FRP reinforced-concrete slabs: a comparative design study

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Title: FRP Reinforced Concrete Slabs: A Comparative Design Study

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Abstract

Durability of concrete structures represents a major challenge today for both existing and new structures. Fibre-reinforced polymer (FRP) composite rebar offers a highly durable alternative to steel rebar. Research on the use of FRP rebar has been conducted over the past 40 years; however, its widespread application is yet to occur. This paper investigates the use of FRP as reinforcement, in particular the application to slab elements and associated performance at serviceability and ultimate limit state. Existing design guidance such as the ACI and CSA standards are reviewed and a number of slab designs with dimensions common in practice are undertaken. A third approach that follows the Eurocode design philosophy is also reviewed. The design of the different slab forms is assessed according to these codes, and results compared with the behaviour predicted by non-linear finite-element analysis. The relative costs of each resulting slab design are also discussed. The findings highlight differences in the relative accuracy of the different code recommendations and indicate the need for further research to exclude potentially non-conservative design. The work presented is intended to provide a contextual comparison of existing design approaches, which will be useful to practitioners involved in design of concrete structures with FRP rebar.

Keywords chosen from ICE Publishing list

Composite structures, Concrete structures, Design methods and aids

List of notation (examples below)

A_f, A_{frp}	Area of longitudinal FRP reinforcement, mm ²
A_s	Area of longitudinal steel reinforcement, mm ²
a	Depth of stress block, mm
b	Width of rectangular cross section, mm
C	Resultant of compressive stresses in concrete, N
c	Depth of neutral axis, mm
c_b	depth of neutral axis at the balanced failure condition, mm
d	Distance from extreme compression fibre to centroid of tension reinforcement, mm
E_f	Design value of modulus of elasticity of FRP, MPa

E_{fk}	Characteristic value of modulus of elasticity of FRP, MPa
E_s	Design value of modulus of elasticity of reinforcing steel, MPa
f'_c	Specified compressive strength of concrete, MPa
f_{cd}	Design value of concrete compressive strength, MPa
f_{ck}	Characteristic value of concrete compressive strength, MPa
f_{ctm}	Mean value of concrete tensile strength, MPa
f_f, f_{frp}	Stress in FRP, MPa Tensile strength of FRP bars, MPa
f_{fu}	Design tensile strength of FRP, considering reductions for service environment, MPa Ultimate tensile strength of FRP bars, MPa
f_{fk}	Characteristic value of tensile strength of FRP reinforcement, MPa
f_{yd}	Design yield strength of steel reinforcement, MPa
f_{yk}	Characteristic yield strength of reinforcement, MPa
$\frac{f_{\beta 0}}{f_{c0}}$	Ratio of the strength in the biaxial compression state to the strength in the uniaxial state
G	permanent action
G_k	Characteristic value of permanent action
I_{cr}	Moment of inertia for cracked concrete section
I_e	Effective moment of inertia, mm ⁴
I_g	Moment of inertia of gross concrete section, mm ⁴
I_t	Moment of inertia for uncracked section transformed to concrete, mm ⁴
K_c	Distances between the hydrostatic axis and the compression and tension meridian in the deviatoric cross section respectively
L	Span, mm
L_x	Element length in the longitudinal direction, mm
M_{cr}	Applied moment causing the occurrence of the first crack, kNm
M_n	Nominal moment capacity, Nmm
M_u	Factored moment at section, Nmm
P	Uniform distributed load applied along the loading line
Q	variable action
Q_k	characteristic value of a single variable action
T	Internal force due to tension in FRP reinforcement, N

α_{cc}	coefficient taking into account the long term effects on the compressive strength and of unfavourable effects resulting from the way the load is applied
α_1	Ratio of average concrete strength in rectangular compression block to depth of the neutral axis
β_1	Concrete strength factor
β	Coefficient accounting for the duration of load and bond Ratio of distance between neutral axis and tension face to distance between neutral axis and reinforcement steel
δ	Deflection, mm
ϵ_{cr}	Strain of concrete at cracking
ϵ_{cu}	Ultimate strain of concrete
ϵ_f	Ultimate strain of FRP reinforcement
ϵ_{frp}	Strain in FRP reinforcement
ϵ_y	Ultimate strain of steel
η	Factor defining effective strength of concrete
η_{env}	Environmental strength reduction factor
γ_c	Partial safety factor for concrete
γ_f	Partial safety factor for FRP
γ_G	Partial factor for Permanent actions
γ_s	Partial safety factor for steel
γ_Q	Partial factor for Variable actions
λ	Factor defining effective height of compression zone
μ	Plastic strain
ν	Poisson's coefficient
ξ	Ratio of the neutral axis depth to the effective depth
ρ_f	Reinforcement ratio
ρ_{fb}	Balanced reinforcement ratio
σ_c	Stress in concrete, MPa
σ_s	Stress in the tension reinforcement in a cracked section, MPa
σ_f	Stress in longitudinal FRP reinforcement, MPa
ϕ	Strength reduction factor

Introduction

1.1. Background

Different applications of fibre-reinforced polymers (FRPs) are available in the civil engineering field, with the use of FRP strips or bars to externally repair or strengthen structural elements being the most well-known and common technique. The technique investigated in this study involves the substitution of traditional steel reinforcement with FRP bars. This is particularly convenient in aggressive environments, where the FRP is able to guarantee better long-term performances than unprotected steel reinforcement (Mertol et al., 2006).

The use of FRP materials for civil engineering applications was considered for the first time in the 1960s to solve the problem of corrosion of highway bridges, and of structures heavily exposed to marine salt or subjected to de-icing salt. However, at that time FRP reinforcing bars were not commercially available and the first applications took place only in the 1980s. Initially, FRP reinforcement was used in bridge decks (Clarke et al., 1998; Weaver, 1995), in facilities for magnetic resonance imaging (MRI) medical equipment, seawall construction (Cunningham and Burgess, 2012; Cunningham et al., 2012; Nanni and Dolan, 1993), substation reactor bases, airport runways and electronics laboratories. During the first half of the 1990s, FRP applications were widespread in Japan; then later, during the early 2000s, China became the largest user of FRP materials. In 1986, the first prestressed FRP highway bridge was built in Germany, and the use of FRP reinforcement started to spread in Europe (ACI, 2006). Among the latest examples in the UK are the new central area coastal defences in Blackpool, completed in late 2010 and the largest single application of FRP at the time. To avoid the corrosion of reinforcement bars and related phenomena, such as spalling or staining, the design team adopted FRP-reinforced precast concrete (Cunningham and Burgess, 2012; Cunningham et al., 2012; also see Figure 1). Stainless steel is usually used for this type of application; however, owing to the market price fluctuation of the material, the design team used the fibre-reinforced composite reinforcement instead. This project proved that the material can be used in large-scale projects and that the geometrical constraint due to the nature of the reinforcement can be overcome with careful detailing. Nowadays, despite the potential of FRP materials for civil engineering applications and the benefits they offer as an alternative to steel rebar, the use of FRP is still not widespread. One of the reasons for this is the relative lack of design codes for FRP structures

(Shave, 2014). In Europe, although there are Eurocodes for the common construction materials, there is not one for FRP structures. Therefore, only specialists who already possess a considerable amount of knowledge and understanding of the material usually embrace the design of such structures, and many designers lack familiarity with the material. It is clear that FRP materials are a possible and effective alternative to steel reinforcing bars for concrete structures, and as their physical and mechanical behaviour is different from that of steel (e.g. see Table 1), a specific guidance on their use is necessary. Another obstacle for the wide-scale uptake of FRP rebar is the absence, unlike for steel rebar, of uniform quality standards. Every manufacturer produces and certifies a different product, which makes the use of the material more complex (Gremel, 2012; Railway Technology, 2007).

Figure 1. Tower headland wave-wall unit reinforcement cage. Straight glass FRP bars at the coffered portion used in combination with steel bars in the ribs (Cunningham and Burgess, 2012)

Table1 Common tensile properties of reinforcing bars, with typical values for fibre volume fractions ranging from 0.5 to 0.7

	GFRP	CFRP	AFRP	Steel
Yield stress (MPa)	-	-	-	276 to 517
Tensile strength (MPa)	483 to 1600	600 to 3690	1720 to 2540	483 to 690
Elastic modulus (GPa)	35.0 to 51.0	120.0 to 580.0	41.0 to 125.0	200
Yield strain	-	-	-	0.14 to 0.25
Rupture strain	1.2 to 3.1	1.2 to 3.1	1.9 to 4.4	6.0 to 12.0

51
52
53

1.2. Aim and Scope

54 The aim of this study is provide a comparison between avail- able codes and assess their
55 prediction against finite-element (FE) models. It is essential to highlight the lack of comparative
56 studies of FRP rebar codes; at the time of writing, there are only a few publications readily
57 available in the literature (Cosenza et al., 1997; International Federation for Structural Concrete
58 (fib) – TG 9.3 (fib, 2007); Pilakoutas et al., 2011).

59 In this study, the performance of FRP as reinforcement and its application to reinforced-
60 concrete (RC) slabs will be assessed. For this purpose, several structural elements are
61 modelled using the commercially available FE software Abaqus, to reproduce the non-linear
62 behaviour of the structure. In addition, different codes are presented and used to design the
63 structural elements that are finally tested using Abaqus. The key point of the paper is to assess
64 the relative accuracy of the code predictions of ultimate strength. This is a critical problem:
65 considering the elevated cost of the material in relation to normal steel rebar, an overly
66 conservative design will not be economic. At the same time, however, to guarantee a safe
67 design, the performance of the designed element and in particular the mode of failure need to
68 be properly investigated, especially under certain environmental and/or load conditions.

69 This study focuses on flexure and the slabs designed in Section 3 were chosen such that
70 flexural failure governs.

2. Codes and Design

2.1. Available codes

73 Several codes on the design of RC structures with FRP are available. Both the USA (ACI, 2003,
74 2006) and Canada (CSA, 2002; ISIS Canada, 2007) have published and revised specific
75 guidelines and codes. The ‘Eurocrete’ research project was started in 1993 and delivered a set
76 of design guidelines in 1996, focusing on the development of FRP rebars for concrete (Clarke et
77 al., 1996). As part of the research project, the first completely FRP reinforced footbridge was
78 installed in the UK in 1996. In the UK, the Institution of Structural Engineers (IStructE) published
79 a technical report providing design guidance in 1999 (IStructE, 1999), following the work done
80 by the Eurocrete project. Other countries such as Italy (CNR, 2006) have published their
81 recommendations, while other inter- national bodies are carrying out experimental studies and

publishing technical reports (fib – TG 9.3 (fib, 2007)). Unlike for other construction materials such as concrete, steel and timber, there is no Eurocode available for RC elements reinforced with embedded FRP rebar. For the scope of this study, a design approach based on the Eurocode philosophy was adopted, drawing on guidance on different European sources following the steps illustrated in Figure 2. Because the fib (see Table 2) document is the closest to the Eurocodes for its international nature, it will be considered as the principal source for the design of the FRP RC elements. For those cases where equations provided by non-European codes are recommended over adapted Eurocode equations, or no adapted Eurocode equations are provided, the Italian National Research Council (CNR) equations were used.

2.2. Ultimate Limit State

Considering a balanced typical steel RC section, with a steel strength to stiffness ratio similar to that of concrete, the neutral axis depth is usually not far above the middle of the effective depth. As for FRP reinforcement, the strength to stiffness ratio is greater by an order of magnitude, and the neutral axis depth is very close to the compressive end, as shown in Figure 3. Therefore for FRP RC elements, if the section is balanced, a large part of the cross-section is subjected to tensile strain at rupture. This causes large flexural deflections and cracks where the concrete is under tension. Owing to the large difference in stiffness between an uncracked and a cracked section, the stress in the bars and the deflection vary substantially between the two sections. The large amount of cross-section subjected to tensile strain can be decreased either by pre-stressing or increasing the amount of reinforcement. However, considering that an FRP RC section with $\rho_f \geq 0.5\%$ is over-reinforced, increasing the amount of reinforcement will lead to higher costs, larger short-term deformations due to the high strains needed and larger long-term deformations owing to the high stress in concrete, which will cause larger creep deformations (Pilakoutas et al., 2011).

Table 2 Overview of existing codes/guides

Organisation	Year
IStructE - Interim guidance on the design of reinforced concrete structures using fibre composite reinforcement (IStructE, 1999).	1999
CSA S806-02 - Design and construction of building components with fiber-reinforced polymers (CSA, 2002).	2002
ACI 440.1R-06 - Guide for the design and	2006

construction of structural concrete reinforced with Fiber-Reinforced Polymer (FRP) bars (ACI, 2015).	
CNR DT 206/2006 - Guide for the design and construction of concrete structures reinforced with fiber-reinforced polymer bars (CNR, 2006).	2006
ISIS Canada Design Manual No.3 - Reinforcing Concrete Structures with Fibre-Reinforced Polymers (ISIS Canada, 2007).	2006
fib TG 9.3 - Frp reinforcement for rc structures (fib - TG 9.3, 2007).	2007

108

109

110 **Figure 2. Design steps diagram according to the modified Eurocode approach**

111
$$\rho_f = \frac{A_f}{bd} \quad (\text{Eq. 1})$$

112 Different codes suggest different equations to calculate the balanced reinforcement ratio ρ_{fb}
 113 and the FRP reinforcement ratio ρ_f . Unlike steel reinforcement, FRP bars do not show a ductile
 114 failure, as shown in Figure 4, and designing for failure due to concrete crushing would generally
 115 be the favoured choice. However, this means that only a small portion of the FRP rebars'
 116 strength is used and the potential of the material is not fully utilised. The various codes provide

different equations to calculate the balanced reinforcement ratio, based on the internal force equilibrium and on the mechanical properties of the materials

119

$$\rho_{fb} = 0.85 \beta_1 \frac{f'_c}{f_{fu}} \frac{E_f \epsilon_{cu}}{E_f \epsilon_{cu} + f_{fu}} \quad (\text{Eq. 2})$$

$$\rho_{fb} = \alpha_1 \beta_1 \frac{\phi_c}{\phi_f} \frac{f'_c}{f_{frp}} \left(\frac{\epsilon_{cu}}{\epsilon_{cu} + \epsilon_{frp}} \right) \quad (\text{Eq. 3})$$

$$\rho_{fb} = 0.81 \frac{(f_{ck} + 8) \epsilon_{cu}}{f_{fk} \left(\frac{f_{fk}}{E_{fk}} \epsilon_{cu} \right)} \quad (\text{Eq. 4})$$

123

124 **Figure 3. Strain distribution for FRP and steel RC balanced section**

125

126 **Figure 4. Stress-strain curves of typical reinforcing fibres**

127 The American Concrete Institute (ACI) code recommends Equation 2, and a similar equation is
 128 proposed by the Canadian Standards Association (CSA) (Equation 3). Both codes replace the
 129 non-linear stress distribution with an equivalent rectangular stress block. Using a similar
 130 approach to Eurocode 2 (BS EN 1992-1-1:2004; BSI, 2004), Pilakoutas et al. (2002) proposed
 131 Equation 4 for FRP beams. The equation takes into account the material variability of concrete
 132 and, to avoid a premature failure, provides higher values of p_{fb} , compared to Equations 2 and 3.

133 **2.3. Moment resistance of FRP RC elements**

134 Fibre-reinforced polymer RC members in flexure can be designed by checking that the nominal
 135 flexural strength (M_n), multiplied by a factor of safety due to the brittle type of failure, exceeds
 136 the factored moment (M_u). From ACI 4401r-06 (ACI, 2006)

$$137 \quad \phi M_n \geq M_u \quad (\text{Eq. 5})$$

138 Different codes give different values for ϕ , all based on the values of p_f and p_{fb} . The ACI, CSA
 139 and fib provide Equations 6, 7 and 8, respectively, to calculate the ultimate moment resistance
 140 of an over-reinforced FRP RC member.

$$141 \quad M_n = A_f f_f \left(d - \frac{a}{2} \right) \quad (\text{Eq. 6})$$

$$142 \quad M_n = C \left(d - \frac{\beta_1 c}{2} \right) \quad (\text{Eq. 7})$$

$$143 \quad M_n = \eta f_{cd} b d^2 (\lambda \xi) \left(1 - \frac{\lambda \xi}{2} \right) \quad (\text{Eq. 8})$$

144 In Eq. 6 the stress distribution is approximated with a rectangular stress block, where a is the
 145 depth of the stress block:

$$146 \quad a = \left(\frac{A_f f_f}{0.85 f'_c b} \right) \quad (\text{Eq. 9})$$

147 The stress level in the FRP can be calculated as follows

$$148 \quad f_f = E_f \varepsilon_{cu} \frac{\beta_1 d - a}{a} \quad (\text{Eq. 10})$$

149 The CSA uses a similar approach with Equation 7, where the moment resistance of the section
 150 can be determined considering the force equilibrium:

$$151 \quad C = \alpha_1 \phi_c f'_c \beta_1 c b = A_{frp} \phi_{frp} f_{frp} = T \quad (\text{Eq. 11})$$

152 where c is the neutral axis depth and ϕ_c the material factor for concrete. The product $\beta_1 c$ is the
 153 depth of the stress block used to approximate the stress distribution, which leads to similar
 154 results to Eq. 6. In Eq. 8 proposed in the fib report (fib – TG 9.3 (fib, 2007)), λ and η are factors
 155 that take into account the strength of concrete. The ratio between the neutral axis depth and the
 156 cross section effective depth ξ , can be calculated as follows

$$157 \quad \xi = \frac{x}{d} = \frac{\epsilon_{cu}}{\epsilon_f + \epsilon_{cu}} \quad (\text{Eq. 12})$$

$$158 \quad \epsilon_f = \frac{-\epsilon_{cu} + \sqrt{\epsilon_{cu}^2 + \frac{4\eta\alpha_{cc}f_{ck}\lambda\epsilon_{cu}}{\gamma_c\rho_f E_f}}}{2} \quad (\text{Eq. 13})$$

159 The stress block used when the failure is provoked by concrete crushing is not applicable when
 160 designing an under-reinforced element and the failure is induced by the FRP rupture. In this
 161 case the concrete compressive strain at failure is unknown and an approximated equivalent
 162 stress block is needed. The ACI code proposes Eq. 14 to calculate the nominal moment
 163 capacity.

$$164 \quad M_n = A_f f_{fu} \left(d - \frac{\beta_1 c}{2} \right) \quad (\text{Eq. 14})$$

165 To avoid a complex analysis with multiple unknowns it is possible and conservative to assume
 166 that the maximum concrete strain is attained, using Equation 15

$$167 \quad M_n = A_f f_{fu} \left(d - \frac{\beta_1 c_b}{2} \right) \quad (\text{Eq. 15})$$

$$168 \quad c_b = \left(\frac{\epsilon_{cu}}{\epsilon_{cu} + \epsilon_{fu}} \right) d \quad (\text{Eq. 16})$$

169 where c_b is the depth of neutral axis at balanced failure condition, and can be calculated as per
 170 Eq. 16. A similar equation is proposed by the CSA Standard. Otherwise, it is necessary to
 171 determine the concrete compressive strain, which can be done carrying out an iterative
 172 procedure as suggested in the fib document (fib – TG 9.3 (fib, 2007))

$$\xi = \frac{x}{d} = \frac{\varepsilon_{cu}}{\varepsilon_f + \varepsilon_{cu}} \quad (\text{Eq. 17})$$

$$F_C = bd\xi \frac{\int_0^{\varepsilon_c} f_{cd} d\varepsilon_c}{\varepsilon_c} = \frac{A_f f_{fk}}{\gamma_f} = F_T \quad (\text{Eq. 18})$$

$$M_n = \frac{A_f f_{fk}}{\gamma_f} \left(1 - \frac{\xi}{2}\right) \quad (\text{Eq. 19})$$

2.4. Reduction factors

When considering the influence of durability on the design, the international codes propose environmental effect factors that reduce the strength of the FRP reinforcement. If refining the criteria used to reduce the material strength, taking into consideration factors as moisture, temperature, time and presence of alkali, a more accurate and less conservative result can be achieved. A comparison of the reduction factors proposed by the existing guidelines is presented in Tables 3 and 4. The fib Bulletin 40 (fib – TG 9.3 (fib, 2007)) proposes a methodology to determine an environmental factor as a function of the environmental conditions chosen by the designer. This provides a factor that has to be considered in the design strength Equation 20 (fib – TG 9.3 (fib, 2007))

$$f_{fd} = \frac{f_{fk0}}{\eta_{env,f}} \quad (\text{Eq. 20})$$

The environmental strength reduction factor η_{env} relates experimental factors and factors depending on the designer's judgement on the type of environment (e.g. moisture percentage). This approach brings benefit to the design, providing an accurate level of safety. It does not imply that the suggested factor would necessarily be less restrictive than those presented in Tables 3 and 4. In fact, as shown in the fib bulletin (fib – TG 9.3 (fib, 2007)) and in Stuart (2014) the environmental strength reduction factor can reduce the bar strength by up to 40%.

Table 3 Reduction factors for tensile strength at ULS

Material	IStructE (1999)	ACI 440.1R-06	CSA S806-02
ULS environment	E-glass: 1/3.6 Aramid: 1/2.2 Carbon: 1/1.8	GFRP: 0.8-0.8 AFRP: 0.8-0.9 CFRP: 0.9-1.0	GFRP: 0.5 AFRP: 0.6 CFRP: 0.75
ULS long term	GFRP: 0.3	-	-

	AFRP: 0.5 CFRP: 0.6		
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Table 4. Reduction factors for tensile strength at SLS

Material	IStructE (1999)	ACI 440.1R-06	CSA S806-02
SLS environment	GFRP: 0.3 AFRP: 0.5 CFRP: 0.6	GFRP: 0.39-0.52 AFRP: 0.44-0.59 CFRP: 0.50-0.65	GFRP: 0.25 AFRP: 0.35 CFRP: 0.65
SLS long term	-	GFRP: 0.14-0.16 AFRP: 0.24-0.27 CFRP: 0.44-0.50	(pre/post tension) GFRP: 0.25-0.30 AFRP: 0.35-0.40 CFRP: 0.65-0.70

2.5. Serviceability Limit States

The principles behind the verification of SLS for steel RC elements can be applied to FRP RC elements (fib - TG 9.3, (fib, 2007)), therefore the provisions in the existing codes of practice can be taken and modified to allow for differences in both short and long-term properties between steel and FRP reinforcement. The stress in concrete, at service load levels, can reach 80 to 100% of the peak stress (Cosenza et al., 1997). This leads to limit the working stress of the concrete and results in a non-economical use of the FRP reinforcement and an unnecessary increase in flexural capacity. Therefore, more research should investigate the SLS concrete stress limits for FRP RC to maximise the efficiency of the reinforcement.

2.5.1. Deflections

Considering two RC members under similar conditions, loading, dimensions, area of reinforcement and with different reinforcement material, steel and FRP, the latter would develop larger deformations due to the lower modulus of elasticity of the (non-carbon) FRP bars. Therefore, even considering that glass-fibre-reinforced polymer (GFRP) bars have a similar price to stainless steel ones, it is more expensive to maintain the same deflection limit when using FRP products (Balafas and Burgoyne, 2012). The Eurocode 2 formula for the calculation

214 of short- and long-term deflection for steel RC can be used for FRP-reinforced members, being
215 opportunely adapted using the factors to allow for the tension stiffening effect, duration of the
216 load and bond (CNR, 2006). ACI 440.1R-06 (ACI, 2006) provides equations to calculate the
217 effective moment of inertia that can be used in the deflection analysis of steel-reinforced
218 concrete beams (Branson, 1968) and opportunely adapted to be used for FRP-reinforced
219 members. The CSA recommends the use of the transformed moment of inertia I_t to calculate
220 the immediate deflection (CSA, 2002). If the service moment exceeds the cracking moment,
221 CSA A23.3-04 recommends the use of the effective moment of inertia, I_e , which is related to the
222 transformed moment of inertia and the moment of inertia for a cracked concrete cross-section
223 (CSA, 2004).

224 **2.5.2. Crack width**

225 The theory used for steel RC can be applied to FRP RC and the same equations can be used,
226 with opportune changes of coefficients and crack width limit, to take into account the different
227 material and the different bond conditions.

228 It can be seen in Table 5 that the crack width limitations for FRP RC elements are more relaxed
229 than for steel ones. This is because FRP bars are corrosion resistant and therefore less severe
230 limits can be adopted (ACI, 2006). In FRP-reinforced structures, the appearance, rather than
231 durability, governs the crack width.

232 **2.6. Design case studies**

233 **2.6.1. Design approach**

234 In the following, the results of the design of three different slab systems with spans commonly
235 seen in practice are presented. These will also be compared with a traditional steel
236 reinforcement solution designed to the Eurocode. For the comparative design studies presented
237 in this work, glass FRP bars were chosen for all of the designs; even if they did not match the
238 stiffness and ultimate strength properties of carbon fibre bars size for size, their competitive
239 price makes them the most popular type on the market. For the design of the slabs, as
240 recommended by the ACI guideline (see Section 2 for overview of existing codes), an over-
241 reinforced condition was preferred over a balanced-reinforced condition. For each slab type the
242 first design was carried out using the modified European approach to work out the slab depth
243 and required reinforcement. Once the properties of the slab were fixed, the design was checked

244 using the American and Canadian codes. To establish the loads considered for the design,
 245 precedence was given to the modified Eurocode approach, and the loads and safety coefficients
 246 recommended by the Eurocodes were used to calculate the factored loads on the slabs. The fire
 247 requirements were not considered in this study, and it was assumed that the elements designed
 248 would be able to carry the loads under fire through an alternative mechanism and load path.
 249 The following slabs will now be analysed

- 250 • One-way RC spanning slab for a multi-storey car park using normal weight (NW) concrete
- 251 • □Two-way RC spanning slab for a swimming pool roof using NW concrete
- 252 • Two-way RC spanning flat slab for a medical facility using light-weight (LW) concrete

253 **Table 5. Crack width limitations for FRP and steel RC elements**

Code	Eurocode 2	CNR (2006)	ACI 440.1R (2006) / CSA (1996)	
Material	Steel	FRP	FRP	
Exposure	Normal	-	Interior	Exterior
w_{max}	0.3mm	0.5mm	0.7mm	0.5mm

254

255 2.6.2. Material properties, Geometry and Loads

256 Tables 6–9 present the properties of the materials used in the models of the slabs. The depth of
 257 the slabs modelled in Abaqus was obtained from the design in accordance with the European
 258 approach. The first slab analysed is a one-way NW RC spanning slab for a multi-storey car
 259 park, simply supported over 7.5 m, as shown in Figure 5. The environment is considered to be
 260 aggressive, classified by BS EN 206:2013 (BSI, 2013) as XD3, hence would require a 40 mm
 261 cover for traditional steel reinforcement. The second slab analysed in this study is part of a
 262 swimming pool roof, assumed to be supported by beams in both directions, as shown in Figure
 263 6. The environmental condition considered is wet and rarely dry, and it is classified by BS EN
 264 206:2013 (BSI, 2013) with the label XD2, which requires a minimum concrete cover of 35 mm.
 265 This is a requirement due to environmental conditions, and hence it applies to the steel
 266 reinforcement only. The third slab presented in this study is a flat slab, with dimensions as
 267 shown in Figure 7. It is assumed to be an internal panel of a larger structure, where slabs are
 268 cast in situ and are continuous along the structure. Tables 10–12 show the loads for each slab
 269 type.

270 **Table 6. Mechanical properties of concrete**

Element	One-way slab	Two-way slab	Flat slab
$f_{ck,cube}$ [MPa]	45	37	28
$f_{ck,cyl}$ [MPa]	35	30	25
f_{cm} [MPa]	43	38	33
f_{ctm} [MPa]	3.21	2.90	2.75
f_{ctk} [MPa]	2.25	2.03	1.80
f_{cd} [MPa]	19.83	17	14.73
E_{cm} [GPa]	34.077	32.837	28.042
ν	0.2	0.2	0.2

271

272 **Table7. Mechanical properties of steel reinforcement**

f_{yk} [MPa]	f_{yd} [MPa]	f_{cd} [MPa]	E_s [MPa]	ε_y [%]
500	434.8	1.15	21000 0	0.2

273

274 **Table 8. Mechanical properties of FRP reinforcement**

f_f [MPa]	E_{cm} [MPa]	ε_f [%]
775	45000	1.7

275

276 **Table 9. Material and load factors (BSI, 2002; 2004 and fib, 2007)**

Load factors		Material factors (EN 2)		
γ_G	γ_Q	γ_C	γ_S	γ_f
1.35	1.5	1.5	1.15	1.25

277

278

279

280 **Figure 5. One-way GFRP slab: elevation (dimensions: mm)**

Figure 6. Two-way GFRP slab: elevation and plan (dimensions: mm)

2.6.3.Results

Summaries of the results of the design of steel-reinforced slabs and GFRP-reinforced slabs are presented in Tables 13–15. The results show that an FRP RC slab can cost from two to four times more than a traditional RC slab. On material cost alone, FRP cannot compete with normal carbon steel rebar, but it certainly can compete with stainless steel rebar. It is worth mentioning that one of the main reasons to choose FRP, in addition to niche applications, is its durability (Hollaway, 2010; Micelli and Nanni, 2004). When considering whole-life costs, it can be an economical solution, assuring a longer useful life for the structure and reducing maintenance costs (Shapira and Bank, 1997). Note that the relative reinforcement cost and total material cost of the slabs were calculated according to Balafas and Burgoyne (2012).

Figure 7. GFRP RC flat slab: elevation and 3D view (dimensions: mm)

Table 10. One-way NW RC spanning slab: loads

Loads	
Self-weight of concrete: 25 kN/m^3	$G = 6.75 \text{ kN/m}^2$
Weight of wearing surface: 0.5 kN/m^2	
Variable action q_k from EC: 2.5 kN/m^2	$Q_1 = 2.5 \text{ kN/m}^2$

Table 11. Two-way NW RC spanning slab: loads

Loads	
Self-weight of concrete:	25 kN/m^3
Slab surface:	1.5 kN/m^2
Snow variable action q_{snow} from Eurocode:	0.4 kN/m^2
Variable action q_k from Eurocode:	0.6 kN/m^2

Table 12. RC flat slab: loads

	Dead load	Live loads
ULS	$\gamma_g G = 8.24 \text{ kN/m}^2$	$\gamma_g q_k = 7.5 \text{ kN/m}^2$
SLS	$G = 6.1 \text{ kN/m}^2$	$q_k = 5.0 \text{ kN/m}^2$
	Distributed load	Linear load
ULS	15.74 kN/m^2	94.4 kN/m

(self-weight of LW concrete: 18 kN/m^3)

3. Overview of FE modelling of RC

3.1. Model validation

The modelling was undertaken using the commercial software Abaqus version 6.12. The model had been previously validated against several experimental studies, including the modelling of traditional steel RC beams and slabs. For brevity, one of the validation studies will be discussed here, further details and additional validation studies can be found in Stuart (2014). One of the validation models simulated the tests conducted at the Universit de Sherbrooke (Quebec, Canada) by Benmokrane et al. (2004) on a GFRP RC one-way slab denoted in the experiment as specimen S-GGB. A schematic drawing of the test set-up is shown in Figure 8, and a plan view of the FE model is provided in Figure 9. The markers used in Figures 10 and 11 show where the concrete reached its maximum compressive strength. The model simulated correctly the mode of failure by concrete crushing. Some discrepancies of the results can be accepted due to reasonable uncertainties related to the material properties, degree of bond (the rebar was assumed to be fully bonded), and the influence of the supports used in the tests. The nature of bond between the rebar and the concrete can be influential in determining the pre-crack stiffness of the concrete. In practice, it is common to assume full bond between rebar and concrete when modelling, however, this ignores the effect of bond slip. Any reduction in bond can result in a less stiff response in the member. Typically, where flexural elements have been modelled in FE using full bond assumptions, the pre-crack early behaviour of the members may be in excess of 30% stiffer than the experimental case (Genikomsou and Polak, 2015; Jendele and Cervenka, 2006). For the case of FRP rebar, it is likely that the effect of bond slip may be more pronounced than for steel rebar in the pre-crack regime, depending on the type of rebar profile, namely, ribbed or smooth, and so on. Recently a number of bond-slip models have been proposed for FRP rebar which offer improved prediction of the early pre-cracking behaviour; a comprehensive review of these can be found in Lin and Zhang (2014). Although the full bond assumption may over-predict stiffness in the early pre-crack regime, studies have shown this to be less significant in the post-cracking regime. Rafi et al. (2007) conducted a series of FE simulations of FRP-reinforced beams collated from various different experiments, in all cases a full bond assumption was adopted. In each case the FE model was shown to provide an accurate prediction of overall member behaviour; a similar conclusion can be drawn from

Figures 10 and 11. Based on the aforementioned, in the current study, a full bond assumption was adopted for expediency.

Table 13. One-way GFRP RC slabs: comparison

	Steel	GFRP		
Code	Eurocode	Eurocodes	ACI	CSA
Thickness	280mm	275mm		
Longitudinal reinforcement	$\phi 16@200\text{mm}$	$\phi 24@100\text{mm}$		
MRd	100kNm	302kNm	223kNm	249kNm
$\sigma_{\text{GFRP, fail}}$	490MPa	321MPa	361MPa	517MPa
Admissible stress ULS	-	332MPa	448MPa	775MPa
$\sigma_{\text{GFRP, SLS}}$	-	79MPa	70MPa	70MPa
Admissible stress SLS	-	233MPa	155MPa	233MPa
Δ	-	35.5mm	38.0mm	39.6mm
ω_{cracks}	-	0.303mm	0.300mm	0.257
$\omega_{\text{cracks, MAX}}$	-	0.3mm	0.5mm	0.5mm
Design governed by:	Deflection	Reinforcement: over-reinforced. Concrete: deflection	Deflection	Deflection
Longitudinal reinf. cost	£210	£780		
Total reinforcement cost	£312	£955		
Total cost	£580	£1326		

Table 14. Two-way GFRP RC slabs: comparison

	Steel	GFRP		
Code	Eurocode	Eurocodes	ACI	CSA
Thickness	200mm	150mm		
Longitudinal reinforcement	$\phi 12@200\text{mm}$	$\phi 20@200\text{mm}$		
MRd	35kNm/m	41kNm/m	31kNm/m	28kNm/m
$\sigma_{\text{GFRP, fail}}$	222MPa	320MPa	358MPa	425MPa
Admissible stress ULS	-	339MPa	448MPa	775MPa
$\sigma_{\text{GFRP, SLS}}$	-	75MPa	67MPa	67MPa
Admissible stress SLS	-	233MPa	155MPa	231MPa
Δ	-	1.5mm	2.0mm	2.0mm
ω_{cracks}	-	Uncracked	Uncracked	Uncracked
Design governed by:	Deflection	Depth of concrete governed by deflection. Amount of reinforcement, governed by ρ_{fb}		
Total reinforcement	£456	£2080		

cost		
Total cost	£982	£2469

Table 15. GFRP RC flat slabs: comparison

	Steel	GFRP		
Code	Eurocode	Eurocodes	ACI	CSA
Thickness	200mm	275mm		
Longitudinal reinforcement Upper layer	φ10@150mm	φ18@100mm		
	φ10@100mm	φ20@100mm		
Longitudinal reinforcement Bottom layer	φ10@150mm	φ20@150mm		
	φ10@150mm	φ18@150mm		
	φ12@100mm	φ20@100mm		
MRd+ @ midspan	131kNm/m	178kNm/m	132kNm/m	175kNm/m
MRd-,MRd+ @ supports	142kNm/m	187kNm/m	139kNm/m	185kNm/m
σGFRP,fail @ midspan	163MPa	313MPa	351 MPa	408 MPa
σGFRP,fail @ supports	212MPa	357 MPa	399 MPa	465 MPa
Admissible stress ULS	-	620 MPa	512 MPa	775 MPa
σGFRP,SLS @midspan	-	48.9 MPa	50.5 MPa	49.8 MPa
σGFRP,SLS @supports	-	52.5 MPa	51.5 MPa	50.7 MPa
Admissible stress SLS	-	233 MPa	155 MPa	233 MPa
Shear	-	42kN	88kN	88kN
Δ	-	4.4-42mm	4.4-56.5mm	4.4-56.5mm
ωcracks,MAX	-	0.36mm	0.5mm	0.5mm
Design governed by:	Deflection	Crack width limit		
Total reinforcement cost	£2310	£8360		
Total cost	£2830	£8950		

3.2. Material Models

3.2.1. Concrete Damaged Plasticity

The constitutive model used in this project is the concrete damaged plasticity (CDP) model. This model was chosen because of its inherent robustness in regard to numerical convergence, particularly where significant non-linear behaviour is in occurrence, (Daud et al., 2017; Genikomsou and Polak, 2015). One of the main advantages of the CDP model is the direct

relation between the parameters that describe the model and their physical interpretation. The recommended values for the parameters describing the CDP model are summarised in Table 16. Further background detail on the FE model can be found in Stuart (2014), Kmiecik and Kaminski (2011) and Simulia (2012). Severe convergence difficulties might be caused in models where softening behaviour and stiffness degradation are exhibited. It is possible to solve those convergence difficulties by using a viscoplastic regularisation of the constitutive equations. The CDP model can allow the stresses to be outside the yield surface. The use of the viscoplastic regularisation, adopting a small value for the viscosity parameter μ compared to the value of the time increment, can significantly improve the rate of convergence of the model without significantly affecting the results. Here μ represents the relaxation time of the viscoplastic system and is the plastic strain evaluated in the inviscid backbone model (Simulia, 2012). A value of 0.0005 was used, based on the validation study (Stuart, 2014).

Figure 8. One-way GFRP RC slab: test set-up, elevation (dimensions: mm)

Figure 9. One-way GFRP RC slab: FE model (dimensions: mm). Considering the symmetry of the set-up, only the portion highlighted on the left-hand side was modelled in Abaqus

Figure 10. One-way GFRP RC slab: moment–strain curves in the bottom rebar at mid-span

Figure 11. One-way GFRP RC slab: load–displacement (at mid-span) curves. Abaqus results compared with experimental results (S-GGB). The marker identifies the point of first onset of concrete failure

3.2.2. Stress strain relations

In the model, additive strain rate decomposition is assumed:

$$\dot{\epsilon} = \dot{\epsilon}^{el} + \dot{\epsilon}^{pl} \quad (\text{Eq. 21})$$

where $\dot{\epsilon}$ is the total strain rate, and $\dot{\epsilon}^{el}$ and $\dot{\epsilon}^{pl}$ are the elastic and plastic parts of the strain respectively. In the CDP model the inelastic strains are used to draw the stress-strain curve for uniaxial compression, for more information refer to the ABAQUS manual (Simulia, 2012).

380 To transform the strains, it is necessary to identify the stress for which the material can be
381 considered as non-linearly elastic. As suggested in the Eurocode 2, this point was assumed as
382 $0.4 f_{cm}$. When detailed laboratory results are not available, it is still possible to plot the concrete
383 stress–strain curve using an appropriate formula. In this study the concrete stress–strain
384 relation adopted is the one given by equation 3.14 from the BS EN 1992-1-1 (BSI, 2004). A
385 weakening non-linear curve for the tension stiffening, shown in Figure 12, was used in the
386 model validation (Wang and Hsu, 2001). The relation is given in Equation 22. The suggested
387 value for ε^* , corresponding to a zero value of the stress, can be assumed equal to ten times the
388 strain at which f_{cm} is reached (Simulia, 2012). A value of 0.01 is recommended.

$$389 \left\{ \begin{array}{ll} \sigma_t = E_c \varepsilon_t & \text{if } \varepsilon_t \leq \varepsilon_{cr} \\ \sigma_t = \left(\frac{\varepsilon_{cr}}{\varepsilon_t} \right)^{0.4} & \text{if } \varepsilon_t > \varepsilon_{cr} \end{array} \right. \quad (\text{Eq. 22})$$

390
391 **Figure 12. Modelling of RC structures and composite structures with concrete strength**
392 **degradation taken into consideration**

393
394 **Table 16 . Concrete damaged plasticity parameters**

Dilation angle	Eccentricity	f_{b0}/f_{c0}	K_C	μ
36	0.1	1.16	0.66 7	0

395
396
397
398

3.3. Steel

The properties of the steel reinforcement used in the models were gathered from associated experimental data. An elastic perfectly plastic stress-strain curve was used. The steel material properties were defined using isotropic linear elasticity up to the yield stress σ_y , using the Young's modulus, E, and the Poisson's ratio, ν .

3.4. FRP

Two major aspects of FRP that are important to consider when modelling the material are: its linear behaviour up to failure and that the physical properties are directionally dependent. The rebar behaviour can be defined as perfectly elastic up to the ultimate stress of the material. By modelling the rebar using wire truss elements it is not necessary to define properties in multiple directions, this is deemed to be a reasonable approximation.

3.5. Elements

Two types of elements were used in this paper: two-node elements to model the reinforcement and eight-node linear brick elements to model the concrete. The interaction between the two materials was considered a perfect bond; as previously mentioned, this is a main factor of uncertainty, considering how much the bond between the concrete and the FRP rebar can influence the behaviour of the element (Achillides and Pilakoutas, 2004; Lin and Zhang, 2014).

4. Design Series FE Models

4.1. Geometry and material properties

The slabs designed in Section 2.6 were then analysed using non-linear FE. The slabs' material properties can be found in Tables 17–19. To reduce the computational time, the symmetry of the slabs was considered. The portion of the slabs modelled in Abaqus is shown in Figures 13, 16 and 18. The loads were applied in increments until failure occurred, in some cases several times the load dictated by the ultimate limit state design case. Mesh sensitivity studies were undertaken together with the model validation to define the optimum mesh density for the different models; other sensitivity tests included the viscosity and tension stiffening parameters (Stuart, 2014). Eight-node linear brick elements with a thickness of 30 mm were used to model the slabs, based on the results from the validation study (Stuart, 2014). Solid-extrude and Wire features were used in Abaqus to model the slabs and the reinforcement, respectively. For the one-way simply supported slab a regular mesh with 200 mm square elements was adopted

(Figures 14 and 15). Similarly, for the two-way spanning slab, square elements with edges not greater than 200 mm were used. However, the density of the mesh was adapted to have a higher number of elements close to the edges and larger elements in areas with smaller stress gradient, as shown on Figure 17. The sizes of the elements ranged from 40 to 200 mm. In the flat slab model a mesh similar to the one adopted for the two-way spanning slab was used (Figure 19). The column supporting the slab was considered to be 1.5 m long on both sides of the slab, tied to its surface and fully fixed at the edges (Figure 20). Common steel was used as longitudinal and transversal reinforcement in the column, with a C30/37 concrete. Regarding the conventions used to present the results, the unit kNm is used for bending moment, MPa when describing stresses and mm for deflections. Black dashed lines are used to identify axes of symmetry.

Table 17. Boundary conditions

One-way GFRP RC slab	Element	U1	U2	UR2	UR3
	Vertical mid-span face	0	free	0	0
	Support	free	0	free	free

(1) x-axis: longitudinal axis, (2) y-axis: vertical axis, (3) z-axis: transverse axis.

Two-way GFRP RC slab	Face	U1	U2	U3	UR1	UR2	UR3
	AB	free	0	free	0	0	free
	AD	free	0	free	0	free	0
	CD	free	free	0	0	0	free
	BC	0	free	free	free	0	0
GFRP RC flat slab	Face	U1	U2	U3	UR1	UR2	UR3
	AB	free	free	free	free	free	free
	AD	free	free	free	free	free	free
	BC	0	free	free	free	0	0
	CD	free	free	0	0	0	free

(1) x-axis: longitudinal axis, (2) y-axis: vertical axis, (3) z-axis: transverse axis.

GFRP	Face	U1	U2	U3	UR1	UR2	UR3
------	------	----	----	----	-----	-----	-----

RC flat slab	AB	free	free	free	free	free	free
	AD	free	free	free	free	free	free
	BC	0	free	free	free	0	0
	CD	free	free	0	0	0	free

(1) x-axis: parallel to AB, (2) y-axis: vertical axis, (3) z-axis: parallel to AD.

Table 18. Concrete properties

	f'_c (MPa)	f_{cm} (MPa)	E_c (MPa)	ν
One-way RC slab	35	43	34007	0.2
Two-way RC slab	30	38	32827	0.2
RC flat slab	20	28	28042	0.2

Table 19. GFRP rebar properties

	Diameter (mm)	Area (mm ²)	f_u (MPa)	E_s (MPa)	ϵ_0	ν
One-way RC slab	9.9	71	775	45000	0.017	0.3
	11.90	104	775	45000	0.017	0.3
	23.40	434	775	45000	0.017	0.3
Two-way RC slab	19.5	298	775	45000	0.017	0.3
	19.5	298	775	45000	0.017	0.3
RC flat slab	17.9	251	775	45000	0.017	0.3
	19.9	311	775	45000	0.017	0.3

4.2. Results

The three models show a brittle failure due to concrete crushing, as predicted by the codes. The shape of the load– displacement curves matches the expectations and the results of the validation models, meaning that the behaviour of the slab was simulated correctly. The load– displacement curves can be divided into two parts

- A first segment where the concrete is carrying most of the load and the slope of the curve is governed by the elastic modulus of the concrete

- A second linear segment that follows the cracking of the concrete and in which the FRP rebar starts to carry an increasing portion of load up to the failure due to concrete crushing.

The results from the analysis in Abaqus are presented next. Reference is made to SLS and ULS as the stages where the factored design load at serviceability limit state and at ultimate limit state, respectively, were applied. The third and final stage presented, referred to as failure, is the stage where the maximum strength of the material is reached under the load applied.

4.2.1. One-way GFRP RC slab

Results from the ABAQUS model presenting the principal stress distribution at different load stages are shown in Figures 21 and 22. When the maximum strength of concrete is reached, the FE analysis continues to redistribute stresses to adjacent material up to a point where there is no more residual strength to be used. In reality, failure occurs when the maximum strength of one of the two materials is reached, in this particular case the strength of the concrete. Localised crushing of concrete first occurred at around 37 kN/m^2 in the mid-span; the magnitude of the loads at SLS and ULS is highlighted in Figure 23. Subsequent to the onset of concrete crushing and given the low stress in the rebar, further loading of the slab was achieved. At these later stages, membrane action is engaged with eventual rebar rupture occurring at around 90 kN/m^2 . Such an enhancement in ultimate strength over the service loads as a result of membrane action is consistent with that observed in tests of traditional steel reinforced slabs (e.g. Gouverneur et al., 2013).

In Figure 23 and other similar load–deflection curves, the deflections/stress under SLS and ULS loads are identified with a triangle and a square, respectively, while the circle represents the first onset of concrete crushing. The bending moment corresponding to the onset of concrete crushing (and similar) was identified as the moment resistance of the slab and compared to the moment resistance calculated using the codes (which is considerably higher than the bending moment under the ULS load). As shown in Figure 24, the stress in the FRP reinforcement is well within the material strength at the first onset of concrete failure.

484

485 **Figure 13. One-way GFRP RC slab – FE model geometry (dimensions: mm). Hatched area**
 486 **represents the portion of the slab modelled in Abaqus**

487

488 **Figure 14. One-way GFRP RC slab model – mesh**

489

490 **Figure 15. One-way GFRP RC slab – FE model geometry, slab outline and reinforcement**
 491 **distribution**

492

493 **4.2.2. Two-way GFRP RC slab**

494 The load displacement curve for the two-way slab is provided in Figure 27, together with the
 495 SLS and ULS loads. As in the one-way slab, the onset of concrete crushing occurs long before
 496 rupture of the FRP. The distribution of the principal stresses in the slab at failure and ULS, is
 497 shown in Figures 25 and 26, respectively. The load displacement curve for the two-way slab is
 498 provided in Figure 27, together with the SLS and ULS loads. The load levels at which those

499 stresses are reached are identified in Figure 28. Post-onset of concrete failure, the deflections
 500 of the slab are far in excess of practical limits. Again, this behaviour is made possible by the
 501 generation of membrane action in the slab.

502

503 **Figure 16. Two-way GFRP RC slab – FE model geometry (dimensions: mm). Hatched area**
 504 **represents the portion of the slab modelled in Abaqus**

505

506 **Figure 17. Two-way GFRP RC slab model – mesh**
 507

508 **4.2.3.GFRP RC Flat slab**

509 The stress distribution in the slab at ULS and SLS is presented in Figures 29 and 30,
 510 respectively, with the points corresponding to relevant load stages identified in Figures 31 and
 511 32. As the slab is considerably over-reinforced, the model is able to redistribute the load multiple
 512 times, reaching a load-carrying capacity much higher than the ULS design load. As for the
 513 previous cases, the onset of concrete crushing occurred while the FRP rebar was still relatively

514 lightly stressed. Extensive residual capacity of the rebar mesh allowed further load-carrying
515 capacity to be achieved, with eventual rebar rupture occurring around 88kN/m^2 .

516

517 **Figure 18. GFRP RC flat slab: one-quarter of the structure – FE model geometry**
518 **(dimensions: mm). Hatched area represents the portion of the slab modelled in Abaqus**

519

520 **Figure 19. Flat GFRP RC slab model – mesh**

Figure 20. GFRP RC flat slab: boundary conditions

4.3. Comparison with the Code predictions

The results obtained are collated and presented in Figure 33, with the results from the FE analysis overlain in light grey. If looking at the results given by the modified Eurocode approach (denoted EN on the diagram), for the bending moment resistance, the one-way slab has a resistance of 302 kNm, while the slab fails under an applied bending moment of 256 kNm according to the FE model, taking onset of concrete crushing as the failure load. Considering the stress in the rebar at ULS, the modified Eurocode approach gives a value of 321 MPa, against the 263 MPa from the FE analysis. Finally at SLS, the Abaqus model gives a rebar stress of 6.5 MPa, against the 79 MPa given by the modified Eurocode design. If looking at the bending moment resistance of the slabs, it is possible to state that the modified European approach (Eurocode) gives potentially non-conservative results for the one-way slab and the flat slab, while the prediction for the two-way slab is fairly accurate. The ACI and CSA give an accurate result for the one-way slab and two-way slab, and potentially non-conservative results for the flat slab. At ULS the code predictions of the stress in the rebar maintains a certain ratio in the different models, the prediction is quite accurate and conservative. The modified Eurocode approach gives lower values for the stress for all the three models. The CSA gives higher values than the other codes, for the one-way slab the stress at ULS is double the stress recorded in the FE model. The stress in the rebar at failure is always below the maximum strength, meaning that the equations proposed to estimate the balanced failure ratio are

reliable. Considering that the SLS governs the design of the three slabs, the amount of reinforcement and the depth of the cross-sections had to be increased considerably, in comparison to the requirements from the ULS. However, the resulting bending moment capacity is still lower or not sufficiently higher than the FE model prediction in some cases. The checks for the SLS give conservative results, which is a priority when designing a structure. However, the codes provide overly conservative results, overestimating the stress more than ten times. It is important to remember that the results are being compared with the FE models, and this discrepancy could be attributed in part to inaccuracy of the numerical strain in the rebar in the pre- cracking phase, as outlined in Section 3.1. When comparing the code prediction with the earlier mentioned experimental tests of Benmokrane et al. (2004), as shown in Figure 34, the code prediction for the stress at SLS is much closer to the experimental results. A possible reason for the underestimation of the stresses in the FE models may be the CDP model, overestimating the resistance of the concrete for low levels of load, in particular the tensile strength (which in practice can be variable), in combination with uncertainties over rebar bond. Observation of the experimental results reveals the same general pattern between code predictions as the designed one- way slab. Both the 7.5 m case study slab and the Benmokrane slab have very similar reinforcement ratios, while the span-to- depth ratios are 27.3 and 16.7, respectively. It should be noted that, in the Benmokrane test, the moment on first cracking in the experiment was 13 kNm; at the SLS design load the corresponding moment is just beyond this at 14.5 kNm. This is in contrast to the 7.5 m case study slab, where the SLS design moment exceeds the theoretical moment on first cracking by almost 100%. Further investigation is needed to find out if this early behaviour is also noticed in other similar applications.

Figure 21. One-way GFRP RC slab: min. principal stresses in concrete at failure (MPa)

Figure 22. One-way GFRP RC slab: max./min. principal stresses in concrete (a) at ULS and (b) at failure (MPa)

Figure 23. One-way GFRP RC slab: Abaqus load-displacement curve

Figure 24. One-way GFRP RC slab: stress in bottom longitudinal reinforcement, at mid-span

Figure 25. Two-way GFRP RC slab: max. principal stresses in concrete at failure (MPa)

Figure 26. Two-way GFRP RC slab: max./min. principal stresses in concrete at ULS (MPa) (both stresses at the top and bottom surfaces of the slab are shown)

Figure 27. Two-way GFRP RC slab: Abaqus load-displacement curve

Figure 28. Two-way GFRP RC slab: stress in the bottom reinforcement, at mid-span

Figure 29. GFRP RC flat slab: min. principal stresses in concrete at ULS (MPa) – plan view

593 **Figure 30. GFRP RC flat slab: vector plot of max./min. principal stresses in concrete, at**
 594 **SLS (MPa) – plan view. The column was removed from the view for clarity**

595 **Figure 31. GFRP RC flat slab: Abaqus load–displacement curve at point C**
 596

597 **Figure 32. GFRP RC flat slab: stress in the bottom reinforcement at point B**
 598
 599

600 5. Conclusions

601 This paper has presented a comparative review of design approaches to FRP bars as a
 602 reinforcement for concrete members, with an introduction to the materials and their properties,
 603 the behaviour of the composite and the available inter- national codes. In order to make the
 604 FRP reinforcement a strong alternative to traditional reinforcement, the performance prediction
 605 from the codes has to be as accurate as possible considering that rough approximations in the
 606 design imply a significant increment of the costs. Considering the existing design codes, a
 607 possible design path was proposed in Section 2, gathering the recommendations from
 608 Pilakoutas et al. (2011), fib and CNR. Following this approach, in addition to the ACI code and

609 the CSA standard recommendations, it was shown how the design of FRP RC elements is
610 governed by the SLS requirements. Also, when designing for a ductile failure the amount of
611 FRP rebar required is elevated. As a consequence, designing the element to be over-reinforced
612 requires an amount of rebar that causes the element price to be uncompetitive in comparison
613 with a similar steel RC element and potentially for some stainless steel grades. The
614 reinforcement required to guarantee concrete crushing can be reduced by reducing the thick-
615 ness of the slab; however, the design at SLS sets a lower boundary to the minimum amount of
616 concrete necessary in order to avoid excessive deflection and cracking. To achieve an efficient
617 design, an accurate prediction of the stress in the concrete at SLS is necessary. More research
618 is needed in this regard. This study has focused on flexure; however, some elements and
619 particularly flat slabs may be vulnerable to shear. The designs presented in this paper were
620 chosen such that flexural failure governs. The results of the CDP model validation show how
621 Abaqus is able to correctly describe the load–deflection curve and mode of failure of the
622 concrete elements. For the purpose of the FE model, the bond between the reinforcement and
623 the concrete was assumed as perfect. However, as emerged from the first part of the study, this
624 assumption does not accurately represent the real interaction between the two components.
625 This can significantly influence the performance of the structural element in the pre-cracking
626 phase. Further research and adoption of appropriate bond-slip models is needed in order to
627 improve the accuracy of future FE models. The codes predictions were compared with the FE
628 analysis results, showing that, for the case of the one-way slab and flat slab, the designs are
629 potentially non-conservative with respect to flexure. Furthermore, a discrepancy between the
630 codes' prediction at SLS and the FEA results was shown. The evidence from this study
631 suggests that the Canadian approach emerges as the most conservative one, while the
632 modified Eurocode approach gives quite accurate, but still potentially non-conservative
633 predictions for the stress at ULS in the rebar. Finally, the findings of this study have raised some
634 doubts regarding the results given by the codes for different GFRP RC slabs. However, the
635 general implication of the results is limited by the small sample size and further research is
636 needed in order to study the response of a greater set of specimens, combining both
637 experimental and analytical work. Further research is needed to investigate the performance of
638 FRP RC elements under fire. Owing to the nature of the material, a substantial amount of

639 reinforcement is needed to guarantee the safety of the structure under fire. This leads to bigger
 640 and heavier elements, which are uneconomical both because of the increased dead load and
 641 because of the amount of reinforcement required to assure a ductile failure of the element. It is
 642 recommended that further research be under- taken to define a possible solution for decreasing
 643 the concrete area without a substantial reduction of the moment of inertia of the cross-section. A
 644 possible approach for this issue could be the combination of hollow-core slabs and FRP
 645 reinforcement. As the SLS design case is critical for the design of FRP RC elements, further
 646 investigation is needed to refine the prediction of the codes especially considering the difference
 647 with the numerical models results. Finally, an internationally recognised manufacturing standard
 648 for the FRP rebar would make use of this material easier for designers, and contribute to the
 649 more widespread use of this technique.

650
 651 **Figure 33. Code comparison: bending moment resistance at mid-span (kNm); stress in**
 652 **the rebar at ULS (MPa) – GFRP bar at mid-span (one- and two-way slabs) and at the**
 653 **column strip for the flat slab; stress in the rebar at SLS (MPa) – GFRP bar at mid-span**
 654 **(one- and two-way slabs) and at the column strip for the flat slab**

655
 656 **Figure 34. Code comparison compared with test results: Benmokrane slab (Benmokrane**
 657 **et al., 2004) – bending moment resistance at mid-span (kNm) (failure due to concrete**
 658 **crushing). Stress in the rebar at ULS (MPa) – GFRP bar at mid-span. Stress in the rebar at**
 659 **SLS (MPa) – GFRP bar at mid-span**

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