = 1 / (dt ^ 2) + xhi * w / dt;
Ax = 1 / (dt ^ 2) - xhi * w / dt;
Bx = 2 / (dt ^ 2);

a2=a/(1+a);
a1=1-a2;
dy2=Vy2_m/(a2*w^2);

vg=IntegrateSignal(ag,dt);

% initialisations
flagPlastic1=No;
flagPlastic2=No;
hasChanged=No;

up1=0;
up2=0;
ductAcc1=0;
ductAcc2=0;

ui=0;
```
Appendix G. Subroutine to Solve the Equation of Motion of Dual Systems

```matlab
uiplus1=0;
uiminus1=0;
vi=0;
viplus1=0;
viminус1=0;
si=1;
si_ant=1;
ur_ant=0;
dres=0;
ddres=0;
fs=0;
fs1=0;
fs2=0;
fr1=0;
fr1_ant=0;
fr2=0;
fr2_ant=0;
umax=0;
vmax=0;
accmax=0;
InputEnergy=0;

% Initiating estimation of the response
r=size(ag,1);
for i=1:r
    if i==1
        ai=0;
    else
        ai=ag(i);
    end
    if sign(vi)>=0
        si=1;
    else
        si=-1;
    end
    if si_ant~=si
        hasChanged=Yes;
    else
        hasChanged=No;
    end
    if hasChanged==Yes
        si_ant=si;
        ur_ant=uiminus1;
        ur=uiminus1;
        fr1_ant=fs1;
        fr1=fs1;
        fr2_ant=fs2;
        fr2=fs2;
        if flagPlastic1==Yes
            flagPlastic1=No;
            ductAcc1=ductAcc1+abs(ui - up1)/dy1;
        end
        if flagPlastic2==Yes
            flagPlastic2=No;
            ductAcc2=ductAcc2+abs(ui - up2)/dy2;
        end
    else
        ur=ur_ant;
        fr1=fr1_ant;
        fr2=fr2_ant;
    end
end
```
fs=0;
faux1=fr1+((w ^ 2)*a1)*(ui-ur);
faux2=fr2+((w ^ 2)*a2)*(ui-ur);
if abs(faux1)<=Vy1_m
    fs1=faux1;
else
    fsiminus1=fs1;
    fs1=sign(faux1)*Vy1_m;
    if flagPlastic1==No
        flagPlastic1=Yes;
        up1=uiminus1+(fs1-fsiminus1)*(ui-uiminus1)/(faux1-
fsiminus1);
    end
end
if abs(faux2)<=Vy2_m
    fs2=faux2;
else
    fsiminus1=fs2;
    fs2=sign(faux2)*Vy2_m;
    if flagPlastic2==No
        flagPlastic2=Yes;
        up2=uiminus1+(fs2-fsiminus1)*(ui-uiminus1)/(faux2-
fsiminus1);
    end
end
fs=fs1+fs2;
uiplus1=(-ai-Ax*uiminus1+Bx*ui-(fs))/Kx;
vplus1=(uiplus1-ui)/dt;
acci=(vplus1-vi)/dt;
VelAbs=vg(i)+vplus1;
AccAbs=ai+acci;
if i>1
    InputEnergy=InputEnergy-((ag(i)+ag(i-1))/2.0)*(ui-uiminus1);
end
uiminus1 = ui;
ui = uiplus1;
viminus1 = vi;
vi = vplus1;
if umax<abs(uiplus1)
    umax=abs(uiplus1);
end
if vmax<abs(VelAbs)
    vmax=abs(VelAbs);
end
if accmax<abs(AccAbs)
    accmax=abs(AccAbs);
end
if i>0.95*r
    dres=dres+uiplus1;
    ndres=ndres+1;
end
u=umax;
v=vmax;
a=accmax;
RD=abs(dres/ndres);
m1=umax/dy1;
m2=umax/dy2;
m1acc=ductAcc1;
m2acc=ductAcc2;
Einp=InputEnergy;
Appendix G. Subroutine to Solve the Equation of Motion of Dual Systems

function vel = IntegrateSignal(x,dt)
% by HECTOR GUERRERO - November 2015
% ag = signal to be integrated
% dt = time step of ag
% returns integrated signal

r=size(x,1);
y=zeros(r,1);
for i=1:r
    if i == 1
        y(1) = 0;
    else
        y(i) = (x(i) + x(i - 1)) * dt / 2 + y(i - 1);
    end
end
vel=y;