The Dynamics of Public Spending and Economics

Development

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Abstract

The objective of the thesis is to provide a theory that explains the stylized facts regarding the trend of taxation policies, public spending and sovereign debt in advanced economies for the past couple of decades. The thesis focuses on distinguishing two types of public spending – productive investment and welfare payment – and develops two different frameworks to examine the importance of the composition of these two types of public spending for economic growth and welfare.

Chapter 2 presents a dynamic political-economy model in which voters decide tax rates and the proportion of public goods expenditure devoted to non-productive (but utility-enhancing) public goods. This non-productive public goods expenditure gives rise to a habit effect - it has to be at least as large as a fraction of last period value to provide utility. The median voter theorem applies. Starting from a steady state without the habit effect, its introduction leads to transitional dynamics that mimic several stylized facts: in particular, countries with higher income tend to have larger government and spend more on welfare programme.

Chapter 3 studies the impact of public deficit on long-run economic growth by distinguishing the different types of government spending: investment and welfare payment. The model in this chapter predicts a non-monotonic or threshold effect in the relationship between public deficit and steady state growth rate. The composition of the public spending (the ratio between productive and non-productive) dictates the “threshold” in the national debt level. Countries which spend more on providing productive public goods could maintain a higher level of national debt that promotes growth.
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Dedication
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Chapter 1

Introduction

The objective of the thesis is to provide a theory that explains the stylized facts regarding the trend of taxation policies, public spending and sovereign debt in advanced economies for the past couple of decades. The thesis focuses on distinguishing two types of public spending – productive investment and welfare payment – and develops two different frameworks to examine the importance of the composition of these two types of public spending for economic growth and welfare.

Historical data on government revenue and expenditure reveals the striking feature that the size of government has expanded in general throughout history, especially during the past few decades and particularly in developed countries. A deeper investigation of the composition of various governments’ expenditure indicates that a major increase in public spending is the consequence of the growing involvement of the state in welfare-related activities. Welfare support has one of the highest growth rates among government expenditures, and accounts for most of the spending differences between developed and developing countries. Although the share of government spending in national income remained at 11-12 percent during the period between 1870 and 1913, the figure started to rise to 27.9 percent in 1960 and 44.9 percent in 1990, despite the fact that there was no major war or economic depression during that period (Tanzi and Schuknecht, 1997).
During the same period, public debt increased in most countries, especially in major developed countries, after the beginning of the great recession, and more recently there have been extensive academic and political debates over the effectiveness of austerity in fiscal policies for debt-burdened economies. Despite its importance and urgency, there is still little consensus on the effect of public debt on economic growth and welfare (see for example, Reinhart and Rogoff, 2010 and Krugman 2013).

1.1 Habit Effect

The model presented in Chapter 2 is intended to explain the positive correlation between the development stages of an economy and its preference for higher tax rate and welfare spending using the argument of habit formation in public goods.

The following stylized facts are summarized from OECD and the Penn World Table data:

1. The share of tax revenue in GDP increased across all developed countries over time.

2. The share of welfare spending in GDP and total government expenditure increased in most developed countries over time.

3. The share of tax revenue in GDP is higher in countries with higher GDP per capita than countries with lower GDP per capita.

4. The share of welfare expenditure in GDP is higher in countries with higher GDP per capita than countries with lower GDP per capita.

Therefore, over time, growth in per capita income tends to be accompanied by a rise in tax rate. A slow-down of growth is accompanied by a decline in the share of productive public spending. This suggests that changes in the tax rate and composition of public spending are important to the growth of per capita income.
These stylized facts have been the subject of two strands of literature. The first studies the optimal level of tax rates and government expenditure when this provides productive factors that raise the productivity of firms. The basic finding is that there exists an inverted U shape relationship between tax rate and growth. At low tax rates, the growth rate increases if the tax rate rises; at high tax rates, the growth rate declines if the tax rate continues to rise. The optimal tax rate which maximizes the growth rate should equal the productivity of public spending (e.g. Barro, 1990; Barro and Sala-i-Martin, 1992; Glomm and Ravikumar, 1992; Glomm and Ravikumar, 1994).

The second set of literature investigates the determination of re-distributive tax policies using political-economy models (e.g. Alesina and Rodrik, 1994; Persson and Tabellini, 1994), where heterogeneous agents have different policy preferences. The winning policy is the one preferred by the median voter, whose identity will depend on the income/wealth distribution of the economy. This literature is able to explain the cross-country observation that countries with higher inequality are normally associated with higher tax and welfare spending. However, these models do not address the time series stylized facts mentioned earlier.

The model in chapter 2 describes an economy where the government provides two types of public goods: one that directly improves the productivity of the economy, the productive public goods (PPG) which goes into firm’s production function; the other directly improves the well-being of the individuals in the society, the non-productive but utility-enhancing public goods (NPG) which goes into households’ utility function.

We argue that there exists a habit effect on NPG. The habit effect is that the level of utility a household derives from NPG depends on the increment of NPG provided by government in this period compared with the previous period. This implies the government can not adjust the level of NPG freely - the benefit that was already given in previous periods is very difficult to withdraw. The recent enormous public pressure against benefit cuts in different countries
may justify this assumption.

This chapter has made several contributions to the existing literature. Firstly, it provides a political-economy model that generates transitional dynamics for tax rates and welfare spending consistent with the above stylized facts. Secondly, this model formalizes a well-known observation in political-economy that welfare spending is difficult to reduce once it has been implemented. This has been described in the literature on governance as a “ratchet effect”, which captures the seemingly irreversible expansion of the size of governments. Finally we describe the introduction of the habit effect to certain types of government expenditure that may create the consequent ratchet effect. Discussion of the habit effect on private consumption is not new, but there has been little written on the habit effect on public goods.

1.2 Sovereign Debt

There has been fundamental change in fiscal policy in many developed countries in recent years amounting to the disappearance of balanced budget regimes in these countries. Instead, deficit financing has arisen as an alternative for many countries (Chalk, 2000). Empirical data suggest that, since the Second World War, public debt has been increasing for all major developed countries. In 2008, Japan—then the second largest economy—had a public debt to GDP ratio of more than 170%; the figure in the US and Euro zone was above 70% (Figures are taken from Organization for Economic Cooperation and Development Economic Outlook, 2008). Given the fact that billions of Dollars and Euros have been spent by governments in order to stabilize the economy since the 2008 financial crisis and the level of their sovereign debt has increased along with a slow down in the GDP growth rate, the ratio between the two has become even larger. Therefore, the question of how government should respond to the increasing public debt attracts more attention from both scholars and policy makers. The most important issue is to find out the effect of public debt on economic growth and welfare and to discover whether
borrowing could rescue the economy from detrimental shock and restore its long-run growth path or whether it just reduces the current pain at the expense of a permanent welfare loss in the long run.

The relationship between debt and growth is more complicated and has always been a focus in the literature of public finance. The core question concerns the real effect of high levels of government debt on the long-run growth rate. Most existing theoretical models yield ambiguous results. Many empirical studies reveal a high level of government debt negatively correlating with growth. In addition, the causality between these two has not yet been convincingly established. Some papers have suggested that there may exist a non-monotone relationship between debt and growth (for example, see Checherita-Westphal, Hallett and Rother, 2014). However, the turning points found by these papers are not so robust to minor changes in data range or statistical methods.

The conventional view of the relationship between government debt and economic growth is that government deficits in general have a positive effect on disposable income, aggregate demand and output in the short run. However, in the long run, assuming the Richardian Equivalence does not hold, the decrease in public saving brought about by a higher budget deficit will not be fully compensated by an increase in private savings and, as a consequence, national savings will decrease, resulting in lower investment at home which will have a negative effect on GDP (Elmendorf and Mankiw, 1999).

On the other hand, some argue that a higher level of debt is always bad for the economy and could lead to a lower growth rate in the long run, as well as in the short run. When public debt is so high, often even the interest payment will become a difficult expenditure for the government, especially for countries suffering from negative economic shocks, like the recent financial crisis. Other factors can also amplify the negative impact of debt. For example, when the uncertainty and negative expectations of the agents toward the high level of debt, such as a
potential rise in inflation and financial repression, is considered (Cochrane (2011 a,b)). In these scenarios, agents will make counter-productive and inefficient decisions to offset the potential future loss. The magnitude of these two forces in reality is defined by empirical studies.

In general, empirical studies find that borrowing has more positive effect when debt is small, and more negative effect when it is large. Many empirical papers have found a non-linear relationship between the level of government debt and growth which is characterized by the existence of a threshold such that the debt displays opposite effects on growth below and above the threshold. Krugman (1988) uses the feature of debt overhang to generate this non-linearity and threshold effect between debt and growth. However, it is doubtful that the argument of debt overhang could be applied to developed countries, where most of the public debt is domestic. Therefore, in order to study the relationship between the public debt and economic growth in developed countries, a model with features that reflect the characteristics of a developed country is more appropriate. Adam and Bevan (2005) has found supporting evidence for this non-linear relationship between fiscal deficits and growth in developing countries as well. A panel data of 45 developing countries suggesting this threshold of the deficit level is around 1.5% of GDP. Reinhart and Rogoff (2010) argue that when gross external debt reaches 60 percent of GDP, a country’s annual growth declines by two percent and, for levels of external debt in excess of 90 percent, GDP growth is roughly cut in half. This provided support for pro-austerity policies in the aftermath of the financial crisis of 2007–2008. Herndon et al. (2014) argue that, when properly calculated, the average real GDP growth rate for countries carrying a debt-to-GDP ratio over 90 percent is not dramatically different from the rate when debt over GDP ratios are lower.

The model in Chapter 3 distinguishes two types of public spending – government investment that improves the level of productive public goods and welfare payment in the form of a direct transfer that increases on agent’s budget for consumption – and develops a simple
dynamic model to examine the importance of the composition of these two types of public spending on debt and economic growth. This model generates a non-monotone relationship between debt and growth. Higher debt could lead to either a higher or lower long-run economic growth rate depending on several other economic variables.

The results shed light on the recent policy debate on taxation, borrowing and public spending intend to cope with the recent economic crisis. Firstly, we have demonstrated the importance of countries’ allocation of their public resources: spending more on investment to increase the productivity of the economy or more on welfare payment to increases the consumption for a given time period. It suggests that the composition of public spending (the ratio between investment and welfare payments) dictates the “threshold” in the national debt level. Countries that benefit from an increase in public deficit are those which spend more on investment. The second economic variable that has impact on the relationship between public deficit and balanced growth rate is the existing level of productive public goods of an economy. There exists a critical level of productive public goods, such that, for countries with higher level of productive public goods, an increase in the ratio of pubic debt to GDP will reduce the balanced growth rate. Therefore, for these countries, higher public debt is detrimental to economic growth. Finally, the model suggests that, for countries with high level of tax, an increase in public debt could increase the balanced growth rate.

The rest of the thesis is organized as follows: Chapter 2 studies the positive correlation between the development stages of an economy and its preference for higher tax rate and welfare spending using the argument of habit formation in public goods; Chapter 3 investigates the relationship between the ratio of pubic deficit to GDP and economic growth by distinguishing types of government spending; finally Chapter 4 concludes and discusses some policy implications.
Chapter 2

The Dynamics of Taxation and Habitual Public Spending

2.1 Introduction

Historical data on government revenue and expenditure reveals the striking feature that the size of government has expanded in general throughout history, especially during the past few decades and particularly in developed countries. A deeper look at the composition of the various government expenditures indicates that the major increase in public spending is the consequence of the growing involvement of the state in welfare related activities. Welfare support has one of the highest growth rates among government expenditures, and accounts for most of the spending differences between developed and developing countries. Although the share of government spending in national income remained at 11-12 percent during the period between 1870 and 1913, the figure started to rise to 27.9 percent in 1960 and 44.9 percent in 1990, despite the fact that there was no major war or economic depression during that period (Tanzi and Schuknecht, 1997).

Using data from the OECD and the Penn World Table, we can summarize the following stylized facts as illustrated in Figures:

1. The share of tax revenue in GDP increased across all developed countries over time.
2. The share of welfare spending in GDP and total government expenditure increased in most developed countries over time. (Figures 2.1.2 and 2.1.3)

3. The share of tax revenue in GDP is higher in countries with higher GDP per capita than countries with lower GDP per capita. (Figure 2.1.4)

4. The share of welfare expenditure in GDP is higher in countries with higher GDP per capita than countries with lower GDP per capita. (Figure 2.1.5)

Figure 2.1.1: TAX REVENUE AS THE SHARE OF GDP 1965-2010
Data Source: OECD.Stat
Figure 2.1.2: Welfare spending as the share of GDP 1980-2010
Data Source: OECD.Stat
Figure 2.1.3: Welfare spending as the share of total government expenditure 1980-2010

Data Source: OECD.Stat
**Figure 2.1.4:** TAX REVENUE AS THE SHARE OF GDP 2007-2011 AVERAGE

Data Source: OECD.Stat
The first three figures show the change of the share of tax revenue in developed countries over time,\textsuperscript{1} and the last two show the current share of tax revenue and the composition of public expenditure across OECD countries.\textsuperscript{2} In general, a positive correlation between the two variables in each figure can be observed.

The above stylized facts show the correlation between per capita income and tax and different components of public spending. They imply that, over time, growth in per capita income tends to be accompanied by a rise in tax rate. A slow-down of growth is accompanied by a decline in the share of productive public spending. They suggest that changes in the tax

\textsuperscript{1}To make their presentation compact, Figures 2.1.1 and 2.1.2 only show OECD average and G7 countries, Figure 2.1.3 shows G7 without Japan because of data availability, similar trends can be found in other OECD countries as well.

\textsuperscript{2}Except Luxemburg
rate and composition of public spending are important to the growth of per capita income. This paper intends to explain the forces behind these correlations.

The existence of a positive correlation between government expenditure and the degree of development was first pointed out by German economist Adolph Wagner. His finding has been refereed as the “Wagner’s law” states that government activities will increase as economies grow, with different pace for different government branches. A stricter version of this law postulates a long-run elasticity of public spending above unity, which means that public goods is a type of luxury goods, so its demand increases as countries become richer. “Wagner’s law” has been tested empirically for various countries with different data sets. The results are mixed, a cointegrating relationship was found in some countries but not in others. In Akitoby et al..(2006), a panel data containing 51 developing countries has been tested which find evidence that in line with Wagner’s Law.

William Baumol offers another explanation for the increase of government expenditure across his work which is known as “Baumol Law” today. Specifically, Baumol (1996) presented a model that features an unbalanced growth path between private and public sector. The model depends on only one essential assumption that sectors differ in their labour productivity growth rate with technological progression. In particular, if we assume public services are characterized by relatively low labour productivity progression rate, than even if the public goods are normal, if the elasticity of substitution between public and private provision is low then we will observe rising public expenditure shares.

Tornell and Lane (1999) provided a theoretical model that shows the existence of powerful interests groups will lead to slow economic growth and a “voracity effect” by which a shock, perversely generates a more-than-proportionate increase in fiscal redistribution and reduces growth. Therefore, the size of the state expands as a result of institutional failure through which interest groups fail to internalize this due to a tragedy of the commons. Hercowitz and
Strawczynski (2004) empirically tested the “voracity effect” using an OECD panel data set covering the 1975 – 1998 periods. The paper finds evidence that support the hypothesis that the government spending/output ratio tends to increase in contractions, and its reduction in expansions is only partial, resulting in a rising ratio over time.

Many recent studies also focus on the impact of demographical changes on the public spending. Imrohoroglu and Kitao (2012) built a general equilibrium model of agents who decide on consumption, saving, labour supply and make claims on social security benefits. After calibrated with US data, the model predicates an increase in the number of retirees claim benefits as well as a rapid decline in the labour participation rate. Both lead to a rise in public spending over output ratio. Hence, population aging could be one of the causes for the expansion of the government size. Another striking change in demographics in last few decades is the increase in female labour participation rate. Cavalcanti and Tavares (2011) developed a growth model based on Galor and Weil (1996) to show the causality between the increase in female labour participation and the share of government spending over GDP. In their model, female prefers government spending as it reduces the cost of performing house hold chores (Rosen, 1996); empirical studies also confirm the positive correlation between these two.

Our paper follows two sets of literature explaining the expansion of government size and the increased welfare expenditure in developed countries. The first studies the optimal level of tax rates and government expenditures when these expenditures provide productive factors that raises the productivity of firms. The basic finding is that there exists an inverted U shape relationship between tax rate and growth. At low tax rates, the growth rate increases if the tax rate rises; at high tax rates, the growth rate declines if the tax rate continues to rise. The optimal tax rate which maximizes the growth rate should equal the productivity of public spending (e.g. Barro, 1990; Barro and Sala-i-Martin, 1992; Glomm and Ravikumar, 1992; Glomm and Ravikumar, 1994). These papers focused on productive public spending only.
The fact that an increasing proportion of government expenditure goes to welfare (stylized fact 2) suggests the need to consider other types of public spending (non-productive but utility enhancing).

There are some papers in this strand of literature that consider more than one type of public spending, for instance Park and Philippopoulos (2003) studies the case where government decides a capital tax rate and the allocation between productive public services, public consumption and redistributive transfers simultaneously. Their model predicts the tax rate should be at all times higher than in the standard Barro-type public finance model, and generates increasing transitional dynamics for the tax rate on productive public services. It also established that the existence of redistributive transfers will bring indeterminacy to the steady states.

The second set of literature investigates the determination of redistributive tax policies using political-economy models (e.g. Alesina and Rodrik, 1994; Persson and Tabellini, 1994). In this type of model, agents who are heterogeneous in their income/wealth have different policy preferences. The winning policy is the one preferred by the median voter, whose identity will depend on the income/wealth distribution of the economy. This literature is able to explain the cross-country observation that countries with higher inequality are normally associated with higher tax and welfare spending. However, these models do not address the time series stylized facts mentioned earlier.

Our paper contributes to the literature in several ways. Firstly, we construct a political-economy model that generates transitional dynamics for tax rates and welfare spending that are consistent with the above stylized facts. Secondly, we formally model a well known observation in political-economy that welfare spending is difficult to reduce once it has been implemented.³

³Our model captures some observations of Alesina (1999) but differs in terms of the underlining mechanisms. Alesina (1999) presents a conceptual model and argues that there exists a so called “vicious cycle” that drives the government tax policies. The cycle begins with government increasing transfer payments, which
This has been described in the literature on governance as a ratchet effect, which captures the seemingly irreversible expansion of the size of governments. Finally, we introduce a habit effect to certain types of government expenditure that may create the consequent ratchet effect. Discussion of the habit effect on private consumption is not new, but there has been little written on the habit effect on public goods.

Before proceeding, we clarify some concepts relating to public goods. Government provides two types of public goods: one that directly improves the productivity of the economy, the productive public goods (PPG); another one directly improves the well-being of the individuals in the society, the non-productive but utility-enhancing public goods (NPG). PPG normally include government provision of infrastructure, for example: building roads, railways, airports, harbors or public R&D expenditures. The NPG we have in mind overlap with most government welfare expenditures, like public health systems, public leisure facilities (local parks, beaches and museums etc.) and public transport.4

In terms of modeling, the PPG enter into the production function as one of the production factors; it is the factor that prevents diminishing returns to the reproducible factor. Following the line of many endogenous growth models, productive government spending is once again the factor that generates endogenous growth.

4Of course, there are public goods that could belong to both groups. For instance, public education is the type of public good that can improve the well-being of individuals. A well established public education system provides more equal opportunities within the society and increases non-materialistic factors in human welfare; both are core components of Amartya Sen’s capability approach to social welfare. On the other hand, the public education system can also improve the productivity of the economy simply because the economy has a higher quality labor force. However, in this model, we assume any government spending has to be one of the two types, PPG or NPG.
NPG directly improve the well-being of individuals in the sense that they enter the household’s utility function. We assume that there are two effects that influence how an individual enjoys the NPG, the wealth effect and habit effect. The wealth effect is that for a given level of NPG provided by government, an individual derives less utility from it if there is an increase in his or her wealth level. The logic behind this comes from two sources: most publicly provided facilities and services have private substitutes of higher quality which are more easily accessed by richer individuals. The poorer an individual is, the more heavily he or she will have to rely on those NPG. Another reason to assume such a wealth effect is that most welfare programmes have some kind of eligibility criteria often linked to an individual’s wealth or income. The richer an individual is, the lower the chance he or she will be eligible for the welfare programmes.

The habit effect is that the level of utility a household derives from NPG depends on the increment of NPG provided by government this period compared with last period. This implies the government can not adjust the level of NPG freely - the benefit that was already given in previous periods is very difficult to withdraw. The enormous public pressure against benefit cut in different countries recently justifies this assumption to some extent.

This paper constructs a political-economy model addressing the empirical observations summarized in the above stylized facts. In our model, the government policy in each period involves setting a tax rate and deciding how to spend the tax revenue, specifically, choosing the ratio of provision between productive public goods non-productive public goods. The policy is determined by majority voting; we show that the median voter theorem holds despite the multi-dimensional policy space. The voters face a policy trade-off: PPG promotes growth which then increases their private consumption and NPG enhances their utility directly. The existence of a habit effect on NPG causes voters to choose a policy that is biased toward NPG. Starting from a steady state without the habit effect, the tax rate and the proportion of NPG will increase monotonically while the growth rate declines.
The rest of the chapter is organized as follows: section 2 introduces the model and derives several propositions; section 3 describes the dynamics and the steady states of the model, finally section 4 concludes and discusses some policy implications.

2.2 The Model

We use a simple endogenous growth model for a closed economy with a continuum of measure one agents. Each agent only lives for one period of time and then reproduces another agent so that the total population is constant. Time is discrete and infinite, with $t = 0, 1, 2...$. There is only one homogenous good\(^5\) that can be used for consumption, or production or left as bequest to their offspring. Firms have access to a technology that uses capital and public goods as inputs to produce this homogenous good in a competitive market. Agents are endowed with different levels of bequests from the previous generation and supply them inelastically to the firm for production. After receiving the return on their investment, they choose to consume and leave bequests according to their preference.

Government in this model spends its tax revenue in two categories, providing productive public goods $G_{p,t}$ and non-productive but utility enhancing public goods $G_{n,t}$ to the economy,\(^6\) financed by a tax on the capital stock $B_t$ with the tax rate $\tau_t$. It is worth mentioning, in this model, the word “capital” refers to all growth-producing assets that can be accumulated. That includes physical capital, human capital and proprietary technologies etc.\(^7\) The capital tax therefore should be interpreted as a broad range of government policy tools that collect resources from the accumulated factor.

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\(^5\)We assume a full depreciation of this good at the end of each period.

\(^6\)The two types of public goods will be referred as PPG and NPG when notations $G_{p,t}$ and $G_{n,t}$ are not used.

\(^7\)Hence, the “bequest” in this model represents agents’ total spending to endow their offspring, that includes consumption, education (increase their human capital) as well as the physical capital passed over to the them.
2.2.1 Agents

In this economy, agents are identical in all aspects except the different amount of bequests \( b^t_i \) inherited from the last generation. The initial distribution of wealth \( b^0_i \) for generation \( t = 0 \) is exogenously given, represents the level of bequest left by the preexisting generation with a continuous cumulative probability distribution function \( \Lambda_0(b^0_i) \), where \( b^0_i \in [b, \tilde{b}] \). The level of capital stock \( B_t \) in this economy in each period \( t \), is the aggregate bequest inherited from the generation \( t - 1 \). That is \( B_t = \int_0^1 b^t_i di \), and because population has been normalized to one, \( B_t = \tilde{b}_t \), where \( \tilde{b}_t \) is the average level of bequest at time \( t \). In order to explicitly obtain the evolution of the bequest distribution, we assume the initial distribution \( \Lambda_0 \) is log-normally distributed, specifically, \( N_0 = \ln \Lambda_0(\mu_0, \sigma^2_0) \) is normally distributed with mean \( \mu_0 \) and variance \( \sigma^2_0 \).

There are a number of merits of using the log-normal distribution. Firstly, it is arguably the empirically best fit distribution for income and wealth and is easy to handle in statistical analysis. Secondly, the most commonly used inequality measurement, the Gini coefficient can be represented by the standard deviation, \( \sigma \) alone under log-normal distribution (Aitchison and Brown, 1957). In this model, it provides a clear evolution path of the bequest distribution. It will be shown later that along the utility maximizing path, the bequests \( b^t_{i+1} \) left by the generation who were born at time \( t \) to generation \( t + 1 \) distribute log-normally with \( \mu_{t+1} \) and \( \sigma^2_{t+1} \) depending on the values of \( \mu_t \) and \( \sigma^2_t \) for all \( t \). Therefore, given the initial values \( \mu_0 \) and \( \sigma^2_0 \), we can compute \( \Lambda_t(\mu_t, \sigma^2_t) \) recursively for all \( t \).

Agents gain utility from: a) present private consumption, b) leaving bequests to their off-spring, and c) non-productive public goods consumption. Utility derived from private consumption and making bequests in an additively separable form is standard in most overlapping generation models with an altruism assumption. In addition to a) and b), including the aggre-
gate level of utility enhancing public goods provision in the agent’s utility function in additively separable form is also common in public goods literature (e.g. Warr, 1983). Thus, agent $i$ born in period $t$ maximizes a utility function as follows:

$$U^i_t = \ln(c^i_t) + \beta \ln(b^i_{t+1}) + \theta \left[ \frac{\ln(G_{n,t} - H_t)}{(b^i_t/B_t)^\varepsilon} \right]$$

(2.2.1)

where $H_t \equiv \eta G_{n,t-1}$, $\eta \geq 0$, $\varepsilon \in [0, 1]$ and $\beta, \theta > 0$.

$c^i_t$ is the level of agent’s private consumption, $b^i_t$ and $b^i_{t+1}$ denote bequests agent $i$ inherited from her parents and is going to leave for her offspring respectively, $G_{n,t}$ is the level of NPG provided by government in period $t$ and $H_t$ is the habit formed from the level of NPG provided in period $t - 1$. The parameters $\beta$ and $\theta$ represent how much utility agents derive from leaving a bequest to their offspring and enjoying the NPG respectively.

The utility from personal consumption is hedonic in log form. The second term $b^i_{t+1}$ represents how much bequest agent $i$ will leave for his or her offspring. In this model, we assume that the agent derives utility from the level of the bequest not the utility of the offspring, because this is much more tractable than any other bequest motives. Similar setups can be found in Banerjee and Newman (1993) and Antunes, Cavalcanti and Villamil (2008).

The third term in the utility function captures the utility agents derive from accessing NPG subject to two effects: the habit effect and wealth effect. The habit effect on private consumption has been well understood in the micro-economic consumption literature (e.g. Carroll and Weil, 1994), in macro-economic literature on asset pricing (e.g. Gali, 1994; Abel, 1999) and in the growth literature (e.g. Ljungqvist and Uhlig, 2000; Overland and Weil, 2000). Little has been studied on the habit effect of pubic goods.

The enormous public pressures against benefit cuts in different countries (even for those countries with heavy sovereign debt) suggest that people form habits on the benefits publicly...
provided by government. In the habit formation literature, the habit stock is defined as a weighted average on past consumption. Utility is determined by the increment between current consumption and this habit stock. In this paper, the habit effect on public goods is modeled by the simplest form – only the level of NPG of the last period matters.\footnote{It is more intuitive to think that agents live two periods, childhood and adulthood. In the first period, she does nothing other than being endowed by her parents. She is able to observe the level of NPG provision in the first period and forms a habit. In the second period, she will demand a level of NPG provision based on the amount she had already costumed to. In this case, for agent $i$ who were in her childhood in $t-1$ and adulthood in $t$, the utility function is:}

\[ U_i^t = \theta \ln(G_{n,t-1}) + \ln(c_i^t) + \beta \ln(b_i^{t+1}) + \theta \left[ \ln(G_{n,t} - H_t) \right] \frac{(b_i^t / B_t)^{\varepsilon}}{\left( b_i^t / B_t \right)^{\varepsilon}} \]

The \textbf{Habit Effect on NPG} is captured by the habit stock, $H_t \equiv \eta G_{n,t-1}$: The utility an individual derives from accessing NPG is determined by the increment between current level of NPG $G_{n,t}$ and the habit stock $H_t \eta \geq 0$ is the parameter measuring how strong this habit effect is, a larger $\eta$ represents a stronger habit effect and \textit{vice versa}.\footnote{The habit effect in this paper is an exogenous parameter. It would be desirable to endogenize it, for example, as a result of (a) political constraint(s). Although such a micro-foundation would allow for a more transparent interpretation of the sources and the strength of the habit effect, it would deviate from the current focus and make the paper much lengthier.}

The term $\frac{b_i^t}{B_t}$ represents the relative wealth position of agent $i$, where $b_i^t$ is the wealth level of agent $i$ at time $t$; and $B_t$ is the average (mean) wealth level of the economy at time $t$. Getachew (2010) has pointed out that the types of public goods the government provides may have disproportionate impacts on individuals with different levels of wealth. Hence he proposes a utility function where such disproportionate impacts are captured by a coefficient on the wealth level of an individual, such that when the coefficient takes positive values poor agents derive more utility from public goods than rich agents, and when it takes negative values, the public goods are pro-rich. This paper follows the idea, the utility individuals derive
from some level of NPG provision varies with the individual’s relative wealth position. In this model, the effect is captured by parameter $\varepsilon$: $0 \leq \varepsilon \leq 1$, hence NPG benefit the poor disproportionately, i.e. for the same level of NPG provision, the poorer an agent is, the more utility he or she derives from it. This is the economic reason why the level of NPG provision could represent the welfare spending: NPG is financed by tax on public and benefits the poor more than the riches.\textsuperscript{10}

The disproportionate impact of government spending on poor and rich is well studied both empirically and theoretically (Calderón and Chong, 2004; Getachew, 2010). A number of areas of government spending are widely considered to have disproportionate impacts on the poor than the rich, such as the provision of public education, health services and/or the provision of public housing, clean water, sanitation and public transport etc. The disproportionate impact may be caused by the lack of access to private substitutes by the poor and to a great extent, the poor are defined as those who lack these basic inputs (World Bank, 1994).

The budget constraint for an agent is:

$$c_t + b_{t+1} = (1 - \tau_t)r_t b_t$$ \hspace{1cm} (2.2.2)

where $\tau_t$ is the tax rate and $r_t$ is the competitive interest rate, both of which will be discussed in detail in later sections.

\textsuperscript{10}It is worth noting that if we assume $\varepsilon > 1$, the non-productive public goods becomes luxury goods in the sense that it benefits rich people disproportionately more. In some other interpretations of NPG, for example environmental protection expenditure where one could argue rich people cares more about the environment.
2.2.2 Technology

The firm has access to a technology that produces the single final good using two inputs\textsuperscript{11} - capital investment and productive public goods as follows\textsuperscript{12}:

\[ Y_t = AG_{p,t}^{\alpha}K_t^{1-\alpha}, \quad 0 < \alpha < 1 \] (2.2.3)

where \( Y_t \) is the total output of the economy, \( K_t \) is the aggregate private investment and \( G_{p,t} \) is the level of productive public goods (PPG) provided by government in time \( t \). \( A \) represents the total factor productivity (TFP), which is assumed to be constant over time. \( \alpha \) is the productivity of the PPG, while \( 1-\alpha \) is the productivity of private investment\textsuperscript{13}.

The firm’s profit is therefore:

\[ \Pi_t = Y_t - r_tK_t \] (2.2.4)

In the equilibrium with perfect competition, the first order condition implies that when the firm maximizes its profit according to equation (2.2.4), the following equation holds:

\textsuperscript{11}“Labor” is omitted because after decomposing human capital, labor only means raw labor which is often assumed to be identical for each individual in macroeconomics models. We can consider there also exists another input: labor, which has been normalized to one so that it has no qualitative contribution to production. In this case, agents also have an extra component in their income: a wage, which we can assume to be fully consumed to satisfy basic needs so it has no qualitative impact on agent’s decision making.

\textsuperscript{12}It is possible to generalize the production function to \( Y_t = AG_{p,t}^{\alpha}K_t^{\kappa} \), where \( \alpha, \kappa \in (0,1); \alpha + \kappa \) can be less than, greater than or equal to 1 in case of decreasing, increasing or constant return to scale respectively. This paper only considers the case of constant return to scale production with respect to the total reproducible factor, as this yields interesting analytical results.

\textsuperscript{13}We need the domain for \( \alpha \) to be \((0,1)\) rather than \([0,1]\) because, in the case where \( \alpha = 0 \), the production function becomes a standard AK production function where output \( Y_t \) displays a linear relationship with respect to private capital stock \( K_t \); public goods contribute nothing to production, so there is no point for the government to provide any public goods. In the case where \( \alpha = 1 \), the output depends solely on the level of public goods, which means the marginal return of private investment is zero, hence the individual receives no income.
\[ r_t = \frac{\partial Y_t}{\partial K_t} = A(1 - \alpha)(\frac{G_{p,t}}{K_t})^\alpha \]  

(2.2.5)

From (2.2.5) the marginal productivity of capital depends on the private capital stock and the level of productive public goods provided by the government.

2.2.3 Government

Government provides PPG which enters the production function, from equation (2.2.3) it prevents the diminishing return to the reproducible factor, thus enabling endogenous growth for the economy. Consistent with the endogenous growth literature, productive government spending is the engine that drives the growth here. Government also provides NPG that enhances agent’s level of utility directly.

The government budget is balanced at any given time,\(^{14}\) so that:

\[ \tau_t B_t = G_{p,t} + G_{n,t}, \quad \tau_t \in (0, 1) \]  

(2.2.6)

Let the share of PPG in total government spending be \( \rho_t \):

\[ \rho_t = \frac{G_{p,t}}{\tau_t B_t}, \quad \rho_t \in (0, 1) \]  

(2.2.7)

Then the level of NPG is given by:

\[ G_{n,t} = (1 - \rho_t)\tau_t B_t \]  

(2.2.8)

Thus, government policies can be characterized as a sequence of pairs \((\tau_t, \rho_t)\), where \( t = 0, 1, 2, \ldots \)

In fact, government policy has three dimensions, the third one being the share of \( G_{n,t} \) in total

\(^{14}\)In reality, governments often violate equation (2.2.6) by issuing government bonds, a case which will be considered in a separate paper.
spending. However, the assumption that the government budget is balanced at all time means the government only needs to choose one share, that for $G_{p,t}$ or for $G_{n,t}$. Without loss of generality, we choose that to be the share of $G_{p,t}$, denoted as $\rho_t$.

### 2.2.4 Market Clearing Condition

The market clearing condition is that the aggregate level of private capital input equals the after tax aggregate bequests for every time period $t$. Formally:

$$K_t = (1 - \tau_t)B_t \tag{2.2.9}$$

Since total population is normalized to one, the aggregate level of bequest $B_t$ always equals the mean. Recall that the initial distribution of $b_0^i$ is assumed to follow a log-normal distribution $\Lambda_0(\mu_0, \sigma^2_0)$, where $\mu_0$ and $\sigma^2_0$ are mean and variance at time $t = 0$, respectively. Thus $B_0 = \exp(\mu_0 + \frac{1}{2}\sigma^2_0)$. Substitute equation (2.2.9) and (2.2.7) into (2.2.5) and redo the first order condition. We have:

$$r_t = A(1 - \alpha)(\frac{\rho_t\tau_t}{1 - \tau_t})^\alpha \tag{2.2.10}$$

### 2.2.5 Household’s Problem and Solution

Households maximize their utility subject to their budget constraints taking $\rho_t$ and $\tau_t$ as exogenous. Therefore, from (2.2.1) and (2.2.2) the agent $i$ born at time $t$ faces the following problem with a given level of initial bequest received, $b_0^i$:

$$\max_{c_t^i, b_{t+1}^i} U_t^i = \ln(c_t^i) + \beta \ln(b_{t+1}^i) + \theta \ln\left[\frac{G_{n,t} - H_t}{(b_t^i/B_t)^\epsilon}\right]$$

subject to equation (2.2.2). Substituting equation (2.2.10) into (2.2.2), the agent’s choice is optimal when the marginal utility of consumption equals the marginal utility of leaving bequest.
to her offspring, and the following equations must hold:

\[(c^*_i t) = \frac{A(1 - \alpha)}{1 + \beta} \left[ (1 - \tau_t)^{1-\alpha} \tau_t^{\alpha} \rho_t^\alpha \right] b^i_t \]  
(2.2.11)

\[(b^*_{t+1} i) = \frac{\beta A(1 - \alpha)}{1 + \beta} \left[ (1 - \tau_t)^{1-\alpha} \tau_t^{\alpha} \rho_t^\alpha \right] b^i_t \]  
(2.2.12)

that is the agent’s optimal choices of consumption \((c^*_i)^*\) and bequest to her offspring \((b^*_{t+1})^*\) depend on the agent specific \(b^i_t\) and the economy-wide government policies \(\tau_t\) and \(\rho_t\).

An important insight from equation (2.2.12) is that all agents in this economy leave bequest to their offspring at the same rate. Hence, according to the property of the log-normal distribution, the new distribution \(\Lambda_{t+1}(\mu_{t+1}, \sigma^2_{t+1})\) is still the log-normal with:

\[\mu_{t+1} = \ln \left\{ \frac{\beta A(1 - \alpha)}{1 + \beta} \left[ (1 - \tau_t)^{1-\alpha} \tau_t^{\alpha} \rho_t^\alpha \right] \right\} + \mu_t \]  
(2.2.13)

\[\sigma^2_{t+1} = \sigma^2_t \]  
(2.2.14)

Another nice property of the log-normal distribution is that the Gini coefficient can be calculated explicitly from the formula \(Gini = 2N \sim \left( \frac{\sigma}{\sqrt{2}} \right) \left( 0, 1 \right) - 1\), where \(\sigma\) is the standard deviation and \(N \sim \left( 0, 1 \right)\) is the standard normal distribution. Thus, the value of \(\sigma\) fully captures the level of inequality for this economy. From equation (2.2.14), \(\sigma\) is a time invariant constant which indicates that the level of wealth inequality stays unchanged in all periods.

2.2.6 Government Policy under Majority Voting

From equations (2.2.11) and (2.2.12), the utility maximizing solution for agent \(i\)'s consumption \((c^*_i)^*\) and bequest \((b^*_{t+1} i)^*\) are increasing functions of her initial wealth, the bequest she received, \(b^i_t\). Substituting (2.2.11) and (2.2.12) into (2.2.1), we have the indirect utility of the agent \(i\) as:
\[ V^i_t(\tau_t, \rho_t, b^i_t) = \ln\left\{ \frac{A(1 - \alpha)}{1 + \beta}[(1 - \tau_t)^{1 - \alpha} - \alpha^\tau_t \rho^i_t b^i_t] \right\} + \beta \ln\left\{ \frac{\beta A(1 - \alpha)}{1 + \beta}[(1 - \tau_t)^{1 - \alpha} - \alpha^\tau_t \rho^i_t b^i_t] \right\} + \frac{\theta}{(b^i_t / B_t)^\varepsilon} \ln[(1 - \rho_t)\tau_t B_t - H_t] \] (2.2.15)

Differentiating the indirect utility function with respect to the tax rate \( \tau_t \) and the share of PPG spending \( \rho_t \):

\[
\frac{\partial V^i_t}{\partial \tau_t} = \frac{\alpha(1 + \beta)}{\tau_t} - \frac{(1 + \beta)(1 - \alpha)}{1 - \tau_t} + \frac{\theta(1 - \rho_t)B_t}{(b^i_t / B_t)^\varepsilon[(1 - \rho_t)\tau_t B_t - H_t]} 
\] (2.2.16)

\[
\frac{\partial V^i_t}{\partial \rho_t} = \frac{\alpha(1 + \beta)}{\rho_t} - \frac{\theta}{(b^i_t / B_t)^\varepsilon} \frac{\tau_t B_t}{(1 - \rho_t)\tau_t B_t - H_t} 
\] (2.2.17)

As the indirect utility function is concave, the tax rate and share of PPG spending that correspond to the maximum \( V^i_t \) require (2.2.16) and (2.2.17) to be zero, which gives us the government policy that agent \( i \) prefers most, the bundle \((\tau^i_t, \rho^i_t)\) that maximizes her indirect utility \( V^i_t \):

\[
\tau^i_t = \frac{(b^i_t / B_t)^\varepsilon}{\alpha} \ln(1 - \alpha)(1 + \beta)(1 - \rho_{t-1})\tau_{t-1} B_{t-1} + \frac{\alpha(1 + \beta)(b^i_t / B_t)^\varepsilon + \theta}{(1 + \beta)(b^i_t / B_t)^\varepsilon + \theta} 
\]

\[
\rho^i_t = \frac{\alpha(1 + \beta)(b^i_t / B_t)^\varepsilon}{\alpha} \ln(1 - \alpha)(1 + \beta)(1 - \rho_{t-1})\tau_{t-1} B_{t-1} + \frac{(b^i_t / B_t)^\varepsilon}{\alpha(1 + \beta)(b^i_t / B_t)^\varepsilon + \theta} \frac{\theta}{(1 - \rho_t)\tau_t B_t - H_t} 
\]

Suppose government policy is reached under simple majority voting and universal suffrage so that each agent is a voter and government policy will be the one that is supported by most of the voters. Each individual prefers the policy that generates the maximum utility for

\[ V^i_t \] is the sum of logarithmic functions and linear transformation of log functions, all of which are strictly concave.

Although the model applies to democratic governments where policies are decided by voting, it is also to some extent applicable to dictatorial regimes as well. As Alesina and Rodrik (1994) point out, "Dictator's
herself. Note that because voters have two issues to decide, namely, \( \tau_t \) and \( \rho_t \), their policy preferences are therefore multi-dimensional, and the celebrated median voter theorem cannot be applied directly. However, the theorem may still apply if the conflicts in multi-dimensional policy space can be projected into a uni-dimensional parameter space. Essentially, this type of projection can occur only if the differences in preferences for government policy results from a single heterogeneous attribute of voters (Persson and Tabellini, 2000). In this model, agents are only different in their initial wealth \( b^t_i \), the preferred government policy for agent \( i \) born in time \( t \) is the bundle \((\tau^t_i, \rho^t_i)\) that maximizes her utility \( U^t_i \) subject to \( b^t_i \). The source of disagreement among voters only comes from the heterogeneity in their initial wealth. Based on the proof in (Persson and Tabellini, 2000), we have the following lemma:

**Lemma 2.1.** Let \( q = (q_1, q_2, \ldots q_n) \) be a finite policy space and \( x^i \) be the heterogeneous attribute of voter \( i \). If the indirect utility function \( V^i(q, x^i) \) for each voter \( i \) satisfies \( V^i(q, x^i) = I(q) + P(x^i)J(q) + Q(x^i) \), where \( P(x^i) \) is monotonic in \( x^i \), \( I(q) \) and \( J(q) \) are common to all voters and \( Q(x^i) \) is any real function that only depends on \( x^i \), then the median voter theorem holds. That is the winning policy \( q^* \) via pairwise voting will coincide with the policy chosen by the voter with median level of \( x^m \).

**PROOF:**

See Appendix A1.

**Proposition 2.2.** The government policy \( (\tau_t, \rho_t) \) will be the bundle preferred by the agent with the median level of income, denoted as \( b^m_t \). That is:

\[
\tau_t = \frac{\alpha(1 + \beta)w + \theta}{(1 + \beta)w + \theta} + \frac{\eta w(1 - \alpha)(1 + \beta)(1 - \rho_{t-1})\tau_{t-1}B_{t-1}}{[(1 + \beta)w + \theta]B_t} \tag{2.2.18}
\]

policy decisions are also influenced by social demands and social conflicts". 


\[ \rho_t = \frac{\alpha(1 + \beta)w}{\alpha(1 + \beta)w + \theta} - \frac{\eta w\alpha(1 + \beta)(1 - \rho_{t-1})\tau_{t-1}B_{t-1}}{[\alpha(1 + \beta)w + \theta]\tau_t B_t} \]  

(2.2.19)

where \( w = (b_t^n/B_t)^\epsilon \) is a constant.

**PROOF:**

See Appendix A2.

As a result of Proposition 1, there are two particular features that follow and are of importance.

**Proposition 2.3.** The tax rate \( \tau_t \) and the share of productive government spending \( \rho_t \) are negatively related as follows:

\[ \rho_t = \frac{\alpha(1 - \tau_t)}{\tau_t(1 - \alpha)} \]  

(2.2.20)

Thus, an increase in \( \tau_t \) implies a decrease in \( \rho_t \) and vice versa.

**PROOF:**

See Appendix A3.

In addition, the growth rates of the economy, defined as \( g_t = \frac{B_{t+1}}{B_t} \) is given by substituting equation (2.2.20) in to (2.2.12):

\[ g_t = \frac{A\alpha(1 - \alpha)^{1-\alpha}\beta(1 - \tau_t)}{1 + \beta} \]  

(2.2.21)

Thus, we have the following proposition:

**Proposition 2.4.** The growth rate in this economy is negatively related to the tax rate \( \tau_t \).

It is worth mentioning that the tax rate \( \tau_t \) in here represents the policy bundle \( (\tau_t, \rho_t) \) and the relationship between \( \tau_t \) and \( \rho_t \) has been established in Proposition 2.\(^\text{17}\) Thus, the increase

\(^{17}\)This relationship also allows us to simplify a two dimensional policy choice of a tax rate and a share of
in \( \tau_t \) means a higher tax rate as well as a lower share of spending in PPG. It is the latter that reduces the growth rate. The result fits well with empirical data such that: as countries become richer over time, their tax rates went up while the growth rates declined. Though we would expect this to happen as convergence occurs in a Solow Growth Model, here the mechanism is different, i.e. the higher tax (lower share of productive spending) is the reason for the declining growth rates.

2.2.7 The Maximum Growth Rate

In this section, we will show that there is an upper bound to the growth rate attainable. This will be a useful benchmark.

As established before, the wealth distribution in this economy is log-normal \( \Lambda_t(\mu_t, \sigma^2_t) \) for all \( t \). Suppose the economy is in equilibrium in time \( t \), and from this point onwards, \( \tau_t \) and \( \rho_t \) become constants \( \tau^g \) and \( \rho^g \) respectively.\(^{18}\) The level of output is given by:

\[
Y_t = A(\rho^g \tau^g)^\alpha (1 - \tau^g)^{1-\alpha} \exp(\mu_t + \frac{1}{2} \sigma^2_t)
\]

Using equations (2.2.13) and (2.2.14), we have:

\[
Y_{t+1} = A(\rho^g \tau^g)^\alpha (1 - \tau^g)^{1-\alpha} \exp\{\ln\left[\frac{\beta A(1 - \alpha)}{1 + \beta}[(1 - \tau^g)^{1-\alpha}(\tau^g \rho^g)^\alpha]\right] + \mu_t + \frac{1}{2} \sigma^2_t\}
\]

\(^{18}\)It will be demonstrated later that time invariant \( \tau \) and \( \rho \) are indeed the case in an equilibrium in which the government maximizes the growth rate.

productive public goods into a choice of tax rate. In this case, the underlining implication is that, for any chosen tax rate, a unique share of PPG spending is simultaneously determined to ensure the median voter achieves the maximum level of utility.
Therefore, the growth rate $g_t$ can be calculated as:

$$g_t = \ln Y_{t+1} - \ln Y_t = \alpha \ln \rho^g + \alpha \ln \tau^g + (1 - \alpha) \ln (1 - \tau^g) + \ln \frac{\beta A (1 - \alpha)}{1 + \beta}$$ (2.2.22)

It is easy to verify that $g_t$ always increases with $\rho^g$ but the relation between $g_t$ and $\tau^g$ is not monotone. The first order derivative of the growth rate with respect to the tax rate is:

$$\frac{\partial g_t}{\partial \tau^g} = \frac{\alpha - \tau^g}{\tau^g (1 - \tau^g)}$$ (2.2.23)

Thus, $\frac{\partial g_t}{\partial \tau^g} > 0$, if $0 < \tau^g < \alpha < 1$; on the other hand, $\frac{\partial g_t}{\partial \tau^g} < 0$, if $0 < \alpha < \tau^g < 1$. Thus, the relation between the growth rate and the tax rate displays as an inverted U-shape curve. An increase in the tax rate will increase growth when the tax rate is low ($0 < \tau^g < \alpha < 1$); and the growth rate declines with a rise in the tax rate if it is high ($0 < \alpha < \tau^g < 1$). Intuitively, this is because for any given value of $\rho^g$, an increase in the tax rate has two opposite effects on the growth rate: it promotes the economic growth due to the fact that a greater $\tau^g$ enables the government to provided a higher level of productive public goods; however, it suppresses the growth rate by crowding out some private investments and reducing the level of income of agents, so less bequest will be passed over to the next generation. The first effect dominates in case the tax rate is low and the second effect dominates if the tax rate is already high. Thus, given $\rho^g$, $\tau^g = \alpha$ produces the maximum possible growth.

The result is consistent with the well-known tax rate from Barro (1990) and Barro and Sala-i-Martin (1992), that the growth maximizing tax rate is constant over time and equals the productivity of the public goods (in our case, it is the productivity of PPG).

From equation (2.2.22), $g_t$ is strictly increasing with $\rho_t$. Thus, $\rho^g = 1$ provides an upper
bound on attainable growth rates. This is natural, since $G_{p,t}$ is by definition the productive spending which increases the output without any distortion. The negative impact on private capital only comes from the capital tax rate $\tau_t$. As a result, for a given tax rate, the government should spend all of its revenue on $G_{p,t}$ if its objective is simply to maximize growth.

Substituting $\tau^g = \alpha$ and $\rho^g = 1$ into equation (2.2.22), the maximum growth rate $g_{\text{max}}$ equals:

$$g_{\text{max}} = \frac{A\beta\alpha^\alpha(1 - \alpha)^{2-\alpha}}{1 + \beta}$$

(2.2.24)

### 2.3 The Dynamics and Steady States of the Model

In this section the impact of the habit effect on transitional dynamics and the long run equilibrium is investigated.

Substituting equation (2.2.20) and (2.2.21) into (2.2.18) produces the following difference equation which gives us the evolution of the tax rate under the assumptions made, as long as $\tau_t \in (0, 1)$;

$$\tau_t = M\frac{\tau_{t-1} - \alpha}{1 - \tau_{t-1}} + N \equiv \phi(\tau_{t-1})$$

(2.3.1)

where $M = \frac{\eta(1+\beta)^2w}{[(1+\beta)(\omega + \theta)A\alpha(1-\alpha)]^{1-\alpha}}$ and $N = \frac{\alpha(1+\beta)w + \theta}{[(1+\beta)(\omega + \theta)]}$.

If there was no habit effect ($\eta = 0$) then the graph of this difference equation collapses to a horizontal line, $\tau_t = N$ for all $t$. In this special case, whatever the initial value, the economy would move straight to a steady state equilibrium with tax rate $N$.

Now suppose that the economy has been in such a steady state up to period $t = 0$, but then a habit effect emerges ($\eta > 0$). How does this change the evolution of the economy thereafter? The following diagram helps to start the answer.
Notice that $\phi(\alpha) = N$ and $\phi(\tau_{t-1})$ is increasing and strictly convex; also $\phi(\tau_{t-1})$ increases as $\eta$ increases if $\tau_{t-1} > \alpha$. For $\eta > 0$ but sufficient small, the typical difference equation graph $\tau_t = \phi(\tau_{t-1})$ is shown as the bold curve in Figure 2.3.1. The evolution of the tax rate is then clear, from the dotted line zig-zags in Figure 2.3.1. $\tau_t$ increases over the time approaching, as $t \to \infty$, the steady state equilibrium value which corresponds to the smaller root of $\phi(\tau) = \tau$, namely:

$$\tau_{ss} = \frac{1}{2} \left[ 1 + N - M - \sqrt{(M - N - 1)^2 - 4(N - \alpha M)} \right] \quad (2.3.2)$$

As $\eta$ increases further there will be a critical value $\bar{\eta}$ say where the difference equation graph becomes the dashed curve in Figure 2.3.1. It is apparent that for $\eta > \bar{\eta}$, the evolution of
the tax rate will be forever increasing, eventually reaching $\tau_t = 1$, at which point the economy collapses, unable to finance such large habit effects. The focus here will be on sustainable habit effects, where the following proposition is a precise statement of the above arguments:

**Proposition 2.5.** There exists $\bar{\eta} > 0$ such that for any $\eta \in (0, \bar{\eta})$, and starting from the steady state tax rate $N$ where there is no habit effect, the appearance of habit effect causes the tax rate $\tau_t$ to increase monotonically over time, converging asymptotically to the steady state tax rate $\tau_{ss}$ in equation (2.3.2).

**PROOF:**
See Appendix A4.

The impact of the habit effect on the evolution of $\rho_t$ and $g_t$ now follows from equation (2.2.20) and (2.2.21):

**Corollary 2.6.** Under the assumption of Proposition 4:

a) $\rho_t$ decreases monotonically over time, converging asymptotically to the steady state value,

$$\rho_{ss} = \frac{\alpha(1 - \tau_{ss})}{\tau_{ss}(1 - \alpha)}$$

b) $g_t$ also decreases monotonically over time, converging asymptotically to the steady state value,

$$g_{ss} = \frac{B_{t+1}}{B_t} = \frac{A\alpha(1 - \alpha)^{1-\alpha}\beta(1 - \tau_{ss})}{1 + \beta}$$

It follows that the emergence of a habit effect on public spending, as modeled here, predicts time series for $\tau_t$ and $1 - \rho_t$ as in Figure 2.3.2:
Figure 2.3.2: TIME PATHS OF THE TAX RATE AND WELFARE SPENDING

The model therefore explains stylized facts from actual time series described in the Introduction. Figure 2.3.2 (a) replicates the increasing pattern seen in Figure 2.1.1, and Figure 2.3.2 (b) does the same for the Figure 2.1.2. Figure 2.1.3 corresponds to the value of $\tau_t(1 - \rho_t)$ in the model, when both $\tau_t$ and $1 - \rho_t$ are increasing, their product increases as well. For the cross-country comparison in Figure 2.1.4 and 2.1.5, suppose countries are on the same convergence paths for $\tau_t$ and $1 - \rho_t$, but richer countries are more advanced along it than poorer countries. Specifically in Figure 2.3.2 (a), suppose at a particular moment of (real) time that a rich country has reached a point in its development that corresponds to $t_R$ in the diagram, whilst a poor country corresponds to $t_P$. Then, clearly, the model predicates $\tau_R > \tau_P$: rich countries have higher tax rates, which is the stylized fact from Figure 4. And the analogous argument for Figure 2.3.2 (b) predicts the corresponding stylized fact in Figure 2.1.5.\(^{19}\)

\(^{19}\)Note that the Proposition 3 states a negative relationship between tax rate and the growth rate; therefore, the model predicts countries with larger government (higher tax rate and lower productive public spending) should have a lower growth rate and GDP per capita in the long horizon. However, this does not contradict with what happens in the transition period when cross-countries comparisons have been carried out. In fact,
There is something else that can be said about the steady state equilibrium regarding the strength of the habit effect.

**Corollary 2.7.** Under the assumption of Proposition 4, for two countries with different habit effects, namely \( \eta_1 \) and \( \eta_2 \), if \( \eta_1 < \eta_2 \) then the steady state tax rate \( \tau_{ss1} < \tau_{ss2} \), and vice versa.

Therefore, a country with weaker habit effect on NPG will converge to a lower steady state tax rate, accompanied by a lower share of NPG spending as well as a high growth rate compare with a country with stronger habit effect.

### 2.4 Numerical Exercises

In this section, we are going to carry out some numerical exercises to test the quantitative importance for the findings of this Chapter. According to Proposition 2.5, there exists a critical value of the habit effect \( \bar{\eta} > 0 \), for countries with higher values of \( \eta \), the tax rate will increase forever. Eventually when \( \tau_t = 1 \), the economy will collapse, which mean it is unable to finance such large habit effects. Hence, our first task is to calculate this critical value \( \bar{\eta} \) under some reasonable assumption on the other parameters.

We know that:\(^{20}\)

\[
\bar{\eta} = \frac{\beta A \alpha^\alpha(1 - \alpha)^{2-\alpha}}{(1 + \beta)^2} \left[ (1 + \beta + 2\theta) - 2\sqrt{\theta(1 + \beta + \theta)} \right]
\]

In order to estimate the value of \( \bar{\eta} \), firstly we need to make some assumptions about the values of \( \alpha, \beta \)and \( \theta \). In our model, \( \alpha \) is the productivity of public goods. Conventionally, we assign 1/3 to the physical capital without distinguishing public or private capital, while 1/3 to labour and human capital. Thus, the choice of \( \alpha \) should be much less than 1/3, two values has

\(^{20}\)The calculation of \( \bar{\eta} \) can be found in Appendix 2.6: A4.
been chosen: 0.17 and 0.1 which represents at most half and least 1/3 productivity contribution by public capital in all physical capital. $\beta$ is the degree of inter-generational altruism, here we follow conventional literature and choose the values to be 0.3 and 0.45 to reflect an annual discount rate of 0.04 in 30 years and 20 years respectively. The parameter $\theta$ represents how much utility agents derive from non-productive public goods. Because the coefficient of private consumption in our utility function is unity, the value of $\theta$ should be less than one and in this exercise, it is fixed to 0.5. Parameter $A$ is the TFP in our model, we assume it equals one in this case. Therefore, the values of $\bar{\eta}$ under the assumptions above are shown in the following table:

![Table 2.4.1: The values of $\bar{\eta}$](image)

To the best of our knowledge, we could not found any empirical studies on the strength of habit effect for different countries. The most relevant study is Alesina and Perotti (1997), which seeks to estimate the persistence in fiscal behaviour. The most important finding from the paper is that composition of fiscal adjustment matters for their likelihood of success and for their macroeconomic consequences. Our paper echos this finding in the sense that under the existence of habit effect of certain type of public goods, the composition of government expenditure matters on their likelihood of stay on a sustainable growth path. However, the actual strength of habit effect remains unclear as an empirical question that deserves further investigation.
2.5 Conclusion

Empirical data on taxation and government spending suggests some stylized facts: Historically, both the share of tax revenue over GDP and the share of welfare spending over GDP have increased in all developed countries. In addition, countries with higher GDP per capita tend to have larger share of tax to GDP ratio and welfare spending to GDP ratio than countries with lower GDP per capita. This paper has studied a political-economy model in which government policies in each period are chosen by rational voters. The model shows how the emergence of a habit effect relating to non-productive public good spending generates dynamics for tax rates and the welfare spending that are consistent with the stylized facts.

The habit effect that drives the results derives from a well known observation in political-economy that welfare policy is difficult to reduce once it has been implemented. Our economy starts from its long run equilibrium with no habit effect, the tax rate and the share of the welfare spending will increase once a habit on NPG has been fostered. In the long run, they will asymptotically converge to a new steady state, providing the habit effect is not too strong. In addition, the levels of the steady state tax rate and share of the welfare spending are positively related with the strength of the habit effect. Thus, holding everything else equal, countries with stronger habit effects will end up in a steady state that features a higher tax rate, lower welfare spending proportionately and lower growth rate.

Hence, this paper provides a theoretical framework to analyze the expansion of government welfare spending and the impact of government policy on economic performance. It adds insights to the current “Austerity vs Stimulus” debate. From the model, in order to promote growth, it is not how much government spends, but where it spends that matters. An increase in PPG spending will generate higher growth; higher spending in NPG will foster a larger habit stock for the future generation.
2.6 Appendix

A1 Proof of Lemma 1

The proof of this lemma is an extension of the proof in Persson and Tabellini (2000). We are going to extend the class of the intermediate preferences utility function. We will show that median voter theorem holds for a more general form of indirect utility function, \( V^i(q, x^i) = I(q) + P(x^i)J(q) + Q(x^i) \), where \( P(x^i) \) is monotonic in \( x^i \), \( I(q) \) and \( J(q) \) are common to all voters and \( Q(x^i) \) is any real function that only depends on \( x^i \).

**Definition.** A Condorcet winner is a policy \( q \) that is unbeatable against any other feasible policies under pairwise voting.

In this economy, the policy \( q \in \mathbb{R}^n \) is a vector in n-dimensional space and all agents are characterized by a single attribute parameter \( x^i \in \mathbb{R} \) capturing their differences, for example: wealth, income, productivities or tastes etc... The distribution of \( x^i \) is bounded from below by \( \underline{x} \), from above by \( \overline{x} \) and described by a strictly increasing cumulative distribution function \( \Gamma(x^i) : [\underline{x}, \overline{x}] \rightarrow [0, 1] \). Then the median voter is denoted by \( x^m = \Gamma^{-1}(\frac{1}{2}) \). Voter \( i \)'s preference over policy is represented by the indirect utility function \( V(q, x^i) \). The aggregation of individual preferences takes place by means of majority rule. The majority preference over any pair of policies \( (q_a, q_b) \) is defined formally as:

\[
q_a \succ q_b \iff N(q_a, q_b) > N(q_b, q_a)
\]

where \( N(q_a, q_b) = \# \{ x^i \in [\underline{x}, \overline{x}] : V(q_a, x^i) > V(q_b, x^i) \} \) is the number of voters who prefer policy \( q_a \) to \( q_b \), similarly, \( N(q_b, q_a) = \# \{ x^i \in [\underline{x}, \overline{x}] : V(q_a, x^i) < V(q_b, x^i) \} \) denotes the number of voters who prefer \( q_b \) to \( q_a \). We call a policy \( q^* \) a Condorcet winner if and only if \( q^* \succ q_i \) for all \( q_i \neq q^* \).
Suppose that every voter has the following indirect utility so that, for any given pair of policies \((q_a, q_b)\), he or she prefers the policy that generates the higher value of \(V^i\).

\[
V^i(q, x^i) = I(q) + P(x^i)J(q) + Q(x^i)
\]

where \(I(q)\) and \(J(q)\) are common to all voters, \(Q(x^i)\) is any real function that only depends on \(x^i\) and \(P(x^i)\) is monotonic in \(x^i\). We assume \(P(x^i)\) is monotonically decreasing with \(x^i\) in this proof; the case where \(P(x^i)\) is monotonically increasing follows similarly.

For any pair of policies \((q_a, q_b)\), define:

\[
\Delta_{a,b}^i = V^i(q_a, x^i) - V^i(q_b, x^i) = I(q_a) - I(q_b) + P(x^i)[J(q_a) - J(q_b)]
\]

Hence, for any voter \(i\), \(\Delta_{a,b}^i > 0\) implies he or she prefers policy \(q_a\) to \(q_b\). Suppose \(q_a\) is the optimal policy chosen by the median voter, \(x^m\). Then, \(\Delta_{a,b}^m > 0\) for all \(q_b \neq q_a\).

For any other voter \(i\):

\[
\Delta_{a,b}^i - \Delta_{a,b}^m \equiv [P(x^i) - P(x^m)][J(q_a) - J(q_b)]
\]

Note that, if \([J(q_a) - J(q_b)] = 0\), \(\Delta_{a,b}^i = \Delta_{a,b}^m > 0\) for all \(x^i\). Thus, everyone prefers policy \(q_a\) to \(q_b\).

If \([J(q_a) - J(q_b)] > 0\), because \(P(x^i)\) is monotonically decreasing with \(x^i\), \(\Delta_{a,b}^i > \Delta_{a,b}^m\) for all voters with \(x^i < x^m\). Since \(x^m\) is the median voter, we must have \(N(q_a, q_b) > N(q_b, q_a)\).

If \([J(q_a) - J(q_b)] < 0\), \(\Delta_{a,b}^i > \Delta_{a,b}^m\) for all voters with \(x^i > x^m\), we also have \(N(q_a, q_b) > N(q_b, q_a)\).
Therefore, \( q^* > q_i \) for all \( q_i \neq q^* \) and the Condorcet winner \( q^* \) coincides with the median voter’s optimal policy choice, \( q_a \).

A2 Proof of Proposition 1

In our model, the government policy is characterized as \((\tau_t, \rho_t)\), so \( q = (\tau_t, \rho_t) \); the agents only differ in the levels of their bequest, \( b^t_i \); and the indirect utility function for agent \( i \) takes the following form:

\[
V^i(\tau_t, \rho_t, b^t_i) = \ln\left\{ \frac{A(1-\alpha)}{1+\beta} [(1-\tau_t)^{1-\alpha} \tau^\alpha_t \rho^\alpha_t] b^t_i \right\} + \beta \ln\left\{ \frac{\beta A(1-\alpha)}{1+\beta} [(1-\tau_t)^{1-\alpha} \tau^\alpha_t \rho^\alpha_t] b^t_i \right\} \\
+ \frac{\theta}{(b^t_i/B_t)^\varepsilon} \ln[(1-\rho_t)\tau_t B_t - H_t] \\
= I(\tau_t, \rho_t) + P(b^t_i) J(\tau_t, \rho_t) + Q(b^t_i)
\]

where \( Q(b^t_i) = (1+\beta) \ln b^t_i \) is a real function of \( b^t_i \); \( P(b^t_i) = \frac{\theta}{(b^t_i/B_t)^\varepsilon} \) is monotonically decreasing with \( b^t_i \); \( J(\tau_t, \rho_t) = \ln[(1-\rho_t)\tau_t B_t - H_t] \) and \( I(\tau_t, \rho_t) = \{(1+\beta) \ln[\frac{A(1-\alpha)}{1+\beta} (1-\tau_t)^{1-\alpha} \tau^\alpha_t \rho^\alpha_t] + \ln(1+\beta)\} \) are common to all voters. Therefore, the pair of \((\tau_t, \rho_t)\) that maximizes the utility of the voter with median level of bequest will be the Condorcet winner in this economy.

Finally we show that \( w \) is a constant. Recall that the bequest is always log-normally distributed. The ratio \( b^m_t/B_t \), that is the median over mean ratio will always equal to \( \exp(-\sigma^2_t/2) \), where \( \sigma^2_t \) is the variance of the wealth distribution at period \( t \). From equation (2.2.14), the variance is a constant, so we know \( b^m_t/B_t = \exp(-\sigma^2/2) \) for all periods and \( \sigma^2 \) is the variance for the initial wealth distribution. Also, the strength of the wealth effect \( \varepsilon \) is assumed to be constant, \( w \) is therefore a constant that represents the wealth effect on NPG.
A3 The Relationship Between $\tau_t$ and $\rho_t$

Recall that

$$\tau_t = \frac{w\eta(1-\alpha)(1+\beta)(1-\rho_{t-1})\tau_{t-1}B_t}{[w(1+\beta)+\theta]B_t} + \frac{\alpha w(1+\beta)+\theta}{w(1+\beta)+\theta} \tag{2.6.1}$$

$$\rho_t = \frac{\alpha w(1+\beta)}{\alpha w(1+\beta)+\theta} - \frac{w\eta\alpha(1+\beta)(1-\rho_{t-1})\tau_{t-1}B_t}{[\alpha w(1+\beta)+\theta]\tau_t B_t} \tag{2.6.2}$$

$$w = (b_t^n / B_t)\varepsilon$$

Divide (2.6.1) by $\tau_t$ on both sides:

$$\frac{\tau_{t-1}}{\tau_t} = \left\{1 - \frac{\alpha w(1+\beta)+\theta}{[w(1+\beta)+\theta]B_t}\right\} \left\{\frac{w(1+\beta)+\theta}{w\eta(1-\alpha)(1+\beta)(1-\rho_{t-1})B_t}\right\} \tag{2.6.3}$$

Rearrange (2.6.2):

$$\frac{\tau_{t-1}}{\tau_t} = \left[\frac{\alpha w(1+\beta)}{\alpha w(1+\beta)+\theta} - \rho_t\right] \left\{\frac{[\alpha w(1+\beta)+\theta]B_t}{w\eta\alpha(1+\beta)(1-\rho_{t-1})B_t}\right\} \tag{2.6.4}$$

Equate (2.6.3) and (2.6.4), multiply both sides by $\frac{1-\alpha}{w(1+\beta)+\theta}$ and after some manipulation, it produces: $\rho_t = \frac{\alpha(1-\tau_t)}{\tau_t(1-\alpha)}$.

Hence $\frac{d\rho_t}{d\tau_t} = -\frac{\alpha}{(1-\alpha)\tau_t^2} < 0$.

A4 The Values of $\eta$

The task is to find out when the following quadratic equation:

$$\tau^2 + (M - N - 1)\tau + N - \alpha M = 0 \tag{2.6.5}$$
where \( M = \frac{\eta(1+\beta)^2}{(1+\beta+\theta)\beta A^\alpha(1-\alpha)^{1-\alpha}} \) and \( N = \frac{\alpha(1+\beta)+\theta}{1+\beta+\theta} \), has a unique real root. This happens when 
\[(M - N - 1)^2 - 4(N - \alpha M) = 0 \]
holds, that is

\[
M^2 + 2(2\alpha - N - 1)M + (N - 1)^2 = 0 \quad \text{(2.6.6)}
\]

Let \( M = k\eta \) in equation (2.6.6), we have:

\[
k^2\eta^2 + 2k(2\alpha - N - 1)\eta + (N - 1)^2 = 0 \quad \text{(2.6.7)}
\]

Thus,

\[
\eta = \frac{-2k(2\alpha - N - 1) \pm \sqrt{4k^2(2\alpha - N - 1)^2 - 4k^2(N - 1)^2}}{2k^2} \quad \text{(2.6.8)}
\]

\[
= \frac{1}{k} \left[ -(2\alpha - N - 1) \pm 2\sqrt{(\alpha - 1)(\alpha - N)} \right]
\]

Substitute the values of \( k \) and \( N \) into equation (2.6.8):

\[
\eta = \frac{\beta A\alpha^a(1-\alpha)^{1-\alpha}}{(1+\beta)^2} [(1 - \alpha)(1 + \beta + 2\theta) \pm 2(\alpha - 1)\sqrt{\theta(1 + \beta + \theta)}]
\]

Without loss of generality, we assume \( \eta_1 < \eta_2 \):

\[
\eta_1 = \frac{\beta A\alpha^a(1-\alpha)^{2-\alpha}}{(1+\beta)^2} [(1 + \beta + 2\theta) - 2\sqrt{\theta(1 + \beta + \theta)}] \\
\eta_2 = \frac{\beta A\alpha^a(1-\alpha)^{2-\alpha}}{(1+\beta)^2} [(1 + \beta + 2\theta) + 2\sqrt{\theta(1 + \beta + \theta)}]
\]

From equation (2.6.5), \( \tau = \frac{1+N-k\eta}{2} \) when there is a unique real root. Substitute both \( \eta_1 \) and \( \eta_2 \), we have \( \frac{1+N-k\eta_1}{2} > \alpha \) and \( \frac{1+N-k\eta_2}{2} < \alpha \). Recall that the initial value \( \tau_0 = N > \alpha \), thus,
η_1 is the value we are interested in.

Finally, because \( \frac{\beta A(1 - \alpha)^{2-\alpha}}{(1+\beta)^2} > 0 \) and \((1 + \beta + 2\beta) - 2\sqrt{\theta(1 + \beta + \theta)} > 0\), \(\frac{\beta A(1 - \alpha)^{2-\alpha}}{(1+\beta)^2}[(1 + \beta + 2\beta) - 2\sqrt{\theta(1 + \beta + \theta)}] = \eta_1 > 0\).

Therefore, the critical value \( \bar{\eta} = \eta_1 = \frac{\beta A(1 - \alpha)^{2-\alpha}}{(1+\beta)^2}[(1 + \beta + 2\beta) - 2\sqrt{\theta(1 + \beta + \theta)}] \).
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Chapter 3

The Dynamics of Sovereign Debt and the Composition of Public Spending

3.1 Introduction

Public debt has been increasing in most countries in recent decades, especially in major developed countries since the beginning of the great recession and there has been extensive academic and political debate over the effectiveness of austerity in fiscal policies for debt-burdened economies. Despite its importance and urgency, there is still little consensus on the effect of public debt on economic growth and welfare (see for example, Reinhart and Rogoff, 2010 and Krugman 2013). This paper distinguishes two types of public spending – productive public goods and non-productive but budget-enhancing public transfer – and develops a simple dynamic model to examine the importance of the composition of these two types of public spending on debt and economic growth.

Figure 3.1.1 demonstrates an increasing trend of the debt to GDP ratio in the major economies of the Western Europe in the post 1945 period up to the eve of the global financial crisis, where the horizontal axis is the time and the vertical axis represents the percentage of a country’s public debt over GDP. Figure 3.1.2 shows the evolution of debt to GDP ratio of UK and US, dated back from 1880 until the very recent 2011. We can observe that sharp
increase during the war time as well as the great depression. After they reached the lowest point in 1980s, this ratio started to increase again, especially after the crisis. A similar increasing trend can be observed in almost all developed countries and many developing countries as well. Chalk (2000) points out that there has been fundamental change in the fiscal policies of many developed countries during the last couples of decades, which is the disappearance of balanced budget regimes in these countries. Instead, deficit financing has arisen as an alternative for many countries.

Figure 3.1.1: Debt to GDP ratio (Western European countries, 1950-2008)
Data source: IMP Datamapper
While the increase of debt burden for many countries is obvious, the relationship between debt and growth is more complicated and has always been a focus in the literature of public finance. The core question concerns the real effect of high levels of government debt on long run growth rate. The results from theoretical models are mixed. Many empirical studies have found a high level of government debt negatively correlates with economic growth.

The conventional view on the relationship between government debt and economic growth is that government deficits in general have a positive effect on disposable income, aggregate demand and output in the short run. However, in the long run, assuming the Richardian Equivalence does not hold, the decrease in public saving brought about by a higher budget deficit will not be fully compensated by an increase in private savings and, as a consequence,
national savings will decrease, resulting in lower investment at home which will have a negative effect on GDP (Elmendorf and Mankiw, 1999).

On the other hand, some argue that higher level of debt is always bad for the economy and could lead to a lower growth rate in the long run, as well as in the short run. When public debt is so high, often even the interest payment will become a difficult expenditure for the government, especially for countries suffering from negative economic shocks, as they have in the recent financial crisis. Public debt may have an impact on the accumulation of physical capital. Azariadis and Reichlin (1996) build a model to explore this link using Arrow-Romer type technology that displays a production externality. In this setup, even the smallest positive amount of public debt will prevent the economy achieving its maximum growth rate. Therefore, zero public debt is the best fiscal policy a government could implement in order to promote growth. Other factors can also amplify the negative impact of debt, when the uncertainty and negative expectations of the agents towards a high level of debt (such as a potential raise of inflation and financial repression) is considered (Cochrane, 2011 a,b). In these scenarios, agents will make counter productive and inefficient decisions to offset potential future loss.

In addition, the causality between debt and growth has not yet been convincingly established. Some papers have suggested that there may exist a non-monotone relationship between debt and growth (for example, see Checherita-Westphal, Hallett and Rother, 2014). However, the turning points found by these papers are not so robust to minor changes in data range or statistical methods.

In general, empirical studies find that borrowing has more positive effects when debt is small, and more negative effect when it is large. Many empirical papers have found a non-linear relationship between the level of government debt and growth which is characterized by the existence of a threshold such that the debt displays opposite effects on growth below and above the threshold. Krugman (1988) uses the feature of debt overhang to generate this
non-linearity and threshold effect between debt and growth. However, it is doubtful that the argument of debt overhang could be applied to developed countries, where the majority of the public debt is domestic. Therefore, in order to study the relationship between public debt and economy growth in developed countries, a model with features that reflect the characteristics of a developed country is more appropriate.

Adam and Bevan (2005) has found supporting evidence for this non-linear relationship between fiscal deficits and growth in developing countries as well. A panel data of 45 developing countries suggesting this threshold of the deficit level is around 1.5% of GDP. The thresholds involve not only a change of slope but also a change of sign for its impact on growth. The paper emphasizes on the importance the composition of public expenditure (i.e: how to allocate the deficit) and how the deficit are financed (through changes in borrowing or seigniorage). The differences in these characteristics of an economy will result in a different threshold level of deficit.

Reinhart and Rogoff (2010) argue that, when gross external debt reaches 60 percent of GDP, a country’s annual growth declines by two percent and for levels of external debt in excess of 90 percent, GDP growth is roughly halved. These findings provided support for pro-austerity policies in the aftermath of the financial crisis of 2007–2008. Herndon et al. (2014) argue that, when properly calculated, the average real GDP growth rate for countries carrying a debt-to-GDP ratio over 90 percent is not dramatically different from a situation in which when debt over GDP ratio is lower.

Checherita-Westphal, Hallett and Rother (2012) develop a theoretical model in which, over the business cycle, debt can only be issued to finance public investment and the optimal level of public debt is determined by the public to private capital ratio that maximizes economic growth. They highlight the importance of debt-related fiscal rules and derive growth maximizing public debt ratios. Kumar and Woo (2010) explore the impact of high public debt
on long-run economic growth and suggest that, on average, a 10 percent increase in the initial
debt-to-GDP ratio is associated with a slowdown in annual real per capita GDP growth of
around 0.2 percentage points per year, with the impact being somewhat smaller in advanced
economies.

Annicchiarico and Giammarioli (2004) introduce a simple fiscal policy rule and show that
the fiscal rule displaying time invariant parameters may produce non linear dynamic processes
of adjustment of the fiscal ratios as well as endogenous fluctuations in the rate of growth of
the economy. In addition the transitional process towards fiscal targets critically depends on
the adjustment tool chosen by the fiscal authorities to implement the rule. It shows that in
the steady state, the fluctuations of the growth rate and debt to GDP ratios are determined by
government fiscal policies. Specifically, countries with higher initial public debt to GDP ratios
will experience more intensive fluctuations of growth rate in the long run.

In order to study the effectiveness of austerity measures implemented by many govern-
ments to stop the spreads of public debt, Born, Muller and Pfeifer (2014) use a new panel data
set with 31 emerging and developed countries to estimate the relationship between government
consumption cuts and their economic activities. The results suggest a non linear relationship
between the two that depends on the economic environment when austerity took place. Auster-
ity is more likely to be beneficial in the long run if the economy is not under fiscal stress during
the time of cuts and the public debt will in fact rise when austerity policies are implemented
during times of fiscal stress. These findings are counter intuitive as austerity measures are of-
ten popular during bad times while expansionary policies are usually proposed in benign times.
However, even the authors are cautious about such conclusions as policy recommendations
because the study reveals only a short-term relationship between the two; adverse long-term
implications are also possible.

Some theoretical papers assume a government deficit rule from actual legislation. In
Brauninger (2005), the government follows a constant budget deficit ratio. Under such assumption, the model generates a critical level of public deficit ratio such that two steady states will exist if the deficit ratio stays below the critical level. One of them is locally stable and the other is unstable. At the steady state, the capital, output and the level of public debt all grow at the same constant rate and this rate is lower if the government deficit ratio is higher. The steady state does not exist if the deficit is above the critical level, in this case, the growth rate of capital will decline continuously and become zero when time goes to infinity.

Many papers have considered the possibility of relative economic performance explaining why some countries are able to maintain a higher level of public debt than others. U.S, with large amount of public debt and relatively strong economic performance among developed countries, has been a focal point of many studies. Engel and Rogers (2006) investigate whether the optimizing behavior will lead to the large current account deficit of the U.S given its characteristics. A novel feature of their model is that the expected discounted present value of U.S’s future share of world GDP relative to its current share of world GDP determines its long run current account deficit. The results suggest that the current account deficit is close to its optimal levels under some reasonable assumptions about future U.S. GDP growth relative to the rest of the developed countries. Therefore, the paper considers the robust performance of the U.S. economy relative to the rest of the advanced countries is a major factor that has led to its current account deficit. If in reality, the relative future U.S GDP growth deviates from the current assumption, this level of deficit could put the country on the path to ruin.

Futagami and Shibata (2003) also predict the existence of a critical level of the initial volume of government debt and the ratio of primary budget deficits to GDP. In their model, if the initial level of government budget deficit is not large, then at most two steady states with different balanced growth rates can exist. The higher growth rate steady state is associated with a low ratio of public debt to GDP and the low growth steady state is associated with
a high ratio of public debt to GDP. In addition, the level of government consumption has completely opposite effects on the long run growth rate at these two steady states: an increase in government consumption is bad for high growth steady state but good for the low growth steady state. Therefore, government could affect the long-run growth rate as well as agent’s utility by adjusting the level of public spending. However, these changes cannot be Pareto improving.

Futagami, Iwaisako and Ohdoi (2008) investigate the relationship between the public debt and long run growth with productive government spending. In this model, the government has two methods of financing expenditure: one is levying income tax from the agents and the other is running a budget deficit by issuing government bonds. Assuming the government attempts to maintain the ratio of public debt to GDP as a constant, then there will be two steady states, one is associated with high growth rate and the other with low growth rate. In addition, the paper has demonstrated that there is significant difference between the methods of finance the government could adopt in different steady state. Specifically, in the low growth steady state, debt financing is more beneficial as an increase in government bond will raise its growth rate while an increase in tax rate will reduce the growth rate; in the high growth steady state, there exists a tax rate that maximizes the growth rate while an increase in government bond always reduces the growth rate. When interpreting countries in high growth steady state as advanced economies and countries in low growth steady state as less developed economies, the results have policy implications that less developed countries would do better to use debt financing rather than tax financing to raise growth rate. In contrast, developed countries should use tax financing rather than bond financing but must be cautious when choosing the tax rate.

Ghosh et al (2013) investigates the limit of public debt rise without compromising fiscal solvency, using a stochastic ability-to-pay model to determine sovereign defaults. If the government is unable to increase its primary balances at least as fast as the increases of public
debt, it will display “fiscal fatigue” to its lenders who are assumed to be risk-neutral. In such case, lenders will not allow the debt to be rolled-over. Thus, the debt limit is endogenized as a government policy choice. Using empirical data for 23 developed countries in the years 1970-2007, Ghosh et al (2013) suggest that governments do have their fiscal reaction function with these features when their fiscal policies were made.

Greiner (2011) studies the impact of three different budgetary rules regards their convergence to a balanced growth path and long run growth rates. The three budgetary rules considered in this paper are: 1) a rule of balanced budgets; 2) a budgetary rule where public debt grows in the long run but more slowly than all other economic variables and 3) a budgetary rule where public debt grows at the same rate as all other economic variables providing it satisfies the inter-temporal budget constraint. In their model, the government spends on welfare enhancing public goods with externalities of capital and elastic labor supply. The results suggest that the balanced budget rule and the rule where public debt grows more slowly than all other economic variables produces the same balanced growth rate which is higher than the growth rate under the rule where public debt grows at the same rate as all other economic variables. In aspects such as welfare and stability, the rule of having public debt growth at the same rate as all other economic variables performs worst.

In essence, many factors matter for a country’s growth and debt performance and there is a non-monotone relationship between debt and growth. Moreover, there is no single threshold for debt ratios that can delineate the “bad” from the “good”.

In this paper, we propose that higher debt could lead to either a higher or a lower long run economic growth rate depends on several factors. Firstly, we distinguish the way countries allocate their public resources, which means whether countries spend more on investment that increases the level of productive public goods and so enhances productivity of the economy or
more on welfare payment that increases consumption for a given time period\textsuperscript{1}. Productive public goods normally include government provision of infrastructure that increases the productivity of the economy. For example: building roads, railways, airports, harbors or public R&D expenditures. The government welfare payment usually refers to the direct transfer to citizens as well as expenditure on public health systems, public leisure facilities (local parks, beaches and museums) and public transport.

The theoretical model in this paper predicts a non-monotonic or threshold effect in the relationship between public deficit and economic growth. The composition of the public spending (the ratio between productive and non-productive) dictates the “threshold” in the national debt level. Countries which spend more on providing productive public goods could maintain a higher level of national debt that promotes growth.

This paper aims to shed light on the recent policy debate on taxation, borrowing and public spending intend to cope with the recent economic crisis. The chapter contributes to the literature in the following three ways: 1) it argues that dependent on the productivity of public capital and the fiscal rules governments follow, there is not a fixed optimal or a critical value of debt to GDP ratio. 2) the composition of public spending is the key in understanding and determining the impact of a larger public debt to GDP ratio on the long run growth rate. A larger share of expenditure in providing productive public goods allows a country to have a higher debt to GDP ratio and increases the growth rate in the long run. 3) so our answer to the current debate on public debt is that the emphasis should be put on where it is spent, rather than how much should be borrowed.

The rest of the chapter is organized as follows: section 3.2 introduces the economy and

\textsuperscript{1}We think that a model allowing non-productive public spending is more appropriate for developed countries because of the positive correlation between the level of GDP per Capita and the share of welfare spending over total government spending. In other words, rich countries tend to spend more on welfare than on investment.
the setup of the model; section 3.3 describes the steady state of the model and derives several propositions that justify our answers to the empirical question; finally section 3.4 concludes and discusses some policy implications.

3.2 The model

In this chapter, we use a simple endogenous growth model for a closed economy with a continuum of rational and identical agents who maximize their utility over an infinite time horizon. The total population is constant and normalized to unity. Time is continuous and infinite. There only exists one homogeneous good that can be used for consumption and production. Firms have access to a technology that uses both private capital and public goods as inputs to produce this homogeneous good in a competitive market. Agents own the firm and make investment and consumption decisions to maximize their utilities subject to the inter-temporal budget constraint. In time $t$, the income of an agent contains three parts: 1) the after tax revenue of the firm, $(1 - \tau)Y_t$, 2) the interest payment for holding government bond, $rD_t$, and 3) government transfer payment, $W_t$.

The government in this model makes two types of public spending, $S_t$: 1) government investment, $I_t$, which increases the level of the stock of productive public goods, $G_t$; and welfare payment, $W_t$, which is a direct transfer payment that increase the budget of the agent. Government spending is financed by a tax on the firm’s output, $Y_t$ with the tax rate $\tau_t$; and borrowing from the private sector, where $D_t$ represents the level of outstanding government debt. It is worth mentioning, in this model, private capital refers to all growth-producing assets that can be accumulated by an individual. That includes physical capital, human capital and proprietary technologies.

The firm is owned by the agent who has access to a technology that uses both private capital and public goods to produce output in a competitive market. The interest rate of private
capital is determined endogenously when a firm maximizes its profit. Because both goods and capital market are competitive, the interest rate of the private capital also equals the interest rate paid by the government to its creditors who hold the government bonds.

### 3.2.1 Agents

The representative agent in this economy lives infinitely and gains utility from private consumption. Thus, an agent born in period $t$ maximizes her utility function as follows:

$$\max_C \int_0^\infty e^{-\beta t} [\ln(C_t)] dt \quad (3.2.1)$$

The coefficient $\beta > 0$ denotes the household’s rate of time preference, $C_t$ is the levels of household’s private consumption in period.

The budget constraint for an agent is:

$$\dot{K}_t + \dot{D}_t = rD_t + (1 - \tau)Y_t - C_t + W_t \quad (3.2.2)$$

where $\tau$ is the tax rate and $r_t$ is the competitive interest rate, which will be discussed in detail in later sections.

### 3.2.2 Technology

The firm has access to a technology that produces the single final good using two inputs - capital investment and public goods as follows:

$$Y_t = AG_t^\alpha K_t^{1-\alpha}, \, \, 0 < \alpha < 1 \quad (3.2.3)$$

where $Y_t$ is the total output of the economy, $K_t$ is the private capital that has been invested in the firm and $G_t$ is the level of public goods provided by government. $A$ represents the total
factor productivity (TFP), which is assumed to be constant over time. \(\alpha\) is the productivity of the public goods, while \(1 - \alpha\) is the productivity of private capital.

The firm’s profit is therefore:

\[
\Pi_t = (1 - \tau)Y_t - r_t K_t
\]  \hspace{1cm} (3.2.4)

In the equilibrium with perfect competition, the first order condition implies that when the firm maximizes its profit according to equation, the following equation holds:

\[
r_t = \frac{\partial \Pi_t}{\partial K_t} = A(1 - \tau)(1 - \alpha)G_t^\alpha K_t^{-\alpha}
\]  \hspace{1cm} (3.2.5)

Therefore, from Equation (3.2.5) the marginal productivity of capital depends on the value of TFP, the tax rate, the level of private capital and public goods as well as their relative productivity contribution in production.

3.2.3 Government

In time \(t\), the government spending \(S_t\) contains three parts: 1) government investment, \(I_t\), that increases the level of public goods in the economy, the welfare spending, \(W_t\), in the form of a direct transfer payment to the agent and the interest payment, \(r_t\) of the outstanding sovereign debt, \(D_t\). Productive public goods, \(G_t\), is a production factor and transfer payment, \(W_t\), enhances agent’s budget with leads to higher consumption directly. From equation (3.2.5), the higher the level of public goods the higher interest rate will the agent get, thus a larger budget for private consumption also leads to a higher utility level. Government finances its spending through a taxation, \(\tau\), on the aggregate output, \(Y_t\), and a borrowing \(\dot{D}_t\). Specifically, the government budget constraint is the following:
\[
\dot{D}_t = r_t D_t + S_t - \tau Y_t
\]

(3.2.6)

Where \(S_t\) is the aggregate level of government spending, in the economy, it includes the government investment and welfare payment. Let \(\rho\) be the proportion of government spending used for investment, we have the following:

\[
I_t = \rho S_t, \ \rho \in (0,1)
\]

(3.2.7)

thus, the welfare payment, \(W_t\) equals \((1 - \rho)S_t\).

Assuming zero depreciation, the level of productive public goods, \(G_t\), evolves according to the following:

\[
\dot{G}_t = I_t
\]

(3.2.8)

The inter-temporal budget constraint to ensure that the government does not run a Ponzi Scheme is: \(\lim_{t \to \infty} e^{-rt} D_t = 0\).

In addition, we assume the government follows the so called “Golden Rule of Public Finance” which is a guideline for the operation of fiscal policies in a country. It states that the government will only allow borrowing to invest for future gain but not to spend for current consumption over business cycles. This implies that, at any given time, the government deficit, \(\dot{D}_t\), should be less than or equal to the amount of public investment, \(I_t\). Therefore, we define the term \(\sigma\) which represents the ratio between the government deficit and investment in any given time period, so that:

\[
\sigma = \frac{\dot{D}_t}{I_t}, \ \sigma \in [0,1]
\]

(3.2.9)

When \(\sigma = 0\), the government has no deficit, the level of public debt stays constant; when
\( \sigma = 1 \) government borrow at same amount as its investment, so that the level of debt will grow at the same rate as the accumulation of productive public goods. It can be said that the model in Checherita-Westphal, Hallett and Rother (2012) is a special case of our model when all government spending provides productive public goods and the public deficit grows at the same rate of government spending. (That is when \( \rho = 1 \) and \( \sigma = 1 \) holds.) The result in Checherita-Westphal, Hallett and Rother (2012) is driven by the implicit assumption that the increase in debt level is always reaches the level of government investment in public goods in each period. i.e borrowing equals investment in public goods, \( \sigma = 1 \). (while the golden rule is \( \sigma \leq 1 \)). The turning point is not that of debt but that of the productivity of public goods. i.e \( \rho = 1 \). The current paper discusses the scenario that \( \rho \in (0, 1) \).

3.2.4 Agent’s Problem and Solution

In order to study the steady states of this economy, we begin with equation (3.2.1). The agent chooses her rate of consumption (which determines the rate of capital accumulation and the level of the government bond she is willing to hold due to constraint) to maximizes her inter-temporal utility subject to her budget constraints, equation (3.2.2), taking \( \rho \) and \( \tau \) as exogenous. Thus the Hamiltonian function for this economy reads as the following:

\[
H_t = \ln(C_t)e^{-\beta t} + \mu_t[r_tD_t + (1 - \tau)Y_t + W_t - C_t] \tag{3.2.10}
\]

The necessary conditions for a path to be optimal are

\[
\frac{\partial H_t}{\partial C_t} = 0, \\
\frac{\partial H_t}{\partial D_t} = -\dot{\mu}_t,
\]
\[
\lim_{t \to \infty} D_t \mu_t = 0
\]

The last one is the called the transitivity condition, which is equivalent to the no-Ponzi game condition.

The above conditions gives rise to the following equations:

\[
e^{-\beta t} \frac{c_t}{c_t} = \mu(t),
\]

\[
r_t \mu_t = -\dot{\mu}_t
\]

\[
\lim_{t \to \infty} D_t \frac{e^{-\beta t}}{c_t} = 0
\]

Therefore, solving the these equations and substituting equation (3.2.5), the growth rate of consumption, \(\gamma_C\) equals the following:

\[
\gamma_C = \frac{\dot{C}}{C} = -\beta + (1 - \tau)(1 - \alpha)AK^{-\alpha}G^\alpha
\]  
(3.2.11)

Use the budget constraint of the agent, equation (3.2.2) and that of the government, equation (3.2.6), the growth rate of private capital is obtained as:

\[
\gamma_K = \frac{\dot{K}}{K} = AK^{-\alpha}G^\alpha - C/K - (S/G)(G/K)
\]  
(3.2.12)

From equations (3.2.6), (3.2.7), (3.2.8) and (3.2.9), the growth rate of public debt and of public goods are determined as:
\[
\gamma_D = \frac{\dot{D}}{D} = \frac{\sigma}{\rho} (G/K)(K/D)(\dot{G}/G) 
\] (3.2.13)

\[
\gamma_G = \frac{\dot{G}}{G} = \frac{\rho}{1-\sigma} \left[ \tau A(G/K)^{a-1} - \frac{\sigma}{\rho} (1 - \tau)(1 - \alpha)A(G/K)^{a-1}(D/K) \right] 
\] (3.2.14)

In order to simplify further analysis on this system, we follow convention in the endogenous growth literature, normalizing the variables by capital so that we define the following terms: 
\[g = G/K, \quad c = C/K \text{ and } d = D/K.\]

3.3 The Balanced Growth Path

We define a balanced growth path (BGP) as the trajectory where all the variables grow at the same rate. In particular, we have \( \gamma_C = \gamma_K = \gamma_D = \gamma_G = \gamma \), where \( \gamma \) is Balanced Growth Rate. Differentiating equation (3.2.14) with respect to the public deficit, \( \sigma \), we obtain:

\[
\frac{\partial \gamma}{\partial \sigma} = \frac{\tau A \rho g^{a-1} - (1 - \tau)(1 - \alpha)Ag^a}{(1 - \sigma)^2}.
\]

Therefore, we have the following Propositions:

**Proposition 3.1.** On the Balanced Growth Path (BGP), there exists a critical value of the share of productive public spending over total government spending, that is \( \bar{\rho} = \frac{(1 - \tau)(1 - \alpha)g}{\tau} \). Countries with \( \rho > \bar{\rho} \), the Balanced Growth Rate, \( \gamma \), is higher, if the public deficit, \( \sigma \), is higher; countries with \( \rho < \bar{\rho} \), the Balanced Growth Rate, \( \gamma \), is higher, if the public deficit, \( \sigma \), is smaller. When \( \rho = \bar{\rho} \), the value of \( \sigma \) has no effect on \( \gamma \).

Proposition 3.1 provides the first dimension to our answer to the empirical puzzle shown at the beginning of this chapter that there is no clear relationship between the level of deficit
and the growth rate. It demonstrates that the relationship between the balanced growth rate and the public deficit is not monotone. Specifically, if the government spends more on the provision of productive public goods, an increase in the public deficit will raise the balanced growth rate. Conversely, if the government spends a larger proportion of its budget on welfare payment with equivalently less government investment in productive public goods, increasing the public deficit will reduce the balanced growth rate. Depending on how countries spend their budget, higher deficit will be beneficial for countries that invest for the future but detrimental for countries that borrow just to consume.

The reason for this happening is the public deficit in this model has two opposing effects on $\gamma$: when $\rho$ is high ($\rho > \bar{\rho}$), the positive effect of $\sigma$ on $\gamma$ dominates, and the increase in spending leads to more provision of productive public goods. However, when $\rho$ is small ($\rho < \bar{\rho}$), the negative effect dominates. That is, when countries spend more on welfare payment, the increase in public deficit brings a larger burden of interest payment. When $\rho = \bar{\rho}$, the two effects cancel out each other, so that the level of public deficit has no impact on the balanced growth rate.

**Proposition 3.2.** On the Balanced Growth Path (BGP), there exists a critical value of $\bar{g} = \frac{\rho \tau}{(1-\tau)(1-\alpha)}$, the ratio between productive public capital and private capital, such that: for countries with $g < \bar{g}$, the Balanced Growth Rate, $\gamma$, is higher, if the public deficit, $\sigma$, is higher; and for countries with $g > \bar{g}$, the Balanced Growth Rate, $\gamma$, is higher, if the public deficit, $\sigma$, is smaller. When $g = \bar{g}$, the value of $\sigma$ has no effect on $\gamma$.

When a country does not have adequate infrastructure (the productive public goods are in a state of under-provision), borrowing will increase growth. Effectively, this is the same as a comparison between the marginal benefit of borrowing (marginal rate of productive public goods) and the marginal cost of borrowing (interest rate). Lack of spending on public goods
imposes severe constraints on economic activity in a country. When productive public goods are under-provided, the country suffers from weaknesses in terms of access to productive public goods such as physical infrastructure and thus bears a high opportunity cost if not borrowing. Borrowing allows allocation of resources to meet the needs in infrastructure investment and other types of public goods that increase the productivity of the economy.

**Proposition 3.3.** On the Balanced Growth Path (BGP), there exists a critical value of the tax rate, $\bar{\tau} = \frac{(1-\alpha)\rho}{\rho + (1-\alpha)g}$, such that: for countries with $\tau > \bar{\tau}$, the Balanced Growth Rate, $\gamma$, is higher, if the public deficit, $\sigma$, is higher; and for countries with $\tau < \bar{\tau}$, the Balanced Growth Rate, $\gamma$, is higher, if the public deficit, $\sigma$, is smaller. When $\tau = \bar{\tau}$, the value of $\sigma$ has no effect on $\gamma$.

This proposition suggests that borrowing is beneficial if the country has already had a high tax rate. In this model, the government has two channels to finance its spending, taxation and deficit. Each has different associated cost. When the tax rate is high, the cost of finance through an even higher tax rate overcomes the cost of borrowing, which makes increasing the level of public deficit a more efficient way of financing. However, for countries with a low tax rate, its financial needs are better fulfilled through the channel of taxation, increasing the level of deficit thus has negative impact on the balanced growth rate.

**Proposition 3.4.** On the Balanced Growth Path (BGP), there exists a critical value of $\bar{\alpha} = 1 - \frac{\rho\tau}{\rho (1-\tau)}$, the share of productive public spending over total government spending, such that: for countries with $\alpha > \bar{\alpha}$, the Balanced Growth Rate, $\gamma$, is higher, if the public deficit, $\sigma$, is higher; and for countries with $\alpha < \bar{\alpha}$, the Balanced Growth Rate, $\gamma$, is higher, if the public deficit, $\sigma$, is smaller. When $\alpha = \bar{\alpha}$, the value of $\sigma$ has no effect on $\gamma$.

The logic behind this proposition is obvious, as the positive effect of $\sigma$ on $\gamma$ comes from the provision of productive public spending, $\dot{G}$, when the relative productivity of $\dot{G}$ is high enough, increasing the level of public deficit could increase the balanced growth rate.
3.4 Numerical Exercises

In order to test the quantitative importance for the findings of this Chapter, some numerical examples will be exercised in this section. From Proposition 3.1, we have:

\[ \bar{\rho} = \frac{(1 - \tau)(1 - \alpha)g}{\tau} \]

which defines the critical value of the composition of government expenditure. For an economy with \( \rho > \bar{\rho} \), when the public deficit increases, its government spends more on the provision of productive public goods which will raise the balanced growth rate. We are going to estimate this value under some reasonable assumptions on other parameters. First, \( \tau \) is defined as the aggregate tax burden of an economy, the single tax that includes everything. This could be seen as the ratio between tax revenue over GDP of a country. The average value for this ratio in our data set from 33 major economies is 34.14%, and the ratio for US is 25.65%, these will be the two values of \( \tau \) in our estimation (\( \tau = 0.25 \) and \( \tau = 0.35 \)). The parameter \( \alpha \) is the productivity of public goods \( G \), here we follow the numerical section in Chapter 2 and test two values for \( \alpha \) : 0.1 and 0.167, which represents 1/10 and 1/6 productivity contribution respectively. The parameter \( g \) is the ratio between public capital and private capital, \( G/K \). The value for \( g \) is very straightforward, assuming the resources will be allocated efficiently in the balanced growth path, thus when \( \alpha = 0.1 \) (public capital making 1/10 productivity contribution), the ratio between public and private capital should be 1:9, i.e. \( g = 0.111 \). The values of \( \bar{\rho} \) under the assumptions above are shown in the following table:
Table 3.4.1: The values of $\bar{\rho}$

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\alpha$</th>
<th>$0.1$</th>
<th>$0.167$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.25$</td>
<td>$\rho=0.30$</td>
<td>$\rho=0.50$</td>
<td></td>
</tr>
<tr>
<td>$0.35$</td>
<td>$\rho=0.18$</td>
<td>$\rho=0.31$</td>
<td></td>
</tr>
</tbody>
</table>

3.5 Conclusion

In the past half century, most developed countries have experienced dramatic increases in their national debt level. More governments rely on the deficit to finance their spending needs. The financial crisis has made the bad situation worse, where billions of dollars have been spent by the government in order to stabilize the economy. Although the stimulus plan usually does increase growth for the short run, the debate on the real effect of debt on long run growth has never stopped.

In this paper, we have provided a theoretical model that generates a non-monotonic relationship between the public deficit and the long run growth rate of the economy. In particular, the model predicts the existence of several critical values which dictate whether the increase in government deficit brings positive or negative effects on the growth rate of a balanced growth path. Firstly, There is a critical value for the proportion of the public investment in overall government spending, denoted as $\bar{\rho}$. Such that, for countries more willing to invest ($\rho > \bar{\rho}$), increase in public deficit will increase steady state growth rate on the balanced growth path and therefore benefit the economy. Conversely, countries that spend more on welfare payment ($\rho < \bar{\rho}$) and maintain a higher level of public deficit will reduce the balanced growth rate and so harm the economy in the long run. Another insight from the model is that borrowing is beneficial to long-run growth rate if the country does not have adequate public goods. Because
the positive growth effect of borrowing comes from the provision of public goods, its magnitude depends on the marginal productivity of public goods. When the level of public goods is low \((g < \bar{g})\), increasing the borrowing to finance a higher level of public investment that produces more public goods could increase the long run-growth rate. Finally, the model predicts the threshold value of the tax rate, \(\bar{\tau}\), such that, for countries with high tax rate \((\tau > \bar{\tau})\), increasing the government deficit will increase its balanced growth rate; conversely, higher government deficit will lead to a lower balanced growth rate for countries with low tax rate \((\tau < \bar{\tau})\).
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Chapter 4

Conclusion

In this crisis, people often forget the importance of growth, and focus on cutting spending or on stimulating consumption. What we need in crisis is not only to increase liquidity or stimulate consumption but, more importantly, to make meaningful investments in the economic future, especially in infrastructure, which can reduce the transaction costs of the economy such as public education and clean sustainable energy. “This will both promote growth and generate tax revenues, lowering debt to gross domestic product ratios in the medium term and increasing debt sustainability.” (Stiglitz, 09/08/2011 Financial Times: “How to make the best of the long malaise”)

Some suggest tax rise and austerity as a way to reduce debt, some suggest that government should borrow more and spend more to generate growth. However, both approaches miss key points. If government spending is on subsidizing consumption, it may be able to increase utility temporally, but this would only mean smooth consumption and will have very little impact on growth. However, if more money is spent on investment in productive public goods, it will have a growth effect in the long run. The key is not the size of government spending but the target of the expenditure.

Chapter 2 has established that the existence of a habit effect on non-productive public goods could generate a transitional dynamic that mimics several stylized facts: in particu-
lar, countries with higher income tend to have larger government and spend more on welfare programmes. The government always maintains a balanced budget, uses its tax revenue to provided productive public goods (PPG) and non-productive public goods (NPG). The habit effect is that the level of utility an agent derives from NPG depends on the increment of NPG provided by government in the current period compared with the previous period. This implies that government can not adjust the level of NPG freely - the benefit that was already given in previous periods is very difficult to withdraw. The enormous public pressure against any benefit cut in different countries (even those on the verge of bankruptcy like the recent crisis in Greece) may justify this assumption. One could consider that the habit effect on NPG is a non-monetary debt burden inherited by the government in which the strength of habit is the “interest rate” the government needs to pay. A country with stronger habit means its government faces a higher level of interest rate, which has to have a higher steady state tax rate and lower share of productive spending.

Empirical data on taxation and government spending suggest some stylized facts: Historically, both the share of tax revenue over GDP and the share of welfare spending over GDP have increased in all developed countries. In addition, countries with higher GDP per capita tend to have larger share of tax to GDP ratio and welfare spending to GDP ratio than countries with lower GDP per capita. Chapter 2 studied a political-economy model in which government policies in each period are chosen by rational voters. The model shows how the emergence of a habit effect relating to non-productive public good spending generates dynamics for tax rates and the welfare spending that are consistent with the stylized facts.

The habit effect that drives the results derives from a well-known observation in political economy that welfare policy is difficult to reduce once it has been implemented. Our economy starts from its long run equilibrium with no habit effect, the tax rate and the share of the welfare spending will increase once a habit on NPG has been fostered. In the long run, they
will asymptotically converge to a new steady state, providing the habit effect is not too strong. In addition, the levels of the steady state tax rate and share of the welfare spending are positively related to the strength of the habit effect. Thus, holding everything else equal, countries with stronger habit effects will end up in a steady state that features a higher tax rate, lower welfare spending proportionately and lower growth rate.

Hence, Chapter 2 provides a theoretical framework to analyze the expansion of government welfare spending and the impact of government policy on economic performance. It adds insights to the current “Austerity vs Stimulus” debate. From the model, in order to promote growth, it is not how much government spends, but where it spends that matters. An increase in PPG spending will generate higher growth; higher spending in NPG will foster a larger habit stock for future generations.

Many advanced economies have experienced dramatic increases in their public debt levels. More governments rely on the deficit to finance their spending needs. The financial crisis has made the bad situation worse, where billions of dollars have been spent by the government in order to stabilize the economy. Although the stimulus plan usually does increase growth for the short run, the debate on the real effect of debt on long-run growth has never stopped.

In Chapter 3, we have provided a theoretical model that generates a non-monotonic relationship between the public deficit and the long run growth rate of the economy. In particular, the model predicts the existence of several critical values which dictate whether the increase in government deficit exerts a positive or negative effect on the growth rate of a balanced growth path. Firstly, there is a critical value for the proportion of the public investment in overall government spending such that, for countries more willing to invest, increase in public deficit will increase steady state growth rate on the balanced growth path and therefore benefit the economy. Conversely, countries that spend more on welfare payment and maintain a higher level of public deficit will reduce the balanced growth rate and so harm the economy in the
Another insight from the model is that borrowing is beneficial to long-run growth rate if the country does not have adequate public goods. Because the positive growth effect of borrowing comes from the provision of public goods, its magnitude depends on the marginal productivity of public goods. When the level of public goods is low, increasing the borrowing to finance a higher level of public investment that produces more public goods could increase the long-run growth rate. Finally, the model predicts the threshold value of the tax rate, such that, for countries with high tax rate, increasing the government deficit will increase its balanced growth rate; conversely, higher government deficit will lead to a lower balanced growth rate for countries with low tax rate.